HYDRAULICS OF WELLS

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and

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CER47-52DFP42

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ACKNOWLEDGEMENTS

Dr. Naji Kadir, Paul Riley, and Stanley Marsh, while graduate students at Utah State Agricultural College, rendered valuable assistance in checking the manuscript and in solutions of the problems.

Special acknowledgement is given to C. H. Zee, a graduate student in irrigation engineering, and to Dr. Robert O. Bock, associate professor of physics, both of Colorado A & M College, for making available unpublished data on the seepage face for gravity wells.

Prof. C. E. Jacob, University of Utah, examined the manuscript and encouraged the authors in their plans for its publication.

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SYNOPSIS

This bulletin is concerned largely with new developments for the study of steady flow or equilibrium ground-water flow conditions. Actually it is difficult to develop and maintain steady flow of ground water. The derivation of nonequilibrium formulas for ground-water flow by Theis (1935), Jacob (1947), and others in recent years is recognized as an important contribution to ground-water hydraulics.

A brief summary of the literature on the hydraulics of steady-flow wells is presented. Several deficiencies of existing formulas and practices are discussed and suggestions for improvement made.

For unconfined ground-water flow into wells, the existence of the seepage surface at the well and the relationship of the magnitude of such a surface to the other elements of the well system are but little understood. Hansen in 1949 developed dimensionless parameters which enabled specific test data to be plotted in general terms. His work has been extended here by adding both the theoretical solutions made by Yang in 1948 using "relaxation" methods, and the unpublished data obtained by Zee in 1951 using a combination of the electrical and membrane analogies. These solutions enable the investigator to estimate the magnitude of the seepage face for a wide range of specific cases of unconfined flow into a well.

Dupuit's classical solution for unconfined ground-water flow into a well is based on the assumption that all of the discharge flows horizontally into the zone of influence from outside the region under consideration. For a drainage well to relieve lands waterlogged by surface irrigation, flow enters the region of influence by vertical percolation of water falling on or applied to the overlying land surface so that the flow toward the well increases as it is approached. A theoretical solution is presented herein for this condition.[1] The geometry of the region of influence for this type well of radius, r_w, is the same as for a well of the Dupuit type having the same drawdown but with a transformed radius, r_w', equal to $r_w e^{n/2}$ in which n is the ratio of the discharge originating from vertical percolation to the total discharge of the well. With this transformation, the seepage surface for the unconfined, vertically-recharged well may be found by using the same curves as for the horizontally-recharged well.

Critical evaluation of the commonly-used formulas for confined and unconfined ground-water flow into wells leads to the conclusion that they are indeterminate. They are made determinate in practice by introducing the radius of influence, r_e, usually as an arbitrary value, a concept which is somewhat vague at best, if not illogical.

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1. See list of symbols and definitions, pages 43-48.

The variables in these formulas include the soil permeability, k, the drawdown, D_w, the thickness of water-bearing material, t, the radius of the well, r_w, the discharge, Q, and the radius of influence, r_e. The independent variables are D_w, k, t, and r_w. Q and r_e are dependent and mutually interdependent. An additional independent variable, q, describing the unit rate at which the influence cone is replenished with water from an external source, is necessary to complete the analysis. These five independent variables are sufficient to determine fully the flow into a well for any particular system.

The discharge of the well may be expressed in the dimensionless form, Q/kr_w^2, called the discharge number. Dimensional analysis shows that the discharge number is functionally related to the dimensionless quantities D_w/r_w, t/r_w, q/k. By introducing certain approximations specific functional relationships between these quantities are developed for particular types of well systems. The quantity Q/kr_w^2 is of special significance since, as Hansen (1949) has shown, it equals the ratio of the Froude and Reynolds numbers and therefore expresses the relative importance of the viscous and gravity forces. The discharge number also describes, in relative terms, the geometry of the flow in the medium surrounding the well.

Care must be used in calculating the effectiveness as defined by Wenzel (1942) for wells in unconfined-flow systems or values much too small will result. If the piezometric head is measured at the bottom of the permeable stratum instead of at the water table, reasonable results may be expected. Normal procedure results in considering the head represented by the height of the seepage face as lost head, so that wells in unconfined systems normally appear to be somewhat less effective than similar wells in confined systems. Actually, other things being equivalent, a well in an unconfined system is inherently somewhat more efficient in utilizing available specific energy than a similar well in a confined system.

Each of the five zones for unconfined flow, four of which apply to confined flow, have their own important distinguishing characteristics. Knowledge of the peculiarities of each zone will enable the practicing engineer to avoid many of the pitfalls encountered in a blind use of theoretical formulas.

Seven illustrative numerical examples are given. Dimensionless quantities presented herein may be used for any system of units providing the same system is used throughout.

HYDRAULICS OF WELLS

Dean F. Peterson, Jr.,[2] Orson W. Israelsen,[3]
and Vaughn E. Hansen [4]

INTRODUCTION

Pumping ground water for irrigation and drainage and also for domestic water supplies and industrial purposes in the 17 western arid-region states, is of major importance to the public welfare. Such remarkable progress has been made in pumping ground water during the past decade that each year, according to recent estimates, nearly 20 million acre-feet are pumped from the ground-water reservoirs of the western states.

California farmers and industries pump more than 10 million acre-feet each year--53 percent of the volume for the 17 states--Arizona nearly 18 percent; New Mexico and Texas each about 9 percent; Colorado and Idaho together about 9 percent; and the other 11 western states combined only 4 percent. Utah's ground-water reservoirs are used little as yet; the volume of pumped water each year being only 1 percent of the volume pumped in the 17 states. In some Utah areas pumping ground water for drainage purposes is even more important than pumping for irrigation. When the water can be pumped for drainage and used for irrigation, wells become of prime importance.

Western progress in pumping has developed numerous problems with reference to ground-water reservoir capacities, annual recharge of pumped water supplies, pumping lifts, and costs. Public interest in the hydraulics of wells has substantially increased because of the basic importance of this branch of science to water economy in arid regions.

Darcy's Law

The flow of ground water under conditions of saturation has been widely discussed by many authors including Babbitt and Caldwell (1948), Casagrande (1937), Gardner and Israelsen (1940), Hansen (1949), Jacob (1947), Kirkham (1940), Hubbert (1940), Muskat (1946), Taylor (1948), and Wenzel (1942). Through isotropic soils the velocity of flow is expressed by the empirical equation developed by Darcy in 1856,

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$$V = k \frac{h}{l} \tag{1a}$$ [5]

In the vector form:

$$\vec{V} = k \text{ grad } h = k \nabla h, \tag{1b}$$

and in the differential form:

$$V = k \frac{\partial h}{\partial s} . \tag{1c}$$

The ratios h/l, ∇h, and $\partial h/\partial s$ each represent the hydraulic gradient and V is the velocity of flow.

Soils are usually stratified and therefore the permeability varies with the direction of flow. Further, both sedimentation and pressure of overlying soil materials cause flat particles to be orientated with their longest dimensions horizontal, resulting in a nonisotropic condition with respect to permeability, even though ordinary stratification is not present. Often, however, flow will be parallel to one of the principal directions of permeability, and in this instance k in equation 1 may be treated as constant if measured in the direction of flow. In this report only isotropic cases are treated.

Equation 1 simply states that the direction of flow is parallel to the direction of greatest hydraulic gradient and that the volume of discharge through unit area per unit of time is proportional to the hydraulic gradient.

Equation of Continuity and Laplace's Equation

If flow is steady and the fluid is incompressible, from the law of conservation of matter the net flow into and out of any elementary volume of space is zero. This may be expressed mathematically by the equation of continuity:

$$Q = A V \tag{2a}$$

or in the differential form

$$\frac{\partial V_x}{\partial X} + \frac{\partial V_y}{\partial Y} + \frac{\partial V_z}{\partial Z} = 0 \tag{2b}$$

where X, Y, and Z are the Cartesian coordinates; or in the vector form:

$$\text{div } \vec{V} = 0 \tag{2c}$$

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5. Darcy's equation is sometimes written with a minus sign on the right, $V = -k\, h/l$, to denote that the flow is in the direction of decreasing head.

Substituting in equation 2 the velocity, V, from equation 1 gives Laplace's equation which may be applied to isotropic soils and may be written in the vector form:

$$\nabla^2 h = 0 \tag{3}$$

or in the form:

$$\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial Y^2} + \frac{\partial^2 h}{\partial Z^2} = 0 \tag{4}$$

For steady flow of ground water toward a well, cylindrical coordinates are more convenient, and equation 4 may be written:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{1}{r^2}\frac{\partial^2 h}{\partial \theta^2} + \frac{\partial^2 h}{\partial Z^2} = 0 \tag{5}$$

where r, θ, and Z are the cylindrical coordinates.

The cylindrical components of velocity are given by:

$$V_r = k\frac{\partial h}{\partial r},\ V_\theta = \frac{k}{r}\frac{\partial h}{\partial \theta},\ \text{and } V_z = k\frac{\partial h}{\partial Z} \tag{6}$$

If a mathematical function or expression satisfying equation 3 and reducing to the known values of h and V at the boundaries can be found, this function will describe the hydraulic head, h, at every point in the region of flow.

FLOW OF GROUND WATER INTO WELLS

Confined-Flow Systems

If a permeable water-bearing stratum is bounded above and beneath by impermeable layers (fig. 1), and if the drawdown in the well is less than the vertical distance from the static water table to the top of the permeable stratum, the flow is designated as confined flow.

Simple Case Solution

For a simple case the confined system may be readily solved. Assume

(1) the thickness of the permeable stratum, t, is uniform;
(2) the permeable stratum is horizontal;
(3) the well penetrates the entire depth of the permeable stratum;
(4) the elevation of the piezometric surface at the uniform maximum radial distance of the region of influence, r_e, has the constant value, h_e.

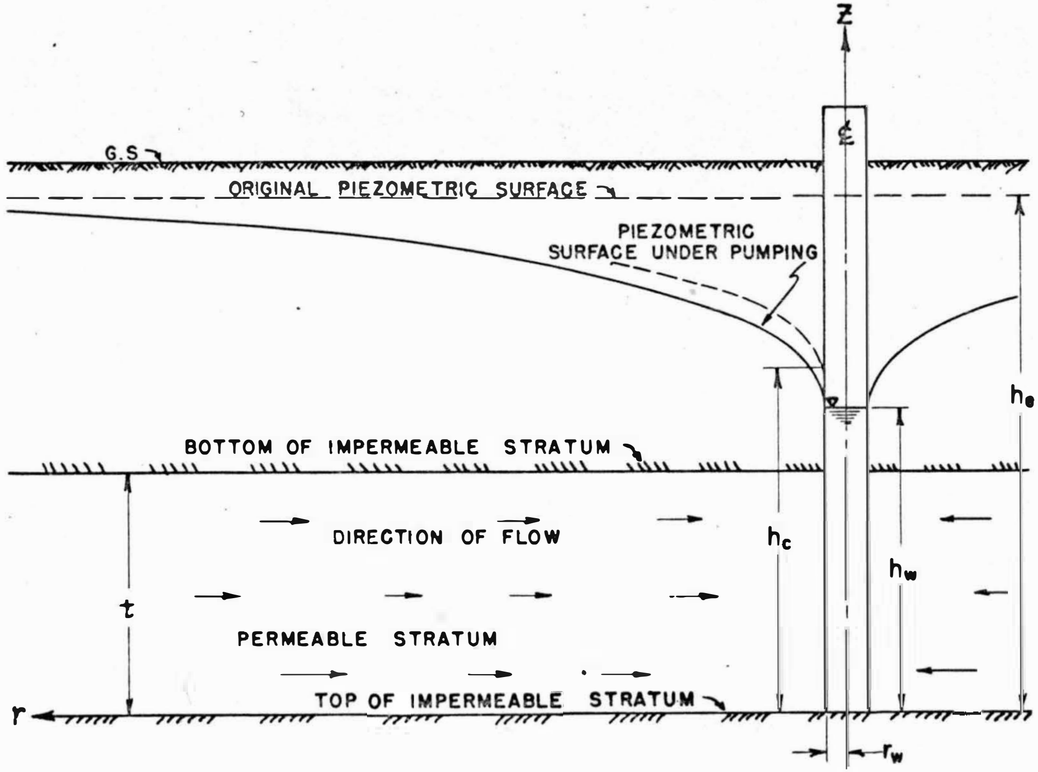

Fig. 1. Flow of confined ground water to a well

Confined flow under these assumptions is horizontal, radial, and symmetrical. There is no component of velocity in either the direction z or θ, and thus $\partial h/\partial z$ and $\partial h/\partial \theta$ and all higher partial derivatives of h with respect to z and θ are equal to zero. Consequently, equation 5 becomes

$$\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0 \qquad (7)$$

with boundary conditions as follows: at $r = r_w$, $h = h_w$; and at $r = r_e$, $h = h_e$.

