T H E S I S

A NEW ZERO TEMPERATURE COEFFICIENT QUARTZ OSCILLATOR PLATE

Submitted by

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WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR

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CREDITS none

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Chapter I

INTRODUCTION

Although the piezoelectric effect was discovered in 1881, it did not find extensive practical application for nearly 40 years until the development of the science of radio communication brought a need for some method of accurately controlling the frequencies of electron tube oscillators. The discovery in 1921 that the mechanical vibrations of piezoelectric crystals could be used for this purpose resulted in a revived interest in the long neglected field of piezoelectricity. This interest has continued at an accelerating rate up to the present time.

Thus far, no principle has been proposed which approaches the effectiveness of a piezoelectric crystal as a frequency stabilizing device. This superiority arises from the extremely sharp resonance of the mechanical vibration of a crystal resonator as compared with electrical resonators consisting of inductances and capacitances. Of the many piezoelectric crystals, quartz has the widest application in the field of radio communications. This is due to the fact that quartz is relatively abundant in nature and combines almost ideal elastic qualities with good mechanical strength. But most important of all, resonators can be cut from the quartz crystal in such a way that their mechanical frequencies are nearly independent of temperature, at least over limited temperature ranges. This particular property of quartz has been the basis of numerous investigations which have led to the development of several types of quartz resonators for various applications.

As more services demand space in the radio spectrum, it has become necessary to control, more closely, the operating frequencies of the transmitters. This has resulted in the necessity for limiting the temperature variation of the quartz crystal units which control the transmitters and receivers. This limitation has usually been accomplished by enclosing the crystal unit in a small oven which maintains a temperature somewhat higher than the expected ambient temperature. When the highest expected temperature does not exceed about 70°C, the AT type of resonator is used since it has a zero temperature coefficient at this temperature. In this case the oven thermostat is set to maintain a temperature of approximately 70°C.

In many radio applications, however, the ambient temperature reaches as high as 100°C where the frequency of the AT cut changes rapidly with temperature. Consequently there has arisen a need for a quartz resonator which has a zero temperature coefficient of frequency at a higher temperature.

In spite of the rather large amount of literature upon the subject of piezoelectricity, there is much confusion in the field. This is because each investigator has tended toward using his own particular system of notation and expression. Therefore, attempts to correlate the data of different workers have proven extremely difficult as have all attempts to use these results as a basis for further investigations.

Recently the Committee on Standards of Piezoelectricity of the Institute of Radio Engineers (21) devised a system which it recommended for all future investigations. The purpose of this recommendation is to establish more uniformity and help to clarify existing confusion in the field.

This study will use, whenever possible, the recommended system of notation and expression and, because of the need for clarification, will present the theoretical analysis more completely than might seem necessary. It is hoped that this will prove an asset in further work of this kind.

<u>The problem</u>

Is it possible to obtain a quartz crystal resonator, with useful properties, by multiple rotations about the crystallographic axes?

<u>Problem analysis</u>

1. What are the frequency-temperature characteristics of oscillator plates obtained by multiple rotations?

2. How is the piezoelectric coupling affected by the rotation?

3. What are the electrical properties of quartz plates obtained in this manner?

4. How does multiple rotation affect the frequency spectrum?

<u>Delimitation</u>.--This study is limited to those quartz plates which vibrate in a thickness shear mode of motion and, in particular, to a plate which has a zero temperature coefficient.

<u>Definitions</u>

1. A resonator refers to any piezoelectric device which has a natural mechanical frequency or frequencies, and is vibrated electrically at or near this frequency.

2. A temperature coefficient, unless otherwise specified, expresses the change in frequency with

respect to a change in temperature, this ratio being divided by the frequency.

3. In this study, both a plate and a cut are terms used to denote resonators which vibrate in a thickness mode only and which may be either circular or rectangular.

4. Frequency stability refers to the ability of the resonator to maintain a nearly constant frequency when some parameter is varied. Unless otherwise stated this parameter is the temperature.

5. The activity is a measure of the grid current in the oscillator whose frequency is being controlled by the resonator. It is an indication of the quality of the resonator.

6. A rotated plate or cut refers to a plate whose normal is not parallel to one of the crystallographic axes. A single rotated plate is a cut whose normal is perpendicular to one of the crystallographic axes. A double rotated plate refers to a plate whose normal is not perpendicular to any of the axes.

7. Contouring refers to the process of changing the surface of a plate from plane to convex. The degree of contour refers to the amount of convexity.

8. The frequency spectrum of a given resonator is the representation of the different vibrational

modes or resonance points of the resonator over a given range of frequencies.

Chapter II

REVIEW OF LITERATURE

In order to have a suitable background for this particular investigation it was necessary to have a rather thorough knowledge of previous work on other types of resonators. For this reason this particular chapter has not been restricted only to those resonators which vibrate in a thickness mode but, rather, has been extended to include most of the notable achievements in this particular phase of the field of piezoelectricity.

The mechanical vibration of a quartz crystal resonator was first used to control the frequency of a vacuum tube oscillator by Cady (7) and Nicholson (20) in 1921. Both of these investigators used X-cut plates in which the normal to the plate is parallel to the X or electrical axis. This type of oscillator plate is characterized by a complicated frequency spectrum which results in a tendency for the oscillator to jump from one frequency to another. The X-cut plate also has an unfavorable temperature coefficient. In 1927, Cady, (4) discovered that a Y-cut plate might also be used as a frequency-controlling resonator. The frequency spectrum of this type of plate, whose normal is parallel to the Y or mechanical axis, is somewhat better than the X-cut but the frequency stability is worse.

The first types of zero temperature coefficient resonators were the coupled types which obtain their low temperature coefficient by virtue of the interaction between two modes of motion. Marrison (15) in 1929 described the first of these coupled types. This particular resonator is in the shape of a ring with a shear vibration coupled to a flexure motion. The shear has a positive temperature coefficient while the flexure has a negative coefficient. Due to the coupling the resulting coefficient is nearly zero over a limited temperature range. In 1929, also, Lack (12) described a cut, devised by Heising, which accomplished the same effect by coupling a positive temperature coefficient shear vibration with the harmonic of a low frequency flexure vibration. However, neither of these resonators has proven of much value since they are difficult to adjust and possess several spurious frequencies near the desired mode.

The next low temperature coefficient resonators were of the long-bar type. Matsumura in 1932 and Kansaki in 1933, as pointed out by Cady (5:453), achieved low coefficient resonators of this type by inclining the length of an X bar at an angle of 20° in the Y Z plane and at the same time having a definite ratio of length to width. Mason (17), in a study reported in 1934, used a similar method to obtain low temperature coefficient resonators.

In 1934 it was discovered independently by both Koga in Japan and Straubel in Germany, as pointed out by Cady (5:454) and also Bechmann (2) in Germany, and Lack, Willard, and Fair (13) in New York, that zero temperature coefficient plates could be obtained by rotating the Y-cut a given angle about the X axis. It was shown by each of these investigators that the zero temperature coefficient of these plates results from the fact that the coefficient changes from positive to negative as the plate is rotated about the X axis. It was also shown that there are two different positions where the coefficient becomes zero. These resonator plates, known as the AT and BT, vibrate in a pure shear mode of vibration and therefore, are more nearly independent of undesired spurious modes than either the X-cut or Y-cut. The AT and BT were the first high frequency resonators obtained by rotation about any of the crystallographic axes.

Hight and Willard (9) in 1937 described another set of zero coefficient plates. These plates, the CT and DT, vibrate in a low frequency face shear and their positions in the Y-Z plane are approximately 90° from the BT and AT, respectively.

In 1935, Bechmann, as shown by Cady (5:144), investigated the possibility of obtaining zero temperature coefficient resonators by means of multiple rotations about the crystallographic axes. Bechmann calculated the temperature coefficients, elastic constants, and piezoelectric constants for plates vibrating in thickness modes in several different orientations. Further work along this line was carried out by Koga (11) in 1936. He worked out the numerical values of the elastic coefficients for a large number of orientations. However, neither Bechmann nor Koga appears to have experimentally determined any zero temperature coefficient plate which might be commercially useful.

Mason (18), in a study reported in 1940, stated that a series of zero temperature coefficient angles were calculated by using c'_{66} as the elastic constant determining the frequency. Mason further reported that a few of these plates were measured and found to have zero temperature coefficients but they also had a much more complicated frequency spectrum than the AT or BT crystal plates. This investigator used the results of Bechmann to graphically show the orientation of a series of zero coefficient plates.

<u>Summary</u>

There have been several investigations into the possibilities of obtaining zero temperature coefficient oscillator plates by multiple rotations about the crystallographic axes. However these investigations seem to have been of a theoretical nature with little emphasis upon actually determining a particular plate for a given purpose. The experimental work that has been done points to the idea that the complexity of the frequency spectra of double rotation plates renders them less useful than the present oscillator plates, with no advantages to justify the additional complexity.

It might be expected that the theoretical results already available in this particular field could be used as a basis for experimental work. However the many discrepancies existing in the notation render this approach almost futile. In order to correlate various results and form a systematic basis for further work on this subject, it has been necessary to carry out a complete theoretical analysis.

Chapter III

THEORETICAL ANALYSIS AND APPLICATION

The calculation of a zero temperature coefficient plate, which vibrates in a thickness shear mode of motion, involves a theoretical consideration of thickness vibrations in general. The expressions obtained through this consideration can then be applied to a particular type of thickness vibration. By experimental procedure it is then possible to determine whether the properties of the resonator thus calculated make it useful for the desired purpose.

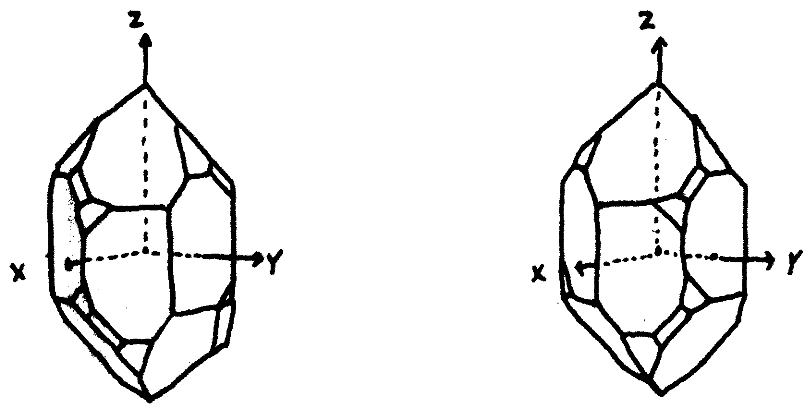
Axial system and notation

As pointed out in the introduction this study will conform, whenever possible, with the notation and axial system recommended by the Committee on Piezoelectric Standards.

The axial system which will be used is a right handed orthogonal system for both right and left quartz as shown in Fig. 1. It can be seen from Fig. 1 that this system requires a different definition of the positive X axis in the two forms of quartz. The positive X axis is defined as that axis which upon extension becomes negatively charged for right quartz and positively charged for left quartz.

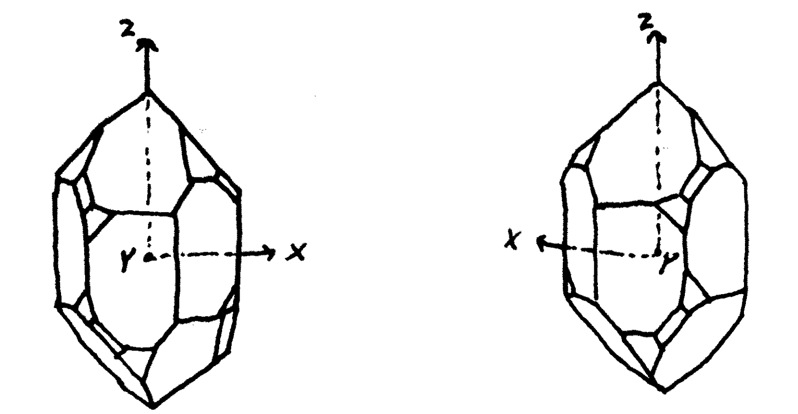
The more common axial system is shown in Fig. 2. As indicated, this system defines a right-handed axial system for right quartz and a left-handed system for left quartz. This is the axial arrangement in which the values and signs of the elastic and piezoelectric constants were initially determined. In this system the piezoelectric coefficients e_{11} and e_{14} are positive for both right and left hand quartz. The elastic coefficients assume the same sign as those commonly quoted. Comparison of Fig. 1 with Fig. 2 shows that if e_{11} is positive in Fig. 2 it becomes negative for right quartz and remains positive for left quartz in the axial system of Fig. 1. The coefficient e_{14} remains positive for right quartz and changes its sign to negative for left quartz. All signs of the elastic coefficients remain the same except for C_{14} which now must be defined as negative for both forms. Because of the symmetry of quartz the numerical values of the coefficients are, of course, the same in both systems. For this reason no further distinction between the two forms of quartz need be made in this study and all of the following discussion, unless specifically stated otherwise, is based upon the axial system of Fig. 1.

Fig. 1.--Right handed axial system



Left Quartz Right Quartz

Fig.2--Dual axial system



Left Quartz Right Quartz

$$-X_x = e_{11}E_x = -S_1$$

$$-Y_z = e_{14}E_x = -S_5$$

The Committee of Piezoelectric Standards has also recommended a notation for the elastic variables, stress and strain. By this notation the tensor components of stress are denoted by T_{ij} while the tensor components of strain are denoted by S_{ij} where

$$i = 1, 2, 3 \text{ and } j = 1, 2, 3.$$

But since $T_{ij} = T_{ji}$, only six of the nine components are independent and the stress components are written as T_p where

$$p = 1, 2, 3, 4, 5, \text{ or } 6.$$

An analgous situation exists for the strain components.

