MAINTENANCE STATION LOCATION
THROUGH
OPERATIONS RESEARCH
AT THE
WYOMING STATE HIGHWAY DEPARTMENT

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ABSTRACT

"Maintenance Station Location Through Operations Research at the Wyoming State Highway Department" by R. W. Hayman and C. A. Howard, 1970.

The purpose of the investigation reported in this paper is to develop mathematical models by means of operations research techniques to optimally allocate scarce resources used in highway maintenance.

Various maintenance functions are investigated according to amount of expenditure and their adaptability to operations research techniques. From this investigation, snow removal and sanding are selected for further study.

The effects of snowfall rate, travel time, allowable service response time, travel distance, operating costs, stockpile location and vehicle velocity upon maintenance station location are investigated.

Computer models for snow removal and for sanding are developed. A discussion of the constraints and the objective function for each model is presented.

The models are applied to a test area located in north-central Wyoming. The results are presented for various values of critical parameters.

It is concluded that highway maintenance activities can be analyzed in this manner and it is recommended that other maintenance functions be investigated.

BACKGROUND FOR THE STUDY

The typical roadway maintenance station in the State of Wyoming is charged with the general and total upkeep of those assigned roadways from the time of their completion to their eventual replacement or reconstruction. Table 1 lists the major activities and their budgets for the calendar year 1969.

Item	Dollars Spent in 1969	Cumulative Dollars	Percent of Total Dollars	Cumulative Percent of Total Dollars
1. Snow Removal	$1,550,000	$1,550,000	23.7%	23.7%
2. Surface				
a. Motor Grader Patch	$1,011,000			
b. Tight Blade	311,000			
c. New Surface	57,500			
d. Miscellaneous	1,600			
	$1,381,900	$2,931,000	21.1%	44.8%
3. Seal	677,800	3,608,800	10.3%	55.1%
4. Hand Operations				
a. Fencing				
b. Snow Fence				
c. Hand Patch				
d. Litter Control				
	$ 610,200	$4,219,500	9.3%	64.4%

Table 1. 1969 Roadway Maintenance Budget for Critical Activities. (1)

The remainder of the budget is distributed among lesser activities (as measured by budget) such as signing, lighting, centerline painting, etc.

Over the years, maintenance equipment and procedure has improved, along with other elements of the highway industry. Even so, it is a rare case that the stations themselves have been abandoned or relocated as service requirements and techniques have changed. Modern highway management has recognized the need for a reevaluation of the maintenance system, particularly with respect to the locations of the stations themselves. Population characteristics were changing and it was felt that the present requirements for servicing the central portion of the State required particular attention.

A formal study of this problem was initiated in the spring of 1970. The study was charged with identification of parameters and policies which affect the question of location of the maintenance station and, thereafter, to design and build a mathematical optimization model which incorporated these critical issues. The model was to be used, by management, to investigate the effects of policy, operational procedure and cost structure on the question of station location. As an additional function, the model was to be used in examining the effect of inaccuracies in cost and performance data. That is, how much inaccuracy may be allowed in physical data before critical decisions are affected? The results of this effort are reported in this paper.

The principal issue is the specification of required <u>locations</u> of maintenance bases in order to provide the required services in the most economical fashion. The study was allowed to assume that any existing station could be removed and new facilities could be constructed in any case justified by economics and service requirements.

Initially, the study was to be directed to the west-central portion of the State. However, the methodology of the study and particular techniques developed for producing a solution were to be applicable, whenever possible, to any other region within the State.

The problem was eventually reduced to two mathematical models which are optimized according to the standard techniques of Mathematical Programming. Computer programs were developed which convert familiar physical parameters, as they apply to any specific case, to the problem form required by the solution methodology.

SELECTION OF THE OBJECTIVE FUNCTION

The technical development of the study revolved around the determination of a program objective; this was made as a conjunctive decision between management and Operations Research mechanician. The objective selected was: Define the locations of the required maintenance stations, within the boundary of the study, such that the sum of operational and depreciation costs is an absolute minimum.

To those acquainted with optimization technology, the selection of this program objective is no particular surprise. However, during the selection process, many other alternatives were considered; among the more notable were the possibilities of maximizing various service benefits.

IDENTIFICATION OF CRITICAL MAINTENANCE ACTIVITIES

Typical annual budgets were analyzed to determine the current patterns of expenditures, classified according to the various maintenance activities. Subsequently, an effort was made to associate each of the activities with some fraction of the cost of the maintenance station itself. For example, some fraction of the physical plant exists only for the purpose of housing and upkeep of the snowplow units; in fact, this particular fraction was 20% to 30%. This task was completed in a very subjective manner and remains open to debate.

The next step in attempting to define critical activities was to define, for each of the major maintenance operations, the manner in which operating costs varied as a function of location of the operating base for the activity. The most obvious variable was the amount of travel required to reach a work site from a particular base station.

The strategy in all of this is to reduce operation costs for each activity through the most favorable location of a base for the activity. Cost saving, if any, could be converted to construction and upkeep of new facilities. Since the optimization objective is to minimize the sum of operating and depreciation costs, we are looking for that configuration of a physical system that produces a savings in operating expenses which is at least as great as the cost of building and upkeep of the required group of physical facilities.

Most of the standard maintenance functions enjoyed little or no operational savings as a function of location of the operating base. In fact, only one set of activities promised to generate sufficient savings to pay for its share of the physical facility - and this was the snow removal and sanding program. Accordingly, it was determined that the mathematical models for the optimization need only consider this set of activities, together with their proportionate share of the cost of the physical plant.

DEFINITION OF PROGRAM CONSTRAINTS

The primary source of information used for definition of program constraints was the "Policy and Procedure Directive 70-1," Maintenance Division, Wyoming State Highway Department. (2) A portion of this document is included in the Appendix to this paper and describes the expected quality of the snow removal program for each of several roadway classifications.

For purposes of the optimization study, maintenance services were divided into two types: sanding and plowing; separate models were developed to optimize these services independently. In addition to optimizing station location and vehicle assignments, the sanding model provides the optimum locations for stockpiles of sanding material.

Constraints on the Sanding Operation

The language of the "Policy and Procedure Directive" was abstracted to provide four controls, stated nominally as follows:

1. Sanding must begin before the snow has accumulated to 1/4 inch depth on the roadway.

2. For some design storm, all roadways entitled to sanding services must be entirely sanded before the snow accumulates to some stated depth depending on the "Class of Service" assigned to each roadway (see Policy and Procedure Directive for definition of Class A through E service).

3. Sanding shall be performed continuously until the entire facility has been sanded or until the snow has accumulated to the maximum depth associated with Constraint 2.

4. Sanding material shall be applied to the entire driving surface at the application rate of 2000 pounds per two-lane mile.

Four additional program specifications were identified through interview with Maintenance Department personnel:

5. The traditional concepts of maintenance district boundaries were to impose no restriction on station location or equipment work assignment.

6. Any sanding unit could be assigned to any work location within the geographic domain of the model.

7. There is no restriction on the number of sanding units assigned to any base station.

8. Within each of the service classifications, A through E, provision should be made for service priorities on the basis of relative traffic density.

Several of these statements are either ambiguous or require further interpretation before they can be paraphrased in mathematical terms. The necessary discussion is given in the subsequent paragraphs.

Of primary importance is the definition of a design storm. It is not to be expected that Specification 2, above, could be met for all storm situations. Consequently, the maximum storm intensity to be accomodated (the design storm) was defined to be a continuous snowfall occurring at the rate of 1/2 inch per hour. For any storm of higher intensity, all roadways could not be completely sanded before the snow depth has accumulated to the limiting depth, at which time sanding would be terminated and plowing begun.