Equation 7 is in the form of Euler's equation so that when integrated and boundary values substituted, the result is

$$\frac{h - h_w}{\ln r/r_w} = \frac{h_e - h_w}{\ln r_e/r_w} \qquad (8)$$

from which

$$h = h_w + \frac{h_e - h_w}{\ln r_e/r_w} \ln \frac{r}{r_w} \qquad (9)$$

At a distance, r, from the well, combining the equation of continuity in the form Q = AV with that of Darcy yields:

$$Q = 2\pi rt\ k \frac{dh}{dr} \qquad (10a)$$

Differentiating equation 9 with respect to r, and substituting in equation 10a the value for dh/dr thus obtained, gives

$$Q = 2\pi t\ k \frac{h_e - h_w}{\ln r_e/r_w} \qquad (10b)$$

The same equation can be obtained by integrating equation 10a between the limits r_e and r_w and h_e and h_w.

Eliminating $(h_e - h_w)$ from equations 9 and 10b and solving for $(h - h_w)$ gives

$$h - h_w = \frac{Q}{2\pi tk} \ln \frac{r}{r_w} \qquad (11)$$

Likewise, this same equation can be obtained by integrating equation 10a between the limits r and r_w and between h and h_w. The total drawdown, D_w, which equals $(h_e - h_w)$ as obtained from equation 10b is

$$D_w = \frac{Q}{2\pi tk} \ln \frac{r_e}{r_w} \qquad (12)$$

Theoretical Difficulties

Equation 12 presents some theoretical difficulties which are of considerable practical importance. In order to write equation 12 the investigator must assume that the piezometric surface was level at elevation h_e prior to pumping and that its elevation is unaffected at distances in excess of r_e. Thus, h_e becomes also the elevation of the static water level in the well. These assumptions imply no flow toward the well from beyond the distance r_e, a condition which cannot exist if the flow into the well is steady. The quantities D_w, k, t, and r_w are independent variables, but Q and r_e are mutually dependent. Equations 8 and 10a are, therefore, indeterminate except by pumping experiment to determine Q, even though D_w, k, t, and r_w may be known. They may be used to determine k in the field if measurements of h are taken at one or more values of r greater than r_w. By assuming r_e one may estimate Q for a particular value of D_w by using equation 12 if k, r_w, and t are known; conversely k may be estimated if Q, r_w, and t are known. Selection of r_e determines the shape of the cone of depression and its profile in a vertical plane. These equations ignore a major factor in well hydraulics; that is the rate at which the region of influence is replenished. In practice an attempt is made to overcome this deficiency by assuming some arbitrary value, r_e. Actually the concept of a radius of influence is fallacious and misleading in practical well hydraulics.

Unconfined-Flow Systems and the Seepage Surface

If there is no impermeable stratum overlying the permeable water-bearing aquifer (fig. 2), or if the drawdown is greater than the depth from the static water level to the bottom of the confining stratum, the flow system may be classified as unconfined or partially confined. The problem of developing a rational equation to find the drawdown is much more complicated because the position of the boundary of the region of flow is unknown. In addition to this difficulty, the top surface of the flow region intersects the well at an elevation somewhat greater than the elevation of the water in the well. The necessity for the existence of the resulting seepage surface of height AB (fig. 2) between the water surface in the well and the free surface of the flow region AE, is demonstrated by Muskat (1946). This may readily be inferred from a consideration of Kozeny's solution for a porous dam on impermeable foundation reported by Taylor (1948) and others which is the corresponding case in Cartesian coordinates and in which the existence of the seepage surface is mathematically demonstrated. The universal economy of nature invariably results in dynamic systems involving minimum expenditure of energy. The development of the free surface at an elevation above the water level in the well is in accordance with this principle. A real seepage surface always exists around a well in an unconfined system while pumping. As the drawdown in the well increases the seepage surface approaches a maximum as h_w approaches zero.

For the symmetrical gravity or unconfined system (Wyckoff 1932) the following boundary conditions apply (see fig. 2).

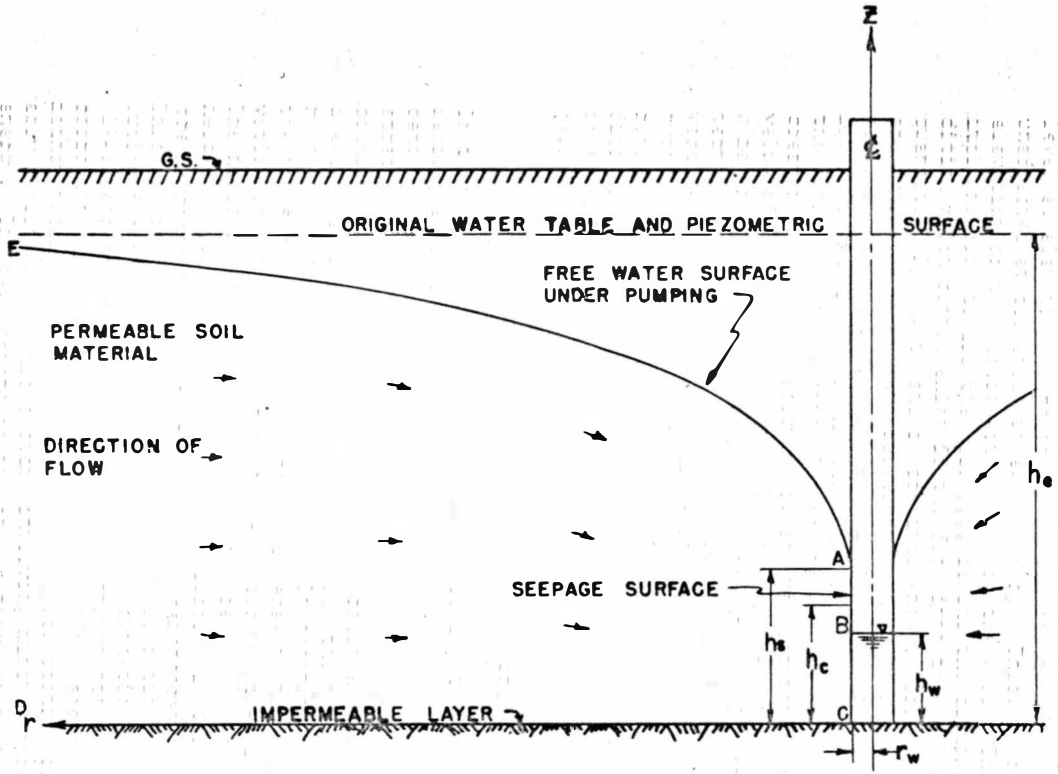

Fig. 2. Flow of unconfined ground water to a well in permeable soils overlying impermeable material

(1) $h = h_w$ along BC ($r = r_w$ and $0 \leq z \leq h_w$)
(2) $h = z$ along AB ($r = r_w$ and $h_w \leq z \leq h_s$)
(3) $\partial h/\partial z = 0$ along CD ($z = 0$ and $r_w \leq r \leq r_e$)
(4) $h = h_e$ along DE ($r = r_e$ and $0 \leq z \leq h_e$)
(5) $h = z$, $p = 0$, $\partial h/\partial N = 0$ at the piezometric surface, AE, where $h_s \leq z \leq h_e$

At any point along the seepage surface, AB, the pressure head is zero (atmospheric) and the piezometric head equals the elevation head. This is also true along the free surface, AE. The component of velocity normal to the free surface, AE, must be zero which implies $\partial h/\partial N = 0$ where N denotes a direction normal to the surface AF. Flow along AE must become parallel in direction to the vertical surface of seepage at point A (fig. 2) (Muskat 1946).

Dupuit Solution

Dupuit, in 1863, introduced the assumptions that flow through any concentric cylinder at radius, r, was horizontal and that the hydraulic gradient at all points on the cylinder surface was equal to the slope of the free surface at its intersection with the cylinder. Since, for steady flow, the entire discharge of the well must pass through each concentric cylinder at any radius, combining the equation of continuity with Darcy's equation yields:

$$Q = 2\pi rhk \frac{dh}{dr}\ ^{6/}$$

which may be integrated to obtain

$$\frac{Q}{\pi k} \ln r = h^2 + C$$

By introducing the boundary conditions at r_w, the constant may be evaluated to obtain

$$\frac{Q}{\pi k} \ln \frac{r}{r_w} = h^2 - h_w^2 \qquad (13)$$

or substituting $h = h_e$ when $r = r_e$

$$\frac{Q}{\pi k} \ln \frac{r_e}{r_w} = h_e^2 - h_w^2 \qquad (14)$$

Equation 14, while not rigorously correct, has been shown by experiment (Wyckoff 1932) to give correct values for discharge.

The equation for the free surface

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6. Equation 9 cannot be used here for evaluating dh/dr as done before since it is valid only for confined flow.

$$h^2 = \frac{h_e^2 - h_w^2}{\ln r_e/r_w} \ln \frac{r}{r_w} + h_w^2 \qquad (15)$$

is obtained by eliminating Q from equations 13 and 14.

The Dupuit solution for the position of the free surface gives quite accurate results at fairly large distances from the well but is incorrect in the neighborhood of the well where the assumptions made in developing equation 13 are less applicable. Perhaps the most serious objection to this solution is that the seepage surface, AB, is ignored. Like the solution for the confined case, equations 13, 14, and 15 are indeterminate except by experiment. Again the usual procedure is to introduce the radius of influence. The same objections are made to this device as for the confined case.

Recent Solutions and the Discharge Number

Yang (1949) and Hansen (1949) both report the 1932 experiments of Wyckoff, Botset, and Muskat (1932) using a sand tank and a sector model of a radial-flow system. These experiments show that equation 15 does not give the correct location of the free surface. On the other hand, the distribution of piezometric head on the surface of the underlying impermeable stratum, CD, (fig. 2) was found to be accurately represented by the Dupuit equation. Hansen (1949) demonstrates theoretically that this must be true. Neglecting flow through the capillary zone, Wyckoff, Botset, and Muskat found that the Dupuit equation gave the correct discharge. They noted the existence of the seepage face, but no expression for the position of the free surface was proposed.

Babbitt and Caldwell (1948), using both electrical and sand models, reached the same general conclusions as Wyckoff, Botset, and Muskat on the validity of the Dupuit equation. Babbitt and Caldwell plotted the percent of drawdown of the free surface at any distance, r, to the drawdown of the free surface at the well, $100(h_e - h)/(h_e - h_s)$, against the ratio r/r_e and found the shape of the free-surface curve to be independent of the physical dimensions of the system. The following equation was proposed for the position of the free surface:

$$h_e - h = \frac{2.3\ Q\ C_x}{\pi k h_e} \log \frac{r_e}{0.1\ h_e} \qquad (16)$$

where C_x is the ratio of the drawdown of the free surface at any distance, r, to the maximum possible drawdown when the well is discharging at the maximum.

