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THE DISSIMILARITY BETWEEN SPACIAL AND VELOCITY-WEIGHTED CONCENTRATIONS

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Abstract

Theoretical and measured differences between spacial and velocity-weighted concentrations show that (1) the spacial concentration is normally the greater and increases as the velocity and concentration gradients increase, (2) that velocity-weighted concentration must be used to compute sediment discharge, and (3) that spacial concentration must be used to compute the pressure or specific weight on the bed.

INTRODUCTION

Sediment concentration data in natural streams are used mostly to determine the amount of solids, either by weight or volume, moving with streamflow. Such concentration data must be discharge weighted; that is, it must be a mean of velocity-weighted concentrations at many points in the stream cross section. The depth-integrating suspended samplers give a velocity-weighted concentration when a uniform vertical transit rate is used at evenly spaced verticals in the cross section. The discharge-weighted concentration is also obtained by traversing the nappe of flow from a flume at a uniform transit rate with an interception device having a uniform width of slot.

Concentration resulting from a spacial collection procedure is defined as the relative quantity of sediment contained in an immobilized prism of water-sediment mixture over a specific area of the channel. The essential distinction between the two concentrations is that one is based on sediment and water discharged through a cross section and the other on sediment and water in motion above an area of stream bed at a particular instant.

The difference between the two concentrations (velocity-weighted and spatial) has been understood, to some degree, by a few sedimentologists for several years. Much confusion still exists, however, and the quantitative differences between the concentrations has seldom been determined even approximately. The purposes of this paper are to explain the differences and some uses of these concentrations and to show differences between the two concentrations determined experimentally by use of plastic pellets transported by water in a small flume.

THEORETICAL DIFFERENCES

Differences between the velocity-weighted and spatial concentrations can be evaluated theoretically by consideration of the following four equations:

$$\frac{U_y}{U_*} = 5.75 \log \frac{30.2\ yx}{K_s} \tag{1}$$

This is the Prandtl-von Karman relation used by Einstein (1950) where U_y is the average point velocity at distance y above the stream bed and

U_* = shear velocity, $\sqrt{g\ DS}$

g = acceleration due to gravity

D = depth of flow

S = slope of the energy grade line

x = a corrective parameter

k_s = grain roughness

For the distribution of sediment concentration with respect to depth

$$\frac{C_y}{C_a} = \left[\frac{D-y}{y} \cdot \frac{a}{D-a}\right]^z \tag{2}$$

where C_y = concentration of particles at distance y above the bed

C_a = concentration of particles having the settling velocity w at distance a from bed

z = $w/0.40\ U_*$

The concentration C, obtained by velocity-weighted samples, and water discharge in the sampled zone Q_m is used to compute the sediment discharge per unit of time q_s through the sampled zone

$$q_s = Q_m C \tag{3}$$

Equation 3 is also equivalent in concept to the sediment discharge q_s

determined by the relation

$$q_s = \int_a^D C_y U_y \, dy \qquad (4)$$

where C_y and U_y are defined by equations (1) and (2).

Figure 1 shows the approximate variation of velocity, sediment concentration, and sediment discharge with depth. It is apparent from this illustration and from the equations that the mean concentration as weighted with velocity will be considerably lower than the spatial concentrations. The relatively low velocity-weighted concentration is due to the integration with depth ranging from low concentration and high velocity near the stream surface to high concentration with low velocity near the streambed. The spatial concentration on the other hand would be the mean of the concentration from top to bottom and thus be considerably greater than the velocity-weighted concentration. Under the rare condition when all of the sediment being transported is very fine, the concentration may be uniform from top to bottom which would result in equal spatial and velocity-weighted concentrations.

Stream flow data show that neither U_y or C_y behave spatially or temporally consistent as defined in the equations therefore, a sample of sediment or velocity at a point in a stream vertical or cross section can not define the concentration or velocity distribution in such a vertical or cross section. In order to overcome this difficulty, the "precise method," Report No. 1 (1940), p. 63, was developed which

> "involves collection of a relatively large number of point sediment samples simultaneously with velocity measurements. Sufficient data are collected to construct accurate vertical velocity and sediment distribution curves, the corresponding abscissas of which are multiplied to obtain a sediment-velocity curve. The area under this curve represents the sediment discharge in the vertical."

