PROBABILITY DISTRIBUTIONS OF WIND-PRESSURE FLUCTUATIONS ON BUILDINGS

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NOMENCLATURE

Cp	instantaneous pressure coefficient, $\frac{P-P_o}{\frac{1}{2}\rho U^2}$
Cp_{mean}	mean of instantaneous pressure coefficient over time T
Cp_{rms}	root-mean-square of pressure coefficient about its mean
Cp_{max}	maximum (most positive) value of Cp over time T
Cp_{min}	minimum (most negative) value of Cp over time T
m_r	rth moment of the spectrum S(n)
n	frequency, Hertz
P	instantaneous pressure at a pressure tap
P_o	mean static pressure in ambient flow
p(x)	probability distribution
Q(x)	cumulative probability distribution
S(n)	power spectrum of a fluctuating signal
T	time interval
U	gradient wind velocity
x	pressure fluctuation variable, $\frac{Cp-Cp_{mean}}{Cp_{rms}}$
η	pressure peak fluctuation variable, $\frac{Cp_{max}-Cp_{mean}}{Cp_{rms}}$ or $\frac{Cp_{min}-Cp_{mean}}{Cp_{rms}}$
ν	spectral characteristic of fluctuating pressure, $(m_2/m_o)^{\frac{1}{2}}$
ρ	air density

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by J.A. Peterka[2] and J.E. Cermak[3]

INTRODUCTION

Rational design of glass and cladding on structures requires a knowledge of the peak pressures expected to act on the structure during its lifetime. The peaks of concern may be either inward or outward acting. Fluctuations in pressure are caused by turbulence in the flow approaching the structure and by flow disturbances generated by the structure itself. The instantaneous pressure acting at a particular point on a structure is thus a function of wind magnitude and direction, roughness characteristics of the local and distant upwind area, overall building shape and local disturbances to the flow on the structure such as mullions or exposed columns. Because of the random nature of wind direction and amplitude, the local pressure also fluctuates in a random manner. Knowledge of the statistical characteristics of these pressure fluctuations is required to predict the peak values likely to occur in a given time period.

Techniques have been developed to predict the peak wind loading values on a structure. Davenport (7,8) provided a peak value theory for wind loads based in part on earlier statistical work of Cartwright

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and Longuet-Higgins (2) and Rice (11). His analyses produced a probability distribution for peak pressures assuming a Gaussian distribution for the wind turbulence structure and a Gaussian response for the pressure fluctuations. Peak loads for design purposes are usually obtained by simpler techniques. In practice, design pressures are generally obtained from a gust factor approach. Davenport (9) and Vellozzi and Cohen (12) demonstrate a rational approach to wind loads for the entire structure using a gust factor. These techniques have been incorporated into the wind loading provisions of the proposed American National Standards Institute code (1). Inherent in the procedures is the assumption of a Gaussian pressure distribution of local pressures acting on the structure in response to a Gaussian distribution of velocity in the turbulent flow about the structure. In the absence of actual measurements of pressure fluctuations, and with the knowledge that wind approaching the structure reasonably approximates a Gaussian distribution (in the absence of structures immediately upstream which the design procedures are not intended to handle), the assumptions made regarding pressure fluctuations seemed reasonable.

More recent evidence of the nature of pressure fluctuations on structures tends to support the concept of a Gaussian pressure distribution at least on the windward face. Dalgliesh (6) reported probability distributions of peak pressures measured on a 45-story office building in Montreal. His pressures were obtained mostly on the positive-pressure upwind side of the structure. There pressure distributions agreed rather well with Davenport's theoretical analysis (8). He reported only one distribution for a negative pressure on the leeward side of the structure. This data, significantly, did not agree well with the theory. In another study, Cermak and Sadeh (5) reported spectra for pressure fluctuations on the upstream face of a model building placed in a wind tunnel suitable for modeling atmospheric flows. Similarity in shape of the spectra for approach flow and for pressure fluctuations again tended to confirm the assumption of similarity in statistical characteristics, at least on the upstream face. The important point in these studies was that agreement with theory was shown only for the windward side of the structure. The leeward pressure distributions were virtually ignored.

With the advent of wind tunnels capable of modeling the wind forces on structures (4), the capability exists for obtaining large amounts of data at a number of locations on a structure for many wind directions at reasonable cost. Thus a more extensive analysis of peak pressures can be performed than was possible in the past. Data obtained during wind tunnel tests by the authors on the Federal Reserve Bank, Richmond, Virginia (10) with supporting data from wind-tunnel tests on two additional structures permitted the experimental determination of the probability distributions of the fluctuating pressure and the probability distribution of the pressure peaks. The purpose of this paper is to present the results of that investigation and to discuss some implications of the findings relative to the design pressures selected by building codes.

EXPERIMENTAL MEASUREMENTS

The Fluid Dynamics and Diffusion Laboratory at Colorado State University has developed wind-tunnel facilities to model wind effects on structures. Three large wind tunnels have been designed specifically to model atmospheric flows and their influence on structures, dispersion of pollutants and other wind-related phenomena. Wind-tunnel determination of pressures on structures is an efficient and economical method of selecting design loads, particularly for local pressures on cladding and glass lites. On a large structure, the cost of the structure skin represents a considerable investment and the potential losses associated with wind damage are high. The cost of a conservative design to protect against wind damage may be reduced through wind-tunnel tests of local mean and fluctuating pressures on a small scale model.

The data analyzed in this paper originated in a wind-tunnel study (10) of the Federal Reserve Bank, Richmond, Virginia, Figure 1. The structure is 456 ft high and nominally 150 ft square in cross section with small corner projections as shown in Figure 2. The area upwind of the building for the wind directions considered here was rolling terrain with relatively low structures--an approach which would be classified as suburban or type B as designated by the ANSI standard (1). A "Lucite" model of the structure was installed in the wind tunnel with approximately 1500 ft of upwind terrain and structures modeled in detail. Upstream from the modeled area, randomized roughness elements approximating the field roughness were used to develop the proper approach mean velocity and turbulence characteristics. The total length of wind-tunnel test section from entrance to model was 84 ft.

