POROSITY IN THE VICINITY OF CONTAINER WALLS

by

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SYNOPSIS

Porosity is greater in the immediate vicinity of container walls than in the body of a porous medium. If the ratio of particle diameter to container diameter is large enough, significant errors will be made in measured porosities. Additional error will be made in porosity measurements if the ratio of particle diameter to height of container occupied by the porous medium is large. Preliminary measurements indicate that there will be an error of about 7.3 percent in observed values of porosity when both of the above ratios are 0.1 .

The porosity in the vicinity of the container walls decreases as the ratio of particle diameter to container diameter decreases. As this ratio approaches zero, the shell porosity approaches the shell porosity of the bottom and top (flat) walls of the container, which is about 0.444 for uniform diameter spheres with a random packing. The equations that have been developed to predict the observed porosity for uniform diameter spheres as a function of the above ratios give calculated values that are within experimental error of the observed values.

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The effect of porosity errors on surface area per unit volume and permeability are treated quantitatively. In addition the results of this paper have been applied to nonspherical particles successfully.

Notation: The symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in Appendix I.

INTRODUCTION

Franzini[2] derived an expression for the measured porosity of a porous medium composed of uniform diameter spheres that can be stated as follows:

$$\epsilon_M = \epsilon_C + \left(\epsilon_S - \epsilon_C\right)\left(\frac{d}{D}\right)\left(2 - \frac{d}{D}\right) \tag{1}$$

Where

ϵ_M = measured porosity, dimensionless

ϵ_C = core porosity, dimensionless

ϵ_S = shell porosity, dimensionless

d = sphere diameter, cm

D = diameter of cylindrical container, cm

In the derivation of Equation 1, Franzini considered the medium to be made up of two volumes, a core of diameter (D-d) and an outer shell of thickness d/2. Franzini determined experimentally that ϵ_S increased with d/D. It should be noted that $\epsilon_M = \epsilon_C$ if $d/D = 0$ and $\epsilon_M = \epsilon_S$ if $d/D = 1$.

Although it may appear that the choice of the thickness of the outer shell is somewhat arbitrary, consider the expression developed if the thickness of the outer shell is chosen to be d instead of d/2:

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[2] "Permeameter Wall Effect," by J. B. Franzini, Transactions, American Geophysical Union, Vol. 37, No. 6, December, 1956, pp. 735-737.

$$\epsilon_M = \epsilon_C + \left(\epsilon_S - \epsilon_C\right)\left(4\frac{d}{D}\right)\left(1 - \frac{d}{D}\right) \qquad (2)$$

In this case $\epsilon_M = \epsilon_C$ for both $d/D = 0$ and $d/D = 1$. Because it is desirable for ϵ_C to be independent of d/D, Equation 2 is unsatisfactory and will not be used.

For spheres, ϵ_C would be expected to be somewhere between 0.4764 (orthogonal packing) and 0.2595 (rhombic packing)[3]. The average of these two values is 0.3680 and it has been determined experimentally[4] that for random packing of spheres, ϵ_C is approximately 0.37.

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[3] Fair, G. M. and Geyer, J. C. Water Supply and Waste-Water Disposal. New York: John Wiley and Sons, Inc., 1954, page 217.

[4] Closure of "Turbulent Flow in Porous Media", by J. C. Ward, Journal of the Hydraulics Division, ASCE, Vol. 92, No. HY 3, May, 1966, 10 pages, in press.

DIAMETER-TO-HEIGHT RATIO

If one also takes into account the height, H, of the porous medium in the cylinder, the medium can be considered to be made up of three volumes: a core volume of diameter (D-d) and height (H-d), an outer circular shell of thickness d/2 and height (H-d), and two flat shells of diameter D and thickness d/2. Because ϵ_S increases with d/D, it is reasonable to assume that the minimum value of ϵ_S, which will be designated ϵ_{S0}, will be the porosity of the flat shells at the top and bottom of the container. The porosity of the outer shell will be ϵ_{S0} when d/D = 0. The above leads to the following expression for ϵ_M:

$$\epsilon_M = \epsilon_C \left(1 - \frac{d}{H}\right) + \left(\epsilon_S - \epsilon_C\right) \left(\frac{d}{D}\right) \left(2 - \frac{d}{D}\right) \left(1 - \frac{d}{H}\right) + \epsilon_{S0} \frac{d}{H} \tag{3}$$

where H = height of the porous medium in the cylindrical container, cm

ϵ_{S0} = minimum value of ϵ_S, dimensionless. If d/H = 0, Equation 3 is the same as Equation 1. If d/H = 1, $\epsilon_M = \epsilon_{S0}$. By choosing the volumes in this manner, ϵ_S is a function of d/D only.

EVALUATION OF ϵ_{S0}

The first step necessary for using Equation 3 is the experimental evaluation of ϵ_{S0}. This can be expidited by defining α and β so that

$$\alpha \equiv \frac{d}{D} \left(2 - \frac{d}{D}\right) \left(1 - \frac{d}{H}\right) \tag{4}$$

and

$$\beta \equiv \alpha + \frac{d}{H} \qquad (5)$$

where α and β are dimensionless.

Substituting Equation 4 into Equation 3, one obtains:

$$\epsilon_M = \epsilon_C \left(1 - \frac{d}{H}\right) + \left(\epsilon_S - \epsilon_C\right) \alpha + \epsilon_{S0} \frac{d}{H} . \qquad (6)$$

As $d/D \longrightarrow 0$, $\epsilon_S \longrightarrow \epsilon_{S0}$, and therefore

$$\epsilon_M \longrightarrow \epsilon_C \left(1 - \frac{d}{H}\right) + \left(\epsilon_{S0} - \epsilon_C\right) \alpha + \epsilon_{S0} \frac{d}{H} . \qquad (7)$$

Substituting Equation 5 into Equation 7, one obtains:

$$\epsilon_M \longrightarrow \epsilon_C + \left(\epsilon_{S0} - \epsilon_C\right) \beta . \qquad (8)$$

Therefore a plot of ϵ_M versus β, will give a straight line of slope $(\epsilon_{S0} - \epsilon_C)$ for those points with the lowest d/D ratios.

