FROM CANYONS TO CANALS: APPLYING REGULATED RIVER RESEARCH TO CANAL BANK ANALYSIS

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ABSTRACT

Numerous studies have analyzed river bank dynamic porewater responses to regulated flows. This research has been found to be critical to understanding not only river inflows and outflows from groundwater sources, but also bank failures as a result of flow scheduling. Although the success of these models comes largely from further developing advancements in other related fields, likewise transfer of the research to other related fields has been slow. In response, this paper extends a recently developed analytical porewater pressure response model, utilized to advise flow scheduling in the Grand Canyon, to analyze irrigation canal leakage and resulting large scale groundwater reactions. The new model directly accounts for canal bank geometry, driving upstream and / or downstream water tables, and time varied irrigation flow schedules given by any piecewise continuous function. This model can be used to analyze both near and far hydraulic effects, executes quickly, and is easy to implement on any spreadsheet program. The model showed good agreement between predicted and measured canal leakage and resulting downstream water table changes for the Interstate Canal in Nebraska. Recommendations are made for further uses of the model.

INTRODUCTION

Regulating rivers through controlled dam flows can cause tremendous geomorphological effects, often expressed through numerous streambank failures. These failures can cause unchecked lateral bank migration, thalweg reorienting and even avulsions, resulting in an unintended, unnatural, and uncontrolled restructuring of the entire riparian area. The adverse geomorphologic consequences of river regulation have been well documented at the Glen Canyon Dam, located on the Colorado River within the Grand Canyon. In particular, the riverbank stability has been found to be particularly sensitive to loading conditions such as the river stage fluctuations and the resulting porewater pressure changes.

In response, an analytical model of saturated flow in a deep streambank was derived by Travis (2010). This solution is capable of analyzing any one of numerous periodic river stage conditions, such as those expected downstream from a hydroelectric dam or due to natural hydrologic events.

Like riverbanks, unlined canals are both significantly affected by and significantly contribute to groundwater conditions. Leakage results in lost revenue, unregulated

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groundwater contributions, and is driven by both canal flow schedules and existing water table elements. Seepage into the canal can adversely affect water quality and complicate canal design (Swamee, Mishra, and Chahar, 2004). Both leakage and seepage can cause loss of bank stability (Lorenzo et al., 2003; Thomas, Iverson, and Burkart, 2009). Recent modeling efforts include Lal et al. (2010) who successfully applied an analytical solution to sinusoidal canal flows to improve aquifer property measurements; and Li, Boufadel, and Weaver (2008) who utilized a numerical solution to account for unsaturated flow in canal banks.

In this paper, the Grand Canyon porewater pressure model is extended to account for groundwater effects of canal leakage. This new model collapses to the well known one dimensional solution utilized by Lal et al. (2010) model for sinusoidal canal flows, horizontal water table conditions, and vertical banks. Verification is obtained using the detailed field measurements of the Interstate Canal in Nebraska reported by Harvey and Sibray (2001).

POREWATER RESPONSE MODEL

Porewater response modeling is an established application of the basic laws of saturated groundwater flow. Indeed, from the well accepted observation of Henry Darcy in 1856 that groundwater flow is proportional to hydraulic head (Darcy, 1856), the complete governing equations of dynamic seepage flow can be immediately derived (see Mays and Todd, 2005). And while groundwater flow remains a highly active area of research, current efforts on the subject tend to focus on application specific finite difference / element algorithms, rather than pursuing analytical solutions to the governing equations (e.g. Boutt, 2010; Haitjema et al., 2010; others).

Unfortunately, numerical flow solutions become difficult to achieve for periodic, tidal type, loading conditions, since porewater pressure distributions are dependent on their history, and it is not clear what constitutes reasonable initial conditions of periodic fluctuations. One approach to resolving the initial condition problem is to run the finite element model through sufficient cycles that risk response also becomes periodic. The alternative approach is to iteratively adjust the initial conditions until they are in agreement with those at the end of the period. Either method would be expected to significantly increase computing time.

A resolution to this challenge is to derive an analytical model general of the porewater response to periodic adjacent water stages. Figure 1 shows the simplified bank geometry, defining x (m) the horizontal coordinate, y (m) the vertical coordinate, w the bank width, b (m) the bank height, and z(t) the adjacent stage (in meters) as a function of time t (sec). The definition and units of the soil variables are shown in Table 1, which also includes the specific values for the example application shown subsequently.

