Technical Report

DIFFUSION OF HEAT FROM AN INSTANTANEOUS POINT SOURCE IN A TURBULENT BOUNDARY LAYER

by

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ABSTRACT

DIFFUSION OF HEAT FROM AN INSTANTANEOUS POINT SOURCE IN A TURBULENT BOUNDARY LAYER

Diffusion of heat from an instantaneous point source located in a thick turbulent boundary layer over a wind-tunnel floor was investigated. A technique was developed for production of heat spots and detection of temperature fluctuations downstream from the point of release. As a heat source a short length of platinum-iridium wire 0.0004 in. in diameter and approximately 1/10 in. long was used which was heated by a short pulse of electric current. A high-response resistance thermometer was employed for detection of temperature fluctuations. The output of the resistance-thermometer bridge was amplified and applied to an oscilloscope with a "memory screen". The instantaneous temperature profiles of the convected heat spots were displayed on the screen and readings of the maximum temperatures were taken. From about 100-120 readings, mean maximum temperatures were calculated.

Data were taken at different distances up to 5 in. from the source with the source placed at 2-1/2 in. and 5-7/8 in. from the wall. The velocity of air was kept at 15 ft/sec ($R \simeq 16 \times 10^6$).

The obtained horizontal distributions of heat spots are very close to Gaussian curves. The vertical distributions show a skewness. The skewness is such that the greater spread occurs at the side of the greater value of the mean velocity. The skew distribution obtained was compared with the Hinze's skewed temperature distribution and the agreement is satisfactory.

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LIST OF SYMBOLS

Symbol	Definition	Dimension*
x,y	Coordinate system with the heat source as the origin	L
X,Y	Random Lagrangian particle position	L
$\overline{X^2},\overline{Y^2}$	Mean square displacement of a particle	L^2
T,t	Time	T
u',u	Fluctuating velocity component in x-direction	LT^{-1}
v	Fluctuating velocity component in y-direction	LT^{-1}
$\bar{u},\bar{U}$	Mean velocity in x-direction	LT^{-1}
$\sqrt{\overline{u^2}},\sqrt{\overline{v^2}}$	Root-mean square value of the velocity fluctuation	LT^{-1}
u_o,v_o	Fluctuating velocity components at the point of the source	LT^{-1}
$\bar{U}_o$	Mean velocity at the point of the source	LT^{-1}
$R_{(\tau)},R_{(t)}$	Lagrangian correlation coefficient	-
ε	Rate of energy dissipation	L^2T^{-3}

Any other symbol not listed here is explained whenever it appears.

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* L = length, T = time

Chapter I

INTRODUCTION

Diffusion is one of the fundamental processes of turbulence. The ability to disperse fluid particles that initially are grouped together is the most important property of turbulent motion. This is demonstrated by the general statistical tendency of neighboring fluid elements to become more and more widely separated as time proceeds and the associated physical quantity thus becomes diffused. Turbulent diffusion affects such a wide array of phenomena as the dispersion, in the earth's atmosphere, of pollen, bacteria and viruses, radioactive materials,and vulcanic dust. It also affects contamination of air by smoke and gases produced by industrial enterprises and transport; transfer of moisture; dispersion of a cloud of dye or a solute in a fluid, dispersion of objects floating on the water's surface, etc. Of course, besides the turbulent diffusion, we have on many occasions (diffusion of heat and gases, for example) the effect of molecular diffusion too. But this process is far slower than turbulent diffusion and in a turbulent field it plays a minor role only.

Despite the fact that turbulent diffusion has an obvious importance, progress toward a proper understanding of it has not been very great. In the case of turbulent diffusion, besides the difficulties similar to all problems of turbulent motion, there is the added difficulty that we must follow the motion of marked fluid and must therefore employ some kind of Lagrangian description of the motion.

A complete solution of a problem of purely turbulent diffusion consists (4), if we ignore the effect of molecular diffusion, of the determination of the probability, at time t, that the marked fluid is to be found simultaneously at n given points in space, where n has an integral value. This is the complete statistical specification of the spatial distribution of the marked fluid at time t. Thus, a complete solution can be expected only if there is complete knowledge of the statistical functions describing the turbulent motion. In practice, usually, we are content with a knowledge of a small number of statistical parameters which are physically useful, such as the probability of finding marked fluid at any point (one particle analysis) or the mean dispersion of the cloud of marked fluid about its center of mass (two particle analysis).

An analytical start on this problem was not made, however, until the now classic work of Taylor in 1921 (26) on diffusion by continuous movements. Not only did this paper lay a groundwork for the study of turbulent diffusion but it also represented a forward step in the ideas essential to development of a general statistical theory of turbulence. G.I. Taylor demonstrated that the characteristics of transport processes are related to the Lagrangian statistical properties of turbulent motion. He developed his theory in terms of the correlation between the value of the speed of any particle at any instant and the value of the speed of the same particle after a time interval t. It has been formulated for the simple case of homogeneous turbulence. In the early days of turbulence investigation, knowledge and insight into the mechanism of turbulent flow were rather poor and a solution for such a simplified case was not of much help in coping with many engineering and meteorological problems.

In order to make a solution of such problems possible, phenomenological theories were introduced. These theories have been applied primarily in describing the distribution of mean values of a quantity (momentum, heat, mass, etc.) by the effect of turbulence. The diffusive action of turbulence is considered to result in an eddy viscosity or eddy heat conductivity from which the distribution of mean values can be calculated, just as, in the kinetic theory of gases, molecular-transport processes result in a viscosity and heat conductivity. Among these theories, those based upon the concept of a "mixing length" (introduced by Taylor and independently by Prandtl) have proved to be most fruitful. This is not so much because they describe correctly the mechanisms underlying turbulent-transport processes as because they have resulted in useful and practical semi-empirical relations. Though later studies have shown that the physical picture based upon the concept of a mixing length cannot be correct in all details, the mixing-length theories still prove to be most useful to engineers.

The essentially random nature of turbulent motion suggests an analogy between turbulent diffusion and molecular diffusion. The turbulent analogue replaces molecular collisions by successive turbulent exchanges between fluid elements. The existence of very large eddies however, does not justify such an analogy. The usual concept of a diffusion coefficient can in general be at best a crude first approximation, because the variation of the statistical properties of interest in turbulent diffusion occurs over scales comparable to that of the scale of the turbulent motion itself. The analogy in the molecular case would be variations at scales comparable to the mean free path. Some recent analyses (8) however, have proposed the exploitation of the similarities between Brownian motion and turbulent diffusion.

Since Taylor introduced his theory of diffusion by continuous movement in 1927, knowledge about the mechanics of turbulent motion has been improved remarkably. However, progress in the study of Lagrangian statistical functions and a real understanding of them was, as a result of many difficulties, unsatisfactory for years. Therefore, it was assumed that Taylor's approach could be useful only in the extremely simple case of homogeneous turbulence which represents a mathematical idealization that does not bear any similarity to the majority of situations encountered in practice.

The situation, however, has changed considerably in this respect in recent years. In 1957 Batchelor (2) indicated that in the case of free turbulent shear flows (turbulent jets, wakes, and mixing layers) the functional form of a number of important Lagrangian statistical properties can be determined by means of the application of a general similarity hypothesis. In 1958 Batchelor (3) indicated that the direct application of dimensional arguments to Lagrangian statistical properties in a logarithmic boundary layer permits a simple and natural explanation of the law of decrease, along ground level, of the maximum mean concentration of contaminants released at ground level from a line or point source. Cermak (5) and Gifford (12) have further extended the theory to the case of a turbulent boundary layer with density stratification, with an arbitrary height of the source and when x is not necessarily large.

In the following chapter a short survey of the theoretical background pertinent to the problem considered in this thesis is given. Neither phenomonological theories nor diffusion in turbulent shear flow for long diffusion times are discussed. Theoretical analysis of heat diffusion from a point source considering only short diffusion times is taken up in Chapter III.

Chapter II

THEORETICAL BACKGROUND

A. Diffusion in Homogeneous Turbulence

In the kinetic theory of gases, relations for the diffusion of molecules are derived on the basis of the concept of random motion of the molecules. The random nature of turbulent motion suggests the application of similar considerations to the diffusion of fluid particles in a turbulent fluid field. The motions of the molecules are discontinuous, and in the simplified theory they preserve their properties completely during collisions. In a turbulent flow field, however, the motions of the fluid particles are continuous; moreover, it is possible that, because of the intense interaction between the particles, there may occur between them a continuous exchange of a transferable property.

Taylor (26) extended the considerations used in the molecular diffusion studies to the diffusion in turbulent flow, taking into account the continuous movements of the fluid particles by considering the path of a marked fluid particle during its motion through the flow field. Stemming directly from the Taylor theory of diffusion by continuous movement the so-called one-particle and two-particle analysis have been developed. Under these two headings the survey of theories on diffusion in homogeneous flow is presented.

One particle analysis

If at zero time (t = 0) the fluid particle at the origin (x = 0) is passively tagged, the problem is to relate the future statistical distribution of tagging material to some more accessible statistical properties of the turbulence. Then the mean concentration field at time t is precisely the probability density function of the random Lagrangian particle position. Though we cannot predict this probability density function due to some mathematical difficulties (it has been found experimentally to be Gaussian), the primary measure of dispersion is the mean square displacement, $\overline{X^2}$, and it can be predicted by the theory. Formally, it can be written as:

$$\overline{X^2} = \int X^2 P(X,T)\, dX$$

where P(X,T) is the probability density function of the displacement X, after time T. Or, because

$$X = \int_0^T u(t)\,dt$$

it follows that:

$$\frac{1}{2} \frac{d}{dt} \overline{X^2} = \overline{Xu(T)} = \int_0^T \overline{u(t)u(T)}\,dt \quad . \qquad (2.1)$$

For homogeneous and stationary turbulence, the average properties are uniform in space and steady in time, and by introducing the Lagrangian correlation coefficient:

$$R(\tau) = \frac{\overline{u(t)u(T)}}{\overline{u^2}} \; ; \quad \tau = T-t$$

Eq. (2-1) becomes:

$$\frac{1}{2} \frac{d}{dT} \overline{X^2} = \overline{u^2} \int_0^T R(\tau) d\tau$$

where $R(\tau)$ and $\overline{u^2}$ are both independent of the time origin. Hence,

$$\frac{1}{2} \overline{X^2} = \overline{u^2} \int_0^T dt' \int_0^{t'} R(\tau) d\tau$$

or, by carrying out a partial integration

$$\overline{X^2} = 2\overline{u^2} \int_0^T (T-t) \; R(t) dt \; . \qquad (2.2)$$

For very small values of T, such that $R(T)$ is very close to unity (the implication being that the velocity of the particle does not change appreciably over the time interval T). we get:

$$\overline{X^2} = \overline{u^2} \; T^2 \; .$$

Hence, diffusion proceeds proportionally with time. At the other extreme, when T is so large that $R(T) \simeq 0$ we get:

$$\overline{X^2} = 2 \; \overline{u^2} \; T \int_0^T R(t) dt \qquad (2.3)$$

because $2\overline{u^2} \int_0^T t \; R(t) dt << 2 \; \overline{u^2} T \int_0^T R(t) dt$.

The integral (2-3) is a measure of the time scale of diffusion, and the equation,

$$D = \overline{u^2} \int_0^\infty R(t)\,dt \qquad (2.4)$$

may be regarded as a diffusion coefficient, since for molecular diffusion $\overline{X^2} = 2DT$.

Thus, the concept of a diffusion coefficient is justified for large values of time of diffusion. Schubauer (23) and later others have observed the error law for the distribution of temperature, and it would appear that the phenomena of turbulent diffusion can be described by an adequate diffusion coefficient associated with turbulent motion. However, the above analysis shows that the use of a diffusion coefficient is valid only for large distances downstream. Taylor pointed out that the Gaussian distribution of temperature near the source is not associated with a usual diffusion coefficient but should be accounted for by Gaussian distribution of the fluctuating velocity. On the other hand, after a long period of time, no matter what the frequency distribution of the fluctuating velocity may be, the distribution of temperature must necessarily fit the Gaussian distribution just as in the process of molecular diffusion.

Eq. (2-4) can be written as

$$D = \sqrt{\overline{u^2}}\left[\sqrt{\overline{u^2}} \int_0^\infty R(t)\,dt\right] = \sqrt{\overline{u^2}}\,\Lambda_L \qquad (2.5)$$

where Λ_L may be interpreted as Lagrangian integral scale, because $\int_0^\infty R(t)\,dt$ is a measure of the time scale of diffusion.

From Eq. (2-5) it could be concluded that the diffusion for long periods of time is determined by the large-scale motions with slow fluctuations.

To demonstrate this, we introduce the Lagrangian energy-spectrum function $E_L(n)$:

$$R(t) = \frac{1}{\overline{u^2}} \int_0^\infty dn\ E_L(n) \cos 2\pi nt \qquad (2.6)$$

into Eq. (2-2):

$$\overline{X^2} = 2\overline{u^2}\int_0^T (T-t)\ R(t)dt = 2\int_0^\infty dn\ E_L(n) \int_0^T dt(T-t) \cos 2\pi nt$$

$$= 2\int_0^\infty dn\ E_L(n)\ \frac{1 - \cos 2\pi nt}{4\pi^2 n^2} \qquad (2.7)$$

For small value of T:

$$\overline{X^2} \simeq T^2 \int_0^\infty dn\ E_L(n)$$

Hence, for small diffusion time, all frequency components of the motion contribute to $\overline{X^2}$.

For large values of T, rewriting Eq. (2-7) as,

$$\overline{X^2} = \frac{T}{\pi} \int_0^\infty d\ \frac{1-\cos\theta}{\theta^2}\ E_L(\frac{\theta}{2\pi T}) \ ; \ \theta = 2\pi nT$$

we get:

$$X^2 \simeq \frac{T}{\pi}\ E_L(o) \int_0^\infty d\theta\ \frac{1-\cos\theta}{\theta^2} = \frac{T}{2}\ E_L(o)$$

Thus, only the small-frequency components of the motion of the fluid particle contribute to $\overline{X^2}$; or, what is the same, for large T the diffusion of a single fluid particle is determined mainly by the large-scale motions of the turbulence.

It must be emphasized, however, that if the Lagrangian frequency spectrum happens to have a large enough maximum at high frequency, this region may dominate the dispersion for quite awhile (8).

Two-particle analysis

The above considerations on the diffusion of a single marked fluid particle may be applied to the motion of the center of mass of a lump of fluid. But to study the deformation of such a cloud of particles, the relative motion of two fluid particles of the cloud must be considered.

In the following survey only the essential principles and final results of this analysis are given.

At very small values of T, when the velocities of the two particles have not had time to change appreciably, the particle trajectories will be approximately straight lines. At very large values of T, when the particles have separated so widely that their velocities are uncorrelated,

$$\overline{y^2}_{T\to\infty} \to 2\overline{Y^2}$$

where y is the separation of the pair of particles and $\overline{Y^2}$ is the dispersion of a single particle. Thus, the mean square separation of a pair of particles tends (for a long time) to a value which is exactly twice the mean square displacement of particles released serially from a fixed position.

If the separation y lies in the range of scales of turbulence for which the universal equilibrium theory holds, there is only one parameter--the rate of energy dissipation ε --characterizing the properties of the turbulent motion. Thus, dimensional arguments show that,

$$\overline{y^2} \propto \varepsilon T^3$$

or

$$\frac{d\overline{y^2}}{dT} \propto \varepsilon T^2 \propto \varepsilon^{\frac{1}{2}}(\overline{y^2})^{2/3}$$

This means that the dispersive effect becomes larger and larger as the particles separate further and further from each other, the diffusion coefficient increasing as the 4/3 power of the separation.

An important case is that of two particles with an infinitesimal initial separation, i.e., so small that the rate of strain is substantially the same for each particle at all times. Such analysis can be used to examine the deformation of a fluid element--material lines, surfaces, and volumes. The results are of practical importance and they indicate, for example, that vectors such as magnetic field in a conducting fluid are convected by fluid and increase in intensity in proportion to the length of the vector line, which is made up of line elements. Similarly, the gradient of a convected scalar increases in proportion to the area of the surfaces of equal intensity. This increase continues until the scale of fluctuations becomes so small that molecular diffusion destroys the gradients as fast as they are created by the stretching of the turbulence (4).

B. Interaction between Molecular and Turbulent Diffusion

In almost all the literature on the theory of turbulent diffusion the effect of molecular diffusion is ignored. It is indeed true that molecular diffusion is a feeble process by comparison with turbulent diffusion, but the interaction of the two processes is often important. The action of the turbulence is to draw out the cloud of marked fluid and to keep the gradient of concentration of marked fluid large, despite the smoothing effect of molecular diffusion. In that way this interaction results in the acceleration of molecular diffusion"--i.e., much faster dispersion of a cloud of contaminants and accordingly much faster decrease of the mean concentration (4).

Saffman (21,22) has analyzed the diffusion of heat spots in isotropic turbulence. For small values of time he has shown that the interaction of the process of molecular diffusion with the stretching effect of the turbulence increases the rate of cooling of the spot or equivalently its size, but that the interaction decreases the spreading of the heat relative to the point of release.

Starting with the following equation,

$$\frac{\partial \theta}{\partial t} + \vec{u}\, \nabla\theta = k\nabla^2\theta$$

where $\theta(\vec{x},t)$ denotes the temperature, k is the molecular diffusivity and $u(\vec{x},t)$ is the turbulent velocity field, the mean dispersion of the spot (or variance of the temperature) in an arbitrary direction, relative to its centroid, can be obtained as

$$2kt + \frac{2}{9} kt^3w^2 + \ldots \tag{2.8}$$

where w^2 is the mean-square-vorticity. Thus, the size of the spot is increased by the turbulence.

For the mean-square instantaneous wake width, behind a line source, Townsend (29) obtained

$$2kt + \frac{5}{9} kt^3w^2 + \ldots \tag{2.9}$$

which implies further that the rate of cooling of the wake is accelerated by the turbulence.

Considering the dispersion of the heat spots relative to a fixed origin (essentially the one particle analysis) for small values of the time from release, it can be shown (22) that

$$\overline{X_1{}^2} = \overline{Y_1{}^2} - kt^3w^2 + k\mathcal{O}(t^4) \tag{2.10}$$

where Y_1 denotes the displacement of the fluid particle which was originally coincident with the heat spot.

Thus, the effect of molecular diffusion is to make the dispersion of the centroid (X_1) relative to the point of release, less than that of the originally coincident fluid particle.

The dispersion of the spot relative to its centroid is given by (2-8), so that the dispersion (in the 1-direction) of the spot relative to the point of release is

$$\overline{Y_1{}^2} + 2kt - \frac{1}{9} kt^3 w^2 \left\{ 1 + \mathcal{O}\left[(1+\frac{k}{\nu}) (w^2)^{\frac{1}{2}} t\right] \right\} \qquad (2\text{-}11)$$

The physical significance of the error term is that the size of the spot should be small compared with the length scale of the small eddies and the time should be small compared with the time scale of the small eddies.

If the effects of the molecular and turbulent diffusion were independent and additive, the dispersion would be $\overline{Y_1{}^2} + 2kt$. The extra terms in Eq. (2-11) are due to the interaction. The striking feature is that the interaction is negative (in the initial stages at least) and reduces the spreading of the heat relative to a fixed origin.

The above analysis has theoretical validity for small times only. Saffman has analyzed the interaction between molecular and turbulent diffusion for large times and has derived an estimate of the spreading for large times (22).

So far, there has not been any quantitative proof either for Eq. (2-11), for short diffusion time, or for the asymptotic expression Saffman derived for large times.

Mickelsen (20) measured the lateral dispersion of both helium and carbon dioxide in air from a continuous point source. The data showed that for long diffusion time the molecular diffusion makes only an independent contribution to the total dispersion.

C. Diffusion in Turbulent Shear-Flow

The analyses reviewed in the previous sections are strictly valid only for homogeneous turbulence, and the extension to sheared flow is very difficult, because the Lagrangian properties there depend on the initial position and subsequent trajectory of the particle. This increases greatly the effort required to obtain a comprehensive set of measurements, and their relation to the statistics of particle motion is more difficult to interpret. Very few measurements of diffusion from line sources are available, and all of them refer to diffusion times so small that they can provide information only about the initial behavior of the diffusion.

In order to provide a framework for the interpretation of measurements, Batchelor and Townsend (4) have developed that part of the theory of diffusion in shear flows that refers to small diffusion times.

All experiments measure the temperature distribution over a plane parallel to a heat source. The problem in turbulent diffusion that corresponds to this experimental arrangement is not the distribution of particle displacement after a fixed diffusion time, but the distribution in a plane at right angle to the mean stream after a fixed displacement parallel to the stream. Only if the turbulent fluctuations are very small compared with the mean velocity is it possible to assume that the time for a given displacement parallel to the mean flow is constant and to apply results of an analysis based on a fixed time for diffusion.

The effects of molecular diffusion, for very small diffusion times, are assumed as independent of turbulent diffusion and additive.

If movement of a fluid particle in the Ox direction between the time t_o at which it passes through the source position and time t is expressed as

$$X(t) = \int_{t_o}^{t} u(t')dt'$$

and if it is developed into a Taylor series if $t-t_o$ is small, the particle displacement along Oy after this time can be obtained, for small values of X, as

$$Y(X) \simeq \frac{v}{u} X \qquad (2.12)$$

Thus, for small values of X the distribution of marked particles over the detection plane is identical with the distribution of $\frac{v}{u} X$. If, in addition, the fluctuations of u are small compared with the mean, the particle distribution is identical with the distribution of v

(except for the constant factor $X/\bar{u}$), but, when this is not true, there are very substantial differences as Hinze and Van der Hegge Zijnen (14,15,16) have pointed out. This may be shown by computing moments of the probability distribution of v/u and expressing them in terms of moments of the joint distribution of v and u.

With $u = \bar{u} + u'$, the first moment is

$$\frac{1}{X}\ \bar{Y} = \overline{\left(\frac{v}{u}\right)} = - \frac{\overline{u'v}}{(\bar{u})^2} + \frac{\overline{u'^2 v}}{(\bar{u})^3} - \dots$$

This equation shows that the median of the particle distribution is displaced from the mean-flow streamline through a distance $- \frac{\overline{u'v}}{(\bar{u})^2}\ X$ (a more detailed discussion on this is given in the next chapter).

