

DISSERTATION

EDUCATION, CARE, AND FERTILITY IN SOUTH KOREA: ESSAYS ON HOUSEHOLD
CONSTRAINTS AND SOCIAL PRESSURE

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Jackie Dajin Young

Department of Economics

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Doctoral Committee:

Advisor: Daniele Tavani

Co-Advisor: Elissa Braunstein

Sammy Zahran

Ray Miller

Renzo Cavalieri

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ABSTRACT

EDUCATION, CARE, AND FERTILITY IN SOUTH KOREA: ESSAYS ON HOUSEHOLD CONSTRAINTS AND SOCIAL PRESSURE

This dissertation studies how family decisions about education, care, and fertility are shaped by social pressure and household constraints. Across the three chapters, I focus especially on South Korea, where very low fertility has coincided with intense educational competition, high private education spending, and persistent gendered expectations around childcare. The dissertation combines empirical evidence and theoretical modeling to examine how peer behavior, parental time allocation, and education-related status competition can raise the perceived cost of children and contribute to lower fertility.

The first chapter, “Can Parents Overspend in their Children’s Education? Peer Effects in Korea’s Shadow Education Market,” studies peer effects in household tutoring expenditures using the Korean Education Longitudinal Study. I focus on the transition from middle school to high school, when students in many equalization areas were assigned to general academic high schools by district lottery and re-sorted into new schools and peer groups. The results show that peer exposure affects household tutoring spending, with the clearest increases occurring when students enter the top quartile of the peer-spending distribution. More moderate upward moves produce weaker and less stable effects, while downward transitions are associated with sharper reductions in spending. These findings suggest that high-spending peer environments can intensify positional pressure and shape household education investment in Korea’s shadow education market.

The second chapter, “How Many Parents Does It Take? Parental Time Allocation and the Effectiveness of Fertility Subsidies,” develops an overlapping-generations model of parental childcare time, fertility, and human capital investment. The chapter asks whether greater paternal involvement necessarily reduces mothers’ childcare burden. When maternal and paternal time are sub-

stitutes, father involvement can relax the maternal time constraint. When they are complements, however, greater father involvement can raise mothers' childcare time as well, lowering fertility and shifting resources toward child quality. The chapter also shows that fertility subsidies may have limited or unintended effects when they do not change the underlying organization of care.

The third chapter, "Rat Race, Educational Signaling, and Fertility," develops a model linking peer pressure in education spending to fertility and long-run demographic outcomes. Households choose consumption, fertility, and shadow education investment, but perceived child quality depends partly on spending relative to peers. This creates overinvestment in education and lowers fertility. The chapter then embeds this mechanism in a dynamic framework in which population and human capital evolve jointly. Calibrating the model to Korea suggests that shadow-education competition creates a modest but meaningful fertility distortion and contributes to excessive education spending.

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DEDICATION

*To Jordan and Lincoln,
for giving me the courage to begin,
and the strength to keep going.*

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Chapter 1

Introduction

Family decisions about children are often modeled as private choices shaped by income, preferences, prices, time constraints, and policy incentives (Becker, 1965). Yet these decisions are also deeply social. Parents make choices about education, care, and fertility while observing other households, responding to institutional expectations, and navigating gendered norms about who should provide care and what children require to succeed (Folbre, 2001; Goldscheider, Bernhardt, & Lappegård, 2015; McDonald, 2000). These pressures can raise the perceived cost of children beyond direct financial expenses. They can also create gaps between household behavior and socially desirable outcomes.

This dissertation studies these dynamics in the context of South Korea, a setting where questions about education, care, and fertility are especially urgent. South Korea combines very high educational attainment with intense educational competition and large household expenditures on private tutoring, often described as shadow education (S. Kim & Lee, 2010; Ministry of Data and Statistics, 2025b; OECD, 2024). At the same time, Korea has experienced one of the most dramatic fertility declines in the world (World Bank, 2025). Prior work has linked this decline in part to education fever, status competition, and the rising cost of raising successful children (Anderson & Kohler, 2013; S. Kim, Tertilt, & Yum, 2024). These pressures operate alongside other important demographic and social changes, including delayed marriage, declining marriage rates, and persistent gender inequality in family life (E. H.-W. Kim, 2017; McDonald, 2000; Raymo & Park, 2020; Raymo, Park, Xie, & Yeung, 2015). The country has also expanded family policy and encouraged greater paternal involvement in childcare, yet fertility has remained persistently low (Y. Lee, 2022; Ministry of Data and Statistics, 2025a; S.-M. Park, Cho, & Choi, 2010). These patterns suggest that the cost of children cannot be explained only by income or formal policy incentives. It is also shaped by social expectations surrounding educational success, parental responsibility, and the division of care within the household.

The Korean case highlights a broader economic problem. When childrearing becomes tied to relative position, family decisions may generate externalities that are not fully internalized by individual households. A parent may increase tutoring spending because other parents spend more, even if the academic returns are modest. A father may spend more time with children without necessarily reducing the mother's role as the default caregiver or organizer of care. A household may choose to have fewer children not only because children are expensive in an absolute sense, but because raising children is perceived as requiring increasingly intensive educational and parental investment. In each case, families make decisions within constraints shaped by income, time, institutions, and social expectations, but the resulting equilibrium may involve excessive spending, persistent gendered burdens, and lower fertility.

Across the three chapters, I examine how social pressure and household constraints interact to shape parental behavior. The central argument is that families may face not only resource constraints, but also positional and relational constraints. In education, parents may feel pressure to increase spending when other families spend more, even when the academic returns to private tutoring are modest or unevenly distributed (Bray, 2014; Byun, 2014; Kang & Park, 2021; Ryu & Kang, 2013). In care, greater paternal involvement may not automatically reduce mothers' childcare burden if parental time inputs are complementary rather than substitutable (Craig, 2006; Daminger, 2019; Data2X, 2025). In fertility, education-related status competition may raise the perceived cost of childbearing and contribute to lower fertility (Anderson & Kohler, 2013; S. Kim et al., 2024). Together, these mechanisms help explain why policies that target only the direct cost of children may have limited effects when they do not address the social and institutional environment in which household decisions are made.

Chapter 1, "Can Parents Overspend in their Children's Education? Peer Effects in Korea's Shadow Education Market," provides empirical evidence on peer effects in private tutoring expenditures. Using the Korean Education Longitudinal Study, I focus on the transition from middle school to high school, when students are reshuffled into new schools and peer groups. This transition creates useful variation in peer exposure because students encounter new reference groups at a

stage when families are making important decisions about high-school academic preparation. The results show that households increase their own tutoring expenditures when students enter higher-spending peer environments, especially when they move into the top quartile of the peer-spending distribution. The evidence points to a nonlinear and asymmetric peer effect: families respond most strongly when peer spending becomes visibly high, suggesting a rat-race dynamic in which parents spend partly to avoid falling behind. This chapter provides the empirical foundation for the dissertation by showing that peer pressure in education spending is not only a theoretical concern, but also visible in household behavior.

Chapter 2, “How Many Parents Does It Take? Parental Time Allocation and the Effectiveness of Fertility Subsidies,” turns from educational spending to the organization of childcare within the household. This chapter develops an overlapping-generations model of parental time allocation, fertility, and human capital investment. The model asks whether greater paternal involvement necessarily relaxes the maternal time constraint. When maternal and paternal childcare time are substitutes, father involvement can reduce mothers’ childcare burden and lower the effective cost of children. However, when parental time inputs are complements, greater father involvement can also raise mothers’ childcare time, tightening the household time constraint and lowering fertility. This distinction matters because policy discussions often treat father involvement as an unambiguous solution to unequal care burdens. The chapter shows that fertility subsidies may have limited or even unintended effects when they do not change the underlying organization of care.

Chapter 3, “Rat Race, Educational Signaling, and Fertility,” connects the education-spending channel to fertility and long-run demographic outcomes. The chapter develops a model in which households choose consumption, fertility, leisure, and shadow education investment, while perceived child quality depends partly on spending relative to peers. This positional component creates a rat-race externality: households increase education spending in response to the reference group, but when many households do so, relative position does not necessarily improve. As a result, decentralized households overinvest in shadow education relative to the social optimum, raising the perceived cost of children and reducing fertility. A central feature of the chapter is

that the strength of peer pressure is not fixed. Instead, the peer-effect parameter is linked to a university-admissions tournament, so that education competition becomes more intense when cohort size increases and more students compete near the admissions margin. The chapter first uses a static model to build intuition for this rat-race mechanism and then embeds it in a simple dynamic framework in which population and human capital evolve jointly. In this dynamic setting, population affects the intensity of peer pressure, while fertility and education spending shape future population and human capital. Calibrating the model to Korea suggests that shadow-education competition creates a modest but meaningful fertility distortion and contributes to excessive education spending. This chapter brings together the dissertation's central themes by showing how private education competition can affect not only individual household choices, but also aggregate fertility and long-run demographic dynamics.

The dissertation contributes to the literature on household decision-making, fertility, education, and care by emphasizing the role of social externalities. First, it contributes to the literature on shadow education by providing evidence that household tutoring expenditures respond to peer environments, particularly at the upper end of the peer-spending distribution. Second, it contributes to research on care and fertility by showing that the effect of paternal involvement depends on whether mothers' and fathers' childcare time operate as substitutes or complements. Third, it contributes to macroeconomic and demographic models of low fertility by linking education-related status competition to household fertility choices and long-run population dynamics. Across these chapters, the dissertation treats education, care, and fertility not as separate topics, but as connected parts of the same household decision problem.

In the Korean context, these mechanisms are especially important because educational competition and low fertility are not separate puzzles. They are connected through the perceived cost of raising children in a highly competitive and socially demanding environment. If parents believe that children require large private investments to succeed, and if mothers remain primarily responsible for organizing and providing care, then the cost of children remains high even when formal

subsidies reduce some direct expenses. The burden is therefore not only financial. It is also social, temporal, and relational.

The broader implication is that fertility policy cannot be evaluated only through cash transfers, subsidies, or formal childcare support. These tools matter, but their effectiveness depends on whether they reduce the deeper constraints households face. If parents continue to experience childrearing as an escalating educational race, and if mothers remain the default organizers of care even when fathers do more, then the private cost of children may remain high. Policies aimed at low fertility may therefore need to address not only affordability, but also the social pressures and institutional norms that make parenting feel increasingly costly.

Chapter 2

Can Parents Overspend in their Children's Education? Peer Effects in Korea's Shadow Education Market

2.1 Introduction

South Korea's education system is widely praised for high test performance and a highly educated population. It is also defined by intense competition and large private spending on out-of-school study, often called "shadow education" (S. Kim & Lee, 2010). Participation is widespread, and household spending on shadow education is large relative to income and GDP (S. Kim & Lee, 2010). Recent work shows that participation and expenditure remained high through the COVID-19 policy cycle, which suggests that shadow education is a structural feature of the system rather than a short-term response (Piao & Hwang, 2021). This culture of human capital investment may support high average achievement, but it raises concerns about student well-being, equity, and the size of the academic gains purchased (Bray, 2014).

The literature describes this "education fever" as a social equilibrium shaped by competitive admissions to elite universities, high private returns at the top of the hierarchy, and strong parental preferences for quality over quantity of children (S. Kim & Lee, 2010). In Korea, the prestige premium associated with top universities is closely linked to occupational status and networks, which intensifies status competition and demand for supplementary study (Y. Choi & Park, 2016). These incentives scale up to the macro outcomes. Calibrated models imply that, absent status externalities in education, fertility would be about 28 percent higher (S. Kim et al., 2024).

What do households actually buy, academically, with this spending? When identification is careful, estimated average returns are modest. Using an IV strategy, Ryu and Kang (2013) finds small effects; even a 10 percent increase in spending moves a student's test rank by less than

one percentile point. Results from Korean and international microdata vary by subject, format, and dosage (Byun, 2014). Reviews conclude that selection and measurement choices account for much of the variation, so broad claims of large average effects are difficult to sustain (Bray, 2014; Byun, 2014).

Distributional evidence sharpens the picture. Using Korean middle-school data and modern distributional methods, Kang and Park (2021) show that average gains are small, while impacts are concentrated in the upper half of the score distribution. This raises equity concerns even when mean effects are modest. It also helps explain the persistence of spending: families may be responding to positional pressures rather than to a production function that delivers uniform learning gains (Bae & Choi, 2024).

Peers are a natural channel for these pressures. Exploiting random classroom assignment and an IV approach in the Gyeonggi Education Panel, T. Kim, Jang, and Kim (2022a) find that a 10-percentage-point rise in classmates receiving tutoring increases girls' own take-up by 7.1 percentage points, with smaller effects for boys. In related analyses with school fixed effects, the same peer exposure is associated with higher depressive symptoms, reduced leisure, and greater test-related stress (T. Kim, Jang, & Kim, 2022b).

This paper asks a simple question: how much do peers shift after-school spending when peer groups reset? I study the Korean Education Longitudinal Study (KELS; 2005 cohort), a nationally representative panel that follows one cohort from middle school into high school and beyond. In KELS, Year 1 is 7th grade, Year 2 is 8th grade, Year 3 is 9th grade at the end of middle school, and Year 4 is 10th grade at the start of high school. I focus on the transition from Year 3 to Year 4, when classmates are re-sorted and local reference groups change. I begin by documenting co-movement in simple linear models, then allow for nonlinearity and concentrate on transitions across peer-spending quartiles.

At the time of data collection (2007–2008), many students in the Korean Education Longitudinal Study cohort entered high school under South Korea's high-school equalization system. In most metropolitan and provincial "equalization" (*pyeongjunhwa*) areas—including Seoul, Busan,

and other major cities—admission to general academic high schools was determined largely by computerized assignment within each school district, rather than by school-specific entrance exams or academic records. Only a small share of students applied to selective special-purpose or vocational schools, which lay outside the equalization system. This structure substantially reshuffled middle-school classmates into new high-school peer groups. I therefore use the high-school transition as a source of useful variation in peer exposure, while recognizing that the setting is not a pure randomized experiment because families may still sort across districts, school types, tracks, neighborhoods, and tutoring markets.

A key challenge is endogeneity because peer exposure is not random. Families choose schools, tracks, and neighborhoods based on aspirations, information, and access to providers, and these factors also affect shadow education demand. Classmates can face correlated time varying shocks such as the entry of new providers, shifts in school workload, or cohort counseling, so peer means may move with local conditions even if households adjust spending at different times. Simple regressions can therefore mix peer influence with selection, common shocks, mean reversion, and measurement error.

The design addresses these concerns in three parts. First, I focus on the move from middle school to high school and measure peers as the school $\times$ year leave-one-out mean, which avoids mechanically including the student's own spending in the peer measure. Second, I estimate first-difference models, $\Delta Y = Y_4 - Y_3$, to difference out time-invariant student traits and fixed factors. Third, I estimate effects using propensity-score matching (PSM) and augmented inverse-probability weighting (AIPW) with rich pre-treatment covariates: baseline total and subject-specific spending, Year 3 scaled test scores, household income, parents' education, sex, disability, region, and private-school status. Design diagnostics include balance and overlap statistics, weight dispersion, winsorized outcomes, bootstrap standard errors, value-added specifications using Year 4 levels with Year 3 controls, and a placebo that replaces the outcome with the change in household income. I also examine whether the main Top 25 percent result is sensitive to alternative comparison groups, including eligible students, students who remained in the same peer-spending

quartile, and students who did not experience downward peer transitions. Additional threshold checks around the Top 25 percent cutoff are used to assess whether the main pattern reflects a broader upper-tail response rather than a mechanical feature of one cutoff. These checks do not eliminate all possible unobserved sorting or school-level shocks, but they reduce concerns that the results are driven by observable imbalance, outliers, a particular outcome specification, or the exact definition of the comparison group. I therefore interpret the estimates causally under conditional ignorability and overlap given observed Year 3 covariates.

The results point to a threshold pattern rather than a smooth gradient. Larger movements into higher peer-spending groups are associated with larger increases in own spending. The most stable effects appear when students enter the top quartile of peer spending, while upper-decile estimates provide suggestive evidence of even stronger responses at the very top of the distribution. Changes at the top half are weaker or more model-dependent. By subject, responses are largest in Math, followed by English, and are smaller or not statistically significant in Korean in most specifications. A value-added specification yields similar conclusions, and the income placebo points away from an income-driven explanation. Together, these findings quantify how high-intensity peer environments shift private investment in shadow education, raising important questions about positional pressure and household spending decisions.

2.2 A Toy Model

To interpret the empirical results, I introduce a minimal benchmark that isolates the peer channel. The aim is not to match every feature of the data but to clarify how a household's spending choice s_i responds to the peer reference level s_{-i} . In the data that follow, s_i will be household spending on shadow education, and s_{-i} will be an externally defined peer reference (constructed as a leave-one-out cohort mean). The model delivers clean comparative statics that guide the empirical exercises.

To appreciate the inefficiency of shadow education spending driven by peer effects, consider the following extremely simplified model. Household $i = \{1, \dots, N\}$ has a linearly homogeneous

Cobb-Douglas utility function defined over consumption c_i and the human capital of its children, h'_i , where $\alpha \in (0, 1)$ denotes the weight of consumption in the household's preferences. Thus, the utility function is $u_i(c_i, h'_i) = \alpha \ln c_i + (1 - \alpha) \ln h'_i$. Human capital increases with education spending s_i relative to peer education spending s_{-i} . The extent to which peer education spending matters is captured by a parameter $\phi_{-i} \in (0, 1)$, to allow for heterogeneity among households in the way they react to peer group spending.¹ We assume that $h'_i = (s_i - \phi_{-i}s_{-i})$. The household earns income y_i , so that the budget constraint is $c_i + s_i \leq y_i$. From the first-order conditions, we find the following choices of consumption and education spending as a function of income and peer spending:

$$c_i = \alpha y_i - \alpha \phi_{-i} s_{-i} \quad (2.1)$$

$$s_i = \alpha \phi_{-i} s_{-i} + (1 - \alpha) y_i \quad (2.2)$$

Equation (2.2) motivates the empirical analysis at the heart of this paper. To close the model, assume that households in the same group best-respond to one another, so that $s_i = s_{-i}$. Then,

$$c_i = \frac{\alpha(1 - \phi_{-i})}{1 - \alpha \phi_{-i}} y_i \quad (2.3)$$

$$s_i = \frac{1 - \alpha}{1 - \alpha \phi_{-i}} y_i \quad (2.4)$$

A social planner internalizes the peer spending externality by imposing $s_i = s_{-i}$ when solving the household problem. Accordingly, the efficient consumption and education spending are:

$$c_i^* = \alpha y_i \quad (2.5)$$

$$s_i^* = (1 - \alpha) y_i \quad (2.6)$$

¹The reference parameter is indexed by the reference group, as opposed to the individual household, given that the results of the empirical analysis suggest that peer effects do in fact vary as the reference group varies.

And we have that

$$s_i - s_i^* = \left[\frac{\alpha\phi_{-i}(1-\alpha)}{1-\alpha\phi_{-i}} \right] y_i > 0 \quad (2.7)$$

$$c_i - c_i^* = - \left[\frac{(1-\alpha)\alpha\phi_{-i}}{1-\alpha\phi_{-i}} \right] y_i < 0 \quad (2.8)$$

So that household i overspends on education and underspends on consumption. The extent of the misallocation is both household- (through heterogeneity in income) and group- specific, through the potential heterogeneity in the reference group.

Equations (2.1)–(2.2) yield two testable predictions. First, holding income fixed, higher peer spending shifts own spending up in proportion to $\alpha\phi_{-i}$, potentially heterogeneous given the reference group. Second, in the symmetric case (2.4), private spending exceeds the planner benchmark (2.6) whenever $\phi_{-i} > 0$, implying overspending relative to the social optimum. These predictions motivate the empirical focus on what happens when students are exposed to a new peer group—during the middle-to-high school transition—and on movements into higher-intensity peer environments.

In the empirical analysis below, I estimate how shifts in the peer reference level s_{-i} are associated with changes in own spending s_i . I use within-student changes and peer measures that exclude the individual, together with estimators that combine matching with outcome models, to limit bias from selection and common shocks. The linear setup is a guide for interpretation, but allowing for heterogeneity in the peer response provides the theoretical background for the stronger responses observed when students enter the most elite peer groups.

2.3 Data

I use the Korean Education Longitudinal Study (KELS; 2005 middle-school cohort), a nationally representative student panel that tracks individuals from 7th grade into adulthood. The analysis focuses on the transition from Year 3 (9th grade) to Year 4 (10th grade), from the end of middle school to the start of high school in South Korea when classmates, schools, and local

reference groups are re-sorted. This reset is arguably when peer exposure should move shadow-education choices. Although not purely random, the middle-to-high school reshuffle is plausibly quasi-random with respect to unobserved household taste for tutoring once I condition on rich Year-3 covariates; students change classmates and schools, and the within-school class assignments are made administratively rather than by parents. This institutional reshuffling provides the variation I exploit in the identification strategy.

The primary outcome variable is household expenditure on private tutoring (shadow education) for the observed student, reported as total out-of-school study expenditure. I analyze the total and the breakdown of spending by subject (Korean, English, and Math). All monetary amounts are reported in units of 10,000 KRW.²

I also define *Any tutoring* as a binary indicator equal to 1 if total spending is positive. For each student i in school s and year t , peer exposure is the leave-one-out mean of classmates' (same grade in same school) tutoring expenditure,

$$\text{peer}_{it} = \frac{1}{N_{st} - 1} \sum_{\substack{j \in s,t \\ j \neq i}} \text{Spend}_{jt},$$

where N_{st} is the number of students observed in school s during year t . This measure is constructed at the school $\times$ year level. For descriptives, I sort students into *year-specific* quartiles of peer_{it} (Q1–Q4). For the transition analysis, I focus on changes in peer exposure between Year 3 and Year 4.

Family and student covariates include household income (measured in 10,000 of KRW), parents' average education (years), male, and disability. School context is summarized with mutually exclusive metropolitan, medium-sized city, and rural indicators, with “rural” defined residually when the metropolitan and medium-city indicators are both zero, plus private-school status. Where

²Using WDI GDP–PPP conversion factors for Korea of 770.09 KRW per international dollar in 2007 and 785.72 KRW per international dollar in 2008, 10,000 KRW is approximately \$12.99 in 2007 and \$12.73 in 2008 PPP-adjusted dollars.

available, I use scaled test scores in Korean, English, and Math (mean ≈ 500 , SD ≈ 60) to describe baseline achievement in Year 3. For all binaries, tables report “Yes = 1.”

I retain students observed in Year 3 and/or Year 4 with non-missing outcomes and core covariates. The Year-3 descriptive sample contains $N = 6,908$ students (Table 1). The Year-4 descriptive table (Table 2) is constructed analogously using the Year-4 cross-section. Monetary descriptives use untrimmed values; robustness checks later report results under standard trimming rules (e.g., 1st/99th percentiles).

Table 2.1: Year 3 Descriptive Statistics by Peer Spending Quartile

	All	Q1 (Low)	Q2	Q3	Q4 (High)
Male (Yes = 1)	0.52 (0.50)	0.50 (0.50)	0.51 (0.50)	0.60 (0.49)	0.48 (0.50)
Disabled (Yes = 1)	0.04 (0.19)	0.03 (0.18)	0.04 (0.20)	0.04 (0.19)	0.04 (0.20)
Parents’ avg. education (years)	12.87 (2.22)	12.03 (2.18)	12.51 (2.11)	13.11 (2.04)	13.81 (2.15)
Household income (10,000 KRW)	434.11 (647.25)	355.64 (625.52)	393.20 (604.31)	442.23 (599.94)	545.13 (734.63)
Has religion (Yes = 1)	0.75 (0.44)	0.77 (0.42)	0.76 (0.43)	0.71 (0.45)	0.74 (0.44)
Metro area (Yes = 1)	0.46 (0.50)	0.36 (0.48)	0.44 (0.50)	0.47 (0.50)	0.57 (0.49)
Medium city (Yes = 1)	0.45 (0.50)	0.41 (0.49)	0.43 (0.50)	0.52 (0.50)	0.43 (0.49)
Rural (Yes = 1)	0.09 (0.29)	0.23 (0.42)	0.13 (0.33)	0.01 (0.10)	0.00 (0.00)
Private school (Yes = 1)	0.20 (0.40)	0.13 (0.33)	0.21 (0.40)	0.30 (0.46)	0.16 (0.37)
Any tutoring (Yes = 1)	0.58 (0.49)	0.43 (0.50)	0.57 (0.50)	0.62 (0.49)	0.69 (0.46)
Tutoring exp. total (10,000 KRW)	25.23 (40.03)	15.70 (44.54)	20.15 (27.08)	26.75 (33.56)	38.32 (47.72)
Tutoring exp. Korean (10,000 KRW)	4.26 (9.68)	2.75 (7.74)	3.80 (7.43)	4.56 (8.58)	5.95 (13.44)
Tutoring exp. Math (10,000 KRW)	10.78 (18.91)	6.59 (21.39)	8.15 (12.04)	11.47 (16.06)	16.91 (22.52)
Tutoring exp. English (10,000 KRW)	10.19 (17.67)	6.36 (21.12)	8.20 (12.06)	10.73 (15.08)	15.46 (19.61)
Korean score (scaled)	499.69 (61.72)	488.16 (63.98)	494.02 (61.10)	502.86 (59.87)	513.81 (58.84)
English score (scaled)	524.95 (64.03)	506.52 (58.32)	517.41 (59.73)	529.27 (63.95)	546.78 (66.84)
Math score (scaled)	510.30 (63.09)	494.87 (57.17)	502.31 (59.31)	518.23 (65.06)	526.04 (65.61)
Observations	6908	1727	1728	1726	1727

Standard deviations in parentheses

Year 3 (Table 2.1). In the full Year-3 sample, 52% of students are male and 4% report a disability. Parents’ average education is 12.9 years. Mean household income is 434 (10,000 KRW).

School location is 46% metropolitan, 45% medium-sized city, and 9% rural; 20% attend private schools. Any tutoring equals 58%. Mean total tutoring expenditure is 25.2 (10,000 KRW). Scaled test scores center around 500 with $SD \approx 60$. Across peer-spending quartiles, there is a strong monotone gradient: any-tutoring rises from 0.43 (Q1) to 0.69 (Q4); total spending rises from 15.7 to 38.3; and scaled achievement is higher in higher-peer quartiles (e.g., English: 506 $\rightarrow$ 547). These patterns indicate that the peer quartiles capture meaningful socioeconomic and academic sorting already in Year 3.

Table 2.2: Year 4 Descriptive Statistics by Peer Spending Quartile

	All	Q1 (Low)	Q2	Q3	Q4 (High)
Male (Yes = 1)	0.52 (0.50)	0.53 (0.50)	0.52 (0.50)	0.53 (0.50)	0.51 (0.50)
Disabled (Yes = 1)	0.04 (0.19)	0.04 (0.20)	0.04 (0.20)	0.03 (0.16)	0.03 (0.18)
Parents' avg. education (years)	12.84 (2.19)	11.87 (2.15)	12.60 (2.11)	13.02 (1.93)	13.82 (2.07)
Household income (10,000 KRW)	420.20 (606.08)	307.87 (515.04)	416.53 (650.31)	448.89 (635.61)	506.92 (608.18)
Has religion (Yes = 1)	0.74 (0.44)	0.77 (0.42)	0.76 (0.43)	0.74 (0.44)	0.72 (0.45)
Metro area (Yes = 1)	0.45 (0.50)	0.33 (0.47)	0.34 (0.48)	0.56 (0.50)	0.60 (0.49)
Medium city (Yes = 1)	0.37 (0.48)	0.36 (0.48)	0.37 (0.48)	0.37 (0.48)	0.38 (0.49)
Rural (Yes = 1)	0.18 (0.38)	0.31 (0.46)	0.29 (0.45)	0.07 (0.26)	0.02 (0.15)
Private school (Yes = 1)	0.47 (0.50)	0.42 (0.49)	0.48 (0.50)	0.50 (0.50)	0.51 (0.50)
Any tutoring (Yes = 1)	0.53 (0.50)	0.22 (0.42)	0.52 (0.50)	0.67 (0.47)	0.75 (0.43)
Tutoring exp. total (10,000 KRW)	26.61 (38.10)	8.59 (23.37)	22.93 (33.92)	31.27 (35.60)	44.22 (46.20)
Tutoring exp. Korean (10,000 KRW)	3.52 (9.46)	1.20 (5.80)	3.10 (9.44)	3.95 (8.50)	5.84 (12.53)
Tutoring exp. Math (10,000 KRW)	12.64 (19.40)	3.82 (10.79)	10.83 (16.86)	14.88 (19.03)	21.47 (23.77)
Tutoring exp. English (10,000 KRW)	10.45 (17.26)	3.57 (10.65)	9.00 (15.19)	12.44 (17.53)	16.91 (20.85)
Observations	5941	1488	1485	1483	1485

Standard deviations in parentheses

Year 4 (Table 2.2). Year-4 descriptive statistics reproduce the same qualitative results by peer quartile (higher peer spending is associated with higher participation and higher expenditure). Levels differ modestly compared to Year 3, consistent with re-sorting at high-school entry and changes in school mix (e.g., private-school share and rural representation). The quartile gradients remain steep, which motivates studying changes in peer exposure at the transition.

Figure 2.1 shows transitions between Year 3 and Year 4 peer-spending quartiles. For each student, I assign a Year-3 quartile using the school-year leave-one-out peer mean of tutoring expenditure, recompute the peer mean in Year 4, and reassign quartiles; the figure displays the distribution of moves across these categories. A substantial share of students enter different peer environments

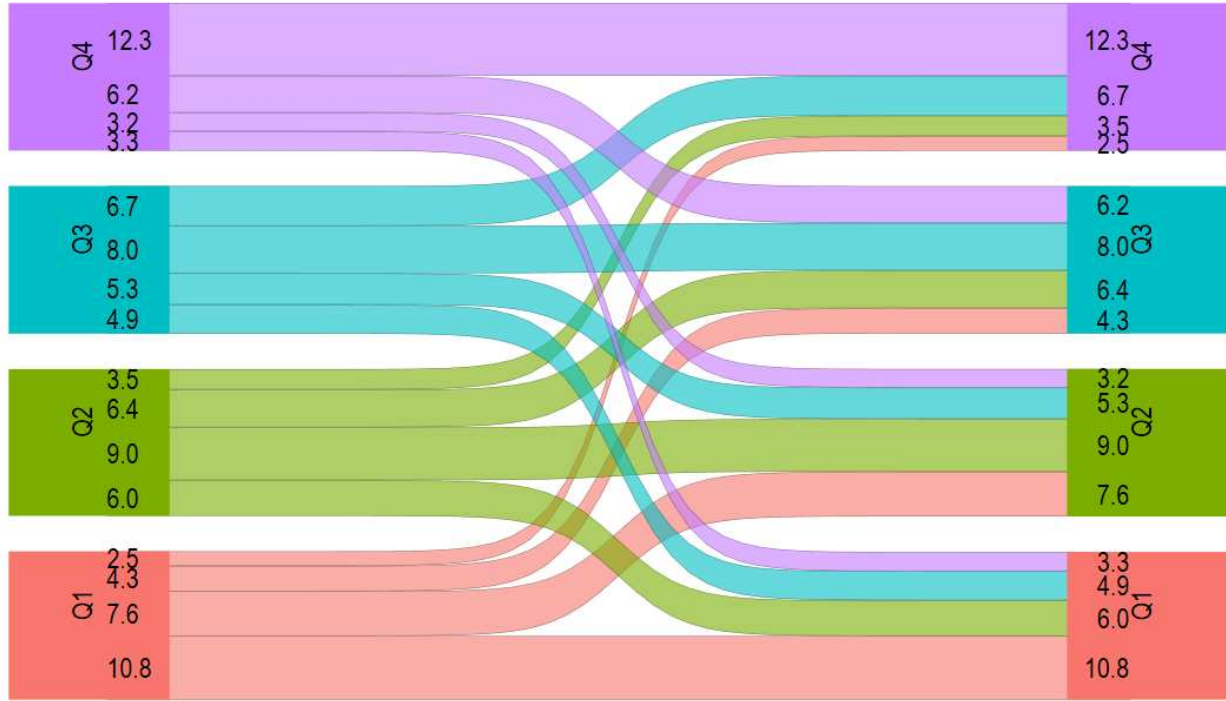


Figure 2.1: Transitions across peer-spending quartiles from Year 3 (rows) to Year 4 (columns). Off-diagonal indicates re-sorting at high-school entry.

at high-school entry. The empirical strategy leverages this peer-group change at the middle school to high school transition.

2.4 Initial and motivating statistics

Before turning to the causal design, I begin with a set of basic OLS descriptions using the Year 3 and Year 4 panel. The outcome is the student's change in tutoring expenditure, measured in 10,000 Korean won-units ($\approx$ \$12.99 (2007) PPP). Peer exposure is the leave-one-out mean within school $\times$ year, computed separately in Year 3 and Year 4. I condition on the student's baseline spending level in Year 3 and include a common set of covariates (household income, parents' average education, metro and medium-size city indicators, male, and disability). These specifications summarize how own spending co-moves with shifts in the peer environment at the high school transition, though they are descriptive rather than causal.

Let y_{ist} denote student i 's tutoring expenditure in school s and year $t \in \{3, 4\}$, and let the peer mean that excludes i be

$$p_{s \setminus i, t} = \frac{1}{N_{st} - 1} \sum_{j \in \mathcal{S}_{st} \setminus \{i\}} y_{jst}.$$

Define $\Delta y_i = y_{is4} - y_{is3}$ and $\Delta p_i = p_{s \setminus i, 4} - p_{s \setminus i, 3}$. The baseline OLS model is

$$\Delta y_i = \alpha + \beta \Delta p_i + X_i' \gamma + \varepsilon_i, \quad (2.9)$$

where X_i collects covariates and includes y_{is3} . The coefficient β is the marginal association between peer shifts and own shifts at the transition, conditional on starting levels and observables.

To let the data speak without forcing a single slope, I replace the continuous regressor Δp_i with equal frequency terciles of the peer change. The construction is simple. First, set $x_i = \Delta p_i$. Second, split the empirical distribution of x_i at its 33rd and 67th percentiles to form three groups: Low, Mid, and High. Mid sits around the center of the Δp distribution and serves as a convenient reference group. Third, estimate

$$\Delta y_i = \alpha + \lambda_L D_i^L + \lambda_H D_i^H + X_i' \gamma + \varepsilon_i. \quad (2.10)$$

with Mid omitted. The two reported coefficients are mean differences in Δy_i relative to Mid. A positive λ_H means that students whose peers increased spending the most raised their own spending more than the middle group. A negative λ_L means that students whose peers decreased spending the most reduced their own spending relative to the middle group. I prefer terciles here because they balance transparency and precision: they reveal the shape across the distribution, reduce the influence of extreme peer changes, and keep the control set identical to the linear model so that the columns are comparable.

To allow the slope to vary flexibly while keeping coefficients easy to read, I estimate a piecewise-linear spline in $x_i = \Delta p_i$ with knots at the 33rd and 67th percentiles,

$$\Delta y_i = \alpha + \beta x_i + \theta_1(x_i - \kappa_1)_+ + \theta_2(x_i - \kappa_2)_+ + X_i' \gamma + \varepsilon_i, \quad (2.11)$$

where κ_1, κ_2 are the tercile cutpoints and $(z)_+ = \max\{z, 0\}$. This basis yields a continuous fitted curve and a clear slope interpretation: the local slope is β for $x_i < \kappa_1$, $\beta + \theta_1$ for $\kappa_1 \leq x_i < \kappa_2$, and $\beta + \theta_1 + \theta_2$ for $x_i \geq \kappa_2$. In Table 2.3 column (3), I report these three segment slopes (computed from linear combinations of the basis coefficients), which correspond to the low, middle, and high segments of Δp_i . A joint Wald test of $\theta_1 = \theta_2 = 0$ tests whether a single global slope suffices. Pairwise tests allow us to compare segment slopes to identify where the slope is steepest. Heteroskedasticity-robust standard errors are used, and the covariate set matches the linear and binned models.

Because tutoring expenditures may be heavily tailed, I replicate the linear changes specification in (2.9) after winsorizing the outcome at the first and 99th percentiles. This reduces the influence of a small number of extreme changes while preserving the sample and the control set. Stability of estimates across the winsorized and non winsorized columns indicates that a few large observations are not driving the findings.

For total tutoring expenditure (Table 2.3), the pattern is consistent across specifications. In the linear model, the estimated slope is positive but modest: own spending rises as peers' mean spending increases. The tercile specification (with the middle bin as the reference group) sharpens this result: students in the high peer-change group raise their spending relative to the middle group, whereas those in the low group reduce it; both differences are statistically significant. The piecewise linear spline shows the steepest marginal effect in the central third of the peer-change distribution and flatter segments in the lower and upper thirds, indicating diminished sensitivity in the tails. Estimates are virtually unchanged after 1/99 winsorizing, so the results are not driven by a few extreme observations. Overall, the ordering is Low < Mid < High in changes to own shadow-education spending.

