THESIS

LONG RANGE FIBER NOISE CANCELLATION

Submitted by

Noah Helburn

Department of Physics

In partial fulfillment of the requirements

For the Degree of Master of Science

Colorado State University

Fort Collins, Colorado

Summer 2024

Master's Committee:

Advisor: Christian Sanner

Sam Brewer
Jesse Wilson

ABSTRACT

LONG RANGE FIBER NOISE CANCELLATION

Optical atomic clocks are beyond timekeeping applications an increasingly important tool for testing fundamental physics and pushing the quantum science frontier. Being able to compare remote optical clocks by sharing coherent laser light between them opens exciting scientific perspectives. Optical fibers are almost ideal guides for sending light over long distances, but they induce phase noise on the light travelling through them. We demonstrate an actively phase-stabilized optical fiber link over a distance of 80km. A fractional frequency instability of 7×10^{-15} at 1 s and 6.3×10^{-18} after 1800 s was achieved.

TABLE OF CONTENTS

Chapter 1

Introduction

When performing high precision laser spectroscopy, a stable frequency reference is essential for obtaining accurate results within short measrurement intervals. Often times, labs need an entire optical table to set up these types of references and it is therefore desirable to share such a reference among remote labs, e.g., between JILA/NIST in Boulder [1–4] and CSU [5] in Fort Collins, which have a separation of roughly 80 km. Several optical atomic clocks are operated in Boulder, among them strontium and ytterbium lattic clocks, and an aluminum ion clock. In the microwave regime, transmitting frequency signals over long distances is rather straightforward [6]. The intuitive way to share an optical frequency reference between different locations separated by tens of kilometers is by using an optical fiber where the main perturbation introduced is phase noise accumulated when travelling through it predominantly caused by environmental perturbations. This can induce frequency noise in the light out of the remote end by several hundred Hz to kHz, which severely limits a coherent frequency transfer. To solve this spectral broadening issue caused by this accumulated frequency noise, active phase stabilization using electrical feedback is necessary.

1.1 Fiber Optic Cables

If we want an accurate and precise frequency transfer, the type of fiber we use is especially important. Fiber optic cables come in two types, single-mode and multi-mode, and to make sure we get the same frequency on both ends of the fiber, single-mode fiber are preferable. If we were to use a multi-mode fiber meaning several different modes could be transferred through the fiber, we would experience various dephasing issues [7].

Optical fibers have a 3 layered structure where in the center there is an 8-10 μm core to limit the amount of transverse modes able to be transmitted through the fiber. Then on the outside of that is a 125 μm cladding which importantly has a smaller refractive index than the core material so total internal reflection is able to occur. Then the final layer is a cover that can be manufactured to a typical 2-3 mm diameter total or 900 μm if a smaller cable diameter is desired.

Similarly, if we want to transfer light over long distances there are only certain wavelength windows we can use where we sustain the least amount of losses propagating through the fiber. Over the last 50 years, fibers centered at 1310 nm and 1550 nm have come to be the standards for long haul fiber optics. In this experiment, fibers centered at 1550 nm were chosen as they have a slightly lower $\frac{db}{km}$ loss than 1310 nm (1310 nm light has roughly double the loss as compared to 1550 nm light). Typically, since 1310 nm laser light is cheaper and easier to produce it is used for fiber lengths less than 10 km while 1550 laser light is used for anything longer because of its smaller losses [8].

Chapter 2

Experimental Setup

2.1 Fiber Splicing

A common technique for connecting fiberized components is using fiber splicing. This allows us to order all parts separately and assemble them together into one optical setup. In this case, we are using non polarization maintaining fibers.

The way fiber splicing is achieved is by using a precise fiber cutter as well as an arc splicer. First, two pieces of fiber must be stripped with fiber strippers until their core is exposed (inner most part of the fiber). An extremely straight cute must be performed to ensure that the end of the fiber is completely flat faced so that the welding of the two fibers will not change the properties of the newly created fiber.

Once both fibers have been stripped and cut properly, an arc splicer is used to fuse the two fibers into a continuous one. The two fibers are locked into place and the splicing machine using two cameras, one from the top view and one from the side view, lines up the two fiber cores with each other. A large arc voltage is then applied which fuses the two cores into each other. The result is the two fibers/components are now connected with no impact on the new fiber properties. There are small losses caused by this process on the order of 0.01 dB but over the course of the whole experiment shown in 2.1 in the following section, we observe around 0.3 dB loss in total of optical power. This corresponds to about 30 splices required to build the setup.

2.2 Schematic

For this experiment, the only infrastructure that was in place beforehand were the two 20km fibers from here to the WWV radio station. The goal is to design an optical setup where the instantaneous frequency that's measured on one end of the long fiber is exactly the same frequency measured on the other end. This implies that the noise accumulated from travelling in the fibers is actively "cancelled out". Figure 2.1 depicts the fully assembled setup where the red lines indicate electrical connections, the black lines represent optical paths, and the blue box shows what infrastructure is at WWV.

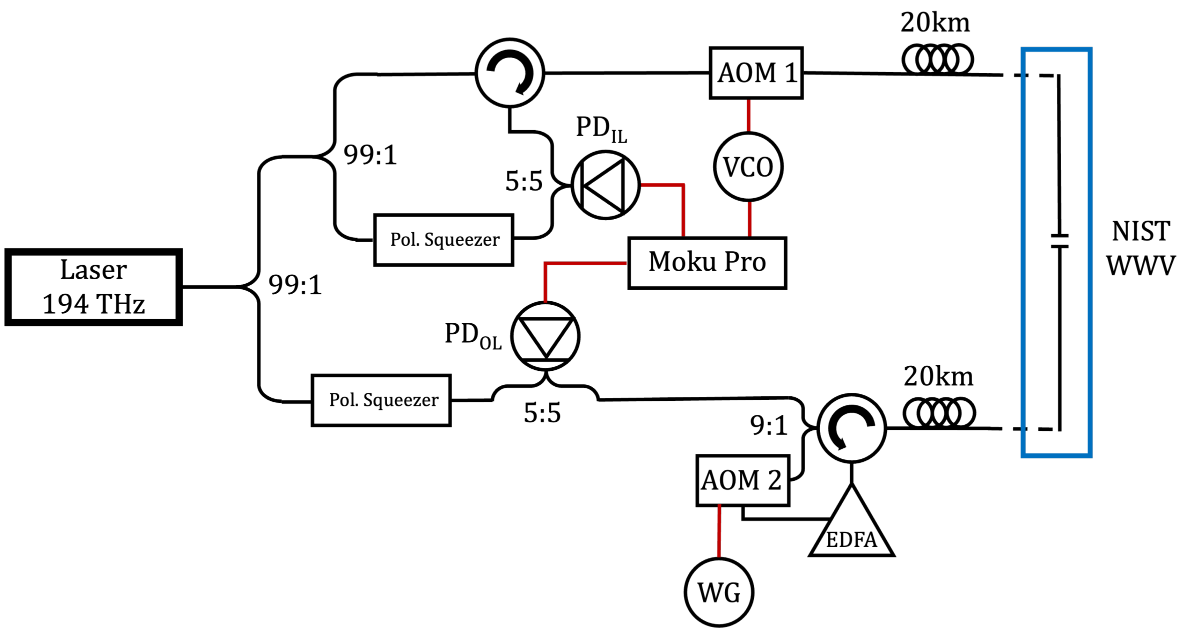

Figure 2.1: Experimental setup to detect frequency and phase noise and to provide optical path length stabilizing feedback where black lines are fiber connections while red lines are electrical connections. VCO, Voltage Controlled Oscillator. AOM, Acoustic Optic Modulator. EDFA, Erbium Doped Fiber Amplifier. Circles with arrows are Optical Circulators.