Needless to say, the "precise method" was too laborious for routine investigations of sediment movement in streams. The velocity-weighting technique, commonly called depth-integrating, was developed to facilitate sediment data collection and analysis, Report No. 6 (1952), p. 14

> "The design of the depth-integrating sampler was predicated on the hypothesis that an integrated sample of the water-sediment mixture existing at the place and time of sampling would be obtained if the filling rate were such that the velocity at the point of intake is equal to the local stream velocity while the sampler is moved at a uniform vertical speed in the stream."

The spatial concentration may be used to determine the actual load (weight per unit area) in transport over the stream bed. It is also the correct concentration for determining the specific weight of the water sediment complex. The spatial concentration can be obtained in stream flow by averaging the concentration of a number of equally spaced point samples in the stream cross section. In the laboratory Bagnold (1955) used a mechanical device for isolating a rectangular slug of flow in a flume (figure 2). It is generally difficult to obtain a representative sample of spatial concentration because of the unsteady motion of particles near the bed. Most point samplers do not operate closer than 0.3 or 0.4 foot of the bed. The mechanical-isolation device cannot distinguish between stationary and moving particles and thus cannot be used on an alluvial bed.

EXPERIMENTAL DIFFERENCES

In order to show the difference between the velocity-weighted and spatial concentrations, a series of runs were completed in an 8-in. wide recirculating flume. Plastic pellets with a specific gravity of 1.04 to 1.06 and with a median diameter of about 3 mm were used for the sediment grains. Sampling was accomplished with a mechanical-isolation device only when the velocity was sufficient to insure that all sediment over the bed was in motion, even though some particles were moving very slowly. The variation of sediment concentration and velocity with depth appeared to approximate that indicated by equations (1) and (2) and as illustrated in figure 1.

Total sediment concentration was sampled for six different particle concentrations with respect to the total amount of sediment circulating in the flume. At two of these concentrations samples were obtained at three widely different velocities. To maintain a given magnitude of concentration in circulation, an equivalent volume of sample was returned to the flume for each sample removed. The velocity-weighted samples were obtained by collecting all particles discharged at the tail end of the flume in a nylon-mesh basket. The duration of the sampling ranged from 8 to 15 seconds. The samples for spatial concentration were obtained by quickly isolating the water and sediment moving in a 1-ft length of the flume by use of a sharp-edged frame with wire mesh ends, similar to the sampler illustrated in figure 2. The flow was stopped immediately after isolating the sample and the particles were removed by siphoning from the known volume of water.

The concentrations in ppm by weight for both methods of sampling are summarized in Table 1. This table shows the concentration means, the standard deviation, and the limit of probable error of measurement at the 90 percent confidence level for each run. The sediment concentration ranges from a velocity-weighted value of 85,000 ppm and a spatial value of 110,600 ppm on the flume bed for run no. 1 to a velocity-weighted value of 3,440 ppm and a spatial value of 5,250 ppm on the flume bed for run no. 6. The error of measurement is appreciably less at the tail box because of the greater time of sampling; hence, only 3 samples were taken for each run at the tail box, whereas 5 were obtained on the flume bed for most runs.

Table 1. --Concentration by weight of velocity-weighted and spacial samples of plastic pellets from 8-inch flume.

Flume run	Mean velocity ft./sec.	Velocity-weighted concentration, ppm (At tailbox)			Spacial concentration, ppm (On flume bed)		
		Mean	Stand. deviation	Limits of prob. error*	Mean	Stand. deviation	Limits of prob. error*
#1	2.09	85,000	1,350	2,290	110,600	6,920	6,830
#2	1.72	63,200	6,950	11,700	92,100	10,300	12,200
#3	1.66	41,300	1,900	3,220	58,700	7,310	7,050
#4	1.18	20,900	890	2,610	40,300	4,450	7,530
#4	1.67	24,800	1,140	2,690	29,400	5,140	4,920
#4**	1.67	24,800	1,140	2,690	21,400	950	1,161
#4	2.46	24,400	750	1,280	26,800	2,500	4,220
#5	1.81	10,800	560	940	13,400	2,760	3,260
#6	1.13	3,500	130	220	5,760	890	850
#6	1.42	3,440	260	440	5,250	970	1,140
#6	2.87	3,890	105	180	3,870	670	640

* Limit of probable error of measurement at 90 percent confidence level.

