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HYDRAULICS OF WELLS

by Dean F. Peterson, Jr., M. ASCE

IRRIGATION AND DRAINAGE  
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## HYDRAULICS OF WELLS\*

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### INTRODUCTION

Groundwater is extensively used, not only for irrigation in the Western United States and in other arid and semi-arid regions, but all over the world for domestic and industrial purposes. The basis for development of this resource is a long history of practical experience plus relatively recent developments of theory.

The study of well hydraulics is a fascinating subject and has occupied the attention of many capable investigators. Like all engineering sciences, however, it is not an exact science and the mathematical assumptions which often have to be made represent only limiting cases of real conditions. For instance, one interested in theoretical well hydraulics usually finds he must postulate at least some of the following conditions, which are not usually fully correct:

- 1) The natural groundwater regime affecting an aquifer remains constant in time.
- 2) The permeability is uniform and isotropic throughout the formation.
- 3) The porosity and thickness of the formation is uniform.
- 4) The well penetrates fully and the hydraulic head is the same at all points on the well boundary.
- 5) The initial piezometric surface is level.
- 6) Only laminar flow exists in the region affected by the well.
- 7) The region from which the well draws is infinite in extent.

In spite of the necessity for introducing some of these unrealities, excellent progress has been achieved by students of theoretical well hydraulics. Useful tools for taking into account specific deviations of natural conditions from the idealized conditions assumed have also been devised in many instances. Well hydraulics, especially in recent years, has contributed much toward a better and more economic development of the important groundwater resource. This paper will attempt to review the general development of theoretical well hydraulics and to comment on certain aspects of the application of these developments to practical engineering problems.

### Flow Formulas

#### Equilibrium Case

Dupuit, 1863,<sup>(1)</sup> is credited with first combining Darcy's Law of laminar flow through sands and gravels with the statement of continuity in order to derive an equation for well discharge. Dupuit assumed complete axial

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symmetry, steady flow through an infinitely extending aquifer, and ignored the effects of curvilinear motion. He deduced

$$Q = \pi K \frac{(h_2^2 - h_1^2)}{\ln r_2/r_1}, \quad (1)$$

where,  $Q$  is the discharge, ( $L^3T^{-1}$ ),  $K$  is the permeability or, more properly, the hydraulic conductivity, ( $LT^{-1}$ ), and  $h_1$ , and  $h_2$  are the elevations of the water surface above a level, impermeable bed bounding the lower side of the aquifer at radial distances  $r_1$  and  $r_2$ . See Fig. 1. All quantities in Eq 1 must be expressed in compatible units or the necessary conversion constants introduced.

As stated in Eq 1, Dupuit's equation implies a well discharging from a water-table condition. However, all of the common steady-flow formulas for fully penetrating wells stem from the Dupuit concept and may readily be deduced by algebraic manipulation of Eq 1. Considering Eq 1,

$$h_2^2 - h_1^2 = (h_2 - h_1)(h_2 + h_1) = (s_1 - s_2)(2m),$$

where  $s_1$  and  $s_2$  are the drawdowns from the original water table or piezometric surface and  $m$  is the average thickness of the water-transmitting formation. Direct substitution yields

$$Q = \frac{2\pi Km(s_1 - s_2)}{\ln r_2/r_1}. \quad (2)$$

Eq 2 may also be used to describe a well discharging from an artesian aquifer of thickness  $m$ .

Common practice of the Geological Survey is to introduce the term transmissibility  $T$  into these equations. Transmissibility is the product  $Km$  and is widely used in characterizing an aquifer. Making this substitution in Eq 2 yields

$$T = \frac{Q \ln r_2/r_1}{2\pi(s_1 - s_2)}. \quad (3)$$

In Geological Survey practice  $T$  is ordinarily expressed in gallons per day per foot and  $Q$ , in gallons per minute. Length units are in feet. Introducing the necessary unit conversion constants into Eq 3 yields

$$T = \frac{527.7 Q \log_{10} r_2/r_1}{s_1 - s_2}. \quad (4)$$

Equations 1 through 4 are useful for determining  $K$  or  $T$  of water-bearing formations using pump tests. This procedure is useful in engineering investigations of dam foundations and land drainage as well as in water supply work. These equations are also useful in predicting well yields. Ordinary practice is to plot drawdown against  $\log r$ , then the quantity  $(s_1 - s_2)/\log_{10} r_2/r_1 = \Delta s$  is simply the drop in piezometric surface for one log cycle of radii from the well center.  $T$  is directly proportional to  $Q$  and inversely proportional to  $\Delta s$ , that is  $T = 527.7 Q/\Delta s$ .

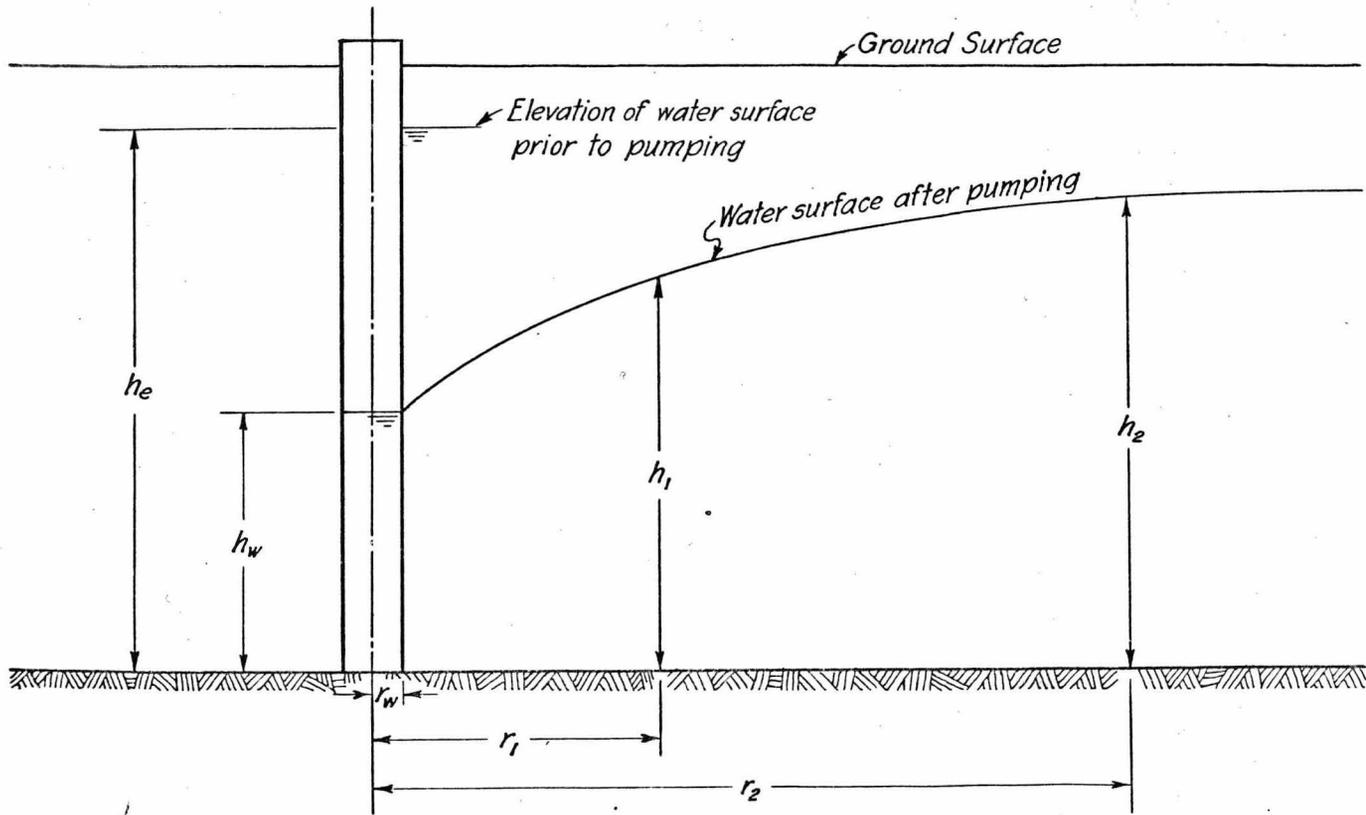


Fig. 1. Definition Sketch for Dupuit Equation.