Starting with the laser, we used a 1550nm or 194THz fiber laser as the light source with 40mW coming out. The light is then split using a 99:1 ratio fiber splitter where 99% of the light continues on to the input fiber and the other 1% goes into one port of a 50:50 fiber coupler for the out of loop photodiode (PD_{OL}). The second 99:1 splitter does the same action as the first except for the in loop photodiode (PD_{IL}). The two squeezers are there to align the polarization for the two arms of the 50:50 couplers to get the maximum contrast on the PD. The light then goes through channel 1 of the optical circulator which feeds the light from channel 2 into AOM 1, modulated at 80MHz. From there, the light travels through the 20km fiber to WWV where it is connected via a fiber adapter to the other 20km fiber which sends the light back to the lab where in total it travels 40km round trip. This incoming light from WWV enters channel 2 of the second circulator which is then split by a 90:10 fiber splitter from the output of channel 3. 90% of the light goes to AOM 2 while the other 10% mixes with the reference arm of the 50:50 coupler for PD_{OL}. From AOM 2 (also modulated at 80MHz) the light goes into an Erbium Doped Fiber Amplifier (EDFA) where the power is amplified to go back through both 20km fibers making the light coming back to the in loop photodiode take a 80km trip. The signal from both photodiodes are fed through 2 RF Amplifiers before being fed into input 1 and 2 into the Moku Pro.

The In Loop photodiode detects a modulated signal at 240 MHz (Passes through AOM 1 twice and AOM 2 once) which is then mixed down with an external mixer to 100 MHz. This is fed into Input 1 of the Moku Pro where it is used as the input signal for a phase/frequency servo canceling the optical path length noise passing through 40 km of fiber. The Out of Loop photodiode detects a 80 MHz signal (passes through AOM 1 once) which is then fed into Input 2 of the Moku Pro where it will then be used to assess the stability of the optical link.

2.3 Moku Pro

The Moku Pro works as a box emulating many different electronic devices using an FPGA as well as Analog I/O circuitry. For this experiment the Phasemeter was used as a phase-to-voltage converter, the PID controller was used to control the gain parameters of the error signal sent to the VCO, and the Spectrum Analyzer was used to determine phase noise levels. They were connected as shown in 2.2.

2.3.1 Phasemeter

The phasemeter in essence is a digital tracking oscillator, phase locking to the provided input signal. Using an adjustable scaling factor, a voltage output dependent on the acquired phase or frequency offset between the tracking oscillator and a reference frequency is generated and subsequently used as an error signal. When locking using the In Loop photodiode, the phase offset feature was used (instead of outputting a voltage based on frequency offset). The phasemeter keeps

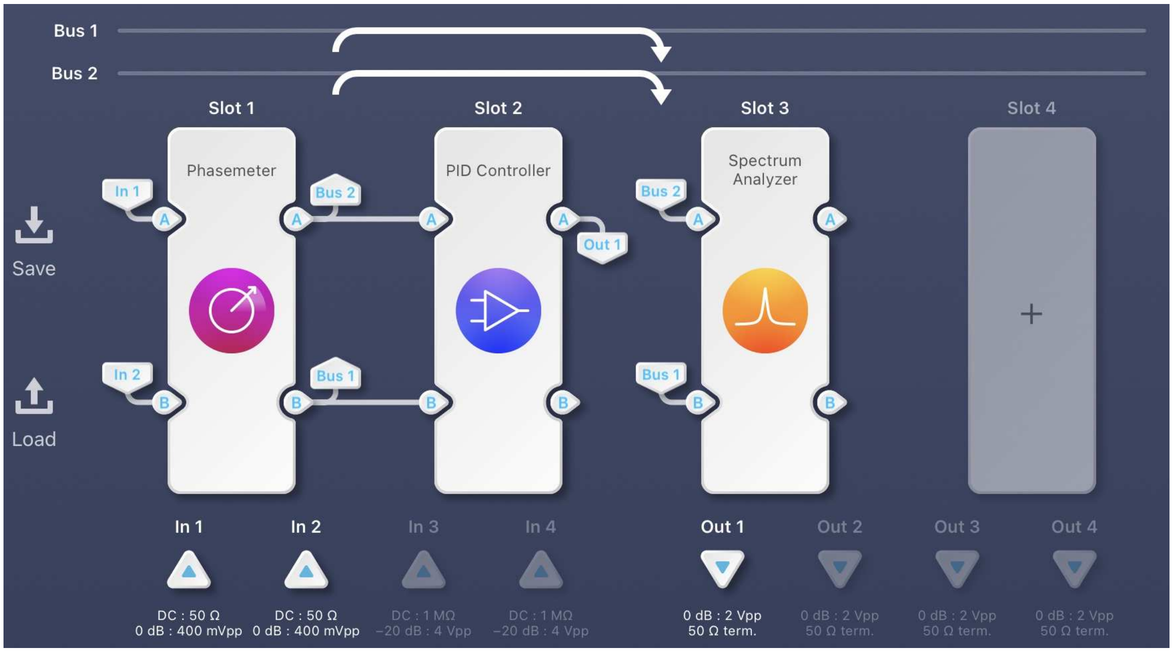

Figure 2.2: Internal connections of the Moku instruments used for optical path length stabilization. Inputs from both photodiodes go into the Phasemeter where their outputs both enter the PID Controller as well as the two inputs of the Spectrum Analyzer. Output A of the PID controller goes to the VCO which steers AOM 1.

a rolling sum of all the phase gained or lost and so can track and unwrap phase changes beyond 2π radians.

2.3.2 PID Controller

The Moku's PID controller takes the error signal output from the Phasemeter and converts it to the VCO signal steering AOM 1. The phase offset output from the Phasemeter only offers Proportional gain to our signal so used in conjunction with a PID controller, allows for Integral gain. Integral gain gives stronger feedback for frequencies closer to DC.

The output of the PID Controller went directly to the VCO, which controls the frequency going to AOM 1 to cancel out the phase fluctuations over the course of the 80km fiber. The phase noise from the fiber is observed as a frequency shift of our light and so the frequency modulation mode is used to account for this. The Frequency Modulation Deviation (FMDev) setting on the VCO defines the frequency tuning range and therefore effectively contributes to the servo loop's proportional gain. The FMDev is set to around 100 kHz.

2.3.3 Spectrum Analyzer

The Spectrum Analyzer (SA) was used mainly for taking Phase Power Spectral Density plots to determine the effectiveness of the phase cancellation. The output of the Phasemeter is fed into the SA where a scaling in units of $\frac{mV}{cyc}$ is manually set. The SA then displays the phase noise vs frequency centered at the reference frequency (either 100MHz or 80MHz). The y-axis is given in units of $\frac{V}{\sqrt{Hz}}$ but this can be re-scaled to have units of $\frac{cyc}{\sqrt{Hz}}$ by dividing the y values by the phasemeter scaling and the resulting plot is the Phase Power Spectral Density.

