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ARTIFICIAL ROUGHNESS STANDARD FOR OPEN CHANNELS

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Abstract--In the formulas in present use the resistance coefficient has been considered constant for a particular type of material in a particular state of upkeep. The resistance coefficient, however, is not a constant for a given channel but varies with both velocity and depth. From the nature of these formulas in present use, it may be seen that the variable influence of viscosity is not taken into consideration.

Because of the need for more accurately determining the discharges in open channels a standard for roughness for open channels such as that which exists for pipes is desirable. The problem of developing a reproducible standard for roughness in open channels has previously received very little systematic study. Although several investigations have been made, each was made with a different type of roughness.

This study was conducted for the purpose of determining whether an artificial roughness standard could be established for a channel with a boundary so rough that the viscous effects were negligible. The equation

$$C = 26.65 \log_{10} (1.891\ d/a)$$

was established and is applicable over at least a fourfold variation in d/a and nearly a twofold variation in the resistance coefficient.

Although the conveyance of water in open channels is one of the earliest of engineering achievements, as yet no formulas for determining discharges have been developed that are without important limitations. The difficulty has been in determining the extent to which the flow is retarded for different boundary and flow conditions. In the discharge formulas in present use, the resistance coefficient is considered to be a constant for a particular type of material in a particular state of upkeep without regard to the other variables. Many investigators have shown that this is not the case as the resistance coefficient may vary as much as 50 pct for different depths in the same channel because of the effects of viscosity and shape. In the case of alluvial channels, this variation in resistance coefficient may also be effected by the formation of dunes on the bed of the channel.

It has become increasingly evident that an artificial standard for roughness in open channels, such as that which exists for pipes, is needed. Pipe roughness has been referred to an artificial standard in order to determine a relative roughness for each type of boundary material having various surface conditions. Using this standard as a basis, formulas have been developed experimentally so that the discharge through pipes can be predicted with considerable accuracy.

The problem of determining an artificial standard for roughness in open channels has hitherto received very little systematic study. Several investigators have made studies using artificial roughness, each using a different type of roughness for the purpose of determining the retardation of flow for the particular type of roughness chosen.

This study was for the purpose of choosing an artificial roughness and testing it in a range where the channel would be so rough that viscous forces would be relatively insignificant.

Previous developments--Two formulas for computing discharges in open channels are now widely used. The first of these is the Chezy formula which may be stated as

$$Q = CA\sqrt{RS} \qquad (1)$$

where Q is the discharge, C is the Chezy resistance coefficient, A is the cross-section area, R is the hydraulic radius, and S is the slope of the channel. The Manning formula which is more widely used in the United States at the present time is

$$Q = (1.486/n)\ AR^{2/3}\ S^{1/2} \qquad (2)$$

where n is the Manning coefficient of resistance. In both cases the resistance coefficient is considered to be constant for a particular type of boundary material in a particular state of repair regardless of the flow conditions.

In 1860, BAZIN [1888] and Darcy [GANGUILLET and KUTTER, 1889] made the first studies of artificial roughness in open channels. Experiments were conducted using transverse wood strips nailed to the bottom and sides of a channel to produce the roughness. Experiments were also conducted using gravel glued to the sides of the channel. No attempt was made to correlate the data obtained using artificial roughness with that available for natural roughness.

KEULEGAN [1938] applied the modern principles of fluid flow in circular pipes to the problem of turbulent flow in open channels by theoretical development. The formula which was developed by using the assumption that resistance to flow in a rough channel is the same as to flow in a rough pipe with the same hydraulic radius and degree of roughness, may be stated as

$$C/\sqrt{g} = 6.25 + 5.75 \log_{10} (R/k) \qquad (3)$$

The roughness k is the diameter of equivalent NIKURADSE [1933] sand particle.