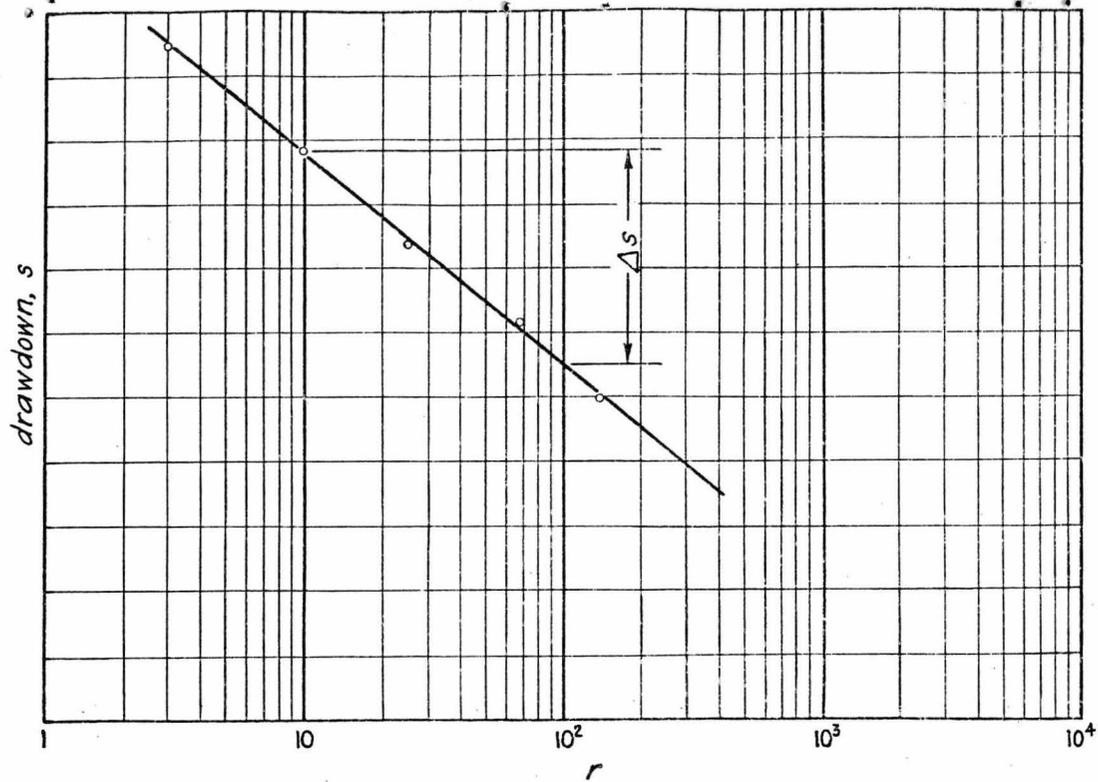


Fig. 2. Solution for  $\Delta s$ .

If  $s_1 - s_2 = D$  is considered to be the drawdown of the well,  $r_1$  corresponds to the well radius  $r_w$ ; and  $r_2$  becomes the radius to a point where the functioning of the well does not disturb the existing piezometric surface, originally assumed as level. This latter distance is ordinarily termed the radius of influence. Obviously an equilibrium well cannot exist in a finite, level, piezometric field. The concept of a radius of influence accordingly implies equilibrium between the flow discharged by the well and the flow replenished by nature to the region influenced by the well. The radius of influence is simply a length parameter describing the size of the influence region in a naturally replenishing field.

Israelsen, Hansen and the writer<sup>(8)</sup> in 1950 postulated a general functional relationship for steady-state wells expressed in the dimensionless form

$$\frac{Q}{K r_w^2} = F(D/r_w, m/r_w, I), \quad (5)$$

where  $I$  represents the unit natural replenishment. These authors wrote expressions for natural seepage flow intercepted by the influence region of a well. Eliminating the radius of influence by substituting these expressions into the well discharge equations, Eqs 1, 2, and 3, approximate functional relationships were obtained for fully-penetrating wells in both artesian and unconfined aquifers having natural piezometric slopes  $i_n$ ; and for a well in an unconfined aquifer replenished vertically at a uniform rate. Details of these solutions appear in the reference. For the unconfined case, this procedure results in the equation

$$\frac{Q}{K h_e^2} = \frac{\pi \left[ 1 - \left( \frac{h_w}{h_e} \right)^2 \right]}{2.303 \log \left[ \frac{1}{2} \left( \frac{Q}{K h_e^2} \right) \left( \frac{h_e/r_w}{i_n} \right) \right]}, \quad (6)$$

where  $h_e$  is the original height of the water surface above the impermeable stratum, Figure 1; and  $i_n$  is the natural slope of the water table. For the confined case,

$$\frac{Q}{K D m} = \frac{2 \pi}{2.303 \log \left[ \frac{1}{2} \left( \frac{Q}{K D m} \right) \left( \frac{D/r_w}{i_n} \right) \right]}. \quad (7)$$

For the case of a well in an unconfined aquifer replenished both by vertical seepage at the uniform rate  $q_v$  (volume per unit area per unit of time) and by horizontal seepage into the influence zone, a procedure embracing similar assumptions to those of Dupuit in deriving Eq 1 results in, approximately,

$$Q = \frac{\pi K (h^2 - h_w^2)}{\ln \frac{r}{r_w} - \frac{n}{2} \left( \frac{r}{r_e} \right)^2}. \quad (8)$$

In Eq 8,  $n$  is the proportion of total discharge  $Q$  originating by vertical replenishment and  $r_e$  is the radius of influence. If  $n = 1$ ,

$$\frac{Q}{K h_e^2} = \frac{\pi \left[ 1 - \left( \frac{h_w}{h_e} \right)^2 \right]}{1.151 \log \left[ \frac{1}{\pi} \left( \frac{Q}{K h_e^2} \right) \left( \frac{h_e^2 r_w^2}{q_w / K} \right) \right] - \frac{1}{2}} \quad (9)$$

Equations 6, 7, and 9 are not in the identical symbolic form of Eq 5. However  $m$  and  $D$  may be defined in terms of  $h_e$  and  $h_w$  for Eqs 6 and 9.  $I$  is represented by  $i_n$  in Eqs 6 and 7 and by  $q_w/K$  in Eq 9.

The relationships expressed by Eqs 6, 7, and 9 are shown graphically by Figures 3, 4, 5, and 6. These should be useful in predicting the performance of wells for the limiting case when the discharge is in equilibrium with recharge by nature.

Letting  $h = h_e$  and  $r = r_e$ , Eq 9 may be rewritten in the form

$$Q = \frac{\pi K (h_e^2 - h_w^2)}{\ln \left[ e^{-n/2} \left( \frac{r_w}{r_e} \right) \right]} \quad (10)$$

where  $e$  is the naperian base. Equation 10 is interesting because it implies that if a transformation  $r_w = r'_w e^{-n/2}$  is introduced, the resulting flow system of a vertically replenished aquifer drained by a well of radius  $r_w$  will be geometrically similar to that of a Dupuit system drained by a well of radius  $r'_w$ . Equation 10 is subjected to the same theoretical approximations as Eq 1 and in addition to the difficulty that the phreatic surface is not actually a streamline as for the Dupuit case. In spite of these approximations, Eqs 9 and 10 are believed to be useful in the design of relief drainage systems using pumped wells. For such systems the average vertical replenishment is apt to be uniformly distributed over the influence area.

#### Non-Equilibrium Case

In 1935, Theis, <sup>(11)</sup> utilizing mathematical methods developed in the study of heat flow, proposed a means for analyzing the non-steady flow of water into a well. Because the steady-flow or equilibrium condition is a limiting case in well hydraulics, rather than the rule, this advance has led to much greater understanding of well performance and to improved well engineering. It takes into account the water drawn from storage in the aquifer,--as the result of unwatering in the water table case, or from volume changes associated with changes in pressure under pumping in the artesian case. An additional aquifer characteristic, the storage coefficient  $S$  was introduced. This coefficient, in common units, equals the volume of water released in one square foot of area tributary to the well owing to a unit decrease in piezometric head. For the water table case this becomes the specific yield.