The values of the piezoelectric coefficients as well as the values of the elastic and temperature coefficients are given in Table 1. These are the values which were used for all of the calculations in this study. The piezoelectric and elastic coefficient values are those recommended by Cady (5:137) while the temperature coefficients are those experimentally determined by Mason (19:103). These latter values are said to be accurate to two percent between the temperatures of 20° and 60°C.

The effective elastic coefficient

The general theory of the propagation of plane waves in an anistropic medium was first given by Green, as shown by Love (14:299). Green showed that for any

Table 1.--THE VALUES OF THE ELASTIC, PIEZOELECTRIC AND TEMPERATURE COEFFICIENTS

ELASTIC COEFFICIENTS

$C_{11} = 87.5 \times 10^{10}$ dynes/cm^2

$C_{33} = 107.7 \times 10^{10}$ dynes/cm^2

$C_{44} = 57.3 \times 10^{10}$ dynes/cm^2

$C_{12} = 7.6 \times 10^{10}$ dynes/cm^2

$C_{13} = 15.1 \times 10^{10}$ dynes/cm^2

$C_{14} = -17.2 \times 10^{10}$ dynes/cm^2

$C_{66} = 39.9 \times 10^{10}$ dynes/cm^2

PIEZOELECTRIC COEFFICIENTS

$e_{11} = 5.2 \times 10^{4}$ statcoulombs/cm^2

$e_{14} = 1.2 \times 10^{4}$ statcoulombs/cm^2

TEMPERATURE COEFFICIENTS

$tc_{11} = -46.5 \times 10^{-6}$ dynes/cm^2/degree

$tc_{33} = -204 \times 10^{-6}$ dynes/cm^2/degree

$tc_{44} = -166 \times 10^{-6}$ dynes/cm^2/degree

$tc_{12} = -3{,}300 \times 10^{-6}$ dynes/cm^2/degree

$tc_{13} = -69.7 \times 10^{-6}$ dynes/cm^2/degree

$tc_{14} = 90.2 \times 10^{-6}$ dynes/cm^2/degree

$tc_{66} = 164 \times 10^{-6}$ dynes/cm^2/degree direction of propagation there are in general three possible types of waves, each with a different velocity and therefore a different frequency. He also pointed out that the three vibration directions are mutually perpendicular.

The form of the theory to be considered in this paper is due to Christoffel, as used by Bechmann (2). Christoffel applied the considerations of Green in a manner which makes the theory directly applicable to quartz.

If damping terms are neglected, the general equation of motion of a small elemental volume of an elastic solid undergoing stress is

$$\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}$$

where ρ is the density, t the time, and u the component of displacement in the x direction. Similar expressions represent v and w, the y and z components of the displacement. The three equations of motion are then given by the following relationships where the T_p are the stress components which produce the respective displacements, u, v, and w.

$$\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \qquad (1)$$

$$\frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}$$

If it is assumed that the electric field and the temperature are constant then the generalized form of Hooke's law relating the stresses with the strains is given by the following symmetrical tensor.

$$\begin{aligned}
T_1 &= c_{11} S_1 + c_{12} S_2 + c_{13} S_3 + c_{14} S_4 + c_{15} S_5 + c_{16} S_6 \\
T_2 &= c_{12} S_1 + c_{22} S_2 + c_{23} S_3 + c_{24} S_4 + c_{25} S_5 + c_{26} S_6 \\
T_3 &= c_{13} S_1 + c_{23} S_2 + c_{33} S_3 + c_{34} S_4 + c_{35} S_5 + c_{36} S_6 \qquad (2) \\
T_4 &= c_{14} S_4 + c_{24} S_2 + c_{34} S_3 + c_{44} S_4 + c_{45} S_5 + c_{46} S_6 \\
T_5 &= c_{15} S_1 + c_{25} S_2 + c_{35} S_3 + c_{45} S_4 + c_{55} S_5 + c_{56} S_6 \\
T_6 &= c_{16} S_1 + c_{26} S_2 + c_{36} S_3 + c_{46} S_4 + c_{56} S_5 + c_{66} S_6
\end{aligned}$$

Upon application of the conditions of symmetry in quartz the stress tensor is reduced to

$$
\begin{aligned}
T_1 &= c_{11} S_1 + c_{12} S_2 + c_{13} S_3 + c_{14} S_4 \\
T_2 &= c_{12} S_1 + c_{11} S_2 + c_{13} S_3 + c_{14} S_4 \\
T_3 &= c_{13} S_1 + c_{13} S_2 + c_{33} S_3 \\
T_4 &= -c_{14} S_1 + c_{14} S_2 + c_{44} S_4 \\
T_5 &= c_{44} S_5 - c_{14} S_6 \\
T_6 &= -c_{14} S_5 + c_{66} S_6
\end{aligned}
\tag{3}
$$

If u, v, and w are the components of the particle displacement along the X, Y and Z axes then the strains, S_p are given by

$$
\begin{aligned}
S_1 &= \frac{\partial u}{\partial x} & S_4 &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\
S_2 &= \frac{\partial v}{\partial y} & S_5 &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\
S_3 &= \frac{\partial w}{\partial z} & S_6 &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}
\end{aligned}
\tag{4}
$$

and the stress tensor becomes

$$
\begin{aligned}
T_1 &= C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y} + C_{13}\frac{\partial w}{\partial z} - C_{14}\frac{\partial w}{\partial y} - C_{14}\frac{\partial v}{\partial z} \\
T_2 &= C_{12}\frac{\partial u}{\partial x} + C_{11}\frac{\partial v}{\partial y} + C_{13}\frac{\partial w}{\partial z} + C_{14}\frac{\partial w}{\partial y} + C_{14}\frac{\partial v}{\partial z} \\
T_3 &= C_{13}\frac{\partial u}{\partial x} + C_{13}\frac{\partial v}{\partial y} + C_{33}\frac{\partial w}{\partial z} \\
T_4 &= -C_{14}\frac{\partial u}{\partial x} + C_{14}\frac{\partial v}{\partial y} + C_{44}\frac{\partial w}{\partial y} + C_{44}\frac{\partial v}{\partial z} \\
T_5 &= C_{44}\frac{\partial u}{\partial z} + C_{44}\frac{\partial w}{\partial x} - C_{14}\frac{\partial v}{\partial x} - C_{14}\frac{\partial u}{\partial y} \\
T_6 &= -C_{14}\frac{\partial u}{\partial z} - C_{14}\frac{\partial w}{\partial x} + C_{66}\frac{\partial v}{\partial x} + C_{66}\frac{\partial u}{\partial y}
\end{aligned}
\tag{5}
$$

The third equation of motion as given in Eq. 1 is given by

$$\frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2} \tag{6}$$

Substitution of the proper stresses from Eq. 5 into Eq. 6 results in the following equation of motion.

$$
\begin{aligned}
&\frac{\partial}{\partial x}\left[C_{44}\frac{\partial u}{\partial z} + C_{44}\frac{\partial w}{\partial x} - C_{14}\frac{\partial v}{\partial x} - C_{14}\frac{\partial u}{\partial y}\right] \\
&+\frac{\partial}{\partial y}\left[-C_{14}\frac{\partial u}{\partial x} + C_{14}\frac{\partial v}{\partial y} + C_{44}\frac{\partial w}{\partial y} + C_{44}\frac{\partial v}{\partial z}\right] \\
&+\frac{\partial}{\partial z}\left[C_{13}\frac{\partial u}{\partial x} + C_{13}\frac{\partial v}{\partial y} + C_{33}\frac{\partial w}{\partial z}\right] = \rho\frac{\partial^2 w}{\partial t^2}
\end{aligned}
\tag{7}
$$

If S represents the directed distance from any plane to the origin, as shown in Fig. 3, then the equation of the plane is given by

$$S = lx + my + nz$$

where l, m, and n are the respective direction cosines of S.

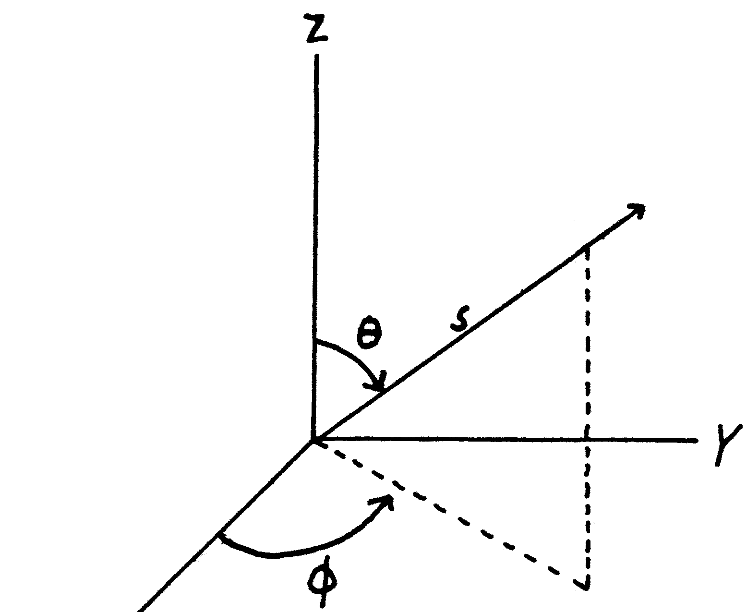
The axial variables x, y, and z can now be expressed in terms of the arbitrary direction variable,

$$\begin{aligned} &C_{44}\, ln \frac{\partial^2 u}{\partial S^2} + C_{44}\, l^2 \frac{\partial^2 w}{\partial S^2} - C_{14}\, l^2 \frac{\partial^2 v}{\partial S^2} \\ &- C_{14}\, lm \frac{\partial^2 u}{\partial S^2} - C_{14}\, lm \frac{\partial^2 u}{\partial S^2} + C_{14}\, m^2 \frac{\partial^2 v}{\partial S^2} \\ &+ C_{44}\, m^2 \frac{\partial^2 w}{\partial S^2} + C_{44}\, mn \frac{\partial^2 v}{\partial S^2} + C_{13}\, ln \frac{\partial^2 u}{\partial S^2} \\ &+ C_{13}\, mn \frac{\partial^2 v}{\partial S^2} + C_{33}\, n^2 \frac{\partial^2 w}{\partial S^2} = \rho \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{8}$$

Combination of terms results in the following expression for the motion equation.

$$\begin{aligned} &\left[C_{44}\, ln - 2C_{14}\, lm + C_{13}\, ln\right] \frac{\partial^2 u}{\partial S^2} \\ &+ \left[C_{14}\, m^2 - C_{14}\, l^2 + C_{44}\, mn + C_{13}\, mn\right] \frac{\partial^2 v}{\partial S^2} \\ &+ \left[C_{44}\, l^2 + C_{44}\, m^2 + C_{33}\, n^2\right] \frac{\partial^2 w}{\partial S^2} = \rho \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{9}$$

Fig. 2.--Orientation of an arbitrary plane.



$$l = \cos\phi \sin\theta$$
$$m = \sin\phi \sin\theta$$
$$n = \cos\theta$$

When the same conditions are applied to the other two equations of motion, the three equations are expressed in abbreviated form as follows.

$$
\begin{aligned}
\rho \frac{\partial^2 u}{\partial t^2} &= \Gamma_{11} \frac{\partial^2 u}{\partial s^2} + \Gamma_{12} \frac{\partial^2 v}{\partial s^2} + \Gamma_{13} \frac{\partial^2 w}{\partial s^2} \\
\rho \frac{\partial^2 u}{\partial t^2} &= \Gamma_{12} \frac{\partial^2 u}{\partial s^2} + \Gamma_{22} \frac{\partial^2 v}{\partial s^2} + \Gamma_{23} \frac{\partial^2 w}{\partial s^2} \\
\rho \frac{\partial^2 w}{\partial s^2} &= \Gamma_{13} \frac{\partial^2 u}{\partial s^2} + \Gamma_{23} \frac{\partial^2 v}{\partial s^2} + \Gamma_{33} \frac{\partial^2 w}{\partial s^2}
\end{aligned} \tag{10}
$$

where the moduli, Γ_{ij}, are given by

$$
\begin{aligned}
\Gamma_{11} &= C_{11} l^2 + C_{66} m^2 + C_{44} n^2 - 2C_{14} mn \\
\Gamma_{12} &= [C_{12} + C_{66}] lm - 2C_{14} ln \\
\Gamma_{13} &= [C_{13} + C_{44}] nl - 2C_{14} lm \\
\Gamma_{22} &= C_{66} l^2 + C_{11} m^2 + C_{44} n^2 + 2C_{14} mn \\
\Gamma_{23} &= C_{14} [m^2 - l^2] + [C_{13} + C_{44}] mn \\
\Gamma_{33} &= C_{44} [l^2 + m^2] + C_{33} n^2
\end{aligned} \tag{11}
$$

If the component displacements u, v, and w are expressed in terms of the resultant displacement, D, then

$$u = \alpha D$$
$$v = \beta D$$
$$w = \gamma D$$

where α , β , and γ are the respective direction cosines.