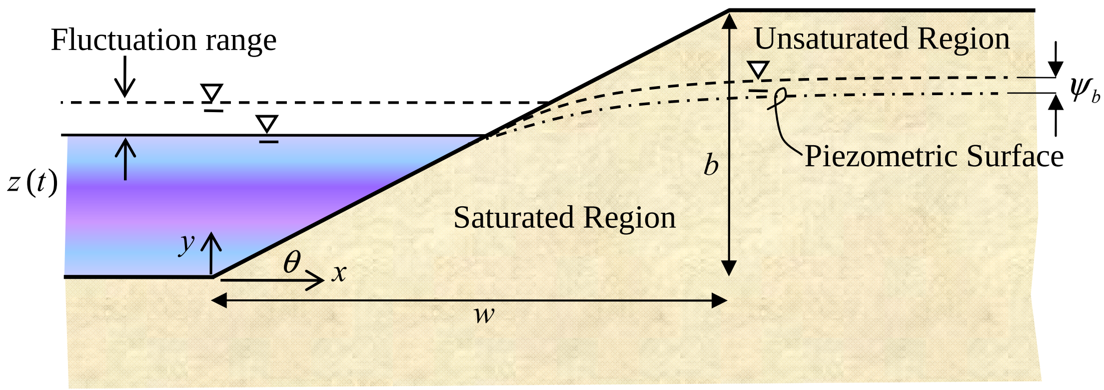

Figure 1. Canal Bank Model

Saturated Flow

The saturated region in the riverbank is described by the two-dimensional Richards equation for saturated flow (Fredlund and Rahardjo, 1993):

$$\frac{d^2 h_s}{dx^2} + \frac{d^2 h_s}{dy^2} = \frac{s_s}{k_s}\frac{dh_s}{dt} \tag{1}$$

where $h_s(x,y,t)$ (m) is the hydraulic head in the saturated region, s_s (m^{-1}) is the specific storativity, and k_s (m/sec) is the saturated hydraulic conductivity. The bank is assumed to be homogeneous and all of the soil properties are assumed to be constant. The origin is located at the interface of the sandbar with the river base.

Equation (1) is simplified by introducing the composite variable u, where

$$u = x - y / \tan\theta \tag{2}$$

resulting in

$$\frac{d^2 h_s}{du^2} = \frac{s_s \sin^2\theta}{k_s}\frac{dh_s}{dt} \tag{3}$$

Limiting the solution to periodic functions, the time constraint is the periodic condition:

$$h_s(u,t) = h_s(u,t+p) \tag{4}$$

The boundary conditions for h_s are

$$\left.\frac{dh_s}{du}\right|_{u\to\infty} = h_\infty^{/} \tag{5}$$

$$h_s\left(0,t\right) = z\left(t\right) \tag{6}$$

The first boundary condition requires that the hydraulic head equation converge to a known water table gradient $h_\infty^{/}$ (m/m) as u increases, whereas the second accounts for periodic river stage changes over time.

The solution to Equation (3) is

$$h_s\left(u,t\right) = h_0 + h_\infty^{/} u + \sum_{n=1}^{\infty} e^{-\varepsilon_n u}\left[S_n \sin\left(\eta_n t - \varepsilon_n u\right) + C_n \cos\left(\eta_n t - \varepsilon_n u\right)\right] \tag{7}$$

where h_0 (m) is the average of the $z(t)$ function over the time period; S_n (m) and C_n (m) are constants; n is an integer; and η_n (sec^{-1}) and ε_n (m^{-1}) are

$$\eta_n = \frac{2n\pi}{p} \tag{8}$$

$$\varepsilon_n = \sin\theta\sqrt{\frac{2s_s n\pi}{k_s p}} \tag{9}$$

Through standard methods for Fourier application, Equation (7) can be applied to any periodic stage function at $u = 1$. In particular, for sinusoidal $z(t)$ and $\theta = 90^o$, Equation (7) becomes one dimensional and collapses to the well known solution for tidal driven groundwater fluctuations (e.g. Furbish, 1997); and successfully applied by Lal et al. (2010) to describe canal effects. Expansions of the tidal solution by Fourier series were also developed by Nielsen (1990) for sloping beaches, wherein he utilized perturbation to derive similar equations but expressed in terms of x only and did not account for the long range water table gradient.