EXPERIMENTAL RESULTS

The experimental results are tabulated in Table 1. The first four columns are the actual experimental data. The observed value of ϵ_M was determined with water. The value of β was calculated from the data in the second and third columns using Equations 4 and 5.

Figure 1 is a plot of the observed value of ϵ_M in the fourth column versus the value of β in the sixth column of Table 1. Each

TABLE 1 - EXPERIMENTAL RESULTS

d, cm	d/D	d/H	ϵ_M		β	ϵ_S
			observed	calculated		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.6	0.0746	0.0341	0.388	0.387	0.173	0.482
0.6	0.100	0.0190	0.393	0.392	0.205	0.489
0.6	0.121	0.0245	0.400	0.398	0.246	0.495
0.6	0.167	0.0258	0.415	0.413	0.324	0.517
0.6	0.231	0.0328	0.443	0.436	0.427	0.548
0.5	0.0623	0.0282	0.385	0.384	0.146	0.478
0.5	0.0834	0.0158	0.385	0.388	0.173	0.459
0.5	0.101	0.0240	0.386	0.393	0.211	0.445
0.5	0.139	0.0211	0.400	0.404	0.274	0.482
0.5	0.192	0.0281	0.417	0.422	0.365	0.577
0.4	0.0497	0.0228	0.379	0.380	0.118	0.452
0.4	0.0667	0.0125	0.385	0.384	0.140	0.480
0.4	0.0806	0.0166	0.393	0.387	0.169	0.506
0.4	0.112	0.0172	0.397	0.394	0.214	0.500
0.4	0.154	0.0230	0.404	0.408	0.300	0.488
0.3	0.0374	0.0171	0.377	0.378	0.0892	0.450
0.3	0.0500	0.0095	0.379	0.380	0.106	0.456
0.3	0.0604	0.0126	0.381	0.382	0.128	0.456
0.3	0.115	0.0166	0.396	0.396	0.230	0.489

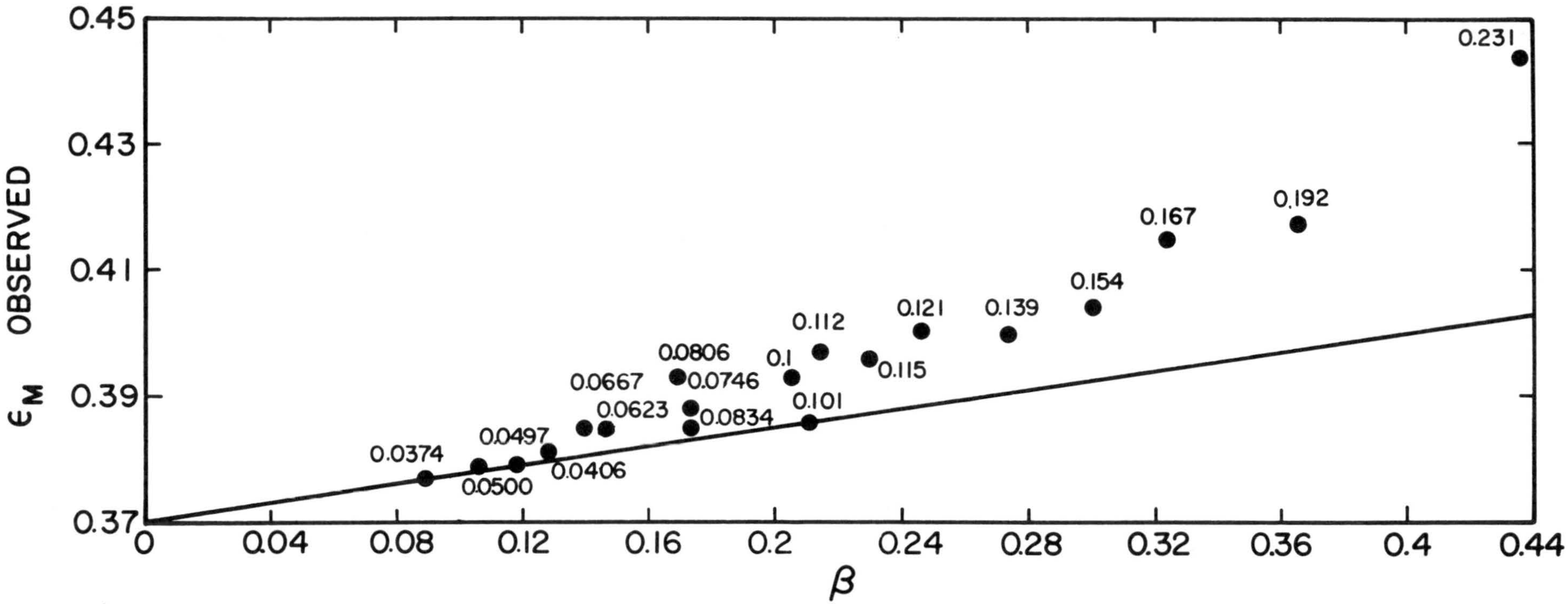

FIG. 1 ϵ_M OBSERVED VERSUS β

point is labeled with the appropriate value of d/D given in the second column of Table 1. The value of ϵ_{S0} obtained from the straight line shown, using Equation 8, is 0.444. It should be noted that, in general, as the slope of the line is increased, it passes through successively higher values of d/D (the steepest line passes through the largest value of d/D).

