THESIS

A DIGITAL MODEL OF A STREAM-AQUIFER SYSTEM ON THE SOUTH PLATTE RIVER NEAR STERLING, COLORADO

Submitted by

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ABSTRACT OF THESIS

A DIGITAL STREAM-AQUIFER MODEL

A digital finite difference groundwater model was developed for a stream-aquifer system in northeastern Colorado. The general mathematical model and computer program used in the study were developed by the Groundwater staff at Colorado State University. The model uses a finite difference approximation of the basic non-linear equation describing transient flow in a saturated porous media. The twenty square mile study area includes a nine mile reach of the South Platte River and a portion of Prewitt Reservoir. Simulated water table elevations were compared with historic water level data for the period of November, 1969 to November, 1971. A satisfactory match of simulated and historic water table elevations was obtained for the second year of analysis. Due to inadequate data on the existing aquifer conditions near the reservoir, problems were encountered in simulating the monthly reservoir seepage. As a result, use of the model is limited to certain conditions of reservoir storage.

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I. INTRODUCTION

An increasing demand for water will require improved management of all our water resources. Proper management of groundwater aquifers will be an important part of that objective. In the future, groundwater reservoirs will probably play an even greater role in water conservation due to the high cost of surface reservoirs and the limited number of available sites for surface reservoirs. In addition, the advantage of an almost negligable evaporation loss from a groundwater reservoir will have to be given more consideration in the coming years.

Proper management of groundwater aquifers is important in preventing costly annoyances such as waterlogging and overdrafts. Waterlogging can result in severe crop damage while an overdraft may be detremental by requiring the replacement of pumps, requiring the lowering of pump bowls, causing wells to dry up, allowing salt water intrusion, or causing damaging land subsidence.

Management of a groundwater aquifer may be more difficult if the aquifer is in hydraulic connection with a stream. In recent years, water administrators have recognized the necessity of an integrated approach to stream-aquifer systems management. In 1969, the State of Colorado passed legislation requiring an integrated appropriation, use, and administration of groundwater and surface water if the groundwater is tributary to the surface water. A recent report of the National Water Commission (review draft) stated that "eliminating arbitrary distinctions between groundwater and surface water should be an overriding objective" in the revision of water law (Corker, 1972). Managers of stream-aquifer systems may be required to estimate the exchange of water between the stream and aquifer for each management alternative.

Management of a stream-aquifer system can be aided by use of digital computer models. The models will predict the response of aquifer systems to various hydrologic stresses and can be used to analyze proposed management and operation schemes such as alternate patterns of pumping, artificial recharge, or redistribution of surface water. Finite difference models are the most common mathematical simulation models currently being proposed for use in stream-aquifer management. With the finite difference model, the water table aquifer of a study area is represented by a system of finite grids and a mass balance equation is written for each grid for each time increment. The mass balance equation uses a finite difference approximation of the basic non-linear, second-order, partial differential equation describing transient flow in a saturated porous media. The water table elevation for each grid at the end of the desired time increment is computed from the resulting system of mass balance equations. The computed water table elevations for each grid are then used as known values in the system of equations for the next desired time level and the process is repeated. The flow between adjacent grid blocks during a specified time increment may be computed by use of the respective grid water table elevations at the end of that particular time increment.

Recently enacted laws requiring the conjunctive use of surface water and groundwater provide impetus for the development of simulation models for use in stream-aquifer management. This report will present the development of a digital finite difference simulation model for an existing stream-aquifer system. The specific objectives are:

1) Review representative literature pertaining to digital finite difference groundwater models.

2) Select a study area containing a stream-aquifer system where management of the system could be aided by use of a model.

3) Develop a digital finite difference simulation model of the selected study area.

4) Operate the model for a selected historic time period and, if necessary, adjust model parameters until a reasonable match is attained between the simulated and historic water table elevations.

5) Evaluate the results.

Representative literature pertaining to digital finite difference groundwater models is reviewed in Chapter 2.

II. REVIEW OF LITERATURE ON DIGITAL FINITE DIFFERENCE MODELS

The methods and solutions of finite difference equations have been available for some time and are discussed by many authors including Southwell (1946), Crank and Nicholson (1947), Allen (1954), Crank (1956), Kunz (1957), Todd (1962), Richtmeyer and Morton (1967), and Brakensiek (1967). Since finite difference techniques require a large number of arithmetic calculations, numerical finite difference solutions were not very practical until after the advent of the digital computer. Numerical solutions to flow problems were first used by the petroleum industry in the early 1950's. Some of these developments are discussed by Peaceman and Rachford (1955), Douglas and Peaceman (1955), and Douglas and Rachford (1956).

Stallman (1956) was one of the first to apply numerical methods to problems in groundwater hydrology. He saw a need for regional analyses of groundwater aquifers and described a method of using field measured head differentials to calculate aquifer permeability distributions. The technique utilized a finite difference approximation of the general differential equation describing two-dimensional nonsteady flow in a nonhomogeneous aquifer. Data were gathered from a well-field of 21 uniformly spaced wells. It was noted that even though the technique would only compute relative values, an aquifer system could be described in absolute terms if one absolute value is known.

Fiering (1964) used finite difference methods to predict the response of a groundwater reservoir to pumping stresses. Fiering pointed out that analytic methods of computing a transient drawdown surface are unmanageable for cases having a large number of wells in irregular patterns. It was also noted that the Theis solution would not permit irregularities or cyclic variation in the pumping pattern, seasonal recharge from rain, leakage, or surface irrigation, and spatial variation in permeability and porosity. He proposed a model that allowed these irregularities. The model used an iterative implicit technique to solve the finite difference approximations of mass balance equations. The model required excessive amounts of computer time for aquifers with large transmissibility values and small storage coefficients.

Eshett and Longenbaugh (1965) presented a general mathematical model to simulate transient groundwater movement. The basic partial differential equation for transient saturated flow was written in finite difference form for each grid of the model. The resulting system of equations was solved by two methods: the Gaussian elimination procedure and the alternating direction implicit procedure. The model could handle an impermeable boundary or a constant head boundary, as in the case of a fully penetrating hydraulically connected river. Input data included the space coordinates, the initial water table elevation, the impermeable bedrock elevation, the storage coefficient, and the hydraulic conductivity of each grid. A net groundwater withdrawal (or recharge) was also calculated for each grid by considering consumptive use, evapotranspiration, pumping,precipitation, and deep percolation of canal seepage and applied surface water. Output included the water table elevations and estimates of the net volumetric exchange of water between the stream and aquifer at the desired time levels.

A hypothetical 50 grid model was used to examine the validity of the program and to compare the two solution techniques. The numerical results of both solution techniques coincided with analytic solutions.

The computing time for a 50 grid model was approximately the same for both the Gaussian elimination technique and the alternating direction implicit procedure, although the latter technique required significantly less computer storage.

Bittinger, Duke, and Longenbaugh (1967) discuss the development and use of a finite difference model similar to that presented by Eshett and Longenbaugh. Vertical-inflow boundaries as in the case of a leaky aquifer, and horizontal-inflow boundaries as required by problem segmentation were also proposed. They included a general discussion of the development, verification, and use of digital finite difference models in groundwater management. Several specific applications were briefly described.

Stettner (1968) evaluated the adequacy of the mathematical model presented by Eshett and Longenbaugh (1965) in predicting aquifer response. The aquifer response of the mathematical model was compared to the response of a physical porous media model that used glass beads as the medium and a mixture of glycerine and water as the fluid. Stettner concluded that the mathematical model would give acceptable solutions to problems of aquifer response as long as the slope of the water table is moderate, the rate of drawdown is not too large, and the capillary fringe is small compared to the total saturated thickness.

Pinder and Bredehoeft (1968) developed a mathematical model that was designed to handle vertical leakage as well as nonhomogeneous anisotropic porous media and irregular boundary conditions. The finite difference approximations to the flow equations were solved using the alternating direction implicit procedure. The computer program was verified by a comparison of the digital model results with analytical solutions for problems in homogeneous and isotropic aquifers. The model was used to analyze an aquifer at Musquodoboit Harbour, Nova Scotia. An electric analog model of the area was constructed as a check of the overall performance of the digital model. The two solutions compared favorably. Transmissibility values and recharge boundary gradients of the digital model were adjusted until the results matched field pumping test data. The digital model was considered to be a valid representation of the aquifer and was used to evaluate the aquifer as a water supply for the village of Musquodoboit Harbour.

Trescott, Pinder, and Jones (1970) investigated the feasibility of developing a particular aquifer system as a supplementary water source for the town of Antigonish, Nova Scotia. A digital finite difference model solved by the alternating direction implicit technique was instrumental in the investigation. The one-half square mile study area located at the mouth of the Rights River on Antigonish Harbour was analyzed with a 50 foot grid spacing. Impermeable boundaries were assumed for three sides of the study area and the harbor side was treated as a constant head boundary with the same thickness and permeability as the river at that end. In order to attain good correspondence between the model and pumping test data, it was necessary to reduce the permeability of the river bed to one-third of its original value and treat the storage coefficient as a function of time and space. The authors mentioned several factors that were not considered in the model including seasonal groundwater recharge and depletion, variation of the river stage, inflow from the bedrock, and seasonal alteration of the river bed permeability due to siltation or scour.

Young and Bredehoeft (1972) developed a digital computer simulation model to analyze alternative management policies for the conjunctive use of groundwater and surface water systems. The model is comprised of a hydrologic model to predict the response of a stream-aquifer system to applied hydrologic stresses and an economic model to predict the response of irrigation water users to the resulting state of the system. A 50 mile reach of the South Platte River in northeastern Colorado was selected as the study area. The hydrologic part of the model utilized finite difference techniques with solutions by the alternating direction implicit procedure.

The review of literature indicates the feasibility of using digital finite difference techniques to model stream-aquifer systems. The work of Stettner (1968) indicated that the mathematical model presented by Eshett and Longenbaugh (1965) would give acceptable solutions in predicting aquifer response. A mathematical model similar to that presented by Eshett and Longenbaugh (1965) will be used in this study.

The specific applications in the reviewed literature were for a study area of less than one square mile or more than 100 square miles. Effective management of stream-aquifer problems involving alternate patterns of pumping, artificial recharge, or redistribution of surface water, generally requires a study area greater than one square mile, yet in many cases, it is not necessary to analyze an area of 100 square miles. It would be of interest to demonstrate the capability of the digital finite difference approach in developing a model for a study area of about twenty square miles.

The applications in the reviewed literature did not consider the case of having an off-stream surface water storage reservoir adjacent to the study area. It would be of value to analyze some of the problems involved in developing a digital finite difference model for a study area containing an off-stream reservoir with significant seepage.

III. DESCRIPTION OF THE STUDY AREA

Selected Location

The study area is located along a nine mile reach of the South Platte River at the Logan-Washington County line in northeastern Colorado as shown in figure 1. The reasons for selecting this area for a stream-aquifer model study are:

1) The area qualifies as a stream-aquifer system since the South Platte River is considered to be in hydraulic connection with the alluvial fill aquifer of the river valley.
2) The area is irrigated with both groundwater and surface water and as a result, management of the stream-aquifer system is difficult and could be aided by use of a simulation model.
3) The State Engineer of Colorado, as required by State law, is currently studying the stream-aquifer management problems of this area and a simulation model could be of value in the State of Colorado study.
4) Geologic and hydrologic data are available for this area.
5) The area includes Prewitt Reservoir, an off-stream surface water reservoir that has significant seepage into the alluvial aquifer.

The rectangular 9 mile by 2.3 mile study area includes the alluvial aquifer on only the south side of the South Platte River. The lack of hydrologic data on the north side and the desire to keep the model reasonably small led to this decision. The river within the study area reach is a meandering stream with several low flow channels. The nine mile length of river was approximated as a straight line to serve as the northwest boundary of the stream-aquifer study.

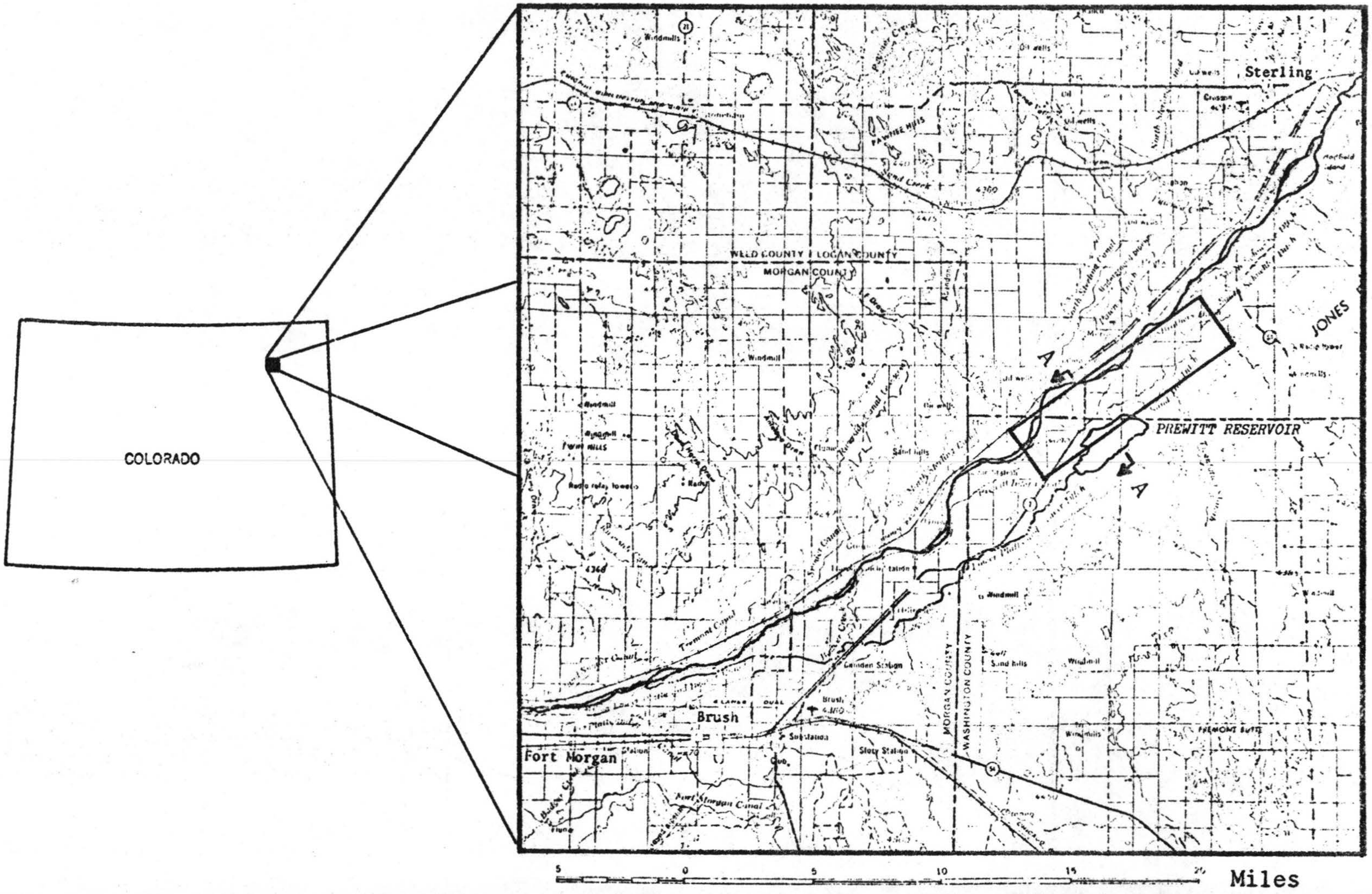


Figure 1. Location Map. (Cross-section A-A given in Figure 2.)

