TECHNOLOGY ASSESSMENT OF IRRIGATION SCHEDULING AND CROP RESPONSE

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I. INTRODUCTION

During the 1970 decade, the United States has undergone a dramatic shift in the political philosophy for irrigation planning and management, from one of continually seeking to augment the existing supplies of irrigation water to one of better utilization and management of those supplies already developed.

This change in emphasis has been neither complete, nor without controversy, primarily because the relevant science and technology implicitly required for the rational evaluation of the new philosophy is largely undeveloped.

It is the objective of this technology assessment to evaluate the requirements which must be met if the new philosophy is to serve national objectives most effectively, to assess the current state of the related science and technology, and to recommend research and development programs which will be required.

The basic issues are the amount, timing and uniformity of the application of irrigation water to the fields and the consequences thereof. These issues are important to a number of questions relevant both to good farm water management and to the implications of various proposed national water policies.

To be able to assess the adequacy of modified technologies as generally implied by the terms "irrigation scheduling" and "crop response" it will be useful to review some of the specific purposes to be served by these technologies, whether in their current state of the art or as they may be developed.

In the discussion of purpose, it is essential to bear in mind that the most important characteristic of water for virtually all uses is not the volume of water per se, but rather the assurance or reliability that any given volume will be available when needed, where needed and with satisfactory quality characteristics. The economic or social value of an acre foot of water is essentially zero if one cannot count on it being available when and where required. Indeed one can demonstrate mathematically that, if the productivity of water is a linear function only of volume used by the plants (as staunchly asserted by some), then dams and reservoirs, including the soil reservoir, are completely unnecessary insofar as maximizing the mathematical expectancy of total production, regardless of the magnitude of the change variations in streamflow or rainfall.

In this sense, the basic issue of irrigation scheduling is that of providing an adequate reliability that water of appropriate quality will be available to the roots of the plants being irrigated when needed and where needed. This statement includes uniformity of application (where needed). Good farm water management is a trade-off decision between the improvement of this reliability and the cost (broadly defined) of doing so.

Among the purposes which have been set forth in scientific and political arguments for improvements in farm irrigation management and the related scientific information on crop response to water are included (1) conservation of water, (2) improvement in productivity, (3) development of drought strategies,

(4) improved crop quality, (5) reduced water costs, (6) improvement of return flow water quality, (7) area-wide assessment of potential crop production as the growing season progresses (crop reports), (8) evaluation of the economic potential of supplementary irrigation (or conversion from dry-farming to irrigated agriculture), (9) a more practical scientific definition of "drought hardiness" and (10) genetic development of drought hardiness characteristics of crop varieties.

Unfortunately, each of these purposes is easily misinterpreted when considered as a generalized response to various water problems, hence some discussion of each is in order so that the requirements of the technology and the related scientific information can be assessed in a proper context.

A. Water Conservation

As a fundamental principle of physics, water is a substance which must always be conserved unless converted back to its elemental constituents (hydrogen and oxygen). Thus in a strict physical sense, water management can not conserve water. If we modify the term to "liquid water" conservation, then water evaporated is "lost" and a physical significance to the term exists. However, for the production of crops, it can be shown that with no "losses" due to the liquid to vapor there will be no production at all.

The term water conservation must therefore be given a much more restricted meaning than that which comes to the mind of the average layman. The meaning is not at all easy to define. It is not a simple matter of using less water, unless indeed one is unconcerned about the resultant productivity. Rather it involves as a minimum a complex matter of getting as much productivity as possible with a given amount of water (water use efficiency).

Even this concept of conservation involves some misconceptions. Before the amount of water not used because of better water use efficiency in one field can be presumed to be "conserved" it is necessary to determine what happens to it subsequently. If it can then be made available (economically, politically, legally, etc.) for other beneficial purposes not otherwise possible, it is at least conservable (although not necessarily conserved).

Thus the same specific management practice applied to the same crop, soil and climate might result in water conservation in some cases, yet conserve nothing in another case. It is an issue which depends as much or more on the subsequent history of the water as it does on the management practice.

In fact subsequent history of the water after the introduction of a "good" water management practice could result in a significant decrease in the total productivity of the available water of a stream. This occurs whenever the "waste" liquid water (surface runoff or deep percolation) then passes through natural channels to become the supply of other farmers, cities, etc., but otherwise would bypass the irrigators downstream. In a significant number of instances the water supplies of the latter must rely on the existing practices which, from a single farm water use efficiency point of view, must be considered wasteful, yet overall are quite effective.

Where deep percolation to a good quality groundwater reservoir occurs, it could actually result in water conservation in yet another sense. When a management practice which prevents deep percolation, would result in the subsequent immediate runoff of the water to the sea, the deep percolation "loss" in the groundwater may well be recoverable for making up a deficiency in a subsequent dry year.

There is a further issue, however. As stressed earlier, quantity or volume of water lost or saved is secondary to the reliability of its availability when and where needed. To improve this reliability, water resource systems are designed and built. These systems, being largely fixed entities, have a capacity characteristic which defines the largest amount of water that can be made available at any instant in time or over a specific interval of time. This capacity in turn involves two determining factors. One is the flow rates (volumes per unit time) that can be transmitted to the locations where needed. The other is the reserve storage capacity which determines when and for how long water will be available to transmit through the transmission system.

If, over a period of time, the water use requirements increase significantly at a given geographic area serviced by a water conveyance system, that demand may exceed the conveyance capacity. Obviously, if the waste volumes are not reused by other geographic units, either reduced conveyance losses or more productive use will result in conservation of conveyance capacity even if it does not reduce true, unretrievable, losses.

A similar statement can be made with respect to storage capacity conservation as a means of providing water when needed. Excessive non-productive losses early in a dry period can result in depletion of storage capacity, whereas, reduction of these losses could, but not necessarily would, prolong the availability of water over time.

The conclusion is that improved technology for irrigation management in some cases can result in conservation of water, and of water storage and conveyance capacities. Where true, improved water management can be a viable alternative to further project construction. Whether it is or is not the best alternative in such cases is a matter of economic (plus social, legal and political) evaluation.

At the same time, as indicated above, practices generally considered as improved water management can result in a decrease in volumes of useful water and/or in conveyance and storage capacity for the hydrologic units as a whole, and may require construction of additional storage and conveyance facilities in order to maintain the current productivity levels.

For river systems which are already extensively developed, such reuse of water volumes and use of natural hydrologic conveyance and storage systems frequently exist and in fact may dominate. For example virtually no usable water escapes from the Colorado River systems in the form of liquid water, in spite of rather low estimated overall conveyance and irrigation efficiencies (the major use). Obviously reuse and natural conveyance and storage by natural systems are major factors for this river which cannot be ignored in any discussion of water conservation management options.

This is not to argue that the Colorado River and similar waters are effectively used. Rather it is to warn that in such systems, conservation must be analyzed in a much more sophisticated way than simply mandating "improved efficiency." Substantial volumes of water diverted from the river are non-productively evaporated. Of the total liquid water brought into this river system by precipitation, it is estimated that only about 5% ever reaches the stream. Most of this is the runoff from the high mountain areas. Of the watershed evaporative loss, perhaps as much as 3/4 is used by "non-productive" evapotranspiration (in the economic use sense). Of that which does reach the river and is diverted one can only guess at the quantity non-productively lost to the liquid water system. The estimates of the amounts which might then be prevented by better water management is even more speculative. The hydrologic-vegetative-geological system involved is virtually unknown.

B. Improvement in Productivity

Numerous examples have been cited where improved water management resulted in substantial improvements in agricultural productivity[1]. As can be argued from the subsequent discussion of the scientific relationships between soil water and plant growth, much of this improvement follows from improved uniformity of applications of irrigation water over the field. While in part reflected by the water application efficiency (ratio of the volume of water held in the root zone to the volume of water applied to the field), uniformity is not synonymous with water application efficiency. Poor uniformity can produce poor water application efficiency but, an improvement in poor efficiency could result in less uniformity as easily as more uniformity.

The root zone of a field constitutes a rather forgiving buffer which temporarily stores, and to a much lesser extent, distributes water to the roots of the plants. In general, the productivity of any given plant will be adversely affected if it cannot draw water from the soil as fast as the climatological factors (net radiation, temperature, humidity, wind speed and turbulence, etc.) require it. It also appears (but is less well documented) that excessive amounts of water in the root zone may also decrease the productivity of the plants. Certainly for most crops prolonged water-logging (saturated root zone) will seriously curtail production, frequently by killing the plants.

Even where water-logging does not occur due to good subsoil drainage, the deep percolation associated with over irrigation will leach nitrogen and <u>some other fertilizers</u> below the root zone.

[1] The author was consulted (too late) by a California farmer who had literally destroyed 40 acres of mature high quality peach trees by over irrigation late in the previous growing season. The trees were killed when the roots were destroyed by the lack of oxygen which resulted from nearly a year of continuous waterlogging.

In some cases, the buffering effect of the root zone water storage capacity acts to mitigate the effects of non-uniformity of any one irrigation (e.g., where the specific non-uniformities are chance determined and are different for each irrigation). In too many cases, however, the non-uniform pattern is preserved through successive irrigations and the effect is cumulative.

In addition to these general effects, research conducted since 1950 (approximately) by the Agricultural Research Service, USDA, and others, has conclusively demonstrated that the magnitude of the adverse effects of soil moisture deficiencies on productivity is very much influenced by the point in time during the growing season at which the adverse effects occur.

This introduces the scheduling issue along with the uniformity issue. Both are affected significantly by the irrigation technology being used. A specific irrigation method will tend to produce essentially the same uniformity distribution from one irrigation to the next for the same depth of water application. Thus areas of deficiency or excess tend to be perpetuated. To obtain the most economical production, there is a corresponding requirement for over-irrigation (usually less damaging) to provide more water for the otherwise under irrigated areas. The result is a greater "waste" of water but substantially better productivity from the field.

In such cases, the frequently encountered admonition, simply to apply less water in the name of conservation, can be not only counterproductive but in fact disastrous to the farm enterprise.

The productivity issue is further complicated by the fact that both the uniformity and efficiency of water application by all available irrigation methods are affected to a greater or lesser extent by the amount of water (rate and depth of application) to be provided in an irrigation. Once designed and built, the irrigation method is fixed. One cannot shift at will between sprinkler, drip, furrows, borders, basins, etc. The amount needed at any irrigation is determined by factors beyond the control of the irrigator. Sometimes (as in rotation delivery systems) the amount and frequency of the available supply are beyond his control as well.

The stage of crop growth will also modify the uniformity and efficiency of any given irrigation. The advance-recession characteristics of furrow and border irrigation systems are greatly influenced by whether the flow occurs over a freshly disturbed soil or through dense or sparse vegetative cover.

Whatever the technological procedures currently designed into the farm irrigation system (and frequently his external supply), a modification to a more uniform, more productive irrigation system represents a major capital expenditure on the part of the owner. For the most part these expenditures are irreversible. No part thereof may be salvaged if the future proves it to be uneconomical.

Thus before he can be convinced that a technological change is in order, the farmer must be able to evaluate the economic consequences thereof, particularly the effects on productivity. These cannot be generalized but are rather very site specific even to individual portions of the fields. There are no charts or tables which tell the farmer the amount of productivity to be expected for any amount, uniformity or timing he might choose, let alone what the uniformity of any new system might be.

In some cases the farmer's irrigation system is so bad that improvement is clearly in order. However, if it is that obvious, the farmer has, in all probability, already made the change if it is feasible for him to do so.

Thus it is apparent that, if productivity is of concern, substantially better scientific information and analytical methods must be made available to allow an evaluation of the alternatives for change.

C. Development of Drought Strategies

In an earlier section, the importance of reliability of water availability was stressed. Unfortunately in water resources there is no such thing as a perfectly reliable supply. There is always a non-zero probability that the actual availability at any point in time (or place) may be less than that anticipated.

Such situations, where the amounts of water from whatever source are less than that counted upon by the users, are droughts[2] in a socio-economic definition of the term. The absolute magnitude of the available supply is irrelevant to this definition. The recent "drought" in eastern U.S. still provided more water and more frequently available water than a good water year in the high plains. There are few droughts on the Mohave Desert and and virtually none in the central Sahara.

Thus regardless of the hydrologic characteristics of any region, drought can, and will occur whenever realization of supply is less than anticipated.

In irrigated areas, storage and conveyance facilities (including groundwater) permit decisions to be made concerning when and where the shortage will be allowed to occur. Such decisions constitute a drought strategy. There are, of course, many other possible elements of a drought strategy which may prove useful, but the discussion here is limited to modification of when and where deficiencies will exist.

[2]This socio-economic definition of drought is used here rather than the more commonly encountered definition based on purely physical concepts, since the idea of "drought strategy" inherently assumes there is some socio-economic purpose to be served.

This question of where may involve detail as small as specific plants (e.g., in orchards) or as large as entire sectors (agriculture vs. urban areas). Regardless of scope, an essential element in the development of such drought strategies is the necessity to evaluate the consequences of alternative courses of action.

To accomplish this evaluation, as well as to assist in the initial conception of possible courses of action, there must be reasonably accurate scientific models by means of which the productivity effects of alternate time and amounts of irrigation water applied can be estimated.

In context with drought strategies, the comments made earlier regarding conservation of water and uniformity of application are all relevant. Attempts to decrease the amount of water "wasted" can in fact decrease the amounts productively used and increase the amounts wasted, all at increased economic cost.

The reliability issue which creates drought is easily misinterpreted from still another point of view. Suppose the strategy successfully implemented involves "conservation" in the sense of accomplishing the same ends with a reduced supply. In reality then, the drought strategy succeeds only because the "waste" was built into the system, thus providing a reserve for use in emergency. However, if the "saving" is routinely utilized in normal periods, then the reserve will have been destroyed and the impact of the next drought may be far more severe than would otherwise have been the case.

To a lesser extent the same is true of many other elements of drought strategy if they become general practice during non-drought periods.

Whatever the strategy, its utility is to be found in its impact on objectives, particularly productivity, and not in the amounts of deficit allocated or otherwise accommodated. This implies substantially more scientific information (research) and probably better irrigation technologies than exist at the present time.

D. Improved Crop Quality

Quality of production is also influenced by soil moisture availability and irrigation procedures. The effects have been shown to be favorable in most instances but adverse in others. For a very few crops, soil moisture deficits at some phases of growth have been shown to increase both the quantity and/or quality of the desired product. More commonly non-optimal sequences of deficit and plenty cause serious quality deterioration (e.g., fruit split). In some cases anti-transpirants have proved useful to minimize adverse effects of adequate moisture after an unavoidable shortage.

Probably the most adverse effects have come from ill-advised attempts to improve product quality by over-irrigation (see footnote 1).

Although the impacts on quality are known to exist, quantitative information by means of which these impacts could be evaluated is virtually non-existent.

E. Reduced Water Costs

Although commonly proposed as an incentive for improved irrigation management, in most cases there is little scope for reducing water costs. The resource is the right of use, in the form of a water right, which is usually owned by the water users. The ownership may be private or mutually held with others. In such cases, charges related to water are often fixed and virtually independent of the actual volumes used by the owners. Reductions in use simply mean that the fixed costs must be distributed over smaller volumes of water, with the total costs to be collected unaffected.

Where pumping must occur and its cost is significant, some cost reduction can result from less use. However, for other water systems, energy is generated in the conveyance system (e.g., the Los Angeles Aqueduct) hence reductions in use could significantly increase net costs.

F. Improvement of Return Flow Quality

Virtually all water used for irrigation contains dissolved solids which, if present in sufficient quantities, produce deleterious effects on its use. In order to maintain a salt balance in the root zone, thereby preventing the accumulation of harmful quantities, leaching is essential.

This leaching water of course retains virtually all of the harmful salines in the original irrigation water. To the extent that it passes on to the water supply of downstream users, the nominal quality of that downstream supply is inferior to what it would have been had the leachate not been returned for further use.

The normal farming operations may also contribute to the total dissolved solids in the leaching water, either by mineral salts added or by increased weathering and solution actions. Likewise, in passing through the soil, the percolating leachate may either dissolve additional salts or, if sufficiently concentrated, actually deposit them.

These observations have led to suggestions that the deposition of salts can be encouraged in the lower root zone portions of the soil if the amount of leaching water provided is reduced to a point where precipitation of salts will occur. Obviously this concept requires a very precise control of the irrigation water if it is to succeed without salt damage to the crop. This precision of control is not even approached with current irrigation technology.

Even if possible, the reduction of dissolved solids will not in every case be useful. While it is common practice to quantify irrigation water quality characteristics by the term "total dissolved solids" (TDS), the problems created by the dissolved solids frequently are determined by concentrations of individual ions rather than by the total. The minimum leaching concept, if successful, does not reduce sodium or chloride, both of which are common sources of problems for further use. The calcium salts, which can be reduced, contribute to adverse effects only in terms of osmotic pressure effects. In some cases, where the soils to be irrigated have a fairly large clay fraction, the calcium salts may in fact have to be added back to the water to avoid serious reductions in soil permeability. The author has seen corn, irrigated daily with water which had only 100 ppm TDS, which was seriously wilted for lack of soil moisture. When calcium sulfate was added to the irrigation water, the problem ceased. The figure 100 ppm TDS may be compared with 800-1000 ppm TDS Colorado River water which has been successfully used since the turn of the century.