Fluctuating pressure data was obtained from the model by means of 1/16 in. piezometer taps connected by short lengths of plastic tubing through a specially designed pressure switch to a high-response differential pressure transducer. The frequency response of the pressure measurement system was sufficiently high (greater than 100 cps) that all significant information was obtained. The reference side of the pressure transducer was connected to the static side of a pitot tube placed in the flow above the structure so that all pressures were referenced automatically to the ambient pressure in the simulated atmospheric flow above the building.

For each wind direction of interest, 272 individual pressures covering all four sides, top, and corner projections were measured. Measurement was accomplished by processing the fluctuating signal from the pressure transducer through a mini-computer and analog-to-digital converter onto digital magnetic tape. Each pressure port was sampled at 250 samples per second for a total of 4080 samples (16 seconds of data). The data was then analyzed on the Colorado State University CDC 6400 computer.

The computer processing first converted all data in the pressure coefficient form,

$$Cp = \frac{P-P_o}{\frac{1}{2}\rho U^2} \qquad (1)$$

where $P-P_o$ represents the transducer-measured pressure difference and $\frac{1}{2}\rho U^2$ is a reference dynamic pressure associated with the gradient wind velocity above the earth's boundary layer. These pressure coefficients are directly applicable to the full-scale structure as shown by an appropriate analysis of the modeling criteria. Four additional types of pressure coefficient were computed for each pressure port: Cp_{mean}, the average of the 4080 Cp data values collected for each pressure port; Cp_{rms}, the root-mean-square of the fluctuations of Cp about Cp_{mean}; Cp_{max}, the largest Cp (most positive) value in the 4080 data samples; Cp_{min}, the smallest (largest negative) Cp value in the 4080 data samples. The computer analysis was also capable of determining the frequency distribution of Cp values for any desired pressure port by sorting values into appropriate bins.

PROBABILITY DISTRIBUTIONS

For the purpose of analyzing the probability distributions for pressure fluctuations on the structure, two wind directions were selected for study. A wind azimuth of 350 degrees relative to true north was selected to provide a glancing wind on the upwind faces and an azimuth of 310 degrees was selected to provide wind normal to the upwind face, Figure 2. Both wind directions studied showed relatively stable separation points on the structure which did not vary significantly with time. For each wind direction, probability distributions were calculated for each of the 282 pressure ports on the structure using 30 bins to subdivide the range of fluctuations. Each distribution consisted of 4080 data samples.

The distributions were found to fall into two distinct categories: those associated with direct wind impingement on the structure with generally positive mean pressures ($Cp_{mean} > -0.1$) and those associated with separated regions with negative pressures ($Cp_{mean} < -0.25$). In the region between -0.1 and -0.25, representing a small percentage of ports, distributions were a combination of positive and negative distributions. Figure 3 shows the probability density associated with the positive Cp_{mean}. The data shown reflects 13 locations for one wind direction. All positive pressure distributions examined exhibited the same characteristics. Points shown were selected to sample the entire upwind area. The data is normalized in the usual way by extraction of the mean and division by the standard deviation. Comparison with a standard normal distribution indicates the data do, in fact, follow a Gaussian distribution. It is evident that local pressures on the upwind face of the structure follow essentially the same distribution as expected in the approach flow.

Probability densities for 21 pressure taps on the lee side of the structure are plotted in Figure 4. The taps were again selected to sample the entire separated region including the separated region over the building roof. Significant deviations from the Gaussian distribution are immediately evident. The most important deviation occurs on the tail of the curve for the larger negative values of the distribution. Large numbers of points are in evidence past 6 standard deviations from the mean indicating a much higher probability for values in this region than a normal distribution would predict. This finding is particularly significant since the largest loads on a structure's cladding is usually due to the negative pressures in sensitive areas of the structure - near corners and roof lines. The other evident feature of the distributions plotted in Figure 4 is the higher peak of the distribution near zero which is shifted slightly to the positive side. It can thus be concluded that in the regions of the structure exposed to separated flow, the pressure would be close to its mean value and also very negative more frequently than would be predicted by a Gaussian distribution.

In order to determine the extent to which the frequency of large negative values exceed the normal distribution, probability for the negative side of the distribution was displayed on a logarithmic scale, Figure 5. Fifty pressure taps were included in the graph. The distribution for each tap was based on 10 bins in order to increase resolution of the data for the small probabilities. Despite the considerable scatter due to the small number of data points collected in each bin, it is evident that the probability of a pressure at the 4 standard deviation level is 15 to 20 times that predicted by a normal distribution. If it is assumed that all distributions with $Cp_{mean} < -0.25$ are similar, a single distribution using the 4080 data samples from all 156 available taps with $Cp_{mean} < -0.25$ for wind direction 350 degrees can be calculated. That curve is shown by the circles and dashed lines. That data indicates the probability of a pressure at 6 standard deviations from the mean is about 4 orders of magnitude larger than predicted by the Gaussian distribution.

Concern about the validity of the above assumption led to additional analysis. Long time-records were available from two pressure taps on the lee side of two structures which had been studied in the wind tunnel for wind loading. These records were in excess of 200,000 data points each. Alternate building 1 was placed in a city environment but without any tall structures immediately upstream - an approach in the B category based on the ANSI Standard. Alternate building 2 was placed in the wind tunnel with no roughness of any kind on the floor upstream. Thus while the building was in a shear flow, the turbulence level of the approach flow was low. In Figure 6, the probability densities for these two pressure taps are plotted. In addition, the cumulative distributions for the Richmond building for 350 degree and 310 degree azimuth winds are plotted for comparison. Reasonable agreement between the curves is evident. It should be noted that the distribution for the smooth approach fell somewhat below those with higher approach turbulence. Based on this data, it may be concluded that the distributions of pressure fluctuations throughout the separated region on the structure of large negative pressures is orders of magnitude larger than a prediction based on a normal distribution.