Climate

The area is semiarid with an average annual rainfall of approximately 14 inches. In an average year, about 70% of the annual precipitation occurs during the months April through August (U.S. Weather Bureau). Most of the summer precipitation is in the form of erratically distributed thundershowers which result in drought periods for some areas each year. The average temperatures for July and January are approximately 74 and 25 degrees Fahrenheit, respectively. Normally July has the maximum average temperature and January the lowest. The length of the growing season is about 145 days (Bjorklund and Brown, 1957).

Land Use

Due to a limited and irregular rainfall, irrigation plays an important role in the agricultural economy of the lower South Platte River valley. Aerial photos of the study area, provided by the USDA Agriculture Stabilization Office, Sterling, Colorado, indicate that the 13,000 acre study area has approximately 7,800 acres of cultivated agricultural land. For the purposes of this study, it was assumed that all 7,800 acres of cultivated agricultural land are irrigated. This is a reasonable assumption since the major crops in the study area, sugar beets, corn, and alfalfa, normally require irrigation. The remaining 5,200 acres of the study area includes marshes, swamps, river channels, barren areas, and a portion of Prewitt Reservoir. The major crops are irrigated with both surface water from the South Platte River and groundwater from the underlying aquifer. Surface water is diverted from the river and conveyed to the irrigated acreage by the South Platte Ditch and the Davis Brothers Ditch. The Davis Brothers Ditch irrigates less than 900 acres of the 7,800 irrigated acres in the study area.

Lands irrigated by the South Platte Ditch also have some rights to the South Platte River water stored in Prewitt Reservoir. Prewitt Reservoir has a capacity of 32,800 acre-feet and storage rights of 32,300 acre-feet. The average annual volume of water diverted into Prewitt Reservoir is 40,200 acre-feet and the average annual volume of water released from Prewitt Reservoir to the various ditches is 10,500 acre-feet. This gives an average annual seepage and evaporation loss of 29,700 acre-feet (Bittinger and Associates, 1969).

Geology

The South Platte River valley typically consists of an alluvium of Pleistocene to Recent age deposited on a bedrock of Late Cretaceous time. The alluvium fills a trough from 2 to 10 miles in width and is generally comprised of a heterogeneous mixture of clay, sand, and gravel, or lenses of these materials. In the study area, this alluvium ranges in depth from about 40 feet to 270 feet. The alluvium is in contact with the relatively impermeable Pierre shale formation of Late Cretaceous age. The Pierre shale bedrock in the study area has a deeply eroded stream channel which was formed during the lifting of the Rocky Mountains in Early Tertiary time (Bjorklund and Brown, 1957).

Hydrogeology

It has been reported that before irrigation was practiced in the valley, the South Platte River was an intermittent stream which was generally dry during the summer. At present, due to the deep percolation of applied irrigation water and the seepage from reservoirs and canals, the South Platte River is a perennial stream (Bjorklund and Brown, 1957). The water table aquifer of the study area generally slopes diagonally downstream and toward the river and is in hydraulic connection with the river. There is horizontal underflow into the study area at the upstream end and along the boundary opposite the river boundary. Horizontal outflow occurs at the downstream end of the study area. In addition, water generally discharges from the aquifer into the river. The aquifer is recharged by deep percolation of precipitation, applied surface water, and applied water from pumping, and by seepage from the South Platte Ditch and Prewitt Reservoir. Leakage to or from the Pierre shale bedrock is considered insignificant.

Seasonal and annual fluctuations of the water table are dependent on the ratio of the volume of surface water to the volume of groundwater used for irrigation. In general, the water table will rise if the volume of surface water diverted for irrigation is greater than the volume of groundwater pumped for irrigation, and if more groundwater than surface water is used for irrigation, the water table will be lowered. However, the exact relationship depends upon the respective percentages of surface water and groundwater that percolate to the water table. Since unlined ditches are used to convey surface water to the irrigated acreage of the study area, the percentage of diverted surface water that percolates to the water table will normally be greater than the percentage of pumped groundwater that percolates to the water table.

Typical depths to the water table in the study area are 15 to 20 feet below land surface. The permeability of the aquifer ranges from approximately 170 to 1100 feet per day. A typical cross-section of the study area through Prewitt Reservoir is shown in figure 2. The indicated bottom of Prewitt Reservoir is based on the average areal depth of water in the reservoir when the reservoir storage is at

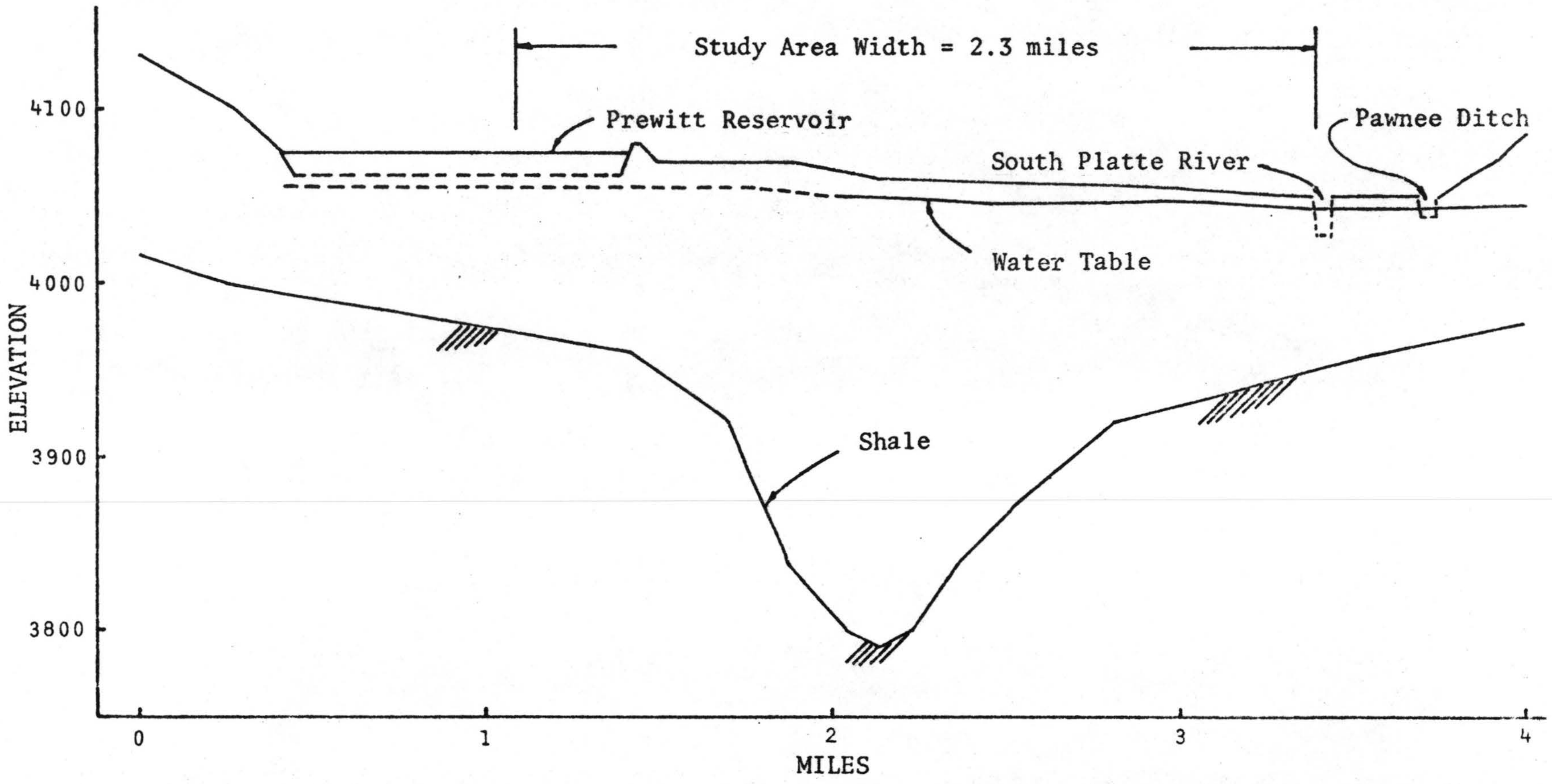


Figure 2. Cross-section A-A of study area. (See Figures 1 and 4 for location of cross-section.)

maximum capacity. This average depth of water of 18.0 feet was computed from the maximum reservoir capacity of 32,800 acre-feet and the reservoir area of 1,825 acres. The indicated elevation of the water table beneath the reservoir was estimated from results of the model study.

IV. DESCRIPTION OF THE MATHEMATICAL MODEL

Theoretical Development

The basic non-linear partial differential equation describing two-dimensional transient flow in a saturated porous medium may be derived from the mass continuity equation and Darcy's law and written as (Jacob, 1950):

$$\frac{\partial}{\partial x}\left(K_x h \, \Delta y \, \frac{\partial H}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(K_y h \, \Delta x \, \frac{\partial H}{\partial y}\right)\Delta y = S \, \frac{\partial h}{\partial t} \, \Delta x \Delta y + q \qquad (1)$$

where,

h = saturated thickness of aquifer (L)

H = water table elevation above a datum (L)

K = hydraulic conductivity (L/T)

S = storage coefficient (dimensionless)

q = net groundwater withdrawal (L^3/T)

x,y = space dimensions (L)

t = time dimension (T) .

Although equation 1 has no general solution, a finite difference approximation of this equation will allow a numerical solution. Equation 1 written in implicit, central finite difference form is as follows:

$$[A \, H_{i,j-1} + B \, H_{i,j+1} + C \, H_{i-1,j} + D \, H_{i+1,j} - (A+B+C+D+E) \, H_{i,j}]^{t+\Delta t}$$

$$= q - E \, H^{t}_{i,j} \qquad (2)$$

where,

$$A = \frac{2\ K_{i,j} \cdot K_{i,j-1} \cdot \Delta y_{i,j} \cdot \Delta y_{i,j-1} \cdot h_{i,j-1/2}}{\Delta y_{i,j} \cdot K_{i,j} \cdot \Delta x_{i,j-1} + \Delta y_{i,j-1} \cdot K_{i,j-1} \cdot \Delta x_{i,j}}$$

$$B = \frac{2\ K_{i,j} \cdot K_{i,j+1} \cdot \Delta y_{i,j} \cdot \Delta y_{i,j+1} \cdot h_{i,j+1/2}}{\Delta x_{i,j} \cdot K_{i,j+1} \cdot \Delta y_{i,j+1} + \Delta x_{i,j+1} \cdot K_{i,j} \cdot \Delta y_{i,j}}$$

$$C = \frac{2\ K_{i,j} \cdot K_{i-1,j} \cdot \Delta x_{i,j} \cdot \Delta x_{i-1,j} \cdot h_{i-1/2,j}}{\Delta y_{i,j} \cdot K_{i-1,j} \cdot \Delta x_{i-1,j} + \Delta y_{i-1,j} \cdot K_{i,j} \cdot \Delta x_{i,j}}$$

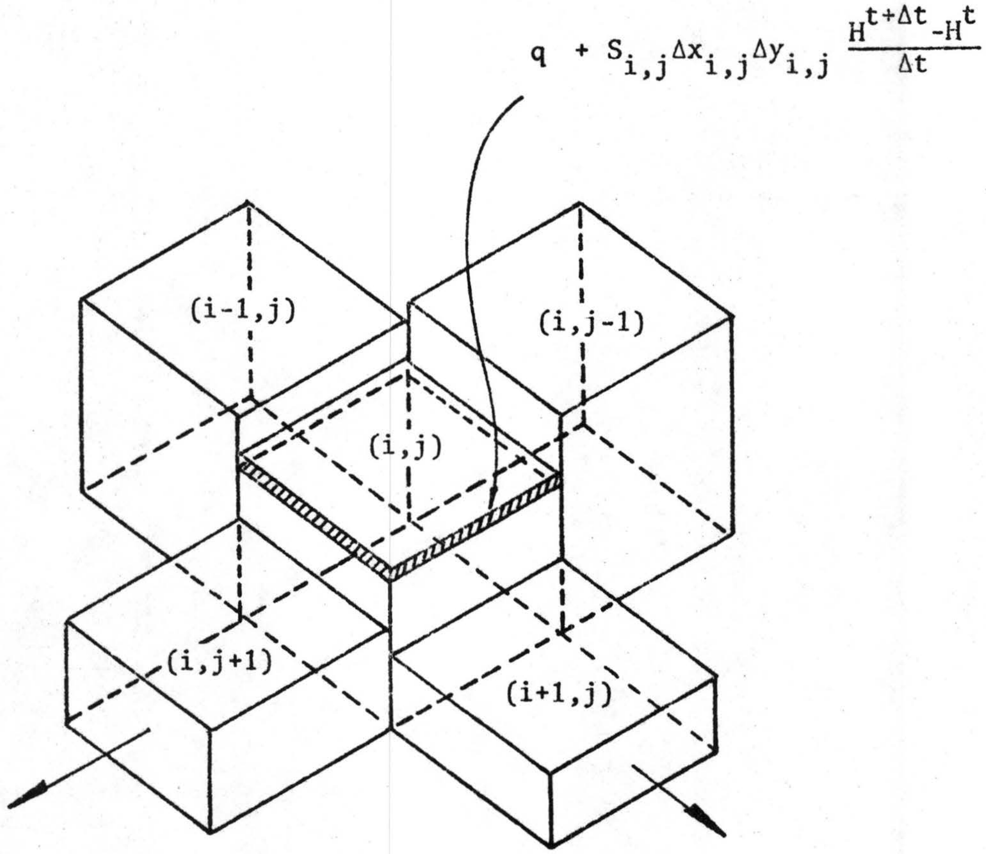
$$D = \frac{2\ K_{i,j} \cdot K_{i+1,j} \cdot \Delta x_{i,j} \cdot \Delta x_{i+1,j} \cdot h_{i+1/2,j}}{\Delta y_{i,j} \cdot K_{i+1,j} \cdot \Delta x_{i+1,j} + \Delta y_{i+1,j} \cdot K_{i,j} \cdot \Delta x_{i,j}}$$

$$E = \frac{S_{i,j} \cdot \Delta x_{i,j} \cdot \Delta y_{i,j}}{\Delta t} \quad .$$

The subscript notation refers to a particular grid block in a five grid system as indicated in figure 3. The superscript t refers to the starting time or previous time level and Δt is the time increment. Equation 2 is written for each grid in the study area for each designated time increment. The system of equations for the first time increment is solved simultaneously for the values of $H_{i,j}$ at the end of the time increment. These computed values of $H_{i,j}$ are then used as initial values in the system of equations representing the next time increment.