Sound irrigation water management on the farm may also have another beneficial effect. In some instances the irrigated area lies over a groundwater basin with very high salt content, far above the allowable maximum levels. The leaching water percolates down to these waters and either produces a rise in the water table which soon creates problems (with or without salt) or it causes the highly saline waters, by displacement, to flow into nearby streams with a corresponding significant degradation of quality. Although eventually the displacement will "freshen" the saline groundwater, this is a process of centuries. In the meantime, the extra salt, in its effect, reduces the amount of the fresh water in the system that can be used productively in irrigated agriculture.

G. Areawide Assessment of Potential Crop Production

At the present time, considerable effort goes into regional and national crop production forecasting based on acreage sown and weather conditions to the date of the forecast.

These forecasts involve a significant amount of judgment regarding the actual impacts of the adverse weather conditions, particularly the effects of rainfall and the resulting soil moisture. To obtain accurate forecasts it would be desirable to be able to estimate those adverse impacts more scientifically. To do this requires the development of appropriate crop production models (and their parameters) which will take into account the stage of growth in which the soil moisture deficits occur as well as the magnitudes of the deficits.

H. Evaluation of the Economic Potential of Supplemental Irrigation

Historically irrigation was confined to those areas where the natural precipitation was so chronically deficient as to clearly justify the capital expenditures required. Recently there has been a rapidly expanding introduction of supplemental irrigation systems into areas previously considered to be adequately watered from rainfall.

Such systems have appeared in virtually every state of the Union, indicating that there may be a tremendous potential whose realization will depend on a satisfactory evaluation of the economic results and the financial resources required.

Obviously the economic potential depends upon the amount of additional productivity (and security of income) which might be produced. This in turn depends upon the management decisions, such as seeding rates, fertilizer applied, etc., and on the magnitudes and timing of the drought events, or short supply conditions, which may occur.

There presently exists sufficient scientific knowledge and methodologies to estimate the probabilities of short supply conditions for virtually any location in the U.S. There is also a reasonably satisfactory scientific basis for estimating the results of policies on seeding rates, fertilizers, weed and pest control measures, etc. What is lacking is a science and methodology for assessing the expected impact of the drought events, should they occur, on the final marketable production.

The benefits are not simply due to the relative increases in production with and without supplemental irrigation with all other factors constant. Rather it is the ability of the farmer to utilize fertilizer, weed and pest control and his limited land more effectively. Without this control, he must take a more conservative policy and utilize less of these significant factors of production.

There is another very important aspect related to supplemental irrigation vis-a-vis State, regional and national water policies.

As a very crude model (which might invalidate the speculative conclusions presented here), most major regions, or watersheds, in the rain-fed agricultural areas of the U.S. appear to have approximately 70 to 80 percent of the precipitation lost by evapotranspiration, and only 30-20 percent appears as streamflow volume, using a long time basis.

The basic purpose of supplemental irrigation is to eliminate soil moisture stress. As indicated in the next section, the currently accepted measure of this is the ratio of actual transpiration to that which could potentially be transpired if the crop were adequately watered, t_p, then crop production should be maximum (with some notable exceptions).

If we assume that the long term average production effect is a linear function of the accumulated ratio t_a/t_p as defined above as a rough approximation, and if it is further concluded that the potential for increased yield is on the order of 20% (it could be even more than this), with full supplemental irrigation the total seasonal evapotranspiration will increase by at least 20% approximately.

If in addition most of the area of the watershed is in agriculture production, this would imply a decrease in streamflow equal to 0.2 x 0.80 (or 0.20 x 0.70) or 16% to 14% of the total precipitation. If streamflow is presently 20% this is a reduction to only 4% of total precipitation--a reduction of 4/5 of the present annual streamflow. If it is 30% the reduction goes to 16% for a reduction of nearly 1/2. These are potentially serious reductions of streamflow for these regions, which should be evaluated long before the practice is initiated.

Obviously there are many ramifications to this analysis which could make these numbers either less or even more severe. In any case, it would seem prudent for water policy purposes to assess the degree to which supplemental irrigation might be economically feasible in any planning region or river basin. If feasible a further assessment should be made of the resulting implications for surface water availability and groundwater availability. Any required protective control measures should be initiated before the practice becomes so widespread as to produce de facto rights and or irreversible adverse consequences to the participating farmers and/or others currently depending on those same water supplies.

I. Definition of Drought Hardiness

With or without irrigation development, the United States will continue to have large areas of land under cultivation in rain-fed areas where drought is chronic or at least a serious problem. For such areas, varieties and types of crops to be planted by the farmers should presumably be "drought hardy," a term frequently mentioned but never defined in a way to permit the best suited variety to be selected.

This is largely because the response of the plants to moisture stress depends on when during the growing season the deficit will occur. As in the case of the supplemental irrigation problems, it is possible to estimate the relative frequency (probability) of rainfall amounts, at any point in time, hence likewise for the soil moisture deficits. Drought hardiness, in its conceptual definition, should refer to its ability to produce under the stress conditions which might be encountered. Obviously the same variety could be drought hardy in one location with one set of soil moisture probabilities during the various phases of growth and yet be drought susceptible in another location with a different distribution of the same total water availability.

Again, the key to the resolution of the problem lies in the development of the crop response relationship to the distribution of soil moisture deficiencies.

J. Genetic Development

One interesting possibility is the genetic development of crop varieties which are "drought hardy" for specific frequently encountered conditions. Little has been done in this direction because of the long time periods required to experience enough drought events (even artificially created) to be able to classify the variety as good or poor for any locality. If the characteristics of the crop response could be observed in controlled experiments, substantial time would be saved in genetic selection and testing. This might make programs of genetic development for drought hardiness economically feasible, depending on the research cost involved in obtaining the response characteristics.

II. SCIENCE REQUIREMENTS AND THE STATE OF THE SCIENTIFIC MODELING

The term "modeling" has unfortunately been restricted in the modern computer age to be synonymous with mathematical modeling for computer analysis. Such a definition is in reality much too restrictive. It leaves out a very much larger set of descriptive, experimental and "mind's eye" or judgmental models all of which are absolutely essential to the creation and use of mathematical models.

For purposes of this discussion, a model is defined to be a conceptualization of an actual or hypothetical system which retains the essential characteristics of that system for a purpose.

Such conceptualizations may, and often do, exist only in the mind or thought processes of the modeler. Others may be expressed qualitatively through symbols (e.g., words, pictures, sketches, graphics, or even abstract symbols as in mathematics), through analogous physical representation (including some toys) or through sets of experiments dynamic or static. The above definition is a model of the essential characteristics of the concepts we refer to as "models" in the general sense.

However, to meet the concept of a model, whatever form is used it must identify and reflect the essential characteristics of that which is to be modeled, in context with purpose. An accurate model of a given system for one purpose might be rather inaccurate for another purpose and vice versa.

Science may be considered to be a set of models of various systems by means of which knowledge and understanding of these systems may be efficiently transferred from one human mind to another. Obviously, the purpose to which the transferred knowledge and understanding is to be put determines the accuracy and utility of the models which constitute a science.

There are an almost infinite number of purposes for science, from satisfaction of curiosity to generating new and better science, to stimulating the conception of potential problem solving strategies, and to evaluation of those potential strategies for problem solving implementation.

In this report, we are concerned with the state of scientific modeling (all forms) of phenomena related to the use of water for the production of agricultural products, and in particular the conception and evaluation of potential problem-solving strategies related to the management of irrigation water.

Furthermore, since irrigation is an established technology, the focus must be on change. That is, prediction of the favorable and adverse changes which might result from changes in technology and/or management decision with respect to the technologies available.

Because of this purpose, many major scientific contributions for the purpose of extending our general scientific modeling capability will probably not receive attention which would be due on scientific merit alone. This is particularly true of much of the research in plant physiology. Its importance to the current state of modeling for water management purposes is absolutely critical. However, for our purposes, the criterion is the degree to which this basic knowledge has been assimilated into working models for water management.

The problems cited in Section I above, all will require essentially the same scientific knowledge and understanding for their resolution. In its most general form the requirement is for mathematical, descriptive and/or experimental models which will relate both the managerial decisions taken with respect to conservation and use of water, and natural factors (e.g., rainfall), to the desired (and undesired) consequences on agricultural productivity over a substantial period of time. This relationship is commonly referred to as the "production function." If it were known with sufficient accuracy and detail, then given the magnitudes and time of occurrence of the natural factors and the managerial decisions, the resulting magnitudes of the various products of agriculture (seed, fruit, forage, cellulose, sugar, etc.) could be determined.

Obviously this "production function" would need to take into account an embarrassingly large number of causative factors that include all climate characteristics, all soil physics, all soil chemistry, all plant physiology, all plant nutrition, all plant pathology, a bit of hydraulic engineering, and a substantial amount of thermodynamics not to mention physically damaging effects such as wind, hail, torrential rain, floods, etc.

As a result of basic physiological research combined with field experimentation, the modeling required for our purposes can probably best be treated under four major sub-topics directed toward answering the following questions:

1. Given a set of management decisions (a strategy) what are the resulting soil, soil moisture and aerial environment to which the plants will be exposed?

2. Given a set of soil, soil moisture and aerial environments, what are the relationships between these environments and the evaporation and transpiration (evapotranspiration) of water from the soil, through the plants, to the atmosphere?

3. Given the resultant evaporation and transpiration what will be the resultant effects on the rate of growth related to all significant physiological functions during any particular phase of growth.

4. Given the growth effects on the various significant physiological functions of the plants, what is the combinatorial effect thereof on the final production of economically significant plant parts at harvests.

The answers to these four questions, if and when available, provide the scientific linkage between the prospective water management decisions (and other decision components) and the consequences thereof. A satisfactory state of scientific modeling can be said to exist when the set of models involved (still including descriptive and other non-mathematical models) are sufficiently accurate as to allow the a priori estimation of the change in productivity resulting from a change in management strategy (decision set). As is well-known, a much greater degree of accuracy is required to predict changes than is required to predict the magnitudes per se.

A. Management Decisions and Soil-Aerial Environment

As indicated earlier, the principal water management decisions are directed toward providing an assurance that water will be available where needed and when needed. These two issues (where and when) involve the water supply and district distribution systems (reservoirs and aqueducts) and the farm supply and distribution systems. The former are reasonably well understood, but still require substantial research to develop the most effective planning, implementation and management, particularly the operation of reservoirs for risk avoidance and canal operation and maintenance to minimize losses. These issues, however, will be considered to be outside the scope of the present review.

The farm supply and distribution system modeling also leaves something to be desired, particularly with respect to low cost technologies for achieving uniformity of application of irrigation water over the surface of the fields (assurance of where available). Methods of achieving high uniformity (e.g., computerized sprinkler system design) are available but with today's agricultural economy they are not low cost. Except for these deficiencies, concepts of mass balance, together with experimental results, appear reasonably satisfactory for estimating the consequences of management decisions on the quantities of water which would be available in the root zone.

Almost all irrigation technologies apply water to the soil surface. The soil characteristics then result in the soil being refilled to its field capacity from the top down. That is, very little water passes downward until the uppermost soil pore volumes have been filled to the maximum level possible with essentially complete gravitational drainage. This permits a fairly accurate estimate of where in the soil profile any particular volume of applied water will reside (until removed by evapotranspiration).

The normal resultant aerial environment will be primarily determined by the plant-soil-water mechanisms which operate on the soil water. The exceptions would involve application of water as mists (or as the equivalent effect) deliberately to modify the vapor and entrained water in the air. Such cases are rare.

In summary, except for better modeling of the water application processes to provide more accurate indices of uniformity, and for inspiring new and better (lower cost or more effective) irrigation technologies, scientific modeling of these processes is not the limiting element in the problems addressed here.

B. Soil Water-Evaporation Relationships

The earliest irrigation experiments consisted primarily of three or four "treatments," in which the total seasonal amount of irrigation water was varied and the resulting production observed [Beckett and Huberty, 19].

Unfortunately, the unexplained variations often exceeded the "explained" variations in resulting productivity. Although it appeared that the volume of irrigation water applied had a very significant effect, it was virtually impossible to express this as a cause-effect model with any accuracy. The only possible exceptions were the experiments conducted by Huberty (and others) with irrigated alfalfa on well managed farms.

Attention then turned to an examination of the mechanisms by means of which water was utilized by the plant in producing the desired products. This lead to major contributions to our understanding of soil-water physics. In particular, it has lead to conceptual and rudimentary[3] physical models of the mechanism by means of which water is transferred from the pores of the soil, through the plant, to the atmosphere as "transpiration" (actually plants transpire gases other than water as well).

In general, it is agreed that an equivalent hydraulic conduct system can be considered to exist between the water held in the pores of the soil and the atmosphere. The energy required to drive water through the system comes largely from heat energy provided by sunlight, air movement and a very small amount of stored heat in the soil.

The driving gradients (potentials) which force the water through the system are believed to be well understood and characterized by potential theory. Continuity relationships, however, are still subject to unknown variations in the equivalent cross sectional area of the "conduits," particularly those affected by the dynamic physiological processes associated with growth and the factors affecting growth over time, unfortunately including the water flow itself.

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[3]The term "rudimentary" is applied here to any mathematical model for which the deviation of the prediction from the experienced values is inadequate for the purposes of making the prediction. Since our purpose involves the prediction of <u>changes</u> due to <u>changes</u> in management, a higher order of magnitude of accuracy is implied, than is generally recognized as "good" by plant-soil-water scientists.

Despite these problems, a near concensus has been reached that the net effect of all these processes was an actual transpiration (water) loss, which under certain poorer soil moisture conditions would be less than that which would have been expected to occur had better soil moisture conditions existed at that same point in time and phase of growth.

The ratio of these two rates of transpiration, with and without soil moisture stress, might then be used as an index of the soil moisture conditions as they might affect plant growth. Unfortunately it is a very difficult ratio to observe directly by experiment, or even theoretically by mathematical models. Consequently the ratio of actual evapotranspiration to potential evapotranspiration is more generally used. Since the term evapotranspiration includes both soil evaporation and transpiration through the plant system, it is directly related to the total soil-water balance over short periods of time.

Unfortunately, not all authors are clear regarding the definition they use for "potential evapotranspiration." In most cases it is a value calculated from energy conservation considerations or empirically related to other directly observable climatic factors such as temperature, radiation, vapor pressure, windspeed, etc., or from the evaporation from a free water surface in a standardized pan. The index, however, should involve the potential evapotranspiration from the plant-soil-water system at the same point in time and condition of achieved growth as used for the numerator of the ratio. This would involve corrections for degree of crop cover and height and condition of the crop canopy.

Mathematical modeling is fairly well advanced for the prediction of the amount of water which potentially could be evaporated by the crop if soil-water conditions are fully adequate. In addition, a number of models have been developed to estimate the actual amount of water which would be evaporated under less than fully adequate conditions.

Modeling for prediction of the actual evapotranspiration has followed two principle avenues. The first is a strictly "black box" empirical relationship which appears to exist between the relative remaining available soil moisture Θ/Θ_{MHC} and the relative rates of evapotranspiration e_a/e_p. Here available soil moisture is defined to be the volume of water held in the root zone soil after a deep well drained irrigation which in turn can be extracted therefrom by the plants before they "permanently" wilt, i.e., wilt and do not recover without the addition of more water to the soil.

Originally, Veihmeyer and Hendrickson [1957] in numerous studies of fruit trees and vines came to the conclusion that the rates of actual evapotranspiration to the potential evapotranspiration under well watered conditions was unity and constant so long as the remaining soil moisture was above the permanent wilting point (i.e., zero remaining available water). This is illustrated by curve numbered 1 in Figure 1.

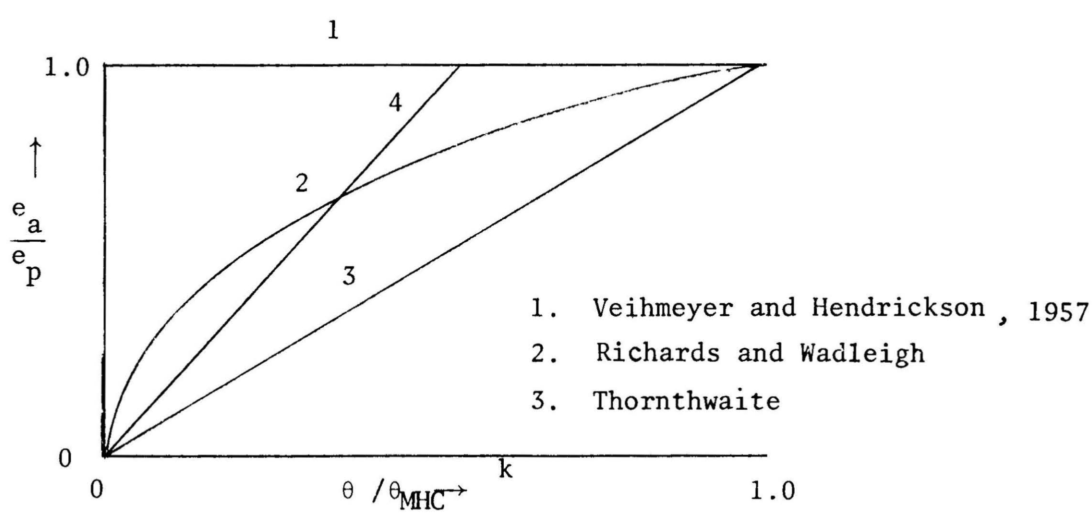

Figure 1. Conceptual relationships between relative ET and soil moisture.

About the same time Thornthwaite proposed that the ratio should be directly proportional to the remaining available soil moisture[4]. This hypothesis is illustrated by curve number 2 in Figure 1.

On the basis of considerable experimentation, Richards and Wadleigh proposed an intermediate function such as curve 3, with the exact position determined by the relationship between available soil moisture and its energy potential or "soil suction."

