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DISSERTATION

DISENTANGLING RISK FROM STABILITY PREFERENCES: A NEW PERSPECTIVE TO ANALYZE TECHNOLOGY ADOPTION DECISIONS

Submitted by

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In partial fulfillment of the requirements
for the Degree of Doctor of Philosophy

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ABSTRACT OF DISSERTATION

DISENTANGLING RISK FROM STABILITY: A NEW PERSPECTIVE TO ANALYZE TECHNOLOGY ADOPTION DECISIONS

This research investigates the intertemporal decision-making process under risk by exploring technology adoption. Previous studies may have attributed too much importance to risk by failing to account separately for variability in expected income. My view of the problem is that adoption is influenced not only by risk and risk aversion, but also by the variability of the expected income and a preference for income stability. The approach used to construct the adoption model is a generalized version of the Expected Utility model, the Generalized Expected Utility (GEU) approach, in which the coefficient of risk aversion is disentangled from the Elasticity of Intertemporal Substitution (EIS). The EIS accounts for stability preferences as it measures the willingness to trade between the consumption/income of one period for another.

A two-period and a multi-period adoption model were developed. The two-period model was used to perform an exhaustive sensitivity analysis form which important conclusions with respect to the individual's behavior under different conditions and preferences parameter were obtained. The multi-period model was used to examine the impact of different stability levels and minimum income on stability premiums. The stability premium is the amount that an individual would be willing to pay to trade an uneven income path for an even path. The results of this analysis were used to investigate the implications of considering preferences for stability on policy and financial tools by doing an application of multi-period model in forestry adoption. The results indicate that there is a good opportunity to increase the adoption rate in tree plantation if stability is considered and incorporated in designing financial instruments that will meet the preferences of the farmers.

The approach developed in this dissertation to analyze and explain technology adoption decisions opens a new line for research in intertemporal decision-making. The findings generated also can be applied to other areas of decisions such as inventory management, management of the farm annual production, or investment.

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I want to thank specially my adviser and friend, Dr. Hoag, for his support and advise. When things became difficult for me, he always had the right word to comfort me.

Finally, I want to say to my husband Juan: "Thanks for being here for me. I know it has not been easy for you and I will always be grateful for letting me follow my dreams."

DEDICATION

Quiero dedicar este trabajo a mi esposo Juan por su permanente apoyo y comprensión, a mis hijas Javiera y Francisca por su cariño siempre reconfortante, y a mis padres Peky y Carlos por haberme abierto un camino.

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CHAPTER ONE

INTRODUCTION

There are more than five decades of research about the influence of risk on farm management decisions involving inventory, investments, and technology adoption. In general, the results of these studies support the importance of risk in farmers' decisions. While growth in knowledge and management have been significant, there still are many questions without answers. One of these areas is modeling risk in an intertemporal setting, including individual behavior, serial correlation of farming outcomes, and the role of long run versus short-run risk (Just, 2000). My purpose is to address individual behavior over time by integrating concepts from a developing literature about intertemporal preferences (i.e. Hall, 1989 and Van der Ploeg, 1992) with traditional risk analysis.

This research investigates the intertemporal decision-making process under risk by exploring technology adoption. Technology adoption constitutes a good area to study intertemporal decisions because it is, in many cases, a long run choice with a high degree of irreversibility. Moreover, past literature has recognized risk as a central factor in farmers' decision to adopt certain technologies. In a general sense, Carcamo et al. (1994), Parks (1995), Reeves et al. (1993), and Reppeto (1987) conclude that the primary barriers to technology adoption in developing countries are credit restrictions, the stochastic nature of income or risk, and low expected profits. In particular, Amacher, Hyde and Rafiq (1993), and FAO (1986) believe that risk attitude and uncertain returns are the most critical factors in agroforestry adoption.

Certainly, risk appears to be a critical factor when predicting adoption decisions. However, previous studies may have attributed too much importance to risk by failing to account separately for variability in expected income. My view of the problem is that adoption is influenced not only by risk and risk aversion, but also by the variability of the expected income and a preference for income stability. Income stability (or aversion to intertemporal substitution) and aversion to risk are two different components in individuals' decisions that traditionally have been entangled in a single concept called risk. The following hypothetical example shows this difference.

Let us assume that a farmer needs to decide between two technologies A and B that have the same Present Value (PV). In figure 1.1 panel A and B show the PV, the expected net returns for four periods (denoted by a star) and the density function associated with each annual return for both technologies. The black four-point stars are the realization (observable data) of net returns for each of the four years. Based on the observable information, one could incorrectly conclude that the distribution of the net returns is the density function adjacent to the vertical axis. According to these density functions, technology B would have a higher variance than technology A. However, looking at the true distribution of the annual net returns, technology B is actually less risky (the density function for each set of annual returns has a lower variance). The confusion is generated by a failure to separate variability into an expected and non-expected component. The expected component of the variability is determined by the variability across the expected value of the net returns, while the non-expected or stochastic component is determined by the variance around the yearly expected return. Technology B has a higher "expected variability" causing a higher variability in the net returns.

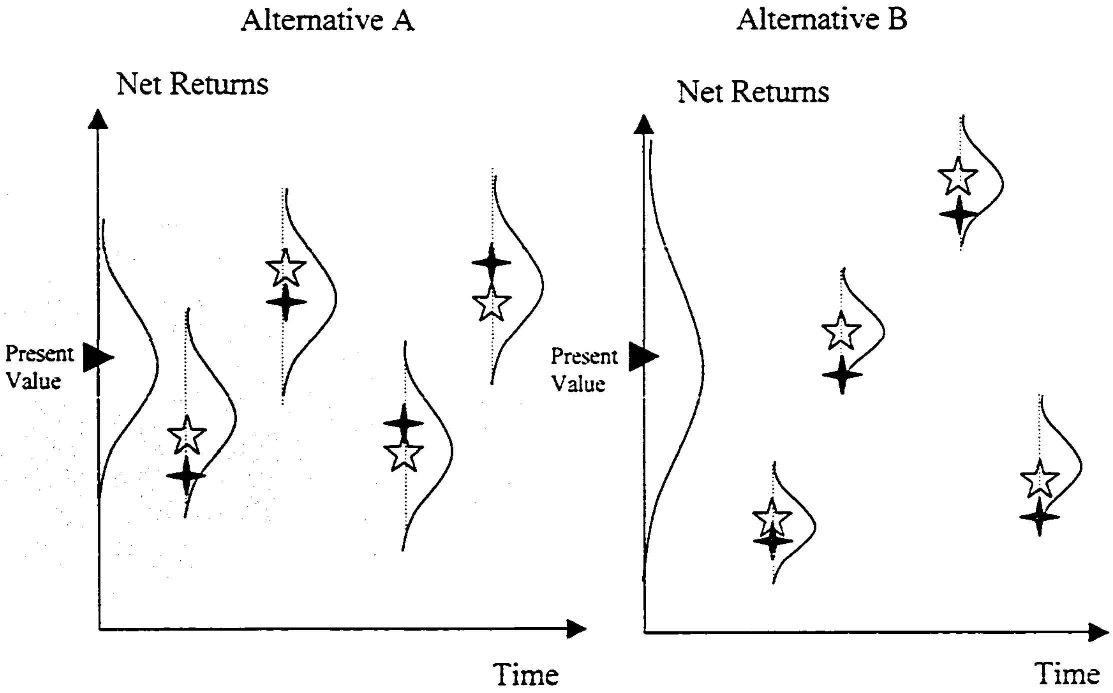

Figure 1.1: Comparison of productive alternatives with different expected returns and variance.

A farmer's preference for technology A is not explained by risk aversion. This farmer is willing to accept a higher risk in order to have a more stable income over time. If in the same situation, another farmer decided to adopt technology B, we could conclude that his/her aversion for risk is higher than his/her preference for stability.

From this example we can conclude that predicting farmers' behavior requires not only an accurate definition of the joint distribution of income or net returns[1], but also a good understanding of farmers' decision motives. If the objective function does not

---

[1] Holt and Aradhyula (1990) is a good example of efforts in this direction.

accurately capture the individual's preferences for both components, prediction with respect to farmers' choices will be biased.

The general purpose of this study is to contribute to the risk literature by incorporating into the analysis distinct preference parameters for risk and temporal substitutability. Related work has been done in asset pricing (Consumption Capital Asset Pricing Models as described in Campbell, MacKinlay, and Lo 1997) and for explaining consumption-savings decisions, which showed that the individuals have a preference for stability that can not be fully captured by risk aversion alone. In this context, risk preferences are called safety preferences, and intertemporal substitutability is called stability preference (Weil, 1990).

Goal

A first goal is to develop a theoretical framework to demonstrate the roles of safety and stability preferences in farmers' decisions about technology adoption. A second goal is to apply the conclusions derived from this theoretical framework to develop financial and policy tools that will encourage the adoption of desirable long run technologies.

Objectives

1. Define the differences between income stability and risk.
2. Develop a measure for income stability.
3. Develop a model that disentangles risk and income stability to predict technology adoption decisions.

4. Generate trade-off measures for income stability and income, and income stability and risk.
5. Develop one or two examples of financial tools to encourage adoption of long-run technologies.

Organization of the Dissertation

The main contribution of this dissertation is the development of a new approach to analyze and predict technology adoption decisions. In order to achieve this goal, a theoretical model that is able to disentangle safety and stability preferences is developed. This model is used to describe the individual's behavior for different combinations of safety and stability preferences and for different production alternatives. The conclusions of this exercise are used to construct financial tools that match the different types of individuals' preferences. These tools could also be used to provide new policy alternatives to encourage the adoption of certain technologies, such as tree plantation in deforested areas. The dissertation is organized as follows:

Chapter Two presents the background for this research. The first section presents a review of the literature on the influence of risk on three different decisions areas: consumption-saving, inventory management, and technology adoption. The second section develops the state of the art of the Generalized Expected Utility model - the theoretical framework used in the study - and explains the role of each of the components of the model.

Chapter Three presents the methodology and steps that will be followed to develop and explore the technology adoption decision model that is presented in this dissertation. Chapter Four explains the concepts of deterministic and stochastic variability presented in this introduction, and proposes distinct measuring devices for both variability components.

Chapter Five develops and analyzes a two-period model to explain technology adoption decisions based on the Generalized Expected Utility approach that will be explained in Chapter Two. The first section of the Chapter is dedicated to the model development. The second section contrasts the Generalized Expected Utility model with the traditional Expected Utility model. The final section of the Chapter presents the results of an analysis of the sensitivity of the parameters and variables involved in the model.

Chapter Six extends the two-period model presented in Chapter 5 to a multi-period model. This Chapter contains two sections. The first section develops the model and the second section presents the sensitivity analysis on the model.

Chapter Seven is dedicated to summarize the most important results of the two and multi-period model as well as to investigate their implications for constructing policy and financial tools.

Finally, Chapter Eight presents an application of the multi-period model. This application is based on a farmer's decision of producing an annual crop versus the alternative of planting trees. The objective of the application is to show how the model developed here can be used in explaining technology adoption decisions and how different policy tools and financial instruments could be constructed to encourage the adoption of long run technologies.

This work is expected to contribute to both, applied and theoretical economics. On one hand, the results generated in this study are expected to add to the explanations of farmers' technology choices. It is expected that the findings will be applied in policy and financial programs to improve the adoption of technologies that would help improve peoples' lives and reduce environmental degradation. On the other hand, the theoretical model developed here is expected to open a window to analyze the decision-making process in an intertemporal setting. This research may encourage an expansion of the literature about stability in inventory, savings, and long-run investments, as well as risk analysis literature.

CHAPTER TWO

BACKGROUND

Decision-making Process

Farmer behavior, decision-making processes and risk have been analyzed from different points of view and under different conditions. Most of this work has concentrated on consumption and investment behavior - technology adoption, input decisions and inventory management, studying how farmers react to risk and other constraints affecting his/her final choice. In this section, I review decisions and risk from three different perspectives: consumption-saving, inventory management and technology adoption. The purpose of including an analysis in three different areas is to show that consumption and/or income smoothing play a role in intertemporal decisions.

Most of the studies reviewed for these three decision areas use the household production model to account for consumption and investments needs (e.g. Mullen et.al., 1988 and Rozenswieg and Wolpin, 1993). Investment and consumption are strongly related on farms. Deciding the level of consumption implies deciding the level of investment, and the consequences can be seen in farm growth, farm practices, and debt payment. As Phimister (1993) recognizes, household production models are useful because of the potential interaction between consumption and future production through investment.

This modeling approach is important because it constitutes the basis for most of the research on farm decisions.

Consumption-saving

A review of the literature about consumption behavior reveals that the Marginal Propensity to Consume (MPC) for farm households is lower than the MPC from non-farm households (Mullen et. al., 1988; Langemeier and Patrick, 1990; Chen et al, 1999). Since the 1960's, studies have concluded that MPC on farms is " 0" or close to "0", suggesting that, at the margin, increases in income are saved. This result can lead to two interpretations. One is that income is highly volatile and that the variability in income could be considered as transitory or temporary and would therefore not change consumption in the long run. This interpretation is based on Friedman's Permanent Consumption Theory, in which transitory income does not affect permanent consumption (Mullen et al, 1988). Another interpretation for a low MPC is that income risk has a significant effect on precautionary savings. Related to this, Chen et al. (1999) analyzed the impact of risk in explaining consumption behavior and how it affects savings motives. Using a time additive Expected Utility (EU) framework they conclude that risk is an important variable in explaining consumption and the motive for saving, leading to a reasonable doubt about the relevance of a traditional life-cycle permanent income hypothesis. Moreover, Phimister (1995), who studied consumption behavior under credit constraints and income uncertainty, failed to accept the life-cycle hypothesis under perfect capital markets. In conclusion, it can be said that the literature on consumption decisions suggests that farmers use saving and debt to smooth consumption across time, and as Chen et al. (1999) noticed, farmers use precautionary saving as a risk management strategy.

Inventory Management

Saving is not the only strategy that farmers use to smooth consumption. A second group of strategies involves inventory management in which risk has been used as one of the explanatory variables. Rosensweig and Wolpin (1993); Thijssen (1996); and Bierlen et al. (1998) have shown how the stochasticity of income has an important effect on inventory and durable input use, and how inventories are used as a source of resources to smooth cash flow, and consequently, consumption.

Rosensweig and Wolpin (1993) use a time additive EU function subject to credit constraints to explain how bullock inventory is used in India to smooth consumption when income is lower than the average. The research indicates that the sale of bullocks in interregional markets increases when income is low and decreases when income is high.

Thijjsen (1996) explored farmers' investment behavior under different specifications of expectations – rational expectation and static expectations. From his conclusions it can be derived that investment is very sensitive to changes in the price of output, capital cost and technology. Furthermore, Bierlen et al. (1998) analyzed the impact of cash flow variability in feeder cattle and beef cow inventories under credit constraints. Their results indicate that farms absorb financial shocks using inventories. That means that inventory investment is more sensitive to cash flow during weak periods.

Despite the specification and objective of the three inventory studies presented here, they share the same conclusion: inventory management is used to smooth income over time.

Technology Adoption

The third strategy used to smooth income over time is technology. The role of risk and risk aversion in technology adoption decisions has been extensively discussed in the literature, and in general it can be concluded that farmers have a tendency to adopt less variable cash flow practices. Amacher, Hyde and Rafiq (1993), and FAO (1986) found that risk attitude and uncertain returns are the most critical factors affecting adoption. Batz et al. (1999) agrees that risk is a critical factor. The approach used is based on a utility maximization model subject to different technology characteristics. The characteristics included in the model are relative profitability, relative risk, initial cost, relative investment, and relative complexity of the technology. Regressions with different combinations of characteristics as independent variables and adoption as the dependent variable were run, finding that the model that includes relative risk, investment and complexity performs well and all coefficients were significant.

The same results were obtained by Chavas and Holt (1996). Their model goes further by jointly testing preferences and technology characteristics to identify the relationship among these factors. The model assumes that the decision-maker has preferences according to the von Neumann-Morgansten utility function and that production is a function of inputs and a set of parameters. Three measures of risk aversion were calculated: the Arrow-Pratt absolute risk aversion coefficient, the relative risk aversion coefficient, and the downside risk aversion coefficient. The authors established that all three are significant in the decision-making process. The conclusion in this case is that farmers are risk averse and exhibit decreasing absolute risk and downside risk aversion.

Agroforestry studies, which are relevant here because of the relatively longer time horizon of return than those found in agriculture, are more inconclusive with respect to the role of risk in adoption. On one hand, agroforestry systems will increase risk because of the uncertainty associated with tree establishment derived from drought conditions and poor planting material, and weak markets for tree products. On the other hand, agroforestry increases income diversification, thus reducing risk (Current et.al., 1995).

Empirical studies about the impact of risk on agroforestry adoption do not completely agree with respect to the final effect of risk on the adoption of soil conservation systems. Shively (1998) finds that consumption risk associated with agroforestry, especially during the initial period, influences soil conservation decisions. Through a traditional household approach subject to a safety first constraint, Shively was able to model how variability affects the adoption of intercropping trees. The model is clear in explaining the role of risk in decisions; however, its limitation is that reduced form solutions present a constant impact of risk with respect to different levels of profits.

According to all authors cited, risk consistently appears as one of the most relevant variables in explaining technology adoption. Although, as shown in the introductory Chapter, it is important to define what kind of variability is influencing more the relative aversion to adopt certain technologies. Each technology has a certain level of risk or stochastic variability, but each technology also has an expected income that varies from period to period. Which kind of variability is more prevalent in adoption decisions? The question is not trivial since different strategies can be designed to encourage the adoption of desirable technologies depending on the answer to this question. If the important factor in defining adoption is risk, then strategies orientated to reduce or shift risk, like insurance or direct subsidies, could be the answer. If, on the other hand, the predictable variability explains adoption better, then other strategies that smooth income can be used with higher success and lower cost.

Conclusion

In analyzing the conclusions of different decision studies, one can observe that final choices are highly influenced by the distribution of money availability over time, in which an even distribution is preferred over a variable distribution. This conclusion is also observed from the results of a case study published in 1998. Based on interviews of eighteen Swedish farmers, Ölhmer (1998) found that farmers do demonstrate preferences for stability or more even cash flow. The results from the interviews reveal that most farmers valued a set of private cash withdrawals per year more than they value high profits.

The purpose of this section of the literature review was to establish a common point: lower income variability (smooth income) is an important objective in farmers' decisions. However, income variability has two possible sources: stochastic or risk and deterministic. The next section will show how to disentangle preferences for stability (an even, deterministic income path) and safety (risk) into an intertemporal maximizing utility model.

Risk Aversion and Intertemporal Elasticity of Substitution

Once the problem has been stated and the arguments to support the hypothesis have been presented, it is necessary to present an approach that will address a solution for the problem. This problem can be restated as inability of past adoption decisions models to account for distinct preferences for risk (safety) and substitutability across time (stability). The independence of these two components of the individual's preferences can be addressed by a generalization of the traditional EU Theory that has the flexibility to consider risk aversion and stability preferences separately. In this section, I will review the traditional Expected Utility theory and explain how, in a temporal setting, it is constrained to consider stability and safety in a single parameter. Finally, I present a state of the art of the Generalized Expected Utility (GEU) theory, the model approach used in the following Chapters.

Expected Utility Theory

The assumption that the decision–maker maximizes expected utility has been used frequently in model specifications. More specifically, time additive expected utility maximization has been used in several temporal studies such as Chavas and Holt (1996), Rosenswieg and Wolpin (1993) and Chen et al. (1999). The development of the EU theory started from the initial work of Von Neuman and Morgenstem, "Theory Games and Economic Behavior" (1944), that established a set of axioms that allows preferences to take into account risky situations and give a consistent ranking. The result of the EU Theory can be summarized in the following equation:

$$U(\tilde{x}) = E(u(\tilde{x})) \qquad (2.1)$$

Where U is Utility index, u is utility function, $\tilde{x}$ is any random argument[2] that can be described with a specific probability function, and E is the expectation operator.

Let us define a lottery L as a random alternative formed by the prizes x and y that is described as: $p \circ x \oplus (1\text{-}p) \circ y$. This notation means that the individual receives the prize x with probability p and the prize y with probability (1-p). Then the set of axioms supporting equation 2.1 is:

1. Complete ordering axiom:

   a. For lotteries L1 and L2, it is true that: $L1 \succ L2$, $L2 \succ L1$, or $L1 \sim L2$[3].

   b. If $L1 \succ L2$ and $L2 \succ L3$, then $L1 \succ L3$

2. Continuity axiom:

   If $x \succ y$ and $y \succ z$, then there must a probability p such that:

   $p \circ x \oplus (1\text{-}p) \circ z \sim y$

3. Independence or substitution axiom:

   If $x \sim y$, then $p \circ x \oplus (1\text{-}p) \circ z \sim p \circ y \oplus (1\text{-}p) \circ z$

4. Unequal utility axiom:

   If $x \succ y$, then $p \circ x \oplus (1\text{-}p) \circ y \succ q \circ x \oplus (1\text{-}q) \circ y$ for probability p > probability q.

A fifth axiom is added to this set in order to allow for compound lotteries:

---

[2] Typically, wealth, income or consumption are used as arguments for the utility function.

[3] $\succ$ means preferred and $\sim$ means indifferent.

5. Reduction of compound lotteries or complexity axiom:

$$q\circ(p\circ x \oplus (1\text{-}p)\circ y) \oplus (1\text{-}q)\circ y \sim qp\circ x \oplus (1\text{-}qp)\circ y$$

If the preferences of a decision-maker represented by the ordering (L, $\succ$) satisfy axioms 1 to 4, then there is a utility function u – unique to any linear transformation, defined on the lottery L, that satisfies the expected utility property as follows[4] (Varian, 1992):

$$u(p\circ x \oplus (1\text{-}p)\circ y) = p\cdot u(x) + (1\text{-}p)\cdot u(y) \qquad (2.2)$$

A special class of decision-makers is defined as risk averters. Someone that is risk averse is an individual for which a certain prospect x_0 has a higher utility than a gamble that offers a prize $(x_0 + h)$ with probability 0.5 and $(x_0 - h)$ with probability 0.5 (Arrow, 1971):

$$u(x_0) > 0.5\cdot u(x_0 + h) + 0.5\cdot u(x_0 - h) \qquad (2.3)$$

In terms of the utility functional form, an individual is risk averse if the second derivative of u(x) is negative - $u''(x) < 0$ – or, in other words, if u(x) is concave.

Two measures of the degree of risk aversion are typically used: Relative Risk Aversion Coefficient (RRA) and Absolute Risk Aversion Coefficient (ARA). These Aversion Coefficients are measured as follows:

$$\begin{aligned} RRA &= -\frac{x\cdot u''(x)}{u'(x)} \\ ARA &= -\frac{u''(x)}{u'(x)} \end{aligned} \qquad (2.4)$$

---

[4] Equation 2.1 is a generalization of equation 2.2, and therefore they have the same meaning.

RRA is the elasticity of the marginal utility of the argument x and it is designed to capture aversion to a bet expressed in relative terms[5], while ARA measures the preference of an individual for more fair bets expressed in absolute terms (Arrow, 1971).

The following table presents commonly used functional forms to model the EU and the characteristics of these forms:

Table 2.1. Utility functions: different functional forms

Functional Form	Characteristic
$U = 1 - e^{-\beta \cdot X}$ $\beta \geq 0$	Constant ARA coefficient, β.
$U = X^c$ $0 \leq c \leq 1$	Decreasing ARA coefficient
$U = \ln(X)$	Decreasing ARA coefficient
$U = \frac{X^\delta}{\delta}$ $\delta \neq 0$	Constant RRA coefficient, (1-δ).

The fifth axiom allows a compound lottery to be reduced to a binary choice, and constrains the utility function to be additive across outcomes and states and to be linear in the probability terms. The following example can illustrate the situation. Let us assume that an individual needs to choose between two income streams represented by lotteries A

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[5] A relative term bet would be in percentage of wealth or income.

and B. Both lotteries are designed for a three period sequence and are shown in Figure 2.1.

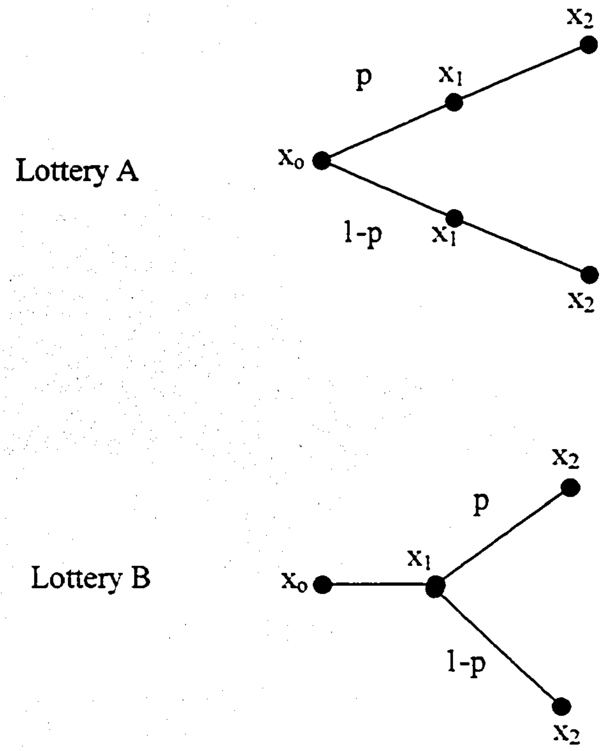

Figure 2.1. Comparison of time resolution

In this example, if the reduction axiom is assumed, the individual should be indifferent between A and B since for both lotteries, each stream of the tree has the same prize and probability of occurrence. However, lottery A differs from lottery B in that uncertainty resolves earlier for lottery A. That is, in lottery B an individual does not know his consumption path until period 1. A decision-maker that is averse to risk would prefer earlier resolution in order to make a better allocation of the prizes, making lottery A preferable for this example. There have been some limitations to this axiom pointed out in the literature. Kreps and Porteus (1978, 1979) establish that when the reduction axiom is abandoned, non-indifference with respect to the time of the resolution can be captured. Moreover, in a temporal analysis of risk this axiom will impose the restriction that the coefficient of relative risk aversion (RRA) will be the reciprocal of the elasticity of intertemporal substitution (EIS) (Selden, 1978). The EIS measures the individual's willingness to trade present for future consumption, and in this sense it constitutes a measure of the desire for stability in the consumption path. A low EIS is related to a high desire for stability, while a high EIS relates to a low desire for stability. To show this effect, consider the standard utility model used to incorporate intertemporal stochastic decisions:

$$U(\tilde{x}) = E\sum_{t=0}^{T}\rho^{t}u(\tilde{x}_{t}) \qquad (2.5)$$

Using a utility functional form with constant RRA such as:

$$u(x) = \frac{x^{\delta}}{\delta} \qquad for \delta \neq 0$$

It can be shown that the RRA coefficient (1-δ) is the reciprocal of the elasticity of intertemporal substitution (EIS), which is equal to $(1\text{-}\delta)^{-1}$ (See Appendix I). This additional and not obvious consequence of assuming the timeless expected utility function makes preferences depend only on one parameter for two distinct characteristics of the individual taste.