Solving Equation 6 for ϵ_S, one obtains:

$$\epsilon_S = \frac{\epsilon_M + \epsilon_C (\beta - 1) - \epsilon_{S0} \frac{d}{H}}{\alpha} \tag{9}$$

Equation 9 was used to calculate the values of ϵ_S given in the seventh column of Table 1 using the values of ϵ_M, β, and d/H given in columns four, six, and three respectively, along with Equation 5 and values for ϵ_C and ϵ_{S0} of 0.370 and 0.444 respectively. Because ϵ_S is a function of d/D only, it is possible to determine the functional relationship by plotting the calculated values of ϵ_S in the seventh column of Table 1 versus the values of d/D listed in column 2. This has been done in Figure 2. The numbers above or beside each plotted point are the sizes of the uniform diameter spherical flint glass beads in millimeters. Although the size of the spheres should have no effect, it appears that the best results were given by the 6 and 3 mm beads.

There are undoubtedly several reasons for the wide scatter of the points in Figure 2. Listed in decreasing order of importance they appear to be:

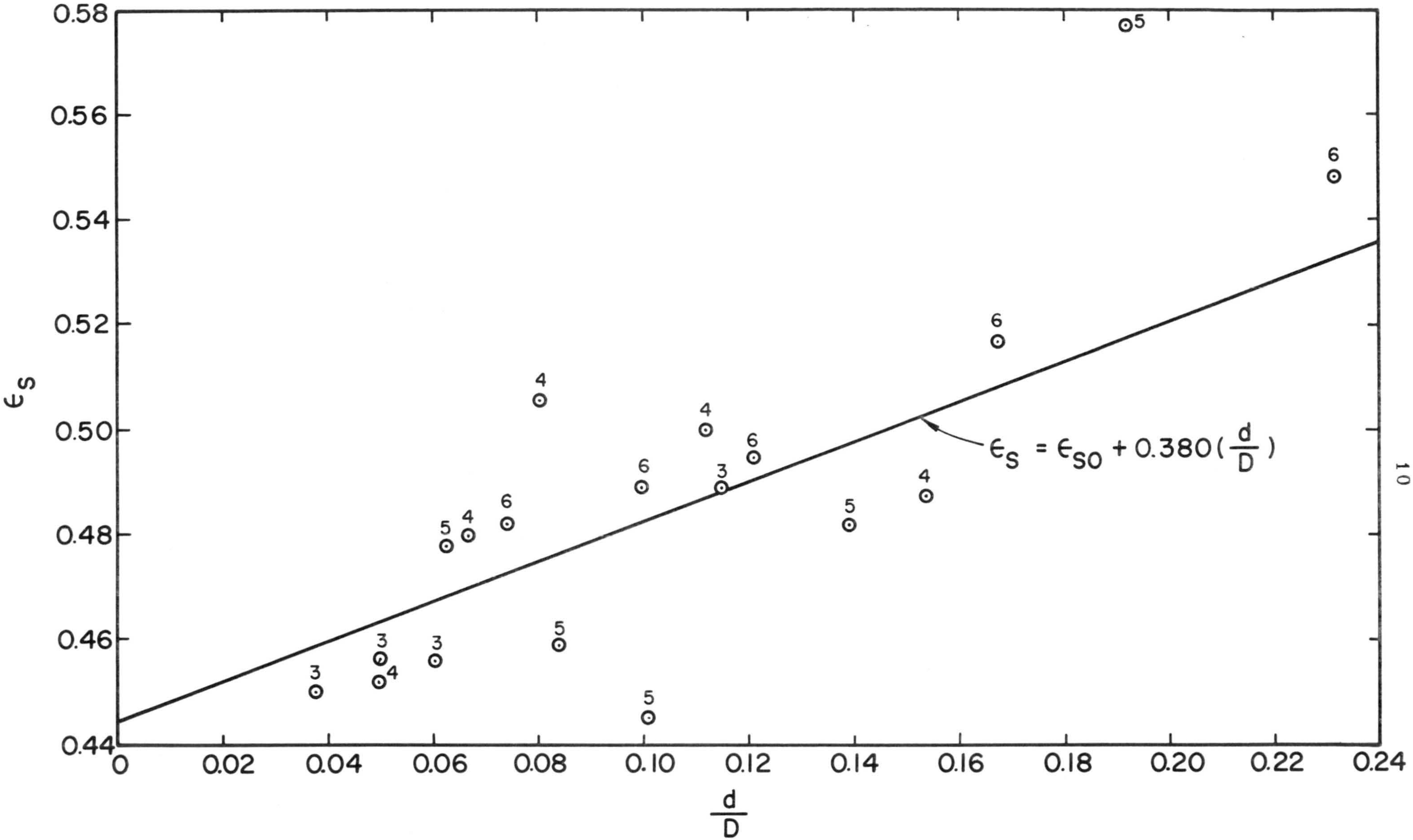

FIG. 2 ϵ_S VERSUS $\frac{d}{D}$

(1) Error in the evaluation of ϵ_{S0} .

(2) Experimental error in ϵ_M .

(3) Possible slight error in the value of ϵ_C .

(4) Error in the value of d .

(5) Error in the value of D .

(6) Error in the value of H .

(7) Other .

At any rate there is no justification for any but the simplest possible functional form which is:

$$\epsilon_S = \epsilon_{S0} + C\left(\frac{d}{D}\right) \tag{10}$$

where C = constant, dimensionless. C was evaluated by solving Equation 10 for C:

$$C = \frac{\epsilon_S - \epsilon_{S0}}{d/D} = \frac{\epsilon_S - 0.444}{d/D} \tag{11}$$

The values of ϵ_S and d/D listed in columns 7 and 2 respectively of Table 1 were used to obtain an average value of C of 0.380, and the standard deviation of C was determined to be ± 0.050 .

Combining Equations 5, 6 and 10, one obtains:

$$\epsilon_M = \epsilon_C + \beta\left(\epsilon_{S0} - \epsilon_C\right) + C\left(\frac{d}{D}\right)\alpha \tag{12}$$

Substituting in the values of ϵ_C, ϵ_{S0}, and C previously obtained, Equation 12 becomes:

$$\epsilon_M = 0.370 + 0.074\ \beta + 0.380\ a \left(\frac{d}{D}\right) \tag{13}$$

Equations 5 and 13 along with the values of d/D, d/H, and β given in columns 2, 3, and 6 of Table 1 respectively were used to calculate the values of ϵ_M given in column 5 of Table 1.

