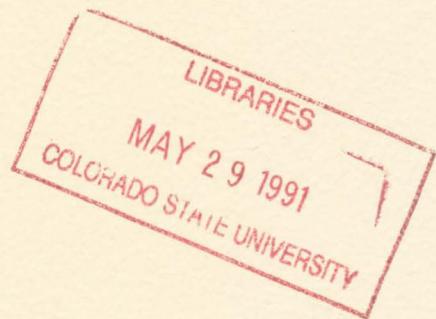


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Polynomial Disaggregation Procedures

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Abstract

In air quality monitoring, often aerosol samples are collected using various filter methods in regular sampling cycles. Some aerosol species samples may be collected at 12 hour cycles while others may be collected at 6 hour cycles or 24 hour cycles, etc. If some species are collected at 6 hour intervals and others at 12 hour intervals, then a statistical analysis of these data are often carried out after aggregating the 6 hour data to produce 12 hour data so that all data correspond to the same time cycle. It is of some interest to investigate the alternative possibility of disaggregating the 12 hour data to obtain "6 hour (pseudo) data," and then performing statistical analyses on the 6 hour scale. In this report we investigate this possibility by studying how well a certain class of disaggregation procedures is able to disaggregate aggregated data. The performance of the methods considered are evaluated using real data collected as part of the WHITEX study.

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1 Polynomial Disaggregation Procedures.

Suppose that for a sequence of values y_1, y_2, \dots, y_{2n} the sums

$$\begin{aligned} s_1 &= y_1 + y_2 \\ s_2 &= y_3 + y_4 \\ &\vdots \\ s_n &= y_{2n-1} + y_{2n} \end{aligned} \tag{1}$$

are available. A disaggregation procedure is a method for “recovering” the values y_1, y_2, \dots, y_{2n} given the sums s_1, s_2, \dots, s_n . Clearly, a unique solution does not exist as the system of equations (1) amounts to n equations in $2n$ unknowns. For this reason, additional information or assumptions are necessary. Here, it will be assumed that

$$\begin{aligned} &\text{any set of } 2k \text{ consecutive values } y_r, y_{r+1}, \dots, y_{r+2k-1} \text{ from the } y \text{ sequence} \\ &\text{lie “approximately” on a polynomial curve of degree } k-1. \end{aligned} \tag{2}$$

This assumption leads to useful solutions for the values of y_1, y_2, \dots, y_{2n} in (2.1). Such a procedure will be called a polynomial disaggregation procedure of degree $k-1$. We now derive explicitly polynomial disaggregation procedures of degree $k-1$ for $k=1, 2, 3, 4, 5$.

Case 1: $k=1$.

This implies that every pair of adjacent y values are “approximately” equal. In particular $y_1 \approx y_2$ so that the equation $y_1 + y_2 = s_1$ gives $\hat{y}_1 = \hat{y}_2 = s_1/2$. Similarly, $\hat{y}_3 = \hat{y}_4 = s_2/2$. In general, we get $\hat{y}_{2i-1} = \hat{y}_{2i} = s_i/2$, for $i=1, 2, \dots, n$.

Case 2: $k=2$.

Assumption (2) implies that every sequence of four adjacent values lies “approximately” on a straight line:

$$y_i \approx a + bi, \text{ for some } a, b,$$

where a and b will depend on the particular set of four values being considered. In particular, consider y_1, y_2, y_3 , and y_4 . In this case, we have for some a and b ,

$$\begin{aligned} y_1 &\approx a + b & y_3 &\approx a + 3b \\ y_2 &\approx a + 2b & y_4 &\approx a + 4b. \end{aligned}$$

Hence, $s_1 = y_1 + y_2 = 2a + 3b$ and $s_2 = y_3 + y_4 = 2a + 7b$. This yields the solutions:

$$a = \frac{7s_1 - 3s_2}{8} \quad b = \frac{s_2 - s_1}{4}.$$

Thus, we have the following estimates:

$$\begin{aligned}\hat{y}_1 &= a + b = \frac{5s_1 - s_2}{8} & \hat{y}_3 &= a + 3b = \frac{s_1 + 3s_2}{8} \\ \hat{y}_2 &= a + 2b = \frac{3s_1 + s_2}{8} & \hat{y}_4 &= a + 4b = \frac{-s_1 + 5s_2}{8}.\end{aligned}$$

Now consider sliding a “window” that is four values wide across the y sequence—sliding it to the right two values each time. This generates the following “windows:”

$$\begin{aligned}y_1, y_2, y_3, y_4 \\ y_3, y_4, y_5, y_6 \\ \vdots \\ y_{2n-5}, y_{2n-4}, y_{2n-3}, y_{2n-2} \\ y_{2n-3}, y_{2n-2}, y_{2n-1}, y_{2n}.\end{aligned}$$

Note that there will be $n - 1$ “windows.”

Case 2A: Consider using each “window” to estimate the first two points that it contains. Then, for $1 \leq r \leq n - 1$,

$$\left. \begin{aligned}\hat{y}_{2r-1} &= \frac{5s_r - s_{r+1}}{8} \\ \hat{y}_{2r} &= \frac{3s_r + s_{r+1}}{8}\end{aligned} \right\} \text{ using the window } \underbrace{y_{2r-1}, y_{2r}}_{s_r}, \underbrace{y_{2r+1}, y_{2r+2}}_{s_{r+1}},$$

and

$$\left. \begin{aligned}\hat{y}_{2n-1} &= \frac{s_{n-1} + 3s_n}{8} \\ \hat{y}_{2n} &= \frac{-s_{n-1} + 5s_n}{8}\end{aligned} \right\} \text{ using the window } \underbrace{y_{2n-3}, y_{2n-2}}_{s_{n-1}}, \underbrace{y_{2n-1}, y_{2n}}_{s_n}.$$

Note that all four values in the last window are estimated from the information provided by that window. In all other windows, only the first two values are estimated.

Case 2B: Consider using each “window” to estimate the last two points that it contains. Then,

$$\left. \begin{aligned}\hat{y}_1 &= \frac{5s_1 - s_2}{8} \\ \hat{y}_2 &= \frac{3s_1 + s_2}{8}\end{aligned} \right\} \text{ using the window } \underbrace{y_1, y_2}_{s_1}, \underbrace{y_3, y_4}_{s_2},$$

and, for $2 \leq r \leq n$,

$$\left. \begin{aligned}\hat{y}_{2r-1} &= \frac{s_{r-1} + 3s_r}{8} \\ \hat{y}_{2r} &= \frac{-s_{r-1} + 5s_r}{8}\end{aligned} \right\} \text{ using the window } \underbrace{y_{2r-3}, y_{2r-2}}_{s_{r-1}}, \underbrace{y_{2r-1}, y_{2r}}_{s_r}.$$

Here, note that all four values in the first window are estimated from the information provided by that window, but in subsequent windows only the last two values are estimated.

Case 2C: There is an asymmetry associated with the disaggregation procedures in cases 2A and 2B which may be unsatisfactory. One may wonder why the two middle values from each window are not estimated. The reason for this is that, when aggregated, the disaggregated data are not guaranteed to reproduce the original sums. Since this could be viewed as an undesirable property, only polynomial disaggregation procedures that preserve the original sums will be considered. To overcome the asymmetry we consider using the average of the two disaggregation procedures—cases 2A and 2B. Then,

$$\left. \begin{array}{l} \hat{y}_1 = \frac{5s_1 - s_2}{8} \\ \hat{y}_2 = \frac{3s_1 + s_2}{8} \end{array} \right\} \quad \begin{array}{l} \text{using the window} \\ \underbrace{y_1, y_2}_{s_1}, \underbrace{y_3, y_4}_{s_2}. \end{array}$$

For $2 \leq r \leq n-1$,

$$\left. \begin{array}{l} \hat{y}_{2r-1} = \frac{s_{r-1} + 8s_r - s_{r+1}}{16} \\ \hat{y}_{2r} = \frac{-s_{r-1} + 8s_r + s_{r+1}}{16} \end{array} \right\} \quad \begin{array}{l} \text{using the window} \\ \underbrace{y_{2r-3}, y_{2r-2}}_{s_{r-1}}, \underbrace{y_{2r-1}, y_{2r}}_{s_r}, \underbrace{y_{2r+1}, y_{2r+2}}_{s_{r+1}}. \end{array}$$

Also,

$$\left. \begin{array}{l} \hat{y}_{2n-1} = \frac{s_{n-1} + 3s_n}{8} \\ \hat{y}_{2n} = \frac{-s_{n-1} + 5s_n}{8} \end{array} \right\} \quad \begin{array}{l} \text{using the window} \\ \underbrace{y_{2n-3}, y_{2n-2}}_{s_{n-1}}, \underbrace{y_{2n-1}, y_{2n}}_{s_n}. \end{array}$$

Case 3: $k = 3$.

Assumption (2) implies that each sequence of six adjacent y values lies “approximately” on a quadratic:

$$y_i = a + bi + ci^2, \text{ for some } a, b, c,$$

where a , b , and c will depend on the values of the particular set of six values being considered. In particular, consider y_1, y_2, \dots, y_6 . In this case, we have for some a , b and c ,

$$\begin{array}{ll} y_1 \approx a + b + c & y_4 \approx a + 4b + 16c \\ y_2 \approx a + 2b + 4c & y_5 \approx a + 5b + 25c \\ y_3 \approx a + 3b + 9c & y_6 \approx a + 6b + 36c \end{array}$$

Hence, $s_1 = 2a + 3b + 5c$, and $s_2 = 2a + 7b + 25c$, and $s_3 = 2a + 11b + 61c$. This yields the solutions:

$$\begin{aligned} a &= \frac{19s_1 - 16s_2 + 5s_3}{16} \\ b &= \frac{-9s_1 + 14s_2 - 5s_3}{16} \\ c &= \frac{s_1 - 2s_2 + s_3}{16}. \end{aligned}$$

Thus, we have the following estimates:

$$\begin{aligned} \hat{y}_1 &= \frac{11s_1 - 4s_2 + s_3}{16} & \hat{y}_4 &= \frac{-s_1 + 8s_2 + s_3}{16} \\ \hat{y}_2 &= \frac{5s_1 + 4s_2 - s_3}{16} & \hat{y}_5 &= \frac{-s_1 + 4s_2 + 5s_3}{16} \\ \hat{y}_3 &= \frac{s_1 + 8s_2 - s_3}{16} & \hat{y}_6 &= \frac{s_1 - 4s_2 + 11s_3}{16}. \end{aligned}$$

Now consider sliding a “window” that is six values wide across the y sequence—sliding it to the right two values each time. This generates the following “windows:”

$$\begin{aligned} &y_1, y_2, y_3, y_4, y_5, y_6 \\ &y_3, y_4, y_5, y_6, y_7, y_8 \\ &\vdots \\ &y_{2n-7}, y_{2n-6}, y_{2n-5}, y_{2n-4}, y_{2n-3}, y_{2n-2} \\ &y_{2n-5}, y_{2n-4}, y_{2n-3}, y_{2n-2}, y_{2n-1}, y_{2n}. \end{aligned}$$

Note that there will be $n - 2$ “windows.” Consider using each “window” to estimate the two middle values that it contains. (The first window will estimate the first two values, also. Similarly, the last window will estimate the last two values, as well.) Then a disaggregation procedure would be:

$$\left. \begin{aligned} \hat{y}_1 &= \frac{11s_1 - 4s_2 + s_3}{16} \\ \hat{y}_2 &= \frac{5s_1 + 4s_2 - s_3}{16} \end{aligned} \right\} \quad \begin{aligned} &\text{using the window} \\ &\underbrace{y_1, y_2, y_3, y_4, y_5, y_6}_{s_1, s_2, s_3}. \end{aligned}$$

For $2 \leq r \leq n - 1$,

$$\left. \begin{aligned} \hat{y}_{2r-1} &= \frac{s_{r-1} + 8s_r - s_{r+1}}{16} \\ \hat{y}_{2r} &= \frac{-s_{r-1} + 8s_r + s_{r+1}}{16} \end{aligned} \right\} \quad \begin{aligned} &\text{using the window} \\ &\underbrace{y_{2r-3}, y_{2r-2}, y_{2r-1}, y_{2r}, y_{2r+1}, y_{2r+2}}_{s_{r-1}, s_r, s_{r+1}}. \end{aligned}$$

Also,

$$\left. \begin{array}{l} \hat{y}_{2n-1} = \frac{-s_{n-2} + 4s_{n-1} + 5s_n}{16} \\ \hat{y}_{2n} = \frac{s_{n-2} - 4s_{n-1} + 11s_n}{16} \end{array} \right\} \quad \begin{array}{l} \text{using the window} \\ \underbrace{y_{2n-5}, y_{2n-4}}_{s_{n-2}}, \underbrace{y_{2n-3}, y_{2n-2}}_{s_{n-1}}, \underbrace{y_{2n-1}, y_{2n}}_{s_n}. \end{array}$$

Case 4: $k = 4$.