For the case of an infinitely extending aquifer and constant discharge,

$$s = \frac{Q}{4\pi T} \int_{r^2 S / 4Tt}^{\infty} \frac{e^{-u}}{u} du \quad (11)$$

This integral is known as the exponential integral and tables of its values have been published. <sup>(10)</sup> It has also been called the well function,  $W(u)$ , where the argument  $u$  is the lower limit,  $r^2 S / 4Tt$ . Figure 7 shows  $W(u)$  as a function

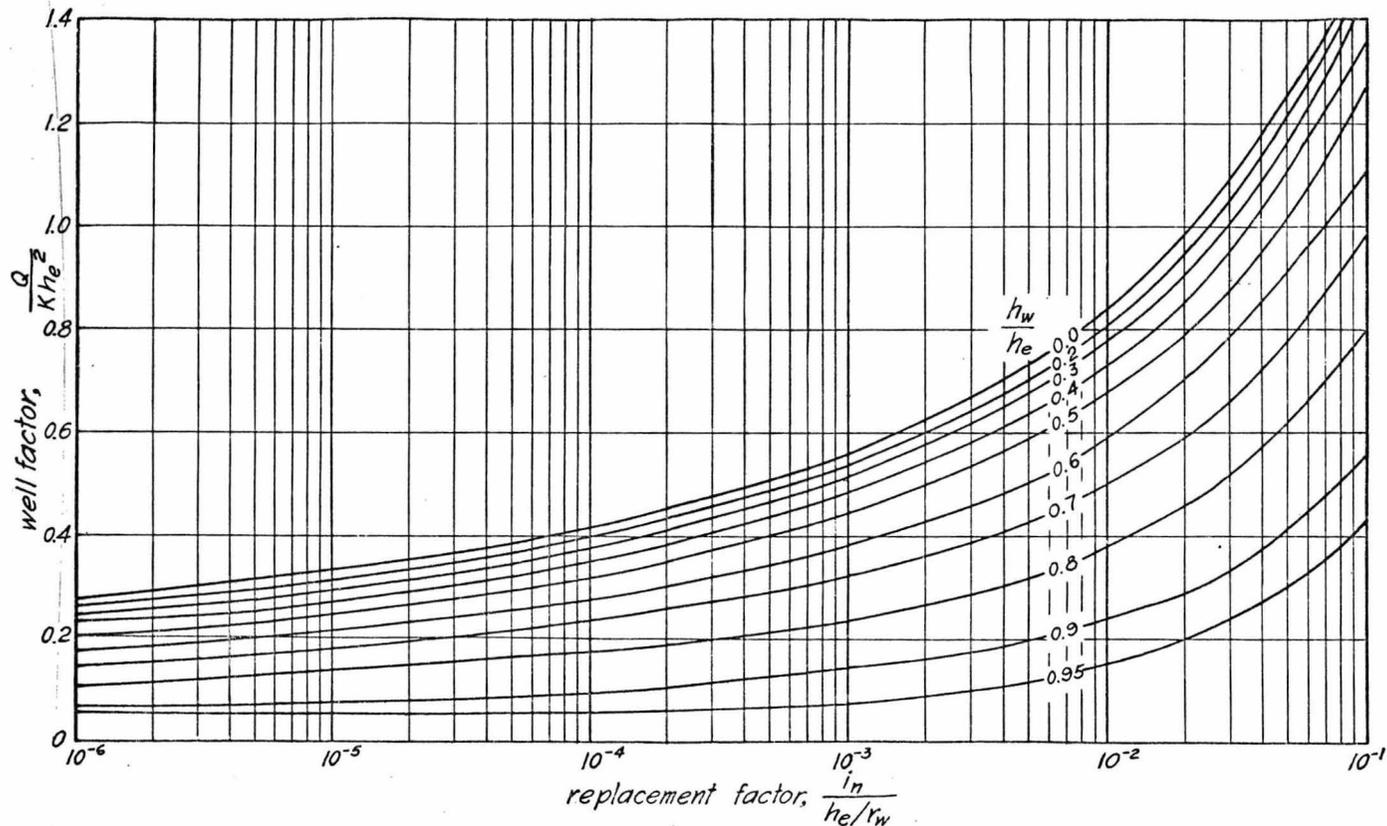


Fig. 3. Horizontally-recharged Well in Unconfined System.

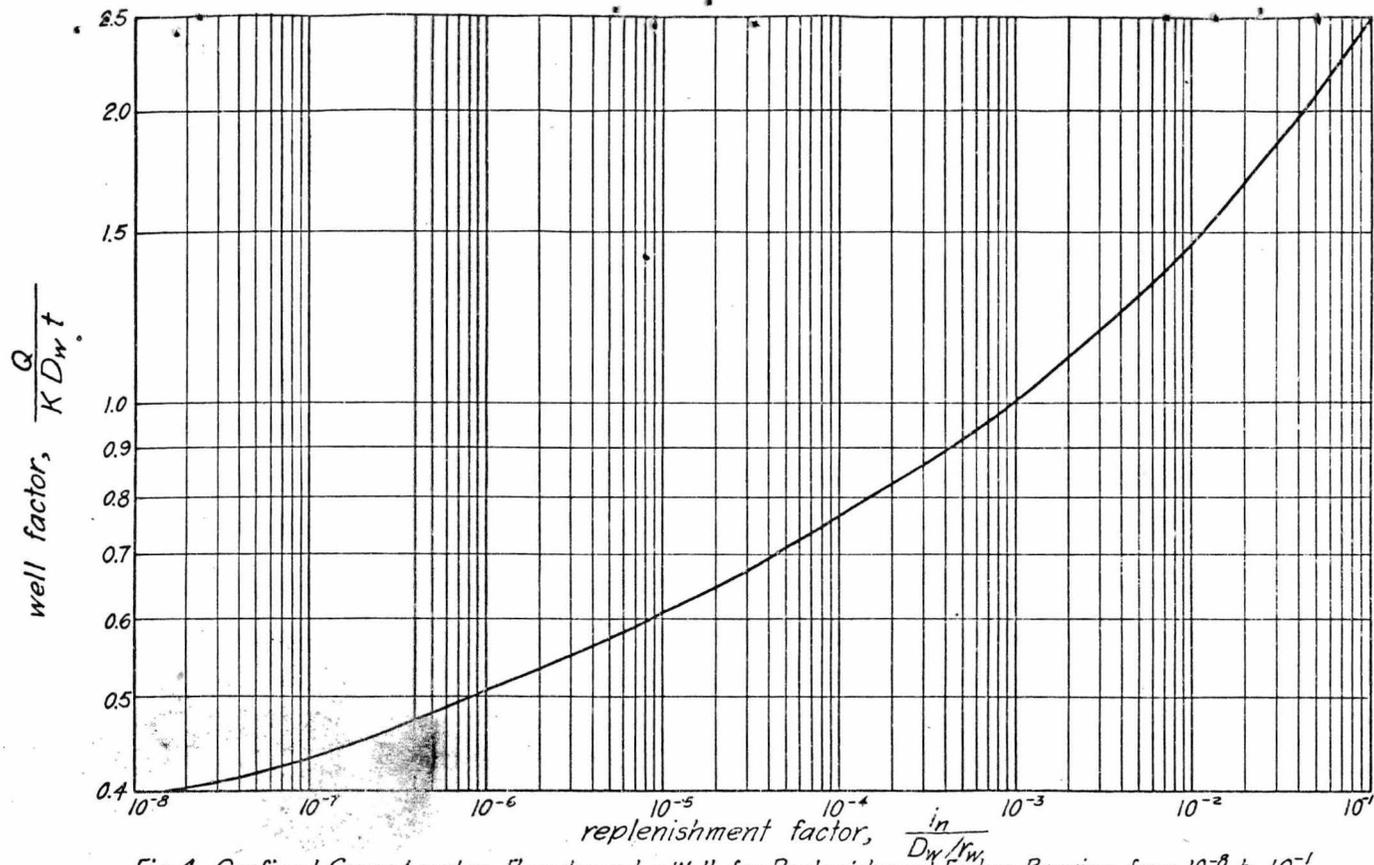


Fig. 4. Confined Ground-water Flow toward a Well for Replenishment Factors Ranging from  $10^{-8}$  to  $10^{-1}$ .