POWELL [1946, 1950] made a series of experiments using square steel strips across the sides and bottom of a channel. With the fundamental formula given by Keulegan, he developed the equation

$$C = 1.263J + 17.9\,\beta_k + 41.2 \log_{10} (R/k) \qquad (4)$$

where $J = \sqrt{g}\,(a_r - 2.5)$, a_r being a function of k; and β_k is a function of the shape of the cross section. Through further simplification which included changing the scale by which the roughness was measured, the equation

$$C = 42 \log_{10} (R/\epsilon) \qquad (5)$$

was presented, where ϵ is the measure of roughness. This equation applies to that zone where the channel is so rough that the resistance to flow is caused by the roughness alone. In the transition zone between the rough and smooth channels the resistance depends both on the viscous effects and the relative roughness. For this condition, Powell developed the equation

$$C = -42 \log_{10} (C/Re + \epsilon/R) \qquad (6)$$

where Re is the Reynolds number.

The method of estimating the resistance coefficient for pipes is given by ROUSE [1943]. It is noted that the roughness of each type of pipe material is given in terms of k, which is the equivalent Nikuradse sand roughness. In that section in which the pipe is very rough the equation which determines the Darcy-Weisbach resistance coefficient f is given as

$$1/\sqrt{f} = 2 \log (D/k) + 1.14 \qquad (7)$$

where D is the diameter of the pipe.

Theoretical analysis--For the purpose of making a preliminary analysis of the problem, the forces which are acting on the free body as shown in Figure 1 must be considered, where τ_o is the shear along the boundary and W^P is the wetted perimeter.

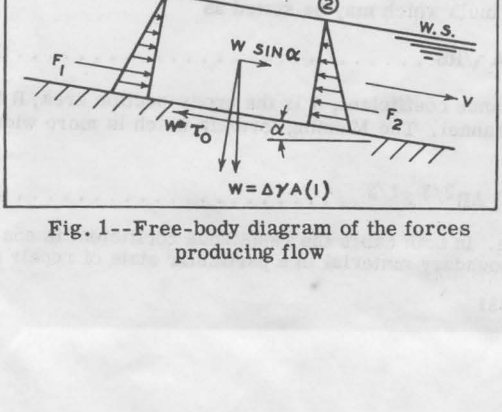

Fig. 1--Free-body diagram of the forces producing flow

If the flow is uniform and steady, the sum of the forces is equal to zero so that

$$\Sigma F = F_1 + W \sin\alpha - F_2 - W^P \tau_o = 0 \qquad (8)$$

$$F_1 = F_2 \qquad (9)$$

and

$$\tau_o = (A/W^P)\,\Delta\gamma \sin\alpha = R\,\Delta\gamma \sin\alpha \qquad (10)$$

Although it is possible to solve for the shear by analytical means, the use of dimensional analysis shows more clearly the influence of the variables involved. The variables which govern the intensity of shear along the boundary may be stated as

V = velocity of flow in the channel
k = roughness of the channel in terms of the equivalent Nikuradse sand roughness
sf = factor expressing the shape of the channel
R = hydraulic radius
ρ = density of the fluid
μ = fluid viscosity
$\Delta\gamma$ = difference in specific weight of air and water

The general relationship that exists may be stated as

$$\tau_o = \phi_1 (V, k, sf, R, \rho, \mu, \Delta\gamma) \quad (11)$$

Choosing V, R, and ρ as repeating variables, dimensional analysis yields

$$\tau_o/\rho V^2 = \phi_2 (R/k, sf, V/\sqrt{(\Delta\gamma/\rho)\, R}, VR\rho/\mu) \quad (12)$$

$$\tau_o = \rho V^2 \phi_2 (R/k, sf, V/\sqrt{(\Delta\gamma/\rho)\, R}, VR\rho/\mu) \quad (13)$$

in which $V/\sqrt{(\Delta\gamma/\rho)\, R}$ is a Froude number Fr and $VR\rho/\mu$ is a Reynolds number Re.