Hansen (1949) proposed the empirical equation

$$C_x = -0.3 \log \frac{r}{r_e} \qquad (17)$$

for values of r/r_e greater than 0.05. Substituting equation 17 in equation 16 gives

$$(h_e - h) = \frac{0.69Q}{k\ h_e} \log \frac{r_e}{0.1\ h_e} \log \frac{r_e}{r} \qquad (18)$$

which indicates that the elevation of the free surface is a linear function of log r for any value of Q. It should be remembered that equation 18 is based upon data obtained near the well. Hence, the free surface can be inferred as being a linear function of log r only in the vicinity of the well.

The values of C_x given by Babbitt and Caldwell are expressed in terms of the drawdown of the free surface curve extended to the center of the well. Yang (1949) points out that the curve must theoretically become tangent to the well casing at an elevation of h_s above the bottom of the well and cautions against the use of equation 9 in the region close to the well.

Hansen (1949) conducted additional experiments using sand models and gave particular attention to correction for the effects of capillarity. He points out the desirability of expressing the constant of integration in terms of the radius of the well instead of the radius of influence which is an indefinite and hypothetical value at best. Hansen gave particular attention to the extent of the seepage surface and using dimensional analysis developed the relation

$$\frac{Q}{kr_w^2} = F\left(\frac{h_s}{r_w}, \frac{h_w}{r_w}\right) \tag{19}$$

The fractional parameters in equation 19 are dimensionless. Using available experimental data the curves shown by fig. 3 were developed.

Yang (1949) applied a relaxation method of numerical calculation proposed by Southwell (1940) to theoretical solutions of equation 3 for six particular cases. A great amount of time is required to complete a solution by this method which involves successive approximations.

THE SEEPAGE SURFACE FOR UNCONFINED SYSTEMS

By using the results of Yang's work, Hansen's curves may be extended over a much wider range of values of Q/kr_w^2. Since the Dupuit equation has been found to be correct for the calculation of discharge, though not for the free surface, one may write, by rearrangement of equation 14

$$\frac{Q}{kr_w^2} = \frac{\pi(h_e^2 - h_w^2)}{r_w^2 \ln r_e/r_w} \tag{20}$$

Equation 20 makes possible the calculations of the discharge number, Q/kr_w^2, from the theoretical computations of Yang (1949). Values of h_s/r_w and h_e/r_w found by Yang may thus be plotted against Q/kr_w^2.

In 1951, C. H. Zee at Colorado A & M College, using the membrane analogy to form the free boundary to a water medium and the electrical analogy for fluid flow, determined values of the discharge number, Q/kr_w^2, and the ratios h_s/r_w and h_e/r_w for 28 additional points. These data of Yang and Zee, together with the information compiled by Hansen, form the basis for the curves of fig. 4. Because of the wide range of values of Q/kr_w^2, fig. 4 is plotted on semilogarithmic paper. The use of these resulting curves is illustrated by example 1 presented at the end of the bulletin.

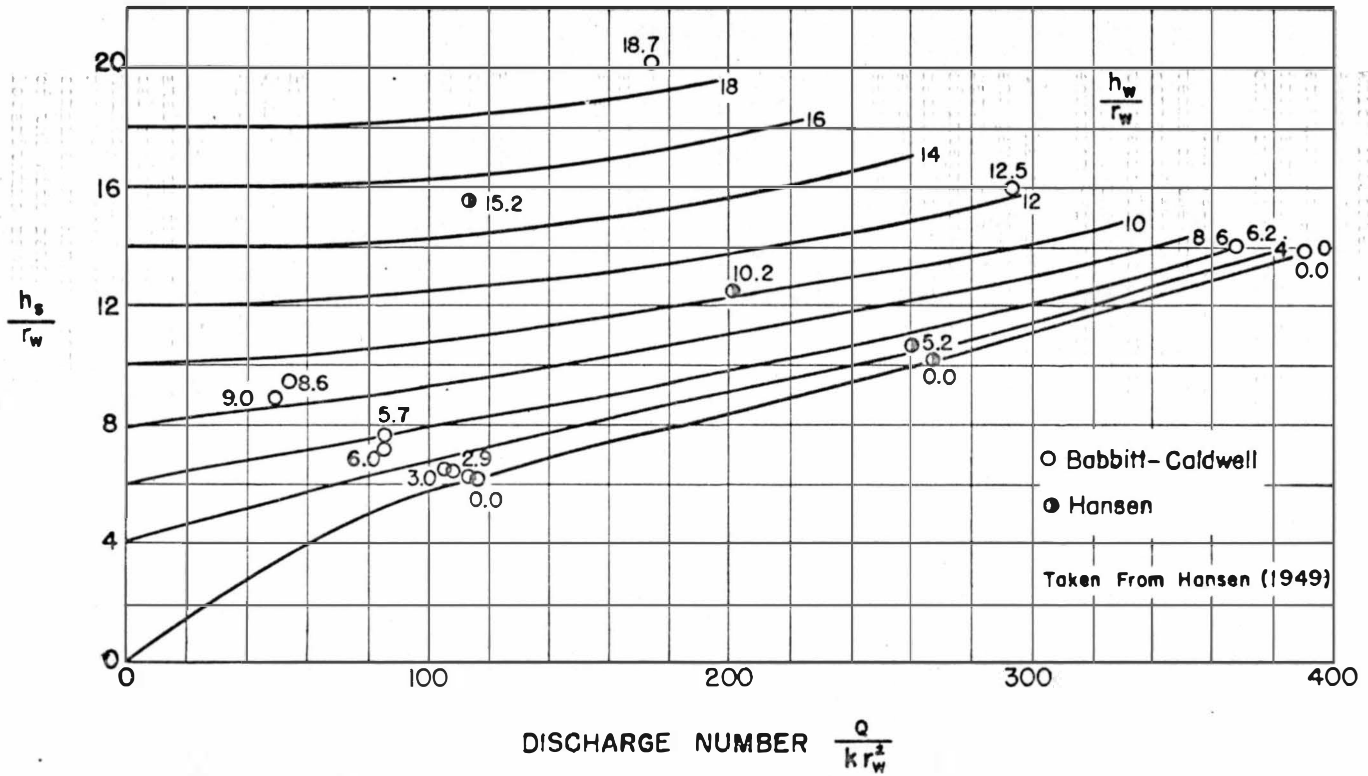

Fig. 3. The relation of the ratio h_s/r_w to discharge numbers of small magnitudes

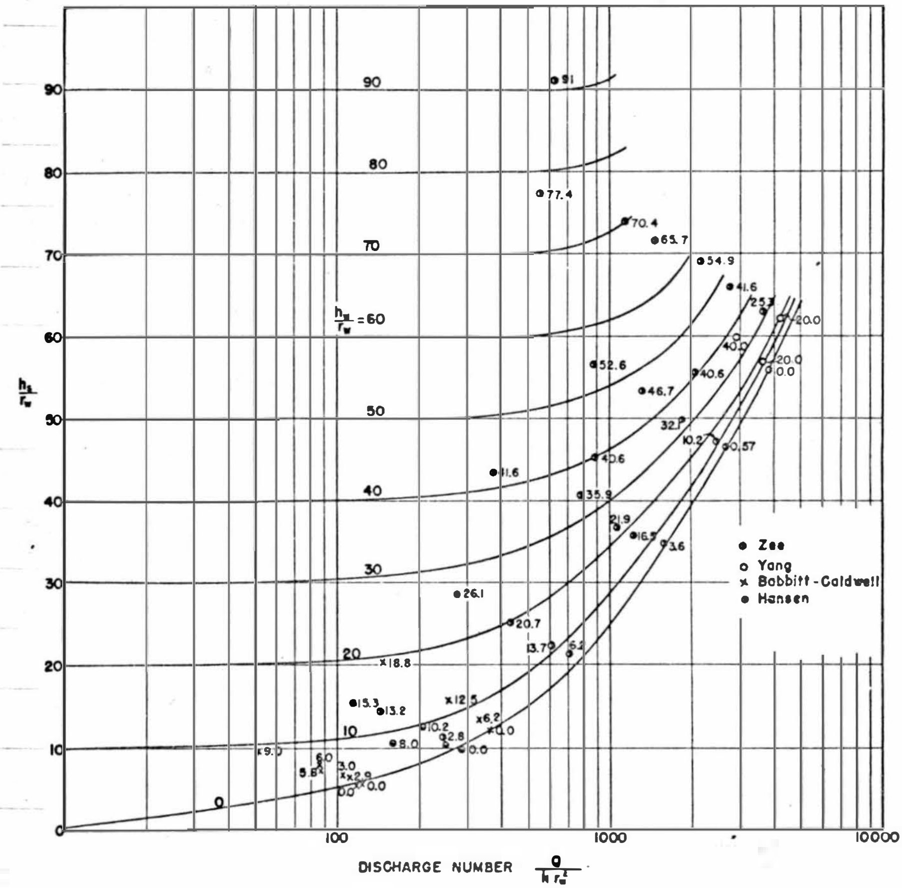

Fig. 4. The relation of the ratio h_s/r_w to discharge numbers of large magnitudes

The height of the seepage surface, AB, (fig. 2) should be of particular interest to those designing wells in unconfined aquifers, especially if such wells are for drainage purposes. In the case of example 1 the 30-foot drawdown at the well results in lowering the water table a maximum distance of only six feet. However, wells in unconfined strata are more efficient in utilizing available energy to cause discharge than wells in confined strata. The maximum effectiveness of the well of example 1 calculated as proposed by Wenzel (1942) cannot exceed 6/30 (100), or 20 percent.

UNCONFINED FLOW REPLENISHED BY VERTICAL PERCOLATION

Development of Equations

If a valley fill is fully drained by pumped wells, little of the discharge of the wells comes from water moving horizontally into the region of influence. Water replenishing the soil pore space within the cones of depression percolates vertically downward to be intercepted by the influence region of each well.

The flow, Q_r, through any concentric cylinder at radius r, if the Dupuit assumptions are made, is

$$Q_r = Q - \pi q_v (r^2 - r_w^2) \tag{21}$$

Since r is large in relation to r_w, equation 21 may be written

$$Q_r = Q - \pi q_v r^2 \tag{22}$$

From equation 22 and from the equation of continuity and Darcy's law

$$Q - \pi q_v r^2 = 2\pi r h k \frac{dh}{dr}$$

which may be integrated to give

$$\frac{Q}{\pi k} \ln r - \frac{q_v r^2}{2k} = h^2 + C \tag{23}$$

Evaluating C when $r = r_w$, $h = h_w$ gives

$$\frac{Q}{\pi k} \ln \frac{r}{r_w} - q_v \frac{(r^2 - r_w^2)}{2k} = (h^2 - h_w^2) \tag{24}$$

and at the radius of influence

$$\frac{Q}{\pi k} \ln \frac{r_e}{r_w} - q_v \frac{(r_e^2 - r_w^2)}{2k} = (h_e^2 - h_w^2) \tag{25}$$

by the approximation

$$r_e^2 - r_w^2 = r_e^2$$

and noting that by definition

$$n = \frac{\pi r_e^2 q_v}{Q} \tag{26}$$

equation 25 may be written in the forms

$$Q = \frac{\pi k \, (h_e^2 - h_w^2)}{2.303 \log r_e/r_w - (n/2)} \tag{27}$$

and

$$q_v = \frac{\dfrac{nk \, (h_e^2 - h_w^2)}{r_e^2}}{2.303 \log r_e/r_w - (n/2)} \tag{28}$$

From equations 24 and 26, and ignoring the term $(r_w/r_e)^2$ because of its negligible effect, the equation for the drawdown curve will be given by the relationship

$$Q = \frac{\pi k \, (h^2 - h_w^2)}{2.303 \log r/r_w - n/2 \, (r/r_e)^2} \tag{29}$$

Substituting the value of Q from equation 27 in equation 29 gives

$$h^2 = (h_e^2 - h_w^2) \, \frac{2.303 \log r/r_w - n/2 \, (r/r_e)^2}{2.303 \log r_e/r_w - n/2} + h_w^2 \tag{30}$$

Validity of Equations

No experimental data are available for checking the validity of the equations for the unconfined flow toward a well in a system replenished by vertical percolation. The seepage surface which exists at the well has been neglected in the theoretical development. Also the assumption that the flow is entirely horizontal through a vertical cylinder at distance r has been introduced as was done in the classical Dupuit analysis. By inference, equation 30 cannot be expected to give reliable results for the position of the free water surface near the well; however, equation 27 should give reliable values for discharge. Equation 29 may be expected to give reliable results if the measurements of piezometric head are made at points where the assumption of horizontal radial flow through vertical potential surfaces applies. This is true along the impermeable boundary, CD, (fig. 2) and is closely approximated at greater distances above CD as the radius increases.