Most recent studies, including studies of mechanisms of stomatal openings, together with the degree of experimental precision required to fix any curve in Figure 1 reliably, suggest that an adequate representation can be obtained by using a curve such as curve 4 in Figure 1. There is considerable evidence that the relative evapotranspiration remains constant at a value of unity until somewhere near 1/2 to 3/4 of the available soil moisture has been depleted. Bo connecting the point (k,1), when k is $1/2 \leq k \leq 3/4$ to the point 0,0 as indicated, a practical relationship is established. If this is used in a soil moisture mass balance calculation, the actual remaining soil moisture, as calculated is not significantly different whether $k = 1/2$ or $k = 3/4$ or $k = 0.6$.

Furthermore, so long as the k value chosen is kept consistent when calculating the ratio e_a/e_p, the only error introduced is a small distortion of the resultant relationship between e_a/e_p and the achieved growth. This is readily compensated by small adjustments in the empirical coefficients, which must be experimentally determined in any event.

This procedure will be used most often for future predictions of responses to yet to be taken management decisions.

[4]Available soil moisture is usually defined as the amount of water which the plants are capable of extracting from a fully irrigated and drained soil before the plants permanently wilt (i.e., wilt and do not recover without an addition of water to the soil).

The alternative procedure represents all attempts to model actual or relative evapotranspiration by modeling the processes by which it occurs, or to relate evapotranspiration to some observable characteristics of the plant such as relative leaf temperature (relative to fully evaporating leaf surfaces) or leaf water potential or turgor pressure.

Most of the internal mechanism models for the relative evapotranspiration ratio, are based on the phenomena associated with small pores in the epidermis of the leaves (and to a lesser extent, stems) of growing plants. These pores are called stomata. Associated with each pore are "guard cells" which open and close in response (at least) to light intensity and internal water stresses in the plant in a complex way.

If it is assumed that the tendency to open or close due to all such stimuli except for water stress are fixed to be some prespecified pattern over the growing period of interest, then the magnitude of the water stresses are presumed to result in a specific degree of change in closure of the stomata openings. If no water stress exists, then the area is presumably maximum (for the given other conditions). Under maximum severe stress, the stomata will close to the maximum extent possible.

These assumptions are not strictly true. The degree of closure of the stomata appear to be determined by the difference in osmotic pressure of the guard cells and that of the surrounding plant tissue thus should be relatively independent of soil water potential until the latter is sufficiently low to cause internal structural and solute changes. This appears to be the case since effects are not generally observed until half to three quarters of the available soil moisture has been removed from the soil.

Even so, since most of the water vapor loss of plants occurs through the stomata by diffusion to the atmosphere, the rate of actual transpiration at any time should be closely related to the total area of the stomatal openings for any given climate-energy condition.

Usually it is assumed that the relative evapotranspiration should be proportional to the total area of opening of the stomata since this would appear to be the primary difference under stress and non-stress conditions for water. It is then further assumed that the degree of opening is controlled to be different from that under non-stress equivalent conditions by the level of stress. It follows then that since water related stresses in the plant tissue are created by soil moisture deficit conditions that a direct mechanism has been established which can be modeled.

To a certain extent this is true. However, plant water stresses (e.g., those reflected by, say, leaf water potentials) are not the only factors controlling the stomatal openings. In fact according to Allaway and Milthorpe [1976] it has been known since 1856 that it is not the osmotic pressure in the leaves which alone controls the opening, but rather a difference in osmotic pressure between that in the leaf cells surrounding the stomata and that in the guard cells.

Scholander et al. [1965] states:

> "It is generally recognized that stomata do not respond directly to to leaf water potential until a critical threshold is reached, following which stomata close over a narrow range of potentials... . However, there is no unique leaf water potential causing stomatal closure."

This is, of course, compatible with the assumptions about soil moisture threshold stated earlier.

These observations are quite compatible with the "black box" models illustrated in Figure 1.

Although somewhat discouraging for direct mechanistic models linking soil moisture parameters to relative evapotranspiration, the relative ease with which leaf water potential can be observed as the crop is growing has justified continued research to find a suitable relationship. Soil moisture balance methods are costly, slow and not overwhelmed with accuracy.

Figure 2 represents the result of a model to relate evapotranspiration from critical leaf water potential [Slabbers, 1980]. The unexplained variance may or may not be experimental error. On the basis of the comments of Schrader [1980] and Allaway and Milthorpe [1976] cited earlier it is probably due to other, as yet unidentified factors influencing the relationship.

In summary, the estimation of the relative evapotranspiration is being pursued along two rather different procedures. One is a "black box" or empirical relationship between remaining available soil water (or its energy potential) and the relative evapotranspiration. The other attempts to define and describe the internal mechanisms by means of which these same two variables are related.

The former remains crude and subject to calibration of an important parameter which may not be constant from one crop to another or from one soil to another. However, with consistency in its use, the error will tend to be corrected in the next relationship, that of relative evapotranspiration and plant growth.

The second or mechanistic approach has the greatest promise, but that promise would appear as yet unfulfilled, at least until the unexplained variation of actual and predicted values is significantly reduced.

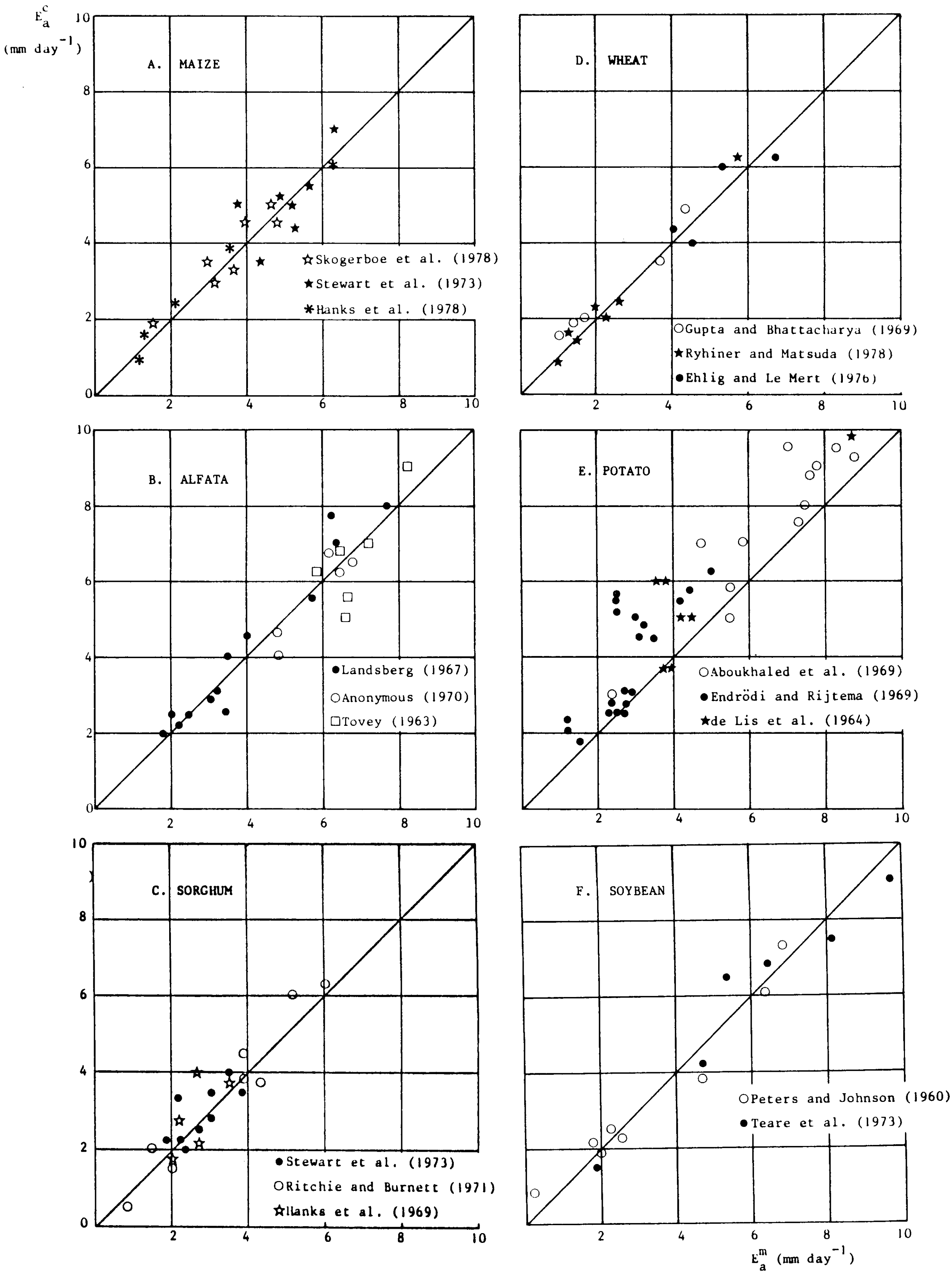

Fig. 2. Comparison between measured (E_a^m) and calculated (E_a^c) actual evapotranspiration for maize A, alfalfa B, sorghum C, wheat D, potato E, and soybean F [Slabbers, 1980].

C. Evapotranspiration and Plant Growth

The primary linkage between the evapotranspiration and corresponding plant growth is also assured to be forced in the action of the stomata, particularly the open areas.

This statement is a serious oversimplification, since in fact growth responds to a large number of factors, one of which is stomata opening and another is the moisture potentials (stresses) resulting from the complex flow of water at whatever rate or level of stress the latter may have.

However, recognizing the oversimplification a general descriptive model can be developed which will give considerable insight to any possible relationship between relative evapotranspiration and relative growth rate and from this to the interpretation of accumulative relative evapotranspiration and accumulated growth for any particular physiological function.

The stomatal openings provide the pathway for both the diffusion of water vapor from the interior of the plants and for diffusion of CO_2 from the atmosphere to the plant for photosynthesis. Since the ability of the stomatal pores to store CO_2 is limited, it follows that the rate of photosynthesis is reflected by the flow of CO_2 through the stomata, hence the total stomatal area would also be related to the rate of photosynthesis as long as CO_2 uptake is a limiting factor (as it appears to be the case under all natural conditions).

These two observations taken together, suggest strongly that the rate of photosynthesis and the rate of water transpiration, since they are controlled by the same area perpendicular to the flow, ought to be proportional. That is if P is the time rate of photosynthesis and t is the time rate of transpiration.

$$P = Kt \tag{1}$$

Note that K is simply an instantaneous "constant" of proportionality and which is determined in turn by all of the other factors affecting photosynthesis and all the other factors affecting transpiration at that instant (or small interval) in time, τ. Since these other factors are indeed quite dynamic, it is to be expected that $K = K(\tau)$, an unknown function of time.

To avoid this problem, Equation 1 has not been generally used. Rather, the concept has been introduced of relative photosynthesis and relative transpiration (both with respect to conditions of no soil moisture related stresses). This results in the $K(\tau)$ being eliminated in constructing the relative ratio (with the subscript representing the unstressed condition).

$$\frac{P}{P_p} = \frac{t}{t_p} \qquad (2)$$

This is an instantaneous relationship valid only for very short time intervals. However, if the ratios were known for each of a series of m time intervals.

$$\sum_1^m \frac{P}{P_p} = \sum_1^m \frac{t}{t_p} \qquad (3)$$

Unfortunately, it is difficult to observe t and t_p for very short time intervals, since the volumes of water involved are small and direct observations of rate of transpiration do not as yet appear feasible[5]. Consequently relationship (3) has been modified to

[5]An interesting possibility is to observe relative temperatures of partially transpiring leaf surfaces compared to fully transpiring by remote sensing, a technique reviewed earlier.

$$\frac{\sum^{m} P}{\sum^{m} P_p} = \frac{\sum^{m} t}{\sum^{m} t_p} \tag{4}$$

The validity of this relationship depends on the degree of constancy of P_p and t_p over the interval of time indicated by the summation. Thus theoretically it should be used only for relatively short periods of time, during which the mean values of t_p and P_p do not change significantly (on the order of a week or two weeks).

Even with these simplifications problems exist because of the difficulty in separating evaporation from transpiration in the measured evapotranspiration. Since evaporation is considered to be small compared to transpiration, the usual practice has been to substitute evapotranspiration for transpiration in equation (4).

In the current situation most investigators use the accumulative relative evapotranspiration as the index of soil moisture deficit conditions, with an implicit (and sometimes explicit) recognition that a correction term should be applied, particularly for the early season when the plant canopy may not effectively cover the surface, and/or for very frequent irrigations which keep the soil surface moist.

Attempts to relate plant growth rates to other variables have been met with considerably less success than the relationship to accumulated relative evapotranspiration.

For example, Figure 3 [Watts 1974] illustrates one experimental relationship between leaf water potential and leaf growth rate. The controlled environment (light and dark) shows some consistency, but with considerable unexplained variation (20% at -4 bars L.W.P.). However, under field conditions (squares) there is no apparent connection with work cited earlier by Scholander, 1965, and others.

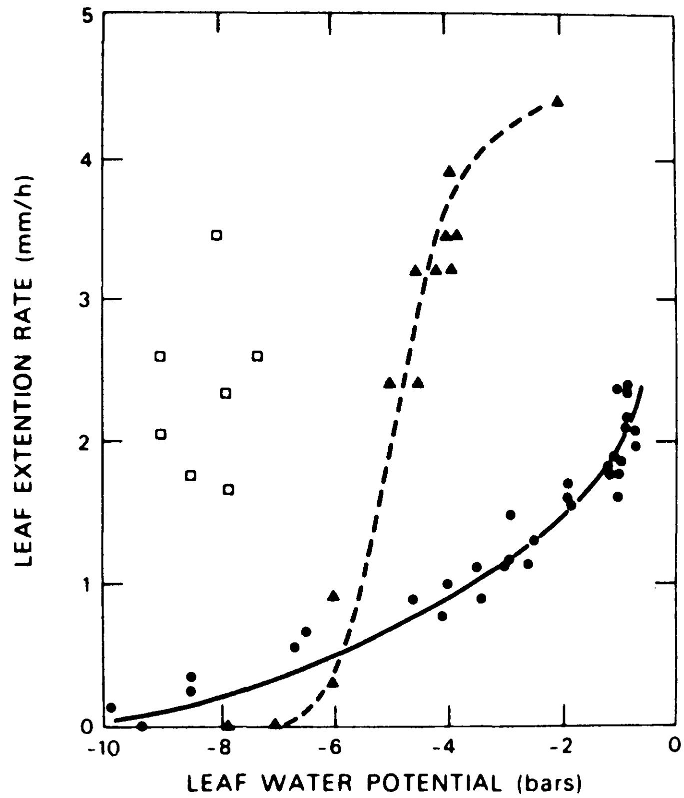

Fig. 3. Relationship between leaf extension and leaf water potential for corn growth in the field (□), or grown in controlled environment in the dark at 28°C(●), or in the light at 30°C(▲). From Watts (1974).

A corollary to the use of leaf water potential or a similar plant indicator is the possibility of using remote sensing of the temperatures of a growing crop canopy, referenced to that of a growing crop canopy of the same variety which is kept "well watered" (well above the critical Θ/Θ_{MHC}. In theory, the temperature of the canopy transpiring less than its potential should rise until a new equilibrium of heat transfer is reached. If all other things were equal, the temperature rise would be determined by the change in heat transferred as latent heat of the evapotranspiration.

Finally the accumulated evapotranspiration (actual and potential) could be measured on a regular basis in the crop and in a well watered standard plot. This in effect is an experimental model.

Except for the use of Figure 1 curve 4 (or similar) together with potential evapotranspiration models, the above alternatives are not available for a priori estimation of the effects of proposed, but yet to be implemented, management decisions. The complete mechanistic methods, if they can be perfected, would allow a more definitive relationship between available soil moisture and evapotranspiration rates.

The discussion to this point reflects relative growth and relative evapotranspiration. However, in many, if not most, agricultural crop production systems only a part of the total photosynthetic growth is harvested. For root crops the tops are much less important and of the "top" crops many involve harvest only of fruit or seed.

Thus from a mechanistic point of view, there remains the problem of modeling the relationship between the evapotranspiration and the growth related to the particular physiological functions which are in turn significantly related to the parts of the plant which will be harvested. Some of these are direct and others indirect, thereby adding to the difficulty of the modeling problem.

This is a large order. Ritchie [1980] points out that the "physiological process of plant extension is more sensitive to water deficits than are stomatal regulating processes." This would argue for a rather nonlinear relationship between relative evapotranspiration and rate of growth.

Even if the questions of evapotranspiration and photosynthesis are resolved, Schrader [1980] contends that "... it is difficult to document that economic yield and photosyntheses are indeed related in any direct way."

Schrader [1980] also states "We simply do not understand enough about the internal control mechanisms of photosynthesis in plants. ... How does a plant sense and respond to an environmental change? What controls the partitioning of photosynthate into different compounds (e.g., amino acids and sugars), to different physiological processes (e.g., respiration vs. leaf growth), and to differential sinks (e.g., roots vs. developing seed?"

Such warnings must be heeded, but at the same time progress must be made. Accordingly, modeling has been based largely on an assumption that the total photosynthetic products are apportioned to various internal growth purposes by some unknown internal "rule" or mechanism whose net effect can be represented by a fraction k of the total photosynthesis. This fraction is in turn determined by the current state of achieved growth. Thus if appropriate states or stages of growth can be defined as temporal units in the growing season, the corresponding k values might be considered to be constant for these briefer periods of time even if rather different during other stages of growth.