Probability distributions for pressure taps with Cp_{mean} values between -0.1 and -0.25 did not fall clearly into either positive or negative type distributions. Some fit the positive mean characteristics some fit the negative mean distribution and some seemed to have characteristics of each. Since only a few points occurred in this regime, a conservative treatment would place the points with the negative distribution for peak-pressure prediction.

A difficulty arises in attempting to predict the probable return period for a particular pressure loading level using the distributions presented. While it is possible to predict the total length of time for which the pressure is expected to exceed a given value during a given design period, it is not possible to predict how many times the pressure will exceed the pressure level which is the important feature required to predict the likely return period for cladding failure.

A probability distribution for the maximum peak value expected during a given time period T has been obtained by Davenport (8). The analysis was intended to apply to all aspects of a structure subjected to gust loading: turbulent velocity fluctuation, fluctuating aerodynamic force and structure response. While the analysis is not specifically aimed at local pressure forces, the implication of the theory is that local pressures should also follow the same distribution. This type of probability distribution provides a better measure of frequency of occurrence of failure loads than that discussed previously. The assumed probability distribution for the pressure fluctuations is

$$p(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}x^2\right). \qquad (2)$$

The cumulative probability distribution is the probability $Q(x)$ of the function exceeding some value x

$$Q(x) = \int_x^\infty p(x)\,dx. \qquad (3)$$

The probability distribution for the largest peak in time T is

$$p(\eta) = \eta\nu T \exp[-\frac{\eta^2}{2} - \nu T \exp(-\frac{\eta^2}{2})] \quad (4)$$

where $p(\eta)d\eta$ is the probability that the maximum peak will lie between η and $\eta + d\eta$ in time T, and

$$\nu = (m_2/m_o)^{\frac{1}{2}} \quad (5)$$

$$m_r = \int_0^\infty n^r S(n)\,dn \quad (6)$$

in which S(n) is the power spectrum of the random function at the frequency n.

To compare the available experimental data from the model with that from the theory, it was again assumed that the distributions for all pressure distributions in either of the two pressure regimes studied were similar. The largest negative peak in each 4080 sample record for negative means and largest positive peak for positive means were recorded. These values formed data to plot a frequency distribution of the peak value. From both wind directions, 376 peaks from data with negative means and 125 peaks from data with positive means were obtained. These data are shown in Figures 7 and 8. In order to plot the theoretical distribution, a value of ν was required. Because a better spectrum could be obtained from a long time series, a spectrum was formed for positive and negative mean data from one of the alternate buildings used for the study. Appropriate moments of the spectrum were taken to form ν. The theoretical distributions are shown also in Figures 7 and 8.

Figure 7 shows that the data for the windward side of the building follows the theoretical distribution reasonably well. The peaks are fairly narrowly confined with very few extending above 4 to 5 standard deviations from the mean. The data in Figure 8 do not fit the theoretical curve at all well. The peak of the experimental data is displayed to higher values and the larger peaks tail off slowly to values of 9 standard deviations from the mean. This result is not surprising in light of the probability distribution for the fluctuating pressure presented earlier. The result is significant, however, in the prediction of the largest values of pressure to use for design purposes.

The data presented by Dalgliesh (6) for fluctuating pressures on a full-scale structure are mutually supportive with the present data. His data for positive pressures, was predicted reasonably well by Davenport's theory. The single case presented for a negative mean did not agree as well with the theory showing larger probability values for large multiples of standard deviations from the mean. Dalgliesh also reported large negatively oriented "spikes" in his data in the separated region which were the features which reached the large negative values. These same "spikes" were also visible in strip chart records of the wind-tunnel pressures for separated regions.

DISCUSSION

The design of structural cladding or glass lites relies on some type of gust factor. The ANSI wind loading standard uses a basic velocity pressure for parts and portions of the structure which is higher than that for overall structure design to account for the high correlation of wind fluctuation and pressure response on a small area. Built into the basic pressure loading is a factor to account for the peaks in the pressure above the mean pressure. This factor, however, does not include the tendency of the negative peaks to spread to larger values than would be expected from a Gaussian distribution of pressure fluctuations. This difference is absorbed in the design safety factor and in the conservative nature of the pressure coefficients specified. Thus the real safety factor may be lower than desired, particularly in the wind sensitive areas of the structure. Fortunately, for much of the wake region, the rms pressure fluctuations are sufficiently small that even 9 standard deviations from the mean does not represent a large pressure loading.

The present study does not provide sufficient information to provide a fully useful procedure for augmenting code-based design. The approach conditions studied apply only to a limited range of structures and approach conditions. Evidence in Figure 6 suggests that the largest peak negative values are reduced somewhat as the approach turbulence level decreases. No data were obtained with the structure in the wake of a large upstream building - a condition which could dramatically alter the results shown. They do provide a significant guide to further investigation of the peak loading for local building features.

The large peak negative values shown by the present data are due to negatively oriented spikes in the pressures. The nature of the flow in the building wake causing these pressure fluctuations is not known. It has been speculated that some form of vortex shedding is responsible. Whatever the cause, the extent of high correlation of these pressure pulses is not known - it is possible that fairly large areas respond to the same pulse. Methods of eliminating the pressure pulses by modification of the flow about the structure should be studied as a possible means of lowering the loading on sensitive areas of the structure.