The two simplest non-dimensional parameters specifying the form of a probability distribution are the skewness and flatness factors. Batchelor and Townsend (4) have derived expressions for these factors assuming (a) that the fourth-order velocity products are related to the second-order products as they would be if the joint-probability distribution function for the velocity components were normal, and (b) that the odd-order products $\overline{u'v^2}$ and $\overline{u'v^4}$ are less than $\overline{v^3}$ and $\overline{v^5}$ in the ratio $\overline{u'v}/\overline{v^2}$

They have used these derivations to interpret the measurements of heat diffusion in turbulent shear flow. Measurements in jets by Corrsin and Uberoi (9) and by Hinze and Van der Hegge Zijnen (16) have shown that temperature distributions for a source on the axis of the jet are symmetrical and distributions for one off the axis are skew, with negative skewness. From these measurements, using the derived expression for the skewness factor, the skewness factor of v can be inferred. It has been found that the skewness factor of v is positive, as is to be expected from the negative sign of the gradient of the turbulent intensity. The magnitude of the inferred skewness of v is consistent with direct measurements of the same quantity in wakes. Turbulent intensities are substantially less (relative to the mean velocity) in boundary layers than in jets, but the difference between the distributions of temperature and of transverse velocity are still appreciable. Skramstad (Dryden) (10) has measured the vertical distribution of temperature behind a line source of heat in a boundary layer. From the measured temperature distributions, the skewness factors of Y have been computed (after corrections for the effect of conductivity) and the consequent skewness factors of v inferred. It has been found that the difference is everywhere much less than the difference found in jets. Also, $\overline{Y^2}$ has been found to be proportional to X^2 over the range of measurement confirming that Eq. (2-12) holds. The distances from the source were, of course, very small--up to 4 cm.

Chapter III

THEORETICAL ANALYSIS

In homogeneous turbulence the distribution curves show a symmetry with respect to one axis or with respect to a plane--depending on the kind of the source (point or line).

If the mean velocity is not uniform but is some function of the space coordinates, it may be expected that there will no longer be symmetry in the distribution curves. In general, the shape of these distribution curves will be determined by the characteristics of the turbulence and by the spatial distribution of the mean velocity.

The picture of the diffusion process close behind the source and within the integral scale of turbulence is different from that seen at greater distance. If we consider, however, only short diffusion times, an idea of this picture may be obtained mainly by considering the local distribution of the mean velocity and the local turbulence at the source.

The problem investigated in this thesis, with short distances from the source and short diffusion times, falls into this category and an analysis, which follows the work of Hinze (14,15,16), is presented in this chapter.

The considered distances are so short that perfect correlation exists between the velocity at subsequent times of a fluid particle passing the source.

Thus, the assumption is that the Lagrangian correlation coefficient $R_L(t) \simeq 1$ during the time of diffusion considered. For short diffusion times we have

$$Y(t) \simeq v_o t$$
$$X(t) \simeq (\overline{U}_o + u_o)t$$

where the index zero marks the quantity at the point of the source. Hence,

$$Y(t) \equiv Y(x) = \frac{v_o}{\overline{U}_o + u_o} \cdot X = \frac{v_o}{\overline{U}_o + u_o} \cdot x \quad .$$

If $u_o/\overline{U}_o << 1$, the probability-density distribution of $Y(x)$ is identical with that of v_o. But if the relative intensity of turbulence is not that small, this identity no longer holds.

If the average value of Y is taken over a large number of fluid particles that have passed the source at subsequent times t_o, we have

$$\overline{Y}(x) = \frac{x}{T} \int_o^T \frac{v_o(t_o)}{\overline{U}_o + u_o(t_o)} \, dt_o \quad .$$

With the assumption that $u_o(t_o)/\overline{U}_o < 1$, we may develop $(1 + u_o/\overline{U}_o)^{-1}$ into a Taylor series,

$$\frac{\overline{Y(x)}}{x} = \frac{1}{T}\int_0^T \frac{v_o(t_o)}{\overline{U}_o}\left[1 - \frac{u_o(t_o)}{\overline{U}_o} + \frac{u_o^2(t_o)}{\overline{U_o^2}} - \dots\right] dt_o$$

$$= -\frac{\overline{u_o v_o}}{\overline{U_o^2}} + \frac{\overline{u_o^2 v_o}}{\overline{U_o^3}} - \frac{\overline{u_o^3 v_o}}{\overline{U_o^4}} + \dots \quad (3.1)$$

and

$$\frac{\overline{Y^2}}{x^2} = \frac{\overline{v_o^2}}{\overline{U_o^2}} - 2\,\frac{\overline{u_o v_o^2}}{\overline{U_o^3}} + 3\,\frac{\overline{u_o^2 v_o^2}}{\overline{U_o^4}} - \dots \quad (3.2)$$

$$\frac{\overline{Y^3}}{x^3} = \frac{\overline{v_o^3}}{\overline{U_o^3}} - 3\,\frac{\overline{u_o v_o^3}}{\overline{U_o^4}} + 6\,\frac{\overline{u_o^2 v_o^3}}{\overline{U_o^5}} - \dots \quad (3.3)$$

Thus, the skewness factor $S = \overline{Y^3} / (\overline{Y^2})^{3/2}$ of the distribution of Y can be determined from the last two relations.

$$\frac{\overline{Y^3}}{(\overline{Y^2})^{3/2}} = \frac{\overline{v_o^3}}{(\overline{v_o^2})^{3/2}} - 3\,\frac{\overline{u_o v_o^3}}{\overline{U}_o(\overline{v_o^2})^{3/2}} + 3\,\frac{\overline{u_o v_o^2}}{\overline{U}_o} \cdot \frac{\overline{v_o^3}}{(\overline{v_o^2})^{5/2}}$$

$$+ \text{ terms of order } \overline{v_o^2}/\overline{U_o^2} \ .$$

The approximate magnitude of the first-order correction term can be found following the suggestion made by Batchelor and Townsend (4) about relations between various correlation terms appearing in the expression for the skewness factor S (Chapter II). Then,

$$\frac{\overline{Y^3}}{(\overline{Y^2})^{3/2}} = \frac{\overline{v_o^3}}{(\overline{v_o^2})^{3/2}} - 9\,\frac{\overline{u_o v_o}}{\overline{v_o^2}} \cdot \frac{(\overline{v_o^2})^{1/2}}{\overline{U}_o}$$

$$+ 3\,\frac{\overline{u_o v_o}}{\overline{v_o^2}} \cdot \frac{(\overline{v_o^3})^2}{(\overline{v_o^2})^3} \cdot \frac{(\overline{v_o^2})^{1/2}}{\overline{U}_o} + \dots \quad (3.4)$$

Therefore, the skewness factor S can be expressed in terms of the skewness factor $S_v = \overline{v_o^3}/(\overline{v_o^2})^{3/2}$.

From Eq.(3.1) we may obtain one interesting result. If the effect of the triple and higher-order correlations is negligibly small, we get

$$\frac{\overline{Y}}{x} \simeq - \frac{\overline{u_o v_o}}{U_o^2} \tag{3.5}$$

which means that $\overline{Y}/x$ is proportional to the shearing stress in the flow at the point of the source. Thus, the value of the mean of the distribution, since $\overline{Y}/x$ is identical with the mean, is in first approximation proportional to the local turbulent shearing stress.

At the same time, this expression shows us that the median of the particle distribution is displaced from the mean flow streamline through that distance.

In order to calculate the skewed distribution, Hinze assumed a normal joint-probability-density distribution of the turbulence velocity components u and v.

$$F\left(\frac{u}{U}, \frac{v}{U}\right) = \frac{\overline{U^2}}{2\pi\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}(1-R_{uv}^2)^{\frac{1}{2}}}$$

$$\exp\left\{\frac{-1}{2(1-R_{uv}^2)}\left[\frac{u^2}{\overline{u^2}} - \frac{2uvR_{uv}}{\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}} + \frac{v^2}{\overline{u^2}}\right]\right\} \tag{3.6}$$

where

$$R_{uv} = \frac{\overline{u_o v_o}}{\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}} .$$

Consider an element dxdy at the point (x,y). The probability of finding in this element dxdy a fluid particle originating from the source of unit strength at (x = 0, y = 0) is

$$P(x,y)\ dxdy = \frac{1}{\overline{U^2}} \int_{-\infty}^{t} dt_o\ F\left(\frac{u}{\overline{U}}, \frac{v}{\overline{U}}\right) dudv$$

with $du = dx/(t-t_o)$, $dv = dy/(t-t_o)$

$$P(x,y)\ dxdy = \frac{1}{\overline{U^2}} \int_{-\infty}^{t} dt_o\ F\left(\frac{u}{\overline{U}}, \frac{v}{\overline{U}}\right) \frac{1}{(t-t_o)^2}$$

For a given value of x,

$$\frac{dt_o}{(t-t_o)^2} = d\left(\frac{1}{t-t_o}\right) = d\left(\frac{\overline{U}_o+u_o}{x}\right) = \frac{du_o}{x}$$

Hence,

$$P(x,y) = \frac{1}{x\overline{U}} \int_{-1}^{\infty} d\ \frac{u_o}{\overline{U}}\ F\left(\frac{u}{\overline{U}}, \frac{v}{\overline{U}}\right).$$

Substituting the expression (3.6) and carrying out the integration, we get

$$P(x,y) = \frac{1}{2\sqrt{2\pi}\ (\overline{v^2}\ x^2 - 2\sqrt{\overline{u^2}}\ \sqrt{\overline{v^2}}\ R_{uv}xy + \overline{u^2}\ y^2)^{\frac{1}{2}}}$$

$$\mathrm{erfc}\left[-\frac{\sqrt{\overline{v^2}}\ \overline{U}_o\ x\ (1 - \sqrt{\overline{u^2}}/\sqrt{\overline{v^2}}\ R_{uv})}{\sqrt{\overline{u^2}}\left[2(1-R_{uv})(\overline{v^2}\ x^2 - 2\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}\ R_{uv}+\overline{u^2}y^2)\right]^{\frac{1}{2}}}\right]$$

$$\exp\frac{-\ \overline{U_o}^2\ y^2}{2(\overline{v^2}\ x^2 - 2\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}\ R_{uv}\ xy + \overline{u^2}\ y^2)} \qquad (3.7)$$

For short diffusion times the value of erfc in Eq. (3.7) is practically equal to 2. For the normalized distribution, we then obtain

$$\frac{P(\eta)}{P(o)} = \frac{1}{\left[1-2\frac{\sqrt{\overline{u^2}}}{\sqrt{\overline{v^2}}} R_{uv}\,\eta + \frac{\overline{u^2}}{\overline{v^2}}\,\eta^2\right]^{\frac{1}{2}}} \exp \frac{-\overline{U_o^{\,2}}\,\eta^2}{2\overline{v^2}\left[1 - 2\frac{\sqrt{\overline{u^2}}}{\sqrt{\overline{v^2}}} R_{uv}\,\eta + \frac{\overline{u^2}}{\overline{v^2}}\,\eta^2\right]} \qquad (3.8)$$

where

$\eta = y/x.$

For the case of isotropic turbulence ($u = v$, $R_{uv} = 0$),

$$\frac{P(\eta)}{P(o)} = \frac{1}{(1 + \eta^2)^{\frac{1}{2}}} \exp \frac{-\overline{U_o^{\,2}}\,\eta^2}{2\,\overline{v^2}\,(1 + \eta^2)}$$

The last expression indicates that for greater values of η we would have a somewhat greater spread than according to a Gaussian distribution.

Chapter IV

MEASURING EQUIPMENT AND PROCEDURES

The measurements described in this chapter were carried out in the low-speed wind tunnel of the Fluid Dynamics and Diffusion Laboratory at Colorado State University. The main objective of the conducted measurements was to obtain data needed for an analysis of the process of diffusion of heat spots in a turbulent boundary layer. The measured quantities consisted of the mean velocity, the maximum instantaneous temperature of a heat spot, the instantaneous length of a heat spot, turbulent velocity components at the point of the heat source, distances of the source from the wall and of the resistance-wire thermometer (RWT) from the source as well as distances of RWT from the streamline through the source in the horizontal and the vertical direction. A detailed description of these measurements and procedures is given in the following paragraphs.

A. Heat Source and Production of Heat Spots

For the study of diffusion of heat spots, a technique was developed for production of heat spots and detection of temperature distributions downstream from the point of production. A producer of constant current pulses--current generator--was designed and built. As a heat source, a short length of platinum-iridium wire 0.0004" in diameter and approximately 1/10" long was used. The wire can be heated almost instantaneously by a short pulse of electric current and a small heat spot can be produced.

An arrangement with a resistance thermometer as a sensor element was employed for detection of temperature distributions. The output of this circuit was amplified and applied to a "memory scope" (a detailed description of the detection equipment is given in paragraph B).

A considerable amount of time has been spent on the proper choice, design and initial set-up of the needed equipment. Two spark producing systems were built. A system similar to an auto-ignition system with 12 VDC, and a system with 10,000 VAC output were tested. Unfortunately, a few disadvantages and inconveniences were encountered that affect the use of these systems in an investigation of turbulent diffusion of heat. The problem of controlling the heat production, a very important one in an investigating technique that uses "instantaneous heat source", and problems associated with the existence of repelling forces due to ionization of air during a spark discharge, were the most serious ones. Also, there always exists a possibility of radiation effects which may affect measurements close to the heat source.

Because of all these reasons, a system with a short length of platinum-iridium wire, with a controlled current flowing through it, was chosen as a heat source. The wire is 0.0004" in diameter and about 1/10" long. It is heated by a short pulse of electric current and then allowed to cool in the air-stream. The time constant of the voltage pulse is about 0.0006 sec.

The circuit of the current generator is shown in Fig. 5. The voltage pulses are converted into current pulses of up to about 250 mA. The peak temperature of the wire can be computed from the ratio of the peak voltage drop across the wire to the initial voltage drop assuming that the current pulse has an instantaneous rise to its peak value followed by an exponential decay. The characteristic oscillogram of voltage applied to the source wire is shown in Fig. 6.

Preliminary work and qualitative data taken showed from the beginning a very rapid cooling of heat spots. This is considered to be a limiting factor in the use of heat spots to trace diffusion movements of turbulence over large time intervals, i.e., large distances. Therefore, the sensitivity of the sensor element is of the greatest importance. Evaluations done at CSU (6) showed that with the proper electronic equipment a signal corresponding to approximately 0.02°F (difference) could be detected. Preliminary investigation, done with the equipment described in this chapter, showed that, due to distortions and some limitations in amplification of a signal produced by the noise and the very procedure of data taking, a signal corresponding to about 0.15°F (difference) could be detected.

B. Detection of Instantaneous Temperature Profiles

To detect temperature distributions downstream from the source, a 2.5×10^{-5} inch diameter, 90% platinum – 10% rhodium wire was used as a resistance thermometer. The wire, with a detection current of 0.1 milliamps, has a sensitivity of approximately 0.07 millivolts per °F.

The resistance thermometer is an application of the family of resistance-temperature transducers. Such a sensor is especially valuable in the measurement of temperature fluctuations in a thermal layer in which the use of a fast response thermometer is required. It is assumed that the instrument used must follow fluctuations of the order of 10,000 cycles per second and a sensitivity of about one hundredth of a degree centigrade in temperature is desirable. The resistance thermometer is capable of following transient temperature fluctuations and with the proper electronic circuit the frequency response of 10,000 cycles can be obtained. However, the sensitivity of a high frequency instrument is limited up to 0.01°C. An evaluation of a 90% platinum – 10% rhodium wire as a resistance thermometer has been done by Chao and Sandborn (6). The results of their experimental evaluation of the wire sensitivity to temperature fluctuations, Fig. 8, have been used in the analysis of distributions of heat spots. By means of the diagram in Fig. 8, the mean maximum temperature distribution curves obtained with resistance-thermometer wires of different resistances were corrected to correspond to a single value of resistance. In that way an analysis and a comparison of results were made possible.

A Wheatstone bridge was employed to operate the wire at a constant current of 0.1 milliamps. The output of the bridge was amplified and applied to an oscilloscope with a "memory screen". The instantaneous temperature profiles of heat spots passing over the resistant-thermometer wire were displayed--by means of the oscilloscope's triggering circuit--and fixed on the memory screen. After the reading of the peak of a temperature profile was taken, the memory screen was erased and the process repeated. At the same time with the peak temperature reading, the length of an instantaneous profile (expressed in centimeters of the oscilloscope's horizontal scale) was taken.

An effort to obtain data on mean temperatures of passing heat spots by means of an integrator circuit has not shown positive results. This failure was due to extremely small amplitudes of temperature profiles,especially at distances further downstream from the point of release or at the boundary points of plume region. However, additional amplification was not possible because the triggering circuit of the "memory scope" was employed in the process of data taking (display of instantaneous temperature profiles on the memory screen), and amplified noise would have disturbed this process too much. The maximum amplitude of instantaneous temperature profiles and instantaneous horizontal lengths of passing heat spots were read from the "memory screen" directly. From about 100-120 readings, mean maximum temperature and mean cloud length were calculated as an ensemble average. These mean maximum temperatures appear as single points in the plots of lateral and vertical distributions of heat spots, Figs. 14-34.

Data were taken up to 5 in. downstream from the source. There was not much possibility for improvement in this direction. Very fast dissipation of heat spots in turbulent flow and the extremely small amount of heat released at the source ($\sim$1.5W) do not allow detection of diffusing heat at long distances. This is considered as a limiting factor in the use of heat spots to trace diffusion movements of turbulence. Moreover, an "instantaneous" source means a further amplification of these difficulties. Possibilities of increasing the source strength are very limited due to inherent demands to keep the source size as small as possible (to simulate a "point") and to avoid disturbance of the surrounding flow.

As an amplifier, the Keithley Model 103 ultra-low noise preamplifier with a bandwidth of 0.1 cycles to 100 kc and gains of 100 and 1000 was used. The model 103 is of use wherever low-level signals within its bandwidth are to be amplified. Separate low-frequency and high-frequency controls permit the bandwidth to be reduced to improve the signal-to-noise ratio where reduced bandwidth is permissible. At maximum bandwidth setting, frequency response is essentially flat between 3-dc points at a lower frequency limit of 0.1 cycles and a high frequency limit of 100 kc.

As a "memory scope", the Tektronix Type 564 Storage Oscilloscope was used. This is a special-purpose oscilloscope designed to store cathode-ray tube displays for viewing or photographing up to an hour after application of the input signal. In addition, this instrument can be operated as a conventional oscilloscope. The two separate storage screens of the cathode-ray tube provide convenience and versatility for comparison and analysis of waveforms. Display storage is also convenient for detailed viewing and photography of single-sweep displays and extremely low frequency waveforms ($<$60 cps). As has been described, temperature profiles were displayed on the storage screen and the data collected were read directly from the screen.

The movement of the resistance-thermometer wire was possible in both the horizontal and vertical directions. Its holder was fixed on a rod which was connected to a vertically mounted micrometer scale with an accuracy of vertical adjustments up to 1/40". In turn, the micrometer scale was fixed to the rod of a level measuring gauge movable in the horizontal direction with an accuracy of adjustment up to 1/1000 ft. The distances were measured from the point of the heat source to the tip of the resistance-thermometer wire.

C. Sample Size

Preliminary work was done to find out how big the sample size, i.e., the number of readings for a particular point in a cross section downstream from the source, should be. This preliminary investigation was conducted in the free stream flow outside the boundary layer. Its original purpose was to obtain data on diffusion of heat spots in (assumed) homogeneous isotropic turbulence, to compare these data with similar, available experimental data and, as the main objective, to discover the limitations and peculiarities of the technique used and the possibility of improvements in the later work in the boundary layer.

From the oscillograph of an instantaneous temperature profile of a heat spot, the peak temperature was read. By repeating the process, a number of peak temperature values were obtained and mean maximum temperature was computed as the mean of taken values. Preliminary data were taken at different distances from the source with the purpose of finding the number of repeated readings needed for computation of mean values with prescribed accuracy.

By taking about 100 readings at different distances from the source, the mean value of readings $\bar{y}$ and the standard deviation of readings, s, were computed. The following equation was used to compute the sample size necessary to obtain a $100(1 - \alpha)\%$ confidence interval with expected length 2d:

$$n = \left(\frac{s t_{\alpha/2, n-1}}{d} \right)^2$$

where $t_{\alpha/2, n-1}$ is the t deviate for $n-1$ degrees of freedom.

All data, taken with a free stream velocity of 15 ft/sec, were expressed in centimeters representing the height of an instantaneous temperature profile on the oscilloscope screen. For confidence of 80% and with mean maximum temperature specified within $d = \pm 0.2\bar{y}$, where $\bar{y}$ is the mean calculated from the readings taken from the oscilloscope screen, the following sample sizes were obtained:

Distance	n
1"	7
3"	23
6"	60

These values correspond to the measurements along the streamline through the source (i.e., central line). Obviously, different values--larger sample size-would have to be obtained for the points off the central line. For a boundary layer, these sample sizes are naturally larger. However, an investigation similar to the one described has not been conducted in a boundary layer. Instead, as many as 100-120 readings were taken from the "memory screen" for a particular point. Though it is difficult, without an accurate and lengthy investigation, to evaluate the confidence in obtaining mean values, it can be concluded, from the reproducibility of data taken in a range from 2" to 5" from the source, that a confidence of about 80% was obtained--at least along the central line.

D. Wind Tunnel

The investigation of diffusion of heat spots was conducted in a low speed, closed circulation type wind tunnel, Fig. 3. The wind tunnel has a test section of 30 ft, and a cross-sectional area of 6x6 ft^2. Damping screens, located in the 4:1 contraction area, yield a free stream turbulence level of less than 1%. Wind speed in the tunnel can be varied from about 5 to 75 fps. Air is moved by a constant speed axial fan and velocity is adjusted by a remote pitch control of the variable pitch blades of the fan. The heat source was located 27'-3" from the beginning of the test section. A free stream velocity of 15 fps was used for all experiments. Since there is no temperature control in the tunnel, the temperature of air varied through experiments from about 25°C to 34°C. The mean velocity was measured with a pitot tube and the Transonic, Equibar type 120, electronic manometer.

E. Turbulent Measurements

Turbulent quantities, $\overline{u^2}$, $\overline{v^2}$, and $\overline{uv}$, were measured by means of a constant hot-wire anemometer. A single wire was used for the measurement of $\overline{u^2}$ and a cross wire was used for the evaluation of $\overline{v^2}$ and $\overline{uv}$. The hot wire was operated by a constant temperature type servo amplifier, Hubbard 3A, from which the output was fed into a true rms-meter, type Bruel and Kjaer 2409. The output voltage of the Hubbard set is a linear function of velocity. Both hot wires were calibrated against a pitot tube placed in the free stream. The straight line calibration curves--the free stream velocity vs. the output reading of the hot-wire anemometer--were obtained, Figs. 11, 12.

If fluctuations of velocity are small compared with the mean value, the relation between the fluctuation of velocity (u) and that of current (I) is given

$$I = Su$$

where S is the sensitivity of the hot-wire anemometer. For the Hubbard set, the following relation is used for the computation of $\overline{u^2}$:

$$\overline{u^2} = \left(\frac{A}{50}\right)^2 \overline{e^2}$$

where $\overline{e^2}$ is the mean square reading of the output voltage, A is a constant obtained from the calibration curve as the slope of the curve ($A = 1/S = du/dI$), and 50 $[K\Omega]$ is the value of a resistor across which the output voltage is measured.