Table 2.3: Peer exposure and change in own tutoring spend (Y3→Y4)

	(1) Linear	(2) Bins (terciles)	(3) Linear splines	(4) Winsorized 1/99
Δ Peers' mean spending (Y4–Y3)	0.200*** (0.028)			0.180*** (0.025)
Household income	0.004*** (0.001)	0.004*** (0.001)	0.004*** (0.001)	0.003*** (0.001)
Parents' avg. education (years)	2.437*** (0.233)	2.456*** (0.232)	2.425*** (0.232)	1.991*** (0.220)
Metro area	10.385*** (1.199)	9.844*** (1.207)	9.933*** (1.217)	8.401*** (1.149)
Medium city	5.787*** (1.145)	5.390*** (1.147)	5.494*** (1.160)	4.457*** (1.120)
Male (1=yes)	2.309*** (0.879)	2.251** (0.880)	2.300*** (0.877)	1.760** (0.776)
Disabled (1=yes)	-0.398 (2.698)	-0.429 (2.697)	-0.364 (2.696)	-1.030 (2.279)
Baseline total tutoring (Y3)	-0.707*** (0.046)	-0.709*** (0.046)	-0.708*** (0.046)	-0.512*** (0.046)
Low Δ peer (vs Mid)		-4.978*** (1.019)		
High Δ peer (vs Mid)		5.153*** (1.190)		
spline(d_peer): below p33			0.134* (0.070)	
spline(d_peer): p33–p67			0.454*** (0.102)	
spline(d_peer): above p67			0.115** (0.057)	
Baseline Academic Achievement	5.153*** (0.530)	5.013*** (0.528)	4.962*** (0.533)	4.180*** (0.480)
Constant	-22.680*** (2.781)	-22.112*** (2.854)	-24.163*** (2.939)	-19.692*** (2.580)
Observations	5626	5626	5626	5626

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2.4: Peer exposure and change in own Math tutoring spend (Y3→Y4)

	(1) Linear	(2) Bins (terciles)	(3) Linear splines	(4) Winsorized 1/99
Δ Peers' mean spending (Y4–Y3)	0.104*** (0.014)			0.092*** (0.012)
Household income	0.002*** (0.000)	0.002*** (0.000)	0.002*** (0.000)	0.001*** (0.000)
Parents' avg. education (years)	1.156*** (0.118)	1.169*** (0.118)	1.150*** (0.118)	0.919*** (0.108)
Metro area	4.440*** (0.607)	4.212*** (0.613)	4.209*** (0.620)	3.568*** (0.580)
Medium city	2.392*** (0.573)	2.207*** (0.576)	2.241*** (0.582)	1.824*** (0.560)
Male (1=yes)	0.638 (0.446)	0.615 (0.447)	0.633 (0.446)	0.342 (0.402)
Disabled (1=yes)	-0.581 (1.214)	-0.607 (1.208)	-0.566 (1.210)	-0.460 (1.150)
Baseline total tutoring (Y3)				
Baseline Math tutoring (Y3)	-0.729*** (0.037)	-0.729*** (0.037)	-0.729*** (0.037)	-0.524*** (0.036)
Low Δ peer (vs Mid)		-2.518*** (0.499)		
High Δ peer (vs Mid)		2.453*** (0.605)		
spline(d_peer): below p33			0.070** (0.030)	
spline(d_peer): p33–p67			0.230*** (0.050)	
spline(d_peer): above p67			0.063** (0.029)	
Baseline Academic Achievement	2.988*** (0.259)	2.939*** (0.259)	2.895*** (0.262)	2.559*** (0.238)
Constant	-9.487*** (1.464)	-9.239*** (1.506)	-10.237*** (1.541)	-7.631*** (1.329)
Observations	5626	5626	5626	5626

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2.5: Peer exposure and change in own English tutoring spend (Y3→Y4)

	(1) Linear	(2) Bins (terciles)	(3) Linear splines	(4) Winsorized 1/99
Δ Peers' mean spending (Y4–Y3)	0.074*** (0.013)			0.062*** (0.012)
Household income	0.002*** (0.000)	0.002*** (0.000)	0.002*** (0.000)	0.001*** (0.000)
Parents' avg. education (years)	0.880*** (0.104)	0.879*** (0.103)	0.877*** (0.104)	0.742*** (0.097)
Metro area	4.321*** (0.581)	4.097*** (0.593)	4.191*** (0.596)	3.892*** (0.560)
Medium city	2.718*** (0.560)	2.574*** (0.560)	2.638*** (0.569)	2.388*** (0.544)
Male (1=yes)	1.123*** (0.423)	1.085** (0.423)	1.123*** (0.422)	1.115*** (0.377)
Disabled (1=yes)	-0.319 (1.168)	-0.302 (1.170)	-0.288 (1.169)	-0.266 (1.101)
Baseline total tutoring (Y3)				
Baseline English tutoring (Y3)	-0.722*** (0.031)	-0.724*** (0.031)	-0.722*** (0.031)	-0.580*** (0.034)
Low Δ peer (vs Mid)		-2.323*** (0.469)		
High Δ peer (vs Mid)		1.928*** (0.584)		
spline(d_peer): below p33			0.061* (0.032)	
spline(d_peer): p33–p67			0.195*** (0.048)	
spline(d_peer): above p67			0.022 (0.025)	
Baseline Academic Achievement	1.709*** (0.235)	1.598*** (0.235)	1.603*** (0.236)	1.391*** (0.224)
Constant	-8.293*** (1.294)	-7.810*** (1.317)	-8.862*** (1.366)	-7.577*** (1.203)
Observations	5626	5626	5626	5626

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2.6: Peer exposure and change in own Korean tutoring spend (Y3→Y4)

	(1) Linear	(2) Bins (terciles)	(3) Linear splines	(4) Winsorized 1/99
Δ Peers' mean spending (Y4–Y3)	0.022*** (0.008)			0.022*** (0.007)
Household income	0.001*** (0.000)	0.001*** (0.000)	0.001*** (0.000)	0.000** (0.000)
Parents' avg. education (years)	0.483*** (0.060)	0.486*** (0.060)	0.478*** (0.060)	0.414*** (0.052)
Metro area	1.913*** (0.283)	1.811*** (0.282)	1.799*** (0.296)	1.298*** (0.270)
Medium city	0.922*** (0.271)	0.849*** (0.272)	0.843*** (0.281)	0.410 (0.279)
Male (1=yes)	0.742*** (0.239)	0.740*** (0.239)	0.737*** (0.239)	0.496** (0.208)
Disabled (1=yes)	0.635 (0.800)	0.615 (0.802)	0.621 (0.803)	0.098 (0.624)
Baseline total tutoring (Y3)				
Baseline Korean tutoring (Y3)	-0.833*** (0.044)	-0.834*** (0.044)	-0.833*** (0.044)	-0.581*** (0.083)
Low Δ peer (vs Mid)		-0.206 (0.282)		
High Δ peer (vs Mid)		0.791*** (0.301)		
spline(d_peer): below p33			-0.001 (0.022)	
spline(d_peer): p33–p67			0.036 (0.028)	
spline(d_peer): above p67			0.029** (0.014)	
Baseline Academic Achievement	0.602*** (0.137)	0.609*** (0.134)	0.605*** (0.136)	0.573*** (0.117)
Constant	-5.481*** (0.757)	-5.588*** (0.787)	-5.697*** (0.793)	-4.958*** (0.646)
Observations	5626	5626	5626	5626

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Math spending (Table 2.4) follows the same pattern and is often the strongest. The linear slope is larger, the High vs. Mid contrast is more pronounced, and the spline's middle segment is steep. English (Table 2.5) also shows a positive association, typically smaller than Math but still showing increasing pattern in the terciles, with the spline again the highest in the middle segment. Korean (Table 2.6) is the weakest: the linear slope is smaller and less precise, the tercile gaps are narrower, and the spline segments are flatter. The ordering remains the same but less pronounced, which may reflect weaker responsiveness of Korean tutoring to peer shifts or more noise in that category.

These models are helpful for organizing facts, yet they are not a basis for causal claims. The central issue is selection into peer groups and schools at the high school transition. Families sort across schools, tracks, and neighborhoods in ways that correlate with unobserved determinants of shadow education demand such as aspirations, information, or access to cram schools. If those factors also move the peer mean, then changes in peer exposure are endogenous and simple OLS confounds peer effects with selection.

The binned and spline point to clear nonlinearities in the effects of peer spending. In terciles, the High-vs.-Mid contrast is noticeably larger than Low-vs.-Mid, and the ordering $\text{Low} < \text{Mid} < \text{High}$ is stable. In the spline, the slope is steep between the tercile cutpoints and flatter in the tails. Together, these patterns suggest threshold behavior rather than a single smooth gradient. In practice, the most meaningful threshold is the boundary separating the top quartile of the peer distribution from the rest. The methodology used in this paper therefore focuses on discrete transitions into higher peer groups, with special attention to moves into the top quartile and, as a secondary check, into the top half.

The next section pivots from continuous associations to a design based on discrete peer-environment transitions. Guided by the transition matrix in Figure 1, treatment is moving into a higher peer-spending group between Year 3 and Year 4 (e.g., into the top quartile or into the top half). Average treatment effects on the treated are estimated using propensity-score matching (PSM) and augmented inverse-probability weighting (AIPW).

Propensity-score matching constructs a comparison group of students who did not experience an upward peer transition but look similar on observed pre-treatment characteristics: baseline test scores, baseline shadow-education spending, demographics, parents' education and income, school and region indicators. By enforcing covariate balance and restricting attention to common support, matching reduces bias from selection on observables and limits extrapolation. The AIPW estimator combines a propensity-score model with an outcome model and is doubly robust: the estimand is consistent if either model is correctly specified. In practice this reduces model dependence and can improve precision by using the efficient influence function for weighting.

2.5 Identification Strategy

The aim is to estimate how exposure to a higher-spending peer environment at the middle-to-high-school transition is associated with changes in own spending on shadow education, and to interpret this relationship causally under explicit assumptions. Let $T_i = 1$ if student i moves into a higher peer-spending group between Year 3 and Year 4. In the main design, treatment is entry into the top quartile of the peer-spending distribution; a secondary design uses entry into the top half. Let $Y_{i,t}$ denote student i 's own tutoring expenditure in year t . The main outcome is the change in own tutoring expenditure,

$$\Delta Y_i \equiv Y_{i,4} - Y_{i,3}.$$

The primary estimand is the average treatment effect on the treated (ATT): among students who entered a higher peer-spending environment, how much more did their own tutoring spending change relative to comparable students who did not experience that upward peer transition?

2.5.1 Propensity score matching (PSM)

Recall the question: among students who moved into a higher peer-spending group between Year 3 and Year 4, how much more did their own shadow education spending change relative to comparable students who did not experience that upward peer transition? The difficulty is that treated and control students differ in observed covariates X_i , such as baseline spending, test scores,

income, parents' education, city type, and school context. In other words, selection into peer environments is not random. Propensity score matching addresses selection on observables by pairing treated students with comparison students who look similar in their pre-treatment characteristics (Imbens & Rubin, 2015; Rosenbaum & Rubin, 1983). After this balancing step, differences in outcomes are interpreted as treatment effects under two conditions: (i) conditional ignorability, meaning that $(Y_i(1), Y_i(0))$ are independent of T_i given observed covariates X_i measured in Year 3 or earlier; and (ii) overlap, meaning each student had a positive chance of treatment and non-treatment given X_i (Imbens & Rubin, 2015; Rosenbaum & Rubin, 1983). PSM does not create true randomness; it constructs treated and comparison groups that are comparable *given* X_i . The design therefore remains observational, with a causal interpretation that depends on the maintained assumptions of conditional ignorability and overlap.

Under these assumptions, the estimand is the average treatment effect on the treated (ATT),

$$ATT = E[Y_i(1) - Y_i(0) | T_i = 1] = E[Y_i(1) | T_i = 1] - E[Y_i(0) | T_i = 1], \quad (2.12)$$

where $Y_i(1)$ and $Y_i(0)$ denote potential outcomes under treatment and non-treatment. Because $Y_i(0)$ is not observed for treated students, PSM replaces the unobserved counterfactual term $E[Y_i(0) | T_i = 1]$ with the mean outcome for matched controls who share the same propensity score,

$$e(X_i) = \Pr(T_i = 1 | X_i), \quad E[Y_i(0) | T_i = 1] \approx E[Y_i | T_i = 0, e(X_i)]. \quad (2.13)$$

Intuitively, once treated and control students are balanced on $e(X_i)$, outcomes for controls with the same score proxy for the untreated outcomes of the treated.

In practice, the first step is to estimate the propensity score $e(X_i) = \Pr(T_i = 1 | X_i)$ with a flexible logit. Covariates X_i include baseline tutoring levels, subject-specific spending, scaled test scores of Year 3 as the baseline academic achievement, household income, parents' education, sex, disability, region type (metropolitan, medium city or rural), and private-school status. Only pre-treatment information enters X_i . Students are then matched on the estimated score using nearest-

neighbor matching with replacement and a caliper on the logit of the score; observations outside common support are dropped. The ATT is computed as the average over treated students of the difference between each treated outcome and the mean outcome of its matched controls, with standard errors that account for matching (e.g., Abadie & Imbens, 2006). Balance tables and standardized differences (in A.1.1) verify that matching made covariates similar across groups (Stuart, 2010). PSM reduces bias from imbalance and limits extrapolation, but it cannot remove bias from unobserved factors that influence both peer transitions and outcomes (Imbens & Rubin, 2015).

2.5.2 Augmented inverse probability weighting (AIPW)

Augmented inverse probability weighting combines a propensity score model with an outcome regression. Let $e(X_i)$ be the propensity score as above, and let $m_0(X_i) = \mathbb{E}[Y_i \mid T_i = 0, X_i]$ be the outcome model for the untreated potential outcome. For the ATT, a convenient AIPW estimating equation reweights controls by $e(X_i)/(1 - e(X_i))$ and adjusts outcomes by their model-based predictions:

$$\widehat{\text{ATT}} = \frac{1}{n_T} \sum_{i:T_i=1} \{Y_i - m_0(X_i)\} - \frac{1}{n_T} \sum_{i:T_i=0} \frac{e(X_i)}{1 - e(X_i)} \{Y_i - m_0(X_i)\},$$

where n_T is the number of treated students (Bang & Robins, 2005; Lunceford & Davidian, 2004).

AIPW is attractive for three reasons. First, it is *doubly robust*: consistency holds if either the propensity model $e(X)$ or the outcome model $m_0(X)$ is correctly specified (Bang & Robins, 2005). Second, it is typically more efficient when both models are well specified, using the efficient influence function to reduce variance (Lunceford & Davidian, 2004). Third, it is more stable when some students have extreme propensities, because the augmentation subtracts residuals from the outcome model. As with PSM, overlap still matters; diagnostics report the distribution of propensities, effective sample sizes, weight dispersion, and post-weighting balance. Standard errors are computed with a cluster bootstrap in the appendix section. Results do not differ significantly.

PSM and AIPW both target selection on observables and help ensure that treated students are compared with observationally similar control students. PSM is easy to explain and to diagnose through balance tables and overlap plots. AIPW keeps those strengths while adding double robustness and efficiency. Neither design can eliminate bias from unobserved sorting or from shocks that move many students in a school at once. For this reason, I interpret the estimates causally under conditional ignorability and overlap given observed Year 3 covariates, and I accompany the estimates with balance diagnostics, placebo tests, alternative control-group definitions, threshold checks, winsorized outcomes, value-added specifications, and bootstrap standard errors.

2.6 Results

This section reports effects on tutoring expenditure measured in units of 10,000 KRW, which is about \$12.99 PPP in 2007. Subject rows correspond to Math, English, and Korean tutoring. I begin with upward transitions into higher peer spending groups, then turn to downward transitions, gender splits, and robustness. Placebo tests, alternative control-group definitions, and additional robustness checks are discussed in the next section.

Both AIPW and PSM are shown for completeness, but the discussion focuses on AIPW. AIPW is doubly robust, meaning it remains consistent if either the outcome model or the propensity score is correctly specified. It also uses all observations rather than dropping unmatched units, which makes the estimates less sensitive to specific matching rules or caliper choices. In this sample, AIPW produces slightly tighter standard errors and more stable effects across winsorization, suggesting it better captures the underlying relationship rather than noise from a few large movements.³

2.6.1 Upward transitions: Top 25 percent and Top 50 percent

Table 2.7 shows the main upward contrast. Students who move from below the 75th percentile into the top 25 percent of peer spending increase total tutoring by a large and precisely

³Balance checks for the covariates and outcome models are reported in Appendix Table A.1. Results remain similar under alternative link functions and caliper widths.

estimated amount relative to comparable students who do not make that transition. The magnitudes are economically meaningful. An average treatment effect of six to seven units corresponds to roughly 60,000 to 70,000 KRW (\$78–\$91 PPP 2007) per student per month. Winsorized estimates are slightly smaller yet remain significant. By subject, the ordering is stable. Spending in Math increases the most, English follows, and Korean is small and not statistically significant. Re-expressing the outcome in a value-added form using Year 4 levels with Year 3 controls yields very similar totals and preserves the subject ranking (Table A.4, appendix).

Table 2.7: ATT (Delta Y4–Y3), Upward to Top 25 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	6.888*** (1.407)	6.684*** (1.587)	5.818*** (1.180)	5.885*** (1.373)
Math	3.892*** (0.751)	4.703*** (0.951)	3.027*** (0.620)	3.643*** (0.738)
English	2.983*** (0.664)	2.159*** (0.791)	2.565*** (0.575)	1.803** (0.707)
Korean	0.016 (0.361)	0.403 (0.444)	-0.053 (0.307)	0.409 (0.380)
Observations	5626	5626	5626	5626

Figure 2.2 visualizes the same Top 25 percent contrast using AIPW raw estimates: points show ATTs in units of 10,000 KRW ($\approx$ \$12.99 PPP 2007) and whiskers are 95% confidence intervals from robust standard errors. The total effect lies well above zero, spending on Math is largest, English is smaller but clearly positive, and Korean is not significant, which matches Table 2.7.

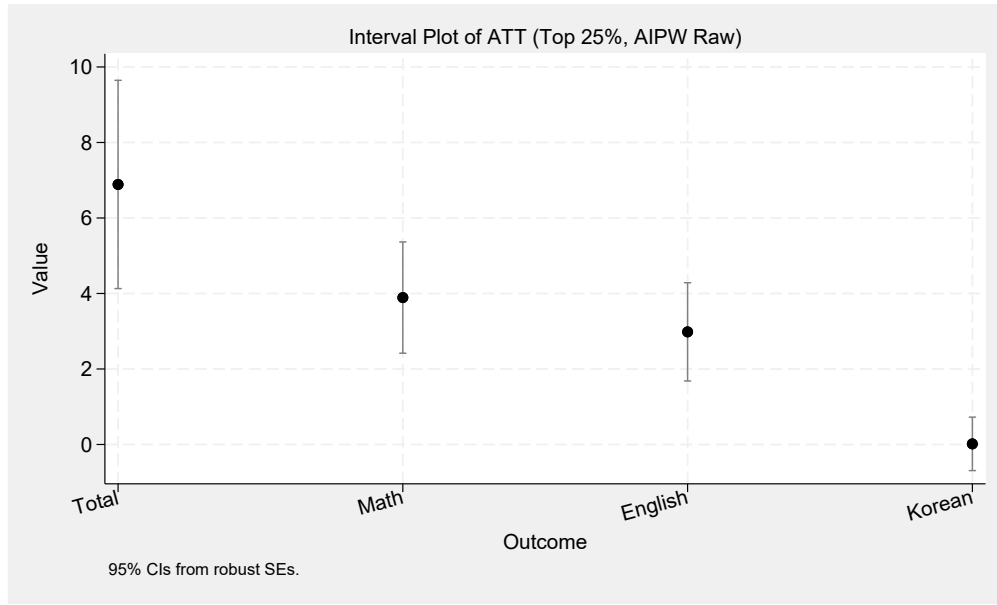


Figure 2.2: Interval Plot of ATT of Top 25 percent transition

Table 2.8 examines a smaller dose. Moving into the top half of the peer distribution produces weaker and more model-dependent results. AIPW totals are small and imprecise, while PSM totals are modestly positive. Subject-level estimates mirror this ambiguity. The contrast with Table 2.7 suggests that moderate peer intensity is not sufficient to shift spending clearly within one year.

The Top 50 percent contrast is consistent with a threshold pattern rather than a linear dose-response. Point estimates for totals are smaller than in Table 2.7, and confidence intervals for AIPW include zero, indicating limited precision at moderate peer intensity. Subject rows mirror this pattern: Math and English are positive in several specifications, but the magnitudes are roughly one third to one half of the Top 25 percent effects and are not uniformly significant. Winsorization leaves the ordering intact while trimming a few large moves, which suggests that the weaker averages are not driven solely by outliers. Taken together, these patterns indicate that entering the upper quartile produces clearer and more economically meaningful increases than entering the top half more broadly.

Table 2.8: ATT (Delta Y4–Y3), Upward to Top 50 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	1.536 (1.174)	2.951** (1.389)	1.697 (1.041)	2.682** (1.228)
Math	0.596 (0.572)	1.001 (0.700)	0.937* (0.534)	1.004 (0.674)
English	0.806 (0.587)	1.223* (0.708)	0.757 (0.485)	1.002 (0.621)
Korean	0.200 (0.299)	0.632 (0.443)	0.226 (0.267)	0.371 (0.383)
Observations	5626	5626	5626	5626

2.6.2 Upward transitions: Top 10 percent

Table 2.9 considers the highest observed intensity. The upper-decile results provide suggestive evidence of stronger responses in very high-spending peer environments, but they should be interpreted more cautiously than the Top 25 percent estimates. In the raw specification, entering the top 10 percent of peer spending is associated with the largest total increase in the study. After winsorization, however, the total effect is smaller and less precisely estimated, indicating that the upper-tail estimate is more sensitive to large spending changes. The subject-level pattern remains informative: Math rises clearly, English is positive in AIPW specifications, and Korean increases in some specifications. Overall, the Top 10 percent estimates are consistent with nonlinear peer pressure at the very top of the distribution, but the Top 25 percent transition remains the main and most stable upward estimate.

Table 2.9: ATT (Delta Y4–Y3), Upward to Top 10 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	12.747*** (3.152)	7.107** (3.048)	5.622* (3.027)	5.144** (2.597)
Math	5.882*** (1.358)	5.868*** (1.516)	3.156*** (1.044)	4.271*** (1.132)
English	4.242*** (1.262)	1.668 (1.233)	1.884* (1.027)	0.787 (1.049)
Korean	1.966*** (0.702)	2.089*** (0.683)	0.917 (0.595)	1.431*** (0.538)
Observations	5626	5626	5626	5626

2.6.3 Downward transitions: Bottom 50 percent and Bottom 25 percent

Downward transitions show large reductions in tutoring spending. Table 2.10 shows that moving to the bottom half of the peer distribution is associated with statistically significant reductions in total tutoring, with declines concentrated in Math and English.

Table 2.10: ATT (Delta Y4–Y3), Downward to Bottom 50 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	-4.962*** (1.315)	-5.483*** (2.087)	-5.116*** (1.124)	-5.587*** (1.732)
Math	-2.552*** (0.620)	-2.616*** (0.816)	-2.590*** (0.561)	-2.567*** (0.743)
English	-2.159*** (0.574)	-2.757*** (0.830)	-1.897*** (0.534)	-2.326*** (0.722)
Korean	-0.213 (0.390)	0.263 (0.462)	-0.479 (0.311)	-0.053 (0.370)
Observations	5626	5626	5626	5626

Table 2.11 shows even larger cuts when students move to the bottom quartile. Total spending falls by roughly ten to thirteen units, which is about 100,000 to 130,000 KRW ($\approx$ \$130–\$169 PPP 2007), and subject-level declines are large in Math and English with a smaller drop in Korean. These results indicate that reductions in tutoring can occur abruptly once students exit high-spending peer environments.

Table 2.11: ATT (Delta Y4–Y3), Downward to Bottom 25 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	-11.183*** (0.933)	-12.758*** (1.405)	-10.533*** (0.844)	-12.364*** (1.305)
Math	-5.175*** (0.435)	-5.276*** (0.623)	-4.932*** (0.413)	-4.978*** (0.558)
English	-4.368*** (0.413)	-4.028*** (0.472)	-4.122*** (0.392)	-3.891*** (0.456)
Korean	-1.620*** (0.259)	-1.025*** (0.328)	-1.432*** (0.214)	-0.924*** (0.264)
Observations	5626	5626	5626	5626

2.6.4 Gender splits

Tables 2.12 and 2.13 report male and female subsamples for the Top 25 percent transition. For male students, total and Math spending increase robustly, while English is sensitive to the estimator and becomes weaker after winsorization. For female students, total, Math, and English all increase, and the estimates are stable across AIPW and PSM with only modest changes after winsorization. The direction of the total response is similar for both groups, while girls show a broader and more consistent English response across specifications.

Table 2.12: ATT ($\Delta Y_4 - Y_3$), Upward to Top 25% — Male students

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	7.544*** (2.258)	7.875* (4.039)	5.826*** (1.817)	4.899* (2.658)
Math	4.118*** (1.170)	4.460*** (1.481)	3.176*** (0.893)	3.564*** (1.237)
English	3.074*** (1.014)	0.779 (1.552)	2.254*** (0.851)	-0.009 (1.256)
Korean	0.222 (0.568)	-0.725 (0.800)	-0.012 (0.480)	-0.779 (0.679)
Observations	2920	2920	2920	2920

Table 2.13: ATT ($\Delta Y_4 - Y_3$), Upward to Top 25% — Female students

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	7.881*** (1.717)	4.304* (2.470)	7.017*** (1.537)	4.122* (2.271)
Math	4.360*** (0.972)	5.185*** (1.828)	3.633*** (0.890)	3.693*** (1.314)
English	3.423*** (0.883)	3.493*** (1.240)	3.278*** (0.787)	3.095*** (1.110)
Korean	0.069 (0.479)	-0.447 (0.829)	-0.159 (0.399)	-0.062 (0.612)
Observations	2706	2706	2706	2706

2.6.5 Revisiting Peer Pressure Parameter ϕ

The toy model in Section 2.2 isolates the peer channel and yields the prediction that, holding income fixed, own spending responds to the peer reference level with slope $\alpha\phi_{-i}$ (equation (2.2)). The empirical results from the Top 25 percent transition provide a natural opportunity to translate the reduced-form magnitudes into an interpretable measure of peer pressure intensity. The aim is not to claim a fully structural estimate, but to ask what value of ϕ would rationalize the observed spending response under the minimal model.

Starting from the best-response condition

$$s_i = \alpha\phi_{-i}s_{-i} + (1 - \alpha)y_i,$$

first differences over the transition period imply

$$\Delta s_i \approx \alpha \phi_{-i} \Delta s_{-i},$$

abstracting from short-run income movements, which the placebo exercises suggest are limited. Let $\widehat{ATT}_s$ denote the AIPW estimate of the effect of entering the Top 25 percent peer environment on own spending (Table 2.7). To implement the back-of-the-envelope calculation, I also estimate the induced change in the leave-one-out peer mean spending under the same treatment definition, denoted $\widehat{ATT}_{peer}$. The model mapping then implies

$$\hat{\phi}(\alpha) = \frac{\widehat{ATT}_s}{\alpha \cdot \widehat{ATT}_{peer}}.$$

Because α is not separately identified in the reduced-form design, Table 2.14 reports $\hat{\phi}$ for a sensitivity set of values $\alpha \in [0.90, 0.97]$. Across this range, the implied peer-pressure parameter lies between 0.234 and 0.252. In the context of the toy model, this means that a one-unit increase in peer spending reduces the effective contribution of own spending by roughly one quarter of a unit. Equivalently, households would need to raise their own spending by about 24 percent of any peer increase to maintain the same effective human capital. The magnitudes therefore suggest a meaningful but not one-for-one positional component in shadow education.

Table 2.14: Peer Pressure Parameter ϕ (Top 25% Transition, AIPW)

α	ϕ
0.90	0.252395
0.92	0.246908
0.94	0.241655
0.95	0.239111
0.96	0.236620
0.97	0.234181

The reduced-form magnitudes are also economically large when expressed relative to income. The AIPW estimate for the Top 25 percent transition implies an average increase of 6.9 units of monthly tutoring expenditure, or approximately 68,900 KRW. Using the sample mean baseline income of roughly 4.34 million KRW per month, this corresponds to about 1.6 percent of monthly household income. Given that baseline tutoring spending is on the order of 5–6 percent of income, entering a high-intensity peer environment raises tutoring expenditures by nearly one quarter to one third of the existing tutoring budget within a single year. In this sense, the peer environment induces a nontrivial reallocation of household resources toward shadow education, consistent with the positional mechanism emphasized in the model.

Appendix Table A.11 provides suggestive evidence that the peer-pressure channel may be stronger in larger peer environments. Splitting students by the size of the destination grade cohort, the estimated Top 25 percent effect is larger in larger grades than in smaller grades. Total tutoring spending rises by 5.6 units of 10,000 KRW in smaller grade cohorts, compared with 7.1 units in larger grade cohorts. The difference is especially visible in Math, where the response rises from 2.8 to 4.2 units. Since the model maps the reduced-form spending response into an implied peer-pressure parameter ϕ , this pattern is consistent with a larger effective ϕ when the reference group contains more students. I interpret this as suggestive rather than structural evidence, since

the table splits the reduced-form response by grade size rather than directly estimating separate peer-spending doses for each group.

Two qualifications are important. First, the back-of-the-envelope estimate of ϕ is tied to the transition-based empirical design and should be interpreted as the implied intensity of peer pressure associated with entering the upper quartile of peer spending during this period, rather than as a universal structural constant. Second, the calculation inherits sampling variation from the peer-dose estimate. For transparency, the table reports the full sensitivity range rather than a single calibrated value, and the interpretation focuses on the economically meaningful interval implied by the data.

2.7 Discussion

2.7.1 Alternative control-group definitions

A concern with the main upward-transition design is that the comparison group may include students whose peer environment deteriorated between Year 3 and Year 4. If upward movers are compared to students who moved downward, the estimated effect could partly reflect negative peer shocks among controls rather than a positive response among treated students. To address this concern, Table 2.15 re-estimates the Top 25 percent transition using several alternative control groups. Column (1) uses the broad comparison group from the baseline specification. Column (2) restricts the sample to students who began below the top quartile in Year 3 and were therefore eligible to enter the top quartile in Year 4. Column (3) compares treated students only to students who remained in the same peer-spending quartile. Column (4) excludes downward movers from the control group.

The estimates remain positive and statistically significant across all four definitions. The total spending effect ranges from 7.5 to 9.0 units of 10,000 KRW, and the subject-level pattern is stable: the response is largest for Math, followed by English, with smaller and less consistent effects for Korean. This suggests that the main Top 25 percent result is not mechanically driven by comparing upward movers to students who experienced downward peer transitions. Instead, the spending

response remains visible even when the control group is restricted to students with more stable peer environments.

Table 2.15: Alternative Control Groups, Upward Transition to Top 25 Percent – AIPW

	(1)	(2)	(3)	(4)
	Baseline	Eligible	Stayers	No down-movers
Total	6.888*** (1.427)	8.490*** (1.706)	7.432*** (2.001)	7.163*** (1.754)
Math	3.892*** (0.762)	4.509*** (0.875)	3.942*** (1.058)	3.954*** (0.899)
English	2.983*** (0.673)	3.338*** (0.826)	2.654*** (0.991)	2.751*** (0.862)
Korean	0.016 (0.367)	0.609 (0.432)	0.838* (0.469)	0.448 (0.438)
Observations	5626	4198	2267	3295
Treated	1042	722	722	722
Controls	4584	3476	1545	2573

2.7.2 Placebo test

The placebo analysis uses changes in household income to test whether simultaneous family shocks or selective mobility can explain the main estimates. If higher spending peer environments were simply correlated with rising family resources, then upward transitions should be accompanied by positive and consistent income changes. Table 2.16 does not show this pattern. For the Top 25 percent contrast, the coefficients are small and imprecise across AIPW, PSM, and their win-sorized versions. For the Top 50 percent contrast, the AIPW estimate is negative and statistically

significant in the raw specification, and the remaining estimates are small. A negative placebo coefficient means that, relative to matched controls, households whose students moved into the Top 50 percent peer group experienced lower measured income growth over the period. This sign goes in the opposite direction of an income driven story. If anything, such a pattern would tend to dampen upward tutoring responses rather than inflate them. These placebo estimates therefore point away from the view that income growth drives the tutoring results.

Table 2.16: Placebo ATT: Parent Income Change (Y4–Y3), Upward to Top 25% and Top 50%

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Top 25%	-22.106	10.812	4.117	20.928
	(21.228)	(39.120)	(9.270)	(15.484)
Top 50%	-31.316**	-33.134	-8.393	-4.650
	(14.258)	(30.535)	(6.585)	(11.676)
Observations	5626	5626	5626	5626

Two considerations are important for interpreting the placebo results. First, income may be measured with error if the responding parent does not report the exact annual amount at the survey date. Such reporting noise would tend to attenuate placebo coefficients toward zero. Second, the placebo is designed to capture contemporaneous income shocks, not changes in how families allocate existing resources. A household could cut other types of consumption to finance tutoring even if total income remains unchanged or falls slightly. In that case, the placebo would correctly show no positive income effect, because the increase in tutoring reflects a reallocation within a fixed budget rather than a wealth effect. This distinction matters for interpretation but does not weaken the main result: the absence of positive income changes points away from rising household income as the explanation for the observed tutoring responses.

Regarding timing, Year 3 corresponds to 2007 and Year 4 corresponds to 2008. The period spans the onset of the global financial crisis. Macroeconomic conditions in that window could depress reported income growth even for some upward movers. The design compares movers to matched controls, so common macro shocks should difference out on average. A bias would require differential shocks across treatment and control groups. This context is therefore consistent with the negative Top 50 percent placebo estimate, but it does not by itself explain the positive tutoring responses documented above.

2.7.3 Further discussion of the results

The full set of estimates points to a nonlinear peer mechanism. Moving into the Top 25 percent of peer spending raises total tutoring by six to seven units of 10,000 KRW. Moving into the Top 50 percent produces weaker and more model-dependent changes. The Top 10 percent estimates provide suggestive evidence of stronger upper-tail responses, especially in the raw specification, although they are more sensitive to winsorization. Math and English are the leading components, and Korean appears more responsive at the very top of the peer-spending distribution. This pattern suggests that once peer intensity becomes very high, families may extend tutoring beyond the core test subjects and treat participation as closer to the default.

Additional origin-quartile checks, reported in Appendix Table A.10, decompose the Top 25 percent transition by students' baseline peer-spending quartile. The estimates remain positive and significant for lower-origin movers, especially when Q1 and Q2 movers are pooled, suggesting that the main result is not driven entirely by students moving from just below the top-quartile cutoff. Because the cell-specific estimates are noisier, I treat these results as supporting evidence rather than as the main basis for interpretation.

Downward transitions are stronger and more pronounced than upward transitions. Falling to the Bottom 50 percent produces clear reductions in total tutoring with declines in Math and English. Falling to the Bottom 25 percent produces very large cuts across subjects, on the order of one hundred to one hundred thirty thousand won in total. The asymmetry suggests that reductions can

occur quickly once students leave high spending environments, whereas increases require time to locate providers, adjust schedules, and build new routines.

Several mechanisms can rationalize both the threshold pattern and the downward asymmetry. Course placement and study networks often change immediately at high school entry. Students who enter less selective schools may face fewer advanced tracks, less tutoring-complementary homework, and weaker study groups, which lowers the perceived return to extra expenditure. Parents may update expectations in an asymmetric way. A negative signal at entry can trigger rapid decline in education spending, while a positive signal produces more gradual increases. Local supply may also matter. High spending neighborhoods arguably have many (and more competitive) providers, while lower spending areas do not. Exiting a thick market reduces access right away. Entering a thick market requires search and enrollment, which can delay spending growth. These mechanisms are not mutually exclusive. They generate testable predictions that can be evaluated by stratifying results by school selectivity, city type, and local tutor density, and by comparing immediate versus lagged responses.

The gender splits reinforce the core picture and connect to prior evidence on subject specific responses in shadow education. Boys show large and stable increases in Math when moving to the Top 25 percent. Their English responses are positive under AIPW and weaker under PSM. Girls show increases in Math and English that are consistent across estimators and after winsorization. One interpretation is that girls adopt language oriented study routines and shared norms more uniformly once they enter high spending peer environments, while boys vary more in English participation across local contexts and providers. The direction of effects is common to both groups. Differences lie in magnitudes and stability.

A final point concerns the size of the effects. At the Top 25 percent threshold, an increase of six to seven units corresponds to roughly sixty to seventy thousand won. At the Top 10 percent threshold, total increases reach twelve to thirteen units before winsorization. The downward Bottom 25 percent case shows cuts that are larger in absolute value than the Top 25 percent gains.

These magnitudes indicate that the peer environment can shift private educational investment by amounts that are meaningful at the household level.

2.7.4 Robustness summary

Appendix tables show that the main conclusions are not artifacts of specification or sampling variability. Bootstrap standard errors with five hundred replications closely track the robust standard errors for both the Top 25 percent and Top 50 percent contrasts (Tables A.2 and A.3). Statistical significance remains for the total estimates and for the main subject components. Value added specifications that use Year 4 levels with Year 3 controls reproduce the ordering seen in the difference form (Tables A.4 and A.5). Effects are strong at the Top 25 percent threshold, weaker or model dependent at the Top 50 percent threshold, and consistent with the first-difference results.

Two additional robustness notes are useful for interpretation. First, winsorization reduces magnitudes modestly but leaves inference unchanged for the totals and for Math and English at the main thresholds. This pattern is consistent with heavy tails rather than outliers driving the results. Second, the similarity of AIPW and PSM at the Top 25 percent threshold, together with their divergence at the Top 50 percent threshold, highlights the role of overlap. Where covariate overlap is stronger and the treatment contrast is sharper, the estimators agree more closely. Where the treatment contrast is weaker, results become more model-dependent. Effective sample sizes, weight dispersion, and standardized differences are reported in the appendix to document these patterns.

A third concern is that leave-one-out peer spending averages may be noisy in small destination school-year cells. Appendix Table A.12 addresses this by re-estimating the main Top 25 percent AIPW specification after requiring the Year 4 school-year cell to contain at least 5, 10, 15, 16, 17, 18, or 19 observed students. The estimates remain positive and statistically significant through the 18-student cutoff, with magnitudes ranging from 5.7 to 9.1 units of 10,000 KRW. This suggests that the main Top 25 percent result is not driven by very small peer groups with noisy leave-one-out averages. At the 19-student cutoff, the point estimate remains positive but becomes imprecise, reflecting the much smaller treated sample and number of destination schools.

Taken together, the placebo evidence, the pattern of estimates across thresholds, and the robustness checks support a peer-effect interpretation of the tutoring responses documented in the Results section. The consistency of findings across estimators at the main Top 25 percent threshold, the absence of supportive placebo effects, and the stability of value-added estimates strengthen the view that these changes in spending reflect behavioral adjustments to peer environments rather than coincident family income shocks or measurement noise.

2.7.5 Broader implications: peer pressure, well-being, and fertility

While these results highlight how peer pressure can amplify educational investment, they also underscore a broader social cost. The pattern of sharp increases at upper thresholds and abrupt cuts when students exit high-spending environments is consistent with a rat-race dynamic, in which relative position rather than absolute learning may drive effort. Prior literature has noted that shadow education markets in East Asia can create cycles of competitive escalation (Bray, 2014; S. Kim & Lee, 2010), and the present findings provide quantitative evidence of how those cycles operate within cohorts. When exposure to high-spending peers triggers large behavioral shifts, the equilibrium outcome may involve excessive household spending without commensurate welfare gains.

Such dynamics carry potential negative externalities beyond expenditure itself. Intense peer comparison and academic competition have been linked in prior research to higher depressive symptoms, reduced leisure, and greater test-related stress among adolescents (T. Kim et al., 2022b). These mechanisms are consistent with the pattern observed here, where tutoring increases once students enter high-intensity peer environments. In the longer run, the same forces that induce households to invest heavily in education may contribute to low fertility, as parents anticipate the high cost of raising children in competitive environments. This connection between peer-driven educational pressure and demographic behavior motivates the analysis in Tavani and Young (2026), which examines how the perceived costs of childrearing under an education rat race affect fertility decisions.

The evidence in this study therefore points in two directions. On one hand, peer environments strongly shape human capital investment, suggesting that information and social norms are powerful channels for educational behavior. On the other hand, when these forces operate through status competition, they can generate aggregate inefficiencies and social stress. Understanding how to mitigate these externalities, by reducing the achievement gap across schools or by decoupling parental status from the intensity of shadow education, remains an important policy challenge.

