

## A MULTI-VARIABLE APPROACH FOR THE COMMAND OF CANAL DE PROVENCE AIX NORD WATER SUPPLY SUBSYSTEM

Yann Viala<sup>1</sup>  
Pierre-Olivier Malaterre<sup>2</sup>  
Jean-Luc Deltour<sup>1</sup>  
Franck Sanfilippo<sup>1</sup>  
Jacques Sau<sup>3</sup>

### ABSTRACT

The Canal de Provence is fully user oriented. Water users can take the water freely without respecting neither rotation nor any sort of priority allocation. Its structure, consisting of main free flow canals and pressurized distribution networks, is well adapted to this strategy. The main canal must be able to face the regime variations coming from this kind of distribution. The current regulation conception first split the whole system into a series of assumed independent sub-systems. The multi-variable aspect is then taken into account by a coordination of the sub-systems adjustment, carrying the discharge correction from downstream to upstream.

The Aix nord branch control presents interesting characteristics such as many different hydraulic entities (free surface canals, reservoirs, pumping stations) and operating constraints (levels in reservoirs, optimization of pumping costs). A real multi-variable approach will allow managing all gate and pump operations and all constraints at the same time, leading to a global optimisation of the whole system.

The MIMO (Mulit Input – Multi Output) model is established from transfer functions, the coefficients of which are deduced from the physical and geometrical characteristics of the system. A Linear Quadratic Regulator is computed and tested on a complete non-linear numerical model of the hydraulic system. The system to be controlled includes many discrete commands (pump operations) that are not managed by a classical optimal control. These commands are treated apart, leading to calculated perturbations that are introduced in the regulator.

---

<sup>1</sup> Société du Canal de Provence – Le Tholonet BP 100, 13603 AIX EN PROVENCE CEDEX 1, France – Tél. : +33 4 42 66 70 00 – Fax : + 33 4 42 66 70 80 – E-mail : [firstname.lastname@canal-de-provence.com](mailto:firstname.lastname@canal-de-provence.com)

<sup>2</sup> CEMAGREF – 361, rue Jean François Breton BP 5095, 34033 MONTPELLIER CEDEX 1, FRANCE – Tél. : +33 4 67 04 63 56 – Fax : +33 4 67 63 57 95 – E-mail : [pierre-olivier.malaterre@cemagref.fr](mailto:pierre-olivier.malaterre@cemagref.fr)

<sup>3</sup> Université Claude Bernard – 43, bd du 11 novembre 1918, 69622 VILLEURBANNE CEDEX, FRANCE – Tél. : +33 4 74 43 12 24 – Fax : +33 4 72 43 12 25 – E-mail : [jacques.sau@univ-lyon1.fr](mailto:jacques.sau@univ-lyon1.fr)

## INTRODUCTION

The control of a canal system is a MIMO control. Usually, in field applications, the design starts from a Single Input Single Output (SISO) design, and the MIMO character is added after by an ad hoc procedure, like coordination between pools. Many studies of MIMO control, in particular optimal control, on canal automation have been done (Corriga 1982, Malaterre 1994 and 1998). A survey can be found in Georges and Litrico (2002). In real cases, the complete control problem must take into account field and operational constraints that are not easily handled by the usual approaches.

The application we present here concerns the first study of a MIMO controller on an existing canal branch. We have chosen a branch with interesting characteristic for the experimentation of this kind of control. The reasons for the selection of the Canal de Provence Aix-Nord branch are the multiplicity of structures of different kind and the existing management constraints.

The Aix-Nord branch is presented in the next section with the hydraulics variables, which are the control, controlled, perturbation and measured variables. In the same section will appear the exploitation constraints, which are important for the control implementation.

A linear modelling of the system is performed in section 3. This modelling starts from transfer functions. Its validity has been verified in the Canal de Provence experience on irrigation canal control. The design of the controller takes place in section 4 and the results in regulation and tracking regarding the Aix-Nord branch are shown in section 5.

The simulations and tests of performance of the controller are carried out on the full non-linear simulation model Sic of the Cemagref, which can model a canal system by solving the Saint-Venant equation with implementation of control structures and perturbations (Sic 1992).