Chapter 3

Noise Measurments

3.1 Laser Noise

The laser used in the interferometer was an NKT Photonics Koheras Adjustik fiber laser centered at 1550 nm (194 THz). The out of box specification for the linewidth of this laser was less than 100 Hz but was measured using a Menlo Systems Frequency Comb to be closer to 1 kHz. This puts a coherence length for this laser at roughly 300km which is on the order of the link length, but can still produce sufficient interference contrast. An additional property of this laser is the apperance of frequency sidebands spaced at 73 kHz as seen in Figure 3.1. When looking at the beatnote made between this laser and the comb, the plus and minus sidebands are coherently following the carrier peak.

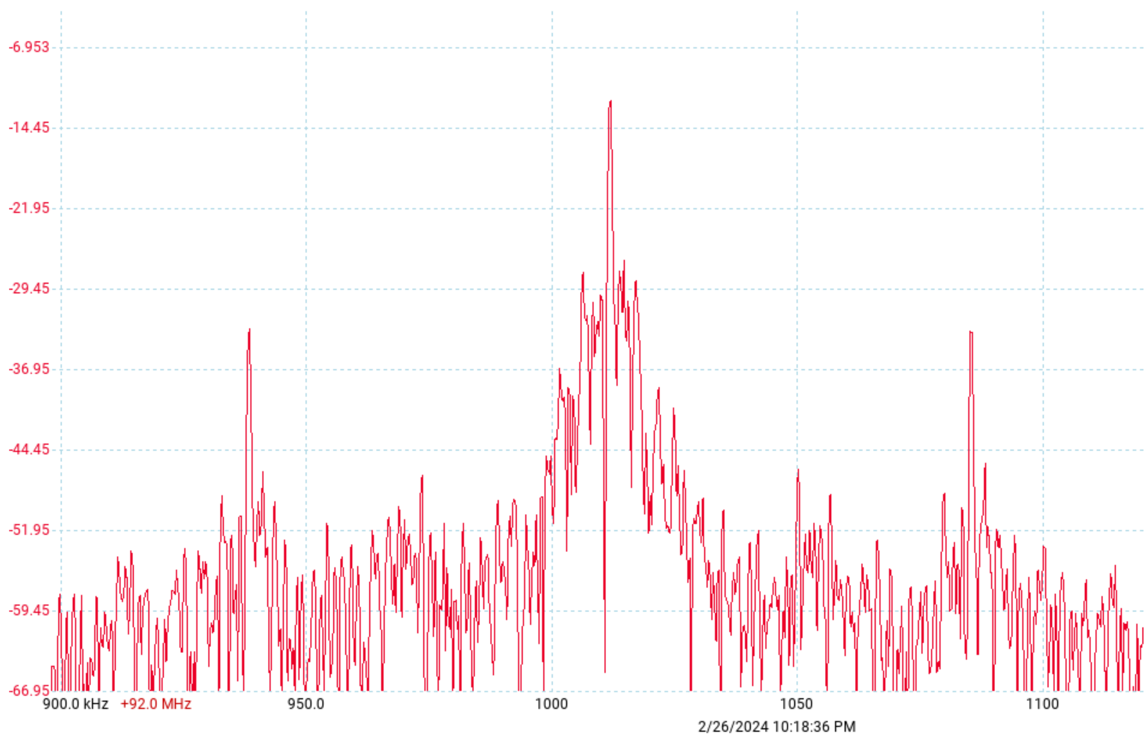

Figure 3.1: Screenshot of Koheras Adjustik laser beatnote with the Frequency Comb. The x axis is in units of kHz while the y axis is in units of dBm. The middle peak is the carrier while the two peaks on either side are the sidebands spaced at 73 kHz.

This unwanted 73 kHz frequency modulation is far outside the achievable servo bandwidth and therefore effectively averages away. On longer timescales, the carrier exhibits a frequency drift of several MHz over the course of a few hours to a day. This slight increases the apparent optical path length noise, but is correspondingly canceled by the servo loop and therefore does not affect the link characterization.

3.2 Fiber Length Measurement

Phase noise imparted on to the light inside the fiber looks like frequency noise when measured on a photodiode. This is due to the fact that a phase change over time corresponds to a frequency, and since the phase is constantly changing at different rates due to environmental factors, you get a different frequency offset overtime from the frequency put into the fiber. This is not unlike a Mach-Zender interferometer with one path having a different optical path length than the other [9]. We can exploit this property to actually measure the length of the long fiber we are sending light through.

For this measurement, a frequency modulation of 1 Hz with ±5 MHz deviations was purposefully induced on to the laser. When we observe the out of loop photodiode signal from light travelling 40 km, this frequency modulation looks like a phase modulation when using the phasemeter to measure the phase overtime of this beatnote. The out of loop signal beatnote's phase was observed for 30 mins and 3.2 below shows a 5 sec piece of the larger observation.

The oscillations seen are due to the frequency modulation of the laser showing up as phase oscillations at the remote end of the fiber. This was similarly done in [10] where light was modulated slowly through a 36.5 km fiber. The period of these oscillations are 1 sec showing these are due to that modulation. To determine a fiber length from this data, we need to determine how a change in length of the fiber corresponds to an addition of phase cycles. First we can think about how a 10 MHz frequency modulation changes the wavelength of the light. We can use the relationship of:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} \tag{3.1}$$

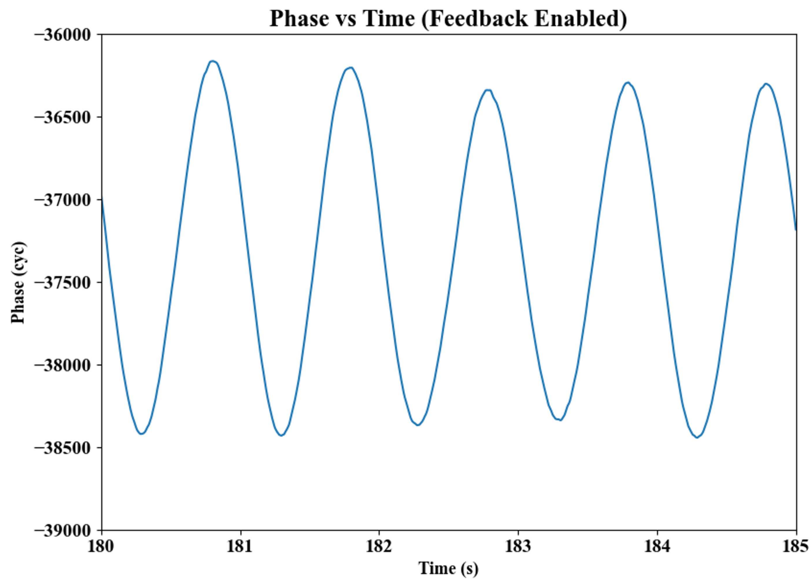

Figure 3.2: Graph of the out of loop photodiode signal with a 1 Hz dither ±5 MHz frequency modulation. This is a 5 sec piece of a 30 min observation

where Δf represents the frequency modulation deviation we apply (10 MHz), $\Delta\lambda$ represents how much the wavelength changes, and λ and f represent the laser wavelength and frequency respectively. Solving for $\Delta\lambda$ gets us a wavelength change of roughly 0.08 pm. Now we just need to convert this change in wavelength to a number of cycles based on the optical path length of the fiber. We can multiply the fractional change in a wavelength by the amount of wavelengths that fit in the fiber. This gets us a number in cycles as each wavelength is equivalent to one cycle. This essentially tells us how many extra wavelengths could fit into this fiber if there was a frequency change of 10 MHz. The equation looks like:

$$\frac{\Delta\lambda}{\lambda} * \frac{nd}{\lambda} = Cycles \tag{3.2}$$

The second term in the equation represents how many wavelengths can fit int the fiber based on its optical path length, d represents the length of the fiber and n represents the index of refraction. To determine the number of cycles corresponding to a 40 km fiber length, we plug in 40 km for d and we get an expected number of cycles to be around 2000.

