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HYDRAULIC HEAD LOSS AT THE INTERFACE BETWEEN UNIFORM SANDS OF DIFFERENT SIZES

Frank N. Leatherwood and Dean F. Peterson, Jr.

Abstract--The hydraulic head loss for flow occurring at an interface between sands or gravels of different sizes was studied by experiment. Dimensional analysis was used in formulating the experiments and in analyzing the data which led to a simple relationship in which the hydraulic head loss at the interface is a function of the mean diameter of the smaller-sized sand or gravel, the Reynolds number for the flow through the smaller-sized gravel, and an empirical constant which appears to depend principally on the mean size and size distribution characteristics of the two sizes of sand or gravel. The experiments provided an opportunity to observe the action of sands or gravels used as filters and observations confirmed the opinion of Terzaghi and others that for uniform-sized sands the ratio of the mean diameter of the filtering sand to that of the sand to be filtered must not exceed approximately five in order that the smaller-sized material be excluded from the pores of the larger-sized material.

Introduction--Filters of sand or gravel are extensively used in many types of engineering works. While most frequently thought of in connection with the treatment of water or sewage, or in chemical engineering, they are of great importance in developing or assuring stability of Earth structures subjected to seepage forces, particularly Earth dams and wells. When used for wells they are usually called "envelopes."

The physical principle which makes filters possible is that, in a packing, the size of the pores is several times smaller than the size of the particles. Filters properly designed must allow the free passage of water but prevent the erosion of the soil or sand particles which are to be filtered. A desirable filter, therefore, is one in which the pores are so small that the filtered material will not enter them but large enough to allow the greatest freedom of percolation. The particles at the outflow face of the filter must be of sufficient mass that they are not eroded under the seepage forces. Often several layers of sand or gravel are required in order to achieve both filtration and stability. The desirable ratio of particle size between filter and filtered material has been investigated by BERTRAM [1939], U. S. WATERWAYS EXPERIMENT STATION [1941], U. S. BUREAU OF RECLAMATION [1947], and others.

Near the boundary or interface between two packings consisting of particles of different sizes the permeability of the filter packing is greatly decreased for a short distance as a result of the partial entry of the small-sized particles into the filter pores. The study herein reported was concerned with the piezometric head loss in this region.

Theoretical analysis--The flow of water through a filter is usually in the viscous range and is described by the law of Darcy

$$V = Ki$$

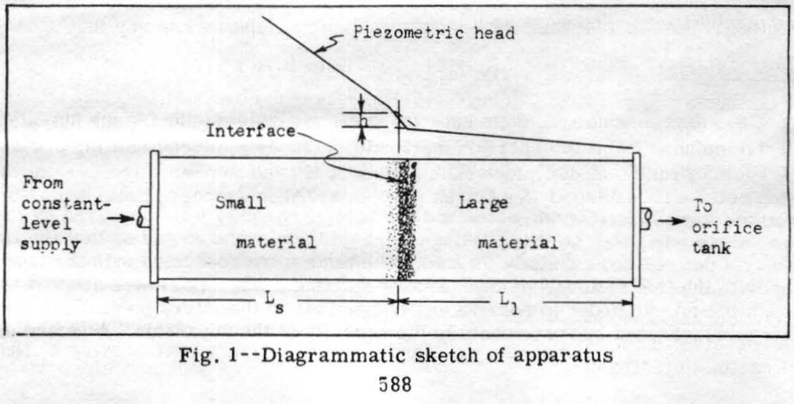


Fig. 1--Diagrammatic sketch of apparatus

where V is the bulk velocity, K is the hydraulic conductivity, and i is the hydraulic gradient. For the purposes of this study, the head loss h assigned to the interface was considered to be an equivalent drop of head in the plane of the interface as shown by Figure 1. It was defined as the vertical distance between the intersection of the upstream and downstream piezometric surfaces with the interface plane. Considering the more important characteristics of the fluid, the packings, and the flow conditions; one may write the symbolic relationship

$$F_1 (\rho, \mu, D_\ell, D_s, \sigma_\ell, \sigma_s, \alpha_\ell, \alpha_s, h, V_\ell, V_s) = 0 \quad \ldots\ldots\ldots\ldots (1)$$

where

ρ is the mass density
μ is the viscosity
D_ℓ, D_s are the mean diameters of the large- and small-sized particles
σ_ℓ, σ_s are the standard deviations of the large- and small-sized particles respectively
α_ℓ, α_s are the porosities of the packings of large- and small-sized particles respectively
V_ℓ, V_s are the upstream and downstream bulk velocities
F designates a functional relationship