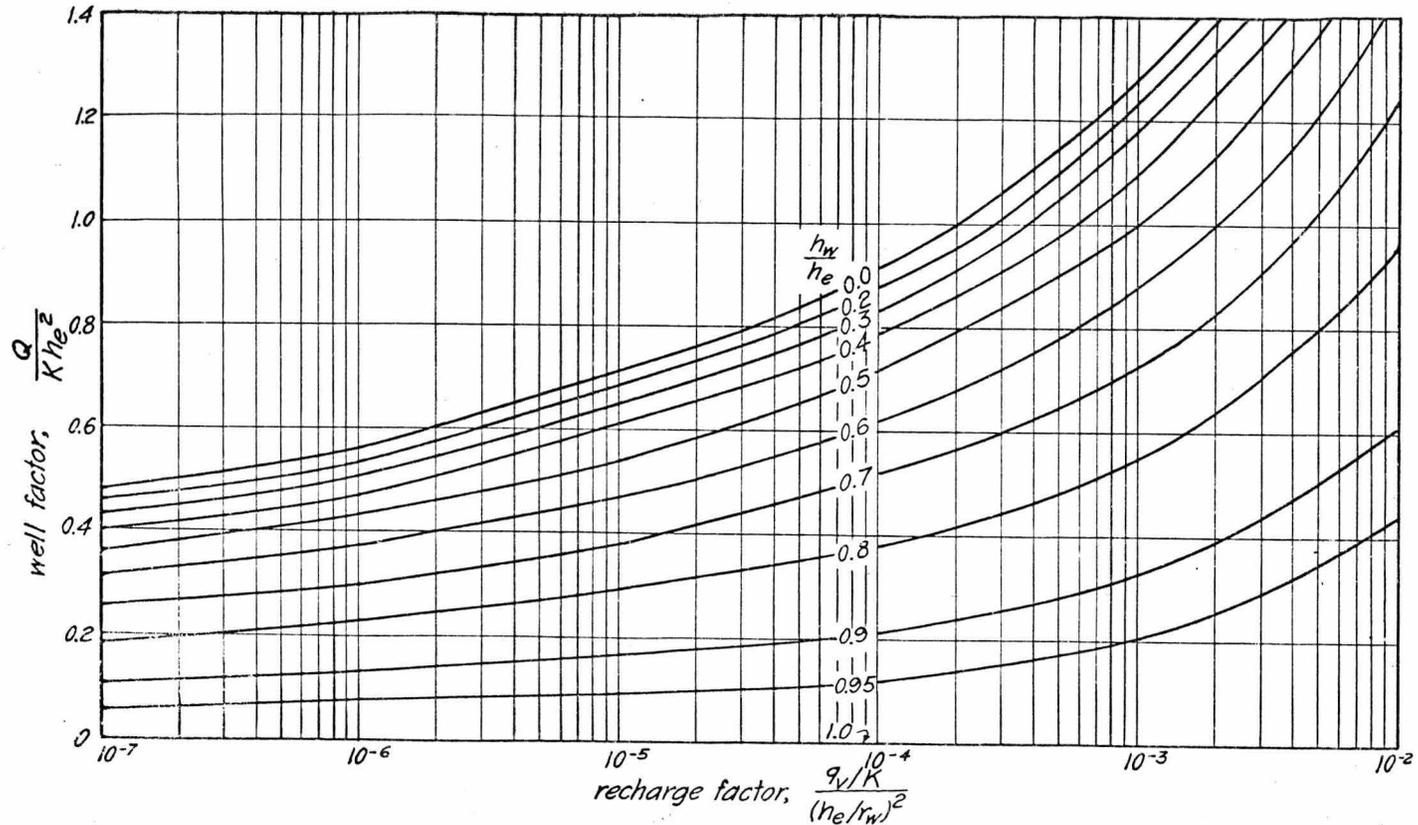


Fig. 5. Vertically Replenished and Unconfined Ground-water Flow for Replenishment Factors Ranging from  $10^{-7}$  to  $10^{-2}$

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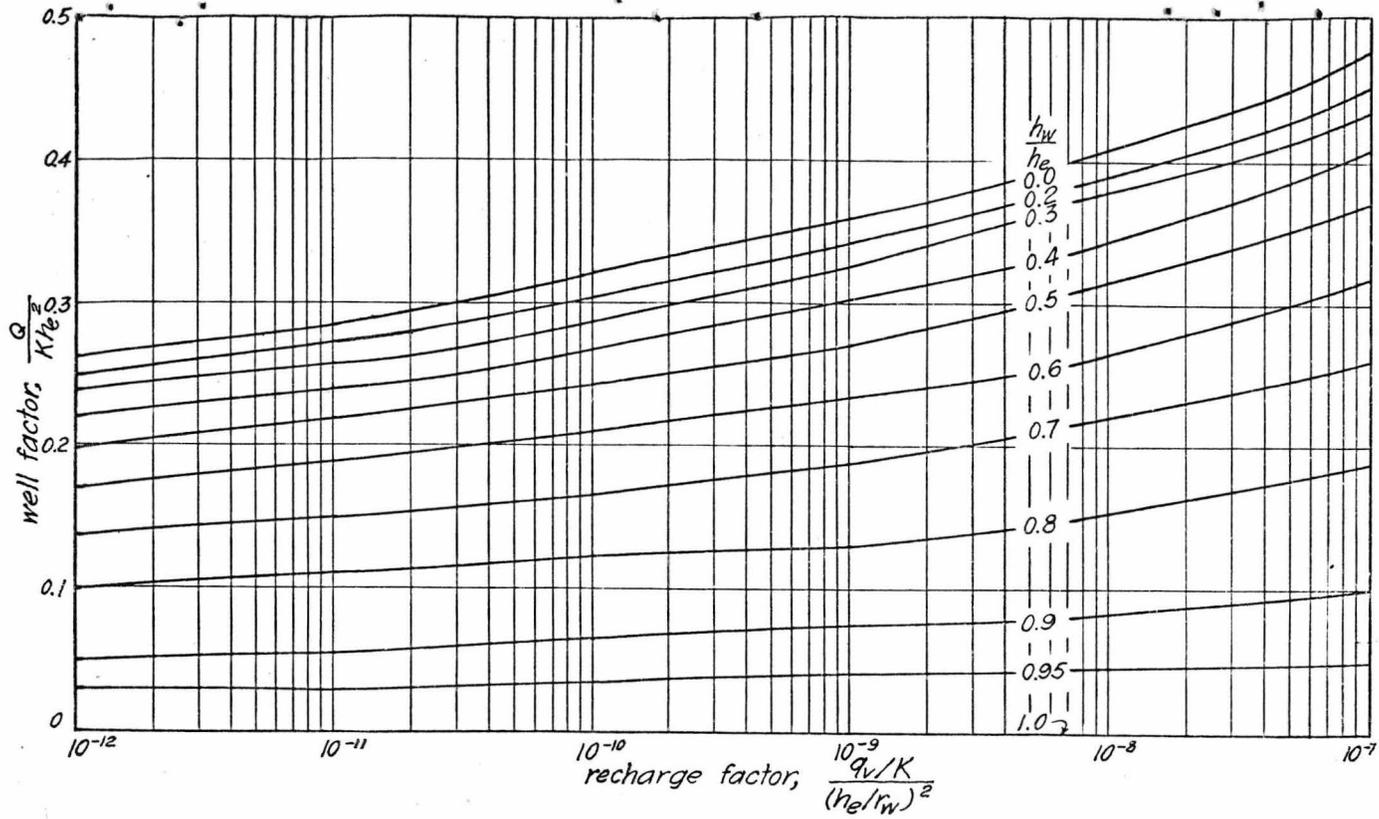


Fig.6. Vertically Replenished and Unconfined Ground-water Flow for Replenishment Factors Ranging from  $10^{-12}$  to  $10^{-7}$ .