When this change of variable is carried out, Eq. 10 reduces to

$$\begin{aligned}
\rho \alpha \frac{\partial^2 D}{\partial t^2} &= \left[\alpha \Gamma_{11} + \beta \Gamma_{12} + \gamma \Gamma_{13}\right] \frac{\partial^2 D}{\partial s^2} \\
\rho \beta \frac{\partial^2 D}{\partial t^2} &= \left[\alpha \Gamma_{12} + \beta \Gamma_{22} + \gamma \Gamma_{23}\right] \frac{\partial^2 D}{\partial s^2} \\
\rho \gamma \frac{\partial^2 D}{\partial t^2} &= \left[\alpha \Gamma_{13} + \beta \Gamma_{23} + \gamma \Gamma_{33}\right] \frac{\partial^2 D}{\partial s^2}
\end{aligned} \tag{12}$$

If the following relationship is assumed

$$\begin{aligned}
\alpha \Gamma_{11} + \beta \Gamma_{12} + \gamma \Gamma_{13} &= \alpha q \\
\alpha \Gamma_{12} + \beta \Gamma_{22} + \gamma \Gamma_{23} &= \beta q \\
\alpha \Gamma_{13} + \beta \Gamma_{23} + \gamma \Gamma_{33} &= \gamma q
\end{aligned} \tag{13}$$

then Eq. 12 reduces to the common wave equation

$$\rho \frac{\partial^2 D}{\partial t^2} = q \frac{\partial^2 D}{\partial s^2} \tag{14}$$

whose general integral indicates a propagation velocity of $v = \sqrt{q/\rho}$ with q the effective elastic coefficient.

The coefficient γ is related to the Γ_{ij} through the following determinant which results from the linear equations, 13.

$$\begin{vmatrix} \Gamma_{11}-\gamma & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12} & \Gamma_{22}-\gamma & \Gamma_{23} \\ \Gamma_{13} & \Gamma_{23} & \Gamma_{33}-\gamma \end{vmatrix} = 0 \tag{15}$$

Upon expansion this determinant yields a cubic equation containing three real and positive roots, γ_1 , γ_2 , and γ_3 , which are functions of the elastic moduli and the direction cosines l, m, and n.

$$\begin{aligned} &[\Gamma_{11}-\gamma][\Gamma_{22}-\gamma][\Gamma_{33}-\gamma] + 2\Gamma_{12}\Gamma_{23}\Gamma_{13} \\ &- [\Gamma_{22}-\gamma]\Gamma_{13}^2 - [\Gamma_{11}-\gamma]\Gamma_{23}^2 - [\Gamma_{33}-\gamma]\Gamma_{12}^2 = 0 \end{aligned} \tag{16}$$

$$\begin{aligned} &\gamma^3 - \gamma^2[\Gamma_{11}+\Gamma_{22}+\Gamma_{33}] + \gamma\left[\begin{matrix}\Gamma_{22}\Gamma_{33}+\Gamma_{11}\Gamma_{33}+\Gamma_{11}\Gamma_{22} \\ -\Gamma_{13}^2-\Gamma_{23}^2-\Gamma_{12}^2\end{matrix}\right] \\ &+ \Gamma_{22}\Gamma_{13}^2 + \Gamma_{11}\Gamma_{23}^2 + \Gamma_{33}\Gamma_{12}^2 + 2\Gamma_{11}\Gamma_{22}\Gamma_{33} = 0 \end{aligned} \tag{17}$$

If the trigonometric method for the solution of a cubic equation is applied to Eq. 17 the three roots are given by

$$q_m = 2\sqrt{H}\ \cos\frac{\Phi + 2n\pi}{3} + \frac{\Gamma_{11} + \Gamma_{22} + \Gamma_{33}}{3} \quad (18)$$

where $n = 0, 1, 2.$
$m = 1, 2, 3.$

$$\cos\Phi = \frac{G}{54\left[H\right]^{3/2}}$$

where

$$H = \frac{1}{9}\left[\Gamma_{11} + \Gamma_{22} + \Gamma_{33}\right]^2 - \frac{1}{3}\left[\Gamma_{11}\Gamma_{22}\left[1 - \frac{\Gamma_{12}^2}{\Gamma_{11}\Gamma_{22}}\right] + \Gamma_{11}\Gamma_{33}\left[1 - \frac{\Gamma_{13}^2}{\Gamma_{11}\Gamma_{33}}\right] + \Gamma_{22}\Gamma_{33}\left[1 - \frac{\Gamma_{23}^2}{\Gamma_{22}\Gamma_{33}}\right]\right]$$

$$G = 2\left[\Gamma_{11} + \Gamma_{22} + \Gamma_{33}\right]^3 - 9\left[\Gamma_{11} + \Gamma_{22} + \Gamma_{33}\right]$$
$$\times\left[\Gamma_{11}\Gamma_{22}\left[1 - \frac{\Gamma_{12}^2}{\Gamma_{11}\Gamma_{22}}\right] + \Gamma_{11}\Gamma_{33}\left[1 - \frac{\Gamma_{13}^2}{\Gamma_{11}\Gamma_{33}}\right] + \Gamma_{22}\Gamma_{33}\left[1 - \frac{\Gamma_{23}^2}{\Gamma_{22}\Gamma_{33}}\right]\right]$$
$$+ 27\,\Gamma_{11}\Gamma_{22}\Gamma_{33}\left[1 - \frac{\Gamma_{12}^2}{\Gamma_{11}\Gamma_{22}} - \frac{\Gamma_{13}^2}{\Gamma_{11}\Gamma_{33}} - \frac{\Gamma_{12}^2}{\Gamma_{22}\Gamma_{33}} + \frac{2\,\Gamma_{12}^2\,\Gamma_{13}^2\,\Gamma_{23}^2}{\Gamma_{11}^2\,\Gamma_{22}^2\,\Gamma_{33}^2}\right]$$

Solution of Eq. 14 for an infinite plate with wave propagation in the thickness direction leads to the

$$f = \frac{1}{2a}\sqrt{q/\rho} \tag{19}$$

where a is the thickness and ρ the density. Since the effective elastic coefficient, q_m , may assume three independent values, there are three frequencies, or three vibrations, which are theoretically possible. Whether all three vibrations can be realized depends upon the value of the appropriate piezoelectric coefficient. In the most general case, all three vibrations are obtainable and any one vibration may be coupled with the others. It is only in special cases, where the couplings are zero, that one independent vibration is realized.

Calculation of a zero temperature coefficient

Even if a resonator were found to have a zero temperature coefficient, there are other conditions which must be satisfied before the given resonator can be considered commercially practical. In addition to the requirements upon the physical performance of the plate, the plane of the plate must be approximately parallel to a set of atomic planes in the crystal so that the orientation of the plate is readily established. Consequently a study of the atomic planes, which might be useful for x-ray orientation, was necessary before any actual calculations were carried out.

One of the stronger atomic planes which might be used for orienting a quartz plate is the $\bar{1}\bar{2}32$ plane, which has orientation angles of $\phi = 70^\circ 54'$ and $\theta = 59^\circ 14'$, as based upon Fig. 3. This particular region is the one that was used in this investigation. The reason for this choice is given later in this chapter.

When the direction cosines given in Fig. 3 are substituted in Eq. 11, the result is of the following form.

$$
\begin{aligned}
\Gamma_{11} &= C_{11}\cos^2\phi\sin^2\theta + C_{66}\sin^2\phi\sin^2\theta \\
&\quad + C_{44}\cos^2\theta - 2C_{14}\sin\phi\sin\theta\cos\theta \\
\Gamma_{12} &= \left[C_{12}+C_{66}\right]\cos\phi\sin\phi\sin^2\theta \\
&\quad - 2C_{14}\cos\phi\cos\theta\sin\theta \\
\Gamma_{13} &= \left[C_{13}+C_{44}\right]\cos\phi\sin\theta\cos\theta \\
&\quad - 2C_{14}\cos\phi\sin\phi\sin^2\theta \\
\Gamma_{22} &= C_{66}\cos^2\phi\sin^2\theta + C_{11}\sin^2\phi\sin^2\theta \\
&\quad + C_{44}\cos^2\theta + 2C_{14}\sin\phi\sin\theta\cos\theta \\
\Gamma_{23} &= C_{44}\left[\sin^2\phi - \cos^2\phi\right]\sin^2\theta \\
&\quad + \left[C_{13}+C_{44}\right]\sin\phi\sin\theta\cos\theta \\
\Gamma_{33} &= C_{44}\sin^2\theta + C_{33}\cos^2\theta
\end{aligned}
\qquad (20)
$$

At the point, $\phi = 70^{\circ}54'$ and $\theta = 59^{\circ}10'$, the coupling moduli Γ_{12} and Γ_{13} vanish and the roots of the cubic equation in γ, become

$$\gamma_1 = \Gamma_{11}$$

$$\gamma_{2,3} = \frac{1}{2}\left[\Gamma_{22} + \Gamma_{33} \pm \sqrt{\left[\Gamma_{22}+\Gamma_{33}\right]^2 + 4\Gamma_{23}^{\,2}}\right] \qquad (21)$$

Consequently if a plate having a zero temperature coefficient lies close to this region, $\gamma_1 = \Gamma_{11}$ will be approximately the value of the elastic coefficient which determines the frequency of the vibration. This value of γ can be used in the following manner to determine the position of a zero temperature coefficient.

Differentiation, with respect to the temperature, of the frequency relationship

$$f = \frac{1}{2a}\sqrt{\gamma/\rho}$$

results in the following expression, since a, γ, and ρ are all functions of temperature.

$$\frac{df}{dt} = \frac{1}{2}\left[\frac{1}{2a}\left(\gamma/\rho\right)^{-\frac{1}{2}}\frac{d\gamma}{dt}\frac{1}{\rho} + \frac{1}{2a}\left(\gamma/\rho\right)^{-\frac{1}{2}}\left(-\gamma/\rho^2\right)\frac{d\rho}{dt} + \left(-\frac{1}{a^2}\right)\frac{da}{dt}\left(\frac{\gamma}{\rho}\right)^{\frac{1}{2}}\right] \qquad (22)$$

$$\frac{df}{dt}\Big/f = \frac{1}{2}\left[\frac{d\gamma}{dt}\Big/\gamma - \frac{d\rho}{dt}\Big/\rho - 2\,\frac{da}{dt}\Big/a\right] \qquad (23)$$

$$\tau_f = -\tau_a - \tfrac{1}{2}\left[\tau_\rho - \tau_q\right] \tag{24}$$

where τ_f is defined as the temperature coefficient of frequency, τ_ρ the temperature coefficient of density, τ_a the temperature coefficient of length, and τ_q the temperature coefficient of elasticity.

Since a zero temperature coefficient of frequency is desired, Eq. 24 may be set equal to zero.

$$\tau_f = 0 = -\tau_a - \tfrac{1}{2}\left[\tau_\rho - \tau_q\right]$$

(25)

or $$2\tau_a + \tau_\rho = \tau_q$$

When this equality is satisfied the result is a zero temperature coefficient. Since both τ_a and τ_ρ are well known it remains to determine whether a value of τ_q can be found which will satisfy relationship (25).

The temperature coefficients of length and density are given by (8),

$$\tau_a = 14.3 - 6.5\cos^2\theta \qquad \tau_\rho = -36.4$$

where θ is again the angle given in Fig. 3. Substitution of these values gives

$$-7.8 - 13\cos^2\theta = \frac{dq}{dt}\Big/q \tag{26}$$

But

$$q = \Gamma_{11} = C_{11}\cos^2\phi\sin^2\theta + C_{66}\sin^2\phi\sin^2\theta + C_{44}\cos^2\theta - 2C_{14}\sin\phi\sin\theta\cos\theta \quad (27)$$

so that Eq. (26) becomes

$$-7.8 - 13\cos^2\theta = \frac{\begin{array}{l} C_{11}\,T_{C_{11}}\cos^2\phi\sin^2\theta \\ + C_{66}\,T_{C_{66}}\sin^2\phi\sin^2\theta \\ + C_{44}\,T_{C_{44}}\cos^2\theta \\ - 2C_{14}\,T_{C_{14}}\sin\phi\cos\theta\sin\theta \end{array}}{\begin{array}{l} C_{11}\cos^2\phi\sin^2\theta + C_{66}\sin^2\phi\sin^2\theta \\ + C_{44}\cos^2\theta - 2C_{14}\sin\phi\sin\theta\cos\theta \end{array}}$$

Substitution of the indicated values from Table 1 results in the following relationship

$$-7.8 - 13\cos^2\theta = \frac{\begin{array}{l} [-46.5][87.5]\cos^2\phi\sin^2\theta \\ -[166][57.3]\cos^2\theta \\ +[164][39.9]\sin^2\phi\sin^2\theta \\ -2[90.2][17.2]\sin\phi\sin\theta\cos\theta \end{array}}{\begin{array}{l} [87.5]\cos^2\phi\sin^2\theta + 57.3\cos^2\theta \\ +[39.9]\sin^2\phi\sin^2\theta \\ -2[17.2]\sin\phi\sin\theta\cos\theta \end{array}}$$

If the angle, ϕ equals 70°54', corresponding to the angle of the $\bar{1}\bar{2}32$ plane, then this relationship is satisfied by θ = 56°06'. Hence a zero temperature coefficient is theoretically available at this point.

The Effective Piezoelectric Coefficient

As pointed out previously, it is necessary to consider the effective piezoelectric coefficient in order to determine whether a particular vibration can be realized.