Assumption (2) implies that each sequence of eight adjacent y values lies “approximately” on a cubic:

$$y_i = a + bi + ci^2 + di^3, \text{ for some } a, b, c, d.$$

where a, b, c , and d will depend on the values of the particular set of eight values being considered. In particular, consider y_1, y_2, \dots, y_8 . In this case, we have for some a, b, c , and d ,

$$\begin{array}{ll} y_1 \approx a + b + c + d & y_5 \approx a + 5b + 25c + 125d \\ y_2 \approx a + 2b + 4c + 8d & y_6 \approx a + 6b + 36c + 216d \\ y_3 \approx a + 3b + 9c + 27d & y_7 \approx a + 7b + 49c + 343d \\ y_4 \approx a + 4b + 16c + 64d & y_8 \approx a + 8b + 64c + 512d. \end{array}$$

Hence,

$$\begin{array}{l} s_1 = 2a + 3b + 5c + 9d \\ s_2 = 2a + 7b + 25c + 91d \\ s_3 = 2a + 11b + 61c + 341d \\ s_4 = 2a + 15b + 113c + 855d. \end{array}$$

This yields the solutions:

$$\begin{array}{l} a = \frac{187s_1 - 233s_2 + 145s_3 - 35s_4}{128} \\ b = \frac{-43s_1 + 90s_2 - 63s_3 + 16s_4}{48} \\ c = \frac{33s_1 - 87s_2 + 75s_3 - 21s_4}{192} \\ d = \frac{-s_1 + 3s_2 - 3s_3 + s_4}{96}. \end{array}$$

Thus, we have the following estimates:

$$\begin{aligned}
\hat{y}_1 &= \frac{93s_1 - 47s_2 + 23s_3 - 5s_4}{128} \\
\hat{y}_2 &= \frac{35s_1 + 47s_2 - 23s_3 + 5s_4}{128} \\
\hat{y}_3 &= \frac{5s_1 + 73s_2 - 17s_3 + 3s_4}{128} \\
\hat{y}_4 &= \frac{-5s_1 + 55s_2 + 17s_3 - 3s_4}{128} \\
\hat{y}_5 &= \frac{-3s_1 + 17s_2 + 55s_3 - 5s_4}{128} \\
\hat{y}_6 &= \frac{3s_1 - 17s_2 + 73s_3 + 5s_4}{128} \\
\hat{y}_7 &= \frac{5s_1 - 23s_2 + 47s_3 + 35s_4}{128} \\
\hat{y}_8 &= \frac{-5s_1 + 23s_2 - 47s_3 + 93s_4}{128}.
\end{aligned}$$

Now consider sliding a “window” that is eight values wide across the y sequence—sliding it to the right two values each time. This generates the following “windows:”

$$\begin{aligned}
&y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8 \\
&y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10} \\
&\vdots \\
&y_{2n-9}, y_{2n-8}, y_{2n-7}, y_{2n-6}, y_{2n-5}, y_{2n-4}, y_{2n-3}, y_{2n-2} \\
&y_{2n-7}, y_{2n-6}, y_{2n-5}, y_{2n-4}, y_{2n-3}, y_{2n-2}, y_{2n-1}, y_{2n}.
\end{aligned}$$

Note that this yields $n - 3$ “windows.”

Case 4A: Consider using each “window” to estimate the third and fourth points that it contains. Then,

$$\left. \begin{aligned}
\hat{y}_1 &= \frac{93s_1 - 47s_2 + 23s_3 - 5s_4}{128} \\
\hat{y}_2 &= \frac{35s_1 + 47s_2 - 23s_3 + 5s_4}{128}
\end{aligned} \right\} \quad \begin{array}{l} \text{using the window} \\ \underbrace{y_1, y_2, \dots,}_{s_1} \underbrace{y_7, y_8}_{s_4} \end{array}.$$

For $2 \leq r \leq n - 2$,

$$\left. \begin{aligned}
\hat{y}_{2r-1} &= \frac{5s_{r-1} + 73s_r - 17s_{r+1} + 3s_{r+2}}{128} \\
\hat{y}_{2r} &= \frac{-5s_{r-1} + 55s_r + 17s_{r+1} - 3s_{r+2}}{128}
\end{aligned} \right\} \quad \begin{array}{l} \text{using the window} \\ \underbrace{y_{2r-3}, y_{2r-2}, \dots,}_{s_{r-1}} \underbrace{y_{2r+3}, y_{2r+4}}_{s_{r+2}} \end{array}.$$

Also,

$$\left. \begin{aligned}
\hat{y}_{2n-3} &= \frac{-3s_{n-3} + 17s_{n-2} + 55s_{n-1} - 5s_n}{128} \\
\hat{y}_{2n-2} &= \frac{3s_{n-3} - 17s_{n-2} + 73s_{n-1} + 5s_n}{128} \\
\hat{y}_{2n-1} &= \frac{5s_{n-3} - 23s_{n-2} + 47s_{n-1} + 35s_n}{128} \\
\hat{y}_{2n} &= \frac{-5s_{n-3} + 23s_{n-2} - 47s_{n-1} + 93s_n}{128}
\end{aligned} \right\} \quad \begin{array}{l} \text{using the window} \\ \underbrace{y_{2n-7}, y_{2n-6}, \dots,}_{s_{n-3}} \underbrace{y_{2n-1}, y_{2n}}_{s_n} \end{array}.$$

Note that the first window is used to also estimate the first two values, and the last window is used to also estimate the last four values.

Case 4B: Consider using each “window” to estimate the fifth and sixth points that it contains. Then,

$$\left. \begin{array}{l} \hat{y}_1 = \frac{93s_1 - 47s_2 + 23s_3 - 5s_4}{128} \\ \hat{y}_2 = \frac{35s_1 + 47s_2 - 23s_3 + 5s_4}{128} \\ \hat{y}_3 = \frac{5s_1 + 73s_2 - 17s_3 + 3s_4}{128} \\ \hat{y}_4 = \frac{-5s_1 + 55s_2 + 17s_3 - 3s_4}{128} \end{array} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_1, y_2, \dots}_{s_1}, \underbrace{y_7, y_8}_{s_4}. \end{array}$$

For $3 \leq r \leq n-1$,

$$\left. \begin{array}{l} \hat{y}_{2r-1} = \frac{-3s_{r-2} + 17s_{r-1} + 55s_r - 5s_{r+1}}{128} \\ \hat{y}_{2r} = \frac{3s_{r-2} - 17s_{r-1} + 73s_r + 5s_{r+1}}{128} \end{array} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_{2r-5}, y_{2r-4}, \dots}_{s_{r-2}}, \underbrace{y_{2r+1}, y_{2r+2}}_{s_{r+1}}. \end{array}$$

Also,

$$\left. \begin{array}{l} \hat{y}_{2n-1} = \frac{5s_{n-3} - 23s_{n-2} + 47s_{n-1} + 35s_n}{128} \\ \hat{y}_{2n} = \frac{-5s_{n-3} + 23s_{n-2} - 47s_{n-1} + 93s_n}{128} \end{array} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_{2n-7}, y_{2n-6}, \dots}_{s_{n-3}}, \underbrace{y_{2n-1}, y_{2n}}_{s_n}. \end{array}$$

Note that the first window is used to also estimate the first four values, and the last window is used to also estimate the last two values.

Case 4C: To overcome the asymmetry involved in cases 4A and 4B, consider using the average of the two disaggregation procedures. Then,

$$\left. \begin{array}{l} \hat{y}_1 = \frac{93s_1 - 47s_2 + 23s_3 - 5s_4}{128} \\ \hat{y}_2 = \frac{35s_1 + 47s_2 - 23s_3 + 5s_4}{128} \\ \hat{y}_3 = \frac{5s_1 + 73s_2 - 17s_3 + 3s_4}{128} \\ \hat{y}_4 = \frac{-5s_1 + 55s_2 + 17s_3 - 3s_4}{128} \end{array} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_1, y_2, \dots}_{s_1}, \underbrace{y_7, y_8}_{s_4}. \end{array}$$

For $3 \leq r \leq n-2$,

$$\left. \begin{array}{l} \hat{y}_{2r-1} = \frac{-3s_{r-2} + 22s_{r-1} + 128s_r - 22s_{r+1} + 3s_{r+2}}{256} \\ \hat{y}_{2r} = \frac{3s_{r-2} - 22s_{r-1} + 128s_r + 22s_{r+1} - 3s_{r+2}}{256} \end{array} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_{2r-5}, y_{2r-4}, \dots}_{s_{r-2}}, \underbrace{y_{2r+3}, y_{2r+4}}_{s_{r+2}}. \end{array}$$

Also,

$$\left. \begin{array}{l} \hat{y}_{2n-3} = \frac{-3s_{n-3} + 17s_{n-2} + 55s_{n-1} - 5s_n}{128} \\ \hat{y}_{2n-2} = \frac{3s_{n-3} - 17s_{n-2} + 73s_{n-1} + 5s_n}{128} \\ \hat{y}_{2n-1} = \frac{5s_{n-3} - 23s_{n-2} + 47s_{n-1} + 35s_n}{128} \\ \hat{y}_{2n} = \frac{-5s_{n-3} + 23s_{n-2} - 47s_{n-1} + 93s_n}{128} \end{array} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_{2n-7}, y_{2n-6}, \dots, y_{2n-1}}_{s_{n-3}}, \dots, \underbrace{y_{2n-1}, y_{2n}}_{s_n}. \end{array}$$

Case 5: $k = 5$.

Assumption (2) implies that each sequence of ten adjacent y values lies “approximately” on a quartic:

$$y_i = a + bi + ci^2 + di^3 + ei^4, \text{ for some } a, b, c, d, e,$$

where a, b, c, d , and e depend on the values of the particular set of ten values being considered. In particular, consider y_1, y_2, \dots, y_{10} . In this case, we have for some a, b, c, d , and e ,

$$\begin{array}{ll} y_1 \approx a + b + c + d + e & y_6 \approx a + 6b + 36c + 216d + 1296e \\ y_2 \approx a + 2b + 4c + 8d + 16e & y_7 \approx a + 7b + 49c + 343d + 2401e \\ y_3 \approx a + 3b + 9c + 27d + 81e & y_8 \approx a + 8b + 64c + 512d + 4096e \\ y_4 \approx a + 4b + 16c + 64d + 256e & y_9 \approx a + 9b + 81c + 729d + 6561e \\ y_5 \approx a + 5b + 25c + 125d + 625e & y_{10} \approx a + 10b + 100c + 1000d + 10000e. \end{array}$$

Hence,

$$\begin{aligned} s_1 &= 2a + 3b + 5c + 9d + 17e \\ s_2 &= 2a + 7b + 25c + 91d + 337e \\ s_3 &= 2a + 11b + 61c + 341d + 1921e \\ s_4 &= 2a + 15b + 113c + 855d + 6497e \\ s_5 &= 2a + 19b + 181c + 1729d + 16561e. \end{aligned}$$

This yields the solutions:

$$\begin{aligned} a &= \frac{1311s_1 - 2154s_2 + 2004s_3 - 966s_4 + 189s_5}{768} \\ b &= \frac{-949s_1 + 2484s_2 - 2574s_3 + 1300s_4 - 261s_5}{768} \\ c &= \frac{242s_1 - 788s_2 + 960s_3 - 524s_4 + 110s_5}{768} \\ d &= \frac{-26s_1 + 96s_2 - 132s_3 + 80s_4 - 18s_5}{768} \\ e &= \frac{s_1 - 4s_2 + 6s_3 - 4s_4 + s_5}{768}. \end{aligned}$$

Thus, we have the following estimates:

$$\begin{aligned}\hat{y}_1 &= \frac{193s_1 - 122s_2 + 88s_3 - 38s_4 + 7s_5}{256} \\ \hat{y}_2 &= \frac{63s_1 + 122s_2 - 88s_3 + 38s_4 - 7s_5}{256} \\ \hat{y}_3 &= \frac{7s_1 + 158s_2 - 52s_3 + 18s_4 - 3s_5}{256} \\ \hat{y}_4 &= \frac{-7s_1 + 98s_2 + 52s_3 - 18s_4 + 3s_5}{256} \\ \hat{y}_5 &= \frac{-3s_1 + 22s_2 + 128s_3 - 22s_4 + 3s_5}{256} \\ \hat{y}_6 &= \frac{3s_1 - 22s_2 + 128s_3 + 22s_4 - 3s_5}{256} \\ \hat{y}_7 &= \frac{3s_1 - 18s_2 + 52s_3 + 98s_4 - 7s_5}{256} \\ \hat{y}_8 &= \frac{-3s_1 + 18s_2 - 52s_3 + 158s_4 + 7s_5}{256} \\ \hat{y}_9 &= \frac{-7s_1 + 38s_2 - 88s_3 + 122s_4 + 63s_5}{256} \\ \hat{y}_{10} &= \frac{7s_1 - 38s_2 + 88s_3 - 122s_4 + 193s_5}{256}.\end{aligned}$$

Now consider sliding a “window” that is ten values wide across the y sequence—sliding it to the right two values each time. This generates the following “windows:”

$$\begin{aligned}&y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10} \\ &y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12} \\ &\vdots \\ &y_{2n-11}, y_{2n-10}, y_{2n-9}, y_{2n-8}, y_{2n-7}, y_{2n-6}, y_{2n-5}, y_{2n-4}, y_{2n-3}, y_{2n-2} \\ &y_{2n-9}, y_{2n-8}, y_{2n-7}, y_{2n-6}, y_{2n-5}, y_{2n-4}, y_{2n-3}, y_{2n-2}, y_{2n-1}, y_{2n}.\end{aligned}$$

Note that there will be $n - 4$ “windows.” Consider using each “window” to estimate the two middle values that it contains. Here, the first window is used to also estimate the first four values, and the last window is used to also estimate the last four values. Then a disaggregation procedure would be:

$$\left. \begin{aligned}\hat{y}_1 &= \frac{193s_1 - 122s_2 + 88s_3 - 38s_4 + 7s_5}{256} \\ \hat{y}_2 &= \frac{63s_1 + 122s_2 - 88s_3 + 38s_4 - 7s_5}{256} \\ \hat{y}_3 &= \frac{7s_1 + 158s_2 - 52s_3 + 18s_4 - 3s_5}{256} \\ \hat{y}_4 &= \frac{-7s_1 + 98s_2 + 52s_3 - 18s_4 + 3s_5}{256}\end{aligned} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_1, y_2, \dots, y_9, y_{10}}_{s_1 \quad \dots \quad s_5} \end{array}$$