## THE HYDRAULIC SYSTEM

### Description of the system

The Aix-Nord branch consists of a 10 km (6.2 mile) long canal, nine reservoirs and five pumping stations. The sub-system we are interested in is shown schematically on Figure 1.

A gate at the output of the regulation pool controls the discharge in the Trevaresse canal. This canal has been recently modernized by the construction of a series of duckbill weirs. This canal feeds the 13,400 m<sup>3</sup> (10.9 acre-foot) Barounette

reservoir, from which a pumping station feeds the 8,500 m<sup>3</sup> (6.9 acre-foot) Collet-Redon reservoir.

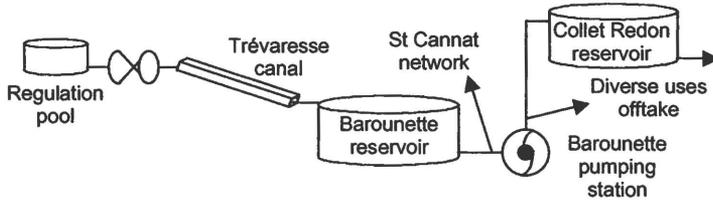


Figure 1. Outline of the hydraulic system

### Hydraulic variables

The hydraulic variables and their notations are the following:

Control variables or inputs:

- The discharge at the output gate of the regulation reservoir,  $u_1$ .
- The discharge of the Barounette pumping station (4 pumps),  $u_2$ .

Controlled variables:

- Volume of the Barounette reservoir,  $y_1$ .
- Volume of the Collet Redon reservoir,  $y_2$ .

Measured variables:

- Discharge downstream of the Trevaresse canal,  $z_1$ .
- Volume of the Barounette reservoir,  $z_2 = y_1$ .
- Discharge at the output of the Barounette reservoir,  $z_3$ .
- Volume of the Collet-Redon reservoir,  $z_4 = y_2$ .

Perturbations:

- Leakage out of the canal,  $w_1$ .
- Outlets upstream of the Barounette pumping station,  $w_2$ .
- Customers outlets downstream of this pumping station,  $w_3$ .
- Output discharge of the Collet Redon reservoir,  $w_4$ , which is measured.

### Constraints

The application shown here deals with many kinds of constraints:

- The Trevaresse canal capacity is 1.5 m<sup>3</sup>/s (53 cfs). However, so as not to empty the canal (important filling time), a minimum discharge of 30 l/s (1.1 cfs) is needed.
- The target volume of the Barounette reservoir is 9,000 m<sup>3</sup> (7.3 acre-foot).

The operation of the Barounette pumping station is the more complex one. This station is made of four parallel pumps, working independently on an on/off basis.

- Concerning the downstream Barounette reservoir, four low levels exist which forbid the running of 1, 2, 3 or 4 pumps.

- A maximum (7,700 m<sup>3</sup>) and a minimum (4,000 m<sup>3</sup>) volume exist for the Collet Redon reservoir. The Barounette pumping station works so that to reach the high level at the end of the off-peak period. During the peak period, a prediction is performed and based on the result, one, two or three pumps are switch on.
- An optimization of energy cost is performed. Without detailing the variation of energy prices, let us simply say that this price varies according to the period in the year and also to the time in each day. The electricity price can vary by a factor of 4 throughout the year.

In this application, only a few of these constraints will be treated.

### THE SYSTEM MODELLING

In order to design a MIMO controller, we need a linear modelling of the whole system. This modelling makes use of:

- The second order transfer function between upstream and downstream discharges of the canal already proposed by Deltour (1988).
- Balance relations between discharges and volumes of the reservoirs

The upstream-downstream discharges transfer function is a second order one with double pole:

$$F(z) = \frac{N}{1 - Dz^{-1}} z^{-r}$$

with

$$D = \exp(-T_e / T)$$

$$N = (1 - D)^2$$

$T_e$  is the control time step and  $T$  is a time constant characterizing the canal,  $r$  is the pure delay expressed in number of time steps. These two last quantities can be related to the global hydraulic delay  $\Delta V / \Delta Q$  by (see Figure 2):

$$\frac{\Delta V}{\Delta Q} = T + rT_e$$

Here  $\Delta V$  is the steady state volume variation due to a  $\Delta Q$  discharge variation. This kind of parameterization is already in use at the "Société du Canal de Provence" since about 30 years and has proven its validity (Rogier 1987, Deltour 1988, Deltour 1998)