Equating (10) to (13) yields

$$R\, \Delta\gamma \sin \alpha = \rho V^2 \phi_2 (R/k, sf, Fr, Re) \quad (14)$$

so that

$$V = \sqrt{(\Delta\gamma/\rho)}\, \phi_2 (R/k, sf, Fr, Re) \sqrt{R \sin \alpha} \quad (15)$$

For small values of α, the slope S is approximately equal to $\sin \alpha$ so that

$$V = \sqrt{g}\, \phi_2 (R/k, sf, Fr, Re) \sqrt{RS} \quad (16)$$

Chezy's formula which is widely used for computing discharges in open channels is

$$V = C \sqrt{RS} \quad (17)$$

From (16) then,

$$C = \sqrt{g}\, \phi_2 (R/k, sf, Fr, Re) \quad (18)$$

or

$$C/\sqrt{g} = \phi_2 (R/k, sf, Fr, Re) \quad (19)$$

Because of the interdependence of R, sf, and depth of flow d as $d = \phi$ (R, sf), the Froude number and relative roughness may be expressed in terms of the mean depth. Further, if the channel to be studied is rectangular in shape, the shape factor sf may be replaced by the parameter B/d, where B is the width of the channel. Therefore,

$$C/\sqrt{g} = \phi_3 (d/k, B/d, Fr, Re) \quad (20)$$

The roughness k is dependent upon the geometric pattern of the type of roughness selected in the case of artificial roughness. In this study it was decided to use baffle plates as shown in Figure 2 so that

$$k = \phi_4 (a, t, b, x, e) \quad (21)$$

where

a = height of artificial roughness
t = thickness of the baffle plates
b = length of the plates
x = longitudinal spacing
e = transverse spacing

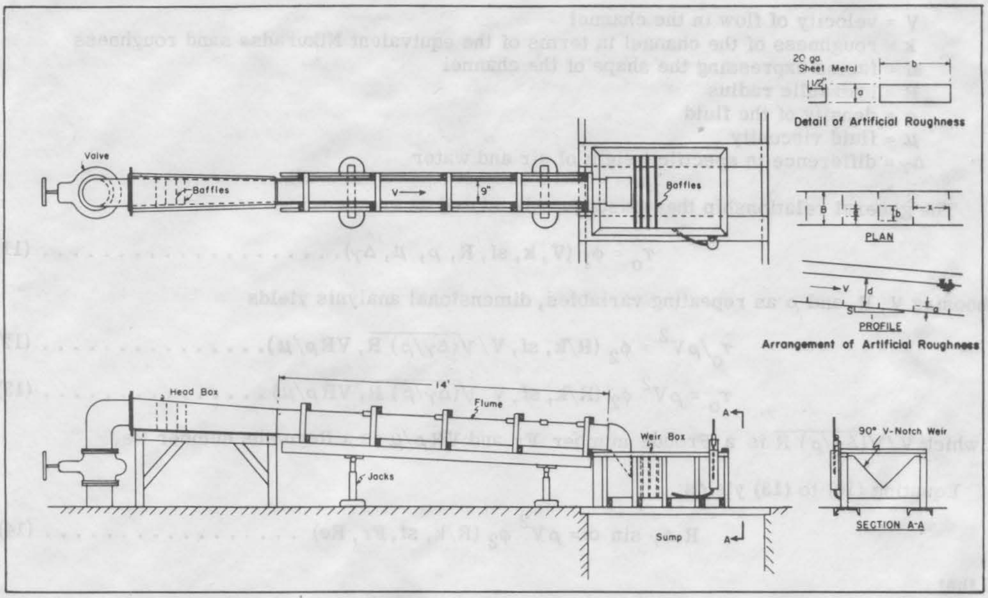

Fig. 2--Layout of experimental equipment

Dividing through by a, so that (21) will be in dimensionless form, results in

$$k/a = \phi_5\ (t/a,\ x/a,\ b/a,\ e/a) \quad \text{.......................} \quad (22)$$

For this particular study the baffle plates used were made of very thin sheet metal and only two heights of roughness were used so t/a could be considered insignificant for all practical purposes. On the basis of previous experiments [ALBERTSON, 1948, 1951; JOHNSON, 1944; POWELL, 1946], maximum roughness should occur for the type of roughness used at a longitudinal spacing-to-height ratio x/a between 2 and 10. For this study this ratio was kept constant at 10. The arbitrary constants describing the roughness b/a and e/a were given values of 4 and 2. Eq. (22) then becomes

$$k/a = C_1 \text{ or } k = C_1\ a \quad \text{.......................} \quad (23)$$

where C_1 is a constant.