The coefficients A , B , C , and D are computed for each grid at the beginning of each time increment and are held constant during the time increment. The term $(h_{i,j-\frac{1}{2}})$ in the equation for coefficient A is the effective saturated thickness between grids $(i,j-1)$ and (i,j) calculated by the following approximation:

$$h_{i,j-\frac{1}{2}} = \text{MAX}\ (H_{i,j}, H_{i,j-1}) - \text{MAX}\ (Z_{i,j}, Z_{i,j-1})$$



$$\frac{\partial}{\partial x}\left(K_x h \Delta y \frac{\partial H}{\partial x}\right)\Delta x = A^t(H_{i,j-1}-H_{i,j})^{t+\Delta t} - B^t(H_{i,j}-H_{i,j+1})^{t+\Delta t}$$

$$\frac{\partial}{\partial y}\left(K_y h \Delta x \frac{\partial H}{\partial y}\right)\Delta y = C^t(H_{i-1,j}-H_{i,j})^{t+\Delta t} - D^t(H_{i,j}-H_{i-1,j})^{t+\Delta t}$$

Figure 3. Finite difference grid notation (adapted from Bittinger et al.)

where Z equals the bedrock elevation above a datum. A similar expression may be written for the h term in the equations for the coefficients B, C, and D. The value for the storage coefficient, S, included in coefficient E, varies spatially but remains constant in time. If the time increment is also constant, coefficient E will remain constant in time for each grid.

The rate of net groundwater withdrawal, q, represents the deep percolation of precipitation and applied surface water, and the rate of net withdrawal by pumping. The extraction of water by phreatophytes or the addition of water by artificial recharge could also be included in the value of q. It is necessary to calculate an average value of q for each grid for each time increment.

Equations 1 and 2 are subject to the Dupuit-Forchheimer assumptions and also assume that the fluid and porous media are incompressible.

Description of the Computer Program

The general mathematical model and corresponding computer program used in this study were developed by the Groundwater Section of the Department of Civil Engineering, Colorado State University.

The program reads in the number of rows and columns for the entire grid system and also for a smaller grid system within the entire grid system, if buffer zones are desired. The time increment of analysis, total time of analysis, and time increment of printout in days are also read as input data.

The following data are initially read for each grid and held constant throughout the total time of analysis.

1) dimensions of each grid in feet.

2) land surface elevation.

3) bedrock elevation.

4) permeability in feet per day.

5) storage coefficient.

6) coefficient for the fraction of each grid that is irrigated.

The initial water table elevations are also read for each grid. Impermeable boundary grids, constant head boundary grids, and grids with horizontal underflow are identified by coding the initial water table elevations. For boundary grids having horizontal underflow, the difference in water elevation between the outermost boundary grid and the next inner grid is held constant throughout the total time of analysis. The constant difference in water table elevation in feet must be read in for all grids having horizontal underflow. The program was modified to read in the elevation of Prewitt Reservoir at the beginning of each time increment. The Prewitt Reservoir grids are identified as constant head boundary grids but in effect changes are permitted at the beginning of each time period.

The program reads in the following hydrologic data:

1) annual phreatophyte extraction from each grid in acre-feet.

2) gross annual pumping withdrawal from each grid in acre-feet.

3) annual precipitation in inches (model assumes a uniform depth of precipitation over the entire study area).

4) annual surface water application in feet (model assumes a uniform application of surface water over the irrigated portion of the study area).

The annual pumping, precipitation, and surface water application are read in for each year of analysis. The annual phreatophyte extraction is read in initially and held constant for the total time of analysis. The program could easily be altered to read in the annual phreatophyte extraction from each grid at the beginning of each year of analysis as in the case of the gross annual pumping withdrawal.

One set of annual distribution coefficients is read in for each of the four types of annual hydrologic data. The coefficients represent the percentage of annual precipitation, surface water application, pumping, and phreatophyte use that occurs during each of the time increments in one year of analysis. The coefficients are read in initially and remain constant throughout the total time of analysis. If the total time of analysis is only one year, then the coefficients may be computed from the respective hydrologic data for that year. If the total time of analysis is more than one year, an average annual distribution is computed from the respective hydrologic data for all the years of analysis. A long-term average annual distribution should probably be used in analyzing future management alternatives. If the annual distribution of one of the hydrologic parameters is highly variable from one year to the next, the use of an average annual distribution could produce erroneous results. Under these circumstances, the model could be operated for only one year at a time or the program could easily be altered to read in the annual distribution coefficients at the beginning of each year of operation.

The program also reads in coefficients that represent the percentage of precipitation and applied surface water that percolates to the water table. Another coefficient is read in to represent the percentage of the gross pumping withdrawal that does not return to the water table.

This percentage of the gross pumping withdrawal will be called the net pumping withdrawal.

The program uses Gaussian elimination to solve the system of equations for each time step. The program output for each time increment includes the following:

1) net withdrawal from each grid in acre-feet.

2) volume of flow in x and y directions at each grid in acre-feet.

3) grid locations that were overdrawn or flooded.

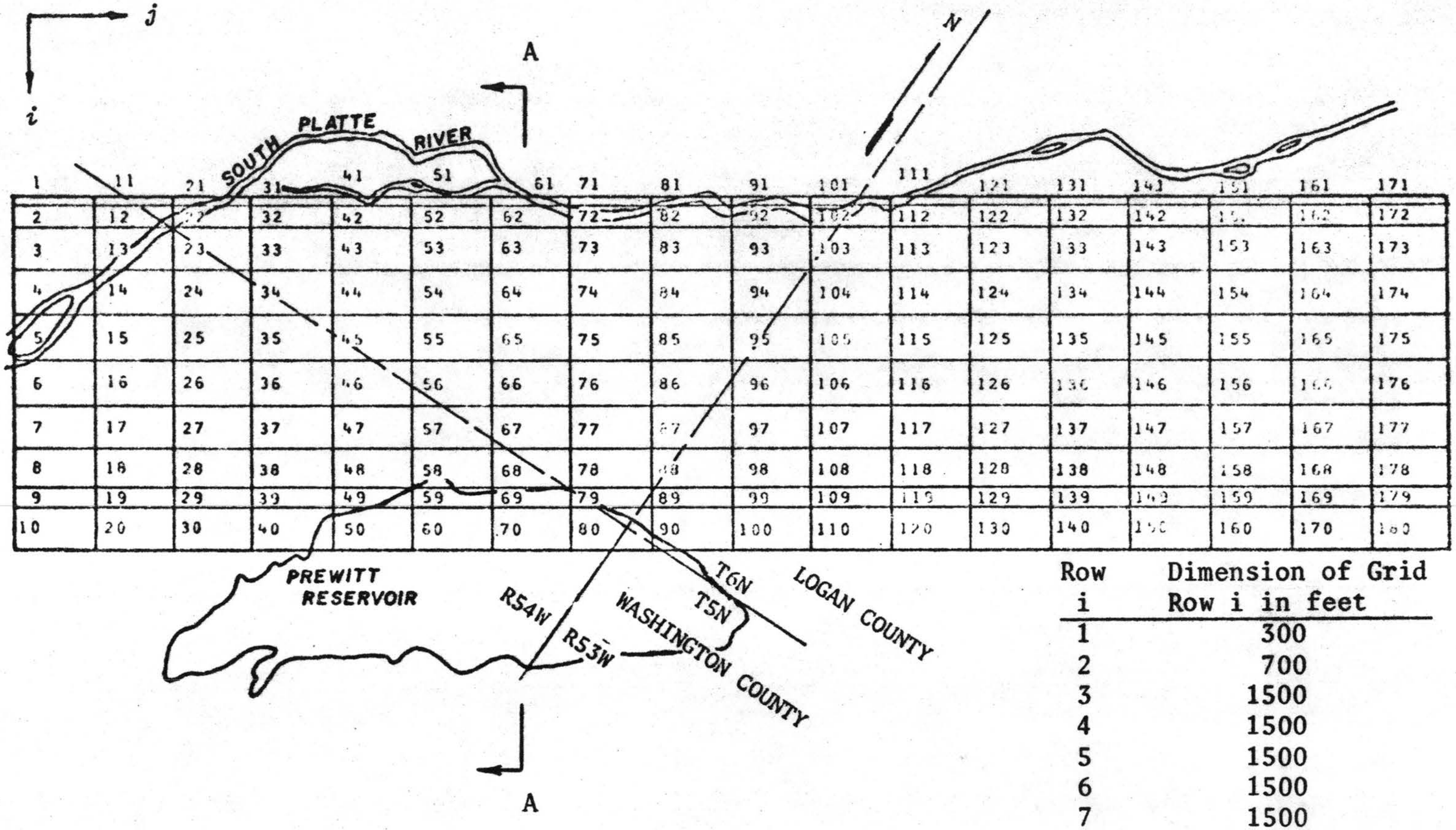
4) volumetric exchange between constant head grids (river or reservoir grids) and aquifer grids in acre-feet.

5) mass balance computations and totals for the entire grid system in acre-feet.

6) water table elevation for each grid at the end of each time increment.

Method of Investigation

The selected grid system for the study area model has 10 grids in a direction perpendicular to the river and 18 grids along the river for a total of 180 grids as indicated in figure 4. The meandering river was approximated as a straight line to serve as the northwest boundary of the study area. The 18 rectilinear grids representing the river have a width of 300 feet. All grid dimensions in a direction parallel to the river are one-half mile. Since flow toward the river is of special interest, the grid dimensions are smaller in this direction to allow greater accuracy. The grids adjacent to the river and reservoir have a width of 700 feet. A smaller grid width was used at these locations in the model in order to more accurately estimate the flow



Row i	Dimension of Grid Row i in feet
1	300
2	700
3	1500
4	1500
5	1500
6	1500
7	1500
8	1300
9	700
10	1500

Figure 4. Grid network for study area. All grid dimensions in j direction are ½ mile. Cross-section A-A given in Figure 2.

from the reservoir grids and the flow to the river grids. Only an interior seven mile length of the study area was considered to be the area of interest. Buffer zones containing two one-half mile grid columns were added at the upstream and downstream ends of the area of interest to give a total study area length of nine miles. Grid numbers 50, 59, 60, 70, and 80 were designated as reservoir grid blocks.

Model parameters for the study area grid network were obtained from data provided by various agencies. The model was operated for a historic two year period in order to compare model results with historic data. The period from November, 1969 to November, 1971 was selected because of the availability of hydrologic data. Groundwater level data for this two year period are available for 22 wells in the study area. The water levels were measured each Autumn. Most of the hydrologic data were only available in monthly values. Accordingly, the model was operated for 24 periods of 30 days each to give a total time of analysis of approximately two years.

For the first computer run of the model, the initial conditions, boundary conditions, and model parameters were estimated from the original basic data provided by various agencies. However, to attain a reasonable match between the simulated and historic water table elevations for the Autumn of 1970 and 1971, adjustments of the initial water table elevations, boundary conditions, and model parameters were required. A discussion of the adjustments that were made may be found in later sections. Adjustments to the model and a final sensitivity analysis required 38 runs and 17 minutes of computing time on a CDC 6400 computer. The cost of a two year run with a time increment of printout of 30 days was approximately four dollars.

Description of the Data Used in the Model Study

Initial Water Table Elevation

The initial water table elevations were obtained from a water table elevation contour map for November, 1969, prepared by the Division of Water Resources, Department of Natural Resources, State of Colorado. Figure 5 contains the study area portion of the contour map. Contour map elevations in the vicinity of the reservoir are denoted by dashed lines which indicate unknown or questionable accuracy. Elevations in other areas of the contour map should be within a foot of actual elevations in November. For simplicity in establishing congruency between the water table elevation data and other monthly hydrologic data, it was assumed that the water table elevations were measured on November 1.