From the calibration curve, Fig. 11, $A = 23.4$. From RMS meter readings, the following values were obtained: $\sqrt{\overline{e^2}} = 2.0V$, for the heat source at 2-1/2", and $\sqrt{\overline{e^2}} = 0.45V$, for the source at 5-7/8". Hence,

$$\overline{u^2} = 0.88, \text{ for } 2\text{-}1/2''$$

$$\overline{u^2} = 0.044, \text{ for } 5\text{-}7/8''$$

Turbulent quantities $\overline{v^2}$ and $\overline{uv}$ were measured by means of a cross wire. The following relations were used for computation:

$$\overline{v^2} = \frac{1}{2\sin^2\alpha}\left(\frac{A_{1,2}}{50}\right)^2\left(\overline{e_1{}^2} + \overline{e_2{}^2}\right) - \overline{u^2}\,\frac{\cos^2\alpha}{\sin^2\alpha}$$

$$\overline{uv} = \frac{1}{2\sin 2\alpha}\left(\frac{A_{1,2}}{50}\right)^2\left(\overline{e_1{}^2} - \overline{e_2{}^2}\right)$$

where $A_{1,2} = A_1 = A_2 = 28.6$ is the slope of the calibration curve, Fig. 12, α and β are angles at which two wires of the cross wire are inclined to the main stream direction, and $\alpha = \beta = 40^\circ$.

With the following values read from RMS meter

$$\sqrt{\overline{e_1{}^2}} = 1.1V, \ \overline{e_2{}^2} = 1.65V, \text{ for } 2\text{-}1/2''$$

$$\sqrt{\overline{e_1^{\,2}}} = 0.5V, \quad \overline{e_2^{\,2}} = 0.55V, \text{ for } 5\text{-}7/8''$$

the values of $\overline{v^2}$ and $\overline{uv}$ are computed as:

$$\overline{v^2} = 0.31, \quad \overline{uv} = -\ 0.258, \text{ for } 2\text{-}1/2''$$

$$\overline{v^2} = 0.1, \quad \overline{uv} = -\ 0.0061, \text{ for } 5\text{-}7/8''$$

These values are used in the next chapter for the computation of vertical and horizontal distribution curves of heat spots.

Chapter V

ANALYSIS OF DATA

A. Vertical Distribution of Heat Spots

With two sets of data taken at several distances from the source in a range from 2" to 5", with the source height of 2-1/2" and 5-7/8" respectively, an analysis has been undertaken to see if any firm conclusion can be made regarding the pattern of diffusion. Due to the nature of the data-taking procedure, as described in Chapter IV, with a finite number of readings used for the computation of average values and a relatively small number of points (about 10) computed for a particular distance in that way, any conclusion about the shape of distribution curves based on only one set of points would appear to be unreliable.

The only profiles in question are the vertical profiles because the horizontal profiles--assuming homogeneity in the lateral direction, at least in the range of measurements--must necessarily be very close to Gaussian curves. Any skewness is mainly due to experimental errors or insufficient number of observations.

To make a conclusion regarding vertical distribution of heat spots possible, the profiles, obtained at different distances from the source, were made to coincide by plotting the ratio $\overline{\theta}/\overline{\theta}_{max}$ as the function of the distance ratio, $\eta = y/x$ (where y is the vertical distance from the streamline through the source and x is the distance from the source in direction of flow). The vertical distributions, for the source placed at 2-1/2" from the wall, are plotted in such a way in Fig. 14. The distribution obtained shows a noticeable and definite skewness.

Similar skewness has been shown in experiments carried out by Skramstad (10) in a turbulent boundary layer, by Corrsin and Uberoi (9) in the turbulent mixing zone of a round free jet, and by Hinze and Van der Hegge Zijnen (16) in the turbulent mixing zone of a plane free jet. All these experiments used a thin heated wire as the line source of heat. The skewness was such that the greater spread occurred at the side of the greater value of the mean velocity. The skewness obtained with the instantaneous point source shows the same characteristics, Fig. 14.

If the spread can be described by means of a constant coefficient of eddy diffusion, the effect of the gradient in the mean velocity would be a skewness just the opposite of that actually observed. For, the time the flow needs to cover a distance from a source is shorter at the side of greater velocity and vice versa. Hence, the skewness will be at the side of smaller mean velocity, i.e., the spread will be wider at that side. The formal objection to the assumption of a constant coefficient of eddy diffusion is that it is not possible to associate an integral scale with a homogeneous shear-flow turbulence.

Dryden (10), in connection with the experiments of Skramstad, pointed out that the skewness cannot be accounted for by the slight variations of mean speed or by the local values of $\sqrt{\overline{v^2}}$ which vary across the wake and that a theory with a single scale diffusion coefficient cannot account for such an effect.

So far the only attempts to explain the observed skewness in the spread of heat in a turbulent shear flow has been made by Corrsin (7) and Hinze (14). Corrsin assumed that the lateral spread at a cross section downstream of the wire was directly determined by the probability distribution of the v-component of the turbulent velocity. In other words, the temperature distribution is simply the probability density of the lateral velocity fluctuation. Hence, the skewness of the y-distribution is caused by the skewness of the v-distribution, and the skewness factor S_y is equal to the skewness factor S_v. However, as is shown in Chapter III, opposite signs of the skewness factors S_y and S_v may be expected.

Hinze followed a different approach to explain the observed skewness of the temperature distribution curve. If, for instance, $d\overline{U}/dy > 0$ and consequently $\overline{uv} < 0$, on the average more fluid particles with negative u will be transported with a positive v through a control plane and vice versa. To positive values of v correspond positive values of y. Hence, for the same value of x, particles with a positive v (positive y) need a longer time to reach this value of x than particles with negative v (negative y). Consequently, the spread of fluid particles for positive values of y must be greater than the spread for negative values of y.

In order to calculate the skewed temperature distribution, Hinze assumed a normal joint-probability-density distribution of the turbulence velocity components u and v. As is shown in Chapter III, Eq. (3.8) can be obtained for the normalized distribution. In Fig. 14, the dashed curve is computed from Eq. (3.8). The turbulent velocity components were measured at the point of the source by means of the constant current hot wire anemometer as it was described in Chapter IV. With the values so obtained, the following values of $R_{uv}(o)$ were computed:

$$\text{for } 2\text{-}1/2'' \quad R_{uv}(o) = \frac{\overline{uv}}{\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}} = -\ 0.494$$

$$\text{for } 5\text{-}7/8'' \quad R_{uv}(o) = \frac{\overline{uv}}{\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}} = -\ 0.07$$

The normalized distribution $P(\eta)/P(o)$ from Eq. (3.8) was then calculated and the result was plotted as a dashed curve in Figs. 14 and 15.

Though, with the method of the measurements used and within its rather restricted accuracy, the agreement looks almost satisfactory, it must be kept in mind that the assumption of normally correlated u and v is probably not justified.

It should be mentioned that no corrections have been made for the effect of molecular diffusion. The error in computation of particular points, due to the nature of statistical averaging, was estimated at about $\pm 20\%$ along the center line, i.e., the streamline through the source. Since the sample size was kept constant throughout the measurements, the accuracy in computation of the points off the central line was probably lower and decreasing sidewise and downstream. This could result in lower values (higher number of "zeroes") for points off the central line and further downstream than should have resulted. Obviously, this kind of uncertainty does not justify any precise corrections that are otherwise required in the case of molecular diffusion due to the very nature of that process.

Another reason for not making any correction for the effect of molecular diffusion was that it is not yet clear whether the contributions of molecular and turbulent diffusion are independent and additive or not, and in which range of distances.

The relation in Eq. (3.7) and, consequently for the normalized distribution, in Eq. (3.8), were derived for the mean values of temperature. A justification for using relations so derived in an analysis of distributions of mean maximum temperatures is given by the measurements of instantaneous lengths of heat spots. As can be seen from Fig. 13, mean instantaneous lengths of heat spots are approximately the same for a particular distance from the source. This indicates that distribution at a particular cross section is due only to the meandering of heat spots. The size and temperature profiles of heat spots are, on the average, the same all over the range of measurement for a given distance. The only difference is in the frequency of occurrence of heat spots at different points of a particular cross section. This was actually observed from the "memory screen" oscillograms. Therefore, a normalized distribution for mean maximum temperatures is the same as one obtained for mean temperatures.

For the vertical distribution of heat spots with the source at 5-7/8", Fig. 15, almost no skewness was found. The reason is that this distance is almost at the edge of the boundary layer and the velocity gradient is extremely small. As it is shown in Chapter IV, $\overline{u^2} = 0.044$ and $R_{uv}(o) \approx 0.07$ only. The distribution calculated from Eq. (3.8) is practically symmetrical. Some data taken at 5-7/8" from the wall show larger scatter and more "irregularities" than the data taken at 2-1/2". This can be explained by the strong intermittency of turbulent flow in the region close to the edge of the boundary layer, though a relatively long time used to obtain data probably smoothed it out.

B. Horizontal Distribution of Heat Spots

The horizontal profiles taken both at 2-1/2" and 5-7/8" from the wall show a reasonable symmetry as was expected under the assumption of homogeneity in the lateral (z) direction. A small skewness in some distributions can be explained, besides the reasons mentioned earlier (inaccuracy of the method of measurement and computation, etc.), by the very fact that the "point" source was not a real point. The holder of the heat source probe, stretched in the E(ast) direction from the source, could affect diffusion of heat spots by wakes produced behind it. This could be the reason for a little wider spread in the W(est) direction which can be noticed in some distributions. The spread in the E direction is consequently smaller because of the faster dissipation of heat. However, it is very difficult to make any firm conclusion of this nature, i.e., regarding small effects and influences on diffusion of heat in turbulent flow. Basic information, for instance, and numerical data concerning characteristics of turbulence can be obtained by investigating the spread of heat behind a source. An instantaneous point source technique, however, cannot be accurate and reliable enough for such purposes.

The spread of heat spots appears to be linear, in both vertical and horizontal distributions, at least in the range of the first few distances. Again, such a conclusion is not based entirely on a firm foundation. An extrapolation has to be done in some distributions in order to fix the "zero" points of the spread. Moreover, some of the distribution curves taken at distances of 4" and 5" show less spread than do the ones taken at distances closer to the source. This is, of course, impossible, and the main reason that such data were obtained is the very fast dissipation of heat spots in turbulent flow. At distances further downstream from the source the turbulent spread of the heat produced by the source was already so great that the temperature rise became almost impossible to detect. Generally speaking, it is possible to make conclusions concerning only general trends of the diffusion pattern (as discussed above) and any further conclusions would be of a speculative nature.

SUMMARY AND SUGGESTIONS

A. Summary

Diffusion of heat from an instantaneous point source was investigated in a turbulent boundary layer. The following results were obtained:

1. The horizontal distributions of heat spots are, within the restricted accuracy of the method of measurement used, very close to Gaussian curves. This is an expected result under the assumption of homogeneity in the lateral direction--at least in the range of measurements.
2. The vertical distributions of heat spots show a skewness. The skewness is such that the greater spread occurs at the side of the greater value of the mean velocity. The obtained skewed distribution was compared with the Hinze's theoretically derived skewed temperature distribution. Though the agreement is satisfactory the assumption of normally correlated u and v is probably not justified.
3. Mean instantaneous lengths of heat spots are approximately the same for a particular distance from the source. This indicates that the dispersion at a particular cross section is affected

only by the random wandering of heat spots. The size and temperature profiles of heat spots are, on the average, the same all over the range of measurements for a given distance. The only difference is in the frequency of occurrence of heat spots at different points of a cross section.

4. The spread of heat spots appears to be linear, at least in the range of the first few distances. However, the data taken are not sufficient for a firm conclusion. An extrapolation has to be done to fix the zero points of some distributions.

B. Suggestions for further study

Diffusion of heat can be used for obtaining basic information and numerical data concerning characteristics of turbulence. This method has been applied successfully to isotropic turbulence. Van der Hegge Zijnen (31) conducted a research in a turbulent boundary layer to investigate its merits in the case of shear flow. Hinze's theory of normally correlated u and v , though probably not entirely correct, offers the possibility of finding with a reasonable degree of accuracy the components of turbulence intensity from experimental diffusion diagrams. This requires, however, such a high degree of accuracy in taking temperature distribution and in locating these profiles with respect to the center line and such a precision in set up that the use of an instantaneous point source for such a purpose would be very difficult.

An instantaneous point source can be used in an investigation on interaction between molecular and turbulent diffusion. Saffman showed analytically that the interaction of the process of molecular diffusion with the stretching effect of the turbulence increases the rate of cooling of heat spots--i.e., their sizes--but that the interaction decreases the spreading of the heat relative to the point of release. So far, there has not been any quantitative proof for Eq. (2.11) and an experiment with an instantaneous point source can be designed to check its validity.

Possibilities for the use of an instantaneous point source of heat seem the most promising in space-time correlation measurements. An experiment truly comparable to the theory could be designed and the space-time correlations would be measured for single heat spots, detecting the same spot simultaneously at more than one place.

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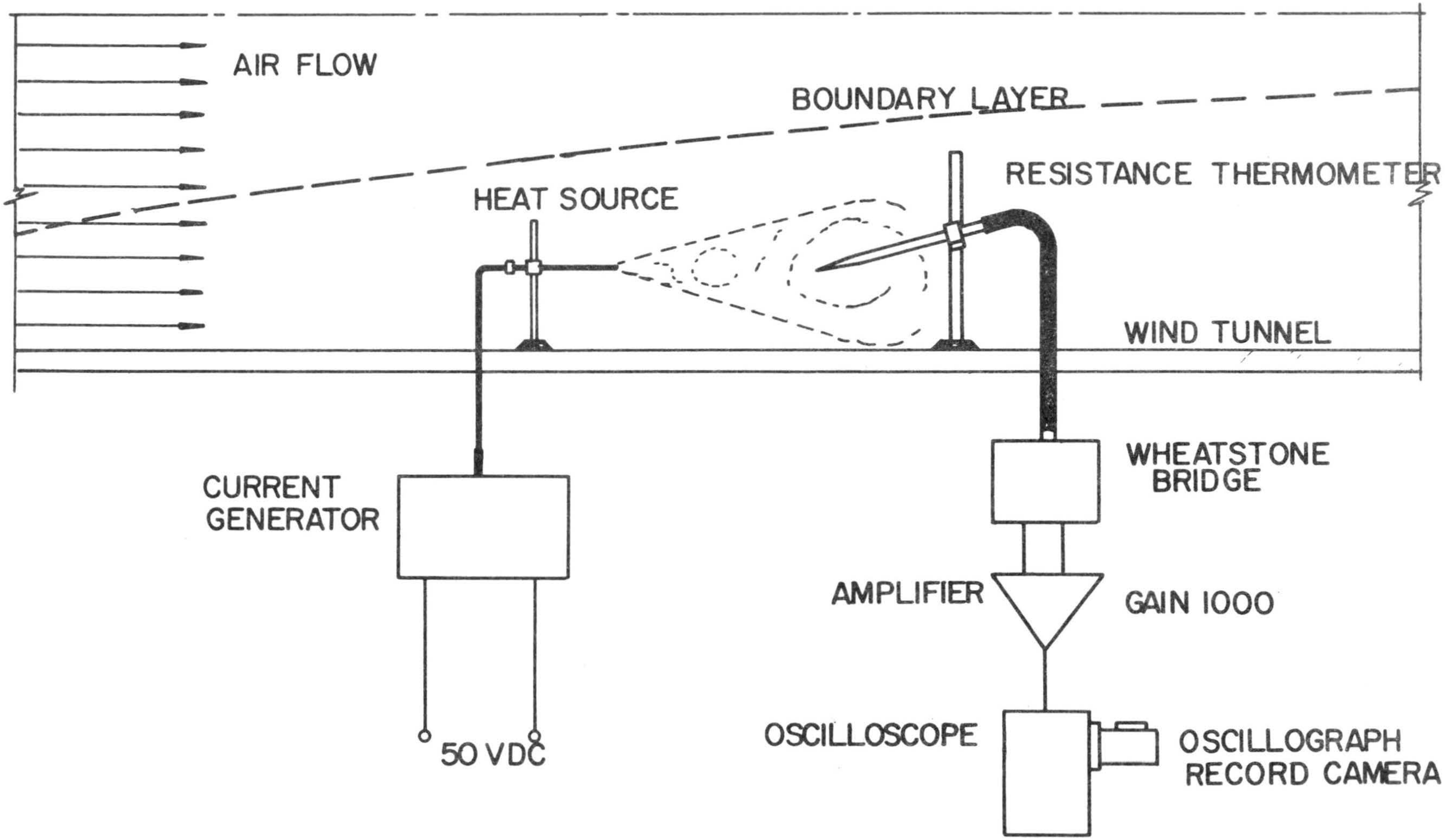


Fig. 1. Schematic of equipment set-up

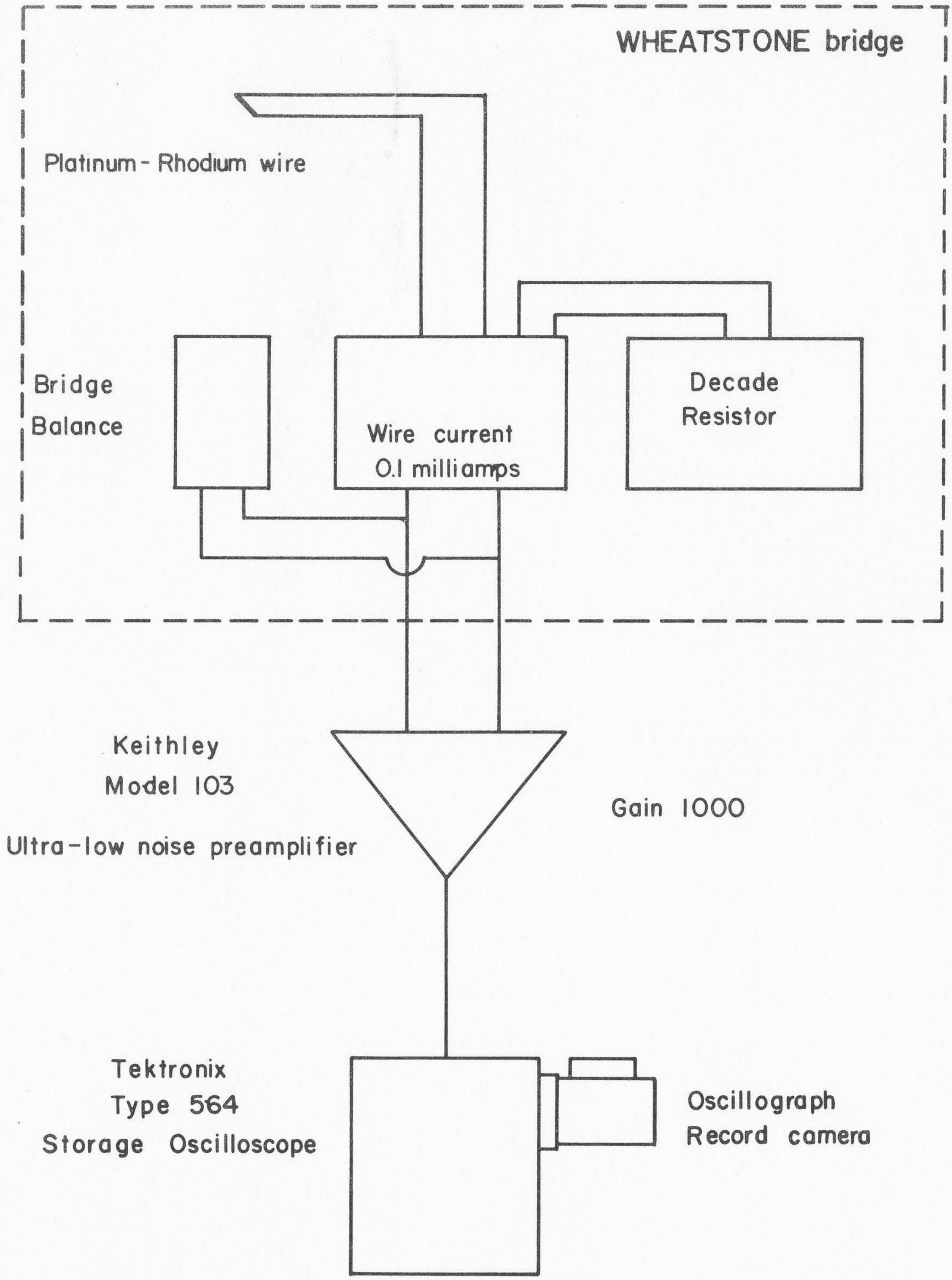


Fig. 2. Block diagram of resistance-thermometer circuit

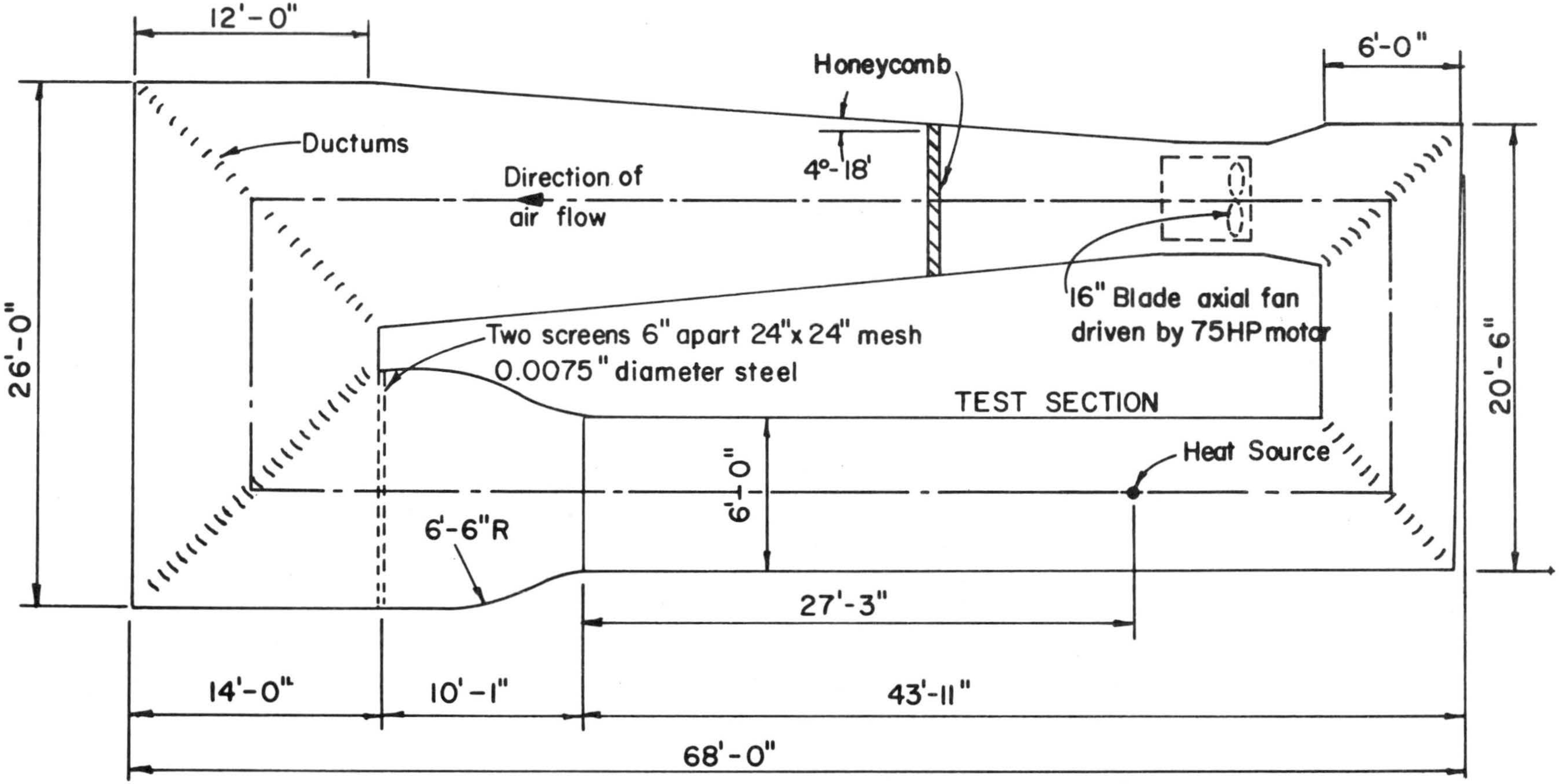
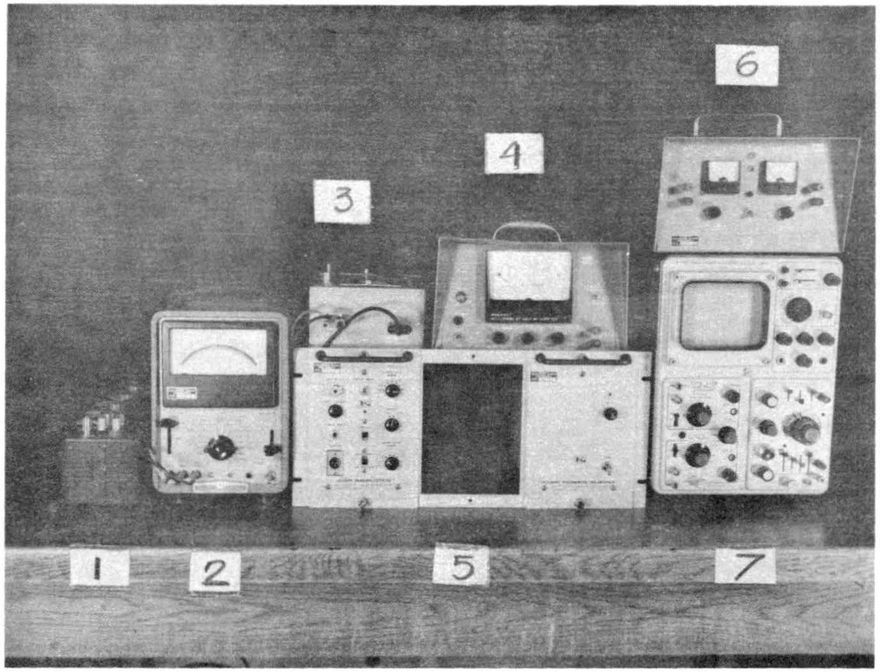