Figure 3 is a plot of the calculated values of ϵ_M versus the observed values of ϵ_M. Despite the scatter in Figure 2, there is reasonably good agreement; the maximum error in the calculated value of ϵ_M (for the points in Table 1) being $\pm$ 0.007 or about $\pm$ 2%. It is conceivable that the experimental error associated with the observed value of ϵ_M could be almost this much. Both of the double-circled points in Figure 3 indicate two points each with the same coordinates.

The points in Figure 3 with an F above them were obtained from Franzini's data[2] (given in Table 2) in the following manner. Because Franzini calculated the values of ϵ_M in Table 2, they were slightly higher, and he estimated the value of ϵ_C to be 0.378. If both sides of Equation 12 are divided by ϵ_C, the result is:

$$\frac{\epsilon_M}{\epsilon_C} = 1 + \beta\left(\frac{\epsilon_{S0}}{\epsilon_C} - 1\right) + \frac{C}{\epsilon_C}\left(\frac{d}{D}\right) a \tag{14}$$

If it is assumed that the ratios ϵ_{S0}/ϵ_C and C/ϵ_C are constant for any value of ϵ_C, then if the values of ϵ_C, ϵ_{S0}, and C previously obtained are substituted into Equation 14, the result is:

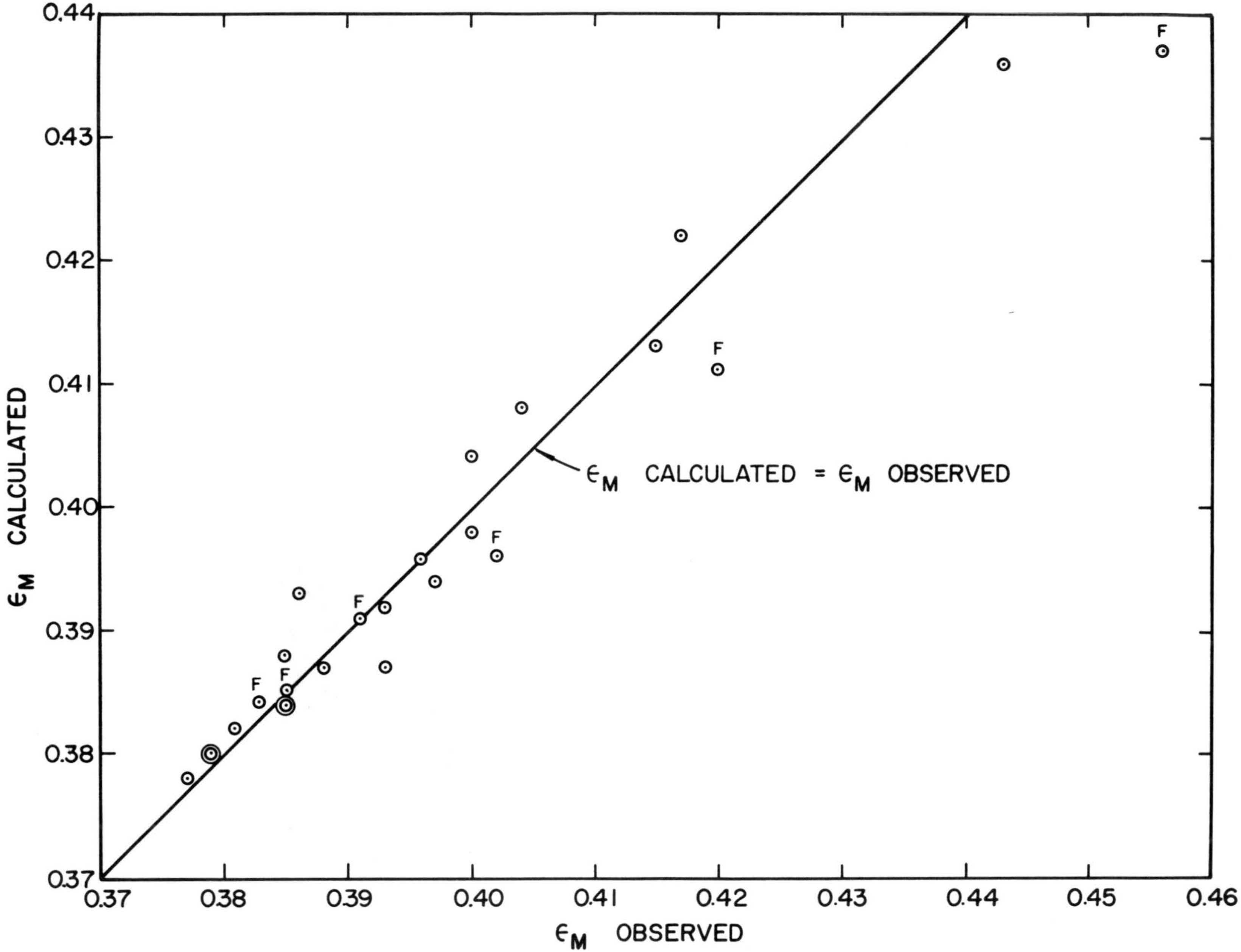

FIG. 3 ϵ_M CALCULATED VERSUS ϵ_M OBSERVED FOR UNIFORM DIAMETER SPHERES

$$\frac{\epsilon_M}{\epsilon_C} = 1 + 0.200\ \beta + 1.027\ \alpha \left(\frac{d}{D}\right). \tag{15}$$

TABLE 2 - FRANZINI'S EXPERIMENTAL RESULTS

d/D (1)	d/H (2)	ϵ_M observed (3)
0.208	0.000469	0.456
0.139	0.000763	0.420
0.0820	0.00338	0.402
0.0595	0.00654	0.391
0.0349	0.0193	0.385
0.0283	0.0293	0.383

Equations 4, 5, and 15 were used to calculate the ratios of ϵ_M/ϵ_C for the values of d/D and d/H listed in columns 1 and 2, respectively, of Table 2. These calculated values of ϵ_M/ϵ_C were then multiplied by 0.378 to obtain the calculated values of ϵ_M plotted in Figure 3. Although there is good agreement at the lower porosities, the divergence between the calculated and observed values of ϵ_M appear to increase as the porosity increases; the calculated values being too low with a maximum error of about 4%.