Note that it does not follow that the total relative accomplishment of a particular growth function during the time interval for the summation is also proportional to the relative total evapotranspiration. This is partly because during any period of time, the growth rate potential is responding to factors different from those which control evapotranspiration potential. Equation (4) can still be used as an index, but the linearity may be lost except for special circumstances.

In particular, the concerns cited earlier, e.g., Schrader [1980], would seem pertinent ("we simply do not understand enough ... what controls the partitioning ... to different physiological processes ..."). For some physiological processes such as pollination, only a small fraction of the total photosynthesis would seem to be required. What is the fraction allocated to pollen and pollination if there is a plant-water stress?

Even so, from an empirical point of view, it should be feasible to model the relationship using coefficients to be determined later. For example, if the growth rate varies linearly with relative evapotranspiration, and the latter varies linearly with soil moisture below the threshold, then the total accumulative impact on rate of productivity of any physiological process due to a continuously increasing deficit index (e.g., $1 - \frac{\sum t}{\sum t_p}$) should increase as the square of the accumulated deficit of soil moisture, not linearly as has been most commonly postulated. The latter would require the effect to be sudden with the rate of productivity dropping to a new constant level. This concept is implied by the use of "stress-day" type indices. Perhaps a number of different mechanisms apply. In any event, the coefficients presumably can still be determined experimentally for any given assumption concerning the probable functional form if enough relevant data is made available.

This possibility is being utilized in a number of plant growth simulation models currently being developed.

Most of plant growth simulation models begin with an auto catalytic assumption regarding rate of growth of leaf area (to which both photosynthesis and water use are later related) [Childs et al. 1977]. For example, if leaf area A is suitably defined, one might assume for any time τ:

$$\frac{dA}{d\tau} = kA \tag{5}$$

Here k is a dynamic constant which is greater than zero when leaf growth begins, and is zero when leaf growth ceases. It is obviously controlled by many factors, including water deficits. However, if it can be assumed to be constant over short intervals of time, this differential equation can be integrated to "simulate" the actual production of photosynthetic surface over time with a corresponding set of k values presumably to be evaluated experimentally such that actual and predicted leaf surface at any time are reasonably compatible.

If growing conditions were always constant this would suffice. Unfortunately k itself must be a function of photosynthesis, which in turn is related to existing leaf area and moisture stresses, as well as a host of other factors including the allocation of the products of photosynthesis to the roots, leaves, pollen, seed and other physiological processes cited by Schrader, 1980. Since these factors are not known the effects must be represented empirically by experimental observations under different moisture stresses at each of the various phases of growth.

To do this, secondary models are introduced which relate photosynthesis to leaf area and other factors or production. In our case this must include a relationship with some moisture stresses, usually represented by leaf water potential as for example, shown earlier in Figure 3 [Watts 1974].

The net photosynthesis is then "allocated" back to leaf production and to other physiological processes such as root development, flowers, pollen, fruit, etc., depending on the current phase of growth. Once again empirical constants (coefficients and functional form) must be determined by some form of calibration experiments. Figure 4 illustrates one such partitioning [Vanderlip and Arkin 1977] for grain sorghum.

Presumably, if all goes well and the correct internal constants are determined as well as the proper form of the internal functions, the net result could provide a good prediction of this final production of any desired element of the plants, be it seed, fruit, root or forage.

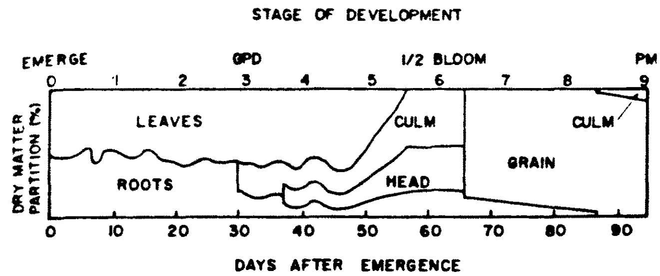

Fig. 4. Partitioning schemé used to determine the relative amount of daily dry matter production going into each plant part throughout the growing season. Days after emergence should be considered as approximate. [From Vanderlip and Arkin,1977].

Figure 5 [Vanderlip and Arkin 1977] and Table 1 [Childs et al. 1977] illustrate both the typical results of the current state of this form of modeling as well as some of the remaining deficiencies to be overcome. There is no intention of being critical of this particular research but rather to illustrate the problems which still remain for all approaches to accurate modeling of a complex multi-stage, multi-variable system such as the production of agricultural crops.

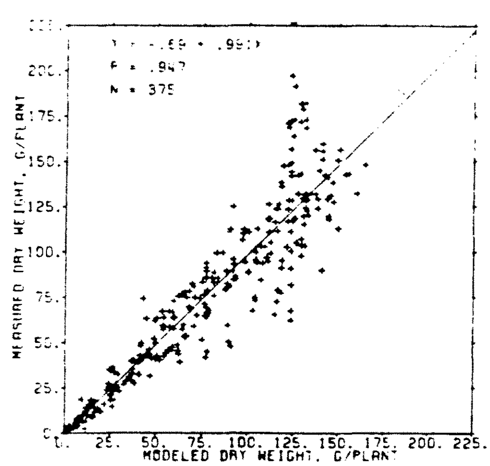

Fig. 5. Modeled and measured dry weights compared for the 21 data sets for which periodic dry matter samplings were available. [From Vanderlip and Arkin, 1977]

TABLE 1. Grain yield in KG/HA (15.5 percent moisture) for five irrigation levels and five years, Mead 1971-1975 [Childs et al.1977].

		Irrigation treatments									
		0.76 cm/day		0.61 cm/day		0.38 cm/day		0.25 cm/day		0.0 cm/day	
Year	Variety	Act.	Pre.	Act.	Pre.	Act.	Pre.	Act.	Pre.	Act.	Pre.
1975	Pioneer 3366	7500	7480*	7260	7480	7050	7410	5230	5680	2490	2620
1974	Pioneer 3388	8680	8360	8800	8340	7670	5840	6470	4400	0	1140
1973	Pioneer 3388	9870	8780	9740	8780	10060	8780	9620	8780	8740	4362
1972	Pioneer 3388	9930	8830	9810	8830	10500	8830	10000	8830	6410	6990
1971	Kekalb 45A	7350	10090	7359	10090	7980	10090	6548	10090	5400	1940

* This treatment in 1975 was used for calibration.

At the risk of attempting to add new ideas not currently well presented in the literature, it can be noted that for each of the internal relationships required in production, the equations selected are "best fit" with substantial deviations between observed and calculated values. From an overall review of the many elemental relationships "found" by various workers, these deviations of experiments are commonly 20% to 40% above and below the predictive equations.

When values of the independent variables of this central tendency are then inserted into a second such relationship (which also shows about the same quality of fit), the resulting deviations will normally be an order of magnitude larger.

As an example in Figure 6 a typical "regression" type relationship between A vs. B and B vs. C are shown. For a given value of observed parameter A intermediate parameter B is "predicted" to be B_3. However, because of unexplained but observed deviations the value of B could be as high as B_1 or as low as B_2. The number of observations near A_1 is small hence the variation could even be larger.

Now if B_3 is used to predict C, the implied value is C_3. However, if B_1 is used the value of C could be as high as C_1. If B_2 is used, it could be as low as C_2. The net effect is to make the estimate of C little more than a pure guess, since virtually the entire experimental range of C is within the uncertainty interval.

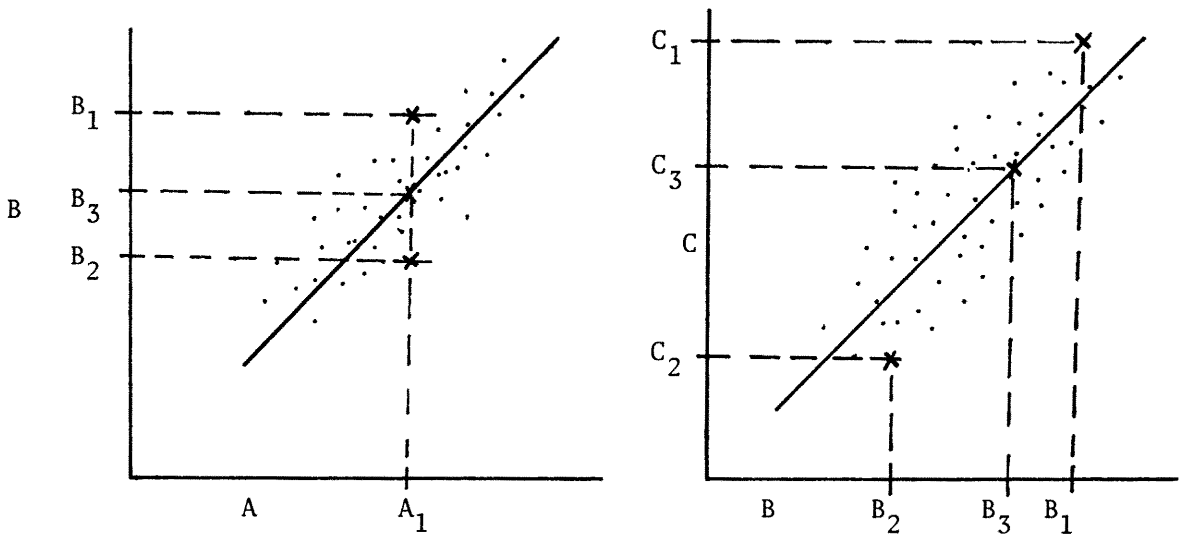

Figure 6. Compounding effects of regression analysis used successively.

Such an analysis suggests that considerably more effort must be placed on reducing the unexplained variances of intermediate models used when a succession of such models is required to go from the initial cause (management decisions) to final impact on the economic products.

This is particularly true when the purpose is to predict changes in economic products from relatively small changes in management practices.

In some cases, the difficulties suggested by Figure 6 can be alleviated by overall calibration. That is, coefficients are adjusted to force the final result to have maximum compatibility between factors of production and final product. In its effect this is equivalent to "black box" modeling described in the following section.

D. Growth Stage Functions and Total Productive Yield

As indicated earlier, work of the ARS-USDA conducted in the 50's established rather conclusively that where the fruit or seeds of a plant were the primary objective of production, the timing of the occurence of a moisture stress of any magnitude was extremely important. That is, each of the physiological phases of growth, if impacted by a given stress, will not have the same effect on the yield of seed or fruit as the others.

If we assume that all of the foregoing research is accomplished, and the impact on the total productivity of any particular physiological stage is fully known, there remains the difficult task of determining what the resulting effect on the desired production element (e.g., grain, forage, roots, fibre, etc.).

The plant growth simulation models described in the preceding section represent one approach to modeling the interrelationships between successive stages of growth to arrive at a predictive estimate of economic harvest expected to be produced. Since the ultimate combinatorial problems were covered, the discussion need not be repeated here.

As an alternative to simulation modeling (actually predating the plant growth simulation models) it is possible to create "black box" models for each of the successive growth phases, then create yet another overall black box model for the final effect. Indeed, possibly because of a lack of success with the various models proposed for this two stage overall model, many, if not most experimental models of water stress vs. crop yield have been presented as a single stage linear relationship of the form:

$$\frac{Y}{Y_o} = k\,\frac{T}{T_o} \qquad (6)$$

or

$$\frac{Y}{Y_o} = k\,\frac{ETA}{ETP} \qquad (7)$$

Several examples of the use of this relationship are shown in Figure 7 [Retta and Hanks 1980].

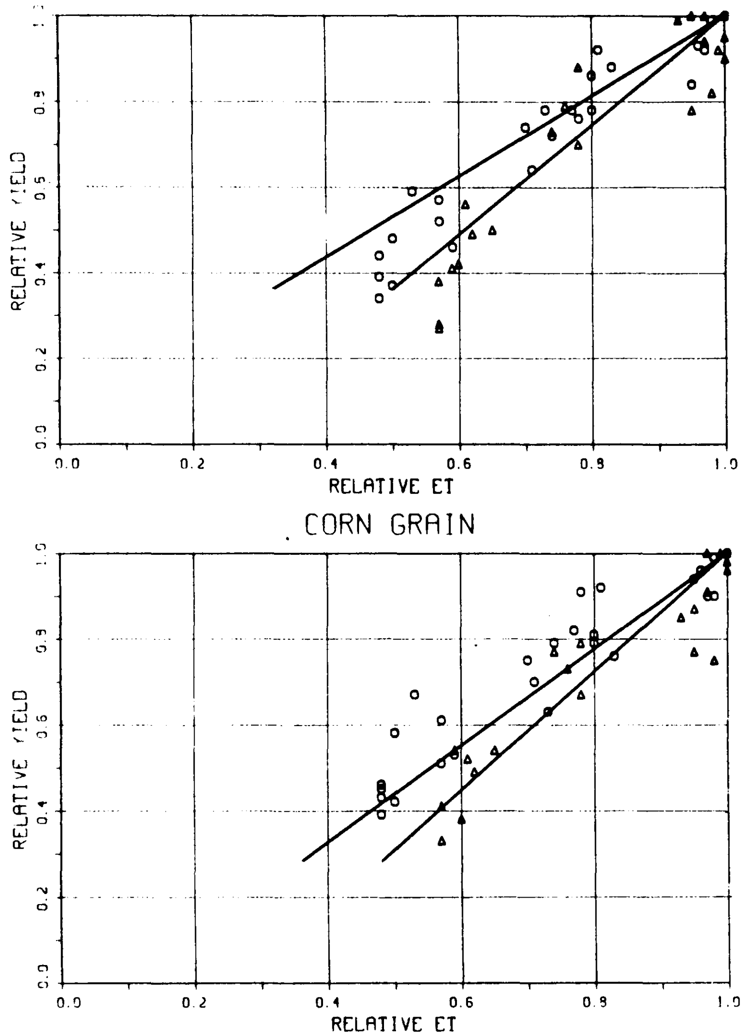

Fig. 7. Typical relationships and data dispersion for the single parameter, accumulative relative evapotranspiration [from Retta and Hanks 1980].

These data may be compared for unexplained variance with those indicated on Figure 5 above, for the plant simulation technique. However, this should not be taken as an indication of superiority of approach. The variance of relationships in Figure 7 probably will remain much the same or more as more and more experimental points are added. However, Figure 5 will obviously get considerably better as the internal mechanisms of plant growth modeling are better understood and modeled.

The reason is that the results illustrated in Figure 7 inherently lump all transpiration deficits into the same parameter regardless of when they occur. The deviations indicated are not experimental errors but rather reflect the differences in the timing and sequence of the deficits indicated.

There are a good many potential experimental "points" to be plotted in the lower right hand side representing "horrible examples" of how not to irrigate. However, since the objectives are "sound water management," their exclusion from the costly experimental program is probably justified for purposes of determining the optimal irrigation practice experimentally.

There also may be additional points upwards and to the left of those shown, representing better management practices than were incorporated in the experiment. These, of course, should be sought even for experimental optimization.

When combined with judgment (i.e., qualitative models), Figure 7 and equivalent functional representations can be very useful. For example, in Chapter 18, American Society of Agricultural Engineers Monograph Number 3, 1980, Table 18.3 is a qualitative model of the most important growth periods as far as timing of a stress is concerned. Figure 18.10 of the same monograph then illustrates where "optimal" and "sub-optimal" regimes will be in a relationship such as Figure 7 above. By providing water during the growth phases suggested in Table 18.3 (shown here as Table 2), the farmer increases the probability that his yield will be on the higher rather than the lower side of Figure 7 for a given total relative evapotranspiration.

TABLE 2. Growing periods in which an adequate water supply is more critical for maximizing Y/IRR

Crop	Growth period most sensitive to water stress	Growth interval in which irrigation produces greatest benefits	References
Sorghum	boot-heading	boot-soft dough	Musick and Grimes, 1961; Herpich, 1974; Hay and Pope, 1975
Wheat	boot-flowering	jointing-soft dough	Robins and Domingo, 1962; Schneider et al., 1960; Salter and Goode, 1967
Corn	tassel-pollination	12 leaf-blister kernel	Robins and Domingo, 1953; Howe and Rhoades, 1955; Salter and Goode, 1967
Cotton	1st bloom-peak bloom	1st bloom-bolls well formed	Levin and Shmueli, 1964; Grimes and Dickins, 1977
Dry beans	flowering-early podfill	axillary bud-podfill	Robins and Domingo, 1956; Salter and Goode, 1967
Potatoes	tuberization	tuberization-maturity	Robins and Domingo, 1956; Delis and Tizio, 1964; Salter and Goode, 1967
Soybean	flowering-early podfill	axillary bud-podfill	Brady et al., 1974; Hay and Pope, 1975; Shaw and Laing, 1966
Sugarbeets	no critical stages	WUE is maximized when water depletion is limited to about 50% available water depletion	Hobbs et al., 1963; Larson and Johnson, 1955; Haddock, 1959
Alfalfa	no critical	WUE is maximized by irrigating to realize full growth potential from start of spring growth until water supply is depleted	Hobbs et al., 1963; Hanson, 1967; Stanberry et al., 1955

There have been several attempts to disaggregate Figure 7 into the component effects due to stresses in various phases of growth. These attempts can be classified as being of two types: (1) Those which postulate that the effects of stress in any one time period should be empirically weighted for importance and the total added and (2) those which postulate a geometric (multiplicative) relationship. The two forms are (taking some liberties to illustrate equivalences.

$$(1)\quad \frac{Y}{Y_o} = \alpha_1(d_1) + \alpha_2(d_2) + \ldots + \alpha_n(d_n) \qquad (8)$$

$$(2)\quad \frac{Y}{Y_o} = a_1(d_1) \cdot a_2(d_2) \cdot \ldots \cdot a_n(d_n) \qquad (9)$$

Note that for small deficits d_i (1) can be shown to be equivalent to (2). Thus (8) has been used with relatively good success for occasional small deficits.