2.8 Conclusion

This study examined how exposure to high-spending peers influences household tutoring decisions during the transition from middle school to high school. Using detailed longitudinal data and quasi-experimental variation in peer composition generated by high-school reshuffling, the analysis found that peer environments have a large and statistically significant effect on shadow education investment under the maintained assumptions of conditional ignorability and overlap. Students who move into cohorts where peer tutoring expenditures lie in the top quartile substantially increase their own spending, while those who move into low-spending peer environments reduce it. The raw estimates are strongest at the very top of the distribution, where exposure to the top decile of peer spenders is associated with large increases across Math, English, and even Korean tutoring, although these upper-tail estimates are more sensitive to winsorization. In contrast, moderate peer changes at the top half of the distribution have small and model-dependent effects. The evidence thus points to a threshold mechanism rather than a smooth gradient: families respond strongly once peer intensity becomes visibly high.

Downward transitions show an asymmetric pattern. Leaving elite, high-spending environments produces sharper and more immediate cuts than the gains observed when entering them. This asymmetry may arise because households revise expectations more quickly when academic prospects decline or because local supply and study networks adjust more rapidly on the downside. In both directions, the results highlight that the peer environment is a powerful driver of educational behavior.

Robustness checks strengthen this conclusion. Bootstrap standard errors confirm that inference is not sensitive to resampling variability, and value-added specifications reproduce the same ordering of effects across thresholds. Placebo tests using changes in household income show no consistent positive pattern, pointing away from simultaneous family income shocks as the primary mechanism. Together, these pieces of evidence strongly support the interpretation that the observed spending responses reflect adjustment to peer environments.

The broader implication is that peer dynamics in shadow education can generate both private incentives and social inefficiencies. High-spending peers set norms that elevate educational effort but can also reinforce positional competition, creating a self-sustaining “rat race” in which households spend more to avoid falling behind. These pressures may impose negative externalities on student well-being, as suggested by prior work linking peers’ private tutoring to higher depressive symptoms, reduced leisure, and greater test-related stress among adolescents (T. Kim et al., 2022b). Peer pressure may also shape long-run demographic behavior. When parents internalize the high costs of competing in such environments, they may adjust fertility choices downward, contributing to persistently low birth rates in advanced East Asian economies (S. Kim et al., 2024).

The analysis here therefore provides both empirical and conceptual groundwork for exploring how educational competition interacts with broader social outcomes. By linking micro-level peer exposure to household spending responses, it highlights the importance of policies that address educational inequality and reduce positional pressure without weakening human capital formation.

Chapter 3

How Many Parents Does It Take? Parental Time Allocation and the Effectiveness of Fertility Subsidies

This chapter is co-authored with Marwil J. D'avila-Fern'andez.

3.1 Introduction

A recurring finding in the literature on intra-household time allocation is that the division of childcare between mothers and fathers is unequal, with mothers bearing the larger share. A common policy intuition in low-fertility settings is that greater paternal involvement can relax mothers' care constraints, lowering the effective cost of childbearing and, at least potentially, supporting higher fertility (e.g. Tamm, 2019; Thomas, Rowe, Williamson, & Lin, 2022; Persson & Rossin-Slater, 2024). This logic is intuitive and has informed family policy in a number of countries facing sustained fertility decline. Yet feminist economics and gender-demographic approaches have long emphasized that additional father-child time need not mechanically displace mothers' responsibility for supervision, planning, and default accountability (Folbre, 2001; Goldscheider et al., 2015; McDonald, 2000).

South Korea offers a striking case through which to examine this premise. Over the past several decades, the country has experienced one of the most dramatic fertility declines ever recorded, reaching a total fertility rate of 0.75 in 2024, while paternal involvement in childcare has also increased. Using comparable Korean Time Use Survey (KTUS) waves, H. Park (2021) shows that fathers' childcare time more than doubled between 1999 and 2014. Yet mothers' childcare time did not fall over the same period. It rose as well, from approximately 130 to more than 200 minutes per day. Although the KTUS underwent changes after 2014 that complicate direct comparisons with later waves, official 2019 estimates continue to show a large maternal care burden. Fathers are doing more than before, but mothers are not doing less. This pattern does not prove complemen-

tarity between parental time inputs, but it is difficult to reconcile with a strong substitution view in which additional father time mechanically displaces mother time. It sits alongside a dramatic rise in educational attainment and persistently high private education expenditure. Falling fertility, rising parental time inputs, and increasing child-quality expenditure driven in part by positional educational competition are difficult to square with models in which paternal and maternal time in childcare are treated as freely interchangeable.

The present paper develops a novel Overlapping Generations (OLG) growth model designed to account for this apparent puzzle. The central innovation is the treatment of the childcare technology. Rather than assuming that parental time inputs are freely substitutable, we model effective childcare time as a CES aggregator of maternal and paternal hours. At the household level, we are able to vary the degree of substitutability between parental inputs and trace its implications for fertility and education investment. At the macro level, we trace the consequences of those optimal allocations for the aggregate dynamics of human capital and population. The model delivers two sets of results. The first concerns the positive implications of the childcare technology. When maternal and paternal time are substitutes, greater paternal involvement eases the maternal time burden, reducing the effective cost of children.

However, when parental time inputs are complements, the opposite occurs. An increase in paternal childcare time necessarily raises maternal childcare time as well, tightening the household time constraint and pushing fertility down. Households respond by reducing the number of children and investing more heavily in each child's education, exacerbating the quantity-quality trade-off. Still, at the aggregate level, while lower fertility results in smaller populations, decreasing returns to education ensure that human capital remains the same across regimes. We argue that the elasticity of substitution between parental time inputs should not be interpreted as a pure preference parameter. It reflects the skills that mothers and fathers bring to child-rearing, which are shaped by gender norms, social expectations, and the intergenerational transmission of household competencies. A father who can perform the same tasks as a mother is closer to a substitute; one who cannot is closer to a complement.

The second set of results concerns the effectiveness of pro-natalist subsidies. We consider two fiscal arrangements. In the first, the government runs a balanced budget at all times and households internalize the implied tax burden, embedding a form of Ricardian equivalence by design. In the second, the tax rate that balances the government's budget is calculated *ex post*, after households have made their fertility and education decisions. Under this arrangement, fertility subsidies have a distortionary effect on household choices. We show that under complementarity, pro-natalist subsidies may generate an unintended anti-fertility bias. The model admits two optimal allocations of education and fertility. In one of them, the subsidy raises the cost of the tax required to finance it in a way that more than offsets the direct incentive to have children. This mechanism provides one possible explanation for why cash-based pro-natalist policies may have limited effects when they do not alter the underlying organization of care and child-investment expectations.

The paper contributes to a broad tradition of OLG models applied to demographic and macroeconomic questions. Their structure has been used to study inflation and savings dynamics (Bernasconi & Kirchkamp, 2000; Sterk & Tenreyro, 2018), optimal monetary policy rules (Crettez, Michel, & Wigniolle, 2002; Hiraguchi, 2014; von Thadden, 2012), rational asset price bubbles (Galí, 2014), the non-neutrality of monetary policy (Braun & Ikeda, 2021; Hu, Ma, Qiao, & Wallace, 2023), and environmental issues (Caravaggio & Sodini, 2023; Dávila-Fernández, Giombini, & Sánchez-Carrera, 2025; John & Pecchenino, 1994). The structure of our model follows de la Croix and Gosseries (2012), abstracting from the environmental component (see also de la Croix & Michel, 2002). Within the strand of gender OLG models that study intra-household time allocation, the closest references are perhaps Agénor and coauthors. An important innovation of our framework is that we do not impose gender-based specialisation in child-rearing. Much of the existing literature assumes that mothers bear the entirety or the majority of the time cost associated with raising children, effectively specialising in this activity within the household (Agénor, 2017). We depart from this assumption, allowing both parents to contribute to child-rearing without exogenous gender-based constraints, and show that the degree of substitutability between their time inputs is

central to understanding both fertility outcomes and the effectiveness of pro-natalist policy (see also Agénor & Agénor, 2023; Agénor, Ozdemir, & Moreira, 2021).

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature and documents the stylised facts from South Korea that motivate the analysis. Section 3 develops the baseline model and analyses the two polar cases of perfect substitutability and perfect complementarity, differentiating between two different treatments of the government budget. Section 4 extends the model to study distortionary fertility subsidies. Section 5 concludes with some final considerations and avenues for future research.

3.2 Motivation and literature review

A large literature in household economics has studied how parents allocate time within the family and whether changes in that allocation can alter mothers' constraints and fertility behaviour. In a seminal contribution, Becker (1965) models time allocation through household production and specialisation, with spouses dividing market and non-market work according to their relative productivities. The bargaining literature that emerged in the 1980s and 1990s constitutes, in part, a critique of this unitary framework. Early formulations by Manser and Brown (1980) and McElroy and Horney (1981) model household decisions as the outcome of negotiation between individuals with distinct preferences. Lundberg and Pollak (1993) and Lundberg and Pollak (1996) further emphasise the role of separate spheres, noncooperative threat points, and the institutional structure of marriage. Subsequent work has extended these foundations into collective, bargaining, and intertemporal models of household behaviour (see Chiappori & Mazzocco, 2017 for a comprehensive review).

While the present paper is informed by that tradition, our primary goal is not to treat household bargaining or the division of domestic labour in full generality. Our focus is narrower on the economics of *childcare*, more specifically, whether greater paternal involvement meaningfully reduces the effective time burden that children place on mothers. This distinction matters because childcare differs from broader domestic labour in several economically important ways. It is more

time-sensitive, less easily postponed, and more tightly bound to norms and skills related to parental responsibility. For instance, although a parent can defer housework, childcare, especially for young children, is far less deferrable. Moreover, an observed increase in fathers' time with children need not imply a comparable reduction in mothers' burden if mothers retain primary responsibility for supervision, planning, cognitive labor, and default accountability (Craig, 2006; Daminger, 2019). For that reason, evidence from the broader domestic-labor literature does not map cleanly into the question we study here. Our interest is not in whether household labor becomes more equal in some broad sense, but in whether mothers' and fathers' *care inputs* are substitutable in a way that lowers the effective cost of childbearing.

3.2.1 A closer look at the care literature

Using U.S. time-diary data, Bianchi (2000) and Sayer, Bianchi, and Robinson (2004) document that both mothers' and fathers' time with children increased over time, even as maternal labour force participation rose. Bianchi, Robinson, and Milkie (2006) reach a similar conclusion. Fathers increased their childcare time from a low base, while mothers remained the primary providers of unpaid care. Crucially, the growth in paternal involvement has not translated into one-for-one reductions in maternal unpaid labour. The historical pattern that emerges is one in which fathers do more than before, but mothers continue to bear a large share of the care burden.

Paternal participation and paternal responsibility are not the same thing. Raley, Bianchi, and Wang (2012) show that mothers are more likely to be relieved when fathers take on solo or routine care responsibilities rather than simply spending more time with children. This distinction is crucial for interpreting aggregate trends. Time-use data may show more father-child time without establishing whether fathers have displaced maternal effort in the dimensions of care that matter most for mothers' daily constraints. In that sense, the relevant margin is not simply paternal presence, but whether fathers absorb tasks that would otherwise remain mothers' responsibility.

Evidence from policy settings points in a similar direction. A number of studies have exploited paternal leave reforms as quasi-natural experiments in fathers' care involvement. For instance, Pat-

naik (2019) shows that reserving parental leave exclusively for fathers increases take-up and has persistent effects on paternal involvement in childcare. Studying Germany, Tamm (2019) finds that fathers' leave-taking raises their time in childcare and housework, providing evidence of some substitution between parental time inputs. More recently, Persson and Rossin-Slater (2024) show that fathers' workplace flexibility around childbirth improves maternal postpartum health, suggesting that paternal temporal availability generates meaningful household spillovers even when the mechanism is not directly observed in time-use data. These studies confirm that paternal involvement can matter for mothers' constraints. However, they do not support a strong substitution view in which additional father time fully displaces the maternal burden. A more careful reading is one of partial relief, conditional on the type of care fathers provide and the institutional setting in which families make decisions.

Recent work also suggests that policy design alone does not guarantee large changes in the division of care. Jørgensen and Søgaaard (2024) show, using Danish administrative data, that parents respond strongly to parental-leave incentives but also display a willingness to pay for gender-traditional leave allocations, with fathers taking little or no leave. Similarly, Rosenqvist (2024) finds that a Swedish cash bonus rewarding a more equal division of leave had only modest effects on within-couple leave differences and no average effect on later earnings gaps or childcare responsibility. At a broader level, Kleven, Landais, Posch, Steinhauer, and Zweimüller (2024) observes that the large long-run expansion of parental leave and childcare policies in Austria had virtually no impact on gender convergence in earnings. These recent contributions temper a simplistic policy optimism. Family policy can affect behavior on some margins, but deeper gender asymmetries in care are often remarkably persistent (see also Thomas et al., 2022).

The literature on bargaining and gender norms provides additional elements on why such persistence arises. Bittman, England, Sayer, Folbre, and Matheson (2003) argue that gender trumps money. Women reduce their housework as their relative earnings rise, but only up to a point, beyond which normative expectations constrain further equalisation. Bertrand, Kamenica, and Pan (2015) provide complementary evidence that gender identity norms shape household behaviour

when wives approach or exceed their husbands in earnings. More directly relevant to childcare, Ichino, Olsson, Petrongolo, and Skogman Thoursie (2019, 2026) argue that the degree of substitutability between maternal and paternal childcare time depends on prevailing gender norms. Changes in relative prices or policy incentives need not produce large reallocations of care if couples remain attached to a norm of maternal primacy. This literature is especially relevant for our framework because it suggests that the effective technology of care within the household is not purely technical in nature. It is socially mediated, and the parameter governing substitutability in our model can be read as capturing that social dimension.

Feminist economics makes a related point from a different angle. Folbre (2001) emphasises that care is not well understood if treated as a standard household commodity because it is bound up with responsibility, supervision, and social obligation in ways that market goods are not. Braunstein (2015) and Braunstein, van Staveren, and Tavani (2011) further stress that unpaid care is central to social reproduction and remains unequally allocated across gender. These contributions do not provide a direct model of parental time substitution. Rather, they motivate why observed hours may be an incomplete measure of the true care burden. Care is embedded in responsibility, coordination, and social reproduction, so the same measured hour may carry different implications depending on who remains the default accountable parent. Even as fathers' observed time with children rises, mothers may continue to bear a disproportionate burden if they remain responsible for ensuring that care is actually delivered (for OLG examples using a related argument, see Agénor, 2017; Agénor & Agénor, 2023).

3.2.2 South Korea as a case example

With these considerations in mind, we turn to the Korean case. Fig. 3.1 indicates that while fathers' childcare time more than doubled between 1999 and 2014, rising from less than 25 minutes per day to around 60 minutes, mothers' childcare time did not fall over the same period. It actually rose from approximately 130 to more than 200 minutes per day. Fathers are doing more than before, but mothers are not doing less.

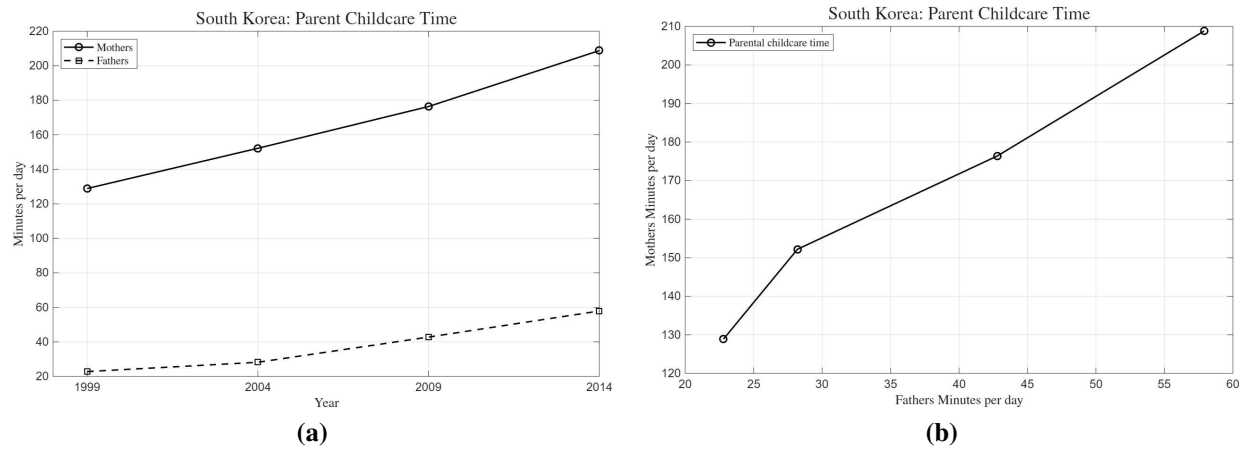


Figure 3.1: Mothers' and fathers' childcare time in South Korea. Source: H. Park (2021) using Korean Time Use Survey (KTUS) estimates for married parents with at least one child under school age.

Such trajectories are not merely an artifact of the pre-2014 sample. Official 2019 time-use summaries point in the same direction. Among parents of children under age 10, mothers devoted 3 hours and 13 minutes per day to childcare, compared with 1 hour and 2 minutes for fathers.⁴ In dual-earner households, the gap in unpaid work is similarly pronounced: wives spent more than three times as much time on unpaid work as husbands. Taken together, the evidence does not establish complementarity by itself, but it is difficult to reconcile with a strong-substitution interpretation in which additional father time mechanically displaces mother time. If fathers' time displaced mothers' time one-for-one, rising paternal involvement should reduce mothers' childcare time or at least hold total parental childcare time roughly constant. In the comparable 1999–2014 series, however, total parental childcare time rose, and the absolute gap between mothers' and fathers' childcare time widened.

The apparent time-allocation puzzle is embedded in a broader set of transformations. Over the past six decades, Korea has experienced one of the most dramatic fertility declines ever recorded. Fig. 3.2 (a) shows that the total fertility rate fell from 6 in the 1960s to below 2 in the 1980s,

⁴The *Korean Time Use Survey* revised its activity classification after 2014. The 2019 revision incorporated ICATUS 2016, refined caregiving categories, and changed the definition of “presence of others,” which is especially relevant for measuring passive or supervisory care. These changes complicate direct comparison with earlier waves. For this reason, Fig. 3.1 is limited to the comparable 1999–2014 waves used by H. Park (2021). The 2019 survey nonetheless confirms that the broader pattern remains: mothers of children under age 10 continued to devote substantially more time to childcare than fathers.

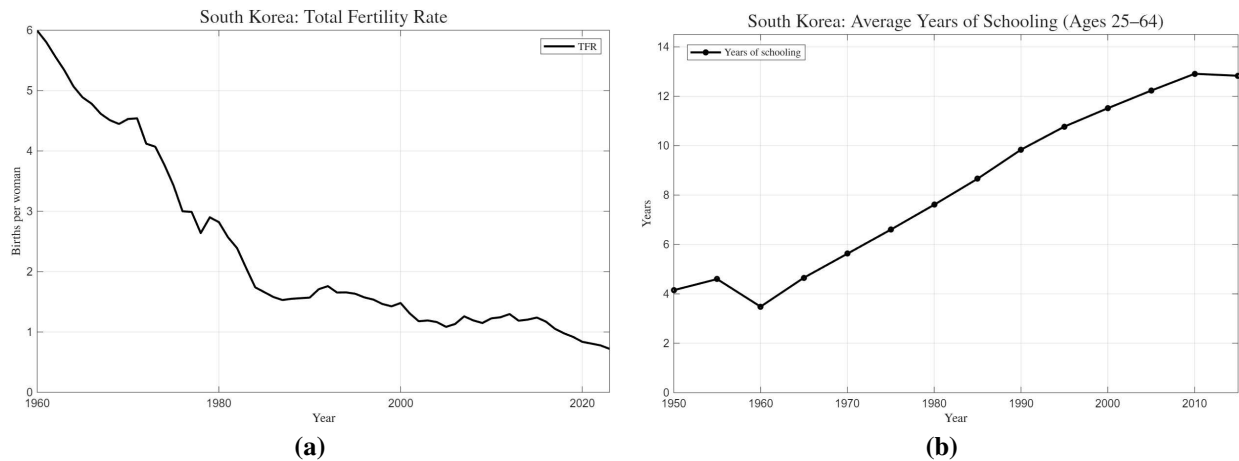


Figure 3.2: Total fertility rates and average years of schooling in South Korea. Panel (a) covers the period 1960–2024 and uses data from the World Bank annual series through 2023 and 2024 official birth statistics from Statistics Korea. Panel (b) relies on Barro-Lee Educational Attainment Data.

reaching 0.75 in 2024. Over the same period, panel (b) reports that educational attainment rose sharply, with average years of schooling jumping from 4 to 13. These two trends are connected. As schooling expanded and returns to education remained high, the standard for what it means to raise a child well shifted upward. Parents are not simply deciding whether they can afford another child. They are deciding whether they can provide enough time, attention, and investment for another child to meet increasingly demanding expectations. Such pressures are visible in current private education spending. In 2024, total private education expenditures reached 29.2 trillion won, with a participation rate of 80.0 per cent and average monthly spending of 474,000 won per student (Ministry of Data and Statistics, 2025b). In this context, private education spending should be understood not only as investment in human capital, but also as part of a positional competition in which private incentives may exceed social returns (Anderson & Kohler, 2013; S. Kim et al., 2024).

The policy response to these trends has centred on two main instruments. First, governments have promoted paternal involvement in childcare, premised on the idea that redistributing the care burden within the household will free up time for mothers and make additional children more feasible. Second, they have expanded fertility subsidies and other transfers designed to reduce the

financial cost of having children. Neither instrument, on its own, appears sufficient to address the organization of care and child-investment expectations that shape the effective cost of fertility.

South Korea's related literature

Three strands of literature speak directly to the patterns documented above. The first looks specifically at parental time allocation. H. Park (2021) documents that both mothers' and fathers' childcare time increased between 1999 and 2014 in South Korea, although mothers remained the primary caregivers throughout. His key point is not simply that fathers are doing more than before. It is that their increased involvement has not resulted in a reduction in mothers' childcare time in the aggregate trend. That is the pattern the model is trying to speak to.

The second asks whether greater support from husbands affects fertility intentions or actual births. Several papers suggest that it does. S.-M. Park et al. (2010) find that paternal investment in childcare and housework is positively associated with women's intention to have a second child, especially among employed women. Along similar lines, Yoon (2017) also finds that husbands' support for housework and childcare increases the likelihood of a second birth, but the result is not linear. In her estimates, the effect follows a threshold pattern. Only the highest level of husbands' daily support is clearly associated with second births. E. H.-W. Kim (2017) obtains a similar pattern for intended second births. Women whose husbands contribute more domestic labor are more likely to have a second child, and the association is stronger when the husband's contribution is larger.

While this literature is useful, one should approach it carefully. Those studies do not imply that any increase in fathers' time will materially reduce the cost of another child. If anything, some of the evidence suggests the opposite. Support seems to matter when it is large enough to change the household's actual division of labor, not when it moves only slightly. That is important for interpreting the aggregate Korean trend. Fathers' childcare time has risen, but the observed increase may still be too small to meaningfully reduce mothers' burden.

Moreover, the evidence is also not uniformly pro-natalist. Y. Lee (2022) shows that fathers' parental leave in South Korea can reduce intentions for additional children rather than increase

them. In her mixed-methods analysis, fathers with leave experience were less likely to report intentions for another child, and the interview evidence suggests that leave made the difficulty of childcare more visible in a setting where broader support for combining work and care remains weak. A recent meta-analysis still finds a positive association between husbands' involvement in parenting and fertility intentions, but the estimated effect is modest rather than large (E. Kim & Yi, 2024). Overall, this literature suggests a more qualified interpretation. Fathers' involvement may matter, but it is unlikely to matter in a simple mechanical way.

The third strand concerns the well-known quantity-quality tradeoff. Anderson and Kohler (2013) argue that Korea's low fertility is tied in part to the pressure parents feel to raise successful and competitive children in an environment of intense educational competition. S. Kim et al. (2024) formalize this idea through status externalities in education and show that competition in educational investment can depress fertility. This part of the literature matters here because it helps explain why time allocation inside the household may be so important. When each child is expected to require high levels of care and investment, the way mothers' and fathers' time combine becomes central to the cost of fertility.

Though the micro evidence suggests that greater support from fathers can matter for fertility, aggregate time-use evidence points to a more complicated relationship. Over time, fathers have done more childcare, yet mothers have not done less, and fertility has remained extremely low. We do not read this as evidence that fathers' involvement is irrelevant. Rather, we take it as evidence that the type of paternal involvement matters. Additional father time may reduce mothers' burden when it substitutes for maternal care in routine, solo, and responsibility-bearing tasks. But when father time is complementary to mother time, or when mothers remain the default accountable parent, greater paternal involvement may increase total parental care without substantially lowering the effective cost of childbearing. The model we develop in the next section addresses this problem by putting the substitutability of parental time at the center of the analysis. If parental time is highly substitutable, greater paternal involvement could reduce the effective time cost of another child. If parental time is complementary, or only weakly substitutable, then more father time may still leave

mothers with most of the burden. In that case, fertility can remain low even when fathers are more involved than before, especially when parents also face strong pressure to invest heavily in each child.

3.3 Substitutes or complements?

We consider an OLG closed economy where agents live for two periods: childhood and adulthood. There is an initial endowment of human capital per person (k_0) and an adult population size (N_0) living on a fixed quantity of land (L). Households face a fundamental trade-off between the quantity and quality of offspring, as raising more children reduces the time and resources available for each child. The structure of the model follows very closely de la Croix and Gosseries (2012), abstracting from the environmental component and instead bringing intra-household time allocation to the center of the stage (see also de la Croix & Michel, 2002). Adults allot their time between work and child-rearing, with parental time further disaggregated between fathers (θ_d) and mothers (θ_m). We abstract from leisure and domestic labor to focus on the parental care technology, implicitly assuming equal bargaining power within the household.⁵

As the intertemporal optimisation problem is specified at the household level, we rule out the possibility of divorce and couples are assumed to die together.⁶ Labor income depends on both hours worked and individual human capital. Investment in children's education and the quality of the parental workforce determine the accumulation of human capital across generations, and physical space and parental effort impose additional constraints on fertility. The government intervenes

⁵This assumption does not undermine the relevance of the literature on asymmetric household bargaining power (e.g. Lundberg & Pollak, 1993; 1996; for OLG models in which mothers bear the whole or most of the cost of rearing children, see Agénor, 2017; Agénor & Agénor, 2023). Rather, our aim is to maintain the most parsimonious structure possible in order to isolate the mechanism of interest. Explicitly incorporating asymmetric bargaining would reinforce, rather than overturn, the results obtained here.

⁶Marriage rates and the timing of marriage are also important for understanding fertility decline, particularly in South Korea where childbirth remains overwhelmingly concentrated within marriage. Recent work shows that Korea's fertility decline has coincided not only with delayed marriage but also with a broader retreat from marriage itself (see J. H. Lee, Park, & Chin, 2025, Raymo & Park, 2020; for broader context, Raymo et al., 2015). We do not claim that the channel studied here is the dominant one, but rather that it constitutes a meaningful mechanism linking intra-household time allocation to fertility decisions. Extending the framework to incorporate marriage formation and divorce is left for future research.

through a fertility subsidy financed by an income tax, but households retain discretion over how to allocate resources to their children.

At each period t , there is a new adult generation of size N that derives utility from consumption (c), the number of children (n), and their quality, which is captured by their future human capital (k). That is:

$$U = \ln c_t + \ln n_t k_{t+1}$$

where, for simplicity, we assume consumption and being parents provide the same utility, though there is a trade-off between the number and quality of the offspring.

The household budget constraint states that their consumption plus education spending per child (e) must be equal to their net income:

$$c_t + n_t e_t = (1 - \tau) y_t + B(n_t) \quad (3.1)$$

where $B(\cdot)$ is an increasing function in n that stands for the fertility subsidy, and $\tau > 0$ is the tax rate that funds the latter. The government has no direct control over how households will spend the subsidy.

We consider two alternative treatments of the government budget. In both of them, the budget is balanced at all times so that fertility subsidies are fully funded by an income tax. However, in the first case, the household fully internalizes Eq. (3.2), while in the second case, it does not. We begin with the treatment in which this constraint is fully incorporated into the household optimization problem. This assumption simplifies the analysis, allowing us to show in a more transparent way the mechanism driving our main results on parental time being substitutes or complements. Thus, we have:

$$B(n_t) = \tau y_t \quad (3.2)$$

so that the fertility subsidy is fully funded by income taxes. In the second treatment, which we discuss in Section 4, the government adopts an ex post approach: it chooses the tax rate required to balance the budget after households have made their allocation decisions. This setup allows us

to isolate more clearly the potentially distorting effects of fertility subsidies on the quantity and quality of children.

Households also face a time constraint. They can allocate their time either to working or non-working activities. In the context of our model, given that we abstract from leisure, adults either work or parent:

$$h_t + \theta_t = 1 \quad (3.3)$$

where h is the number of hours worked and θ stands for the number of hours dad and mom put together to raise the offspring, so that:

$$\theta_t = \theta_{d,t} + \theta_{m,t} \quad (3.4)$$

An important distinction of our model relative to previous studies on parental time allocation (e.g. Agénor et al., 2021; Heintz & Folbre, 2022; Agénor & Agénor, 2023) is that we do not impose gender-based specialization in child-rearing. In much of the literature, mothers are assumed to bear the entirety or the majority of the time cost associated with raising children, effectively specializing in this activity within the household (Agénor, 2017). By contrast, we allow both parents to contribute to child-rearing without exogenous gender-based constraints. Such a simplification is not intended to deny the existence or importance of gender differences in parental time use, but rather to highlight that the main mechanism driving our results operates independently of any assumed gender discrimination.

We assume the adult population is self-employed, so output equals their income immediately. The productivity of each hour of work is determined by the worker's quality:

$$y_t = h_t k_t \quad (3.5)$$

From which it follows that there are two possible channels through which income can increase. Either at the extensive margin through longer working hours or by increasing the quality of labour. Still, the latter depends on decisions made by the previous generation.

Empirical evidence suggests that higher-educated parents have a stronger preference for children's skills or perceive greater returns to investing in those skills (Cunha, Elo, & Culhane, 2022). Thus, we assume future human capital is driven by education spending and the present quality of the workforce:

$$k_{t+1} = Ae_t^\alpha k_t^\beta \quad (3.6)$$

where $A > 0$ is a technological constant, and $0 < \alpha + \beta < 1$ so that we have decreasing returns. While the assumption rules out endogenous growth in the current setup, the framework is sufficiently general to be extended to incorporate it.

Producing offspring requires time-effort, and space:

$$n_t = \frac{\Theta_t}{k_t} \left(\frac{L}{\phi N_t} \right)^\gamma$$

where Θ corresponds to the *effective* time units necessary to raise a child, L/N stands for land or physical space per adult, $\phi > 0$ is a scaling parameter, and $0 < \gamma < 1$ captures its decreasing returns. The term Θ/k captures the increasing opportunity cost of having children for young adult cohorts with higher human capital. To simplify the algebraic steps, let us normalise L , so that the expression above can be rewritten as:

$$k_t n_t = \frac{\Theta_t}{\phi^\gamma N_t^\gamma} \quad (3.7)$$

The variable Θ consists of a key innovation of our model because it allows for the distinction between actual parental hours and effective time dedicated to raising a child. There is an important distinction between inputs into child development that require direct parental involvement, including activities such as reading, sports, or cultural outings, and external childcare services in

which children are cared for independently of their parents (see Caucutt, Lochner, Mullins, & Park, 2026). We focus on the former, acknowledging that while parents may allocate a certain number of hours to childcare, these hours do not translate one-to-one into effective input. Effectiveness depends on factors such as parental coordination, their skills and effort. By explicitly modeling Θ as the effective time units necessary to raise a child, we are able to capture how variations in parental contributions and the complementarity or substitutability of their time affect the quantity-quality trade-off in fertility decisions. Thus, suppose the following CES function:

$$\Theta_t = (\theta_{d,t}^\rho + \theta_{m,t}^\rho)^{\frac{1}{\rho}} \quad (3.8)$$

where ρ is the substitution parameter. If $\rho = 0$, the elasticity of substitution between the time dad and mom dedicate to their child is equal to one. When $\rho = 1$, they are perfect substitutes. As $\rho \rightarrow -\infty$, parents' time becomes complementary to each other.

Note that ρ should not be interpreted purely as a preference parameter over the combination of parental time inputs in childcare. It also embodies social and institutional norms, and the set of skills that mothers and fathers bring to child-rearing tasks. High substitutability reflects a situation in which both parents are capable of performing the same household tasks interchangeably. As the elasticity of substitution falls, parents become increasingly specialised. This is not reducible to preferences, as it depends on the intra-household cultural transmission of skills across generations. To give a practical example, if a father can cook but cannot drive, and a mother can drive but cannot cook, the two become highly complementary. If both parents can cook and drive, they are substitutes.

Substituting Eq. (3.8) into (3.7) and rearranging, we can write the time allocated by the mother as a function of the number of children and the time the father spends with them:

$$\theta_{m,t} = (k_t^\rho n_t^\rho \phi^{\rho\gamma} N_t^{\rho\gamma} - \theta_{d,t}^\rho)^{\frac{1}{\rho}} \quad (3.9)$$

We choose to write θ_d on the right-hand side of the expression to explicitly capture the idea that mothers adjust their time allocation in response to the hours fathers dedicate to childcare. This is how societal norms enter the problem and are reflected in the technology. Notice that for $\rho > 0$, higher fertility requires mothers to dedicate more time to non-working activities, but this effect is balanced by the time fathers allocate to raising their offspring. Thus, there is a substitution between parental labour in child-rearing. However, for $\rho < 0$, an increase in θ_d necessary increases θ_m . Such a difference will serve as the transmission channel driving our main results.

By substituting Eqs. (3.9) into (3.4), and the resulting expression into (3.3), we can rewrite the time constraint in terms of the number of hours worked as:

$$h_t = 1 - \theta_{d,t} - \left(k_t^\rho n_t^\rho \phi^{\rho\gamma} N_t^{\rho\gamma} - \theta_{d,t}^\rho \right)^{\frac{1}{\rho}} \quad (3.10)$$

Having children reduces the time parents can devote to work, but the effect is larger the lower the substitutability between mothers and fathers. If we substitute Eq. (3.10) into our production function in (3.5), the choice of h is translated into production:

$$y_t = \left[1 - \theta_{d,t} - \left(k_t^\rho n_t^\rho \phi^{\rho\gamma} N_t^{\rho\gamma} - \theta_{d,t}^\rho \right)^{\frac{1}{\rho}} \right] k_t \quad (3.11)$$

Making use of Eq. (3.11) and the government budget constraint in (3.2), we can rewrite the household budget constraint (3.1) in terms of consumption as:

$$c_t = \left[1 - \theta_{d,t} - \left(k_t^\rho n_t^\rho \phi^{\rho\gamma} N_t^{\rho\gamma} - \theta_{d,t}^\rho \right)^{\frac{1}{\rho}} \right] k_t - n_t e_t \quad (3.12)$$

Thus, the household intertemporal problem consists in:

$$\begin{aligned} & \max_{c_t, n_t, e_t} \ln c_t + \ln n_t k_{t+1} \\ & \text{s.t.} \\ & c_t = \left[1 - \theta_{d,t} - \left(k_t^\rho n_t^\rho \phi^{\rho\gamma} N_t^{\rho\gamma} - \theta_{d,t}^\rho \right)^{\frac{1}{\rho}} \right] k_t - n_t e_t \end{aligned} \quad (3.13)$$

where agents choose their consumption, the quantity and quality of their offspring. Substituting the constraint into the objective function in (3.13), and making use of Eq. (3.6), we can rewrite the lifetime choice as:

$$\begin{aligned} \max_{n_t, e_t} \ln \left\{ \left[1 - \theta_{d,t} - \left(k_t^\rho n_t^\rho \phi^\rho N_t^{\rho\gamma} - \theta_{d,t}^\rho \right)^{\frac{1}{\rho}} \right] k_t - n_t e_t \right\} \\ + \ln n_t + \ln A + \alpha \ln e_t + \beta \ln k_t \end{aligned} \quad (3.14)$$

so that we end up with an unconstrained optimisation problem in which the choice variables are strictly related to the quantity–quality trade-off.

Along the dynamic equilibrium path, the First Order Conditions (FOC) are:

$$\begin{aligned} \frac{- \left[1 - \left(\frac{\theta_{d,t}}{k_t n_t \phi N_t^\gamma} \right)^\rho \right]^{\frac{1-\rho}{\rho}} k_t - e_t}{\left[1 - \theta_{d,t} - \left(k_t^\rho n_t^\rho \phi^\rho N_t^{\rho\gamma} - \theta_{d,t}^\rho \right)^{\frac{1}{\rho}} \right] k_t - n_t e_t} + \frac{1}{n_t} = 0 \\ - \frac{n_t}{\left[1 - \theta_{d,t} - \left(k_t^\rho n_t^\rho \phi^\rho N_t^{\rho\gamma} - \theta_{d,t}^\rho \right)^{\frac{1}{\rho}} \right] k_t - n_t e_t} + \frac{\alpha}{e_t} = 0 \end{aligned} \quad (3.15)$$

The 2-dimensional nonlinear dynamic system is defined by Eq. (3.6) and the evolution of the adult population, both of which depend on the households' optimal choices.

$$k_{t+1} = A e_t^{*\alpha} k_t^\beta \quad (3.16)$$

$$N_{t+1} = n_t^* N_t$$

where e^* and n^* describe the optimal allocations satisfying (3.15). Given the strong nonlinearities in the first-order conditions, our strategy will be to analyze the model both analytically and through numerical simulations by focusing on two extreme cases: first, when parental time is perfectly substitutable, and second, when it is fully complementary. For each adult generation, the initial

human capital and population size are treated as exogenous. Conditional on these values, we can determine households' optimal choices for fertility and educational investment. As each generation passes, these decisions feed back into the aggregate human capital and population size, shaping the dynamic evolution of the economy over time. We will discuss both micro and macro dimensions for each case. Fig. 3.3 shows the main elements of the model.