The equations relating the polarization components with the strains are given by

$$\begin{aligned} P_x &= e_{11} S_1 + e_{12} S_2 + e_{13} S_3 + e_{14} S_4 + e_{15} S_5 + e_{16} S_6 \\ P_y &= e_{21} S_1 + e_{22} S_2 + e_{23} S_3 + e_{24} S_4 + e_{25} S_5 + e_{26} S_6 \quad (28) \\ P_z &= e_{31} S_1 + e_{32} S_2 + e_{33} S_3 + e_{34} S_4 + e_{35} S_5 + e_{36} S_6 \end{aligned}$$

As shown previously in Fig. 3 an arbitrary plane is defined by

$$s = lx + my + nz$$

Since the thickness is in the S direction, the polarization in the S direction can be expressed in terms of the axial polarization components.

$$P_s = l P_x + m P_y + n P_z$$

so that

$$\begin{aligned} P_s = & \; l\left[e_{11} S_1 + e_{12} S_2 + e_{13} S_3 + e_{14} S_4 + e_{15} S_5 + e_{16} S_6\right] \\ & + m\left[e_{21} S_1 + e_{22} S_2 + e_{23} S_3 + e_{24} S_4 + e_{25} S_5 + e_{26} S_6\right] \quad (29) \\ & + n\left[e_{31} S_1 + e_{32} S_2 + e_{33} S_3 + e_{34} S_4 + e_{35} S_5 + e_{36} S_6\right] \end{aligned}$$

If the variables, x, y and z, are changed to the artibrary direction variable, , Eq. 4 becomes

$$
\begin{aligned}
S_1 &= \ell \frac{\partial u}{\partial s} & S_4 &= n\frac{\partial v}{\partial s} + m\frac{\partial w}{\partial s} \\
S_2 &= m \frac{\partial v}{\partial s} & S_5 &= n\frac{\partial u}{\partial s} + \ell\frac{\partial w}{\partial s} \\
S_3 &= n \frac{\partial w}{\partial s} & S_6 &= m\frac{\partial u}{\partial s} + \ell\frac{\partial v}{\partial s}
\end{aligned}
\tag{30}
$$

If the strains, S_p, in Eq. 30 are substituted into Eq. 29, the resulting polarization, P_s , becomes a function of the direction cosines and the displacement components.

$$
\begin{aligned}
P_s = \ell \Big[& \ell_{11}\, \ell\frac{\partial u}{\partial s} + \ell_{12}\, m\frac{\partial v}{\partial s} + \ell_{13}\, n\frac{\partial w}{\partial s} + \ell_{14}\, n\frac{\partial v}{\partial s} \\
& + \ell_{14}\, m\frac{\partial w}{\partial s} + \ell_{15}\, n\frac{\partial u}{\partial s} + \ell_{15}\, \ell\frac{\partial w}{\partial s} \\
& + \ell_{16}\, m\frac{\partial u}{\partial s} + \ell_{16}\, \ell\frac{\partial w}{\partial s} \Big] \\
+ m \Big[& \ell_{21}\, \ell\frac{\partial u}{\partial s} + \ell_{22}\, m\frac{\partial v}{\partial s} + \ell_{23}\, n\frac{\partial w}{\partial s} + \ell_{24}\, n\frac{\partial v}{\partial s} \\
& + \ell_{24}\, m\frac{\partial w}{\partial s} + \ell_{25}\, n\frac{\partial u}{\partial s} + \ell_{25}\, \ell\frac{\partial w}{\partial s} \\
& + \ell_{26}\, n\frac{\partial u}{\partial s} + \ell_{26}\, \ell\frac{\partial v}{\partial s} \Big] \\
+ n \Big[& \ell_{31}\, \ell\frac{\partial u}{\partial s} + \ell_{22}\, m\frac{\partial v}{\partial s} + \ell_{23}\, n\frac{\partial w}{\partial s} + \ell_{34}\, n\frac{\partial v}{\partial s} \\
& + \ell_{34}\, m\frac{\partial w}{\partial s} + \ell_{35}\, n\frac{\partial u}{\partial s} + \ell_{35}\, \ell\frac{\partial w}{\partial s} \\
& + \ell_{36}\, m\frac{\partial u}{\partial s} + \ell_{36}\, \ell\frac{\partial v}{\partial s} \Big]
\end{aligned}
\tag{31}
$$

When the component displacements, u, v, and w, are expressed in terms of the resultant displacement, D, the polarization becomes

$$P_s = l^2 e_{11}\alpha\frac{\partial D}{\partial s} + lm\, e_{12}\beta\frac{\partial D}{\partial s} + lm\, e_{13}\gamma\frac{\partial D}{\partial s}$$
$$+ ln\, e_{14}\beta\frac{\partial D}{\partial s} + lmn\, e_{14}\gamma\frac{\partial D}{\partial s} + ln\, e_{15}\alpha\frac{\partial D}{\partial s}$$
$$+ l^2 e_{15}\gamma\frac{\partial D}{\partial s} + lm\, e_{16}\alpha\frac{\partial D}{\partial s} + l^2 e_{16}\beta\frac{\partial D}{\partial s}$$
$$+ \cdots\cdots\cdots\cdots\cdots$$

so that

$$P_s = \epsilon\frac{\partial D}{\partial s}$$

where

$$\begin{aligned}\epsilon = {} & \alpha\Big[e_{11}l^2 + e_{26}m^2 + e_{35}n^2 + [e_{25}+e_{36}]mn \\ & \qquad + [e_{31}+e_{35}]nl + [e_{21}+e_{16}]ml\Big] \\ & + \beta\Big[e_{16}l^2 + e_{22}m^2 + e_{34}n^2 + [e_{32}+e_{24}]mn \\ & \qquad + [e_{14}+e_{35}]nl + [e_{12}+e_{26}]ml\Big] \\ & + \gamma\Big[e_{15}l^2 + e_{24}m^2 + e_{33}n^2 + [e_{34}+e_{23}]mn \\ & \qquad + [e_{13}+e_{35}]nl + [e_{14}+e_{25}]lm\Big]\end{aligned} \qquad (32)$$

Since P_s represents the polarization in the thickness direction, s, and $\frac{\partial D}{\partial s}$ represents the arbitrary strain then the coefficient of this strain is, by definition, the effective piezoelectric coefficient.

When the conditions of symmetry in quartz are applied to Eq. 32, the piezoelectric coefficient, ϵ, reduces to the following expression.

$$\epsilon = \alpha\left[e_{11}\left[l^2 - m^2\right] + e_{14} m n\right] + \beta\left[-e_{14} n l - 2 e_{11} m l\right] \qquad (33)$$

Eq. 33 gives the piezoelectric coefficient for any thickness vibration in any orientation in quartz. To determine whether any particular vibration can be excited, it is necessary to substitute the direction cosine values. If the resulting ϵ differs from zero then this vibration can be excited.

When the direction cosines l, m, and n as given by Fig. 3 are substituted into Eq. 33, the resulting effective piezoelectric coefficient is given by

$$\epsilon = \alpha\left[e_{11}\left[\cos^2\phi \sin^2\theta - \sin^2\phi \sin^2\theta\right] + e_{14}\sin\phi\cos\theta\sin\theta\right]$$
$$+ \beta\left[-e_{14}\cos\phi\cos\theta\sin\theta - 2 e_{11}\sin^2\theta\cos\phi\sin\phi\right] \qquad (34)$$

In order to calculate the actual value of this piezoelectric coefficient, it is first necessary to determine the values of the vibration direction cosines α, β and γ. These are obtained from Eq. 13 and the direction cosine relationship.

$$\begin{aligned} \alpha\Gamma_{11} + \beta\Gamma_{12} + \gamma\Gamma_{13} &= \alpha q \\ \alpha\Gamma_{12} + \beta\Gamma_{22} + \gamma\Gamma_{23} &= \beta q \\ \alpha\Gamma_{13} + \beta\Gamma_{23} + \gamma\Gamma_{33} &= \gamma q \end{aligned} \qquad (35)$$

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

When the appropriate values of the moduli, ij, calculated from Eq. 20, are substituted into the preceeding equations the following direction cosine values of this particular vibration are obtained

$$\alpha = .9956 \qquad \beta = -.0862 \qquad \gamma = .0182$$

The effective piezoelectric coefficient, given by Eq. 34, is then determined.

$$\epsilon = 1.5\text{x}10^4 \text{ statcoulombs/cm}^2$$

<u>Analysis of the range of low temperature coefficient</u>

The calculation of a zero temperature coefficient plate is based upon elastic coefficients and temperature coefficients which have been experimentally determined at a given temperature. Consequently these coefficients are only applicable at that temperature or over a limited temperature range about the given temperature. Therefore when a zero temperature coefficient is calculated there is no indication of the length of the range over which the coefficient is zero or very low. However a semi-quantitive analysis can be applied to explain the difference in this range between plates which have nearly similar expressions for their effective elastic coefficient.

As derived earlier, the effective elastic coefficient, q_1, for a plate with the orientation of ϕ = 70°54' and θ = 59°10' is given by

$$q_1 = C_{11} \cos^2\phi \sin^2\theta + C_{66} \sin^2\phi \sin^2\theta + C_{44} \cos^2\theta - 2C_{14} \sin\phi \cos\theta \sin\theta$$

By using analgous considerations as those which led to the preceeding equation it can be shown that the effective elastic coefficient for the AT plate, which has orientation angles of ϕ = 90° and θ = 54°45', is given by

$$q_2 = C_{66} \sin^2\theta + C_{44} \cos^2\theta - 2C_{14} \sin\theta \cos\theta$$

where the subscripts, 1 and 2, are used to differentiate between the two plates. Differentiation of both of these expressions with respect to the temperature results in equations which show the change in q due to a change in the temperature.

$$\frac{dq_1}{dt} = C_{11} t_{C_{11}} \cos^2\phi \sin^2\theta + C_{66} t_{C_{66}} \sin^2\phi \sin^2\theta + C_{44} t_{C_{44}} \cos^2\theta - 2C_{14} t_{C_{14}} \sin\phi \cos\theta \sin\theta \quad \text{(a)}$$

and

$$\frac{df_2}{dt} = C_{66}\,tC_{66}\sin^2\theta + C_{44}\,tC_{44}\cos^2\theta - 2C_{14}\sin\theta\cos\theta \qquad \text{(b)}$$

Reference to Table 1 shows that tc_{11} and tc_{44} are negative while tc_{14} and tc_{66} are positive. However, because of the negative sign of the C_{14} term, the term involving C_{66} is the only positive term in either equation.

The following data are taken from the paper of Atanasoff and Hart (1) who have determined these elastic coefficients at temperatures of 0°C and 100°C.

	0°C	100°C
C_{11}	87.7×10^{10} dynes/cm^2	87.3×10^{10} dynes/cm^2
C_{66}	40.5×10^{10} dynes/cm^2	41.2×10^{10} dynes/cm^2
C_{44}	55.9×10^{10} dynes/cm^2	55.0×10^{10} dynes/cm^2
C_{14}	17.8×10^{10} dynes/cm^2	17.9×10^{10} dynes/cm^2

From this data it is seen that the values of both C_{11} and C_{44} decrease as the temperature increases. Since the two terms involving these particular coefficients are already negative, the result is a lesser negative value at the higher temperatures. The value of C_{66} increases and, since this term is already positive, the result is a greater positive value. Similar considerations show that the C_{14} term has a larger negative value at the higher temperatures. Thus, the first three variations tend to make the resulting change in Eq. (a) and Eq. (b) more positive while only the C_{14} term tends to make the resultant change more negative. A comparison of the respective values of the elastic coefficients shows that the C_{14} term is so small compared with the other terms, that the final tendency in $\frac{dg}{dt}{}^{\prime 12}$ is to become more positive at the higher temperatures. This explains the reason for the accelerating increase in frequency as the temperature increases. More important, however, this analysis indicates the possibility of obtaining a plate having a zero temperature coefficient at a higher temperature than the AT plate. This possibility is seen from a comparison of Eq. (a) and Eq. (b). The two expressions differ by the presence of the trigonometric functions of ϕ and C_{11} term in Eq. (a). The term involving C_{11} is multiplied by $\cos^2\phi$ which makes this term very small as compared with the others. Therefore, if this term is neglected, the resulting expression of Eq. (a) differs from that of Eq. (b) by the presence of the trigonometric function of the angle ϕ. Since both expressions tend to become more positive at higher temperatures, the presence of the function of ϕ in Eq. (a) makes this resulting change less than that of Eq. (b). Consequently a higher temperature must be reached before the frequency of a plate with an effective coefficient given by g_1 starts increasing. If a zero temperature coefficient plate could be found in the region of $\phi = 70°54'$ and $\theta = 59°10'$, then this plate could be expected to have the zero coefficient at a temperature higher than that of the AT. It was for this reason that this particular region was chosen for the investigation.

Since the frequency expression

$$f = \frac{1}{2a}\sqrt{g/\rho}$$

also involves both the thickness, a, and the density, ρ, it might be expected that changes in these two parameters due to changes in temperature, would render the preceeding analysis erroneous. This, however, is not the case since the density change is independent of the orientation while the change in the thickness is dependent only upon the angle θ. Since this angle is nearly the same in both cases, the respective changes in thickness, and therefore the resulting changes in frequency are approximately the same.