For $3 \leq r \leq n - 2$,

$$\left. \begin{aligned}\hat{y}_{2r-1} &= \frac{-3s_{r-2} + 22s_{r-1} + 128s_r - 22s_{r+1} + 3s_{r+2}}{256} \\ \hat{y}_{2r} &= \frac{3s_{r-2} - 22s_{r-1} + 128s_r + 22s_{r+1} - 3s_{r+2}}{256}\end{aligned} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_{2r-5}, y_{2r-4}, \dots, y_{2r+3}, y_{2r+4}}_{s_{r-2} \quad \dots \quad s_{r+2}} \end{array}$$

Also,

$$\left. \begin{array}{l} \hat{y}_{2n-3} = \frac{3s_{n-4}-18s_{n-3}+52s_{n-2}+98s_{n-1}-7s_n}{256} \\ \hat{y}_{2n-2} = \frac{-3s_{n-4}+18s_{n-3}-52s_{n-2}+158s_{n-1}+7s_n}{256} \\ \hat{y}_{2n-1} = \frac{-7s_{n-4}+38s_{n-3}-88s_{n-2}+122s_{n-1}+63s_n}{256} \\ \hat{y}_n = \frac{7s_{n-4}-38s_{n-3}+88s_{n-2}-122s_{n-1}+193s_n}{256} \end{array} \right\} \begin{array}{l} \text{using the window} \\ \underbrace{y_{2n-9}, y_{2n-8}, \dots, y_{2n-1}, y_{2n}}_{s_{n-4} \quad \cdots \quad s_n} \end{array}$$

2 Comment on the Procedures

On examination, one will notice a relationship among some of the procedures. In particular, cases 3 and 2C yield the same estimates for y_3, \dots, y_{2n-2} , and cases 5 and 4C yield the same estimates for y_5, \dots, y_{2n-4} . As proposed, case 2C is the average of cases 2A and 2B, and case 4C is the average of cases 4A and 4B. For these reasons, one might consider using only polynomial disaggregation procedures of even degree. In the applications in this paper, cases 2C and 4C will not be considered since they are only different from cases 3 and 5, respectively, at the beginning and end of the sequence.

Since it is possible that the disaggregation procedure will yield negative solutions, a modification to the described procedures will be needed if the individual terms in the y sequence are assumed to be nonnegative. Let y_1, \dots, y_{2n} denote the disaggregated y sequence. The modification used in this paper is:

If $y_{2k-1} < 0$, then set $y_{2k-1} = 0$ and $y_{2k} = s_k$, and if $y_{2k} < 0$, then set $y_{2k-1} = s_k$ and $y_{2k} = 0$, for $k = 1, \dots, 2n$.

Another problem one might encounter is missing values. The way they were handled, here, was to break the data up into complete sequences. For example, if the data consisted of s_1, \dots, s_{10} , where s_8 was missing, then the data would be broken into two complete sequences: s_1, \dots, s_7 , and s_9, s_{10} . Missing values cause a difficulty in the sense that one must examine each complete sequence to see which cases of the polynomial disaggregation procedures can be applied. In the previously mentioned example, only cases 1, 2A, and 2B could be applied to the second complete sequence since the sequence is of only two sums.

3 Artificial Examples

Four artificial y sequences of 100 values were generated, and then aggregated to get 50 sums. The disaggregation procedures were used on each set of sums to "recover" the original values of the y sequence. This was done mainly as a preliminary investigation step only and no general conclusions can be drawn from these limited results. A more in depth study will be beneficial. A brief description of the models used to generate the y sequences follows:

Table 1: Correlation between Actual and Recovered Data for Artificial Data Sets.

Model	Case						
	1	2A	2B	3	4A	4B	5
AR(1)	0.899570	0.890959	0.896676	0.918599	0.906304	0.908505	0.905130
MA(1)	0.735753	0.772545	0.731759	0.786077	0.760003	0.772304	0.763941
Sin(t)	0.879573	0.906997	0.908833	0.955458	0.968981	0.968533	0.986296
Sin(t/20)	0.999715	0.999999	0.999999	1.000000	1.000000	1.000000	1.000000

Autoregressive Process. The y sequence was generated using a first order autoregressive process, AR(1), given by:

$$y(t) = \frac{1}{2}y(t-1) + z(t),$$

where $z(t) \sim NID(0, 1)$, and $t = 1, \dots, 100$.

Moving Average Process. The y sequence was generated using a first order moving average process, MA(1), given by:

$$y(t) = z(t) + \frac{1}{2}z(t-1),$$

where $z(t) \sim NID(0, 1)$, and $t = 1, \dots, 100$.

Trigonometric Models. Two y sequences were generated using the following relations:

$$y(t) = \sin(t/20)$$

$$y(t) = \sin(t),$$

for $t = 1, \dots, 100$.

The correlation between the actual and “recovered” y sequence, as well as the root mean square error,

$$RMSE = \sqrt{\frac{\sum(y - \hat{y})^2}{n}},$$

where \hat{y} denotes the value of y “recovered” by the disaggregation procedure, were calculated for each y sequence generated. The results are summarized in Tables 1 and 2.

As can be seen from these examples, the disaggregation procedures work best for y sequences that are smooth: This is what one would expect. It is interesting to note that, for some examples, increasing the order does not yield a significantly better set of recovered

Table 2: RMSE between Actual and Recovered Data for Artificial Data Sets.

Model	Case						
	1	2A	2B	3	4A	4B	5
AR(1)	0.545102	0.569264	0.554476	0.493243	0.528942	0.522687	0.532873
MA(1)	0.626905	0.588157	0.633091	0.572321	0.604064	0.589352	0.599681
Sin(t)	0.337317	0.300278	0.297431	0.212244	0.175985	0.177245	0.121933
Sin(t/20)	0.017104	0.000917	0.000905	0.000033	0.000002	0.000002	0.000000

values. For each data set, plots of the actual data values versus the disaggregated values for each order are given in the appendix.

In a further test of the procedures, the same four artificial y sequences were aggregated twice to get 25 sums. The disaggregation procedures were used on each set of sums twice to “recover” the y sequences. (This would be useful if one wants to disaggregate 24 hour data to obtain “6 hour (pseudo) data.”) Hence, we are assuming for a sequence of values y_1, y_2, \dots, y_{4n} , that the sums

$$\begin{aligned} s_1 &= y_1 + y_2 + y_3 + y_4 \\ s_2 &= y_5 + y_6 + y_7 + y_8 \\ &\vdots \\ s_n &= y_{4n-3} + y_{4n-2} + y_{4n-1} + y_{4n} \end{aligned}$$

are available. The disaggregation procedures are applied to the sums, s_1, \dots, s_n , to recover the sums

$$\begin{aligned} \hat{s}_1 &= y_1 + y_2 \\ \hat{s}_2 &= y_3 + y_4 \\ &\vdots \\ \hat{s}_{2n} &= y_{4n-1} + y_{4n}. \end{aligned}$$

Then, the disaggregation procedures are applied to the sums, $\hat{s}_1, \hat{s}_2, \dots, \hat{s}_{2n}$, to recover the y sequence, y_1, \dots, y_{4n} .

The correlation between the actual and “recovered” y sequences and the root mean square error are summarized in Tables 3 and 4. For each data set, plots of the actual data values versus the recovered values for each order are given in the appendix.

It is clear from Tables 3 and 4 that the resemblance between the actual and recovered values gets worse when using the disaggregation procedures twice. However, for the smoother examples, the procedures perform well. Here, it is interesting to note the degradation in performance when one goes to higher orders, for the first three artificial data sets.

Table 3: Correlation between Actual and Recovered Data for Twice Aggregated Artificial Data Sets.

Model	Case						
	1	2A	2B	3	4A	4B	5
AR(1)	0.765637	0.731539	0.753123	0.770482	0.728481	0.723534	0.668960
MA(1)	0.417748	0.394650	0.276728	0.327402	0.304301	0.240814	0.204894
Sin(t)	0.467211	0.281098	0.295267	0.361950	0.242390	0.233924	0.201307
Sin(t/20)	0.998574	0.999983	0.999983	1.000000	1.000000	1.000000	1.000000

Table 4: RMSE between Actual and Recovered Data for Twice Aggregated Artificial Data Sets.

Model	Case						
	1	2A	2B	3	4A	4B	5
AR(1)	0.802813	0.859050	0.825745	0.796589	0.872400	0.879168	0.990155
MA(1)	0.841023	0.852569	0.904861	0.880660	0.894682	0.922833	0.947530
Sin(t)	0.626861	0.703700	0.698035	0.667130	0.716293	0.719657	0.732503
Sin(t/20)	0.038222	0.004228	0.004120	0.000286	0.000037	0.000036	0.000004

4 Page Data Set

The performance of the different disaggregation procedures was evaluated using the data from the WHITEX study. In particular, the data used are those collected at Page, and are part of the WHITEX data base file (Source: Kristi Gebhart, NPS, Air Quality Division, CIRA Building, Foothills Campus, Colorado State University.).

The Page data set consists of samples of 27 different aerosol species. The samples were collected at either 6, 12, or 24 hour cycles over a period of days. The data were aggregated, and the disaggregation procedures were then applied. The correlation and RMSE between the actual and recovered values for the variables in the Page data set are given in Tables 5 and 6.

Table 7 ranks the performance of the procedures–based on RMSE–for each variable. As can be seen from the table, the performance of the procedures varies widely, depending on the variable. As a test to see if there is a difference in the performance of the procedures, Friedman's rank test (as available on MINITAB) was applied to the RMSE's using the seven procedures as treatments and the 27 variables as blocks. The results are summarized in Table 8. Based on these results, it would appear that there is a difference in the procedures. In this instance, case 3 has performed the best.

In a further evaluation of the performance of the procedures, three of the variables were aggregated twice, and then disaggregated twice, using the steps discussed in section 3. The variables chosen were ones that had the fewest number of missing values. The correlation and RMSE between the actual and recovered values for the variables in the Page data set that were aggregated twice and then disaggregated twice are given in Tables 9 and 10.

Tables 9 and 10 would indicate that the disaggregation procedures do not perform nearly as well when applied twice. Further, it appears that the procedures achieve worsening results as the order is increased. Just a small number of variables have been examined, so no general conclusions can be drawn regarding disaggregating data twice.

5 Conclusion.

The evidence, here, suggests that polynomial disaggregation procedures can be useful. Further, it appears that only the even degrees need be considered. These preliminary results would also indicate that one's intuition is correct: The higher the degree, the better the results.

For the Page data, the case $k = 3$ (i.e. degree 2) yielded the best results—not case $k = 5$ (i.e. degree 4). The reason for this is due largely to the presence of missing values within the Page data set. To use case $k = 5$, there must be at least 10 adjacent y values—hence, 5 sums—that are not missing, and to utilize the higher degrees to their fullest potential, it is best to have large blocks of complete data. Otherwise, one is using a small number of

Table 5: Correlation between Actual and Recovered Data for the Page Data Set.

Model	Case						
	1	2A	2B	3	4A	4B	5
fbrc	0.896966	0.863555	0.899356	0.888726	0.875605	0.894511	0.888978
fsrc	0.776335	0.771986	0.774129	0.759582	0.762057	0.778467	0.791836
fzrc	0.645784	0.649856	0.702139	0.665220	0.670403	0.698417	0.625379
fpbc	0.730975	0.758433	0.662260	0.724167	0.743156	0.682904	0.710986
mcd4c	0.926913	0.913538	0.926789	0.939646	0.897479	0.889418	0.831476
mcdmwc	0.996152	0.997153	0.997114	0.997797	0.997686	0.997773	0.997913
mfwcdc	0.913141	0.928883	0.939779	0.955940	0.921612	0.917507	0.842570
qtrlc	0.921285	0.933845	0.942270	0.940398	0.936710	0.941155	0.927753
qnepc	0.881259	0.899474	0.886584	0.901951	0.904782	0.900152	0.905942
qtemc	0.968189	0.902450	0.991509	0.965408	0.918635	0.983378	0.955491
qrhc	0.936240	0.900276	0.960955	0.950098	0.918387	0.964716	0.949920
qwsc	0.884294	0.899970	0.898701	0.916662	0.911865	0.914065	0.918072
qwdc	0.747979	0.732810	0.708498	0.746203	0.733826	0.710679	0.724263
sso2c	0.736398	0.800754	0.700838	0.741678	0.769434	0.691382	0.722729
wycpmc	0.829511	0.792348	0.811715	0.816342	0.793497	0.811165	0.797463
nsoilc	0.861038	0.892722	0.870950	0.923569	0.925597	0.913262	0.905725
nomhc	0.827736	0.778942	0.865005	0.839996	0.815228	0.859770	0.839841
bso4c	0.963061	0.961829	0.976944	0.972570	0.969512	0.976814	0.979460
cocc	0.775758	0.791800	0.821192	0.825966	0.819822	0.821545	0.824700
cecc	0.885971	0.884256	0.887144	0.898834	0.899231	0.892480	0.896730
eno3c	0.872472	0.930202	0.776287	0.872929	0.912308	0.823465	0.866604
ffpmc	0.921329	0.918433	0.946548	0.941125	0.936602	0.947193	0.942728
ffec	0.868889	0.893688	0.857817	0.885179	0.889795	0.865077	0.871419
fcuc	0.882919	0.859719	0.863468	0.878825	0.861168	0.889243	0.885369
fznc	0.924409	0.907972	0.888261	0.912380	0.905646	0.895358	0.893984
fasc	0.743923	0.768247	0.612673	0.707090	0.729228	0.628963	0.696880
fsec	0.848295	0.816993	0.884004	0.862507	0.843470	0.869869	0.860252

Table 6: RMSE between Actual and Recovered Data for the Page Data Set.