It is a physical impossibility to begin sanding on all parts of any roadway on some command. Accordingly, Specification 1 must be interpreted at less than literal value. This condition was redescribed to mean that the sanding units would be deployed to their work assignments before the snowfall has accumulated to 1/4 inch depth.

Constraints 1 and 2, taken together with the definition of the design storm, mean that all sanding must be completed within a specific time period, following the beginning of the storm. For example, for a Class A roadway, the maximum snow depth allowed, during sanding, is 2 inches. Therefore, for the 1/2 inch/hr design storm, the sanding must be completed with a 4 hour period measured from the storm beginning.

In order to provide service priorities with each Class of Service (Constraint 8), allowable service times are reduced for high priority roadways.

No particular effort was given, in this study, to establishing a procedure for setting service priorities on the basis of traffic density and no specific policies were found to exist within the Department. However, the optimization model does allow for examination of the effect of policy by perturbation of the allowable service time parameter.

Other elements given in the list of program constratins, 1 through 8, are taken at their face value. No other arguments were imposed on the solution to the problem. The solution is categorically guaranteed to meet these specific requirements, assuming that equipment performance and cost data are correct.

Constraints on the Plowing Operation

Once again, the "Policy and Procedure Directive" provided the major guidelines in assembling performance descriptions:

1. Plowing operations shall begin when snowfall has accumulated to some minimum depth established for each Class of Service to be provided.

2. For Class A Facilities, sufficient equipment shall be deployed, and remain in continuous service, in such a way that the roadway be kept "bare."

3. The roadway is defined to mean normal driving lanes and passing lanes.

4. For Class B Service, plowing shall be continuous throughout the storm and sufficient equipment shall be made available that the entire roadway may be cleared "soon after the storm subsides."

5. For Class C Service, sufficient equipment shall be available to clear the entire roadway "soon after the end of storm."

In addition to the foregoing constraints, the conditions 5 through 8, as applied to the sanding program, were imposed on the plowing program:

6. Maintenance station boundaries impose no restriction of the problem solution.

7. Any plow could be assigned to any roadway within the geographic area considered.

8. There are no limitations on the number of plow units that can be based at any given maintenance station.

9. Service priorities may be applied to any of the facilities falling within the Class of Service A through C.

As in the case of the sanding program, several of the program constraints, as nominally identified above, require translation to more specific form.

First of all, the design storm used for the plow model is a continuous 1-1/2 inch of snowfall per hour. It should be pointed out that the duration of the snow storm is not a factor that the model is required to consider. Service specifications, as outlined, require either continuous service, with an associated continuous result (keep the road bare), or desirable service levels to be achieved following the end of the storm. A collection of equipment designed to keep the roadway bare for one hour will also keep it bare throughout a design storm of indefinite duration. The essential difference between servicing a one-hour storm and a 100-hour storm would be the personnel required to operate the equipment. The 100-hour storm is obviously more costly to service, but the cost differences are independent of the location of the maintenance stations and are therefore of no consequence to the current study.

Constraint 2, as given above, was taken directly from the "Policy and Procedure Directive" and needs considerable restructuring in order to be at all realistic. If the definition of "bare roadway" is that absolutely no snow is allowed at accumulate at any point on the driving surface, then a continuous circulation of plows, moving end-to-end, would not fulfill the requirement for the design storm. A more reasonable requirement would be to limit the <u>average</u> snowfall accumulation to some minimum depth, chosen such that traffic could negotiate the roadway at all times. A satisfactory statement on the average depth requirement is a matter for continued debate; for purposes of this study, the maximum average accumulation of snowfall was taken to be two inches, and a four-inch accumulation is the absolute maximum allowed to accumulate at any point on the roadway. The computer programs which solve the models are designed to accept these numbers as input data which may be varied at will to determine effect on the final solution.

Constraints 4 and 5 state that the entire roadway shall be cleared "soon after the storm subsides." Obviously, a strict definition of the word "soon" must be supplied.

The general scheme employed is shown in Table 2.

	Class of Service		
Priority	A	B	C
1	2	4	10
2	3	6	14
3	4	8	18

Table 2. Allowable Time for Clearing Roadway Based on Roadway Priority.

It should be emphasized that the numbers reported in Table 2 are not the result of existing departmental policy. For purposes of the optimization study, the emphasis was placed on developing the mechanics of providing for these management features. Again, such data are treated as input parameters.

DEVELOPMENT OF THE OPTIMIZATION MODELS

The "Policy and Procedure Directive" given in the Appendix represents a relatively new philosophy for snow control programs in the State of Wyoming. Previous policy was directed almost exclusively to snow control by plowing. It had been suspected that the quality of service could be upgraded at little or no increase in maintenance cost through a more extensive application of abrasive/liquefacient material to the roadway (hereinafter referred to as sanding). In Wyoming, it has been found that the roadway frequently may be maintained in satisfactory driving condition through the application of sanding material, with no plowing required. Some storm situations require both sanding and plowing and some require plowing exclusively. Accordingly, it was decided to build two models - one which optimizes station locations according the the sanding requirements and one to optimize according to plowing requirements. Management hoped that the station location solutions would be the same in both cases. However, any differences found between final solutions will have to be weighed against the cost of the two maintenance functions. The two models are summarized in the following two sections.

General Performance Data for Both Models

The standard vehicle used for both models was a four cubic year dump truck equipped with a snowplow and slip-in sanding unit. The current hourly use charge including operation is $15.00. The four cubic yard capacity equates to six tons of sanding material. Six miles of two-lane roadway may be covered with one load of sand at an application rate of one ton per centerline mile of roadway.

The average working velocity for both sanding and plowing operations was set at 24 m.p.h. which accounts for adverse driving conditions, nominal delays and loading. A velocity of 40 m.p.h. was assigned to travel from base to work site.

Sanding Model

The normal work pattern followed in the cases of a general storm is a simple progression down the roadway with return to the nearest stockpile for reloading when the trucks are empty.