See Travis (2010) for examples and Fourier series application.

Unsaturated Flow

The saturated solution h_s is valid only when $h_s \geq y + \psi_b$, where ψ_b (m) is the pressure head at the air entry value (a negative value). When this condition is violated (e.g. $h_s < y + \psi_b$), flow is governed by the unsaturated head.

Travis (2010) analyzed the unsaturated head for short time periodic loading by simplifying the governing equation through a scale comparison of the unsaturated hydraulic conductivities and specific storativities as presented in several key studies. In general, however, this approach is not valid for applications to canals, where the timeframes of interest are much longer – on the order of months rather than hours. It is suspected that the long timeframes can be utilized in a similar matter to simplify the unsaturated analysis.

The present work follows simply ignores unsaturated flow, and thereby introduces some degree of error. This approach is weakly defensible by noting that it is consistent with numerous other studies. Future work will account for unsaturated flow by utilizing potential simplifications to the governing equations and insights provided by Li, Boufadel, and Weaver (2008).

Leakage

Leakage at the canal is governed by Darcy's law, $q = k_s ia$, where q (m^3/sec) is the leakage (negative for seepage), i is the hydraulic head gradient (m/m) and a (m^2) the flux area. Specific to the defined canal geometry, the leakage is given by

$$q = -2k_s \ell b^2 \tan\theta \left. \frac{dh_s}{du} \right|_{u=0} \tag{10}$$

where ℓ is the length of the canal. Note that Equation (10) accounts bank flows on both sides.

The fluctuating component of the leakage may be obtained by utilizing Equation (10) with the derivative (with respect to u) of Equation (7), and applying the derived formula at the bank. The result is

$$q = \left\{ -h_\infty^{/} + \sum_{n=1}^{\infty} \varepsilon_n \left[\left(S_n + C_n\right)\cos\left(\eta_n t\right) + \left(S_n - C_n\right)\sin\left(\eta_n t\right) \right] \right\} 2k_s \ell b^2 \tan\theta \tag{11}$$

The gradient term in Equation (11) must be carefully considered. Because the leakage effect is local, assuming a large scale gradient would be incorrect. Instead, since it is expected that leakage will descend freely through the aquifer until encountering the water table, the long range hydraulic gradient is simply $h_\infty^{/} = -1$ (m/m). Thus, the correct leakage formula is

$$q = 2k_s \ell b^2 \tan\theta \left\{ -1 + \sum_{n=1}^{\infty} \varepsilon_n \left[\left(S_n + C_n\right)\cos\left(\eta_n t\right) + \left(S_n - C_n\right)\sin\left(\eta_n t\right) \right] \right\} \tag{12}$$

On average, the trigonometric terms will average to zero, resulting in the simple formula

$$q = -2k_s \ell b^2 \tan\theta \tag{13}$$

CASE STUDY: INTERSTATE CANAL

Parameters

Harvey and Sibray (2001) describe a detailed, long term field study of leakage from Nebraska's Interstate Canal. The data from the Interstate Canal is particularly applicable given the potential that the leakage from the base of the canal may be limited, because of sand intrusion into the highly fractured aquifer (Harvey and Sibray, 2001). Thus, bank leakage may account for a large portion of the measured effects.

Downstream groundwater conditions appear to be governed by University Lake, located approximately 900 m downstream at an elevation approximately 14 m below the nearest point on the canal (see Figure 2).

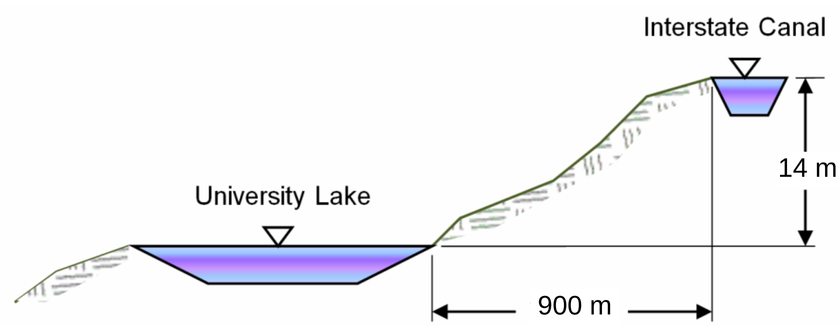

Figure 2. Interstate Canal Schematic

As shown in Table 1, Harvey and Sibray (2001) report hydraulic conductivities from a number of sources. Both horizontal (k_h) and vertical (k_v) values are shown, along with the median and mean values. Anisotropy is high, a complication resolved by determining an effective hydraulic conductivity.