Chapter IV

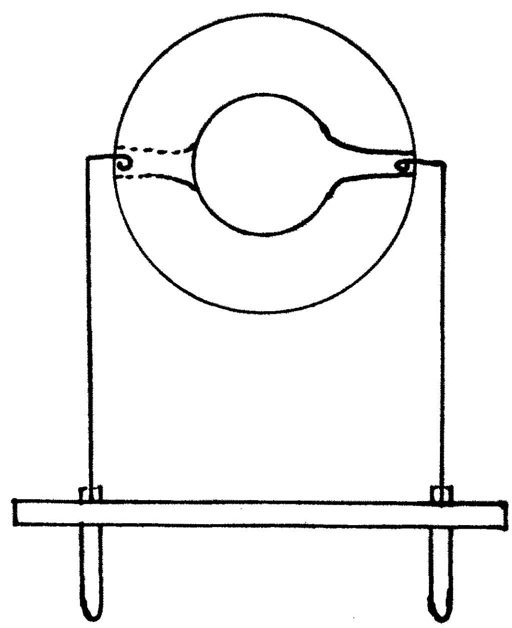
EXPERIMENTAL PROCEDURE

Although theoretical considerations led to the possibility of obtaining a plate with a higher zero temperature coefficient than that of the AT plate, experimental work was still necessary to determine whether the particular cut had useful properties. Among the properties which had to be determined experimentally were the electrical parameters, the range of temperature in which the temperature coefficient was nearly zero, and the freedom of the oscillator plate from coupling with spurious modes.

Preparation and mounting

After the proper orientation was achieved, several blanks of quartz were cut and circular plates were made from these blanks. These plates were lapped until the thickness was approximately .018 inches. The plates oscillated with a frequency of about $4x10^6$ cycles/sec. The diameters of the plates were approximately .55 to .60 inches. After the plates were contoured, they were plated with aluminum and wire mounted as shown in Fig. 4.

Fig. 4.--Wire mounted oscillator plate



<u>Temperature coefficient and activity</u>

The range of temperature over which the crystal units have low temperature coefficients was determined by plotting the frequency against the temperature. The crystal unit was placed in an oven whose temperature was varied from -55°C to 130°C. The frequency was measured at five degree intervals as the temperature was varied through this range. The activity of the crystal unit was observed and also plotted as a function of the temperature.

<u>Frequency spectrum</u>

In order to determine the presence of any spurious modes which might interfere with the behavior of the oscillator plate, it was necessary to record the frequency spectrum over a wide frequency range about the desired mode of vibration. A block diagram, which indicates the method for recording the spectrum,is shown in Fig. 5. The crystal unit was placed in a drive oscillator and the frequency varied mechanically at a constant rate. As the crystal unit approaches a resonance frequency, the current through the unit increases and there is a larger voltage drop across the resistor R_2. As indicated in Fig. 5, this changing Voltage is rectified and recorded by a Speedomax recording potentiometer. Since an increase in current corresponds to an increase

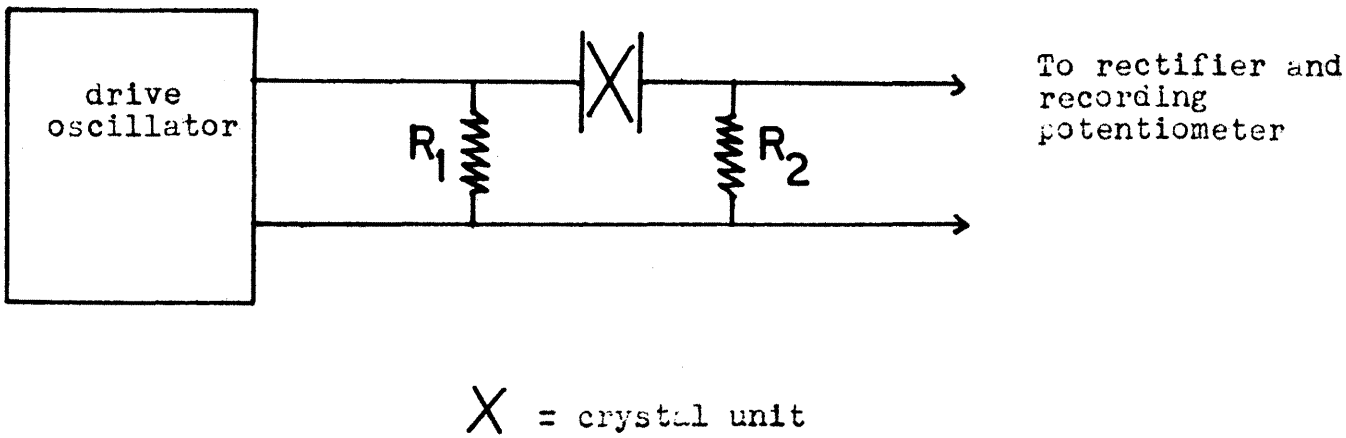


Fig. 5.--Method of recording frequency spectra

in the admittance of the crystal unit, the resulting record on the Speedomax recorder can be interpreted as the resonance curve i.e. admittance plotted against frequency. The resistors R_1 and R_2 must be much smaller than the resistance of the crystal unit so that the effective resistance is very nearly that of the crystal unit and so that these two resistors will short circuit any stray reactances and insure the series resonance operation of the crystal unit.

Electrical parameters

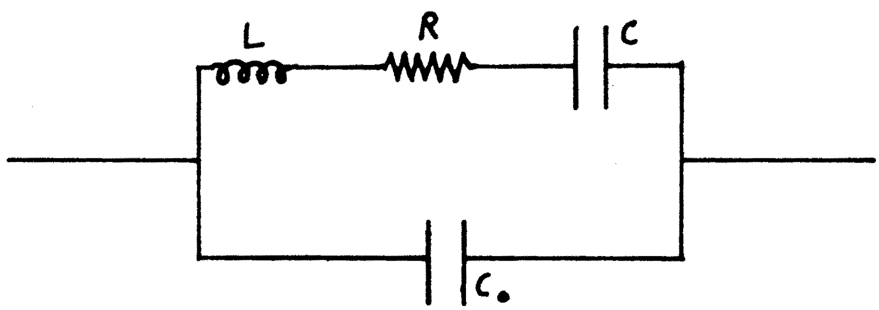
The equivalent circuit of a resonator, which vibrates in a mode free of coupling, can be represented by the circuit shown in Fig. 6.

The inductance L and the capacitance C are the electrical analogues of the mass and compliance of the quartz plate and R is associated with the energy losses in the plate and its mountings.

C_o is the capacitance of the unit when considered as an ordinary condenser with the quartz plate as the dielectric between two electrodes. This particular parameter was measured by the substitution method.

The other electrical parameters, L, R, and C were measured with the Crystal Impedance meter which was designed specifically for this purpose by the Signal Corps Engineering Laboratories (10).

Fig. 6.--The equivalent circuit of a crystal resonator.



Chapter V

PRESENTATION AND DISCUSSION OF DATA

<u>The low temperature coefficient</u>

As pointed out in Chapter III the value of the effective elastic coefficient used in the calculations was that of the modulus $\sqrt{11}$. This value was derived for the point, ϕ = 70°54' and θ = 56°06' and, therefore, at any other point, $\sqrt{11}$ will not necessarily be the actual effective elastic coefficient. Consequently an error in the final results should be expected since the calculations did not yield a zero temperature coefficient at this exact point. This error was demonstrated experimentally when the zero temperature coefficient was obtained at an angle of 55°43' instead of the calculated position of 56°06'. A more significant indication of the error involved in the approximation is obtained by calculating the value of the actual q from Eq. 18. When the trigonometric values are substituted into this relationship the result is an effective elastic coefficient of

$$q = 33.0 \times 10^{10} \text{ dynes/cm}^2$$

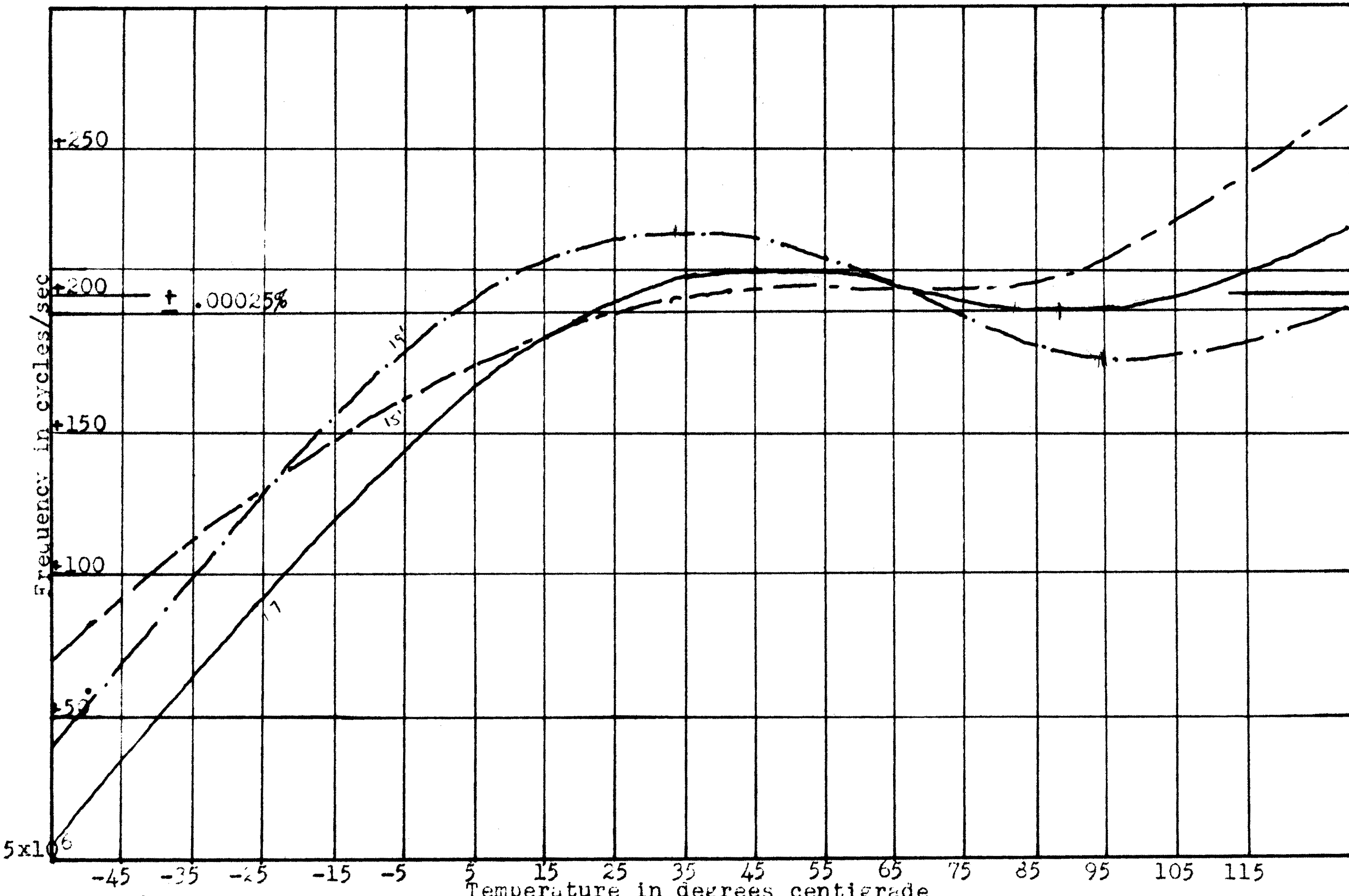
as compared to the approximate value of the elastic coefficient of

$$\gamma = 33.3 \times 10^{10} \text{ dynes/cm}^2$$

However since the experimental values of the basic elastic coefficients, C_{ij}, can be considered accurate to only two or three percent, the approximation would seem to be justified.

Fig. 7 shows typical frequency-temperature characteristics for these plates. Plates which were cut at slightly different angles are shown to illustrate the dependence of the shape of the curve upon the angle of cut. As indicated in this figure, the best frequency stability for the limited range of 40°C to 90°C is that for a plate with an angle θ of 55°45'. The plate with an orientation angle of θ = 55°43' gives the best frequency stability over the wider region of 30°C to 110°C and this plate also shows the best stability at temperatures of around 100°C. Consequently a plate with angles of 70°54' and 55°43' seems to give the best performance for the desired purpose.