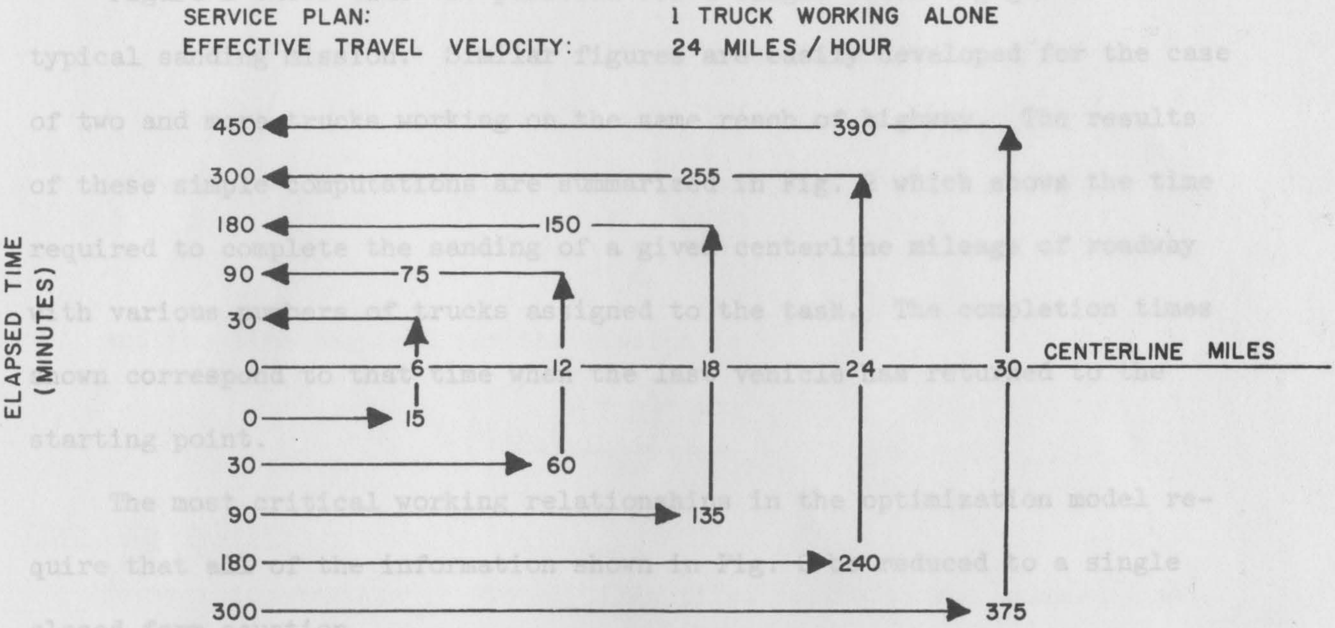


FIGURE 1 CENTERLINE MILES COVERED IN TIME

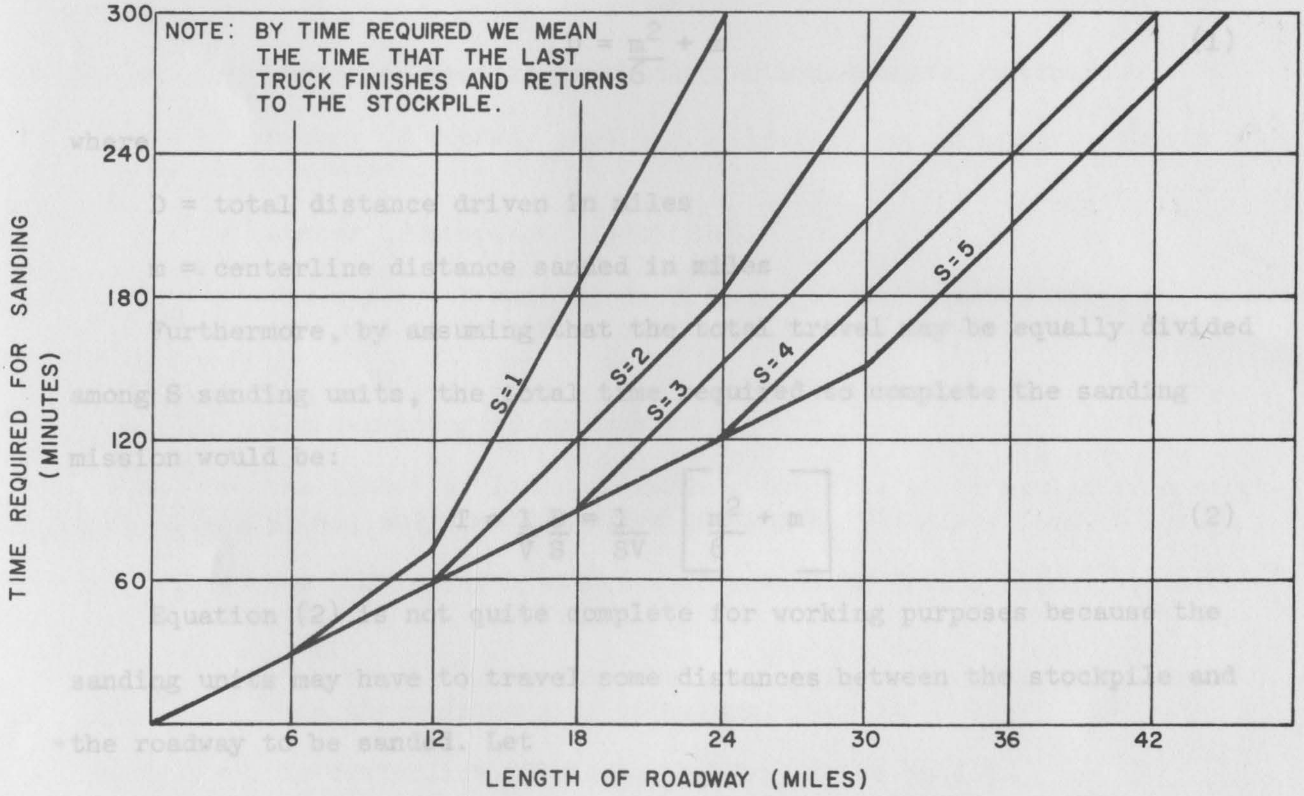


FIGURE 2 SANDING COMPLETION TIME

Figure 1 shows time vs. position for a single truck engaged in a typical sanding mission. Similar figures are easily developed for the case of two and more trucks working on the same reach of highway. The results of these simple computations are summarized in Fig. 2 which shows the time required to complete the sanding of a given centerline mileage of roadway with various numbers of trucks assigned to the task. The completion times shown correspond to that time when the last vehicle has returned to the starting point.

The most critical working relationships in the optimization model require that all of the information shown in Fig. 2 be reduced to a single closed form equation.

In the case of a single sander the relationship between the total miles driven and the centerline miles sanded may be suitably represented by a quadratic, given as Eq. (1)

$$D = \frac{m^2}{6} + m \qquad (1)$$

where

D = total distance driven in miles

m = centerline distance sanded in miles

Furthermore, by assuming that the total travel may be equally divided among S sanding units, the total time required to complete the sanding mission would be:

$$T = \frac{1}{V}\frac{D}{S} = \frac{1}{SV}\left[\frac{m^2}{6} + m\right] \qquad (2)$$

Equation (2) is not quite complete for working purposes because the sanding units may have to travel some distances between the stockpile and the roadway to be sanded. Let

d = "dead haul" travel distance in miles between the stockpile and the beginning of the roadway section to be sanded.

n = number of trips required to sand "m" miles of roadway.

Using Eq. 1, the total distance to be traveled is:

$$D = 2nd + \left[\frac{m^2}{6} + m\right] \qquad (3)$$

and the time required for the mission is:

$$T = \frac{2n\left[d + \frac{m}{2}\right] + 6n}{VS} \qquad (4)$$

Time Constraints

Several constraints previously identified relate to the elapsed time allowed to complete the sanding operation. With proper interpretation and specification of parameters, these conditions may be satisfied with the following development. The necessary terminology is identified at the outset.

s = number of propsed locations for maintenance stations.

r = number of roadway sections to be serviced within the domain of the model.

p = number of proposed stockpiles.

S_{ij} = the number of sanding units based at maintenance station i and assigned to work from stockpile j.

S_{jk} = the number of sanding units assembled at stockpile j to effect the servicing of roadway k.

n_{jk} = the number of loads of sanding material to be hauled from stockpile j and distributed on roadway k.

t_{ij} = the time spent by the S_{ij} in traveling from station i to stockpile j.

T_k = time available to complete the sanding of roadway k measured from the beginning of the storm.

M_k = the centerline mileage of roadway to be sanded.

d_{jk} = the "dead haul" travel distance in traveling from stockpile j to roadway k.

The basic strategy is to deploy S_{ij} sanding units from station i to stockpile j. Thereafter, n_{jk} loads of material are to be hauled from stockpile j to roadway k, using S_{jk} sanding units.

In the execution of this strategy, the time of arrival of the sanding vehicles at a particular stockpile will vary, depending on the origin station for the trucks. In other words, t_{ij} will vary with i for any j. Temporarily, assume that the t_{ij} are the same for all i and some particular j. Call this average time $\bar{t}_{ij}$. Then the time available for productive work on a given roadway, measured from the beginning of a storm, is:

$$(\text{Time Available})_k = \left[T_k - \bar{t}_{ij}\right] \qquad (5)$$

Now S_{jk} sanding units will be deployed from stockpile j to roadway k and will distribute n_{jk} loads of sanding material. The time required for this may be computed from equation (4).