In summary, the reduction axiom in the temporal EU model carries two important anomalies: first, it does not recognize the individual's preference for the timing of the resolution of a gamble and second, it will cause stability and safety preferences to be represented by only one parameter.

The response to these anomalies leads to the birth of the GEU approach, which provides a model that considers independence between the risk aversion parameter and the EIS.

Generalized Expected Utility Theory

At the end of the seventies, the validity of the reduction axiom was not only questioned by Kreps and Porteus (1978, 1979), but also by Lucas (1978) and Selden (1978). They point out that, under the reduction axiom, risk and time[6] preferences can not be treated as two independent components in the individual's tastes. Their conclusion was that risk aversion and EIS will remain entangled in the utility index as long as uncertainty is inserted in each period of the utility function. The solution of the problem is to look for an aggregate representation of the utility index that provides independence for both coefficients, risk attitudes and EIS.

Kreps and Porteus (1978, 1979) and Selden (1978) provide the starting point in this direction. Kreps and Porteus recognize as a primary problem in the temporal EU model the temporal treatment of risk as well as its inability to account for the timing of

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[6] In early papers, the Elasticity of Intertemporal Substitution (EIS) was used as a measure of time preferences. Later Weil (1993) starts using EIS as a measure of stability preferences.

the resolution of uncertainty. They provide a set of axioms that will allow for "dating" the uncertainty in the probability distribution. Through a "temporal consistency" axiom they construct a preference structure in which the time of resolution may be relevant[7].

Selden (1978) presents the two period ordinal-certainty equivalent (OCE) model as one of the earliest versions of the GEU model. In this representation, the EIS is independent from risk aversion, thereby pointing in the same direction as Kreps and Porteus. The main objective of the OCE model was to separate risk and time preferences. In this sense, the contribution of the OCE representation is that it provides a way to account for pure risk preferences over an uncertain prospect at the same time that it accounts for time preferences, measured by the EIS, related to a certain prospect[8]. The proposed model can be simplified as follows:

$$\begin{aligned} W &= T(c_1, \beta\hat{c}_2) \\ \hat{c}_2 &= u^{-1}E(u(\tilde{c}_2)) \end{aligned} \qquad (2.6)$$

Where W is the utility index for the certain prospect, u is the utility function for the uncertain prospect, T is a concave transformation, c_1 is the certain current consumption level, $\hat{c}_2$ is the certainty equivalent of random period 2 consumption and β is the personal discount factor. The utility index is conformed only by certainty values leaving risk aversion preferences out of the aggregation of utilities over time. Therefore, the risk aversion parameter will be related only to the certainty equivalent of period 2

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[7] This paragraph intends to give a flavor of what Kreps and Porteus proposed as a solution of the EU theory deficiency. Further analysis of the axiomatization of the model was found irrelevant for the purpose of the model that will be developed later.

[8] Selden called this modeling approach a representation of preferences over "certain x uncertain" consumption pairs.

consumption, while the EIS is given by the parameter included in the transformation function.

To show explicitly how preferences for risk and time are represented in the OCE model, consider the following functional form for the utility index from the certain prospect:

$$W = -\frac{1}{\delta_1}(c_1^{-\delta_1} + \beta \cdot \hat{c}_2^{-\delta_1}) \qquad for\delta_1 \geq -1$$

The parameter δ_1 captures time preferences through a constant EIS equal to $(1+\delta_1)^{-1}$. Now consider the following functional form for the uncertain prospect with constant relative risk aversion coefficient:

$$u(c_2) = -\frac{c_2^{-\delta_2}}{\delta_2} \qquad for\delta_2 \geq -1 \qquad (2.8)$$

The parameter δ_2 defines risk preferences for which RRA coefficient is equal to $(1+\delta_2)$. The formulation of the model clearly leads to a mathematical independence between δ_1 and δ_2. In particular, notice that when $\delta_1 = \delta_2$ the OCE representation collapses into the traditional EU model, making it just a special case of the OCE representation. The following is the reduced form combining equation 2.7 and 2.8 when $\delta_1 = \delta_2 = \delta$:

$$W = -\frac{1}{\delta}(c_1^{-\delta} + \beta \cdot E(c_2^{-\delta})) \qquad (2.9)$$

Kreps and Porteus's axiomatization for a multi-period setting and Selden's two-period OCE model constitutes the first attempt to relax the Von Neuman-Morgenstern reduction axiom and to include timing of uncertainty and time preferences. Later development of this preference treatment resulted in the GEU theory[9].

Further development of these early works was motivated by their limitations. On one hand, the Kreps and Porteus approach is limited in deriving empirical, testable relations (Attanasio and Weber, 1989). On the other hand, the OCE representation exhibits intertemporal inconsistencies when extended to a multi-period context (Hall, 1988 and Attanasio and Weber, 1989). The application of a multi-period OCE representation would imply considering a "naïve consumer who continually ignores that plans formulated at any given time will generally not be carried out in the future" (Epstein and Zin, 1991).

The deficiencies explained above led Epstein and Zin (1989, 1991) to formulate a generalization of the conventional time-additive EU model that built on Kreps and Porteus' preference structure. The model assumes a representative agent who lives for an infinite period of time and receives utility from the consumption of a single good. This individual's lifetime utility at period t is defined as:

$$U_t = W(c_t, u[\tilde{U}_{t+1}| I_t]) \qquad (2.10)$$

This representation has two key assumptions. First, the individual forms a certainty equivalent of a random future utility using his risk preferences. This certainty equivalent is expressed by the term $u[\tilde{U}_{t+1}| I_t]$, where $\tilde{U}_{t+1}$ is the future random utility, I_t is the information set available at period t to form the certainty equivalent and u is a function that contains the individual's risk preferences to define the certainty equivalent.

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[9] It is also called Non-expected Utility model.

The second assumption is that the current period lifetime utility U_t is obtained by combining the certainty equivalent and current consumption c_t through an aggregator function W as in Koopman's terminology[10]. This approach also provides a consistent separation of the risk aversion parameter and the EIS, allowing for independence of preferences for both concepts.

The approaches developed since Kreps and Porteus and Selden, have opened a gate to explore the impact of savings decisions on asset pricing, and the impact of taxes, assets returns, and other sources of income on consumption through a flexible choice ranker. However, the important issue for the purposes of the model that is developed in this dissertation is to recognize the role that EIS (stability preferences) and risk aversion (safety preferences) have on utility, and finally, on the consumption choice. With respect to this issue, Selden (1979) concluded that "...what matters is the degree to which he insists first and second- period consumption *go together* (i.e. are complements). If he views them as complements, then on the one hand increased capital risk will cause him to increase his savings, and on the other hand he will save more in an uncertain setting than in a certain one." This is the conclusion of applying the OCE representation to explaining savings decisions, which leaves a flavor of the importance of EIS as a measure of the desire for stability in the consumption stream.

Moreover, Weil (1990) makes explicit the terms of stability and safety when referring to EIS and the risk aversion coefficient respectively. Weil noticed that the certainty equivalent of lottery B, in an example similar to the shown in Figure 2.1, fluctuates less than of lottery A; therefore, an individual with a higher aversion to

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[10] Koopman (1960) showed the advantage of defining preferences for advance timing entirely in terms of a utility function called the prospective utility.

intertemporal substitutability than risk aversion would prefer lottery B. On the other hand, an individual with higher risk aversion would prefer lottery A. Thus, depending on his/her preference for safety and stability, the choice can be either lottery A or B.

The Generalized Expected Utility approach has been used over the last two decades in a variety of applications to explain consumption and saving behavior. Attanasio and Weber (1989) conclude that distinguishing between risk aversion and elasticity of intertemporal substitution may be fruitful in explaining the impact of the rate of returns in consumption growth.

Hall (1988) analyzed intertemporal substitution in consumption using the real interest rate as an explanatory variable. The sample used to test the model was U.S. consumption data from the twentieth century. His result indicates that, in general, people have a low EIS (close to 0.1 and significant) or, in other words, people have a high desire to smooth consumption over time.

Van der Ploeg (1992) analyzed the impact of a tax cut on consumption levels for different degrees of risk aversion and EIS. His conclusions support the idea that different combinations of these parameters will cause different reactions on the level of consumption. The lower the EIS, the lower the increase in consumption derived from a tax cut. On the other side, the lower the risk aversion the higher the increase in consumption. The net effect will depend on the size of each of the parameters. Obstfeld (1992) uses the GEU model to evaluate a risky consumption path. His conclusion reveals that unless risk aversion and EIS are fully separated, attempts to measure the welfare cost derived from an increase in risk may lead to false conclusions.

Weil (1993) studied the determinants of precautionary savings through the scope of the GEU approach. He found that a prudent behavior is reinforced by larger income risk, more persistence in the income process, stronger aversion to risk, weaker distaste for intertemporal substitution, and higher interest rates, validating the GEU approach.

Finally, Lence (2000) applied the GEU model to estimate temporal preferences among farmers in the U.S. He constructs the model using consumption levels and the rate of return from farm assets. The results show that, contrary to the findings in the general population, farmers do not have a high desire to smooth consumption over time[11]. He showed that the risk aversion preference was more important in the final decision, although risk aversion was decreasing over the period covered in his analysis.

Regardless of the result, all the papers reviewed concluded that considering a GEU instead of the traditional EU model improves the predictions and explanation of consumption changes derived from changes in risk. It can be concluded that the flexibility permitted by the GEU model captures more accurately the impact of variability on the choices of an individual. EIS, or stability preferences, will capture the impact of systematic variability, while risk aversion or safety preference, will capture the impact of unexpected variability through the certainty equivalent of future utility. This desirable property of preference representations can be extended to other applications of the decision-making process.

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[11] Lence found that the EIS could be close to 9. Comparing this estimate to the EIS estimated by Hall (1988) we can observe a high discrepancy.

Concluding Remarks

In this Chapter, I showed that farmers use a variety of strategies to smooth net returns over time, and that most of the studies attribute this result to high risk aversion and a high level of risk within the farm business. However, looking at similar cases through the scope of the GEU presented here, it can be seen that the strategies adopted to smooth income may be explained in part by stability preferences rather than risk aversion. In particular, in technology adoption decisions the influence of farmers' time preferences could be significant. I believe that the extension of the GEU model to technology adoption decisions can present a new perspective for the analysis of such decisions, and that the conclusions can improve the understanding of the impact that different instruments and policies designed to encourage the adoption of some technologies may have on the effective adoption rate.

CHAPTER THREE
METHODOLOGY

In Chapter Two, the background for this research was presented and two main ideas were developed. First, farmers adopt different strategies to smooth income over time, showing that they dislike "variability". Second, in a intertemporal setting, variability can be extended to stability - variability of a certain prospect across time - and safety - variability of an uncertain prospect in a point in time. The modeling approach to capture these two components of variability is the Generalized Expected Utility (GEU) model. It disentangles stability and risk preferences in the individual's choice. This property of the GEU model generates decisions that not only incorporate aversion to risk, but also incorporate a desire for a stable expected income across time. Recalling the example presented in Chapter One, I showed how both variability components would affect the farmers decision for technology A and B. Farmers that prefer alternative A would have a higher preference for stability, while farmers that prefer alternative B would have a higher aversion to risk than preference for stability.

Turning now to the goal of this dissertation, the purpose is to develop a framework to show how farmers' adoption decisions are influenced by risk and stability preferences. The methodology to address this objective consists of the development of a theoretical model that uses the GEU framework to represent the farmers' preferences subject to a set of production alternatives that incorporates a stability variable independently from a risk variable. The reduced form solution of this model is analyzed for different combinations of risk and stability preferences to explore their impact on the farmer's final choice. This exercise will give us a better understanding of the implications of the model for different types of individuals, as well as its implications for developing policy tools to encourage the adoption of long run technologies.

The proposed methodology includes the following steps:

Step 1: Define and measure stability and risk in the net returns path. This step is important because so far, stability and risk have been identified as two components of variability, but I have not developed a measurement device that can be used to empirically generate these two components.

Step 2: Develop a two-period GEU to explain adoption decisions. The result of this step will be a production alternative ranker that will contain the relevant variables – net returns, stability and risk - and the relevant parameters – stability preferences and risk aversion. The reduced form solution is analyzed in terms of its sensitivity with respect to the parameters and variables of interest to give a sense of the direction of the impact of the components of the model.

Step 3: Compare the traditional Expected Utility model to the GEU model proposed in Step 2. In comparing both approaches, five production alternatives with different net return patterns (different combinations of total net returns and stability) are ranked using the GEU and the EU model.

Step 4: The GEU model developed is analyzed by simulating different types of individuals. Each individual will have a different combination of risk (3 levels) and stability (5 levels) preferences. The combination generates 15 different types of individuals to be explored as shown in the following table:

Table 3.1. Simulated preference types

Low ← Stability preferences → High

↑ Low

Risk aversion

↓ High

Type 1: Low risk Low stab.	Type 2: Low risk Med-low stab.	Type 3: Low risk Moderate stab.	Type 4: Low risk Med-high stab.	Type 5: Low risk High stab.
Type 6: Med. Risk Low stab.	Type 7: Med. risk Med-low stab.	Type 8: Med. risk Moderate stab.	Type 9: Med. risk Med-high stab.	Type 10: Med. risk High stab.
Type 11: High risk Low stab.	Type 12: High risk Med-low stab.	Type 13: High risk Moderate stab.	Type 14: High risk Med-high stab.	Type 15: High risk High stab.

The objective of this section is to derive trade-off measures between net returns and stability. These measures will be presented through indifference curves, which provide an opportunity to estimate an elasticity of substitution between net returns and stability. After completing the exercise for the 15 types of individuals identified above, the trade-off across individuals with the same risk level will be compared, as well as the trade-off across individuals with the same degree of stability preference.

The trade-off measure allows to estimate the willingness to pay, or stability premium, for more stability for each type of individual. Finally, the trade-off between net returns and risk will be analyzed. The exercise will be performed for an even distribution of the net returns as well as an uneven distribution. The impact of a low versus a high risk level will be also examined.

Step 5: The two –period GEU model is extended to a multi-period GEU model and its reduced form solution is analyzed in terms of its sensitivity with respect to the parameters and variables of interest to give a sense of the direction of the impact of each component of the model.

Step 6: The multi–period GEU model developed is analyzed by simulating different types of individuals. The individual types are presented in Table 3.1. The objective of this section is to derive stability premiums that different types of individual would pay for an even distribution for the net returns. The stability premiums across individuals with the same risk level will be compared, as well as the premiums across individuals with the same degree of stability preference.

Figure 3.1 shows a graphic representation of the steps included in the research methodology explained above. The results and conclusions of each of the steps described above will be presented in the following three Chapters of this dissertation. Step 1 is related to the two-period as well as the multi-period model, and therefore it is covered in Chapter Four by itself. Chapter Five, will include the steps related to the development and analysis of the two-period model, steps 2 to 4, while Chapter Six will present steps 5 and 6.

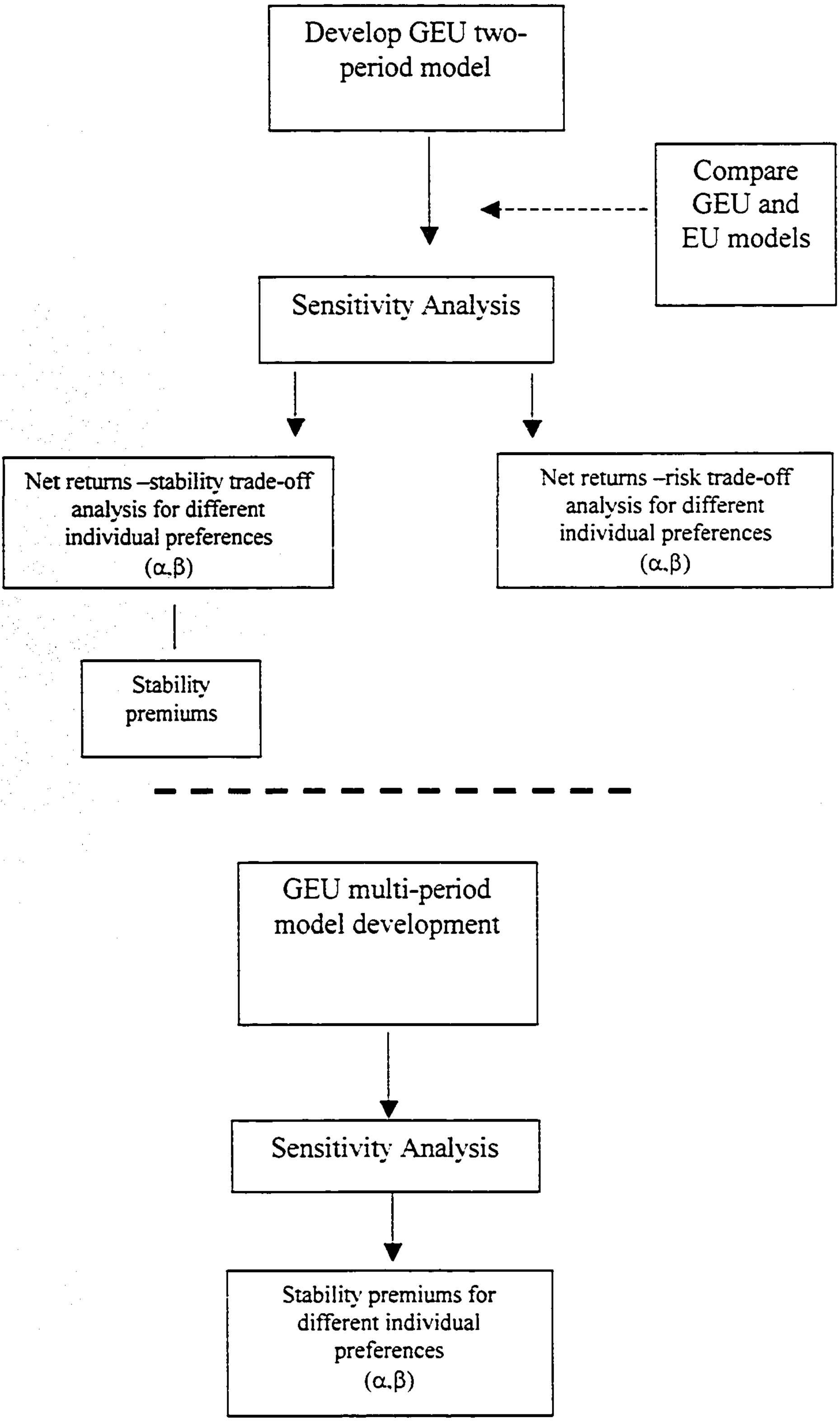

Figure 3.1. Steps included in the research development and analysis

CHAPTER FOUR

DISENTANGLING RISK FROM STABILITY

The first step in introducing independence in stability and risk is to define and measure them properly. An accurate prediction of farmers' behavior requires not only a good description of farmers' decision motives, but also an accurate definition of the joint distribution of income or net returns. If the net income representation does not accurately capture the stability and the risk component, prediction with respect to farmers' choices will be biased.

Past research has usually considered risk through the sample variance estimator, or by using a regression model from which the variance of the residual is a measure of risk. In both cases, there is a missing estimate of variability. In the first case, risk and stability are clearly confused in a single measure that does not represent either of them. In the second case, although the estimation of risk could be accurate, the analysis considers only the risk component, thereby overlooking the power that stability may have in the final decision. The difference between both variability components needs to be clear and adequately measured to capture a good prediction of the individual's choice.

To explain and define the difference between stability and risk let us consider the potential net income path for T periods of an investment alternative. The expected income is represented by I_t and it has been discounted in order to remove the time effect from the analysis. For further simplification, it will be assumed that I_t follows a Normal distribution such that the income stream $I_1, ..., I_T$, represented by the vector **I,** follows a multivariate Normal distribution of the form:

$$\mathbf{I} \sim MVN(\boldsymbol{\mu}, \mathbf{V})$$

Where μ is the Tx1 vector of expected values and V is the TxT variance covariance matrix associated to the random vector **I**.

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \cdots \\ \mu_T \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \sigma_1^2 & \text{cov}_{12} & .. & \text{cov}_{1T} \\ \text{cov}_{21} & \sigma_2^2 & .. & .. \\ .. & .. & .. & \text{cov}_{(T-1)T} \\ .. & .. & \text{cov}_{T(t-1)} & \sigma_T^2 \end{bmatrix}$$

The stability component is related to the variability of the expected values μ_t, while the risk associated to each outcome I_t is represented by σ^2_t. The covariance indicates the "creeping" effect of one period's risk into another period's risk, which is ignored here.

Figure 4.1 is an example of the net income path that could be observed for an investment alternative for a period of four years. The bars show the net income that is expected for each year. The normal distribution attached to the bars represents the stochastic nature of the net income, and the variance of each distribution is the risk associated with the yearly income.

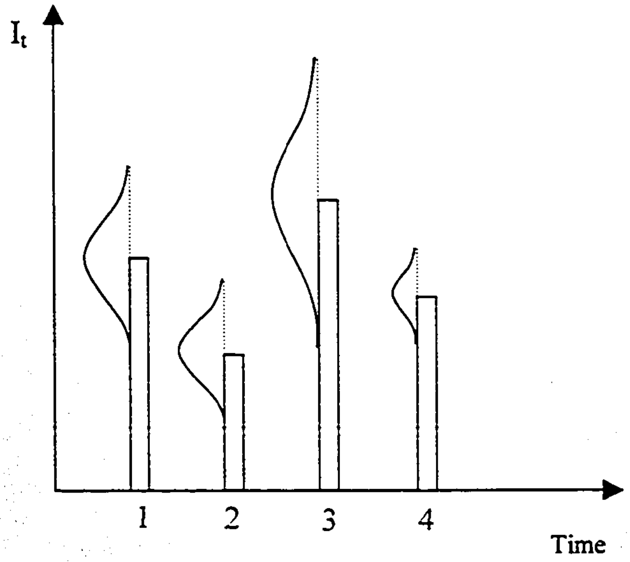

Figure 4.1. Example of net income path

Stabilitv Measure

To measure stability, I propose to use the average of the expected values over time as a reference point. Using the properties of the normal distribution, we can say that the average expected income across time, mean of μ_t, is calculated as follows:

$$\overline{\mu} = \frac{1}{T} * \sum_{t=1}^{T} \mu_t \qquad (4.1)$$

The expected values were previously discounted, therefore this average could be considered as the discounted average income for each alternative, equivalent to dividing the Net Present Value (PV) of the alternative by the number of years:

$$\overline{\mu} = \frac{PV}{T} \qquad (4.2)$$

The average of the net expected income is used as the reference point to estimate the deterministic stability of income (SI) for which the following index is proposed:

$$VI = \{2(1-\frac{1}{T}) - \frac{\sum_{t=1}^{T} \left|\mu_t - \overline{\mu}\right|}{\sum_{t=1}^{T} \mu_t}\} / 2(1-\frac{1}{T}) \qquad (4.3)$$

This stability index is a relative measure of stability that takes on values between 0 and 1, where 0 means perfect unevenness and 1 is perfect evenness. Figure 4.2 (a) shows the expected income pattern for an index SI = 1, and Figure 4.2 (b) and (c) show two alternative income paths for SI = 0.

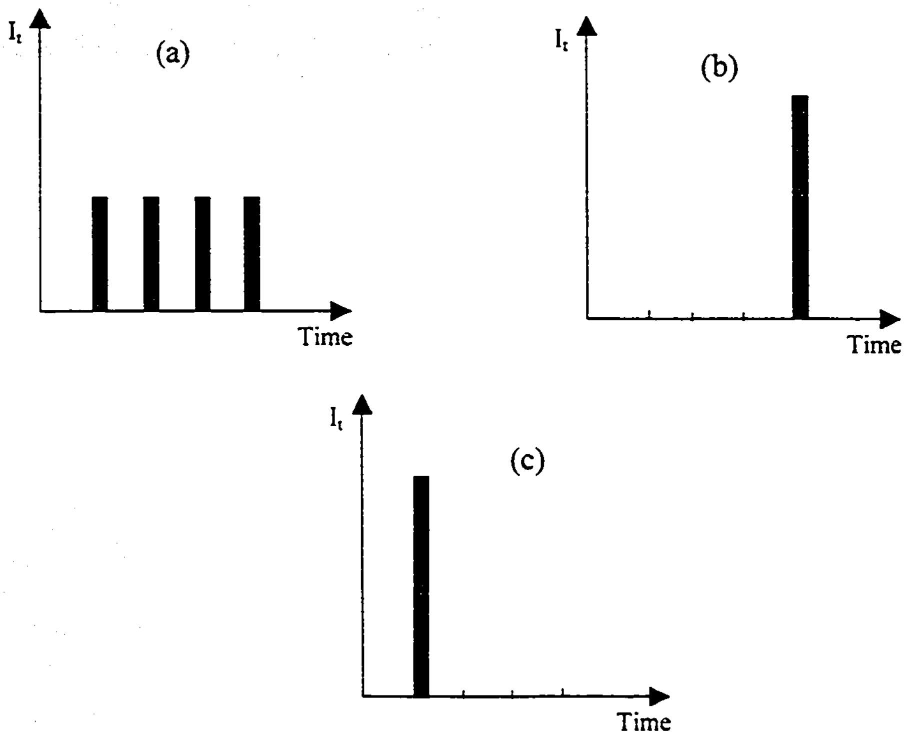

Figure 4.2. Patterns for different Stability Index

This index was adapted from the diversity index proposed by Önal (1997) to measure diversity of an ecosystem. The properties of the stability index are the following:

- The index can take values between 0 and 1.
- The index is zero for the most extreme pattern, where for all years, except for one, the profit is zero. On the other extreme, a perfectly even pattern takes an index value of 1.
- The higher the variability, the smaller the index.
- It is a relative measure and therefore different alternatives can be compared in terms of the index.