Equation (2), however, has not fared so well, at least as reported in the literature [Stewart et al. 1977]. Figure 7 shows the apparent predictive capability of equation (2) when expressed in the form

$$\frac{Y}{Y_o} = \pi_i \left(\frac{e_a}{e_p}\right)_i^{\lambda_i} \tag{10}$$

Alkazzaz in his thesis [1980], however, using the same data as used for Figure 7 and the form

$$\frac{Y}{Y_o} = \pi_i \left[1 - (1 - k_i) \left(\frac{e_a}{e_p}\right)_i\right]_i^2 \tag{11}$$

gave the results shown in Table 3.

While both instances represent far too few data points to assert the superiority of (10) over (11) let alone validity of equation (9) it does serve to illustrate a very important concept which should be kept in mind for all attempts at mathematical modeling.

Equation (10) appears to follow logically from the earlier discussion. The relative evapotranspiration $(e_a/e_p)_i$ for any short period i was shown to be a good index of relative photosynthesis hence of relative production. That being true, the representation of that relationship by a power function with exponent λ_i indicating relative importance seems to be the logical next step.

However, such an assumption fails to take into account the known characteristics (mathematical properties) of the required functions. First, the power function form requires the zero of the resulting production to be at $e_a/e_p = 0$, when in reality, for some growth periods (such as "hard dough" stage of wheat or "dent" stage of corn) even zero e_a/e_p if it could be achieved cannot bring the production of grain already accomplished back to zero. For other stages, even a non-zero (but low) e_a/e_p could

TABLE 3. Summary of Predicted and Observed Corn Grain Yields [from Al-Kazzaz 1980]

	Predicted Corn Grain Yield T/ha	Observed Corn Grain Yield T/ha	Percent Deviation
Davis	7.18	6.6	8
	7.14	7.2	0.8
	7.02	7.7	8
	8.7	8.7	0
	8.0	8.5	5.8
	10.4	10.5	0.9
	9.4	9.7	3
	11.7	11.4	2.6
	11.05	10.9	1.3
	12.0	11.9	0.8
	12.0	11.5	4.3
	12.0	12.0	0
	12.0	11.2	7
	12.0	12.0	7
	12.0	11.5	4.3
	12.0	11.0	9
Logan	4.4	4.8	8
	6.5	6.8	4.4
	6.6	6.6	0
	6.4	6.6	3
	6.0	6.6	0.9
	6.6	6.6	0
	6.2	6.2	0
	6.4	6.6	3
Fort Collins	4.8	4.5	6
	5.4	5.2	3
	5.3	5.1	3
	6.1	5.7	7
	6.2	6.1	1
	6.1	5.9	3
	6.2	6.3	1
	6.1	5.7	7
Yuma	1.96*	0.6	226
	3.40	1.1	209
	3.55	2.9	22
	3.96	3.8	4
	4.06	4.5	9
	1.36	0.2	580
	1.96	0.6	226
	2.86	0.2	1330

*Predicted corn grain yield values for this site were predicted using the parameters of the other three sites.

result in death of the plants before any useful grain is produced. Again (0,0) is not an appropriate point.

The point (1,1) is an appropriate point and the function does pass through that point. However, it appears that the derivative (rate of change) of the function at (1,1) should be zero. With the power function form given it is not zero but rather $1/\lambda_i$.

On the other hand equation (11) although much more complex meets these mathematical requirements and the exponent 2 used therein is compatible with the assumed linear instantaneous rate relationship below the critical point shown in Figure 1 (which results in a power of 2 for the integrated effect).

A somewhat different sequential simulation modeling approach is that of Morgan et al. [1980]. This approach assumes (as do later "black box" combinatorial models) that the process of production of any desired final product can be modeled for each stage or function of growth as a function of a soil moisture deficit index similar to equation (10). This is then used as a state variable to estimate the next growth period relationship between the soil moisture index and the accomplishment of that next growth phase. This is continued as a dynamic development until the final state is reached. The results are shown in Figure 8.

Again there is substantial variance which may be due to the choice of the functional form of the soil moisture stress index used (see the discussion above).

E. Systems Engineering Models

Systems engineering is defined in Hall-Dracup, 1970, as the art and science of selecting from a large number of feasible alternatives, involving

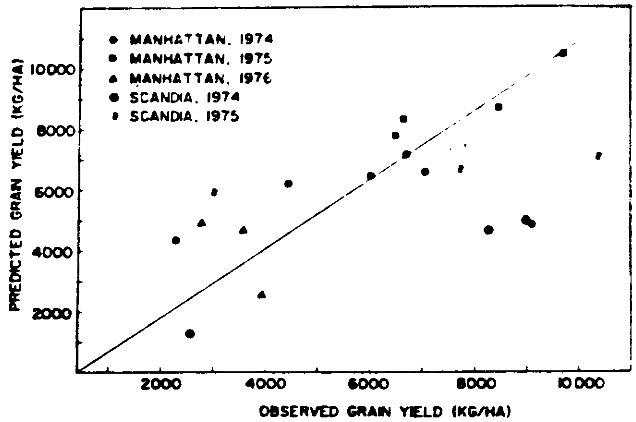

Fig. 8. Compared predicted and actual yields (from Morgan et al., 1980)

substantial engineering content, that particular set of actions which will best accomplish the overall objectives of the decision makers, within the constraints of law, morality, economics, resources, political and social pressures, and laws governing the physical, life, and other natural sciences.

In this broadly defined sense, the resolution of any of the various problems cited in the introduction will involve "systems engineering" or its agricultural equivalent. The foregoing discussion of the scientific aspects is sufficient to insure that there are indeed a large number of possible combinations to be reviewed as potential decision sets and furthermore, that the interrelationships between decisions and results (impact on objectives) are not apt to be simple.

It would also appear that the "objective function" relationship between the decisionable "causes" and the resultant effects, desirable and undesirable will be much more complicated than the form of objective functions generally encountered in textbooks on mathematical optimization.

Thus there clearly is a need for additional research, obviously very much problem-oriented, in the science and art of optimization under these conditions, and as related to the problems cited in the Introduction.

The Hall-Butcher multiplicative model equation (9) [1968] was in fact introduced as a strawman hypothesis to illustrate a possible optimization technique under severely limited circumstances (complete knowledge of the seasonal water supply volume). It was not intended as a proper representation of a production function (it was not therein justified by logic or experimentation), but rather as an illustration that even such unusual functions could be handled with what proved to be a three dimensional dynamic programming procedure.

Moore, 1961, created a similar strawman about 1960, in this case using the linear-additive model (equation (8)) to illustrate how linear programming could be used to optimize the use of water. Others have followed this lead because of the widespread knowledge of, and acceptance of linear programming (hardly a satisfactory justification if the production function is not in fact linear).

Subsequently, in the Joint U.S.-India Team Report on Soil and Water Management in India, 1970 (out of print), Hall utilized a slight modification of the Hall-Butcher model to estimate an appropriate seasonal drought strategy for wheat and sorghum. While illustrative, the $a_i(d_i)$ functions used were crude estimates based on a very limited data sample, hence the results were hardly useful, let alone conclusive of an appropriate dry season strategy for India.

Dudley (1969, 1979) and others in Australia have also followed up on Moore's original work with linear programming. As indicated earlier, so long as the deficits are not too large, the linearization of the production function in the vicinity of full production is a reasonable approximation and, to this extent the results of linear programs used are also reasonable except for the fact that L.P. (being linear) tends to seek solutions at one extreme or the other, hence the magnitudes of the derivatives of the true function will deviate from the assumed values (the linear coefficients) at the maximum amount for most "solutions."

All of the above models were directed to the single problem of maximizing yield from a known but deficient supply. As indicated in the introduction this is but one aspect of one of the many problems related to these functions, i.e., the "drought strategy" problems under conditions of certainty and full control over when that supply should be released for use. Thus it is obvious that there is a large amount of "systems engineering" type of research still to be accomplished before mathematical models for selecting the best practical answers to the spectrum of problems cited can be produced.

In the meantime the proper judgmental optimization (as described above using ASAE Monograph Number 3, 1980) should not be discounted. The term "systems engineering" is used rather than "systems analysis" or any of the many other names used for mathematical optimization modeling since it is quite likely that a large element of qualitative, judgmental optimization will be required if satisfactory results are to be obtained.

However, there is another much more important role for systems engineering in the scientific knowledge picture. As indicated earlier there are difficulties in determining the proper functions to be used when there are a large number of causative agents and only one (or a few) consequences thereof.

One of the strengths of optimization is its ability to determine the "least error" fit of a set of experimental observations of causes to the corresponding experimental observation of effects. The most common of these applications is "linear regression." However, nonlinear regression can often be accomplished (albeit with more computational difficulty).

Even when the functional form of f is known, above, it is difficult to determine the set of parameters (e.g., $\underline{c}$ = set of coefficients and exponents) since the resulting equations from taking the derivatives and setting them equal to zero are nonlinear and perhaps even non-algebraic.

The case where the production function was a multiplicative form, equation (9) above, was undertaken by Alkazzaz in his thesis (CSU 1980). The problem, although simplified by assuming the exponent was exactly 2, is still more difficult. Obviously better "optimization methodologies" for function determination would be of considerable help in defining the many internal and overall functional relationships which, as discussed earlier must be refined to a much greater extent if a satisfactory science is to be achieved.

F. Experimental Results

It would be an impossible task to review and chronical the results of the thousands of irrigation experiments which have been conducted since the turn of the Century, all of which were directed toward the definition of the cause-effect relationship between soil moisture levels and crop production. No attempt will be made to do so. Instead an attempt will be made to review the strengths and weaknesses of these experimental programs as a means of aiding the direction of future research.

To begin, experimentation is essential and considerably more experimental work must be accomplished before the scientific requirements described above can hope to be met. Although it was not desirable to interrupt the flow of ideas involved in the discussion of various theories, all of these theories have evolved gradually from the experimental work, both published and unpublished. The path of evolution to the present state is long and involved.

It has included many false starts, many setbacks and many small steps forward, each contributed by many individuals and, through discussion and interactions, many groups of individuals.

In each case, the scientists responsible for the experimental design used their own interpretations of what all the previous experiments indicated, even in their failure to provide answers. Some of these guesses provided further insight. Others only produced frustrations. In all too many cases the inconclusiveness of previous results was confirmed but without shedding additional light on the theoretical nature of the problems.

Progress can probably best be summarized by describing the several "eras" in the experimental developments to date. The first of these, as might be expected, consisted of little more than varying the amount of irrigation water and observing the resulting production. The important contribution of these experiments was a demonstration that the phenomena involved were by no means simple and, furthermore the quantity of water provided was an extremely poor and inconsistent indicator of effect. It seemed clear then that a study of the interaction of the plants and the soil moisture would be essential.

The second phase of irrigation research focused on this concept. It resulted in the Veihmeyer-Hendrickson (1957) theory recited earlier and the alternative theories presented by Thornthwaite (1948) and by Richards and Wadleigh (1952). While no conclusive theory was produced, this era produced a very significant science of soil moisture in the root zone of actively growing plants. At the same time, it illuminated the experimental difficulties of making satisfactory soil moisture measurements for theoretical interpretations. These implied need for knowledge of the conditions in specific pores of the soil rather average conditions over a large number of such pores. Although remembered by some as an era of bitter controversy, it is now recognized for its basic contributions upon which later experiments and theories could be more rationally defined.

The third phase of irrigation research may be properly identified with the discovery by ARS-USDA scientists that time was a very significant factor in the interaction of soil moisture stress and productivity. This introduced intensive experimentation in which attempts were made to produce differential stresses at various important phases of growth for major crops such as wheat (representing similar small grains), sorghum and corn.

Unfortunately the experimentation has not yet produced definite qualitative results. Even the most recent papers [Morgan, Biere and Kanemasu, 1980] show as much or more unexplained variance as encountered previously.

The fourth phase may be identified with the development of simulation models during the past decade. This work is exciting whether incorporated into models of the form of equation (8) or (9) or by-passing these for direct accumulative prediction. It represents the aggregation of the knowledge produced by many different disciplines into a structured whole. Experimentation remains the key to this structure as well as its components.

III. SUMMARY OF THE STATE OF THE ART AND SCIENCE

From a qualitative or descriptive science point of view there is a reasonable concern that the relationship between soil moisture levels, as a distributed or variable function over time, and the final yield of economic products from agricultural crops is far from simple. However, despite this complexity, certain apparent facts have emerged from the many experiments and their interpretation and reinterpretation.

1. As the moisture levels in the root zone soil decrease with time after irrigation or rainfall, the rate of use of water by the plants, relative to that which would otherwise have been used with high moisture levels, will decrease. Furthermore, this decrease is not linear but the decrease becomes increasingly pronounced as the moisture levels fall. At the soil moisture level at which plants "permanently wilt" (i.e., do not revive without the additional water added to the soil), the rate of loss is negligible compared to the potential rate under good moisture conditions.

2. This "permanent wilting point" occurs at rather different absolute levels of soil moisture for different soils. However, the equivalent soil moisture potential (or soil suction pressure) is the same for all soils for all practical purposes.

3. Because the CO_2 for plant growth and the H_2O from respiration must pass through the same area of the open stomata, the instantaneous crop growth rate and water use rate should be essentially proportional. This has resulted in a reasonable concensus that cumulative relative growth in any phase of growth is directly proportional to the cumulative relative evapotranspiration.

4. Because the several phases of growth are related to final production through rather different mechanisms, stresses in some phases of growth may be much more critical than others. This has been experimentally verified in a qualitative sense.

5. Application of irrigation water (or rainfall) to the soil surface does not result in a distribution of water in the root zone in proportion to the stress or to the amount previously removed by the roots, but rather replenishes the upper levels of the soil to field capacity, from the top down, to the extent of the volume of water applied. However, it is uncertain whether bringing a few inches of soil to field capacity results in a full relief of the stress or merely a fractional mitigation thereof.

6. The feeder roots of plants do not completely permeate all the available pores in which water may be stored. The density of these roots (i.e., number per unit volume) decreases with increasing depth below the surface, except perhaps for the surface soil subject to air drying and/or cultivation disturbance.

7. The evapotranspiration, because of the low capacity of the plants themselves to store heat, is essentially equal to the corresponding net energy transfer to the plants. This means that plants whose evapotranspiration is below that for no stress conditions must decrease the relative net energy transport. The only apparent mechanism for this is an increase in temperature and/or an increase in reflectance.

On the deficiency side, substantially more accurate models (refinements and new models) for:

a. The quantification of the mechanism for the transfer of water from the soil to the plant's guard cells and stomatal cavities.

b. The quantitative mechanism relating soil moisture stress to stomatal opening.

c. The distributive mechanism which results in differential moisture removal at different depths in the root zone.

d. As a summary of all of the above, a quantitative mechanism for predicting the actual evapotranspiration from plants with any given distribution of remaining soil moisture.

e. Evaluating the interaction between either reduced relative evapotranspiration or moisture stress levels and the resultant impact on the relative accomplishment of any particular phase of growth. The possible exception is the purely vegetative growth phase (leaves and stems).

f. A suitable quantitative model reflecting the relationship between the relative accomplishment of the various phases of growth on the yield of economic product.

Corollary to each of these deficiencies, there is also the inadequacy of current data interpretation methods to evaluate any reasonable hypothesis regarding any of these missing relationships. As indicated earlier, linear regression (including multi-factor linear regression) is the only parameter identification tool in general use and few of the missing relationships appear to be linear. Mitigation of this inadequacy, is clearly a necessity if the other missing relationships are to be found.

In the process of researching these questions it is quite likely that still other relationships and parameters will be found to be significant and will require attention as well. For example, if soil moisture at any specific phase of growth is a significant factor, it would seem obvious that fertilizer concentrations, salt concentrations, root zone oxygen levels, etc., may also be significant. However, the nature of the soil-water reservoir is such that changes in soil moisture levels must inevitably affect the concentration of salts, fertilizers, oxygen, etc., to the extent that these are not buffered by low solubility, other chemical release, or other mechanisms of ready equilibrium replacement. These are complicating factors which hopefully can be separable and considered as integrated levels.

IV. CONCLUSIONS AND RECOMMENDATIONS

The problems whose resolution are dependent on satisfactory achievement of a suitable science for evaluating the impact on crop production under various possible water management options are serious and will become more serious with passage of time. Such knowledge is vital to the rational assessment of various water policy options presently under serious political considerations.

The processes whose nature is to be modeled quantitatively and qualititatively have proved to be complex and relatively unyielding. No conclusive results are available which could be used for quantitative prediction. At the same time considerable progress has been made in gaining insight into the physical-biological nature of the elements of the system concerned. If approached with a carefully designed and managed experimental and theoretical research program, prognosis for success in the near future is relatively high.

The key words in the previous optimistic statement are "carefully designed and managed." Past experimental research has been very extensive and a number of theories have been available for more than a decade. Yet the two have not as yet been reconciled and, as a guess, about 75 to 90 percent never will be. In retrospect, for these experiments, the proper variables were not observed or were not properly observed, unpredictable climate factors loused up the experiments, or (usually most important) budget considerations severely limited the number of variables and the range of their values which could be observed and incorporated.