Model	Case						
	1	2A	2B	3	4A	4B	5
fbrc	0.000378	0.000430	0.000371	0.000389	0.000411	0.000380	0.000389
fsrc	0.000083	0.000085	0.000084	0.000087	0.000087	0.000084	0.000081
fzrc	0.000131	0.000124	0.000116	0.000122	0.000121	0.000117	0.000127
fpbc	0.001166	0.001125	0.001304	0.001191	0.001156	0.001270	0.001219
mcd4c	0.004509	0.005010	0.004617	0.004309	0.005670	0.005886	0.007568
mcdmwc	0.076565	0.065872	0.066325	0.057955	0.059399	0.058269	0.056413
mfwcdc	0.003430	0.003172	0.002927	0.002574	0.003252	0.003334	0.003979
qtr1c	0.006883	0.006331	0.005931	0.006024	0.006279	0.006063	0.006841
qnepc	0.003918	0.003622	0.003835	0.003581	0.003531	0.003611	0.003511
qtemc	1.873578	3.293800	0.973715	1.952973	3.002634	1.359799	2.217906
qrhc	5.486683	6.873435	4.321379	4.871890	6.210885	4.112450	4.879755
qwsc	2.158179	2.016628	2.029336	1.848634	1.897445	1.874627	1.832595
qwdc	72.573957	74.686716	77.663610	72.813640	74.562771	77.400051	75.699459
sso2c	4.547551	4.153350	4.942119	3.840328	3.740391	4.243564	4.149390
wycpmc	0.808982	0.889017	0.848894	0.837499	0.887231	0.850326	0.880807
nsoilc	0.069391	0.060912	0.066358	0.054240	0.053539	0.057666	0.060298
nomhc	0.403555	0.461689	0.368089	0.429949	0.459748	0.404742	0.430164
bso4c	0.227663	0.235870	0.183906	0.206762	0.226215	0.197615	0.194042
cocc	0.349038	0.341427	0.318961	0.315196	0.320005	0.318596	0.316102
cecc	0.163961	0.165800	0.163770	0.155377	0.155155	0.160031	0.156979
eno3c	0.087500	0.065848	0.115324	0.087404	0.073326	0.102720	0.090065
ffpmc	0.678068	0.689962	0.561951	0.589076	0.610305	0.558704	0.581040
ffec	0.004421	0.003943	0.004522	0.004088	0.004009	0.004412	0.004314
fcuc	0.000207	0.000226	0.000224	0.000210	0.000225	0.000202	0.000205
fznc	0.000442	0.000482	0.000530	0.000470	0.000488	0.000514	0.000517
fasc	0.000342	0.000328	0.000414	0.000363	0.000351	0.000405	0.000368
fsec	0.000604	0.000660	0.000533	0.000577	0.000613	0.000562	0.000581

Table 7: Ranks Disaggregation Procedures for Each Variable Based on RMSE.

Model	Case						
	1	2A	2B	3	4A	4B	5
fbrc	2	7	1	4.5	6	3	4.5
fsrc	2	5	3.5	6.5	6.5	3.5	1
fzrc	7	5	1	4	3	2	6
fpbc	3	1	7	4	2	6	5
mcd4c	2	4	3	1	5	6	7
mcdmwc	7	5	6	2	4	3	1
mfwcdc	6	3	2	1	4	5	7
qtrlc	7	5	1	2	4	3	6
qnepc	7	5	6	3	2	4	1
qtemc	3	7	1	4	6	2	5
qrhc	5	7	2	3	6	1	4
qwsc	7	5	6	2	4	3	1
qwdc	1	4	7	2	3	6	5
sso2c	6	4	7	2	1	5	3
wycpmc	1	7	3	2	6	4	5
nsoilc	7	5	6	2	1	3	4
nomhc	2	7	1	4	6	3	5
bso4c	6	7	1	4	5	3	2
cocc	7	6	4	1	5	3	2
cecc	6	7	5	2	1	4	3
eno3c	4	1	7	3	2	6	5
ffpmc	6	7	2	4	5	1	3
ffec	6	1	7	3	2	5	4
fcuc	3	7	5	4	6	1	2
fznc	1	3	7	2	4	5	6
fasc	2	1	7	4	3	6	5
fsec	5	7	1	3	6	2	4

Table 8: Results of Friedman's Test Applied to RMSE Using the Disaggregation Procedures as Treatment.

	Case	N	Est. Median	Sum of RANKS
S = 13.73 d.f. = 6 p = 0.034	1	27	0.05945	121.0
S = 13.76 d.f. = 6 p = 0.033 (adjusted for ties)	2A	27	0.05954	133.0
	2B	27	0.05942	109.5
	3	27	0.05922	79.0
	4A	27	0.05940	108.5
	4B	27	0.05940	98.5
	5	27	0.05942	106.5
Grand median = 0.05941				

Table 9: Correlation between Actual and Recovered Data for Twice Aggregated Variables.

Model	Case						
	1	2A	2B	3	4A	4B	5
2qtemc	0.865953	0.869504	0.873498	0.873376	0.870641	0.872604	0.869722
2wycpmc	0.721602	0.637625	0.740485	0.689560	0.616241	0.616967	0.541133
2eno3c	0.630901	0.694111	0.535496	0.580634	0.614136	0.529689	0.53960

Table 10: RMSE between Actual and Recovered Data for Twice Aggregated Page Data Sets.

Model	Case						
	1	2A	2B	3	4A	4B	5
2qtemc	3.744821	3.698430	3.645279	3.646992	3.683382	3.657245	3.695570
2wycpmc	1.028034	1.161978	0.999443	1.145701	1.298056	1.296348	1.570445
2eno3c	0.137439	0.127599	0.151537	0.150944	0.14644	0.158390	0.158048

“windows” to “recover” a large number of y values. Polynomial disaggregation procedures will work best with data that has no missing values.

These results are preliminary. It would be beneficial to carry out a more rigorous study of the proposed procedures using a broad variety of artificial data sets without any missing values. It would also be of interest to further examine the repeated (2 or more times) use of polynomial disaggregation procedures on single data sets.

6 Minitab Macros.

In order to implement the disaggregation procedures, several macros were written for use with Minitab. A brief description of how to use them follows:

orderk.mtb for $k = 1, 2A, 2B, 3, 4A, 4B, 5$. These programs assume that the aggregated data (the sums) are in C1 and that K9 = n. Using the disaggregation procedure called for by k , it puts the disaggregated data (estimated y sequences) in C2. K1 and K8 are used in the computations.

chkdisag.mtb. This program assumes that K2 = source column number for the y sequence and that K9 = n. It aggregates the y sequence, and performs all of the disaggregation procedures. The original y sequence is put in C3 and the disaggregated data are put in C4 - C10. In its computations, the following are used: C1, C2, K1, K8, and K10.

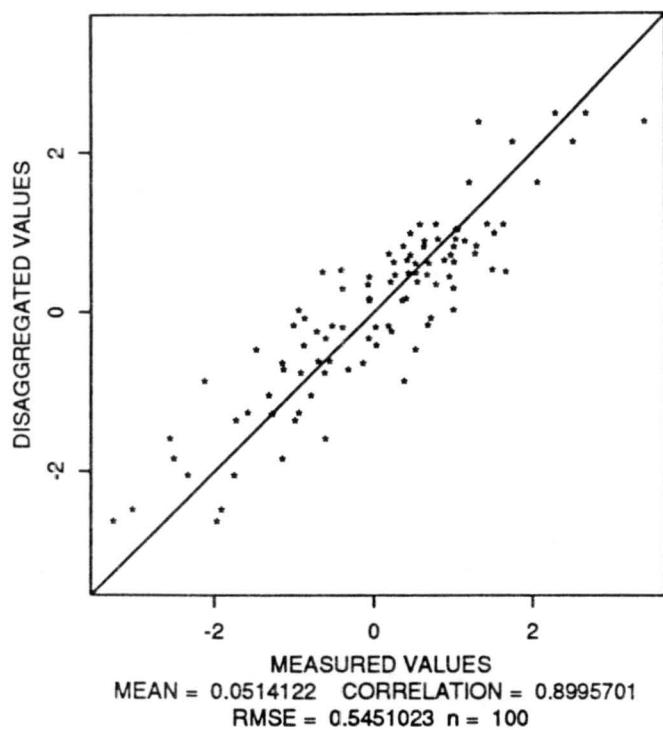
2upto4.mtb. Here, it is assumed that C1 contains the twice disaggregated data, and that K9 = the number of entries in C1. The program performs all of the disaggregation procedures twice, and stores the estimated y sequences in C30 - C36. The following are used in its computations: C2, C4, K1, K8, and K11.

In the programs discussed above, the following are called in iterative loops: Kone.mtb, ktwoA.mtb, ktwoB.mtb, kthree.mtb, kfourA.mtb, kfourB.mtb, kfive.mtb, and sums.mtb.

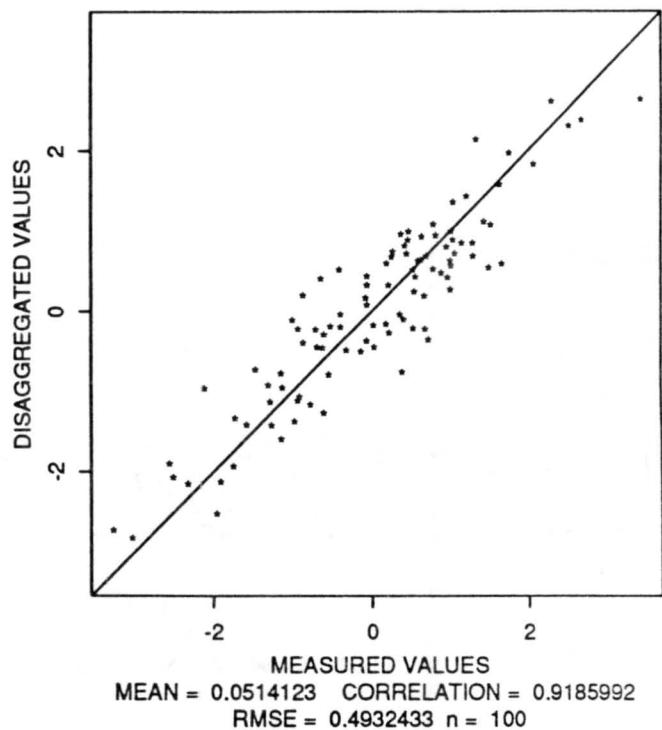
Appendix

Plots of observed data versus disaggregated data.

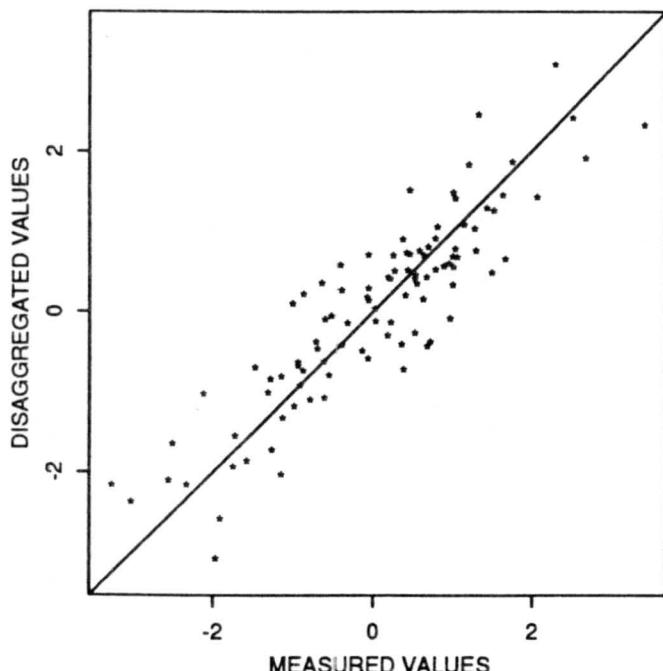
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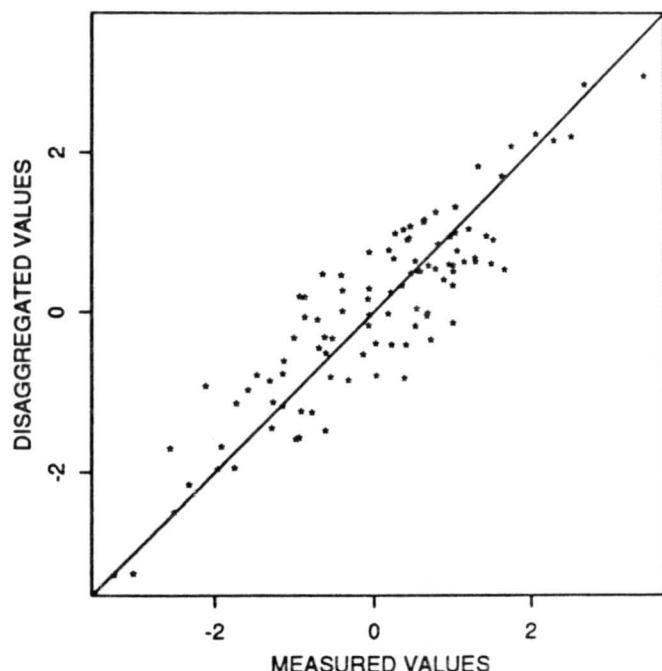
ORDER 2 DISAGGREGATION FOR AR1