First estimates of the initial water table elevation for each grid block were made by superimposing the grid network on the contour map and taking the elevation at the center of the grid. Operation of the model with these initial water table elevations resulted in having the water table elevations of many of the grids either rise or fall excessively during the first few time increments. The average total rise or fall during the first three time increments was approximately one or two feet. It should be noted that the excessive rise or fall was not due to an applied hydrologic stress since during the first several months of operation, there is no pumping or applied water and only insignificant amounts of precipitation. The jump in water table elevations was partially due to the fact that the center of a grid does not always represent the appropriate water table elevation for a large grid block. As a result, the first estimates of the water table elevations contained random errors. Furthermore, there were probably additional random

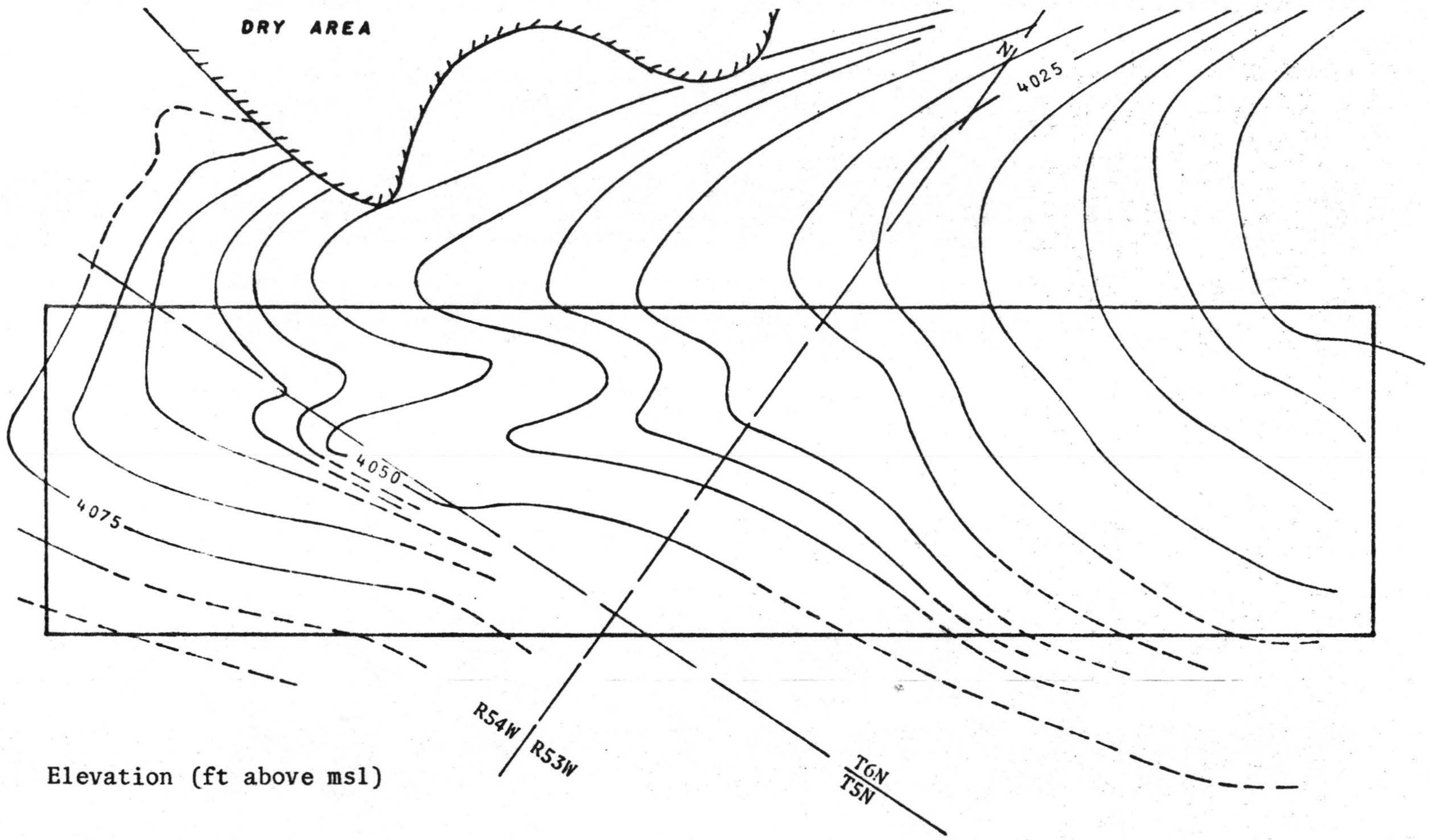


Figure 5. Water table elevation contour map of study area. (Taken from a map prepared by the Division of Water Resources, State of Colorado.)

errors in the original water table elevation basic data. The random errors in the first estimates of the water table elevation for each grid resulted in an unstable system that tended to approach equilibrium with time. The amount of time needed to reach equilibrium depends on the characteristics of the porous media. The effect of random errors in initial water table elevations is further discussed by Bibby (1971). In order to reduce the random error in the estimates of the initial water table elevations, the grid water table elevations at the end of three months of operation were used as initial water table elevations. Additional initial water table elevation adjustments required for boundary grids with horizontal underflow will be explained in the discussion of results.

The final estimates of the initial water table elevation for each grid are indicated in Table 1. A constant head boundary grid is signified by adding 30,000 to the elevation of the grid. A boundary grid with horizontal underflow is signified by adding 20,000 to the elevation of the grid. The 18 constant head grids representing the South Platte River are in the far right-hand column of Table 1. The five constant head grids on the left side of Table 1 represent a portion of Prewitt Reservoir.

Final Historic Water Table Elevations

Historic groundwater level data for 22 wells in the study area are available in the Colorado Water Conservation Board Basic-Data Release No. 26 of 1972. This publication includes the depth to water for the years 1968 through 1972. Water levels for 1972 were measured in November and water levels for the four preceding years were measured each Autumn. Groundwater levels in the study area for the years 1969, 1970, and 1971

TABLE 1

INITIAL WATER TABLE ELEVATIONS FOR GRID NETWORK[1]

24084.000	24081.800	24079.700	24076.100	24072.900	24069.200	24066.100	24063.000	24060.500	34059.000	1
24081.800	4080.000	4078.100	4075.000	4072.100	4069.100	4066.100	4063.000	4060.500	34059.000	2
24079.200	4077.100	4075.200	4072.700	4069.800	4066.900	4064.000	4061.000	4059.500	34059.000	3
24077.000	4074.000	4071.700	4068.900	4066.200	4063.500	4060.800	4057.400	4054.000	34052.000	4
34078.600	4066.900	4065.900	4064.000	4062.100	4060.300	4057.800	4053.900	4050.100	34048.000	5
34078.600	34078.600	4061.000	4059.500	4058.000	4055.900	4053.000	4049.300	4046.500	34045.000	6
34078.600	4056.700	4056.100	4054.700	4052.600	4050.300	4048.000	4045.200	4043.100	34042.000	7
34078.600	4054.500	4053.200	4051.100	4048.800	4046.600	4044.300	4041.900	4040.500	34040.000	8
24063.000	4057.000	4053.400	4049.800	4046.700	4043.500	4040.800	4037.900	4035.300	34034.000	9
24056.800	4052.700	4050.400	4047.300	4044.000	4040.300	4037.300	4034.700	4033.100	34032.500	10
24052.000	4048.300	4045.600	4042.400	4039.000	4035.800	4033.000	4031.100	4029.900	34029.500	11
24046.000	4042.700	4040.100	4036.800	4033.500	4030.600	4028.300	4026.900	4026.000	34025.500	12
24038.800	4036.100	4034.000	4031.000	4028.100	4025.800	4024.300	4023.600	4022.300	34021.500	13
24032.600	4030.500	4028.800	4026.300	4023.800	4021.900	4020.500	4019.600	4018.600	34018.000	14
24029.800	4027.200	4025.100	4022.400	4019.800	4017.600	4016.100	4015.100	4014.400	34014.000	15
24026.900	4024.200	4022.100	4019.300	4016.400	4014.300	4012.900	4011.600	4010.800	34010.500	16
24025.300	4022.000	4019.700	4016.700	4013.800	4011.600	4009.900	4008.100	4006.500	34005.600	17
24023.000	24021.700	24019.000	24015.300	24012.100	24009.800	24008.700	24007.000	24005.600	34003.900	18

[1]Values for the 18 river grids are in the far right-hand column. Constant head grids are signified by adding 30,000 to the elevation of the grid. Constant gradient boundary grids are signified by adding 20,000 to the elevation of the grid. Elevations in feet above msl.

are given in Table 2. The water table elevations in Table 2 were calculated by use of the depths to water and the approximate land surface elevations. Since the land surface elevations are only accurate to the nearest foot or so, only the relative change in water table elevation at each well is accurate.

As an approximate check of the correspondence between sources of data, the estimated water table elevations at the 22 wells in the Autumn of 1969 given in Table 2 were compared with the elevations at the respective well locations on the water table elevation contour map of November, 1969 given in figure 5. The mean difference is 0.07 feet with a standard deviation from the mean of 1.53 feet.

Bedrock Elevation

The bedrock elevations were obtained from a bedrock elevation contour map prepared by the Division of Water Resources, Department of Natural Resources, State of Colorado. Figure 6 contains the study area portion of the contour map. The elevation at the center of each grid block was assumed to be the proper bedrock elevation of the grid. The bedrock elevations for the study area grid network are given in Table 3.

Permeability

The permeabilities were obtained from a transmissivity map prepared by the Division of Water Resources, Department of Natural Resources, State of Colorado. The study area portion of the map is shown in figure 7. The saturated thickness of November, 1969, as computed from the State of Colorado water table and bedrock contour maps (figures 5 and 6), was used to estimate the permeability of each grid. The permeabilities were computed from the equation:

$$K = \frac{T}{7.48h}$$

TABLE 2
HISTORIC GROUNDWATER LEVELS IN STUDY AREA

Grid No.	Well Location T-R-S	Depth[1] of Well	Land[1] Surface Elev.	Depth to Water[1] 1969	1970	1971	Water Table Elev.[2] 1969	1970	1971
16	5-54-4	91	4082	13.58	13.69	12.91	4068.4	4068.3	4069.1
18	5-54-10	-	4092	13.85	13.80	13.69	4078.2	4078.2	4078.3
32	6-54-34	70	4060	7.60	7.65	7.84	4052.4	4052.4	4052.2
34	6-54-34	88	4066	9.00	8.99	8.93	4057.0	4057.0	4057.1
36	5-54-3	111	4074	9.43	9.70	9.95	4064.6	4064.3	4064.0
45	6-54-35	60	4060	10.83	10.92	10.92	4049.2	4049.1	4049.1
47	5-54-2	100	4085	12.77	16.49	13.63	4072.2	4068.5	4071.4
53	6-54-34	72	4058	8.30	8.11	8.11	4049.7	4049.9	4049.9
54	6-54-35	-	4060	8.82	8.38	8.28	4051.2	4051.6	4051.7
63	6-54-26	22	4055	6.24	6.25	5.98	4048.8	4048.8	4049.0
75	6-54-25	70	4056	11.41	11.27	11.43	4044.6	4044.7	4044.6
88	6-53-31	112	4059	10.58	11.12	11.59	4048.4	4047.9	4047.4
94	6-53-30	110	4050	16.03	14.45	13.85	4034.0	4035.6	4036.2
108	6-53-30	88	4053	9.45	9.78	10.25	4043.6	4043.2	4042.8
113	6-53-19	105	4041	11.14	11.01	11.39	4029.9	4030.0	4029.6
115	6-53-30	28	4041	8.37	8.13	8.92	4032.6	4032.9	4032.1
118	6-53-29	110	4060	17.08	16.97	17.63	4042.9	4043.0	4042.4
134	6-53-17	105	4032	15.45	14.90	15.19	4016.6	4017.1	4016.8
137	6-53-20	100	4058	31.68	28.55	29.16	4026.3	4029.4	4028.8
156	6-53-16	80	4038	24.25	23.45	24.21	4013.8	4014.6	4013.8
162	6-53-9	103	4020	15.18	14.74	15.47	4004.8	4005.3	4004.5
168	6-53-22	92	4055	35.60	36.22	36.23	4019.4	4018.8	4018.8

[1]Colorado Water Conservation Board Basic-Data Release No. 26, 1972. Water levels were measured each Autumn.
[2]Water table elevations were calculated using the given approximate land surface elevations. Only relative differences at each well are accurate.

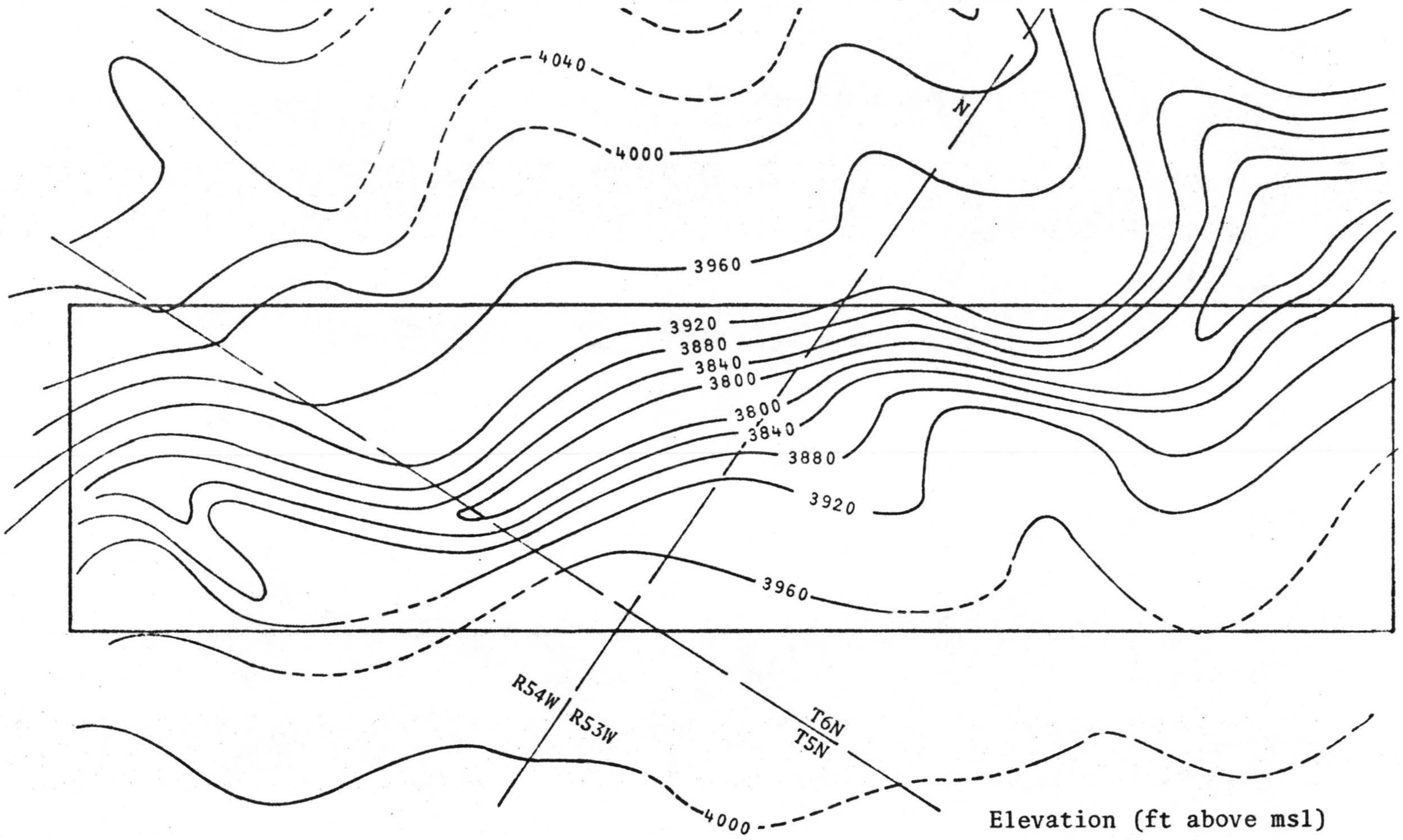


Figure 6. Bedrock elevation contour map of study area. (Taken from a map prepared by the Division of Water Resources, State of Colorado.)