If experiments are conducted at a standard value of porosity, α_ℓ and α_s may be eliminated; and further if they are conducted using packings in a tube of constant cross-section $V_\ell = V_s = V$. The remaining variables may be related in the expression

$$h/D_s = F_2 (VD_s \rho/\mu, D_\ell/D_s, \sigma_\ell/D_s, \sigma_s/D_s) \quad \ldots\ldots\ldots\ldots (2)$$

Since the materials used in these experiments were of nearly uniform size, screened between two standard sieve sizes; the authors assumed that the variables σ_ℓ/D_ℓ and σ_s/D_s would not be important. Experiments indicated that the size of the large particles had little influence, however, and that h was influenced largely by the size of the small particles, Reynolds number $VD_s \rho/\mu$, and some combination of the uniformity parameters σ_ℓ/D_ℓ and σ_s/D_s.

Apparatus and materials--The permeameter in which the packings were placed was a six-inch diameter plastic tube 24 inches long to which wooden plates could be attached at the ends. The length of the tube was perforated for a number of piezometer taps which were connected to a manometer bank. Water from the city mains of Fort Collins was supplied from the 2000-gallon storage tank in which a constant level was maintained by an overflow pipe. The effluent water was discharged into a small tank equipped with orifices which were calibrated to determine the rate of flow. A photograph of the apparatus and a diagrammatic sketch are shown in Figure 2.

All of the sands and gravels were screened from gravel deposits of the Cache la Poudre River. The seven sizes used may be described in terms of Tyler sieve numbers as 40-60, 30-40, 20-30, 14-20, 10-14, 5-8, and 3-4 mesh respectively. The 10-14 and 14-20 samples were used as both fine and coarse material. In addition one specimen having graded particle size was used. Gradation curves were obtained for the various samples and are shown in Figure 3. The standard deviations of particle size were computed by the formulas

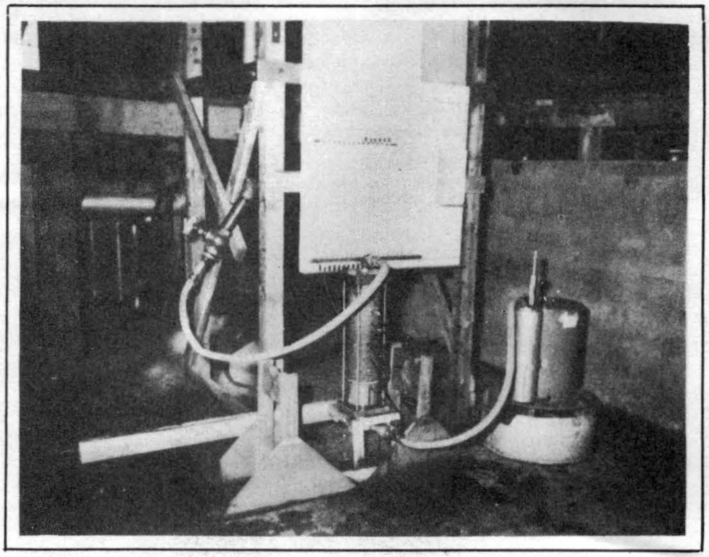
For 3-4 and 5-8 mesh: the standard deviation = Range of particle sizes/ $\sqrt{12}$

For other than 3-4 and 5-8 mesh: the standard deviation = $(D_{90} - D_{10})/2.6$

Various size combinations used were as follows

Coarse material		Fine material
(3-4)-mesh	with	(5-8)-, (10-14)-, (14-20)-mesh
(5-8)-mesh	with	(10-14)-, (14-20)-, (20-30)-, (30-40)-, (40-60)-mesh
(10-14)-mesh	with	(14-20)-, (20-30)-, (30-40)-, (40-60)-mesh, Graded
(14-20)-mesh	with	(20-30)-, (30-40)-, (40-60)-mesh