Fig. 3. Small wind tunnel



1. Decade resistor
2. Voltmeter
3. Wheatstone bridge
4. Current generator
5. Amplifier
6. Power supply
7. Storage oscilloscope

Fig. 4. Heat source and resistance-thermometer equipment

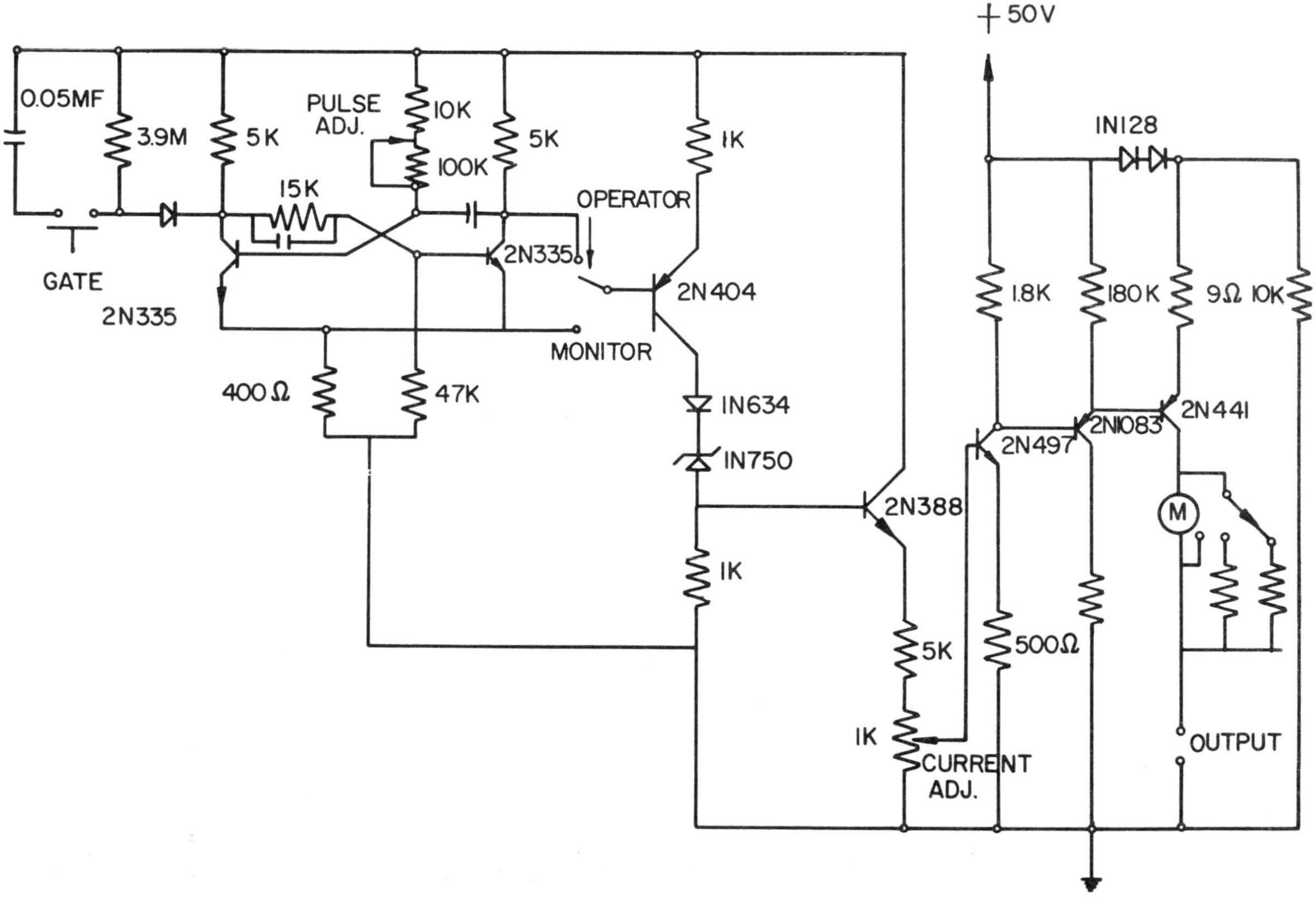


Fig. 5. Current-generator circuit

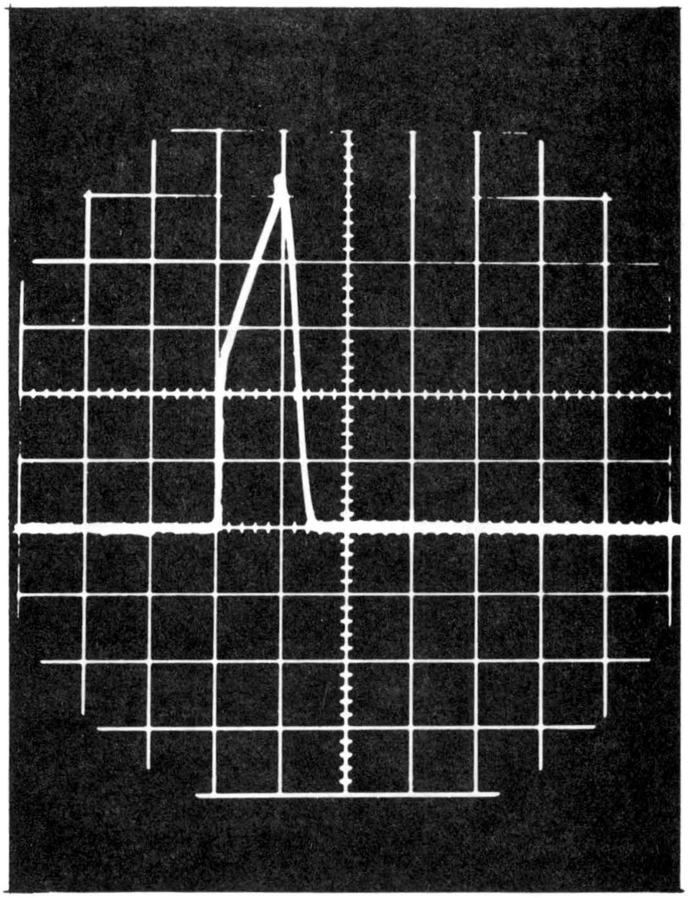


Fig. 6. Voltage pulse
(0.5 V/cm, 1 m sec/cm)

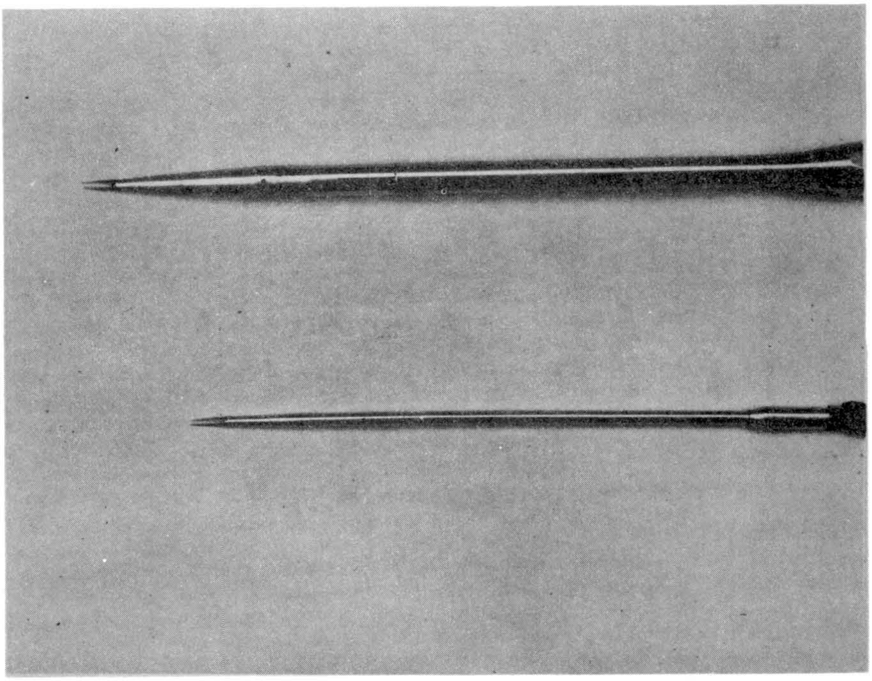


Fig. 7. Resistance-thermometer and heat source

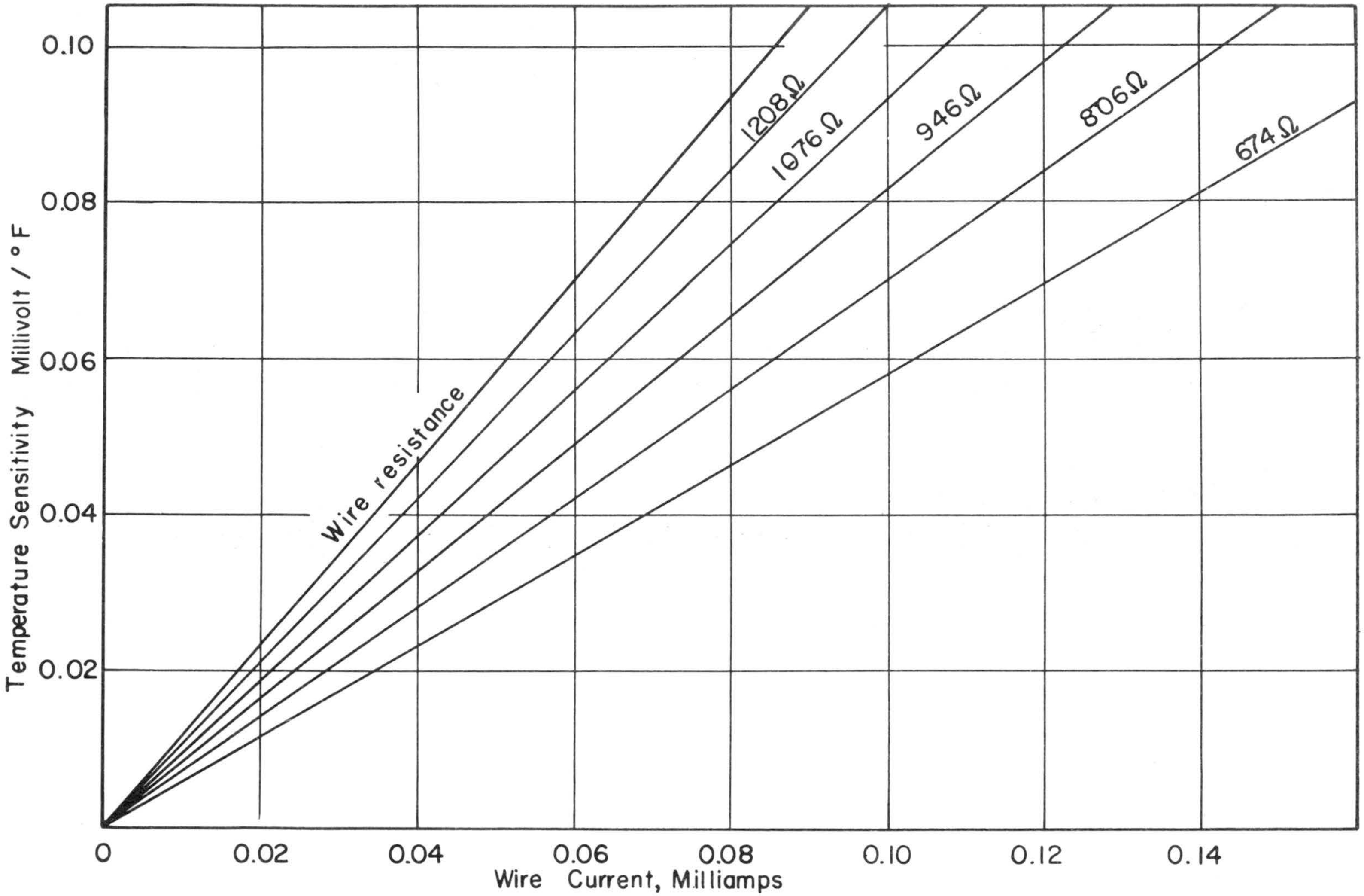


Fig. 8. Sensitivity of resistance-thermometer (90% platinum, 10% rhodium wire) to temperature fluctuations

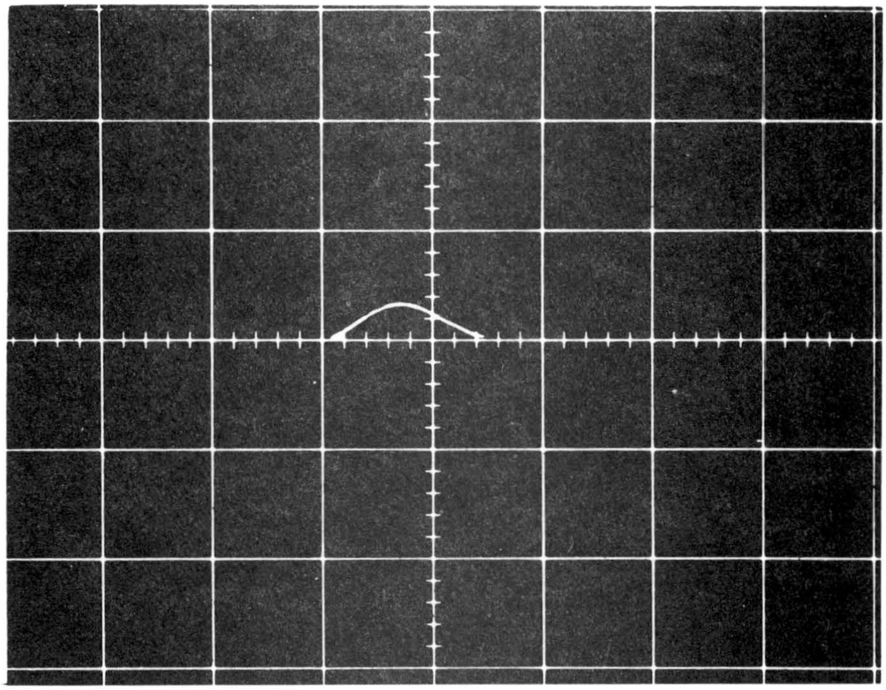
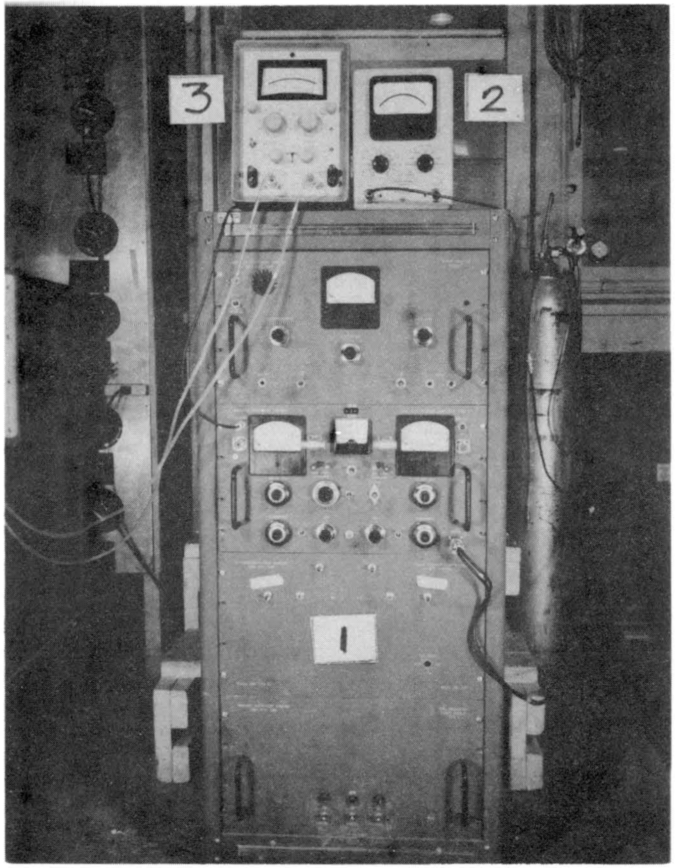


Fig. 9. Instantaneous temperature profile (0.5 V/cm, 1 m sec/cm)