To get a measured value of fiber length, all we need to do is extract the phase excursions for one oscillation in the graph which corresponds to the right side of the above equation. The oscillations are all slightly different heights but all correspond to on average 2000 cycles peak to peak. Measuring the peak to peak height of the third oscillation in 3.2 gives us a cycle excursion of 1989 cyc. Plugging this in and solving for d we get a fiber length of 39.8 km which gives a great confirmation our system is working correctly.

3.3 Brillouin Scattering

An important effect to consider with optical fibers is Stimulated Brillouin Scattering (SBS). This involves the incoming light exciting inelastic acoustic modes in the walls of the fiber creating a phonon wave (vibration) in the fiber, an outgoing light wave that propagates in the same direction as the incoming light, and a backward propagating light wave as described by Kobyakov [11]. The phonon and the backwards propagating light wave take an amount of power away from the forward light wave resulting in a significant decrease in optical power out the other side of the fiber. A measurement was conducted to compare the backward propagating optical power to the input light initially entering the fiber shown in Figure 3.3.

This graph has a small linear region at low powers but then increases quadratically with the input power after a few mW. For a narrow-linewidth laser source and km-length optical fibers, Brillouin scattering is expected to occur at optical input power in the milliwatts range [11], consistent with the reported measurement. Given the SBS induced constraints, the fiber link is operated with an optical input power of 8 mW.

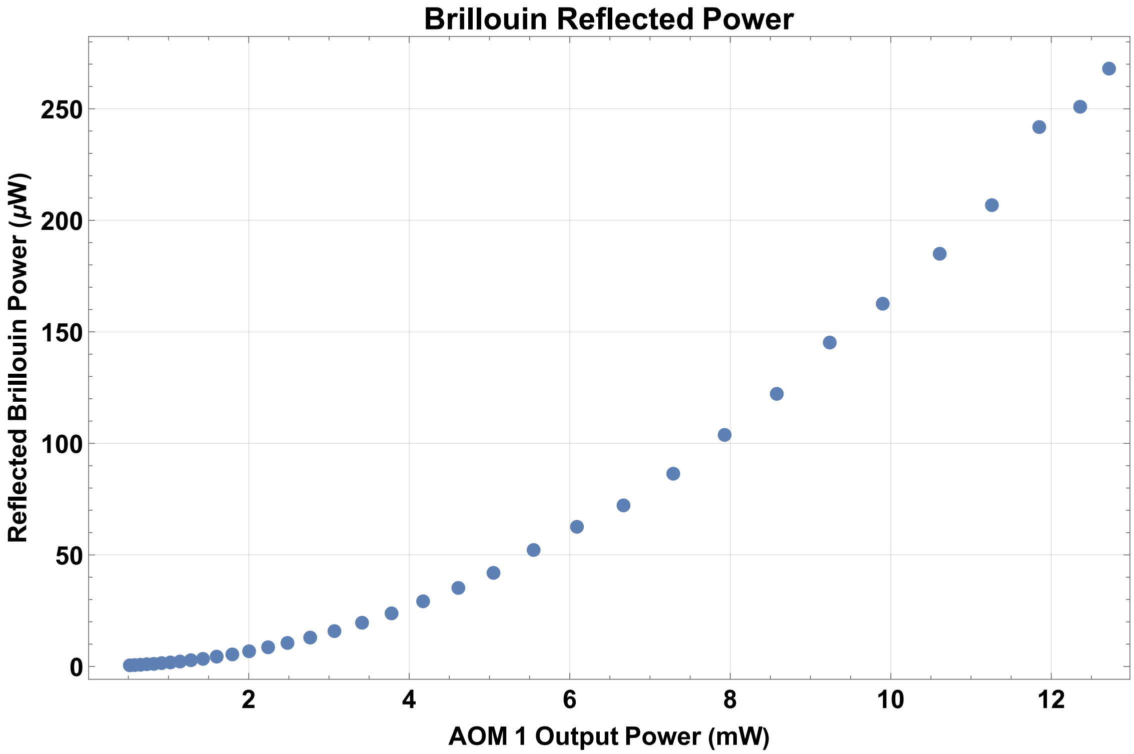

Figure 3.3: This graph shows the back reflected optical power vs the input optical power. The light going into the long fiber was coming directly out of AOM 1 to control how much power was allowed through. The remote end of the fiber was disconnected and a power meter was placed at the location of the in loop photodiode. These powers are converted to their real value after passing back through AOM 1 and the splitter.

Chapter 4

Active Phase Cancellation

4.1 Method

To perform the phase cancellation we use a combination of a phase locked loop (Phasemeter) and a PID Controller to send a signal to the VCO. First we send the out of loop photodiode signal into the Phasemeter which essentially acts as a phase locked loop detecting the accumulated phase of the signal. We then set a scaling to tell the Phasemeter how many volts it should output per cycles gained. A positive cycle count means a positive output voltage while a negative cycle count means a negative output voltage. We have determined the optimal scaling to be 5 $\frac{mV}{cyc}$ based on testing of the gain settings with the error signal outputted.

This signal is then sent to the PID Controller where Proportional and Integral gain is applied to this signal. The P gain was set to 20 dB and the Integral corner frequency was set to 100 Hz to get the best lock without high frequency oscillations. This PID Controller signal was then outputted from the Moku to the VCO (Rigol Function Generator) where FM was performed.

There is however a limit on how high of a frequency we can perform feedback on. Because of the length of the fiber, light takes a significant amount of time to travel the length of it (0.4 ms for 80 km) which therefore means there will be a delay on the signal we perform feedback on going into the fiber to the signal we receive coming out of the fiber. This cutoff frequency is calculated to be around 625 Hz for this length of fiber.

4.2 Phase Evolution

After discussing the noise in the system, we can now look at the phase measurements obtained from the out of loop photodiode. Like explained in 2, the Phasemeter on the Moku is able to determine the phase accumulated by the interferometer beatnote. When no active feedback is being performed to cancel the phase from the long fiber, the phase over time of this signal quickly accumulates excursions of several thousand cycles because of environmental perturbations which can be seen in Figure 4.1. This can range from acoustic vibrations from wind or cars driving by to thermal expansion from a changing lab or outdoor temperature.