For this particular study C_1 was assumed to have a value of one so that k = a and(20) becon es

$$C/\sqrt{g} = \phi_6\ (d/a,\ B/d,\ Fr,\ Re) \quad \text{..............} \quad (24)$$

This equation is valid for all values and combination of values of relative roughness, relative width, Froude number, and Reynolds number. However, the Froude number is important only if surface waves or large irregularities are involved. For this study, large irregularities would not be involved and although surface waves possibly would exist, they should not be of sufficient magnitude that the Froude number would be of importance. This assumption simplifies (24) to

$$C/\sqrt{g} = \phi_7\ (d/a,\ B/d,\ Re) \quad \text{....................} \quad (25)$$

For large values of the Reynolds number or when the roughness is relatively large, the laminar sub-layer is destroyed so that the viscous effects are negligible and the Reynolds number may be omitted. Then

$$C/\sqrt{g} = \phi_8\ (d/a,\ B/d) \quad \text{......................} \quad (26)$$

When the resistance on the bottom is very great compared with the resistance on the sides, the effect of relative width becomes unimportant and B/d may be omitted.

$$C/\sqrt{g} = \phi_9 \ (d/a, Re) \quad (27)$$

For large values of the Reynolds number and when the resistance on the bottom is great compared to that on the sides, the problem reduces to

$$C/\sqrt{g} = \phi_{10} \ (d/a) \quad (28)$$

When the viscosity is of major importance compared with relative roughness, then d/a is unimportant and

$$C/\sqrt{g} = \phi_{11} \ (B/d, Re) \quad (29)$$

If the relative width is great and the roughness is relatively small, then the function reduces to

$$C/\sqrt{g} = \phi_{12} \ (Re) \quad (30)$$

On the basis of the theoretical development and previous research, an equation in logarithmic form should apply in determining the resistance coefficient for wide, rough channels. This equation should have the form of

$$C/\sqrt{g} = K_1 + K_2 \log_{10} (d/a) \quad (31)$$

where K_1 and K_2 are constants to be determined experimentally.

Range of variables--In order to cover the widest possible range of variables with the limitations imposed by the equipment available, the following range was placed on those variables involved.

a = 1/2 and 1 inch
d = 2 to 8.5 inches
S = 0.001 to 0.04
B = 9 inches
V = 0.30 to 5.0 ft/sec (approx.)
d/a = 4 to 17
B/d = 1.1 to 4.5
$Re = 3.5 \times 10^3$ to 2.5×10^5 (approx.)

Experimental equipment and procedure--The laboratory equipment used for this study consisted of a water-supply system, an adjustable slope flume to accommodate the artificial roughness, and a weir box to measure the discharge. The test section of the flume, which was made of wood, was 9 inches wide by 10 1/2 inches deep and 14 ft long. Upstream from the test section was an entrance section six feet long in which an attempt was made to establish the velocity distribution before the flow entered the test section. Adjustable jacks were used to set the flume at the desired slope. The artificial roughness was in the form of small sheet metal angles 1/2 and one inch high which were fastened to the floor with an arrangement as shown in Figure 1.

In order to adjust the depth of flow in the flume, the downstream end was constructed so that small slats could be placed across the flow to form a tailgate. To measure the depth of flow a movable point gage was used which could be placed at any one of six measuring points distributed along the length of the flume. The discharge was measured by a 90° V-notch weir placed at the end of the flume.

To begin an experiment a certain size of roughness was installed in the flume and the desired slope was set by adjusting the jacks and checking the drop for a known horizontal distance with a surveyor's level and rod.