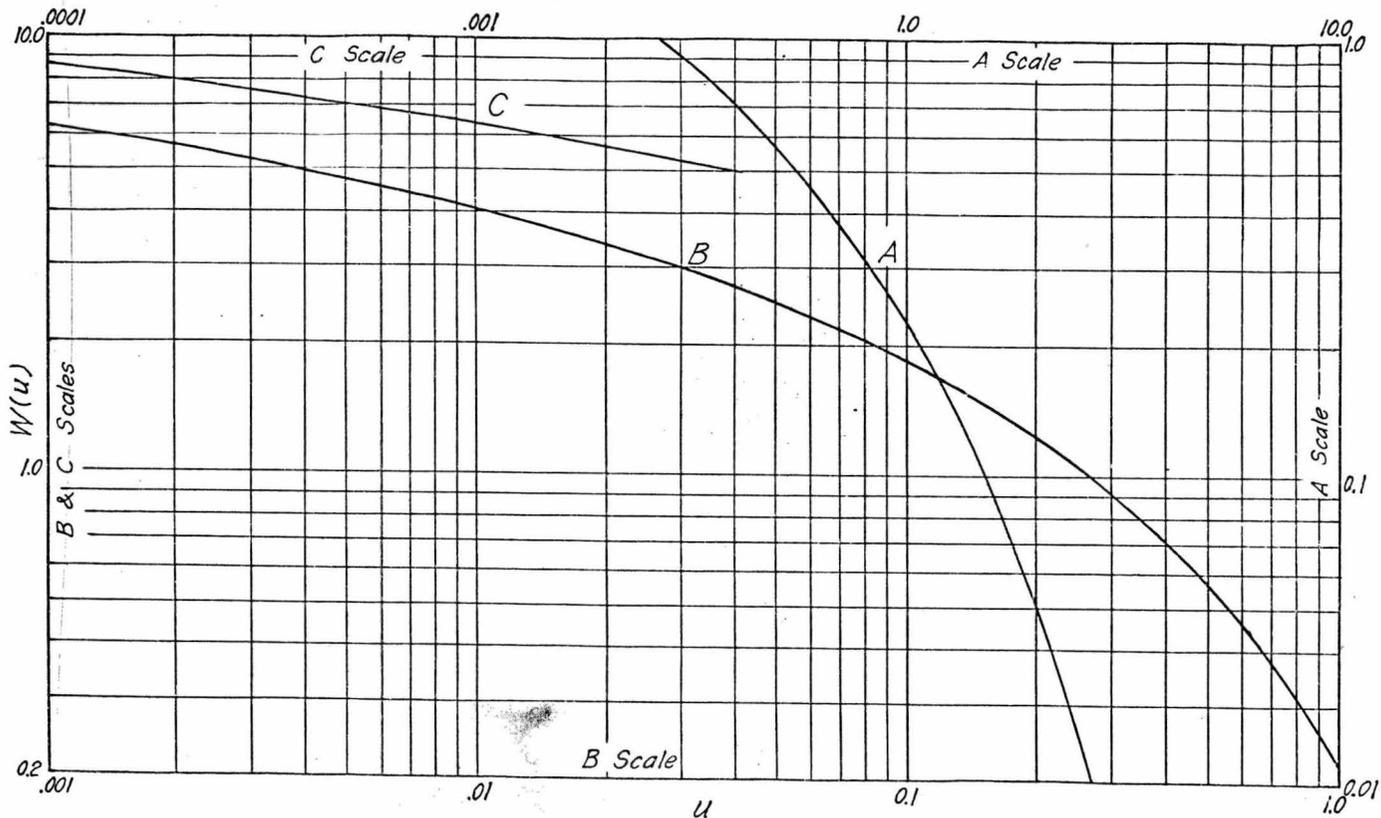


Fig. 7. Well Function Curves.

of  $u$ . For constant discharge,  $u$  is proportional to  $r^2/t$  and  $s$  is proportional to  $W(u)$ . By plotting observed values of  $s$  vs  $r^2/t$  to the same scale, respectively, as  $W(u)$  and  $u$  one obtains a curve similar to that of Fig. 7. This may be matched with the curve of Fig. 7 and a specific match point chosen for which values of  $u$ ,  $W(u)$ ,  $s$ , and  $r^2/t$  may be noted. Rewriting Eq 11,

$$s = \frac{Q}{4\pi T} W(u), \quad (12)$$

and recalling,

$$r^2/t = 4Tu/S, \quad (13)$$

one may substitute these specific values to solve for  $S$  and  $T$  for the formation.

The exponential integral may be expressed by an infinite series. For small values of  $u$  only the first two terms are significant and  $W(u) \cong -0.5772 - \ln u$ . Substituting in Eq 12 gives

$$s = \frac{Q}{4\pi T} (\ln \frac{1}{u} - 0.5772). \quad (14)$$

Taking into account the value of  $u$  from Eq 13 for a well at a particular radial distance one can demonstrate from Eq 14 that

$$s_2 - s_1 = \frac{Q}{4\pi T} \ln \frac{t_2}{t_1}; \quad (15)$$

or in Geological Survey units,

$$s_2 - s_1 = \frac{264Q}{T} \log_{10} \frac{t_2}{t_1}. \quad (16)$$

In using Eqs 15 and 16,  $s$  may be plotted against  $\log t$  until the points fall on a straight line. Data from the straight line portion of the curve are used to solve Eq 16 for  $T$ , see Fig. 8. By extending the straight line to a value of  $t$ ,  $t_0$ , when drawdown is zero, one may combine Eqs 13 and 14 to give a solution for  $S$ ,

$$S = 2.25 T t_0 / r^2. \quad (17)$$

The work of Theis is based on a boundary condition of constant discharge. In 1952 C. E. Jacob and S. W. Lohman<sup>(5)</sup> published a solution for the case of unsteady flow to a well at constant drawdown as follows.

$$s = \frac{Q}{\pi} \frac{2JG(\alpha)}{r^2}. \quad (18)$$

In Eq 18,  $\alpha = Tt/Sr_w^2$  and  $G(\alpha)$  is an irregular integral, which could be evaluated only by numerical methods. Values of  $G(\alpha)$  were also published. For

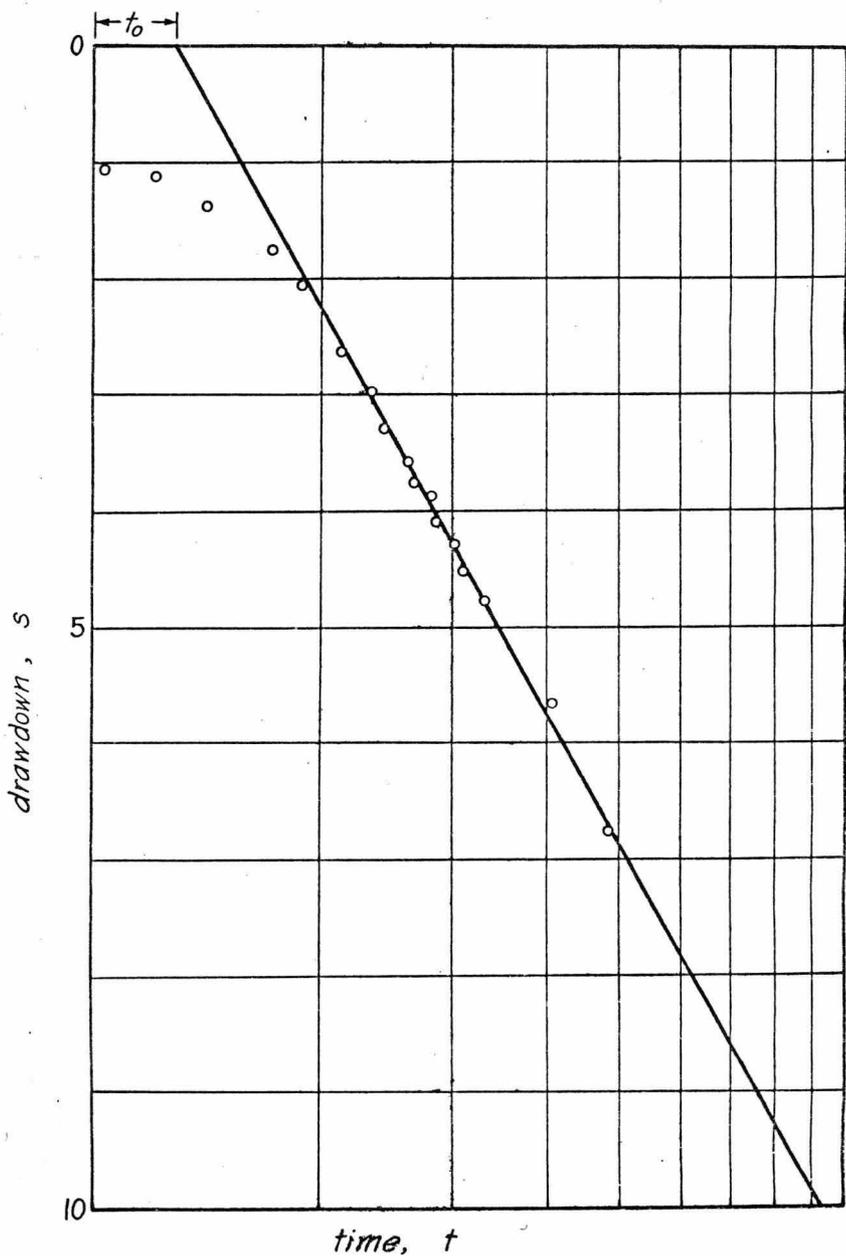


Fig. 8. Drawdown in a Well as a Function of Time.

large values of  $\alpha$ ,  $G(\alpha)$  was found to approach  $2/W(1/4 \alpha)$ . The solution of Jacob and Lohman is particularly applicable to analysis of flowing artesian wells.

The non-equilibrium formula may be applied to wells drawing from water table conditions if proper correction is made to allow for the decrease in effective thickness owing to lowering of the water table. Jacob<sup>(4)</sup> has shown that a correction of  $s^2/2m$  should be subtracted from the numerical values of observed drawdown,  $s$ . Failure to apply this correction results in rather considerable errors in the computed values of  $T$  and  $S$ .

Over the past two decades, the non-equilibrium formula has been widely used with excellent results. Even where an equilibrium state may be possible, at the beginning of pumping the condition is always one of non-equilibrium. As time increases and the cone of influence becomes larger, the condition of the region near the well more closely resembles an equilibrium state because proportionately less of the flow through that part of the region comes from storage. Whether some sort of equilibrium may eventually occur or not depends on the natural hydrology of the aquifer.