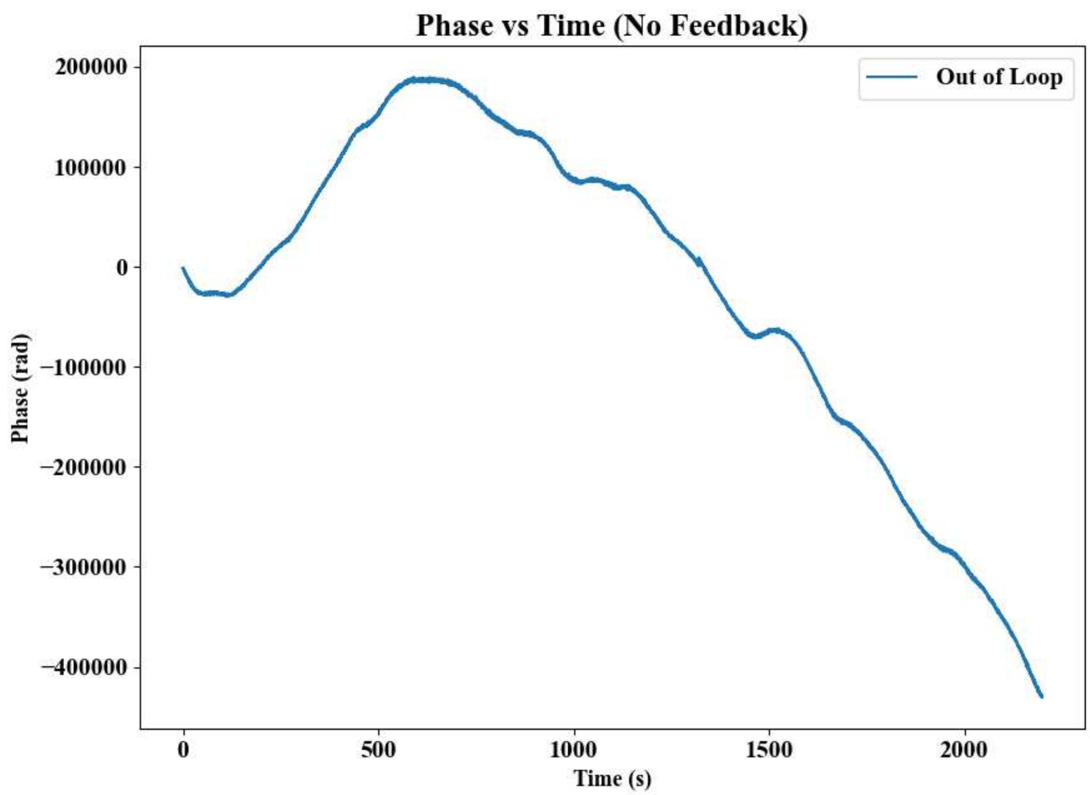

Figure 4.1: This was a 35 min run where the x axis is the time elapsed in seconds and the y axis are the amount of accumulated cycles. Over a short amount of time the signal can dephase by >600,000 radians.

This is in stark contrast to when the active phase cancellation is enabled. When the feedback is enabled, the part of the fiber up to the 9:1 splitter at AOM 2 is stabilized meaning there is a small length of fiber from there to the out of loop photodiode which can still show cycle drifts on the out of loop signal. This is seen in Figure 4.2 where the in loop signal is constant around 0 radians while the out of loop signal drifts several hundred radians over the course of several hours.

This out of loop drift is something we can attempt to remedy by isolating the fiber setup better. This can be done by covering the setup to eliminate noise from stray air currents in the lab as well

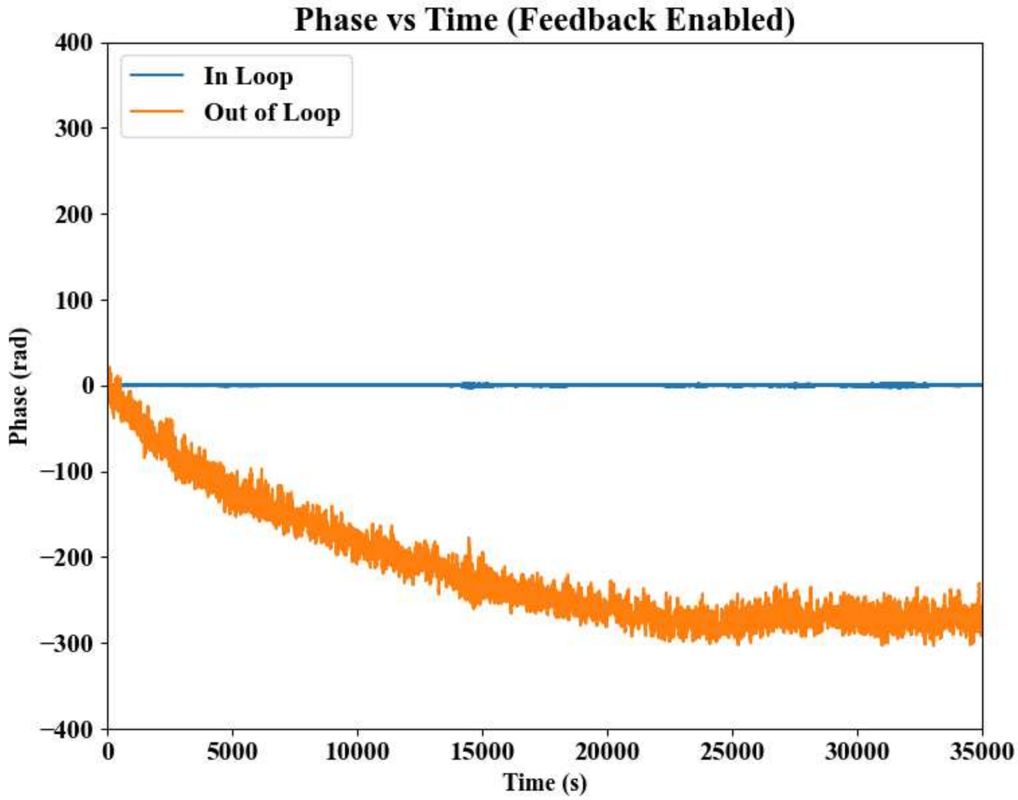

Figure 4.2: This run was 12 hours with the phase cancellation feedback enabled but was truncated at 9.7 hours (35,000 s). The out of loop signal drifts several hundred radians over the course of the run due to thermal instability of the un-stabalized portion of the fiber.

as help temperature stabilize the fibers so no thermal expansion occurred. Over night when this data was taken, the temperature in the lab stabilized and stray air currents were at a minimum due to no one being in the lab which is why at around 20,000 seconds the phase levels out at a constant value (around -300 radians).

4.3 Power Fluctuations

To be able to detect these beatnotes on the Phasemeter they need to be at significant enough power as to keep track of their frequency. This is not something that can be kept constant as this setup utilizes non polarization maintaining fiber. This means that over time, the polarization of the light in the fiber will drift due to those same environmental factors. The squeezers are there to match the polarization from the reference arm to the polarization of the light coming from the long fiber. In the phase over time plot from the run shown in Figure 4.2, at around 36,000 s the out of loop signal phase jumps abruptly which is then shown in Figure 4.3.