Following Todd and Mays (2005) the effective hydraulic conductivity in the direction of University Lake is given in terms of the vertical and horizontal values by

$$k_s = \left[k_h^{-1} \cos^2 \beta + k_v^{-1} \sin^2 \beta \right]^{-1} \tag{14}$$

where β is the angle from the horizontal between the canal and the lake (0.6°). From Equation (14), the k_s based on both the mean and median values is approximately 10^{-2} m/sec, wherein rounding to the order of magnitude was applied to reflect the significant uncertainty of this approximation.

Likewise for the leakage calculation, where the gradient is governed by local conditions, applying Equation (14) results in estimates for k_s of 1 m / day for the median conditions and 10 m / day based on average values.

Table 1. Reported Hydraulic Conductivities in the Study Region

	k_h (m/day)	k_v (m/day)	Source	Comment
	6912	0.86	Sibray and Zhang (1994)	Reported average
	864	0.86	Barash and Ralston, (1991)	Reported lower bound
	3456	17.30	Barash and Ralston, (1991)	Reported upper bound
Mean:	**3744**	**6.34**		
Median:	**3456**	**0.86**		

Canal water surface elevations (WSE) were not reported by Harvey and Sibray (2001), but flows were. Assuming normal flow conditions, the WSE for the periods of record were converted from the reported flows, and are shown in Figure 3.

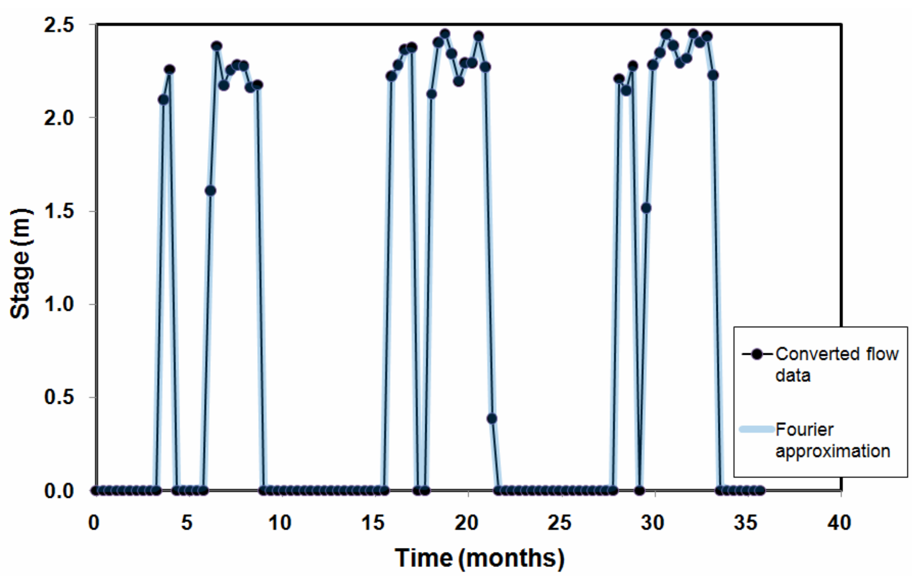

Figure 3. Converted Flow Data and Corresponding Fourier Approximation

For an unconfined aquifer, the specific storativity may be approximated as the porosity divided by the aquifer depth. Thus, assuming a porosity and depth of the shallow aquifer of 40% and 4 m respectively, s_s was estimated to be 10^{-1} m^{-1}.

The coefficients of the Fourier series were evaluated based on the converted flow data. The series was evaluated out to 500 terms which resulted in excellent convergence (Figure 3).

Model execution took about 10 seconds on an Excel spreadsheet.

Fluctuation Predictions

Figure 4 compares the predicted and recorded water table elevations at monitoring well 12, located just upstream of the University Lake (approximately 900 meters from the nearest point on the canal). The predicted elevations were based on hydraulic head estimates projected from the average water surface elevation at the canal (elevation 1277.7).