It would be presumptious of the author of this review to recommend the details of experimental design and theory evaluation. These must be left to the scientists involved. Thus the author's own suggestions and opinions in this regard will be suppressed as much as possible. Rather the recommendations will be directed primarily to the issues of research management conducive to the development of "carefully designed and managed" experiments and theories.

There is an obvious implicit recommendation that all of the unsettled scientific questions should be researched until satisfactory results are in hand. It is tempting to urge that most of the research attention be directed to items (e) and (f) of the research needs described in Section III[1]. After all, these represent the ultimate research requirement. If they are answered, other issues are relatively unimportant. However, a more satisfactory insight into the other mechanisms will probably be essential to the effective design of the experiments directed to these two research needs, and for the interpretation of the results in mathematical model form.

At least for the present and until otherwise conclusively demonstrated, all six research questions in Section III would appear to be highly interrelated and mutually interdependent for satisfactory results.

Beyond these obvious subject matter recommendations, there are some additional research management questions to be addressed.

Perhaps the most important of these is the relaxation of budget restrictions which have generally limited any one experimental program to relatively few different treatments. As indicated earlier, it is impossible to find even five points on each of 7 functions with a budget limitation to

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[1](e) A suitable quantitative model for the interaction between relative evapotranspiration during a growth phase and the relative achievement of that growth phase.
(f) A model of the relationship of such relative achievement of growth phase and the ultimate economic products.

half a dozen or a dozen trials, even if all functional forms were correctly known a priori.

A corollary management recommendation is for the deliberate development of research teams composed of carefully selected scientists. As indicated earlier there are too many interacting elements to expect any one expert to be able to integrate the design, execution and interpretion of the necessary experiments, with the development and testing of the corresponding theories. Each of us would like to think that we can but several decades of experience would suggest otherwise.

Those research teams should include both plant-soil-water scientists of various related expertise, and scientists familiar with parameter identification techniques and optimization concepts. The latter in particular should have much more than a passing acquaintance with plant-soil-water science. The team members must also be capable of developing and testing alternative mathematical forms that the relationships might, and in fact ought to assume, given what is already known. For example, it is not sufficient to assume that, since we believe relative production is closely related to relative evapotranspiration, that this relationship can be represented by a simple power function (as most of us, including the author, have done). Rather, the functional form must be true to what we know or expect regarding where the origins of coordinates should be, how the rate of change should vary at the known points, in which direction should the functions be convex or concave (if not linear), and similar qualitative characteristics.

Note that there must be more than one such research team created. It is unlikely, from a purely stochastic point of view, that any one group of scientists will by chance discover the key points and issues that will assure success. How many such teams may be needed is indeterminant and rather dependent on the specific members of the teams formed and an assessment of their chances for success.

Finally, if the team approach is to succeed, research managers responsible for promotion and retention must modify their existing so-called "objective" measures of performance. By and large these are based on clearly identified individual achievement rather than on group success. It is quite unlikely, under the existing rewards system, that any rising scientist with potentially good ideas will donate them to the group effort unless he can be assured that his own economic and professional stature will be enhanced at least to the same extent that it would have been had he kept on the path of individual research.

This requirement will not be met by a pronouncement (even in writing) that group effort will be equally rewarded as if it were individual effort. The concept of individual contributions as the sole index of scientific stature (and of promotion) is too strongly imbedded in the fabric of science for such a pronouncement to be credible. Indeed, it has probably been more enthusiastically embraced by the scientists themselves than by the research program managers, however happy the latter may have been to avoid the difficult task of more subjective, but more accurate, evaluations of personnel.

Under these circumstances, it would appear that the more appropriate management technique will be one of assuring that all individual contributions are appropriately memorialized in scientific publications, each involving at most two or three coauthors. This should include alternative and even contradictory hypotheses and/or interpretations which, under "teamwork" are often suppressed to maintain an illusion of university. Such hypotheses and interpretations are essential to arriving at sound conclusion and, even if subsequently disproved or abandoned, still constitute indispensable contributions to the final success.

In some instances, particularly where group discussion, including controversy results in better understanding and forces more precise thinking, it will be difficult to identify, al fin, who was responsible for which results. In such cases, credit still must be apportioned judiciously and fairly if the objectives are to be met.

The problems related to the research issues discussed here are significant, and fully justify the necessary research. "Try it and see what happens" will be a costly, if not catastrophic, alternative to formal research. While the subject area is clearly complex, there has been generated sufficient insight to give a reasonably high probability of success, particularly if a well designed, well managed research program is deliberately created and adequately supported. While it will clearly be expensive in cost, the do-nothing alternative clearly will be far more costly.

APPENDIX A

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APPENDIX B

REVIEW OF SOME OF THE RELEVANT LITERATURE

Robins, J.S. and Domingo, C.E., 1953, Some Effects of Severe Moisture Deficits at Specific Growth Stages in Corn

The authors reported the results of an experimental study of the effects on yield and plant development of severe soil moisture deficits at specific growth stages of corn rather than the "response to some arbitrary moisture level." Six moisture treatments were studies.

The authors came to the following conclusions:

1. Timing or the growth stage at moisture stress and length of the stress were important.

2. Moisture deficits at tasseling or pollination for 1 or 2 days resulted in 22% yield reduction.

3. Moisture deficits for 6 to 7 days at tasseling resulted in 50% yield reduction.

4. Reduction in yield when moisture stress occurs after fertilization appears to be related to the grain maturity at time of the stress.

5. Moisture stress after maturity did not lead to yield reductions.

No mathematical model was proposed.

Denmead, O.T. and Shaw, R.H., 1960, The Effect of Soil Moisture Stress at Different Stages of Growth on the Development and Yield of Corn

The authors presented the results of a study on the effects of soil moisture stress at the vegetative, silking and ear stages of growth of corn on the yield. Moisture stress at vegetative and silking, vegetative and ear, and silking and ear were also reported. Stressing was applied by allowing soil moisture to be depleted to wilting point. They also varied the duration of the stress periods, and also studied stress effects on stalk height, cob length, leaf area, and assimilation.

Although no mathematical model was hypothesized or enounciated afterwards, the authors came to the following conclusions:

1. Grain yield was affected more than any other plant characteristic by moisture stress at all stages of growth.

2. Stress at silking was most harmful, reducing grain yield by 50%, than stress at any other single growth stage.

3. Effects produced by various stress combinations was additive on plant elongation.

4. Early stress indirectly affects yield by reducing the assimilation surface.

5. There may be a tendency for stress imposed in any one stage to harden the plant against damage from stress at a later stage as far as grain yield is concerned.

Musick, J.T. and Grimes, D.W., 1961, Water Management and Consumptive Use by Irrigated Grain Sorghum in Western Kansas

A quadratic functional relationship between yield and available soil moisture was derived from varying irrigation treatments during five stages of growth over a three year period 1957-1959 at Garden City, Kansas. Reductions in yield were shown to be significant when available soil moisture dropped below 30%. Effects of one preplanting irrigation and dry fallow farming on yields were compared to the maximum yield value of the varied available soil moisture experiments.

Moore, C.V., 1961, A General Analytical Framework for Estimating the Production Function for Crops Using Irrigation Water

Recognition of the importance of timing of irrigation to the potential growth of crops. Assumed use of water by crops, evapotranspiration, was independent of soil moisture conditions and assumed additive model whereby growth in any period is independent of growth in any other period--simply factor of evapotranspiration rate.

Herron, G.M., Grimes, D.W.., and Musick, J.T., 1963, Effects of Soil Moisture and Nitrogen Fertilization of Irrigated Grain Sorghum on Dry Matter Production and Nitrogen Uptake

Dry matter production as a function of growth stage and for various amounts of applied nitrogen and varying soil moisture conditions were reported. Effect of deficient nitrogen levels during particular growth stages shown to effect dry matter production but functional relationship not specified, nor combinatorial effect of deficient soil moisture and deficient nitrogen level on production.

Musick, J.T., Grimes, D.W. and Herron, G.M., 1963, Irrigation Water Management and Nitrogen Fertilization of Grain Sorghums

A graphical relationship between relative yield and available soil moisture was presented for various applied nitrogen treatments over a three year period, 1957-1959. Yield data shows a significant decrease in yield when soil moisture during the period of root to dough stage of grain was depleted below approximately 25% available in the top 4 feet.

Jensen, M.E. and Sletten, W.H., 1965, Evapotranspiration and Soil Moisture-Fertilizer Interrelations with Irrigated Grain Sorghum in the Southern Great Plains

Effects of various soil moisture levels and nitrogen treatment were reported on grain sorghum yields for a four year study, 1956-1959, at Bushland, Texas. Relative yield versus relative seasonal evapotranspiration was graphically represented. Yield decreases were shown for decreases in available soil moisture below 20%.

Jensen, M.E. and Sletten, W.H., 1965, Evapotranspiration and Soil Moisture-Fertilizer Interrelations with Irrigated Winter Wheat in the Southern Great Plains

Effects of various soil moisture levels and nitrogen treatments were reported on winter wheat for a three year period 1956-1958 at Bushland, Texas.

Musick, J.T., Sletten, W.H., 1966, Grain Sorghum Irrigation-Water Management on Richfield and Pullman Soils

Experimental effort at determining a functional relationship for relative yield of grain sorghum to available soil moisture for two different soil types and for different numbers of irrigations on a seasonal basis. Assumed a linear function between the two variables.

Flinn, J.C. and Musgrave, W.F., 1967, Development and Analysis of Input-Output Relations for Irrigation Water

Used the concept of the ratio of actual to potential evapotranspiration to be a function of the soil moisture. Postulated an additive crop growth model that was a function of stress free days, i.e., when actual evapotranspiration does not limit growth due to deficient soil moisture conditions.

Grimes, D.W., Dickens, L., Anderson, W. and Yamada, H., 1967, Irrigation and Nitrogen for Cotton

A three dimensional graphical relationship between lint yield, applied nitrogen and applied seasonal irrigation water was presented. On a seasonal basis optimal combinations of nitrogen and irrigation can be determined and excessive amounts of nitrogen were shown to decrease yields.

Zaslavasky and Buras, 1967 Crop Yield Response to Nonuniform Application of Irrigation Water

Theoretical model. Objective was to determine nonuniformity in area, not amount of water or timing of water.

Grimes, D.W., Yamada, H. and Dickens, W.H., 1968, Functions for Cotton Production from Irrigation and Nitrogen Fertilization Variables: I Yield and evapotranspiration

A quadratic function relating relative lint yield of cotton to accumulative seasonal evapotranspiration was derived from experimental data at two locations in the San Joaquin Valley, California.

Musick, J.T., 1968, Irrigating Grain Sorghum with Limited Water

Results from experiments at Bushland, Texas, with varying irrigation treatments (within season) and corresponding sorghum yield were reported for 1956-1959 and 1963-1965 periods. Effect of eliminating preplant irrigation on sorghum yield was also reported.

Hall, W.A. and Butcher, W.S., 1968, Optimal Timing of Irrigation

The authors postulated that the magnitude of the losses in crop yield may depend almost as much on when the soil-moisture deficiency occurs as it does on the total magnitude of the shortage. If the maximum yield of a crop when soil moisture conditions were the best possible under any other given conditions was Y_{max}, then the resulting yield under an imperfect soil-moisture condition in only one growth period i may be expressed as

$$Y = a_i Y_{max}$$

where

$a_i = a_i(w_i)$ and w_i = soil moisture level in period i

If moisture stress occurs in more than one growth period then

$$y = a_1(w_1) \cdot a_2(w_2) \cdot a_3(w_3) \ldots a_n(w_n) Y_{max}$$

Since the yield function meets two important conditions: (when $w_i = w_f$ and there was no adverse moisture condition (w_f = moisture level at field capacity), $a_i = 1$ and $y = Y_{maxj}$ if $a_i(w_i) = 0$ for any i, y = 0) the authors feel that the yield function was reasonably accurate.

Having advanced this postulate the authors proceeded to develop a dynamic programming procedure for determining the optimal seasonal distribution of water at each point in the overall production function. If the objective was to maximize the expected yield from a unit of land but the total water supply was limited and inadequate to meet the desired soil-moisture demand, the optimization problem may be set up as

$$y = \max\ [a_1(w_1) \cdot a_2(w_2) \ldots a_n(w_n) \cdot Y_{max}]$$

or after logarithmic transforms and substitutions as

$$Y = A_1(w_1) + A_2(w_2) + \ldots + A_n(w_n) + Y_{max}$$

$$\sum_{i+1}^{n} x_i \leq q_o$$

$$w_p \leq w_i \leq w_f \quad , \quad w_{i+1} = w_i + \eta x_i - e_i + \phi_i$$

and $a_i \geq 0$

where $Y = \log y$, $A_1(w_1) = \log a_1(w_1)$, etc.

w_p, w_f = permanent wilting point and field capacity respectively

x_i = quantity of water to be applied to the soil in period i

η = efficiency of water application

e_i = evapotranspiration in period i

ϕ_i = precipitation in period i

q_o = total irrigation water available at the beginning of season

The solution of the problem was x_i^*, the decision variables that maximize both the logarithm of the yield and the yield also. If the quantity of available water was less than adequate for maximum yield production and cost of water application was costly and related to the quantity applied, then an optimum quantity that maximizes the returns and an optimal application policy for the quantity of water will exist. The authors then put forth a model that determines the optimal policies for application of the limited quantity of water. The recursive relation for this dynamic programming problem was given as

$$R_i(q_i,w_i) = \max[P\ a_i(w_i + \eta x_i - e_i + \phi_i) \cdot Y_{i+1}(q_{i+1},w_{i+1}) - $$
$$- c_i(x_i) - C_{i+1}(q_{i+1},w_{i+1})]$$

subject to

$$0 \leq x_i \leq q_i$$
$$0 \leq q_i \leq q_o$$

and $w_p \leq w_i \leq w_f$

with $Y_{i+1}(q_{i+1},w_{i+1}) = a_{i+1}(x_{i+2}) \cdot Y_{i+2}(q_{1+2},w_{i+2})$

$$C_{i+1}(q_{i+1},w_{i+1} = c_{i+1}(x_{i+1}) + C_{i+2}(q_{i+2},w_{i+2})$$

$$q_{i+2} = q_{i+1} - x_{i+1}$$

and $$w_{i+2} = w_{i+1} + \eta(x_{i+1}) - e_{i+1} + \phi_{i+1}$$

The authors affirmed that they tested the feasibility of the method by a three time period hand calculation and a ten-time period computer solution that gave good results.

In their conclusion, the authors stressed that optimum use of limited irrigation water implies not only an efficient system but also timing of irrigation to conform to critical growth stages of the crop. The methodology developed by them, they argued, will permit a farm manager to determine the time and quantity of irrigation that will maximize his total net returns allowing for cost of irrigation water. The method as take account of the adverse effects of soil moisture deficiency as well as cases in which a moisture deficiency improves net return. However, they cautioned that more information, preferably in a dimensionless form needed on the nature of critical periods, characteristics of commercial crops, and the adverse effects of the associated soil-moisture deficiencies.

Jensen, M.E., 1968, Water Consumption by Agricultural Plants

The author distinguished between two general types of crops when dealing with effects of inadequate soil water on yields: (a) grain crops that have a determinate type of flowering and (b) grasses that can tolerate severe stress for upto a week in the growing season and recover almost with little loss in total dry matter production following application and maintenance of adequate soil moisture.

Where other factors except soil moisture are non-limiting, the marketable product of a determinate crop, according to the author, could be linearly related to soil moisture by

$$\frac{Y}{Y_o} \simeq \prod_{i=1} \left(\frac{W_{et}}{W_{oi}}\right)_i^{\lambda_i}$$

where

$\frac{Y}{Y_o}$ = relative yield of the marketable product

Y_o = yield when soil moisture is not limiting

$(W_{et}/W_{oi})_i$ = total evapotranspiration during a given stage of physiological development

and

λ_i = relative sensitivity of crop to water stress during i^{th} growth stage

The author indicated that λ for specific growth stages would depend primarily on the sensitivity of plant growth to water stress during each growth period. The consequence of his yield functionwas that a marketable farm crop may not be linearly related to the total water use when plants were stress. He site observation from Jensen and Sletten (1965) in which it was found that reducing water use by 20% resulted in grain sorthum yield reduction of 35% and delaying irrigation so that yields reduced 70% reduced water use 40%. From a three growth period analysis he made on grain sorghum, the author affirms the adequacy of the multiplicative function to represent the effect of water stress on yields of a determinate crop.

For indeterminate crops he related the dry matter produced when soil moisture was limiting to soil moisture by

$$\frac{Y}{Y_o} \simeq \frac{\sum_{i=1}^{n} \lambda i (W_{et})_i}{\sum_{i=1}^{n} \lambda i (W_{oi})_i}$$

Thus whereas the effect of water stress on yield was dependent of the stress on the previous stages in determinate crops, for indeterminate crops, the effect on one stage is independent of that in other growth stages. The author sites DeWit (1958) as generally substantiating this function in his analysis.

References:

Jensen, M.E. and W.H. Sletten, 1965. Evaporation and soil moisture-fertilizer interations with irrigated grain sorghum in the Souther High plains, U.D. Department Agriculture Conserv. Res. Rept. 5, 27 pp.

DeWit, C.T., 1958. Transpiration and crop yields, Mededel. Inst. Biol. Schneik. Onderzoek, Landbouwgewassen, Wageningen, 59, 88 pp.

Grimes, et al., 1969, Functions for Cotton-I Yield and ET, II Yield and Quality

Combination of hypothesized response function and experimental work to determine parameters. All done on seasonal basis-ETP as function of pan evaporation and the ETA by water balance. Correlated ET vs. yield.

