International Association

For Hydraulic Research

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SYNOPSIS

A study of total bed-material discharge in alluvial channels, partially based on existing theories, has been conducted.

The bedload discharge was obtained by utilizing the energy-work relation of the fluid and the bed material. An equation for velocity distribution was obtained by integrating the Reynolds equations using Prandtl's hypothesis of mixing length.

Applying the basic equation for the distribution of suspended load by O'Brien, the suspended bed-material discharge was investigated in terms of bedload discharge. Then, the total bed-material discharge was obtained simply by adding the bedload discharge and the suspended bed-material discharge.

The results through checking with available laboratory and field data appeared to be mutually consistent and satisfactory.

RESUME

Une étude du débit solide total dans les fleuves alluvionnaires, basée en partie sur des théories existantes, a été entreprise.

Ledebit solide de fond a été obtenu en utilisant la relation travail-énergie du fluide et du materiau du lit. Une formule donnant la repartition des vitesses a été obtenue en intégrant les equations de Reynolds et en utilisant l'hypothèse de la longuer de mélange de Prandtl.

Le débit solide en suspension a été etudié en fonction du débit solide de fond en appliquant la formule fondamentale pour la repartition des solides en suspension d'O'Brien. Puis le debit solide total a simplement été calcule en additionnant les debits solides de fond et en suspension.

La verification de ces calculs avec les resultats disponibles de mesures effectuées en laboratoires et in situ semblent mutuellement sonsistants et satisfaisants.

1. Introduction

The cross section of alluvial channels is constantly changing in time because of the unsteady nature of the water and bed-material discharge. Any change in water discharge results in a pertubation in the sediment discharge with the consequence that the bed of the stream is either scoured or filled. Over a long time period the net scour and fill may be zero. Thus there may be no net change in the cross section. But during this time period the cross section will be constantly varying with time. Knowledge of this time variance of the cross section is very important to many aspects of hydraulic engineering such as design of channels, flood-relief ways, and highway-bridge crossings.

A recent study has developed a method of predicting bed-material transport for the steady case. This method by proper programing for computer analysis may be used to determine the sediment transport as a function of time. Thus by proper routing of the water and sediment discharge through a cross section the variance with time of this cross section can be determined.

In this study the principal factors investigated included:

a. The classification of the graded sands which comprise the bed material of natural alluvial channels.
b. The development of the velocity distribution which applies to open channel flow.
c. The subdivision of the total bed-material load into components based upon significant forces to which the particles are subjected.
d. Relations for estimating suspended bed-material discharge, bedload discharge, and hence, total bed-material discharge.
e. Application of the total bed-material discharge relations to laboratory and field conditions.

The laboratory data used for the investigation were collected by the U. S. Geological Survey (1) at Colorado State University, Fort Collins, Colorado, the field data were given by Bishop (2) and were also tabulated in Chang's (3) dissertation.

Notation: The letter symbols used are defined and presented alphabetically at the end of the paper.

2. Determination of Sediment Size

Numerous methods have been used to define a representative mean diameter of bed material. These may be divided into two groups: (a) methods based on physical size, and (b) method based on fall velocity. The second method was developed because the physical size of a sediment particle is not an adequate measure of the behavior of the particle moving in a fluid. This idea was further extended to finding the representative mean diameter by considering the energy dissipated by the relative motion between a particle and a fluid.

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(1) Simons, D. B. and Richardson, E. V., 1961, Studies of flow in alluvial channels--basic data from flume experiments. Colorado State University, Civil Engineering Dept., Rept. CER61EVR31.

(2) Bishop, A. A., 1961, Sediment transport in alluvial channels, a critical examination of Einstein's theory. Ph.D. dissertation, Colorado State University, Fort Collins, Colorado.

(3) Chang, F. M., 1962, An investigation of total sediment discharge in a alluvial channels. Ph.D Dissertation, Colorado State University, Fort Collins, Colorado.