CONCLUSIONS

Based on data presented in this paper, several conclusions can be formulated:

1. Probability densities of pressure fluctuations fall into two basic classes--one for $Cp_{mean} > -0.1$ and another for $Cp_{mean} < -0.25$.
2. Probability densities in the class for $Cp_{mean} > -0.1$ are nearly Gaussian.
3. Probability densities in the class for $Cp_{mean} < -0.25$ (flow separation regions) are skewed such that the probability for large negative fluctuations of 6 standard deviations is 4 orders of magnitude greater than for a Gaussian distribution.
4. Probability densities for the maximum positive peak pressure in time T for $Cp_{mean} > -0.1$ agree well with a theoretical prediction based on a Gaussian pressure fluctuation distribution.
5. Probability densities for the maximum negative peak pressure in time T for $Cp_{mean} < -0.25$ do not agree well with a theoretical prediction based on a Gaussian pressure fluctuation distribution.
6. There is an indication that the statistics of laboratory and field pressure fluctuation distributions behave in a similar manner.
7. Space correlations of the large negative peak fluctuations are not known--since this information is needed for design purposes, further research is required.

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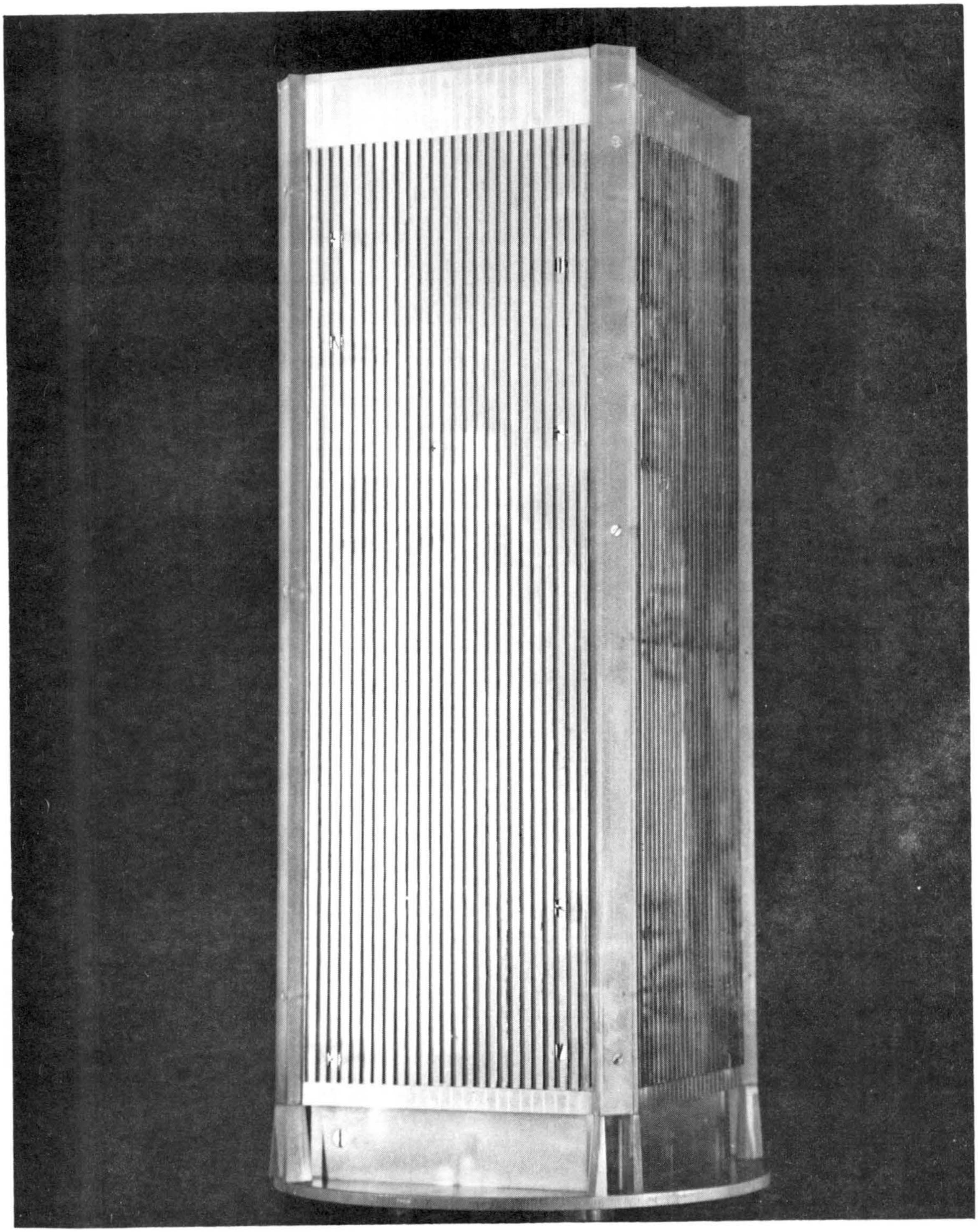