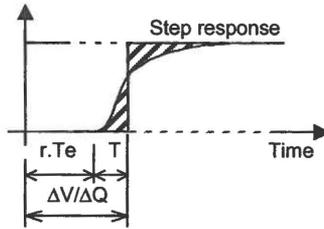


Figure 2. Characteristic times of a canal

The pure delay itself is given by the time needed for a small perturbation to travel from upstream to downstream in the canal:

$$rT_e = \frac{L}{v + c}$$

with  $L$  the length of the canal,  $v$  the flow speed and  $c$  the celerity of hydraulic waves. The great advantage of this transfer function is that it is not the result of black-box identification and depends only on geometrical and physical features of the canal, with no free parameter. Figure 3 shows the downstream response to an upstream input of pseudo-random binary series (PRBS) type around a 700 l/s discharge value. This type of response is a good test for a system modelling because the rich spectral content of a PRBS (Landau 1993, Ljung 1999). In Figure 3 the result is compared with a Sic modelling of the canal. The agreement is very good.

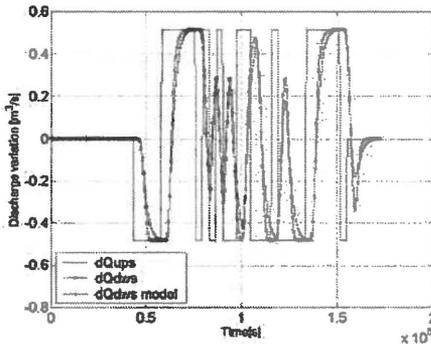


Figure 3. Comparison of downstream response discharge to an upstream PRBS  
dQdws is the Sic result

The modelling of the reservoirs is simpler since they are basically integrators. A discrete time approximation can lead to the following discharge-volume transfer function:

$$F(z) = \frac{T_e}{2} \frac{1+z^{-1}}{1-z^{-1}}$$

The above expressions represent a careful modelling of the hydraulic system, which is required for an efficient control design.

### DESIGN OF THE CONTROL

#### Optimal or LQG control

In order to design a MIMO LQG controller, we shall need a state space description of the system. Starting from the above transfer function description, a minimal state space realisation can be achieved using the specialised routines of Matlab. However, since we need null static error in regulation, integrator type variables must be added (Malaterre 1994, 1998). The state equations then take the classical form:

$$\begin{aligned} X^+ &= A_s X + B_u u + B_w w \\ y &= C_y X + D_{yu} u + D_{yw} w \end{aligned}$$

$X$ ,  $u$ ,  $y$  and  $w$  are respectively state, control action, controlled variables and perturbation vectors.  $A_s, B_u, B_w, C_y, D_{yu}, D_{yw}$  are matrices of appropriate dimensions. The optimal control is obtained by the minimization of a criterion:

$$J = \frac{1}{2} \sum_{k=0}^{N-1} \left[ (X(k) - X_c(k))' Q_X (X(k) - X_c(k)) + (u(k) - u_c(k))' R (u(k) - u_c(k)) \right]$$

$X_c$  and  $u_c$  are the wanted set point trajectories for  $X$  and  $u$ , respectively. An extensive development of the design of the control can be found in Åström and Wittenmark (1997), Malaterre (1994, 1998), Georges and Litrico (2002). The command  $u$  is obtained under the form:

$$u = -KX + H$$

where the gain matrix  $K$  is, in our application, solution of the asymptotic Riccati equation, and the pre-filter  $H$  is dependant on the open-loop or anticipatory part of the control.