The equivalent diameter, d , is defined as the diameter of a particle which may represent a group of particles such that the product of the number of the particles and the rate of work done by a particle having this equivalent diameter is equal to the summation of the rate of work done by all individual particles. The equivalent diameter can be obtained by computing equivalent fall velocity, ω , by

$$\omega = \sum_{i=0}^{P_t} p_i \omega_i \Big/ \sum_{i=0}^{P_t} p_i \tag{1}$$

then d can be determined from the relationship between fall velocity and the diameter of particle (4).

3. Velocity Distribution in Open Channels

The Reynolds equation for two dimensional, steady uniform, open channel flow is expressed as

$$\rho g \operatorname{Sin} a + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \rho \frac{\partial}{\partial y}\left(\overline{u' v'}\right) \tag{2}$$

Applying Prandtl's mixing length hypothesis with boundary condition $\left(\mu \frac{u}{y}\right)_{y=0} = \tau_0$ and Eq. 2 can be simplified for the region excluding the thin layer near the boundary as

$$\frac{du}{d\xi} = \frac{U_*}{\kappa \xi} \sqrt{1.0 - \xi} \tag{3}$$

where

$$\xi = y/d$$

$$U_* = \sqrt{\tau_0/\rho}$$

$$\tau_0 = \gamma D \operatorname{Sin} a$$

Integrating Eq. 3, the velocity distribution formula is

$$\frac{U - u}{2U_*/k} = \ln\left(\frac{\sqrt{\xi}}{1 - \sqrt{1-\xi}}\right) - \sqrt{1-\xi} - \frac{1}{3} \tag{4}$$

The analysis of laboratory data shows the relation between Karman's coefficient and Reynolds number $\frac{U_* d}{\gamma}$ as given in Fig. 1.

4. Total Bed-Material Discharge

Sediment transported by fluid, from consideration of the difference in the mechanics of transport, is generally divided into bedload and suspended bed-material load. In two dimensional flow, the total bed-material discharge q_t is

$$q_t = \int_0^a c_b u_b \, dy + \int_a^D c_s u_s \, dy \tag{5}$$

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(4) U. S. Inter-Agency Subcommittee on Sedimentation, 1957. Some fundamentals of particle size analysis. U. S. Inter-Agency Rpt. No. 12, U. S. Geol. Survey, Minneapolis, Minn.

a. Bed-load discharge

Applying du Boys (5) analysis which assumed that the propulsive movement of the granular bed material varied gradually and uniformly from a miximum at surface level to zero at some depth below this level, the thickness of the bedload layer, a , is

$$a = j \frac{\tau_0 - \tau_c}{(1-e)(\gamma - \gamma)\tan\phi} \tag{6}$$

The bedload discharge, q_b , is expressed as

$$q_b = \int_0^a c_b \, u_b \, dy \tag{7}$$

Using mean values, Eq. 7 may be written as

$$q_b = M_1 \, c_m \, U_b \, a \tag{8}$$

where

$$M = \int_0^a c_b \, u_b \, dy \, / \, c_m U_b \, a$$

C_m is the mean concentration of the bedload layer and probably is closely related to the concentration of the stationary bed which has a value of $(1 - e)\gamma_s$, hence

$$C_m = M_2 (1 - e) \gamma_s \tag{9}$$

where M_2 = constant

Combining Eqs. 8 and 9, the bedload discharge per unit width of channel is

$$q_b = M (1-e) \gamma_s \, a \, U_b \tag{10}$$

Using a concept given by Bagnold, R. A. (6), with respect to the movement of the bedload, the system is regarded as a fluid dynamic transporting machine and Eq. 10 can be expressed as

$$q_b = K_b \frac{\gamma_s}{(\gamma_s - \gamma)\tan\phi} (\tau_0 - \tau_c) \, U \tag{11}$$

b. Suspended bed-material discharge

The basic equation for the distribution of suspended bed-material is

$$\epsilon_s \frac{dc}{dy} + \omega c = 0 \tag{12}$$

Applying Prandtl's mixing length hypothesis and Eq. 3, Eq. 12 can be integrated as