1. Hubbard set
2. True rms-meter
3. Electronic manometer

Fig. 10. Turbulence measuring equipment

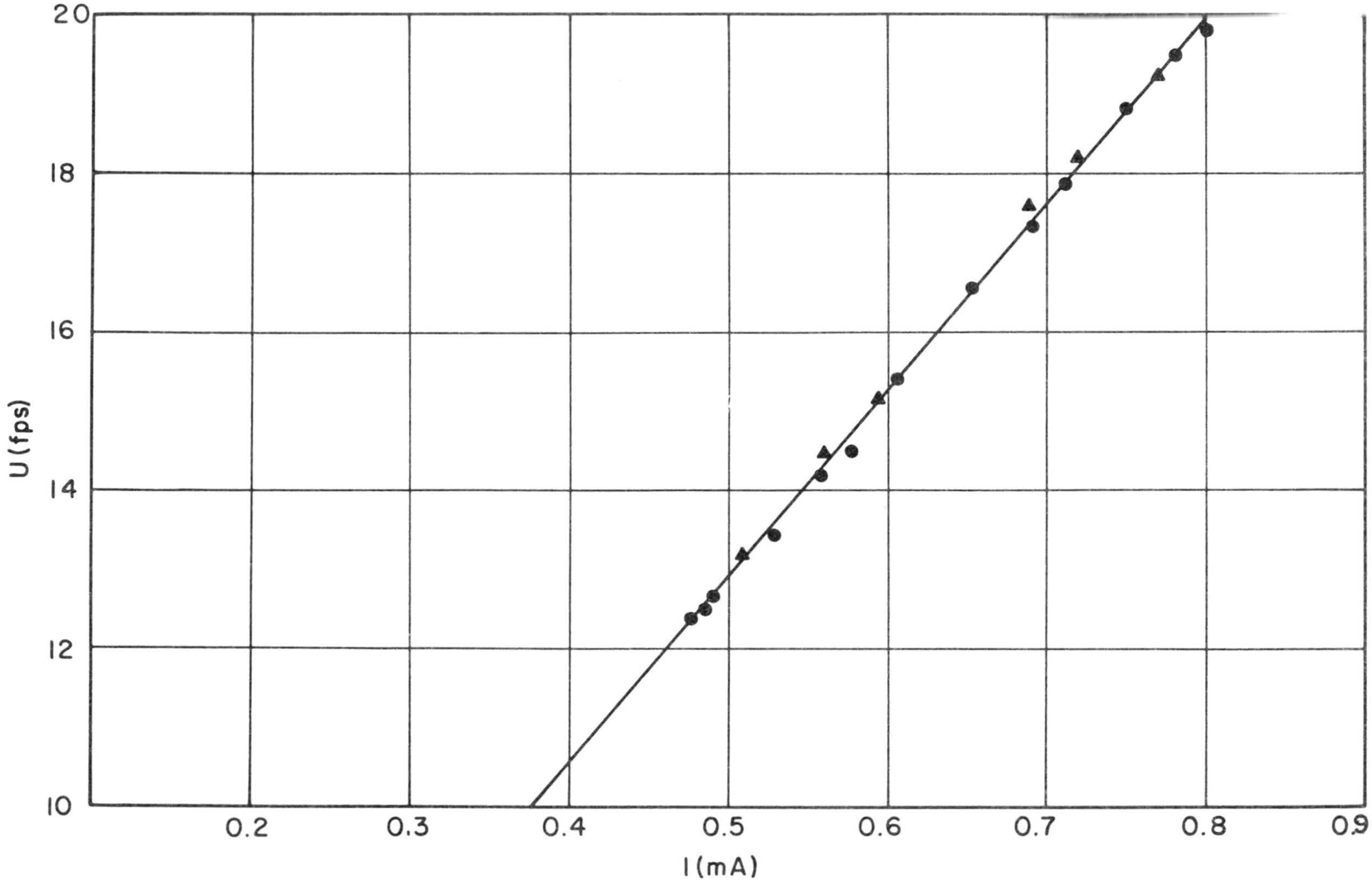


Fig. 11. Calibration curve of hot wire

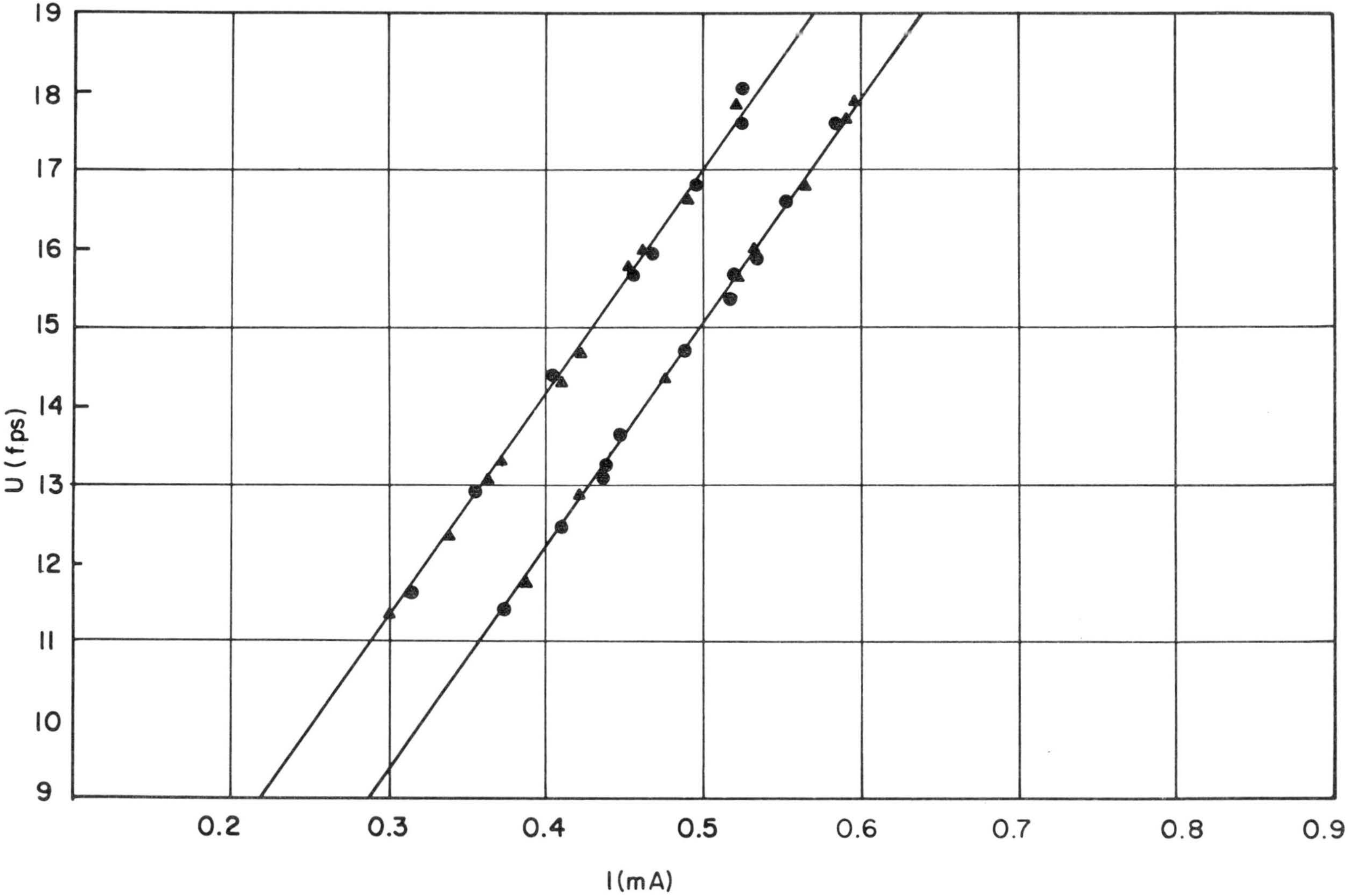


Fig. 12. Calibration curves of cross wire

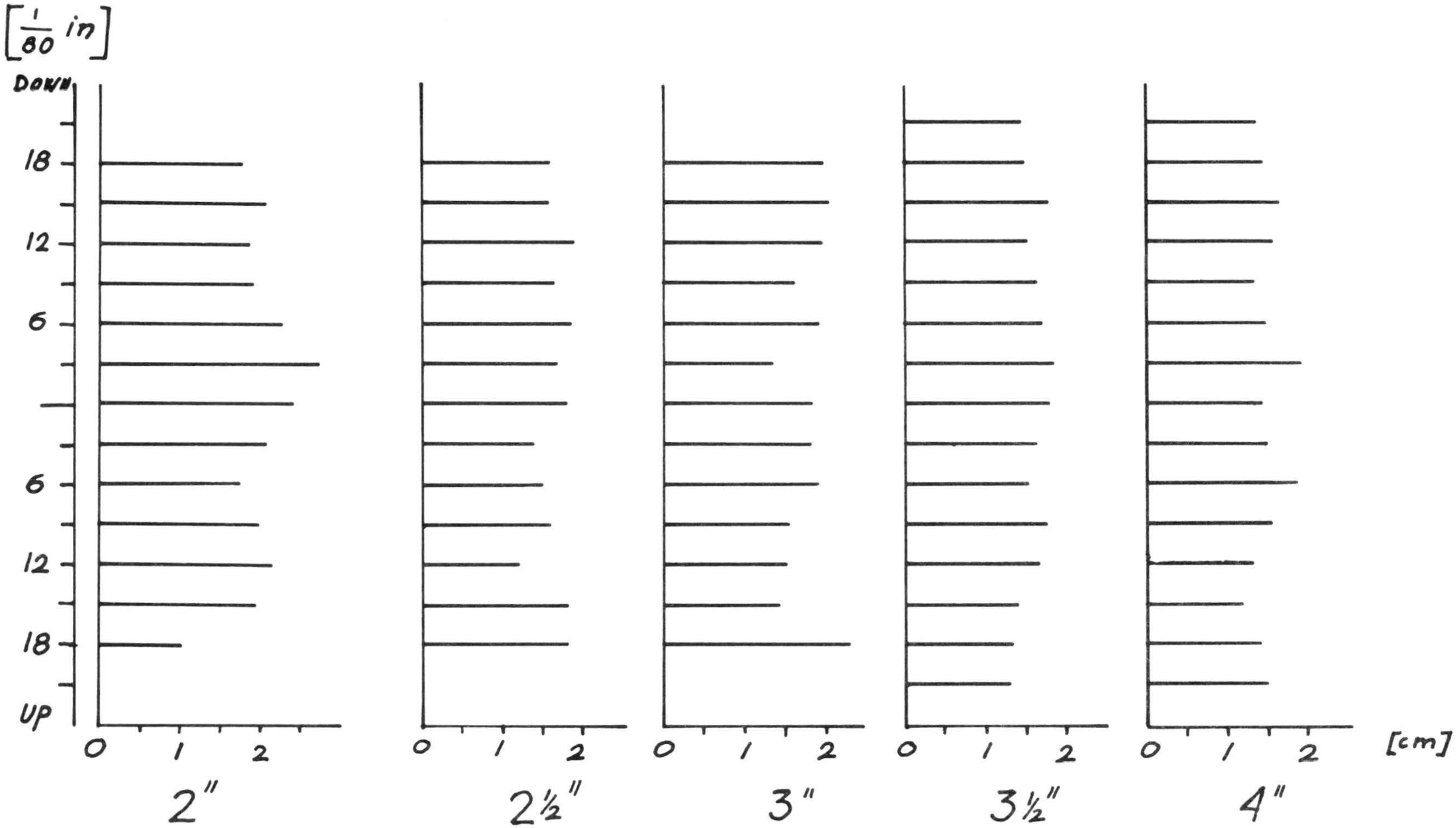


Fig. 13. Average instantaneous lengths of heat spots at different distances from the source

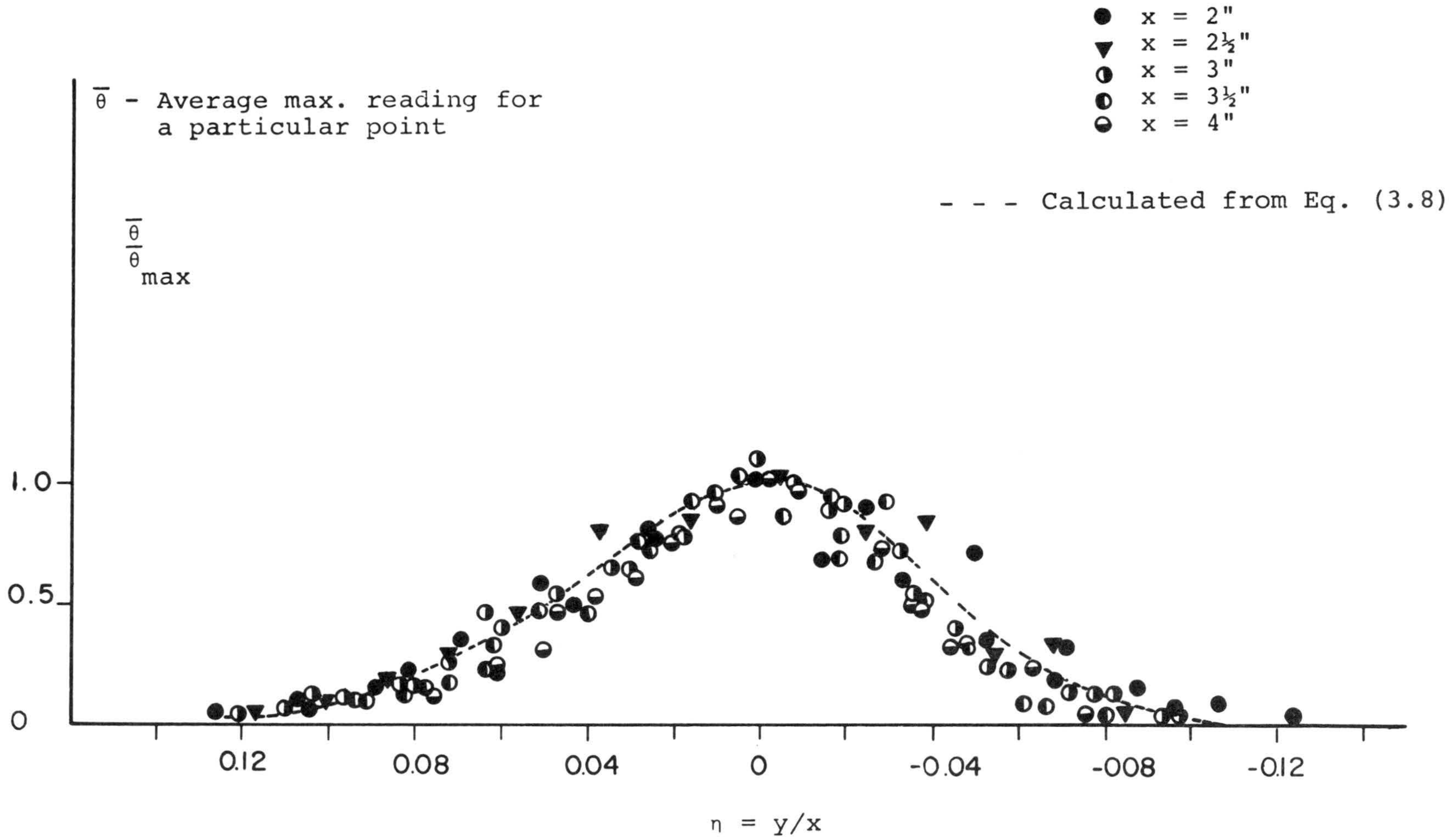


Fig. 14. Vertical distribution of heat spots from an instantaneous point source in a boundary layer.
Heat source: 2½" from the wall
Velocity $\bar{U}$: 15 (ft/sec)

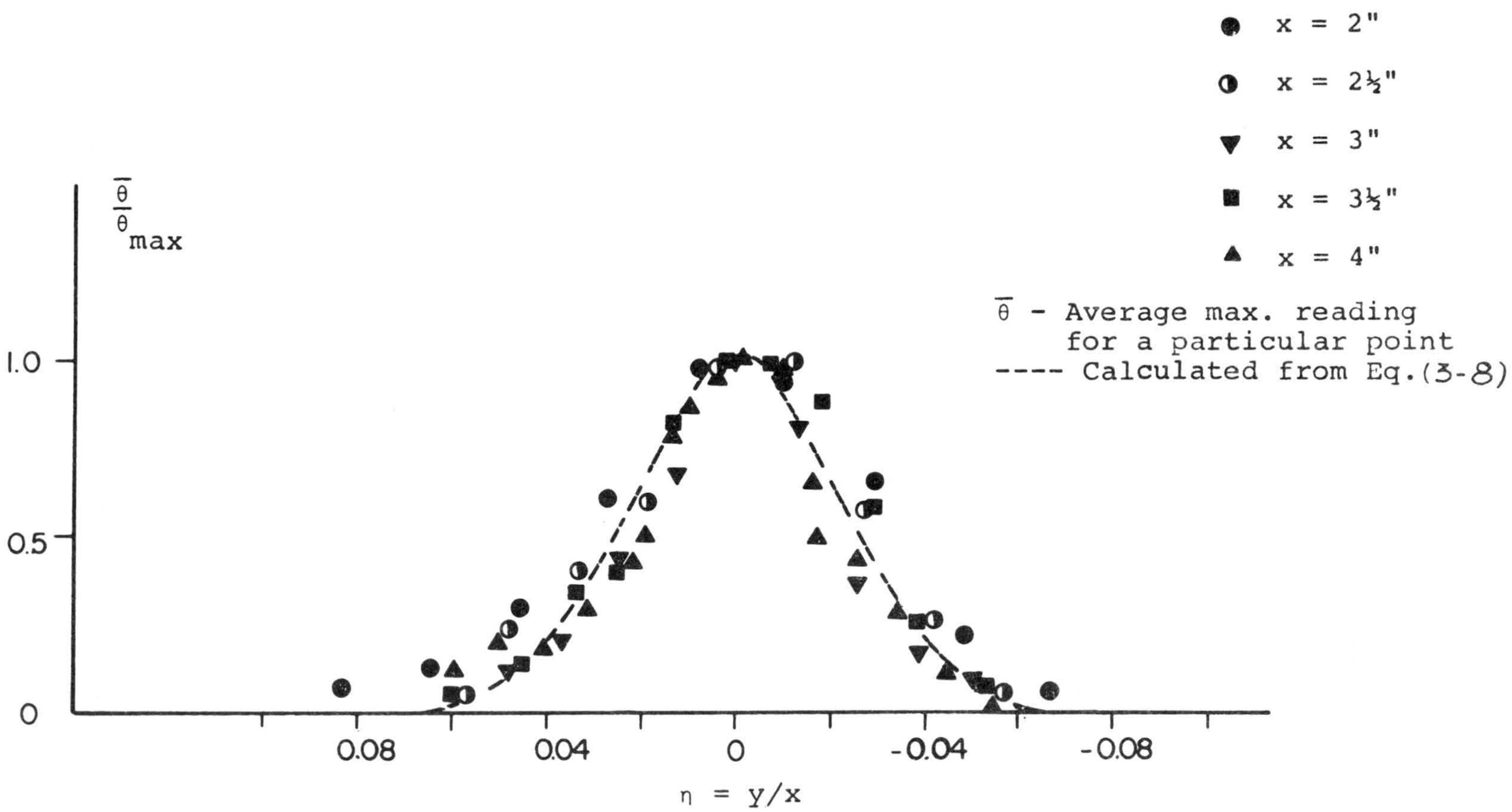


Fig. 15. Vertical distribution of heat spots from an instantaneous point source in a boundary layer.
Heat source: 5-7/8" from the wall, Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

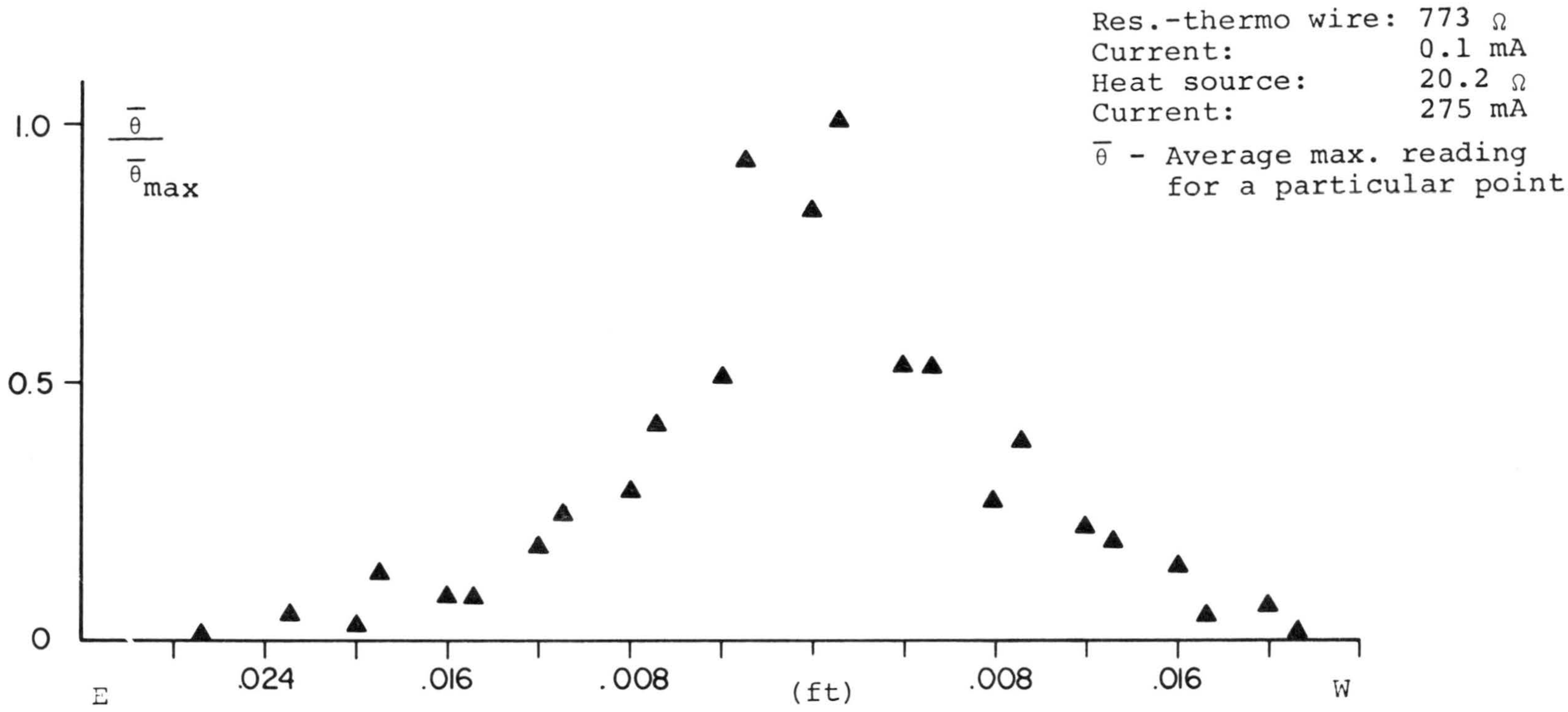


Fig. 16. Lateral distribution of heat spots.
Heat source: 2½" from the wall
Distance: 2" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

Res.-thermo wire: 773 Ω
Current: 0.1 mA
Heat source: 20.2Ω
Current: 275 mA
$\bar{\theta}$ - Average max. reading for a particular point

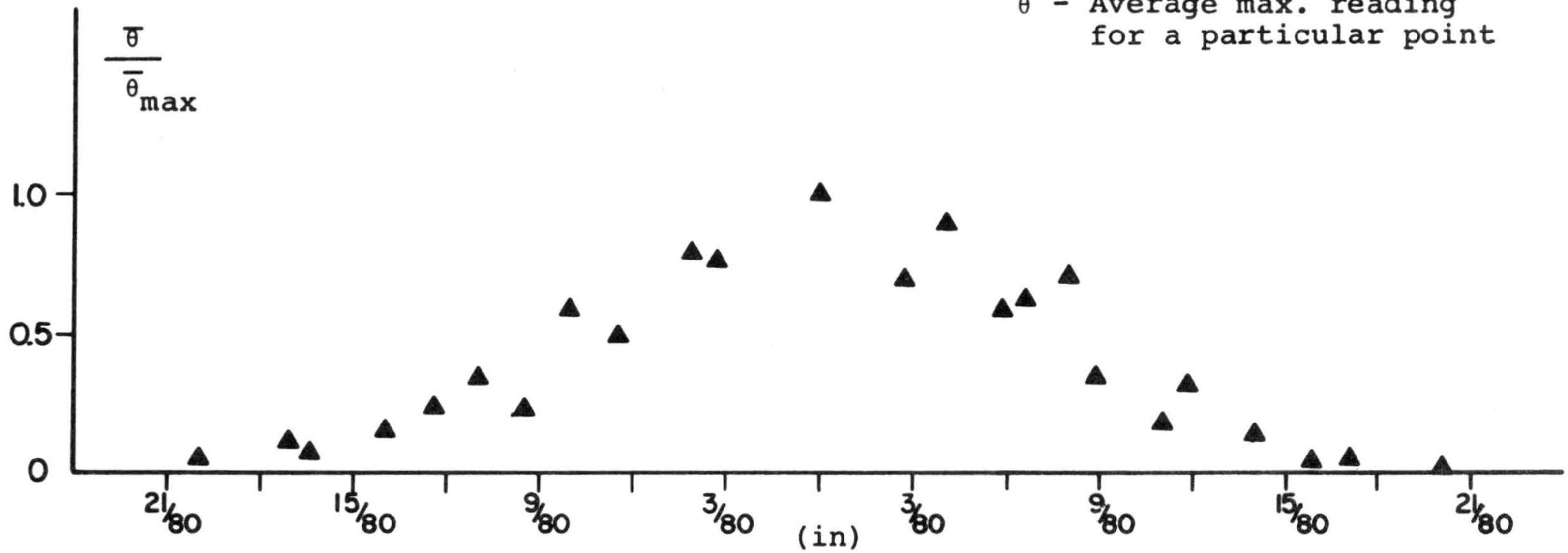


Fig. 17. Vertical distribution of heat spots.
Heat source: 2½" from the wall
Distance: 2" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