TABLE 3

BEDROCK ELEVATIONS FOR GRID NETWORK[1]

3945.000	3935.000	3925.000	3890.000	3845.000	3900.000	3953.000	4004.000	4020.000	4028.000	1
3950.000	3935.000	3922.000	3857.000	3840.000	3867.000	3930.000	3985.000	4020.000	4038.000	2
3920.000	3880.000	3880.000	3884.000	3840.000	3890.000	3943.000	3992.000	3998.000	4001.000	3
3918.000	3905.000	3895.000	3870.000	3860.000	3920.000	3965.000	3977.000	3986.000	3990.000	4
3921.000	3900.000	3890.000	3835.000	3890.000	3939.000	3953.000	3970.000	3985.000	3992.000	5
3951.000	3921.000	3890.000	3800.000	3864.000	3922.000	3939.000	3952.000	3962.000	3970.000	6
3967.000	3960.000	3930.000	3843.000	3785.000	3842.000	3896.000	3929.000	3940.000	3945.000	7
3972.000	3966.000	3961.000	3927.000	3858.000	3783.000	3837.000	3890.000	3927.000	3935.000	8
3971.000	3965.000	3959.000	3938.000	3902.000	3825.000	3792.000	3868.000	3920.000	3932.000	9
3965.000	3960.000	3950.000	3937.000	3920.000	3848.000	3792.000	3858.000	3915.000	3928.000	10
3961.000	3951.000	3940.000	3924.000	3896.000	3860.000	3832.000	3820.000	3890.000	3913.000	11
3959.000	3948.000	3938.000	3921.000	3910.000	3902.000	3882.000	3788.000	3842.000	3876.000	12
3961.000	3952.000	3946.000	3940.000	3936.000	3928.000	3880.000	3803.000	3902.000	3921.000	13
3977.000	3972.000	3967.000	3960.000	3943.000	3923.000	3838.000	3860.000	3922.000	3927.000	14
3963.000	3955.000	3944.000	3930.000	3915.000	3880.000	3798.000	3802.000	3853.000	3880.000	15
3952.000	3943.000	3934.000	3921.000	3891.000	3860.000	3810.000	3765.000	3760.000	3772.000	16
3962.000	3955.000	3947.000	3934.000	3922.000	3896.000	3870.000	3841.000	3807.000	3765.000	17
3976.000	3971.000	3966.000	3961.000	3949.000	3930.000	3906.000	3880.000	3856.000	3845.000	18

[1]Values for the 18 river grids are in the far right-hand column. Elevations in feet above msl.

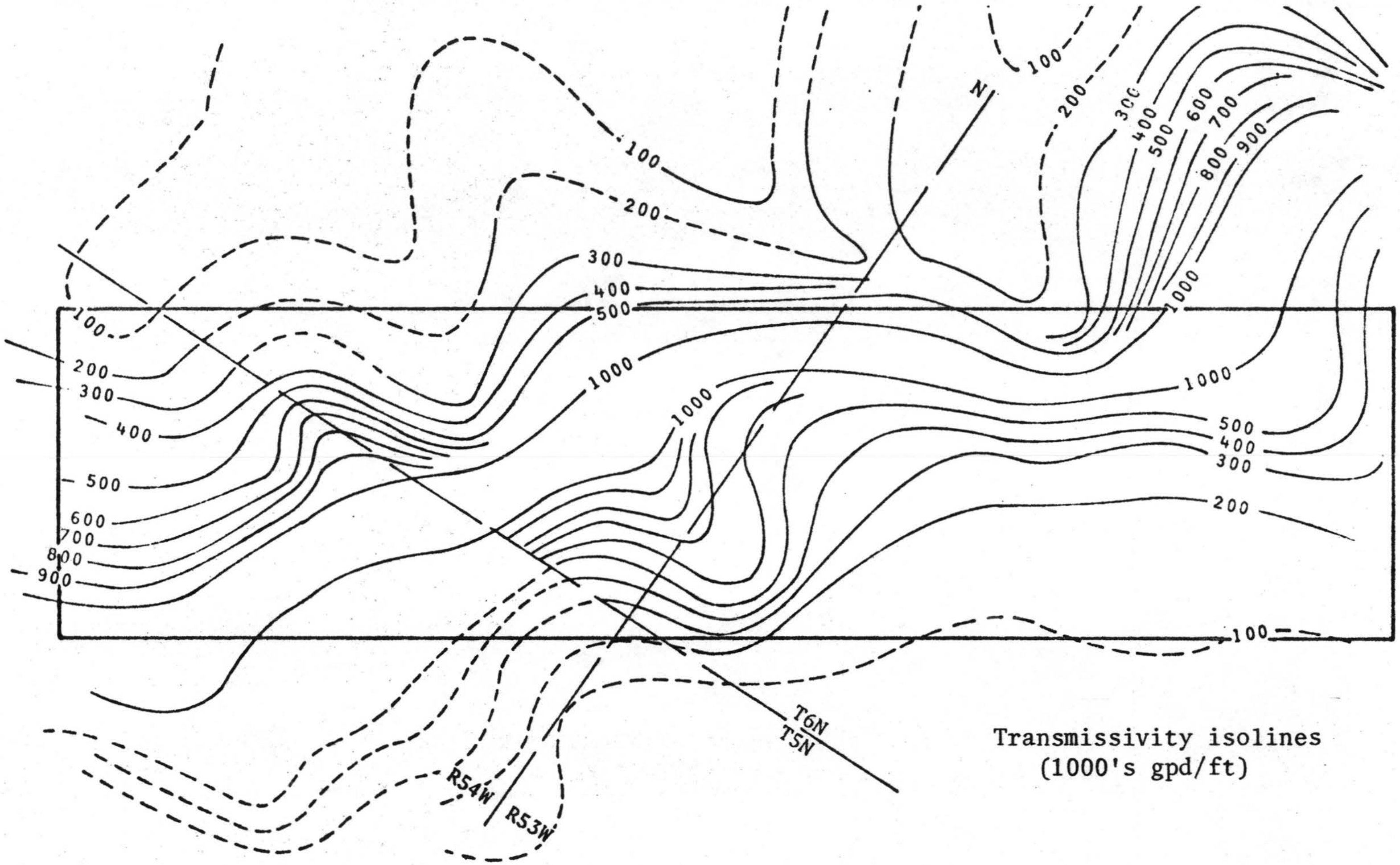


Figure 7. Transmissivity map of study area. (Taken from a map prepared by the Division of Water Resources, State of Colorado.)

where,

K = permeability in feet/day.

T = transmissivity in gpd/foot.

h = saturated thickness in feet.

Permeabilities for the grid network are presented in Table 4. A permeability of approximately 500 feet per day was originally computed for the five reservoir grids. Adjustments to the model required that the permeability of the reservoir grids be reduced to 10 feet per day. An explanation will be given in the discussion of results.

Storage Coefficient

Bjorklund and Brown (1957) discuss the results of pumping tests in the South Platte River valley. Several tests indicated an average storage coefficient of 0.17 . This figure did not include the normal increase of the storage coefficient during continuous pumping. They assumed an average storage coefficient of 0.20 for their particular project area. Since data on storage coefficients in the study area are unavailable, a uniform value of 0.19 was assumed for all grids in the study area.

Precipitation

Precipitation data for Sterling, Colorado, located 15 miles northeast of the study area, were assumed to be applicable. The annual distribution of precipitation was computed from the 1969-1971 two-year average of the monthly precipitation values at Sterling. The two years of data and the corresponding annual distribution are given in Table 5. It was assumed that 20% of the precipitation would reach the groundwater table. For employment of the model in future management decisions,

TABLE 4

PERMEABILITY VALUES FOR GRID NETWORK[1]

922.000	861.000	716.000	460.000	299.000	354.000	400.000	385.000	320.000	319.000	1
1013.000	903.000	716.000	421.000	297.000	281.000	297.000	334.000	416.000	593.000	2
830.000	675.000	686.000	594.000	353.000	386.000	450.000	581.000	539.000	484.000	3
747.000	782.000	751.000	668.000	587.000	793.000	936.000	651.000	598.000	561.000	4
10.000	626.000	698.000	576.000	825.000	960.000	540.000	456.000	459.000	477.000	5
10.000	10.000	631.000	524.000	727.000	588.000	361.000	355.000	334.000	356.000	6
10.000	491.000	560.000	590.000	379.000	659.000	620.000	561.000	514.000	510.000	7
10.000	432.000	616.000	739.000	658.000	516.000	643.000	794.000	828.000	726.000	8
464.000	668.000	850.000	924.000	757.000	522.000	541.000	782.000	988.000	918.000	9
431.000	734.000	810.000	736.000	625.000	403.000	348.000	760.000	1133.000	964.000	10
240.000	262.000	328.000	382.000	370.000	346.000	386.000	631.000	955.000	915.000	11
182.000	206.000	236.000	258.000	326.000	391.000	542.000	557.000	727.000	673.000	12
170.000	210.000	232.000	268.000	363.000	532.000	922.000	608.000	557.000	535.000	13
267.000	300.000	328.000	365.000	430.000	554.000	731.000	420.000	376.000	353.000	14
242.000	260.000	264.000	259.000	291.000	337.000	549.000	625.000	578.000	628.000	15
219.000	234.000	243.000	243.000	263.000	343.000	524.000	541.000	533.000	562.000	16
262.000	279.000	311.000	318.000	413.000	555.000	659.000	716.000	672.000	605.000	17
370.000	408.000	463.000	560.000	626.000	643.000	571.000	463.000	388.000	361.000	18

[1]Values for the 18 river grids are in the far right-hand column. Permeabilities are given in feet per day.

TABLE 5. MONTHLY PRECIPITATION AT STERLING, COLORADO[1] IN INCHES

YEAR	NOV.	DEC.	JAN.	FEB.	MAR.	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	TOTAL
1969-1970	.13	.19	.15	T	.45	1.59	2.75	2.31	1.48	.02	.92	1.10	11.09
1970-1971	.46	.22	.40	.46	1.54	2.87	3.17	2.27	.64	.81	2.19	.54	15.57
TWO YEAR AVERAGE	.30	.20	.28	.23	.99	2.23	2.96	2.29	1.06	.41	1.56	.82	13.33
TWO YEAR AVERAGE FRACTION OF ANNUAL	.02	.02	.02	.02	.07	.17	.22	.17	.08	.03	.12	.06	1.00

[1]Climatological Data for Colorado, U. S. Weather Bureau.

the two-year average annual distribution of precipitation should be replaced with a long-term average annual distribution.

Applied Surface Water

The average depth of surface water applied to the total irrigated acreage was estimated for each year of operation. Estimates were based on the South Platte Ditch diversion records of the Division of Water Resources, Department of Natural Resources, State of Colorado. The monthly diversions and annual distribution of diversions for 1970 and 1971 are given in Table 6. The diversion records indicate that the annual diversions were for the irrigation of 5774 acres. As noted in Chapter 3, aerial photos indicate there are 7800 acres of cultivated agricultural land in the study area and it was assumed that all 7800 acres are irrigated. Assuming the diversion record acreage to be accurate, there is approximately a 2000 acre difference between the study area irrigated acreage and the acreage irrigated by the South Platte Ditch diversions. Surface water for the additional 2000 acres could have been delivered from another canal system or Prewitt Reservoir. For the 5774 acres, the computed average depths of diverted surface water in 1970 and 1971 are 2.12 and 1.58 feet, respectively. Since data are unavailable for the surface water application on the 2000 acres, the average depths of diverted surface water applied to the 5774 acres were assumed to have been uniformly applied to all 7800 acres of agricultural land in the study area. The application of an average depth of diverted surface water assumes that canal seepage (including seepage from the South Platte Ditch) is uniformly distributed over the 7800 acres. It was assumed that 30% of the applied surface water would percolate to the water table. This percentage includes the

TABLE 6

SOUTH PLATTE DITCH DIVERSIONS[1]

MONTH	1970 DIVERSION Ac.-ft.	1970 FRACTION OF ANNUAL	1971 DIVERSION Ac.-ft.	1971 FRACTION OF ANNUAL	TWO YEAR AVERAGE FRACTION OF ANNUAL
APRIL	0	.00	0	.00	.00
MAY	2290	.19	206	.02	.105
JUNE	1420	.12	2220	.24	.18
JULY	4152	.34	2442	.27	.305
AUGUST	2450	.20	1944	.21	.205
SEPT.	1660	.13	2080	.23	.18
OCT.	256	.02	230	.03	.025
TOTAL					
Ac.-ft.	12,228	1.00	9122	1.00	1.00
Ac.-ft./Ac.	2.12		1.58		

[1]Diversion records of the Division of Water Resources, Department of Natural Resources, State of Colorado. Annual diversions were for the irrigation of 5774 acres.

uniformly distributed canal seepage. The coefficients representing the irrigated portion of each grid block are shown in Table 7. These coefficients were estimated from the aerial photos provided by the USDA Agriculture Stabilization Office, Sterling, Colorado.

Pumping

Annual volumes of pumping were estimated for 27 irrigation wells in the study area. The pumping estimates are based on gross electrical power deliveries to irrigation pumping plants and an estimated average energy consumption per acre-foot of water pumped. Power consumption and pumping plant horsepower data were provided by the Morgan County Rural Electric Association, Fort Morgan, Colorado, and the Public Service Company of Colorado, Sterling, Colorado.