Risk Measure

The stochastic variability or risk is given in this model by the variance covariance matrix **V**. Since **V** has not been specified, the variance of the net income can take any form. It is simplest to assume that incomes across time are uncorrelated and have constant variance. A more complex assumption with respect to V is to consider correlation across years and that the variance of I_t is not constant. In general, this pattern implies a heteroskedastic process on the residual that can take endless forms. Some useful alternatives are presented below:

Alternative 1: Simple linear trend

$\sigma^2_t = \beta_0 + \beta_1 * t + \varepsilon_t$

Alternative 2: Autoregressive process

$\sigma^2_t = \beta_0 + \Sigma\ \beta i\ {}^*\sigma^2_{t-i}\ + \varepsilon_t$

Alternative 3: GARCH process (Generalized Autoregressive Conditional Heteroskedasticity)

$\sigma^2_t = \beta_0 + \Sigma\ \beta i\ {}^*\sigma^2_{t-i}\ + \Sigma\ \alpha i\ {}^*\varepsilon_{t-i}\ \varepsilon_t$

(Holt and Aradhyula, 1990)

The availability of data and empirical situation in question would define which variance to use. The implications of the choice will be apparent only when deriving results from the model and making predictions.

CHAPTER FIVE

TWO-PERIOD GENERALIZED EXPECTED UTILITY MODEL

This Chapter develops a simple two-period model to explain technology adoption decisions that accounts for independence between stability and risk preferences. The objective of the model is to show how these two components influence the individual choice for technologies with different distributions of net returns over time. The Chapter is divided in three sections. The first one develops a two-period GEU model applied to technology adoption decisions. The second section presents a traditional EU model to explain adoption with different distributions. At the end of the second section, GEU and the EU are models are compared, in order to demonstrate the differences between both approaches. Finally, the third section simulates different combinations of stability and risk preferences to determine the trade-off preference among net returns, stability, and risk for different types of individual preferences. The simulation section gives a better understanding about the influence of stability and risk preferences, the personal discount factor, and the risk level on the individual decisions. The aim of the exercise is to have a broad comprehension of the behavior of a decision-maker when deciding among enterprise alternatives with different net returns paths.

GEU Two-Period Model – Technology Adoption

The GEU framework is used in this section to explain the individual's behavior with respect to technology adoption decisions. As explained in Chapter One, I believe that adoption is influenced not only by risk and risk aversion, but also by the variability of the expected income and a preference for income stability. The GEU model can capture preferences for these two sources of variability, making them independent from each other. To develop the model, I first present the individual's preference in a GEU context using Selden's OCE representation. Then, I present the production alternatives that constitute the possibility frontier for the decision-maker. In a third part of this section, a graphic representation of the individual's optimization problem is presented, analyzing the optimal solution for different scenarios. Finally, the reduced form solution for this model is presented and analyzed in terms of its sensitivity with respect to the different parameters and variables, and in terms of the first order conditions (FOC) that guarantee an optimal adoption solution for the individual.

Individual's Preferences

The model considers a representative individual in an economy in which one source of utility exists: a single consumption good. The individual seeks to maximize his/her utility function across his/her lifetime, which is reduced to the current period 1 and the future period 2. The OCE representation used in this model, assumes that stability preferences are constant across time and represented by an aggregator function with a constant EIS of the form:

$$U(C_1, C_2) = -\frac{1}{\alpha}(C_1^{-\alpha} + \rho \cdot \hat{C}_2^{-\alpha}) \qquad (5.1)$$

U(•) is aggregating certain current consumption C_1 and the certainty equivalent of a random future consumption C_2 as used by Selden (1978). This utility index accounts for the individual's stability preference for a certain prospect[12]. "ρ" is, under certainty, the personal discount factor ($0< \rho <1$) and represents personal time preferences[13]. The parameter α ($-1 < \alpha < +\infty$) is related to the EIS as follows:

$$EIS = \frac{1}{\alpha + 1}$$

Graphically, the EIS defines the curvature of the utility index U(•). Lower EIS values (associated with higher α values) generate greater concavity in the index reflecting a low desire to substitute consumption across time. On the other hand, a high EIS implies a high willingness to substitute consumption across time and therefore represents a low degree of stability preferences. It is important to notice that the EIS is defined for the deterministic consumption path, since the preferences defined in 5.1 are expressed in terms of certainty equivalents[14].

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[12] In Chapter Two, page 20 it was explained how the OCE representation accounts separately for preferences for a certain prospect and uncertain prospect.

[13] It is important to notice that Selden (1978) and Kreps and Porteus (1978, 1979) incorrectly called stability preferences "time preferences". This name was abandoned later because it did not represent time preferences but preferences for substitutability across time. In this sense, "time preferences" is used here in its correct sense.

[14] At this point, there might be some confusion with respect to the role of the personal discount factor and the EIS. The personal discount factor reflects the issue that people weight the future less than the present time. An exposition of the possible reasons why people exhibit such behavior can be found in Trostel and Taylor (2001). On the other side, the EIS represents a different concept of the individual's behavior with respect to time. According to Hall (1988), the EIS corresponds literally to the elasticity of substitution of the consumption ratio to the corresponding relative prices.

Preferences with respect to the uncertain prospect have an exponential function with a constant Absolute Risk Aversion (ARA) coefficient:

$$u(C_t) = 1 - e^{-\beta \cdot C_t} \qquad (5.2)$$

Where β is the ARA coefficient.

Equation 5.2 is set equal to the certainty equivalent as follows:

$$E(e^{-\beta \cdot C_2}) = e^{-\beta \cdot \hat{C}_2} \qquad (5.3)$$

After some algebraic manipulations, the certainty equivalent of C_2 can be isolated and expressed as:

$$\hat{C}_2 = \frac{\ln E(e^{-\beta \cdot C_2})}{-\beta} \qquad (5.4)$$

Replacing expression 5.4 into equation 5.1, the utility index can be expressed as:

$$U(C_1, C_2) = -\frac{1}{\alpha}(C_1^{-\alpha} + \rho \cdot [\frac{\ln E(e^{-\beta \cdot C_2})}{-\beta}]^{-\alpha}) \qquad (5.5)$$

In this final equation, the preference representation for risk and intertemporal substitution (stability preferences) are independent from each other, which is necessary to assure that variability is disentangled properly as two different attributes in the individual's taste (Weil, 1990).

Production Alternatives

After defining and explaining the objective function of the model, it is helpful to present the alternatives that will define the production possibility frontier for an individual. The central assumption of the model is that an individual has to select an alternative among an infinite set of possible productive alternatives. Each alternative has the same expected present value (PV), but each differs in the distribution of the net returns across time. Given these characteristics, the net returns for period 1 and 2, I_1 and I_2, can be represented by:

$$\begin{aligned} I_1 &= PV \cdot b \\ I_2 &= (PV \cdot (1-b) + \varepsilon) \cdot \frac{1}{r} \end{aligned} \qquad (5.6)$$

PV is the present value of the infinite alternatives, b is the portion of present value earned in period 1, r is the market discount factor[15], and ε is the stochastic component of the net income in period two. For simplicity it is assumed that $\varepsilon \sim N(0,\sigma^2)$.

In this model the vector $[I_1, I_2]$ defines the technology variable. This vector describes the net present value of the alternative, the distribution of the cash flow across time, and the risk level involved; therefore, it presents all the characteristics that are

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[15] The market discount factor is capitalizing period two net income and transforming it into current terms. Remember that r = 1/(1+i), where i is the market return rate, therefore 1/r = (1+i) the capitalization factor. This step is necessary, since in equation (5.5) consumption is expressed in current terms.

relevant[16] when deciding which technology to adopt. Equation 5.6 has both components of variability clearly defined. On one side, risk is represented by σ^2; on the other side, the expected variability is represented by "b". For this simple two-period model the parameter b will define the relative stability of the income path. Using this parameter, the stability index[17] presented in Chapter Four is calculated as follows[18]:

$$SI = 1 - \frac{|b - 1/2|}{1/2}$$

Graphic Representation of the Optimization Problem

The production alternatives and the utility index presented in the previous sections constitute two elements of the individual's optimization problem. To have a better understanding of the individual's alternatives and possible choices, let us consider a graphic exposition of the production alternatives as a production possibility frontier. To construct the frontier, consider that an individual may choose the distribution of the PV between period 1 and 2, as described in the introduction. For this purpose let us assume that PV = 100 monetary units and that the market discount factor r = 0.95. Assume also that the risk level is 0 in order to keep the scenario simple.

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[16] It is important to notice that only the economic aspect of the technology is being modeled. Therefore, when referring to the "relevant characteristics" only this constrained perspective is considered.

[17] In this simple two-period model the stability index introduced earlier is not as useful as in a multiperiod model in which the distribution of the income path would be defined by more than one parameter. For simplicity, in the further development of this two-period model the parameter b will be used to indicate stability instead of the stability index.

[18] It is important to notice that SI is calculated for the net income path in present terms rather than in current terms.

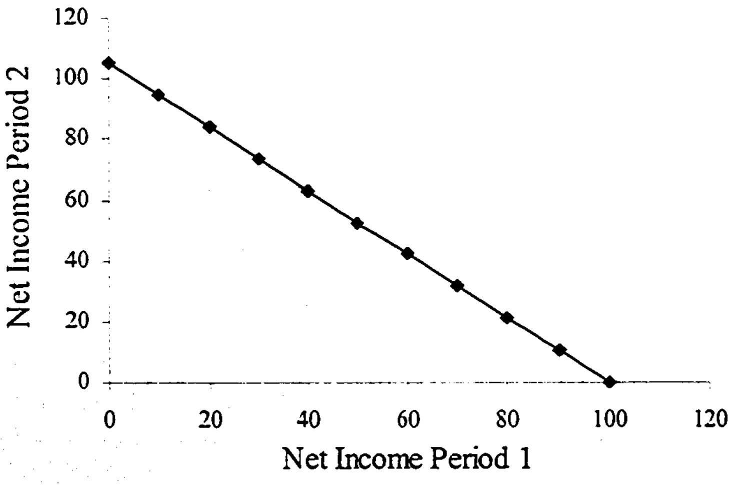

Figure 5.1. Production possibility frontier

As shown in Figure 5.1, equation 5.6 describes a linear possibility frontier with a slope equal to – 1/r (the negative of the inverse of the market discount factor). The PV can be allocated between period 1 and 2 in infinite ways, and the allocation variable is "b" in equation 5.6.

The individual's preferences follow a Constant Elasticity of Substitution (CES) of the form shown in equation 5.1. Depending on the value of α, which defines EIS, this function will take different shapes[19] that would affect the optimal allocation of the PV between period 1 and 2. The limits for α were established as -1 and $+\infty$. These limits define the two extreme cases for this function. At one extreme, when $\alpha = -1$ the CES function corresponds to the case of perfect substitutes, for which equation 5.1 is reduced to: $U = C_1 + \rho \cdot C_2$. At the other extreme, when $\alpha \rightarrow +\infty$ The CES function takes the form of fixed proportions, for which the utility index is represented by: $U = \min(C_1, \rho \cdot C_2)$. The

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[19] See Nicholson (1998) pages 89 and 90 for an explanation of this particular CES function.

implications of these two extreme cases on the optimal allocation can be better explained through Figures 5.2 and 5.3. The solutions showed in these figures make an important assumption: that the individual can not borrow from one period to another, making consumption in period 1 equal to the net income in period 1 and consumption in period 2 equal to net income in period 2.

If consumption in periods 1 and 2 are perfect substitutes, or in other words, if the EIS $\rightarrow \infty$, an individual is not motivated to smooth consumption or income over time. In this case, as shown in Figure 5.2, the indifference curve is a straight line, with a slope equal to $-1/\rho$ (the negative of the inverse of the personal discount factor). Since the possibility frontier also was shown to be a straight line with a slope equal to $-1/r$, the optimal solution is a corner solution defined by the difference between the market and the personal discount factor. If $r > \rho$, then the individual will allocate all the PV in period 2 ($b = 0$), as shown in Figure 5.2. If $r < \rho$, then the individual will allocate all the PV in period 1 ($b=1$). Finally, if $r = \rho$, then the individual would be indifferent among any of the infinite alternatives presented by the possibility frontier. This solution implies that the individual's criteria to choose among different alternatives is the present value PV, as it is considered in the traditional Cost-Benefit analysis, for which the Net Present Value (NPV) and the Internal Rate of Return (IIR) are the relevant criteria for investment decisions.

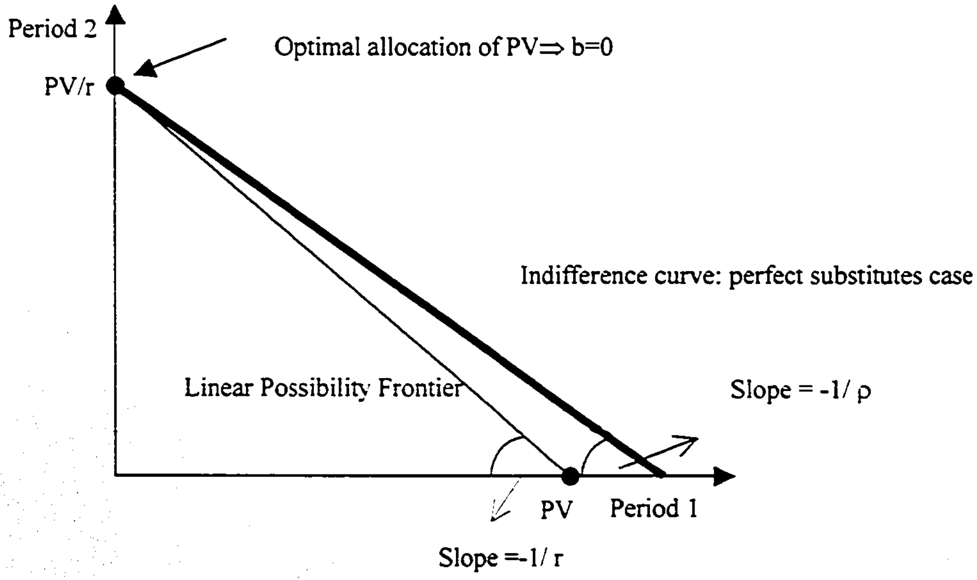

Figure 5.2. Optimal allocation: perfect substitutes case

The case of perfect complements is the other extreme, as shown in Figure 5.3. The case of perfect complements is equivalent to an EIS = 0, in which the indifference curves take the form of fixed proportions. The allocation of the PV will be defined by the proportion determined by $(C_1, \rho{\cdot}C_2)$. The relation between period 1 and period 2 is $1/\rho$, leading to an even allocation across time. Moreover, if it is assumed that $r = \rho$, then the optimal distribution of PV is perfectly even, meaning that $b = 0.5$.

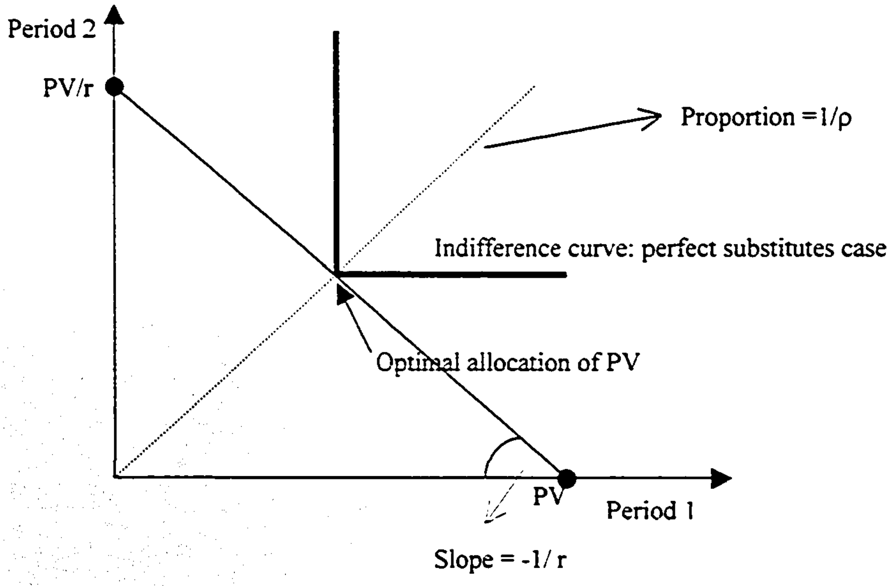

Figure 5.3. Optimal allocation: perfect complements case

The extreme cases presented above showed two types of individuals. The first type (EIS → ∞) does not have as a personal objective to smooth income over time, seeking only a maximum lifetime PV. This individual is not willing to trade PV for a more even income path. On the other extreme, an individual with an EIS = 0 has as a personal objective to have a perfectly even income path, and is not willing to trade more PV for a less even income path. Our typical individual is probably in a situation in between these two extreme cases, seeking to accomplish two objectives when making the choice of which technology to adopt: "between the highest PV possible and the most even income path possible". Since, both components, PV and stability, are part of the individual's objective, then it will be possible to define some degree of trade-off between the variables PV and b that will depend upon his/her preferences for stability represented by the parameter α in equation 5.1.

The next section will present a mathematical representation of the optimization problem given the individual's preferences and production alternatives presented earlier.

Optimization problem

The previous section showed a graphical representation of the optimization problem. The individual's preferences were simulated using the extreme cases of the given functional form. The production possibility frontier was presented with income in period 1 and period 2, given a constant present value, a flexible allocation across time, and no risk. The following mathematical representation presents the same optimization problem with present value, stability and risk as variables.

Mathematical representation of the optimization problem:

Equation 5.7 presents the optimization problem that combines the individual's preferences defined in equation 5.5, and the production alternatives defined in equation 5.6:

$$U(C_1,C_2) = -\frac{1}{\alpha}(C_1^{-\alpha} + \rho \cdot [\frac{\ln E(e^{-\beta \cdot C_2})}{-\beta}]^{-\alpha}) \qquad (5.7)$$

$$\begin{aligned} C_1 &= I_1 = PV \cdot b \\ C_2 &= I_2 = (PV \cdot (1-b) + \varepsilon) \cdot \frac{1}{r} \end{aligned} \qquad (5.7a)$$

In this simple model, it is assumed that the individual is completely constrained by the income he/she can generate and cannot use capital markets to change the distribution of his/her income between period 1 and 2. Knowing that $C_1 = I_1$ and $C_2 = I_2$, the problem can be rewritten as:

$$U(PV,b,\varepsilon) = -\frac{1}{\alpha}\{(PV \cdot b)^{-\alpha} + \rho \cdot [\frac{\ln E(e^{-\beta \cdot \frac{1}{r} \cdot (PV \cdot (1-b)+\varepsilon)})}{-\beta}]^{-\alpha}\} \qquad (5.8)$$

In order to obtain a solution of equation 5.8 that can be analyzed algebraically, the first step is to solve for the certainty equivalent in period 2. The exponential representation of risk preferences involves the expectation of a non-linear function. A closed form of this expectation can be obtained by inspection[20] (See Appendix II). After taking the expectation and some algebraic manipulations, the certainty equivalent is reduced to the following expression:

$$\hat{I}_2 = \frac{1}{r}(PV \cdot (1-b) - \tfrac{1}{2} \cdot \frac{\beta}{r} \cdot \sigma^2) \qquad (5.9)$$

where $\hat{I}_2$ is the certainty equivalent of I_2[21]. Replacing the certainty equivalent $\hat{I}_2$ in equation 5.8, I obtain:

$$U(PV,b,\sigma^2) = -\frac{1}{\alpha}\{(PV \cdot b)^{-\alpha} + \frac{\rho}{r} \cdot [PV \cdot (1-b) - \tfrac{1}{2} \cdot \frac{\beta}{r} \cdot \sigma^2]^{-\alpha}\} \qquad (5.10)$$

---

[20] The function for which the expectation is needed is similar to the Moment Generating Function (MGF) for a Normal distribution. Therefore, the same inspection procedure used to obtain the closed form for the MGF can be applied to the certainty equivalent function.

[21] It can be noted that the certainty equivalent expression found is the same as the result that could have been obtained using a second order Taylor approximation. This could be an interesting alternative if the assumption of normality in income does not hold.

Equation 5.10 presents a simple function that can be analyzed in terms of the variables and parameters determining the individual's satisfaction level. Generically, the satisfaction level (utility) is given by:

$$Utility = U(PV, b, \sigma^2 : \alpha, \beta)$$

The function shows that the utility depends on three variables PV, b, and σ (present value, stability level and risk level) and on two preference parameters α and β (stability preference and risk preference respectively). Three variables create a three dimensional plane to define preferences, in which the preferences parameters will define the degree of convexity of the preference surface. To explore the preference surface, we can constrain one or two of the variables to remain constant and investigate the impact of one variable on the individual's satisfaction, or the trade-off between two variables. The following are some combinations that can be explored:

$$Utility = U(PV : \overline{b}, \overline{\sigma}^2, \alpha, \beta)$$
$$Utility = U(b : \overline{PV}, \overline{\sigma}^2, \alpha, \beta)$$
$$Utility = U(\sigma^2 : \overline{PV}, b, \alpha, \beta)$$
$$PV = f(b : \overline{\sigma}^2, \alpha, \beta, \overline{U})$$
$$PV = f(\sigma^2 : \overline{b}, \alpha, \beta, \overline{U})$$
$$b = f(\sigma^2 : \overline{PV}, \alpha, \beta, \overline{U})$$

In the Sensitivity Analysis section, I will present the results from the Present Value – stability and the Present Value –risk trade-off relationships. As an example, and to follow the graphical example presented in the previous section, I develop here the mathematical analysis of the impact of the stability level (b) on the individual's utility. The analysis assumes that the PV and risk are constant following the expression:

$$Utility = U(b : \overline{PV}, \overline{\sigma}^2, \alpha, \beta)$$

As previously mentioned, the objective of the model was to determine the choice of the individual among alternatives with different distributions. According to how the production alternatives were defined, the variable to be optimized in equation (5.10) is b.

Optimizing utility with respect to b, given all other variables and parameters constant, implies that:

$$\frac{\partial U(b : \overline{PV}, \overline{\sigma}^2, \alpha, \beta)}{\partial b} = 0$$

For which the solution is:

$$(PV \cdot b)^{-(1+\alpha)} = (\frac{\rho}{r})^{-\frac{1}{1+\alpha}} \cdot (PV - \frac{\beta}{2 \cdot r} \cdot \sigma^2)^{-(1+\alpha)} \qquad (5.11)$$

The terms on each side of equation 5.11 can be viewed as the marginal utility of income for each period. In that sense, the interpretation of this result is that the individual finds the optimal distribution of PV when the marginal utility of the current income equates to the discounted marginal utility of the risk-adjusted future income. Solving equation 5.11 for b yields the following expression:

$$b = \frac{(\frac{\rho}{r})^{-\frac{1}{(1+\alpha)}} \cdot (PV - \frac{\beta}{2 \cdot r} \cdot \sigma^2)}{PV \cdot (1 + (\frac{\rho}{r})^{-\frac{1}{(1+\alpha)}})} \qquad (5.12)$$

If ρ = r, the following is the optimal solution for b:

$$b = \frac{1}{2} - \frac{\frac{\beta}{2 \cdot r} \cdot \sigma^2}{2 \cdot PV} \qquad (5.13)$$

Expression 5.13 shows that the optimal solution for b does not depend on the preferences for stability, or in other words does not depend on α. The optimal production alternative has a perfectly even distribution adjusted for the level of risk and risk aversion. If ρ = r and $\sigma^2 = 0$, then the optimal distribution of the PV implies b = 0.5, or in other words, the optimal distribution is perfectly even.

The optimal solution may be surprising at first, but becomes clear after further investigation. According to the GEU model, it would have been expected that the optimal distribution of income depended on the parameter α; however, the final solution for b shows that regardless of the stability preferences, the optimal distribution is an even distribution. The reason is that the utility index that defined the stability preferences is concave with respect to consumption (or income after solving the model) implying that the individual, regardless the level of α, is always averse to any variability. Therefore, if b is the only variable that has to be chosen, an even pattern will always be preferred. In this sense, α only defines the degree of aversion to expected variability (or the degree of stability preferences) and it would influence the individual's choice when they are offered a set of alternatives that show a trade-off between PV and distribution of the PV[22].

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[22] Later in this Chapter, I present a complete analysis of the trade-off between PV and distribution for different degrees of stability preferences.

EU Two-period Model – Technology Adoption

In this section, I develop a model to explain technology adoption decisions using the traditional Expected Utility (EU) framework. The objective of this exercise is to show how different representations of preferences affect an individual's choice of productive alternatives. After developing the EU model, I compare it to the GEU model. This comparison consists of ranking 5 production alternatives with different net income paths. This section is developed as follows: The individual's preferences under the EU approach are presented, the set of production alternatives is defined, and the reduced form solution for the individual's utility is presented. The last part is dedicated to the comparison of the GEU and EU models.

Individual's Preferences

As before, I assume that there is a representative individual in an economy where one source of utility exists: a single consumption good. The individual seeks to maximize his/her utility function across his/her lifetime, which is reduced to the current period (1) and the future period (2). The utility index for random consumption C_1 and C_2 is additive in time of the form[23]:

$$U(C_1, C_2) = E(u(C_1) + \rho \cdot u(C_2)) \qquad (5.16)$$

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[23] To construct the EU model, the typical representation was followed. See for example Chavas and Holt (1996) and Rozenzweig et. al. (1993).

E is the expected operator and "ρ" (0 <ρ < 1) is the personal discount factor. As in the GEU model, the individual's risk preferences are assumed exponential with a constant Absolute Risk Aversion (ARA) parameter β:

$$u(C_t) = 1 - e^{-\beta \cdot C_t} \tag{5.17}$$

Substituting equation 5.17 in equation 5.16 yields 5.18:

$$U(C_1, C_2) = E[(1 - e^{-\beta \cdot C_1}) + \rho \cdot (1 - e^{-\beta \cdot C_2})] \tag{5.18}$$

Using the properties of the Expected operator, 5.18 can be rewritten as:

$$U(C_1, C_2) = (1 + \rho) - E(e^{-\beta \cdot C_1}) - \rho \cdot E(e^{-\beta \cdot C_2}) \tag{5.19}$$

Equation 5.19 represents the new form of preferences for our representative individual.

Production Alternatives

The set of production alternatives is defined (as in the GEU model) in equation 5.6 as follows:

$$\begin{aligned} I_1 &= PV \cdot b \\ I_2 &= (PV \cdot (1 - b) + \varepsilon) \cdot \frac{1}{r} \end{aligned} \tag{5.20}$$

I_1 and I_2 are net income for period 1 and 2. PV is the present value of the infinite alternatives, b is the portion of present value earned in period 1, r is the market discount factor[24], and ε is the stochastic component of the net income in period two. As before, it is assumed that $\varepsilon \sim N(0,\sigma^2)$.