EFFECT OF DIAMETER RATIO ON OBSERVED PERMEABILITY

It is known[4] that

$$k = \frac{\epsilon^3}{KT(1-\epsilon)^2}\left(\frac{V}{A}\right)^2 \tag{16}$$

where k = permeability, cm^2

ϵ = porosity, dimensionless

K = constant that depends on the shape of the cross section of flow, dimensionless

T = tortuosity, dimensionless

V = total volume of the solid portion of a porous medium, cm^3

A = total surface area, cm^2

K is exactly 3 for a cross section formed by closely spaced parallel plates and is exactly 2 for a circular cross section. K is approximately 2.36 ± 0.11 for unconsolidated porous media. The tortuosity of fully saturated isotropic unconsolidated porous media is about 2.

Now

$$a_V = \frac{A}{V_v} \tag{17}$$

where a_V = surface area per unit of total volume, cm^2/cm^3

V_v = total bulk volume of the porous medium, cm^3

Also

$$V_v = \frac{V}{(1 - \epsilon)} \tag{18}$$

Combining Equations 16, 17, and 18, one obtains:

$$k = \frac{\epsilon^3}{K\,T\,a_V^{\;2}} \tag{19}$$

The surface area of wall per unit volume of cylinder is:

$$a_{VW} = \frac{4}{D} \tag{20}$$

where a_{VW} = surface area of wall per unit volume, cm^2/cm^3

The total surface area per unit volume is:

$$a_{VM} = a_V + a_{VW} \tag{21}$$

where a_{VM} = total surface area per unit volume, cm^2/cm^3

For uniform diameter spheres,

$$\frac{V}{A} = \frac{d}{6} \tag{22}$$

and

$$a_V = \frac{6\,(1 - \epsilon)}{d} \tag{23}$$

The ratio of a_{VM} to a_V is:

$$\frac{a_{VM}}{a_V} = 1 + \frac{2}{3}\left(\frac{1}{1 - \epsilon}\right)\left(\frac{d}{D}\right) \tag{24}$$

Assuming that the product $K\,T$ remains constant, the measured permeability should be approximately:

$$k_M = \frac{\epsilon_M^3}{K\,T\; a_{VM}^2} \tag{25}$$

where k_M = the observed permeability, cm^2. The ratio of k_M to k then should be approximately:

$$\frac{k_M}{k} = \left(\frac{\epsilon_M}{\epsilon}\right)^3 \left(\frac{a_V}{a_{VM}}\right)^2 \tag{26}$$

It is clear that ϵ in Equation 26 is exactly equal to ϵ_C. In a typical permeameter, H is usually greater than D and therefore d/H is usually less than d/D. Furthermore, it is possible to eliminate the effects due to d/H altogether by taking pressure readings in between the two ends of the porous medium. For these reasons the ratio of ϵ_M to ϵ in Equation 26 can be obtained by setting d/H = 0 in Equation 4 and 5 with the result that:

$$\beta = \alpha = \frac{d}{D}\left(2 - \frac{d}{D}\right)\left(\text{for } \frac{d}{H} = 0\right) \tag{27}$$

so that Equation 15 becomes

$$\frac{\epsilon_M}{\epsilon} = 1 + \frac{d}{D}\left(2 - \frac{d}{D}\right)\left(0.200 + 1.027\,\frac{d}{D}\right) \tag{28}$$

Table 3 gives the calculated values of three ratios as a function of the diameter ratio. The values in the first column are the number of spheres that could be placed in a single horizontal layer for the d/D ratio given in the second column. It is apparent that the values of d/D of practical significance will, in all probability, be less than 0.414.

The value of ϵ_M/ϵ given for d/D = 1 was calculated from the fact that $\epsilon_M = 1/3$ for d/D = 1. The value of ϵ used in all cases was 0.370. All the other values of ϵ_M/ϵ were calculated using Equation 28.

TABLE 3 - ϵ_M/ϵ, a_{VM}/a_V, and k_M/k AS A FUNCTION OF d/D FOR UNIFORM DIAMETER SPHERES WITH d/H = 0

Number of spheres in a single layer (1)	$\frac{d}{D}$ (2)	$\frac{\epsilon_M}{\epsilon}$ (3)	$\frac{a_{VM}}{a_V}$ (4)	$\frac{k_M}{k}$ (5)	$\left(\frac{a_{VM}}{a_V}\right)\left(\frac{k_M}{k}\right)$ (6)
1	1.000	0.900	2.000	0.183	0.366
2	0.500	1.535	1.769	1.160	2.050
3	0.465	1.482	1.682	1.152	1.940
4	0.414	1.410	1.573	1.131	1.780
	0.300	1.259	1.373	1.057	1.451
	0.200	1.146	1.232	0.992	1.222
	0.100	1.057	1.092	0.993	1.084
	0.010	1.004	1.015	0.983	0.997
	0.001	1.000	1.000	1.000	1.000

All the values of a_{VM}/a_V were calculated using Equation 24 substituting ϵ_M for ϵ in that equation. ϵ_M was determined by multiplying the ratio ϵ_M/ϵ given in the third column of Table 3 by 0.370.

The values of ${}^{k}M/k$ in the fifth column of Table 3 were calculated using Equation 26 and the values of ${}^{\epsilon}M/\epsilon$ and ${}^{a}VM/{}^{a}V$ listed in columns three and four respectively.