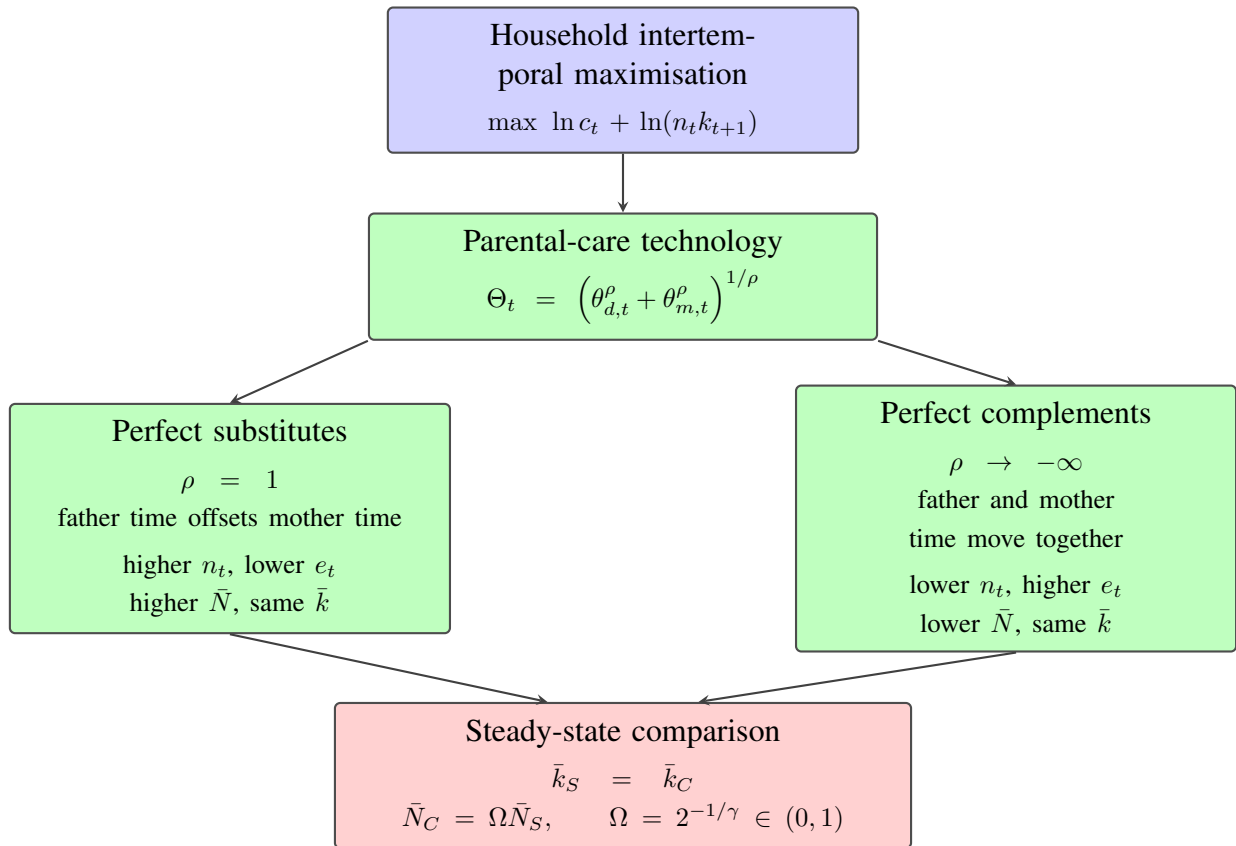


Figure 3.3: Main elements of the model. The parental-care technology determines whether maternal and paternal time are perfect substitutes or perfect complements, which in turn shapes fertility, education, and the steady-state comparison.

3.3.1 Perfect substitution ($\rho = 1$)

When there is perfect substitution between the time of dad and mom, the FOC become:

$$\begin{aligned} \frac{-k_t^2 \phi N_t^\gamma - e_t}{(1 - \theta_{d,t} - n_t k_t \phi N_t^\gamma + \theta_{d,t}) k_t - n_t e_t} + \frac{1}{n_t} &= 0 \\ \frac{-n_t}{(1 - \theta_{d,t} - n_t k_t \phi N_t^\gamma + \theta_{d,t}) k_t - n_t e_t} + \frac{\alpha}{e_t} &= 0 \end{aligned} \quad (3.17)$$

Provided that $k_t > 0$, the optimal fertility and education choices are:

$$\begin{aligned} n_t^* &= \frac{1 - \alpha}{2k_t \phi N_t^\gamma} \\ e_t^* &= \left(\frac{\alpha}{1 - \alpha} \right) k_t^2 \phi N_t^\gamma \end{aligned}$$

Both expressions are intuitive and consistent with standard economic reasoning. When a household faces a macroeconomic environment characterized by a high aggregate human capital stock and a relatively large population with respect to the fixed quantity of land, both the opportunity and space costs of raising children increase. As a result, the optimal fertility rate declines. At the same time, these higher costs create a stronger incentive for parents to invest in the quality of each child, leading to an increase in optimal education investment.

Substituting the optimal choices into (3.16), the dynamic system is given by:

$$k_{t+1} = A \left[\left(\frac{\alpha}{1 - \alpha} \right) k_t^2 \phi N_t^\gamma \right]^\alpha k_t^\beta \quad (3.18)$$

$$N_{t+1} = \left(\frac{1 - \alpha}{2\phi k_t} \right) N_t^{1-\gamma}$$

In steady-state, $k_{t+1} = k_t = \bar{k}$ and $N_{t+1} = N_t = \bar{N}$. This results in the following equilibrium conditions:

$$\begin{aligned}\bar{k} &= A \left[\left(\frac{\alpha}{1-\alpha} \right) \bar{k}^2 \phi \bar{N}^\gamma \right]^\alpha \bar{k}^\beta \\ \bar{N} &= \left(\frac{1-\alpha}{2\phi\bar{k}} \right) \bar{N}^{1-\gamma}\end{aligned}\tag{3.19}$$

Thus, we can state and prove the following Proposition regarding the existence of a unique non-trivial equilibrium solution.

Proposition 1. *When the time allocated by father and mother to parenting are perfect substitutes, the dynamic system (3.18) has a unique economically meaningful equilibrium point $P_S = (\bar{k}_S, \bar{N}_S)$ defined and given by:*

$$\begin{aligned}\bar{k}_S &= \left(A^{\frac{1}{\alpha}} \frac{\alpha}{2} \right)^{\frac{\alpha}{1-\beta-\alpha}} \\ \bar{N}_S &= A^{\frac{-1}{(1-\beta-\alpha)\gamma}} \underbrace{\left(\frac{1-\alpha}{2\phi} \right)^{\frac{1-\beta-2\alpha}{(1-\beta-\alpha)\gamma}}}_{\text{Fertility}} \underbrace{\left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{(1-\beta-\alpha)\gamma}}}_{\text{Education}}\end{aligned}$$

Proof. See Appendix A.1. □

Notice that it is possible to disaggregate the equilibrium population size into its fertility and education components. Both are linked to the returns from education spending on human capital for the future generation (α). Regarding the local stability of the steady-state solution, we are now ready to prove the following Proposition.

Proposition 2. *The unique non-trivial equilibrium $P_S = (\bar{k}_S, \bar{N}_S)$ of the dynamic system (3.18) is locally stable provided that:*

$$1 - \frac{\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N}}{\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A}} - \frac{1}{A\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta} < -\frac{\partial n^*}{\partial \bar{N}} \bar{N} < 1 - \frac{A\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N}}{1 + A\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta} \quad (3.20)$$

$$A\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta > 1$$

where

$$\begin{aligned} \frac{\partial n^*}{\partial \bar{k}} &= -\frac{1-\alpha}{2\bar{k}^2 \bar{N}^\gamma}, & \frac{\partial n^*}{\partial \bar{N}} &= -\frac{\gamma(1-\alpha)}{2\bar{k} \bar{N}^{\gamma+1}}, \\ \frac{\partial e^*}{\partial \bar{k}} &= \frac{2\alpha}{1-\alpha} \bar{k} \bar{N}^\gamma, & \frac{\partial e^*}{\partial \bar{N}} &= \frac{\alpha\gamma}{1-\alpha} \bar{k}^2 \bar{N}^{\gamma-1} \end{aligned}$$

Proof. See Appendix A.2. □

The Proposition above identifies two conditions that must hold jointly for the equilibrium to be locally stable, each with a clear economic interpretation. The first concerns the responsiveness of fertility decisions to population size, a relationship mediated by land availability. If households are unresponsive to changes in available space, they do not sufficiently adjust their fertility decisions as land becomes scarce, and the system loses its self-correcting properties. But the opposite extreme is equally destabilising. Households that are too sensitive to land scarcity may sharply curtail fertility at the first sign of congestion, pushing the economy toward demographic collapse. Therefore, stability requires that this elasticity falls within a corridor bounded by a floor and a ceiling. Too little responsiveness and the population dynamic diverges from above; too much and it collapses from below.

The second condition concerns the intergenerational transmission of human capital. For the equilibrium to be stable, optimal education investment by the current generation must be sufficiently responsive to the human capital stock inherited from the previous one. If parental education

has no meaningful effect on how much the current generation invests in its children's schooling, the intergenerational bridge sustaining the education dynamic is broken, and the system again becomes unstable. The two conditions reflect a deeper feature of the model. Fertility and education decisions are made by young adults who respond to state variables, namely population size and the aggregate human capital stock, that were determined by the choices of the generation before them. Stability requires that both margins of adjustment, fertility and education investment, be calibrated so that this inter-generational feedback loop neither explodes nor collapses.

3.3.2 Perfect complements ($\rho = -\infty$)

Perfect substitutability and perfect complementarity represent the two boundaries of the substitution spectrum. Having analysed the first, we turn to the second. When the time of dad and mom are complements, the FOCs become:

$$\begin{aligned} \frac{-2k_t^2\phi N_t^\gamma - e_t}{(1 - 2n_t k_t \phi N_t^\gamma) k_t - n_t e_t} + \frac{1}{n_t} &= 0 \\ \frac{-n_t}{(1 - 2n_t k_t \phi N_t^\gamma) k_t - n_t e_t} + \frac{\alpha}{e_t} &= 0 \end{aligned} \tag{3.21}$$

If $k_t > 0$, the optimal fertility and education choices are:

$$\begin{aligned} n_t^* &= \frac{1 - \alpha}{4k_t\phi N_t^\gamma} \\ e_t^* &= 2 \left(\frac{\alpha}{1 - \alpha} \right) k_t^2 \phi N_t^\gamma \end{aligned}$$

The optimal allocations under perfect complementarity differ strikingly from those obtained under perfect substitutability. Fertility is half as large, and education investment is twice as high. The intuition follows directly from the structure of the effective parental time function (Θ). Under perfect complementarity, fathers and mothers must contribute jointly to childcare. An increase in paternal involvement necessarily requires a corresponding increase in maternal time at home. This

crowds out time that would otherwise be devoted to market work, putting downward pressure on household income. Faced with this tighter constraint, households optimally respond by reducing the number of children. The resulting loss in utility from lower fertility is then offset by investing more heavily in the quality of each child.

Substituting once more the optimal allocations into (3.16), the dynamic system is given by:

$$k_{t+1} = A \left[2 \left(\frac{\alpha}{1-\alpha} \right) k_t^2 \phi N_t^\gamma \right]^\alpha k_t^\beta \quad (3.22)$$

$$N_{t+1} = \left(\frac{1-\alpha}{4\phi k_t} \right) N_t^{1-\gamma}$$

In steady-state, $k_{t+1} = k_t = \bar{k}$ and $N_{t+1} = N_t = \bar{N}$. This results in the following equilibrium conditions:

$$\bar{k} = A \left[2 \left(\frac{\alpha}{1-\alpha} \right) \bar{k}^2 \phi \bar{N}^\gamma \right]^\alpha \bar{k}^\beta \quad (3.23)$$

$$\bar{N} = \left(\frac{1-\alpha}{4\phi \bar{k}} \right) \bar{N}^{1-\gamma}$$

We proceed by stating and proving the following Proposition regarding the existence of a unique non-trivial equilibrium solution.

Proposition 3. *When the time allocated by father and mother to parenting are perfect complements, the dynamic system (3.22) has a unique economically meaningful equilibrium point $P_C = (\bar{k}_C, \bar{N}_C)$ defined and given by:*

$$\bar{k}_C = \bar{k}_S \quad \bar{N}_C = \Omega \bar{N}_S$$

where

$$\Omega = \frac{1}{2^{1/\gamma}} \in (0, 1)$$

Proof. See Appendix A.3. □

Perfect complementarity in parenting time produces, at the aggregate level, the same level of human capital as before, but a lower $\bar{N}$. The decline in population is intuitive, since fertility is lower in this case. The human capital result, however, is less obvious. It follows from our assumption of decreasing returns to education spending. Additional schooling expenditures raise human capital, but at a diminishing rate.

Proposition 4. *The unique non-trivial equilibrium solution $P_C = (\bar{k}_C, \bar{N}_C)$ of the dynamic system (3.22) is locally stable provided that (3.20) is satisfied, and we further have:*

$$\begin{aligned} \frac{\partial n^*}{\partial \bar{k}} &= -\frac{1 - \alpha}{4\bar{k}^2 \bar{N}^\gamma}, & \frac{\partial n^*}{\partial \bar{N}} &= -\frac{\gamma(1 - \alpha)}{4\bar{k} \bar{N}^{\gamma+1}}, \\ \frac{\partial e^*}{\partial \bar{k}} &= \frac{4\alpha}{1 - \alpha} \bar{k} \bar{N}^\gamma, & \frac{\partial e^*}{\partial \bar{N}} &= \frac{2\alpha\gamma}{1 - \alpha} \bar{k}^2 \bar{N}^{\gamma-1} \end{aligned}$$

Proof. See Appendix A.4. □

The economic interpretation of the stability conditions mirrors that of the perfect substitutes case in broad structure but differs quantitatively. As before, stability requires that the elasticity of optimal fertility with respect to population size falls within a bounded corridor. We also need the education investment to respond sufficiently to the aggregate human capital stock inherited from the previous generation. The novelty lies in how tight those requirements are. Under perfect complementarity, the admissible corridor for the fertility elasticity is half as wide as in the substitutes case. Both the floor and the ceiling are halved. At the same time, the minimum degree of responsiveness required from education investment is twice as large. Complementarity, by binding

maternal and paternal time together, amplifies the consequences of any misalignment in household decisions. When parents cannot substitute for one another, the system has less room to absorb deviations from the equilibrium path, and stability demands more precisely calibrated responses on both margins.

3.3.3 A numerical comparison

To provide a more concrete view of the model's properties, we complement the analytical results with a set of numerical experiments. Parameter values are loosely informed by de la Croix and Gosseries (2012) and chosen to yield equilibrium values broadly consistent with South Korea's demographic reality. These two sources serve only as reference points to discipline the calibration strategy; the parameter values used in the simulations are not structural estimates and do not necessarily coincide with either source. Throughout the analysis, k can be interpreted as years of schooling, and N as the population size in millions.

Fig. 3.4 summarises the main findings in the (k, N) space. The degree of substitutability between parental time inputs shapes educational investment within a generation but leaves aggregate human capital broadly unchanged at the macro level. This result follows from the presence of decreasing returns to skill accumulation, which absorb differences in per-child investment across time. Population size, however, tells a different story. Perfect substitutability between parental time inputs supports higher fertility, which compounds across generations into a substantially larger equilibrium population. Complementarity, by contrast, depresses fertility and leads to a smaller steady-state population.

In Fig. 3.5, we compare the perfect substitution and perfect complementarity cases in terms of how mothers and fathers allocate their time and the implications for steady-state output. Under substitutability, panel (a) shows that as fathers devote more time to childcare, mothers are able to reduce theirs correspondingly, leaving total parental time unchanged, as indicated by the continuous black line. Panel (b) confirms that this reallocation has no effect on output, which remains stable across all combinations of θ_d and θ_m . The picture changes markedly under complementar-

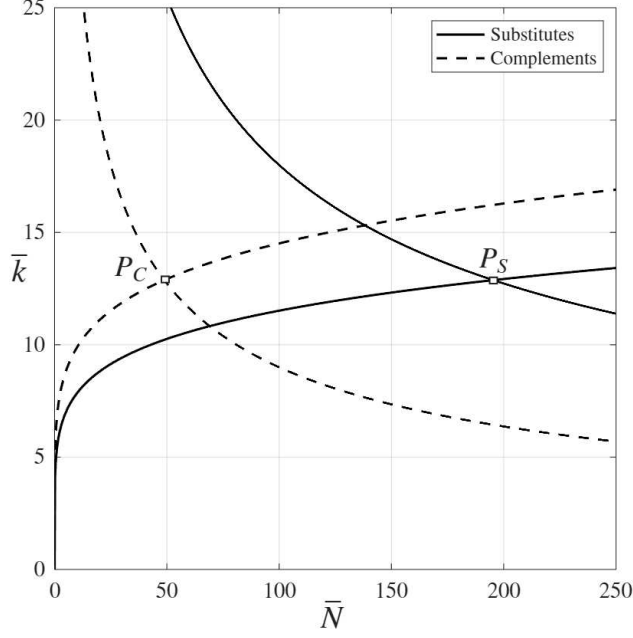


Figure 3.4: Comparing equilibria when dad and mom’s time allocated to parenting are perfect substitutes vs complements. Parameters $A = 3.75$, $\alpha = 0.1$, $\beta = 0.5$, $\gamma = 0.5$, $\phi = 0.0025$.

ity. Two insights emerge. First, there exists a minimum level of θ_d below which the model breaks down. Below that, households prefer not to have children, and the economy eventually collapses. Beyond this threshold, panel (a) shows that any increase in fathers’ childcare time necessarily requires mothers to increase theirs as well, marked by the dotted black line. This crowds out time from market work, reducing output as shown in panel (b), and exacerbates the quantity-quality trade-off. Faced with tighter time and income constraints, households respond by investing more heavily in each child’s education while further reducing fertility.

We proceed by reporting the sensitivity of the equilibrium to changes in the return to education (α) in Fig. 3.6. Under perfect substitutability, panel (a) shows that raising α from 0.1 to 0.15 generates a modest increase in equilibrium schooling of around two years, but is accompanied by a sharp contraction in population size. Under perfect complementarity, panel (b) shows a human capital response of similar magnitude, but the population adjusts by comparatively little. This asymmetry reflects a fundamental difference in the flexibility of the two regimes. When parental time inputs are substitutable, the household can more freely reallocate time across margins, allowing the sys-

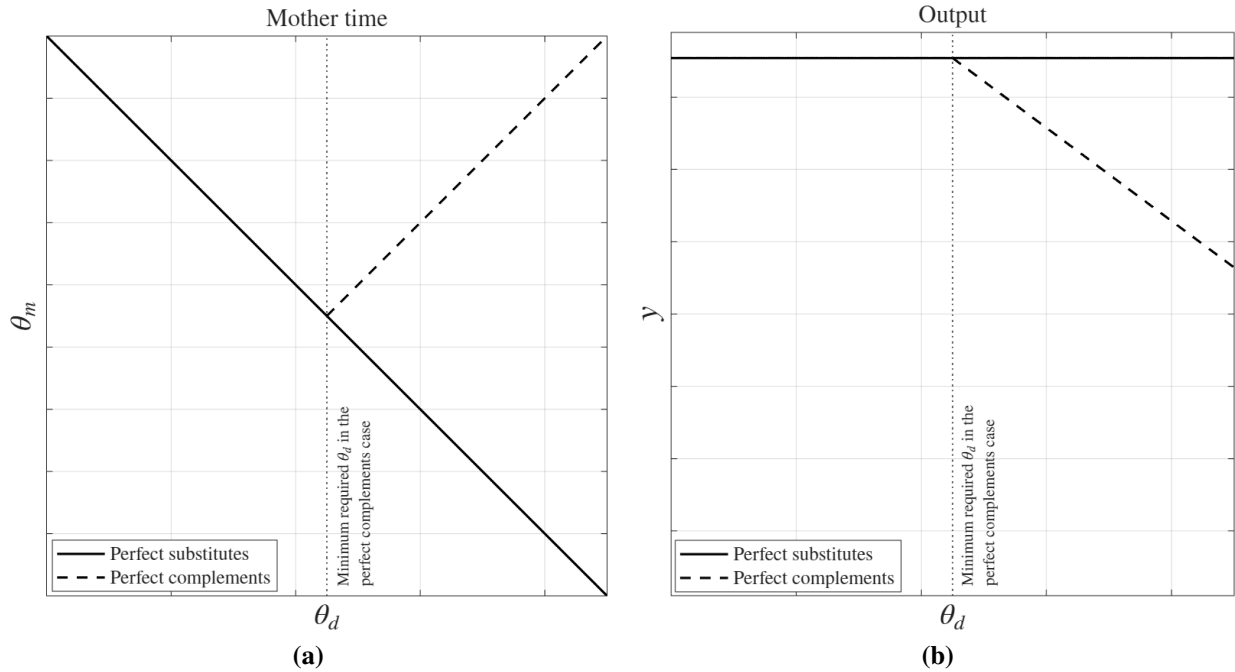


Figure 3.5: A comparison of intra-household time allocation and the corresponding output implications.

tem to respond more strongly to changes in structural parameters. Complementarity constrains that reallocation.

3.4 Distortionary taxation and fertility subsidies effectiveness

The results derived so far rest on a modeling choice that, while analytically convenient, limits the scope for fiscal policy to play any independent role. By assuming that households fully internalize the implied tax burden, the model embeds a form of Ricardian equivalence by design. Fertility subsidies are offset by anticipated taxes, leaving allocation decisions unchanged. We now look at the more realistic case where the representative household does not take the government's balanced budget into account. Households now respond to the subsidy as a transfer without fully considering its cost, creating scope for fertility subsidies to distort the quantity and quality margins of childbearing. Whether those distortions work in the intended direction, raising fertility, or against it, is the question this section addresses.

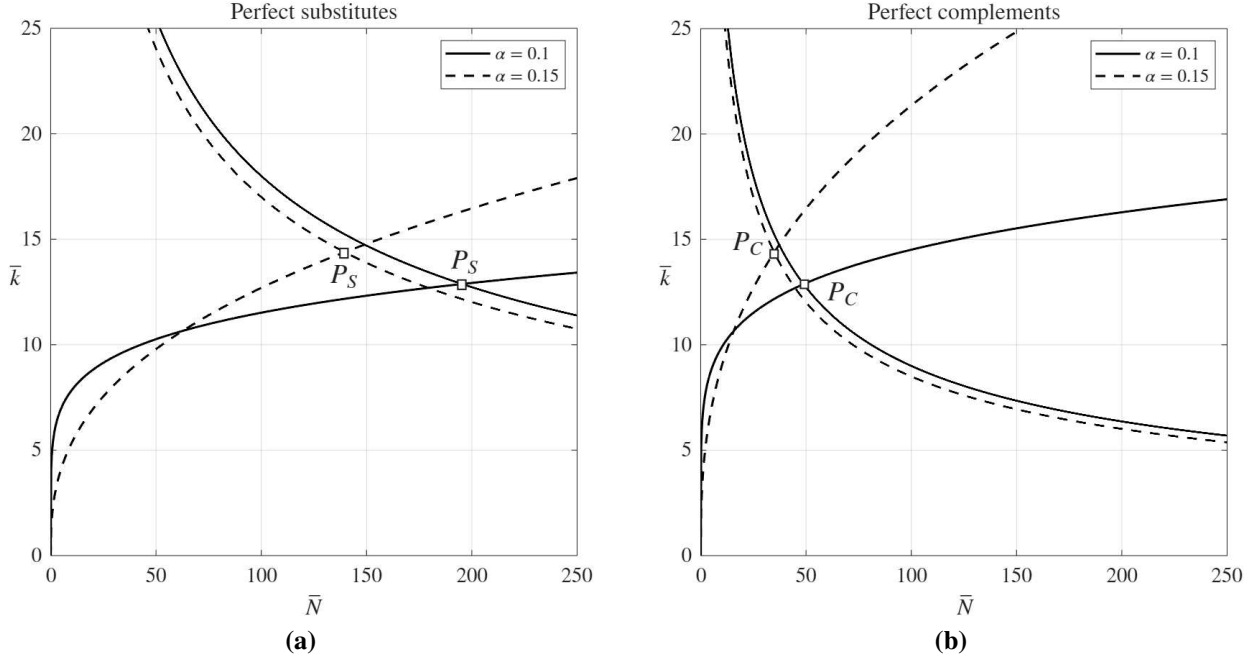


Figure 3.6: Comparing equilibria for different education returns. Parameters $A = 3.75$, $\beta = 0.5$, $\gamma = 0.5$, $\phi = 0.0025$.

From the household perspective, the tax rate τ and the subsidy schedule $B(\cdot)$ are taken as exogenous. Thus, the household budget constraint becomes:

$$c_t + n_t e_t = (1 - \tau)y_t + B(n_t), \quad (3.24)$$

where $B'(n) > 0$ and $\tau \in (0, 1)$. Following our previous strategy, we will focus on the two polar cases for parental time allocation.

3.4.1 Perfect substitution ($\rho = 1$)

When parenting time is perfectly substitutable, from Eq. (3.10), the offspring production or fertility condition implies working hours are:

$$h_t = 1 - k_t n_t \phi N_t^\gamma.$$

which in turn results in an output:

$$\begin{aligned} y_t &= h_t k_t \\ &= k_t - k_t^2 \phi N_t^\gamma n_t. \end{aligned} \quad (3.25)$$

Define $X_t \equiv k_t^2 \phi N_t^\gamma$ as an auxiliary variable, such that $y_t = k_t - X_t n_t$ and $\partial y_t / \partial n_t = -X_t$.

Using Eq. (3.24), consumption can be written as:

$$c_t = (1 - \tau)(k_t - X_t n_t) + B(n_t) - n_t e_t.$$

Therefore, the first-order conditions are:

$$-\frac{n_t}{c_t} + \frac{\alpha}{e_t} = 0 \quad (3.26)$$

$$\frac{1}{c_t} [-(1 - \tau)X_t + B'(n_t) - e_t] + \frac{1}{n_t} = 0$$

Combining both expressions in (3.26) and making use of the budget constraint (3.24), the optimal fertility choice and education spending are:

$$n_t^* = \frac{(1 - \alpha)[(1 - \tau)k_t + B(n_t^*)]}{2(1 - \tau)X_t - (1 + \alpha)B'(n_t^*)} \quad (3.27)$$

$$e_t^* = \frac{\alpha}{1 - \alpha} [(1 - \tau)X_t - B'(n_t^*)]$$

The two expressions above reveal the intended consequences of the fertility subsidy. A higher $B(\cdot)$ and B' leads households to increase the number of children. Given the trade-off between the quantity and quality of offspring, this is accompanied by a reduction in resources allocated to education. As shown in Section 3, however, a lower e^* does not necessarily imply a smaller human

capital stock in the steady state, which at first sight makes a case in favour of the subsidy. The critical step is how the government balances its budget.

Endogenising the tax rate

To simplify the algebra and without loss of generality, suppose the subsidy takes a linear form:

$$B(n_t) = bn_t, \quad 0 < b < 1,$$

where b corresponds to the fraction of the cost of each child covered by the government.

Along the equilibrium path, the government budget constraint (3.2) implies:

$$bn_t = \tau_t y_t$$

so that the government will choose the tax rate that balances the budget after fertility and production decisions have been made. Making use of Eq. (3.25), the endogenous tax rate becomes:

$$\tau_t = \frac{bn_t}{k_t - X_t n_t}, \quad (3.28)$$

where recall $X_t = k_t^2 \phi N_t^\gamma$.

Substituting Eq. (3.28) into the fertility optimal condition (3.27) eliminates τ_t and delivers a quadratic condition in n_t :

$$2X_t^2 n_t^2 + k_t [(\alpha - 3)X_t + (1 + \alpha)b] n_t + (1 - \alpha)k_t^2 = 0. \quad (3.29)$$

The economically meaningful solution satisfies $n_t \geq 0$ and $k_t - X_t n_t > 0$.

Dynamic system with endogenous taxation

Equation (3.29) yields two real roots. Denoting them by $n_{1,t}^*$ and $n_{2,t}^*$, we have:

$$n_{i,t}^* = \frac{k_t [(3 - \alpha)X_t - (1 + \alpha)b \pm \sqrt{\Delta_t^s}]}{4X_t^2}, \quad i = 1, 2$$

where

$$\Delta_t^s = [(3 - \alpha)X_t - (1 + \alpha)b]^2 - 8(1 - \alpha)X_t^2.$$

Hence, there are up to two economically meaningful candidate fertility choices, provided that $\Delta_t^s \geq 0$, $n_{i,t}^* \geq 0$, and $k_t - X_t n_{i,t}^* > 0$.

Substituting the endogenous tax rate (3.28) into the second expression in (3.27), the education allocation becomes:

$$e_{i,t}^* = \frac{\alpha}{1 - \alpha} \left(\frac{k_t(X_t - b) - X_t^2 n_{i,t}^*}{k_t - X_t n_{i,t}^*} \right).$$

Thus, our two-dimensional dynamic system with distorting taxes becomes:

$$k_{t+1} = A \left[\frac{\alpha}{1 - \alpha} \left(\frac{k_t(X_t - b) - X_t^2 n_{i,t}^*}{k_t - X_t n_{i,t}^*} \right) \right]^\alpha k_t^\beta, \quad (3.30)$$

$$N_{t+1} = n_{i,t}^* N_t.$$

where $n_{i,t}^*(k_t, N_t)$ denotes the feasible root of (3.29). We can state and prove the following Proposition regarding the existence of a non-trivial equilibrium solution. Unlike the benchmark case in Section 3, distortionary taxation implies there might be more than one optimal allocation (n^*, e^*) , with steady-state implications for our dynamic system.

Proposition 5. *When paternal and maternal parenting time are perfect substitutes, and the prenatal subsidy is linear, $B(n_t) = bn_t$, there exists a threshold value $\bar{b} > 0$ for which:*

1. *when $0 < b < \bar{b}$, two optimal allocations, (n_{S1}^*, e_{S1}^*) and (n_{S2}^*, e_{S2}^*) , satisfy the household intertemporal maximisation problem;*
2. *when $b = \bar{b}$, (n_{S1}^*, e_{S1}^*) and (n_{S2}^*, e_{S2}^*) collapse into one double root;*
3. *when $b > \bar{b}$, there is no economically meaningful optimal allocation.*

For each pair (n_{S1}^*, e_{S1}^*) and (n_{S2}^*, e_{S2}^*) , the dynamic system (3.30) admits a unique economically meaningful non-trivial equilibrium solution $P_{S1} = (\bar{k}_{S1}, \bar{N}_{S1})$ and $P_{S2} = (\bar{k}_{S2}, \bar{N}_{S2})$.

Proof. See Appendix A.5. □

The proposition establishes the existence of equilibrium, but there is an important subtlety in how it is obtained. For small values of the subsidy (b), the household FOCs admit two distinct solutions for fertility and education investment. When these are substituted into the dynamic system, they give rise to two equilibrium points. This multiplicity does not reflect a property of the dynamic system itself, but rather the fact that two different optimal household plans are consistent with the same set of structural conditions. As b increases, the two solutions approach one another and eventually collapse into a single allocation. Beyond that point, no interior solution to the household problem exists, and the dynamic system admits no equilibrium. Such a result provides an early indication of the risks associated with high subsidy rates, and suggests that intermediate values of b may also give rise to complications. Both cases are examined in more detail in our numerical experiments later on.

Proposition 6. *For each optimal allocation (n_{Si}^*, e_{Si}^*) , $i = 1, 2$, the corresponding unique economically meaningful non-trivial equilibrium solution $P_{Si} = (\bar{k}_{Si}, \bar{N}_{Si})$ of the distorted dynamic system (3.30) is locally stable provided that:*

$$1 - \frac{\alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial N} \bar{k}^\beta \frac{\partial n_i^*}{\partial k} \bar{N}}{\alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial k} \bar{k}^\beta + \frac{\beta}{A}} - \frac{1}{A \alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial k} \bar{k}^\beta + \beta} < -\frac{\partial n_i^*}{\partial \bar{N}} \bar{N} < 1 - \frac{A \alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial N} \bar{k}^\beta \frac{\partial n_i^*}{\partial k} \bar{N}}{1 + A \alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial k} \bar{k}^\beta + \beta} \quad (3.31)$$

$$\alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial k} \bar{k}^\beta + \frac{\beta}{A} > 1$$

where

$$\begin{aligned}\frac{\partial n_i^*}{\partial \bar{k}} &= -\frac{F_{S,k}}{F_{S,n}}, & \frac{\partial n_i^*}{\partial \bar{N}} &= -\frac{F_{S,N}}{F_{S,n}}, \\ \frac{\partial e_i^*}{\partial \bar{k}} &= \frac{\alpha}{1-\alpha} G_{S,k}, & \frac{\partial e_i^*}{\partial \bar{N}} &= \frac{\alpha}{1-\alpha} G_{S,N},\end{aligned}$$

with

$$\begin{aligned}F_{S,n} &= 4X^2 n_i^* + \bar{k} [(\alpha - 3)X + (1 + \alpha)b], \\ F_{S,k} &= \frac{8X^2 n_i^{*2}}{\bar{k}} + [3(\alpha - 3)X + (1 + \alpha)b] n_i^* + 2(1 - \alpha)\bar{k}, \\ F_{S,N} &= \frac{\gamma X n_i^*}{\bar{N}} [4X n_i^* + (\alpha - 3)\bar{k}],\end{aligned}$$

and

$$G_S(\bar{k}, \bar{N}) = \frac{\bar{k}(X - b) - X^2 n_i^*}{\bar{k} - X n_i^*}, \quad X = \bar{k}^2 \phi \bar{N}^\gamma,$$

where

$$\begin{aligned}G_{S,k} &= \frac{\left[3X - b - \frac{4X^2 n_i^*}{\bar{k}} - X^2 \frac{\partial n_i^*}{\partial \bar{k}}\right] (\bar{k} - X n_i^*) - [\bar{k}(X - b) - X^2 n_i^*] \left[1 - \frac{2X n_i^*}{\bar{k}} - X \frac{\partial n_i^*}{\partial \bar{k}}\right]}{(\bar{k} - X n_i^*)^2}, \\ G_{S,N} &= \frac{\left[\frac{\gamma X}{\bar{N}} (\bar{k} - 2X n_i^*) - X^2 \frac{\partial n_i^*}{\partial \bar{N}}\right] (\bar{k} - X n_i^*) + [\bar{k}(X - b) - X^2 n_i^*] \left[\frac{\gamma X n_i^*}{\bar{N}} + X \frac{\partial n_i^*}{\partial \bar{N}}\right]}{(\bar{k} - X n_i^*)^2}.\end{aligned}$$

Proof. See Appendix A.6. □

Despite its apparent algebraic complexity, the local stability of the fixed point has an analogous interpretation to Proposition 4. We continue to have a corridor of stability for the fertility elasticity with respect to the population size, and education needs to respond to the size of aggregate human capital.

3.4.2 Perfect complements ($\rho = -\infty$)

Turning to the case when parenting time is complementary, the time constraint implies:

$$h_t = 1 - 2k_t n_t \phi N_t^\gamma.$$

so that output becomes:

$$\begin{aligned} y_t &= h_t k_t \\ &= k_t - 2k_t^2 \phi N_t^\gamma n_t. \end{aligned} \tag{3.32}$$

Making use of the auxiliary variable $X_t \equiv k_t^2 \phi N_t^\gamma$, we have $y_t = k_t - 2X_t n_t$ and $\partial y_t / \partial n_t = -2X_t$. Using Eq. (3.24), consumption is therefore:

$$c_t = (1 - \tau)(k_t - 2X_t n_t) + B(n_t) - n_t e_t.$$

The FOCs of the household intertemporal optimisation problem become:

$$-\frac{n_t}{c_t} + \frac{\alpha}{e_t} = 0 \tag{3.33}$$

$$\frac{1}{c_t} [-2(1 - \tau)X_t + B'(n_t) - e_t] + \frac{1}{n_t} = 0$$

Combining both expressions in (3.33) and making use of the budget constraint (3.24), the optimal fertility choice and education spending are:

$$n_t^* = \frac{(1 - \alpha)[(1 - \tau)k_t + B(n_t^*)]}{4(1 - \tau)X_t - (1 + \alpha)B'(n_t^*)} \tag{3.34}$$

$$e_t^* = \frac{\alpha}{1 - \alpha} [2(1 - \tau)X_t - B'(n_t^*)]$$

The first-order conditions under perfect complementarity deliver expressions that are structurally similar to those obtained in the substitutes case. The key difference lies in the implied allocations. Optimal fertility is half that of the previous case, while education investment is higher, though by less than a factor of two.

Endogenising the tax rate

Recalling the government budget constraint $bn_t = \tau_t y_t$ and that $y_t = k_t - 2X_t n_t$, we can solve for the tax rate:

$$\tau_t = \frac{bn_t}{k_t - 2X_t n_t}. \quad (3.35)$$

Substituting Eq. (3.35) into the fertility optimal condition (3.34) yields:

$$8X_t^2 n_t^2 + k_t [2(\alpha - 3)X_t + (1 + \alpha)b]n_t + (1 - \alpha)k_t^2 = 0. \quad (3.36)$$

and the economically meaningful solution satisfies $n_t \geq 0$ and $k_t - X_t n_t > 0$.

Dynamic system with endogenous taxation

Equation (3.36) also yields two real roots. Denoting them by $n_{1,t}^*$ and $n_{2,t}^*$, we have:

$$n_{i,t}^* = \frac{k_t [2(3 - \alpha)X_t - (1 + \alpha)b \pm \sqrt{\Delta_t^c}]}{16X_t^2}, \quad i = 1, 2$$

where

$$\Delta_t^c = [2(3 - \alpha)X_t - (1 + \alpha)b]^2 - 32(1 - \alpha)X_t^2.$$

Thus, there are up to two economically meaningful fertility choices, provided that $\Delta_t^c \geq 0$, $n_{i,t}^* \geq 0$, and $k_t - 2X_t n_{i,t}^* > 0$.

Let $n_{i,t}^*(k_t, N_t)$ denote the feasible root of (3.36). Substituting Eq. (3.28) into the education investment optimal condition (3.34), it follows:

$$e_{i,t}^* = \frac{\alpha}{1 - \alpha} \left(\frac{k_t(2X_t - b) - (2X_t)^2 n_{i,t}^*}{k_t - 2X_t n_{i,t}^*} \right).$$

Thus, our two-dimensional dynamic system with distorting taxes becomes:

$$k_{t+1} = A \left[\frac{\alpha}{1-\alpha} \left(\frac{k_t(2X_t - b) - (2X_t)^2 n_{i,t}^*}{k_t - 2X_t n_{i,t}^*} \right) \right]^\alpha k_t^\beta \quad (3.37)$$

$$N_{t+1} = n_{i,t}^* N_t.$$

We proceed by stating and proving the following Proposition regarding the existence of a non-trivial equilibrium solution.