Figure 1. Wind Tunnel Model

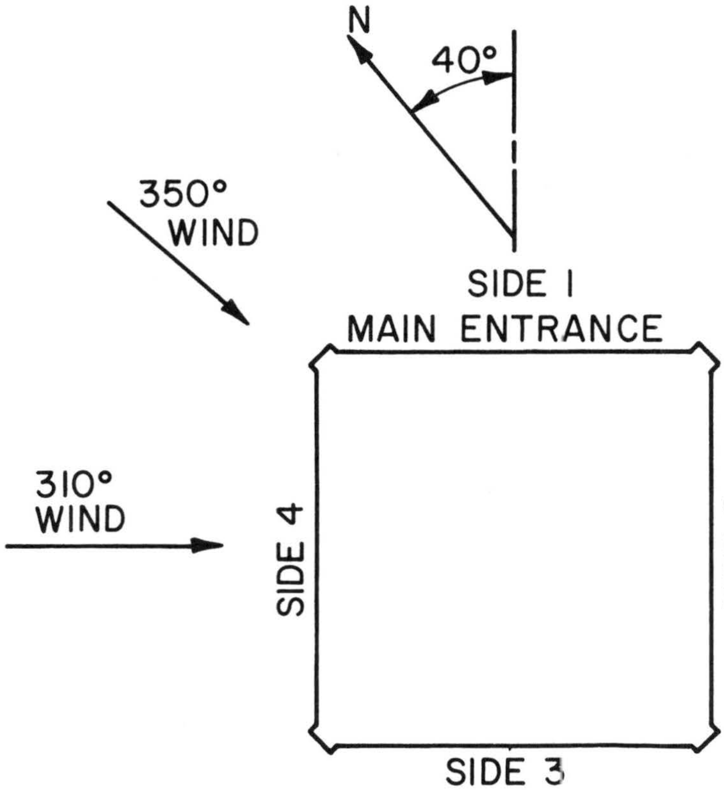

Figure 2. Building Cross Section

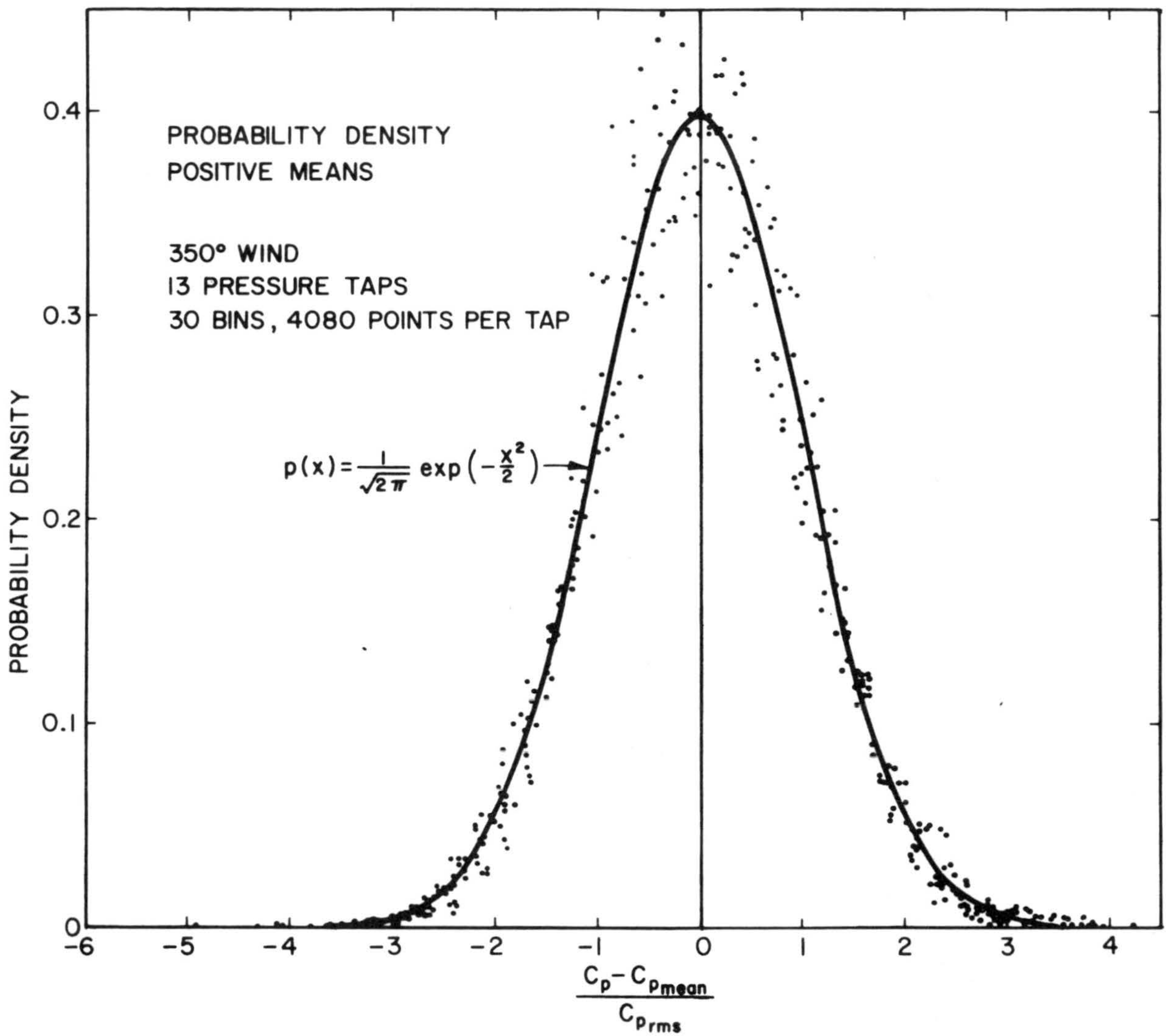

Figure 3. Probability Density for Pressure Fluctuations with $Cp_{mean} > -0.1$