Determination of Seepage Surface

Equation 27 may be rewritten

$$Q = \frac{\pi k \, (h_e^2 - h_w^2)}{2.303 \log \dfrac{r_e}{e^{n/2} \, (r_w)}} \tag{31}$$

By the transformation, $r_w = r_w'/e^{n/2}$, equation 31 reduces to the form of equation 14. The resulting geometry of the transformed well is the same as for the Dupuit case having the same discharge but for a well of radius equal to $e^{n/2}\ r_w$. For the case when n equals 1, the well system is replenished entirely by vertical percolation, and $r_w' = 1.643\ r_w$. The experimental curves of fig. 4 may be used for finding the height $(h_s - h_w)$ of the seepage surface above the water surface in the well by introducing the above transformation for the case of a well in a system replenished by vertical percolation. Use of the foregoing analysis for the purpose of designing an agricultural drainage well of this type is illustrated in example 2.

EFFECT OF REPLENISHMENT

A condition of steady flow implies that the total replenishment of the influence cone equals the discharge of the well. The shape of the influence cone and the discharge, Q, for steady flow for any system for which the well penetrates the full depth of the permeable stratum, must be, in general, a function of the drawdown, D_w, the well radius, r_w, the permeability, k, the thickness of the water-bearing stratum, t,[7] and the rate of replenishment to the influence cone, q. These last five variables are sufficient to determine fully the flow for any particular system. The weakness of existing formulas for well discharge is that they do not contain the rate of replenishment. They are, therefore, indeterminate unless an actual test is made. The foregoing statement may be expressed mathematically by

$$F\ (D_w,\ r_w,\ k,\ t,\ q,\ Q) = 0 \tag{32}$$

where F designates an unknown function. Equation 32 involves only the physical dimensions of length (L) and time (T). Choosing r_w and k as repeating variables one obtains by dimensional analysis

$$F_1\ \left(\frac{D_w}{r_w},\ \frac{t}{r_w},\ \frac{q}{k},\ \frac{Q}{k r_w^2}\right) = 0 \tag{33}$$

or

$$\frac{Q}{k r_w^2} = F_2\ \left(\frac{D_w}{r_w},\ \frac{t}{r_w},\ \frac{q}{k}\right) \tag{34}$$

as a generalized functional relation for flow into a well.

An examination of equation 34 reveals that the dimensionless parameter, Q/kr_w^2, depends only on the geometry of the well (r_w, D_w) and the hydrology of the ground-water system (k, t, q). In other

---

7. For confined systems the thickness of the water-bearing stratum is represented by t. For unconfined systems the value $(h_e + h_w)/2$ is a measure of t. Note that equation 12 reduces to equation 14 by substitution $t = (h_e + h_w)/2$.

words, the combination of the well and its hydrologic environment determines Q/kr_w^2 which may be considered a measure of the discharge expressed in dimensionless terms. This parameter, bearing special importance in the hydraulics of wells, has been demonstrated by Hansen (1949) to be the ratio of the Froude number to the Reynolds number. As such it indicates also the ratio of the viscous forces to the gravity forces. It is here referred to as the discharge number denoted by the symbol N with appropriate subscripts depending upon the nature and direction of the flow and the sources of replenishment.

One should note that consideration of replenishment gives some reality to the measuring of the radius of influence. It is a characteristic length describing the area within which the total replenishment equals the discharge, Q. The following analysis introduces the rate of replenishment and develops theoretically, using certain approximations, the functional relations expressed by equation 34 for the three cases of steady flow discussed previously.

Unconfined System Recharged by Horizontal Flow

Denoting the discharge number for the unconfined case by the symbol, N_{uh}, and substituting $D_w = h_e - h_w$, equation 34 may be rewritten

$$N_{uh} = F_3 \left(\frac{h_e}{r_w}, \frac{h_w}{r_w}, \frac{q}{k}\right) \tag{35}$$

Equation 20 may be rewritten in the form

$$N_{uh} = \frac{Q}{kr_w^2} = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{\ln r_e/r_w} \tag{36}$$

so that all length dimensions are expressed in terms of the well radius, r_w. The right side of equation 36 contains only linear dimensions; thus the numerical value of N_{uh} defines the shape of the influence region. The factor q/k may be taken as the natural slope, i_n, of the free water table in the region, if no water comes into the cone of influence by vertical percolation. Under conditions of steady flow

$$Q = 2r_e\, k\, h_e\, i_n \tag{37}$$

Solving equation 37 for r_e and substituting in equation 36 yields

$$N_{uh} = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{2.303 \log \left(\frac{N_{uh}\, r_w}{2\, i_n\, h_e}\right)} \tag{38}$$

In view of the fact that q/k is inherent in N_{uh}, equation 38 contains all of the parameters required by equation 35 and defines the function F_3 for the case at hand. One may observe that N_{uh} depends upon the radius of the well, the initial and final depths of water in the well, and the natural slope of the water table. Equation 38 may be solved implicitly for $N_{uh}/(h_e/r_w)^2$, the well factor, in terms of $i_n/(h_e/r_w)$, the replenishment factor, for various ratios of h_w/h_e.[8] The resulting graphs of the functions are plotted in fig. 5.

Large values of N_{uh} indicate deep, narrow drawdown cones of influence while small values indicate broad, shallow cones. Both flows into the zones of influence and low permeability cause the cones to become narrow and steep, whereas slow replenishment and high permeability cause them to be broad and shallow in terms of the radius of the well.

Confined Systems

The discharge number for the confined system will be designated by N_c. From equation 10b

$$N_c = \frac{Q}{kr_w^2} \quad \frac{2\pi(D_w/r_w)\,(t/r_w)}{\ln r_e/r_w} \tag{39}$$

where t is the thickness of the permeable stratum. If i_n is the natural slope of the piezometric surface the flow into the zone of influence is approximated by

$$Q = 2r_e\, k\, t\, i_n \tag{40}$$

Solving equation 40 for r_e and substituting in equation 39 gives

$$N_c = \frac{2\pi(D_w t/r_w^2)}{2.303 \log\left(\frac{N_c\, r_w}{2\, i_n\, t}\right)} \tag{41}$$

Equation 41 satisfies the functional relationship of equation 34 and makes possible computation of values of $i_n/(D_w/r_w)$ for various values of $N_c/(D_w t/r_w^2)$ to form the curves of fig. 6.

Unconfined System Totally Replenished by Vertical Percolation

Let $N_{uv} = Q/kr_w^2$ define the discharge number for this system. Combining equations 26 and 27 for n = 1 yields

$$N_{uv} = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{2.303 \log\left(\frac{1}{r_w}\sqrt{\frac{Q}{q_v \pi}}\right) - 1/2} \tag{42}$$

---

8. Algebraic manipulation of equation 38 for the case of zero drawdown (h_w/h_e = 1.0) yields $Q = 2\, i_n\, k\, h_e\, r_w$ instead of zero. The value of Q in this instance is the same as for a well with a radius of influence equal to the radius of the well.