Fig. 8 compares a typical curve for this plate with that of an AT cut. These plates have the same frequency and both plates are cut at angles which give the best frequency stability over their respective temperature ranges. It is seen from Fig. 8 that the shapes of the curves are nearly identical but that the



$\theta = 55°41'$ — · —
$\theta = 55°43'$ ——
$\theta = 55°45'$ — —

Fig. 7.--Frequency-temperature characteristics

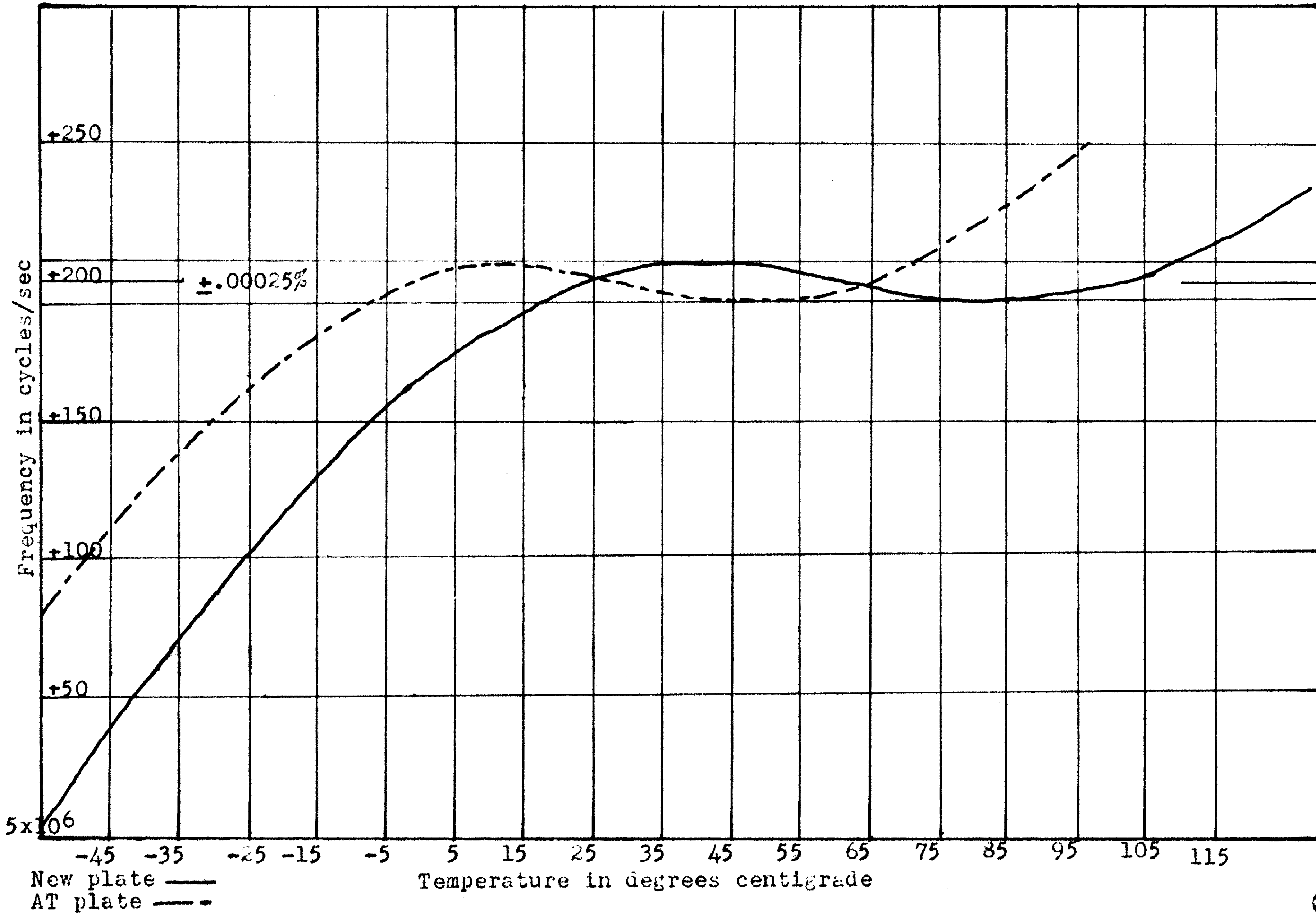


Fig. 8.--Comparison between the frequency-temperature characteristics of the new plate and the AT plate

low coefficient range of this new plate has been translated upward about 30 degrees. Both of these plates will maintain a frequency which only varies about $\pm .00025$ percent over a temperature range of approximately 70 degrees. The respective temperature ranges of the two plates, over which the frequency variation is only $\pm .00025$ percent are indicated in Fig. 8.

It is evident that the zero temperature coefficient range in this cut is extended to a higher temperature than in the AT plate. Consequently the analysis given in Chapter III, which predicted this possibility, justifies the choice of this particular region for the investigation.

<u>Frequency spectrum and activity</u>

From Chapter III the cubic equation in ζ was given by

$$\left[\Gamma_{11}-\zeta\right]\left[\Gamma_{22}-\zeta\right]\left[\Gamma_{33}-\zeta\right]+2\,\Gamma_{12}\,\Gamma_{23}\,\Gamma_{13}$$
$$-\left[\Gamma_{22}-\zeta\right]\Gamma_{13}^{2}-\left[\Gamma_{11}-\zeta\right]\Gamma_{23}^{2}-\left[\Gamma_{33}-\zeta\right]\Gamma_{12}^{2}=0$$

If the coupling moduli, 12, 13, and 23, were zero the three coefficients would be given by

$$\zeta_1 = \Gamma_{11}$$
$$\zeta_2 = \Gamma_{22}$$
$$\zeta_3 = \Gamma_{33}$$

and the resulting vibrations would be independent of each other. In this ideal case the first vibration represents a shear in the XY plane, the second a shear perpendicular to the XY plane, and the third a longitudinal vibration in the thickness direction.

In this new plate all three vibrations are present and, since the coupling moduli are not exactly zero, each vibration is coupled slightly with the others. However the shear motion utilized is the only vibration which has a zero temperature coefficient and this mode also has a much higher piezoelectric coefficient so that there is no tendency for the plate to oscillate in either of the other modes of vibration.

The frequency spectrum shown in Fig. 9 is typical of these circular plates before any contouring is done. The vibrational modes at the higher frequencies are the inharmonic overtones which are present in all thickness vibrations. With one exception the resonance curve for this fundamental mode was free of undesired spurious vibrations for the particular temperature at which these measurements were made. It is assumed that this one spurious resonance is probably due to coupling with the harmonic of a low frequency shear mode. When the plates were contoured the undesired mode vanished as indicated in Fig. 10. Since contouring highly damps a face shear vibration, this assumption would seem to

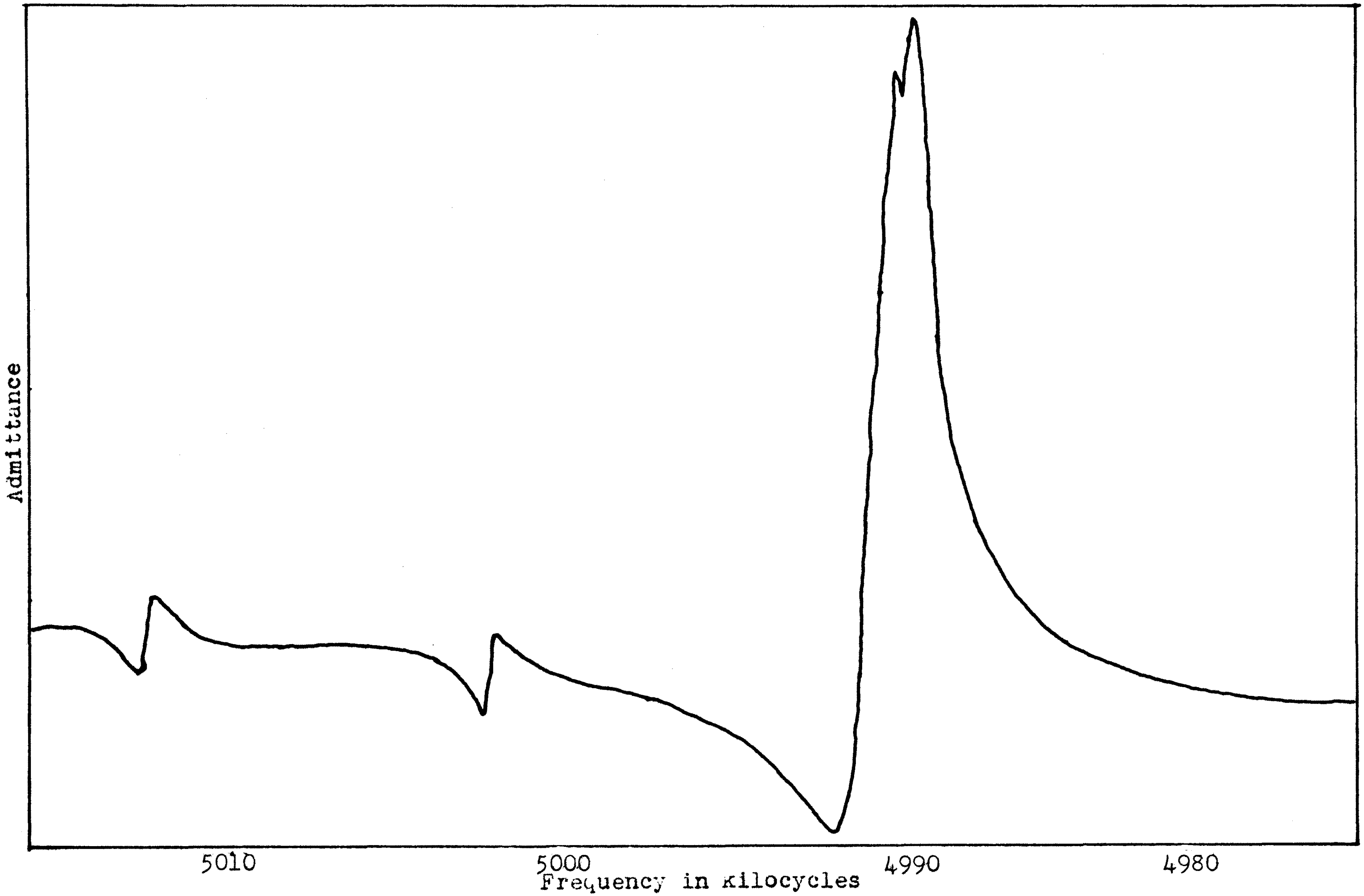


Fig. 9.--Frequency spectrum of a flat plate

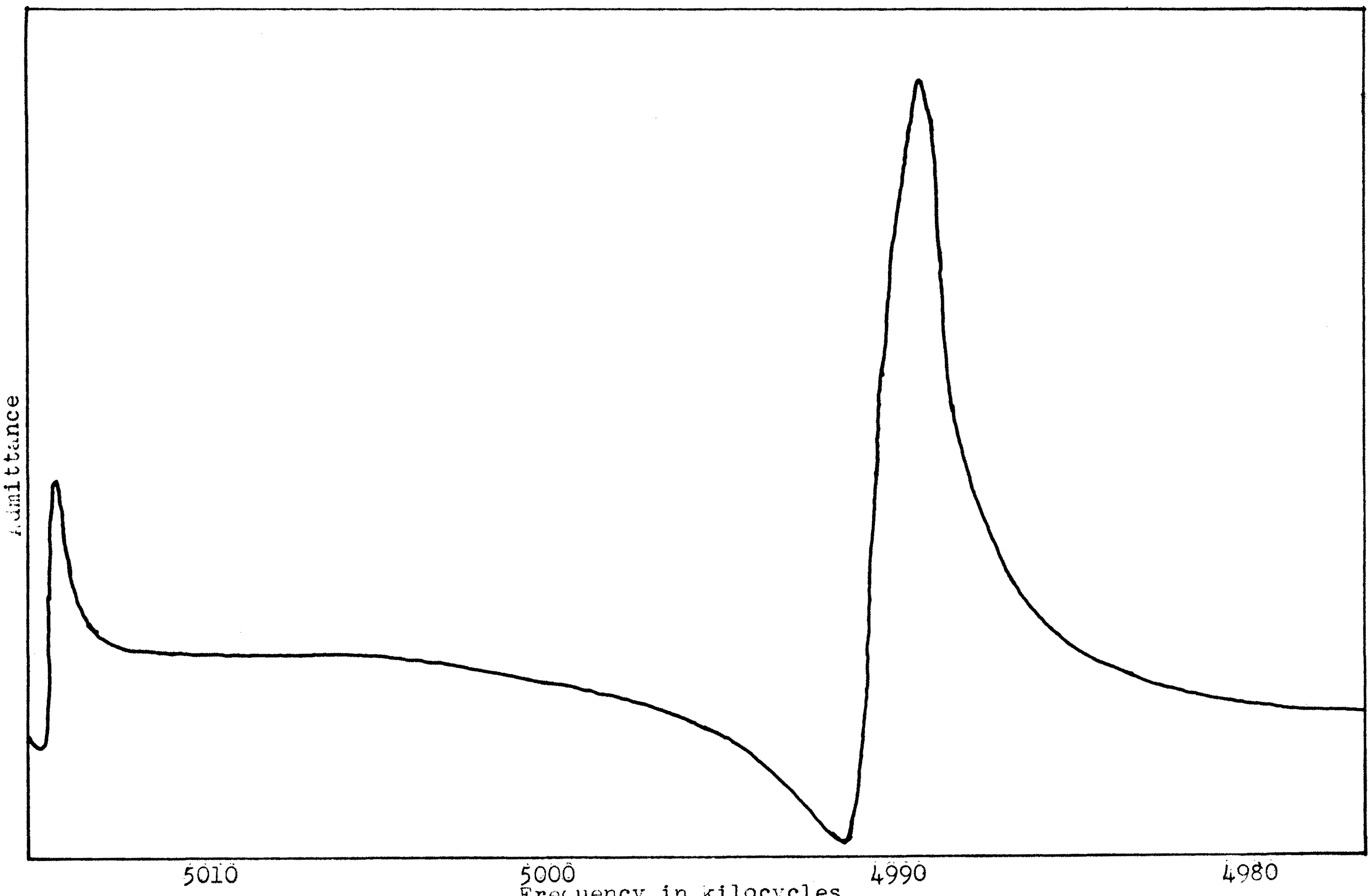


Fig. 10.--Frequency spectrum of a contoured plate

be valid.

The activity of these crystal units is practically constant over wide temperature ranges if the crystal unit is vibrated at low amplitudes. A typical curve for these plates, with activity plotted against temperature is shown in Fig. 11. Since the activity over a wide temperature range is a good measure of the presence of interfering modes, these plates would seem to be more nearly independent of undesired coupling with dimensional modes than are the AT plates.

The frequency constant

As pointed out earlier in this paper, the frequency of an infinite plate which vibrates in a thickness mode is given by

$$f = \frac{1}{2a}\sqrt{g/\rho}$$

from which

$$f = \frac{H}{a}$$

where

$$H = \frac{1}{2}\sqrt{g/\rho}$$

and is defined as the frequency constant.