Solution for Utility Index:

Again, the individual seeks to maximize his/her utility subject to the set of productive alternatives. Using equations 5.19 and 5.20, the model can be stated as follows:

$$U(C_1,C_2) = (1+\rho) - E(e^{-\beta \cdot C_1}) - \rho \cdot E(e^{-\beta \cdot C_2})$$

$$C_1 = I_1 = PV \cdot b \tag{5.21}$$

$$C_2 = I_2 = (PV \cdot (1-b) + \varepsilon) \cdot \frac{1}{r}$$

After solving the expectation the solution of the equation is the following:

$$U(PV,b,\sigma^2) = (1+\rho) - (e^{-\beta \cdot PV \cdot b}) - \rho \cdot (e^{-\frac{\beta}{r}(PV \cdot (1-b) - \frac{\beta}{2 \cdot r} \cdot \sigma^2)}) \tag{5.22}$$

As explained in Chapter Two, equation 5.22 is the solution of the preferences after replacing the restriction into the objective function. In this expression, the individual has risk aversion and stability preferences that are inversely related. In this setting, a high risk aversion level is equivalent to a low EIS, and therefore, high stability preferences.

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[24] The market discount factor is capitalizing period two net income and transforming it to current terms. This step is necessary, since equation (5.19) consumption is expressed in current terms.

On the other hand, a low risk aversion is equivalent to a high EIS and low stability preferences.

A combination of low risk aversion and high stability preference, or any other combination that does not fit into a specific inverse relationship between both components of preferences, is not allowed for this individual.

Comparing GEU and EU Representations

Five different income paths are compared under the GEU and EU models developed in the previous sections. In order to give to the exercise the necessary variety, the productive alternatives that yield different combinations of PV and distributions that range from high PV and low stability to low PV and high stability. The two variables used to characterize each income path are PV, and the variable b from equation 5.6 as a measure of stability. According to this characterization, five income alternatives were developed for examination as shown in Table 5.1:

Table 5.1. Production alternatives considered to compare the GEU and EU models

Alternative	Scenario[a]	Present Value	b – stability
1	LPPS	200 m.u.[b]	0.5
2	LPMU	200 m.u.	0.7
3	MPU	300 m.u.	0.1
4	HPHU	500 m.u.	0.9
5	VHPVHU	700 m.u.	0.99

[a] LPPS= low present value-perfect stability, LPMU=low present value-moderate stability, MPU=moderate present value-unstable, HPHU=high present value-highly unstable, VHPVHU=very high present value-very highly unstable.
[b] m.u. = monetary units.

The comparison of the five alternatives presented above was done through the reduced form solutions obtained for the GEU and EU models presented in equations 5.10 and 5.22.

GEU reduced form solution from equation 5.10:

$$U(PV,b,\sigma^2) = -\frac{1}{\alpha}\{(PV \cdot b)^{-\alpha} + \frac{\rho}{r} \cdot [PV \cdot (1-b) - \tfrac{1}{2} \cdot \frac{\beta}{r} \cdot \sigma^2]^{-\alpha}\}$$

EU reduced form solution from equation 5.22:

$$U(PV,b,\sigma^2) = (1+\rho) - (e^{-\beta \cdot PV \cdot b}) - \rho \cdot (e^{-\frac{\beta}{r}(PV \cdot (1-b) - \frac{\beta}{2 \cdot r} \cdot \sigma^2)})$$

The resulting ranking will give an indication of the ability of each model to capture different aspects of the individual's preferences. For this purpose, different levels of risk aversion and stability preferences were considered. Three levels of risk aversion - low, moderate, and high - and two levels of stability preferences - low and high were used.

The levels of stability preferences were chosen using citations that mention likely values for EIS. Hall (1988) suggested that the EIS in general should be low (which is equivalent to having high stability preference) and close to 0.1, which would be equivalent to considering a value for α equal to 10. I chose a high value for EIS (or low stability preferences) of α equal to -0.9 (EIS = 10) to generate enough contrast for comparison purposes.

To determine the value of β, the Absolute Risk Aversion (ARA) parameter, a more complicated strategy was necessary. The literature, and in particular the agricultural economics literature, is full of estimations of risk aversion parameters. See Antle (1987), Bosch et.al. (1987), Bernardo (1988), Thompson et.al. (1990), and Saha et.al. (1994). However, it is well known that the estimate is highly sensitive to the kind of risk aversion parameter that is being estimated (relative or absolute), the functional form used for the utility function, the level of the argument used for the utility function, and the estimation technique. Consequently, risk aversion estimates used in other studies may not be compatible for the scenario that is used in this exercise.

I estimated reasonable values for β by relating it to the risk premium, which can be found from a generalized version of the certainty equivalent presented in equation 5.9:

$$\hat{I} = E(I) - \tfrac{1}{2} \cdot \beta \cdot \sigma^2 \qquad (5.23)$$

The formula has been generalized to be applicable to any situation. E(I) is the expected value of the random income I, β is the Absolute Risk Aversion (ARA) parameter that comes from and exponential form of risk preferences as presented in equation 5.2, and σ^2 is the variance of the random income I. The second component of the right hand side is the risk premium, and is the element used to estimate values for β consistent with low, moderate and high levels of risk aversion.

Jullien and Salanie (2000) estimated preferences for risky prospects under different functional forms using racetrack bettors. For a traditional EU model using exponential risk preferences, they estimated a risk premium equal to 2.8% of the standard deviation (σ) of the gamble. Therefore, a $\sigma = 10$ would yield a β equal to 0.0056. Because of the characteristics of the group under study, this estimate of β is considered a low level of risk aversion. Consequently, the moderate and high levels of risk aversion were estimated using higher percentages of risk premiums. For moderate risk aversion, I used a risk premium of 5% of σ, leading to a β value of 0.01, and for the high risk aversion level I used a β value of 0.02, which correspond to a premium of 10% of σ[25]. Finally, to complete the set of assumptions necessary to rank the income alternatives, I considered a σ^2 equal to 100 for all the alternatives in analysis,[26] and a market and personal discount factor equal to 0.95 ($r = \rho = 0.95$).

The results of the comparison exercise are presented in table 5.2:

Table 5.2. Rankings under the GEU and EU models at different levels of stability preferences and risk aversion

	GEU Ranking		**EU Ranking**		
	High stability $\alpha=10$ (all levels of risk aversion)	**Low stability $\alpha=-0.9$ (all levels of risk aversion)**	**Low risk aversion $\beta=0.0056$**	**Moderate risk aversion $\beta=0.01$**	**High risk aversion $\beta=0.02$**
1st alternative	LPPS[a]	HPHU	HPHU	HPHU	LPPS
2nd alternative	LPMU	MPU	VHPVHU	LPPS	LPMU
3rd alternative	HPHU	VHPVHU	MPU	LPMU	HPHU
4th alternative	MPU	LPPS	LPPS	MPU	MPU
5th alternative	VHPVHU	LPMU	LPMU	VHPVHU	VHPVHU

[a] LPPS= low present value-perfect stability, LPMU=low present value-moderate stability, MPU=moderate present value-unstable, HPHU=high present value-highly unstable, VHPVHU=very high present value-very highly unstable.

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[25] These levels of risk aversion were tested under the EU model and they appear to provide reasonable results.

[26] A σ^2 that varies according to the expected value of income in year two was also considered. It generated the same ranking shown in Table 5.2.

The ranking generated by the GEU model using an α value of 10 is consistent with the choice of an individual with a very high concern about stability. In this case, the most preferred income path has the lowest PV, and most even income distribution, implying that the individual is willing to trade a high amount of income for a better distribution. The lowest alternative in this ranking is the alternative with the highest PV and worst distribution. This confirms the high value that stability has in this type of individual's preference. At the other extreme, the ranking generated by the GEU model using a low stability preference ($\alpha = -0.9$) is consistent with a low willingness to trade PV for more stability. The most preferred alternative in this case has a high PV and a low stability, implying that this individual is willing to trade some stability for a higher PV. The two worst alternatives for this type of individual present the lowest PV (PV = 200 u.m.).

The rankings generated by the EU model under different levels of risk aversion reveal that the risk aversion coefficient has a similar effect to the EIS in the GEU model. In Chapter 2 it was explained in detail how, in the traditional EU framework, the EIS was constrained to be the reciprocal of the Relative Risk Aversion (RRA) parameter. The exercise proved that in the EU models, the parameter β would play a more important role, discriminating across time rather than by risk level. The ranking generated by a high risk aversion parameter ($\beta = 0.02$) is the same as the ranking generated by the GEU model using an α level equal to 10 (low EIS). On the other hand, the EU model with a low level of risk aversion ($\beta = 0.0056$) gives a similar ranking to the GEU model that uses an $\alpha = -0.9$.

The intention of this simple exercise is to establish the difference between the GEU and the EU approaches. The results however, tell as much about the similarities between both approaches too. The EU model yields the same rankings as the GEU model when choosing the appropriate level of risk aversion. For a low level of risk aversion, it gives a ranking consistent with a GEU model with low EIS. For a high level of risk aversion, it gives a ranking consistent with a GEU model with high EIS. These results lead to the conclusion that in an intertemporal EU model, the risk aversion parameter is evaluating alternatives for the deterministic and stochastic component rather than the stochastic variability only. In other words, β is more a "variability" aversion parameter than a risk aversion parameter. This conclusion supports the need for a model that truly reflects risk aversion and stability preferences independently and by unique parameters. This independence not only allows researchers to analyze a more broad set of individual types, with combinations of stability and risk aversion not permitted by the EU approach, but also allows us to properly identify stability preferences with the stability of the deterministic income path, and risk preferences with the stochastic component of the income.

In the next section, a sensitivity analysis of the GEU model is performed. The section intends to explore the two- period GEU model in depth to give us a sense about the different levels of trade-off between stability and PV that can be generated by having different levels of stability preferences. The analysis will give a better understanding of the adoption model developed here as well as the premiums that different types of individuals are willing to pay for more stability.

Sensitivity Analysis

As explained in Chapter Three, the sensitivity analysis consists of analyzing the two-period GEU model by simulating different types of individuals. The main purpose of this analysis is to understand and measure the impact of different characteristics on the individual's behavior with respect to decisions that include alternatives with different income paths (stability and risk). The ultimate goal is to use this information to construct financial instruments that are attractive to individuals' preferences, and that will encourage them to adopt long run technologies when they are socially preferred.

Fifteen different types of individuals with different combinations of risk (3 levels) and stability (5 levels) preferences are analyzed in this section. The degrees of risk preferences used in the exercise are rated as low, moderate, and high. The values of the Absolute Risk Aversion (ARA) parameter β assigned to each level of risk aversion are the same that were used in the section that compares the GEU to the EU model:

Low level of risk aversion $\longrightarrow$ $\beta = 0.0056$

Moderate level of risk aversion $\longrightarrow$ $\beta = 0.01$

High level of risk aversion $\longrightarrow$ $\beta = 0.02$

The five levels of stability preferences were chosen from the findings of Hall (1988). According to the author, estimates of the EIS over the postwar period are quite precise and support the conclusion that the EIS is low (which implies that stability preference is high) and unlikely to be much higher than 0.1, or $\alpha = 10$. The other four levels of stability preferences were chosen to match different degrees of low and moderate levels of stability preferences. The following are the selected values:

Low level of stability preferences → $\alpha = 0.5$ (EIS = 0.67)
Moderate level of stability preferences → $\alpha = 2$ (EIS = 0.33)
Moderate level of stability preferences → $\alpha = 5$ (EIS = 0.16)
High level of stability preferences → $\alpha = 10$ (EIS = 0.09)
High level of stability preferences → $\alpha = 50$ (EIS = 0.01)

Combining the different levels of stability and risk preferences yields fifteen types of individuals as shown is Table 5.3:

Table 5.3. Simulated preferences types for the Present Value –stability analysis

Low ← Stability preferences → High

Low ↑ Risk aversion ↓ High

Type 1: $\beta = 0.0056$ $\alpha = 0.5$	Type 2: $\beta = 0.0056$ $\alpha =2$	Type 3: $\beta = 0.0056$ $\alpha =5$	Type 4: $\beta = 0.0056$ $\alpha =10$	Type 5: $\beta = 0.0056$ $\alpha = 50$
Type 6: $\beta = 0.01$ $\alpha = 0.5$	Type 7: $\beta = 0.01$ $\alpha =2$	Type 8: $\beta = 0.01$ $\alpha =5$	Type 9: $\beta = 0.01$ $\alpha =10$	Type 10: $\beta = 0.01$ $\alpha = 50$
Type 11: $\beta = 0.02$ $\alpha = 0.5$	Type 12: $\beta = 0.02$ $\alpha =2$	Type 13: $\beta = 0.02$ $\alpha =5$	Type 14: $\beta = 0.02$ $\alpha =10$	Type 5: $\beta = 0.02$ $\alpha = 50$

The first part of this section is dedicated to deriving trade-off measures between the PV and stability for the different types of individuals. After completing the exercise for the 15 types of individuals identified above, the trade-off across individuals with the same risk aversion level will be compared, as well as the trade-off across individuals with the same degree of stability preference.

In the second part of this section, the willingness to pay or premium for more stability is estimated for each type of individual. Finally, a third part will explore the trade-off between risk and PV. Only nine types of individuals are necessary for comparison. In this case, the combinations resulting from the three levels of risk aversion ($\beta = 0.0056$, $\beta = 0.01$, and $\beta = 0.02$) and stability preferences represented by $\alpha = 0.5$, $\alpha = 5$, and $\alpha = 10$.

PV – stability Trade-off Analysis

The first step in determining the trade-off between PV and stability is to generate the isoutility function (indifference function) between PV and stability (measured by b). This isoutlity function shows us combinations of PV and stability that yield the same utility level. Equation 5.10 defined the reduced form solution of the utility index, and could be used to generate the isoutility functions by isolating the variable PV in terms of all the other parameters and variables. The complexity of the functional form in equation 5.10 made the derivation of a closed form solution for the isoutility function difficult. A second best solution in this case is to use an analytical method to generate the isoutility functions. The procedure consists of estimating the utility level for specific preferences, levels of PV, stability in income and risk level, and tracing the curve out by hand [27].

For illustrative purposes, the steps followed to obtain the final isoutility function are detailed using the individual type 8 ($\alpha = 5$, $\beta = 0.01$). The other parameters and variables used to determine the initial utility level are: $\rho = r = 0.95$, $\sigma^2 = 100$ m.u., the initial PV = 100 m.u., and the initial b = 0.5, which defines a perfectly even distribution of the PV between period 1 and 2. This initial value gives us the starting point to generate the combinations of PV and b that conform to the isoutility curve. PV was increased successively by 10 units, until reaching 250 m.u. The isoutility is presented in Table 5.4 and graphed in Figure 5.4. The table and graph show that the model behaves as expected. The pairs of PV and b present a clear decreasing trade-off between both variables, following the law of diminishing marginal utility. The result of this exercise is clearly a good start for the trade-off analysis; however, it only gives us indications with respect to the direction of the trade-off. In order to have a more flexible tool to analyze the behavior of the two–period GEU model, I needed an explicit function for the isoutility curve. The data collection presented in Table 5.4 will be used to generate this function.

In generating the isoutlity function, the estimators for statistical linear models are used. Specifically, regressions were run using Ordinary Least Squares (OLS) estimators. The objective of this exercise was to find the function that generates the best fit with the data presented in Table 5.4. OLS estimators fill the requisite in a sense that they minimize the sum of the errors between the estimated and the real values.

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[27] To obtain pairs of PV and b that yield the same utility level, the PV is increased sequentially. The tool "Goal Seek" in Excel was used to generate the pairs of PV and b.

Table 5.4. Combinations of PV and b for preference type 8

PV	B
100	0.500
110	0.405
120	0.365
130	0.335
140	0.310
150	0.289
160	0.271
170	0.255
180	0.241
190	0.228
200	0.217
210	0.206
220	0.197
230	0.188
240	0.180
250	0.173

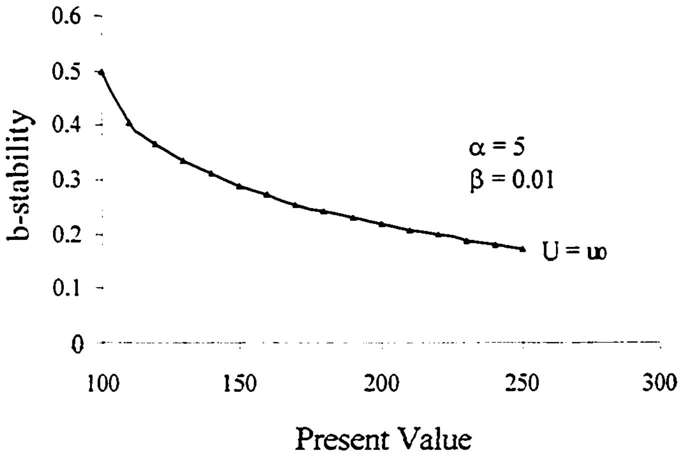

Figure 5.4. Isoutility curve for an individual with moderate stability and risk preferences (α = 5, β = 0.01)

Different functional forms were tested. The function's choice was based on graphical inspection of the fitted and actual curves, and on the R^2 as a measure of goodness of fit. For the individual type 8, the isoutility function that present best his/her trade-off between PV and b is the following:

$$PV = 8.41 + \frac{41.52}{b} \qquad (5.23)$$

The R^2 for this equation is 0.99, showing a high accuracy of equation 5.23 to represent the actual trade-off. A generic expression for the isoutility function is given in equation 5.24:

$$PV = a_0 + \frac{a_1}{b} \qquad (5.24)$$

Once the isoutility function is determined, the elasticity of substitution between PV and stability ($\eta_{PV,b}$) is estimated. In this case, $\eta_{PV,b}$ gives us a relative measure of the willingness to trade between PV and stability and is used as an indication of the trade-off between these two variables. For the function defined in 5.24, the elasticity of substitution is:

$$\eta_{PV,b} = \frac{\partial PV / PV}{\partial b / b} = -\frac{a_1}{PV \cdot b} \qquad (5.25)$$

The procedure explained above is repeated to generate an isoutility function and the elasticity measure for each type of individual defined in Table 5.3. The following Figure presents a comparison of the isoutilities for individuals with different preferences for stability and a moderate risk aversion (β = 0.01).

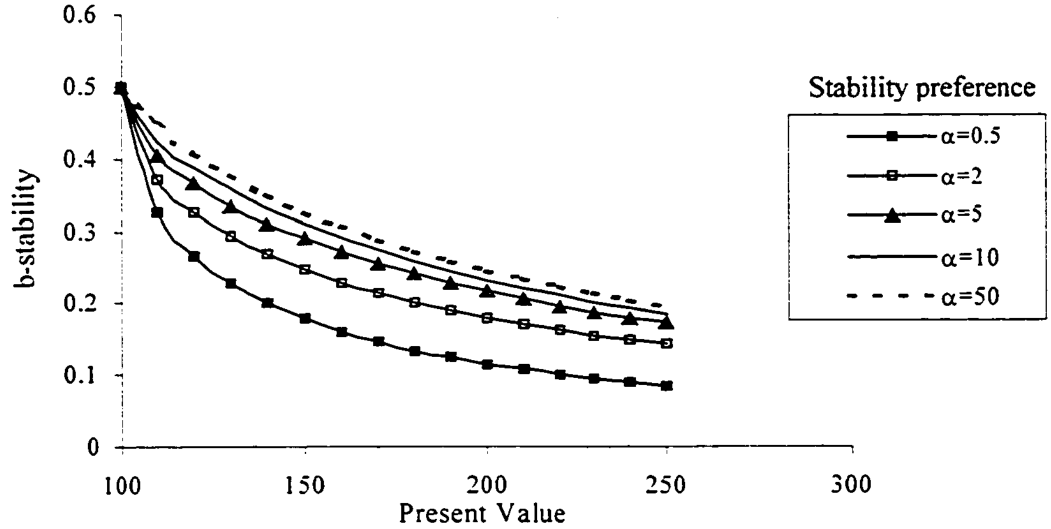

Figure 5.5. Isoutility curves for Present Value (PV) – stability (b) trade-offs given a stability preferences (α) of 5 and risk preferences (β) of 0.0056, 0.01 and 0.02.

Comparing the curves presented in Figure 5.5 it can be seen that the higher the stability preferences (the higher the value of α) the flatter the isoutility curve is, indicating a higher trade-off between PV and stability. If the individual's stability preference is high, the willingness to trade PV for more stability is also high (or the willingness to trade stability for an extra unit of PV is low). As will be presented later, the elasticity of substitution for different stability preferences shows the same conclusion.

The functional form presented in equation 5.24 was used to fit the isoutility function in the remainding fourteen cases. In all of them, the goodness of fit was very high with a R^2 ranging from 0.99 to 0.999. The results are presented in terms of the elasticity of substitution for each function. Since the elasticity varies across the curve (see equation 5.25), the elasticities were estimated for the point in the function that intersects a 45° ray from the origin.

Table 5.5 presents the estimated elasticity of substitution for the fifteen types of individuals.

Table 5.5. Elasticity of substitution between Present Value and stability for 15 types of preferences at a 45° ray from the origin

Stability Preferences

		$\alpha = 0.5$	**$\alpha = 2$**	**$\alpha = 5$**	**$\alpha = 10$**	**$\alpha = 50$**
Risk Aversion	**$\beta = 0.0056$**	-0.612	-0.867	-0.960	-0.984	-0.997
	$\beta = 0.01$	-0.612	-0.866	-0.960	-0.983	-0.996
	$\beta = 0.02$	-0.610	-0.865	-0.958	-0.982	-0.994

Table 5.5 shows that individuals with higher preferences for stability present a higher trade-off (higher elasticity in absolute terms) between PV and stability. In other words, this type of individual is willing to give up more PV for an extra unit of stability than individuals with a lower stability preference, or would need an higher increase in PV for a unit less of stability. This result is entirely consistent with the behavior that was expected from the definition of an individual with high degree of stability preferences.

Another important aspect to notice is that the elasticity of substitution $\eta_{PV,b}$ does not change greatly across different degrees of risk aversion. These results can be clearly seen in the following two figures. Figure 5.6 shows how the elasticity changes across different degrees of stability preferences for each of the three levels of risk aversion, and Figure 5.7 presents, for each degree of stability preference, how the elasticity changes when varying the degree of risk aversion.

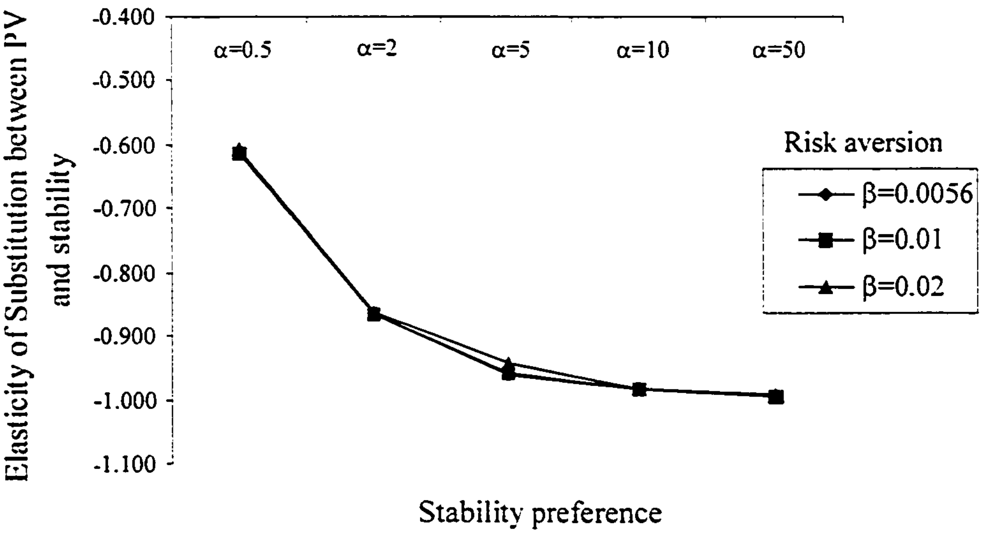

Figure 5.6. Elasticity values for stability preferences (α) of 0.5, 2, 5, 10, and 50 in combination with risk preferences (β) of 0.005, 0.01, and 0.02.

Figure 5.6 presents the same pattern for the elasticity of substitution across different degrees of stability preferences for all three degrees of risk aversion. Figure 5.7 shows a flat curve when presenting the impact of risk aversion on the elasticity pattern. This reinforces the conclusion that the impact of risk aversion on the substitutability between PV and stability is close to zero.

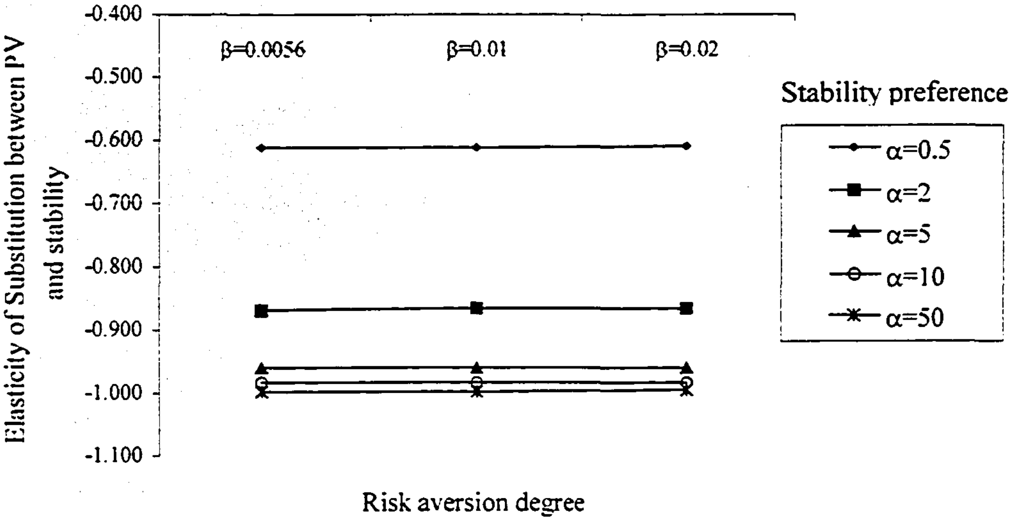

Figure 5.7. Elasticity values for risk preferences (β) of 0.005, 0.01, and 0.02 in combination with stability preferences (α) of 0.5, 2, 5, 10, and 50.

In Chapter Two, I explained that one of the characteristics of the GEU approach is that it can independently capture the impact of risk and stability preferences. These results reveal the ability of the model to capture this independence between the stability and risk aversion parameters.

The last conclusion that can be inferred from the figures, is that for stability preferences with an $\alpha = 5$ and higher, the relative trade-off between PV and the variable b becomes more stable, meaning that individuals with this degree of preferences and higher will probably be willing to trade a similar percent of PV for an extra 1% increase of stability. This result is certainly a useful conclusion that can be used later to characterize groups of individuals for the purpose of constructing financial instruments.

To continue with the analysis of the trade-off between PV and stability, since $\eta_{PV,b}$ varies along each isoutility, I compare the elasticity for different types of individuals at different points of the function. With this purpose, the elasticity at the points that intersect a 22.5°, 45°, and 67.5° angle ray from the origin were compared and graphed in Figure 5.8. The comparison was made for types 6 to 10 (moderate risk aversion and the five different degrees of stability preferences).