Hanks, Gardner and Florian, 1969, Plant Growth-Evapotranspiration Relations for Several Crops in the Central Great Plains

ET vs. yield function based on experimental data on seasonal basis (no data given). No hypothesis of seasonal (or growth periods) production deficits due to water (ET) deficit advanced.

Stewart, J.I. and Hagan, R.M 1969, Predicting Effects of Water Shortage on Crops

The authors reviewed some existing water-crop yield functions and gave some interpretations to published functions, and also discussed some factors that influence the shape of crop yield-water use functions. Most of their analysis was done using alfalfa and wheat and grain sorghum.

The authors recommend the use of ET instead of total applied water in crop production functions because of variations in water use efficiency. Because yield could refer to a major vegetative growth, a storage organ, a reproductive organ or a chemical constituent, distinction should be made between them; water stress could affect growth and yield differently.

For alfalfa, hay (yield) vs. ET function started from the origin, but the exact form, convex, concave or linear for different years were different in the functions examined by the authors. Wheat production functions did not start at the origin, but had an intercept on the ET axis. The final shape of the curve was still under study. Sorghum production function tended to be a straight line.

Factors that influenced the shape of crop yield-water use functions were identified as:

1. differences in crop varieties which exhibit marked differences in yield potentials and responses to water stress.
2. Nonhomogeniety among the members of a given plant population.
3. Nonhomogeniety of field conditions, especially at the extremes--where yield response is just beginning to occur and in the neighborhood of maximum response.
4. Fertility status of the soil.
5. Weather changes from year to year.
6. Sudden weather changes.
7. Planting time.
8. Critical growth stages.
9. Plant populations and row spacings, and
10. Diseases and pests as well as lodging effects.

The concluding remarks were that crop yield-water use functions augmented by water supply information-ET, estimates of maximum yield per acre---would provide answers pertinent to allocation of a below normal water supply and that the decision maker can make more intelligent plans for use of water if information on production functions were provided.

Hogg, H.C., Davidson, J.R. and Chang, J., 1969, Economics of a Water-Yield Function for Sugarcane

The main objective of the authors was to explore the economic aspects of a water-deficiency model that would isolate water and sugarcane yield in Hawaii. But they also pointed out the increasing need for efficient use of water in agriculture, the need for information that will permit estimation of the economic contributions of irrigation to crop production and the incorporation of these into the decision making process. An operationally feasible procedure should be simple, computationally manageable and allow for the effect of soil moisture depletion on yield and for moisture requirements during different stages of plant maturity.

The authors report the results of an experiment in Oahu, Hawaii involving six levels of water treatments of sugarcane. Using water-balance method, they developed a procedure for determining water-yield relationships for sugarcane given by

$Y_a/Y_p = f(E_a/E_p)$ where Y_a = actual yield

Y_p = potential yield

E_a/E_p = effective pan factor EPF = actual ET/potential ET

They also defined EPF = $R_e\{+ (NxS)\}/E_p$

where R_e = effective rainfall, N = no. of irrigation rounds, S = soil moisture storage. From the experimental data the final relation is

$$Y_a/E_a = .2222 + 2.6631(EPF) - 1.7889(EPF)^2$$

The authors then derived a relation for maximum profit that would be produced for any given combination of factor and product prices.

Jensen, M.E., Robb, D.C. and Franzoy, C.E., 1970 Scheduling Irrigation Using Climate-Crop-Soil Data

Although the paper dealt mainly with irrigation scheduling, the authors are of the view that the most important factor affecting irrigation efficiencies and crop yields is the irrigation scheduling in time and amount. The importance is magnified when water supply is short and costs are high. Over irrigation may result in waterlogged soils, which reduces yields and generally results in increased costs for water, fertilizer and drainage.

Dudley, N.J., Howell, D.T. and Musgrave, W.F., 1971(a), Optimal Intraseason Irrigation Water Allocation

Downey, L.A., 1972, Water-Yield Relations for Nonforage Crops

The author reported his findings on the examination of the published papers of many workers. His objectives were a) to investigate the relationship between evapotranspiration, moisture stress duration, and crop yield for crops where the reproductive organ or associated part is the useful portion like cereals, cotton and tubers; and b) to suggest reasons for variations in water-yield functions reported by workers.

From his analysis of published works, the author concluded:

1. That while yield is proportional to ET for forage crops giving a linear relation that passes through the origin, for nonforage crops a certain minimum ET is required before any yield can be obtained. The yield-ET curve does not pass through the origin in nonforage crops.

2. There tends to be a linear functional relationship between yield and the severity of moisture stress if the soil moisture stress is constant. No yield is obtained if the crop is grown under a constant stress of -800J K_g^{-1}.

3. If the stress is not constant as in real field conditions the magnitude of the yield reduction is no longer a simple function of ET, but depends on the severity and duration of the stress and particularly on the physiological stage at which stress occurs.

Stress in the early stage of development of the grain in corn is the most severe, producing a 90% yield reduction for a 10% reduction in ET. Generally, water stress at anytime from flowering to maturity is undesirable and gives inefficient use of water says the author.

4. The reasons for differences in yield at the same stress by many workers is the stage at which the stress is applied, the neglect of deep drainage by some workers, errors in the measurement of ET not accounted for and the effect of nonuniform stress on yield. According to the author, the measurement of the degree of moisture stress within the crop (i.e., leaf water potential depression and its duration) "...has rarely been measured...," and experiments to measure effect on yield of withholding water at various stages of growth are valuable to both irrigation planners and farmers.

Dudley, N.W., 1972, Optimal Interseasonal Water Allocation

Formulated stochastic dynamic programming problem where randomness of evapotranspiration is considered as system input. Assumed additive model similar to Flinn and Musgroves'.

Hiler and Howell, 1972, Crop Response to Trickle and Subsurface Irrigation

Experimental attempt at determining the water savings (efficiency) of trickle versus subsurface irrigation. Using Van Bavel's equation for ETP over season they irrigated every third day to bring the soil water content to field capacity thus measuring amount of water needed for two irrigation methods.

Bucks, Erie, French, 1972, Limiting Quantities and Varying Frequencies of Trickle Irrigation on Cotton

Experimental attempt at comparing trickle versus furrow irrigation. Application of water was a percent of the consumptive use of cotton (particular species) obtained over five year observation period. Both irrigation methods used same percentage irregardless of soil moisture at particular time and applied the water at 3/6/12 day frequencies.

Stewart, J.I. and Hagan, R.M., 1973, Functions to Predict Effects of Crop Water Deficits

The goals of the authors in the 4-year experimental research reported were two fold: (1) to develop rapid and accurate functional relationships between yield (Y) of principal irrigated crops and ET that can be applied at any time and in any location, such that Y vs. ET generated will define the upper limit of Y for any given ET deficit level, and also provide Y the lower limit of ET required for any level of Y. The functions would depend on crop and evaporative conditions alone, and assumes simplicitly 100% irrigation efficiency and an optimal timing of irrigation, and (2) to develop functions capable of generating rapidly before the start of the irrigation season, irrigation programs which will maximize Y or any other required objective function at any given water supply level.

Although the main variable input was water, four parameters associated with it, irrigation depth (IRR), soil water (SW), growing season rainfall (R) and evapotranspiration (ET)) were determined. The plant growth response is the yield (Y). The authors indicated that seasonal total ET deficit determines the minimum reduction in yield below maximum Y, while the precise pattern of times and intensities of deficits related to the sequencing of growth stages determines further reduction in yield. However, they do not consider the quantitative knowledge of the growth-stage effects essential in determining optimal irrigation programming. Their reason was that an optimal irrigation program will try to maximize Y or sequence occurrences in ET deficit "...so that the smallest deficits are scheduled in the most sensitive growth stages while death in the productive tissues is avoided in any stage..."

Results with the above assumption gave a linear relationship

$$Y = 3553 + 593\ ET \quad (r = .99;\ R^2 = .98,\ N = 15)$$

between Y and ET. But Y vs. field water supply (FWS), had a convex shape and was given by

$$Y = -4042 + 779\ FWS - 10.5\ FWS^2 \quad (r = .987,\ R^2 = .97,\ N = 15)$$

where $FWS = ASWP + R + IRR$ and $ASWP$ = available soil water at planting.

The authors indicate that IRR contains ET and non-ET uses. If IRR efficiency = $\frac{ET(\text{from IRR})}{IRR} \times 100$ then both Y_{vs} ET and Y_{vs}. FWS would coincide at 100% IRR efficiency.

The authors nonetheless warn that Y_{vs}ET may vary with crop type and variety and reviewed many research reports in which the relation is both linear and curvilinear. Since a generalized function was not possible, the authors put forward a brief concept for modeling a computer program to output water production functions and optimal irrigation programs.

Minhas, B.S., Parikh, F.S. and Srinivasen, T.N., 1974, Toward the Structure of a Production Fraction for Wheat Yields with Dated Inputs

Using data from experiments on wheat in Delhi and alfalfa in Ohio developed a mathematical model of timed water usage by crops as function of evaporative demand and soil moisture levels-timed in sense of growth stages of crop. From this relationship developed a mathematical model to relate growth to relative evapotranspiration during growth periods:

$$Y_a = a \prod_i [1 - (1 - \frac{ET_{ai}}{ET_{pi}})^2]^{b_i} \qquad i \text{ is growth stage}$$

where a and b_i are parameters to be estimated, Y_a is actual yield
ET_{ai}/ET_{pi} is relative evapotranspiration (actual to potential) during growth stage i

Concluded that model fit sparse data available reasonably well and suggested more experiments to gather data and further variations on production function.

Stewart, J.I., Hagan, R.M. and Pruitt, W.O., 1974, Functions to Predict Optimal Irrigation Programs

The authors set forth a methodology for prediction of optimal irrigation programs at any given level of irrigation water supply, which may be stated as follows in corn production:

1. For mild seasonal ET deficit (up to 10%) it is best to impose it wholly in the vegetative period.

2. Moderate seasonal ET deficits (10-25%) should be distributed through two growth periods one of which must be the vegetative period.

3. Severe ET deficits (25-50%) require distribution through all three major growth periods. Tolerable limits are about 40% in the vegetative period, 1.5 x vegative period deficit in the pollination period, and after it exceeds 25% about 80% in the grain period.

Lewis, R.B., Hiler, E.A. and Jordean, W.R., 1974, Susceptibility of Grain Sorghum to Water Deficit at Three Growth Stages

The authors hypothesized that "extent of yield reduction depends not only on the magnitude of the deficit but also on the stage of growth." In the paper they reported the results of an experimental study to determine the yield response of grain sorthum to a single period of known soil-water deficit during the boot, boot through bloom and milk through soft dough stages of growth. Deficit of soil moisture was applied by keeping the soil moisture potential at -12.9, -13.0 and -12.4 atmosphere at the stages respectively.

The results by the authors are that stress during late vegetative stage gave a yield reduction of 17% while stress at boot through bloom, and milk through dough stages resulted in 34% and 10% respectively. The authors also point out that growth stages are delineated in a nonuniform manner, the growth periods determination was difficult and largely subjective. They also expressed the views that the development of uniform water deficits at all growth stages was a necessary prerequisite for the rational use of quantitative data arising from studies like this one.

Morey, R.V., Gilley, J.R., Bergsrund, F.G. and Dirkzwager, L.R., 1975, Yield Response of Corn Related to Soil Moisture

Research reported by the authors was experimental and applied to a Minnesota need. It presented a three year study on corn aimed at evaluating yield response of corn to controlled sprinkler irrigation and to determine relationships between yield and soil moisture for corn. No model was proposed initially. First year moisture treatments were based on moisture depletion levels. The second and third year treatments were based on chosen fixed intervals and amounts of irrigation. Yield results showed significant differences in the second and third year between irrigated treatments. These differences were attributed to the differences in the June-July-August transpiration ratios by the authors. The authors also concluded that the June-July-August transpiration ratios were the most accurate predictor of corn yield in Minnesota. A linear regression analysis performed on the data relating relative yield and June-July-August transpiration ratio gave a fit with $R^2 = 0.9$.

Stewart, J.I., Misra, R.D., Pruitt, W.O. and Hagan, R.M., 1975, Irrigating Corn and Grain Sorghum with a Deficit Water Supply

The authors speculate that there was more to limited irrigation with planned deficits which hold some exciting prospects for increased world food production levels and stability as well as increased profits for individual farmers. The objectives of the three-year corn and four-year grain sorghum projects they reported were to develop a methodology for estimating water production functions and associated irrigation programs and to present them in a form usable to economists, engineers and policy makers for use in the development, allocation and management of water resources. They gave eight ways in which water production functions and irrigation programs, optimized with specified dates and depths of irrigation to maximize profits or water use efficiency would lead to improved solutions of design, planning and policy matters.

The authors reviewed some important findings and hypotheses of other researchers: (1) growth stages of corn in which water deficits occur may have an effect on the relative yield response, (2) moisture stress in corn during pollination reduces corn yield more than in any other stage, (3) water deficits in two or more time periods may reduce yield in a multiplicative rather than additive fashion and (4) there is a linear relation between grain sorghum and stress day index (SDI).

The authors did not advance any hypothesis at the beginning of the report, nor did they develop and yield function after reporting the results of the field experiments. But eight irrigation treatments were made in each season and the results observed.

Results and Observations on Grain Sorghum:

1. Irrigating grain sorghum in the grain filling period does not improve yield.
2. ET deficits in the vegetative period during tiller to boot stage result in major yield reductions which are only partially recoverable by irrigation afterwards.
3. Percentage reduction in yield below Y_{max} is essentially directly proportional to percentage seasonal ET deficit.
4. ET sequencing does not affect grain sorthum as much as it affects corn.

Results and Observations on Corn

1. ET deficits reduce corn yield in two ways: (1) seasonal total ET and (2) timing or sequencing of ET deficits occurances with respect to crop growth stages which insome cases may exceed the primary seasonal ET deficit effect.
2. Corn yields were highest when there was little or no ET deficits in the pollination period.
3. ET deficit in the vegetative stage, "conditioned" the crop for deficit at the pollination stage making yield reduction less than if the crop had no ET deficit in the earlier vegetative stages.

4. The "water production function" calculated showed that for each % point of seasonal ET deficit optimally sequenced with respect to growth periods, there is an inevitable primary loss of yield equal to 1.2% of Y_{max}. Sub-optimal ET deficit sequencing will lead to secondary loss of yield over and above primary loss. Secondary losses are avoidable.

Comparing corn and grain sorghum, the authors indicated that when water was plentiful for optimum ET deficit sequencing, corn utilizes more water at a higher water use efficiency and would produce more grain than sorghum. However, if water assumed was less than 38% of the required ET, grain sorghum should yield more and at a higher efficiency.

They concluded that: (a) maximizing crop production with limited irrigation water require quantitative information on differential yield responses to given levels of water deficits in each major growth period; (b) a quantitative expression of the ET deficits impact on yield was required to predict yield and the measure selected by the authors was "yield reduction ratio" which is the percentage yield reduction below the maximum possible, resulting from each percentage point of seasonal ET deficit and (c) a mathematical yield prediction model was contemplated--possibly multiplicative--but was not written or developed. The authors rather gave guidelines for irrigating corn and grain sorghum with limited water based on their funding.

CID-Utah Water Research Laboratory, 1977, Optimizing Crop Production Through Control of Water and Salinity Levels in the Soil

A comparison of three models (Stewart model, Hanks model, and Hall-Butcher model) which attempt to correlate a measure of deficiency in crop yield with a measure of deficiency (or oversupply) in water usage in a relationship of maximum crop yield related to optimum water supply was made. The crop chosen was corn with at least three distinct growth phases (for the Hanks model five stages were specified). Experimental data were gathered from four stations --Davis, CA, Fort Collins, CO, Logan, UT, and Yuma, AZ--over a two year period - 1974 and 1975. Measurements and/or estimations of crop water requirements, ET_M or potential evapotranspiration, and actual evapotranspiration, ET_A for the four sites and two years. An identification of the parameters in each of the models was then made from experimental data at each site and year for varying amounts of applied irrigation water and consequently varying water deficiencies.

Stewart (1972, 1976) presents two models--one for seasonal effects and one to account for water deficiencies during growth periods. Essentially the models are:

$$1) \quad Y_A = Y_M(1 - \beta_o(\frac{ET_M - ET_A}{ET_M}))$$

and

$$2) \quad Y_A = Y_M(1 - (\frac{ET_M - ET_A}{ET_M}))$$

where $\beta = \sum_i \frac{\beta_i(ET_{Mi} - ET_{Ai})}{ET_M - ET_A}$ where i is the growth stage

Y_A is actual yield (used both dry matter and grain in calibration)

Y_M is maximum potential yield

β, β_o, and β_i are parameter values called sensitivity factors

ET_M is maximum evapotranspiration

ET_A is actual evapotranspiration

Hanks (1973, 1974) also presents two models for the same purposes as Stewart:

$$1) \quad Y_A = (\frac{T_A}{T_M}) Y_M$$

and

$$2) \quad Y_A = Y_M(\pi_i (\frac{T_{Ai}}{T_{Mi}})^{\lambda_i}) \quad \text{where i is the growth stage}$$

where the symbols are essentially the same as Stewart's and λ_i are parameters specified by Hanks (1974) previously. Hanks (1974) and Childs and Hanks (1974) separate the ET_A and ET_M into actual transpiration T_A and maximum (potential) transpiration T_M using pan evaporation measurements and use transpiration estimates as the model basis.