The density of quartz is 2.65 gm/cm^3 and the value of g for this cut is 33 dynes/cm^2. Hence the frequency constant, H, is

$$H = 69.5 \times 10^3 \text{ inches/sec.}$$

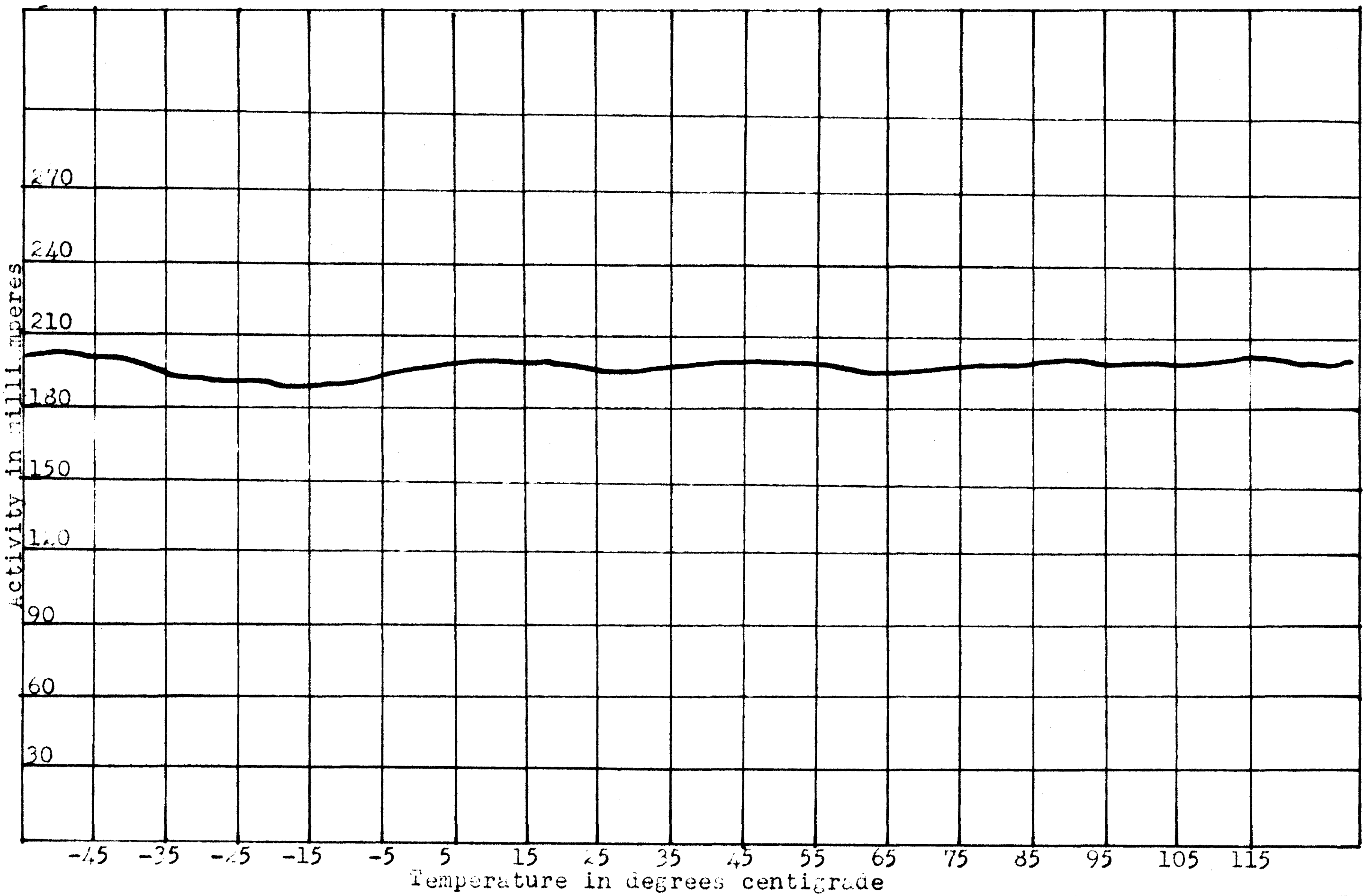


Fig. 11.--Activity vs. temperature

Since the frequency constant of the AT plate is 66×10^3 inches/sec, a given thickness will result in a higher frequency in this new plate.

The experimental value of the frequency constant for this particular shear vibration of the new cut was also determined. This value of $H = 69.7 \times 10^3$ inches/sec compares very favorably with the calculated value given above.

The frequency constants of the other two modes of vibration in this plate were determined both experimentally and theoretically with favorable agreement in both cases also. The other shear mode of motion has a frequency constant of approximately 77×10^3 inches/sec which represents a frequency about 10 percent higher than that of the mode which is utilized. The longitudinal mode has a frequency constant of approximately 121×10^3 inches/sec. and the frequency is about twice that of the desired shear vibration.

<u>Electrical parameters</u>

The values of the electrical parameters of this new plate are of the same order of magnitude as those of the AT with similar contouring and dimensioning. For a plate which has a contour of 2.5 diopters over one third of its radius and which has plating over the non-contoured area, typical parameter values are

$$L = 70 \times 10^{-3} \text{ henries}$$

$$C = .015 \times 10^{-12} \text{ farads}$$

$$R = 15 \text{ ohms}$$

The values of the Q which are given by

$$Q = \frac{2\pi f L}{R}$$

are of the order of 135,000 to 150,000.

Summary

Since the immediate purpose of this investigation was to attempt to find an oscillator plate which might satisfactorily replace the AT for operation at higher temperatures, this chapter has been both a discussion of the data of this new plate and a comparison of the two types of resonators. The preceeding has shown both advantages and disadvantages of this new plate as compared with the AT.

The main advantage of this new resonator is, of course, that it has a zero temperature coefficient at a temperature higher than that of the AT. The new plate is also more nearly independent of undesired spurious modes than the AT cut, a characteristic which results in a nearly constant activity over a wide temperature range. A third advantage is the higher frequency constant which results in a higher frequency for a given thickness.

The two disadvantages of this new plate are the more difficult orientation and the lower activity because of a weaker piezoelectric coefficient.

Plate notation

The Committee on Standards of Piezoelectricity has also recommended a system of notation for defining the orientation of a resonator in the quartz crystal. However, since this notation involves an axial system which defines the resonator in terms of its edges and rotations about its edges and since, of necessity, the normal to the plate was used in this investigation, the system adopted by this Committee was not used. However, in keeping with their recommendation this particular resonator will be defined in terms of the adopted notation.

Under this system the plate is assumed to be rectangular and is defined by successive rotations about its edges. Its initial orientation, before any rotations take place, can assume any one of six orientations. This initial orientation is that orientation in which the thickness, length, and width of the resonator fall along the X, Y, and Z axes but not necessarily respectively. However this initial position must be definitely stated. The general orientation is defined in the following manner.

A. The first two letters indicate the initial position.

(a) The first letter is x, y, or z and indicates the direction of the plate thickness before any rotations have been made.

(b) The second letter, also x, y, or z, indicates the direction of the length before any rotations.

(c) These two letters completely specify unnotated plates.

B. The remaining letters indicate the edges of the plate which were used as the axis of rotation.

(a) The third letter is t, l, or w according to whether the thickness, length, or width direction is the axis of the first rotation.

(b) The fourth letter denotes the edge used for the second rotation.

(c) The fifth letter denotes the edge used for the third rotation.

C. These letters are followed by a list of rotation angles ϕ , θ , ψ ; angles negative in sense are indicated as such by a negative sign.

(a) A positive angle means a rotation counter clockwise as seen looking toward the origin from the positive end of the axis of rotation.

(b) The positive ends of the axes t, l, and w are the ends that initially pointed in the positive direction of the three coordinate axes.

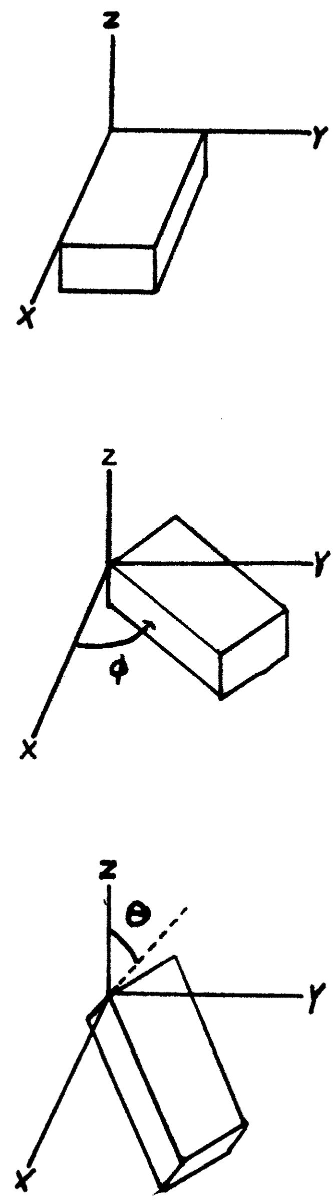
To illustrate this method and, at the same time, to follow the recommended notation, this new plate will be defined as outlined above. If the initial position is chosen, as shown in Fig. 12-a, with the thickness in the Z direction and the length in the X direction, then the first two letters are zx. The plate is then rotated about the thickness by an angle of $70^{\circ}54'$ as shown in Fig. 12-b. The second rotation is about the width by an angle of $34^{\circ}17'$ as indicated by Fig. 12-c. Then the final orientation of this plate is given as

$$zxtw\ 70^{\circ}54'/34^{\circ}17'$$

Recommendations for further study

A study of theoretical interest which has arisen as a direct result of this investigation would be the extension of the analysis of the low temperature coefficient range as given in the theoretical analysis. This extension could be accomplished by using the effective elastic coefficient of the BT plate as well as those of the AT and this new cut in the following manner. The average rates of change, with respect to the temperature, of the elastic temperature coefficients tc_{66}, tc_{44}, and tc_{14} have been calculated by Koga, as given by Cady (5:139). The fourth temperature coefficient

Fig. 12.--Orientation of the new plate



(a)

(b)

(c)

tc_{11}, involved in the elastic coefficient expression of the new cut could be obtained with the aid of the above data. If the results of Atanasoff and Hart, which are given in Chapter III, are applied along with Koga's results, it might be possible to approximately predict the shape of the frequency-temperature curve in the region of the zero temperature coefficient.

Since only a particular region in the quartz crystal was investigated in this work, there are numerous possibilities of obtaining other zero temperature coefficient plates. As pointed out in the theoretical analysis it would seem to be possible to vary the region of the temperature coefficient to suit a desired purpose.

An analysis of the work of Bechmann, as given by Cady (5:144) suggests another possible study. At an angle of $\phi = 80^\circ$ and $\theta \doteq 120^\circ$, one shear vibration has a positive temperature coefficient nearly equal to a negative temperature coefficient of the other shear mode. These results also show that the frequency constants of the two modes are nearly equal at this point. If the piezoelectric coefficients of the two modes were also nearly identical, the coupling between the vibrations might possibly result in a zero temperature coefficient-plate. The equations derived in this study would be directly applicable to such an investigation.

Another study which might prove useful commercially, would be to determine the value of this new oscillator plate for use as a harmonic crystal plate.

Chapter VI

SUMMARY

The AT crystal units being used at present in commercial radio sets have a zero temperature coefficient at approximately 70°C. Very often these sets reach temperatures as high as 100°C where the frequency of the AT crystal unit changes rapidly with temperature. Due to the increasing emphasis upon frequency stability there has arisen a need for a quartz oscillator plate which has a zero temperature coefficient at these higher temperatures.

The Christoffel method for thickness vibrations was used in this study to determine a new quartz oscillator plate by multiple rotations about the crystallographic axes.

After the position of the desired plate was determined, measurements were taken to determine its properties. The electrical parameters-inductance, capacitance, and resistance - of the circular, contoured, wire mounted plates were determined with the Crystal Impedance meter. The frequency spectra of the crystal units were recorded by a Speedomax recording potentiometer. The temperature coefficient and activity characteristics were obtained by plotting changes in frequency and activity due to changes in temperature.

It was found that this plate has a favorable low temperature coefficient for the purpose desired. The shape of the frequency-temperature curve is similar to that of the AT plate, but the range of the low coefficient is moved up about 30° in temperature.

If cut at the proper angle this quartz plate has a frequency variation of about ± .0002% within the temperature range of approximately 30° to 110°C. The activity of this resonator is somewhat lower than that of the AT unit. However, the freedom from undesired spurious modes is better than the AT plates. If vibrated at low amplitude this new crystal unit maintains a remarkably constant activity over the entire temperature range of -55°C to 130°C.

Although harder to produce because it is a multiple rotated plate, this cut seems to be more desirable than the AT plate if operation at temperatures higher than 70°C is desired.

A P P E N D I X

The original data upon which this thesis is based are on file in the Department of Physics of Colorado A&M College.

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ABSTRACT OF

THESIS

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A NEW ZERO TEMPERATURE COEFFICIENT QUARTZ OSCILLATOR PLATE

Submitted by

Walter Richard Ives, Jr.

In partial fulfillment of the requirements

for the Degree of Master of Science

in Physics

Colorado

Agricultural and Mechanical College

Fort Collins, Colorado

March, 1951

ABSTRACT

Of the several piezoelectric crystals which might be used as resonators to control the frequency of electron tube oscillators, quartz has the most extensive application. The most important reason for this widespread use of quartz is that resonators can be cut from the quartz crystal in such a way that their mechanical frequencies are nearly independent of temperature, at least over limited temperature ranges.