The figure shows that $\eta_{PV,b}$ is relatively stable along isoutilities that present a high degree of stability preferences, which would be isoutlities for $\alpha = 5$, $\alpha = 10$, and $\alpha = 50$. For isoutilities with lower degree of stability preferences, the elasticity presents more variability along the curve. This conclusion will be useful when designing financial instruments, because regardless the level of PV or the distribution of the PV between period 1 and 2 of a particular alternative, the relative willingness to trade will be similar.

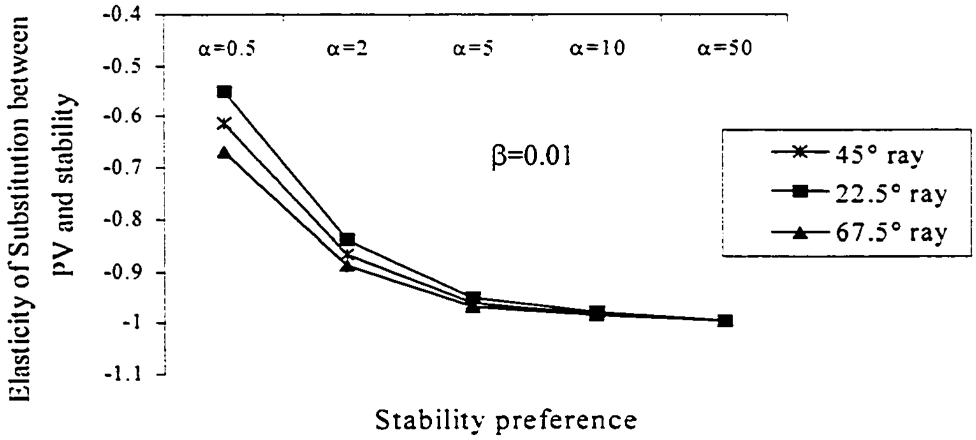

Figure 5.8: Elasticity of substitution at intersection points of the isoutility curve with 45, 22.5, and 67.5 line for stability preferences (α) of 0.5, 2, 5, 10, and 50 and risk aversion (β) of 0.01.

The following two tables present the elasticity for the 15 types of individuals at the intersection with the 22.5° and 67.7° angle lines.

Table 5.6. Elasticity of substitution between Present Value and stability for 15 Types of preferences at a 22.5° ray from the origin

Stability Preferences

Risk Aversion

	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$	$\alpha = 50$
$\beta = 0.0056$	-0.550	-0.839	-0.951	-0.980	-0.996
$\beta = 0.01$	-0.549	-0.839	-0.951	-0.979	-0.995
$\beta = 0.02$	-0.548	-0.837	-0.949	-0.978	-0.993

Table 5.7. Elasticity of substitution between Present Value and stability for 15 types of preferences at a 67.5° ray from the origin

Stability Preferences

Risk Aversion

	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$	$\alpha = 50$
$\beta = 0.0056$	-0.669	-0.890	-0.967	-0.987	-0.997
$\beta = 0.01$	-0.668	-0.889	-0.967	-0.986	-0.997
$\beta = 0.02$	-0.667	-0.888	-0.966	-0.985	-0.995

Finally, the impact on the PV- stability trade-off of a higher risk level will be explored. Initially, I considered a risk level of $\sigma^2 = 100$ m.u. For the exercise this level is considered low. A $\sigma^2 = 625$ m.u., which corresponds to a standard deviation of $\sigma = 25$ is used as a high risk level. This standard deviation implies that there exists a 95% probability of having a realization between 0 and 100 m.u., considering that the expected value is 50 m.u. I compare the elasticity of substitution between PV and stability ($\eta_{PV,b}$) for the two levels of risk, for different degrees of stability preferences and a risk aversion (β) of 0.01, in Figure 5.9. Figure 5.10 shows the elasticity of substitution at a high and low level of risk for different stability preferences and a risk aversion level of 0.02. Only individuals with moderate and high risk aversion level were compared, since the impact on individuals with low risk aversion is minimal. The elasticity of substitution was estimated at the intersection point with the 45° ray from the origin.

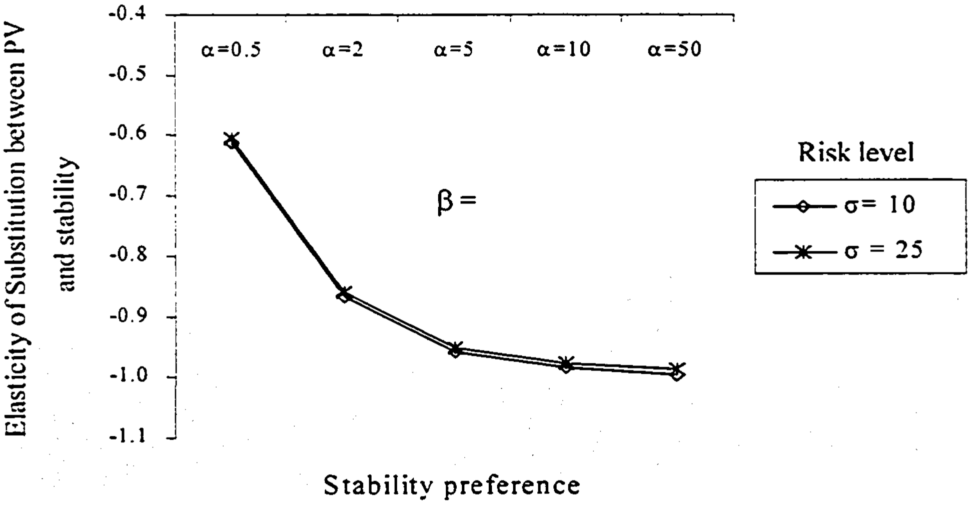

Figure 5.9. Elasticity of substitution for stability preferences of 0.5, 2, 5, 10, and 50 and risk aversion of 0.01 at low and high risk levels.

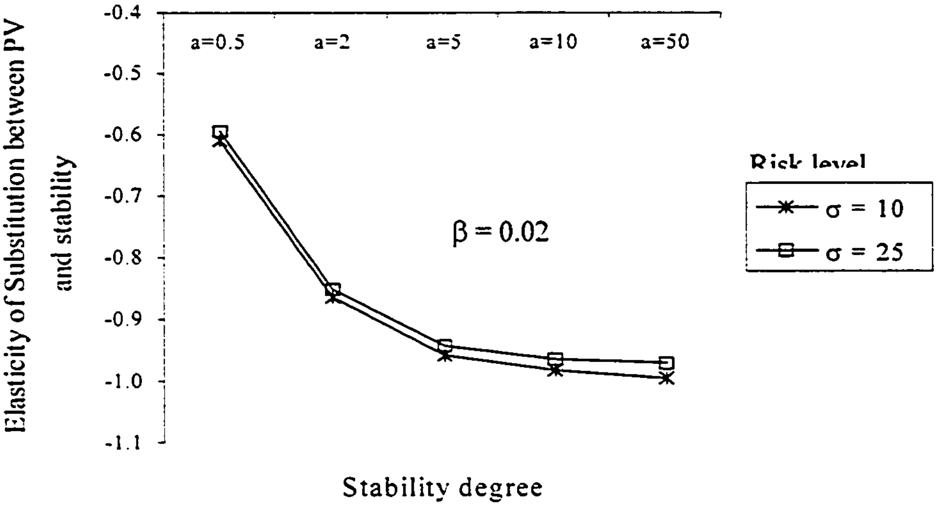

Figure 5.10. Elasticity of substitution for stability preferences of 0.5, 2, 5, 10, and 50 and risk aversion of 0.02 at low and high risk levels.

From the figures, it can be concluded that the impact of the risk level on the elasticity of substitution is low, even for those with high degree of risk aversion. These results further confirm the independence between risk and risk aversion and stability preferences.

Stability Premium

In risk analysis, the risk premium is defined as the difference between the expected value of the risky prospect and its certainty equivalent. The certainty equivalent and the risky prospect would give the individual the same level of expected satisfaction or utility, and on this basis, both values can be compared. In terms of stability, the premium can be defined as the difference between the PV of an uneven prospect and its "equivalent PV". In this case, the term "equivalent PV" is the PV that gives the individual the same level of utility with a perfectly even distribution compared to the initial uneven prospect. In other words, the uneven prospect (PV_0, b_0) and the "equivalent PV" (PV', 0.5) are two points on the same isoutility. The premium is then estimated by:

$$\text{Stability Premium} = PV_0 - PV'$$

The procedure to generate the premiums is to determine the PV', using as reference the utility level that the combination (PV_0, b_0) yields to the individual, and then estimate the difference between PV_0 and PV'. To estimate the premiums, a generalized solution is not possible since each situation will require a specific isoutility function, and although a generic function exists, its parameters need to be estimated for each particular situation.

The following table shows the stability premiums for different income alternatives for individuals with different degrees of stability preferences and a moderate degree of risk aversion ($\beta = 0.01$). The other parameters and variables used to determine the initial utility level are: $\rho = r = 0.95$, and $\sigma^2 = 100$ m.u. The alternatives are characterized by its present value PV and distribution variable b as an ordered pair (PV, b).

Table 5.8. Stability premiums for different income alternatives at different stability preferences and at a risk aversion β of 0.01

Alternatives	**Stability Preferences**			
(PV, b)	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$
(100, 0.45)	0.67	1.34	2.59	4.3
(100, 0.4)	2.87	5.59	9.91	13.87
(100, 0.35)	6.64	12.47	19.79	24.45
(100, 0.3)	12.07	21.57	30.74	35.15
(100, 0.2)	28.52	44.61	53.52	56.56
(1000, 0.45)	7.47	14.74	28.50	47.07
(1000, 0.4)	30.06	58.35	103.05	143.54
(1000, 0.35)	68.34	128.01	202.51	249.38
(1000, 0.3)	123.14	219.59	312.23	356.42
(1000, 0.2)	288.51	450.69	540.08	570.76
All values are expressed in m.u.				

Table 5.8 shows the trend for the stability premiums under different stability preferences and for different initial stability levels. The premiums are expressed in absolute values. The first comment that can be made with respect to the table is that the premiums for same stability preferences are proportional to the PV level. For example, the premiums for the alternatives (100, 0.2) and (1000, 0.2) for an individual with $\alpha = 0.5$

are 28.52 m.u. and 288.52 m.u. respectively. This implies in both cases an approximate 28% premium over the initial PV.

A second important conclusion is that the premiums for higher stability preferences ($\alpha = 5$ and $\alpha = 10$) and in cases of more extreme unevenness ($b = 0.3$ and $b = 0.2$) tend to converge to a unique number. In the table, the shadowed area highlights this phenomenon. It can be said that, on one hand, the higher the initial unevenness of the prospect, the more similar the level of the premium will be for different degrees of stability preferences. On the other hand, the higher the stability preferences, the more similar the premium will be for different prospects.

This conclusion is certainly consistent with the results found in the previous section. In that section, it was concluded that the elasticity of substitution between PV and stability (b) become more stable for higher degrees of stability preferences, and more similar to each other.

It is also important to notice the magnitude the premiums can have for a high degree of stability preferences and high level of unevenness for an alternative. For the alternative (100, 02), which implies obtaining a 20 m.u. income in period 1 and 80 m.u. in period 2, the premium that the individual is willing to pay for a perfectly even income path is 56.56 m.u. This premium implies that the individual would receive with certainty 21.42 m.u. each period.

The impact on the stability premium of different degrees of risk aversion was not analyzed since it was already shown that the risk aversion has little impact on the PV – stability trade-off.

The analysis of the stability premium is somewhat surprising. The high values that an individual is willing to pay for stability leave some questions about the performance of the model. In some cases, a person's premium would leave him/her with barely more than the minimum income each period. For example, for the different stability preferences of $\alpha = 5$ and $\alpha = 10$, an individual would be better off receiving 21 m.u. the first and second period instead of 20 m.u the first period and 80 m.u. the second. For these levels of stability preferences, the minimum income in the stream is driving the utility level for the individual. This result is close to the case of prefect complements analyzed in the previous section of this Chapter, in which the minimum value of the stream is defining the optimal solution for the individual. Then the question is: Do we believe that individuals behave as in the case of perfect complements when allocating income across time? According to the literature, the EIS is likely to be close to 0.1 ($\alpha = 9$). Maybe, the results of the model with an $\alpha = 0.5$ or $\alpha = 2$ are more credible in this case.

This question is relevant if we believe that the behavior of the individual can be captured by equation 5.10. If not, then the final question is: how much is the functional form chosen for the preferences for stability and risk affecting the results? Perhaps, another type of concave transformation[28] or another type of risk preferences form would have a smoother impact on the final premiums.

PV - risk Trade-off Analysis

In this section, the impact of the risk level on the individual's behavior is analyzed, and in particular the willingness to trade PV for extra units of risk level is

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[28] See equation 5.1.

estimated. The objective is to elicit the traditional profit-risk indifference curve using the GEU model, and to analyze the implications on the profit-risk trade-off when disentangling preferences for stability and risk. In order to generate the PV- risk isoutility curve the same analytical procedure as in the preceding section is used. As before, the complexity of the reduced form solution of equation 5.10 does not allow me to generate an algebraic solution for the isoutility function; therefore, a different analysis strategy needs to be implemented. The steps followed in this case are similar to before and are described as follows: The first step is to generate pairs of risk levels and PVs that yield the same utility level. The exercise is repeated then for different types of individuals in order to be able to make comparisons of the trade-offs across individuals with different degrees of risk aversion and stability preferences. The following combinations of risk preferences and stability preferences are used in this section:

Table 5.9. Simulated preference types

Low ← Stability preferences → High

↑ Risk aversion ↓

Type 1: $\beta = 0.0056$ $\alpha =0.5$	Type 2: $\beta = 0.0056$ $\alpha =5$	Type 3: $\beta = 0.0056$ $\alpha =10$
Type 4: $\beta = 0.01$ $\alpha =0.5$	Type 5: $\beta = 0.01$ $\alpha =5$	Type 6: $\beta = 0.01$ $\alpha =10$
Type 7: $\beta = 0.02$ $\alpha =0.5$	Type 8: $\beta = 0.02$ $\alpha =5$	Type 9: $\beta = 0.02$ $\alpha =10$

High

To generate the pairs of PV and risk level, risk, measured as standard deviation σ, is increased progressively from $\sigma = 0$ (no risk) to a $\sigma = 30$. Then for each level of risk a PV that yields the same utility as the initial reference (PV = 100, $\sigma = 0$) is found. The values for the remaining parameters and variables are: $\rho = r = 0.95$ and $b = 0.5$ (perfect even distribution). The following figure shows the different isoutlity curves generated by individual's type 2, 5 and 8 ($\beta = 0.0056$, $\beta = 0.01$, $\beta = 0.02$ and $\alpha = 5$).

The figure clearly shows the impact of the degree of risk aversion on the PV- risk trade-off. The higher the risk aversion, the lower is the willingness to trade risk for PV. In other words, individuals with higher risk aversion would need a bigger increase in PV to accept more risk. Figure 5.11 shows the impact of the stability preferences on the PV-risk trade-off for an individual with moderate risk aversion.

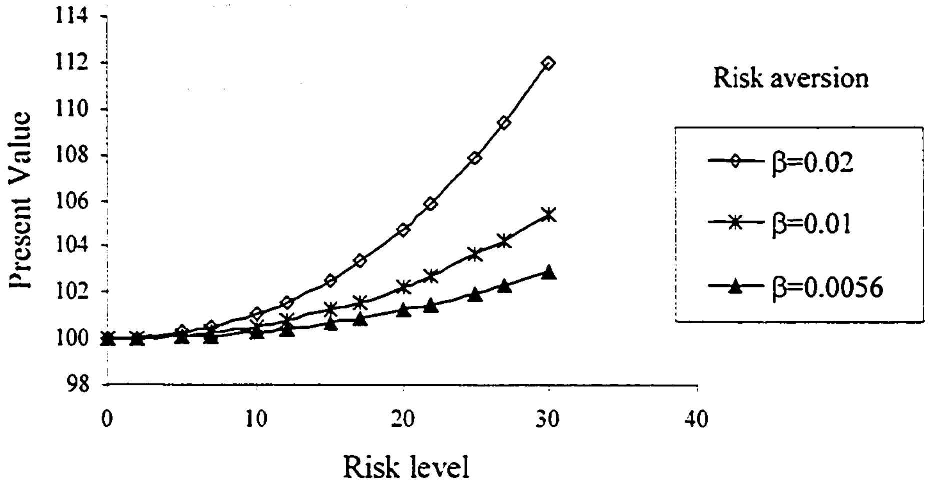

Figure 5.11. Isoutility curves for Present Value (PV) – risk level (standard deviation) trade-offs given a stability preference (α) of 5, risk aversion (β) of 0.0056, 0.01, 0.02, and a stability level b of 0.5.

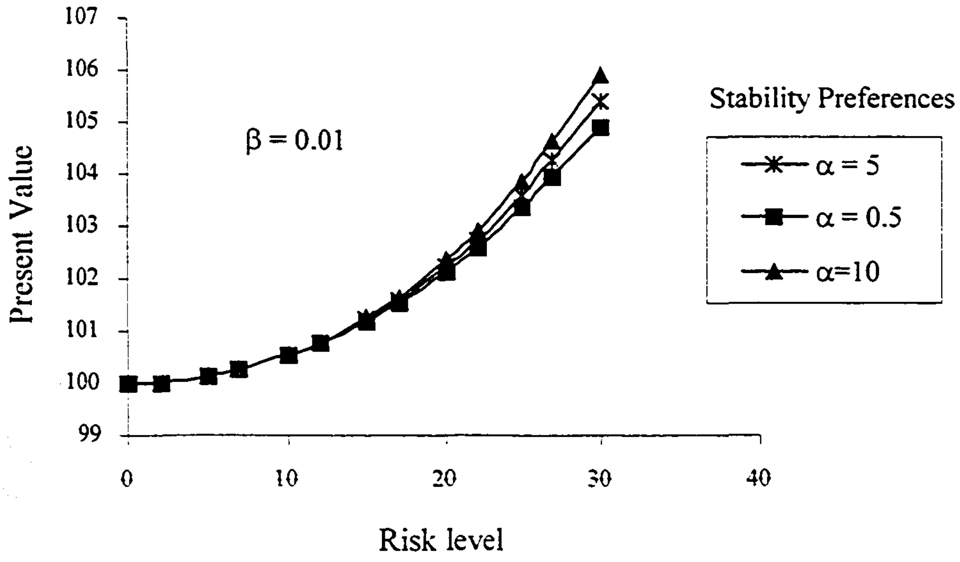

Figure 5.12. Isoutility curves for Present Value (PV) – risk level (standard deviation) trade-offs given a risk aversion (β) of 0.01, stability preference (α) of 0.5, 5, and 10, and a stability level b of 0.5.

As it was expected, the impact of the stability preferences on the PV – risk trade-off is not important, leading to almost the same PV – risk relationship for individuals with different degrees of stability preferences and with the same risk aversion. This conclusion is what was expected from the GEU model since risk aversion and stability preferences were defined as two distinct and independent components of the individual's behavior that would affect decisions independently from each other.

The information generated in the first step is important, but as it was also noticed when generating the PV – stability trade-off, is limited for further analysis. As before the data generated in the first step are used to estimate an isoutilty function using Ordinary Least Squares estimators. The functional form that fits the best the data in this case is a quadratic form for which the R^2 is, in all cases, 0.99 or higher. Equation 5.26 shows a generic PV – risk isoutility function.

$$PV = c_0 + c_1 \cdot \sigma + c_2 \cdot \sigma^2 \qquad (5.26)$$

The trade-off measure used in the analysis is the elasticity of substitution ($\eta_{PV,\sigma}$) as a relative measure of the willingness to trade PV for risk. The elasticity for this isoutility function is shown in equation 5.27.

$$\eta_{PV,\sigma} = \frac{\partial PV / PV}{\partial \sigma / \sigma} = \frac{c_2 \cdot \sigma + 2 \cdot c_3 \cdot \sigma^2}{PV} \qquad (5.27)$$

Table 5.10 shows the elasticity of substitution for the 9 types of individuals analyzed in this exercise.

Table 5.10. Elasticity of substitution between Present Value and risk for different preference types at a risk level of σ = 10

Stability Preferences

		α = 0.5	α = 5	α = 10
Risk Aversion	**β = 0.0056**	0.006	0.006	0.006
	β = 0.01	0.011	0.011	0.012
	β = 0.02	0.021	0.023	0.025

The table shows a significant impact of the risk aversion degree on the elasticity value, although an almost zero impact of the degree of the stability preferences. This situation is the same observed in Figures 5.11 and 5.12 and it is the reverse of what was observed with the PV-stability trade-off. Stability preferences do not influence the risk trade-off and risk preferences do not affect the stability trade-off. The two following figures present this issue graphically. Figure 5.13 shows the elasticity across different degrees of risk aversion and Figure 5.14 across different degrees of stability preferences.

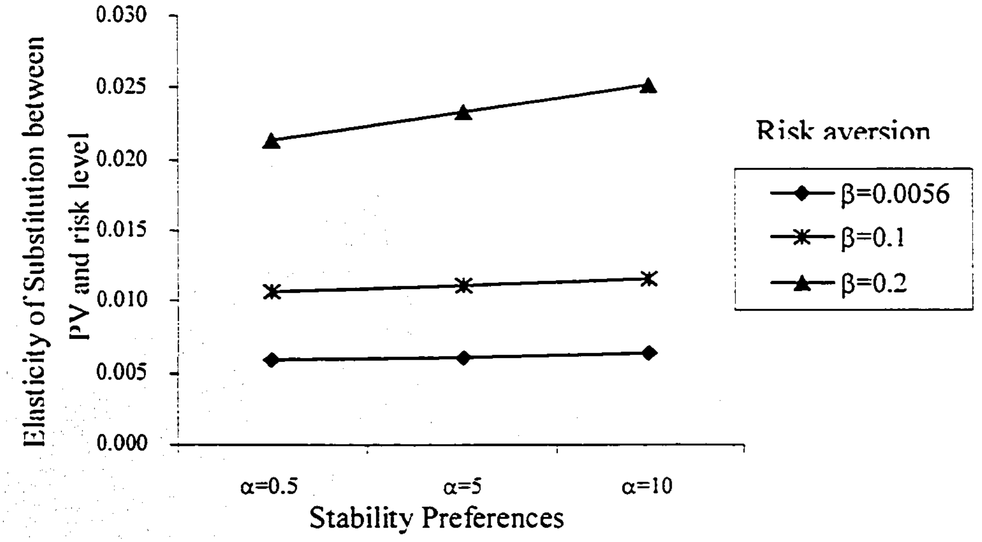

Figure 5.13. Elasticity values for risk aversion β of 0.0056, 0.01, and 0.02 combined with stability preferences of 0.5, 5, and 10.

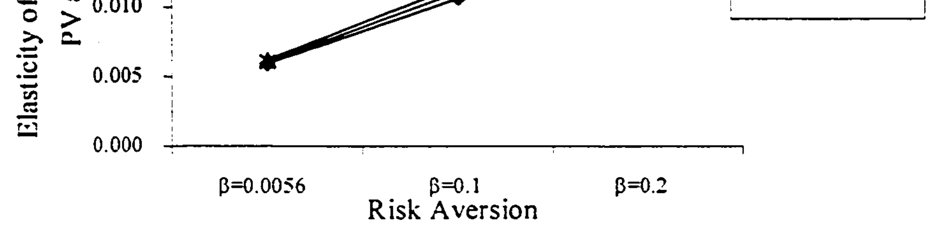

Figure 5.14. Elasticity values for stability preferences of 0.5, 5, and 10 combined with risk aversion β of 0.0056, 0.01, and 0.02.

Equation 5.27 presents the formula for the elasticity, and according to the expression the elasticity varies with the level of risk. In other words, it varies along the isoutility curve. To analyze how much the elasticity varies along the curve, the elasticity at three levels of risk ($\sigma = 10$, $\sigma = 20$, and $\sigma = 25$) was calculated. Figure 5.15 and Tables 5.11 and 5.12 show these results.

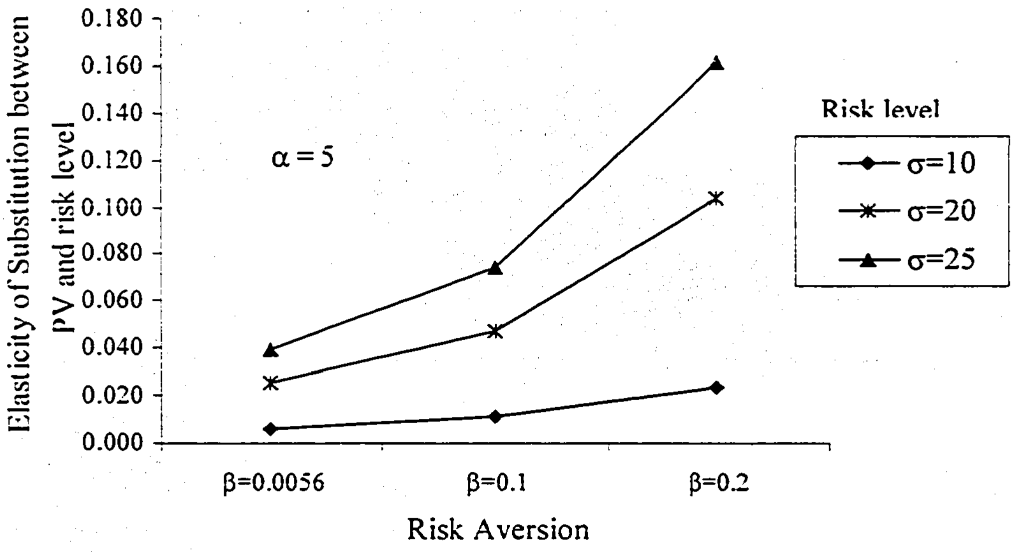

Figure 5.15: Elasticity values for risk aversion (β) of 0.0056, 0.01, 0.02 at risk levels measured in standard deviation (σ) of 10 m.u., 20 m.u., and 25 m.u.

Table 5.11. Elasticity of substitution between Present Value and risk for different preference types at a risk level of $\sigma = 20$

Stability Preferences

Risk Aversion

	$\alpha = 0.5$	$\alpha = 5$	$\alpha = 10$
$\beta = 0.0056$	0.024	0.025	0.027
$\beta = 0.01$	0.043	0.047	0.052
$\beta = 0.02$	0.087	0.104	0.118

Table 5.12. Elasticity of substitution between Present Value and risk for different preference types at a risk level of $\sigma = 25$

		Stability Preferences		
		$\alpha = 0.5$	$\alpha = 5$	$\alpha = 10$
Risk Aversion	$\beta = 0.0056$	0.037	0.040	0.043
	$\beta = 0.01$	0.066	0.074	0.082
	$\beta = 0.02$	0.133	0.161	0.185

The preceding figure and tables lead to the conclusion that for lower risk aversion, the elasticity becomes more stable, while for higher degrees of risk aversion the elasticity is significantly changing along the isoutility curve.