All of the values given in Table 3 are plotted in Figure 4. According to the ${}^{k}M/k$ curve in Figure 4, there will be negligible error in k for any value of d/D less than about 0. 24, but this assumes that the product KT remains constant. It is quite likely that this is not the case and the product KT probably decreases from about 4. 72 to somewhere in the vicinity of its minimum possible value of 2 for laminar flow in a circular pipe. In other words, the ratio $(KT)/(KT)_M$ could conceivably increase from 1 to roughly 2 as d/D increases from 0. 001 to 1. As a very crude estimate of the variation of the ratio $(KT)/(KT)_M$, it could be assumed that it is roughly equivalent to ${}^{a}VM/{}^{a}V$. If this is approximately the case, then the actual variation in permeability would be given roughly by the product $(k_M/k)\,({}^{a}VM/{}^{a}V)$.

If the criteria of a permissible error of 1% is arbitrarily chosen, then Figure 4 gives the following maximum allowable values of d/D:

(1) for porosity measurements, d/D should be $\leqq$ 0. 02

(2) for surface area measurements, d/D should be $\leqq$ 0. 005

(3) for permeability measurements, d/D should be $\leqq$ 0. 025

Franzini[2] recommended that for permeability measurements, d/D should be less than about 0. 025. Other values that have been suggested are 0. 05 by Rich[5] and even as high as 0. 1.

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[5] Rich, L. G. Unit Operations of Sanitary Engineering. New York: John Wiley and Sons, Inc. , 1961, page 142.

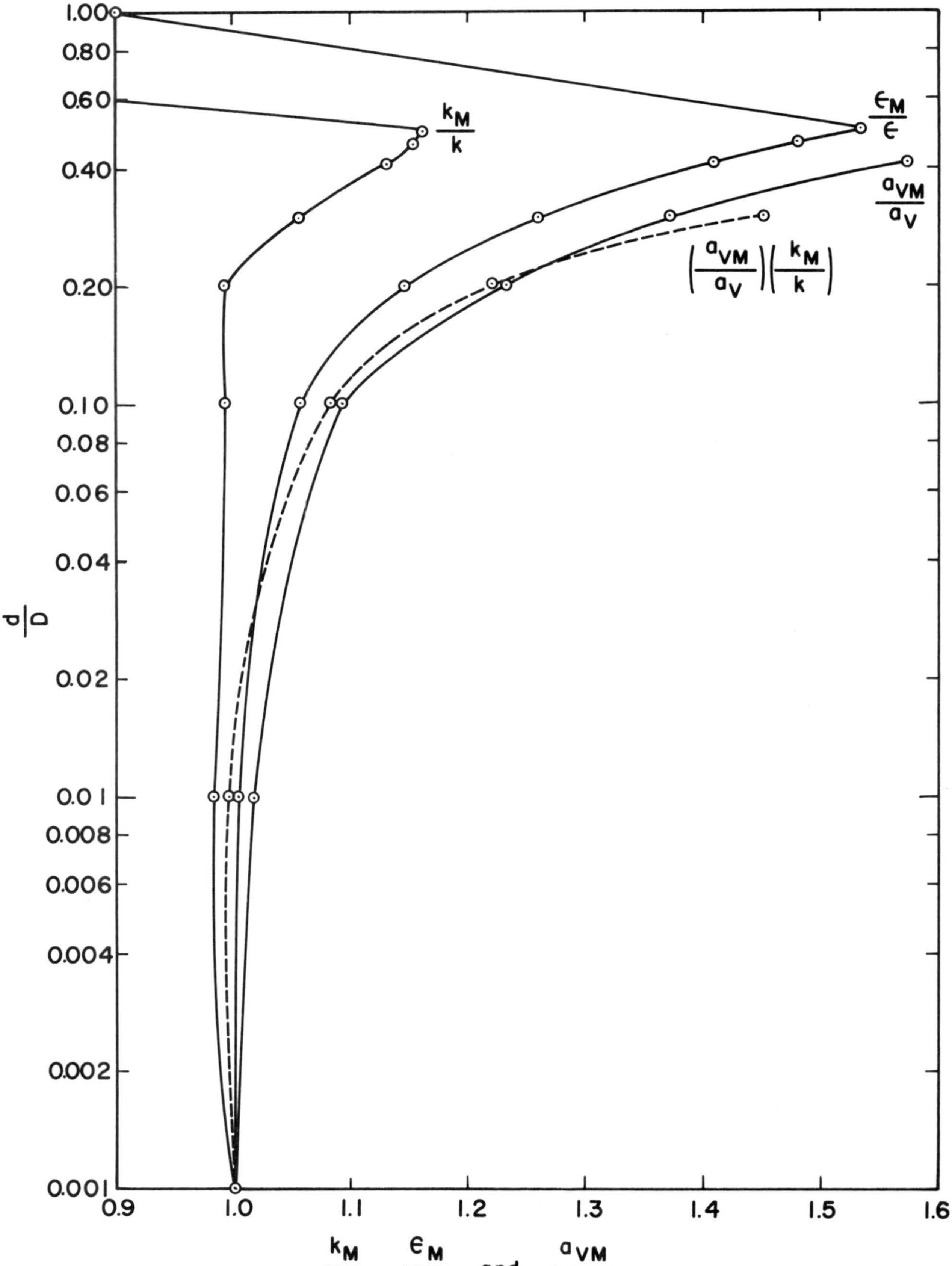

FIG. 4 $\frac{k_M}{k}$, $\frac{\epsilon_M}{\epsilon}$, AND $\frac{a_{VM}}{a_V}$ VERSUS $\frac{d}{D}$ FOR UNIFORM DIAMETER SPHERES WITH $\frac{d}{H} = 0$

BERL SADDLES AND RASCHIG RINGS

In order to partially check the validity of Equation 15 for media other than uniform diameter spheres, uniform size media with a porosity much greater than 0.37 were used with the following properties:

Media	a_V, cm^2/cm^3	ϵ (estimated)	Nominal size, cm
Berl saddles	8.98 (for 0.635 cm size)[6]	0.540	0.6
Raschig rings	9.80 (calculated)	0.582	0.6

The Raschig rings had an outside diameter of 0.6 cm, a length of 0.6 cm, and a wall thickness of 0.1 cm.