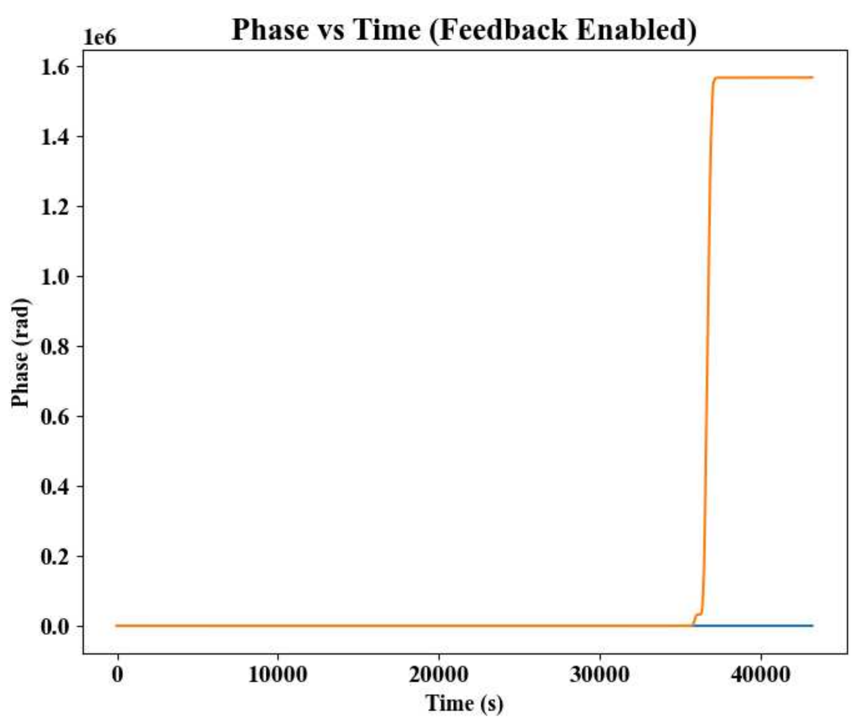

Figure 4.3: This full run was 12 hours with the phase cancellation feedback enabled.

This is due to the beatnote from the out of loop photodiode shrinking too much in power (by about 10 dB). The amount of time this takes is about 10 hours so a readjustment of the squeezers sometime before this is necessary so the Phasemeter does not lose track of the beatnote. Similar power degradations were also observed by Foreman and Ludlow [12] with a fiber of 32km where they observed a decrease in power by 5-10 dB over the course of 1 day.

Chapter 5

Feedback Results

5.1 Phase Power Spectral Density

A Power Spectral Density (PSD) is used to characterize the spectral distribution of the observed fiber phase noise. The trend is typically low frequencies have higher phase noise since many environmental drifts happen at very low frequencies and higher frequency phase noise tends to drop off quickly. PSDs were obtained by sending the output of the Phasemeter to the Spectrum Analyzer. The SA had y axis units of $\frac{V^2}{Hz}$ while the signal being sent from the Phasemeter was in units of $\frac{V}{cyc}$. A quick conversion can by done by squaring the the scaling of the Phasemeter then dividing the y-axis unit on the SA by this squared value. This then gets you units of $\frac{rad^2}{Hz}$. The out of loop Phase PSD for both feedback enabled and disabled is shown in Figure 5.1.

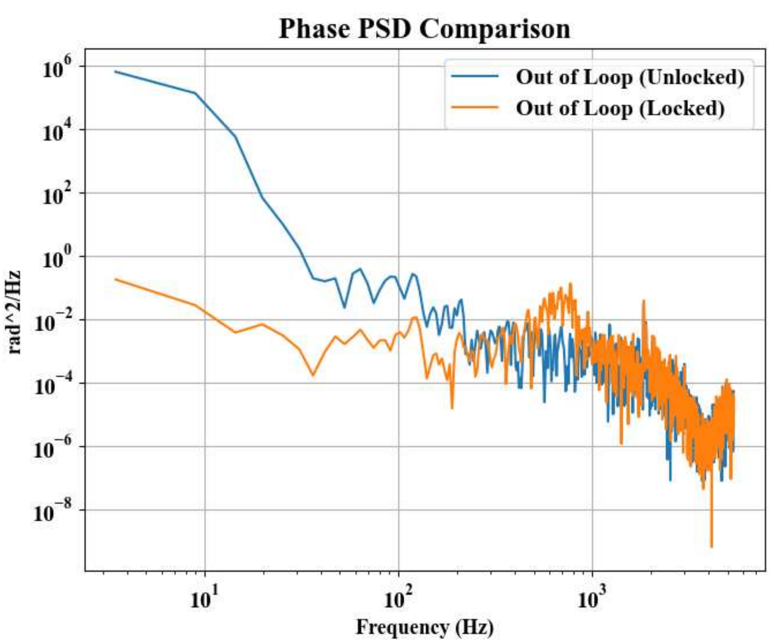

Figure 5.1: The range of this spectra was 1 Hz to 8 kHz. The phase cancellation reduces phase noise below 625 Hz by a factor of 10^6 while keeping the same noise floor for both the unlocked (feedback disabled) and locked (feedback enabled) spectra the same after 625 Hz.

The peak at around 625 Hz is the servo bump which only shows up in the plot where feedback is enabled. The frequency this peak is at depends on the delay time frequency the light encounters to travel through the long fiber which was shown to be 625 Hz. All frequencies to the right of the servo bump are unaffected by feedback which is why they display no change when it is enabled.

This is in excellent agreement with several other similar experiments. This includes a 146 km fiberlink experiment from PTB [13] which had a decrease of the same magnitude and a 251 km link at NIST Boulder [14] which had a decrease of 10^5. The phase noise at low frequencies in this case were an order of magnitude or two higher than the Phase noise in these two experiments because of the unstabilized portion of fiber from our out of loop photodiode.

5.2 Allan Deviations

An Allan Deviation (ADEV) is meant to show the fractional frequency instability at different amounts of averaging time. For frequency transfers using fibers, it is imperative that at longer averaging times the frequency uncertainty decreases.

The ADEV data was acquired by using the Phasemeter on the Moku to count the frequency of both the in loop and out of loop photodiodes. By setting a sampling rate, one can pick how small of an averaging time to start at. In this case, a sampling rate of 37 $\frac{sam}{s}$ was used which yields the smallest averaging time of about 30 ms. Using the AllanTools package in python, an Allan Deviation can be extracted from this frequency vs time data. In Figure 5.2 the Allan Deviation for the in loop and out of loop signal is plotted for the 80 km fiber. The slopes of these runs average down as $\frac{1}{\tau}$ which corresponds to white phase noise being the dominate noise source shown in both Dix-Matthews and Jie Liu frequency transfers [15, 16].