#### Observer construction

The above control law assumes that the state vector  $X$  is known, which is almost all the time unrealistic. Most frequently, certain combination of states, or observed variables  $z$ , are effectively measured:

$$z = C_z X + D_{zu} u + D_{zw} w.$$

From variables  $z$ , the state vector  $X$  can be reconstructed, (Åström and Wittenmark 1997, Malaterre 1994,1998). Then  $X$  is replaced by the reconstructed one,  $\bar{X}$ . Due to unknown perturbations, a state Kalman filter including a perturbation observer is designed:

$$\bar{X}^+ = A_z \bar{X} + B_u u + B_w \bar{w} + L(z - \bar{z})$$

where  $\bar{w}$  is the perturbation vector estimation:

$$\bar{w}^+ = \bar{w} + L_w(z - \bar{z})$$

$L$  and  $L_w$  matrices can be computed through the minimization of the reconstruction error (Kalman filter)

## APPLICATION TO THE AIX-NORD BRANCH

### Tuning of the parameters

The scenarios we tested are without prediction on perturbations, which represent the most difficult cases for the control. Tuning parameters are the various  $Q$  and  $R$  matrices appearing in the criterions of the control and of the Kalman filter of the observer. These matrices are chosen diagonal, with diagonal elements according to the Bryson rule (Bryson 1975, Larminat 1993):

$$R_{ii} = (1 / \text{sup}(u_i))^2$$

$$Q_{ii} = (1 / \text{sup}(y_i))^2$$

where  $\text{sup}(u_i)$  and  $\text{sup}(y_i)$  are the on-field physical magnitude of the corresponding control actions and controlled variables value. These values serve as starting points for trial and error refinement of the parameters.

### A regulation scenario

We simulate first a regulation scenario, where unpredicted step  $w_2$  and  $w_3$  perturbations occur and no  $w_1$  perturbation, as shown in Figure 4. The purpose of this test is to appreciate the ability of the controller to reject perturbations.

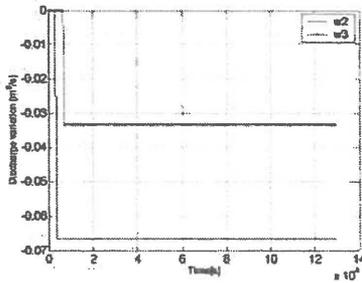


Figure 4. Step perturbations  $w_2$  and  $w_3$

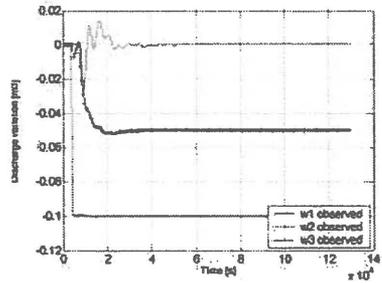


Figure 5. Reconstructed perturbations

The performance of the observer can be seen in Figure 5 where reconstructed perturbations are displayed. The step perturbations are rapidly recovered. One can notice also that  $w_1$  is reconstructed with a low value and returns rapidly to zero. Figures 6 and 7 display respectively the volume variations in the reservoirs and the control action variables. As can be seen, the regulation performances appear satisfactory.

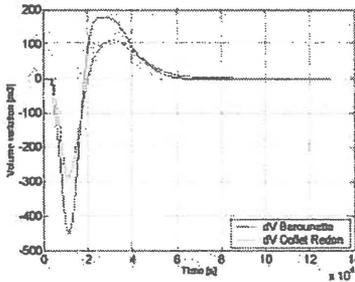


Figure 6. Reservoir volumes variations

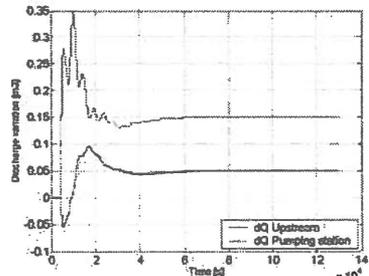


Figure 7. Control action variables

**A Tracking Scenario**

Tracking scenario must be tested since, as we explained in the description of the hydraulic system, set points for the reservoirs can change through time due to conditions like the variation of demand or variations of energy prices.

Figure 8 shows the response to an instantaneous set point change, with same parameter set as the regulation case. The new set points are reached rapidly with no important overshooting.

The control variables variations are shown in Figure 9. As expected, in the transient duration period, these variables vary more rapidly than in the regulation case, even if they remain operationally consistent.