Proposition 7. *When paternal and maternal parenting time are perfect complements, and the pro-natal subsidy is linear, $B(n_t) = bn_t$, there exists a threshold value $\bar{b} > 0$ for which:*

1. *when $0 < b < \bar{b}$, two optimal allocations, (n_{C1}^*, e_{C1}^*) and (n_{C2}^*, e_{C2}^*) , satisfy the household intertemporal maximisation problem;*
2. *when $b = \bar{b}$, (n_{C1}^*, e_{C1}^*) and (n_{C2}^*, e_{C2}^*) collapse into one double root;*
3. *when $b > \bar{b}$, there is no economically meaningful optimal allocation.*

For each pair (n_{C1}^*, e_{C1}^*) and (n_{C2}^*, e_{C2}^*) , the dynamic system (3.37) admits a unique economically meaningful non-trivial equilibrium solution $P_{C1} = (\bar{k}_{C1}, \bar{N}_{C1})$ and $P_{C2} = (\bar{k}_{C2}, \bar{N}_{C2})$, such that:

$$\bar{k}_{Ci} = \bar{k}_{Si} \quad \bar{N}_{Ci} = \Omega \bar{N}_{Si}$$

where

$$\Omega = \frac{1}{2^{1/\gamma}} \in (0, 1)$$

$$i = 1, 2$$

Proof. See Appendix A.7. □

We are able to confirm our previous result that, in equilibrium, the steady-state level of human capital does not depend on the degree of substitutability between parental time inputs. Although education investment is lower under perfect substitutability, decreasing returns to skill accumulation equalise human capital across the two regimes in the long run. The same does not hold for population size. Complementarity in parental time leads to a smaller steady-state population through a mechanism mediated by land availability. In terms of the local stability properties of the equilibrium point, we state and prove the following Proposition.

Proposition 8. *For each optimal allocation (n_{Ci}^*, e_{Ci}^*) , $i = 1, 2$, the corresponding unique economically meaningful non-trivial equilibrium solution $P_{Ci} = (\bar{k}_{Ci}, \bar{N}_{Ci})$ of the distorted dynamic system (3.37) is locally stable provided that (3.31) is satisfied, and we further have:*

$$\begin{aligned} \frac{\partial n_i^*}{\partial \bar{k}} &= -\frac{F_{C,k}}{F_{C,n}}, & \frac{\partial n_i^*}{\partial \bar{N}} &= -\frac{F_{C,N}}{F_{C,n}}, \\ \frac{\partial e_i^*}{\partial \bar{k}} &= \frac{\alpha}{1-\alpha} G_{C,k}, & \frac{\partial e_i^*}{\partial \bar{N}} &= \frac{\alpha}{1-\alpha} G_{C,N}, \end{aligned}$$

with

$$\begin{aligned} F_{C,n} &= 16X^2 n_i^* + \bar{k} [2(\alpha - 3)X + (1 + \alpha)b], \\ F_{C,k} &= \frac{32X^2 n_i^{*2}}{\bar{k}} + [6(\alpha - 3)X + (1 + \alpha)b] n_i^* + 2(1 - \alpha)\bar{k}, \\ F_{C,N} &= \frac{2\gamma X n_i^*}{\bar{N}} [8X n_i^* + (\alpha - 3)\bar{k}], \end{aligned}$$

and

$$G_C(\bar{k}, \bar{N}) = \frac{\bar{k}(2X - b) - 4X^2 n_i^*}{\bar{k} - 2X n_i^*}, \quad X = \bar{k}^2 \phi \bar{N}^\gamma,$$

where

$$G_{C,k} = \frac{\left[6X - b - \frac{16X^2 n_i^*}{k} - 4X^2 \frac{\partial n_i^*}{\partial k}\right] (\bar{k} - 2Xn_i^*) - [\bar{k}(2X - b) - 4X^2 n_i^*] \left[1 - \frac{4Xn_i^*}{k} - 2X \frac{\partial n_i^*}{\partial k}\right]}{(\bar{k} - 2Xn_i^*)^2},$$

$$G_{C,N} = \frac{\left[\frac{2\gamma X}{N} (\bar{k} - 4Xn_i^*) - 4X^2 \frac{\partial n_i^*}{\partial N}\right] (\bar{k} - 2Xn_i^*) + [\bar{k}(2X - b) - 4X^2 n_i^*] \left[\frac{2\gamma X n_i^*}{N} + 2X \frac{\partial n_i^*}{\partial N}\right]}{(\bar{k} - 2Xn_i^*)^2}.$$

Proof. See Appendix A.8. □

3.4.3 Numerical illustrations

Similar to what we did with our baseline model, we provide a numerical example to compare the two main cases studied here. We use de la Croix and Gosseries (2012) and South Korean data as a loose reference for the exercise, which should be interpreted only qualitatively for illustrative purposes. Fig. 3.7 confirms that the FOCs are U-shaped in n so that there are two optimal possible allocations: one with low (n_1^*) and the other with high (n_2^*) fertility. Notice that across cases, perfect substitution delivers significantly higher fertility rates, and often n_{S1}^* is higher than n_{C2}^* .

The quadratic relationship results from two antagonising forces that arise precisely because households do not internalise the government budget constraint in their intertemporal optimisation. On one hand, a higher tax rate τ , required to balance the public budget ex post, reduces household disposable income, leading families to adjust their fertility downward. On the other hand, a higher τ finances a more generous subsidy per child, making additional children less costly and pushing fertility upward. The net effect depends on which force dominates, and, in turn, that depends on where the economy lies along the fertility distribution. When households are at the low-fertility optimal allocation, the marginal utility of an additional child is relatively high, and the subsidy effect prevails, successfully raising fertility. However, when households are at the high-fertility allocation, the marginal utility of another child is low, and the income effect of the higher tax dominates. In this case, it is optimal for households to reduce the number of their children in

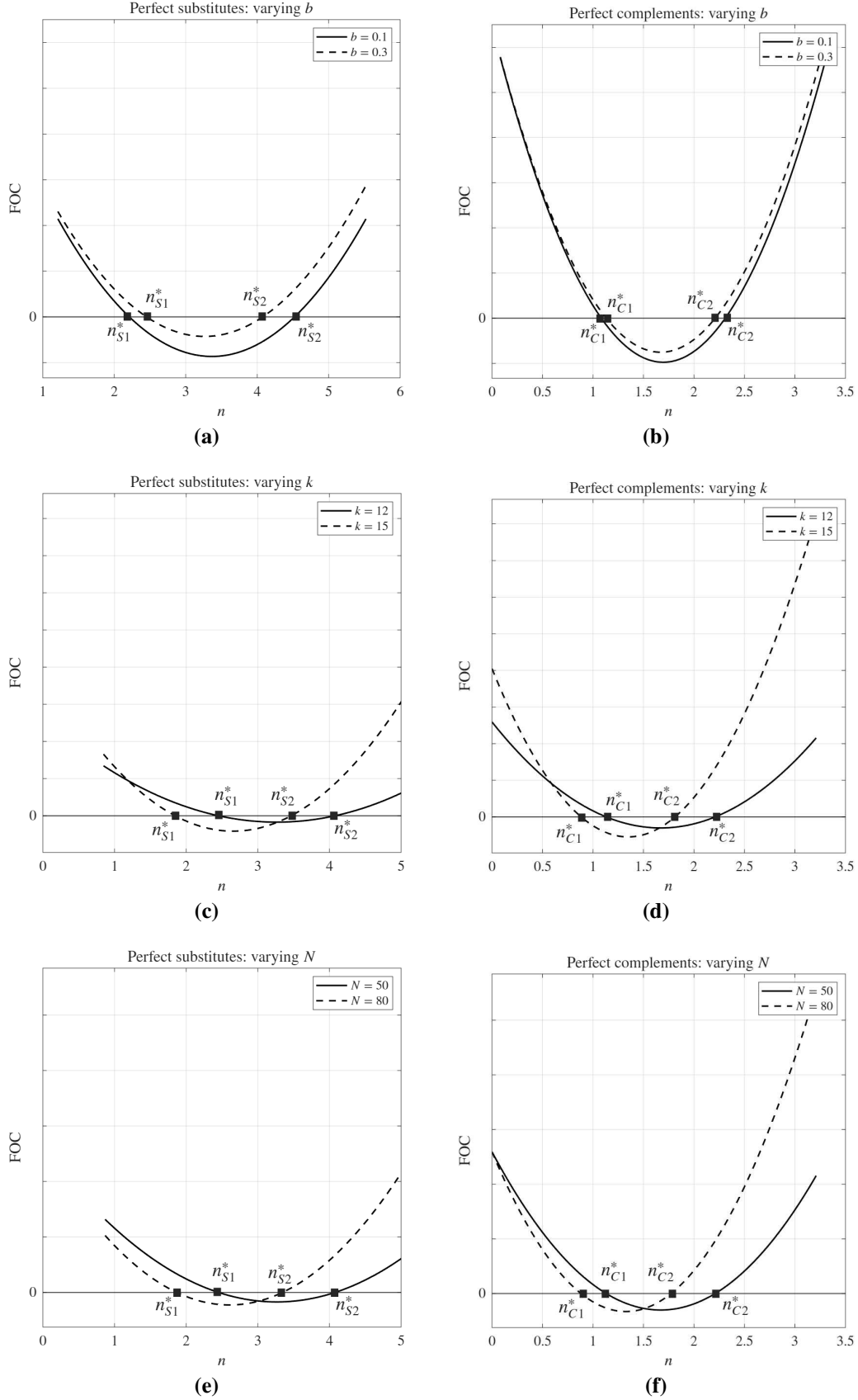


Figure 3.7: Comparing equilibria for different education returns. Parameters $A = 3.75$, $\beta = 0.5$, $\gamma = 0.5$, $\phi = 0.0025$.

response to the subsidy. The quadratic structure of the first-order condition is thus a direct reflection of this tension between the fiscal cost and the direct incentive embedded in the pro-natalist policy.

We show in panels (a) and (b) how optimal fertility allocations respond to an increase in the pro-natalist subsidy. The direction of the effect is not uniform. Whether the policy raises or reduces fertility depends on whether households find themselves at n_1^* or n_2^* . The subsidy achieves its intended purpose only when the economy operates at the lowest point. Under perfect complementarity, panel (b) shows that households are nearly unresponsive to the policy. In fact, fertility barely moves despite a substantial increase in b from 0.1 to 0.3.

Panels (c) and (d) report the response of optimal fertility to a change in aggregate human capital, which is equivalent to tracing the optimal reaction to an exogenous increase in the human capital endowment of the incoming adult generation. In both regimes, the effect is a strong reduction in fertility. The mechanism is straightforward. Higher human capital raises the opportunity cost of time spent outside the labour market, so that a higher k leads households to optimally choose fewer children. Panels (e) and (f) document a similar negative relationship between population size and fertility, though the underlying channel is distinct. A larger population reduces the land available to support additional children, tightening the space constraint faced by households. This shifts the first-order condition parabola to the left, implying lower optimal fertility at any given level of human capital.

We conclude this exercise by reporting the time series of human capital and population under perfect substitutability and perfect complementarity. Fig. 3.8 illustrates the quantity-quality trade-off in children. Continuous lines correspond to the case in which households invest more heavily in education, accepting lower fertility as a result. Dotted lines represent the opposite case, with substantially higher population but considerably lower human capital. The comparison of panels (a) and (b) brings us back to the central message of the paper. Complementarity in intra-household childcare allocation necessarily leads to a smaller population, with no corresponding impact on aggregate human capital.

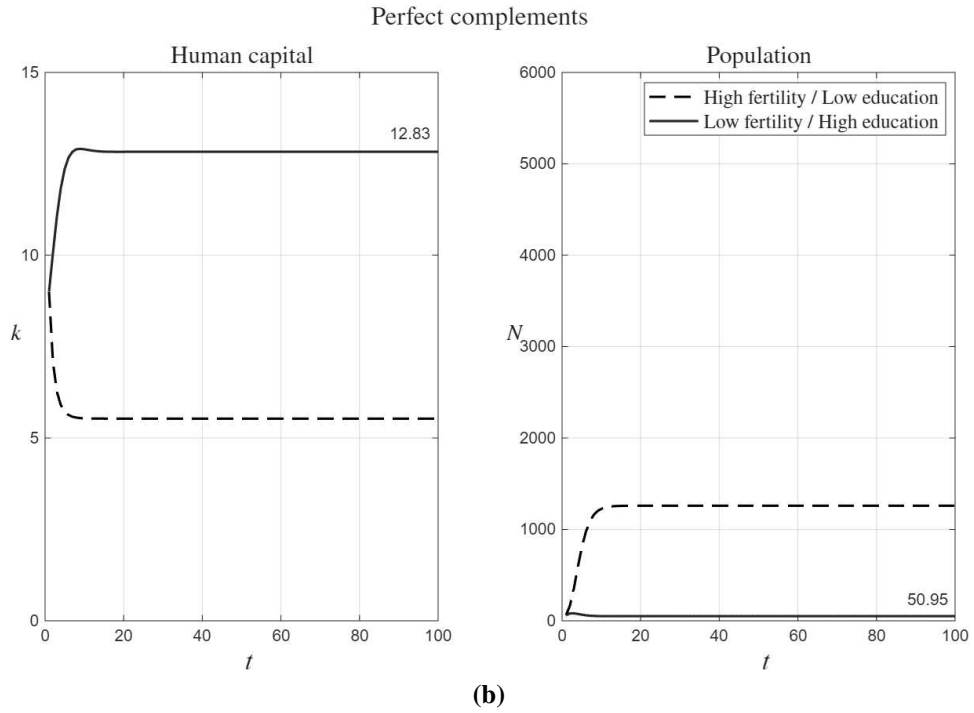
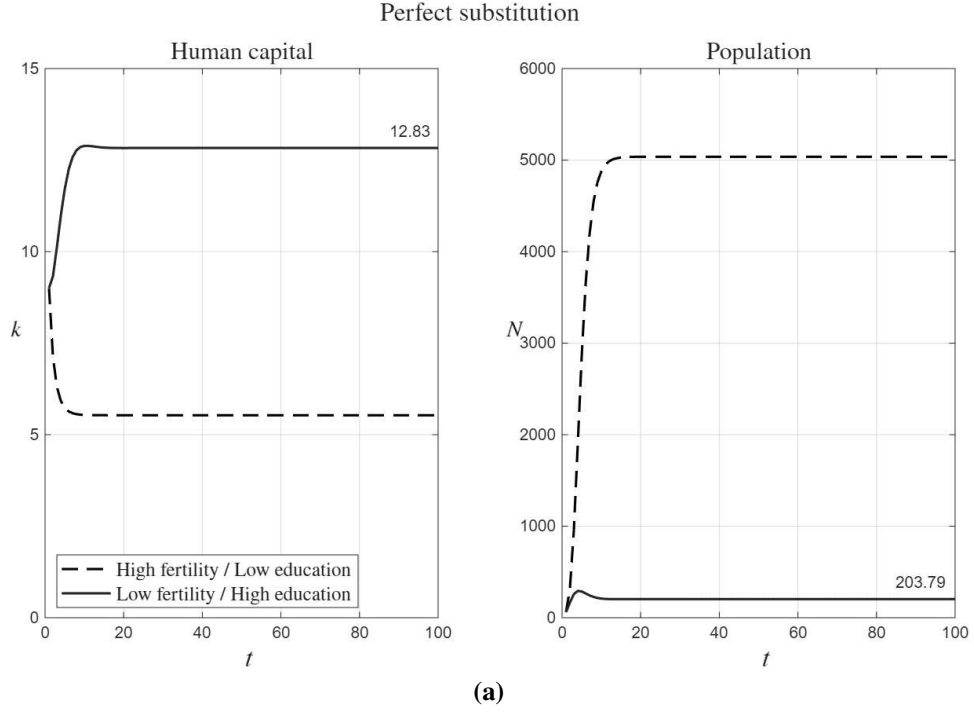


Figure 3.8: Comparing equilibrium points of human capital and population size. Parameters $A = 3.75$, $\beta = 0.5$, $\gamma = 0.5$, $\phi = 0.0025$, $b = 0.1$. In the perfect complements case, $\bar{k}$ is 12.83 and $\bar{N}$ is 50.95, close to Korea's current levels, 12 and 51.6, respectively.

3.5 Final considerations

There has long been an apparent consensus in the literature on intra-household allocation and fertility that greater paternal involvement in childcare alleviates maternal time constraints, enabling mothers to increase their labour supply or leisure. The Korean evidence challenges that view. Increases in fathers' childcare time have coincided not with a reduction, but with a further increase, in the time mothers devote to home production.

This paper developed an OLG growth model to study a mechanism that at least partially explains the puzzle. The core argument rests on the structure of the childcare technology. When parental time inputs are substitutes, the conventional result holds: greater paternal involvement eases the maternal time burden. When they are complements, the opposite occurs, and increased paternal involvement raises maternal childcare time rather than reducing it. The elasticity of substitution between parental time inputs should not be interpreted purely as a preference parameter. It also reflects the set of skills that mothers and fathers bring to child-rearing. High substitutability captures a situation in which both parents are capable of performing the same household tasks interchangeably. As the elasticity falls, parents become increasingly specialised, with each unable to replicate what the other does. The simple example of cooking and driving illustrates the point. Two parents who can both cook and drive are substitutes; a father who can cook but not drive, paired with a mother who can drive but not cook, become complements.

The model was further used to assess the effectiveness of fertility subsidies under two alternative fiscal arrangements. In the first, the government maintains a balanced budget at all times, and households fully internalise the implied tax burden in their optimisation problem in the form of Ricardian equivalence by design. In the second, the budget is balanced *ex post*, after households have made their fertility and education decisions, so that the subsidy has a distortionary effect on household choices. It was shown that pro-natalist subsidies may generate an unintended anti-fertility bias, working against their stated purpose. Numerical experiments confirmed that the model is consistent with additional stylised facts, including the quantity-quality trade-off between fertility and investment in human capital.

Several avenues remain open for future research. The framework developed here deliberately abstracts from the literature on domestic labour and intra-household bargaining. This was done to prioritize parsimony to isolate the mechanism of interest. Of course, this need not be a permanent restriction. The childcare technology introduced in this paper could serve as a natural building block for models that explicitly incorporate asymmetric bargaining power between partners, or that allow for the possibility of divorce. Integrating these elements would enrich the framework and shed further light on how the distribution of power and risk within the household shapes fertility and human capital outcomes.

Chapter 4

Rat Race, Educational Signaling, and Fertility

This chapter is co-authored with Daniele Tavani.

4.1 Introduction

Education has long been central in East Asian societies, and South Korea provides one of the clearest examples. South Korea's education attainment levels are at the very top of the OECD. Among 25-34 year-olds, about 71% have completed college-level education, compared to roughly 48% on average across OECD countries. Among all young adults in Korea, only about 1% of them do not finish high-school, which is the lowest share in the OECD (OECD, 2024). High educational attainment comes with strong status pressure. University admission is competitive and employers place heavy weight on school prestige, so small university rank differences can matter. That pressure helps explain the steady demand for after-school study and private tutoring ("shadow education") (OECD, 2024). Hence, the competitive pressure regarding academic achievement is also an important feature of Korea's educational system.

In fact, a large proportion of students attend after-school institutions such as *hagwons* or private tutoring sessions in addition to their regular schooling. This heavy emphasis on supplementary education is often referred to as *education fever* (Seth, 2002). It reflects not only concern for children's academic achievement but also social expectations and family reputation. Parents often feel compelled to invest in shadow education even when the direct benefits for test performance are limited (?). Peer behavior reinforces the cycle. When a student is exposed to a new cohort with higher average spending on shadow education, families who would otherwise be content with baseline spending feel pressure to match. In (Young, 2026), it shows that the response is not linear: moving into a higher-spending group triggers larger jumps than marginal changes within the middle of the distribution with downward moves showing quicker cutbacks. These threshold

patterns are what we mean by “rat race” dynamics, and they line up with the institutional history that made prohibition hard and regulation only partly effective (H. Choi & Choi, 2015, 2016).

These investments have important demographic and economic consequences. As households allocate more resources to private education, the costs associated with raising children rise. This has been linked to Korea’s rapid decline in fertility. Anderson and Kohler (2013) argue that rising education costs are a major contributor to East Asia’s “fertility puzzle,” where fertility has fallen to very low levels despite relatively high income and health. South Korea is the clearest example: in 2019, the total fertility rate fell below one child per woman, at 0.92 (Doepke, Hannusch, & Kindermann, 2022). Although part of this decline is related to falling marriage rates, the number of children among married couples has also dropped sharply since the 1960s (Raymo & Park, 2020).

A second issue concerns whether these large educational outlays are justified by the outcomes from supplemental education—whether or not the additional investment is efficient, in other words. A number of studies suggest that the marginal returns of private tutoring are modest at best. ? use multiple empirical strategies to show that the effects of private tutoring on middle school exam performance are limited. ? finds similar evidence, pointing to possible over-investment in shadow education with little measurable improvement in learning outcomes. Kang (2007) estimates that a 10 percent increase in private tutoring expenditure raises test scores by about 0.56 percentile points, a modest effect that he compares to estimates for public school expenditures.

One explanation is that such investments serve as signals of parental effort or family status rather than productive human capital accumulation. S. Kim et al. (2024) argue that education investment in Korea is shaped by status externalities: parents care not only about their children’s absolute human capital, but also about their children’s position relative to others. Building on this insight, this paper focuses on the peer-spending channel through which such positional concerns operate. The empirical evidence in Young (2026) shows that households increase tutoring expenditure when their children are exposed to higher-spending peer environments, especially when they move into the upper quartile of the peer-spending distribution. This pattern is consistent with a rat-

race mechanism: parents spend more not simply because tutoring raises absolute achievement, but because falling behind peers may reduce their child’s relative standing in a competitive education system.

Importantly, the pressure exerted by peer groups in determining household spending on tutoring programs and the like also affects household budgets and time allocation. Given the household’s resource constraints, as parents devote more resources to tutoring, they inevitably will have to divert resources away from consumption or savings. In addition, long work hours and limited childcare options have turned shadow education into a form of childcare for many families (S.-K. Lee, 2011). This dual role, as both an educational input and a form of after-school supervision, further complicates the relationship between shadow education and fertility. Gendered social norms intensify these dynamics. Mothers, in particular, often face asymmetric child penalties and expectations that they remain the primary caregivers, even as they also participate in the labor market (Bertrand, 2020). These intra-household dimensions suggest that education spending interacts with broader institutional and cultural constraints, rather than operating only through a simple income-cost channel.

The paper closest to ours is S. Kim et al. (2024), who build a quantitative heterogeneous-agent model of status externalities in Korean education and find that fertility would be 28% higher without the externality. Our paper complements their analysis in three ways. First, we provide a tractable analytical framework—extending the Doepke and Tertilt (2016) benchmark—that delivers closed-form equilibrium allocations and sharp comparative statics, making the mechanism transparent. Second, we embed the static rat race in an overlapping-generations structure where population and human capital evolve jointly, allowing us to study the long-run demographic and growth consequences that a static framework cannot address. S. Kim et al. themselves note that their optimal policy “may not do the same for future generations as it lowers their human capital”—our dynamic model addresses exactly this intergenerational tradeoff. Third, we micro-found the peer-effect parameter ϕ through a university-admission tournament, linking it endogenously to cohort size and closing the dynamic system.

This paper builds on these insights by developing a model that links household decisions about education, fertility, and labor supply to macroeconomic outcomes. Households choose consumption, fertility, leisure, and shadow education investment that is impacted by the peers' shadow education spending as the representative household cares about the perceived human capital of their children. This positional element creates a rat-race dynamic where parents overspend on education in order to keep up with peers, which in turn leads to lower fertility. The result is inefficient allocation of resources with long-term demographic consequences. We first analyze a static partial-equilibrium model to build intuition and characterize the rat-race externality. We then extend the framework to an overlapping-generations setting in which peer-effect intensity depends on population size, generating a two-dimensional dynamical system in human capital and population. The dynamic model delivers a unique interior steady state, clarifies the conditions under which the rat race leads to demographic decline, and shows that a time-varying Pigouvian tax on education spending decentralizes the efficient allocation.

4.2 A Baseline Human Capital Rat Race Model

To build intuition, consider the following partial equilibrium model with a representative unitary household, that extends the benchmark model by Doepke and Tertilt (2016, DT in what follows). The household utility function depends on consumption (c), number of children (N'), and the children's human capital, h' . We assume a simple, linearly homogeneous Cobb-Douglas utility function:

$$u(c, h', N') = \alpha \ln c + \beta \ln h' + (1 - \alpha - \beta) \ln N' \quad (4.1)$$

The rat race in education spending is introduced by assuming that the human capital of the household's offspring depends on the household's spending on education s , relative to other households' spending, $\bar{s}$. The extent to which the household is sensitive to education spending by its peer group is captured by the parameter $\phi \in (0, 1)$. In particular, we posit that :

$$h' = \gamma(s - \phi\bar{s}), \gamma > 0 \quad (4.2)$$

We also assume, following the standard DT model, that fertility depends on leisure time l , to capture the fact that some fraction b of the household's leisure time will be spent in child rearing.

$$N' = bl \quad (4.3)$$

Denoting by w the income earned during non-leisure time, the household's budget constraint is:

$$c + s \leq w(1 - l), \quad (4.4)$$

The household then chooses consumption, leisure, and spending on human capital given the budget constraint and other households' spending on education.

From the first-order conditions, we find the following choices of c, l, s given w and $\bar{s}$:

$$c(w, \bar{s}) = \alpha (w - \phi \bar{s}) \quad (4.5)$$

$$s(w, \bar{s}) = \phi \bar{s} + \beta (w - \phi \bar{s}) \quad (4.6)$$

$$l(w, \bar{s}) = (1 - \alpha - \beta) \left(\frac{w - \phi \bar{s}}{w} \right) \quad (4.7)$$

These choices can be all thought of as best-responses to other households' educational spending.

4.2.1 Symmetric Equilibrium

A symmetric equilibrium allocation is a choice of consumption, leisure time, and spending on education that maximizes the household's preferences under the budget constraint, and such that $s = \bar{s}$. The symmetric requirement closes the model. The equilibrium allocation is:

$$c^{eq} = \alpha \left[\frac{1 - \phi}{1 - \phi(1 - \beta)} \right] w \quad (4.8)$$

$$s^{eq} = \left[\frac{\beta}{1 - \phi(1 - \beta)} \right] w \quad (4.9)$$

$$l^{eq} = (1 - \alpha - \beta) \left[\frac{1 - \phi}{1 - \phi(1 - \beta)} \right] \quad (4.10)$$

The interesting comparative statics exercise is to analyze how consumption, educational spending, and leisure each respond to changes in the extent of peer effects ϕ . We have that:

$$\begin{aligned}\frac{\partial c}{\partial \phi} &= -\frac{w\beta}{[1 - \phi(1 - \beta)]^2} < 0 \\ \frac{\partial s}{\partial \phi} &= \frac{\beta w(1 - \beta)}{[1 - \phi(1 - \beta)]^2} > 0 \\ \frac{\partial l}{\partial \phi} &= -\frac{\beta(1 - \alpha - \beta)}{[1 - \phi(1 - \beta)]^2} < 0\end{aligned}$$

These results are intuitive. As the strength of peer effects increases, leisure and consumption decrease, while spending on education increases. As leisure decreases, fertility also decreases. Figure 4.1 illustrates these comparative statics for a simple parameterization, normalizing income to $w = 1$.

4.2.2 Welfare Analysis

A benevolent social planner chooses consumption, leisure, and education spending to maximize the household's utility under the budget constraint, but internalizes the rat race externality $s = \bar{s}$, so that:

$$h' = \gamma(1 - \phi)s \tag{4.11}$$

Given log-utility, the role of peer effects drops out of the maximization program. The first order conditions together with the budget constraint yield the efficient allocation as follows:

$$c^* = \alpha w \tag{4.12}$$

$$s^* = \beta w \tag{4.13}$$

$$l^* = 1 - \alpha - \beta \tag{4.14}$$

Comparison to the decentralized allocation found in 4.8, 4.9, and 4.10 shows that the efficient allocation corresponds to the case where $\phi = 0$. As long as $\phi > 0$, the household will increase its educational spending at the expense of consumption and leisure and, through the latter, fertility. In

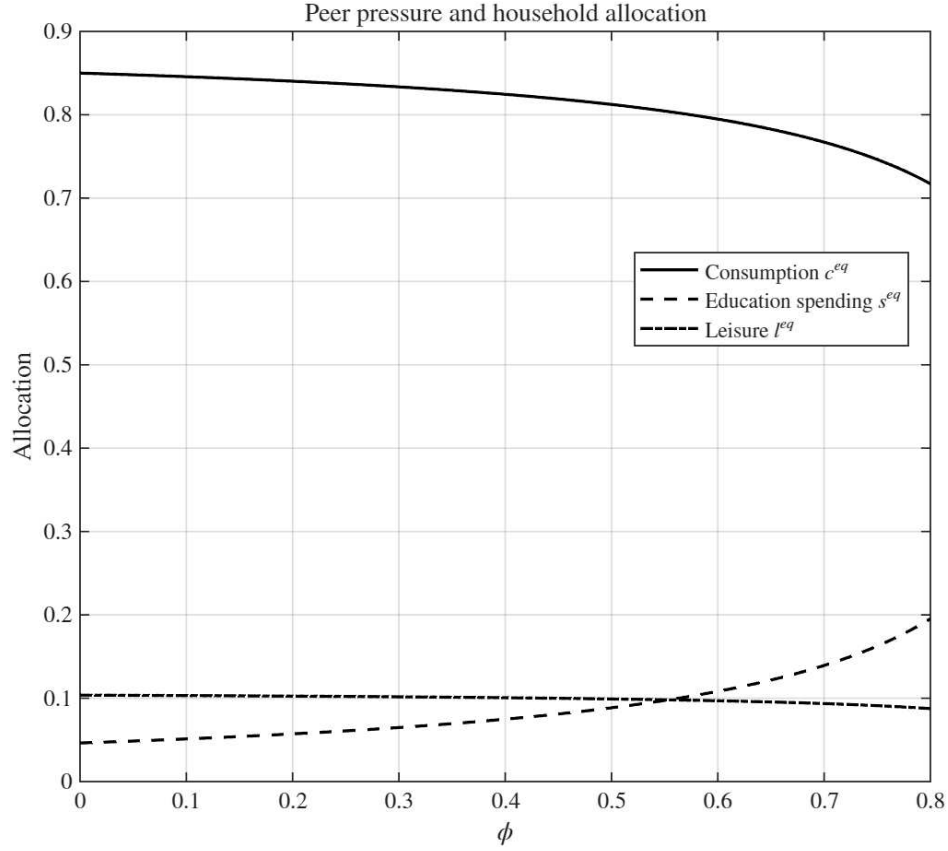


Figure 4.1: Peer pressure and household allocation. As the peer-effect parameter ϕ increases, households devote a larger share of income to education spending and reduce both consumption and leisure. Since fertility is tied to leisure, stronger peer pressure lowers fertility.

fact, simple inspection of equations (4.8), (4.9), (4.10) as compared to their efficient counterparts (4.12), (4.13) and (4.14) reveals that:

$$c^{eq} < c^*$$

$$s^{eq} > s^*$$

$$l^{eq} < l^*$$

Figure 4.2 illustrates this wedge between the decentralized and efficient allocations for education spending and leisure.

This simple model highlights the adverse effect on fertility arising from inefficiently high spending on education. The inefficiency arises because of the role played by spending by peers. Households react by reducing leisure in order to finance additional spending on education, at the expense of consumption. In addition, the reduction in leisure time lowers fertility.

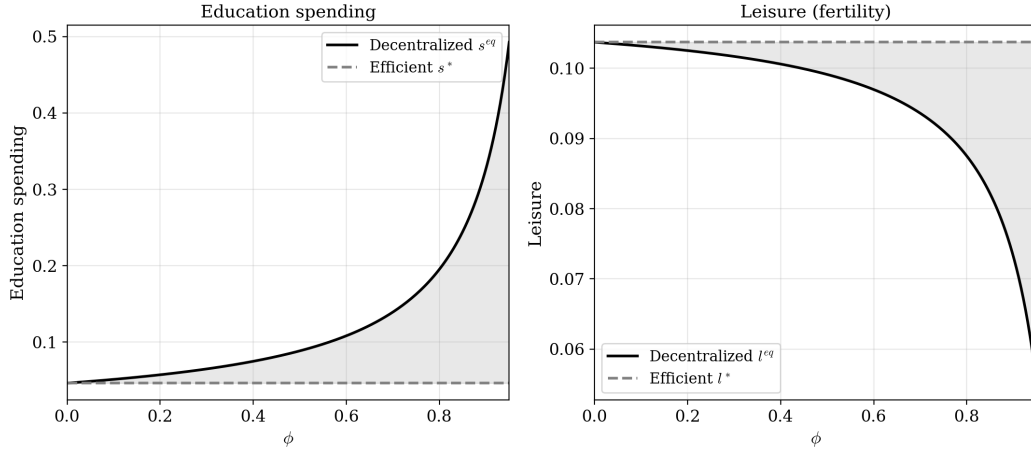


Figure 4.2: Decentralized and efficient allocations. As peer pressure increases, the decentralized economy allocates more resources to education spending and less time to leisure relative to the efficient allocation.

4.2.3 Decentralization

The efficient allocation can be decentralized by distortionary taxation τ on education spending. The tax is rebated as a lump-sum amount σ to the household. The government balanced budget requirement is then $\sigma = \tau s$. The budget constraint becomes:

$$c + s(1 + \tau) \leq w(1 - l) + \sigma \quad (4.15)$$

The key problem concerns distorting spending on human capital, s , so as to achieve the efficient allocation. The household's best-response to peer spending $\bar{s}$ when the tax is included is:

$$s(\bar{s}; w, \tau) = \frac{\beta}{\beta + (1 - \beta)(1 + \tau)} w + \frac{(1 - \beta)(1 + \tau)}{\beta + (1 - \beta)(1 + \tau)} \phi \bar{s} \quad (4.16)$$

which, imposing the equilibrium condition $s = \bar{s}$, yields:

$$s(\tau, w) = \frac{\beta}{\beta + (1 - \beta)(1 + \tau)(1 - \phi)} w \quad (4.17)$$

Decentralizing the efficient allocation (4.13) requires to set $(1 + \tau)(1 - \phi) = 1$, which solves for the optimal tax rate on education spending as follows:

$$\tau^* = \frac{\phi}{1 - \phi} \quad (4.18)$$

As is intuitive, the tax rate increases (at an increasing rate) in the extent of peer effects. If this tax is levied, spending on education reverts to its efficient level, and so do consumption and leisure. Thus, a Pigouvian tax on tutoring outlays prices the positional harm ϕ . With $\tau^* = \phi/(1 - \phi)$, the tax decentralizes the planner's allocation, reducing status competition while preserving productive investment. S. Kim et al. (2024) find that a sizeable education tax of 22%, together with a moderate pro-natal transfer, is welfare-maximizing in their model.

4.3 University Admissions Tournaments and Population-Dependent Peer Effects

The static model of Section 4.2 treats the peer-effect parameter ϕ as exogenous. Before turning to the dynamic extension, we provide a micro-foundation for ϕ that links it to cohort size (population in an aggregate model) N . This will generate an interaction between human capital accumulation and population size that will enable studying a dynamical system in these two state variables, as done in Section 4.4 below. The mechanism we propose is a university-admission tournament: children compete for ranked slots based on a noisy signal of preparation, and the intensity of competition at the admission margin increases with the number of applicants. This generates a reduced-form perceived human capital function of the form $\tilde{h}' = \gamma(s - \phi(N)\bar{s})$, where ϕ is increasing in N , which provides the closure for the dynamic model.

The tournament approach builds on the rank-order tournament literature initiated by Lazear and Rosen (1981, LR henceforth), who show that compensation schemes based on ordinal rank—rather than individual output—can implement efficient effort levels. In their framework, a worker's payoff depends on relative performance against competitors, and the intensity of competition is governed

by the prize spread and the number of contestants. We adapt this structure to university admissions: the “prize” is the wage premium associated with a higher-ranked university tier, and the “effort” is parental spending on shadow education. The key departure from the standard LR setup is that we are interested not in the optimal design of the tournament, but in how the equilibrium sensitivity to relative performance varies with the number of participants.

This question connects to two strands of the subsequent literature. The first is the analysis of multi-prize contests by Moldovanu and Sela (2001) and Moldovanu, Sela, and Shi (2007). Moldovanu and Sela characterize equilibrium effort in all-pay auctions with multiple ranked prizes and incomplete information about ability, showing that the optimal prize structure depends on the distribution of contestant types. Moldovanu et al. extend the framework to settings where agents care directly about their relative position (status), and show that the definition of status categories can be used to influence effort. The university-tier structure in our model provides a close status partition: the SKY/in-Seoul/regional hierarchy defines discrete status categories, and parents exert costly effort (education spending) to place their children in higher categories. The second strand is the positional competition literature. Hopkins and Kornienko (2004) model consumer choice when utility depends on rank in the distribution of a positional good, and show that in symmetric Nash equilibrium all agents overspend on the status good—a result they describe as “running to keep in the same place.” The education rat race in our model is a specific instance of this phenomenon: each family overspends on tutoring to maintain relative standing, yet in equilibrium no one’s rank improves. Frank and Cook (1995) apply the winner-take-all logic explicitly to educational prestige, arguing that small differences in university rank translate into large differences in labor market outcomes, driving an arms race in educational investment.

Our contribution relative to this literature is to derive a specific, microfounded relationship between the intensity of positional competition ϕ and the size of the competing cohort N , and to embed it in a dynamic model where population is itself endogenous. The tournament model delivers $\phi(N)$ as a reduced-form object whose properties— $\phi' > 0$, $\phi(0) = 0$, $\phi \in [0, 1]$ —follow directly from structure of the admission cutoff. Accordingly, the extent of peer effects ϕ serves as

the bridge between the static rat-race externality and the dynamic population-human capital system analyzed in the next section.

4.3.1 Setup

There are M university tiers indexed by $j = 1, \dots, M$, with tier 1 the most prestigious. Each tier offers a fixed number of seats. A child's labor market wage depends on the tier of university attended: tier j yields wage w_j , with $w_1 > w_2 > \dots > w_M$. We denote the wage premium from moving one tier up by $\Delta_j = w_j - w_{j+1}$, and assume that the premium structure can be summarized by a representative premium $\Delta > 0$.

In a cohort of size N , each child i produces a publicly observed signal

$$\sigma_i = s_i + \varepsilon_i, \quad \varepsilon_i \stackrel{i.i.d.}{\sim} F, \quad (4.19)$$

where s_i is the education spending chosen by child i 's parent and ε_i is an idiosyncratic component (innate ability, luck on exam day, etc.) drawn from a continuous distribution F with density f , mean zero, and finite variance τ^2 . Children are assigned to university tiers in order of their signal: the top signals get tier 1 seats, the next group gets tier 2, and so on.

4.3.2 The Marginal Return to Spending

Consider a representative admission cutoff—the signal level σ^* that separates tier j from tier $j+1$.