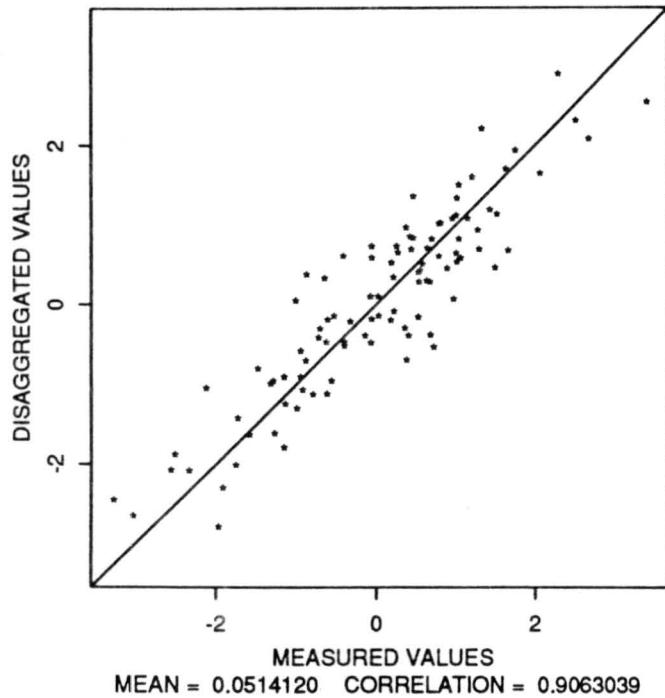
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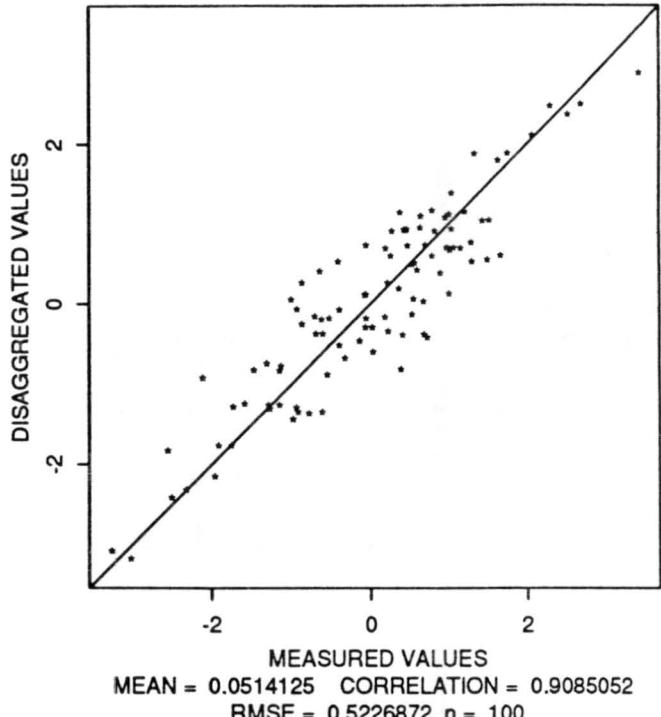
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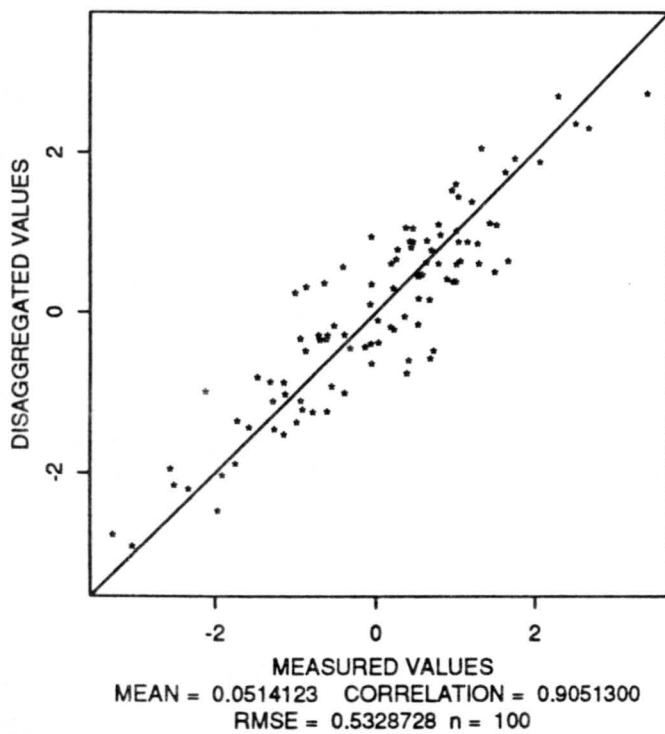
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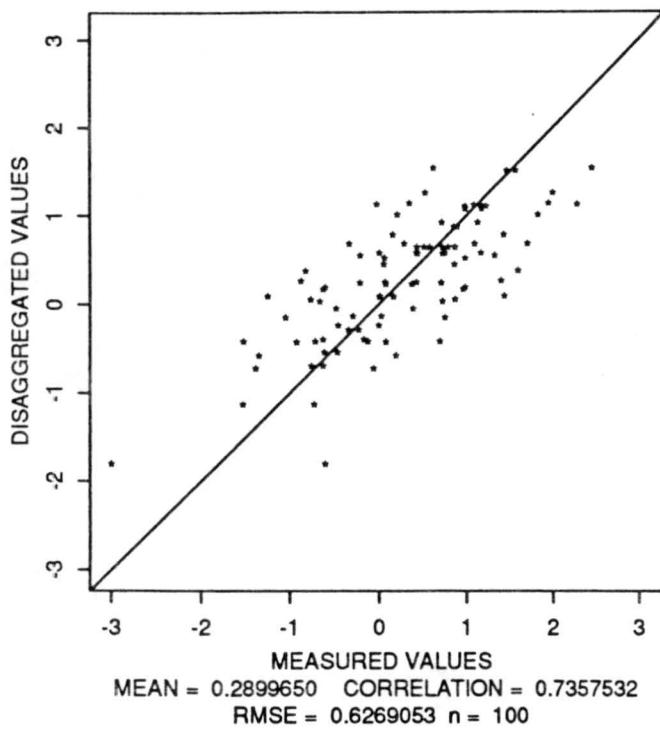
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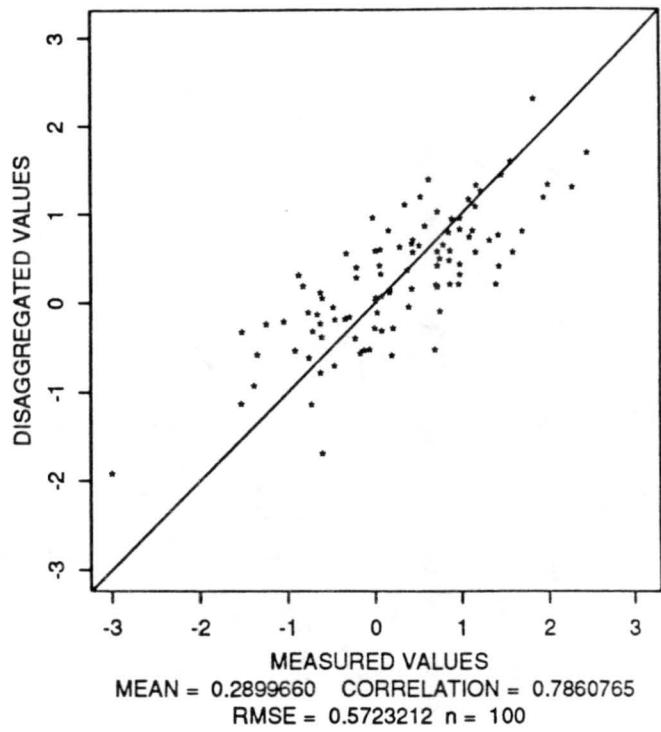
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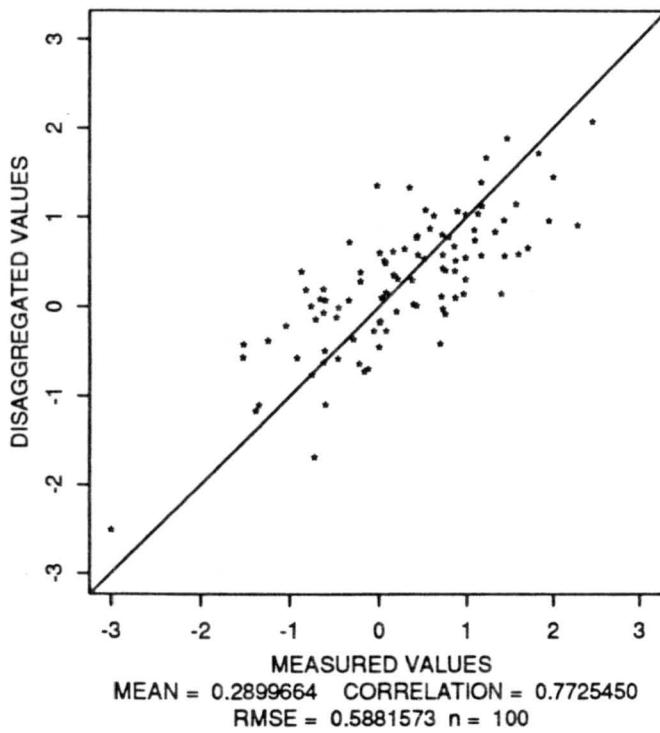
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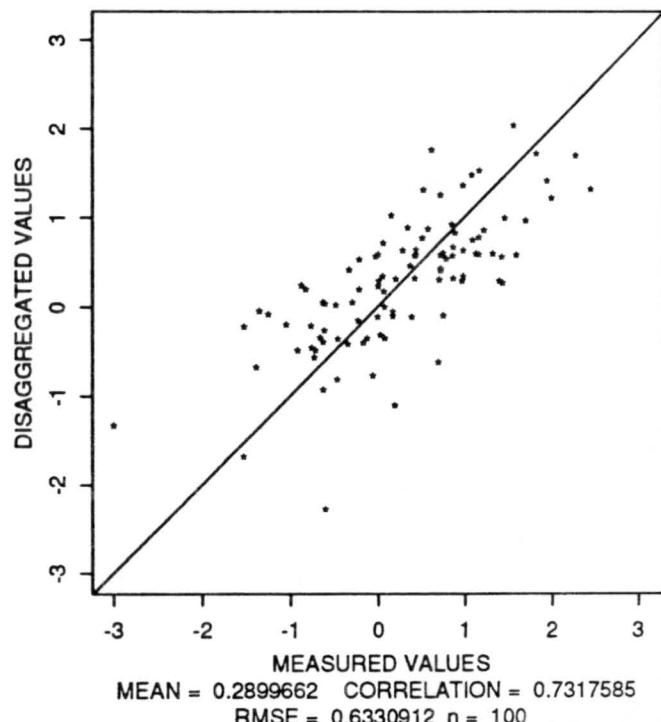
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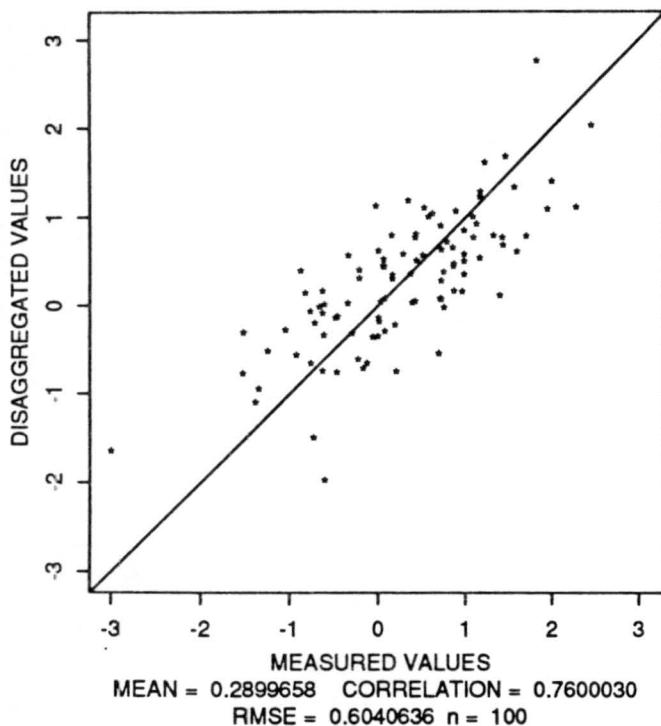
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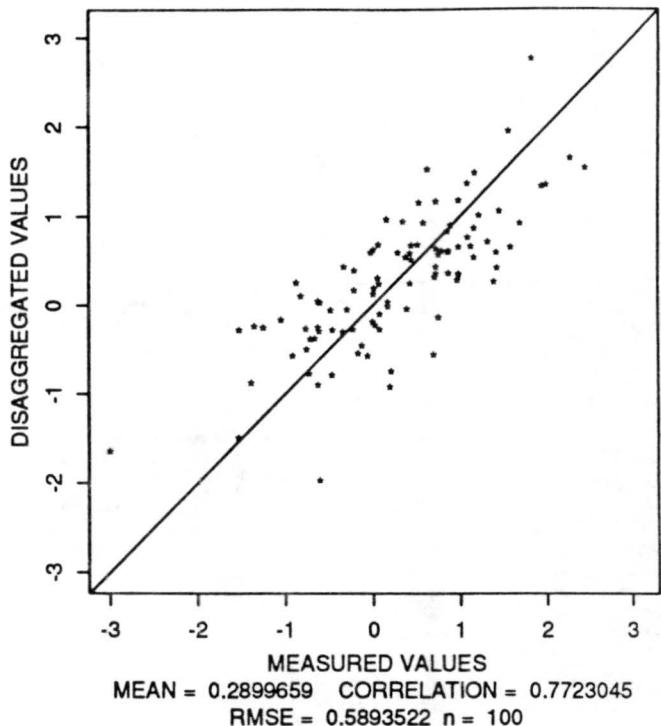