The amount of energy required to lift one acre-foot of groundwater to the land surface can be estimated from the total pumping lift, the pump shaft horsepower, and the well discharge. These data were available for 23 of the 27 wells in the study area. The energy consumption per acre-foot of water pumped to the land surface was estimated for each of the 23 wells. The estimates were based on the pump shaft horsepower in 1969 and well yield data from the Colorado Water Conservation Board Basic-Data Release No. 17 of 1964. Assuming an overall pump efficiency of 55% , the total lift (static head plus drawdown) was computed from the equation:

$$h + s = 2180 \frac{Hp}{Q}$$

where h is the static head in feet, s is the drawdown in feet, Hp is the pump shaft horsepower, and Q is the discharge in gpm. For the

TABLE 7

COEFFICIENTS REPRESENTING IRRIGATED PORTION OF EACH GRID BLOCK[1]

.930	1.000	1.000	1.000	.230	0.000	0.000	0.000	0.000	0.000	1
.320	1.000	1.000	1.000	.960	.450	.030	0.000	0.000	0.000	2
0.000	.280	.650	.990	1.000	1.000	.950	.490	.110	0.000	3
0.000	0.000	0.000	.530	1.000	1.000	1.000	1.000	.460	0.000	4
0.000	0.000	0.000	.580	1.000	1.000	1.000	1.000	.500	0.000	5
0.000	0.000	0.000	.400	1.000	1.000	1.000	.980	.900	0.000	6
0.000	0.000	0.000	.550	1.000	1.000	.890	.300	0.000	0.000	7
0.000	0.000	0.000	.630	1.000	1.000	.920	.030	0.000	0.000	8
0.000	0.000	.130	1.000	1.000	1.000	.760	0.000	0.000	0.000	9
0.000	0.000	.450	1.000	1.000	1.000	.740	0.000	0.000	0.000	10
0.000	0.000	.330	1.000	1.000	1.000	.990	.270	0.000	0.000	11
.260	.810	.940	1.000	1.000	1.000	1.000	.990	.830	0.000	12
.020	.190	.370	.500	1.000	1.000	1.000	1.000	1.000	0.000	13
0.000	0.000	.280	.880	1.000	1.000	1.000	1.000	.880	0.000	14
.120	.750	1.000	1.000	1.000	1.000	1.000	1.000	.520	0.000	15
.030	.240	.530	1.000	1.000	1.000	1.000	1.000	1.000	0.000	16
0.000	0.000	0.000	.430	1.000	1.000	1.000	1.000	1.000	0.000	17
0.000	0.000	0.000	.210	.540	.930	1.000	1.000	1.000	0.000	18

[1]Values for the 18 river grids are in the far right-hand column. Coefficients are expressed as decimal fractions of the respective grid block area.

same efficiency of 55% , the energy consumption per acre-foot of water pumped, C , was computed from the equation:

$$C = \frac{h+s}{.54}$$

where C has units of kilowatt-hours per acre-foot.

The estimated average energy consumption for all 23 wells is 66 Kwh per acre-foot. It should be noted that many of the well yields may have been estimated by a well driller at the time of drilling and some of these estimates could be in error. In addition, the yield of a well tends to decrease with age due to the plugging up of aquifer pores or the well screen. If all of the well yields were decreased by 15% , the average energy consumption per acre-foot would increase by 15% , to a value of 76 Kwh per acre-foot.

Bjorklund and Brown (1957) present a study of the electric power consumption of irrigation wells in the South Platte River valley based on data supplied by Morgan County REA and pumping tests. The average energy consumption for 53 wells from Brush to Sterling was estimated to be 74.3 Kwh per acre-foot. Brush is located approximately 18 miles southwest of the study area.

The average energy consumption for the wells in the study area was assumed to be 75 kilowatt-hours per acre-foot of water pumped. The computations for the value of C and the estimated annual volumes of pumping for 1970 and 1971 are given in Table 8. Most of the electrically powered irrigation pumping plants in the study area are believed to be included in Table 8. The number of non-electric pumping plants in the study area was not determined.

TABLE 8

ESTIMATED ANNUAL VOLUME OF PUMPING FOR 27 WELLS IN STUDY AREA

Grid No.	Name	Ser.[3] Co.	Location T-R-S	Pump[1] Elev.	Water[2] Table Elev.	Static Head h	Kwh[3] Used 1969	Pump[3] Hp	Q[4] gpm	Total Lift h+s	Draw-down s	Spec. Cap. Q/s	Kwh per Ac-Ft	Kwh[3] Consumed 1970	Kwh[3] Consumed 1971	Ac-Ft[5] Pumped 1970	Ac-Ft[5] Pumped 1971
16	Higgason	REA	5-54-4	4084	4069	15	4560	20	-	-	-	-	-	3060	4810	41	64
16	Fritzler	REA	5-54-3	4085	4070	15	37130	40	2500	35	20	120	65	8580	24990	114	333
18	Fritzler	REA	5-54-10	4094	4078	16	26100	36	-	-	-	-	-	11060	17930	147	239
32	Henderson	REA	6-54-34	4062	4052	10	7040	23	2250	22	12	190	41	5770	19130	77	255
34	Henderson	REA	6-54-34	4068	4057	11	16940	26	2200	26	15	150	48	9400	11410	125	152
36	Grigsby	REA	5-54-3	4076	4065	11	11430	23	3000	17	6	500	31	8330	10250	111	137
37	Grigsby	REA	5-54-2	4076	4066	10	20070	27	-	-	-	-	-	14250	12700	190	169
45	Shino	REA	6-54-35	4062	4049	13	8710	27	969	60	47	20	111	2520	6440	34	86
47	Fritzler	REA	5-54-2	4087	4072	15	7260	22	1800	27	12	150	50	1950	6550	26	87
54	Shino	REA	6-54-35	4062	4051	11	7580	22	2000	24	13	150	44	8500	9000	113	120
55	Lutin	REA	6-54-35	4064	4047	17	9260	24	2016	26	9	220	48	3080	10810	41	144
75	Schott L.	REA	6-54-25	4058	4045	13	8320	16	1300	27	14	90	50	4180	3370	56	45
86	Curlee	REA	6-54-25	4053	4045	8	16070	39	1950	44	36	50	81	13320	11160	178	149
88	Ostermiller Farms	REA	6-53-31	4061	4048	13	12810	29	2000	32	19	110	59	5410	15680	72	209
94	Bartlett & Gaines	REA	6-53-30	4052	4034	18	22190	34	2300	32	12	190	59	8840	26830	118	358
108	Helmut	REA	6-53-30	4055	4044	11	10500	14	1200	25	14	90	46	3390	8080	45	108
113	Probst	PSC	6-53-19	4043	4030	13	33600[6]	40	1800	-	-	-	-	24720	33840	330	451
118	Schott M.	REA	6-53-29	4062	4043	19	8240	31	2200	31	18	120	57	8480	25930	113	346
134	Hettinger	PSC	6-53-17	4034	4017	17	8220	25	2250	24	7	320	44	10280	9880	137	132
136	Karg	REA	6-53-21	4050	4022	28	39030	49	2000	53	25	40	98	45460	48710	606	649
137	Karg	REA	6-53-20	4060	4026	34	16852	36	2000	39	5	400	72	18589	18625	248	248
145	South	PSC	6-53-20	4043	4021	22	9620	30	1800	36	14	130	67	3620	14280	48	190
156	Smart	PSC	6-53-16	4040	4014	26	6248	30	1000	65	39	30	120	17650	10430	223	139
157	Smart	REA	6-53-21	4054	4020	34	3370	29	1200	53	19	60	98	4830	8350	64	111
162	Schott H.	PSC	6-53-9	4022	4005	17	3380	15	1500	22	5	300	41	9210	13420	123	179
168	Smart	REA	6-53-22	4057	4019	38	3180	37	1200	67	29	40	124	74670	54670	995[7]	729[7]
173	Jones	PSC	6-53-10	4022	4008	14	11540	30	2000	33	19	110	61	9600	11260	128	150

[1]Land surface elevation from USGS 7½ min. quadrangle plus 2 feet.
[2]Water table elevation contour map for November, 1969, Division of Water Resources, Dept. of Natural Resources, State of Colorado.
[3]Data from power company serving each well as noted. Wells are served by the Morgan County Rural Electric Assoc. of Fort Morgan, Colorado (REA), and the Public Service Company of Colorado, Sterling, Colorado (PSC).
[4]Well yields from Colorado Water Conservation Board Basic-Data Release No. 17, 1964.
[5]Based on an estimated average energy consumption of 75 Kilowatt-Hours per acre-foot of water pumped.
[6]Two wells connected on one meter.
[7]Began using feed sprinkler system in 1970. Estimate of annual pumping is invalid.

The estimated annual volume of pumping is assumed to have been pumped entirely from the grid in which the well is located. If a well location fell on the boundary between two grid blocks, the volume of pumping from that well was arbitrarily assigned to one of the grid blocks. The net pumping withdrawal was assumed to be 75% of the gross pumping withdrawal. This means that 25% of the gross pumping withdrawal is assumed to percolate to the water table of the grid in which the well is located.

The annual distribution of pumping was estimated from the 1971 monthly kilowatt-hour consumption for 17 wells in the study area as indicated in Table 9.

The well in grid number 168 was connected to a sprinkler system in 1970. The additional energy consumption of a sprinkler system invalidates the estimates of annual pumping for this well. The energy consumption of a power plant connected to a sprinkler system is estimated to be 3 times that of the other wells or 225 Kwh per acre-foot. The annual volumes of pumping for this well in 1970 and 1971 are estimated to be one-third of the respective values given in Table 8. With this adjustment, the total annual volumes of pumping in the study area in 1970 and 1971 are 3840 and 5493 acre-feet, respectively.

It would be of interest to determine if the summation of the estimated total gross pumping withdrawal and estimated total applied surface water is approximately equal to the gross irrigation water requirement of the irrigated acreage of the study area. If it is assumed that the estimated total annual volumes of pumping are distributed over all 7800 acres of irrigated land in the study area, the estimated average depths of applied water from pumping in 1970 and 1971 are 0.49 and 0.70 feet

TABLE 9. ESTIMATED ANNUAL DISTRIBUTION OF PUMPING VOLUMES

NAME	KILOWATT-HOUR CONSUMPTION IN 1971[1]					
	MAY	JUNE	JULY	AUG.	SEPT.	TOTAL
Bartlett & Gaines	0	7710	8410	9380	1330	26830
Curlee	0	2400	5950	1320	1490	11160
Fritzler	960	3820	9350	9610	1250	24990
Fritzler	0	1770	8890	7210	60	17930
Grigsby	1440	4680	3790	2470	320	12700
Grigsby	770	2960	4000	1920	600	10250
Henderson	50	7430	4830	5010	910	19130
Henderson	1130	3120	2630	3440	1090	11410
Higgason	0	1020	1580	1900	310	4810
Karg	0	8005	4958	2198	3464	18625
Karg	4820	9830	14200	16830	3030	48710
Ostermiller Farms	0	2960	5340	5360	2020	15680
Mary Schott	1610	8720	5040	9290	1270	25930
Shino	680	3120	2900	2160	140	9000
Smart	1180	8990	26650	14980	2870	54670
Smart	0	1140	1860	3620	1730	8350
Uhler	0	4040	6240	5100	1100	16480
TOTAL	12640	81715	116618	102698	22984	336655
Fraction of Annual	.04	.24	.35	.30	.07	1.00

[1]Data from the Rural Electric Association of Fort Morgan, Colorado.

respectively. The estimated average depths of applied surface water in 1970 and 1971 are 2.12 and 1.58 feet, respectively. Adding these average depths of applied water from pumping and applied surface water, the estimated total average depths of application in 1970 and 1971 are 2.61 and 2.28 feet, respectively. Assuming a crop distribution of 1/3 corn, 1/3 sugar beets, and 1/3 alfalfa, and using the Jensen-Haise technique, a recent study estimated the average farm headgate requirement of this area to be 2.5 acre-feet per acre (Bittinger and Associates, 1969). The estimated total average depths of application in the study area include the seepage from the South Platte Ditch (canal seepage is assumed to be uniformly distributed over the irrigated acreage) whereas the estimated headgate requirement of 2.5 feet does not include this seepage. If the South Platte Ditch seepage was 20% of the headgate delivery, the gross irrigation water requirement would be 3.0 feet. If this were the case, the estimated total average depths of application in the study area would appear to be slightly low.

Phreatophytes

Since there is only a small scattered acreage of phreatophytes in the study area, the consumptive use of phreatophytes was considered to be insignificant.

Reservoir Elevation

Monthly gage heights and storage volumes for Prewitt Reservoir are available in the reservoir report of the Division of Water Resources, Department of Natural Resources, State of Colorado. By interpretation of contour lines on a USGS 7½ minute quadrangle map, the maximum reservoir elevation was estimated to be 4080. By use of this maximum reservoir elevation and the given maximum storage capacity of 32,800 acre-feet, the gage height data were converted to reservoir elevation data as indicated in Table 10. It was assumed that the gage heights were read on the first day of each month. Gage heights for January and November of 1971 were missing and therefore assumed.

River Elevation

The South Platte River grid elevations indicated by the initial water table elevations of Table 1 were held constant throughout the total time of analysis. The monthly flows during the 1970 and 1971 water years, at a USGS gaging station at Balzac, Colorado, about 3 miles upstream of the study area, were generally higher than average but also appeared to be less variable than most years. A more extreme peak with a duration of approximately 15 days did occur during June of 1970. Under conditions of prolonged extreme river discharge, the river grids of the model should probably be converted to variable constant head grids.

Land Surface Elevation

The land surface elevation of each grid was estimated from a USGS 7½ minute quadrangle map. Land surface elevations for the grid network are presented in Table 11.

TABLE 10

MONTHLY ELEVATION OF PREWITT RESERVOIR[1]

Month	1969-70 Storage Volume (Ac-Ft)	1969-70 Gage Height (feet)	1969-70 Elev.	1970-71 Storage Volume (Ac-Ft)	1970-71 Gage Height (feet)	1970-71 Elev.
Nov.	29300	26.80	4078.6	-	-	-
Dec.	26770	25.70	4077.5	26200	25.45	4077.2
Jan.	24100	24.50	4076.3	-	24.30[2]	4076.1
Feb.	21680	23.35	4075.1	21380	23.20	4075.0
Mar.	26540	25.60	4077.4	19778	22.40	4074.2
Apr.	29060	26.70	4078.5	27900	26.20	4078.0
May	26990	25.80	4077.6	29060	26.70	4078.5
June	27900	26.20	4078.0	29530	26.90	4078.7
July	26650	25.65	4077.4	29060	26.70	4078.5
Aug.	21790	23.40	4075.2	19390	22.20	4074.0
Sept.	12570	18.30	4070.1	11150	17.35	4069.1
Oct.	13690	19.00	4070.8	19390	22.20	4074.0
Nov.	26990	25.80	4077.6	-	25.80[2]	4077.6
Mean (Dec.-Nov.)			4076.0			4075.9
Std. Dev.			2.8			2.8

[1]Gage height and storage volume data are from the reservoir report of the Division of Water Resources, Dept. of Natural Resources, State of Colorado. Elevations were computed by use of the gage height and storage for July 1, 1969 (G.H. = 27.40 and storage = 30,720), a given maximum capacity of 32,800 acre-feet, and a maximum reservoir elevation of 4080.

[2]Data unavailable for this month. Gage height was assumed.