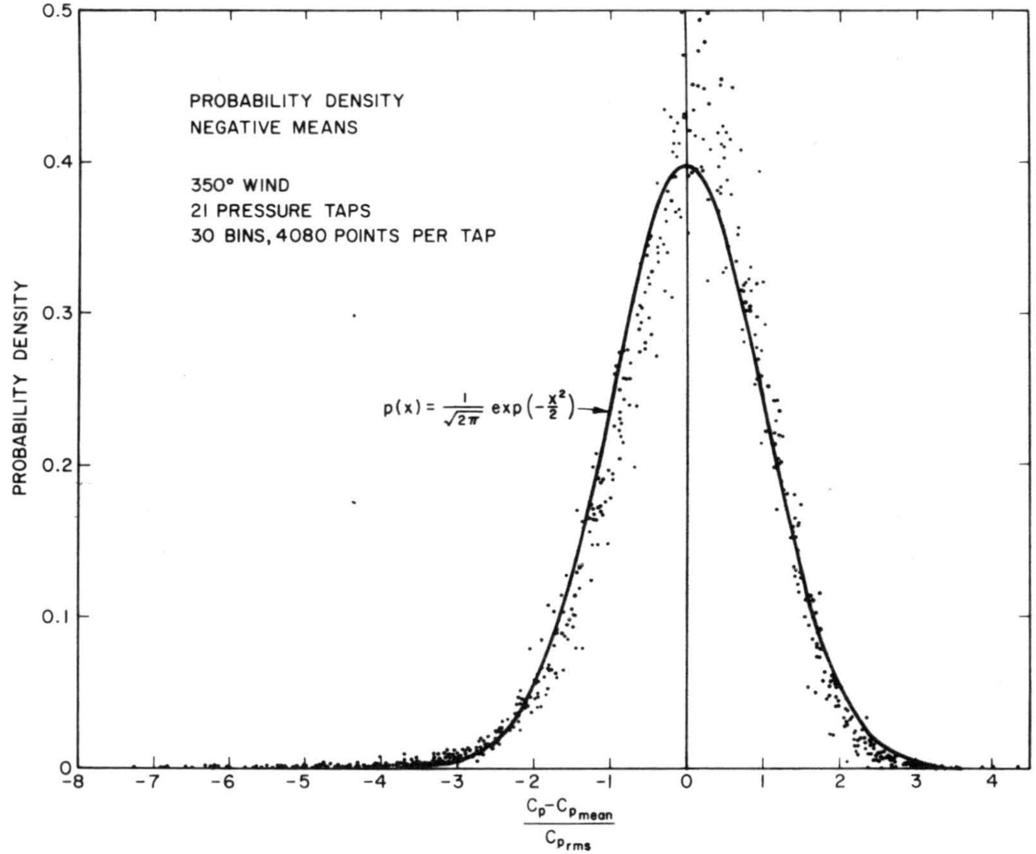

Figure 4. Probability Density for Pressure Fluctuations with $Cp_{mean} < -0.25$

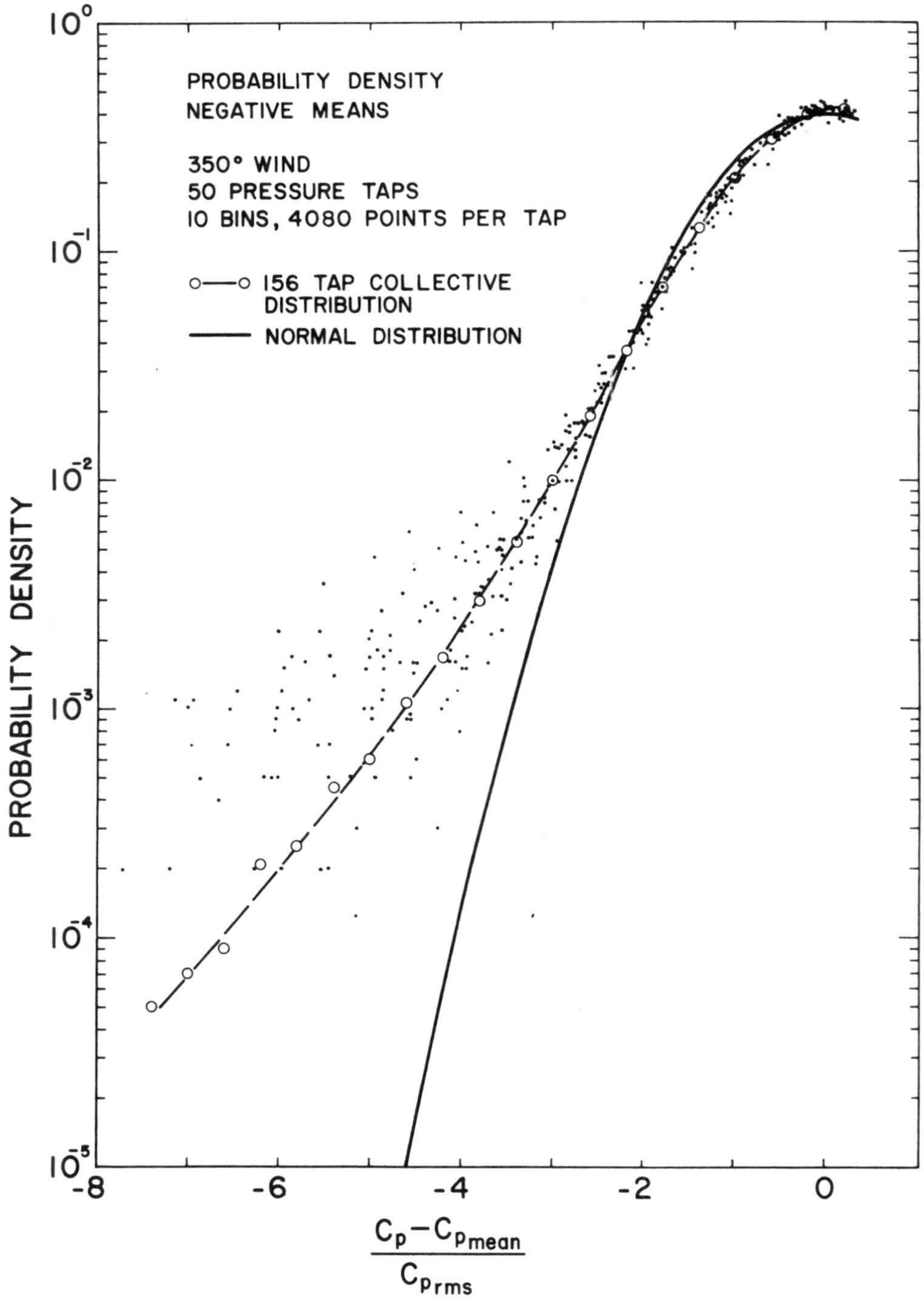

Figure 5. Negative Tail of the Probability Density for Pressure Fluctuations with $Cp_{mean} < -0.25$