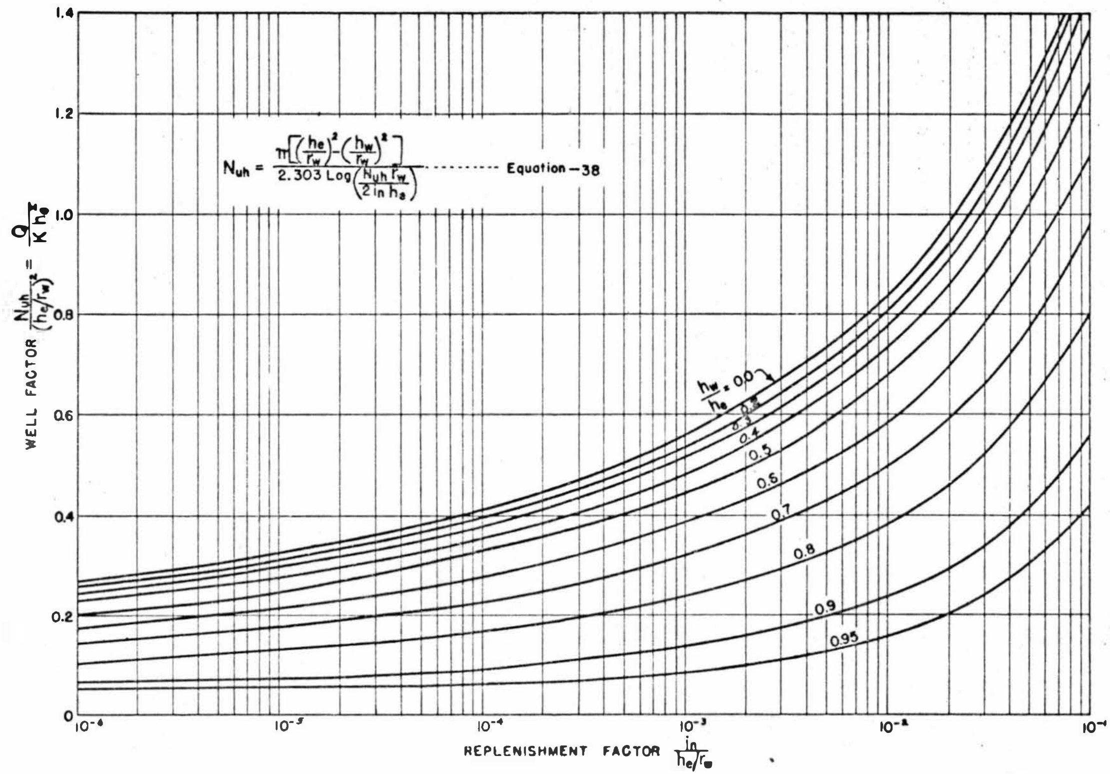

Fig. 5. Horizontally-recharged well in unconfined system

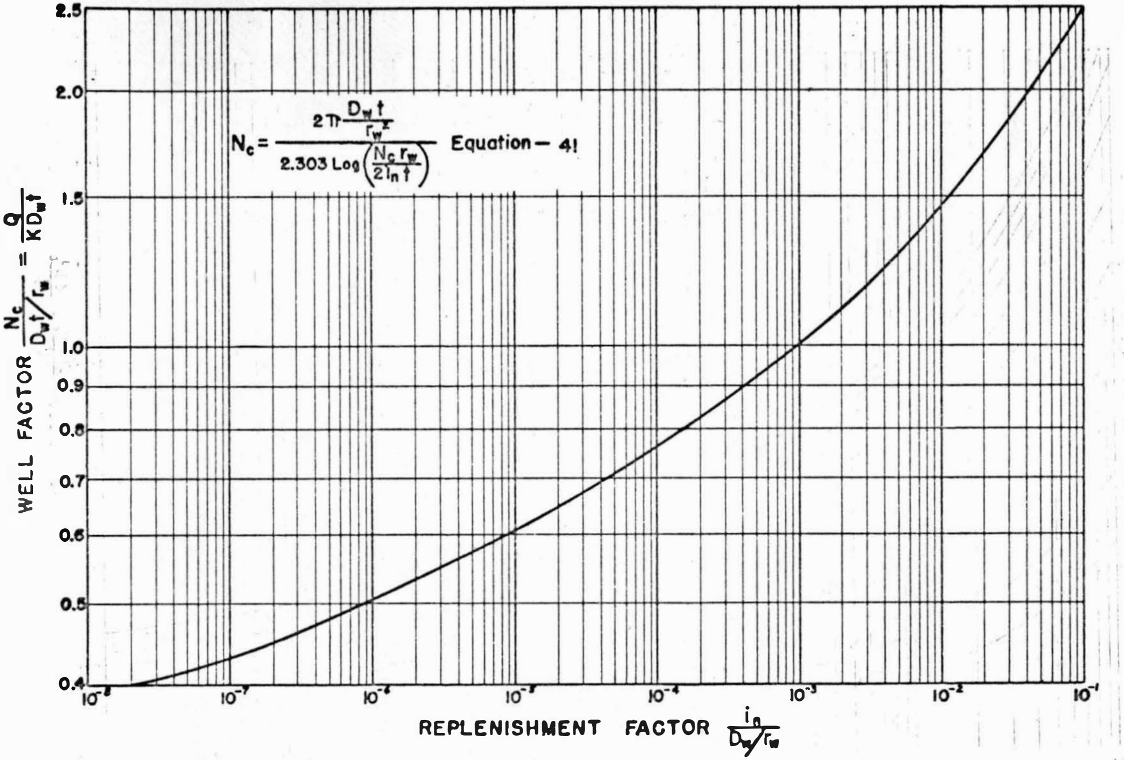

Fig. 6. Confined ground-water flow toward a well for replenishment factors ranging from 10^{-8} to 10^{-1}

Substituting $Q = N_{uv} k r_w^2$ in the denominator gives

$$N_{uv} = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{1.151 \log\sqrt{\left(\frac{N_{uv}\ k}{q_v\ \pi}\right)} - 1/2} \tag{43}$$

which satisfies the relationship required by equation 34. Equation 43 may be rewritten in the form

$$\frac{N_{uv}}{\left(\frac{h_e}{r_w}\right)^2} = \frac{\pi\left[1 - \left(\frac{h_w}{h_e}\right)^2\right]}{1.151 \log\left[\frac{N_{uv}}{\left(\frac{h_e}{r_w}\right)^2} - \log\frac{(\pi q_v)/k}{\left(\frac{h_e}{r_w}\right)^2}\right] - 1/2} \tag{44}$$

The ratio h_e/r_w depends entirely on the dimensions of the well while q_v/k is the ratio of the unit replenishment to the permeability. From equation 44, $N_{uv}/(h_e/r_w)^2$ can be computed for values of $(q_v/k)/(h_e/r_w)^2$ for various drawdown ratios, h_w/h_e. Graphs of the results are presented in figs. 7a and 7b.

Although the value of q_v will no doubt change at various times during the year and with the seasons, equation 42 should be valuable in planning the design of a system of drainage wells as well as in developing the ultimate ground-water supply of a closed ground-water basin of the unconfined type.

SIGNIFICANCE OF THE DISCHARGE NUMBER

If the value of the discharge number, Q/kr_w^2, is known, the effective radius of influence may be established by consideration of the hydrologic factors and the well radius. For the three sources of ground water and classes of flow discussed herein the following formulas may be used.

$$N_c = F\left(\frac{r_e}{r_w}, \frac{t}{r_w}, i_n\right) \tag{45}$$

$$N_{uh} = F\left(\frac{r_e}{r_w}, \frac{h_e}{r_w}, i_n\right) \tag{46}$$

$$N_{uv} = F\left(\frac{r_e}{r_w} \ \frac{q}{k}\right) \tag{47}$$

For the confined case, equation 11 can be written as

$$\frac{N_c r_w}{t} = \frac{2\pi\frac{h - h_w}{r_w}}{2.303 \log\frac{r}{r_w}} \tag{48}$$

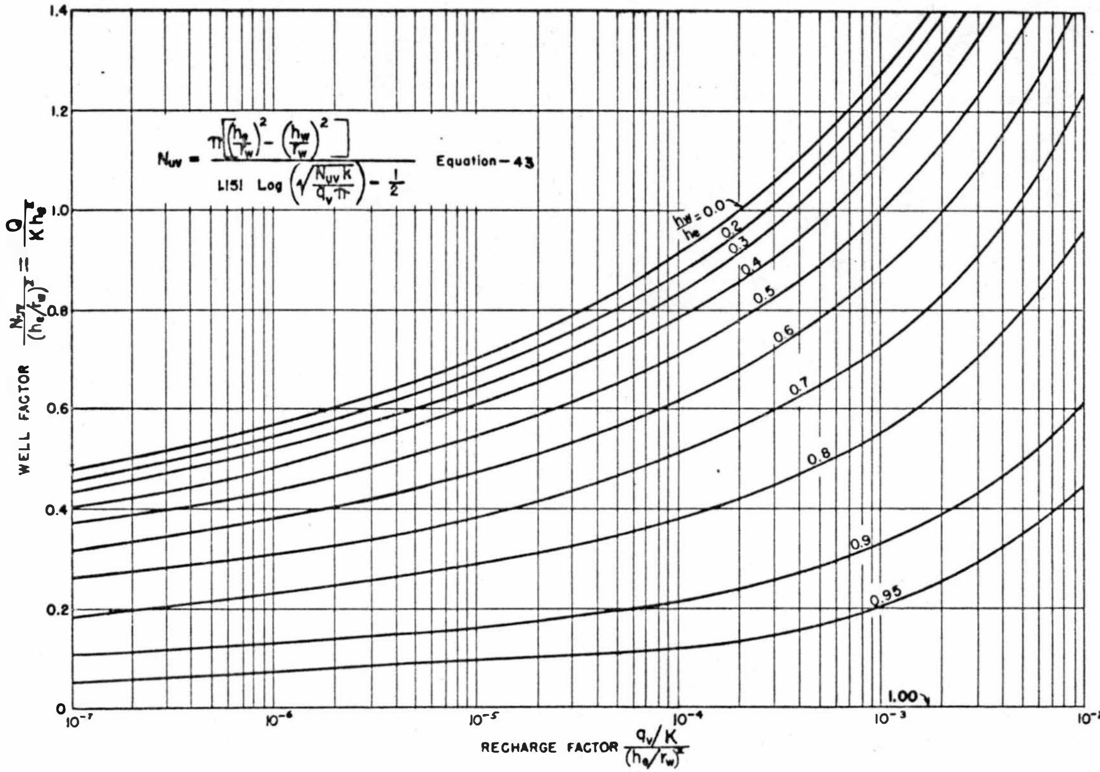

Fig. 7a. Vertically replenished and unconfined ground-water flow for replenishment factors ranging from 10^{-7} to 10^{-2}

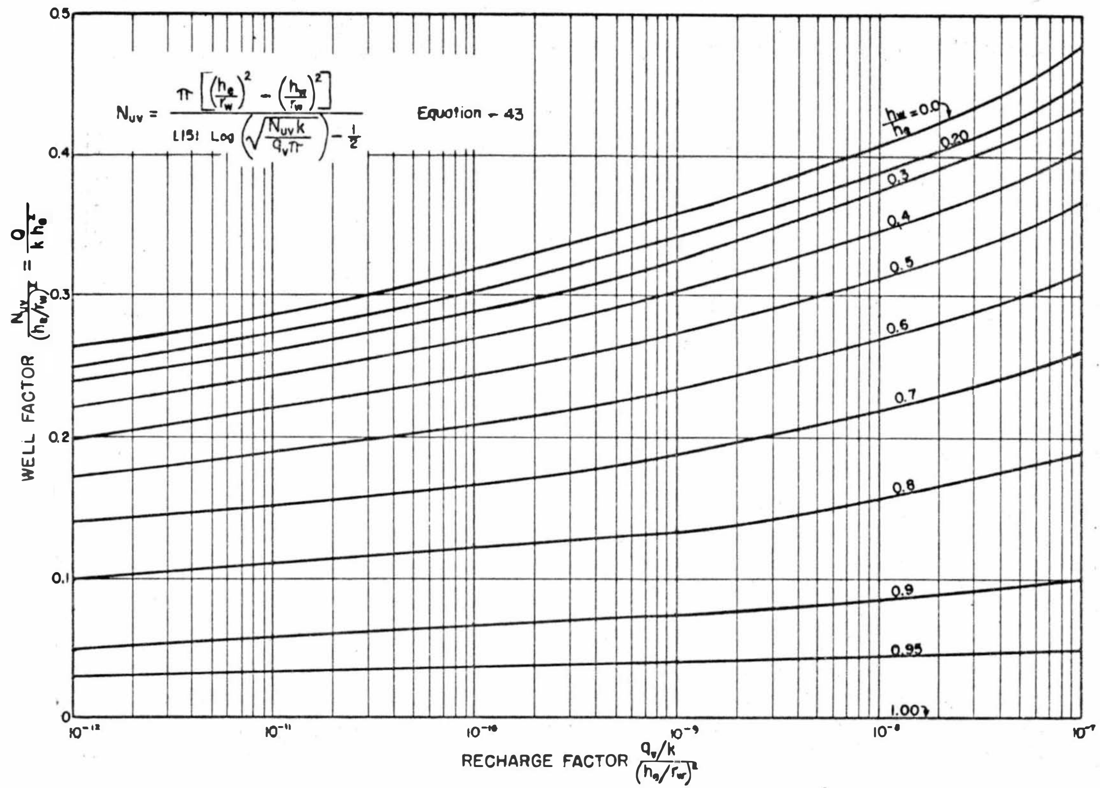

Fig. 7b. Vertically replenished and unconfined ground-water flow to a well for replenishment factors ranging from 10^{-12} to 10^{-7}

This relationship is shown by fig. 8. In using fig. 8 enter known values of $N_c r_w/t$ with the values of D_w/r_w to intersect the graph for $N_c r_w/t$. The resulting abscissa is r_e. The value of $(h - h_w)/r_w$ may be found by entering with the value of r and proceeding vertically to intersect the proper curve for $N_c r_w/t$. The resulting ordinate gives the value of $(h - h_w)/r_w$. For example, suppose $N_c = 2{,}640$, $t/r_w = 30$, and $D_w/r_e = 74$. Then $N_c r_w/t = 88$. Entering with D_w/r_e to intersect $N_c r_w/t = 88$ gives $r_e/r_w = 190$. At $r/r_w = 110$, $(h - h_w)/r_w = 66.2$. Interpolation for intermediate values of $N_c r_w/t$ may be easily done because the value of the ordinate along the heavy vertical line at $r/r_w = 524$ is equal to the value of $N_c r_w/t$ through that point. Thus, to find the line for $N_c r_w/t = 88$, draw a line through the ordinate 88 on the heavy vertical line and the origin. The relationships for the other two systems are much more complex and further study is needed in order to present them fully.