In order to accomplish the service within the specified time frame, it is required that:

$$2n_{jk}\left[d_{jk} - \frac{M_k}{2}\right] + 6n_{jk} \leq \left[T_k - \bar{t}_{ij}\right] \quad VS_{jk}; \qquad (6)$$

$$j = 1, \ldots, p \text{ and } k = 1, \ldots, r$$

The variables in this constraint are the n_{jk} and S_{jk} of which there are a total of 2 x p x r. The set of these constraints consists of p x r separate inequalities.

The substitution of the travel time averages $\bar{t}_{ij}$ for the actual t_{ij} must now be reconciled. The answer for the substitution is iterative programming. The reason for the substitution lies in the fact that to have kept proper account would have increased the model size. The number of variables and constraints would have been multiplied by "i" in the process, without gaining substantially in the accuracy of the model or in the amount of physically useful information derived. As it happens, there are very few cases where more than one station services from a given stockpile and it was not difficult to iterate to the correct combination.

Model Efficiency Constraints

Although not required to satisfy the theoretical behavior of the model, two time constraints were developed for the purpose of accomplishing substantial reduction in the size of the model as it applies in any particular case. We are able to state, without interacting with other constraints in the model, that

$$\left[T_k - t_{ij}\right] \geq q_1; \; i = 1, \ldots, s \text{ and } j = 1, \ldots, p \text{ and for any } k \qquad (7)$$

and

$$\frac{d_{jk}}{V} \geq q_2; \qquad j = 1, \ldots, p \text{ and } k = 1, \ldots, r \qquad (8)$$

The relations (7) state that the time spent in traveling from station i to stockpile j, t_{ij}, must be something less than the time alloted to sanding the roadways to be services from stockpile j. If this is not the case, then no sanding unit should be deployed from station i to stockpile j. In other words, if $\left[T_k - t_{ij}\right] < q_1$ then $S_{ij} = 0$. Similarly, if too much time is consumed by deadhaul in servicing roadway k from stockpile j, there will be none left for the useful work. The time for one way passage over the deadhaul distance is $\left[d_{jk}/V\right]$ and must actually be consumed $2n_{jk}$ times. Consequently, if $\left[d_{ij}/V\right] < q_2$ then the quantities S_{jk} and n_{jk} could be set to zero. Actually, instead of setting the S_{ij}, S_{jk} and n_{jk} to zero, they are discarded before being built into the final form of the optimization model. The q_1 and q_2 numbers were conservatively chosen so that potentially valid variables were not discarded.

The remaining constraints required for the optimization model follow quite simply.

Work Quantity Constraint

The worst storm situation must be used as a basis of argument; this would be a storm which covers the entire domain of the model at any given time. This situation is not, in fact, unusual for the area of interest. The set of constraints (6) do not, by themselves, require that the total roadway system must be covered; they merely provide that any work undertaken must be completed within a given time frame. The requirement for total sanding coverage will be given in terms of the number of loads of material required to service each and every roadway in the system. In order to cover any roadway k with the type of equipment being used, the number of loads of material required are:

$$n_k = \frac{M_k}{6} \tag{9}$$

These materials may (theoretically) be transported from any stockpile within the system. Accordingly, it is required that:

$$\sum_{j=1}^{p} n_{jk} \geq \frac{M_k}{6} \; ; \; k = 1, \ldots, r \tag{10}$$

Constraints (6) and (10) together ensure that all roadways are sanded within the required time allotment.

Equipment Continuity Constraint

So far we have used S_{jk} units to deliver n_{jk} loads of material from stockpile j to roadway k. It remains to assemble the correct number of sanding units at any stockpile. We do this by dispatching the required number of units from the various maintenance stations. Therefore,

$$\sum_{j=1}^{s} S_{ij} \geq \sum_{k=1}^{r} S_{jk}; \; j = 1, \ldots, p \qquad (11)$$

is required.

Constraint Summary and Observations

The sets of constraints (6), (10) and (11) complete the constraint requirements for the model.

Recall that:

s = the number of proposed maintenance stations,

p = the number of proposed stockpiles, and

r = the number of proposed roadway sections.

The total number of variables in the problem are (s x p) + (2 x p x r). There are (p x r) constraints in the set (6); r constraints in set (10); and p constraints in the set (11). The rejection of candidate variables based on the relationships (7) and (8) is the only hope of making a practical solution for a geographic area of any size.

In the section of this paper giving original definition to the constraints on the problem, there were 8 requirements given. All of these conditions are satisfied through the modeling constraints (6), (10) and (11). The connections between verbalization and model will be given no further discussion here.

The constraint models are all linear in the variables S_{ij}, S_{jk} and n_{jk}, although no particular attempt was made to cause this. In the solution of the model, the variables S_{jk} and S_{ij} were not restricted to integer values.

Now the total number of trucks required at any stockpile would be

$$S_j = \sum_{k=1}^{r} S_{jk} \; ; \; j = 1, \ldots, p \qquad (12)$$

and the total trucks required from any maintenance station would be

$$S_i = \sum_{j=1}^{r} S_{ij} \ ; \ i = 1, \ldots, s \qquad (13)$$

After the final summations of (12) and (13), one must round upward to the nearest integer value.

A final point concerns the original objective of defining the most favorable (economical) locations of the maintenance stations thenselves. The station locations are hidden in the variables S_{ij}. If the final solution to the model gives

$$S_i = \sum_{j=1}^{p} S_{ij} = 0 \text{ for any } i \qquad (14)$$

then a station is not required at location i. Similarly, if

$$S_j = \sum_{k=1}^{r} S_{jk} = 0 \text{ for any } j \qquad (15)$$

then a stockpile would not be required at location j.

The Objective Function

Any procedure used to amortize the cost of stations against the various maintenance activities is highly debatable. At the present time, standard departmental accounting procedure does not attempt to do this, at least in any direct way. Even so, management does associate the station costs with the maintenance program; consequently, a direct procedure for accomplishing this was adopted in the optimization study.

A study was made of existing maintenance stations to establish some "average" cost of construction and upkeep and subsequent physical allocation to the snow removal activity. It was conservatively estimated that 25% of the physical composition of a typical maintenance station was devoted to provision of the trucks, plows and sanders used in the snow removal program.

The following additional data was assumed:

Typical maintenance station cost = \$400,000

Amortization Period = 25 years

Average number of sander/plow units assigned to each station = 4

Average number of sanding/plowing missions per year = 40

On the basis of this data, and assuming no interest charges for the investment, the station cost was amortized over each sanding/plowing mission as follows:

$$\text{Cost per mission} = \frac{(.25)(400{,}000)}{25\text{x}4\text{x}40} = \$25$$

Now, the optimization model will attempt to locate the various stations in such a way that savings in operating costs will recover the largest possible fraction of the amortized station costs. According to these arguments, the lowest total cost, including operation and amortization, will define the optimum station locations.

Where the optimization model considered a new station possibility, the \$400,000 station cost was assumed. The 25 year costs associated with existing stations were estimated fractions of the new station costs and attempted to account for anticipated major repairs, remodeling, etc.

Since this station amortization procedure is a new concept for the department, the numbers used are gross estimates and should be challenged for validity. The computer programs for solving the optimization model have been designed so that these numbers may be easily revised and the problem re-solved. In the investigation to date a range of amortization costs were studied for effect on the final solution.