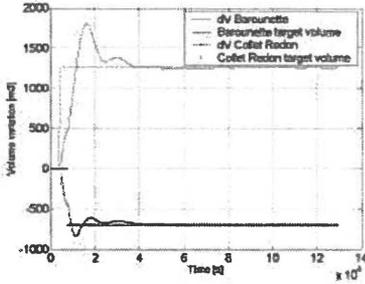


Figure 8. Reservoir volumes variation in the tracking scenario

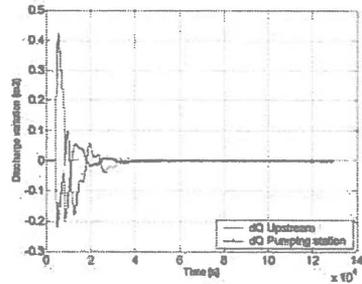


Figure 9. Input control variables

Finally it is interesting to look at the reconstructed perturbations, even if, in this scenario, it is not a crucial point. No perturbations are present. Figure 10 shows that, indeed, after a transient period, where the reconstructed perturbations vary around zero, they vanish in the steady state.

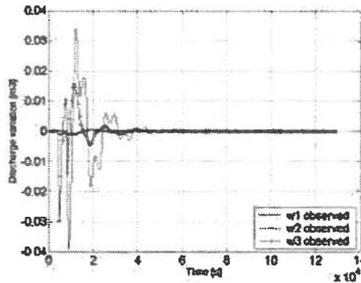


Figure 10. Reconstructed perturbations in the tracking scenario

## CONCLUSION

The optimal control we have proposed aim to take into account the whole multivariable character of an irrigation canal control. This needs first a careful design of the system. However, for control purposes model is required to be as simple as possible. The modelling we proposed in section 3 fulfills these conditions. The controller appears to work well. Nevertheless, prior to field implementation, one must make sure that it provides a benefit in relation to the actual SISO automation. Further studies are planned since the field operators are

not, in general, specialists of system control. In the next step, discontinuous values of discharge in the pumping station will be incorporated in the control.

### REFERENCES

Åström K.J. and Wittenmark B. (1997), *Computer-Controlled Systems, Theory and Design*, Prentice-Hall International.

Corriga G., Fanni A., Sanna S., Usai G., (1982), *A Constant Volume Control Method for Open Channel Operation*, International Journal of Modelling and Simulation, 2(2), 108.

Deltour J.L. (1988), *La regulation des Canaux d'Irrigation*, Rapport de DEA, Société du Canal de Provence et d'aménagement de la Région Provençale.

Deltour J.L., Sanfilippo F. (1998) *Introduction of Smith Predictor into Dynamic Regulation*, ASCE Journal of Irrigation and Drainage engineering, 124(1):47-55, Jan/Feb 1998.

Bryson A.E., HO Y.C. (1975), *Applied Optimal Control*, Hemisphere, Washington, D.C.

Georges D. and Litrico X. (2002), *Automatique pour la Gestion des Ressources en Eau*, Systèmes Automatisés, Hermes, Paris.

de Larminat Ph. (1993) *Commande des systèmes linéaires*, Automatique - coll Traité des Nouvelles Technologies, Hermes, Paris.

Landau I.D. (1993), *Identification et commande des systèmes*, Automatique - coll Traité des Nouvelles Technologies, Hermes, Paris.

Ljung L. (1999), *System Identification, Theory for the user*, Prentice Hall PTR.

Malaterre P.O. (1994), *Modelisation, Analysis and LQR Optimal Control of an Irrigation Canal*, Ph.D. Thesis, LAAS-CNRS-ENGREF-Cemagref, Etude EEE n° 14.

Malaterre P.O. (1998), *PILOTE: Linear Quadratic Optimal Controller for Irrigation Canals*, ASCE Journal of Irrigation and Drainage engineering, 124(1):187.

Rogier D., Coeuret C., Brémond J. (1987), *Dynamic Regulation on the canal de Provence*. In Darell D.Zimelman, editor, Planning, operation, rehabilitation and automation of irrigation water delivery systems – p 180-191, Portland, Oregon, July 1987.

Sic, (1992) *User's guide and Theoretical Concepts*, Cemagref Publications.