1. A parent who raises s_i by a small amount ds shifts the child's signal distribution upward, increasing the probability that σ_i exceeds σ^* . The marginal increase in the probability of clearing the cutoff is

$$\frac{\partial \Pr(\sigma_i > \sigma^*)}{\partial s_i} = f(\sigma^* - s_i). \quad (4.20)$$

In a symmetric equilibrium, all parents choose the same $s_i = \bar{s}$, so the cutoff σ^* is determined by the quantile of the noise distribution at the admission threshold. The expected wage gain from

a marginal increase in spending is therefore

$$\left. \frac{\partial E[w_i]}{\partial s_i} \right|_{s_i=\bar{s}} = \Delta \cdot g_N(\bar{s}), \quad (4.21)$$

where Δ is the wage premium, collapsed to a single number given our focus on a symmetric equilibrium, and $g_N(\bar{s})$ is the density of signals near the admission cutoff when all parents spend $\bar{s}$.

The key observation is that g_N depends on the cohort size N . When N applicants compete for a fixed number of tier- j seats, the admission cutoff corresponds to a specific order statistic of N draws from F . As N grows, more signals cluster near the cutoff—the empirical distribution tightens—so the density at the cutoff increases. Formally, for the q -th quantile of the empirical distribution of N i.i.d. draws from F (where $q = M_j/N$ is the admission rate for tier j), the density of the order statistic at the quantile scales as

$$g_N \propto \sqrt{N} \cdot f(F^{-1}(q)). \quad (4.22)$$

This is a standard result from order statistics: the distribution of the sample quantile concentrates around the population quantile at rate $\sqrt{N}$.

4.3.3 From Tournament Returns to Peer-Effect Intensity

The parent's total perceived return to education spending thus has two components: a direct human-capital component γ (the return from absolute spending, as in the baseline model) and a tournament component $\Delta \cdot g_N$ (the return from relative position). The household's effective perceived human capital can be written as

$$\tilde{h}_{i,t+1} = \underbrace{\gamma s_i}_{\text{absolute return}} + \underbrace{\Delta \cdot g_{N_t} \cdot (s_i - \bar{s}_t)}_{\text{tournament return}}. \quad (4.23)$$

The tournament return depends on the *deviation* from peer spending, not on absolute spending: if everyone raises spending by the same amount, the cutoff shifts but no one's rank changes. Factoring out γ and defining $\phi_t \equiv \Delta \cdot g_{N_t} / (\gamma + \Delta \cdot g_{N_t})$, we can write

$$\tilde{h}_{i,t+1} = (\gamma + \Delta \cdot g_{N_t})(s_i - \phi_t \bar{s}_t). \quad (4.24)$$

Since $\ln \tilde{h}$ enters utility, the scaling factor $\gamma + \Delta \cdot g_{N_t}$ does not affect the allocation of s_t relative to consumption and fertility (it is absorbed into the constant γ of the baseline model). What matters is the peer-effect weight

$$\phi(N_t) = \frac{\Delta \cdot g_{N_t}}{\gamma + \Delta \cdot g_{N_t}}, \quad (4.25)$$

which inherits the following properties directly from the tournament structure:

[nosep]

- $\phi(0) = 0$: with no competitors, relative position is irrelevant.
- $\phi'(N) > 0$: more competitors raise g_N (via (4.22)), increasing the tournament weight.
- $\phi(N) < 1$ for all N : the absolute return $\gamma > 0$ always retains positive weight.
- $\lim_{N \rightarrow \infty} \phi(N) = \Delta \cdot f(F^{-1}(q)) \cdot \lim_{N \rightarrow \infty} \sqrt{N} / [\gamma + \Delta \cdot f(F^{-1}(q)) \cdot \lim_{N \rightarrow \infty} \sqrt{N}] \rightarrow 1$: in a very large cohort, the tournament motive dominates.

Proposition 9 (Micro-Foundation for $\phi(N)$). *In a university-admission tournament with N applicants, M ranked tiers, signal noise $\varepsilon \sim F$, and tier premium Δ , the reduced-form peer-effect parameter $\phi(N)$ defined by (4.25) is strictly increasing in N with $\phi(0) = 0$ and $\phi(N) \rightarrow 1$ as $N \rightarrow \infty$.*

Mapping to Korea The tournament structure maps directly onto Korea's university admission system. The tier structure corresponds to the well-documented hierarchy of Korean universities (SKY, in-Seoul, regional) whose graduates face sharply different labor market outcomes. The signal σ_i corresponds to the College Scholastic Ability Test (*suneung*) score, which is the primary

determinant of university placement. The idiosyncratic noise ε_i captures the role of innate ability and test-day variation. The key empirical implication—that peer-effect intensity increases with cohort size—is consistent with the observation that education fever is most intense in Seoul, where cohort density and competition for university slots are highest.

Remark 10 (Reduced-form specification). *For tractability, the dynamic model of the next section uses the parametric form $\phi(N) = 1 - 1/(1 + \xi N)$, where $\xi > 0$ is a scalar. This specification satisfies all the qualitative properties established in Proposition 9. The parameter ξ absorbs the structural tournament parameters (Δ , γ , the shape of F) into a single reduced-form coefficient that can be calibrated to match observed education overspending.*

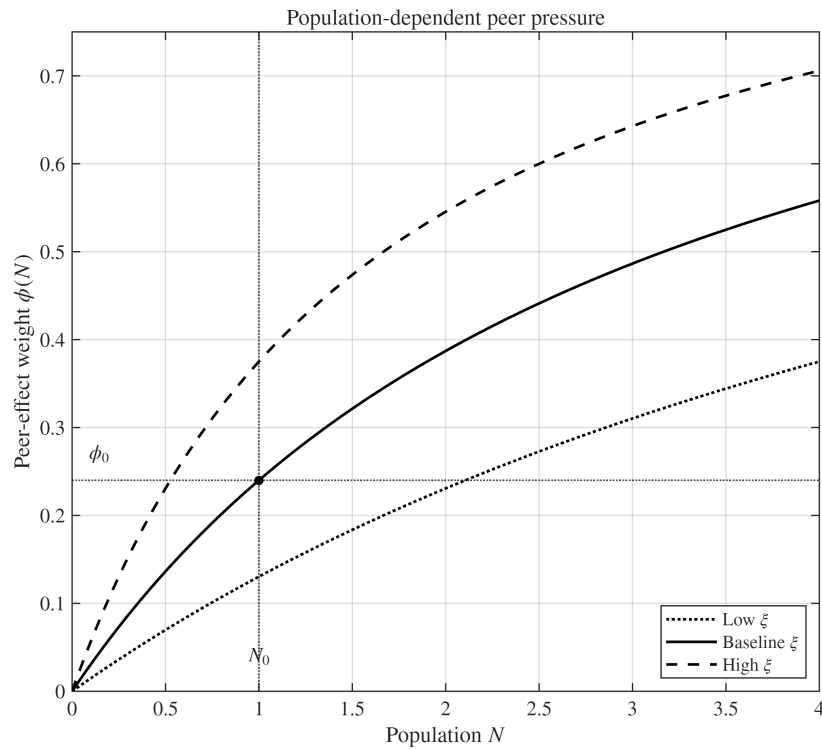


Figure 4.3: Population-dependent peer pressure. Larger cohorts increase the peer-effect weight $\phi(N)$, while higher values of ξ make peer pressure rise more quickly with population size.

4.4 The Full Dynamic OLG Model

The static model in Section 4.2 shows that peer effects in education spending generate a race that depresses fertility through reduced leisure, and Section 4.3 provides a micro-foundation linking the peer-effect intensity ϕ to cohort size through a university-admission tournament. We now embed these ingredients in an overlapping-generations framework to study the joint evolution of human capital and population. The key modeling choices are: (i) the peer-effect intensity ϕ depends on population size, as motivated by the tournament mechanism of Section 4.3; (ii) human capital accumulates through education spending and a knowledge spillover from aggregate population; and (iii) a density-dependent attrition rate combined with endogenous fertility generates well-defined interior steady states for both the decentralized and the planner's economies. The resulting two-dimensional dynamical system in human capital and population admits a unique steady state whose comparative statics with respect to the peer-effect parameter can be studied analytically.

4.4.1 Environment

Time is discrete, $t = 0, 1, 2, \dots$. The economy is populated by overlapping generations of households. Each household lives and makes decisions for one period, corresponding to its young-adult phase, then is replaced by the next generation. This keeps the household problem identical to the static benchmark of Section 4.2 while allowing the economy to feature nontrivial population and human capital dynamics across cohorts.

Demographics. Let N_t denote the size of generation t . Each household chooses fertility n_t (number of children). The generational attrition rate $d(N_t) \in (0, 1)$ is increasing in population size. The law of motion for population is

$$N_{t+1} = (1 - d(N_t) + n_t) N_t. \quad (4.26)$$

Density-dependent attrition. The variable d should be interpreted broadly as a generational attrition rate, not as a biological death rate: it encompasses mortality, emigration, and any mechanism by which households exit the active economy. In a model where each period corresponds to a generation, d near unity reflects near-complete generational turnover, and d can exceed unity when density pressure causes net population loss beyond simple replacement—for example, through out-migration from high-cost metropolitan areas.

The assumption $d'(N) > 0$ captures, in reduced form, the resource pressure that larger populations exert on finite capacity. Density-dependent population regulation is central to the unified growth theory of Galor and Weil (2000) and Galor (2011), and is documented empirically by Ashraf and Galor (2011). In OLG models with endogenous population, density-dependent attrition is standard: Cervellati and Sunde (2005) model interactions between life expectancy and human capital, and Strulik and Weisdorf (2008) use endogenous mortality to generate dynamics consistent with successive growth regimes. The density–mortality link has deep historical roots (Voigtländer & Voth, 2013) and persists in modern urbanizing economies (Jedwab & Vollrath, 2019; Lorentzen, McMillan, & Wacziarg, 2008).

In Korea, the assumption maps onto well-documented cost pressures in dense metropolitan areas. Seoul’s population density exceeds 15,000 people per square kilometer, apartment prices rose 144% over the decade ending in 2024, and the city experienced a net outflow of approximately 861,000 residents over ten years, with 1.74 million citing housing as the primary reason for leaving. Fertility in Seoul has fallen to 0.55 children per woman—roughly half the national rate—and a recent OECD assessment attributes the gap to the combined burden of private education costs and housing prices (OECD, 2024). The assumption $d'(N) > 0$ captures these patterns—housing-driven out-migration, fertility suppression, and tightening cost-of-living pressure—in reduced form.

Formal specification. We assume $d : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is continuously differentiable with: (i) $d'(N) > 0$ for all $N > 0$; (ii) $d(0) \equiv \underline{d} < \nu/b$; and (iii) $\lim_{N \rightarrow \infty} d(N) \equiv \bar{d} > \nu/b$. Conditions (ii) and (iii) ensure that both the decentralized and the planner’s economies admit interior steady states. A

tractable parametric form is

$$d(N) = \underline{d} + (\bar{d} - \underline{d}) \frac{\zeta N}{1 + \zeta N}, \quad \zeta > 0, \quad (4.27)$$

where $\underline{d}$ is baseline attrition, $\bar{d}$ is the upper bound, and ζ governs density sensitivity. The parameter ζ is pinned by requiring the baseline economy to be at a population steady state (see Section 4.4.8 and the details provided in the Appendix).

Human capital. A generation- t household is endowed with human capital h_t , which determines its wage. We normalize the price of effective labor to one, so the household's potential income is h_t .

Preferences. The household has preferences over consumption c_t , the perceived human capital of children $\tilde{h}_{t+1}$, and the number of children n_t :

$$U_t = \alpha \ln c_t + \beta \ln \tilde{h}_{t+1} + \nu \ln n_t, \quad (4.28)$$

where $\alpha, \beta, \nu > 0$ and $\alpha + \beta + \nu = 1$. This is the same utility specification as in the static benchmark model, with $\nu \equiv 1 - \alpha - \beta$ denoting the weight on fertility.

Remark 11 (On the one-period household problem). *The dynamic model features no physical capital and no intertemporal borrowing or lending by households, and human capital is produced via education spending and knowledge spillovers, not via saved resources. Thus, there is no role for a savings decision within the household problem. Introducing an old-age consumption variable would simply carve out a constant share of full income and forward it to a future period without affecting the allocation of the remaining choices. We therefore keep the household problem identical to the static benchmark. The “OLG” structure refers to the demographic layer: cohorts are born, have children, and die, with human capital transmitted across generations via education investment.*

The rat race in education. Shadow education spending by the household is s_t . Perceived child quality depends on the household's spending relative to the peer group's average spending $\bar{s}_t$:

$$\tilde{h}_{t+1} = \gamma(s_t - \phi_t \bar{s}_t), \quad \gamma > 0, \quad (4.29)$$

where $\phi_t \in (0, 1)$ measures the intensity of peer effects.

Population-dependent peer effects. As shown in Section 4.3, the university-admission tournament generates a peer-effect intensity that depends on cohort size. The central closure of the dynamic model is therefore that the rat-race intensity depends on population size:

$$\phi_t = \phi(N_t), \quad \phi' > 0, \quad \phi(0) = 0, \quad \lim_{N \rightarrow \infty} \phi(N) < 1. \quad (4.30)$$

These properties follow from Proposition 9: larger cohorts raise the density of competitors at the admission margin, increasing the return to relative spending. A tractable reduced-form specification that preserves all the qualitative features of the structural model is

$$\phi(N_t) = 1 - \frac{1}{1 + \xi N_t}, \quad \xi > 0, \quad (4.31)$$

where ξ absorbs the structural tournament parameters into a single coefficient, calibrated below.

Time allocation and budget constraint. The household has one unit of time. Raising n_t children costs $b n_t$ units of time, where $b > 0$ is the per-child time cost (corresponding to the time-cost interpretation from the static model, where $n_t = l_t/b$ maps leisure into fertility). The remaining time $(1 - b n_t)$ is supplied to the labor market at wage h_t . Education spending s_t is at the household level. The budget constraint is

$$c_t + s_t + b h_t n_t = h_t. \quad (4.32)$$

Actual human capital transmission. While perceived quality depends on relative spending, the *actual* human capital of the next generation depends on absolute education investment and a knowledge spillover from aggregate population (Jones, 2022):

$$h_{t+1} = A s_t^\eta N_t^\mu, \quad A > 0, \eta \in (0, 1), \mu > 0. \quad (4.33)$$

The scaling parameter A is useful for calibration purposes. The term N_t^μ captures the idea that a larger population generates more knowledge spillovers—more ideas, more specialization, more peer learning—that raise the productivity of education investment. This is the channel through which population size feeds back into human capital, following the insight that ideas and human capital are closely linked, so that A can be treated as a pure scaling constant.

4.4.2 Household Optimization

The household maximizes (4.28) subject to (4.32), taking $\bar{s}_t$, ϕ_t and h_t as given. Substituting $\tilde{h}_{t+1} = \gamma(s_t - \phi_t \bar{s}_t)$ and setting up the Lagrangian:

$$\mathcal{L} = \alpha \ln c_t + \beta \ln [\gamma(s_t - \phi_t \bar{s}_t)] + \nu \ln n_t + \lambda [h_t - c_t - s_t - b h_t n_t].$$

The first-order conditions are:

$$c_t : \frac{\alpha}{c_t} = \lambda, \quad (4.34)$$

$$s_t : \frac{\beta}{s_t - \phi_t \bar{s}_t} = \lambda, \quad (4.35)$$

$$n_t : \frac{\nu}{n_t} = \lambda b h_t. \quad (4.36)$$

Define $\hat{s}_t \equiv s_t - \phi_t \bar{s}_t$ (effective education spending from the household's perspective) and note that $s_t = \hat{s}_t + \phi_t \bar{s}_t$. The budget constraint becomes

$$c_t + \hat{s}_t + b h_t n_t = h_t - \phi_t \bar{s}_t \equiv \Omega_t,$$

where Ω_t is **adjusted full income**—the resources available after the “committed” component of education spending induced by peer pressure.

From the first-order conditions, the standard Cobb-Douglas share results apply over Ω_t :

$$c_t = \alpha \Omega_t, \quad (4.37)$$

$$\hat{s}_t = \beta \Omega_t, \quad (4.38)$$

$$b h_t n_t = \nu \Omega_t. \quad (4.39)$$

These are the household’s best responses to peer spending $\bar{s}_t$. From (4.38), education spending is $s_t = \phi_t \bar{s}_t + \beta(h_t - \phi_t \bar{s}_t)$; from (4.39), fertility is $n_t = (\nu/b)(1 - \phi_t \bar{s}_t/h_t)$. Both expressions reduce to the static best responses (4.6)–(4.7) of Section 4.2 under $l_t = b^{-1} n_t$, i.e., total child-rearing time in the dynamic model corresponds to leisure in the static model.

4.4.3 Symmetric Equilibrium

In symmetric equilibrium, all households are identical, so $s_t = \bar{s}_t$. Imposing this on the best response for s_t :

$$s_t = \phi_t s_t + \beta(h_t - \phi_t s_t) \implies s_t[1 - \phi_t(1 - \beta)] = \beta h_t.$$

Proposition 12 (Symmetric Equilibrium Allocation). *The symmetric equilibrium allocation is:*

$$s_t^{eq} = \theta(\phi(N_t)) h_t, \quad \theta(\phi(N)) \equiv \frac{\beta}{1 - \phi(N)(1 - \beta)}, \quad (4.40)$$

$$n_t^{eq} = \bar{n}(\phi(N_t)), \quad \bar{n}(\phi(N)) \equiv \frac{\nu(1 - \phi(N))}{b[1 - \phi(N)(1 - \beta)]}, \quad (4.41)$$

$$c_t^{eq} = \frac{\alpha(1 - \phi(N_t))}{1 - \phi(N_t)(1 - \beta)} h_t. \quad (4.42)$$

Remark 13. *Equilibrium fertility $n_t^{eq} = \bar{n}(\phi_t)$ depends on h_t only through $\phi_t = \phi(N_t)$. This is a consequence of log-Cobb-Douglas preferences: income and substitution effects cancel exactly,*

so fertility is independent of the level of human capital. The dependence on the state variable N_t comes entirely through the population-dependent peer-effect channel.

Properties of the equilibrium allocation. The education spending share $\theta(\phi)$ is increasing and convex in ϕ , and therefore is increasing in population size N :

$$\theta'(N) = \frac{\beta(1-\beta)}{[1-\phi(1-\beta)]^2} \phi'(N) > 0.$$

Fertility $\bar{n}(\phi)$ is decreasing in ϕ and therefore in population size N :

$$\bar{n}'(N) = -\frac{\nu\beta}{b[1-\phi(1-\beta)]^2} \phi'(N) < 0.$$

As peer pressure intensifies, households spend more on education and have fewer children—exactly the comparative statics from the static model, now indexed by the state variable N_t .

4.4.4 The Dynamical System

Substituting the equilibrium allocation into the laws of motion (4.26) and (4.33), and using $\phi_t = \phi(N_t)$, we obtain the two-dimensional system:

$$h_{t+1} = A [\theta(\phi(N_t))]^\eta h_t^\eta N_t^\mu, \quad (4.43)$$

$$N_{t+1} = [1 - d(N_t) + \bar{n}(\phi(N_t))] N_t. \quad (4.44)$$

Remark 14 (Triangular structure). *The population equation (4.44) is autonomous: N_{t+1} depends only on N_t , not on h_t . The human capital equation (4.43) depends on both h_t and N_t . The system is therefore block-triangular. This means that population dynamics can be analyzed independently, and human capital dynamics can then be studied conditional on the population path. This structure is a consequence of log-Cobb-Douglas preferences, which make fertility independent of income through the balancing out of income and substitution effects, and the closure $\phi = \phi(N)$. The*

triangular structure makes the model tractable, because it allows sequential analysis of the two state variables while still capturing the key feedback from population to human capital through the knowledge spillover N_t^μ .

Population Dynamics

Define the gross population growth factor as

$$G(N) \equiv 1 - d(N) + \bar{n}(\phi(N)). \quad (4.45)$$

A population steady state N^* requires $G(N^*) = 1$, i.e.,

$$\bar{n}(\phi(N^*)) = d(N^*). \quad (4.46)$$

The left-hand side is continuous and strictly decreasing in N (from ν/b to 0, since $\bar{n}(\phi)$ is decreasing in ϕ and $\phi(N)$ is increasing in N). The right-hand side is continuous and strictly increasing in N (from $\underline{d}$ to $\bar{d}$). Since $\nu/b > \underline{d}$ and $0 < \bar{d}$, the two curves cross exactly once.

Figure 4.4 illustrates the population steady-state condition, under the calibration detailed below. The downward-sloping curve gives equilibrium fertility as a function of population; the upward-sloping curve gives the attrition rate $d(N)$. Their intersection determines the unique steady-state population N^* .

The figure shows the self-correcting mechanism behind population dynamics in this model. When population is below N^* , peer pressure is relatively weak and resource pressure is low, so fertility exceeds attrition and population grows. When population is above N^* , peer pressure is stronger and resource pressure is higher, so fertility falls below attrition and population declines. The two forces—rat-race pressure depressing fertility and resource pressure raising attrition—reinforce each other ensuring stability, as shown below.

Proposition 15 (Existence and Uniqueness of the Population Steady State). *If $\underline{d} < \nu/b < \bar{d}$, there exists a unique $N^* > 0$ satisfying $\bar{n}(\phi(N^*)) = d(N^*)$.*

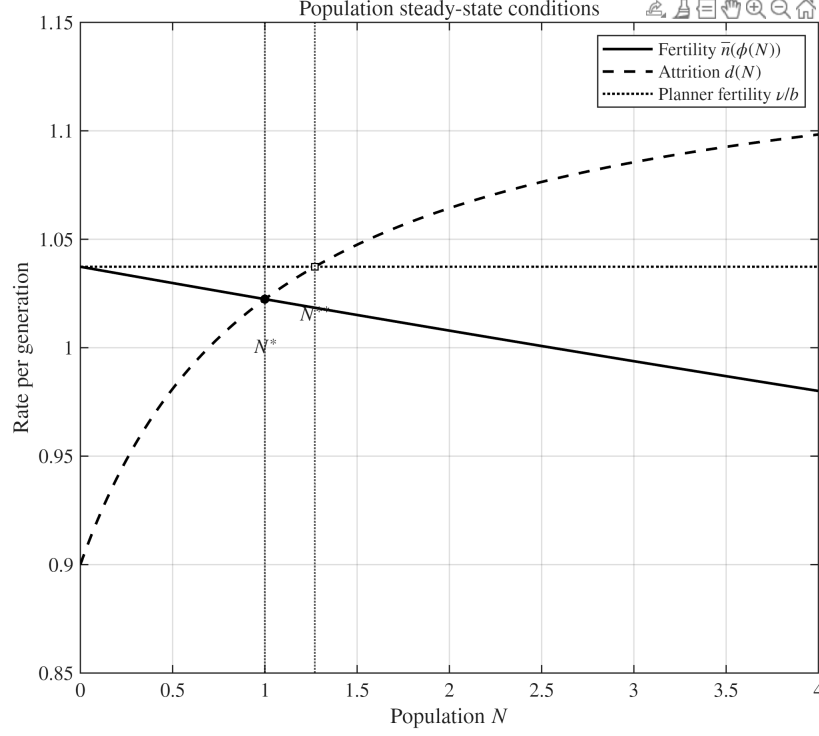


Figure 4.4: Population steady state. Fertility (downward-sloping) declines with population because larger cohorts intensify peer pressure. The attrition rate (upward-sloping) rises with population because of density-dependent resource pressure. The steady-state population N^* is determined at their intersection.

Proof. Define $f(N) \equiv \bar{n}(\phi(N)) - d(N)$. Then f is continuous and strictly decreasing ($\bar{n}'(\phi)\phi' < 0$ and $-d' < 0$), with $\lim_{N \rightarrow 0} f(N) = \nu/b - \underline{d} > 0$ and $\lim_{N \rightarrow \infty} f(N) = 0 - \bar{d} < 0$. By the intermediate value theorem, there exists a unique N^* such that $f(N^*) = 0$. $\square$

Local stability. At the steady state, the derivative of the population map $N \mapsto G(N) \cdot N$ is

$$\left. \frac{dN_{t+1}}{dN_t} \right|_{N^*} = G(N^*) + N^* G'(N^*) = 1 + N^* [\bar{n}'(\phi(N^*))\phi'(N^*) - d'(N^*)].$$

Both $\bar{n}'\phi' < 0$ and $-d' < 0$, so $G'(N^*) < 0$ and the derivative is strictly less than 1. For local stability, we also need it to exceed -1 , which requires $N^*|\bar{n}'\phi' + d'| < 2$. The density-dependent attrition rate *reinforces* stability relative to a constant death rate, because it adds a second restoring force: when population is too high, attrition rises as well as fertility falling.

The intuition has already been stated above: the stability mechanism now has two self-correcting arms. If population exceeds N^* : (i) peer pressure ϕ rises, depressing fertility; and (ii) resource pressure raises the attrition rate. Both forces push population back toward N^* . If population falls below N^* , both forces reverse.

Steady-State Human Capital

Given the steady-state population N^* , define $\phi^* \equiv \phi(N^*)$ and $\theta^* \equiv \theta(\phi^*)$. The human capital steady state solves

$$h^* = A(\theta^*)^\eta (h^*)^\eta (N^*)^\mu,$$

which yields

$$h^* = [A(\theta^*)^\eta (N^*)^\mu]^{1/(1-\eta)}. \quad (4.47)$$

Since $\eta \in (0, 1)$, h^* is uniquely determined.

Proposition 16 (Existence and Uniqueness of the Interior Steady State). *Under the condition $\underline{d} < \nu/b < \bar{d}$, the dynamical system (4.43)–(4.44) admits a unique interior steady state (h^*, N^*) with $h^* > 0$ and $N^* > 0$, given by (4.46) and (4.47).*

Local Dynamics

Conditional on $N_t = N^*$, human capital dynamics follow $h_{t+1} = A(\theta^*)^\eta (N^*)^\mu h_t^\eta$. Since $\eta \in (0, 1)$, this is a concave map in h_t with a unique, globally stable fixed point h^* , and convergence is monotone from any $h_0 > 0$. For the full system where N_t is also in transition, the log-linearized dynamics around (h^*, N^*) are governed by the Jacobian⁷

$$J = \begin{pmatrix} \eta & \mu + \eta \theta'(\phi^*) \phi'(N^*) N^* / \theta^* \\ 0 & 1 + N^* [\bar{n}'(\phi^*) \phi'(N^*) - d'(N^*)] \end{pmatrix}. \quad (4.48)$$

⁷The (1, 2) entry follows from differentiating $\ln h_{t+1} = \ln A + \eta \ln \theta(\phi(N_t)) + \eta \ln h_t + \mu \ln N_t$ with respect to $\ln N_t$, evaluated at the steady state.

Because the system is triangular, the eigenvalues are the diagonal entries:

$$\lambda_1 = \eta \in (0, 1), \quad (4.49)$$

$$\lambda_2 = 1 + N^* [\bar{n}'(\phi^*)\phi'(N^*) - d'(N^*)] < 1. \quad (4.50)$$

Proposition 17 (Local Stability). *If $N^*|\bar{n}'(\phi^*)\phi'(N^*) - d'(N^*)| < 2$, both eigenvalues lie strictly inside the unit circle and the steady state (h^*, N^*) is a locally stable node. The rate of convergence of h_t is governed by η ; the rate of convergence of N_t is governed by λ_2 .*

Remark 18. *The $-d'(N^*)$ term in λ_2 makes it smaller in absolute value than in the constant- d case, strengthening stability. The triangular structure implies that the steady state is either a stable node or a saddle—never an unstable node or a spiral. The condition for a saddle would be $\lambda_2 < -1$, which requires extremely strong peer-pressure responses and is unlikely for realistic parameterizations.*

4.4.5 Comparative Statics

We now analyze how the steady state responds to a uniform shift in peer-pressure intensity, parameterized by ξ in (4.31). An increase in ξ raises $\phi(N; \xi)$ for every $N > 0$.

Effect on steady-state population. The steady-state condition $\bar{n}(\phi(N^*; \xi)) = d(N^*)$ implicitly defines $N^*(\xi)$. By the implicit function theorem,

$$\frac{dN^*}{d\xi} = -\frac{\bar{n}' \cdot \phi_\xi}{\bar{n}' \cdot \phi_N - d'} < 0,$$

since the numerator is positive ($\bar{n}' < 0$, $\phi_\xi > 0$) and the denominator is negative ($\bar{n}'\phi_N < 0$, $-d' < 0$). Stronger peer pressure reduces steady-state population.

Effect on steady-state human capital. From (4.47), $\ln h^* = \frac{1}{1-\eta}[\ln A + \eta \ln \theta^* + \mu \ln N^*]$.

Differentiating with respect to ξ :

$$\frac{d \ln h^*}{d\xi} = \frac{1}{1-\eta} \left[\eta \frac{\theta'(\phi^*)}{\theta^*} \frac{d\phi^*}{d\xi} + \mu \frac{1}{N^*} \frac{dN^*}{d\xi} \right]. \quad (4.51)$$

The first term is positive (stronger peer pressure raises education spending, which increases human capital); the second term is negative (smaller population weakens knowledge spillovers). The net effect depends on the relative magnitudes of η (returns to education) and μ (strength of the scale effect).

Proposition 19 (Ambiguity of Human Capital Response). *An increase in the peer-pressure sensitivity parameter ξ raises steady-state human capital if and only if the direct education-spending channel dominates the population-spillover channel:*

$$\eta \frac{\theta'(\phi^*)}{\theta^*} \frac{d\phi^*}{d\xi} > -\mu \frac{1}{N^*} \frac{dN^*}{d\xi}. \quad (4.52)$$

This is the central tradeoff of the dynamic model. The rat race makes each household invest more in education, which raises human capital per person. But it also reduces fertility, leading to a smaller population and weaker aggregate knowledge spillovers. If the scale effect is strong (large μ), the demographic channel dominates and the rat race is unambiguously harmful—reducing both population and human capital in the long run. If μ is small, higher education spending per household more than compensates for the loss of scale, and h^* rises despite the population decline. Equilibrium education spending at the steady state is $s^* = \theta^* h^*$; the share of income devoted to education, θ^* , is always increasing in peer-pressure intensity even when the level s^* is ambiguous.

4.4.6 Welfare and Decentralization

Consistent with the static welfare analysis in Section 4.2.2, a benevolent planner who internalizes the symmetric-equilibrium condition $s_t = \bar{s}_t$ obtains the efficient within-period allocation

$$s_t^* = \beta h_t, \quad n_t^* = \nu/b, \quad c_t^* = \alpha h_t, \quad (4.53)$$

which corresponds to the decentralized allocation with $\phi = 0$.⁸ For any $\phi_t > 0$, the decentralized economy features overspending on education ($s_t^{eq} > s_t^*$), under-provision of fertility ($n_t^{eq} < n_t^*$), and lower consumption ($c_t^{eq} < c_t^*$). This extends the static welfare result to the dynamic setting.

The planner's steady state. Under the planner's allocation, fertility is constant at ν/b and does not depend on population. The planner's population dynamics are $N_{t+1} = (1 - d(N_t) + \nu/b) N_t$, and the planner's population steady state N^{**} solves

$$d(N^{**}) = \nu/b. \quad (4.54)$$

Since d is increasing with $d(0) = \underline{d} < \nu/b < \bar{d}$, a unique $N^{**} > 0$ exists. The planner's steady-state human capital is

$$h^{**} = [A \beta^\eta (N^{**})^\mu]^{1/(1-\eta)}. \quad (4.55)$$

Proposition 20 (Planner vs. Decentralized Steady State). *The planner's steady-state population exceeds the decentralized steady-state population: $N^{**} > N^*$. The comparison of steady-state human capital is*

$$h^{**} \geq h^* \iff \beta^\eta (N^{**})^\mu \geq (\theta^*)^\eta (N^*)^\mu.$$

Proof. At N^{**} , the planner's fertility equals attrition: $\nu/b = d(N^{**})$. The decentralized fertility at N^{**} is $\bar{n}(\phi(N^{**})) < \nu/b = d(N^{**})$, since $\phi(N^{**}) > 0$. Thus at N^{**} , the decentralized economy

⁸The dependence of the peer-pressure parameter on population is not a concern for the planner's problem, because the stock of current population is not a choice variable—the flow of leisure time is.

is shrinking ($\bar{n} < d$). Since $f(N) \equiv \bar{n}(\phi(N)) - d(N)$ is strictly decreasing with $f(N^*) = 0$ and $f(N^{**}) < 0$, we have $N^{**} > N^*$.

For human capital, $h^{**} = [A\beta^\eta(N^{**})^\mu]^{1/(1-\eta)}$ and $h^* = [A(\theta^*)^\eta(N^*)^\mu]^{1/(1-\eta)}$. Since $\theta^* > \beta$ (the rat race raises the education spending share) but $N^{**} > N^*$ (the planner supports a larger population), the comparison depends on whether the planner's population-spillover advantage outweighs the decentralized economy's education-spending advantage. $\square$

The economic interpretation is the following. The rat race simultaneously inflates the education spending share from β to $\theta^* > \beta$, raising human capital per person, and depresses population from N^{**} to N^* , reducing knowledge spillovers. If the spillover effect is strong (large μ), the planner's larger population generates enough additional knowledge transmission to dominate, and $h^{**} > h^*$. If μ is small, the decentralized economy's overspending on education may raise human capital above the planner's level, even though each household is individually worse off.

Pigouvian taxation The efficient allocation can be decentralized by a tax τ_t on education spending, rebated lump-sum. Following the same logic as in Section 4.2.3, the optimal tax rate in the dynamic model is

$$\tau_t^* = \frac{\phi_t}{1 - \phi_t} = \frac{\phi(N_t)}{1 - \phi(N_t)}. \quad (4.56)$$

Proposition 21 (Dynamic Pigouvian Tax). *The tax rate $\tau_t^* = \phi(N_t)/[1 - \phi(N_t)]$ decentralizes the planner's within-period allocation at each date t . Under the tax, fertility reverts to ν/b , and population converges to the planner's steady state $N^{**} > N^*$. Because ϕ depends on N_t , the optimal tax is time-varying and converges to $\tau^{**} = \phi(N^{**})/[1 - \phi(N^{**})]$ in the long run.*

The time-varying character of the optimal tax is a substantive departure from the static result of Section 4.2.3. It implies that policy should be more aggressive precisely when and where population density makes the rat race most intense—a result that maps onto cross-regional variation within Korea, where education pressure is most severe in Seoul and other dense urban areas. Moreover, the long-run tax rate τ^{**} is higher than the tax that would prevail at the decentralized steady state, because $N^{**} > N^*$ implies $\phi(N^{**}) > \phi(N^*)$.

4.4.7 A-efficiency and the intergenerational welfare comparison.

The welfare comparison between the decentralized and planner’s allocations involves different population paths—the planner supports a larger steady-state population $N^{**} > N^*$. The appropriate welfare criterion for environments with endogenous population is A-efficiency, proposed by Golosov, Jones, and Tertilt (2007): an allocation is A-efficient if there is no feasible alternative that makes all individuals *alive in both allocations* weakly better off, with at least one strictly better off. The advantage of our simple model structure is that, under A-efficiency, the static welfare ranking is preserved: for any generation that exists under both paths, the planner’s allocation dominates (more consumption, more leisure, less wasteful education spending). The decentralized equilibrium is therefore A-inefficient. However, the comparison of steady-state human capital is ambiguous (Proposition 20): when μ is large, the planner’s larger population generates stronger knowledge spillovers, so $h^{**} > h^*$ and the planner dominates on all margins. When μ is small, the decentralized economy’s overspending on education may raise h^* above h^{**} despite the smaller population, creating an intergenerational tradeoff: the current generation overspends and is worse off, but future generations that exist under both paths inherit higher human capital. This tension is precisely the one flagged by S. Kim et al. (2024), who note that their optimal policy “may not do the same for future generations as it lowers their human capital.”

4.4.8 Calibration

We calibrate the model to a conservative Korea-based baseline using the peer-spending estimates from Young (2026) and Korean time-use data. The purpose is not to provide a full quantitative accounting of Korea’s fertility decline, but to discipline the size of the peer-pressure channel and illustrate the model’s mechanisms. Table 4.1 summarizes the parameter values; Appendix A.3.1 provides detailed derivations.

The peer-pressure intensity $\phi_0 = 0.24$ is taken from the top-quartile transition estimates in Young (2026). The tutoring-spending share $\theta_0 = 0.06$ is the baseline share in the KELS sample. Together these pin the education weight $\beta = 0.0463$. The child-cost parameter $b = 0.10$ is

calibrated from the 2019 Korean Time Use Survey. The density-dependent attrition function $d(N)$ uses a parametric form analogous to $\phi(N)$, with baseline attrition $\underline{d} = 0.90$, upper bound $\bar{d} = 1.15$, and density sensitivity ζ chosen so that the decentralized economy is at a population steady state at $N_0 = 1$. This yields the planner's steady-state population at $N^{**} \approx 1.27$, i.e., removing the education externality would support a 27% larger population.

Table 4.1: Baseline calibration

Parameter	Value	Description	Source/Role
<i>Peer pressure and education</i>			
ϕ_0	0.2400	Baseline peer-pressure intensity	Young (2026)
θ_0	0.0600	Baseline tutoring-spending share	Young (2026)
β	0.0463	Education weight	$\theta_0 = \beta/[1 - \phi_0(1 - \beta)]$
ξ	0.3158	Pop. sensitivity of peer pressure	$\phi(N_0) = \phi_0$
<i>Preferences and child cost</i>			
α	0.8500	Consumption weight	Chosen baseline
ν	0.1037	Fertility weight	Residual: $\nu = 1 - \alpha - \beta$
b	0.1000	Time cost per child	Korean Time Use Survey
<i>Density-dependent attrition</i>			
$\underline{d}$	0.9000	Baseline attrition rate	Low-density benchmark
$\bar{d}$	1.1500	Upper bound on attrition	$\bar{d} > \nu/b$
ζ	0.9593	Density sensitivity of attrition	$d(N_0) = \bar{n}(\phi_0)$
<i>Human capital production</i>			
η	0.3000	Education elasticity	Baseline sensitivity
μ	0.0500	Population spillover elasticity	Baseline sensitivity
A	2.3257	Scale parameter	Normalizes $h_0 = 1$
<i>Normalizations and implied values</i>			
N_0	1.0000	Initial population	Normalization
h_0	1.0000	Initial human capital	Normalization
$n^{eq}(\phi_0)$	1.0224	Decentralized fertility	Implied
$n^* = \nu/b$	1.0373	Planner fertility	Implied
N^{**}	1.2700	Planner steady-state population	Implied by $d(N^{**}) = \nu/b$
τ_0	0.3158	Pigouvian wedge at baseline	$\phi_0/(1 - \phi_0)$

At the baseline, the model generates a decentralized–planner fertility wedge of about 1.5 percentage points ($n^{eq} = 1.022$ vs. $n^* = 1.037$). This modest gap reflects the conservative nature

of the calibration: $\phi_0 = 0.24$ captures only the measured tutoring-spending response, not the full range of child-quality investments that peer pressure may distort. The implied Pigouvian wedge is $\tau_0 = 0.316$, indicating that shadow education spending would need to be taxed at about 32% to offset the positional externality at the baseline. The calibration also implies that policies that restrict the supply of tutoring—such as Korea’s 1980 ban—leave the underlying demand for relative standing intact, since the household’s incentive to spend $s - \phi\bar{s}$ above peers is unchanged.