Until now the PV – risk trade-off analysis has been done under the assumption that the distribution of the PV across time is perfectly even (b = 0.5). The following two figures show the impact of the distribution of the PV on the trade-off for an individual with an $\alpha = 5$ and high and low risk aversion. Each figure compares the trend of the PV - risk trade-off for an even cash flow (b = 0.5) and an uneven cash flow (b = 0.3).

Figures 5.16 and 5.17 show that for any degree of risk aversion, the PV – risk trade-off for alternatives that present an uneven distribution of the net returns is not significant. In other words, higher risk levels will have a very small impact on the PV level when the prospect has low stability. PV – risk isoutilities of uneven prospects for individuals with different degrees of stability preferences were also elicited, reaching the same conclusion.

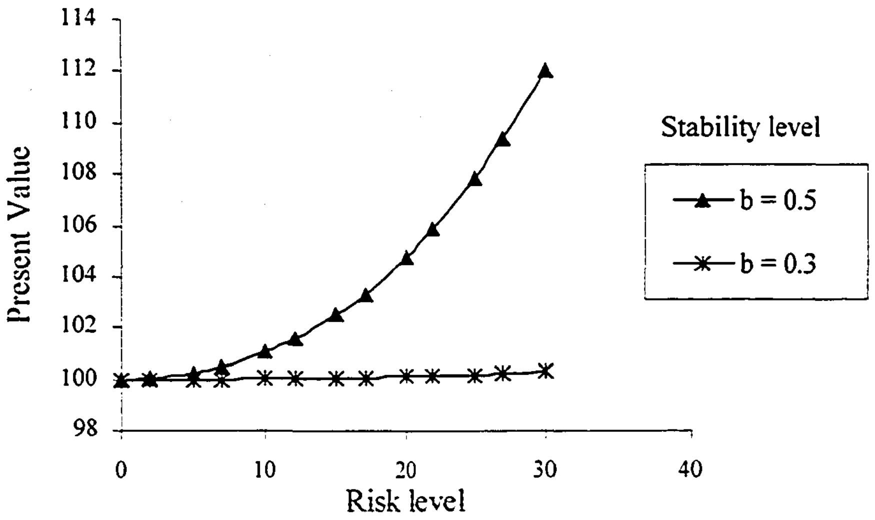

Figure 5.16. Isoutility curves for Present Value (PV) – risk level (standard deviation) trade-offs at stability levels (b) of 0.5 and 0.3 given a risk aversion (β) of 0.02 and stability preferences (α) of 5.

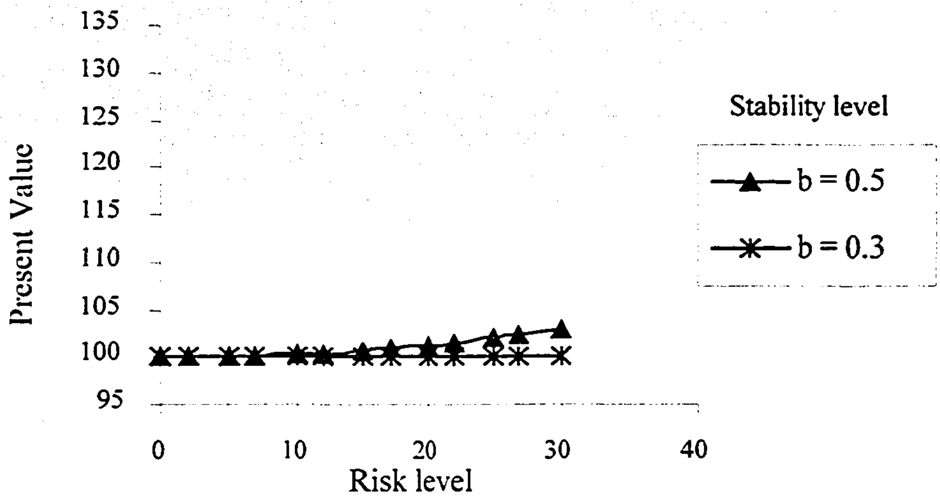

Figure 5.17: Isoutility curves for Present Value (PV) – risk level (standard deviation) trade-offs at stability levels (b) of 0.5 and 0.3 given a risk aversion (β) of 0.0056 and stability preferences (α) of 5.

This phenomenon is quite interesting and leads us to consider a complete new issue with respect to the stability and risk behavior. For any individual, the risk level of an alternative becomes a less important factor in the investment decision compared to stability if the alternative presents an uneven expected path.

A graphic exposition may help in understanding this concept. The following figure shows the alternatives that are being compared. Alternative A shows a distribution of the income represented by a b = 0.3, while Alternative B is perfectly even. The risk level is the same for both to make them comparable.

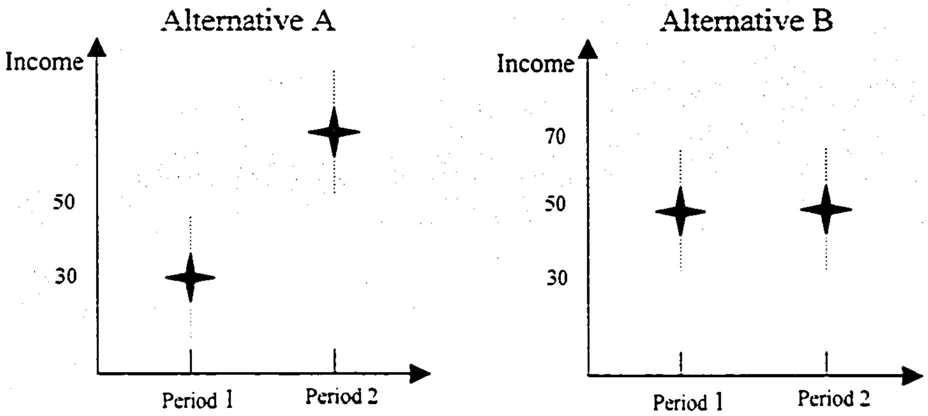

Figure 5.18. Comparing income path for a distribution b = 0.3 and a distribution b= 0.5.

Comparing Alternatives A and B in Figure 5.18, it can be seen that for an uneven path, even if risk is adjusted, the individual would have a highly variable income path. Therefore, the individual is willing to trade only a small amount of PV for less risk. In the case of a perfectly even distribution of the income, the risk level is the only source of variability and adjusting for this source will have a high impact on the total variability of the income path. Therefore, in this case the individual is willing to trade a higher amount of PV in order to achieve stability. Following this explanation for the behavior of the PV

– risk trade-off under different distributions of income, it should be expected that for more stable expected income paths the trade-off would be more significant. Figure 5.19 shows the PV – risk trade-off at three different distributions of the income between period 1 and period 2. The exercise was performed for a stability preferences $\alpha = 5$ and a risk aversion $\beta = 0.02$.

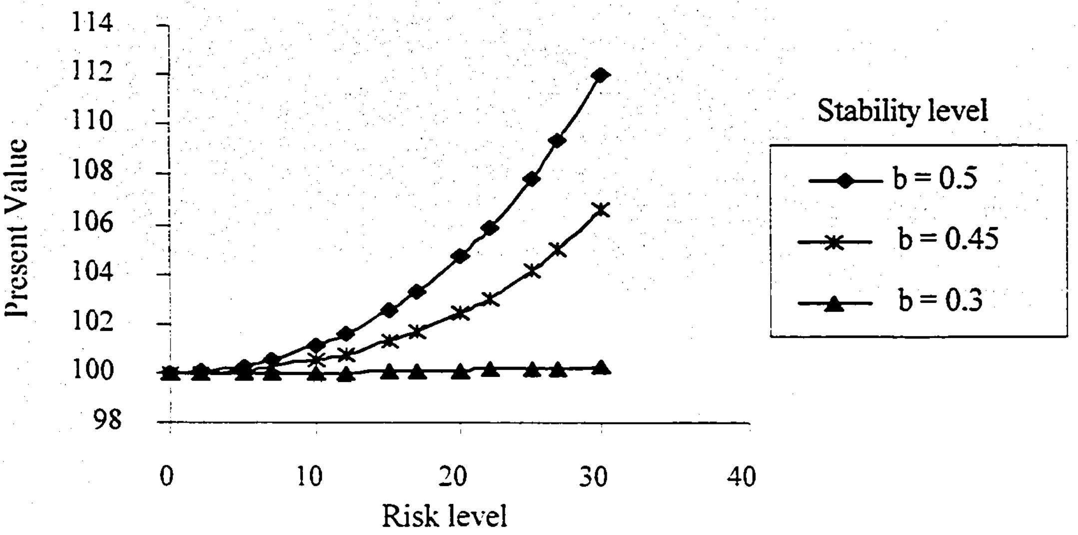

Figure 5.19: Isoutility curves for Present Value (PV) – risk level (standard deviation) trade-offs at stability levels (b) of 0.5, 0.45, and 0.3 given a risk aversion (β) of 0.0056 and stability preferences (α) of 5.

Figure 5.19 confirms the proposition that for higher levels of stability the PV - risk trade-off is more significant. This result is certainly important to understand how the two sources of variability interact in determining the individual's behavior when deciding among different productive alternatives.

Summary

The sensitivity analysis reveals some interesting results with respect to the behavior of individuals with different stability and risk preferences, as well as at different levels of stability and risk. The following is a summary of these findings:

- For high degrees of stability preference ($\alpha = 5$ and above), the elasticity of substitution between PV and stability ($\eta_{PV,b}$) tend to converge to the same number, reflecting that the relative trade-off becomes similar among individuals with high stability preferences.
- The stability premiums also tend to converge to a unique number for α values of 5 and above.
- The elasticity $\eta_{PV,b}$ becomes more stable along the PV –stability indifference curve for higher values of α ($\alpha = 5$ and above).
- When analyzing the Present Value PV - stability trade-off, risk preferences appear not to be important to the elasticity of substitution between PV and stability (b) and to the stability premium.
- The risk level, measured as the standard deviation σ, has a low impact on the PV – stability trade-off. The elasticity of substitution $\eta_{PV,b}$ does not have a significant change when the risk level is changed.
- The stability premiums exhibited for higher stability preference ($\alpha = 5$ and above) were extremely high. For higher stability preferences, the minimum income in the stream highly influenced the overall utility level for the individual. When α values exceed 5, the present and future are similar to prefect complements as described in the previous section of this Chapter.

- Turning the discussion to the PV – risk trade-off, it was observed that the stability preferences have no impact on the elasticity of substitution between PV and risk $\eta_{PV,\sigma}$.
- It could be also concluded that the elasticity of substitution becomes more stable at lower degrees of risk aversion along the PV – risk indifference curve.
- Finally, it is important to highlight the impact that the stability in an income path has on the PV-risk trade-off. It was observed that higher risk levels have a very small impact on the PV –risk trade-off when the prospect has low stability.

CHAPTER SIX

MULTI-PERIOD TIME GENERALIZED EXPECTED UTILITY MODEL

In this Chapter, the two-period model presented in Chapter Five is extended to T periods. Since technology alternatives are long run decisions, a model that can handle a multi-period frame is required. The two-period model is useful to understand and explore the different outcomes that can be expected under different preferences and alternatives, although for application purposes a multi-period framework is more adequate. As before, the objective of the model is to show how stability preferences and risk aversion influence individual choice for technologies with different distributions of net returns over time. The Chapter is divided in two sections. The first one develops the multi-period GEU model applied to technology adoption decisions. The second section presents a sensitivity analysis in which different combinations of stability and risk preferences are simulated to determine the trade-off preference between net returns and stability, and ultimately, the stability premiums that different types of individuals are willing to pay for having an even instead of an uneven cash flow.

GEU Multi-period Model – Technology Adoption

The GEU framework has the property of capturing preferences for stability and risk independently, and as it was shown in Chapter Five has the ability to generate consistent rankings of alternatives with different net revenue patterns. In this Chapter, the GEU model is extended to a multi-period framework for which two alternative approaches can be used. One approach is to extend Selden's GEU version, the OCE representation, into more than two periods. The other approach is the Epstein and Zin (1989, 1991) solution, in which they provide a recursive representation of the GEU model, where the optimal solution is sought by a successive approximation method.

Let us consider the advantages and disadvantages of both approaches. In Chapter Two, I mentioned that the extended Selden's version of the OCE representation generates inconsistencies across time. These inconsistencies were reflected in the behavior of what Zin (1987) referred to as a "naive individual", who does not adjust his/her consumption path to new information and who does not have the ability to abandon the consumption plan in the future. The Epstein and Zin approach addresses the "inconsistency" by using a recursive representation of the utility index. However, there may be situations where the OCE model assumptions are not naïve or constrained. For example, in the case of technology adoption, each period an individual has all the relevant information at the moment of making the decision and this decision is irreversible. In this sense, the "naïve" behavior and the "inability" to change the consumption plan in the future fit with the scenario. Therefore, the multi-period OCE representation is suitable for the technology adoption. Moreover, based on the timing of information, the Epstein and Zin recursive approach and a model using the multi-period OCE representation should yield the same solution.

Individual's Preferences

The model considers a representative individual in an economy in which one source of utility exists: a single consumption good. The individual seeks to maximize his/her utility function across a T-period cycle. The multi-period OCE representation used here assumes, as before, that stability preferences are constant across time and represented by an aggregator function with a constant Elasticity of Intertemporal Substitution (EIS). The utility index takes the form:

$$U\ (C_0 \ldots\ldots C_T) = -\frac{1}{\alpha}(\sum_{t=0}^{T} \rho^{t} \cdot C_t^{-\alpha}) \qquad (6.1)$$

Where C_t is period t consumption, ρ is the discount factor, and α is the stability parameter. The Elasticity of Intertemporal Substitution is as before:

$$EIS = \frac{1}{\alpha + 1}$$

Following the same model development as in the previous Chapter, the preferences with respect to the uncertain prospect have an exponential function with a constant Absolute Risk Aversion (ARA) coefficient β:

$$u(C_t) = 1 - e^{-\beta \cdot C_t} \qquad (6.2)$$

Using expression 6.2, the certainty equivalent of a random consumption can be defined as follows:

$$\hat{c}_t = \frac{\ln E(e^{-\beta \cdot c_t})}{-\beta} \qquad (6.3)$$

Rewriting equation 6.1, using the certainty equivalent of each period's consumption, the utility index can be rewritten as:

$$U(C_0,...,C_T) = -\frac{1}{\alpha}(C_0^{-\alpha} + \sum_{t=1}^{T} \rho^t \cdot (\frac{\ln E(e^{-\beta \cdot c_t})}{-\beta})^{-\alpha}) \qquad (6.4)$$

Equation 6.4 shows the extension for a multi-period model of the OCE preferences assuming a "naïve individual", or as it was explained for this application, assuming that all the relevant information is known at period 0, when the decision is made.

Production Alternatives

The production alternatives are defined as in the two period model, but extended to T periods in the following way:

$$\begin{aligned} &I_0 = PV \cdot b_0 \\ &I_t = (PV \cdot b_t + \varepsilon_t) \cdot \tfrac{1}{r} \qquad (6.5) \\ &\sum_{t=0}^{T} b_t = 1 \end{aligned}$$

PV is the present value of the alternative, b_t is the portion of present value earned in period t, r is the market discount factor[29], and ε_t is the stochastic component of the net income in period t. For simplicity it is assumed that $\varepsilon_t \sim N(0,\sigma^2)$.

As in the two-period model, equation 6.5 has both components of variability clearly defined. On one side, risk is represented by σ^2, on the other side, the expected variability is represented by "b_t". The variable b_t will define how stable the income path is. The stability index presented in Chapter Four is used for b_t as follows[30]:

$$SI = 1 - \frac{\sum_{t=0}^{T} \left| b_t - \frac{1}{T} \right|}{2(1 - \frac{1}{T})} \qquad (6.6)$$

In a multi-period context, the SI constitutes a useful measure of the stability of an income prospect. In contrast with the two-period model in which the stability variable is only one "b", in a multi-period model the distribution of the PV is made through a set of variables: $\mathbf{b} = (b_0, b_1, \ldots, b_T)$. In this case, the whole set **b** is defining the stability of the prospect and therefore, the stability measure of this prospect needs to be a combination of all its elements. As shown in Chapter Four, the properties of the SI make this index a good alternative to measure stability.

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[29] The market discount factor is capitalizing net income from period 2 to T, and transforming it to current terms. This step is necessary since in equation (6.4) consumption is expressed in current terms.

[30] It is important to notice that SI is calculated for the net income path in present terms rather than in current terms.

Optimization Problem

Equation 6.7 presents the optimization problem that combines the individual's preferences defined in equation 6.4, and the production alternatives defined in equation 6.5:

$$U(C_0,\ldots,C_T) = -\frac{1}{\alpha}(C_0^{-\alpha} + \sum_{t=1}^{T} \rho^t \cdot (\frac{\ln E(e^{-\beta \cdot c_t})}{-\beta})^{-\alpha})$$

$$\begin{aligned} &C_0 = I_0 = PV \cdot b_0 \\ &C_t = I_t = (PV \cdot b_t + \varepsilon_t) \cdot \tfrac{1}{r} \\ &\sum_{t=0}^{T} b_t = 1 \end{aligned} \qquad (6.7)$$

In this simple model, it is assumed that the individual is completely constrained by the income he/she can generate and cannot use capital markets to change the distribution of his/her income between period 1 and 2. Knowing that $C_1 = I_1$, $C_2 = I_2,\ldots, C_T = I_T$, the optimization problem can be rewritten as:

$$U(PV, b_t, \sigma^2) = -\frac{1}{\alpha}(PV \cdot b_0^{-\alpha} + \sum_{t=1}^{T-1} \rho^t \cdot (\frac{\ln E(e^{-\beta \cdot (PV \cdot b_t + \varepsilon_t) \cdot \frac{1}{r}})}{-\beta})^{-\alpha} + \rho^T \cdot (\frac{\ln E(e^{-\beta (PV \cdot (1 - \sum b_t) + \varepsilon_t) \cdot \frac{1}{r}}}{-\beta})^{-\alpha})$$

$$(6.8)$$

In order to obtain solution of the function in (6.8) that can be analyzed algebraically, the certainty equivalents of period 1 to T need to be solved. The exponential representation of risk preferences involves the expectation of a non-linear function, and following the same procedure as in the two-period model, a closed form solution of this expectation can be obtained by inspection (See Appendix II). After taking the expectation and some algebraic manipulations, the certainty equivalent of each year income is reduced to the following expression:

$$\hat{I}_t = \frac{1}{r^t}(PV \cdot (b_t) - \tfrac{1}{2} \cdot \frac{\beta}{r^t} \cdot \sigma^2) \qquad (6.9)$$

Where $\hat{I}_t$ is the certainty equivalent of I_t. Replacing the certainty equivalent $\hat{I}_t$ in equation 6.8, I obtain:

$$U(PV, b_t \sigma^2) = -\frac{1}{\alpha}(PV \cdot b_0^{-\alpha} + \sum_{t=1}^{T-1} \rho^t \cdot (PV \cdot b_t - \tfrac{1}{2} \cdot \frac{\beta \cdot \sigma^2}{r^t}) \cdot \tfrac{1}{r^t}^{-\alpha} + \rho^T \cdot (PV \cdot (1 - \sum b_t) - \tfrac{1}{2} \cdot \frac{\beta \cdot \sigma^2}{r^T}) \cdot \tfrac{1}{r^T})^{-\alpha})$$

$$(6.10)$$

Equation 6.10 presents a simple function that can be analyzed in terms of its FOC. Following the same exercise done in Chapter 5, I will optimize the utility index with respect to the vector **b** = (b_0, b_1,…,b_T), assuming that the Present Value and risk level are constant.

Optimizing the utility with respect to b_t implies that:

$$(PV \cdot b_0)^{-(1+\alpha)} = \frac{\rho}{r} \cdot (PV \cdot b_1 - \frac{\beta \cdot \sigma^2}{2 \cdot r^2})^{-(1+\alpha)} = \ldots = \frac{\rho^T}{r^T} \cdot (PV \cdot (1 - \sum b_t) - \frac{\beta \cdot \sigma^2}{2 \cdot r^T})^{-(1+\alpha)}$$

As before, the terms of each side of the equation can be viewed as the marginal utility of the distribution of income for each period. In that sense, the interpretation of this result is that the individual is optimizing when the marginal utility of the current income equates the discounted marginal utility of the risk-adjusted future income.

GEU Model Using a Different Aggregator Function

Until now, the two-period and multi-period models have used a specific Constant Elasticity of Substitution (CES) function. In this section a different functional form of the CES type functions is used to develop the multi-period model in order to compare their performance. The assumptions and components of the model are the same as before, except for the function that aggregates certain consumption into the utility index. The new aggregator of the certain prospect is:

$$U\ (C_0, \ldots\ldots\ C_T) = (\sum_{t=0}^{T} \rho^t \cdot C_t^{(1-\gamma)})^{\frac{1}{(1-\gamma)}} \qquad (6.12)$$

In equation 6.12 the parameter γ is the new stability preferences parameter. The range of values for γ is 0 to ∞. The EIS for this function is:

$$EIS = \frac{1}{\gamma}$$

Preferences with respect to risk are still represented by equation 6.2 and therefore the certainty equivalent of random consumption are expressed as in equation 6.3. The production alternatives were already defined in equation 6.5. The resulting optimization problem of assuming equations 6.11 and 6.3 as aggregator and certainty equivalent, respectively, and assuming the production set as in equation 6.5 is the following:

$$U(C_0,\ldots,C_T)=(C_0^{(1-\gamma)}+\sum_{t=1}^{T}\rho^t\cdot(\frac{\ln E(e^{-\beta\cdot c_t})}{-\beta})^{(1-\gamma)})^{\frac{1}{(1-\gamma)}}$$

$$\begin{aligned}&C_0=I_0=PV\cdot b_0\\&C_t=I_t=(PV\cdot b_t+\varepsilon_t)\cdot\tfrac{1}{r}\\&\sum_{t=0}^{T}b_t=1\end{aligned}\qquad(6.12)$$

After taking the expected values involved in the objective function, and doing the same algebraic manipulations that were done to solve the model in the previous section, the utility index U(•) becomes:

$$U(PV,b_t,\sigma^2)=(PV\cdot b_0^{(1-\gamma)}+\sum_{t=1}^{T-1}\rho^t\cdot(PV\cdot b_t-\tfrac{1}{2}\cdot\frac{\beta\cdot\sigma^2}{r^t})\cdot\tfrac{1}{r^t}^{(1-\gamma)}+\rho^T\cdot(PV(1-\sum b_t)-\tfrac{1}{2}\cdot\frac{\beta\cdot\sigma^2}{r^T})\cdot\tfrac{1}{r^T})^{(1-\gamma)})^{\frac{1}{(1-\gamma)}}\qquad(6.13)$$

In the next section, equations 6.10 and 6.13 are compared for productive alternatives with different income paths. The comparison will be based on the stability premiums that both models would show for individuals with the same EIS and risk preferences.

Sensitivity Analysis

Stability Premium

The sensitivity analysis performed for the multi-period model consists of analyzing the premiums that different individuals would be willing to pay for trading an enterprise alternative with an uneven income for an even income path. The exercise is concentrated in analyzing stability premiums under different conditions, where again the ultimate goal is to generate the necessary information to develop financial instruments that will encourage the adoption of long run technologies.

The stability preferences considered to perform the analysis are $\alpha = 0.5$, $\alpha = 2$, $\alpha = 5$, and $\alpha = 10$. The section analyzed the impact of these degrees of stability preferences, on three different aspects of an income prospect. First, I analyzed how the order in which the income is distributed affects the stability premium. Then, the stability premiums for alternatives with different stability are estimated and analyzed. A final exercise will explore the impact of different risk preferences on the stability premiums.

In order to determine the impact of the distribution of the income across time on the stability premium, three different alternatives are analyzed. According to how the variables where defined in the previous section the vector $\mathbf{b} = (b_0, b_1, \ldots, b_T)$ includes the variables that determine the income stability of each alternative. The three alternatives share the same stability (SI = 0.68), although they differ in the distribution of the cash flow over time. The following are the income paths for the three alternatives:

Table 6.1. Productive alternatives: different ordering

Alternative	PV	Vector b
Alternative 1	100	(0.1, 0.1, 0.1, 0.1, 0.3, 0.3)
Alternative 2	100	(0.1, 0.1, 0.3, 0.3, 0.1, 0.1)
Alternative 3	100	(0.3, 0.3, 0.1, 0.1, 0.1, 0.1)

The risk level, measured as the standard deviation σ, is considered to be 20% of the income for the period. As an example, for Alternative 1, the risk level for period 1 is 2 m.u. (Income level :10 m.u. x 20% = 2 m.u.). The remaining parameters included in the model are the following: the risk aversion parameter β is equal to 0.01, the market and personal discount factors (r and ρ) are 0.95, and for the stability parameter α, the values are 0.5, 2, 5, and 10.

For each of the three alternatives, I estimated the premium that an individual would be willing to pay in order to trade the uneven path for an even path. As shown in Table 6.2, the ordering of the income path does not affect the premium. Later in this section, I show that premiums do vary for the same stability index when different income levels and distributions are assumed.

Table 6.2. Stability premium: impact of ordering

Stability preferences

	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$
Alternative 1	18.69	28.44	34.91	37.47
Alternative 2	18.69	28.44	34.92	37.48
Alternative 3	18.70	28.46	34.94	37.49

Table 6.2 also shows that as the stability parameter α increases, the stability premium increases and starts to converge. More specifically, for $\alpha = 5$ and $\alpha = 10$, the stability premiums are close to each other. This conclusion is the same conclusion reached for the two-period model for which it was discovered that for parameter values over five the trade-off and premiums converged to the same value.

The level of stability of the alternatives is indeed the most important factor when analyzing stability premiums. Therefore, I developed alternatives with different income paths and stability levels, shown in Table 6.3, for further analysis. As in the previous exercise, the risk level, measured as standard deviation σ, is considered as 20% of the income for the period. The remaining parameters included in the model are the following: risk aversion parameter β is equal to 0.01, the market and personal discount factors (r and ρ) are 0.95, and for the stability parameter α, the values 0.5, 2, 5, and 10 were considered.