Examples 3, 4, 5, 6, and 7 illustrate the use of the foregoing equations and analyses.

EFFECTIVENESS OF WELLS

The effectiveness of a well is defined by Wenzel (1942) as

$$E_w = \frac{100\,(h_e - h_c)}{(h_s - h_w)} \tag{49}$$

Because of the head loss through the well casing the effectiveness is less than 100 percent, except for highly developed permeable materials near the well.

Confined System

For a well in a confined system equation 49 has the quality of well efficiency because $(h_e - h_c)$ is actually the power per unit of weight discharge delivered by the well to the fluid outside of the well whereas $(h_e - h_w)$ is the power per unit of weight discharge imparted by the pumps to the water inside the well. The difference between the two values represents the power loss per unit of weight discharge through the boundary of the well. Effectiveness of wells is a widely-used term describing their condition from the point of view of their efficiency as power-transferring devices. The draw-down in the well $(h_e - h_w)$ may be quite easily measured; however, greater difficulty is encountered in the measurements of $(h_e - h_c)$. The head loss for the simple, confined case of a radial flow is a linear function of the logarithm of the radius (equation 11). The elevation of the piezometric surface observed at various distances from the well may therefore be plotted against log r and the resulting straight line extended to the casing to determine h_c of fig. 9.

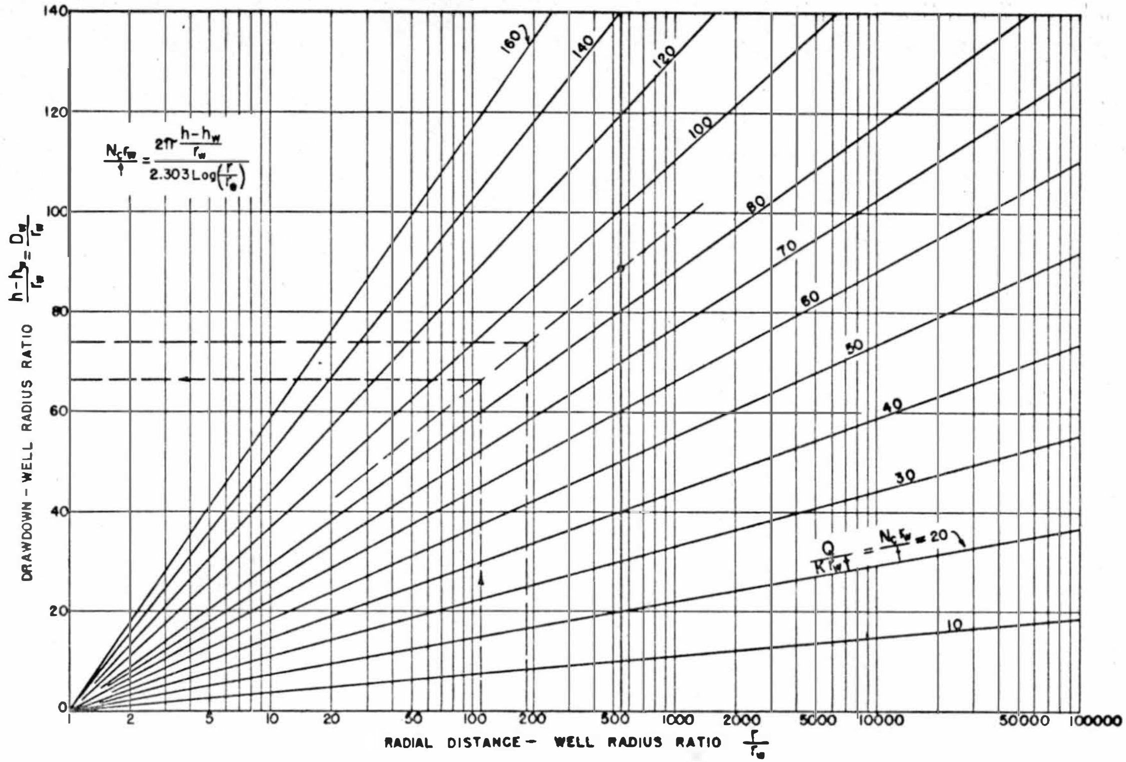

Fig. 8. Drawdown curves for confined ground-water flow to a well for discharge numbers from 10 to 160

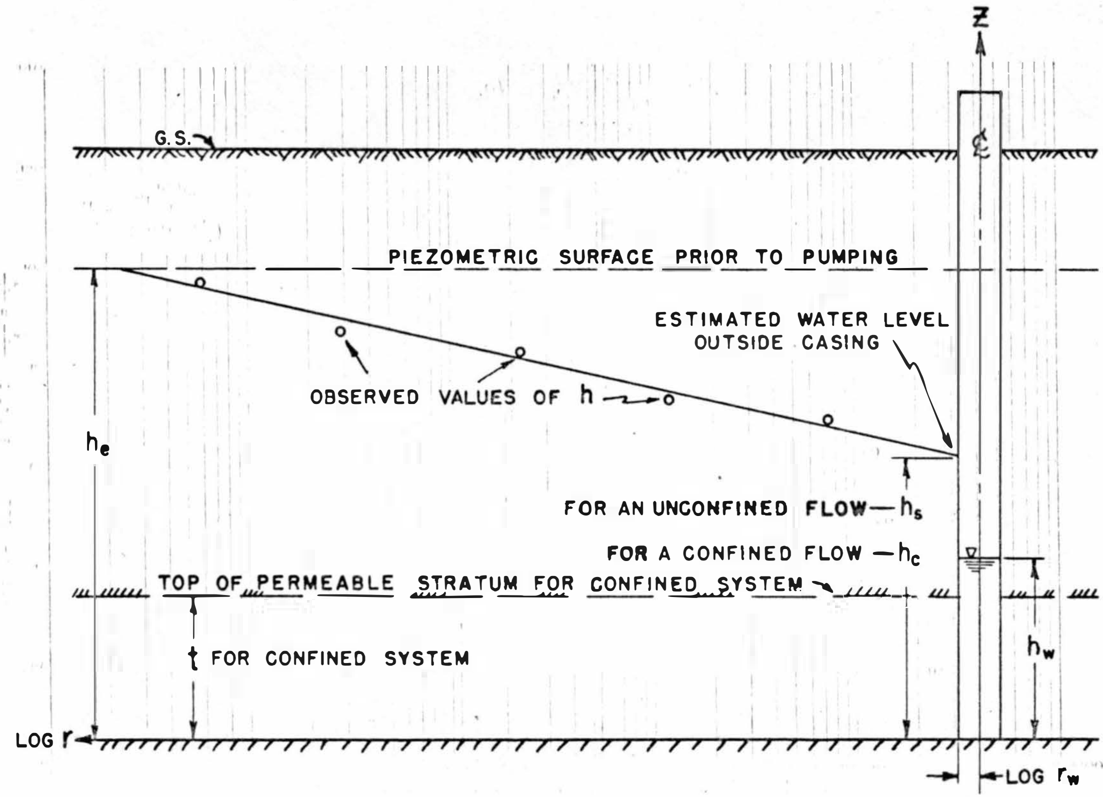

Fig. 9. Illustrating the method of estimating the water level outside a well for both confined and unconfined systems

Unconfined System

For the unconfined case considerable care must be exercised in applying Wenzel's procedure or misleading results will be obtained. One should clearly realize the distinction between h_s and h_c as applied to this type of system. The former, h_s, is a theoretical value for a fully efficient well in an unconfined system and differs from h_w because of hydrodynamic considerations resulting from the minimum energy concept. The latter, h_c, is the effective energy head inside of the well and is equal to h_w plus the hydraulic head losses through the casing and envelope. In a confined flow system h_c is equal to the hydraulic or piezometric head just outside of the well. However, in an unconfined flow system h_c is a hypothetical elevation somewhere between h_w and h_s; $h_c = h_w$ only in a fully efficient well. No suitable method for determining h_c for the unconfined system has yet been developed. If piezometers extend only a short distance into the saturated medium, the elevations represent the level of the free water surface. Thus, the value of h_s rather than of h_c is obtained. The value of effectiveness as given by h_s will, of course, always be too small.

Two methods for determining the value of h_c are suggested: (1) The piezometer pipes should be unperforated with an open end and should be extended to within a short distance of the bottom impermeable stratum. The squares of the differences between the piezometric level and the elevation of the bottom impermeable stratum will form a straight line when plotted against the logarithm of the radial distance from the well if the ends of the piezometers are in zones of essentially horizontal flow. The value of h_c may be found by extending the plotted line to the position of the well casing. (2) Determine the elevation of the free water surface at various radii from the well. As shown by equation 18, this quantity varies linearly as the logarithm of the radius and may be plotted on semi-logarithmic paper and extended to the well as illustrated by fig. 9. Call the resulting elevation h_s and enter fig. 4 with h_s/r_w and Q/kr_w^2 to determine a value of h_w/r_w. The resulting value of h_w determined by this procedure is actually h_c.

Comparison of Unconfined and Confined Systems

The total available energy per unit weight of water for any well system is equal to D_w or $(h_e - h_w)$. Actually, the unconfined system is inherently more efficient in utilizing the available specific energy than is the confined system. This conclusion may be readily deduced. The available specific energy in a confined system is, from equation 10b,

$$h_e - h_w = \frac{Q \ln (r_e/r_w)}{2\pi kt}$$

and in an unconfined system is, from equation 14,

$$h_e - h_w = \frac{Q \ln (r_e/r_w)}{k\pi(h_e + h_w)}$$

For the same value of available energy, permeability, h_w, h_e, h_c, r_w, and r_e,

$$\frac{Q_u}{Q_c} = \frac{h_e + h_w}{2t} \tag{50}$$

Since h_w must be greater than t in order to assure confined flow, and as h_e is always greater than t, it is evident from equation 50 that the ratio Q_u/Q_c will always excede unity. The available energy per unit weight of water causing flow to the unconfined system is, therefore, more efficiently used and produces a discharge greater than that from the confined system.

In all of the examples the well is assumed to be 100 percent effective. If the effectiveness is less than 100 percent, the value of h_c should be used in place of h_w in all cases.

NON-STEADY FLOW

Although the steady-flow formulas presented herein have real value in the design and operation of ground-water development facilities, it is important to remember that the flow of ground water to pumped wells is frequently--perhaps usually--non-steady. Theis in 1935, and Jacob in 1940 developed formulas for estimating non-steady flow and thus made substantial contributions to the advancement of the science of hydraulics of wells.9/

ZONES OF FLOW IN WELL HYDRAULICS

Ground-water flow to wells should be considered in the light of the characteristics of the flow in different regions or zones. A clear understanding of these zones and the nature of the flow within them will be of value in wisely applying the principles set forth in this bulletin. These zones, illustrated in fig. 10, will be considered as follows:

Zone I: Inside the Well

Within the well, flow parameters are readily measurable and the measurements made are of value in defining the flow in other zones. Both of the lengths h_w and r_w are easily and accurately determined.

Zone II: The Well Casing and Adjacent Soil

Within this zone where the so-called well losses occur, it is

9. To those who are especially interested in ground water and pumped wells, a study of the nonequilibrium formulas of Theis and of Jacob, as summarized by Ferris in chapter VII of Hydrology by Wisler and Brater, will be interesting and valuable.