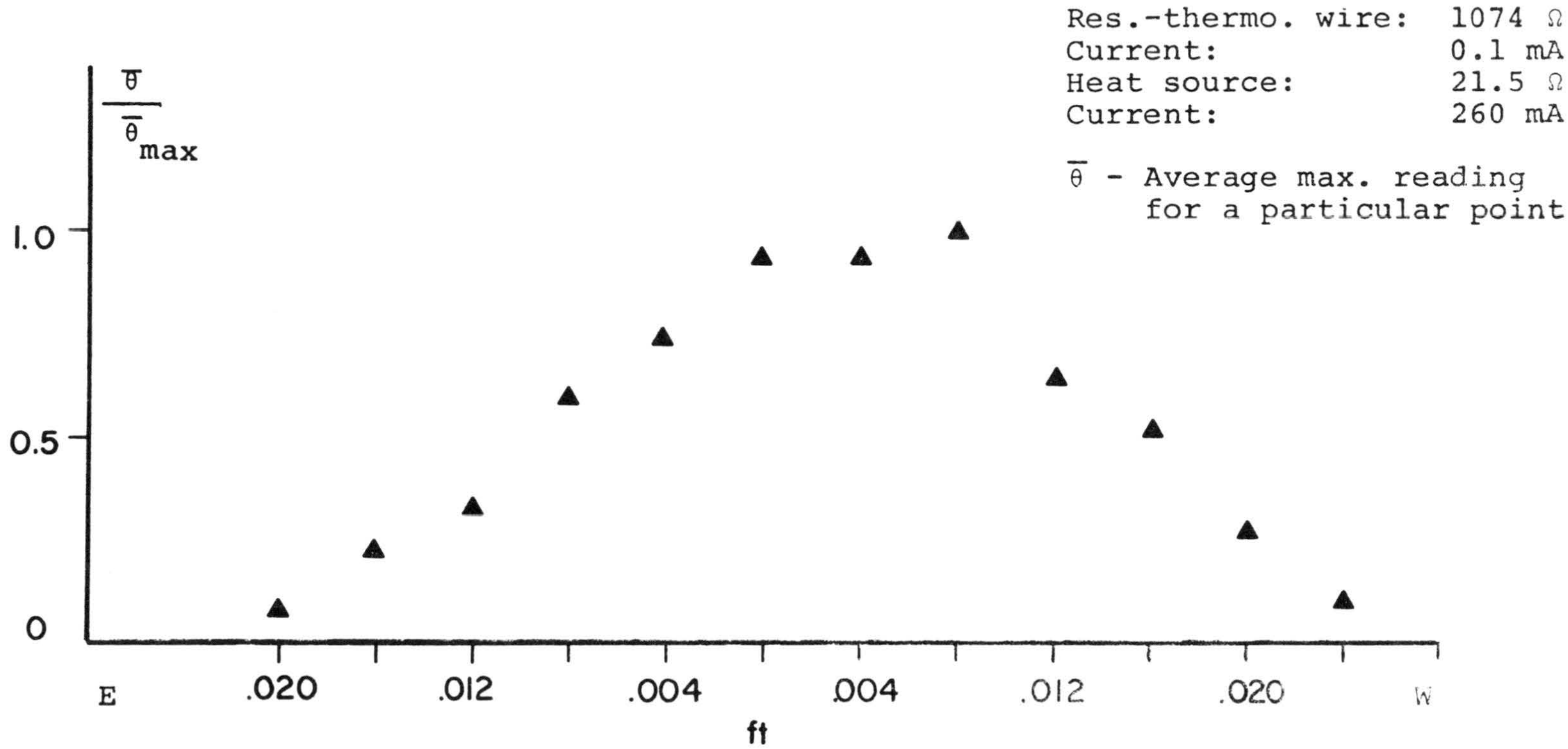


Fig.18. Lateral distribution of heat spots.
Heat source: 2½" from the wall
Distance: 2½" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

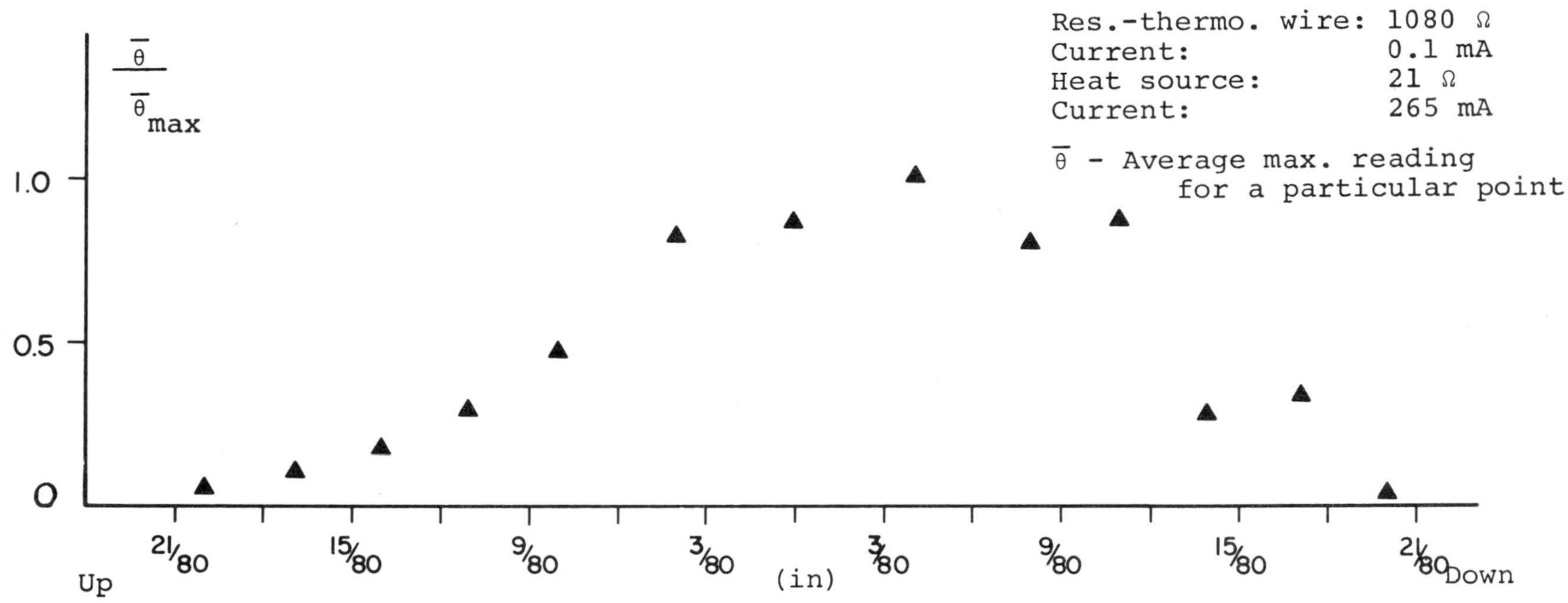


Fig. 19. Vertical distribution of heat spots.
Heat source: 2½" from the wall
Distance: 2½" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

Res.-thermo. wire: 1074 Ω
Current: 0.1 mA
Heat source: 21.5 Ω
Current: 260 mA

$\bar{\theta}$ - Average max. reading for a particular point

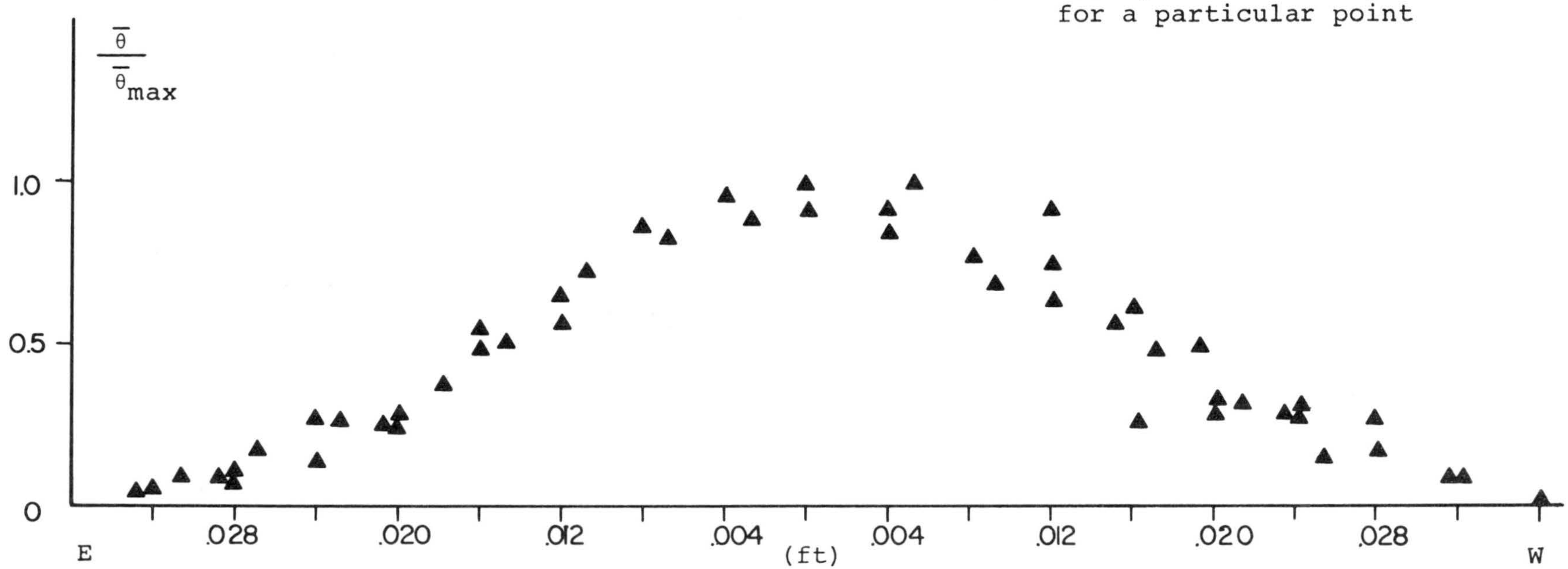


Fig. 20. Lateral distribution of heat spots.
Heat source: 2½" from the wall
Distance: 3" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

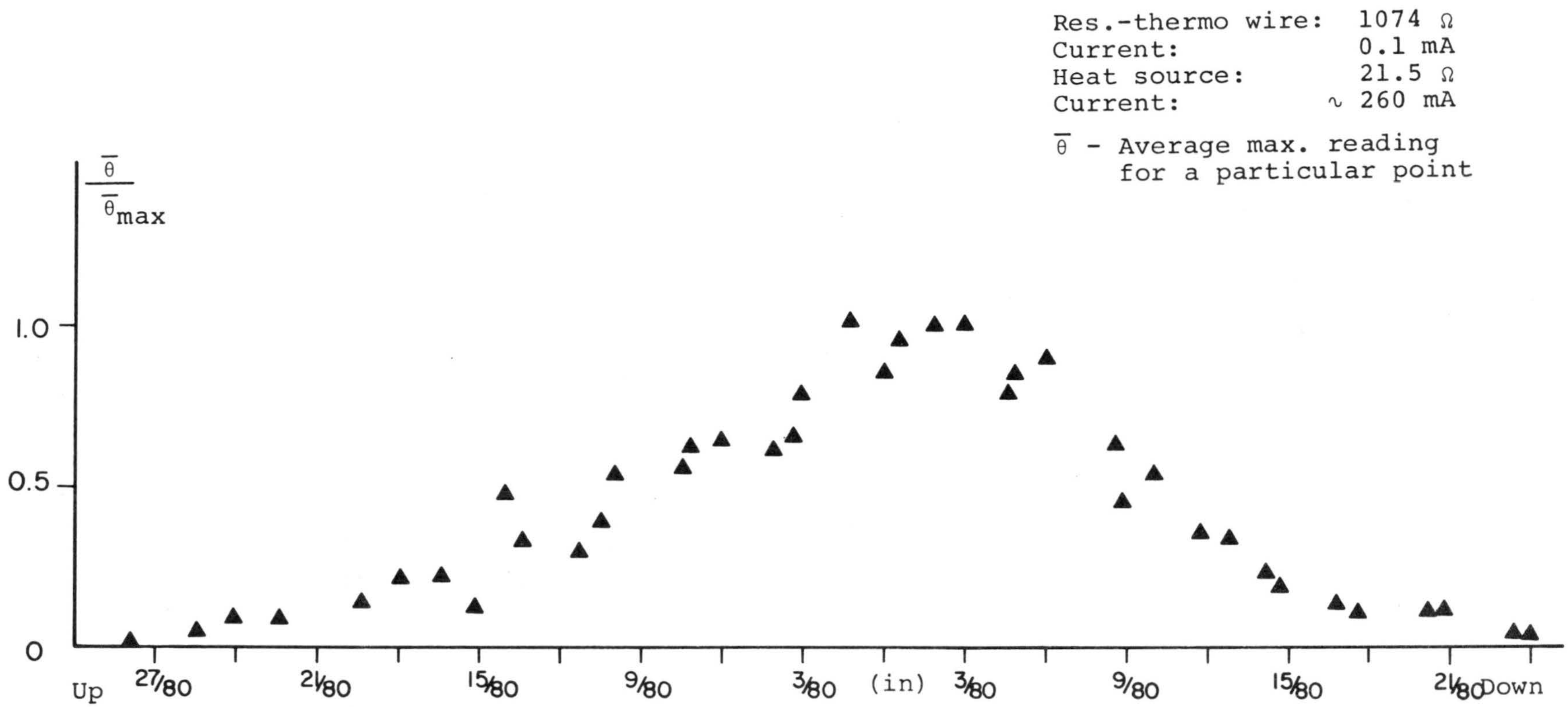


Fig. 21. Vertical distribution of heat spots.
Heat source: 2½" from the wall
Distance: 3" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

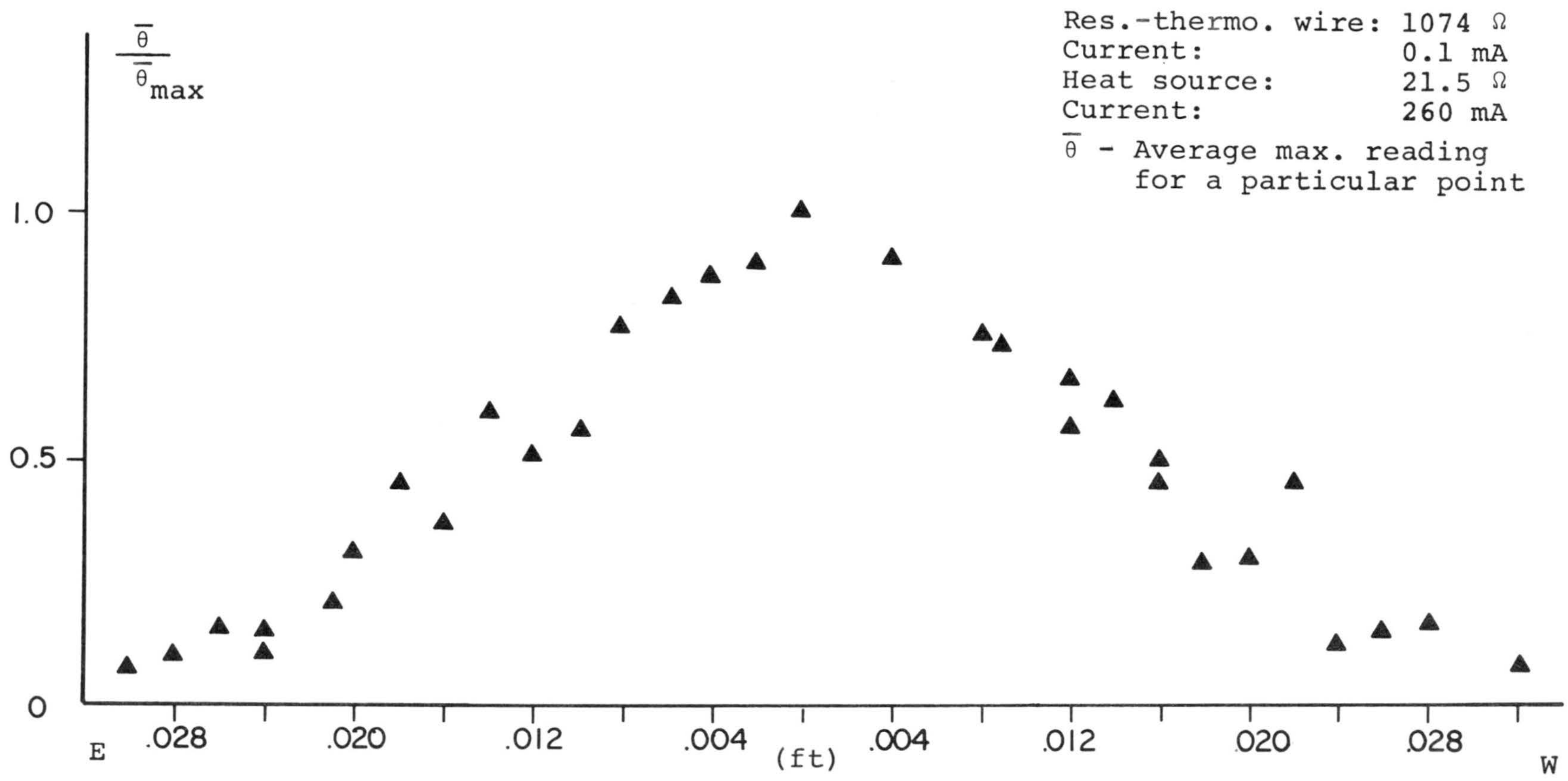


Fig. 22. Lateral distribution of heat spots.
Heat source: 2½" from the wall
Distance: 3½" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

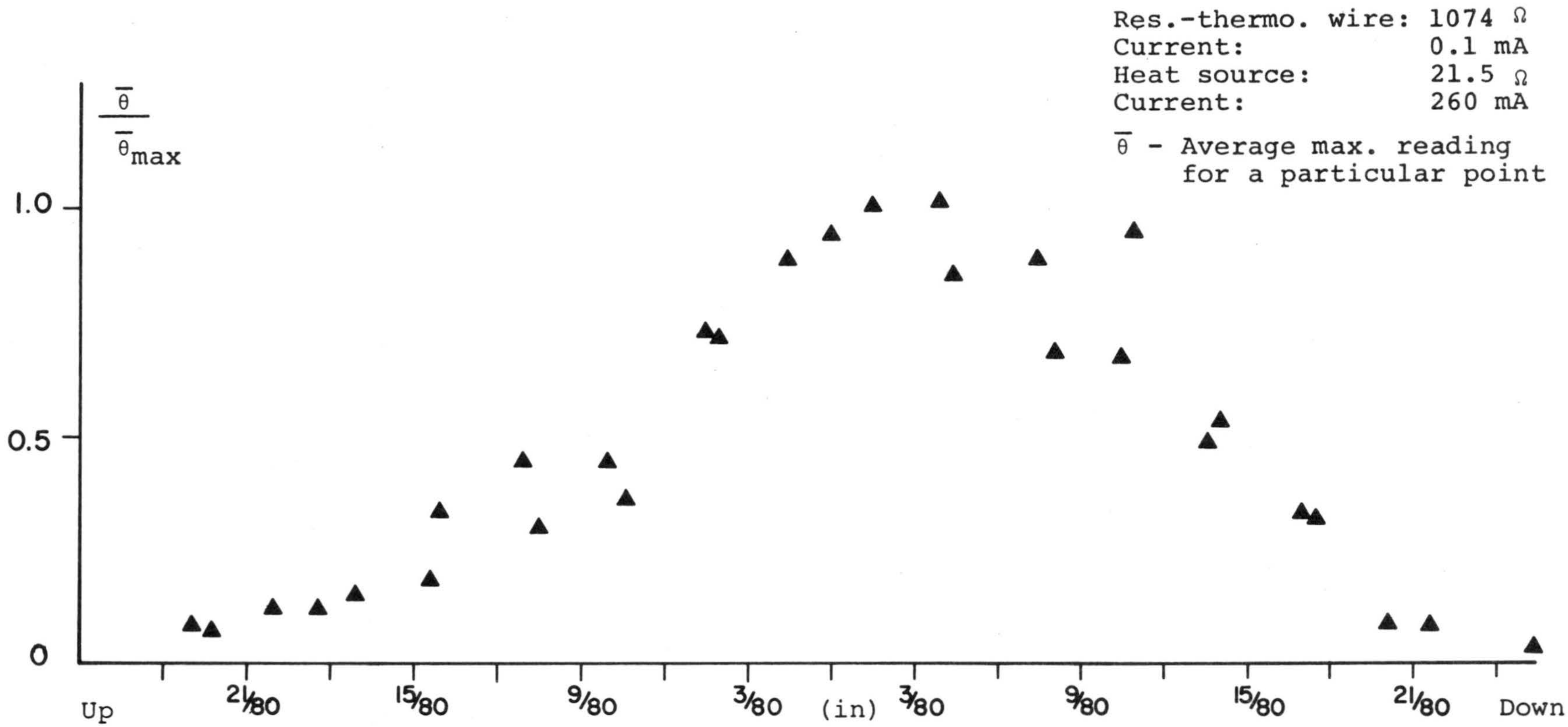


Fig. 23. Vertical distribution of heat spots.
Heat source: 2½" from the wall
Distance: 3½" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

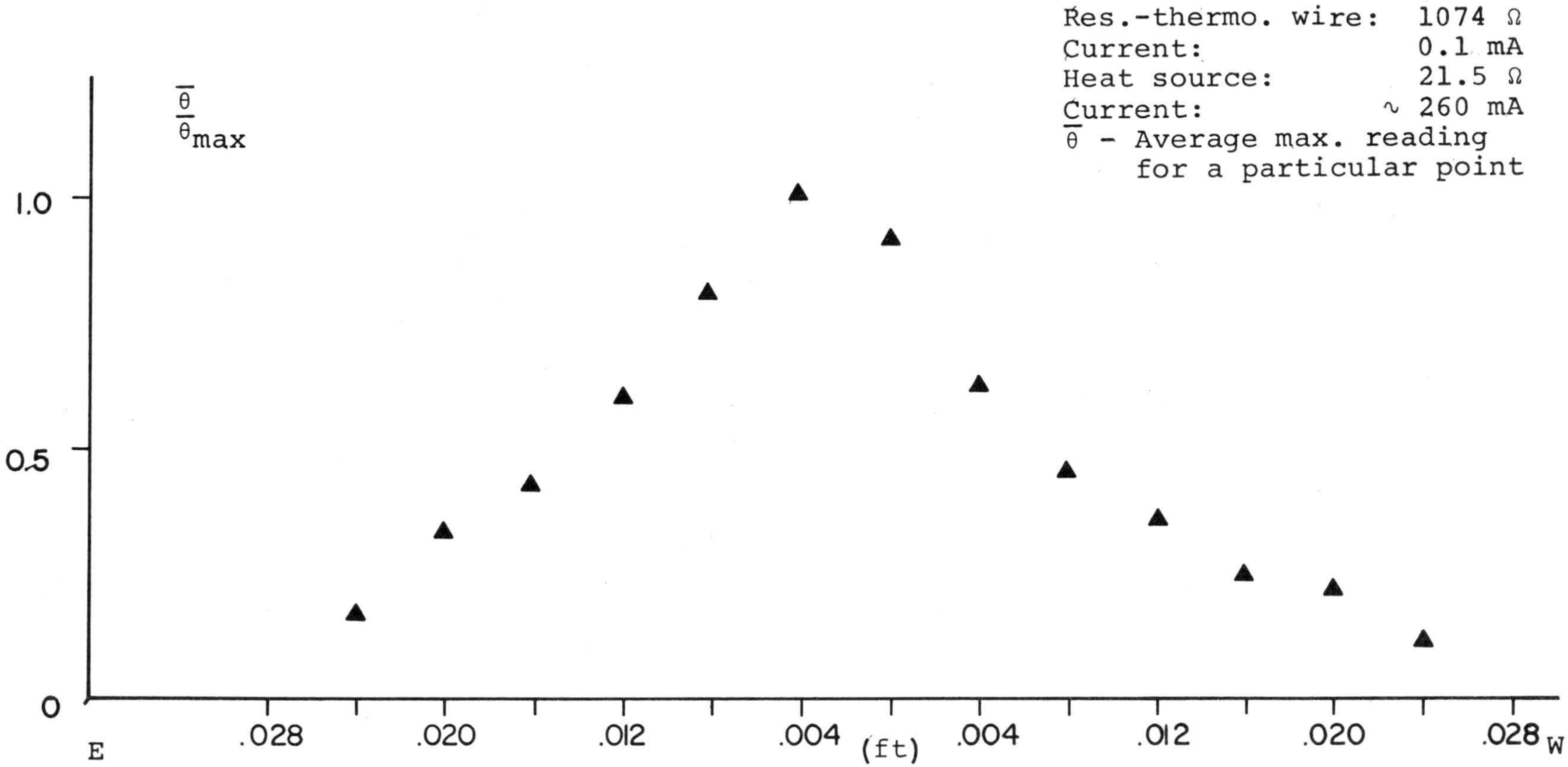


Fig. 24. Lateral distribution of heat spots
Heat source: 2-1/2" from the wall
Distance: 4" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