Equation 28 can be written as:

$$\frac{\epsilon_M}{\epsilon} = 1 + 0.400 \frac{d}{D} + 1.854 \left(\frac{d}{D}\right)^2 - 1.027 \left(\frac{d}{D}\right)^3 \qquad (29)$$

which approaches the following equation as d/D approaches zero:

$$\epsilon_M = \epsilon + 0.400 \, \epsilon \frac{d}{D} \qquad (30)$$

It is clear from Equation 30 (if d/H is negligible) that a plot of ϵ_M versus $1/D$ will approach a straight line as D approaches ∞ and therefore ϵ is the projected value of ϵ_M at $1/D = 0$.

Using the values of ϵ_M and D given in Table 4, this was the method used to determine the values of ϵ given above.

It is known[4] that:

$$D_p = \frac{C}{\phi_s} \frac{V}{A} \qquad (31)$$

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[6] Leva, M. Tower Packings and Packed Tower Design. Akron, Ohio: The United States Stoneware Company, 1953, page 14.

TABLE 4 - BERL SADDLES AND RASCHIG RINGS

Media (1)	D, cm (2)	ϵ_M observed (3)	H, cm (4)	d/D (5)	d/H (6)	β (7)	ϵ_M calculated (8)
Berl saddles	2.58	0.598	17.95	0.204	0.0294	0.385	0.612
d = 0.528 cm	3.60	0.586	23.50	0.146	0.0224	0.287	0.592
	4.95	0.583	24.20	0.107	0.0218	0.220	0.575
	5.97	0.575	31.00	0.0884	0.0170	0.184	0.563
	7.85	0.567	24.60	0.0672	0.0214	0.149	0.567
	∞	0.540	∞	0	0	0	0.540
Raschig rings	2.58	0.656	16.95	0.192	0.0293	0.366	0.664
d = 0.496 cm	3.60	0.634	21.10	0.138	0.0235	0.274	0.635
	4.95	0.625	23.55	0.100	0.0211	0.207	0.618
	5.97	0.610	30.65	0.0831	0.0162	0.173	0.610
	7.85	0.609	30.10	0.0633	0.0165	0.137	0.603
	∞	0.582	∞	0	0	0	0.582

Where

D_p = particle diameter, cm

ϕ_s = particle shape factor, dimensionless.

ϕ_s has a maximum possible value of one for spheres. Further it has been demonstrated[4] that the following empirical equation is valid for $\epsilon \leqq 0.78$ and $1 \leqq \sigma_g \leqq 2$:

$$\phi_s = \frac{1}{\epsilon}\left(\frac{0.198}{\sigma_g} + 0.294\right) - 0.330 \tag{32}$$

Where

σ_g = geometric standard deviation of the particle size distribution of a porous medium, dimensionless.

Combining Equations 17 and 18 gives:

$$\frac{A}{V} = \frac{a_V}{1-\epsilon} \tag{33}$$

and in general it is known[4] that:

$$\frac{A}{V} = \frac{6}{\phi_s} \frac{\sigma_g^{0.5 \ln \sigma_g}}{M_g} \tag{34}$$

Where

M_g = geometric mean size of the particles, cm

Because $\sigma_g = 1$ for Berl saddles and Raschig rings, Equations 31, 32, and 33 were used in combination to obtain D_p which was then used for d. In the general case of nonspherical, nonuniform size particles, Equations 31 and 34 can be combined to give:

$$D_p = \frac{M_g}{\sigma_g^{0.5 \ln \sigma_g}} \tag{35}$$

Equation 35 shows that D_p is a function only of particle size parameters which is desirable if it is to be used for d. It should be noted that the effect of σ_g tends to reduce the value of D_p and thus d because the smaller sized particles will tend to reduce ϵ_S and ϵ_{S0} as shown by Equation 32.

The values of β in Table 4 were calculated using Equation 4 and Equation 5. Equation 15 was used to calculate ϵ_M/ϵ_C and this ratio was multiplied by the appropriate value of ϵ_C (ϵ_C = 0.540 for Berl Saddles and ϵ_C = 0.582 for Raschig rings) to obtain the calculated values of ϵ_M listed in the last (eighth) column of Table 4.

Figure 5 is a plot of ϵ_M calculated versus ϵ_M observed for Berl saddles and Raschig rings. The points with an R above them designate Raschig rings and the points with a B above them designate Berl saddles. The correlation for the Raschig rings was better than that for the Berl saddles because a_V was more accurately determined for the Raschig rings.