The terminology applied to the constraint development is carried over to this section, with the following additions:

C_i = the station amortization costs to be applied to each of the S_{ij} (\$/unit).

C_{ij} = unit time cost in traveling from station i to stockpile j and return (\$/hr).

C_{jk} = unit time cost for trucks plus loaders involved in the sanding mission from stockpile j to roadway k (\$/hr).

P_j = the unit cost of the sanding material, delivered to stockpile j (\$/ton).

d_{ij} = the distance traveled from station i to stockpile j.

V_{ij} = the travel speed from station i to stockpile j.

The total cost of a given sanding mission can be computed in three parts:

Cost = Z = Cost in deployment of the S_{ij} (including station amortization)

+ Cost of delivering the material from stockpile j to roadway k

+ Cost of the sanding material.

Where

Cost of Deployment =

$$\sum_{i=1}^{s}\sum_{j=1}^{p} C_i S_{ij} + \sum_{i=1}^{s}\sum_{j=1}^{p} 2C_{ij}\begin{bmatrix} d_{ij} \\ V_{ij} \end{bmatrix} S_{ij} \qquad (16)$$

Cost of Delivery =

$$\sum_{j=1}^{p}\sum_{k=1}^{r} C_{jk}\,\frac{\left[2n_{jk}\left(d_{jk}+\frac{m_k}{2}\right)+6n_{jk}\right]}{V_{jk}} \qquad (17)$$

Cost of Material =

$$\sum_{j=1}^{p}\sum_{k=1}^{r} P_j n_{jk} \qquad (18)$$

Only two terms in these expressions should require any discussion. The quantity $\left[\frac{d_{ij}}{V_{ij}}\right]$ in (16) is the time required in traveling between station and stockpile. The travel velocity may be appreciably higher than the effective working velocity, but is debatable because of weather conditions. The quantity $\left[2n_{jk}\left[d_{jk} + \frac{m}{2}\right] + 6n_{jk}\right] \div V_{jk}$ in (17) represents the total truck-hours spent in servicing the various roadways, regardless of the actual number of vehicles involved.

Finally, we wish to minimize

$$Z = \sum_{i=1}^{s} \sum_{j=1}^{p} \left\{ C_i + 2C_{ij} \left[\frac{d_{ij}}{V_{ij}}\right] \right\} S_{ij} + \sum_{j=1}^{p} \sum_{k=1}^{r} C_{jk} \qquad (19)$$

$$\frac{\left[2n_{jk}\left[d_{jk} + \frac{m_k}{2}\right] + 6n_{jk}\right]}{V_{jk}} + P_j n_{jk}$$

subject to the constraints given by the relations (6), (10) and (11).

THE PLOWING MODEL

The optimization model for the plowing operation is not as complicated as the one for the sanding operation and will be given brief development as follows:

Considerations in the Type of Facility

The "Policy and Procedure Directive" (Appendix) defines two separate strategies according to classification in type of roadway. Class A facilities are to be kept plowed bare throughout the storm period. Classes B, C and D must be completely cleared within some reasonable period following storm termination. These two types of treatment require different constraints in the optimization model.

Nomenclature

The following symbology is used for constraint development:

S_{ij} = the number of snowplow units dispatched from station i to clear roadway j.

M_j = centerline mileage for roadway j.

V_t = the average plow velocity in reaching the work site.

V_p = the average plow speed under working conditions.

T_j = the allowable time for clearing roadway j.

d_{ij} = the distance in miles from station i to the centroid of length of roadway j.

P_j = the number of plow lanes required to clear the roadway, from shoulder to shoulder.

D = critical snow depth.

R = rate of snowfall used for program design purposes.

t = time, in general.

s = the number of potential maintenance stations.

r = the number of roadway stations to be serviced.

Constraints for Class A facilities

For this type of roadway, the "Policy and Procedure Directive" states that the roadway must be kept bare at all times. In reality, this would be a physical impossibility; the specification was modified to require that the snowfall should not be allowed to accumulate beyond some critical depth, D. For some design storm, the snowfall intensity if defined as R. Then, the snow accumulation, during any time, t, would be:

$$\text{Accumulation} = R\,t \tag{20}$$

Specifically, we wish to know when the snow will accumulate to D, the critical depth. From (20):

$$t = \frac{D}{R} \tag{21}$$

Now the distance a plow will travel, at some working speed V_p, according to (21) is

$$V_p t = V_p \frac{D}{R} \tag{22}$$

Now if a group of plows, all traveling at V_p, were to follow one another down the roadway, and were spaced according to equation (22), the maximum snow accumulation between them would be D. This is the effect desired. Now the total length of roadway to be plowed, for roadway j, would be

$$P_j M_j \tag{23}$$

Therefore, the required number of plows is:

$$S_j = \frac{P_j M_j}{V_p \frac{D}{R}} = \frac{P_j M_j R}{V_p D} \tag{24}$$

For the optimization model:

$$\sum_{i=1}^{s} S_{ij} \geq \frac{P_j M_j R}{V_p D}; j = 1, \ldots, r \tag{25}$$

is required. Furthermore, the S_{ij} defined by (25) must be available for the duration of the storm.

Constraints for Class B, C and D Facilities

These roadways must be cleared within some "reasonable" time following the storm termination; call this time T_j. Any snowplow must be able to reach the work site and complete the assignment in this time. Thus, the time available for work is

$$T_j - \frac{d_{ij}}{V_t} \tag{26}$$

Once on site, the plow must clear M_j centerline miles on roadway j and each mile of centerline requires P_j lanes to be cleared. The total mileage to be cleared is therefore $P_j M_j$. Assume that the mileage may be equally distributed among S_{ij} plows; the mileage assignment for each plow is

$$\frac{P_j M_j}{S_{ij}} \qquad (27)$$

The time required to accomplish this is

$$\frac{1}{V_p}\left[\frac{P_j M_j}{S_{ij}}\right] \qquad (28)$$

and must be accomplished within the time prescribed by relation (26). It is therefore required that:

$$\frac{1}{V_p}\left[\frac{P_j M_j}{S_j}\right] \leq \left[T_j - \frac{d_{ij}}{V_t}\right] \quad ; j = 1, \ldots, r \qquad (29)$$

$$\text{and } S_j = \sum_{i=1}^{s} S_{ij}$$

Model Efficiency Constraints

In order for any S_{ij} to have productive work time available, after reaching the work site, it is required that

$$\left[T_j - \frac{d_{ij}}{V_t}\right] > 0 \qquad (30)$$

A pre-optimization screening process considers (30) for all combinations of i and j. S_{ij} is rejected from the optimization model if

$$\left[T_j - \frac{d_{ij}}{V_t}\right] \leq q_1 \qquad (31)$$

where q_1 is some conservatively chosen time value. This procedure significantly reduces the numbers of the problem variables and, in no way, compromises the final solution.

In the constraint model, the optimization variables are the S_{ij} and there are s x r potential variables. There will be a total of r constraints.

The Objective Function

Once again, the objective is to minimize cost. The debatable feature of the scheme employed is the amortization of the physical plant against the activity. The strategy employed is the same used for the sanding model. The total cost of a single mission is:

Total Cost = Cost of deployment of the S_{ij} to the work site (including station amortization).

+ Cost of operating the S_{ij} during the plowing operation (including the operator).