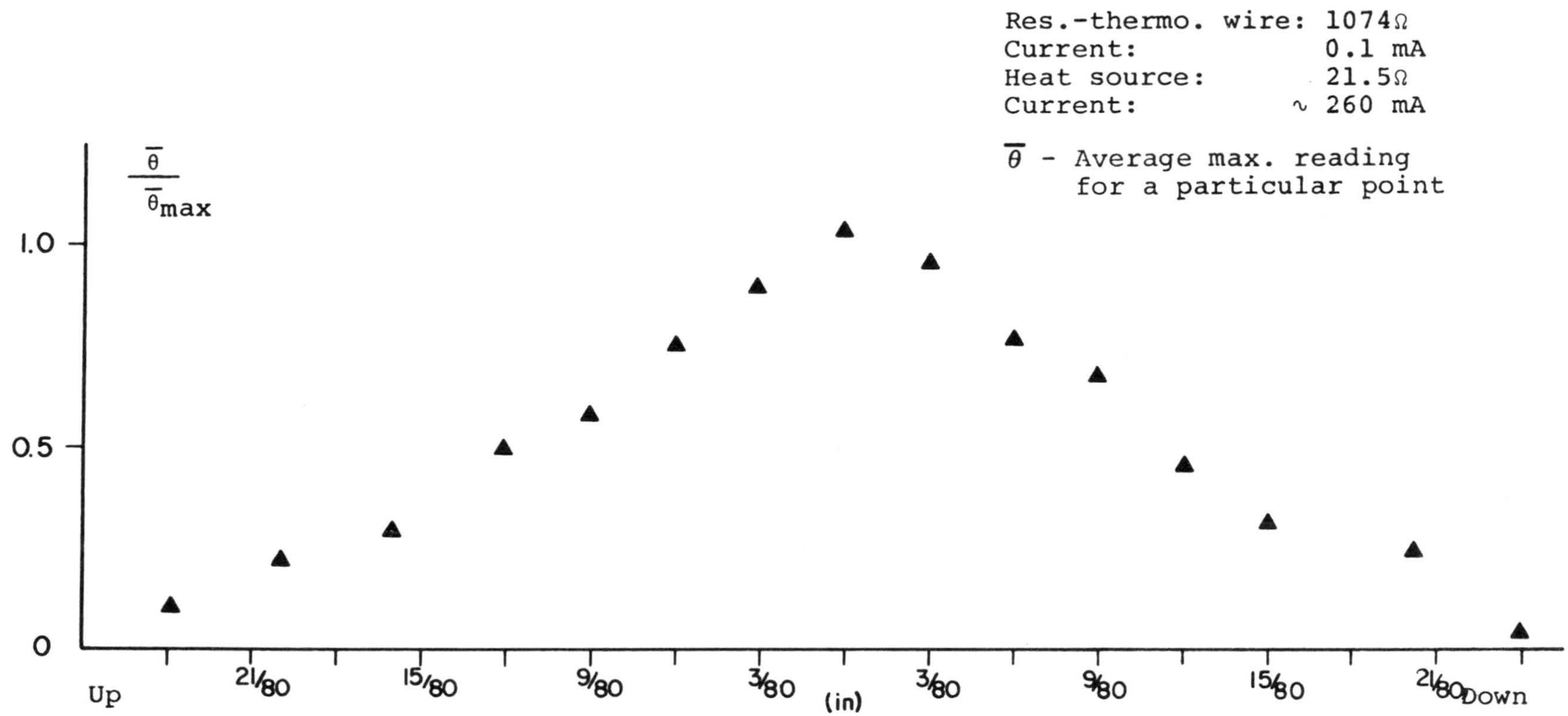
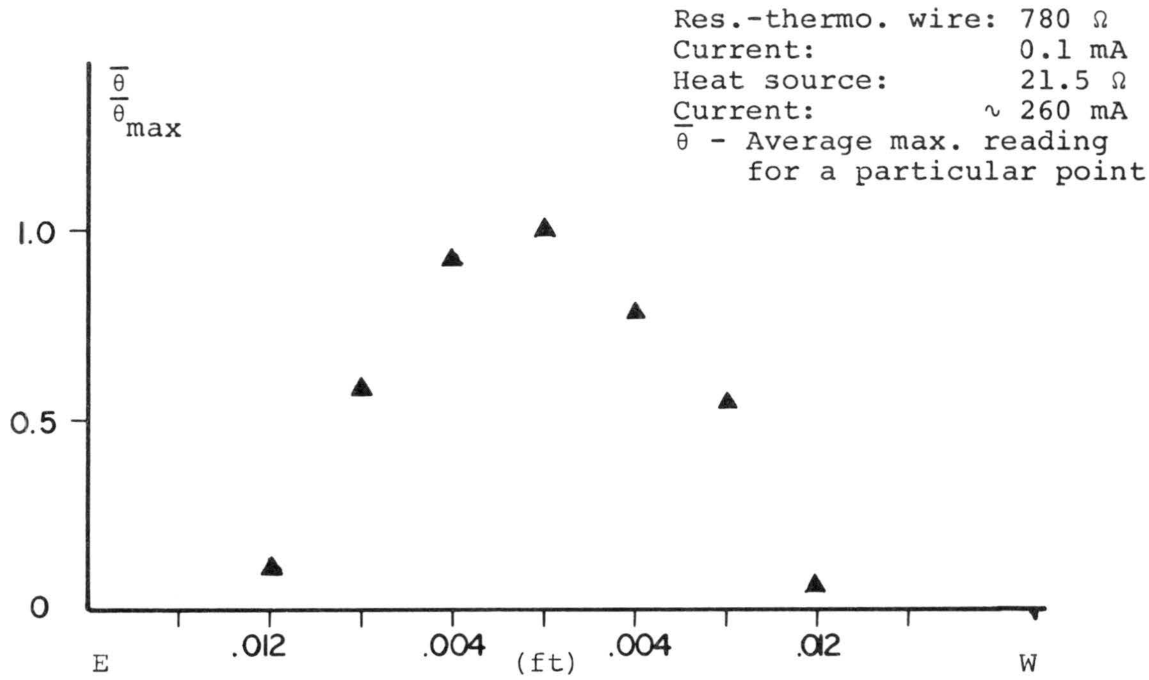


Fig. 25. Vertical distribution of heat spots.
Heat source: 2½" from the wall
Distance: 4" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$



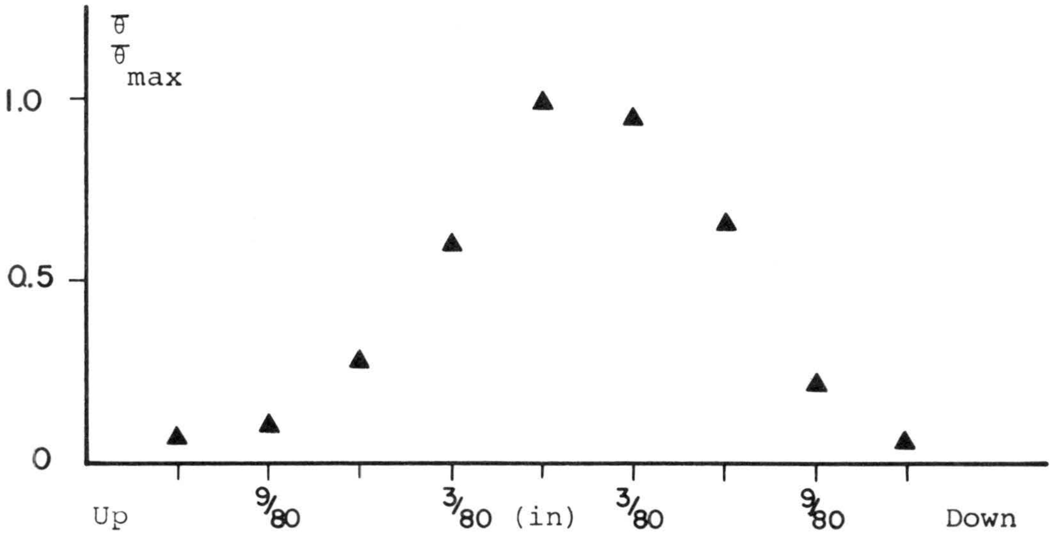
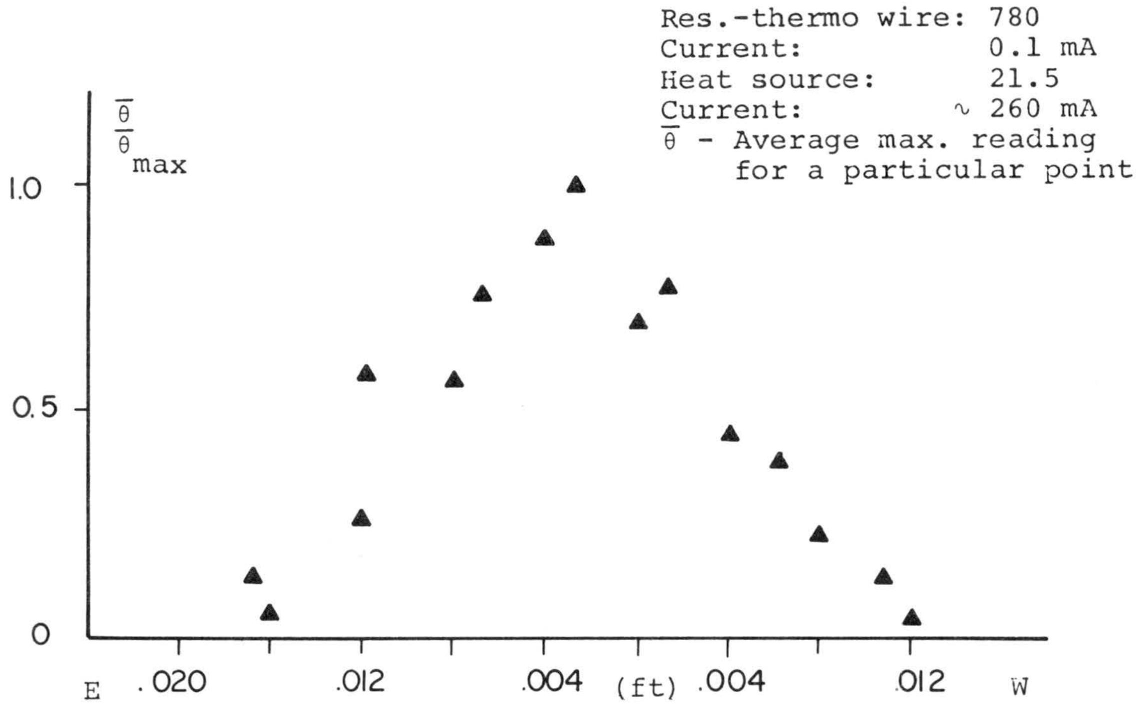


Fig. 26. Lateral and vertical distribution of heat spots
Heat source: 5-7/8" from the wall
Distance: 2" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$



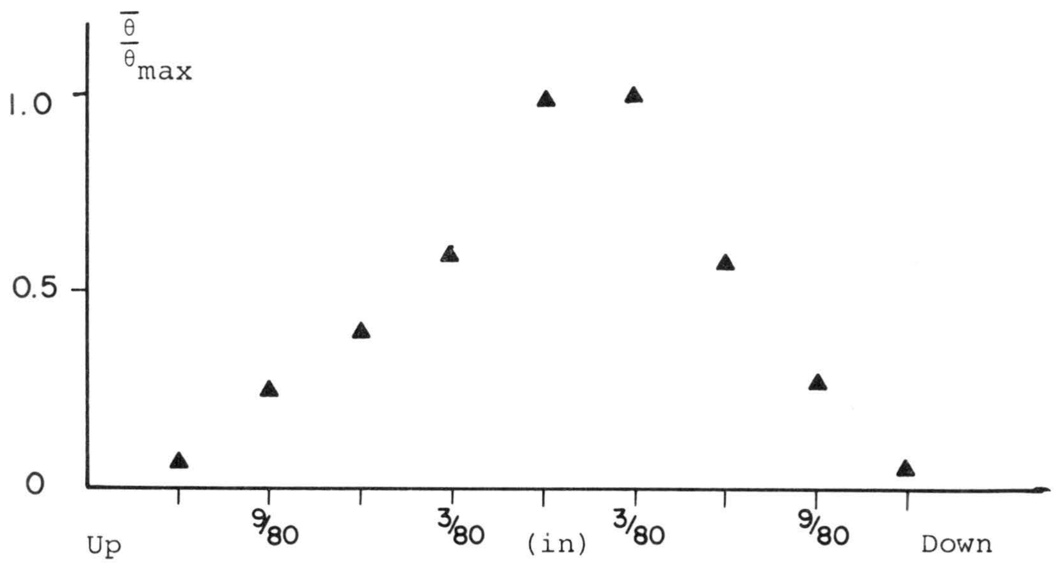


Fig. 27. Lateral and Vertical distribution of heat spots
Heat source: 5-7/8" from the wall
Distance: 2½" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

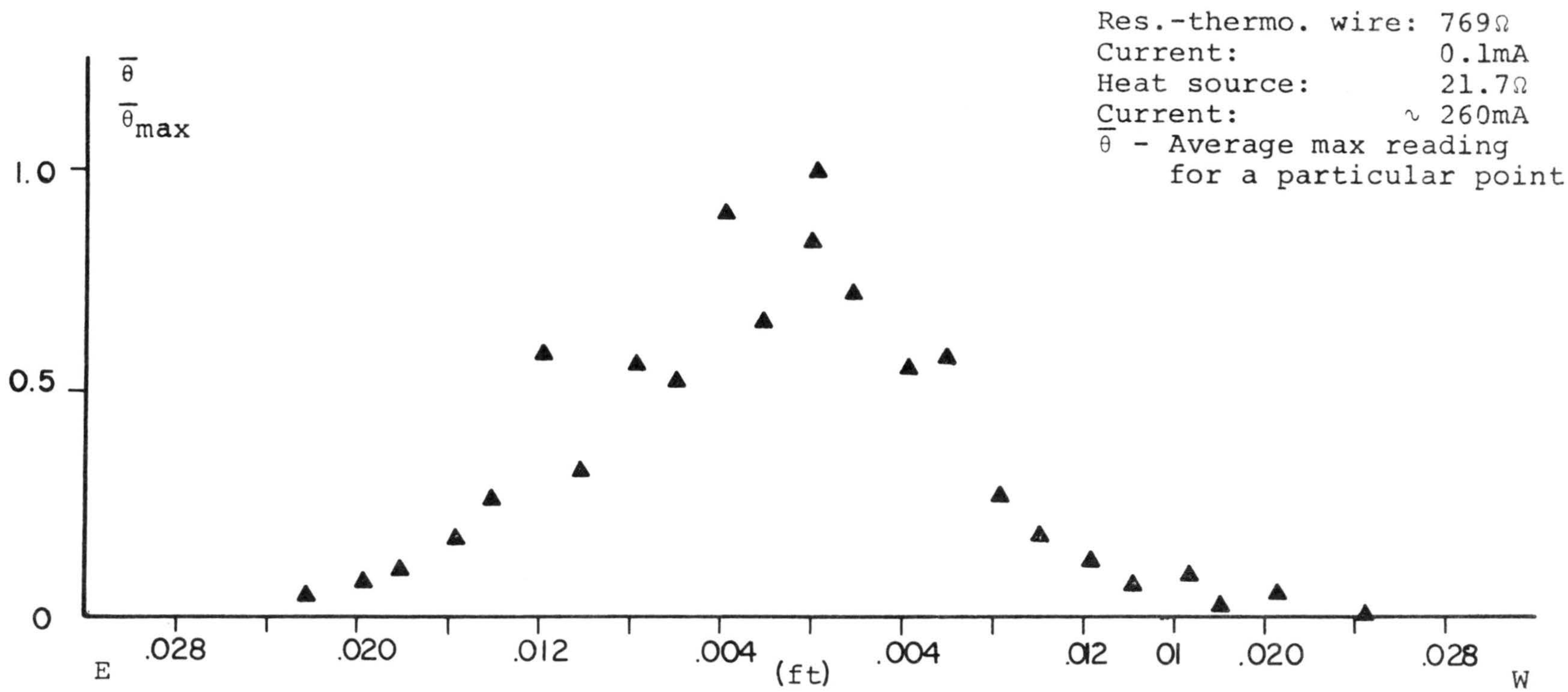


Fig. 28. Lateral distribution of heat spots.
Heat source: 5-7/8" from the wall
Distance: 3" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

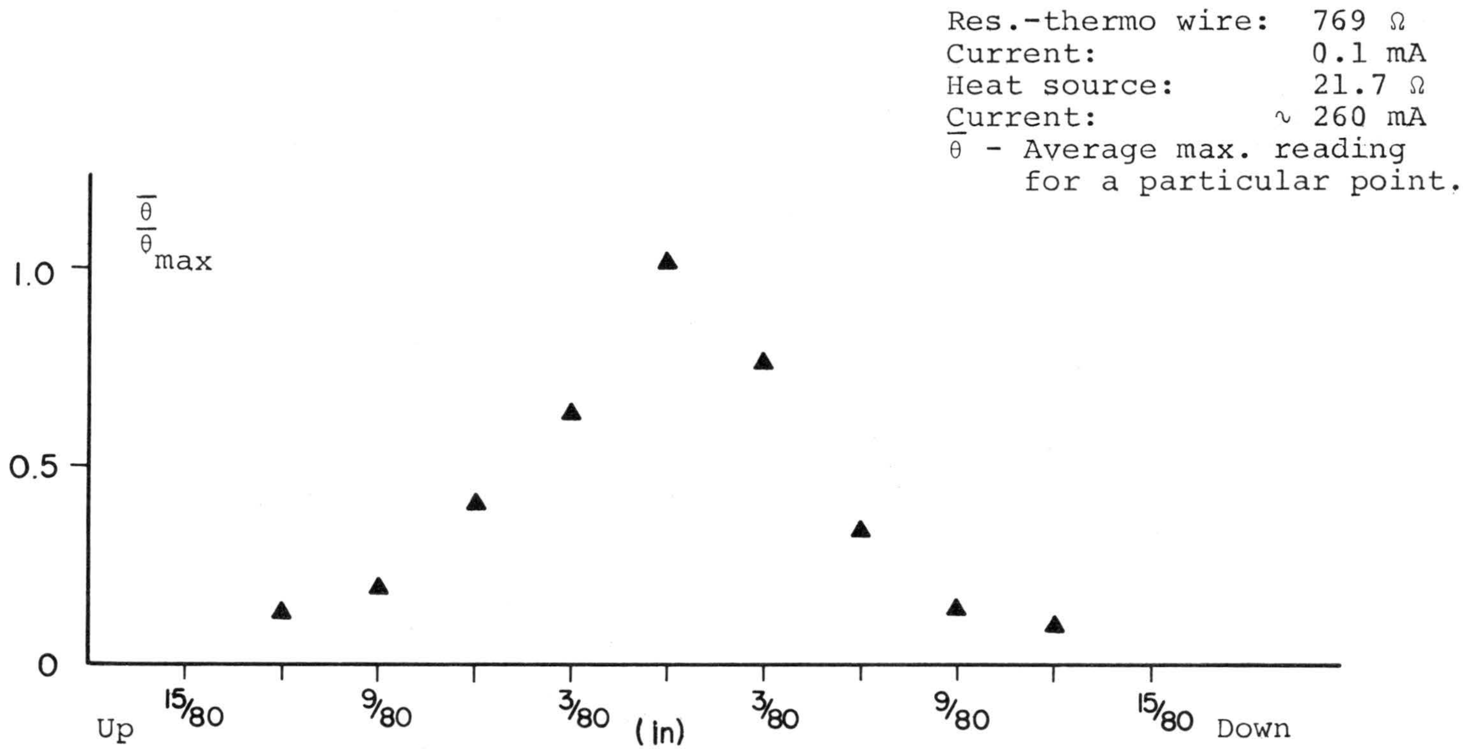
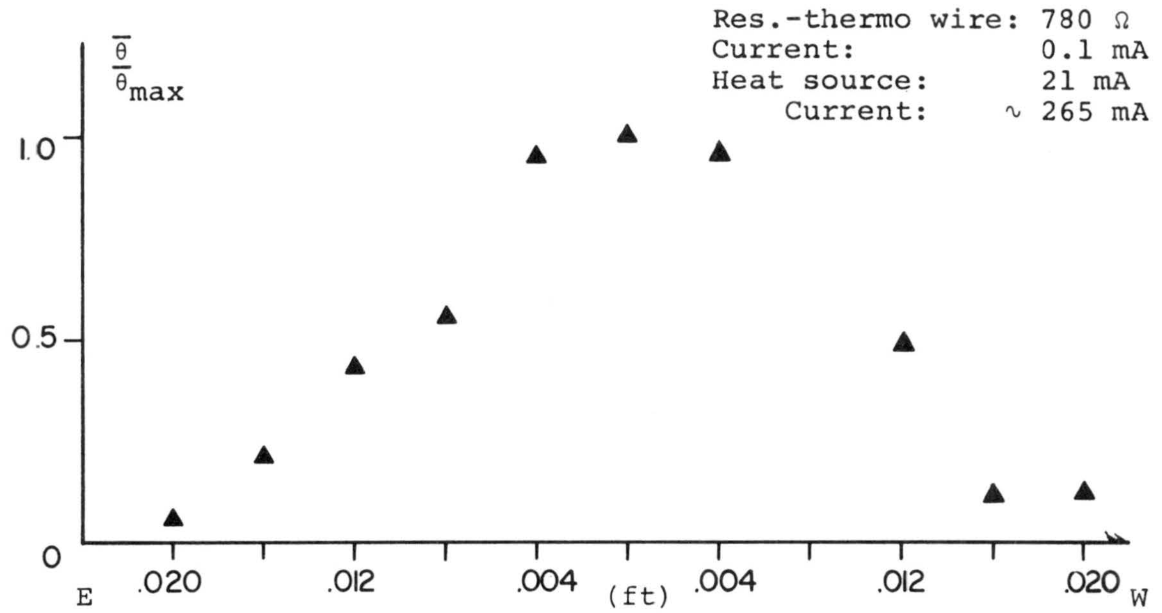