The packings were placed under water and the volume porosity was measured by the water displacement. Care was taken so that packings were uniformly placed and so that the interface plane was clearly defined and carefully located. The tests were conducted with the tube in a vertical position with the fine material on top. Before starting a test, water was allowed to flow slowly upward through the tube in order to remove any trapped air in the interstices. Flow was started at a slow rate and increased by increments to the capacity of the apparatus. A period of 30 minutes



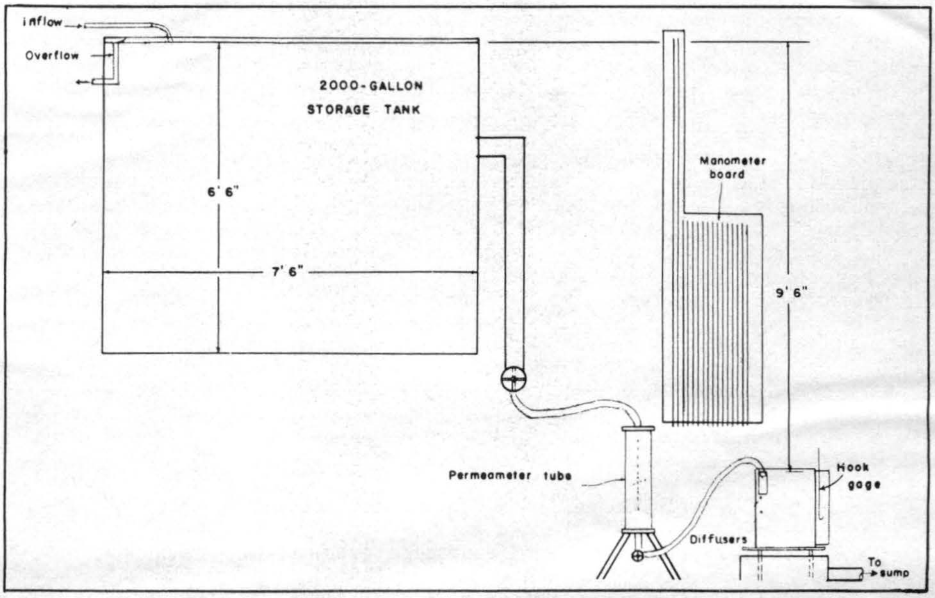


Fig. 2--Photograph and diagram of apparatus

was allowed for each of the first two tests of each series in order for full stability to develop. For succeeding tests in any series the stabilizing period was reduced to 15 minutes. The piezometers were read, the upstream and downstream piezometric surfaces plotted, and h computed for each test.

Head loss at the interface--The data obtained are presented in Figure 4. Curves of h/D_s as a function of Reynolds number for various values of the product of the uniformity parameters for the two sands are shown on this figure. The results form a family of straight lines having a slope of unity when plotted logarithmically. The linear relationship of head loss and Reynolds number indicated that the flow remained in the viscous range for the various tests.