Hall-Butcher (1968) is presented as a multi-growth period model where actual and potential yield is related to soil moisture levels instead of a measure of crop water usage:

$$Y_A = Y_M(C \prod_i (\frac{W_i - W_p}{W_f - W_p})^{b_i}) \quad \text{i is the growth stage}$$

where those common measurements are the same as Stewart's and Hanks models and

W_i is the soil moisture in the i^{th} growth stage

W_p is the water content at permanent wilting point

W_f is the water content at field capacity

C is a model coefficient

b_i are parameters to be determined

Comparison of the models over the range of yield values and various irrigation treatments used at the various sites showed that the Stewart models correlated well at all locations but the Hanks model and Hall-Butcher model were superior at certain locations and much worse at others.

Seginer, 1978, A Note on the Economic Significance of Uniform Water Applications.

Follow-up to Zaslavasky and Buras' work. Areal uniformity of water applications.

Heady, E.O. and Hexam, R.W., 1978, Water Production Functions for Irrigated Agriculture

This text considers the economic applications of production functions, the features of some selected production functions, procedures for estimating production functions and analyses some corn, wheat and cotton experiments.

The authors affirm that production functions are the input-output relationships for crop water demand vs. crop yield in agriculture. Water response functions are not univariate but multivariable functions involving several interacting inputs. They are therefore complex a) in their dynamic nature, b) in their interactions with other biological inputs such as fertilizer, plant variety, pesticides, etc., and c) in conformance with their surrounding soil and climate conditions.

It is often assumed that inputs included in production functions are homogeneous, i.e., non varying with time in their physical and chemical properties. But data from different production units confound input-output relationships because the inputs are really nonhomogeneous. Single-point estimates of production functions are not desirable, according to the authors, since producers attempting to maximize profits must change mix of inputs as the price relationships of inputs and outputs vary. When two or more inputs such as water and fertilizer are incorporated into the production function, marginal rates of substitution of inputs for producing a specified level of output can be estimated. Continuous production functions permit estimation of other concepts such as production elasticities and production supply functions.

The authors tend to favor multivariable dynamic production functions involving at least water and fertilizer and in which time is a variable which takes care of the stage of growth of the plant. A number of forms of some selected production functions were considered, their properties are discussed and their applicability in abstracting plant-water-soil relationships assessed. The authors also advance the idea that yield response to water availability is affected by soil texture not only because of the chemical properties but because it affects the soil moisture holding capacity and the rate plants can extract the water.

Some specific production functions examined are (Y = yield pounds/acre, W = applied water acre-inches, N - pounds fertilizer applied per acre).

1. Two variable Cobb-Douglas or power function

$$Y = aW^bN^c$$

in which a,b,c are parameters

Faults are that Y = 0 if N or W = 0, maximum product is undefined, decreasing total product and in turn negative marginal product is not possible and the least cost proportions in which W and N are used are invariant with respect to output levels. They conclude that Cobb-Douglas functions are less desirable for estimating plant-water-fertilizer relationships.

2. Mitscherlich-Spillman functions

E.A. Mitscherlich in his law of the physiological relationships stated that "yield could be expanded through increasing levels of any single growth factor so long as that growth factor was not present in sufficient amount to produce maximum yield." A two variable form of the Mitscherlich equation which has a considerable theoretical appeal is given by

$$Y = A[1 - e^{-c_1(w+W)}][1 - e^{-c_2(n + N)}]$$

when water and fertilizer are the variables, or

$$Y = A[1 - e^{-c_1x_1} e^{-k_1x_1^2}] \ldots . [1 - e^{-c_nx_n} e^{-k_nx_n^2}]$$

in a general form in which

A = maximum yield when the growth factor or variable input are increased to the limit

x_i = growth factor or variable input in the experiment

c_i = constant representing the "effect factor" of x_i on yield

K_i = injury factor for each x_i

w = available soil moisture when W = 0

n = residual fertilizer in the soil when N = 0.

Spillman's exponential function has features similar to Mitscherlich formulation and is given by

$$Y = A(1 - R_w^W)(1 - R_n^N)$$

R_w and R_n are constants implying that the marginal products for W and N bear a constant proportion to each other and this is its major limitation.

3. Polynomial forms

If f(x) is continuously differentiable the basic polynomial, following Taylor's series expansion, is given by the authors as

$$Y = b_o + b_1x_1 + b_2x_2 + b_3x_1 + b_4x_2^2 + b_5x_1x_2 \ldots$$ for a bivariable input-output relationship.

Some forms examined are the quadratic form and the square root form which have some features accepted by authors as useful in crop yield-water-fertilizer relationships, and the three-halves polynomial function that is a lot more complex. In concluding the examination of these special forms, Hexam and Heady note that the selection of an algebraic form directly imposes restraints on the nature of response allowed in the function.

Some of the issues dealt with by the authors in proceedures for estimating production functions include homogeniety of the experimental units, inter-temporal effects, replication and sources of errors in experimentation. Design of experiments included discussions on variability among experimental units, quality of management inputs, financial and time constraints. The analysis of data and estimation of production functions was also discussed.

The authors then made detailed analysis of corn grain and silage, wheat, cotton and sugar beets experiments which were carried out at nine various sites in Arizona, Colorado, California, Kansas and Texas. The quadratic, square root and three-halves polynomials were "fitted" to the experimental data, and in some cases an exponential fit was tried. The statistical levels of significance for the derived coefficients were 1 percent, 5 percent and 10 percent. Some data had good fits (R^2 = .949) and some were poor (R^2 = .300). The authors also aggregated and synthesized the experimental data into a "generalized" input-output relationships and derived demand functions for water and product supply functions for the crops.

Follett, Benz, L.C., Boering, E.J. and Reichman, G.A., 1978, Yield Response of Corn to Irrigation on Sandy Soils

The authors reported the results of a 3-year experimental study of yield response of corn to irrigation in the coarse and moderately coarse-textured soils of North Dakota, and also analyzed the apparent water use efficiency (AWUE) of the treatments. A theoretical treatment of the effect of probabilities of precipitation on corn yield was also done.

The hypothesis was that yield follows Mitscherlich equation since growth limiting factors follow nonlinear relationships according to the law of diminishing returns. Irrigation water was computed by

$$IW = C(ET_p xK) - P + D$$

where

C = factor for calculating the level for a given irrigation treatment and equals 0, 0.5, 1, 1.5 for the four levels considered.

ET_p = potential evaporation computed using Jansen-Haise equation.

K = crop coefficient.

P = measured precipitation

D = soil water deficit

Irrigation treatments covered periods of 3 weeks before silking to harvest because precipitation was equal or greater than ET calculated during early season each year of the experiments.

Results obtained by the authors are that for 3 year average figures and at 95% significant level, forage yields increased between treatments 1 and 2, 2 and 3, but not 3 and 4 when irrigation, at weekly intervals based on Jansen-Haise ET equations, were given. Grain yields however increased between all the treatments, but the increase was not at significant levels for any given year. They also developed forage and grain yield relations as follows:

Forage: $\text{Log}_{10}(16.48\text{-}Y) = 1.307 - 1.318 \times 10^{-2}X$

Grain: $\text{Log}_{10}(10.150\text{-}Y) = 3.973 - 1.448 \times 10^{-2}X$

where: Y = yield (metric ton/ha. and K_g/ha for forage and grain respectively).

X = water applied (cm).

They concluded from all their studies that

1. Corn and forage yield responses can be described by Mitscherlich equations.

2. AWUE decrease as irrigation was increased.

3. Large yield increases will result from irrigating corn nearly every year.

Barrett, J.W.H. and Skogerboe, G.V., 1978, Effect of Irrigation Regime on Maize Yields

The authors report the results of field experiments in the Grand Valley of Western Colorado which in their view substantiates a form of the crop yield-water use function, and which also demonstrates the effect of different irrigation regimes on the yield of maize. They also reviewed literature on the different conflicting forms of corn-yield water use functions.

The literature reviewed by the authors pointed out that when ET deficit sequencing was optimal (i.e., deficits are timed to cause least possible reduction in yield) the relationship between yield and seasonal ET is well represented by a straight line for maize, grain, sorghum and pinto beans. If the upper bound of yield is related to depth of water applied rather than ET, a curvilinear relationship will result.

Three growth stages (vegetative growth preceeding tasseling, pollination--from tasseling to blister-kernel, grainfilling--from blister kernel to physiological maturity) and eight moisture treatments were investigated. Stressing was achieved by eliminating all irrigation from stressed plots during a growth stage. Both dry matter and grain yield were determined for each moisture treatment and a comparison is made between the relationship of yield and amount of water supplied to plants, and yield and ET.

The authors came to the following conclusions:

1. Dry matter production was directly proportional to ET and timing of deficits had no effect on the relationship.

2. A linear relationship existed between grain yield and dry matter yield given by $y_g = 0.577y_{DM} + 370$ ($r^2 = 0.59$) where y_g = grain yield and y_{DM} = dry matter yield.

3. Severe depression of grain yield was caused by stress during pollination in maize.

4. If deficits have occurred in an earlier vegetative stage, the crop may be somewhat conditioned to stress at pollination, reducing stress impact at the later stage.

5. Maximum ET does not correspond to maximum yield.

6. Considerable scatter in data obtained by many researchers when plotting crop yield vs. water use, is largely a result from time of occurrence of water deficits in relation to stage of growth. There are sensitive periods/stages for each crop in which exaggerated yield reductions will occur if the deficit occurs in that period. If deficits are so timed that they cause least yield reduction for a given quantity of water supplied, the scatter would be reduced.

7. Yield vs. ET was linearly related, the line representing the upper bound on yield for a given ET level.

8. Yield vs. water supply available to the crop was concave downwards. The difference between the linear function and the curvilinear function represented amount of water supplied but not used in evapotranspiration.

Kronti, W.F., 1979, Crop Response to Variable Irrigation

Experimental attempt to determine the response (yield of various corn varieties) to drought conditions. ETA for season calculated. No data or conclusion reached.

Maurer, Watts, Sullivan, Gilley, 1979, Irrigation Scheduling and Drought Stress Condition in Corn

Experimental attempt to 1) identify most efficient irrigation scheduling procedure under limited irrigation supply and 2) test whether a previous period of drought stress conditions corn so that it is less sensitive to a subsequent period of stress. Using four growth periods over a three year study they calculated ETA for growth periods and season (did not specify how) and applied water as percent of full ETP required. No model given nor data included. Their conclusions were inconclusive as to both objectives.

Saxton and Bluhm, 1979, Predicting Crop Water Stress by Soil Water Budgets and Climatic Demands

Computer based simulation model on climate and data analysis of published yield data. Crop stress was function of (not specified) difference between actual and potential ET. With the exception that this model included a routine to account for canopy growth and variation over season of ETA (pan), the model was basically similar to the one developed.

Lynne, G.D. and Carriker, R.R., 1979, Crop Response Information for Water Institutions

The authors presented a theoretical discussion on the nature of crop production functions and the problems in the determination and use of the functions. Production functions must be determined with crop yield (Y) as a function of water input (W), and other inputs (I). So that $Y = F(W,I)$. The marginal value product MVP_W was the relationship of interest and $MVP_W = P_y(\partial Y/\partial W)$ where P_y is the price of a unit of crop yield.

The authors said that crop response information was difficult and extremely costly to obtain because:

1. Crop response to water varies with time and sequence of application. Thus a simple yield versus water relationship is incomplete.

2. Crop response varies with site factors such as soil type, slope, salinity and climate.

3. Weather variability suggests that ultimately such response information should be expressed stochastically, and

4. Shape of the MP_w (marginal product of water, $\partial Y/\partial W$) depends on other production inputs such as fertilizer, plant spacing, management and irrigation technology.

They also discussed some institutional impediments and constraints which they consider cause the neglect or underuse of knowledge on the production functions and affect water demand.

Musick, J.T. and Dusek, D.A., 1980, Irrigated Corn Yield Response to Water

The purpose of the work reported by the authors was to evaluate the effect on corn yields of a limited irrigation practice in the Southern High Plains (Texas) in view of the receeding aquifer and rising pumping costs in the area. One other objective was to develop a functional relationship between yield and seasonal ET.

Three years of study was done and the authors determined that the seasonal ET of adequately irrigated corn in the area was 75-80 cm. Their results were:

1. Grain yields and seasonal ET have a linear regression fit.

2. Yield reductions were much greater than average when plant-water stress occurred near tasseling and continued through pollination.

3. Tasseling-silking stage was the most critical water stress period because it delayed silking relative to pollen shedding and reduced seed set.

4. Vegetative stage was the least sensitive to moisture stress related yield reductions.

5. Threshold ET for grain yield is about 50% of the ET for maximum yield production.

6. Yield reductions associated with reduced water applications are less severe if

(a) on deep soils of high moisture holding capacities and crops of deep roots,

(b) the severity of the stress at any one time is distributed over the entire season,

(c) lower evaporative demand climate limits severity of plant water stress.

The conclusion of the authors was that limited irrigation of corn involved unacceptable high risks and should not be practiced.

Morgan, T.H., Biere, A.W. and Kanemasu, E.T., 1980, A Dynamic Model of Corn Yield Response to Water

The authors asserted that the ultimate objective of an irrigator is to maximize profits from irrigated productions. Irrigation scheduling to meet this objective requires knowledge of the cost of applying each increment of water and the additional revenue resulting from the increased yield associated with each increment of applied water. They are critical of present "static" crop response functions in which harvested yield is correlated to total water use or total water applied during the growing season because such functions do not take into account the dynamics of the continuous growth processes of the plant. Dynamic response functions are needed because irrigation scheduling decisions of when to irrigate depends on the crop's response to water which depends on the crop condition when irrigated. This is a function of the growing conditions previously encountered by the crop.

The authors then proposed a dynamic model which depends on previous growth periods and measures crop response to daily available soil moisture, which is written as:

$$X_t = \Gamma(t)^{\sigma(AM_t)} X_{t-1}$$

where:

- t = time in days since emergence
- X = yield potential except at maturity when it is harvested yield
- Γ = proportional growth in yield potential when soil moisture is not limiting yield
- σ = yield response to soil moisture and a function of AM
- AM = ratio of available soil moisture to available soil moisture in the root zone at field capacity

Thus growth in period t, is dependent on accumulated growth in previous growth period t - 1; absolute growth is not independent of the present condition of the crop. The crop response function σ, specifies the effect that daily available soil moisture has on harvestable yield. It is an implicit function whose boundary values are 1 when AM = 1 (field capacity) and zero when AM = 0 (i.e., permanent wilting point). The relative effect of the level of soil moisture on harvestable yield depends on the plants development and soil moisture because relative growth (Γ(t)), according to the authors, depends on the number of days to maturity.

The authors identified a vegetative and reproducting (silking to maturity) phase for corn. By assuming that potential yield during the reproductive phase can be represented by dry matter accumulation in the cob, silks and grain parts, they used data from Hanway (1971) to estimate Γ, the proportional growth in yield potential and developed a formulation for yield:

$$X_D = (D/T)^{1/D}\Gamma(D)^{\sigma(AM_D)}X_{D-1}$$

which when solved recursively gave

$$X_D = (D/T) \prod_{d=1} \Gamma(d)^{\sigma(AM_d)}X_o$$

and taking logarithmic transforms;

$$\log(X_D\ T/D) = \sum_{d=1} \sigma(AM_d)\log\Gamma(d) + \log X_o$$

where X_D = harvestable grain yield at maturity
T = number of days in reproductive phase of Henway's data
D = number of days in reproeuctive phase of plot
X_o = yield potential stored in the plant at end of the vegetative phase

The logarithmically transformed relation was used to estimate σ, the crop's response to soil moisture. The authors assumed σ was a piecewise linear function of available soil moisture consisting of three segments of the form:

$$(AM_d) = a_i + b_i AM_d \quad \text{for } i = 1,2, \text{ and } 3$$

with AM values in the ranges 0.000 - 0.333, 0.333 - 0.667 and 0.667 to 1.000 respectively. Data from experimental fields at Kansas were then used to estimate a_i and b_i.

To demonstrate the dynamic nature of crop response, the authors reported the results of twelve simulated irrigation schedules of which three were of the accepted rule of thumb practices. The results and findings were:

1. As the number of irrigations increased, simulated yield increases;
2. Proportional growth was highest at the beginning of the reproductive phase and declined as the crop reaches maturity;
3. Soil stress had the greatest effect on yield if it occured proportional growth rate was highest;
4. Yield model was sensitive to time of irrigation;
5. The highest yield did not produce the highest net return;
6. The accepted "rule of thumb" irrigation scheduling does not take advantage of the fact that soil moisture stress early in the reproductive stage reduces yield more than that at a later reproductive phase.

They concluded that a dynamic response model has at least two applications: (a) to analyze present irrigation practices, (b) to analyze dryland cropping strategies where rainfall patterns are known in a stochastic sense. They also think that casting the dynamic response model into a dynamic programming framework to develop an optimal scheduling model could help irrigators establish a criteria on when to irrigate and will provide information on the marginal value of water to be used in water management policy.

Reference: Hanway, J.J., 1971. How a corn plant develops. Extension bulletin, Iowa State University, Ames, June 1971

Al-Kazzaz, S.A.M., 1980, Optimal Parameter Identification for Crop Production Functions

A method to identify parameters for crop production responses to deficiencies of soil moisture in various amounts at various times during the growing period was presented. It uses the Hall-Butcher (1968) model for the production function of corn using experimental data from Fort Collins, Colorado, Logan, Utah, Davis, California, and Yuma, Arizona. The parameter identification is a least squares methodology using the methods of calculus to determine the optimal values.