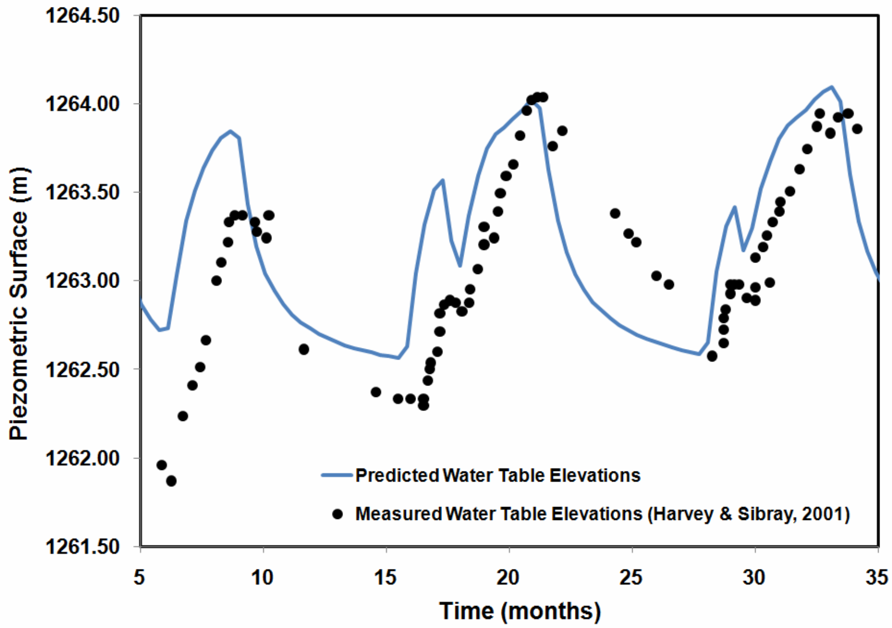

Figure 4. Predicted Versus Measured WSE at Monitoring Well 12

The predicted water table approximates the recorded data with notable similarities and differences. The similarities are:

1. The peak elevations for the second and third seasons are in good agreement;
2. A smaller peak at each season is predicted, consistent with the data;
3. The recorded concavity of the water table recession is predicted;
4. The predicted low water table elevation at the last recorded season is in good agreement with the recorded data.

The differences between the predicted and measured water table elevations are:

1. Although the initial, smaller water table peak at the beginning of each season is predicted, the predicted magnitude is much higher than recorded;
2. The predicted peak elevation at the first season is much higher than recorded, as is the low elevation;
3. The predicted recession limb of the second season is much lower than measured.

Some of the observed differences between the predicted and measured water table elevations result from the assumption of periodic conditions. The result of the periodic assumption is that the best fit is expected near the middle of the time range considered. It is also likely that the predictions would be improved through a more rigorous accounting of the hydraulic conductivity anisotropy and a field measurement of the specific storativity.

Leakage

From Equation (13), the estimated leakage per unit length of the canal when flowing full is about 7 m^3/day/m and 70 m^3/day/m for the median and mean hydraulic conductivities respectively. This compares reasonably with field measurements indicating that $6 \le q \le 35$ m^3/day/m (Harvey and Sibray, 2001; attributed to Ann Bleed).

CONCLUSIONS

By applying the methodology developed for describing riverbank porewater effects of regulated flows, several analytical solutions have been presented to describe leakage and seepage effects in canals. These solutions account for canal bank geometry, driving upstream and / or downstream water table gradients, and time varied irrigation flow schedules given by any piecewise continuous function.

While several measurements of the Interstate Canal verify the derived porewater response model, there are other, potentially more important applications that should be considered. Several examples include:

1. **Canal bank stability**. Bank stability is often critically dependent on the internal seepage processes, particularly for applications such as canals where the adjacent flows are mild. By coupling the derived model with a slope stability program, a powerful analytical tool would be developed.
2. **Canal design**. With a simple analytical method of estimating leakage and seepage, canals could be located and designed to minimize these effects and potentially avoid the necessity of lining part or all of the canal.
3. **Aquifer property measurements.** A useful application of the model presented here is to utilize the Lal et al. (2010) field measurement techniques without the need to change the scheduled flows. Since the model can incorporate any existing flow schedule, the measured data can be compared with predicted results and the aquifer properties thereby calibrated.

While useful, the derived model needs to be expanded in order to adequately account for matric suction effects, which is likely to affect the porewater responses as well as the water retained in the banks, both of which need to be considered for bank stability.

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