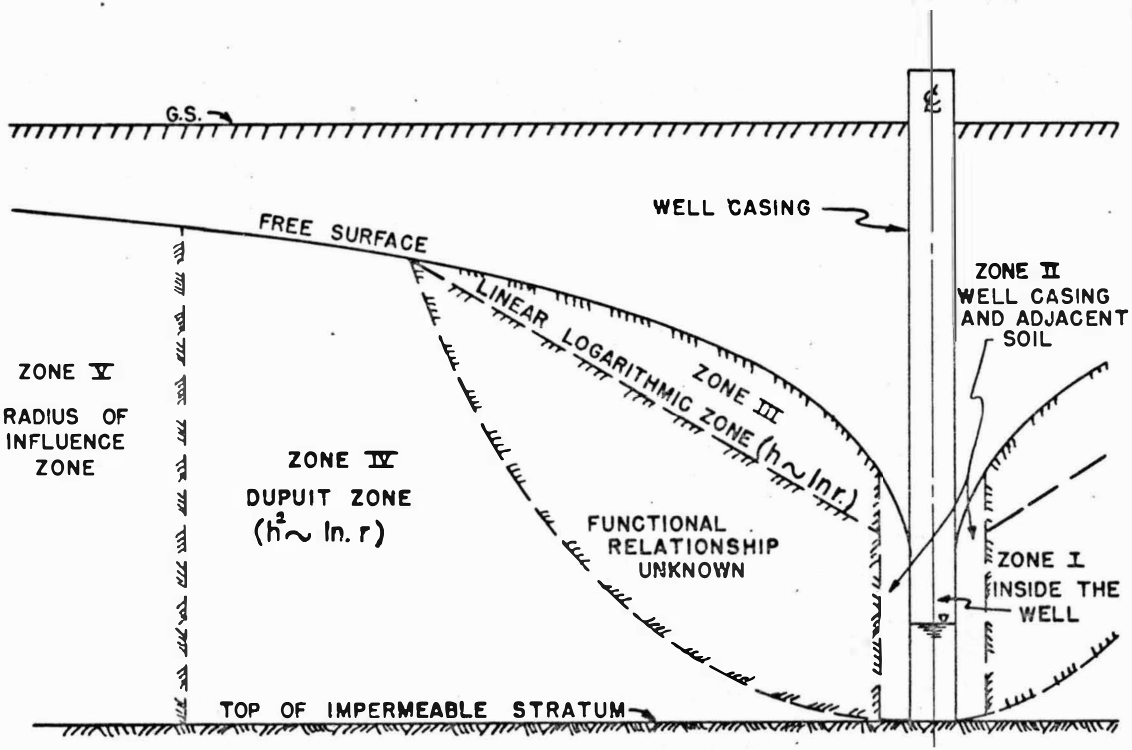

Fig. 10. Relative positions of the major differentiating zones of unconfined flow to a well

extremely difficult to obtain reliable measurements. Furthermore, because of the high velocities and the accelerating flow, measurements must be exactly located to be of any value. It is in this zone also where the effect of local nonuniformities is most pronounced. Hence, a given measurement may not be indicative of the average conditions. Because of these facts, measurements should not be made in this zone for use in predicting the flow conditions in any region other than the exact point at which the measurement was taken.

Zone III: Linear Logarithmic Zone

In confined flow, this zone extends from the region near the well to the so-called radius of influence in region V. In unconfined flow, this zone applies to the free surface near the well where the flow is curvelinear. In this zone the elevation of the piezometric surface in confined flow and the free surface in unconfined flow is proportional to the logarithm of the radius from the well ($h \sim \ln r$). In unconfined flow, measurements of the piezometric head made near the free surface can be used with reliance to extrapolate the free surface to the well casing in order to determine the height of the seepage zone, h_s.

Zone IV: Dupuit Zone for Unconfined Flow

This zone applies only to those regions of unconfined flow where the velocity vector is essentially horizontal. Dupuit's equation ($h^2 \sim \ln r$) developed on the assumption of horizontal flow, applies with increasing accuracy as the flow approaches the horizontal. Consequently, this zone extends all along the base of the aquifer where the flow is horizontal, and includes more and more of the flow as the radius increases, until the so-called radius of influence is approached. Piezometric heads measured in this region will accurately define flow conditions if the Dupuit equation is used.

Zone V: Radius of Influence Zone

This zone is the least accurately defined of any of the zones of flow. For wells being recharged horizontally, an actual radius of influence does not exist. However, for wells receiving their recharge by vertical percolation, the radius of influence has tangible meaning. The developments of this bulletin are based upon assumptions of steady flow; the effects of a departure from steady flow are most pronounced in the radius of influence zone. Piezometric heads measured in this region are of value in approximating the original position of the undisturbed water table. Piezometric heads should not be measured in this zone for the purposes of predicting either the discharge or the permeability of the aquifer.

EXAMPLES

Example 1

Given: A gravity well discharges 2.0 cfs and the permeability of the material is determined as 10^{-3} ft./sec. The original depth of water in the well, h_e, is 50 ft. and the drawdown is 30 ft., making $h_w = 20$ ft. What will be the height of the seepage surface (AB, fig. 2) if the well is 24 in. in diameter?

Solution:

$$\frac{Q}{kr_w^2} = \frac{2.0}{(0.001)\,(1)^2} = 2{,}000$$

$h_w/r_w = 20/1 = 20$, and from fig. 4, $h_s/r_w = 45$. Therefore, $h_s = (1)45 = 45$ ft. and the height of the seepage surface is $(45 - 20)$ or 25 ft. = Ans.

Example 2

Given: Excess irrigation water of 1 ft. depth per year is to be removed by steady pumping. The diameter of the well is 24 in. and the subsoil permeability, k, is 10^{-3} ft./sec. The depth of the saturated permeable overburden, h_e, is 50 ft. and the casing is perforated for the entire depth. What will be the drawdown if an area of radius 2,000 ft. is to be drained by one well? What will be the height of the seepage surface?

Solution:

From equation 28 with n = 1

$$h_e^2 - h_w^2 = \frac{1}{(365)\,(24)\,(3600)}\;\frac{(2000)^2}{(0.001)}\left[2.3 \log \frac{2000}{1} - 1/2\right]$$

$$= (127)\left[(2.303)\,(3.301) - 0.500\right] = 902$$

$$h_w = \sqrt{2500 - 902} = 39.99 \text{ or } 40 \text{ ft.}$$

$\therefore$ Drawdown $(h_e - h_w) = (50 - 40) = 10$ ft. = Ans.

From equation 26 $n = q_v \pi r_e^2/Q$

$$Q = \frac{(2000)^2\,\pi}{(365)\,(24)\,(3600)\,(1)} = 0.4 \text{ cfs}$$

$$r_w' = 1\,\sqrt{e} = 1.643$$

$$\frac{Q}{kr_w'^2} = \frac{0.4}{(0.001)\,(1.643)^2} = 148,\quad \frac{h_w}{r_w'} = \frac{40.0}{1.643} = 24.4$$

and from fig. 4, h_s/r_w = 25.8; therefore, h_s = (1.643) (25.8) = 42.3 ft. and the height of the seepage surface ($h_s - h_w$) = 42.3 - 40.0 = 2.3 ft. = Ans.

Example 3. Yield of Confined-Flow System

Given: A permeable gravel confined aquifer 30 ft. thick at a depth of 100 to 130 ft. below the ground surface has an estimated permeability of 2 x 10^{-3} ft./sec. The natural slope of the piezometric surface is 0.01. The water stands at a depth of 20 ft. below the ground surface in a 12-in. diameter well. Assuming that by pumping the water surface in the well may be drawn down to a depth of 70 ft., what steady water yield may be expected from the well?

Solution:

Equation 41 and the graph of fig. 6 apply.

$$\frac{i_n}{D_w/r_w} = \frac{0.01}{50/0.5} = 0.0001 = 10^{-4}$$

and from fig. 6 $\dfrac{N_c}{D_w t/r_w^2} = 0.764$. Hence, $N_c = \dfrac{Q}{k r_w^2} =$

$(0.764) \dfrac{(50)(30)}{(0.5)^2} = 4584$ and $Q = (4584)(2)(10^{-3})(0.5)(0.5) =$

2.29 cfs = Ans.

Example 4. Permeability of the Confined-Flow System

Given: A confined sand layer 10 ft. thick yields 0.2 cfs steady-flow discharge when pumped under a drawdown of 30 ft. The natural slope of the piezometric surface is 5 ft./hundred and the diameter of the well is 24 in. Estimate the permeability of the sand.

Solution:

$$\frac{i_n}{D_w/r_w} = \frac{0.05}{30/1} = 0.00167$$

and from fig. 6

$$\frac{N_c}{D_w t/r_w^2} = 1.06;\ N_c = 318 = \frac{Q}{k r_w^2}$$

from which $k = \dfrac{0.2}{(318)(1)^2} = \underline{6.29 \times 10^{-4}\ \text{ft./sec.}}$ = Ans.

Example 5

Drawdown Required to Discharge 1 cfs in Confined-Flow System

Given: The permeability of a 20-ft. gravel stratum is estimated at

1.0×10^{-3} ft./sec. The well diameter is 24 in. and i_n is 10 ft./ hundred. What drawdown will be required to produce a discharge of 1.0 cfs?

Solution:

This problem may be solved by trial and error using fig. 6.

$$N_c = \frac{Q}{kr_w^2} = \frac{1.0}{(10^{-3})(1.0)^2} = 1000 \qquad i_n = 0.1$$

Try $D_w/r_w = 20$; $\frac{N_c}{D_w t/r_w^2} = \frac{1000}{(20)(20)} = 2.5$ and $\frac{i_n}{D_w/r_w} = 0.005$,

entering fig. 6 with $\frac{N_c}{D_w t/r_w^2} = 2.5$, the corresponding value of

$\frac{i_n}{D_w/r_w} = 0.10$. Try $D_w/r_w = 50$; $\frac{N_c}{D_w t/r_w^2} = 1.0$, $\frac{i_n}{D_w/r_w} = 0.002$,

entering fig. 6 with $\frac{N_c}{D_w/r_w} = 1.0$ gives $\frac{i_n}{D_w/r_w} = 0.00106$. Therefore, the correct value of D_w/r_w is between 20 and 50 and much nearer 50. Try $D_w/r_w = 40$; $\frac{N_c}{D_w/r_w} = \frac{50}{40} = 1.25$, $\frac{i_n}{D_w/r_w} = 0.0025$,

repeating the preceeding steps gives $i_n/(D_w/r_w) = 0.004$. The correct value of D_w/r_w appears to be about 45, and the required drawdown is therefore estimated to be 45 ft. = Ans.

Example 6a. Yield of Unconfined-Flow System

Given: A ground-water survey of an unconfined-flow system aquifer indicates that: the average depth of water in the permeable layer is 80 ft.; the permeability is 5×10^{-3} ft./sec.; and the natural slope of the water table is 2.5 ft./thousand. If the water level in the well is to be drawn down to 60 ft., what will be the estimated yield of the 12-in. diameter well? Estimate the desirable well spacing.

Solution:

$\frac{i_n}{h_e/r_w} = \frac{.0025}{80/.5} = .0000156$, $\frac{h_w}{h_e} = \frac{80 - 60}{80} = 0.250$, from fig. 5

$\frac{N_{uh}}{(h_e/r_w)^2} = 0.36 \qquad N_{uh} = (0.36)(160)^2 = 8{,}530$

$$\therefore Q = N_{uh}\, r_w^2\, k = (8530)\,(0.5)^2\,(.005) = 10.6 \text{ cfs}$$

$$r_e = \frac{Q}{2k\, h_e\, i_n} = \frac{10.6}{(2)\,(0.005)\,(80)\,(0.0025)} = 5300 \text{ ft.}$$

Wells should be spaced somewhat less than every two miles for greatest economy and full coverage.

Example 6b

Given: The same data as in example 6a but using the gallon as the unit of volume and the minute as the unit of time.