One of the quartz crystal plates being used at present in commercial radio sets is the AT plate which has a zero temperature coefficient at approximately 70°C. Very often radio sets reach temperatures as high as 100°C where the frequency of the AT crystal unit is no longer nearly constant but changes rapidly with the temperature. The present day emphasis upon frequency stability has resulted in a need for a quartz oscillator plate with a zero temperature coefficient at temperatures higher than 70°C.

<u>The problem</u>

Is it possible to obtain a quartz crystal resonator, with useful properties, by multiple rotations about the crystallographic axes?

<u>Problem analysis</u>

1. What are the frequency-temperature characteristics of oscillator plates obtained by multiple

rotations?

2. How is the piezoelectric coupling affected by the rotation?

3. What are the electrical properties of quartz plates obtained in this manner?

4. How does multiple rotation affect the frequency spectrum?

Delimitation.--This study is limited to those quartz plates which vibrate in a thickness shear mode of motion and, in particular, to a plate which has a zero temperature coefficient.

Definitions

1. A resonator refers to any piezoelectric device which has a natural mechanical frequency or frequencies, and is vibrated electrically at or near this frequency.

2. A temperature coefficient, unless otherwise specified, expresses the change in frequency with respect to a change in temperature, this ratio being divided by the frequency.

3. In this study, both a plate and a cut are terms used to denote resonators which vibrate in a thickness mode only and which may be either circular or rectangular.

4. Frequency stability refers to the ability of the resonator to maintain a nearly constant frequency

when some parameter is varied. Unless otherwise stated this parameter is the temperature.

5. The activity is a measure of the grid current in the oscillator whose frequency is being controlled by the resonator. It is an indication of the quality of the resonator.

6. A rotated plate or cut refers to a plate whose normal is not parallel to one of the crystallographic axes. A single rotated plate is a cut whose normal is perpendicular to one of the crystallographic axes. A double rotated plate refers to a plate whose normal is not perpendicular to any of the axes.

7. Contouring refers to the process of changing the surface of a plate from plane to convex. The degree of contour refers to the amount of convexity.

8. The frequency spectrum of a given resonator is the representation of the different vibrational modes or resonance points of the resonator over a given range of frequencies.

Theoretical Analysis

The calculation of a zero temperature coefficient plate, which vibrates in a thickness shear mode of motion, involves a theoretical consideration of thickness vibrations in general. The expressions obtained through these considerations can then be applied to a particular type of thickness vibration to determine its individual characteristics.

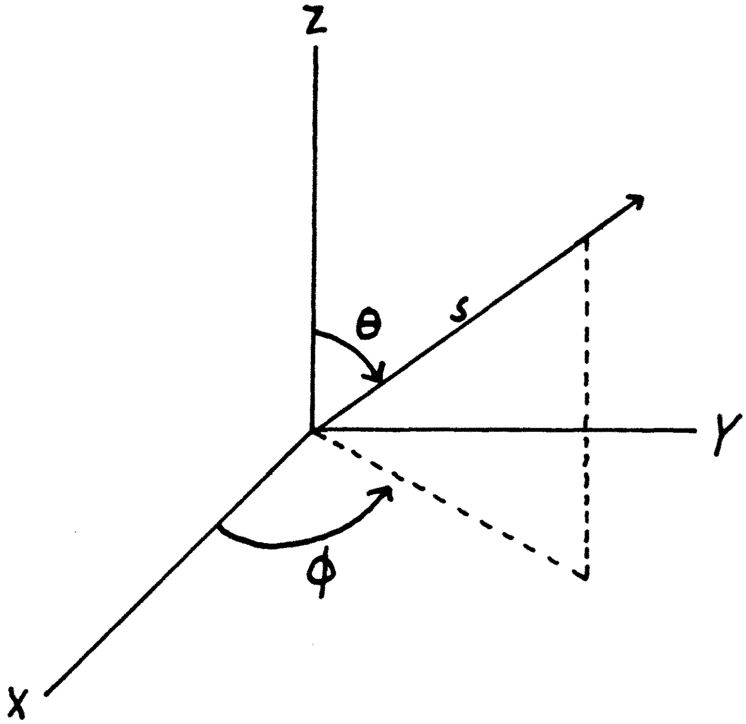
The expression for the effective elastic coefficient of thickness vibrations in general is derived by applying the Christoffel method to the equations of motion of an elemental volume of an elastic solid which is undergoing stress. The resulting expression shows that in the general case there are three possible thickness vibrations and any one vibration may be coupled with the others.

At the point $\phi = 70°54'$ and $\theta = 59°10'$, as based upon the angular system shown in the following figure, one of the vibrations is independent of the others because two of the coupling moduli vanish. By an analysis of the expressions for the effective elastic coefficients for a plate with an orientation of $\phi = 70°54'$ and $\theta = 59°10'$ and the AT plate, it is found that if the former plate has a zero temperature coefficient, then the low coefficient range occurs at higher temperatures than for the AT. Therefore this particular region about $\phi = 70°54'$ and $\theta = 59°10'$ was chosen as the region for investigation.

The equation for the effective elastic coefficient, q, at this point is given by

$$q = C_{11}\cos^2\phi\sin^2\theta + C_{66}\sin^2\phi\sin^2\theta + C_{44}\cos^2\theta - 2C_{14}\sin\phi\cos\theta\sin\theta$$

Fig. 3.--Orientation of an arbitrary plane.



$$l = \cos\phi \sin\theta$$
$$m = \sin\phi \sin\theta$$
$$n = \cos\theta$$

The expression for the frequency of an infinite plate with a wave propagated in the thickness direction is

$$f = \frac{1}{2a}\sqrt{g/\rho}$$

where a is the thickness and ρ the density.

Differentiation of this frequency expression, with respect to the temperature, leads to the following relationship:

$$\frac{df}{dt}\Big/f = \frac{1}{2}\left[\frac{dg}{dt}\Big/g - \frac{d\rho}{dt}\Big/\rho - 2\frac{da}{dt}\Big/a\right]$$

where, for a variable L the quantity $\frac{dL}{dt}\Big/L$ is defined as the temperature coefficient of L. Since the temperature coefficients of the density and the thickness are known, it remains to find the value of $\frac{dg}{dt}\Big/g$ which will result in $\frac{df}{dt}\Big/f$ being zero. In the expression for the effective elastic coefficient, g, which was given earlier, the elastic constants, C_{ij}, are known so it is only necessary to determine the values of the angles, ϕ and θ, which will satisfy the relationship. It is found that for $\phi = 70°54'$, the quantity $\frac{df}{dt}\Big/f$ becomes zero at $\theta = 56°06'$.

To determine whether this particular vibration can be excited, the value of the appropriate piezoelectric coefficient must be determined. The expression for this coefficient is derived in a manner analogous to that used for obtaining the elastic coefficient. The general expression for the piezoelectric coefficient for any thickness vibration in quartz is given by

$$\epsilon = \alpha \left[e_{11} \left[l^2 - m^2 \right] + e_{14} mn \right] + \beta \left[-e_{14} nl - 2e_{11} ml \right]$$

where α , β , and γ are the direction cosines of the particle displacement.

Experimental Procedure

Although theoretical considerations indicate the possibility of obtaining a plate with a zero temperature coefficient at a temperature higher than that of the AT plate, it was necessary to determine experimentally whether the particular cut had useful properties. Among the properties which had to be determined were the electrical parameters, the range of temperature over which the temperature coefficient was nearly zero, and the freedom of the oscillator plate from coupling with spurious modes.

The low temperature coefficient range was determined by plotting the frequency against the temperature. The crystal unit was placed in an oven whose temperature was varied from -55°C to 130°C. The frequency was measured at five degree intervals as the temperature was varied through this range. The activity of the crystal unit was observed and plotted as a function of temperature.

The frequency spectrum was recorded by placing the crystal unit in a drive oscillator and varying the frequency mechanically. A measure of the impedance of the crystal unit at the points of resonance was recorded by a Speedomax recording potentiometer.

The electrical parameters L , R , and C were measured with the Crystal Impedance meter. The shunt capacitance C_0 was measured by the substitution method.

Presentation of data

As predicted by the theoretical analysis this new plate has a zero temperature coefficient at higher temperatures than the AT. Typical frequency-temperature characteristics for the new plate and the AT plate are compared in the following figure. Both of these plates will maintain a frequency which only varies about $\pm$ .00025% over a temperature range of approximately 70°C. The respective temperature ranges of the two plates, over which this frequency tolerance is achieved, are indicated in the figure.

The resonance curves of these plates are extremely smooth and, hence, these new plates would seem

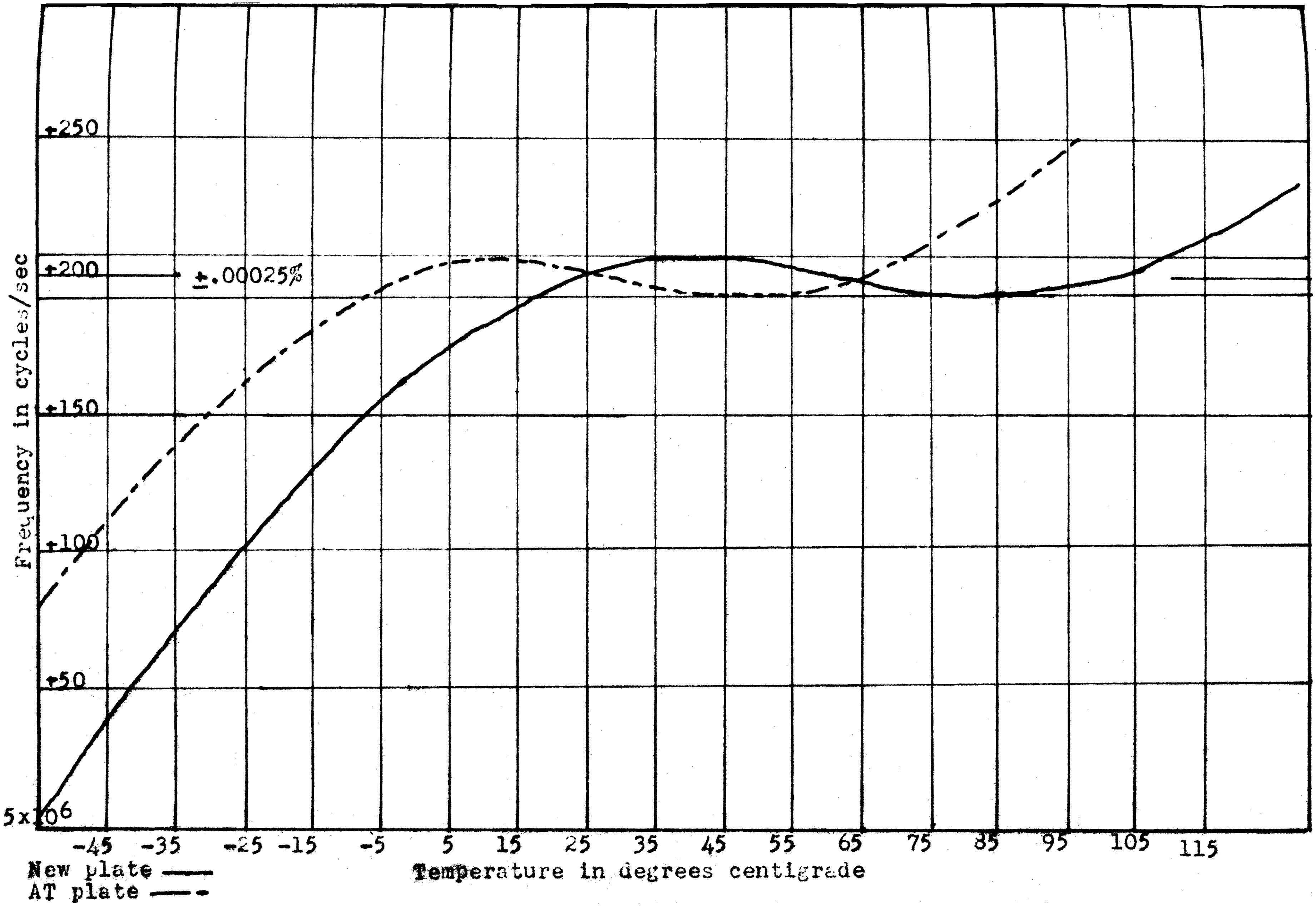


Fig. 8.--Comparison between the frequency-temperature characteristics of the new plate and the AT plate

to be more nearly independent of coupling with spurious modes than is the AT. If vibrated at low amplitudes the new plates maintain a nearly constant activity over the temperature range of -55°C to 130°C.

The values of the electrical parameters are of the same order of magnitude as those of the AT plate.

The piezoelectric coefficient of this cut has a value of 1.5×10^4 statcoulombs/cm^2 which is about one-half that of the AT plate.

The frequency constant for this plate is 69.7×10^3 inches/sec as compared to the frequency constant of the AT which is 66×10^3 inches/sec. Consequently a given thickness results in a higher frequency in the new plate.

Recommendations for further study

Since only a particular region in the quartz crystal was investigated in this work, there are numerous possibilities of obtaining other zero temperature coefficient plates. A specific study of this sort would be to analyse the region where the temperature coefficient of one shear vibration is negative while the temperature coefficient of the other shear mode is positive. The coupling effect between the two modes might result in a plate having a zero temperature coefficient over a very wide range. Another possible analysis would be the extension of the procedure which predicted the low coefficient range at higher temperatures. It would seem possible to actually predict the shape of the curve in the region of the zero temperature coefficient. The equations derived in this study are directly applicable to all of these previously mentioned investigations.

Another study which might prove useful commercially would be the determination of the value of this new oscillator plate for use as a harmonic crystal plate and for use in filter applications.