** Spacial samples collected near headbox of flume.

To illustrate the differences between the two methods of determining concentration, the sampled concentrations are compared in figure 3. As expected, the spatial concentration is greater than the velocity-weighted concentration. The limits of the probable error of concentration measurement (90 percent confidence level) are shown in figure 4 in the form of a rectangle approximately centered over each concentration mean. These rectangles indicate that the variability in the movement of sediment, and consequently the measurement error, is greater for the spatial means than for the velocity-weighted means.

The larger the variation of velocity and sediment concentration with respect to depth (large comparative V near the surface and a large comparative C near the bed) the greater the difference between the two concentrations. As indicated by the fastest velocities sampled for runs 4 and 6 the two methods may be comparable, within limits, when the sediment is thoroughly suspended. In fact, the reverse of the normal was found by comparing spatial concentrations from near the head box with the velocity-weighted concentrations at the tail box. The lower mean concentration of the spatial samples (see Table 1 and figure 3) is caused by an inversion of the normal concentration gradient due to centrifugal force on the sediment particles.

CONCLUSIONS

Velocity and sediment concentration distributions in a cross section of stream flow can be predicted only within rough limits. Theoretical and measured differences between spacial and velocity-weighted concentrations show that:

1. The dissimilarity between the two kinds of concentration widens as the velocity and concentration gradients increase with respect to depth.

2. It is necessary to use a velocity-weighted concentration, obtained by sampling at several points or verticals in the stream cross section, to compute the sediment discharged by the stream. Routine measurements are made with several kinds of depth integrating samplers.
3. The spatial concentration is needed when (a) the actual load or amount of sediment exerting pressure on the bed is required, or (b) when the average specific weight of the sediment-water mixture over the bed is required. Measurement is usually difficult due to inability to distinguish between immobile particles and those moving very slowly.

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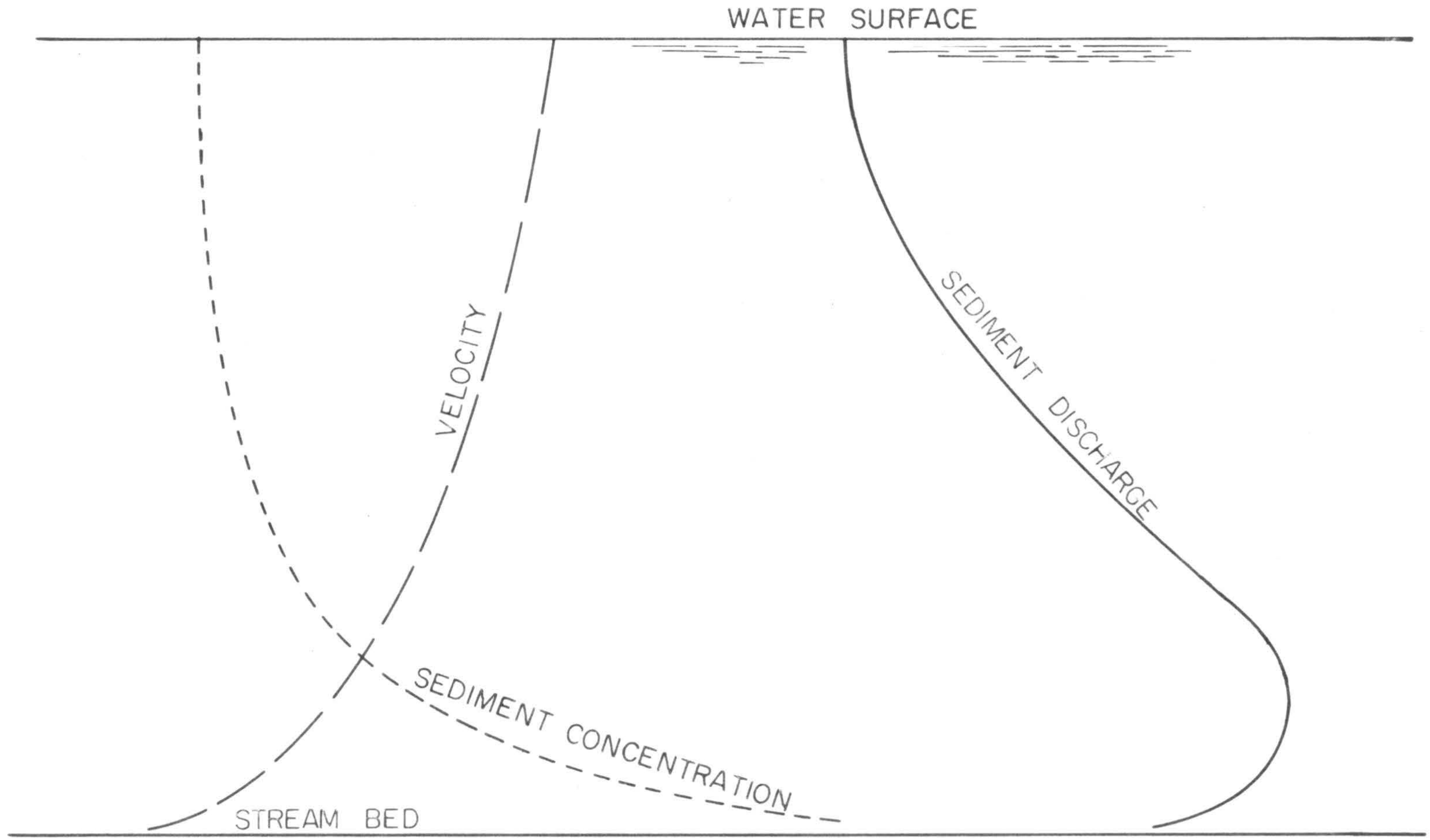