Since the out of loop signal is affected by the unstabilized portion of fiber, the more gradual slope appears since there are fluctuations of the phase on that time scale. We see a fractional frequency uncertainty of 7×10^{-15} and 5×10^{-15} at 1 s for the out of loop and in loop signal respectively. We then average down to 7×10^{-18} and 1.9×10^{-18} at 3600 s for the out of loop and in loop signal respectively. This decreases following white phase noise scaling of $\frac{1}{t}$ which is in line

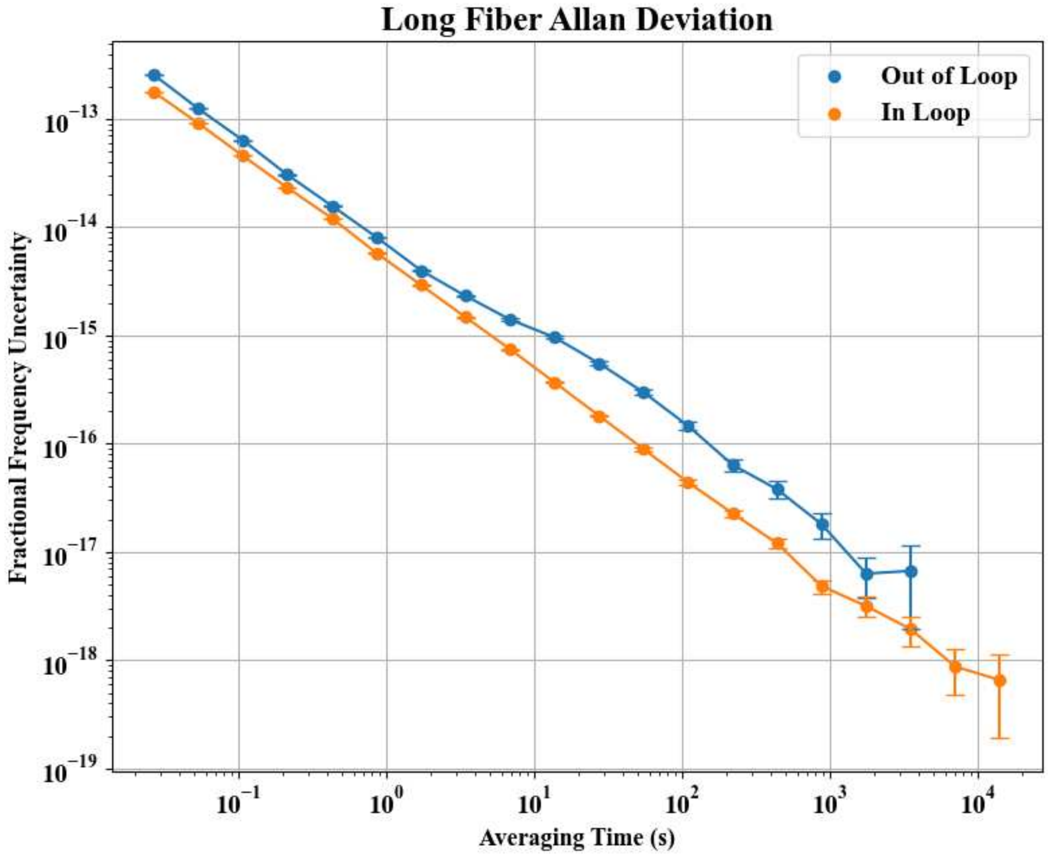

Figure 5.2: This data was taken over the course of 12 hours so an averaging time of 10,000 s could be achieved.

with Grosche and Foreman who achieved 1×10^{-18} [13] and 5×10^{-19} [12] at 3600 s respectively. Far longer fiber links are also showing similar uncertainties with similar averaging times as seen in France with Lopez [17] as well as Ellis with discussion on how to scale distance for frequency transfer for future projects [18].

Along with this result, an Allan Deviation of the noise floor of the interferometer was taken where the 80 km fiber was replaced with a 0.5 m fiber. This represent the limit of how well this setup can do and is shown in Figure 5.3.

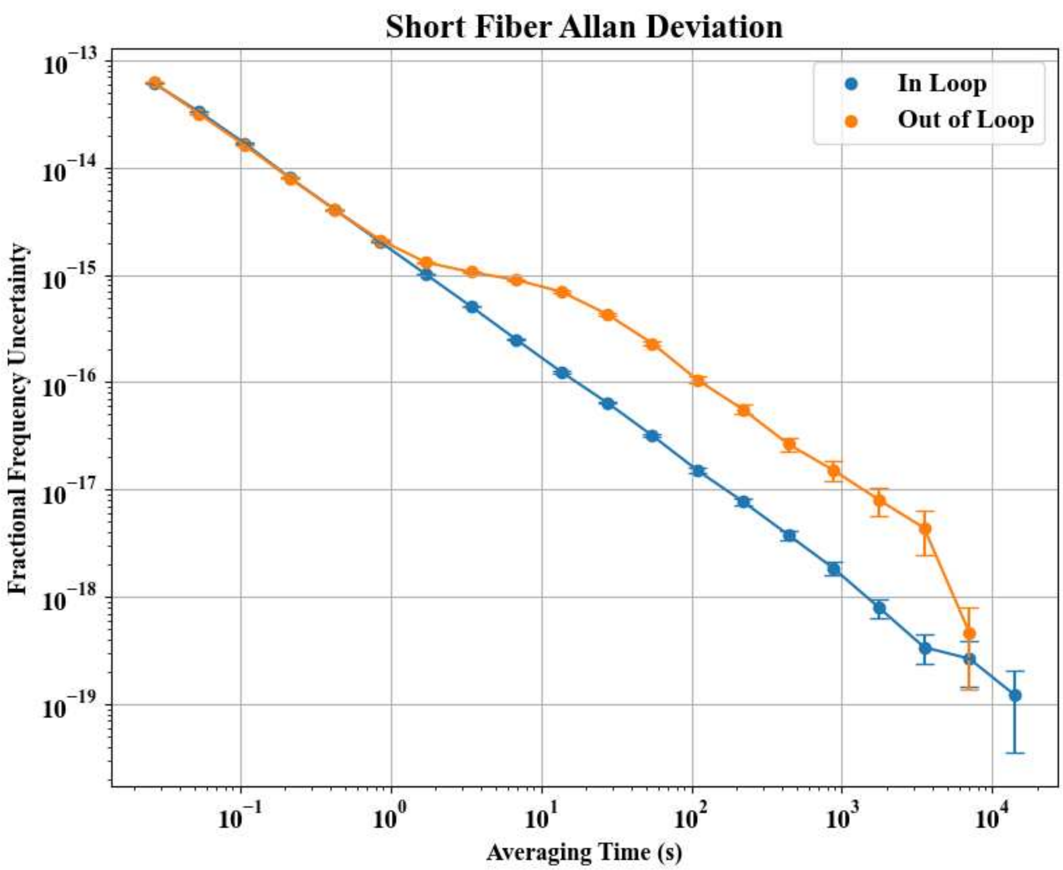

Figure 5.3: This run was 12 hours. The flatter portion of the out of loop signal in the same averaging time range is more pronounced now since the dominant source of noise is now that unstabilized portion of fiber.

The best this interferometer can do is average down to 3×10^{-19} at 3600 s looking at the in loop signal. The noise floor here is an order of magnitude higher than what Grosche and Foreman find. A prime candidate for the discrepency between the in loop and out of loop signals is of the EDFA. Since the EDFA in this setup is unidirectional, the out of loop photodiode signal potentially sees different phase noise than the in loop signal and therefore overtime strays from each other as described in [19].