$$\frac{C}{C_a} = A_1 \left(\frac{\sqrt{\xi}}{1 - \sqrt{1-\xi}} \right)^z \tag{13}$$

where

$$A_1 = \left(\frac{1 - \sqrt{1 - \xi_a}}{\sqrt{\xi_a}} \right)^z$$

$$z = \frac{2\,\omega}{\beta \, U_* K} \quad , \quad \xi_a = \frac{a}{D}$$

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(5) Leliavsky, S., 1955, An introduction to fluvial hydraulics, Constabel and Company, Ltd., London.

(6) Bagnold, R. A., 1960, Sediment discharge and stream power. U. S. Geol. Survey Circ. 421.

Assuming the ratio of the velocity of the suspended bed material to that of the transporting fluid is nearly unity, suspended bed-material discharge per unit width of channel is

$$q_s = \int_a^D c_s \, u_s \, dy = D \int_{\xi_a}^{1} c \;\; u \;\; dy \qquad (14)$$

which by integration, becomes

$$q_s = D C_a \left(U I_1 - \frac{2 U_*}{\kappa} I_2 \right) \qquad (15)$$

where

$$I_1 = A_1 \int_{\xi_a}^{1} \left(\frac{\sqrt{\xi}}{1 - \sqrt{1-\xi}} \right)^z d\xi$$

$$I_2 = A_1 \int_{\xi_a}^{1} \left(\frac{\sqrt{\xi}}{1 - \sqrt{1-\xi}} \right)^z \left[\ln\left(\frac{\sqrt{\xi}}{1 - \sqrt{1-\xi}} \right) - \sqrt{1-\xi} - \frac{1}{3} \right] d\xi$$

The values I_1 and I_2 were numerically integrated.

As an approximation, the concentration, C_a, was assumed as

$$C_a = \frac{q_b}{a \, u} = \frac{q_b}{r_v a u} \qquad (16)$$

where r_v is the ratio of the velocity at $y = a$ to mean velocity.

Therefore, the total suspended bed-material discharge per unit width of channel q_s is

$$q_s = \frac{D \, q_b}{0.8 a U} \left(U I_1 - \frac{2 U_*}{K} I_2 \right) = R_s q_b \qquad (17)$$

where

$R_s = \frac{D}{0.8 a U} \left(U I_1 - \frac{2 U_*}{K} I_2 \right)$ which is the ratio of suspended bed material to bedload.

5. Total bed-material discharge

The total bed-material discharge per unit width of channel q_T may now be calculated from

$$q_T = q_b + q_s = q_b (1.0 + R_s) \qquad (18)$$

or

$$q_T = K_T (\tau_o - \tau_c) \, U (1.0 + R_s) \qquad (19)$$

where

$$K_T = K_b \frac{\gamma_s}{(\gamma_s - \gamma) \tan \phi}$$

The total bed-material discharge coefficient, K_T, is an experimental coefficient which is a function of the bed material, the bed configuration, and flow characteristics. For the flume data, K_T was found to be a function of bed material and $\frac{U}{U_*} \cdot \frac{\tau_o}{\Delta\gamma \, d} \, S$ as shown in Fig. 2. Computations of values of K_T for each of the three natural rivers studied showed they were constants for each river.

The comparison of the measured total bed-material discharge and the value estimated by Eq. 19 for 184 flume runs and for 57 sets of natural river data is given in Fig. 3. For both cases, agreement between measured and computed bed-material discharge is satisfactory. Bishop (2) computed the total bed-material discharge using Einstein's function for the same flume data. For 23% of flume data, the error using Einstein's function exceeds 100%. Comparing the computed total bed-material discharge by the method presented herein with measured total bed-material discharge, only for 43% of the data does the error exceed 100%

5. Summary and Conclusions

A size classification for graded sands based upon the energy value of the particles was developed. Classification considers the gradation of the sand on the basis of the energy needed to suspend each size fraction. Using this method of size classification, the need to subdivide the bed material into fractions in order to compute the total bed-material discharge is eliminated.