Normally, for each slope of the flume, four runs were made with the depth of flow varying from very low to as high as possible. For each run the depth of flow was determined at each of the six measuring points and the discharge obtained with the weir. Since it was necessary that steady uniform flow be maintained, the slats at the downstream end of the flume were used to maintain a water surface that was parallel to the floor of the flume throughout most of its length.

Discussion of results--In the analysis of the problem, it was shown that there are at least 13 relatively important factors in the problem dealing with the effect of artificial roughness on flow

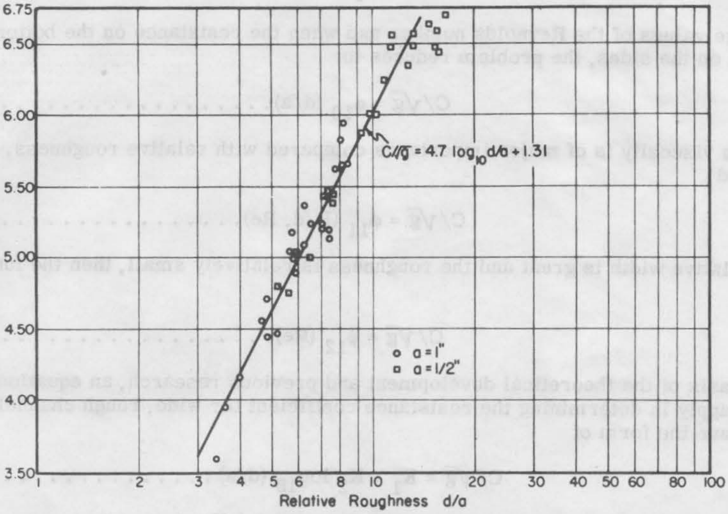

Fig. 3--Variation of resistance coefficient with relative roughness

in open channels. After combining these variables by dimensional analysis and eliminating some from consideration, only three parameters were chosen for study, the resistance coefficient $C/\sqrt{g}$, the relative roughness d/a, and the Reynolds number Re.

Figure 3 is a plot of $C/\sqrt{g}$ as a function of d/a as shown in (28) and (31) for rough channels. The equation of the line shown was found to be

$$C/\sqrt{g} = 1.31 + 4.7 \log_{10} d/a \quad \ldots\ldots (32)$$

or

$$C = 26.65 \log_{10} (1.891\ d/a)$$

The data for the 1/2 and one inch roughnesses agree very well for values of d/a less than 10. For values in excess of 10, the resistance coefficient determined experimentally is less than predicted by (32). The most plausible explanation for this is that the effect of relative width B/d or of viscosity becomes important at this point as shown in (29) and (30). In other words, the channel can not be considered as infinitely wide for values of d/a greater than 10, because at this point the effect of the side walls becomes important. The scatter of the data for values of d/a less than 10 can be attributed to experimental error and to the difficulty of obtaining uniform flow in some of the experiments.

The value of the coefficient in (32) is 26.65 as compared to 32.6 for Keulegan's Eq. (3), and 42 for Powell's Eq. (5). Keulegan used data from natural roughness to check his theoretical equation whereas Powell's equation was obtained by correlating data from his experiments using continuous metal strips for artificial roughness. For the case of Powell's roughness, only horizontal vortices would form because of the bottom roughness and only vertical vortices would form because of the wall roughness. Except at very shallow depths his channel must be considered to be narrow in which the side walls exert a considerable influence on the retardation of flow. Bazin's data, which Keulegan used to check his equation, were taken using a wide channel with gravel placed on the floor to serve as the roughness. Although Bazin's roughness is of a natural type, it does not give the same degree of roughness as that used in this study. Therefore, the comparison between (32) and (3) and (5) must consider the measure and type of roughness. A value of a for any given channel must be determined by first making actual discharge measurements in a rough channel in order to determine the other factors in (32). Then a constant value of a can be determined for a canal of a particular material and state of upkeep.