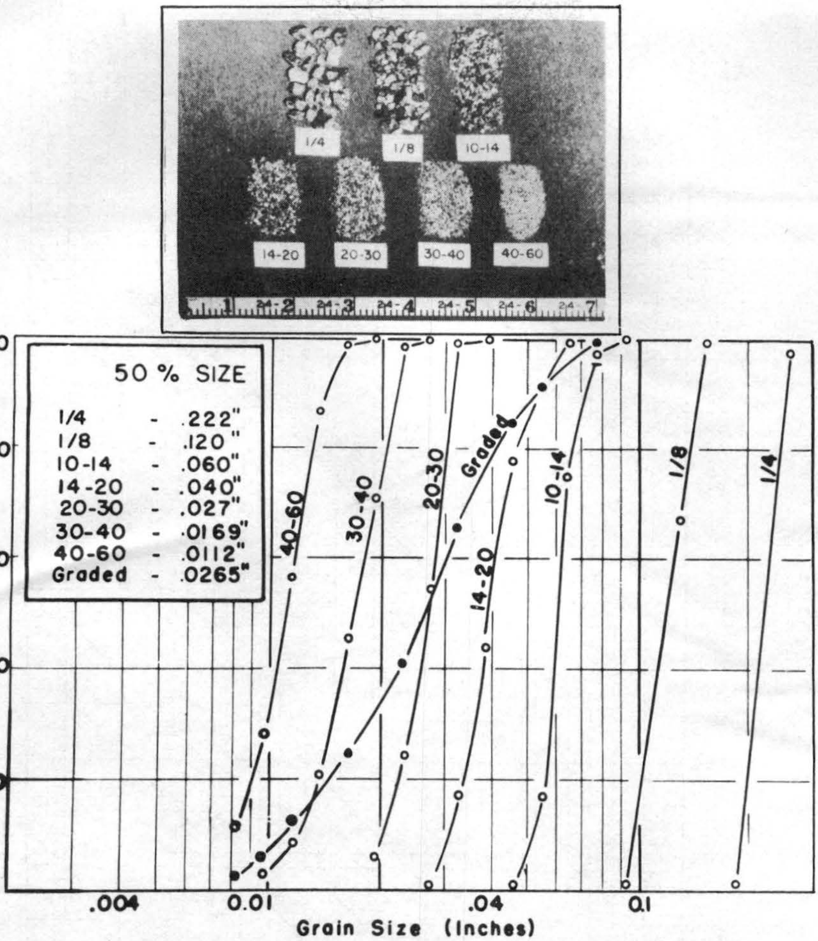


Fig. 3--Photograph of materials and grain-size distribution curves

The equation for head loss at the interface may be written in the form

$$h/D_s = C\,R_e \quad \ldots\ldots\ldots\ldots (3)$$

where C is the value of h/D_s for a Reynolds number $R_e = 1$.

If values of C are plotted logarithmically as a function of $\sigma_s\ \sigma_\ell / D_s\, D_\ell$ (Fig. 5), the points plot approximately on a straight line. Using the method of least squares, the best fit through these points is given by

$$C = 1.02 \times 10^8\,[(\sigma_s\ \sigma_\ell)/(D_s\, D_\ell)]^{4.6} \quad \ldots\ldots\ldots\ldots (4)$$

Eq. (3) should be useful for predicting the piezometric head loss at the interface between uniform sands of different sizes. Reference to Figure 4 should assist the investigator in selecting a value of C to use in this equation. Of major importance is the gradation property of the sand, however, as is shown by (4). This equation was developed principally using sands having a narrow range of grain sizes. Only one material with a fairly wide range of grain sizes was tested, and although C for this test agreed very closely with that predicted by (4), this equation should nevertheless be regarded as restricted to packings in which the grain-size range is relatively narrow.

Stability in filtration--If the ratio of the large-particle diameter to the small-particle diameter was too great, the small particles moved through the pores of the large particles. This