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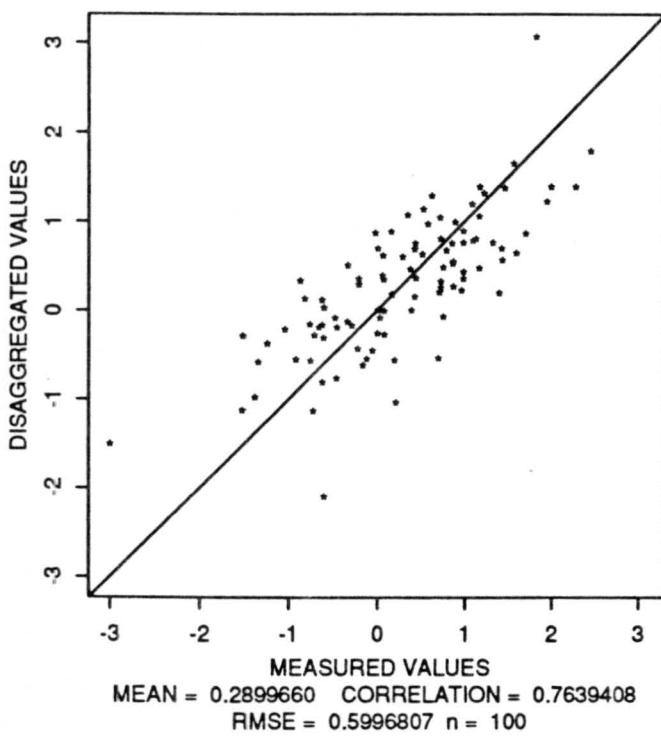
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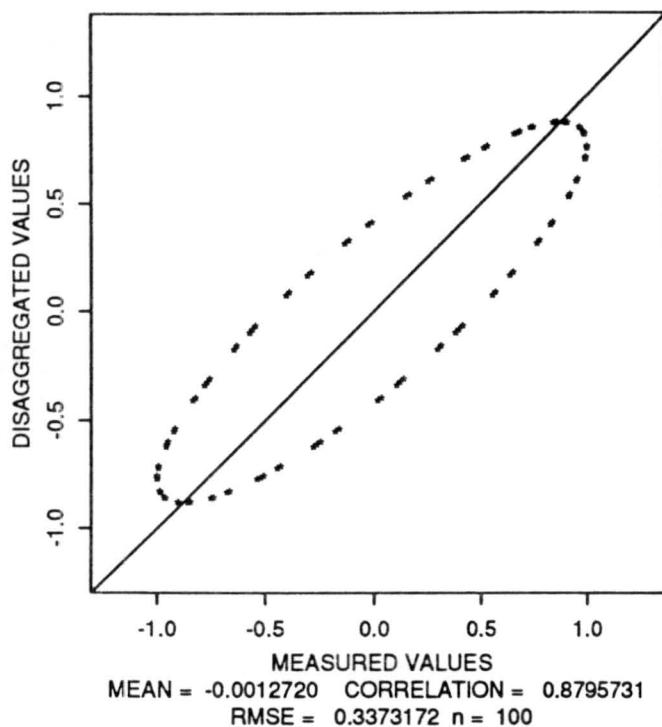
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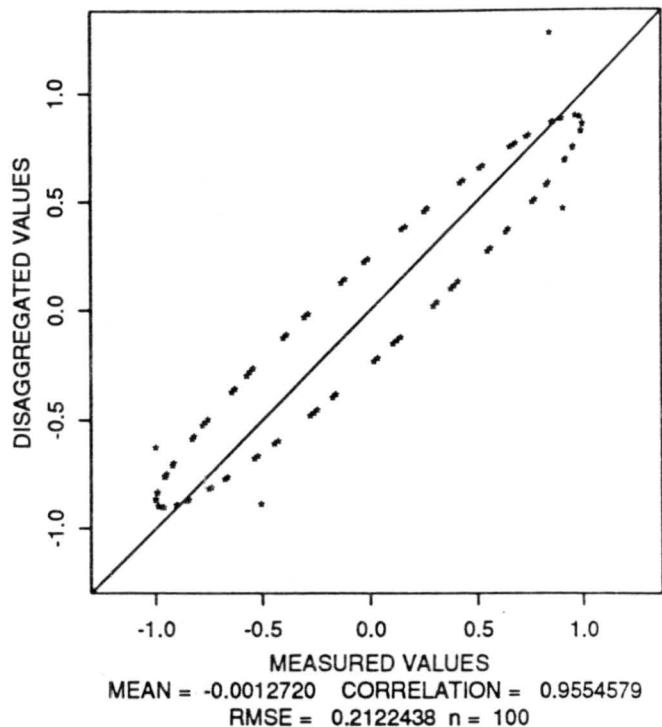
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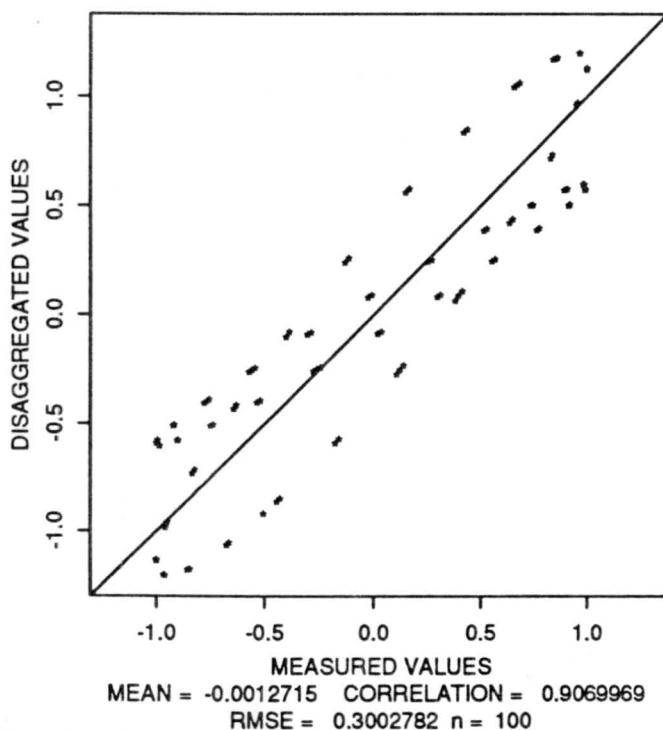
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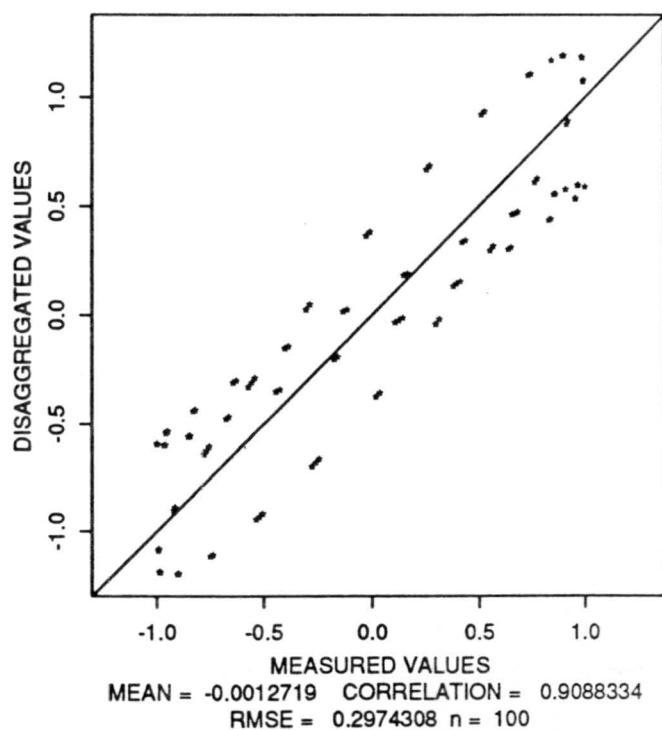
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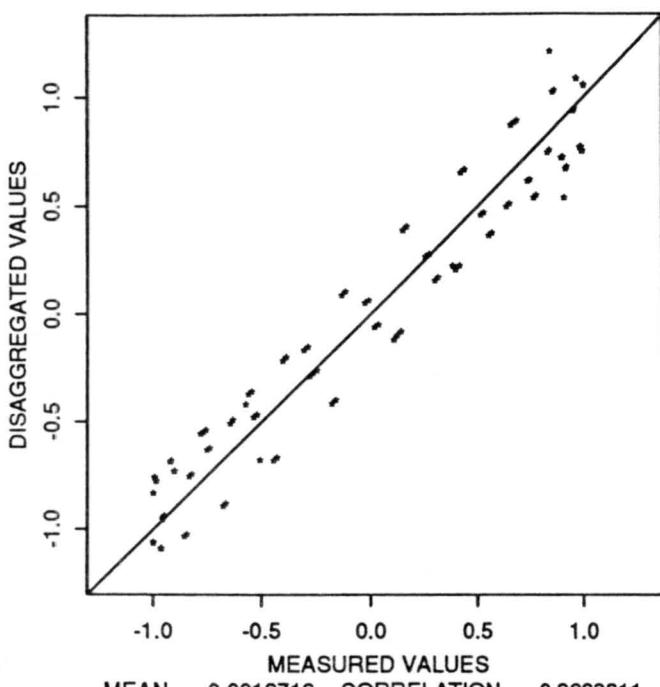
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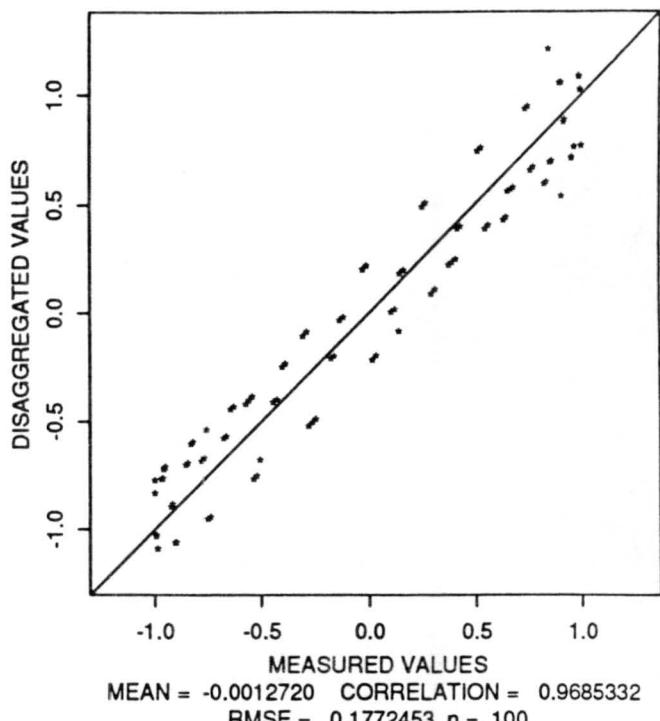
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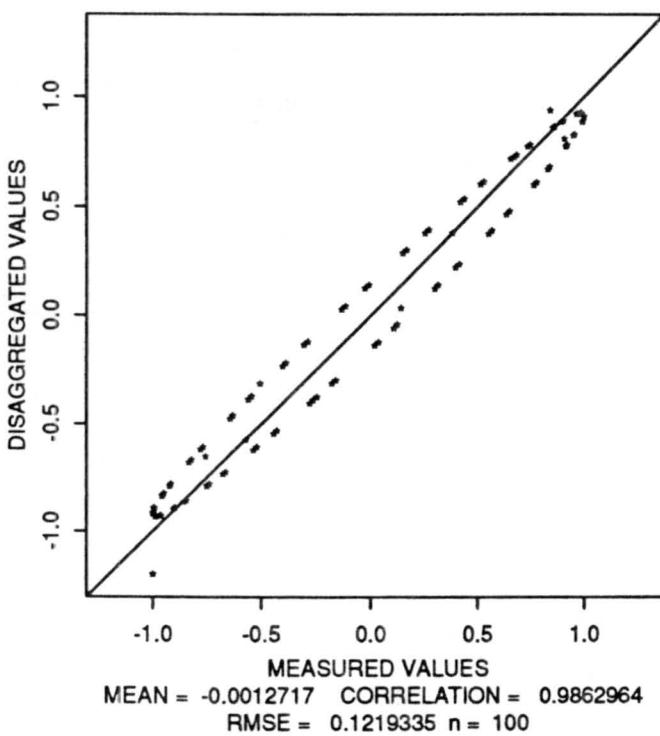
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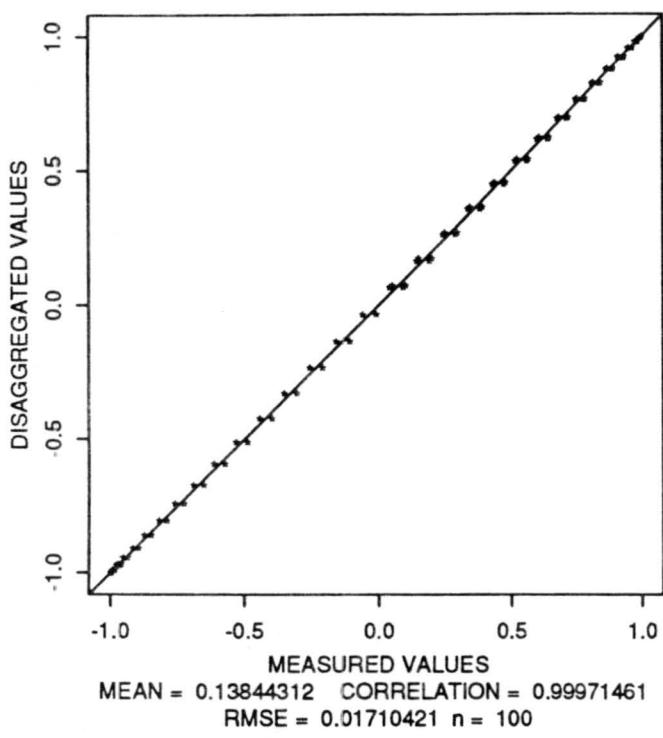