TABLE 11

LAND SURFACE ELEVATIONS FOR GRID NETWORK[1]

4098.000	4096.000	4094.000	4092.000	4080.000	4072.000	4074.000	4075.000	4072.000	4066.000	1
4090.000	4092.000	4091.000	4090.000	4084.000	4076.000	4068.000	4070.000	4070.000	4061.000	2
4092.000	4082.000	4082.000	4085.000	4080.000	4075.000	4072.000	4065.000	4062.000	4056.000	3
4095.000	4090.000	4082.000	4080.000	4073.000	4069.000	4067.000	4064.000	4061.000	4051.000	4
4050.000	4090.000	4087.000	4080.000	4066.000	4062.000	4061.000	4062.000	4060.000	4046.000	5
4050.000	4050.000	4080.000	4070.000	4065.000	4062.000	4059.000	4057.000	4056.000	4043.000	6
4050.000	4070.000	4070.000	4065.000	4060.000	4058.000	4057.000	4051.000	4050.000	4040.000	7
4050.000	4070.000	4070.000	4061.000	4057.000	4054.000	4051.000	4045.000	4042.000	4036.000	8
4070.000	4070.000	4064.000	4053.000	4052.000	4052.000	4050.000	4040.000	4039.000	4033.000	9
4062.000	4060.000	4055.000	4049.000	4049.000	4049.000	4047.000	4037.000	4036.000	4030.000	10
4068.000	4062.000	4056.000	4048.000	4047.000	4046.000	4046.000	4035.000	4032.000	4026.000	11
4063.000	4058.000	4053.000	4048.000	4043.000	4041.000	4040.000	4039.000	4033.000	4021.000	12
4066.000	4060.000	4055.000	4048.000	4043.000	4038.000	4037.000	4032.000	4030.000	4016.000	13
4079.000	4070.000	4065.000	4058.000	4047.000	4039.000	4035.000	4031.000	4027.000	4013.000	14
4074.000	4068.000	4062.000	4056.000	4048.000	4040.000	4033.000	4028.000	4022.000	4010.000	15
4068.000	4062.000	4058.000	4051.000	4040.000	4032.000	4028.000	4027.000	4024.000	4006.000	16
4065.000	4060.000	4050.000	4045.000	4040.000	4034.000	4029.000	4022.000	4020.000	4003.000	17
4080.000	4070.000	4060.000	4040.000	4030.000	4026.000	4021.000	4018.000	4017.000	4000.000	18

[1]Values for the 18 river grids are in the far right-hand column. Elevations in feet above msl.

V. DISCUSSION OF RESULTS

Adjustments to the Model

To attain a reasonable match between the simulated and historic water table elevations for the Autumn of 1970 and 1971, adjustments were required for one initial condition, one model parameter, and one boundary condition. The initial condition adjustment was discussed in the description of the data used in the model study. The grid water table elevations at the end of three time increments were used as initial water table elevations in the model. The permeability of the five reservoir grids was the only model parameter that appeared to require adjustment. However, this was a major adjustment in that the permeability of these grids was reduced from approximately 500 feet per day to 10 feet per day. The constant gradients at boundaries with horizontal underflow were also adjusted.

Prewitt Reservoir was originally assumed to be in hydraulic connection with the water table aquifer. The permeabilities of the reservoir grids were computed from the transmissivity map as in the case of other model grids. However, this resulted in a 5 or 6 foot rise of the water table in the upstream half of the model during a one year period of analysis. The general rise of the water table continued to occur even after the elevations of the reservoir were lowered by as much as 10 feet. It was apparent that the reservoir was not in hydraulic connection with the water table. Evidently, the reservoir was constructed with a low permeability barrier of some type and it is also possible that the bottom has been silted in. The existing condition was simulated by reducing the permeability of the five reservoir grids. As a result, the simulated water table elevation near the reservoir is approximately

20 feet below the mean elevation of the reservoir. Due to the extreme difference in hydraulic head and permeability between the reservoir grid blocks and the adjacent aquifer grid blocks, the reservoir permeability value of 10 feet per day cannot be considered representative of the actual permeability of the porous medium beneath or adjacent to the reservoir. However, by a comparison of simulated and historic water table elevations near the reservoir, the value of 10 feet per day appears to result in a fairly accurate simulation of the amount of seepage from the reservoir to the water table aquifer. The reservoir seepage is further analyzed in another section of this chapter.

The constant gradient for boundary grids having horizontal underflow was originally determined from the water table elevation contour map of figure 5 by taking the difference in elevation between the outermost boundary grid and the next inner grid. Adjustments were required on all three sides of the model having horizontal underflow. Most gradients along the upstream and downstream boundaries were decreased by approximately one foot per mile. Most gradients along the boundary opposite the river were increased by roughly two feet per mile, although it should be noted that the accuracy of the water table elevation contour map along this boundary was unknown or questionable. The constant gradients were adjusted by observation of the change in water table elevation with time at the third grid in from the boundary of the model. Grid numbers 26 , 138 , and 155 are examples of grids that are the third grid in. The response of the water table to changes in the constant gradient was very significant at the third grid in but very minimal at a grid 6 or 7 grids in from the boundary. It was assumed that the variation of the water table elevation at the third grid in should be similar to the known historic variations at nearby grids. The constant gradients were adjusted until this approximate correspondence was achieved.

The constant gradients of the boundary grids adjacent to the reservoir are quite large. This is apparently due to the fact that only a portion of the reservoir was included in the model and an increased gradient is necessary to simulate the additional fringe area seepage due to the unmodeled portion of the reservoir.

Comparison of Simulated and Historic Water Table Elevations

Historic groundwater level data are available for 22 wells in the study area. However, four of these wells are located in grids that are the second grid in on a constant gradient boundary. These four wells were not used in the comparison of historic and simulated values because of the direct effects of a constant gradient approximation at their respective locations in the model. In addition, the historic water levels for the well located in grid numbers 47 , 94 , and 137 were considered extremely abnormal in contrast to the water levels for the remaining wells. The extreme data for these three wells could be due to a termination of pumping or surface water application at a date much earlier or later than normal. Data for these three wells were not considered in the comparison. As a result, only data from 15 of the 22 wells in the study area were used for a comparison of simulated and historic water table elevations.

The results of the comparison are given in Table 12. It is important to note that the model grid differences in water table elevation are for November 1 to November 1 of each year, while the historic differences

TABLE 12

COMPARISON OF SIMULATED AND HISTORIC[1] WATER TABLE ELEVATIONS

Grid	Nov., 1969 to Nov., 1970 Historic Difference (feet)	Model Difference (feet)	Difference Between Model and Historic	Nov., 1970 to Nov., 1971 Historic Difference (feet)	Model Difference (feet)	Difference Between Model and Historic
32	0.0	+0.2	+0.2	-0.2	0.0	+0.2
34	0.0	+0.8	+0.8	+0.1	-0.1	-0.2
36	-0.3	+0.7	+1.0	-0.3	-0.1	+0.2
45	-0.1	+0.7	+0.8	0.0	-0.1	-0.1
53	+0.2	+0.6	+0.4	0.0	-0.1	-0.1
54	+0.4	+0.7	+0.3	+0.1	-0.2	-0.3
63	0.0	+0.3	+0.3	+0.2	-0.1	-0.3
75	+0.1	+0.3	+0.2	-0.1	-0.2	-0.1
88	-0.5	-0.4	+0.1	-0.5	-0.3	+0.2
108	-0.4	+0.6	+1.0	-0.4	-0.5	-0.1
113	+0.1	+0.2	+0.1	-0.4	-0.1	+0.3
115	+0.3	+0.5	+0.2	-0.8	-0.3	+0.5
118	+0.1	+0.5	+0.4	-0.6	-0.7	-0.1
134	+0.5	+0.2	-0.3	-0.3	-0.2	+0.1
156	+0.8	+0.2	-0.6	-0.8	0.0	+0.8
TOTAL	+1.2	+6.1	+4.9	-4.0	-3.0	+1.0
MEAN	+0.08	+0.41	+0.33	-0.27	-0.20	+0.07
STD. DEV.			0.44			0.31
STD. ERROR			0.11			0.08

[1]Model differences in water table elevation are for Nov. 1 to Nov. 1 of each year. Historic differences are from Colorado Water Conservation Board Basic-Data Release No. 26, 1972. Water Levels were measured each autumn at a well located in the grid.

are from water level measurements taken each Autumn at a well located in the respective grid. For the comparison, it was assumed that the water levels were measured each November 1.

From November, 1969 to November, 1970, the mean historic difference in water table elevation was +0.08 feet while the mean model difference is +0.41 feet. This gives a mean difference between the model and historic values of +0.33 feet and a standard deviation of 0.44 feet. For the period of November, 1970 to November 1971, the mean historic difference was -0.27 feet while the mean model difference is -0.20 feet. The mean difference between the model and the historic values is therefore +0.07 feet with a standard deviation of 0.31 feet.

For the first year of operation, there is a significant difference between the model and historic values. This difference appears to be at least partially due to random errors in the initial water table elevations. An additional period of time is needed for the unstable system of initial water table elevations to more closely approach a state of equilibrium. The results of the second year of analysis are considered to be a satisfactory match between the simulated and historic water table elevations. The simulated two-year water table elevation hydrograph for grid number 75 is given in figure 8. Grid number 75 is a typical grid located at approximately the center of the study area.

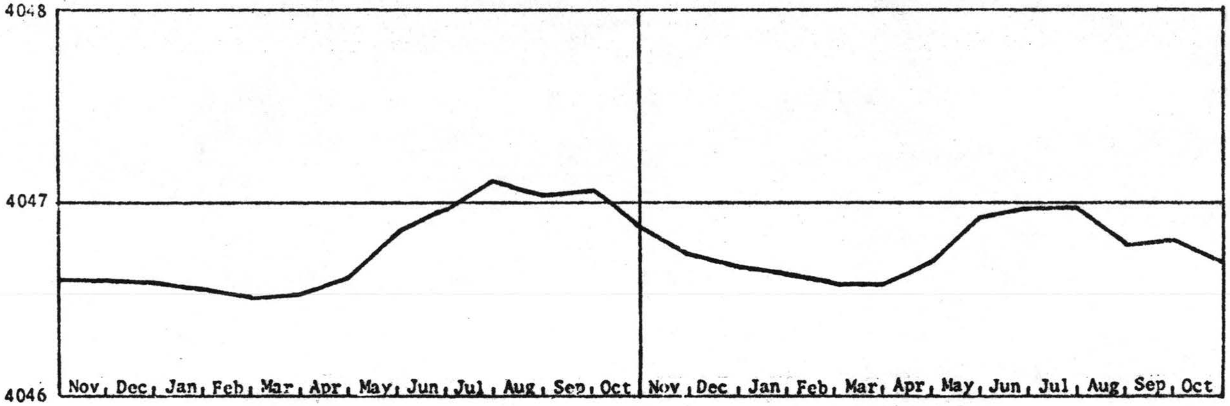
Sensitivity Analysis of Simulated Water Table Elevations

The sensitivity of the simulated water table elevations was analyzed with respect to modifications in the initial water table elevations, the permeability of the aquifer and reservoir grids, the elevation of

SIMULATED TWO-YEAR WATER TABLE ELEVATION HYDROGRAPH FOR GRID NO. 75

ANALYSIS

FIRST YEAR SECOND YEAR



1969-1970 1970-1971

+0.14 = HISTORIC DIFFERENCE AT WELL IN GRID (see table) = -0.16

Figure 8.

the reservoir, the annual volume of pumping, and the magnitude of the constant gradients at boundaries with horizontal underflow.

In general, the initial water table elevations were only important in the results of the first year of analysis. Variation of the initial water table elevation of an individual grid by several feet resulted in insignificant changes in the simulated water table elevations of that grid after 5 or 6 time increments. The general trend of the water table must be initially correct, but random errors are not significant in predicting the spatial distribution of heads after a long time period (Bibby, 1971). It is apparent that a one foot change in the initial water table elevation of a grid will result in a one foot change in the difference in water table elevation of that grid at the end of the first year of analysis.

The model is not very sensitive to alterations of the permeability of the aquifer grids. The permeability of all grids except the five reservoir grids was reduced by 20% . At a typical grid, this resulted in a 0.2 foot increase in the water table elevation at the end of the first year of operation. There was no significant change in the difference in water table elevation for the second year of analysis.

The permeability of the reservoir grids is an important parameter in the simulation of water table elevations for the upstream half of the model. The permeability of the five reservoir grids was increased from a value of 10.0 ft./day to a value of 20.0 ft./day. As a result, the difference in water table elevation at grid number 77 increased 0.94 feet for the first year of analysis and 0.14 feet for the second year of analysis. Grid number 77 is about 2500 feet from the reservoir. As expected, sensitivity to the reservoir permeability decreased as the distance from the reservoir increased. The effect of the permeability change at grid number 115 was almost insignificant. The sensitivity to the change in reservoir permeability at grid numbers 75, 77, and 115 is summarized in Table 13.

Since the permeability of the reservoir grids is only 10.0 feet per day, the water table elevations of the model are not very sensitive to changes in the elevation of the reservoir. When the average elevation of the reservoir during the first year of analysis was reduced by two feet, the water table elevation of grid number 77 at the end of the year was only lowered by 0.17 feet. There were no significant changes in the water table elevation of grid numbers 75 and 115.

To test the significance of the estimated pumping volumes in the model, all annual pumping volumes for the second year of operation were increased by 10% . The water table elevation of a typical grid at the end of the second year was lowered by approximately 0.05 feet.

The sensitivity of the water table to the magnitude of the constant gradient boundaries was analyzed by simultaneously reducing the constant gradient of grid number 7 by 1.0 ft./mile and reducing the constant gradient of grid number 120 by 4.8 ft./mile. The effect on the water table at the third grid in from the altered boundary grid during the first year of operation was very significant. The effect at this same grid during the second year was minimal. The changes in water table elevations at grids near the center of the study area due to the alterations of the two constant gradients were insignificant in both years of analysis. The sensitivity of the water table elevations at grid numbers 27, 75, 115, and 118 is summarized in Table 14.

TABLE 13

SENSITIVITY OF WATER TABLE ELEVATIONS TO RESERVOIR PERMEABILITY[1]

	Grid No. 75 Difference in Water Table Elevation			Grid No. 77 Difference in Water Table Elevation			Grid No. 115 Difference in Water Table Elevation		
	Case 1 K=10.0	Case 2 K=20.0	Case 1 Minus Case 2	Case 1 K=10.0	Case 2 K=20.0	Case 1 Minus Case 2	Case 1 K=10.0	Case 2 K=20.0	Case 1 Minus Case 2
Nov. 1, 1969 to Nov. 1, 1970	-0.05	+0.52	+0.57	-0.33	+0.61	+0.94	+0.15	+0.27	+0.12
Nov. 1, 1970 to Nov. 1, 1071	-0.16	-0.07	+0.09	-0.19	-0.05	+0.14	-0.26	-0.21	+0.05

[1]The permeability of the five reservoir grids was increased from a value of 10.0 ft/day to a value of 20.0 ft/day.