Solution:

In this case $k = 2.24$ gal./sq. ft./min. corresponds to 5×10^{-3} ft./sec. As before $N_{uh} = 8{,}530$ and $Q = (8530)\,(0.5)^2\,(2.24) = 4{,}780$ gpm.

$$r_e = \frac{4780}{(2)\,(2.24)\,(80)\,(.0025)} = 5300 \text{ ft.}$$

Example 7. Design of Drainage Well

Given: During the months May to October, an average depth of 0.5 ft. per month of irrigation water percolates to the ground water. This is to be removed by pumped drainage using 12-in. wells. The depth of the permeable soil is 65 ft. and the water table is to be maintained at a depth of 15 ft. or more below the ground surface. The permeability of the soil materials is estimated at 5×10^{-4} ft./sec. In order to benefit by special power rates pumps will be operated only during the 6-month irrigation season. What discharge may be expected and what should be the well spacing if the average lift (from water surface in the well to the ground surface) is maintained at 50 ft.?

Solution:

$$q_v/k = \frac{(0.5)\,(10^4)}{(30)\,(3600)\,(24)\,(5)}, \text{ and } h_e/r_w = \frac{50}{0.5} = 100$$

$$\therefore \frac{q_v/k}{(h_e/r_w)^2} = \frac{1}{(3)\,(36)\,(24)\,(100)^2} = 3.86 \times 10^{-8}, \text{ and } h_w/h_e =$$

$15/50 = 0.3$. From fig. 7b $(N_{uv})/(h_e/r_w)^2 = \cancel{0.407}\ 0.352$; $\therefore N_{uv} = \cancel{4070}\ 3520 = Q/kr_w^2$. $\therefore$ well discharge, Q, $= (\cancel{4070}\ 3520)\,(5)\,(10^{-4})\,(0.5)^2 = \cancel{0.509}\ 0.44$ cfs.

And, since $Q = q_v \pi r_e^2$,

$$r_e = \sqrt{\frac{Q}{q_v \pi}} = \sqrt{\frac{(\cancel{0.509}\ 0.44)\,(30)\,(3600)\,(24)}{(0.5)\,(\pi)}} = \sqrt{\cancel{840{,}000}\ 726{,}000} = \cancel{918}\ 854 \text{ ft.}$$

If the spacing of the wells is made $2r_e$, there will be a small undrained area because of the circular shape of the influence zone. To assure pumping of all of the water that percolates downward, the boundaries of the three circular areas of diameter $2r_e$ should overlap so that they intersect at a common point. Thus, for an arrangement of three wells the spacing should be made $r_e\sqrt{3}$, and for four wells, $r_e\sqrt{2}$.

In order to find the height of the seepage surface under the above conditions, an equivalent well must be assumed. The equivalent radius of the 12-inch wells of this example would, therefore, be (1.643) (0.5) or 0.821 feet. New values would then be obtained in the solution as follows:

$$\cancel{\text{As before, } q_v/k = \frac{(0.5)(10^4)}{(30)(3600)(24)(5)}}$$

$$Q/kr_w^2 = 0.44/5\times10^{-4}(.821)^2 = 1310$$

$$\cancel{h_e/r_w = \frac{50}{0.821} = 60.9}$$

$$h_w/r_w = 15/0.821 = 18.3$$

$$\cancel{\therefore \frac{q_v/k}{(h_e/r_w)^2} = 1.04 \times 10^{-7}, \text{ and } h_w/h_e = \frac{15}{50} = 0.3.}$$

$$\cancel{\text{Now, from fig. 7b, } \frac{N_{uv}}{(h_e/r_w)^2} = 0.43.}$$

$$\cancel{\therefore N_{uv} = (0.43)(60.9)^2 = 1593 = Q/kr_w^2 \text{ and } h_w/r_w = \frac{15}{0.821} = 18.3.}$$

Now, from fig. 4, $h_s/r_w = \cancel{39.4}\ 37$. $\therefore h_s = (\cancel{39.4}\ 37)(0.821) = \cancel{32.3}\ 30.2$ ft.

The seepage surface is, therefore, $(h_s - h_w) = (\cancel{32.3}\ 30.2 - 15) = \cancel{17.3}\ 15.2$ ft. high.

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SYMBOLS AND DEFINITIONS

Symbols	Definitions	Force-Length-Time Dimensions
A	Area of influence beneath which ground water or piezometric surface contours are modified by pumping, which is πr_e^2 or an equivalent area; also area of saturated soil at right angles to direction of water flow	L^2
C_x	Babbitt-Caldwell variable coefficient	--
D_w	Drawdown or lowering of the water level caused by pumping	L
E_w	Effectiveness of the well	--
e	Base of Naperian logarithms	--
F	An unknown function	--
h	Hydraulic head, p/w + z; this represents the elevation above datum of the free water surface (a piezometric surface) at any distance, r, from the well. (A variable)	L
h_c	Elevation above datum of the water surface in the well plus the loss of head in flow through the well casing	L
h_e	Elevation of either water surface or piezometric surface at maximum radius of circle of influence	L
h_s	Elevation of water surface just outside of a well in an unconfined-flow system	L
h_w	Elevation of water surface in the well while pumping	L
i	Hydraulic gradient for ground-water flow, the ratio expressed by h/l or dh/dl	--
i_n	Natural hydraulic gradient or slope of either the water table or of the piezometric surface	--

Symbols	Definition	Force-Length-Time Dimensions
k	Permeability of soils to water	L/T
l	Length or distance of ground-water flow resulting in loss of hydraulic head, h	L
ln	Logarithm to base e	--
log	Logarithm to base 10	--
N	A distance in the direction normal to the direction of flow	L
N_c	Discharge number, Q/kr_w^2, for a confined system	--
N_{uh}	Discharge number, Q/kr_w^2, for an unconfined system recharged by horizontal flow	--
N_{uv}	Discharge number, Q/kr_w^2, for an unconfined system recharged by vertical flow	--
n	Ratio of the discharge of water from vertical percolation to the total discharge of the well	--
p	Pressure intensity	F/L^2
Q	Discharge from a well under equilibrium conditions	L^3/t
Q_c	Discharge of the confined-flow system	L^3/T
Q_r	Flow radially through a cylindrical surface of radius r	L^3/T
Q_u	Discharge of the unconfined-flow system	L^3/T
q	Flow replenishment for any system, generalized	L/T
q_v	Recharge flow per unit horizontal area for the vertically-replenished system	L/T
r	Radial distance from axis of well. (A variable)	L

Symbols	Definitions	Force-Length-Time Dimensions
r_e	Radius of the circle of influence	L
r_w	Radius of the well	L
r_w'	Equivalent radius of the well, $e^{n/2} r_w$	L
S_f	Free surface. The water surface represented by the cone of depression when pumping from unconfined ground water	L^2
S_s	Seepage surface. The cylindrical area having a height of $(h_s - h_c)$ and a radius slightly greater than r_w	L^2
t	Thickness of water-bearing stratum for any system, general	L
θ	The angle of the planar system of polar co-ordinates	--
V	Velocity of ground-water flow (volume per unit of time per unit gross cross-sectional area)	L/T
w	Weight of water per unit volume; specific weight	F/L^3
z	Elevation above datum	L

Quantities Related to the Hydraulics of Wells

Many quantities not included in the list of symbols are of interest in a study of the hydraulics of wells. Some of these related quantities--not all--are defined here for convenience of the reader. Because of their special interest some of the quantities included in the list of symbols and definitions are here included with amplified definitions.

Aquifer - a geologic formation or structure below the land surface that transmits water in sufficient quantity to supply wells or springs.

Area of influence - the horizontal area beneath which flow is toward the well.

Cone of depression - depression in either the water table in unconfined flow or the piezometric surface in confined flow developed around a well. The periphery of the depression (the ground-water divide) delineates the zone from which ground water flows toward the well.

Confined-flow system - Ground water in an aquifer overlain by material sufficiently impermeable to sever free hydraulic connection with overlying earth materials except at the intake. Confined water moves as a result of a differential pressure resulting from differences in elevation between the intake and discharge areas of the confined water. A confined-flow system is referred to in some of the literature as artesian flow or pressure flow.

Discharge number - a new dimensionless parameter, characterizing the shape of the cone of depression, which depends only on the geometry of the well and the hydrology of the ground-water system. The discharge number, Q/kr_w^2, is defined as the ratio of the well discharge to the product of the permeability of the aquifer times the square of the well radius. It is also the ratio of the Froude and Reynolds numbers, F/R; that is, the ratio of the viscous to gravity forces. Note that inertia forces in F and R cancel.

Drawdown - lowering of the water table in unconfined systems and the lowering of the piezometric surface in confined systems owing to the discharge from the well.

Free surface - a surface of atmospheric pressure. If capillary rise is negligible, the free surface and water table are identical and can be considered as the unbounded upper surface of the unconfined flow.

Ground water - the water in the zone of complete saturation below the water table.

Hydraulic gradient - the rate of change of piezometric or hydraulic head with distance. Hydraulic gradient of ground water records the head consumed by friction in the flow in unit distance since in ground-water flow the velocity heads are generally negligible.

Laminar flow - flow at velocities such that the loss of hydraulic head is proportional to the velocity; in turbulent flow the loss of head is more nearly proportional to the velocity squared. Laminar flow generally occurs at Reynolds numbers less than one. The potential theory and the Laplace equation apply to laminar flow.

Permeability - the capacity of water-bearing earth materials and soils to transmit water when saturated. The permeability is the volume of water flowing through unit cross-section in unit time under unit hydraulic gradient.

Porosity - the ratio of the volume of interstices to the total volume of rock, earth materials, soil, etc., without regard to size, shape, interconnection, or arrangement of openings.

Radial flow - two dimensional fluid flow toward the axis of a confined well with the velocity increasing as the distance from the axis of the well decreases. The vertical velocity at all points is zero.

Radius of influence - the maximum radius from which water moves toward the well.

Reynolds number - a dimensionless number, $N_R = \rho Vd/\mu$, consisting of a ratio of the inertial to viscous forces. The quantities in the above number are usually defined as follows: ρ is the density of the fluid; V is the average or bulk velocity rather than the actual velocity within the pores; d is the diameter of the particle rather than the diameter of the pore; and μ is the viscosity of the fluid. Reynolds number thus computed is not exact since the porosity would effect both the actual velocity and the diameter of the pore for the same values of V and d in the formula, resulting in a possible different flow state for the same apparent value of Reynolds number. However, the Reynolds number as defined above does have considerable utility as a guide in design because of its usefulness in estimating whether a given condition will produce laminar or turbulent flow and consequently whether the head loss will be proportional to the first power of the velocity or more nearly proportional to the second power.

Seepage surface - the outside portion of the well hole (a cylindrical area with a vertical central axis) which is bounded on the lower edge by the water surface in the well and on the upper edge by the intersection of the free-water surface, or cone of depression, with the well casing.

Standing level - the water level in a non-discharging well. The term is used without regard to whether the well is within or outside the area of influence of other wells. If outside the area of influence, the term is equivalent to static level; if within the area of influence, the standing level registers one point on the cone of depression of another well.

Steady flow - flow in which fluid velocity at a given point does not change with time.

Turbulent flow - flow in which the loss in hydraulic head is more nearly proportional to the square of the velocity rather than to the first power of the velocity as in laminar flow. Turbulent flow generally occurs above a Reynolds number of 10.

Unconfined-flow system - flow through an aquifer underlain by a stratum of low permeability. The upper surface of the flowing water is at atmospheric pressure. The water is not confined above by a stratum of low permeability.

Water table - in pervious, granular earth materials, the water table is the upper surface of the body of free water which saturates the material which is sufficiently permeable to permit percolation. In fractured, impermeable rocks and in solution openings, it is the surface at the contact between the water in the openings and the overlying air.