A velocity distribution equation was obtained by integrating the Reynolds equation by using the Prandtl mixing length hypothesis. The theoretical velocity distribution fitted the experimental data for the plane bed configuration very well. Considering the space and time variation of the point velocity when the bed configuration was dunes, the derived velocity distribution curve fit the data as well as could be expected.

An equation was developed to determine the bed-material discharge by dividing it into two parts; the bedload and suspended bed material. The bedload was obtained by applying the work-energy relation between sediment and fluid. The fluid dissipates a part of its energy to transport bedload. The suspended load was obtained by integrating the O'Brien's suspended bed-material distribution equation assuming that the horizontal velocity of suspended bed material is the same as that of the transporting fluid and the sediment transfer coefficient is lineraly proportional to the diffusion coefficient of the fluid.

The total bed-material discharge can be expressed by Eq. 19.

The total sediment discharge coefficient K_T is an experimental coefficient which was found to be a function of the bed material and flow variables for the flume data and was constant for each natural river which was studied. The results through checking with available laboratory and field data appear to be mutually consistent and satisfactory in the sense of sediment transport theory for both flume data and natural river data.

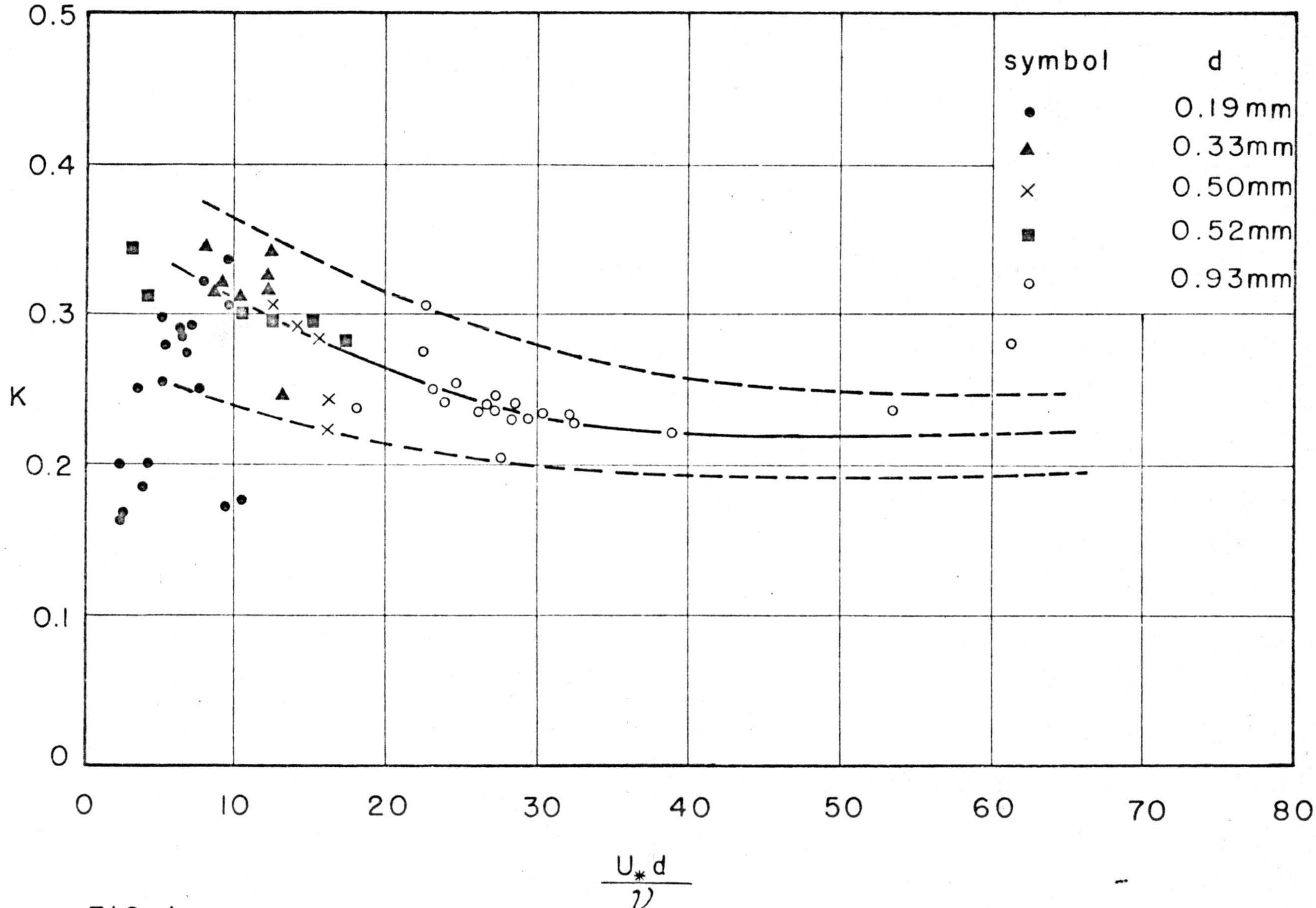
The advantage of the total bed-material discharge formula developed in this paper are: a) because the formula is based on mean velocity of flow, the comparably accurate results can be expected; b) computation is not complicated.