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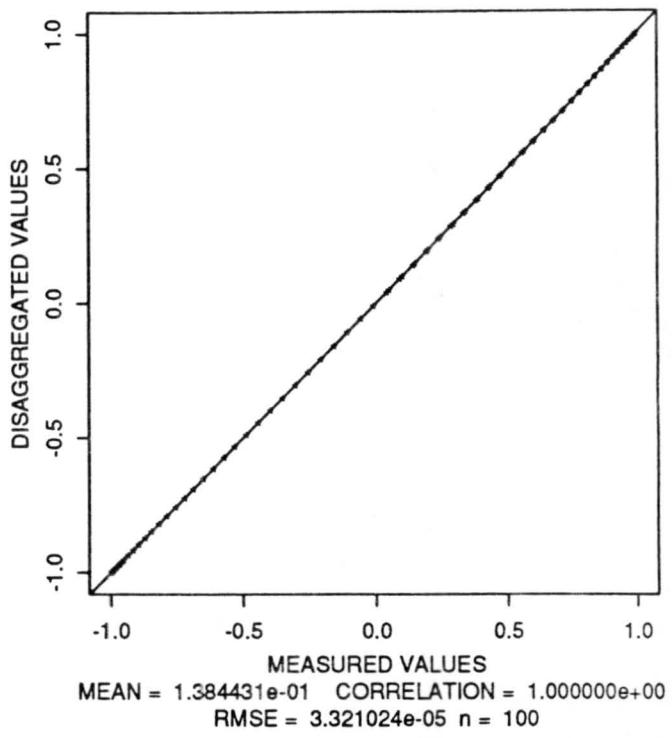
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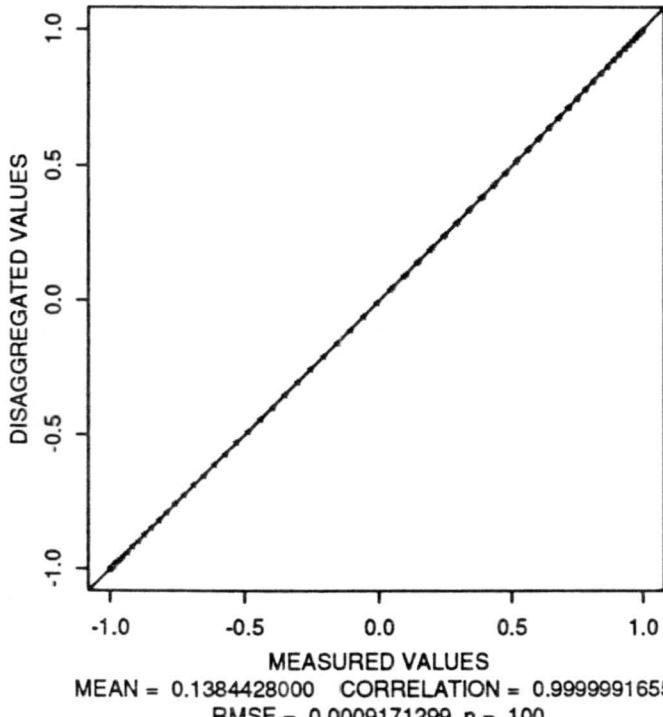
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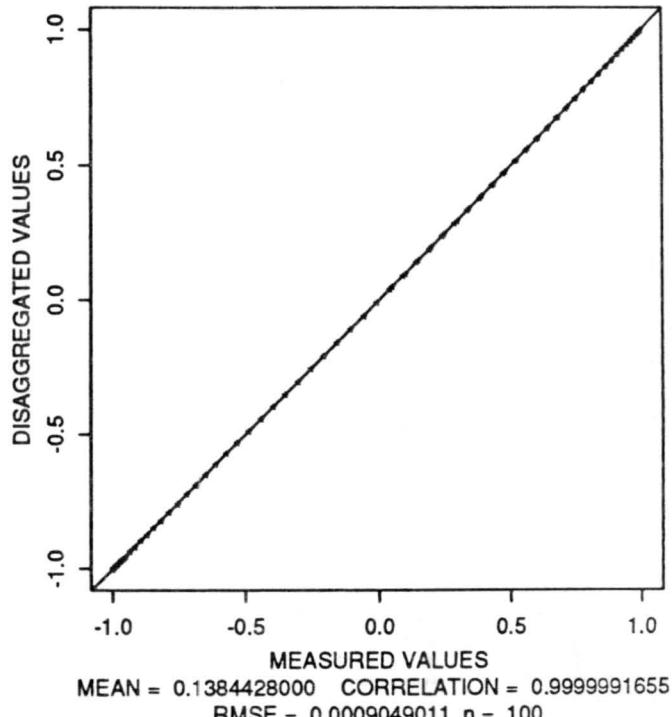
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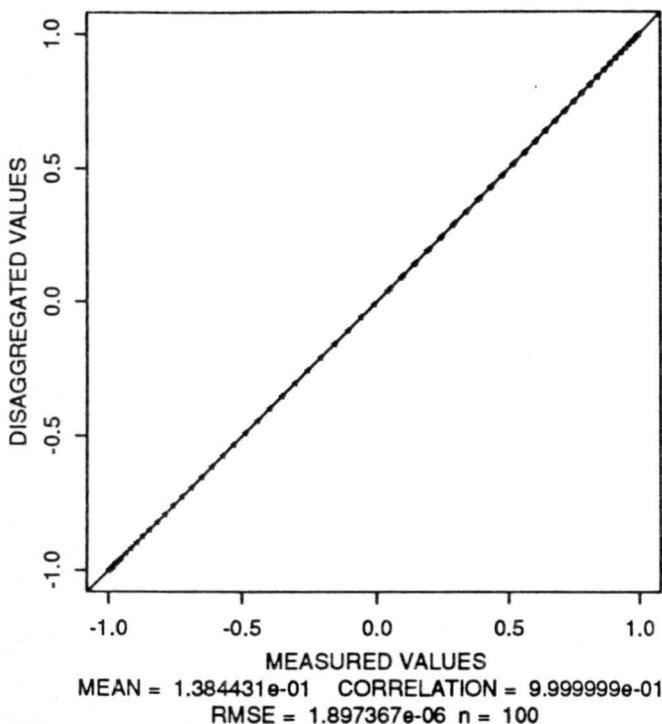
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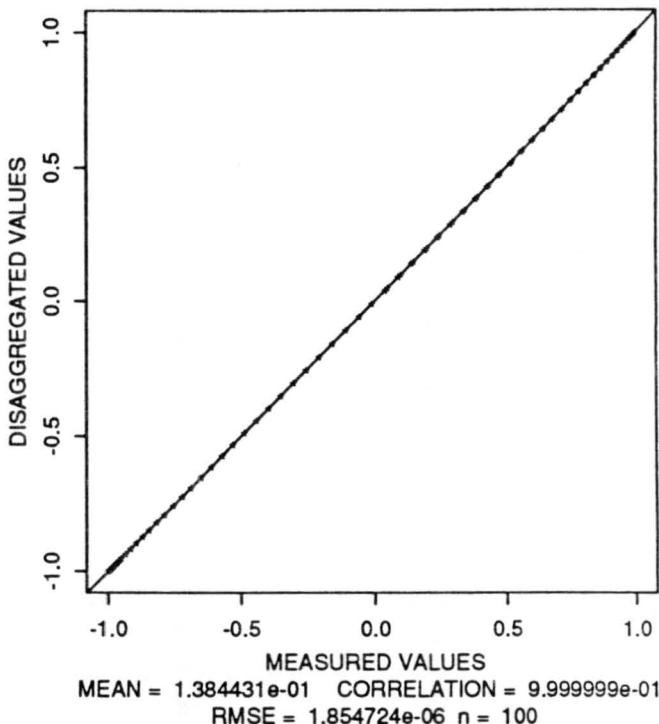
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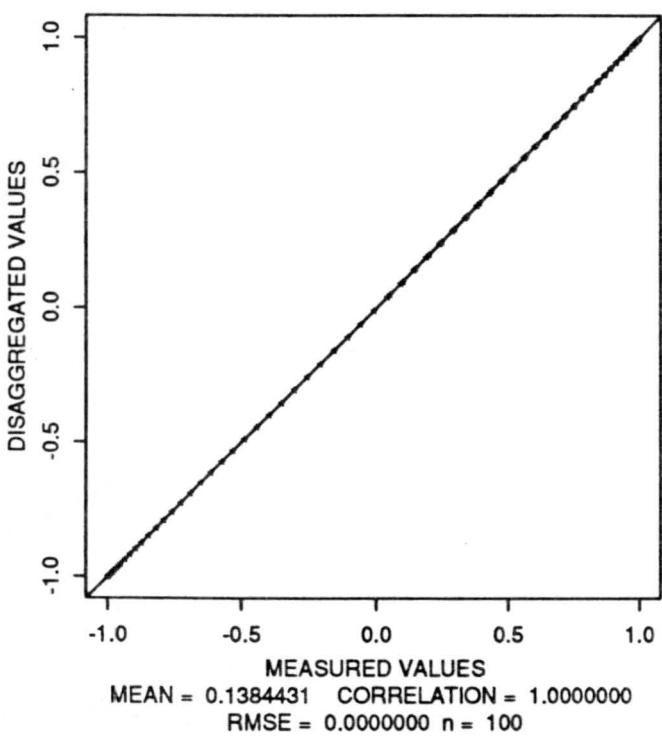
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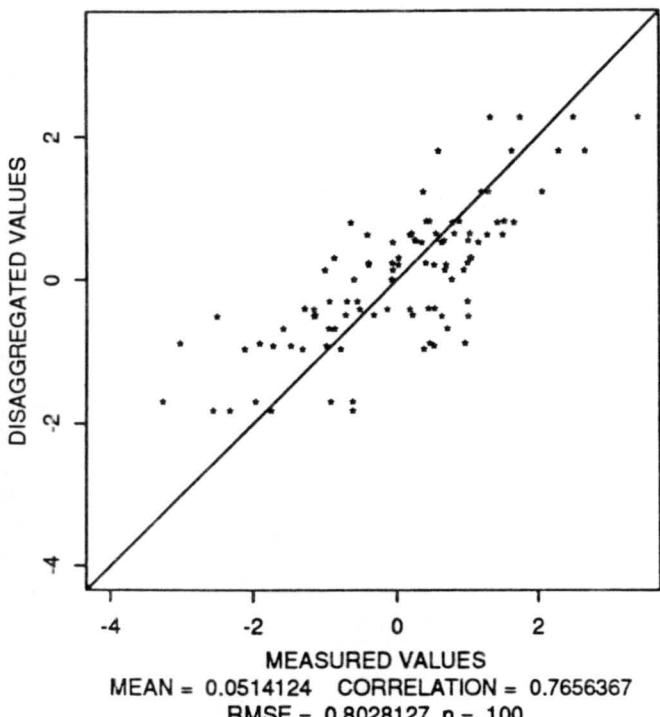
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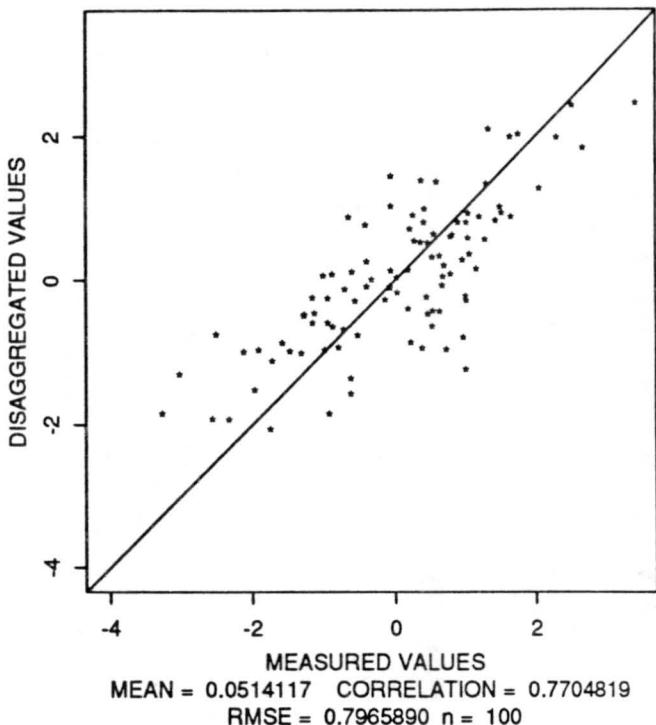
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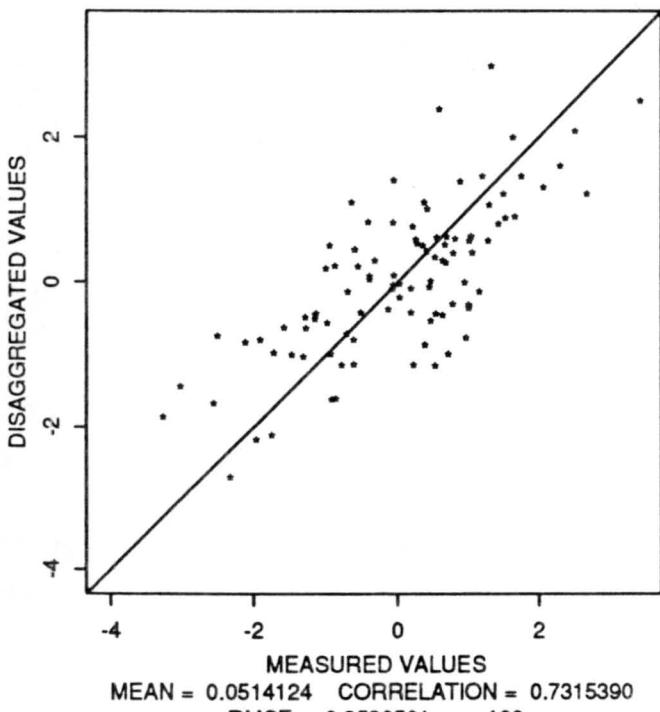