FIGURE 1 RELATIVE GRADIENTS OF VELOCITY, SEDIMENT CONCENTRATION, AND SEDIMENT DISCHARGE WITH STREAM DEPTH

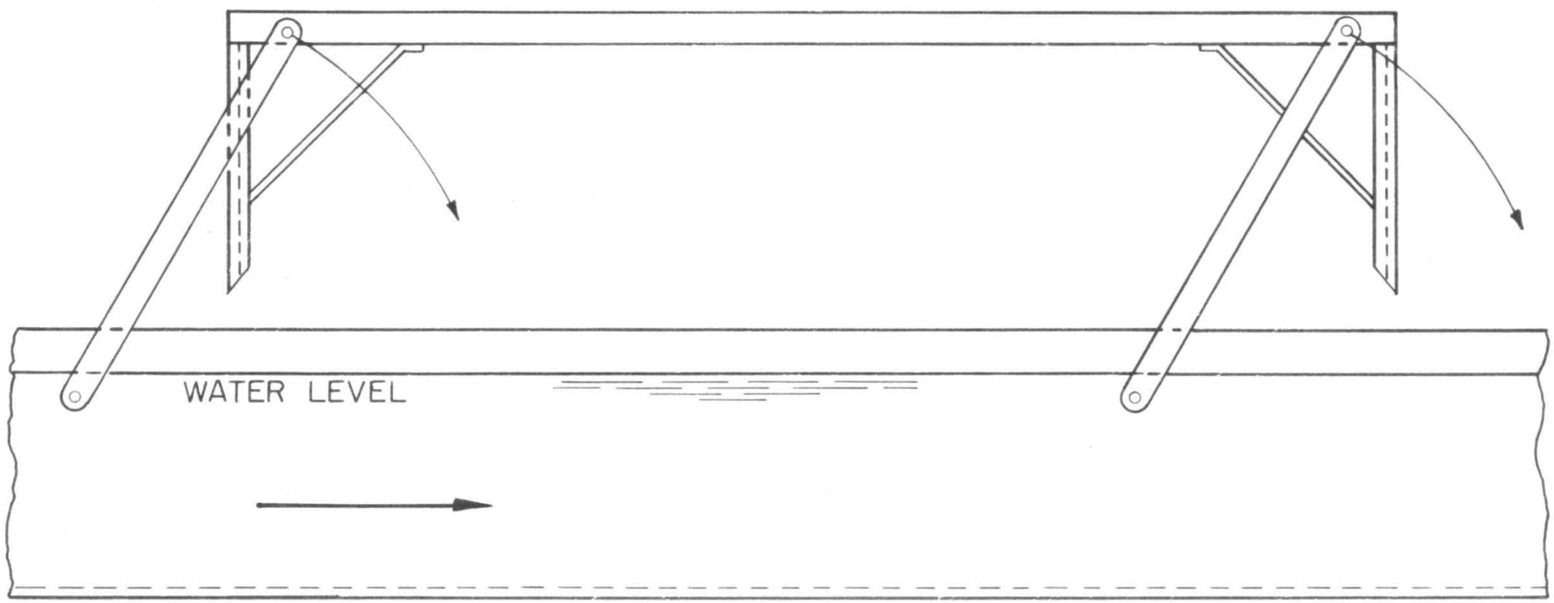

FIGURE 2 A SAMPLING DEVICE FOR DETERMINING THE SPACIAL CONCENTRATION OF SEDIMENT MOVING IN AN OPEN CHANNEL--AFTER BAGNOLD (1955)