Where:

Cost of Development =

$$\sum_{i=1}^{s}\sum_{j=1}^{r} C_i S_{ij} + \sum_{i=1}^{s}\sum_{j=1}^{r} C_{ij}\left[\frac{d_{ij}}{V_t}\right] S_{ij} \tag{32}$$

In the case of the snowplow operation, the operating costs are all the same, once a work site has been reached. Even though this cost is real, it has no relationship to the different choices for the S_{ij}; consequently, the on-site operating costs were not computed. The final form of the objective function is, therefore:

$$\min Z = \sum_{i=1}^{s}\sum_{j=1}^{r}\left[C_i + C_{ij}\left[\frac{d_{ij}}{V_t}\right]\right] S_{ij} \tag{33}$$

where:

C_i = station amortization cost (\$/sanding unit).

C_{ij} = the hourly time costs of snowplow and operator (\$/hr).

APPLICATION AND RESULTS

The general area to which the models were applied and the sites of existing and proposed maintenance stations and stockpiles are shown in

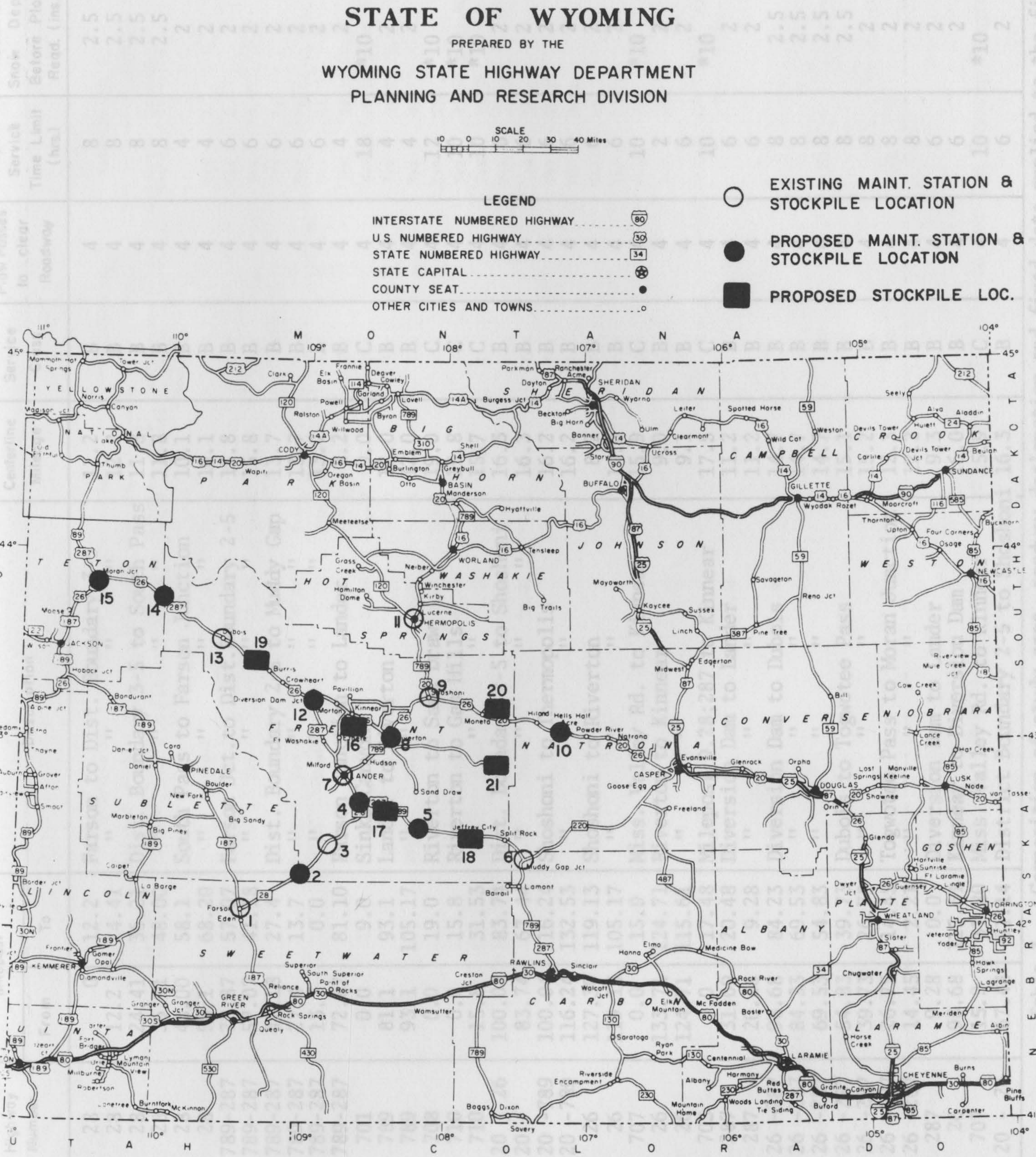
Figure 3. Existing stations were included to see if the model would include or reject these sites.

For both models, there were 15 station sites and 41 roadway sections. For the sanding model, there were 15 stockpiles located at the station sites and 6 additional stockpiles making 21 in all.

There are originally 2037 variables and 917 constraints in the sanding model and 615 variables and 41 constraints in the plowing model. To assemble this much data by hand each time the model is run is a difficult task. To accomplish this, two FORTRAN computer programs or model builders were written. These programs compute all coefficients, reject infeasible combinations of data and assemble the final matrix of coefficients into a form usable as input to the Simplex Algorithm being used to solve the problem. The sanding model was reduced to a problem of 388 variables and 227 constraints and the plowing model was reduced to a problem of 278 variables and 41 constraints.

As a matter of interest the M.I.T. MFOR Simplex Algorithm was used to solve the sanding model and the author's own Simplex Algorithm was used to solve the plowing model. The reason that two different solution algorithms were used is due to available computer space. The M.I.T. algorithm is a column input type of algorithm which operates in a smaller amount of core than most row input types of simplex algorithms. However, there is a great deal of roll-in roll-out required by the M.I.T. algorithm and consequently considerable computer time is consumed.

Several solutions were made for each model in order to examine the effect of variation in critical data. Fixed data applied over the several solutions are given in Table 3. Solutions for two variations of the sanding model are given in Table 4. The results of five variations of the plowing model are given in Table 5.


STATE OF WYOMING
PREPARED BY THE
WYOMING STATE HIGHWAY DEPARTMENT
PLANNING AND RESEARCH DIVISION
SCALE
10 0 10 20 30 40 Miles
LEGEND
INTERSTATE NUMBERED HIGHWAY 80
U.S. NUMBERED HIGHWAY 30
STATE NUMBERED HIGHWAY 34
STATE CAPITAL
COUNTY SEAT
OTHER CITIES AND TOWNS
○ EXISTING MAINT. STATION & STOCKPILE LOCATION
● PROPOSED MAINT. STATION & STOCKPILE LOCATION
■ PROPOSED STOCKPILE LOC.
M O N T A N A
I D A H O
U T A H
C O L O R A D O
S O U T H D A K O T A
N E B R A S K A
YELLOWSTONE NATIONAL PARK
PARK
BIG HORN
SHERIDAN
CAMPBELL
CROOK
JOHNSON
WESTON
TETON
HOT SPRINGS
WASHAKIE
FREMONT
NATRONA
CONVERSE
NIOBRARA
SUBLETTE
LINCOLN
SWEETWATER
CARBON
ALBANY
PLATTE
GOSHEN
LARAMIE
UINTA
CHEYENNE
CASPER
LARAMIE
RAWLINS
SHERIDAN
GILLETTE
BUFFALO
CODY
WORLAND
HERMOPOLIS
LANDER
PINEDALE
JACKSON
KEMMERER
EVANSTON
GREEN RIVER
DOUGLAS
LUSK
TORRINGTON
WHEATLAND
NEWCASTLE
SUNDANCE
BASIN
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43°
42°
41°