Operation of a well depletes underground storage. Actually this increases the capacity of the underground reservoir to utilize available recharge. From an ideal point of view, the management of a system of wells should be such as to deplete underground storage annually in an amount equal to available recharge. Continual pumping in excess of this amount would lead to eventual depletion; pumping less than this amount would mean some of the available recharge waters would be lost owing to lack of storage capacity.

#### Method of Images

By using the common mathematical device of assuming a set of images of the real condition, well hydraulics may take into account the effect of finite geologic boundaries, deviating from the ideal assumed in deriving the various formulas. This may best be illustrated by an example. Consider an aquifer bounded by an impermeable boundary or aquiclude as shown by Figure 9. Assume the aquiclude to be approximately represented by the vertical plane A - B. No flow occurs through this plane. A set of images which makes this a plane of symmetry will provide a zero hydraulic gradient at the required boundary. The real drawdown will be the sum of the drawdowns for the real and the image wells in an infinite aquifer. In the event of two parallel boundaries, two planes of symmetry are necessary and this may be achieved only by postulating an infinite set of images. The actual correction need be made only for the nearest few of the images, the number depending on the degree of accuracy desired.

In the case involving a source of recharge at constant head, a symmetrical, but negative, set of images will provide the necessary boundary condition.

#### Hydraulic Effects of the Well

The foregoing discussion has treated the flow of water through the aquifer under an energy gradient created by a well. The water must also be transferred through the screen and casing or pump column to the point of discharge. Under some circumstances the energy expended in moving the water through the well structure may exceed that used in moving it through the aquifer. Better understanding of hydraulic principles involved in this latter mechanism should lead to improved well engineering. Some of the

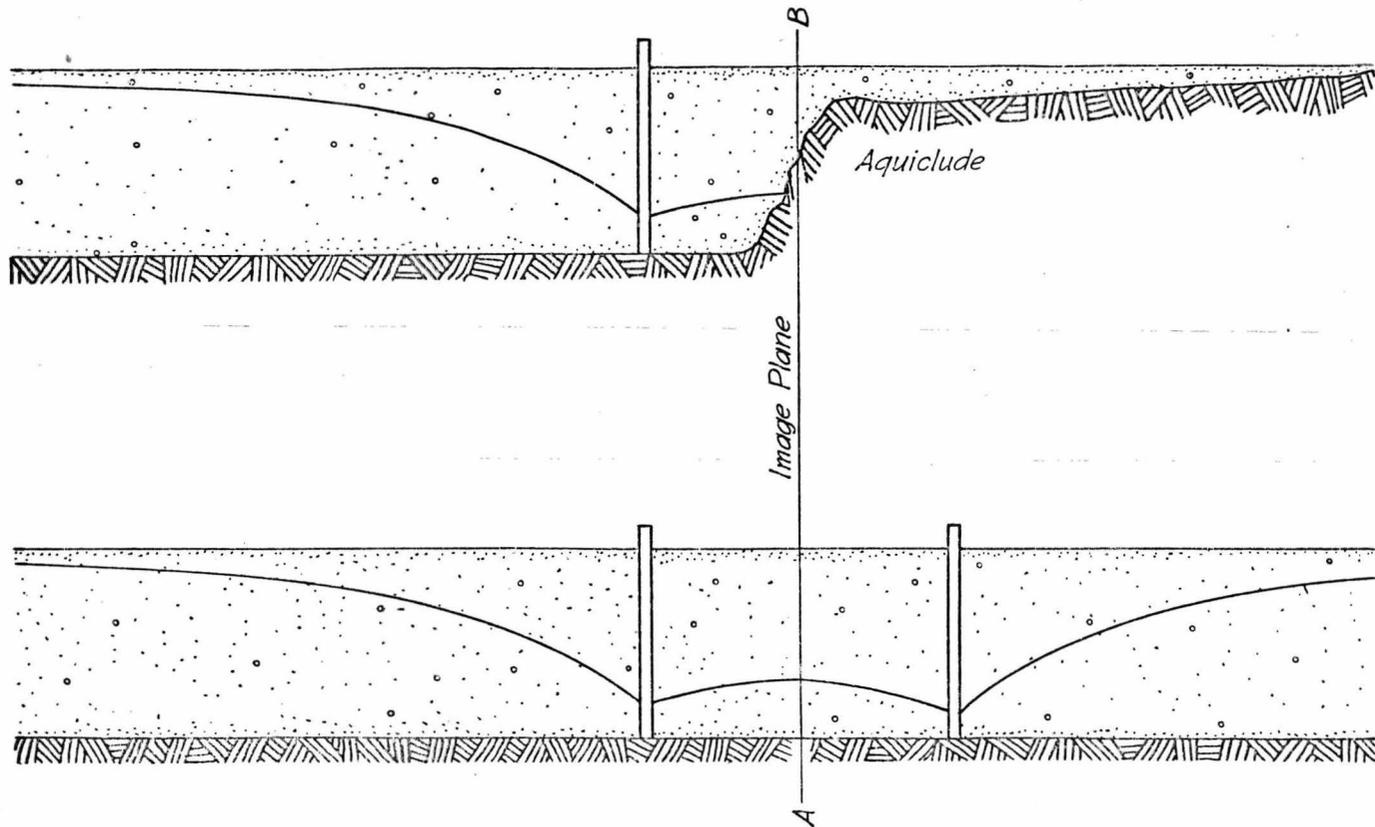


Fig. 9. Method of Images.

considerations which have come to the writer's attention are outlined in the following discussion.

### Specific Capacity

Engineers have designated the ratio of discharge to drawdown specific capacity. If the hydraulic head losses through the screen and casing were zero and the time effect of storage depletion were ignored, the discharge of an artesian well could be expected to be directly proportional to drawdown. This would lead to a constant value of specific capacity corresponding to all values of discharge of an artesian well—a condition usually assumed. For wells in unconfined aquifers, an increase in drawdown at the same time decreases the effective thickness of the aquifer. Thus, even discounting energy losses at the well, the specific capacity would decrease with discharge for the water-table case.

The hydraulic losses through the well cause further non-linearity of the relationship of discharge to drawdown. As mentioned by Jacob,<sup>(3)</sup> flow through the screen and casing usually occurs in the turbulent regime and resulting head losses are thus proportional to  $Q^2$ . Aquifer losses, under conditions of laminar flow, should be proportional to  $Q$  for an artesian well. Thus one may write,

$$s = BQ + CQ^2, \quad (19)$$

where  $s$  is the total drawdown in the well and  $B$  and  $C$  are constants.

Equation 19 can be evaluated approximately by test pumping at two different discharges,  $Q_1$  and  $Q_2$ , and measuring the respective values,  $s_1$  and  $s_2$ . Substitution successively into Eq 19 provides simultaneous equations in  $B$  and  $C$ . The difficulty with this procedure is that it is based on the assumption of steady flow and does not take into account the effect of time depletion of storage. In his excellent paper Jacob<sup>(3)</sup> has outlined a multiple-step draw-down test and procedure which provides a solution for the constants  $B$  and  $C$  and, at the same time, furnishes information leading to determination of the effective radius of a well.

### Effective Radius

The effective radius of a well, as it is used in the formulas of flow, may not be the same as the radius of the screen or hole, especially for wells in uncemented sediments. Development of the well or use of gravel envelopes increases the permeability of the formation immediately surrounding the casing. This effect is the same as increasing the radius. The effective radius is defined by Jacob as "that distance, measured radially from the axis of the well, at which the theoretical drawdown based on the logarithmic head distribution equals the actual drawdown just outside the screen." In the reference previously cited, Jacob gives a procedure for determining this quantity using the results of field tests.

### Seepage Face

For wells in unconfined aquifers, the Dupuit formula implies that the water surface outside of the well intersects the water level in the well at the casing. Even for a hydrodynamically perfect well this cannot be the case—a finite seepage face always exists between the water level in the well and the water surface outside. Hansen,<sup>(2)</sup> in 1949, proposed a relationship of the form

$$\frac{Q}{K r_w^2} = F(h_w/r_w, h_s/r_w) \quad (20)$$

for a well in an unconfined aquifer bounded by a level impermeable stratum at the bottom. In Eq 20,  $h_w$  is the elevation of the water level in the well and  $h_s$  the corresponding elevation outside, both measured from the impermeable stratum. Using experimental data available at that time, Hansen was able to propose curves graphing the functional relationship of Eq 20. Additional information has since been made available by relaxation investigations of Yang<sup>(12)</sup> and analogy investigations of Zee<sup>(13)</sup> which make possible presentation of Figure 10. Figure 10 also shows values of  $h_{115}/r_w$  corresponding to the values of the other parameters where  $h_{115}$  is the height of the water level at a distance of  $115r_w$ . A field test of a well at Mosca, Colorado by Professor W. E. Code and the writer in 1954 correlated very well with the curves of this figure although the actual value of  $h_s$  could not be directly measured. The results of this test appear as point A on Figure 10.