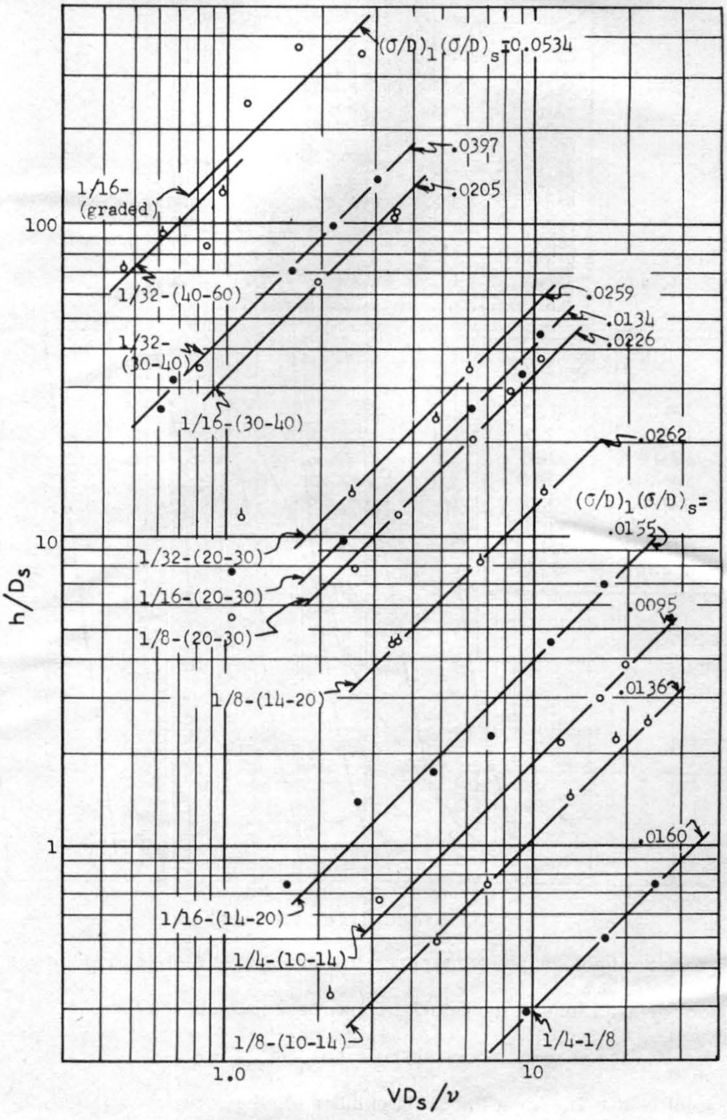


Fig. 4--Head loss parameter as a function of the Reynolds number

resulted in a decrease of head loss at the interface with increasing velocity because of the increase in pore size in the adjacent region as a result of the loss of material. This phenomenon is shown by the curved lines of Figure 6. Previous experiments dealing with the stability of filters have relied on visual inspection to determine whether or not there is movement of the small particles. The technique employed by the investigations reported herein is believed to be more sensitive.

Filter action did not occur if the ratio of the grain size of the filter to that of the filtrate exceeded about five. Criteria for filter action as proposed by this study are

$$\frac{D_{15} \text{ for filter}}{D_{85} \text{ for filtrate}} < 4.1 \qquad \frac{D_{50} \text{ for filter}}{D_{50} \text{ for filtrate}} < 5.3$$

These criteria may be compared with criteria proposed by TERZAGHI [1925], U. S. WATERWAYS EXPERIMENT STATION [1941], U. S. BUREAU OF RECLAMATION [1947], and BERTRAM [1948] which are summarized below:

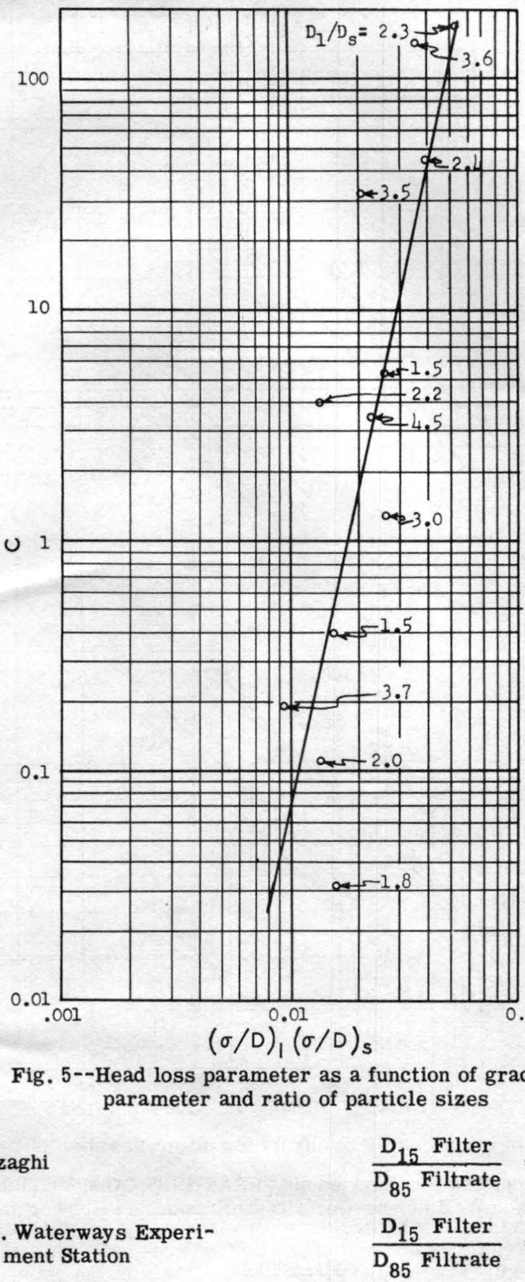


Fig. 5--Head loss parameter as a function of gradation parameter and ratio of particle sizes

Terzaghi $\frac{D_{15}\ \text{Filter}}{D_{85}\ \text{Filtrate}} = 4$

U. S. Waterways Experiment Station $\frac{D_{15}\ \text{Filter}}{D_{85}\ \text{Filtrate}} = 5$

Bertram $\frac{D_{15}\ \text{Filter}}{D_{85}\ \text{Filtrate}} = 9$

U. S. Bureau of Reclamation

Uniform materials $\frac{D_{50}\ \text{Filter}}{D_{50}\ \text{Filtrate}} = 5\text{-}10$

Graded materials $\frac{D_{50}\ \text{Filter}}{D_{50}\ \text{Filtrate}} = 12\text{-}58$

In the foregoing D_{15} is defined as the diameter of the particle for which 15 pct of the material is finer.

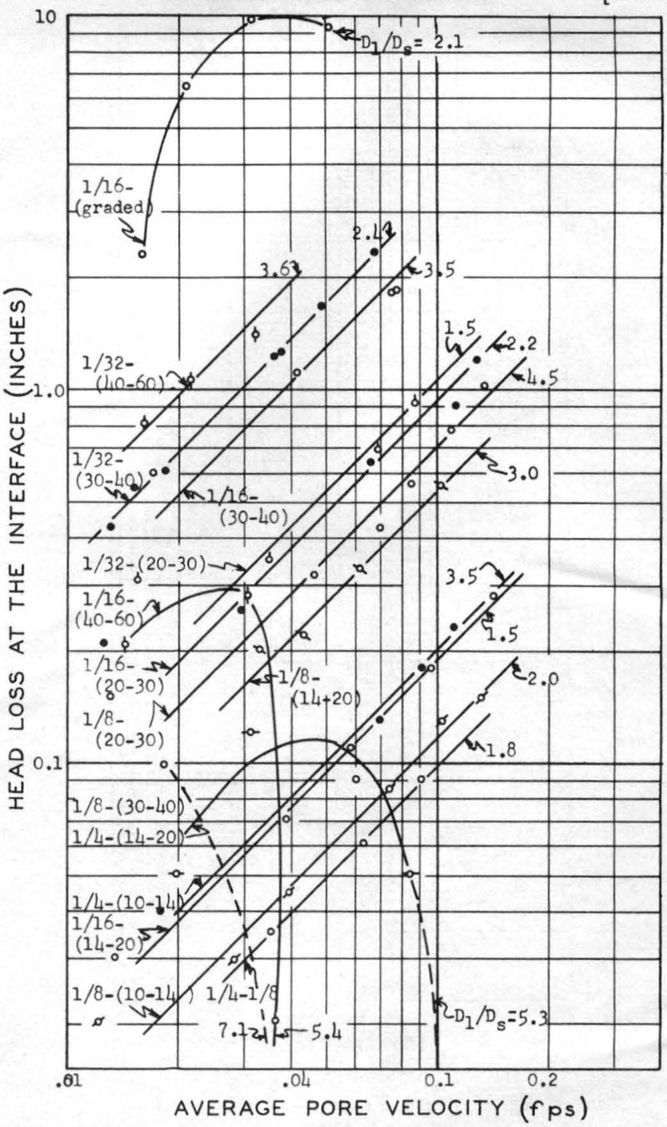


Fig. 6--Variation of head loss with velocity for different ratios of particle sizes

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Colorado Agricultural and Mechanical College, Fort Collins, Colorado

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