TABLE 14

SENSITIVITY OF WATER TABLE ELEVATIONS TO CONSTANT GRADIENT BOUNDARY[1]

	Grid No. 27 Difference in Water Table Elevation			Grid No. 75 Difference in Water Table Elevation			Grid No. 115 Difference in Water Table Elevation			Grid No. 118 Difference in Water Table Elevation		
	Case 1	Case 2	Case 1 Minus Case 2	Case 1	Case 2	Case 1 Minus Case 2	Case 1	Case 2	Case 1 Minus Case 2	Case 1	Case 2	Case 1 Minus Case 2
Nov. 1, 1969 to Nov. 1, 1970	+0.42	+0.18	-0.24	-0.05	-0.10	-0.05	+0.15	+0.01	-0.14	+0.09	-0.71	-0.80
Nov. 1, 1970 to Nov. 1, 1971	-0.14	-0.27	-0.13	-0.16	-0.19	-0.03	-0.26	-0.28	-0.02	-0.65	-0.73	-0.08

[1]The constant gradients of boundary grids 7 and 120 were simultaneously reduced. The constant gradient of boundary grid 7 in Case 1 was reduced by 1.0 ft/mile for Case 2. The constant gradient of boundary grid 120 in Case 1 was reduced by 4.8 ft/mile for Case 2.

Flow to the River

The model computes the volumetric exchange between the river and aquifer for each month of analysis. The computed total net flow into the 7 mile length of river between the buffer zones of the model during the second year of analysis was 32,965 acre-feet. The nearest gaging station downstream of the study area is at Julesburg, Colorado, located approximately 70 miles downstream from the study area. As a result, a check of the computed flow to the river is not readily available. The computed monthly flows for the second year of analysis are given in Table 15.

Reservoir Seepage

The reservoir seepage for December of the second year of analysis was computed from grid flows given by the model. Since the reservoir elevation in December was approximately the same as the average monthly elevation in the second year of analysis, the December flows from the reservoir grids were considered average flows for that year. Since only a portion of the reservoir was modeled, much of the flow from the constant gradient boundary grids near the reservoir must also be considered as reservoir seepage. The flows from the two constant gradient grids on each side of the reservoir are significantly larger than the flows from other constant gradient grids along this boundary. For example, the flow at grid number 90 is 465 acre-feet per month while the average flow at grid numbers 120, 130, 140, 150, 160, and 170 is only 75 acre-feet per month. By these observations, the maximum natural recharge at each grid was estimated to be 100 acre-feet per month. Horizontal underflow across the constant gradient boundary in excess of 100 acre-feet per month was attributed to reservoir seepage. Computations of

TABLE 15

SIMULATED VOLUME OF FLOW TO THE RIVER[1]

DURING THE SECOND YEAR OF ANALYSIS

Month	Flow Ac-Ft.
Nov., 1970	2756
Dec.	2718
Jan.	2696
Feb.	2681
Mar.	2699
Apr.	2773
May	2899
June	2827
July	2749
Aug.	2631
Sept.	2788
Oct., 1971	2748
Total	32965
Mean	2747

[1]Includes net flow into all river grids except buffer zone grids 1, 11, 161, and 171 (total length of river = 7 miles).

the estimated reservoir seepage for December, 1970, are given in Table 16. The estimated seepage for December, 1970, is 1,711 acre-feet which indicates an estimated annual seepage of approximately 21,000 acre-feet. The pan evaporation in this area in 1971 was approximately 5 feet. Assuming this evaporation rate at Prewitt Reservoir, the evaporation loss from the 1,825 acre reservoir in 1971 would have been approximately 9,000 acre-feet. This would give a total seepage and evaporation loss of 30,000 acre-feet. The average annual seepage and evaporation loss based on inflow-outflow computations is 29,700 acre-feet (Bittinger and Associates, 1969).

These computations only serve as a very approximate check of the estimated volume of reservoir seepage entering the water table aquifer of the model. Unfortunately, most of the reservoir seepage is represented in the model as constant gradient horizontal underflow, and the computed monthly reservoir seepage is fairly constant regardless of the variation in reservoir elevation. As indicated in Table 10, the standard deviation of the monthly reservoir elevations during the two years of analysis was only 2.8 feet. If the reservoir elevation is nearly constant, the reservoir seepage should also be nearly constant. As a result, the use of a nearly constant reservoir seepage in the model is considered to be a satisfactory approximation of the reservoir seepage during the two years of analysis used in this study. However, until a further analysis of the reservoir seepage is made, the developed model is considered to have limited value for use in management decisions involving large variations of the reservoir elevation. In addition, the mean reservoir elevation of a proposed management scheme should be

TABLE 16

SIMULATED VOLUME OF RESERVOIR SEEPAGE FOR DECEMBER, 1970

Grid No.	Grid[1] Type	Inflow[2] to Model (Ac-Ft)	Estimated[3] Natural Recharge (Ac-Ft)	Estimated Reservoir Seepage (Ac-Ft)
30	1	428	100	328
40	1	598	100	498
50	2	34	0	34
59	2	130	0	130
70	2	52	0	52
80	2	56	0	56
90	1	465	100	365
100	1	310	100	210
110	1	138	100	38
Total		2211	500	1711

[1]Constant gradient boundary = 1 . Variable constant head boundary = 2 .

[2]Computed by the model for December, 1970 (reservoir elevation = 4076.1). The average reservoir elevation for the second year of analysis was 4075.9 .

[3]Estimated maximum natural recharge from water table above reservoir based on horizontal underflow at similar boundary grids farther away from reservoir. The average monthly horizontal underflow at grids 120, 130, 140, 150, 160, and 170 is 75 acre-feet.

approximately the same as the mean reservoir elevation during the two years of analysis used in this study.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A satisfactory match of simulated and historic water table elevations was obtained for the second year of analysis. However, historic water levels were only available at the beginning and end of the year, and a satisfactory match at the end of the year does not necessarily indicate the model would match throughout the year. There are an infinite number of solutions for the water table elevation hydrograph between the end points of a one year period of analysis. Therefore, the accuracy of the study area model is dependent on the accuracy of the parameters and boundary conditions of the model. Even if the model parameters and boundary conditions in the model happened to be exact, the use of a satisfactory history match as an indicator of the accuracy of the model would still be dependent on the accuracy of the hydrologic input to the model for the one year period of analysis.

Consequently, an independent determination of the accuracy of model parameters, boundary conditions, and hydrologic input is required in order to determine the accuracy of the study area model. The simulated water table elevations were not very sensitive to the aquifer permeability values, and therefore, the permeability values are probably of sufficient accuracy. The total application of irrigation water in the model was compared to the average irrigation requirement of the study area, however, further verification of this hydrologic input would be of value.

Further analyses will be required to determine the accuracy of the boundary conditions in the model. Since the river grids were considered to be at a constant elevation and the permeability of the river bed is not known, the computed flow to the river is of questionable accuracy. The values of horizontal underflow at constant gradient boundaries appeared to be reasonable, but verification of these values would be nearly impossible.

Further analysis of the monthly reservoir seepage is required. Due to inadequate data on the existing permeability and water table conditions surrounding Prewitt Reservoir, only a portion of the reservoir was included in the study area. Although the reservoir seepage is a very important part of the total inflow to the study area, it appears that only about 15% of the total reservoir seepage enters the study area from the portion of the reservoir included in the model. The remaining 85% seems to enter the study area at outlying fringe areas of the reservoir. This is apparently due to the construction of a low permeability barrier along the portion of the reservoir included in the model. It was necessary to model the fringe area reservoir seepage with constant gradient boundaries and therefore, the simulated monthly reservoir seepage is very nearly constant. Due to a nearly constant reservoir elevation during the two years of analysis used in this study, the use of a nearly constant reservoir seepage in the model is considered to be a satisfactory approximation for these two years. However, until a further analysis is made to determine the relationship between reservoir elevation and reservoir seepage, use of the study area model in future management decisions is limited to certain conditions of reservoir storage. The mean reservoir elevation of a proposed management scheme must be approximately the same as the mean reservoir elevation during the two years of analysis used in this study. The annual variation of reservoir elevations should also be similar to that of this study. Since the reservoir storage variable has a stochastic component and cannot be known for future management decisions, the reservoir storage data for the two years of analysis used in this study may be sufficient for analyzing management alternatives such as alternate patterns of pumping or surface water application.

Recommendations

Further verification of the model parameters, hydrologic input, and boundary conditions is recommended. The permeability values in the model could be spot checked by several pumping tests. The accuracy of the estimated total applied irrigation water used in the model could be further verified by estimating the average consumptive use and irrigation requirement in the study area for each year of analysis. In addition, it may be necessary to determine any locations of irrigated acreage in the study area that are only irrigated by pumped water. The acreage irrigated only by pumped water should be eliminated from the 7,800 acres in the model that is assumed to be irrigated with a uniform depth of surface water.

It may be possible to determine the horizontal underflow at the constant gradient boundaries more accurately. For example, the inflow along the upstream boundary could be determined more accurately by extending the study area several miles in an upstream direction. This would eliminate the disadvantage of having an approximate constant gradient inflow boundary located immediately at the area of interest.

The simulated flow to the South Platte River could be approximately checked by a monthly gaging of the river for one year at a point just downstream of the study area. This stream flow data together with the records of the USGS gaging station at Balzac (3 miles upstream from the study area), and other available data on the inflow and outflow between the two points of gaging, could be used to estimate the flow to the river. Since the study area only included one side of the river, the inflow from the alluvial fill aquifer on the opposite side of the river would also have to be estimated.

Additional data are needed on the aquifer conditions in the area surrounding Prewitt Reservoir. It may be necessary to drill several test holes to determine water table elevations near the reservoir. The plans and specifications used in the construction of the reservoir dike would also be of value. If an investigation confirms that the reservoir is not in hydraulic connection with the water table aquifer, as indicated by results of this model study, the reservoir seepage should be modeled as a part of the applied rate of recharge q of the basic flow equation (equation 1 in Chapter 4). The model should be extended to include all of Prewitt Reservoir and the permeabilities of the five reservoir grids used in this study should be returned to their original values of about 500 feet per day. Monthly reservoir inflow-outflow data and evaporation data will be required in order to estimate a monthly value of reservoir seepage to be used in the average monthly value of q . These data could also be used to determine a general relationship between reservoir storage and reservoir seepage. A relationship of this type may be needed in using the model for future management decisions.

REFERENCES CITED

Allen, D. N. and de G., 1954, Relaxation Methods, McGraw-Hill, New York.

Bibby, R., 1971, Statistical Error Analysis of Ground Water Systems, Dissertation, Civil Engineering Department, Colorado State University.

Bittinger, M. W., Duke, H. R., and Longenbaugh, R. A., 1967, Mathematical Simulations for Better Aquifer Management, Publication No. 72 of the IASH Symposium of Haifa, pp. 509-519.

Bittinger, M. W., and Associates, 1969, Development of Model Procedures Pursuant to Provisions of Senate Bill 81, Fort Collins, Colorado.

Bjorklund, J. J., and Brown, R. F., 1957, Geology and Groundwater Resources of the Lower South Platte River Valley between Hardin, Colorado, and Paxton, Nebraska. U. S. Geological Survey, Water-Supply Paper 1378.

Brakensiek, D. L., 1967, Finite Differencing Methods, Water Resources Research, Vol. 3, No. 3, pp. 847-860.

Corker, C. E., 1972, Ground Water Law, Management and Administration, Final Report (review draft), National Water Commission Legal Study 6, p. 146.

Crank, J. and Nicholson, P., 1947, A Practical Method for Numerical Integration of Solutions of Partial Differential Equations of Heat-Conduction Type, Proceedings, Cambridge Philosophical Society, Vol. 43, p. 50.

Crank, J., 1956, The Mathematics of Diffusion, Oxford University Press, London.

Douglas, J., Jr., and Peaceman, D. W., 1955, Numerical Solution of Two Dimensional Heat Flow Problems, J. Am. Inst. Chem. Eng., Vol. 1, No. 4, pp. 505-512.

Douglas, J., Jr., and Rachford, H. H., Jr., 1956, Quasi Numerical Solution of Heat Conduction Problems in Two and Three Space Variables, Trans. Am. Math. Soc., Vol. 82, pp. 421-439.

Eshett, A., and Longenbaugh, R. A., 1965, Mathematical Model for Transient Flow in Porous Media, Colorado State University, CER65RAL-AE59.

Fiering, M. B., 1964, A Digital Computer Solution of Well Field Drawdown, Bulletin Inter. Assoc. Sci. Hydrology, Vol. 9, No. 4, pp. 16-23.

Hurr, R. T., and Luckey, R. R., 1972, Ground-Water Levels in the South Platte River Valley of Colorado, 1968-1972, Colorado Water Conservation Board Basic-Data Release No. 26.

Jacob, C. E., 1950, Flow of Ground Water, in Engineering Hydraulics, ed. Hunter Rouse, John Wiley & Sons, New York.

Kunz, K. S., 1957, Numerical Analysis, McGraw-Hill, New York.

McConaghy, J. A., and Colburn, G. W., 1964, Records of Wells in Colorado, Colorado Water Conservation Board Basic-Data Release No. 17.

Peaceman, D. W., and Rachford, H. H., Jr., 1955, The Numerical Solution of Parabolic and Elliptical Difference Equations, J. Soc. Indust. Appl. Math., Vol. 3, pp. 28-41.

Pinder, G. F., and Bredehoeft, J. D., 1968, Application of the Digital Computer for Aquifer Evaluation, Water Resources Research, Vol. 4, No. 5, pp. 1069-1092.

Richtmyer, R. D., and Morton, K. W., 1967, Difference Methods for Initial-Value Problems, (second edition), Interscience Publishers, New York.

Southwell, R. V., 1946, Relaxation Methods in Theoretical Physics, Oxford University Press, London.

Stallman, R. W., 1956, Numerical Analysis of Regional Water Levels to Define Aquifer Hydrology, Trans. Am. Geophysical Union, Vol. 37, No. 4, pp. 451-460.

Stettner, P., 1968, Aquifer Models: Mathematical and Physical, M.S. Thesis, Civil Engineering Department, Colorado State University.

Todd, J., ed., 1962, Survey of Numerical Analysis, McGraw-Hill, New York.

Trescott, P. C., Pinder, G. F., and Jones, J. F., 1970, Digital Model of an Alluvial Aquifer, Journal Hydr. Div., ASCE, Vol. 96, No. HY 5, pp. 1115-1127.

Young, R. A., and Bredehoeft, J. D., 1972, Digital Computer Simulation for Solving Management Problems of Conjunctive Groundwater and Surface Water Systems, Water Resources Research, Vol. 8, No. 3, pp. 533-556.