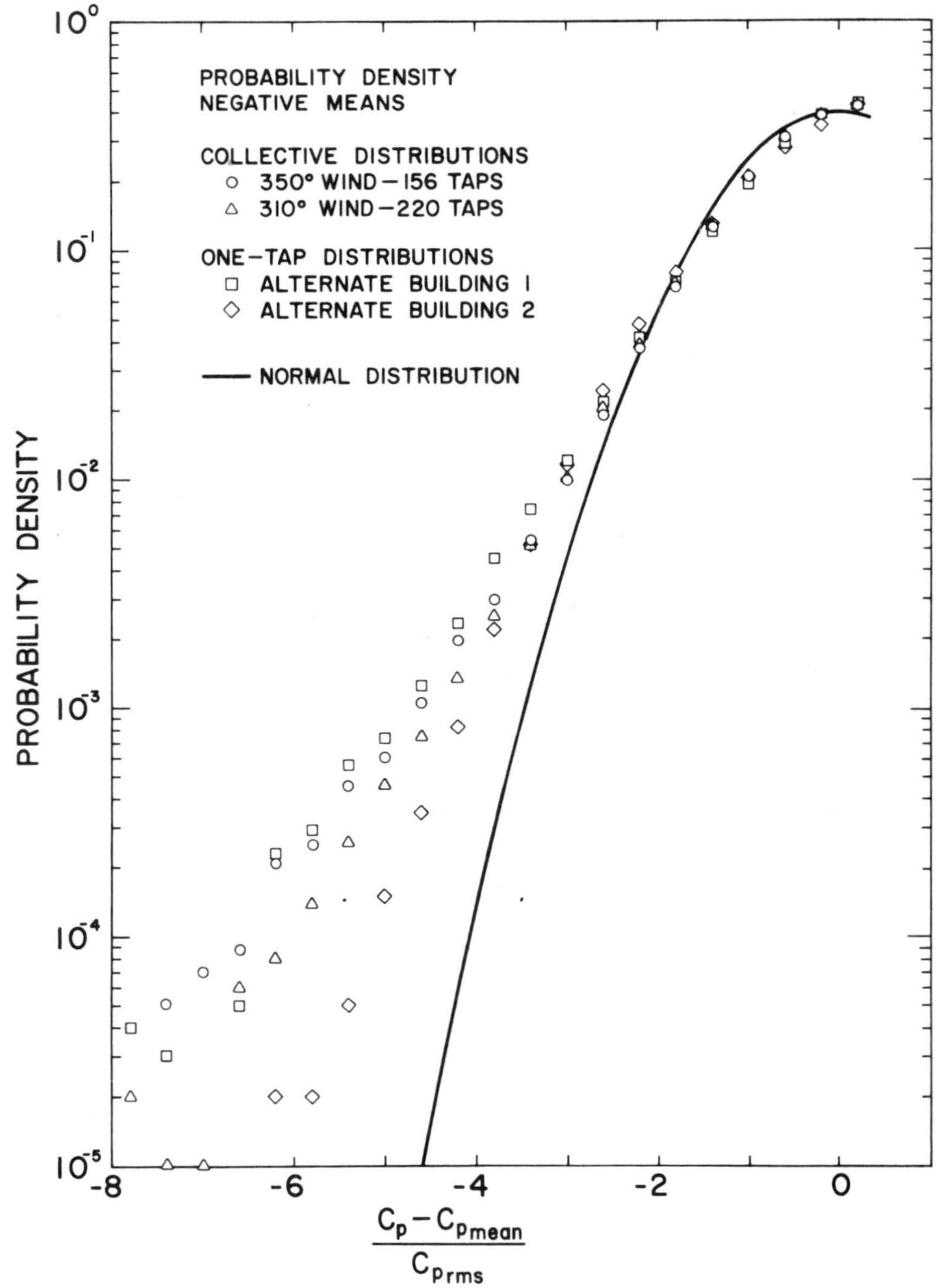

Figure 6. Collective Probability Densities from Regions of Separated Flow

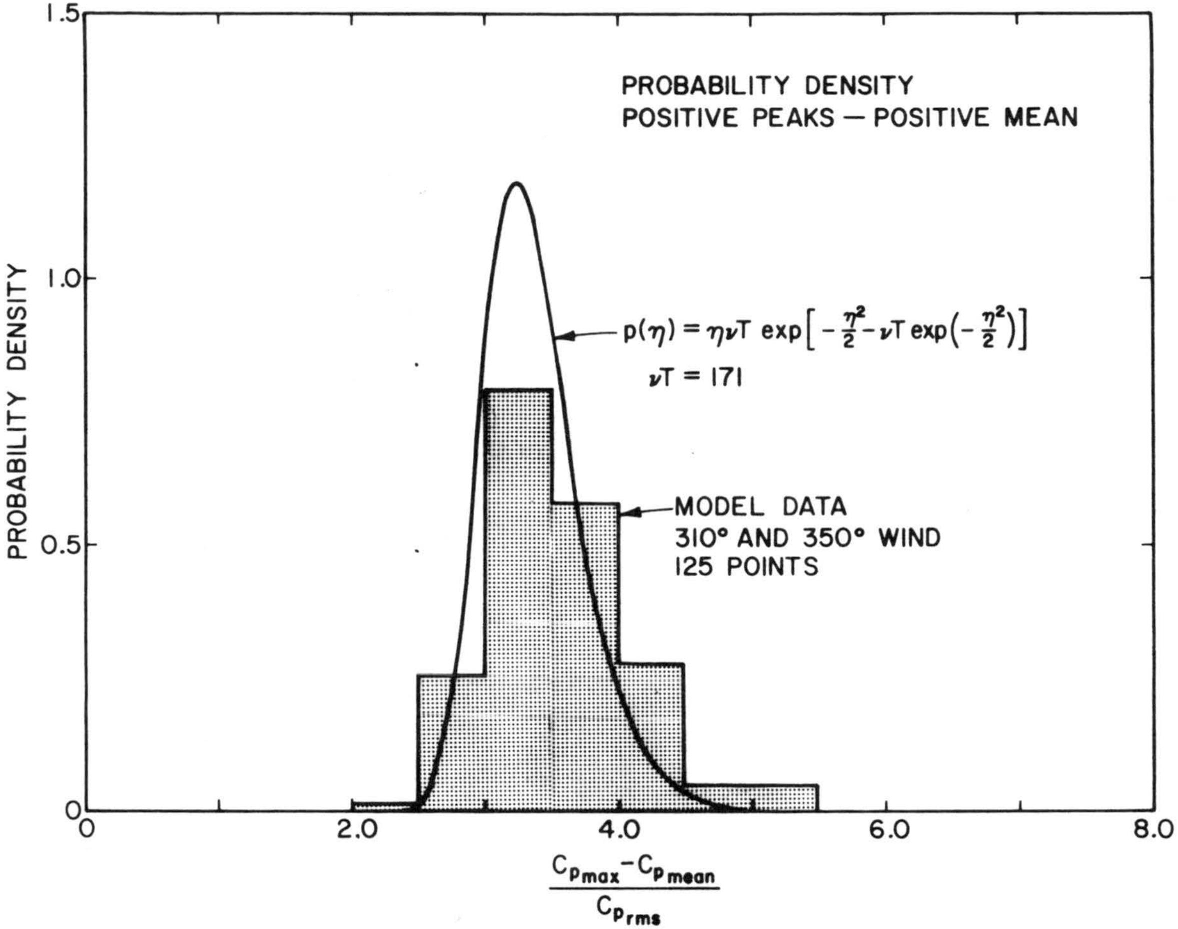