Chapter 6

Conclusion and Future Direction

We have been able to show the efficacy and reliability of an active phase cancellation scheme for an 80 km long fiber. Using a two AOM and spliced fiber scheme a fractional frequency uncertainty for a 1550 nm laser was measured to be 7×10^{-15} at 1 s and 7×10^{-18} at 3600 s with the out of loop signal. The trend of averaging down at a rate of $\frac{1}{t}$ is in agreement with both Grosche at PTB [13] and Foreman at NIST [12]. The order of magnitude offset discrepancy with our allan deviations can be explained by poor isolation of the unstabilized part of the fiber connected to the out of loop photodiode caused by thermal fluctuations of the room and acoustic vibrations in the table or in the air. Construction of a box that is vibrationally isolated from the table would aim to address these problems and should be considered for future improvements. Consideration of power stabilization in the system whether that is through polarization maintaining fibers or polarization scrambling optics would allow for longer operation times without manual input. At the moment this limit is less than 10 hours but with proper stabilization this can get significantly higher to more than several days. Our findings show that with the current setup frequency comparison is not only feasible but reliable and robust, and with few simple modifications would be much more effective with shorter averaging times and smaller fractional frequency uncertainty at 1 s. A long term goal to come after this project is to create an optical clock network between CSU and JILA/Boulder to improve measurements on frequency ratios [20]. These measurements would help make a case for redefining the second away from a microwave transition in Cesium to an optical transition in a single ion clock.

Bibliography

[1] W. H. Oskay, S. A. Diddams, E. A. Donley, T. M. Fortier, T. P. Heavner, L. Hollberg, W. M. Itano, S. R. Jefferts, M. J. Delaney, K. Kim, F. Levi, T. E. Parker, and J. C. Bergquist. Single-atom optical clock with high accuracy. *Phys. Rev. Lett.*, 97:020801, Jul 2006.

[2] Rosenband T, Hume DB, Schmidt PO, Chou CW, Brusch A, Lorini L, Oskay WH, Drullinger RE, Fortier TM, Stalnaker JE, Diddams SA, Swann WC, Newbury NR, Itano WM, Wineland DJ, and Bergquist JC. Frequency ratio of al+ and hg+ single-ion optical clocks metrology at the 17th decimal place.

[3] Gretchen K Campbell, Andrew D Ludlow, Sebastian Blatt, Jan W Thomsen, Michael J Martin, Marcio H G de Miranda, Tanya Zelevinsky, Martin M Boyd, Jun Ye, Scott A Diddams, Thomas P Heavner, Thomas E Parker, and Steven R Jefferts. The absolute frequency of the 87sr optical clock transition. *Metrologia*, 45(5):539, sep 2008.

[4] Baumann E. Fortier T. 20 years of developments in optical frequency comb technology and applications.

[5] N. Huntemann, C. Sanner, B. Lipphardt, Chr. Tamm, and E. Peik. Single-ion atomic clock with 3×10^{-18} systematic uncertainty. *Phys. Rev. Lett.*, 116:063001, Feb 2016.

[6] Bauch A., Achkar J., Bize S., Calonico D., Dach R., Hlavac R., Lorini L., Parker T., Petit G., Piester D., Szymaniec K., and Uhrich P. Comparison between frequency standards in europe and the usa at the 10^{-15} uncertainty level, metrologia.

[7] Liliana Girsang, Angelin Napitupulu, Irmalia Simorangkir, and Dahlan Sitompul. Indepth study of single mode optical fibre. 12 2021.

[8] Singlemode vs multimode optical fibre.

[9] B. Moslehi. Analysis of optical phase noise in fiber-optic systems employing a laser source with arbitrary coherence time. *Journal of Lightwave Technology*, 4(9):1334–1351, 1986.

[10] Ji ří Minář, Hugues de Riedmatten, Christoph Simon, Hugo Zbinden, and Nicolas Gisin. Phase-noise measurements in long-fiber interferometers for quantum-repeater applications. *Phys. Rev. A*, 77:052325, May 2008.

[11] Andrey Kobyakov, Michael Sauer, and Dipak Chowdhury. Stimulated brillouin scattering in optical fibers. *Advances in Optics and Photonics*, 2:1–59, 12 2009.

[12] Seth M. Foreman, Andrew D. Ludlow, Marcio H. G. de Miranda, Jason E. Stalnaker, Scott A. Diddams, and Jun Ye. Coherent optical phase transfer over a 32-km fiber with 1s instability at 10^{-17}. *Physical Review Letters*, 99(15), Oct 2007.

[13] Gesine Grosche, Osama Terra, Katharina Predehl, Ronald Holzwarth, Burghard Lipphardt, F Vogt, Uwe Sterr, and Harald Schnatz. Optical frequency transfer via 146 km fiber link with 10^{-19} relative accuracy. *Optics letters*, 34:2270–2, 09 2009.

[14] N Newbury, Paul Williams, and W Swann. Coherent transfer of an optical carrier over 251 km. *Optics letters*, 32:3056–8, 12 2007.

[15] Gozzard D.R. et al. Dix-Matthews B.P., Schediwy S.W. Point-to-point stabilized optical frequency transfer with active optics.

[16] Jie Liu, Jing Gao, Guanjun Xu, Dongdong Jiao, Long Chen, Linbo Zhang, Haifeng Jiang, Ruifang Dong, Tao Liu, and Shougang Zhang. Experimental study on optical frequency transfer via communication fibers. pages 1–3, 2014.

[17] Olivier Lopez, Adil Haboucha, Bruno Chanteau, Christian Chardonnet, Anne Amy-Klein, and Giorgio Santarelli. Ultra-stable long distance optical frequency distribution using the internet fiber network. *Opt. Express*, 20(21):23518–23526, Oct 2012.

[18] Jennifer L. Ellis, Martha I. Bodine, William C. Swann, Sarah A. Stevenson, Emily D. Caldwell, Laura C. Sinclair, Nathan R. Newbury, and Jean-Daniel Deschênes. Scaling up frequency-comb-based optical time transfer to long terrestrial distances. *Phys. Rev. Appl.*, 15:034002, Mar 2021.

[19] H. Jiang, F. Kéfélian, S. Crane, O. Lopez, M. Lours, J. Millo, D. Holleville, P. Lemonde, Ch. Chardonnet, A. Amy-Klein, and G. Santarelli. Long-distance frequency transfer over an urban fiber link using optical phase stabilization. *J. Opt. Soc. Am. B*, 25(12):2029–2035, Dec 2008.

[20] Boulder Atomic Clock Optical Network, Collaboration, :, Kyle Beloy, Martha I. Bodine, Tobias Bothwell, Samuel M. Brewer, Sarah L. Bromley, Jwo-Sy Chen, Jean-Daniel Deschênes, Scott A. Diddams, Robert J. Fasano, Tara M. Fortier, Youssef S. Hassan, David B. Hume, Dhruv Kedar, Colin J. Kennedy, Isaac Khader, Amanda Koepke, David R. Leibrandt, Holly Leopardi, Andrew D. Ludlow, William F. McGrew, William R. Milner, Nathan R. Newbury, Daniele Nicolodi, Eric Oelker, Thomas E. Parker, John M. Robinson, Stefania Romisch, Stefan A. Schäffer, Jeffrey A. Sherman, Laura C. Sinclair, Lindsay Sonderhouse, William C. Swann, Jian Yao, Jun Ye, and Xiaogang Zhang. Frequency ratio measurements with 18-digit accuracy using a network of optical clocks. 2020.