SYMBOLS

a	thickness of bed load layer
C_a	concentration at $y = a$
C_m	mean concentration of the bed load layer
c_b, c_s	weights of sediment in a unit volume of water-sediment mixture in the bedload zone and suspended load zone
c	concentration at distance y from the bed
D	depth of flow
d	equivalent diameter of particle
e	porosity of bed material
g	the gravitational acceleration
j	experimental constant $j \approx 10$
K_b	experimental constant
K_T	total bed-material discharge coefficient
p	percent by weight of the given size fraction

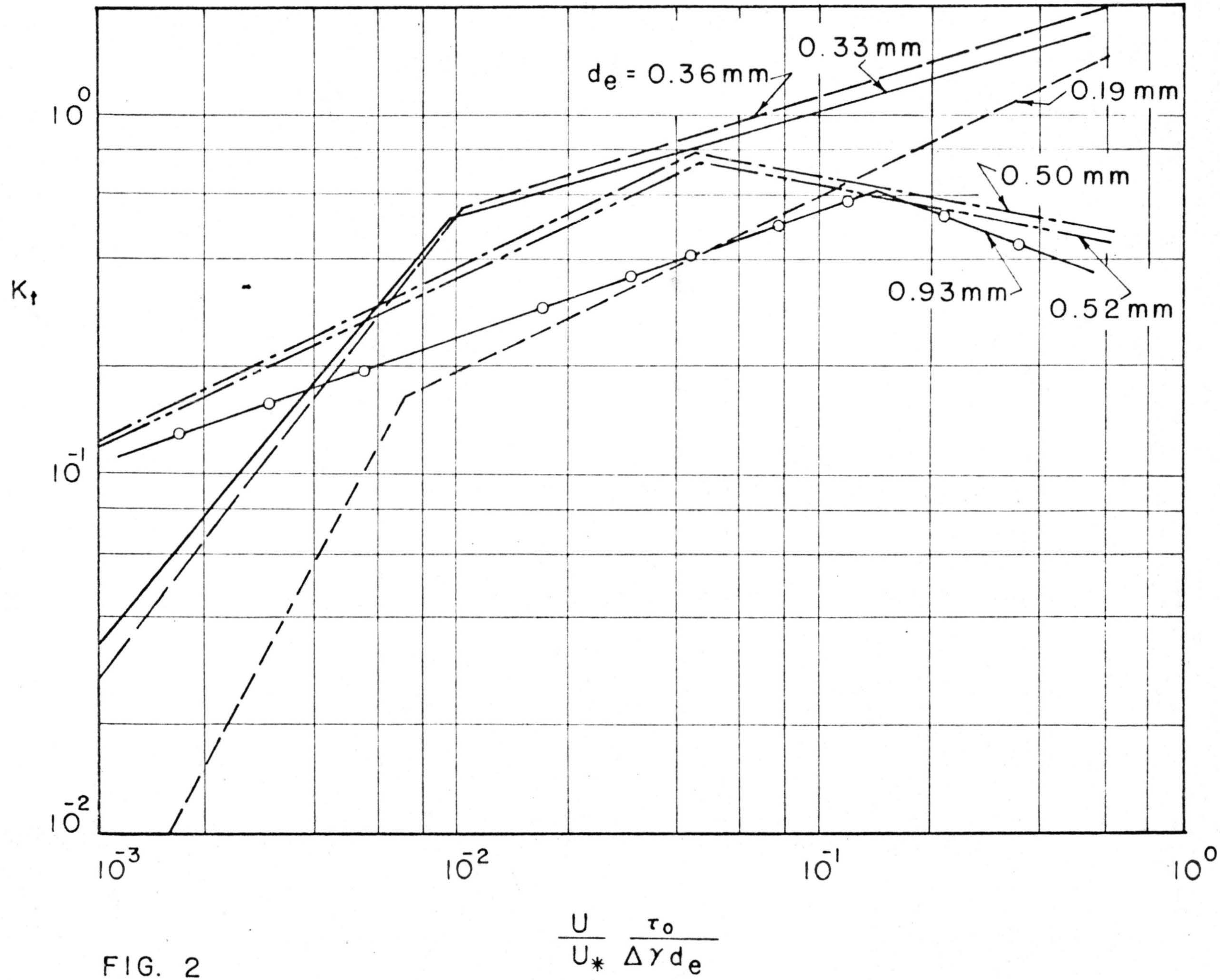
r_v	the ratio of the velocity at $y = a$ to mean velocity of flow. $r_v \approx 0.8$ (Experiment)
U	mean velocity of flow
U_b	mean velocity of bedload
u	velocity at distance y from bed
u_b, u_s	actual velocity of sediment in bedload zone and suspended load zone
u', v'	fluctuating components of velocity in x , y directions
y	distance from bed surface
α	bed slope
β	ratio of the sediment transfer coefficient to the diffusion coefficient of fluid
γ_s, γ	specific weights of sediment and fluid
ϵ_s	sediment transfer coefficient
κ	Karman's coefficient
ξ	relative depth, y/D
μ	dynamic viscosity
ρ_s, ρ	densities of particle and fluid
τ_c	critical tractive force
ϕ	friction angle of bed material in the water
ω	equivalent fall velocity of particle