Table 6.3. Production alternatives: different stability levels

Alternative	PV	Vector b	Stability Index
Alternative 1	100	(0.1, 0.1, 0.1, 0.23, 0.23, 0.23)	0.76
Alternative 2	100	(0.1, 0.1, 0.08, 0.05, 0.103, 0.567)	0.52
Alternative 3	100	(0.01, 0.01, 0.08, 0.2, 0.3, 0.4)	0.52
Alternative 4	100	(0.01, 0.01, 0.01, 0.1, 0.1, 0.77)	0.28
Alternative 5	100	(0.01, 0.01, 0.01, 0.01, 0.01, 0.95)	0.06

The results of the exercise are presented in Table 6.4. The premiums are the amount that an individual is willing to pay for trading the uneven alternative for an alternative with an even income path. The table clearly shows that as the stability level is lower, the stability premium is higher. The most stable alternative, Alternative 1, has the lowest stability premium, while the least stable pattern, Alternative 5, has the highest premium. Analyzing the impact of the stability preference degree, it can be observed that for higher values of α, the stability premium is also higher. This observation is certainly consistent with what was expected and with the results obtained for the two-period model.

Table 6.4. Stability premium: impact of stability level

Stability preferences

	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$
Alternative 1	12.32	21.97	31.23	35.64
Alternative 2	38.66	49.63	58.45	64.10
Alternative 3	74.55	89.57	92.43	93.21
Alternative 4	84.53	91.45	93.01	93.47
Alternative 5	91.61	93.34	93.68	93.80

Another result that is consistent with the conclusions derived for the two-period model is that as the stability level of the alternative becomes more unstable, the premium paid for an even income path becomes more similar for different degrees of stability preferences. Alternative 4, and especially Alternative 5, show this phenomenon. In Alternative 5, the stability premium for a $\alpha = 0.5$ and $\alpha = 10$ are respectively 91.61 m.u. and 93.80 m.u., showing a little difference. In contrast, in Alternative 1 the premiums paid for the extreme degrees of stability preferences are 12.32 and 35.64, showing a higher difference in premium.

Another important aspect in the premium analysis is the difference in premium that an individual would be willing to pay for alternatives that have the same Stability Index (SI) but different pattern of income. Alternatives 2 and 3 exhibit the same SI, 0.52;

however, there is a substantial difference in the premium that would be paid at different degrees of stability preferences. Table 6.5 will help illustrate this issue.

Table 6.5. Equivalent Income per period

	Minimum Income of the path	Average Equivalent Income			
		$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$
Alternative 1	10	14.61	13.01	11.46	10.73
Alternative 2	5	10.22	8.40	6.93	5.98
Alternative 3	1	4.24	1.74	1.26	1.13
Alternative 4	1	2.58	1.43	1.17	1.09
Alternative 5	1	1.40	1.11	1.05	1.03

The second column of Table 6.5 shows the minimum income generated by each of the alternatives, and the remaining columns show the income that the individual would be obtaining if he/she traded the uneven income path that the alternatives offer for an even income path that yields the same utility level. Following the analysis of Alternatives 2 and 3, it can be seen that the difference in premiums is explained by the difference in the minimum income that each alternative offers. Although the alternatives have the same SI, the difference in income path leads to different distribution of the income and difference in the minimum income. It can be seen that there is a high correlation in the equivalent average income and the minimum income.

The confusing results exhibited for alternatives with the same SI but different income paths, leaves questions with respect of the usefulness of the Stability Index as a measure of stability. The stability index does not capture all the dimensions of the distribution of an income path. It only captures the deviations around the average income;

however, it still can discriminate among alternatives with a difference in stability, leading to consistent results as seen in Alternatives 1, 2, 4 and 5. The utility index constructed to capture stability preferences can capture higher dimensions of the distributions since it considered each period's income as arguments of the function instead of the compound information carried by the index. Therefore, the stability index can be useful for giving a sense of the stability of an alternative, but it does not capture all the dimensions of the income path. The index helps us to discriminate among alternatives with different degrees of stability and gives a single valued measure that can be understood and compared. Full information with respect to the income patterns is contained by the vector b, although the multiple dimension of this indicator makes the comparison of stability impossible to handle.

Figure 6.1 shows graphically the impact of the minimum income on the stability premiums.

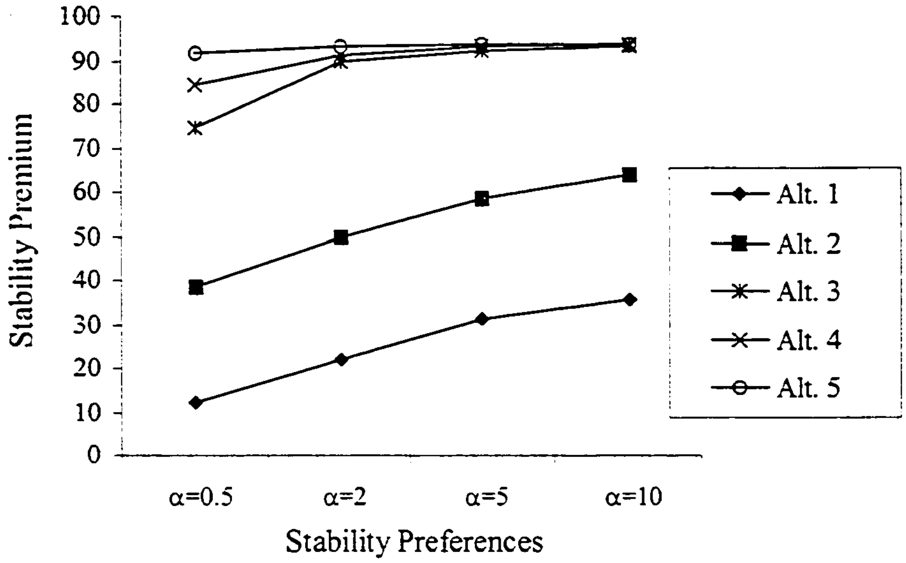

Figure 6.1: Stability premiums for alternatives 1, 2, 3, 4, and 5 given stability preferences (α) of 0.5, 2, 5, and 10 and a risk aversion (β) of 0.01.

For the two-period model, I concluded that the level of risk and the degree of risk aversion were not influential factors on the stability premium. The results of the analysis for the multi-period model with respect to the impact of risk aversion in the stability premium confirm this conclusion. Tables 6.6 and 6.7 present the stability premium for the Alternatives 1 and 5 for the four different degrees of stability preferences and for values of risk aversion $\beta = 0.0056$, $\beta = 0.01$, and $\beta = 0.02$.

Table 6.6. Stability premiums for alternative 1 at different degrees of risk aversion

	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$
$\beta = 0.0056$	12.33	21.99	31.25	64.35
$\beta = 0.01$	12.32	21.97	31.23	35.64
$\beta = 0.02$	12.31	21.94	31.18	35.59

Table 6.7. Stability premiums for alternative 5 at different degrees of risk aversion

	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$	$\alpha = 10$
$\beta = 0.0056$	91.65	93.38	93.73	93.84
$\beta = 0.01$	91.61	93.34	93.68	93.80
$\beta = 0.02$	91.53	93.24	93.59	93.70

After observing the two tables, I conclude that the level of risk aversion does not affect the stability premium paid for trading an uneven path for an even income path. This conclusion is what was expected considering the results with respect to the influence of the risk aversion level on stability premiums obtained for the two-period model. Again, it can be concluded that the independence of both parameters in the individual's taste is reflected in the premiums that he/she is willing to pay for stability.

Comparing Functional Forms

At the end of the previous section I developed a multi-period model using a different aggregator function for the certain prospect. This aggregator also falls into the class of the Constant Elasticity of Substitution (CES) functions, but its functional form is different. In this section, this new aggregator is tested to see if the model shows some sensitivity with respect to the functional form. To make the comparison, I used the stability premium for Alternatives 1, 2, and 3 under both models. The utility indices used for this purpose are presented in equation 6.10 and 6.13. The models were compared at a risk aversion level of 0.01 for the stability preferences of $\alpha = 0.5$, $\alpha = 2$, $\alpha = 5$, and $\alpha = 10$. Since the parametization with respect to stability is different in both expressions, in order to make the comparison for equivalent conditions of stability preferences, the EIS is used as a reference.

The EIS for equation 6.10 is:

$$EIS = \frac{1}{\alpha + 1}$$

The EIS for equation 6.13 is:

$$EIS = \frac{1}{\gamma}$$

The Table below shows the values for EIS and the parameter γ for the values of α used in this exercise.

Table 6.8. Equivalent values for EIS, α, and γ

α	EIS	γ
0.5⇒	0.67⇒	1.5
2 ⇒	0.33⇒	3
5 ⇒	0.17⇒	6
10 ⇒	0.09⇒	11

The results of the comparison show that there is no difference in the functional form chosen for the aggregator function. Both CES functions exhibit the same behavior and generate the same stability premium for the alternatives and parameter values used. It is important to notice that both functions relate to the same class of constant elasticity, and therefore it should be expected that relaxing this assumption will vary the results. The following Table shows the results of this comparison exercise.

Table 6.9. Comparison of aggregator functional forms in terms of stability premiums

	Alternative 1		**Alternative 2**		**Alternative 3**	
EIS	Model A	Model B	Model A	Model B	Model A	Model B
0.67	12.32	12.32	38.66	38.66	74.55	74.55
0.33	21.97	21.97	49.63	49.63	89.57	89.57
0.17	31.23	31.23	58.45	58.45	92.43	92.43
0.09	35.64	35.64	64.10	64.10	93.21	93.21

Summary

The sensitivity analysis for the GEU multi-period model reveals in general the same behavior as the GEU two-period model. The results generated in this section are the following:

- The ordering of the income path does not affect the premium paid for an even income path when analyzing alternatives with the same Present Value (PV) and with the same levels of income per period.
- In the GEU multi-period model, the premiums for higher values of α ($\alpha = 5$ and above) tend to converge to a unique number. The convergence is quicker for income paths with less stability.
- For high stability preferences, such as $\alpha = 10$, the minimum income of the alternative is practically defining the premium that the individual is willing to pay. This result indicates that for high stability preferences the individual behaves as if the incomes in each period are perfect complements.
- Finally, two different aggregator functions of the Constant Elasticity of Substitution (CES) form were tested in terms of the stability premiums. It was concluded that under both functional forms the premiums were exactly the same.

CHAPTER SEVEN

MODELING CONCLUSIONS

The main purpose of this dissertation was to create a new perspective to explain the decision-making process for a risky prospect over time. For decades, the distribution of expected income has been overlooked in explaining choices, and an example of this failure is the traditional cost-benefit analysis that has been used to analyze the eligibility of an investment. However, through the careful analysis of different research conclusions, it can be shown that the distribution of the income flow plays a role on decisions. Savings, inventory management, and technology adoption have been influenced by how the income is distributed over time, in which an even distribution appears to be part of the individual's objectives.

To achieve this main purpose, a generalization of the Expected Utility model was adapted to explain technology adoption decisions. This decision area was found appropriate to study factors affecting intertemporal decisions because it is, in many cases, a long run choice with a high degree of irreversibility. Although some questions remain with respect to the model, I believe that the adaptation successfully incorporates the desire of individuals for a more stable income fulfilling the main objective of this work and opening a new discussion topic in the agricultural economics literature.

In Chapters Five and Six, I presented and analyzed the two-period and multi-period GEU model to explain the technology adoption decision. In this Chapter, I will present the main results derived from the sensitivity analysis and their implications for policy tools. First, I will discuss the results and implications of comparing the GEU and EU model. Then, the main conclusions derived from the two-period model are discussed. The third section summarizes the multi-period model and its implications. Finally, the last section is dedicated to future challenges.

Comparing the EU and GEU Model to Explain Technology Adoption

The comparison between the EU and GEU model performed in Chapter 5 showed that both models generate similar rankings of alternatives that present different combinations of Present Value and stability level. This result leads to the conclusion that in the EU model the risk aversion coefficient plays an important role as a proxy for stability, where a high risk aversion level would imply a high stability preference and vise versa.

Why would be important to use the GEU approach, since both models predict similar results? Three arguments are relevant. First, the EU model cannot discriminate what is causing the ranking, risk aversion and risk level or stability preference and stability level. If the ranking is what is determining the adoption of one alternative over another, then not knowing the source of variability that is preventing the adoption of desirable technologies has important policy implications. Policy- and decision-makers will not be able to provide the appropriate tools that can improve the rate of adoption. The GEU model provides not only the ranking, but also the source of variability that causes the ranking. In this sense, policy- and decision-makers evaluating alternatives using the GEU model will be able to identify the variability source that needs to be intervened. If risk is what is causing low adoption rates, then tools orientated to manage or control risk like insurance, diversification, or forward contracts can be used. However, if stability is identified as the variable limiting adoption, then "income smoothing" tools will be the best incentive.

Second, the EU model is constrained so that the risk aversion coefficient is the inverse of the EIS, or in other words, is directly related to the stability preferences. This limitation does not allow for some combinations in risk and stability preferences (e.g. low risk aversion and high stability preferences or high risk aversion and low stability preferences), constraining the model to apply only to certain types of individuals.

Finally, in this exercise the characteristics of the alternatives and the values chosen for the stability and risk parameters generate consistent rankings between the GEU and EU model. However, the EU model is still limited in the characterization of the individual and in some situations will give incorrect answers related to the GEU model.

Two-period GEU Model: Sensitivity Analysis

The sensitivity analysis from the two-period GEU model yielded several results that have implications in constructing policy tools to encourage the adoption of certain technologies.

Analyzing the Present Value-stability trade-off it was found that the risk aversion degree and the risk level do not have an impact on the trade-off. This result can be explained, in part, by the property of independence between risk and stability. It was also shown that in the Present Value-risk trade-off, the stability preferences do not affect how much Present Value an individual is willing to trade for a unit less of risk. The policy implications of these results are that trying to encourage adoption targeting the wrong tools will not have any impact at all in the adoption rate. If for example, the source of variability that is causing low adoption is stability, a policy tool orientated to reduce risk should have a null impact on the rate of adoption at any level of risk aversion.

Continuing with the sensitivity analysis results, new terms were also created: stability premium and present value (PV) equivalent. To develop these terms the concepts of risk premium and certainty equivalent were borrowed and adapted. The definition of stability premium is related to the definition of the risk premium. It is: the amount of money that an individual with particular stability preferences would be willing to pay for a stable income stream. The PV equivalent is related to the certainty equivalent, and corresponds to the amount of money that makes an even alternative equally desirable to an uneven prospect. Both concepts are affected by the stability preferences of the individual. A high stability preference yields high premium and low PV equivalent as well as high risk aversion yield high risk premiums and low certainty equivalent.

The stability premium analysis showed two results: First, for high degrees of stability preferences and for highly unstable prospects, the stability premiums tend to converge. Second, the stability premiums estimated for individuals with high stability preferences are extremely high and in some cases unrealistic. The premium convergence property can be a helpful result when implementing financial and policy tools using stability premiums. This implies that individuals with high degree of stability preferences can be categorized in homogeneous group, making the management tool easier. On the other side the extreme stability premiums observed for the group of individuals with high stability preferences is certainly explained by the complementarity effect between periods that is generated for high degrees of preferences (close to perfect complements case). The minimum income in this case will determine the stability premium for the whole income stream. In the next section, the multiperiod GEU model analysis, I will return to a discussion about minimum income and its implications for policy instruments.

Finally, the two-period model analysis leads to the result that for any individual the risk level of an alternative becomes less important in the investment decision if the alternative presents a lower level of stability. From this result, I can conclude that variability, deterministic and stochastic, have a consistent impact on consumers, allowing us to say that in general individuals are "variability averse" rather than "risk averse".

The implication of being variability averse is that the individual will focus his/her attention in the variability source that is causing more variation in the outcome. If the stability level is causing most of the variation, then the individual will concentrate his/her effort in getting rid of the expected variability. If risk is the major cause of variability, then this factor will be the main focus.

The policy implications of the conclusion are directed again to identify the source of variability to design the most appropriate policy or financial instrument. Policymakers have to target traditional risk instruments when risk is the source of variability. On the other hand, if stability is the major source, policymakers have to target financial stability. Different tools can be used to generate financial stability. Financial tools are one alternative, in which a loan is designed make a technology alternative with an uneven distribution of income attractive for a group of farmers as it will be shown in the next

Chapter. Another alternative is to design a technology that includes desirable properties for the environment and rural development and that also provides an even cash flow. Following this direction, there have been experiences such as agroforestry systems that provide a more even cash flow compared to pure tree plantation and that also have desirable characteristics of a forest for the environment.

Multi-period GEU Model: Sensitivity Analysis

The results and conclusions derived from the two-period model can be extended to the multi-period GEU model. However, there is one property of the model that I will discuss in this section. It was already shown in the two-periods model that for high stability preferences the minimum income becomes an important factor in the individual's decision. In the multi-period model analysis this phenomenon was even easier to observe. Table 6.5 clearly shows the close relationship between the minimum income of the stream and the average PV equivalent, regardless of the stability level of the alternative.

The minimum income then is a critical factor when evaluating different technologies, and when the stability preference is higher this factor turn out to be more critical. Therefore, increasing the minimum income can increase the adoption rate of technologies in which this factor is limiting the adoption. In particular, technologies that have a smooth cash flow but with an initial investment that makes the technology not attractive to the individual may be promoted by subsidizing the investment. Experiences using these types of incentives to increase adoption rate are the most common as it will be shown in the next Chapter. Another alternative to address this limitation is to allow for borrowing in periods where the income becomes more critical.

Certainly the minimum income factor will have a higher impact when it is close to the minimum subsistence level. In this extreme case, and knowing that no capital mobility is allowed, I would expect that the stability preference of the individual will increase to infinity to become a perfect complement case.

Future Research and Challenges

The models developed in this dissertation can be extended to incorporate or relax other restrictions. In the previous section, the impact of the minimum income and what was expected when the minimum income is close the subsistence level was examine. The impact of the consumption subsistence level can be incorporated to evaluate its impact on the preferences and stability premiums. In this situation, and as it was explained before, the stability preferences should be affected and therefore the formulation of the model would need to be able to account for it.

Another extension of this basic model that is important to develop is relaxing the credit constraint imposed in this model. I expect that the stability premiums estimated in this dissertation will experience a significant drop if the individual has some capital mobility.

Finally, the theoretical work done here leaves an important challenge to the empirical field. Although the model seems to perform well in explaining the importance of stability in the individuals' behavior, its estimation appears to be a hard task because of the nature of data that would be needed. In the first place, trying to statistically disentangle the stochastic and the deterministic variability components of an income prospect would require a large set of panel data. Defining the response variable for the empirical model could also be hard. A binary choice model, in which adoption and no-adoption are the categories for the dependent variable, could be an alternative. However, classifying adoption and no-adoption will not be easy when different degrees of adoption exist.

On the other hand, the complex functional form of the reduced form solution of the model make it hard to recover the initial parameters and have a measure of the Elasticity of Intertemporal Substitution (EIS) and degree of risk aversion. In trying to solve the estimation problems, it seems to me that using data derived from experiments would be a good alternative. In the past, experiments to elicit risk preferences have been successfully done, and the nature of this model makes the experimented approach a good bet for estimation purposes.

The purpose of this Chapter was to group the main results obtained by the different exercises done in Chapters Five and Six, and to discuss the implications that these results have for constructing policy instruments that involved decisions about technologies with different combinations of stability preference, Present Value and risk level. The implications discussed here will be used in the next Chapter, where the multi-period GEU model is applied to address a forestry case in which the objective is to increase the adoption of tree plantation to improve environmental conditions of the area under study.

CHPATER EIGHT

IMPLICATIONS FOR FINANCIAL AND POLICY TOOLS: AN APPLICATION TO FORESTRY ADOPTION

The model approach developed in the previous Chapters has the ability to capture aspects in an investment decision that expand the traditional cost-benefit criterion. The stability of the income path of an investment alternative, as well as the minimum income in the path, are new elements that can be incorporated through this modeling approach into decision analysis.

This different perspective in the decision-making process offers new insights in developing policy tools and financial instruments that will encourage people to adopt practices that they might not otherwise. The objective of this Chapter is to show how the multi-period model developed in Chapter Six can be used to evaluate and construct instruments that offer new incentives. The application is focused on promoting a forest plantation, with an uneven distribution of the cash flow, to replace a traditional productive alternative such as livestock, which typically yields an even cash flow. The Chapter is divided in three sections. First, a brief review of the situation of the world forest sector is given. The second section presents and analyzes a particular example comparing a woodlot alternative and the traditional activity of the area. The third and last section will explore a financial instrument that can contribute to the policy incentives offered currently by international organizations and governments to promote forestry.

Literature Review

Deforestation and resource degradation in developing countries, primarily in the humid tropics, has been a large concern and research objective for the last two decades (Current et.al., 1995; Kindall and Pimentel, 1994). Recent studies have found that the loss of forest cover is occurring at a rate of 17 to 20 million hectares per year, despite efforts from government and international organizations to stop the forest clearing (International Finance Corporation, 2000). According to Liu and Lu (1992), degradation of land resources is occurring in three major ways: slash and burn farming in tropical areas, increased land conversion in temperate climates for settled agriculture, and high rates of inappropriate timer harvesting. The areas most affected by this phenomenon are tropical Africa, the forest areas of Latin America, and Southeast Asia.

The consequences this phenomenon carries are varied and wide. First, it has a local effect on reducing land productivity, leading to an unsustainable ecosystem in the long run. Even in systems with high potential for biological productivity, such as the tropical ecosystem, land productivity has been declining at a high rate through inappropriate and abusive land use (Vergara, 1987). Second, deforestation has a global impact, consisting of an increase of CO_2 and other greenhouse gases in the atmosphere that affect the ozone of the whole earth (Southgate et.al., 1992). Finally, wood is a source of fuel that, under certain prices, could be more profitable than oil or coal. According to Tansini (1994), the only way to meet the current demand for wood and to avoid future shortages will be by stopping the current deforestation rate.

The causes of deforestation have been widely discussed in the literature, and most agree that the primary causes are misdirected government policies, population pressure, and inappropriate land tenure (Angelsen and Kaimowitz, 1999; Sandler, 1993; Southgate 1992). As an example, the approximately 300 million small farmers that live in the tropical and mountain ecosystem, are believed to be the primary source of this land clearing and degradation (Barret, 1999; Southgate, 1990, and Köhlin, 1994). Knerr (1992) explained that the misdirected policies are in the form of agricultural policy, namely producer's incentives and fiscal advantages for agricultural projects.

The alarming deforestation rate that has been experienced over the last two decades has led international organizations and governments to adopt initiatives that would reverse or at least stop this process. For example, in almost all European countries there are policies to support farmers who convert their agricultural land to forestry. The World Bank has also taken actions to contribute to the afforestation of the most affected areas.

In 1991, the World Bank developed a strategic plan to improve the situation in the forest sector, in which it states clearly that its objective for the sector is: "*to reduce deforestation, enhance the environmental contribution of forested areas, promote afforestation, reduce poverty and encourage economic development.*" (International Finance Corporation, 2000).

Another action pointing in the same direction was the 1991 creation of the Global Environmental Facility (GEF), whose objective was to provide new financial mechanisms to protect the global environment. Approximately 16% of the total GEF budget is allocated to projects primarily concerned with protecting or managing forest ecosystems or resources (Campbell and Martin, 2000).

Despite all these efforts and according to the OECD Report, the strategy developed by the Word Bank has failed to combat the loss in forest cover since only a 12% decline in loss has been observed since 1991. This reduction still leaves a significant rate of annual deforestation (International Finance Corporation, 2000).

The reasons why the results of many of these programs are not what was expected are varied and particular to each the situation. In Europe, the main policy tools to promote forestry have been taxation and subsidies. However, financial support for forestry measures is at a low level and only part of the costs of forestry can be covered by public support (Hyttinen, 2001). According to Alhojärvi (2001), international financing institutions such as World Bank, Nordic Investment Bank, or the European Investment Bank, have a lack of sufficient financial instruments for developing these types of projects.

From the experience in Latin America and the Caribbean, it is concluded that a re-evaluation of the policy objectives and instruments is needed to ensure cost-effectiveness and to provide the right support for the emerging farm forestry sector. This should be based upon a solid empirical understanding of actual farmer cost and benefits to forestry (Current and Scherr, 1995). In this sense, Hermelin (2201) also recognizes that policy makers and developers of public woodlot support programs must take into account the socio-economic realities of the "Rural Business Enterprise". The challenge is to not only focus on the production of fiber, but to include the socio-economic aspects of owning and operating a woodlot and its associated activities.

Certainly, the list of possible reasons that explain the partial success of the programs can grow with the cases that are analyzed. However, there are some common points in order to built alternative instruments and tools that would improved past results. Köhlin (1994) believe that the fundamental challenge is to create a favorable incentive structure for investing in soil conservation, agroforestry, and reforestation.

Current and Scherr (1995) conclude that the most successful incentives were those providing minimal in-kind material inputs, combined with technical support for seedling production and planting. They recommend tools that provide minimal in-kind incentives and subsidies, as a temporary measure that will be eliminated as the benefits from tree planting become evident.

In a broader sense, the World Bank recommendations to improve the forest sector recognize the need to continue identifying and operationalizing financial mechanisms for sustaining conservation funding, including trust funds, royalties and fees, conservation easements and tax mechanisms, and reverse funds (Campbell and Martin, 2000).

The policy recommendations listed above agreed in considering that financial support could be a powerful tool to improve the rate of reforestation. Assuming that this conclusion is correct, I propose that constructing financial instruments that meet a farmer's preference for stability in the expected income path might constitute one of the answers that the organizations and governments involved in forest programs are looking for. If the GEU model represents individual adoption behavior, this method may constitute an important guide for developing instruments that could improve past results related to reforestation rate.

In the following section, an example is presented to explain how such an instrument can be designed to provide incentives for tree planting in an area where livestock production is the main activity.

Application: Woodlots versus Livestock

The example presented in this section was taken from a case study of the impact of projects promoting tree planting in Costa Rica done by the Current (1995). In Costa Rica the agricultural sector contributes with 20% of the Gross National product (GNP) and 28% of the national employment. The main agricultural products are banana, coffee and beef. The importance of agricultural and livestock to the national economy has put pressure to clear forests and increase agricultural production. As a consequence, the total forest area dropped from 53% in 1950 to 29% in 1987, the time when most of the reforestation initiatives started.

The government began promoting tree plantations in 1988, offering subsidies to organized, small and medium landowners. The example used here corresponds to the area of Hojancha, which is one of the areas that benefited from these subsidies and support of extension programs. With respect to the results of this program, Current concluded that tree plantations provided benefits to the farmers and the community as a whole; however, questions remain about the level of adoption.

In Hojancha, livestock production is the main economic activity. The dominant farming systems can be classified as follows:

- Landless farmers: who receive their sustenance through labor wage and rent 1 to 3 hectares to produce crops for family consumption.
- Small farmers at the subsistence level: who own 6 to 30 hectares and dedicate approximately 50% of the agricultural gross margin to household consumption.
- Small farms producing cash crops: usually for the production of coffee and vegetables, often owning a few head of cattle.