Figure 4 is a plot (based upon Figure 3) of the resistance coefficient $C/\sqrt{g}$ as a function of the Reynolds number Re for different values of the relative roughness d/a. This plot demonstrates the significant fact that for the roughness tested, the value of C remains constant over the range

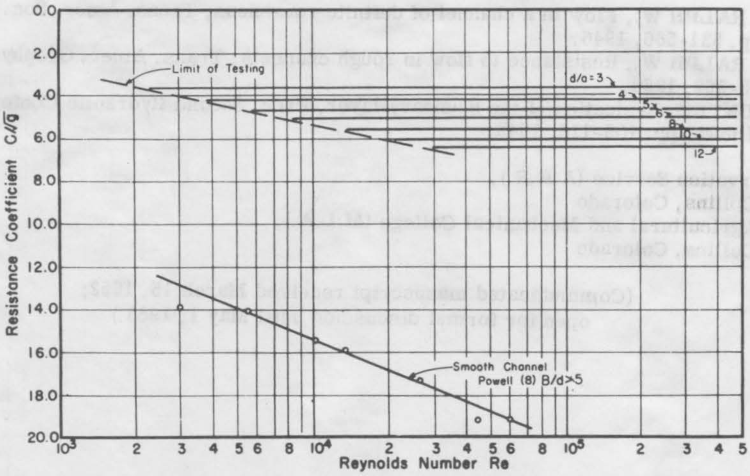

Fig. 4--Variation of resistance coefficient with Reynolds number and relative roughness

of Reynolds number observed for a particular value of d/a. In other words, the channel was acting as a rough channel. As mentioned previously this statement applies only for values of d/a equal to or less than 10. For values greater than 10, a study of Figure 3 leads one to believe that the channel was functioning in the transition zone between a rough and smooth channel or that the relative width has become important.

Also of interest in Figure 4 are the data shown for a smooth channel observed by POWELL [1946]. Based upon the theoretical analysis leading to (27) and (30), one would expect the curves shown for a rough channel for constant values of d/a to approach the curve for a smooth channel asymptotically for the lower ranges of Reynolds number. Similar curves are shown by ROUSE [1943] for pipes.

The foregoing analysis and experimental study demonstrate that a roughness standard such as exists for pipes may be set up for open channels with rough boundaries. The design of artificial roughness used herein could possibly be improved. This particular design was chosen because of ease of construction, ease of duplication in other laboratories, and the belief that the resulting resistance to flow would more nearly duplicate the type of resistance found for natural roughness.

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References

ALBERTSON, MAURICE L., Design of the Loup River bed-load measurement structure, Rep., Div. Eng., Colorado A. and M. College, 15 pp., 1948 (processed).

ALBERTSON, MAURICE L., Turbulence flume to measure bed load, Trans. Amer. Geophys. Union, v. 32, pp. 909-911, 1951.

BAZIN, HENRI ÉMILE, Experiences nouvelles sur l'écoulement en deversoir, Paris, 530 pp., 1888.

GANGUILLET, E., and W. R. KUTTER, A general formula for the uniform flow of water in rivers and other channels, John Wiley, 240 pp., 1889.

JOHNSON, JOE W., Rectangular artificial roughness in open channels, Trans. Amer. Geophys. Union, v. 25, pp. 906-914, 1944.

KEULEGAN, GARBIS H., Laws of turbulent flow in open channels, U. S. National Bureau of Standards Res. Pap. 1151, Res. J., v. 21, pp. 707-741, December, 1938.

NIKURADSE, J., Stromungsgesetze in rauhen Rohren, Forschungsarbeiten auf dem Gebiete des Ingenieur Wesens, no. 361 (4), pp. 1-22, July-August, 1933.

POWELL, RALPH W., Flow in a channel of definite roughness, Trans. Amer. Soc. Civ. Eng., v. 111, pp. 531-566, 1946.

POWELL, RALPH W., Resistance to flow in rough channels, Trans. Amer. Geophys. Union, v. 31, pp. 575-582, 1950.

ROUSE, HUNTER, Evaluation of the boundary layer, Proc. Second Hydraulic Conference, University of Iowa, pp. 105-116, 1943.

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