Figure 4.5 plots the calibrated nullclines. The population nullcline $\bar{n}(\phi(N)) = d(N)$ is independent of human capital; the human-capital nullcline $h(N) = [A\theta(\phi(N))^\eta N^\mu]^{1/(1-\eta)}$ is increasing and concave in population. The dashed curve shows the planner’s human-capital locus, obtained by replacing $\theta(\phi(N))$ with β . The planner’s population steady state N^{**} lies to the right of N^* because efficiently higher fertility (ν/b) requires a larger population to be absorbed by the density-dependent attrition rate.

Figure 4.6 compares the transition paths of the decentralized economy and the planner allocation. The decentralized economy features a higher education-spending share because households do not internalize that relative spending cannot raise all children’s rank at once. This comes at the expense of fertility, which reduces long-run population relative to the planner path.

4.5 Conclusion

This paper shows how Korea’s education race, shaped by peer pressure and social competition, can lead households to overspend on private tutoring at the cost of leisure and fertility. When perceived child quality depends on spending relative to peers, families increase tutoring expenditure to avoid falling behind, even if the academic return is small. The result is an equilibrium where all households devote more resources to shadow education, yet each is worse off than under the socially efficient outcome where the positional motive is removed.

The model also clarifies why Korea’s past attempts to regulate private tutoring had limited success. The shadow education ban in the 1980s focused on restricting supply, but the demand for relative standing remained. Families substituted into informal lessons, longer study hours,

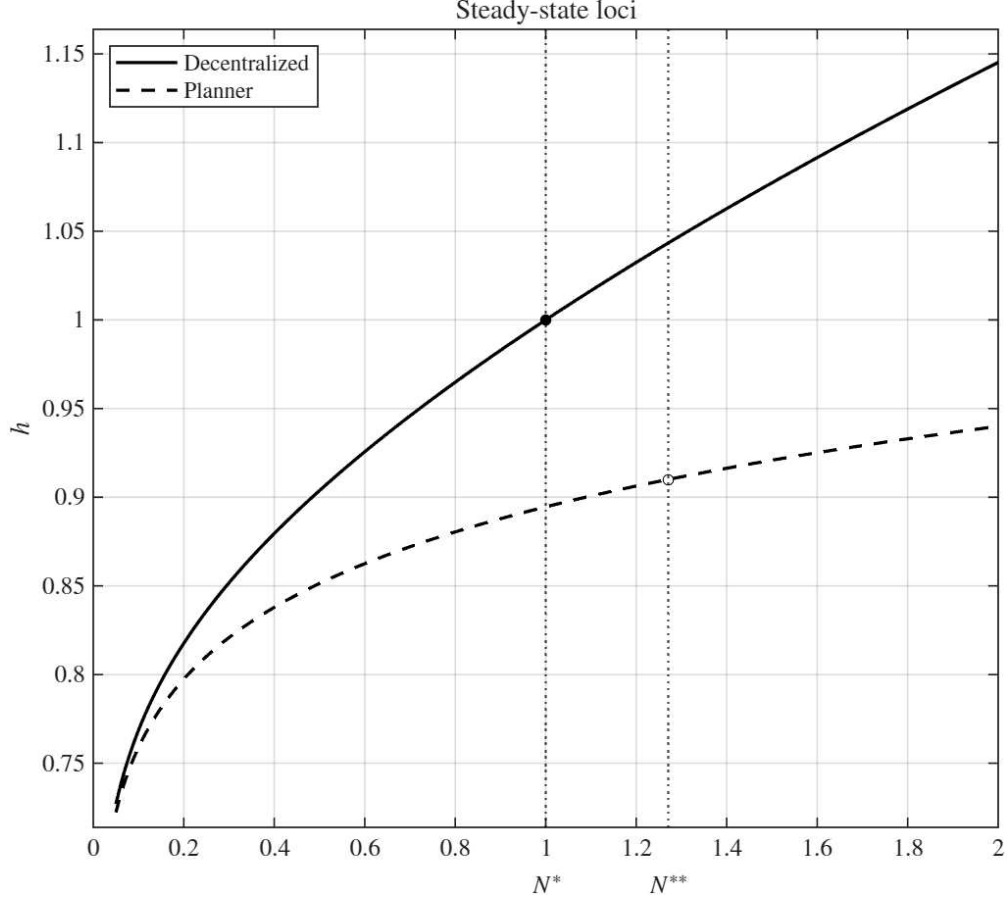


Figure 4.5: Population and human-capital steady-state loci. The solid curve is the decentralized human-capital locus; the dashed curve is the planner's. The decentralized population steady state N^* is where $\bar{n}(\phi(N)) = d(N)$; the planner's population steady state $N^{**} > N^*$ is where $\nu/b = d(N)$.

or even overseas education. Policies that alter incentives—such as a tax on tutoring expenditure that internalizes the externality—better align private and social returns. In the static model, a Pigouvian tax calibrated at $\tau^* = \phi/(1 - \phi)$ restores the efficient allocation by offsetting the positional component of spending. In the dynamic OLG extension, the optimal tax is time-varying, $\tau_t^* = \phi(N_t)/[1 - \phi(N_t)]$: policy should be more aggressive when and where population density makes peer pressure most intense.

The OLG extension developed in Section 4.4 places these static mechanisms in a framework where population and human capital evolve jointly. The key assumption that closes the model is making peer-effect intensity ϕ depend on population size N_t through a university-admission tournament. It also generates a unique interior steady state via a self-correcting mechanism: larger

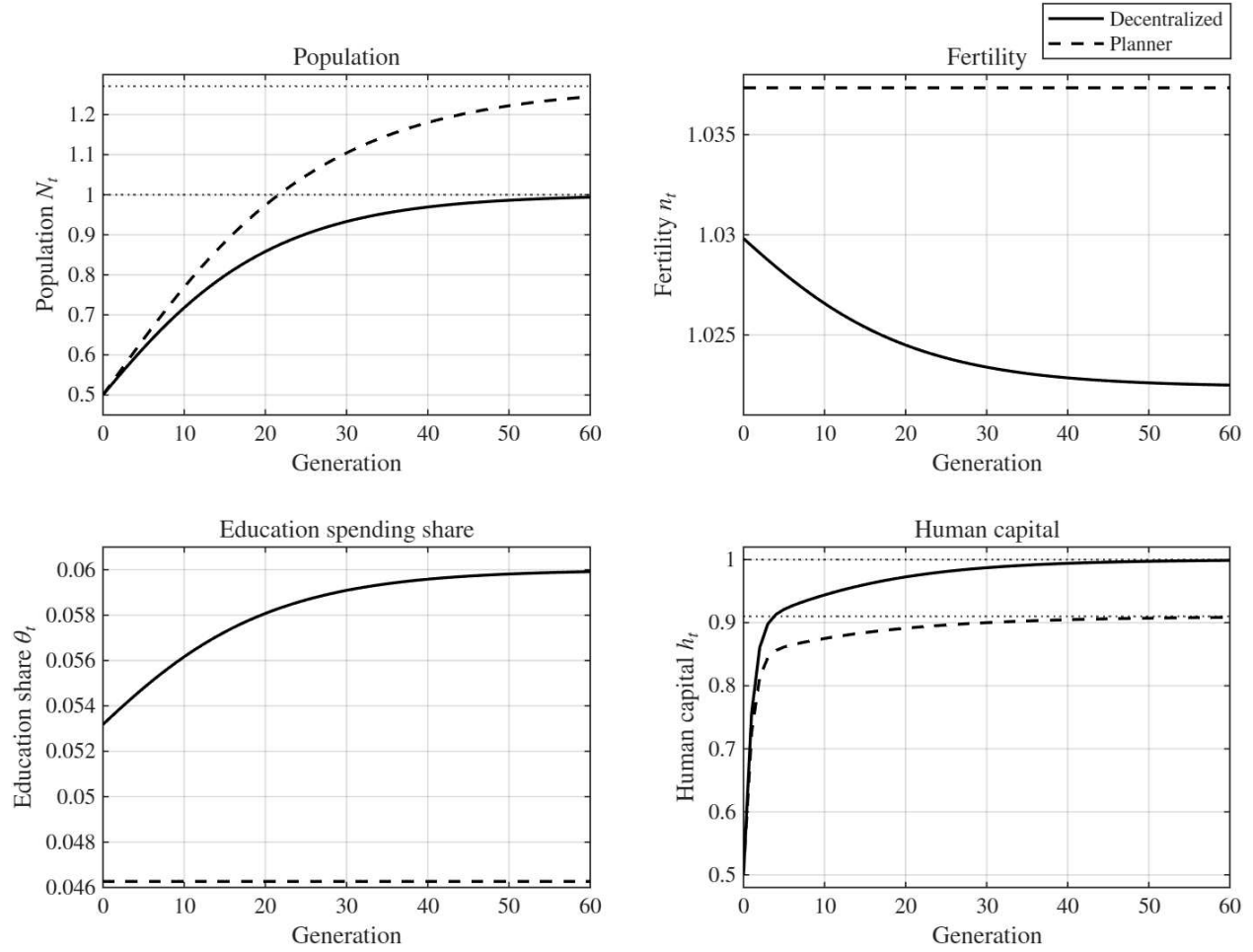


Figure 4.6: Calibrated transition paths: decentralized economy and planner. The figure compares the paths of population, fertility, and human capital when households take peer spending as given versus when the positional externality is removed.

populations intensify the rat race, which depresses fertility toward the death rate, stabilizing population. The dynamic model yields a sharp result with demographic bite. Stronger peer pressure unambiguously reduces steady-state population, but its effect on steady-state human capital is ambiguous: the direct education-spending channel raises h^* , while the weakened knowledge spillover from a smaller population reduces it. When the scale effect is strong, the rat race is unambiguously harmful: it reduces both population and human capital in the long run, a micro-founded version of the “empty planet” scenario emphasized by Jones (2022). When evaluated under A-efficiency (Golosov et al., 2007), the decentralized equilibrium is inefficient for all generations that exist un-

der both the decentralized and the planner's paths, but an intergenerational tradeoff can emerge when the education-spending channel dominates the knowledge-spillover channel.

The broader implication is that peer-driven status competition in education not only misallocates household resources but also contributes to Korea's persistently low fertility. As education becomes both a form of status signaling and a substitute for childcare, the burden of maintaining competitiveness raises the cost of family formation. Addressing these pressures requires policies that recognize the social nature of education spending, rather than simply restricting its form. The Korean case illustrates how even well-intentioned bans can fail when they leave the underlying incentives untouched.

The model also helps interpret Korea's mixed experience with shadow-education regulation. Policies that restrict particular tutoring inputs may reduce some visible forms of spending, but they do not necessarily remove the demand for relative standing. The model instead points to the underlying positional externality: households overspend because each family takes peer spending as given, even though relative rank cannot improve for everyone at once. This suggests that the relevant policy issue is not simply the amount of tutoring, but the social payoff structure that makes tutoring expenditures strategically reinforcing.

Several extensions suggest themselves. First, the closure $\phi = \phi(N)$ could be enriched by letting peer-effect intensity depend on urbanization or density, rather than raw population, with an explicit spatial dimension. Second, introducing heterogeneity across households—for example, in β or b —would allow analysis of the distributional effects of the rat race along the lines of S. Kim et al. (2024). Third, endogenizing the production side through a neoclassical production function with physical capital would add a third state variable and allow the model to speak to savings and investment dynamics. Finally, embedding the model in a small open economy with international migration would allow the rat race to affect brain drain as well as fertility, an increasingly important margin in the Korean context.

References

- Abadie, A., & Imbens, G. W. (2006). Large sample properties of matching estimators for average treatment effects. *Econometrica*, 74(1), 235–267. doi: 10.1111/j.1468-0262.2006.00655.x
- Agénor, P. R. (2017). A computable overlapping generations model for gender and growth policy analysis. *Macroeconomic Dynamics*, 21(1), 11–54. doi: 10.1017/S136510051500036X
- Agénor, P. R., & Agénor, M. (2023). Access to infrastructure and women's time allocation: Implications for growth and gender equality. *Journal of Macroeconomics*, 75, 103472. doi: 10.1016/j.jmacro.2022.103472
- Agénor, P. R., Ozdemir, K. K., & Moreira, E. P. (2021). Gender gaps in the labour market and economic growth. *Economica*, 88(350), 235–270. doi: 10.1111/ecca.12363
- Anderson, T. M., & Kohler, H.-P. (2013). Education fever and the east asian fertility puzzle: A case study of low fertility in south korea. *Asian Population Studies*, 9(2), 196–215. doi: 10.1080/17441730.2013.797293
- Ashraf, Q., & Galor, O. (2011). Dynamics and stagnation in the Malthusian epoch. *American Economic Review*, 101(5), 2003–2041.
- Bae, S. H., & Choi, K. H. (2024). The cause of institutionalized private tutoring in korea: Defective public schooling or a universal desire for family reproduction? *ECNU Review of Education*, 7(1), 12–41. doi: 10.1177/20965311231182722
- Bang, H., & Robins, J. M. (2005). Doubly robust estimation in missing data and causal inference models. *Biometrics*, 61(4), 962–973. doi: 10.1111/j.1541-0420.2005.00377.x
- Becker, G. S. (1965). A theory of the allocation of time. *Economic Journal*, 75(299), 493–517.
- Bernasconi, M., & Kirchkamp, O. (2000). Why do monetary policies matter? An experimental study of saving and inflation in an overlapping generations model. *Journal of Monetary Economics*, 46(2), 315–343. doi: 10.1016/S0304-3932(00)00028-1
- Bertrand, M. (2020). Gender in the twenty-first century. *AEA Papers and Proceedings*, 110, 1–24. doi: 10.1257/pandp.20201126

- Bertrand, M., Kamenica, E., & Pan, J. (2015). Gender identity and relative income within households. *Quarterly Journal of Economics*, 130(2), 571–614. doi: 10.1093/qje/qjv001
- Bianchi, S. M. (2000). Maternal employment and time with children: Dramatic change or surprising continuity? *Demography*, 37(4), 401–414. doi: 10.1353/dem.2000.0001
- Bianchi, S. M., Robinson, J. P., & Milkie, M. A. (2006). *Changing Rhythms of American Family Life*. New York: Russell Sage Foundation.
- Bittman, M., England, P., Sayer, L., Folbre, N., & Matheson, G. (2003). When does gender trump money? bargaining and time in household work. *American Journal of Sociology*, 109(1), 186–214. doi: 10.1086/378341
- Braun, R. A., & Ikeda, D. (2021). *Monetary policy over the lifecycle* (Working Paper No. 2021-20a). Federal Reserve Bank of Atlanta.
- Braunstein, E. (2015). *Economic growth and social reproduction: Gender inequality as cause and consequence* (Tech. Rep.). New York: UN Women. doi: 10.18356/9789210572040
- Braunstein, E., van Staveren, I., & Tavani, D. (2011). Embedding care and unpaid work in macroeconomic modeling: A structuralist approach. *Feminist Economics*, 17(4), 5–31. doi: 10.1080/13545701.2011.602354
- Bray, M. (2014). The impact of shadow education on student academic achievement: Why the research is inconclusive and what can be done about it. *Asia Pacific Education Review*, 15(3), 381–389. doi: 10.1007/s12564-014-9326-9
- Byun, S. Y. (2014). Shadow education and academic success in republic of korea. In *Korean education in changing economic and demographic contexts* (Vol. 23, pp. 39–58). Dordrecht: Springer. doi: 10.1007/978-981-4451-27-7_3
- Caravaggio, A., & Sodini, M. (2023). Environmental sustainability, nonlinear dynamics and chaos reloaded: 0 matters! *Communications in Nonlinear Science and Numerical Simulation*, 117, 106908. doi: 10.1016/j.cnsns.2022.106908
- Caucutt, E., Lochner, L., Mullins, J., & Park, Y. (2026). Child skill production: Accounting for parental and market-based time and goods investments. *Journal of Political Economy*,

- 134(1), 150–209. doi: 10.1086/738480
- Cervellati, M., & Sunde, U. (2005). Human capital formation, life expectancy, and the process of development. *American Economic Review*, 95(5), 1653–1672.
- Chiappori, P.-A., & Mazzocco, M. (2017). Static and intertemporal household decisions. *Journal of Economic Literature*, 55(3), 985–1045. doi: 10.1257/jel.20150715
- Choi, H., & Choi, Á. (2015). *When one door closes: The impact of the Hagwon curfew on the consumption of private tutoring in the republic of korea* (Tech. Rep. No. 2015/32). Institut d’Economia de Barcelona (IEB) / XREAP. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2689777
- Choi, H., & Choi, Á. (2016). Regulating private tutoring consumption in korea: Lessons from another failure. *International Journal of Educational Development*, 49, 144–156. doi: 10.1016/j.ijedudev.2016.03.006
- Choi, Y., & Park, H. (2016). Shadow education and educational inequality in south korea: Examining effect heterogeneity of shadow education on middle school seniors’ achievement test scores. *Research in Social Stratification and Mobility*, 44, 22–32. doi: 10.1016/j.rssm.2016.01.002
- Craig, L. (2006). Does father care mean fathers share? a comparison of how mothers and fathers in intact families spend time with children. *Gender & Society*, 20(2), 259–281. doi: 10.1177/0891243205285212
- Crettez, B., Michel, P., & Wigniolle, B. (2002). Optimal monetary policy, taxes, and public debt in an intertemporal equilibrium. *Journal of Public Economic Theory*, 4(3), 299–316. doi: 10.1111/1467-9779.00099
- Cunha, F., Elo, I., & Culhane, J. (2022). Maternal subjective expectations about the technology of skill formation predict investments in children one year later. *Journal of Econometrics*, 231(1), 3–32. doi: 10.1016/j.jeconom.2020.02.005
- Daminger, A. (2019). The cognitive dimension of household labor. *American Sociological Review*, 84(4), 609–633. doi: 10.1177/0003122419859007

- Data2X. (2025). *Role of time use data in policymaking in south korea*. Country case study. Retrieved from <https://data2x.org/wp-content/uploads/2025/02/South-Korea-TUCS-WR-250225.pdf> (Summarizes official 2019 Korean Time Use Survey results, including parent childcare and unpaid-work statistics)
- Dávila-Fernández, M. J., Giombini, G., & Sánchez-Carrera, E. J. (2025). Climateflation and monetary policy in an environmental OLG growth model. *Environmental and Resource Economics*, 88(11), 2937–2968. doi: 10.1007/s10640-025-00991-1
- de la Croix, D., & Gosseries, A. (2012). The natalist bias of pollution control. *Journal of Environmental Economics and Management*, 63, 271–287.
- de la Croix, D., & Michel, P. (2002). *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*. Cambridge, UK; New York, NY: Cambridge University Press. doi: 10.1017/CBO9780511606434
- Doepke, M., Hannusch, A., & Kindermann, F. (2022). *The economics of fertility: A new era* (Tech. Rep. No. 29948). National Bureau of Economic Research. doi: 10.3386/w29948
- Doepke, M., & Tertilt, M. (2016). Families in macroeconomics. In J. B. Taylor & H. Uhlig (Eds.), *Handbook of macroeconomics* (Vol. 2, pp. 1789–1891). Elsevier. doi: 10.1016/bs.hesmac.2016.04.002
- Folbre, N. (2001). *The Invisible Heart: Economics and Family Values*. New York: The New Press.
- Frank, R. H., & Cook, P. J. (1995). *The winner-take-all society: Why the few at the top get so much more than the rest of us*. New York: Free Press.
- Galí, J. (2014). Monetary policy and rational asset price bubbles. *American Economic Review*, 104(3), 721–752. doi: 10.1257/aer.104.3.721
- Galor, O. (2011). *Unified growth theory*. Princeton: Princeton University Press.
- Galor, O., & Weil, D. N. (2000). Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review*, 90(4), 806–828.
- Goldscheider, F., Bernhardt, E., & Lappegård, T. (2015). The gender revolution: A framework

- for understanding changing family and demographic behavior. *Population and Development Review*, 41(2), 207–239. doi: 10.1111/j.1728-4457.2015.00045.x
- Golosov, M., Jones, L. E., & Tertilt, M. (2007). Efficiency with endogenous population growth. *Econometrica*, 75(4), 1039–1071.
- Heintz, J., & Folbre, N. (2022). Endogenous growth, population dynamics, and economic structure: Long-run macroeconomics when demography matters. *Feminist Economics*, 28(3), 145–163. doi: 10.1080/13545701.2021.1937266
- Hiraguchi, R. (2014). Optimal monetary policy in OLG models with long-lived agents: a note. *Journal of Public Economic Theory*, 16(1), 164–172. doi: 10.1111/jpet.12054
- Hopkins, E., & Kornienko, T. (2004). Running to keep in the same place: Consumer choice as a game of status. *American Economic Review*, 94(4), 1085–1107. doi: 10.1257/0002828042002705
- Hu, G., Ma, G., Qiao, W., & Wallace, N. (2023). Conventional monetary policy in OLG models: revisiting the asset-substitution channel. *International Economic Review*, 64(3), 875–892. doi: 10.1111/iere.12626
- Ichino, A., Olsson, M., Petrongolo, B., & Skogman Thoursie, P. (2019). *Economic incentives, childcare and gender identity norms* (Tech. Rep. No. Working Paper No. 1283). Stockholm: Research Institute of Industrial Economics (IFN).
- Ichino, A., Olsson, M., Petrongolo, B., & Skogman Thoursie, P. (2026). Taxes, childcare, and gender identity norms. *Journal of Labor Economics*. (Epub ahead of print) doi: 10.1086/732773
- Imbens, G. W., & Rubin, D. B. (2015). *Causal inference for statistics, social, and biomedical sciences: An introduction*. Cambridge: Cambridge University Press.
- Jedwab, R., & Vollrath, D. (2019). The urban mortality transition and poor-country urbanization. *American Economic Journal: Macroeconomics*, 11(1), 223–275.
- John, A., & Pecchenino, R. (1994). An overlapping generations model of growth and the environment. *The Economic Journal*, 104(427), 1393–1410. doi: 10.2307/2235455

- Jones, C. I. (2022). The end of economic growth? unintended consequences of a declining population. *American Economic Review*, 112(11), 3489–3527. Retrieved from <https://www.aeaweb.org/articles?id=10.1257/aer.20201605> doi: 10.1257/aer.20201605
- Jørgensen, T. H., & Søgaaard, J. E. (2024). The division of parental leave: Empirical evidence and policy design. *Journal of Public Economics*, 239, 105202. doi: 10.1016/j.jpubeco.2024.105202
- Jun, J., & Eun, K.-S. (2025, March). *Role of time use data in policymaking in south korea* (Country case study). Data2X. Retrieved from <https://data2x.org/wp-content/uploads/2025/02/South-Korea-TUCS-WR-250225.pdf>
- Kang, C. (2007). *Does money matter? the effect of private educational expenditures on academic performance* (Working paper). Department of Economics, National University of Singapore.
- Kang, C., & Park, Y. (2021). Private tutoring and distribution of student academic outcomes: An implication of the presence of private tutoring for educational inequality. *Korean Economic Review*, 37(2), 287–326. doi: 10.22841/kerdoi.2021.37.2.004
- Kim, E., & Yi, J.-S. (2024). Determinants of fertility intentions among south koreans: Systematic review and meta-analysis. *Behavioral Sciences*, 14(10), 939. doi: 10.3390/bs14100939
- Kim, E. H.-W. (2017). Division of domestic labour and lowest-low fertility in south korea. *Demographic Research*, 37(24), 743–768. doi: 10.4054/DemRes.2017.37.24
- Kim, S., & Lee, J.-H. (2010). Private tutoring and demand for education in south korea. *Economic Development and Cultural Change*, 58(2), 259–296. doi: 10.1086/648186
- Kim, S., Tertilt, M., & Yum, M. (2024). Status externalities in education and low birth rates in korea. *American Economic Review*, 114(6), 1576–1611. doi: 10.1257/aer.20220583
- Kim, T., Jang, H., & Kim, J. (2022a). Do peers affect private tutoring engagement in korea? evidence from a quasi-experimental approach. *Asia Pacific Education Review*, 23(2), 271–283. doi: 10.1007/s12564-021-09738-1

- Kim, T., Jang, H., & Kim, J. (2022b). Peers' private tutoring and adolescent depressive symptoms: Quasi-experimental evidence from secondary schools in south korea. *Journal of Adolescent Health, 70*(4), 658–665. doi: 10.1016/j.jadohealth.2021.10.040
- Kleven, H., Landais, C., Posch, J., Steinhauer, A., & Zweimüller, J. (2024). Do family policies reduce gender inequality? evidence from 60 years of policy experimentation. *American Economic Journal: Economic Policy, 16*(2), 110–149. doi: 10.1257/pol.20210346
- Lazear, E. P., & Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy, 89*(5), 841–864. doi: 10.1086/261010
- Lee, J. H., Park, J. S., & Chin, S. (2025). An aggregate economic value perspective on korea's marriage decline: Transitory and secular. *Journal of Demographic Economics, 1*–21. doi: 10.1017/dem.2025.10008
- Lee, S.-K. (2011). Local perspectives of korean shadow education. *Reconsidering Development, 1*(1), 1–18.
- Lee, Y. (2022). Is leave for fathers pronatalist? a mixed-methods study of the impact of fathers' uptake of parental leave on couples' childbearing intentions in south korea. *Population Research and Policy Review, 41*(4), 1471–1500. doi: 10.1007/s11113-022-09697-4
- Lorentzen, P., McMillan, J., & Wacziarg, R. (2008). Death and development. *Journal of Economic Growth, 13*(2), 81–124.
- Lunceford, J. K., & Davidian, M. (2004). Stratification and weighting via the propensity score in estimation of causal treatment effects: A comparative study. *Statistics in Medicine, 23*(19), 2937–2960. doi: 10.1002/sim.1903
- Lundberg, S., & Pollak, R. A. (1993). Separate spheres bargaining and the marriage market. *Journal of Political Economy, 101*(6), 988–1010. doi: 10.1086/261912
- Lundberg, S., & Pollak, R. A. (1996). Bargaining and distribution in marriage. *Journal of Economic Perspectives, 10*(4), 139–158. doi: 10.1257/jep.10.4.139
- Manser, M., & Brown, M. (1980). Marriage and household decision-making: A bargaining analysis. *International Economic Review, 21*(1), 31–44. doi: 10.2307/2526238

- McDonald, P. (2000). Gender equity in theories of fertility transition. *Population and Development Review*, 26(3), 427–439. doi: 10.1111/j.1728-4457.2000.00427.x
- McElroy, M. B., & Horney, M. J. (1981). Nash-bargained household decisions: Toward a generalization of the theory of demand. *International Economic Review*, 22(2), 333–349. doi: 10.2307/2526280
- Medio, A., & Lines, M. (2001). *Nonlinear Dynamics: A Primer*. Cambridge: Cambridge University Press.
- Ministry of Data and Statistics. (2025a, August 27). *Birth statistics in 2024* (Press release). Ministry of Data and Statistics. Retrieved from https://mods.go.kr/boardDownload.es?bid=11773&list_no=439008&seq=2
- Ministry of Data and Statistics. (2025b, March 13). *Private education expenditures survey of elementary, middle and high school students in 2024* (Press release). Ministry of Data and Statistics. Retrieved from https://mods.go.kr/boardDownload.es?bid=11758&list_no=436035&seq=1
- Moldovanu, B., & Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review*, 91(3), 542–558. doi: 10.1257/aer.91.3.542
- Moldovanu, B., Sela, A., & Shi, X. (2007). Contests for status. *Journal of Political Economy*, 115(2), 338–363. doi: 10.1086/518010
- OECD. (2024). *Education at a glance 2024: Korea country note*. OECD. Retrieved from https://www.oecd.org/en/publications/education-at-a-glance-2024-country-notes_fab77ef0-en/korea-republic-of_a362e6bb-en.html
- Park, H. (2021). Diverging gaps in childcare time by parental education in south korea. *Demographic Research*, 44(6), 143–156. doi: 10.4054/DemRes.2021.44.6
- Park, S.-M., Cho, S.-I., & Choi, M.-K. (2010). The effect of paternal investment on female fertility intention in south korea. *Evolution and Human Behavior*, 31(6), 447–452. doi: 10.1016/j.evolhumbehav.2010.07.001

- Patnaik, A. (2019). Reserving time for daddy: The consequences of fathers' quotas. *Journal of Labor Economics*, 37(4), 1009–1059. doi: 10.1086/703115
- Persson, P., & Rossin-Slater, M. (2024). When dad can stay home: Fathers' workplace flexibility and maternal health. *American Economic Journal: Applied Economics*, 16(4), 186–219. doi: 10.1257/app.20220400
- Piao, H., & Hwang, H. (2021). Shadow education policy in korea during the covid-19 pandemic. *ECNU Review of Education*, 4(3), 652–666. doi: 10.1177/20965311211013825
- Raley, S., Bianchi, S. M., & Wang, W. (2012). When do fathers care? mothers' economic contribution and fathers' involvement in child care. *American Journal of Sociology*, 117(5), 1422–1459. doi: 10.1086/663354
- Raymo, J. M., & Park, H. (2020). Marriage decline in korea: Changing composition of the domestic marriage market and growth in international marriage. *Demography*, 57(1), 171–194. doi: 10.1007/s13524-019-00844-9
- Raymo, J. M., Park, H., Xie, Y., & Yeung, W.-j. J. (2015). Marriage and family in east asia: Continuity and change. *Annual Review of Sociology*, 41, 471–492. doi: 10.1146/annurev-soc-073014-112428
- Rosenbaum, P. R., & Rubin, D. B. (1983). The central role of the propensity score in observational studies for causal effects. *Biometrika*, 70(1), 41–55. doi: 10.1093/biomet/70.1.41
- Rosenqvist, O. (2024). Reducing the gender gap in parental leave through economic incentives? evidence from the gender equality bonus in sweden. *Labour Economics*, 89, 102538. doi: 10.1016/j.labeco.2024.102538
- Ryu, D., & Kang, C. (2013). Do private tutoring expenditures raise academic performance? evidence from middle school students in south korea. *Asian Economic Journal*, 27(1), 59–83. doi: 10.1111/asej.12002
- Sayer, L. C., Bianchi, S. M., & Robinson, J. P. (2004). Are parents investing less in children? trends in mothers' and fathers' time with children. *American Journal of Sociology*, 110(1), 1–43. doi: 10.1086/386270

- Seth, M. J. (2002). *Education fever: Society, politics, and the pursuit of schooling in south korea*. Honolulu: University of Hawai'i Press.
- Sterk, V., & Tenreyro, S. (2018). The transmission of monetary policy through redistribution and durable purchases. *Journal of Monetary Economics*, 99, 124–137. doi: 10.1016/j.jmoneco.2018.07.004
- Strulik, H., & Weisdorf, J. (2008). Population, food, and knowledge: A simple unified growth theory. *Journal of Economic Growth*, 13(3), 195–216.
- Stuart, E. A. (2010). Matching methods for causal inference: A review and a look forward. *Statistical Science*, 25(1), 1–21. doi: 10.1214/09-STS313
- Tamm, M. (2019). Fathers' parental leave-taking, childcare involvement and labor market participation. *Labour Economics*, 59, 184–197. doi: 10.1016/j.labeco.2019.04.007
- Tavani, D., & Young, J. (2026). *Rat race, educational signaling, and fertility*. (Colorado State University, mimeo)
- Thomas, J., Rowe, F., Williamson, P., & Lin, E. S. (2022). The effect of leave policies on increasing fertility: a systematic review. *Humanities and Social Sciences Communications*, 9(1), 1–16. doi: 10.1057/s41599-022-01270-w
- Voigtländer, N., & Voth, H.-J. (2013). The three horsemen of riches: Plague, war, and urbanization in early modern Europe. *Review of Economic Studies*, 80(2), 774–811.
- von Thadden, L. (2012). Monetary policy rules in an OLG model with non-superneutral money. *Journal of Macroeconomics*, 34(1), 147–166. doi: 10.1016/j.jmacro.2011.10.004
- World Bank. (2025). *Fertility rate, total (births per woman) – korea, rep*. World Development Indicators. Retrieved from <https://data.worldbank.org/indicator/SP.DYN.TFRT.IN?locations=KR> (Indicator SP.DYN.TFRT.IN; annual series 1960–2023)
- Yoon, S.-Y. (2017). The influence of a supportive environment for families on women's fertility intentions and behavior in south korea. *Demographic Research*, 36(7), 227–254. doi: 10.4054/DemRes.2017.36.7
- Young, J. (2026). *Can parents overspend in their children's education? peer effects in korea's*

A.1 Appendix A

A.1.1 Balance and Standardized Differences

Table A.1: Covariate Balance (AIPW, Upward to Top 25%)

	Raw SMD	Weighted SMD	Raw Var. ratio	Weighted Var. ratio
Household income	0.09	-0.00	1.20	1.08
Parents' avg. education	0.41	-0.03	0.84	0.84
Metropolitan	0.24	-0.02	1.00	1.01
Medium city	-0.06	0.02	0.99	1.01
Male	-0.03	0.00	1.00	1.00
Disability	-0.03	-0.00	0.86	1.00
Baseline spend (Y3)	0.19	-0.03	0.91	0.62
Achievement index (z)	0.45	-0.04	0.96	0.80

Post-adjustment absolute standardized differences < 0.10; variance ratios near 1. Weighted = AIPW reweighting or PSM matching. Thresholds per Stuart (2010).