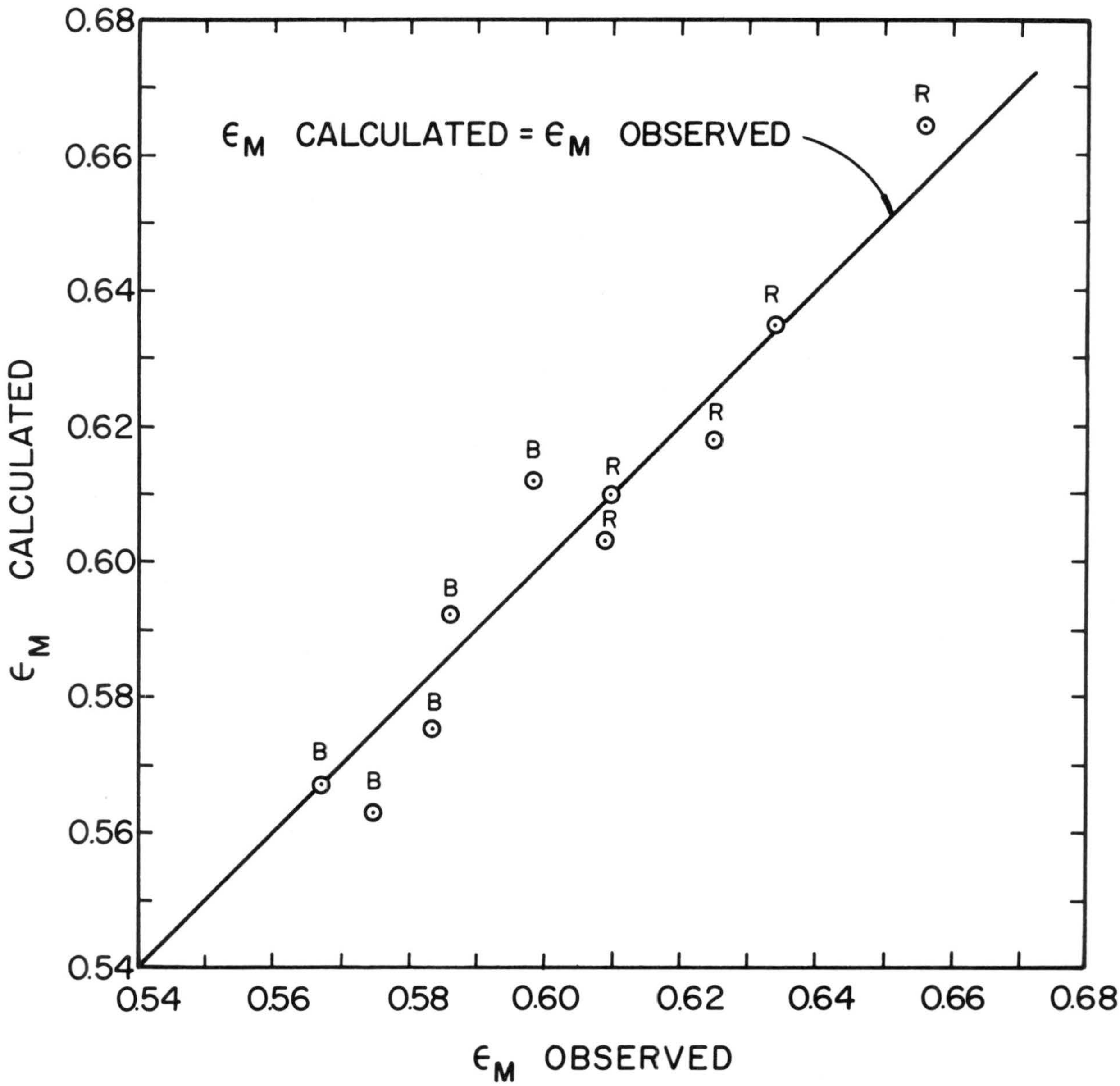

FIG. 5 ϵ_M CALCULATED VERSUS ϵ_M OBSERVED FOR BERL SADDLES AND RASCHIG RINGS

CONCLUSIONS

From the foregoing, it appears that Equation 15 can be restated in its most general form as follows:

$$\frac{\epsilon_M}{\epsilon} = 1 + 0.200\left(\frac{D_p}{H} + \frac{D_p}{D}\left(2 - \frac{D_p}{D}\right)\right)\left(1 - \frac{D_p}{H}\right)\left[0.200 + 1.027\left(\frac{D_p}{D}\right)\right] \quad (36$$

Equation 36 appears to apply to all kinds of unconsolidated porous media. However, since all of the experimental work was done with media with $\sigma_g = 1$ (uniform size) it is believed worthwhile to perform the porosity experiments on media where $\sigma_g > 1$.

Equation 24 can be restated in its most general form by combining Equations 20, 21, 31, and 33 to give:

$$\frac{a_{VM}}{a_V} = 1 + \frac{2}{3}\left(\frac{\phi_s}{1-\epsilon_M}\right)\left(\frac{D_p}{D}\right) \quad (37)$$

Finally, Equation 26 can be written in its most general form by rewriting Equation 25 as follows:

$$k_M = \frac{\epsilon_M^3}{(KT)_M \, a_{VM}^2} \quad (38)$$

and dividing Equation 38 by Equation 19 gives:

$$\frac{k_M}{k} = \left[\frac{(KT)}{(KT)_M}\right]\left(\frac{\epsilon_M}{\epsilon}\right)^3\left(\frac{a_V}{a_{VM}}\right)^2 \quad (39)$$

It is also believed that it would be worthwhile to experimentally evaluate the variation of the ratio $(KT)/(KT)_M$ as a function of D_p/D .

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APPENDIX I - NOTATION

The following symbols have been adopted for use in this paper:

a_V	=	surface area per unit of bulk volume,	L^{-1}
a_{VM}	=	total surface area per unit of bulk volume,	L^{-1}
a_{VW}	=	surface area of wall per unit volume of cylinder,	L^{-1}
A	=	total surface area, of porous medium,	L^2
C	=	constant with a value of 0.380 ± 0.050,	
d	=	sphere diameter,	L
D	=	diameter of cylindrical container,	L
D_p	=	particle diameter,	L
H	=	height of the porous medium in the cylindrical container,	L
k	=	permeability,	L^2
k_M	=	observed permeability	L^2
K	=	constant that depends on the shape of the cross section of flow and has a value between 2 and 3 and is approximately 2.36 ± 0.11 for unconsolidated porous media;	
$(KT)_M$	=	observed value of the product KT which is ordinarily about 4.72 for unconsolidated porous media;	
L	=	fundamental unit of length,	L
M_g	=	geometric mean particle size of the porous medium,	L
T	=	tortuosity and is approximately 2 for isotropic unconsolidated porous media;	
V	=	total volume of solid portion of a porous medium,	L^3
V_v	=	total bulk volume of a porous medium,	L^3

NOTATION - continued:

α = $\frac{d}{D}\left(2 - \frac{d}{D}\right)\left(1 - \frac{d}{H}\right)$;

β = $\alpha + \frac{d}{H}$;

ϵ = porosity = ϵ_C ;

ϵ_C = core porosity = ϵ ;

ϵ_M = measured porosity ;

ϵ_S = shell porosity;

ϵ_{S0} = minimum value of ϵ_S ;

σ_g = geometric standard deviation of the particle size distribution of an unconsolidated porous medium;

ϕ_s = particle shape factor (ϕ_s has a maximum possible value of one for spheres).