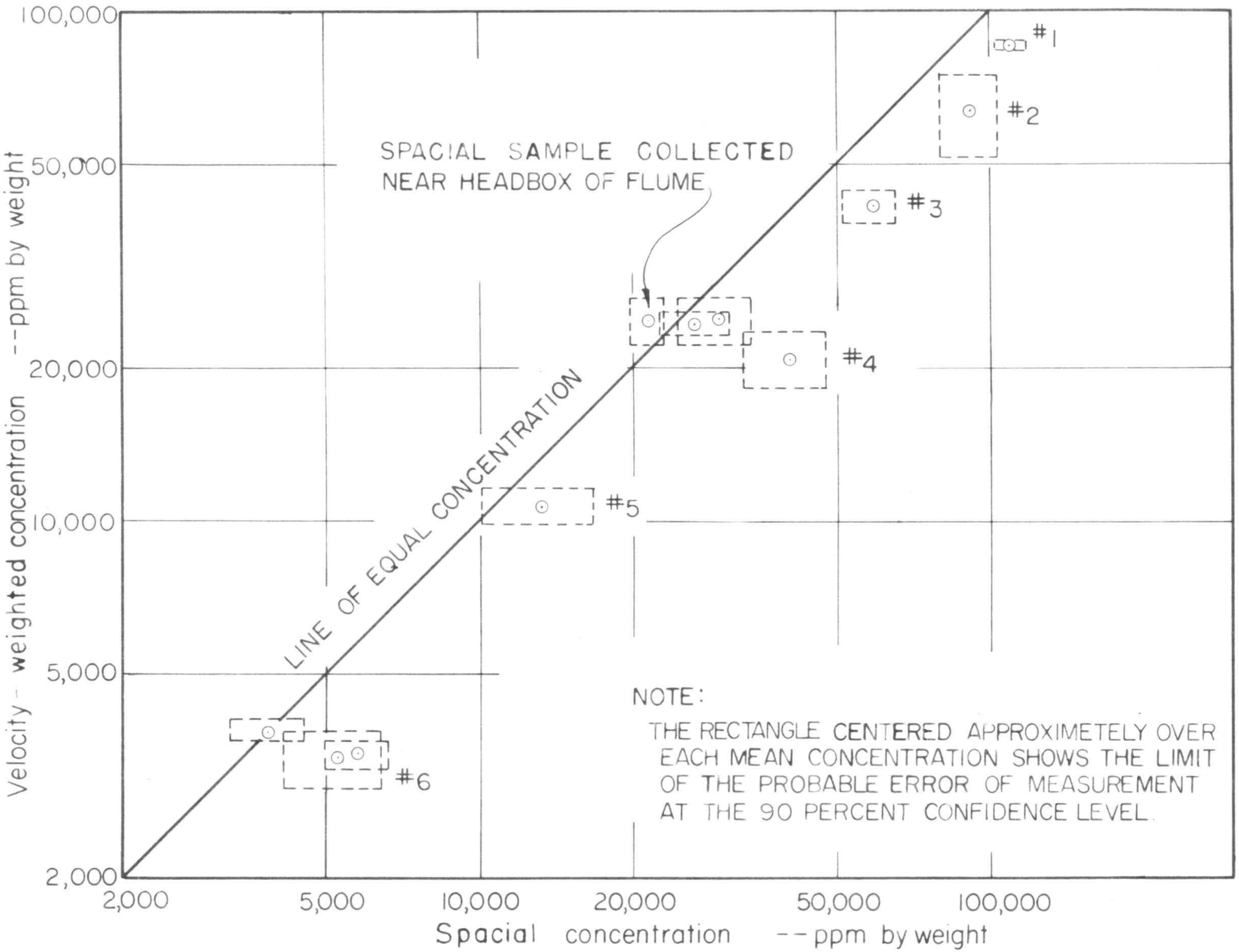

FIGURE 3 RELATIONSHIP OF VELOCITY-WEIGHTED CONCENTRATION TO SPACIAL CONCENTRATION