A.1.2 Robustness checks

Table A.2: ATT (Delta Y4–Y3), Upward to Top 25 percent – AIPW and PSM (Bootstrap SEs, 500 reps)

	(1) AIPW (Raw)	(2) PSM (Raw)	(3) AIPW (Wins.)	(4) PSM (Wins.)
Total	6.888*** (1.875)	6.118** (2.761)	5.818*** (1.543)	5.355** (2.316)
Math	3.892*** (0.965)	3.628** (1.410)	3.027*** (0.804)	2.872** (1.177)
English	2.983*** (0.830)	2.011 (1.306)	2.565*** (0.710)	1.572 (1.106)
Korean	0.016 (0.395)	0.811 (0.725)	-0.053 (0.319)	0.818 (0.535)
Observations	5626	5626	5626	5626

Table A.3: ATT (Delta Y4–Y3), Upward to Top 50 percent – AIPW and PSM (Bootstrap SEs, 500 reps)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	1.536 (1.604)	4.808** (2.322)	1.697 (1.452)	4.322** (2.081)
Math	0.596 (0.726)	1.148 (1.145)	0.937 (0.681)	1.191 (0.997)
English	0.806 (0.745)	1.128 (1.113)	0.757 (0.659)	0.818 (1.000)
Korean	0.200 (0.413)	0.921 (0.689)	0.226 (0.361)	0.635 (0.598)
Observations	5626	5626	5626	5626

A.2 Appendix B

A.2.1 Mathematical Appendix

Proof of Proposition 1

When the time dad and mom dedicate to parenting are substitutes, from the equilibrium conditions, we obtain the following isoquants:

$$\bar{k} = A^{\frac{1}{1-\beta-2\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \bar{N}^\gamma \right]^{\frac{\alpha}{1-\beta-2\alpha}} \quad (\text{A.2.57})$$

$$\bar{k} = \left(\frac{1-\alpha}{2\phi \bar{N}^\gamma} \right) \quad (\text{A.2.58})$$

Combining (A.2.57) and (A.2.58), we have:

$$\frac{1-\alpha}{2\phi \bar{N}^\gamma} = A^{\frac{1}{1-\beta-2\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \bar{N}^\gamma \right]^{\frac{\alpha}{1-\beta-2\alpha}}$$

Table A.4: ATT (Value Added Method), Upward to Top 25 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	6.888*** (1.407)	6.299*** (1.858)	6.346*** (1.184)	6.098*** (1.551)
Math	3.892*** (0.751)	4.330*** (0.860)	3.332*** (0.630)	3.813*** (0.730)
English	2.983*** (0.664)	2.764*** (0.796)	2.722*** (0.578)	2.592*** (0.695)
Korean	0.016 (0.361)	0.460 (0.417)	0.064 (0.297)	0.431 (0.347)
Observations	5626	5626	5626	5626

Table A.5: ATT (Value Added Method), Upward to Top 50 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	1.536 (1.174)	3.460** (1.663)	1.613 (1.051)	3.750** (1.501)
Math	0.596 (0.572)	0.724 (0.720)	0.797 (0.542)	0.735 (0.703)
English	0.806 (0.587)	0.530 (0.740)	0.737 (0.498)	0.423 (0.652)
Korean	0.200 (0.299)	0.186 (0.385)	0.255 (0.266)	0.162 (0.349)
Observations	5626	5626	5626	5626

Table A.6: ATT (Delta Y4–Y3), Upward to Top 30 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	5.381*** (1.282)	5.870*** (2.107)	4.683*** (1.087)	4.777*** (1.633)
Math	3.399*** (0.679)	3.101*** (0.811)	2.823*** (0.575)	2.644*** (0.743)
English	2.193*** (0.609)	1.408* (0.815)	1.949*** (0.535)	1.355** (0.681)
Korean	-0.210 (0.337)	-0.024 (0.453)	-0.210 (0.283)	-0.135 (0.350)
Observations	5626	5626	5626	5626

Table A.7: ATT (Delta Y4–Y3), Upward to Top 40 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	3.893*** (1.173)	2.714 (1.849)	3.619*** (1.017)	2.649* (1.544)
Math	1.995*** (0.593)	2.221*** (0.725)	1.933*** (0.527)	2.000*** (0.670)
English	1.812*** (0.577)	2.490*** (0.718)	1.634*** (0.490)	2.299*** (0.632)
Korean	0.096 (0.310)	-0.275 (0.556)	0.068 (0.261)	-0.069 (0.352)
Observations	5626	5626	5626	5626

Table A.8: ATT (Delta Y4–Y3), Downward to Bottom 30 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	-10.814*** (0.927)	-9.017*** (1.167)	-9.978*** (0.826)	-8.357*** (1.136)
Math	-5.143*** (0.429)	-4.888*** (0.554)	-4.743*** (0.404)	-4.460*** (0.505)
English	-4.226*** (0.408)	-3.203*** (0.502)	-3.850*** (0.385)	-2.961*** (0.475)
Korean	-1.423*** (0.256)	-1.191*** (0.285)	-1.283*** (0.210)	-1.117*** (0.249)
Observations	5626	5626	5626	5626

Table A.9: ATT (Delta Y4–Y3), Downward to Bottom 40 percent – AIPW and PSM (Robust SEs)

	(1)	(2)	(3)	(4)
	AIPW (Raw)	PSM (Raw)	AIPW (Wins.)	PSM (Wins.)
Total	-7.502*** (1.024)	-6.351*** (1.457)	-7.122*** (0.906)	-5.942*** (1.337)
Math	-3.749*** (0.486)	-4.256*** (0.608)	-3.539*** (0.455)	-4.038*** (0.576)
English	-2.955*** (0.455)	-2.858*** (0.611)	-2.685*** (0.429)	-2.575*** (0.565)
Korean	-0.749*** (0.283)	-0.501 (0.390)	-0.691*** (0.233)	-0.612** (0.301)
Observations	5626	5626	5626	5626

Table A.10: Origin-Quartile Heterogeneity, Baseline Controls – AIPW

	(1)	(2)	(3)	(4)
	Q1 to Q4	Q2 to Q4	Q3 to Q4	Q1/Q2 to Q4
Total	4.291 (2.948)	6.132** (2.435)	11.415*** (2.511)	5.225*** (1.923)
Math	3.054** (1.494)	3.079** (1.196)	5.859*** (1.296)	3.017*** (0.965)
English	1.050 (1.322)	1.821 (1.136)	5.007*** (1.235)	1.435 (0.888)
Korean	0.175 (0.717)	1.322** (0.674)	0.442 (0.627)	0.829 (0.506)
Observations	3615	3671	3864	3810
Treated	139	195	388	334
Controls	3476	3476	3476	3476

Table A.11: Heterogeneity by Destination Grade Size: Top 25 Percent Transition

	(1)	(2)	(3)
	All	Smaller grade	Larger grade
Total	6.831*** (1.427)	5.636*** (1.997)	7.098*** (1.953)
Math	3.866*** (0.762)	2.820*** (1.047)	4.157*** (1.056)
English	2.959*** (0.673)	2.637*** (1.018)	2.951*** (0.888)
Korean	0.009 (0.367)	0.108 (0.554)	-0.032 (0.484)
Observations	5606	2811	2795
Treated	1042	404	638
Controls	4564	2407	2157

Table A.12: Minimum Destination Peer Group Size: Top 25 Percent Transition – AIPW

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	≥ 5	≥ 10	≥ 15	≥ 16	≥ 17	≥ 18	≥ 19
Total	7.416*** (1.683)	7.371*** (2.052)	5.979** (2.368)	5.686** (2.599)	8.125*** (2.870)	9.122** (3.969)	14.097 (12.133)
Observations	4457	2659	1510	1378	1226	1080	975
Treated	791	506	257	213	178	148	99
Controls	3666	2153	1253	1165	1048	932	876
Y4 schools	464	175	73	64	54	45	39

Isolating $\bar{N}$, it follows:

$$\begin{aligned}
\frac{1-\alpha}{2\phi} &= A^{\frac{-1}{1-\beta-2\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{1-\beta-2\alpha}} \bar{N}^{\gamma + \frac{\alpha\gamma}{1-\beta-2\alpha}} \\
\left(\frac{1-\alpha}{2\phi} \right) A^{\frac{-1}{1-\beta-2\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{1-\beta-2\alpha}} &= \bar{N}^{\frac{(1-\beta-\alpha)\gamma}{1-\beta-2\alpha}} \\
\bar{N}^{(1-\beta-\alpha)\gamma} &= \left(\frac{1-\alpha}{2\phi} \right)^{1-\beta-2\alpha} A^{-1} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{-\alpha} \\
\bar{N} &= A^{\frac{-1}{(1-\beta-\alpha)\gamma}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{1-\beta-2\alpha}{(1-\beta-\alpha)\gamma}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{(1-\beta-\alpha)\gamma}}
\end{aligned}$$

Looking at the optimal solutions of n and e , notice that we can disaggregate between:

$$\bar{N} = A^{\frac{-1}{(1-\beta-\alpha)\gamma}} \underbrace{\left(\frac{1-\alpha}{2\phi} \right)^{\frac{1-\beta-2\alpha}{(1-\beta-\alpha)\gamma}}}_{\text{Fertility}} \underbrace{\left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{(1-\beta-\alpha)\gamma}}}_{\text{Education}} \quad (\text{A.2.59})$$

Finally, substituting (A.2.59) into (A.2.58), we have:

$$\begin{aligned}
\bar{k} &= \left(\frac{1-\alpha}{2\phi} \right) A^{\frac{\gamma}{(1-\beta-\alpha)\gamma}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{-(1-\beta-2\alpha)\gamma}{(1-\beta-\alpha)\gamma}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha\gamma}{(1-\beta-\alpha)\gamma}} \\
&= \left(\frac{1-\alpha}{2\phi} \right) A^{\frac{1}{1-\beta-\alpha}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{-1+\beta+2\alpha}{1-\beta-\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha}{1-\beta-\alpha}} \\
&= A^{\frac{1}{1-\beta-\alpha}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{\alpha}{1-\beta-\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha}{1-\beta-\alpha}} \\
&= A^{\frac{1}{1-\beta-\alpha}} \left[\left(\frac{\alpha}{2\phi} \right) \phi \right]^{\frac{\alpha}{1-\beta-\alpha}} \\
&= A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha}{2} \right)^{\frac{\alpha}{1-\beta-\alpha}} \\
&= \left(A^{\frac{1}{\alpha}} \frac{\alpha}{2} \right)^{\frac{\alpha}{1-\beta-\alpha}} \\
&= \left(A^{\frac{1}{\alpha}} \frac{\alpha}{2} \right)^{\frac{\alpha\gamma}{(1-\beta-\alpha)\gamma}}
\end{aligned} \tag{A.2.60}$$

Proof of Proposition 2

Recall our dynamic system:

$$k_{t+1} = A e^{*\alpha}(k_t, N_t) k_t^\beta \tag{A.2.61}$$

$$N_{t+1} = n^*(k_t, N_t) N_t$$

where $e^*(k_t, N_t)$ and $n^*(k_t, N_t)$ are optimal education investment and fertility rates obtained from the household intertemporal optimisation problem. In steady-state, $k_t = k_{t+1} = \bar{k}$ and $N_t = N_{t+1} = \bar{N}$, so it follows that:

$$e^{*\alpha}(k_t, N_t) = \frac{\bar{k}^{1-\beta}}{A} \tag{A.2.62}$$

$$n^*(k_t, N_t) = 1$$

The Jacobian matrix is given by:

$$J = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix}$$

where

$$\begin{aligned} j_{11} &= A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) > 0 \\ j_{12} &= A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta > 0 \\ j_{21} &= \frac{\partial n^*}{\partial \bar{k}} \bar{N} < 0 \\ j_{22} &= \frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \geq 0 \end{aligned}$$

Thus:

$$\begin{aligned} \text{tr} J &= A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) + \frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \geq 0 \\ \det J &= A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) \left(\frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} \leq 0 \end{aligned}$$

Making use of (A.2.62), the local stability conditions become (see Medio & Lines, 2001):

(i) $1 + \text{tr} J + \det J$

$$\begin{aligned} &= 1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) + \frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \\ &+ A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) \left(\frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} \\ &= 1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) + \frac{\partial n^*}{\partial \bar{N}} \bar{N} + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} > 0 \end{aligned}$$

$$\begin{aligned}
1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} &> - \frac{\partial n^*}{\partial \bar{N}} \bar{N} - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} \\
1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} &> - \left[1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \right] \frac{\partial n^*}{\partial \bar{N}} \bar{N} \\
- \frac{\partial n^*}{\partial \bar{N}} \bar{N} &< \frac{1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N}}{1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right)} \\
- \frac{\partial n^*}{\partial \bar{N}} \bar{N} &< 1 - \frac{A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N}}{1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right)}
\end{aligned}$$

(ii) $1 - \text{tr} J + \det J$

$$\begin{aligned}
&= 1 - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) - \frac{\partial n^*}{\partial \bar{N}} \bar{N} - n^* (\bar{k}, \bar{N}) \\
&+ A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) \left(\frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} \\
&= 1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) \left(\frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) - 1 \right) \\
&- \left(\frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} \\
&= 1 + A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} - \left(\frac{\partial n^*}{\partial \bar{N}} \bar{N} + 1 \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} > 0
\end{aligned}$$

$$\begin{aligned}
&A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} - \frac{\partial n^*}{\partial \bar{N}} \bar{N} - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} > 0 \\
&\left[A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) - 1 \right] \frac{\partial n^*}{\partial \bar{N}} \bar{N} - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} > 0
\end{aligned}$$

(iii) $1 - \det J$

$$\begin{aligned}
&= 1 - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) \left(\frac{\partial n^*}{\partial \bar{N}} \bar{N} + n^* (\bar{k}, \bar{N}) \right) + A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} \\
&= 1 - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} \\
&\quad - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta e^{*\alpha} (\bar{k}, \bar{N}) \bar{k}^{\beta-1} \right) n^* (\bar{k}, \bar{N}) + A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} \\
&= 1 - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) + A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} > 0
\end{aligned}$$

$$\begin{aligned}
1 - A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} &> A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} \\
- A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) \frac{\partial n^*}{\partial \bar{N}} \bar{N} &> A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} - 1 \\
- \frac{\partial n^*}{\partial \bar{N}} \bar{N} &> \frac{A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right) - A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N} - 1}{A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right)} \\
- \frac{\partial n^*}{\partial \bar{N}} \bar{N} &> 1 - \frac{\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N}}{\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A}} - \frac{1}{A \left(\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A} \right)}
\end{aligned}$$

The sufficient conditions for (i)-(iii) to be satisfied are:

$$1 - \frac{\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N}}{\alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \frac{\beta}{A}} - \frac{1}{A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta} < - \frac{\partial n^*}{\partial \bar{N}} \bar{N} < 1 - \frac{A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{N}} \bar{k}^\beta \frac{\partial n^*}{\partial \bar{k}} \bar{N}}{1 + A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta} \quad (\text{A.2.63})$$

$$A \alpha \frac{\partial e^{*\alpha-1}}{\partial \bar{k}} \bar{k}^\beta + \beta > 1$$

where

$$\begin{aligned}
\frac{\partial n^*}{\partial \bar{k}} &= -\frac{1-\alpha}{2\bar{k}^2 \bar{N}^\gamma}, & \frac{\partial n^*}{\partial \bar{N}} &= -\frac{\gamma(1-\alpha)}{2\bar{k} \bar{N}^{\gamma+1}}, \\
\frac{\partial e^*}{\partial \bar{k}} &= \frac{2\alpha}{1-\alpha} \bar{k} \bar{N}^\gamma, & \frac{\partial e^*}{\partial \bar{N}} &= \frac{\alpha\gamma}{1-\alpha} \bar{k}^2 \bar{N}^{\gamma-1}
\end{aligned}$$

Proof of Proposition 3

When the time dad and mom dedicate to parenting are complements, from the equilibrium the conditions, we obtain the following isoquants:

$$\bar{k} = A^{\frac{1}{1-\beta-2\alpha}} \left[2 \left(\frac{\alpha}{1-\alpha} \right) \phi \bar{N}^\gamma \right]^{\frac{\alpha}{1-\beta-2\alpha}} \quad (\text{A.2.64})$$

$$\bar{k} = \left(\frac{1-\alpha}{4\phi \bar{N}^\gamma} \right) \quad (\text{A.2.65})$$

Combining (A.2.64) and (A.2.65), we have:

$$\frac{1-\alpha}{4\phi \bar{N}^\gamma} = A^{\frac{1}{1-\beta-2\alpha}} \left[2 \left(\frac{\alpha}{1-\alpha} \right) \phi \bar{N}^\gamma \right]^{\frac{\alpha}{1-\beta-2\alpha}}$$

Isolating $\bar{N}$, it follows:

$$\begin{aligned} \frac{1-\alpha}{4\phi} &= A^{\frac{1}{1-\beta-2\alpha}} \left[2 \left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha}{1-\beta-2\alpha}} \bar{N}^{\gamma + \frac{\alpha\gamma}{1-\beta-2\alpha}} \\ \left(\frac{1-\alpha}{4\phi} \right) A^{\frac{-1}{1-\beta-2\alpha}} \left[2 \left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{1-\beta-2\alpha}} &= \bar{N}^{\frac{(1-\beta-\alpha)\gamma}{1-\beta-2\alpha}} \\ \bar{N}^{(1-\beta-\alpha)\gamma} &= \left(\frac{1-\alpha}{4\phi} \right)^{1-\beta-2\alpha} A^{-1} \left[2 \left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{-\alpha} \\ \bar{N} &= A^{\frac{-1}{(1-\beta-\alpha)\gamma}} \left(\frac{1-\alpha}{4\phi} \right)^{\frac{1-\beta-2\alpha}{(1-\beta-\alpha)\gamma}} \left[2 \left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{(1-\beta-\alpha)\gamma}} \end{aligned}$$

Notice we can rewrite it as:

$$\begin{aligned} \bar{N} &= 2^{\frac{-1+\beta+2\alpha}{(1-\beta-\alpha)\gamma}} 2^{\frac{-\alpha}{(1-\beta-\alpha)\gamma}} A^{\frac{-1}{(1-\beta-\alpha)\gamma}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{1-\beta-2\alpha}{(1-\beta-\alpha)\gamma}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{(1-\beta-\alpha)\gamma}} \\ &= 2^{\frac{-1+\beta+\alpha}{(1-\beta-\alpha)\gamma}} A^{\frac{-1}{(1-\beta-\alpha)\gamma}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{1-\beta-2\alpha}{(1-\beta-\alpha)\gamma}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{-\alpha}{(1-\beta-\alpha)\gamma}} \\ &= \underbrace{\Omega A^{\frac{\gamma}{(1-\beta)\gamma+(1-\gamma)\alpha}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{\alpha\gamma}{(1-\beta-\alpha)\gamma}}}_{\text{Fertility}} \underbrace{\left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha\gamma}{(1-\beta-\alpha)\gamma}}}_{\text{Education}} \quad (\text{A.2.66}) \end{aligned}$$

where

$$\Omega = 2^{\frac{-1+\beta+\alpha}{(1-\beta-\alpha)\gamma}} = 2^{-\frac{1}{\gamma}}$$

Because $\gamma > 0$, it follows that $0 < \Omega < 1$.

Finally, substituting (A.2.66) into (A.2.65), we obtain:

$$\begin{aligned} \bar{k} &= \left(\frac{1-\alpha}{4\phi} \right) 2^{\frac{-(-1+\beta+\alpha)\gamma}{(1-\beta-\alpha)\gamma}} A^{\frac{\gamma}{(1-\beta-\alpha)\gamma}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{-(1-\beta-2\alpha)\gamma}{(1-\beta-\alpha)\gamma}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha\gamma}{(1-\beta-\alpha)\gamma}} \\ &= \left(\frac{1-\alpha}{4\phi} \right) 2A^{\frac{1}{1-\beta-\alpha}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{-1+\beta+2\alpha}{1-\beta-\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha}{1-\beta-\alpha}} \\ &= A^{\frac{1}{1-\beta-\alpha}} \left(\frac{1-\alpha}{2\phi} \right)^{\frac{\alpha}{1-\beta-\alpha}} \left[\left(\frac{\alpha}{1-\alpha} \right) \phi \right]^{\frac{\alpha}{1-\beta-\alpha}} \\ &= A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha}{2} \right)^{\frac{\alpha}{1-\beta-\alpha}} \\ &= \left(A^{\frac{1}{\alpha}} \frac{\alpha}{2} \right)^{\frac{\alpha}{1-\beta-\alpha}} \end{aligned}$$

Proof of Proposition 4

Given that our dynamic system can still be specified as in (A.2.61), the local stability conditions for the non-trivial equilibrium point remain the same as in (A.2.63). But now, through direct derivation of the optimal allocations, we have:

$$\begin{aligned} \frac{\partial n^*}{\partial \bar{k}} &= -\frac{1-\alpha}{4\bar{k}^2 \bar{N}^\gamma}, & \frac{\partial n^*}{\partial \bar{N}} &= -\frac{\gamma(1-\alpha)}{4\bar{k} \bar{N}^{\gamma+1}}, \\ \frac{\partial e^*}{\partial \bar{k}} &= \frac{4\alpha}{1-\alpha} \bar{k} \bar{N}^\gamma, & \frac{\partial e^*}{\partial \bar{N}} &= \frac{2\alpha\gamma}{1-\alpha} \bar{k}^2 \bar{N}^{\gamma-1} \end{aligned}$$

Proof of Proposition 5

When the time dad and mom dedicate to parenting are substitutes, from the equilibrium conditions, and recalling that at a non-trivial steady state $\bar{n} = 1$, we obtain the following isoquants:

$$\bar{k} = A^{\frac{1}{1-\beta-\alpha}} \left[\frac{\alpha}{1+\alpha} (1 - \bar{k}\phi\bar{N}^\gamma) \right]^{\frac{\alpha}{1-\beta-\alpha}} \quad (\text{A.2.67})$$

$$\bar{k} = \frac{(1+\alpha)b}{(1 - \bar{k}\phi\bar{N}^\gamma) [1 + \alpha - 2(1 - \bar{k}\phi\bar{N}^\gamma)]} \quad (\text{A.2.68})$$

The first expression follows from the human-capital law of motion together with the fact that, at the steady state, $\bar{c} + \bar{e} = \bar{y}$ and $\bar{e} = \alpha\bar{c}$, so that:

$$\bar{e} = \frac{\alpha}{1 + \alpha} \bar{y}.$$

Under perfect substitution, output is:

$$\bar{y} = \bar{k} - \bar{k}^2 \phi \bar{N}^\gamma = \bar{k} (1 - \bar{k} \phi \bar{N}^\gamma),$$

which yields (A.2.67).

The second expression follows from the corrected quadratic condition (3.29) evaluated at $\bar{n} =$

1. Using $X = \bar{k}^2 \phi \bar{N}^\gamma$, we have:

$$2X^2 + \bar{k} [(\alpha - 3)X + (1 + \alpha)b] + (1 - \alpha)\bar{k}^2 = 0.$$

Substituting back for X and dividing through by $\bar{k}^2 > 0$, it follows that:

$$2\bar{k}^2 \phi^2 \bar{N}^{2\gamma} + (\alpha - 3)\bar{k} \phi \bar{N}^\gamma + (1 + \alpha)\frac{b}{\bar{k}} + (1 - \alpha) = 0.$$

Rearranging:

$$(1 - \bar{k} \phi \bar{N}^\gamma) [1 + \alpha - 2(1 - \bar{k} \phi \bar{N}^\gamma)] = (1 + \alpha)\frac{b}{\bar{k}},$$

from which (A.2.68) follows.

To combine the two equilibrium conditions, define

$$z \equiv 1 - \bar{k} \phi \bar{N}^\gamma.$$

Then (A.2.67) and (A.2.68) become:

$$\bar{k} = A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha z}{1+\alpha} \right)^{\frac{\alpha}{1-\beta-\alpha}} \quad (\text{A.2.69})$$

$$\bar{k} = \frac{(1+\alpha)b}{z(1+\alpha-2z)}. \quad (\text{A.2.70})$$

Combining (A.2.69) and (A.2.70), we have:

$$A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha z}{1+\alpha} \right)^{\frac{\alpha}{1-\beta-\alpha}} = \frac{(1+\alpha)b}{z(1+\alpha-2z)}.$$

Rearranging:

$$A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha}{1+\alpha} \right)^{\frac{\alpha}{1-\beta-\alpha}} z^{\frac{1-\beta}{1-\beta-\alpha}} (1+\alpha-2z) = (1+\alpha)b. \quad (\text{A.2.71})$$

Thus, feasible steady states are associated with the roots z_i , $i = 1, 2$, of (A.2.71) over the interval $z \in (0, \frac{1+\alpha}{2})$.

To study the number of solutions, notice that the left-hand side of (A.2.71) is equal to zero at $z = 0$ and at $z = \frac{1+\alpha}{2}$. Since $0 < \alpha + \beta < 1$, the exponent

$$\frac{1-\beta}{1-\beta-\alpha} > 1,$$

and differentiating the left-hand side of (A.2.71) with respect to z gives:

$$A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha}{1+\alpha} \right)^{\frac{\alpha}{1-\beta-\alpha}} z^{\frac{\alpha}{1-\beta-\alpha}} \left[\frac{1-\beta}{1-\beta-\alpha} (1+\alpha) - 2 \left(\frac{1-\beta}{1-\beta-\alpha} + 1 \right) z \right].$$

Hence, the left-hand side of (A.2.71) is single-peaked over $(0, \frac{1+\alpha}{2})$. Therefore, there exists a threshold value $\bar{b} > 0$ such that (A.2.71) admits two feasible roots if $0 < b < \bar{b}$, one double root if $b = \bar{b}$, and no feasible root if $b > \bar{b}$.

Finally, once a feasible root z_i is known, it follows from (A.2.69) that:

$$\bar{k}_i^b = A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha z_i}{1+\alpha} \right)^{\frac{\alpha}{1-\beta-\alpha}},$$

and since

$$\bar{k}_i^b \phi(\bar{N}_i^b)^\gamma = 1 - z_i,$$

we obtain:

$$\bar{N}_i^b = \left(\frac{1 - z_i}{\phi \bar{k}_i^b} \right)^{\frac{1}{\gamma}}.$$

Proof of Proposition 6

The proof follows the same steps as in Appendix A.2. For each optimal allocation (n_{Si}^*, e_{Si}^*) , $i = 1, 2$, the distorted dynamic system can still be written in the general form:

$$k_{t+1} = A e_i^{*\alpha}(k_t, N_t) k_t^\beta \quad (\text{A.2.72})$$

$$N_{t+1} = n_i^*(k_t, N_t) N_t$$

where $e_i^*(k_t, N_t)$ and $n_i^*(k_t, N_t)$ are the distorted education investment and fertility rates associated with the feasible root n_{Si}^* of (3.29).

At a non-trivial steady state, $k_t = k_{t+1} = \bar{k}$ and $N_t = N_{t+1} = \bar{N}$, so it follows that:

$$e_i^{*\alpha}(\bar{k}, \bar{N}) = \frac{\bar{k}^{1-\beta}}{A} \quad (\text{A.2.73})$$

$$n_i^*(\bar{k}, \bar{N}) = 1$$

Thus, the Jacobian matrix and the local stability conditions are exactly the same as in Appendix A.2. Therefore, each equilibrium point $P_{Si} = (\bar{k}_{Si}, \bar{N}_{Si})$ is locally stable provided that (3.31) is satisfied.

The only difference relative to Proposition 2 is that fertility and education are now determined by distorted policy functions. In particular, distorted fertility is implicitly defined by:

$$F_S(n, k, N) = 2X^2n^2 + k[(\alpha - 3)X + (1 + \alpha)b]n + (1 - \alpha)k^2 = 0,$$

where $X = k^2\phi N^\gamma$. Applying the implicit function theorem:

$$\frac{\partial n_i^*}{\partial k} = -\frac{F_{S,k}}{F_{S,n}}, \quad \frac{\partial n_i^*}{\partial N} = -\frac{F_{S,N}}{F_{S,n}},$$

where

$$\begin{aligned} F_{S,n} &= 4X^2n_i^* + k[(\alpha - 3)X + (1 + \alpha)b], \\ F_{S,k} &= \frac{8X^2n_i^{*2}}{k} + [3(\alpha - 3)X + (1 + \alpha)b]n_i^* + 2(1 - \alpha)k, \\ F_{S,N} &= \frac{\gamma X n_i^*}{N} [4Xn_i^* + (\alpha - 3)k]. \end{aligned}$$

The distorted education rule can be written as:

$$e_i^* = \frac{\alpha}{1 - \alpha} G_S(k, N),$$

where

$$G_S(k, N) = \frac{k(X - b) - X^2n_i^*}{k - Xn_i^*}.$$

Therefore:

$$\frac{\partial e_i^*}{\partial k} = \frac{\alpha}{1 - \alpha} G_{S,k}, \quad \frac{\partial e_i^*}{\partial N} = \frac{\alpha}{1 - \alpha} G_{S,N},$$

where

$$G_{S,k} = \frac{\left[3X - b - \frac{4X^2 n_i^*}{k} - X^2 \frac{\partial n_i^*}{\partial k}\right] (k - Xn_i^*) - [k(X - b) - X^2 n_i^*] \left[1 - \frac{2Xn_i^*}{k} - X \frac{\partial n_i^*}{\partial k}\right]}{(k - Xn_i^*)^2},$$

$$G_{S,N} = \frac{\left[\frac{\gamma X}{N} (k - 2Xn_i^*) - X^2 \frac{\partial n_i^*}{\partial N}\right] (k - Xn_i^*) + [k(X - b) - X^2 n_i^*] \left[\frac{\gamma X n_i^*}{N} + X \frac{\partial n_i^*}{\partial N}\right]}{(k - Xn_i^*)^2}.$$

Finally, making use of the chain rule:

$$\frac{\partial e_i^{*\alpha}}{\partial k} = \alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial k}, \quad \frac{\partial e_i^{*\alpha}}{\partial N} = \alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial N}.$$

Substituting these distorted policy derivatives into the same Jacobian expressions used in Appendix A.2 yields (3.31).

Proof of Proposition 7

The proof follows the same logic as in Appendix A.5, with the same shortcut used in Appendix A.3. When the time dad and mom dedicate to parenting are complements, from the equilibrium conditions, and recalling that at a non-trivial steady state $\bar{n} = 1$, we obtain the following isoquants:

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$$\bar{k} = A^{\frac{1}{1-\beta-\alpha}} \left[\frac{\alpha}{1+\alpha} (1 - 2\bar{k}\phi\bar{N}^\gamma) \right]^{\frac{\alpha}{1-\beta-\alpha}} \quad (\text{A.2.74})$$

$$\bar{k} = \frac{(1+\alpha)b}{(1 - 2\bar{k}\phi\bar{N}^\gamma) [1 + \alpha - 2(1 - 2\bar{k}\phi\bar{N}^\gamma)]} \quad (\text{A.2.75})$$

Now define

$$z \equiv 1 - 2\bar{k}\phi\bar{N}^\gamma.$$

Then (A.2.74) and (A.2.75) become:

$$\bar{k} = A^{\frac{1}{1-\beta-\alpha}} \left(\frac{\alpha z}{1+\alpha} \right)^{\frac{\alpha}{1-\beta-\alpha}} \quad (\text{A.2.76})$$

$$\bar{k} = \frac{(1+\alpha)b}{z(1+\alpha-2z)}. \quad (\text{A.2.77})$$

Thus, the scalar equation determining feasible steady states is exactly the same as in (A.2.71). It follows immediately that the feasible roots z_i , $i = 1, 2$, are the same as in Proposition 5, and therefore:

$$\bar{k}_{Ci} = \bar{k}_{Si}, \quad i = 1, 2.$$

The only difference is in the population term. Under perfect substitution:

$$\bar{k}_{Si} \phi \bar{N}_{Si}^\gamma = 1 - z_i,$$

whereas under perfect complementarity:

$$2\bar{k}_{Ci} \phi \bar{N}_{Ci}^\gamma = 1 - z_i,$$

or equivalently:

$$\bar{k}_{Ci} \phi \bar{N}_{Ci}^\gamma = \frac{1 - z_i}{2}.$$

Since $\bar{k}_{Ci} = \bar{k}_{Si}$, it follows that:

$$\bar{N}_{Ci}^\gamma = \frac{1}{2} \bar{N}_{Si}^\gamma,$$

and therefore:

$$\bar{N}_{Ci} = \frac{1}{2^{1/\gamma}} \bar{N}_{Si} = \Omega \bar{N}_{Si}.$$

Proof of Proposition 8

The proof is identical in structure to Appendix A.6 and closely parallels Appendix A.4. For each optimal allocation (n_{Ci}^*, e_{Ci}^*) , $i = 1, 2$, the distorted dynamic system can still be written as

in (A.2.72). Therefore, the Jacobian matrix and the local stability conditions remain the same as in Appendix A.2. Hence, each equilibrium point $P_{Ci} = (\bar{k}_{Ci}, \bar{N}_{Ci})$ is locally stable provided that (3.31) is satisfied.

The only difference is that fertility and education are now determined by distorted complementary policy functions. In particular, distorted fertility is implicitly defined by:

$$F_C(n, k, N) = 8X^2n^2 + k[2(\alpha - 3)X + (1 + \alpha)b]n + (1 - \alpha)k^2 = 0,$$

where $X = k^2\phi N^\gamma$. Applying the implicit function theorem:

$$\frac{\partial n_i^*}{\partial k} = -\frac{F_{C,k}}{F_{C,n}}, \quad \frac{\partial n_i^*}{\partial N} = -\frac{F_{C,N}}{F_{C,n}},$$

where

$$\begin{aligned} F_{C,n} &= 16X^2n_i^* + k[2(\alpha - 3)X + (1 + \alpha)b], \\ F_{C,k} &= \frac{32X^2n_i^{*2}}{k} + [6(\alpha - 3)X + (1 + \alpha)b]n_i^* + 2(1 - \alpha)k, \\ F_{C,N} &= \frac{2\gamma X n_i^*}{N} [8Xn_i^* + (\alpha - 3)k]. \end{aligned}$$

The distorted complementary education rule can be written as:

$$e_i^* = \frac{\alpha}{1 - \alpha} G_C(k, N),$$

where

$$G_C(k, N) = \frac{k(2X - b) - 4X^2n_i^*}{k - 2Xn_i^*}.$$

Therefore:

$$\frac{\partial e_i^*}{\partial k} = \frac{\alpha}{1 - \alpha} G_{C,k}, \quad \frac{\partial e_i^*}{\partial N} = \frac{\alpha}{1 - \alpha} G_{C,N},$$

where

$$G_{C,k} = \frac{\left[6X - b - \frac{16X^2 n_i^*}{k} - 4X^2 \frac{\partial n_i^*}{\partial k}\right] (k - 2Xn_i^*) - [k(2X - b) - 4X^2 n_i^*] \left[1 - \frac{4Xn_i^*}{k} - 2X \frac{\partial n_i^*}{\partial k}\right]}{(k - 2Xn_i^*)^2},$$

$$G_{C,N} = \frac{\left[\frac{2\gamma X}{N} (k - 4Xn_i^*) - 4X^2 \frac{\partial n_i^*}{\partial N}\right] (k - 2Xn_i^*) + [k(2X - b) - 4X^2 n_i^*] \left[\frac{2\gamma Xn_i^*}{N} + 2X \frac{\partial n_i^*}{\partial N}\right]}{(k - 2Xn_i^*)^2}.$$

Finally, making use of the chain rule:

$$\frac{\partial e_i^{*\alpha}}{\partial k} = \alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial k}, \quad \frac{\partial e_i^{*\alpha}}{\partial N} = \alpha e_i^{*\alpha-1} \frac{\partial e_i^*}{\partial N}.$$

Substituting these distorted policy derivatives into the same Jacobian expressions used in Appendix A.2 yields the result.

A.3 Appendix C

A.3.1 Calibration Details

This appendix provides detailed derivations and discussion for the baseline calibration summarized in Table 4.1.

Peer pressure and education weight

The baseline peer-pressure intensity is set to $\phi_0 = 0.24$, using the midpoint of the range (0.234–0.252) implied by the top-quartile transition estimates in Young (2026). This should be interpreted as a conservative estimate: Young also finds that the peer-spending response is larger in larger destination cohorts, consistent with the model's assumption that ϕ is increasing in cohort size.

The baseline tutoring-spending share $\theta_0 = 0.06$ is the ratio of tutoring expenditure to household income in the KELS sample reported in Young (2026). In the model, the decentralized equilibrium education-spending share is

$$\theta(\phi) = \frac{\beta}{1 - \phi(1 - \beta)}.$$

Evaluating at ϕ_0 and solving for β :

$$\beta = \frac{\theta_0(1 - \phi_0)}{1 - \theta_0\phi_0} = \frac{0.06 \times 0.76}{1 - 0.06 \times 0.24} = \frac{0.0456}{0.9856} = 0.0463.$$

Verification: $\theta(0.24) = 0.0463/[1 - 0.24 \times 0.9537] = 0.0463/0.7711 = 0.0600$. ✓

Preference weights and child cost

The consumption weight is set to $\alpha = 0.85$, and the fertility weight is the residual:

$$\nu = 1 - \alpha - \beta = 1 - 0.85 - 0.0463 = 0.1037.$$

The child-cost parameter $b = 0.10$ is calibrated from Korean time-use evidence. The 2019 Korean Time Use Survey, a nationally representative survey of 12,435 households and 26,091 individuals (Jun & Eun, 2025), reports that mothers of children under age 10 spent 3 hours 13 minutes per day on childcare, while fathers spent 1 hour 2 minutes. Scaling mothers' childcare time by a 100-hour weekly time endowment and assuming that the most intensive childrearing period covers roughly ten of the twenty-five years in a model period gives

$$b_m = \frac{3.217 \times 7}{100} \times \frac{10}{25} = 0.090.$$

Using combined mother–father time gives $b_{m+f} = 0.119$. We set $b = 0.10$ as a conservative baseline between these bounds. Fertility is not used to calibrate b ; it is generated endogenously by the model.

Population normalization and peer-pressure sensitivity

We normalize $N_0 = 1$ and $h_0 = 1$. Given the reduced-form specification $\phi(N) = \xi N/(1 + \xi N)$, the normalization $\phi(1) = \phi_0$ implies

$$\xi = \frac{\phi_0}{1 - \phi_0} = \frac{0.24}{0.76} = 0.3158.$$

The scale parameter in human capital production is set so that $h_0 = 1$ is consistent with the calibrated baseline. Since $h_{t+1} = A\theta(\phi(N_t))^\eta h_t^\eta N_t^\mu$ and $N_0 = h_0 = 1$:

$$A = \theta_0^{-\eta} = 0.06^{-0.30} = 2.3257.$$

Density-dependent attrition

The attrition function uses the parametric form

$$d(N) = \underline{d} + (\bar{d} - \underline{d}) \frac{\zeta N}{1 + \zeta N}, \quad (\text{A.3.1})$$

where $\underline{d}$ is baseline attrition at low population, $\bar{d}$ is the upper bound, and ζ governs density sensitivity.

Interpretation. The generational attrition rate d should not be interpreted as an annual death rate. In the model, a “period” corresponds to a generation (approximately 25 years), and d captures the full exit rate of the parent generation—including mortality, retirement from the active economy, and emigration. Values near 1 reflect near-complete generational turnover, which is the standard OLG interpretation. Values above 1 arise when density pressure causes net population loss beyond simple generational replacement (through, e.g., out-migration from high-cost metropolitan areas or economic exit from the active population).

Calibration. We set $\underline{d} = 0.90$ and $\bar{d} = 1.15$. The lower bound reflects incomplete generational turnover at low population (some overlap between generations); the upper bound ensures that attrition can exceed even the planner’s fertility rate $\nu/b = 1.0373$ at sufficiently high population, as required for the planner’s steady state to exist.

The density sensitivity parameter ζ is pinned by requiring that the baseline economy ($N_0 = 1$) be at the decentralized steady state:

$$d(1) = \bar{n}(\phi_0) = n^{eq}(\phi_0).$$

Computing decentralized fertility:

$$n^{eq}(\phi_0) = \frac{\nu(1 - \phi_0)}{b[1 - \phi_0(1 - \beta)]} = \frac{0.1037 \times 0.76}{0.10 \times 0.7711} = 1.0224.$$

Thus we need $d(1) = 1.0224$:

$$\underline{d} + (\bar{d} - \underline{d}) \frac{\zeta}{1 + \zeta} = 1.0224 \implies 0.90 + 0.25 \cdot \frac{\zeta}{1 + \zeta} = 1.0224.$$

Solving:

$$\frac{\zeta}{1 + \zeta} = \frac{1.0224 - 0.90}{0.25} = 0.4896 \implies \zeta = \frac{0.4896}{1 - 0.4896} = 0.9593.$$

The planner's steady state. The planner's population N^{**} solves $d(N^{**}) = \nu/b = 1.0373$:

$$0.90 + 0.25 \cdot \frac{0.9593 N^{**}}{1 + 0.9593 N^{**}} = 1.0373 \implies \frac{0.9593 N^{**}}{1 + 0.9593 N^{**}} = 0.5492.$$

Let $x = 0.9593 N^{**}$:

$$\frac{x}{1 + x} = 0.5492 \implies x = \frac{0.5492}{0.4508} = 1.2183 \implies N^{**} = \frac{1.2183}{0.9593} = 1.2700.$$

The planner supports a 27% larger steady-state population than the decentralized economy, reflecting the fertility gain from removing the education externality.

Implied fertility wedge and policy implications

Decentralized fertility at the baseline is $n^{eq}(\phi_0) = 1.0224$. The planner's fertility is $n^* = \nu/b = 1.0373$. The gap is modest (about 1.5 percentage points), reflecting the conservative nature of the calibration. The implied Pigouvian wedge is

$$\tau_0 = \frac{\phi_0}{1 - \phi_0} = 0.3158.$$

This number should not be read as a literal policy recommendation, since the calibration uses a narrow measure of tutoring expenditure and abstracts from enforcement, substitution across inputs, and household heterogeneity. Rather, it summarizes the size of the positional distortion implied by the Young (2026) peer-spending estimates.

Comparison with S. Kim et al. (2024)

This calibration differs from S. Kim et al. (2024) in both purpose and interpretation. Their paper develops a richer quantitative model designed to account for broader Korean fertility patterns and heterogeneous household behavior. The resulting fertility wedge is smaller than the counterfactual fertility effects in S. Kim et al.. This difference is not a failure of the calibration; rather, it shows that observed tutoring responses by themselves are unlikely to explain the full Korean fertility collapse. Direct tutoring expenditure is only one component of the broader cost of raising children in a highly competitive education system. In Korea, educational competition may also involve parental time, information search, residential sorting, school choice, and other forms of child-quality investment not fully captured by measured tutoring spending. The sensitivity exercises below vary peer-pressure intensity and child-cost parameters to show how the model behaves in more competitive environments.

Sensitivity

The baseline calibration should be interpreted as a lower bound on the peer-pressure channel. Two margins suggest larger effects:

Broader child-quality investment. If the relevant peer-pressure channel extends beyond measured tutoring expenditure to include parental time, residential sorting, and school-choice costs, the effective ϕ and θ are both larger. Doubling ϕ to 0.48 while keeping θ_0 fixed raises the fertility wedge substantially.

Higher child costs. If b is raised to 0.15 (reflecting combined parental time including supervision, homework help, and transportation to hagwons), the planner's fertility falls to $\nu/b = 0.69$, the decentralized fertility falls further, and the demographic impact of the rat race is amplified.

These sensitivity exercises are not reported in full here but can be explored using the closed-form equilibrium expressions of Section 4.4.3.