FIGURE 3

Section Number	Highway Number	Milepost		Description	Centerline Mileage	Service Class	Plow Passes to clear Roadway	Plow Service Time Limit (hrs.)	Allowable Snow Depth Before Plow Reqd. (ins.)	Service Time Limit for Sanding (min.)
		From	To							
1	28	0	12.2	Farson to Dist. Boundary 3-5	12.2	B	4	8	2.5	240
2	28	12.2	24.41	" " " "	12.2	B	4	8	2.5	240
3	28	24.41	36.21	Dist. Boundary 3-5 to South Pass	11.8	B	4	8	2.5	240
4	28	36.21	48.00	" " " "	11.8	B	4	8	2.5	240
5	28	48.00	58.1	South Pass to Farson Junction	10.1	B	4	4	2	210
6	28	58.1	68.20	" " " "	10.1	B	4	4	2	210
7	789-287	72.87	57.07	Farson Jct.to Dist.Boundary 2-5	15.8	B	4	6	2	300
8	789-287	57.07	41.18	" " " "	15.8	B	4	6	2	300
9	789-287	41.18	27.4	Dist. Boundary 2-5 to Muddy Gap	13.7	B	4	6	2	300
10	789-287	27.4	13.7	" " " "	13.7	B	4	6	2	300
11	789-287	13.7	0.0	" " " "	13.7	B	4	6	2	300
12	789-287	72.87	81.10	Farson Junction to Lander	8.2	B	4	4	2	210
13	701	0.0	9.0	Sinks Canyon	9.0	C	4	18	*10	330
14	789	81.1	93.1	Lander to Riverton	12.0	B	4	4	2	210
15	789	93.1	105.17	" "	12.0	B	4	4	2	210
16	708	0.0	19.0	Riverton to Sand Draw	19.0	C	4	12	*10	330
17	710	0.0	15.8	Riverton to Gas Hills	15.8	C	4	10	*10	330
18	710	15.8	31.53	" " "	15.7	C	4	10	*10	330
19	20 - 26	100.04	83.74	Dist. Boundary 2-5 to Shoshoni	16.3	B	4	6	2	210
20	20 -	83.74	67.44	" " " "	16.3	B	4	6	2	210
21	20 -789	100.04	116.24	Shoshoni to Thermopolis	16.2	B	4	6	2	240
22	20 -789	116.24	132.53	" " "	16.2	B	4	6	2	240
23	26	127.30	119.13	Shoshoni to Riverton	8.2	B	4	6	2	210
24	26	119.13	105.17	" " "	14.0	B	4	6	2	210
25	707	0.0	15.9	Miss. Valley Rd. to Kinnear	15.9	C	4	10	*10	270
26	26	133.71	124.71	Riverton to Kinnear	9.0	B	4	2	2	300
27	26	124.71	115.61	" "	9.1	B	4	6	2	300
28	703	0.0	17.48	Milepost 9.28;287 to Kinnear	17.5	C	4	10	*10	330
29	287	31.69	20.48	Diversion Dam to Lander	11.2	B	4	6	2	270
30	287	20.48	9.28	" " "	11.2	B	4	6	2	270
31	26 -287	98.68	84.23	Diversion Dam to Dubois	14.7	B	4	8	2.5	270
32	26 -287	84.23	69.53	" " "	14.7	B	4	8	2.5	270
33	26 -287	69.53	54.83	" " "	14.7	B	4	8	2.5	270
34	26 -287	54.83	39.73	Dubois to Togwotee Pass	15.1	B	4	8	2.5	270
35	26 287	39.73	26.75	" " "	15.2	B	4	8	2	270
36	26 -287	26.75	14.45	Togwotee Pass to Moran Junction	12.3	B	4	8	2	270
37	26 -287	14.45	2.21	" " "	12.2	B	4	8	2	270
38	287	9.28	0.0	Diversion Dam to Lander	9.3	B	4	6	2	270
39	26	98.68	115.61	Kinnear to Diversion Dam	16.0	B	4	6	2	270
40	707	15.9	31.80	Miss. Valley Rd. to Kinnear	15.9	C	4	10	*10	300
41	20 - 26	67.44	51.14	District Boundary 2-5 to Shoshoni	16.3	B	4	6	2	270

Table 3. Composition of study area, roadway designation and fixed data applied to the final solution

Parameters Used in Study		Solution No. 1			Solution No. 2			Solution No. 1		Solution No. 2		
	Station Number	Amortized Station Costs	Station and Sanders Req'd	Stock-pile Serviced	Amortized Station Costs	Station and Sanders Req'd	Stock-pile Serviced	Stockpile Req'd	Roadway Serviced	Stockpile Req'd	Roadway Serviced	(2) Stockpile Number
1. Proposed Station Location	1	5.00	Yes-1	Same as Solution 2	5.00	Yes-1	1	Yes	Same as Solution 2	Yes	1	1
	2	20.00	Yes-1		30.00	Yes-1	2	Yes		Yes	2,3	2
	3	5.00	Yes		5.00	Yes-1	3	Yes		Yes	4,5	3
	4	20.00	None		30.00	No		Yes		Yes	6,7	4
	5	20.00	Yes		30.00	Yes-1	5	Yes		Yes	8,9	5
	6	5.00	Yes-2		5.00	Yes-2	6,18	Yes		Yes	11	6
	7	5.00	Yes-3		5.00	Yes-3	4,7,16	Yes		Yes	12,13,14,38	7
	8	20.00	Yes-2		20.00	Yes-2	8,21	Yes		Yes	15,16,17,24,26	8
	9	5.00	Yes-2		5.00	Yes-2	9,20	Yes		Yes	20,21,23,25	9
	10	20.00	Yes-1		30.00	Yes-1	10	Yes		Yes	41	10
	11	5.00	Yes-1		5.00	Yes-1	11	Yes		Yes	22	11
	12	20.00	Yes-1		30.00	Yes-1	12	Yes		Yes	29,30,31,39,40	12
	13	5.00	Yes-2		5.00	Yes-2	13,19	Yes		Yes	33,34	13
	14	20.00	Yes-1		20.00	Yes-1	14	Yes		Yes	35,36	14
	15	20.00	Yes-1		20.00	Yes-1	15	Yes		Yes	37	15
								Yes		Yes	27,28	16
								No		NO	None	17
								Yes		Yes	10	18
								Yes		Yes	32	19
								Yes		Yes	19	20
								Yes		Yes	18	21
2. Unit Cost of Sander (dollars/hr.)		10.00			10.00							
3. Unit Cost of Operator (dollars/hr.)		5.00			5.00							
4. Unit Cost of Stockpile (dollars/hr.)								4.80		4.80		
5. Unit Cost of Stockpile Material-delivered to Stockpile (dollars/ton)								2.50		2.50		
6. Effective Sander Working Speed (miles/hr.)				24			24	24		24		

Table 4. Control Data and Associated Optimal Solution for the Sanding Program.