Fig. 29. Vertical distribution of heat spots.
Heat source: 5-7/8" from the wall
Distance: 3" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$



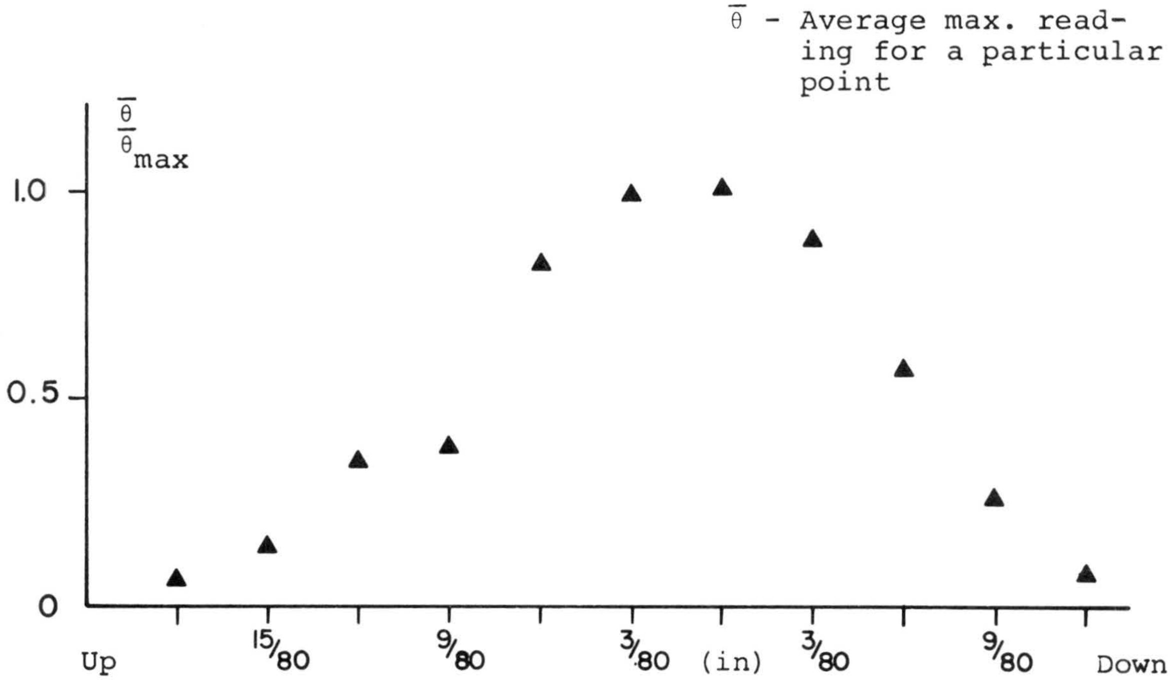


Fig. 30. Lateral and vertical distribution of heat spots
Heat source: 5-7/8" from the wall
Distance: 3½" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

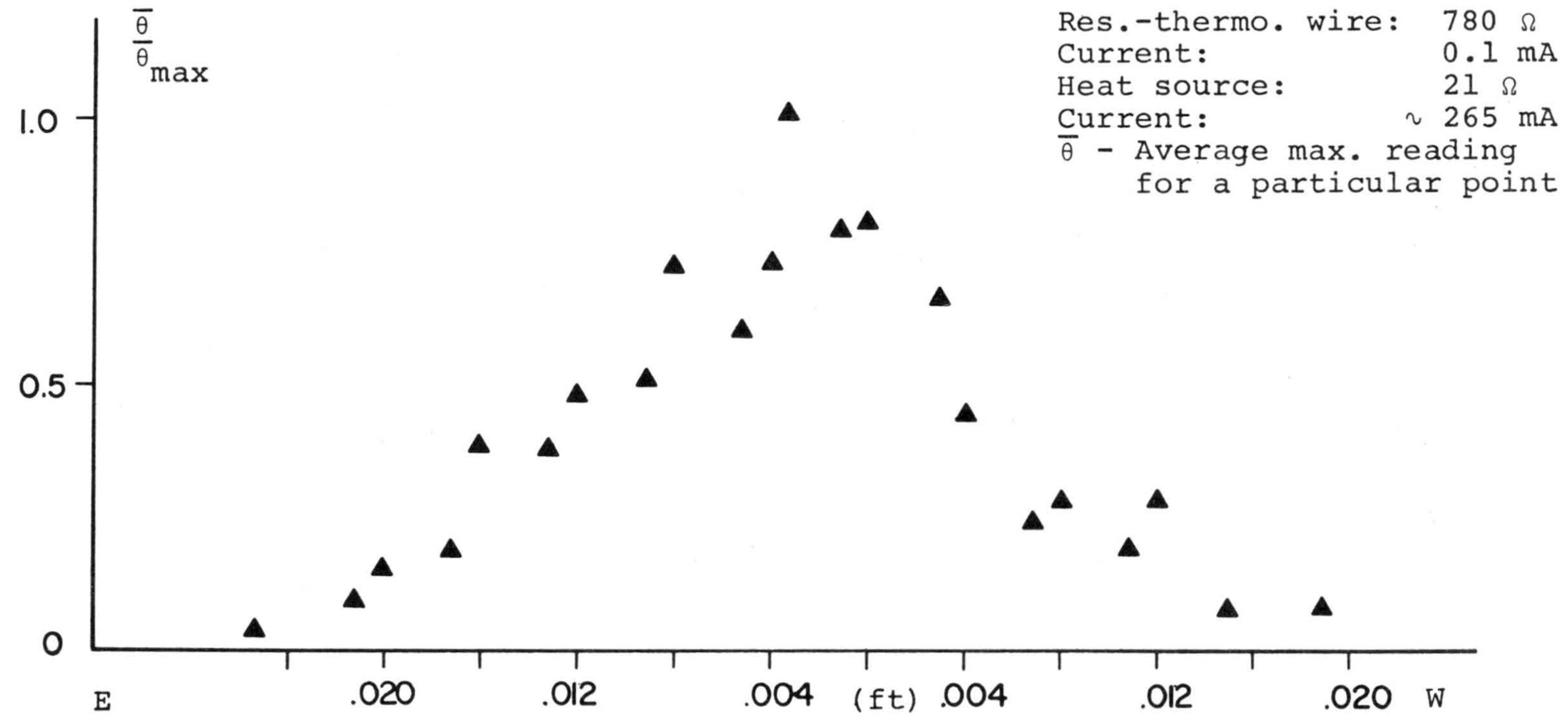


Fig. 31. Lateral distribution of heat spots.
Heat source: 5-7/8" from the wall
Distance: 4" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

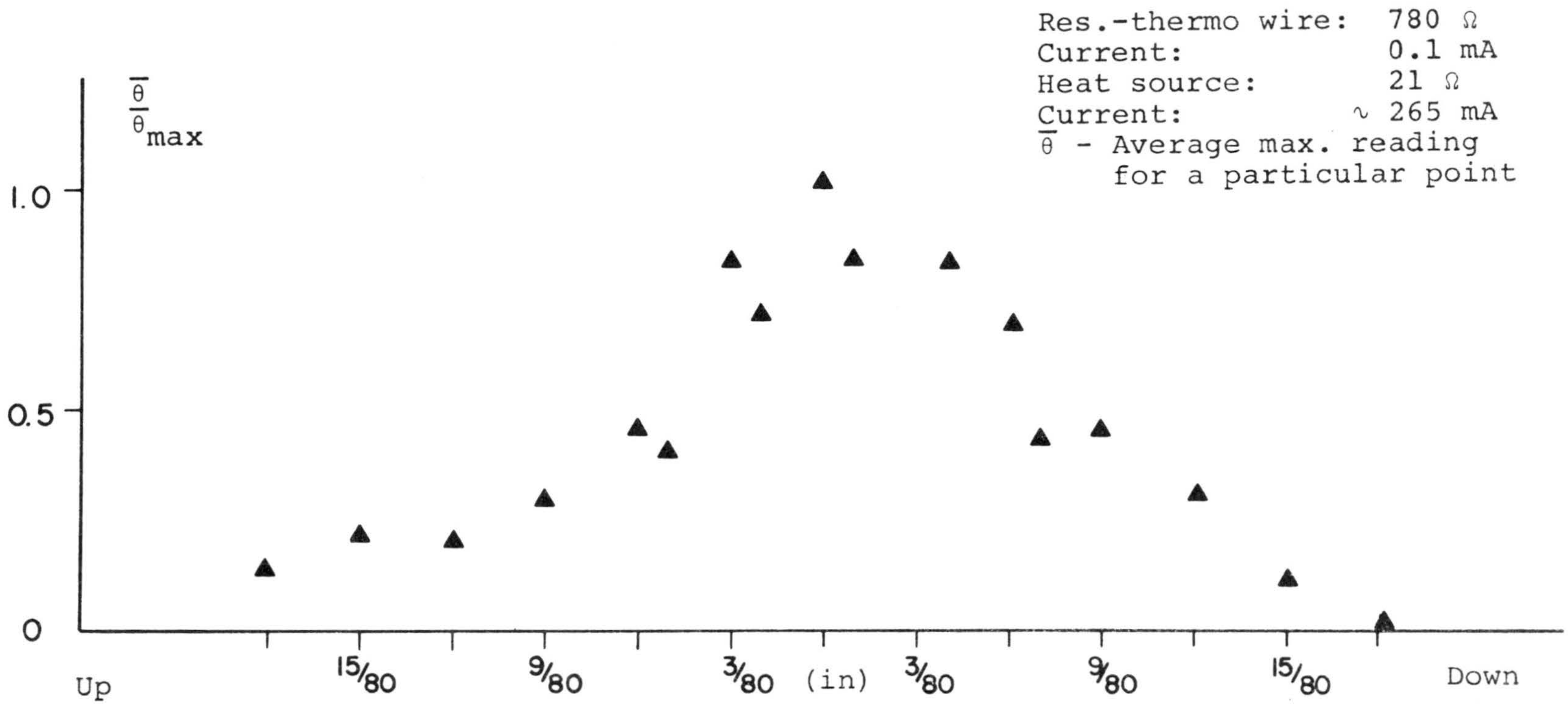


Fig. 32. Vertical distribution of heat spots.
Heat source: 5-7/8" from the wall
Distance: 4" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

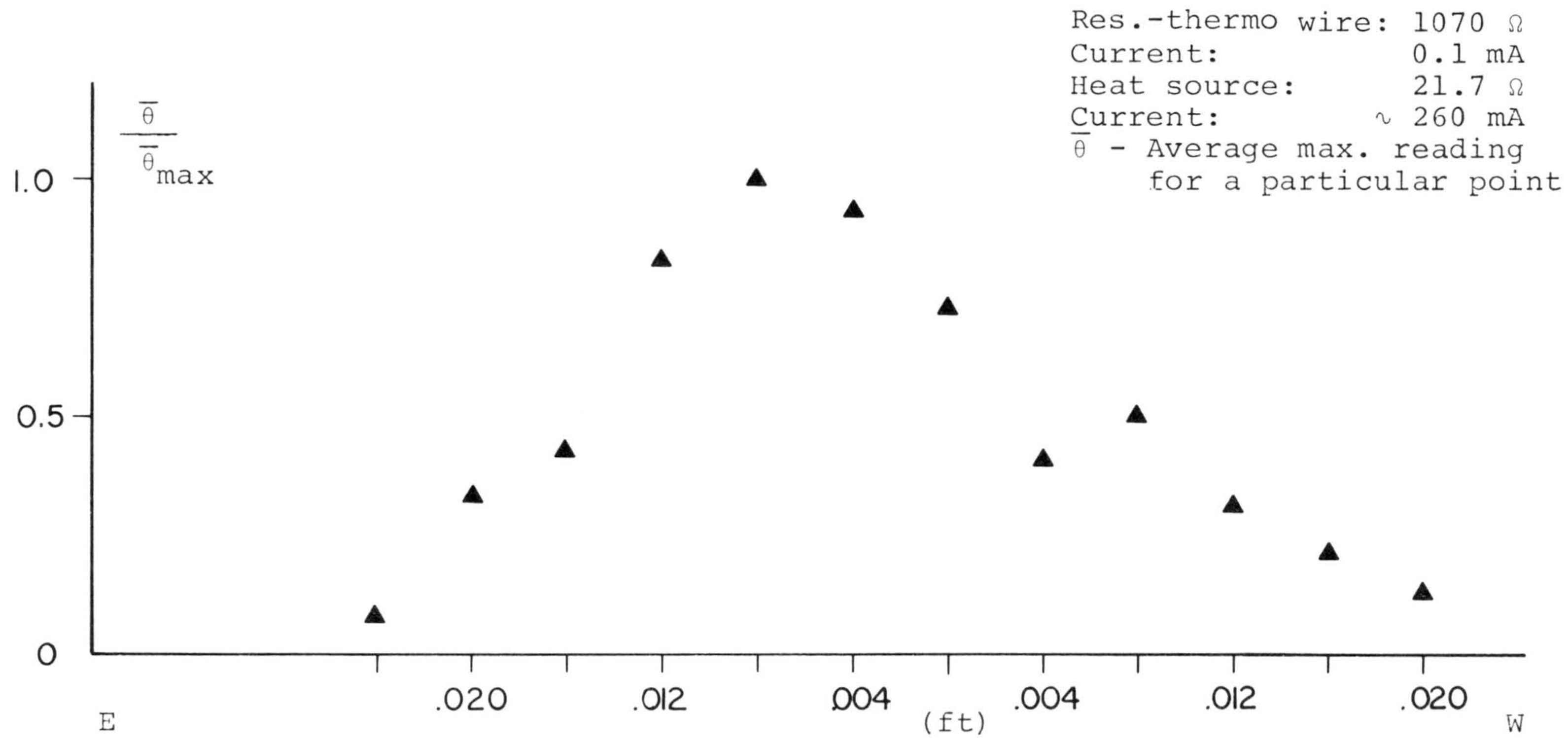


Fig. 33. Lateral distribution of heat spots.
Heat source: 5-7/8" from the wall
Distance: 5" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

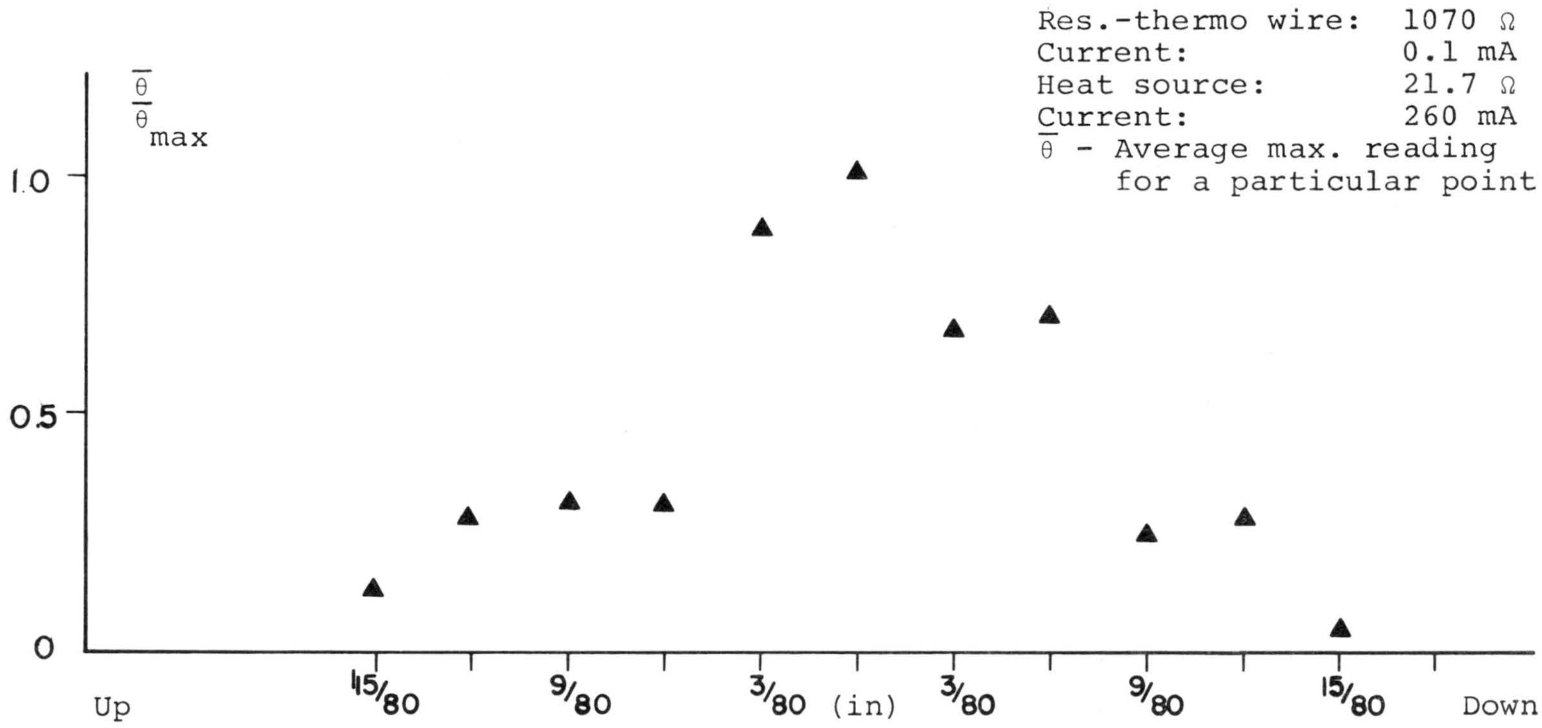


Fig. 34. Vertical distribution of heat spots.
Heat source: 5-7/8" from the wall
Distance: 5" from the source
Velocity: 15 ft/sec, $R_e \sim 16 \times 10^6$

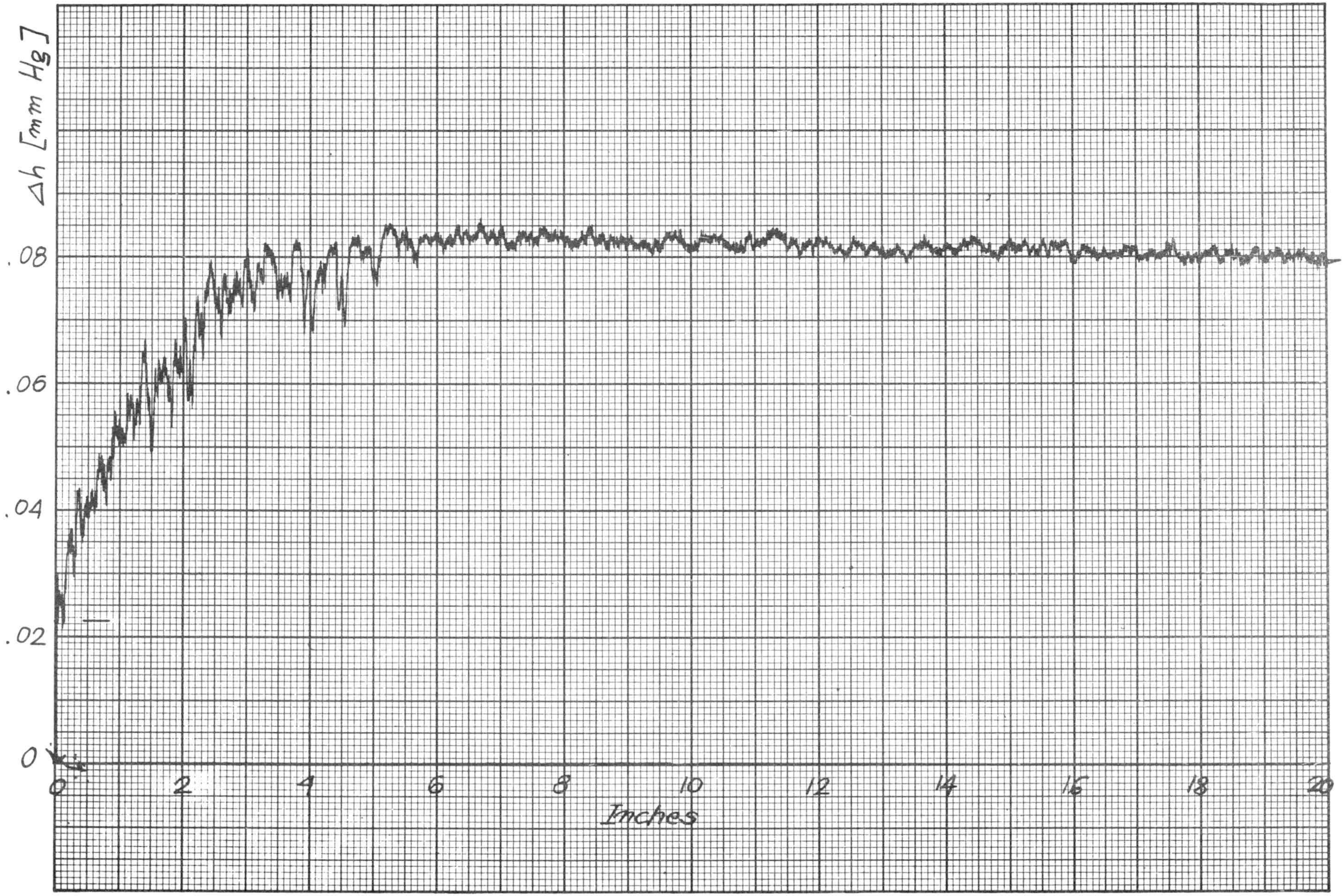


Fig. 35. Velocity profile taken at the heat source location, U = 15 ft/sec

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13. ABSTRACT

Diffusion of heat from an instantaneous point source located in a thick turbulent boundary layer over a wind-tunnel floor was investigated. A technique was developed for production of heat spots and detection of temperature fluctuations downstream from the point of release. As a heat source a short length of platinum-iridium wire 0.0004 in. in diameter and approximately 1/10 in. long was used which was heated by a short pulse of electric current. A high-response resistance thermometer was employed for detection of temperature fluctuations. The output of the resistance-thermometer bridge was amplified and applied to an oscilloscope with a "memory screen". The instantaneous temperature profiles of the convected heat spots were displayed on the screen and readings of the maximum temperatures were taken. From about 100-120 readings, mean maximum temperatures were calculated.

Data were taken at different distances up to 5 in. from the source with the source placed at 2-1/2 in. and 5-7/8 in. from the wall. The velocity of air was kept at 15 ft/sec ($R \simeq 26 \times 10^6$).

The obtained horizontal distributions of heat spots are very close to Gaussian curves. The vertical distributions show a skewness. The skewness is such that the greater spread occurs at the side of the greater value of the mean velocity. The skew distribution obtained was compared with the Hinze's skewed temperature distribution and the agreement is satisfactory.

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Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Diffusion Turbulent diffusion Turbulence Boundary layers Heat and Mass Transfer Fluid Mechanics						

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