LIST OF FIGURES



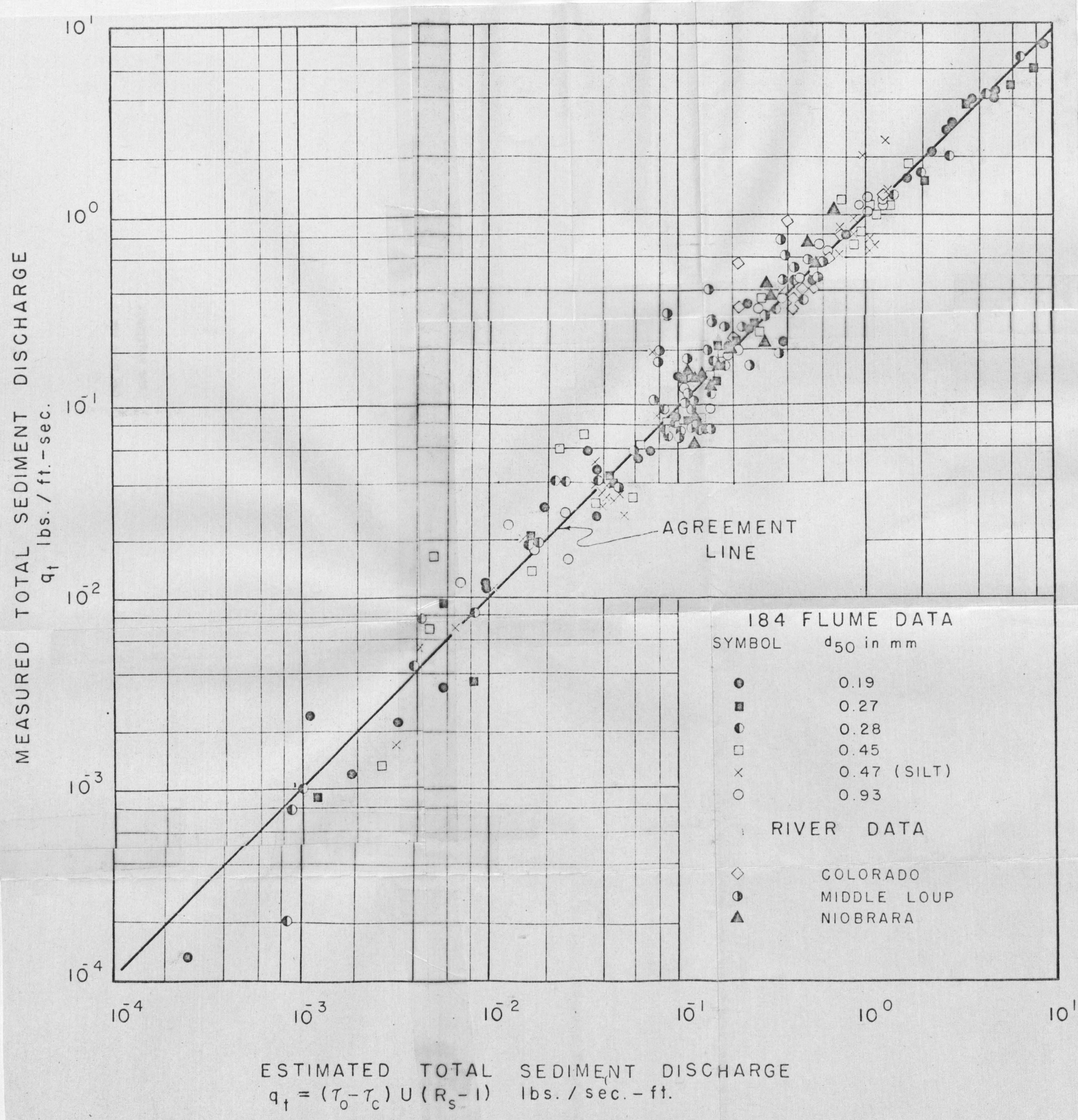
0.5
0.4
0.3
0.2
0.1
0
K
0
10
20
30
40
50
60
70
80
$\frac{U_* d}{\nu}$
symbol
d
0.19mm
0.33mm
0.50mm
0.52mm
0.93mm

FIG. 1



0.33 mm
d_e = 0.36 mm
0.19 mm
0.50 mm
0.93 mm
0.52 mm
K_t
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
$\frac{U}{U_*} \frac{\tau_0}{\Delta\gamma d_e}$

FIG. 2



MEASURED TOTAL SEDIMENT DISCHARGE
q_t lbs. / ft. - sec.
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
AGREEMENT LINE
184 FLUME DATA
SYMBOL d_{50} in mm
0.19
0.27
0.28
0.45
0.47 (SILT)
0.93
RIVER DATA
COLORADO
MIDDLE LOUP
NIOBRARA
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
ESTIMATED TOTAL SEDIMENT DISCHARGE
$q_t = (\tau_0 - \tau_c)\, U\, (R_s - 1)$ lbs. / sec. - ft.

FIG. 3