Parameters Used in Study		Solution No. 1			Solution No. 2			Solution No. 3			Solution No. 4			Solution No. 5		
	Station Number (1)	Amortized Station Costs ($/Mission) (2)	Station and Plows Req'd (3)	Roadway Serviced	Amortized Station Costs	Station and Plows Req'd	Roadway Serviced	Amortized Station Costs	Station and Plows Req'd	Roadway Serviced	Amortized Station Costs	Station and Plows Req'd	Roadway Serviced	Amortized Station Costs	Station and Plows Req'd	Roadway Serviced
1. Proposed Station Location																
Farson	1	5.00	Yes-1	1,2	5.00	Yes-1	1,2	Same as Solution 2	Yes-1	1,2	Same as Solution 2, 3	Yes-1	1,2	5.00	Yes-1	1-3
District Boundary Highway 28	2	20.00	None		30.00	None			None			None		40.00	None	
South Pass	3	5.00	Yes-1	3-5	5.00	Yes-1	3-5		Yes-1	3-5		Yes-1	3-5	40.00	None	
Junction Highways 28,287	4	20.00	None		30.00	None			None			None		30.00	None	
District Boundary 2-5 Highway 287	5	25.00	Yes-1	9	40.00	None			None			Yes-1	9	40.00	None	
Muddy Gap	6	25.00	Yes-1	10,11	40.00	Yes-1	10,11		Yes-1	10,11		Yes-1	10,11	25.00	Yes-1	10,11
Lander	7	5.00	Yes-5	6-8,12-15,27-30,38,40	5.00	Yes-6	6-9,12-15,27-30,38-40		Yes-6	6-9,12-15,27-30,38-40		Yes-6	6-9,12-15,27-30,38-40	5.00	Yes-8	4-9,12-15,27-32,38-40
Riverton	8	20.00	Yes-1	26	30.00	None			None			None		30.00	None	
Shoshoni	9	5.00	Yes-4	16-21,23-25	5.00	Yes-5	16-21,23-26,41		Yes-5	16-21,23-26,41		Yes-5	16-21,23-26	5.00	Yes-5	16-21,23-26,41
District Boundary 2-5 Highway 20,26	10	20.00	Yes-1	41	30.00	None			None			Yes-1	41	30.00	None	
Thermopolis	11	5.00	Yes-1	22	5.00	Yes-1	22		Yes-1	22		Yes-1	22	5.00	Yes-1	22
Diversion Dam	12	20.00	Yes-1	31,39	30.00	None			None			None		30.00	None	
Dubois	13	5.00	Yes-2	32-36	5.00	Yes-3	31-37		Yes-3	31-37		Yes-2	31-36	25.00	Yes-1	33-35
Togwatee Pass	14	25.00	None		40.00	None			None			None		40.00	Yes-1	36
Moran Junction	15	25.00	Yes-1	37	40.00	None			None			Yes-1	37	40.00	Yes-1	37
2. Unit Cost of Plow (dollars/hr)		15.00			15.00			10.00			10.00			10.00		
3. Unit Cost of Operators (dollars/hr)		5.00			5.00			5.00			5.00			5.00		
4. Plow Travel Speed-deadhaul (miles/hr)		40			40			40			30			40		
5. Plow Working Speed-Average (miles/hr)		24			24			24			24			24		
Solution Number		1			2			3			4			5		

Table 5. Control data and associated optimal solution for the plowing program.

CONCLUSIONS

At the present time, no single variation of the optimal solutions reported in Tables 4 and 5 have been accepted as final. In their present form, the optimization models are being used to study the effect of policy, service specification, cost and performance data on the issue of station location and equipment requirements. The models constructed exceed the original expectation in that they are capable of assigning equipment to the roadway network in the most efficient manner, identification of total equipment requirements and specification of the location of sanding material stockpiles.

The models appear to give realistic solutions, based on the fact that certain combinations of input data produce solutions that are consistent with the present configuration of station locations for the specific area studied. However, the information which produces this set of solutions does not necessarily represent the most modern version of cost and policy.

The models show that final solutions are particularly sensitive to service specifications placed on the various sections of roadway. In some cases, changing a roadway from Class B to Class A service will significantly rearrange the configuration of stations and equipment assignment.

In general, the models appear to require less total equipment than is currently assigned to the area under study. The probable reason for this is that the models do not allow for backup equipment required to provide for equipment failure or emergency services.

The more efficient use of scarce resources such as money and materials is paramount in the maintenance of our highway system. Operations research is one method to achieve this goal.

The Wyoming State Highway Department maintains about 5,900 miles of roadway of all types. During 1970, over $2,100,000 was spent on snow removal. If a reduction of 10% can be made in the cost of this one operation $210,000 could be saved each year.

Optimizing techniques must be applied to all major areas of highway maintenance such as sealing, resurfacing and mowing in order to reduce costs. It is hoped that this paper will provide a stimulus to other agencies to develop these needed techniques.

APPENDIX

PERFORMANCE STANDARDS FOR SNOW REMOVAL

Methods and Procedures

Class A Service

1. Apply abrasive material at an approximate rate of 2,000 pounds per mile on Interstate (includes traveled lane and passing lane) and 2,000 pounds per mile on primary type roads. It is recommended that the sand be mixed with approximately 2 to 5% salt (Sodium Chloride). Under certain conditions such as lower temperatures it may be desirable to add approximately 0 to 2% Calcium Chloride.
2. Application of abrasive material may begin immediately, if required, with the beginning of the storm and will continue, with no plowing, until accumulated snow fall on the road surface reaches approximately 2 inches. Strong wind and/or low temperatures may preclude the above procedure.
3. When accumulated snow fall on the road surface reaches approximately 2 inches, begin plowing top of roadway on Interstate and on primary type roads continuously with the objective of maintaining a bare road. Sand icy spots.
4. As the storm subsides, extend plowing to truck climbing lanes and shoulders.
5. At end of storm, provide Class E Service. Do not start Class E Service until Class A, B, or C Service has been provided to all other roads in area.

Class B Service

1. Apply abrasive material at an approximate rate of 2,000 pounds per mile per 2 lanes (includes traveled lane and passing lane) and 2,000 pounds per mile on primary type roads. It is recommended that the sand be mixed with approximately 2 to 5% salt (Sodium Chloride). Under certain conditions, such as lower temperatures, it may be desirable to add approximately 0 to 2% Calcium Chloride.
2. Application of abrasive material may begin when snow depth on the road surface reaches approximately 1/4 inch and will continue, with no plowing, until accumulated snow fall reaches approxumately two inches.

3. When accumulated snow fall on the road surface reaches approximately 2 inches, begin plowing traveled lane continuously with the objective of a plowing frequency to obtain a bare road soon after snow subsides.
4. Continue plowing roadway for duration of storm. As storm subsides, widen plowing as necessary to include truck climbing lanes, etc.
5. At end of storm, provide Class E Service.

Class C Service

1. Begin plowing when accumulated snow fall on the road surface reaches approximately 3 inches or sooner if men and equipment available. Plow roadway continuously with the objective of a plowing frequency to obtain a bare road soon after end of storm.
2. Apply abrasive material at an approximate rate of 2,000 pounds per mile per 2 lanes (includes traveled lane and passing lane) and 2,000 pounds per mile on primary type roads. It is recommended that the sand be mixed with approximately 2 to 5% salt (Sodium Chloride). Under certain conditions such as lower temperatures it may be desirable to add approximately 0 to 2% Calcium Chloride.
3. Continue plowing traveled way for duration of storm. As storm subsides, widen plowing to include truck climbing lanes, etc.
4. At end of storm, provide Class E Service.

Class D Service

1. Close road when snow depth requires. Refer to Maintenance Policy and Procedure Directive 70-2 for proper procedure.
2. Open as soon as feasible in spring.

Class E Service

1. Clean up operations to include removal of any slush from road surface, opening of ramps on low volume interchanges and cleaning of shoulder ridges. To be performed after Class A, B or C Services have been provided on all roads.

   NOTE: Although it is not intended to clean approaches, any snow ridges must be removed to allow users of approaches better access to the highways.

REFERENCES

1. Wyoming State Highway Department, Financial Report January 1, to December 31, 1969, Cheyenne, Wyoming.

2. Wyoming State Highway Department, An Operations Research Study for the Maintenance Division, 1970, Cheyenne, Wyoming.

3. Wyoming State Highway Department, Optimal Solution for Maintenance Station Location, 1971, Cheyenne, Wyoming.