The approach velocity in the neighborhood of an equipotential well is not constant, Figure 11. At the top of the seepage surface, the approach velocity is  $K$ . Theoretically this increases with decreasing elevation and is proportional to the cosecant of the angle of inclination of the streamlines intersecting the seepage face. At the singular point, at the elevation of the water level in the well, this velocity theoretically becomes infinite but, practically, turbulent flow probably occurs. Below the singular point the velocity rapidly decreases to an almost constant value. Figure 11 also shows the relationship between Dupuit's curve and the true water surface.

#### Well Screens

Frequently a large part of the energy imparted through a well is expended in transferring the water through the screen and pump. For this reason, attention should be given to the hydraulic performance of the well structure. While considerable progress has been made, collection of data, especially in the field, is difficult because of rapidly changing flow conditions near the well. Laboratory experiments which simulate field conditions are expensive and arduous. Nevertheless more attention should be given to this important aspect of the problem of well hydraulics.

An important part of the well structure is the screen. Screens of some kind are always required except in cemented sediments. They may range from rough, haphazard, perforations in a steel casing to highly engineered and carefully manufactured screens of specially selected material. The function of a screen is to exclude the natural sediments but to allow the greatest possible flow of water into the well. The factor of longevity influences the choice of screen.

The hydraulic performance of well screens was ably treated by Petersen, Rohwer, and Albertson.<sup>(9)</sup> Water enters the interior of a screen in the form of radial jets at relatively high velocities. The energy of these jets is dissipated and the flow accelerated in the axial direction. From a theoretical consideration of the mechanics involved, these investigators deduced

$$\frac{\Delta h}{V^2/2g} = 2 \frac{\cosh(CL/D + 1)}{\cosh(CL/D - 1)}, \quad (21)$$

where  $\Delta h$  is the hydraulic head loss involved in the screen,  $V$  is the final average velocity along the screen axis ( $Q/A$  where  $Q$  is the well discharge and  $A$  the cross-sectional area of the screen),  $L$  is the axial length of the

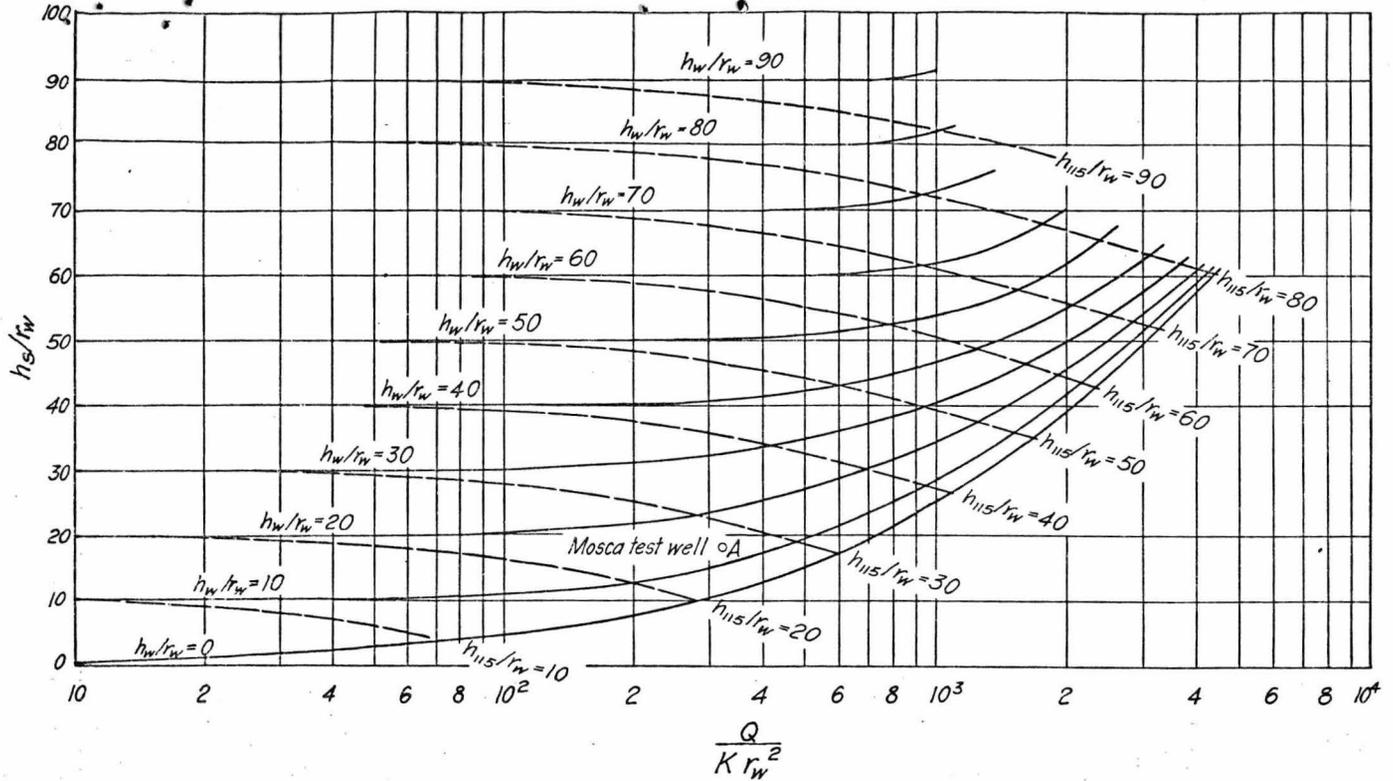


Fig.10. Functional Relationship of Discharge and Geometric Parameters.

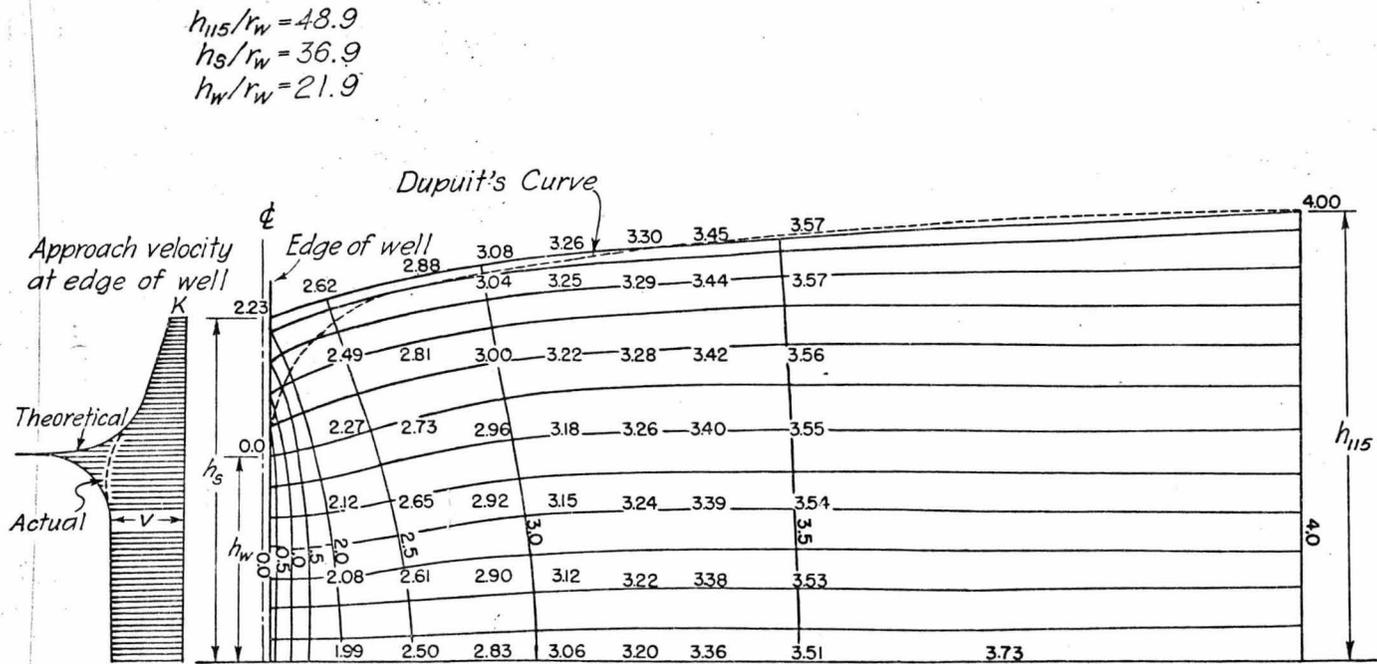


Fig. 11. Radial Flow Pattern for a Gravity Well.

screen and  $D$  is the screen diameter.  $C$  was defined as the screen coefficient,

$$C = 11.31 C_c A_p \quad (22)$$

In Eq 22  $C_c$  is the orifice coefficient of discharge applying to the screen opening and  $A_p$  is the fractional ratio of screen opening to total screen surface. Equation 21 was confirmed by large scale laboratory experiments using specially-made screens with machined openings and not surrounded by gravel, Figure 12. Some coefficients were also determined for certain commercial type screens.