- Medium livestock production: with an average farm size between 30 to 150 hectares.

The forestry alternatives for the area are block planting and trees on boundaries. Block-planting will be examined because it represents the majority of the systems established due to the incentives explained above. The principal species planted is the *Gmelina arborea.* The benefit of this alternative is income generation through the sale of wood, which is high, although the pay back period is long. The species has a cycle of 13 years, at which it can be cut and sold. Table 8.1 describes the characteristics of the forestry alternative and the main activity in the area.

Table 8.1. Description of woodlot and livestock systems in the Hojancha region in Costa Rica[a]

	Woodlot	Livestock
Description	Gmelina arborea 1100 trees/ha. 13 year cutting cycle	Beef cattle for fattening and sale.
Unit of analysis	1 hectare	1 hectare
Net Present Value of net Income (10%)	USS 1,116	USS 658
Pay back period	13 years	1 year
Cash flow distribution	Zero until year 13	Perfectly even
Stability Index	0	1
Risk level (σ)	20% of each period income	20% of each period income

[a] Based on Current (1995)

The table was constructed with the data published in Current (1995). The data were obtained by interviewing farmers who had planted woodlots and have livestock production. However, I made assumptions about the distribution of the income over time and the risk level associated with each activity.

According to the description of the woodlot, it was assumed that the only income obtained was after the cutting it in year 13. For livestock production, a perfectly even distribution of the expected net income was assumed since there is no reason to assume that it should vary across time. Finally, the risk level was assumed proportionally equal for both alternatives and represented by a standard deviation (σ) equal to 20% of the annual net income.

For this exercise, three combinations of woodlot with livestock alternatives were compared to 100% livestock. Table 8.2 shows basic information with respect to the alternatives used in the analysis. The Present Value (PV), estimated at a 10% discount rate, stability index (SI), and the simulated discounted expected income path are presented for each alternative. All the information is based on a unit of 1 hectare.

Table 8.2. Analysis of the proposed woodlot alternatives

ALTERNATIVES	**80% woodlot 20% livestock**	**50% woodlot 50% livestock**	**20% woodlot 80% livestock**	**100% livestock**
Present Value - PV (10%)	S 1022.8	S 883	S 743.2	S 650
Stability Index - SI	0.127	0.368	0.699	1
Present Value advantage[a]	S 372.8	S 233	S 93.2	S 0
Income year 1	S 10	S 25	S 40	S 50
Income year 2	S 10	S 25	S 40	S 50
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
Income year 13	S 902.8	S 583	S 263.2	S 50

[a] The PV advantage is the difference between the PV of the alternative and the lowest PV.

According to the information presented in Table 8.2, the alternatives present an interesting PV-stability trade-off. The alternative that has the highest present value has the lowest stability index, while the alternative with the lowest present value (the current activity) has a perfectly even expected distribution of discounted income[31]. The higher Present Value of the less stable alternatives constitute a PV advantage that can be used to design an incentive to promote the adoption of these alternatives. The PV advantage is presented in the third row of Table 8.2. For example, the alternative 80% woodlot – 20% livestock has a $ 372.8 PV advantage over the alternative 100% livestock, yet the livestock is the dominant system produced in the region.

The plan to analyze this case and to decide the best policy alternative is to first estimate the minimum premium that needs to be paid to the individual for trading the current activity for the woodlot alternatives. This premium constitutes the subsidy that the development organization needs to pay the farmer. The next step in the plan is to analyze the feasibility of a financial instrument as a policy tool.

According to the plan, I estimate the minimum premium needed to make a farmer indifferent between the woodlot alternative and the current activity of 100% livestock. The premiums for stability preferences of $\alpha = -0.5$, $\alpha = 0.1$, $\alpha = 0.5$, $\alpha = 2$, and $\alpha = 5$ are presented in Table 8.3.

In Chapters Five and Six, I concluded that the results from premiums did not vary significantly for α equal to 5 and above, therefore only low values were considered in this exercise. The remaining parameters and variables are: risk level of $\sigma^2 = (0.2 \cdot$ expected

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[31] It is important to remember that the Stability Index SI is estimated using discounted values of current values.

income)^2, a moderate risk aversion degree $\beta = 0.01$, and market and personal discount factors of 0.91 (discount rate of 10%).

Table 8.3. Premium for trading woodlot alternatives for 100% livestock

Stability preferences

	$\alpha = -0.5$	$\alpha = 0.1$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 5$
80% woodlot **20% livestock**	846.39	2830.54	3410.66	3889.6	4009.05
50% woodlot **50% livestock**	170.73	450.52	673.91	813.63	854.73
20% woodlot **80% livestock**	-3.15	70.08	100.56	150.18	170.99

Table 8.3 shows that for higher stability preferences the premiums are higher, and that for alternatives with more stability, premiums are lower. Notice that for the alternative 20% woodlot – 80% livestock, for a preference degree of $\alpha = -0.5$, the premium is negative meaning that the individual would prefer the alternative offered rather than 100% livestock. In all the other cases 100% livestock would be preferred. As explained before, the values shown in Table 8.3 can be considered as the subsidy that the organizations involved in promoting tree plantation would need to pay to farmers to provide the appropriate incentive to promote the adoption of these particular alternatives. The Table shows that for alternatives more intensive in woodlot, extremely high subsidies would be needed and would likely be impossible to implement.

For example, for the alternative 80% trees – 20% livestock even the lowest premium, $ 864, exceeds the Present Value advantage of $ 372.8 ($ 1022.8 - $650).

Another way to look at premiums is to determine at what degree of stability preference the individual would be indifferent between the alternative and the current activity. The following table shows the α value (α - break point) that would make 100% livestock and the three combinations of trees and livestock equally preferable.

Table 8.4. Break point for parameter α when trading woodlot alternatives for 100% livestock

Alternative	α - break point
80% woodlot 20% livestock	-0.782
50% woodlot 50% livestock	-0.701
20% woodlot 80% livestock	-0.450

In all cases, in order to make both alternatives equally preferable, the α values are close to the limit –1. The most uneven alternative (80% woodlot- 20% livestock) has the lowest α - break point, while the more stable alternative (20% woodlot- 80% livestock) has the highest α - break point. Since the α - break point values are close to the lower limit, it can be concluded that there exists a broad group of individuals for whom the alternative 100% livestock is more preferable over combinations of woodlot and livestock.

The information presented above clearly shows that the minimum subsidy that developing organizations need to pay to make woodlot an attractive alternative is in many cases extremely high and would be likely impossible to implement successfully. Following the plan presented earlier, the next step is to analyze the possibility of implementing a financial instrument. Woodlot alternatives offer a PV advantage that can be used for this purpose.

Considering the high impact of stability preferences on the farmers decisions[32], a policy tool that could be successfully implemented to encourage farmer's to adopt technologies with uneven distribution of the cash flows, is a financial instrument that provides farmers with a constant flow over the years, such as a Cumulative Reverse Equity Mortgage. In the next section this instrument is designed and adapted to different situations.

Cumulative Reverse Equity Mortgage: Design and Implementation

The financial instrument designed here consists of a loan that makes the distribution of the income over time perfectly even based on the income flow of an alternative that has an uneven distribution. In the case of the alternatives offered in this exercise, the loan will provide a farmer an amount of money for the first twelve years of the cycle that will be returned the last year, when trees are cut and sold. The amount of money that will be lent each year and that will be returned the last year is estimated using the annual income flow presented in Table 8.2. I am creating the term "Cumulative Reverse Equity Mortgage" (CREM) to mean a standard reverse equity mortgage based on equity that builds over time, accumulates as trees grow. The full equity value is not realized until the end of the cycle.

The first step in designing a CREM is to know the Present Value (PV) equivalent of the alternatives with different degrees of stability preferences. The PV equivalent can be related to the certainty equivalent in risk analysis and can be defined as the amount of

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[32] The high stability premiums exhibited in Table 8.3 also show the importance that stability has on the farmer's decision.

money that would make the uneven alternative equally attractive to an alternative that offers an individual a perfectly even distribution. Therefore, this value constitutes the reference to construct the loan. The individual will at least receive the PV equivalent distributed evenly across time in exchange for the income flow yield by the productive alternative. The following table shows these values:

Table 8.5. Present Value Equivalent for woodlot alternatives at different stability preferences

Stability preferences

	α = -0.5	α = 0.1	α = 0.5	α = 2	α = 5
80% woodlot 20% livestock	355.70	172.61	150.04	135.42	132.21
50% woodlot 50% livestock	544.69	430.41	368.67	338.33	330.33
20% woodlot 80% livestock	652.76	593.98	572.51	553.69	528.43

The values presented in Table 8.5 are what the individual is willing to accept instead of the original alternative if he/she is offered an even distribution of income instead of the original uneven distribution. Notice that these values are lower than the PV yielded by the 100% livestock alternative. If the individual is offered a loan that gives him/her the PV equivalent, he/she will not accept it because livestock is still a better alternative. The reference value to construct the loan is the $ 650 that the livestock offers the individual for a 13-year cycle. It is also important to notice that this value would be the same reference for all the different alternatives at the different degrees of stability preferences considered in the exercise. The only exception is the alternative 20% trees – 80% livestock for individuals with α values of –0.5 and below. These individuals would exchange the flow of the alternative for an even flow that yields a PV of $ 652.76.

Table 8.6 shows the CREM designed for the alternative 80% trees – 20% livestock. To construct the loan I used the reference value S650, which is the lowest amount of money that the lender institution can give farmers to encourage them to adopt the alternative, and therefore, is the amount that would yield the highest return to the lender.

The table can be explained as follows: The second column presents the expected income that the alternative in analysis yields in current terms. The flow was capitalized at a rate of 10%, which is the discount rate used to obtain the present value of the alternative[33]. To construct the CREM, it was necessary to use the current values instead of discounted values to make the estimation of the Internal Rate of Return to the lender institution. The third column presents the assumed risk level (in current terms) for the alternative. The fourth column presents the value that the individual expects to get if he/she takes the loan. To estimate this value the reference PV (S 650) is divided by 13 and then capitalized by 10%, which is the individual's time preference rate. It was already explained that PV below S 650 are no longer attractive for the individual since he/she could be getting S 650 by continuing to produce livestock. Columns five and six present the loan and payments that the individual should be receiving and paying if he/she adopts the alternative offered with the reverse found. Both columns are estimated by taking the difference between the annuity and the expected income of each year. Column five presents the negative difference and column six the positive difference between column four and two. Finally, the seventh column presents the expected flow for the institution

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[33] Remember that it was also assumed that the personal and market discount rates are 10%.

that gives the loan. The Internal Rate of Return was estimated using this flow, which in this case is 19.4 %.

Table 8.6. Cumulative Reverse Equity Mortgage for the alternative 80% trees – 20% livestock using the minimum reference value

Year	Expected Income (current $)	Risk level (current $)	Annuity (current $)	Loan (current $)	Payments (current $)	Bank Flow (current $)
1	10.00	2.00	50.00	40.00	0.00	-40.00
2	11.00	2.20	55.00	44.00	0.00	-44.00
3	12.10	2.42	60.50	48.40	0.00	-48.40
4	13.31	2.66	66.55	53.24	0.00	-53.24
5	14.64	2.93	73.21	58.56	0.00	-58.56
6	16.11	3.22	80.53	64.42	0.00	-64.42
7	17.72	3.54	88.58	70.86	0.00	-70.86
8	19.49	3.90	97.44	77.95	0.00	-77.95
9	21.44	4.29	107.18	85.74	0.00	-85.74
10	23.58	4.72	117.90	94.32	0.00	-94.32
11	25.94	5.19	129.69	103.75	0.00	-103.75
12	28.53	5.71	142.66	114.12	0.00	-114.12
13	2833.37	566.67	156.92	0.00	2676.45	2676.45
				Internal Rate of Return : 19.4%		
Present Value Expected Income (10%)				$ 1022.8		
Present Value Annuity (10%)				$ 655.2		
Personal Discount Rate				10%		

Table 8.7 presents the CREM constructed using as reference value the actual Present Value of the alternative ($ 1022.8), which is the maximum value that the institution can offer the farmer. In this case the return for the lender is 10%.

Table 8.7. Cumulative Reverse Equity Mortgage for the alternative 80% trees – 20% livestock using the maximum reference value

Year	Expected Income (current $)	Risk level (current $)	Annuity (current $)	Loan (current $)	Payments (current $)	Bank Flow (current $)
1	10.00	2.00	78.68	68.68	0.00	-68.68
2	11.00	2.20	86.54	75.54	0.00	-75.54
3	12.10	2.42	95.20	83.10	0.00	-83.10
4	13.31	2.66	104.72	91.41	0.00	-91.41
5	14.64	2.93	115.19	100.55	0.00	-100.55
6	16.11	3.22	126.71	110.60	0.00	-110.60
7	17.72	3.54	139.38	121.67	0.00	-121.67
8	19.49	3.90	153.32	133.83	0.00	-133.83
9	21.44	4.29	168.65	147.22	0.00	-147.22
10	23.58	4.72	185.52	161.94	0.00	-161.94
11	25.94	5.19	204.07	178.13	0.00	-178.13
12	28.53	5.71	224.47	195.94	0.00	-195.94
13	2833.37	566.67	246.92	0.00	2586.45	2586.45
				Internal Rate of Return : 10%		
Present Value Expected Income (10%)				$ 1022.8		
Present Value Annuity (10%)				$ 1022.8		
Personal Discount Rate				10%		

The loans presented in tables 8.6 and 8.7 are the extreme cases, it is clear that the lender can use intermediate references for Present Value. The return that will be generated for the lender will fall between 10% and 19.4%, while the discounted annuity generated for the farmer will fall between $ 50 and $ 78.68 a year.

Knowing the possible range for the rate of return, the lender institution can target a specific return according to its cost of capital. Traditionally, the rate that the lender institution targets covers the rate of a risk-free investment alternative and the risk premium associated to the activity. An additional spread for stability may also be included. Let us consider that the organization implementing the reverse found to encourage the adoption of the alternative 80% trees – 20% livestock has a target return for their capital of 15%. Using this reference, a loan can be constructed that will give the farmer an annuity of S 61.85. The following table shows the loan that targets a 15% return for the organization.

Table 8.8. Cumulative Reverse Equity Mortgage for the alternative 80% trees – 20% livestock for a rate of return of 15%

Year	Expected Income (current S)	Risk level (current S)	Annuity (current S)	Loan (current S)	Payments (current S)	Bank Flow (current S)
1	10.00	2.00	61.85	51.85	0.00	-51.85
2	11.00	2.20	68.03	57.03	0.00	-57.03
3	12.10	2.42	74.83	62.73	0.00	-62.73
4	13.31	2.66	82.32	69.01	0.00	-69.01
5	14.64	2.93	90.55	75.91	0.00	-75.91
6	16.11	3.22	99.60	83.50	0.00	-83.50
7	17.72	3.54	109.56	91.85	0.00	-91.85
8	19.49	3.90	120.52	101.03	0.00	-101.03
9	21.44	4.29	132.57	111.14	0.00	-111.14
10	23.58	4.72	145.83	122.25	0.00	-122.25
11	25.94	5.19	160.41	134.48	0.00	-134.48
12	28.53	5.71	176.45	147.92	0.00	-147.92
13	2833.37	566.67	194.10	0.00	2639.27	2639.27
				Internal Rate of Return : 15%		
Present Value Expected Income (10%)				S 1022.8		
Present Value Annuity (10%)				S 804		
Personal Discount Rate				10%		

Implementing the CREM has benefits for all the parties involved in the process. Implementing the CREM would yield the farmer a PV of S804, higher than the S 650 generated by livestock, and the lender will obtain a 15% of return over its capital.

Moreover, the environment is benefited by increasing the forestation rate, and the economy as a whole by increasing the value of the agricultural production.

Until now, the analysis of the CREM has been based on the expected value of the productive alternative. However, realizations of income could vary according to the risk level of the alternative, carrying this variation to the annuity and/or the loan and payments for the lender institution. In the next exercise, I considered a risk level measured in standard deviation (σ) of 20% of the expected income. This means that there exists only a 2.5% probability that the income will fall below 40% of the expected income. For the alternative 80% trees – 20% livestock it implies that in the first year there is only a 2.5% probability that the income will fall below S 6.

There are three approaches that can be implemented to deal with this risk. The first approach is to let the farmer deal with the risk of the productive alternative giving him with certainty the loan and payments programmed with the expected income. This means that the farmer will receive column five of tables 8.6, 8.7, or 8.8 and pay column six. The other approach is to let the lender deal with the risk of the activity by assuring the farmer the annuity estimated to set the loan. Finally, the third approach is a formula that combines the first two approaches. It is evident that the agent that bears the risk or the biggest share of risk, is also getting the opportunity to make an income higher than the expected value.

Finally, I show the impact on the rate of return of implementing different combinations of trees and livestock. Table 8.9 presents the CREM that will be needed for the three alternatives presented in Table 8.2 using as reference a present value of S 650. As can be seen, all combinations of trees and livestock offer the lender the same rate of return because the expected income in each case is a linear combination of the two alternatives income.

Table 8.9. Comparing the Cumulative Reverse equity Mortgage for different woodlot alternatives

Year	Annuity (current $)	80% trees-20% livest. Bank Flow (current $)	50% trees-50% livest. Bank Flow (current $)	20% trees-80% livest. Bank Flow (current $)
1	50.00	-40.00	-25.00	-10.00
2	55.00	-44.00	-27.50	-11.00
3	60.50	-48.40	-30.25	-12.10
4	66.55	-53.24	-33.28	-13.31
5	73.21	-58.56	-36.60	-14.64
6	80.53	-64.42	-40.26	-16.11
7	88.58	-70.86	-44.29	-17.72
8	97.44	-77.95	-48.72	-19.49
9	107.18	-85.74	-53.59	-21.44
10	117.90	-94.32	-58.95	-23.58
11	129.69	-103.75	-64.84	-25.94
12	142.66	-114.12	-71.33	-28.53
13	156.92	2676.45	1672.78	669.11
80% trees – 20% livestock Present Value Expected Income (10%)			$1022.8	
50% trees – 50% livestock Present Value Expected Income (10%)			$ 883	
20% trees – 80% livestock Present Value Expected Income (10%)			$ 743.2	
Present Value Annuity (10%)			$ 650	
Personal Discount Rate			10%	

Concluding Remarks

In this application, I showed one use of the multi-period GEU model developed in this dissertation. Being able to measure the preferences for a stable income path in evaluating different technology alternatives provides a useful tool for policymakers in constructing the appropriate incentive for specific groups of farmers. The example shows that the use of a CREM may be a powerful tool to promote tree plantation. However, it is important to mention that the management of this type of tool could not be easy because of the length of the technology cycle and a clear incentive for an "adverse selection" or "moral hazard" behavior. A possible solution for the length of the cycle is to generate a technology option that shortens the pay back period of the loan. In this situation, the technology offer would include the reverse found, supplying them as a package. Adverse selection and moral hazard can be minimized by the treatment of the risk component of the alternative or by the contract of collateral. If a high proportion of the risk level is bared by the farmer, or if a collateral obligation is exists, the incentives to misbehave will be reduced.

As shown in this Chapter, using the CREM to encourage tree plantations is one possibility of using the concept of stability and the model that uses stability preferences as an argument. This CREM can also be extended to encourage the adoption of other alternatives or high technology production systems.

Finally, commercial banks can also use this "income smoothing" principle to design short-term loans to manage annual farm production or a loan system that aids inventory management. A one-year period loan can be a profitable instrument for banks and farmers. It can help farmers to smooth their monthly income based on the expected production schedule and help commercial banks to increase their financial offer to the farm sector.

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APPENDIX I

ESTIMATING ELASTICITY OF INTERTEMPORAL SUBSTITUTION

The Elasticity of Intertemporal Substitution (EIS) is defined over the utility index presented in equation 5.1:

$$U(C_1,C_2)=-\frac{1}{\alpha}(C_1^{-\alpha}+\rho\cdot\hat{C}_2^{-\alpha})$$

The EIS can be defined as:

$$EIS=\frac{d\ln({C_1}/{\hat{C}_2})}{d\ln|MRS|} \qquad \text{(AI.1)}$$

Where MRS is the Marginal Rate of Substitution and is estimated as follows:

$$|MRS|=\frac{\partial U(C_1,\hat{C}_2)/\partial C_1}{\partial U(C_1,\hat{C}_2)/\partial\hat{C}_2} \qquad \text{(AI.2)}$$

For the utility index presented above the solution for the absolute value of the MRS is:

$$|MRS|=\frac{1}{\rho}\cdot\left(\frac{\hat{C}_2}{C_1}\right)^{(1+\alpha)} \qquad \text{(AI.3)}$$

The natural logarithm of the absolute value of MRS is:

$$\ln\left|MRS\right| = -\ln(\rho) + (1+\alpha)\ln(\tfrac{\dot{C}_2}{C_1}) \qquad \text{(AI.4)}$$

Taking the total differential of this last expression, I obtained:

$$d(\ln\left|MRS\right| = (1+\alpha)\cdot d\ln(\tfrac{\dot{C}_2}{C_1}) \qquad \text{(AI.5)}$$

Observing the resulting expression in A1.5 it can be seen that it has the two derivatives that define the EIS presented in A1.1. Rearranging the terms in A1.5, the EIS is estimated as follows:

$$EIS = \frac{d\ln({}^{C_1}\!/_{\dot{C}_2})}{d\ln\left|MRS\right|} = \frac{1}{(1+\alpha)} \qquad \text{(AI.6)}$$

APPENDIX II

ESTIMATING THE CERTAINTY EQUIVALENT OF THE RANDOM INCOME

In this Appendix, it will be shown how equation 5.9 was obtained. The certainty equivalent of the random income in period two is defined as follows:

$$\hat{I}_2 = \frac{\ln E(e^{-\frac{\beta}{r}(PV\ (1-b)+\varepsilon)})}{-\beta}$$

Where the expected value ($E(I_2)$) and variance ($V(I_2)$) of the random income are:

$$E(I_2) = (PV(1-b)) \cdot \tfrac{1}{r}$$
$$V(I_2) = \sigma^2 \cdot \tfrac{1}{r^2}$$

The first step in solving the certainty equivalent is to solve the expected value involved in the expression. As it was explained in Chapter 5 and 6, the solution was found by inspection. The following are the steps involved in the solution:

1. Solving expected value:

$$E(e^{-\frac{\beta}{r}(PV(1-b)+\varepsilon)}) = \int_{\infty}^{+\infty} e^{-\frac{\beta}{r}(PV(1-b)+\varepsilon)} \cdot \frac{1}{\sqrt{2\pi\sigma^2/r^2}} \cdot e^{-\frac{1}{2}\frac{((PV(1-b)+\varepsilon)\frac{1}{r}-PV(1-b)\frac{1}{r})^2}{\sigma^2/r^2}} d(PV(1-b)+\varepsilon)\tfrac{1}{r} \quad \text{(AII.1)}$$

In order to make the integral easier to handle let say that:

$$(PV(1-b)+\varepsilon)\tfrac{1}{r}=x$$
$$E(PV(1-b)+\varepsilon)\tfrac{1}{r}=\mu$$

Then the expected value shown in equation A2.1 can be rewritten as:

$$E(e^{-\beta\cdot x})=\int_{\infty}^{+\infty} e^{-\beta\cdot x}\cdot\frac{1}{\sqrt{2\pi\sigma^2/r^2}}\cdot e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2/r^2}}\,dx \qquad \text{(AII.2)}$$

Rearranging terms in the integral it can be obtained:

$$E(e^{-\beta\cdot x})=\int_{\infty}^{+\infty}\cdot\frac{1}{\sqrt{2\pi}\,\sigma^2/r^2}\cdot e^{-\frac{1}{2}\frac{(x^2-2x(\mu-\beta\sigma^2)+\mu^2)}{\sigma^2/r^2}}\,dx \qquad \text{(AII.3)}$$

In the term in parenthesis included in the exponent of equation A2.3, the following terms will be added and subtracted:

$$1.(\beta\sigma^2/r^2)^2$$
$$2.(2\beta\sigma^2/r^2\,\mu)$$

The resulting expression after some manipulations is the following:

$$E(e^{-\beta\cdot x})=\int_{\infty}^{+\infty} e^{-\frac{1}{2}\cdot\frac{(2\beta\sigma^2/r^2\,\mu-(\beta\sigma^2/r^2)^2)}{\sigma^2/r^2}}\cdot\frac{1}{\sqrt{2\pi\sigma^2/r^2}}\cdot e^{-\frac{1}{2}\cdot\frac{(x-(\mu-\beta\sigma^2/r^2))^2}{\sigma^2/r^2}}\,dx \qquad \text{(AII.4)}$$

The first exponential term contain only parameters and therefore it can be taken out of the integral:

$$E(e^{-\beta\cdot x})=e^{-\frac{1}{2}\cdot\frac{(2\beta\sigma^2/_{r^2}\mu-(\beta\sigma^2/_{r^2})^2)}{\sigma^2/_{r^2}}}\int_{\infty}^{+\infty}\cdot\frac{1}{\sqrt{2\pi\sigma^2/_{r^2}}}\cdot e^{-\frac{1}{2}\cdot\frac{(x-(\mu-\beta\sigma^2/_{r^2}))^2}{\sigma^2/_{r^2}}}dx \qquad \text{(AII.5)}$$

The term in the integral defines the likelihood function for a normal distribution that is integrated over the complete range of possible values. Therefore the value of the integral is 1 and the expected value converges to:

$$E(e^{-\beta\cdot x})=e^{-\frac{1}{2}\cdot\frac{(2\beta\sigma^2/_{r^2}\mu-(\beta\sigma^2/_{r^2})^2)}{\sigma^2/_{r^2}}} \qquad \text{(AII.6)}$$

Replacing μ and x by their original values the expected value is:

$$E(e^{-\frac{\beta}{r}PV(1-b)})=e^{-\frac{1}{2}\cdot\frac{(2\beta\sigma^2/_{r^2}(PV(1-b)\frac{1}{r}-(\beta\sigma^2/_{r^2})^2)}{\sigma^2/_{r^2}}} \qquad \text{(AII.7)}$$

2. Solving the certainty equivalent:

After replacing the expression A2.7 in the certainty equivalent I obtain:

$$\hat{I}_2 = \frac{\ln e^{-\frac{1}{2r}(PV\ (1-b)-(\beta\sigma^2/_r)^2}\ )}{-\beta} \qquad \text{(AII.8)}$$

Simplifying equation A2.8 the final result is:

$$\hat{I}_2 = \tfrac{1}{r}(PV\ (1-b) - \tfrac{1}{2}\cdot\beta\,\sigma^2/_r) \qquad \text{(AII.9)}$$