Figure 7. Probability Density for Positive Pressure Peaks for $Cp_{mean} > -0.1$

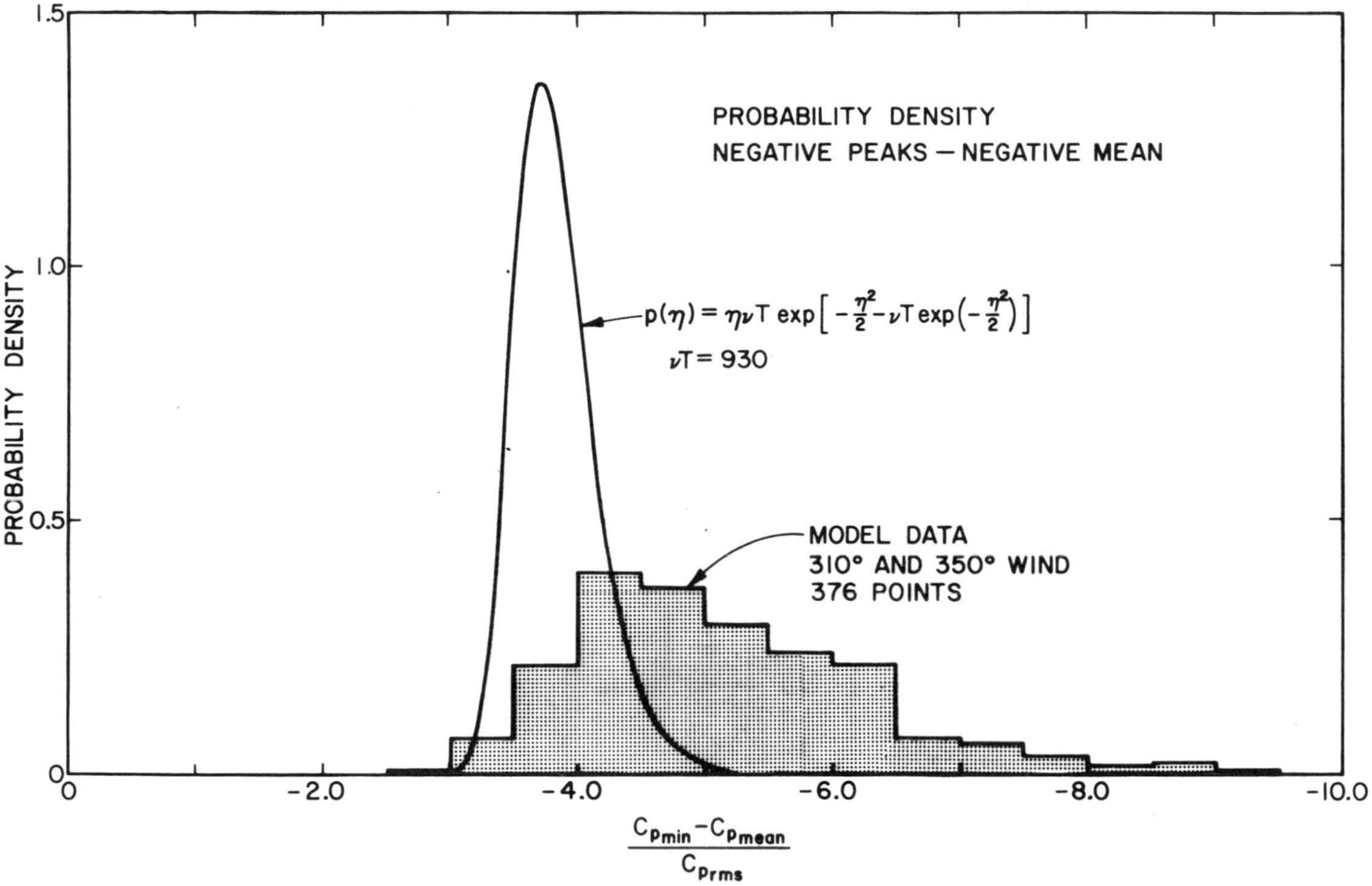

Figure 8. Probability Density for Negative Pressure Peaks for $Cp_{mean} < -0.25$