The loss coefficient,  $\frac{\Delta h}{v^2/2g}$  in Eq 22, approaches 2 for values of  $CL/D$

exceeding about 6. Apparently, for a particular screen of a certain diameter, increasing  $L$  beyond this amount would not result in appreciable decrease in screen losses for a particular discharge. The effect of gravel surrounding the screen was to reduce the screen coefficient but not appreciably for larger sizes of gravel. Where the gravel size was sufficiently small that it effectively reduced the percentage of open area of screen, the screen coefficient was also appreciably reduced. This important consideration needs to be taken into account in transferring the implications of Eq 21 to the field. One should note that Eq 21 does not imply that increasing the length of screen beyond a value corresponding to  $CL/D = 6$  would fail to result in increased discharge, but only that the ratio of screen losses to velocity head along the screen axis, for a particular discharge, will not be improved by lengthening the screen.

The action of a well screen is such that the greatest flow occurs near the discharge point of the screen and flow through the openings decreases with distance from this point. In an actual installation, the discharge point would correspond to the intake for the pump bowls. In solving the conventional equations for flow through aquifers, the well boundary is assumed as a surface of constant head equal to the water level in the well. The effect of screens is to upset this, the head will be least opposite the pump inlet and will increase in both directions from this point.

#### Gravel Envelopes

A well screen is in effect a filter, with an additional characteristic of structural rigidity. In many cases, gravel is installed around the screen in order to improve the filter action. Such an installation is known as a gravel envelope or pack. In other cases a well may be developed in natural materials by vigorous surging so that the finer sediments are removed and pumped out of the well, leaving a natural filter of the coarser materials.

As for a screen, a desirable gravel envelope excludes the sediments of the natural formation and at the same time permits the greatest possible flow of water. Extensive field and laboratory research has been conducted on gravel filters, a considerable amount of which applies to wells. The Johnson Well Screen Company has done extensive work in the development of natural sediments to serve as filters. The size of the slots in the well screen and the size and size distribution of the gravel for the artificial envelope should be selected principally on the basis of the size characteristics of the sediments of the natural formation.

Many criteria relating to the design of filters have been suggested by various investigators. How well these apply to gravel envelopes is not known. During recent years, extensive tests to determine the desirable size

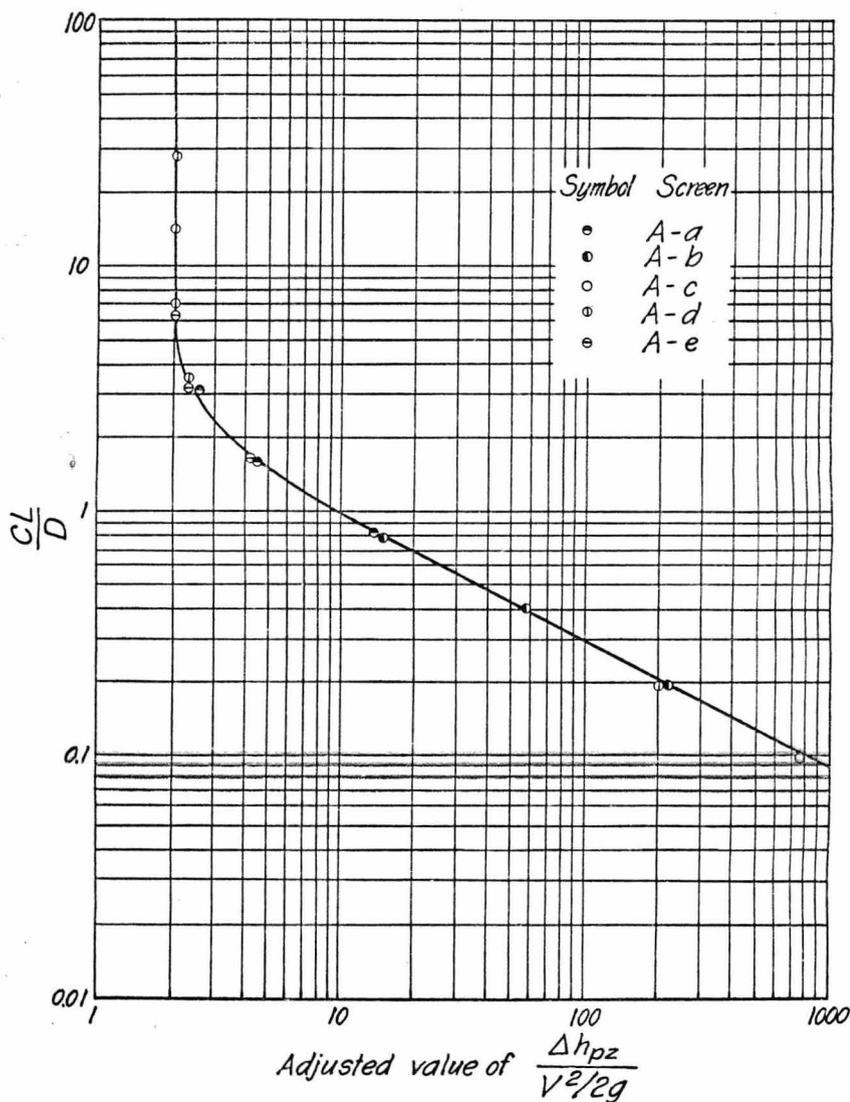


Fig. 12. Adjusted Values of  $\frac{\Delta h_{pz}}{V^2/2g}$  as a Function of  $\frac{CL}{D}$  for Type A Screens.

characteristics for materials for well envelopes have been conducted at Colorado A & M College under the direction of Carl Rohwer. Lockman,<sup>(7)</sup> a student of Rohwer's, reported proposed criteria for filters and well envelopes. A formula proposed by Smith<sup>(7)</sup> is that the ratio of the 50-percent size diameters of the pack and the aquifer, (the P - A ratio), be 5. This was confirmed by Leatherwood,<sup>(6)</sup> who found that instability of the sand occurred for values of this ratio exceeding 5.3. These investigators used gravels essentially uniform in size. Lockman, also using uniform-sized gravels sieved between adjacent size sieves, found, nevertheless that the uniformity of the sand greatly affected the ability of the pack to resist plugging. He proposed a criterion involving the uniformity coefficient (ratio of  $D_{60}$  to  $D_{10}$ ) of the aquifer such that the product of the P - A ratio and the uniformity coefficient be in the range 5 to 8. A uniform-sized pack gravel seemed to give best results. A comprehensive report of these investigations is presently being prepared by Mr. Rohwer and his associates.

### SUMMARY

The work of Dupuit and the formulas involved from the same general concept based on assumed steady flow conditions, have provided a valuable basis for well engineering and for field testing to determine the permeability of underground sediments. Nevertheless, a steady-state flow condition is not the usual case, and most well flow comes about by storage depletion. The formula proposed by Theis for a non-equilibrium well with subsequent adaptations to special conditions has yielded excellent results and contributed greatly to the success of well engineering. Well field developments should be designed to deplete storage only to the extent that subsequent periodical recharge may be expected. A development in which production continuously exceeds available recharge is doomed to eventual depletion, although in some instances the volume of storage is so large that immediate depletion may not be imminent.

A well is, among other things, a mechanism for the transfer of energy to an underground body of water. The energy thus transferred causes flow toward the well. In inefficient wells, energy losses in and near the well may be excessive. Considerable information is available regarding the hydraulics of well components, such as screens, gravel envelopes, etc. and of hydrodynamic details existing near the well. This has contributed greatly to the art of well construction. Much additional research is needed, however, principally in an effort to better understand hydrodynamic conditions near the well and through the well components, and to be able to relate such information to natural geologic conditions and practical well engineering.

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PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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JULY: 457(AT), 458(AT), 459(AT)<sup>c</sup>, 460(IR), 461(IR), 462(IR), 463(IR)<sup>c</sup>, 464(PO), 465(PO)<sup>c</sup>.

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)<sup>c</sup>, 479(HY)<sup>c</sup>, 480(ST)<sup>c</sup>, 481(SA)<sup>c</sup>, 482(HY), 483(HY).

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c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.