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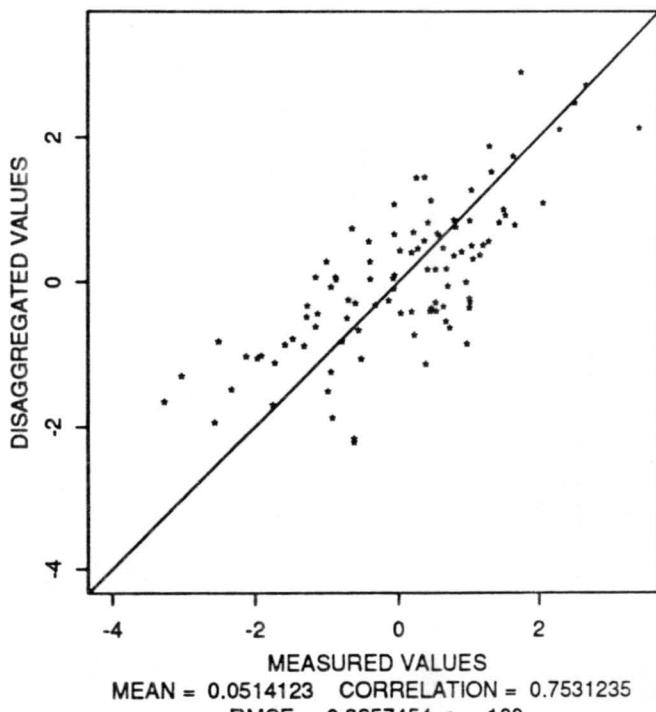
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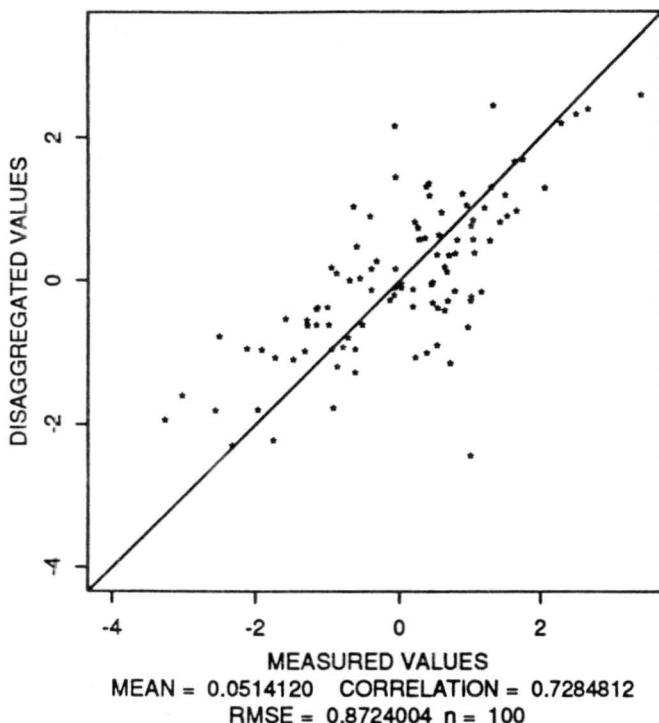
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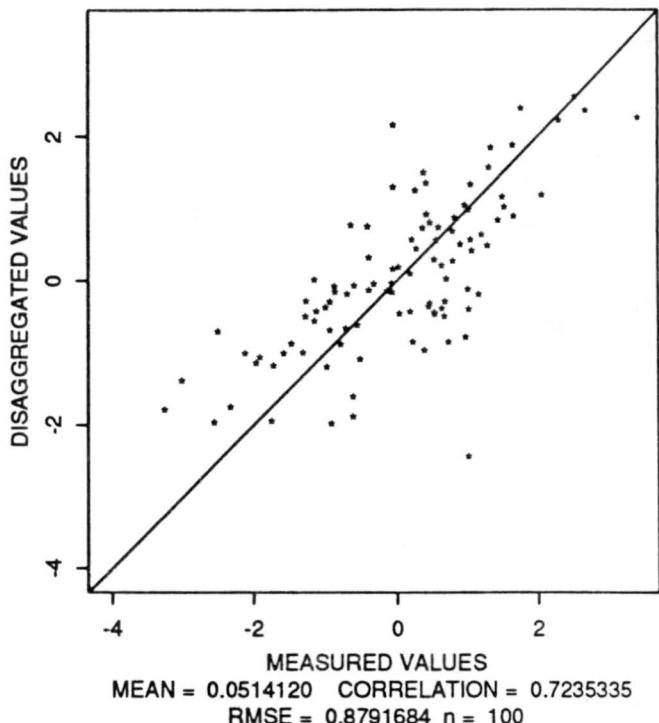
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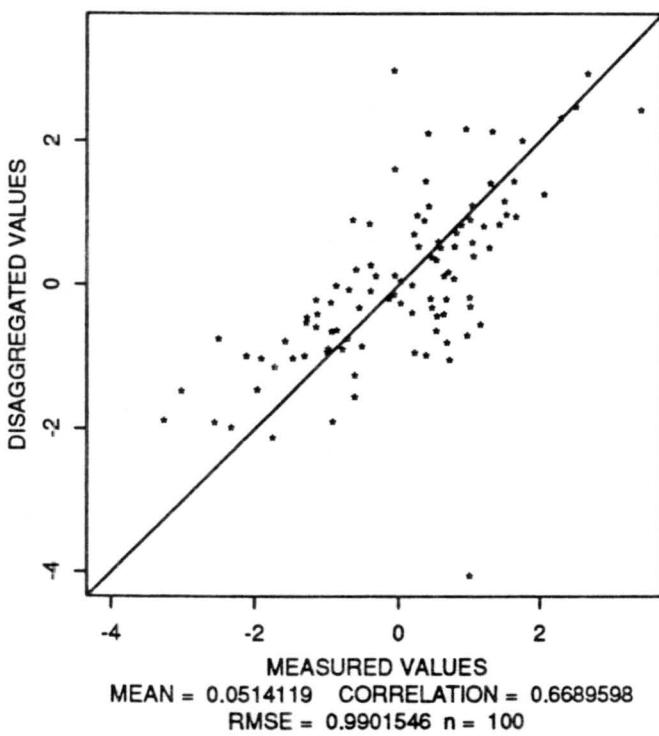
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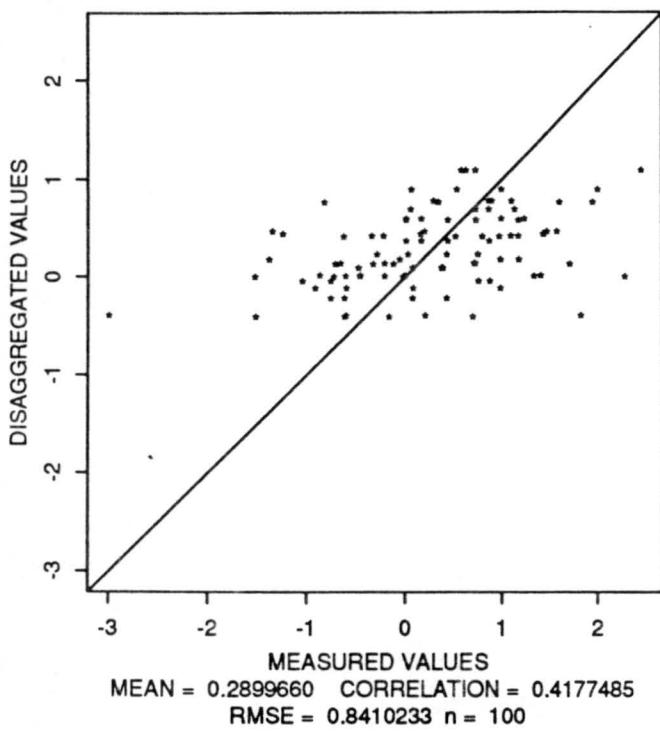
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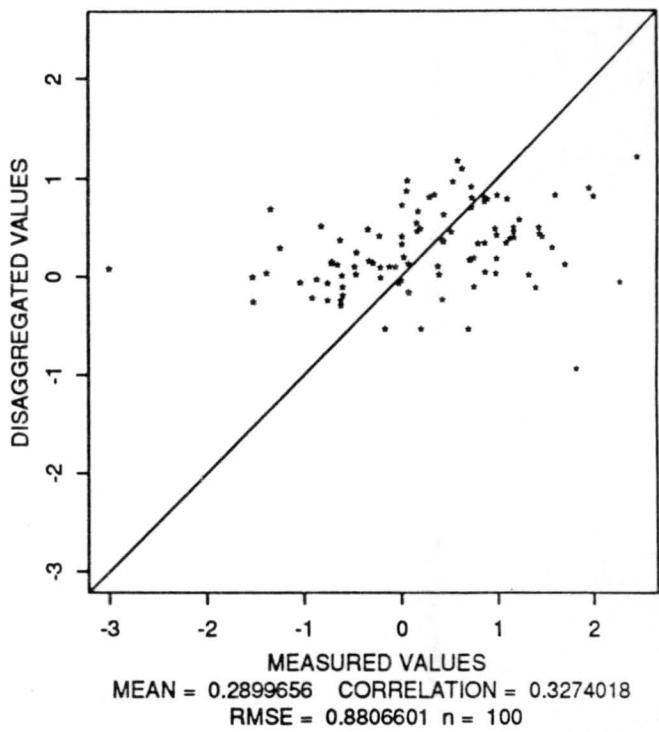
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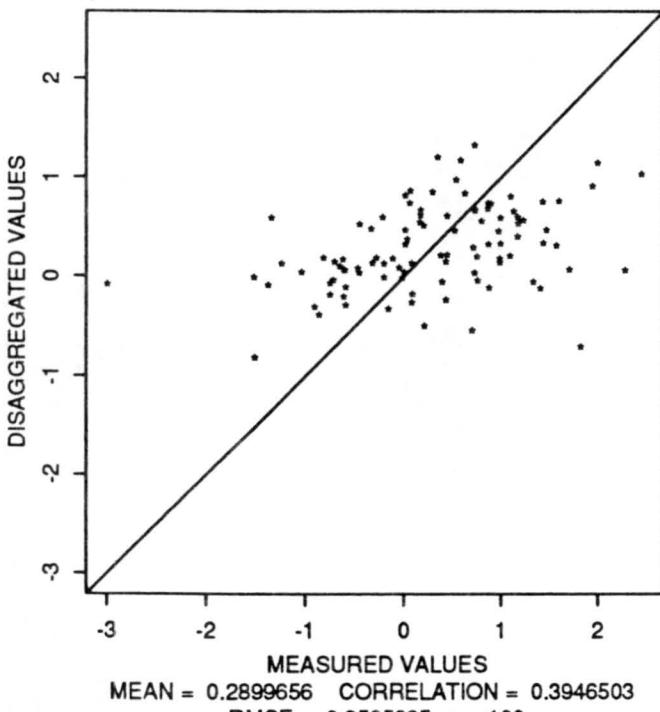
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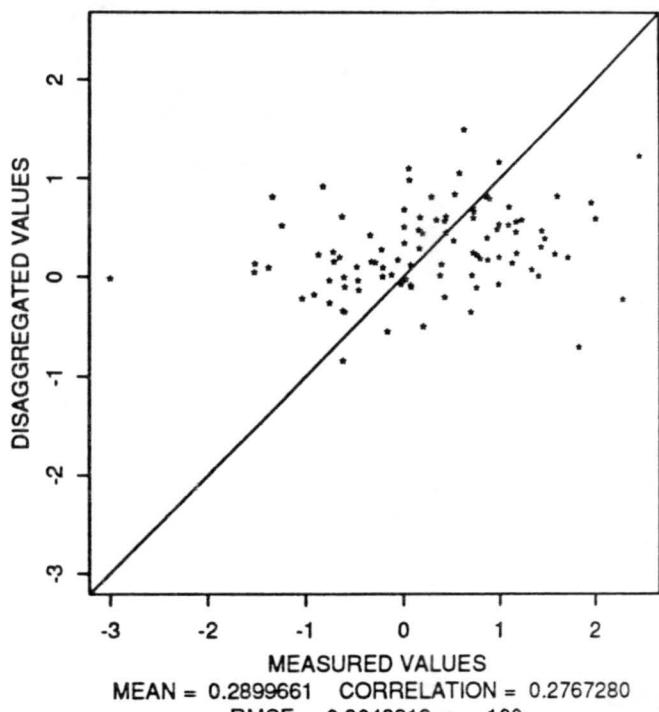
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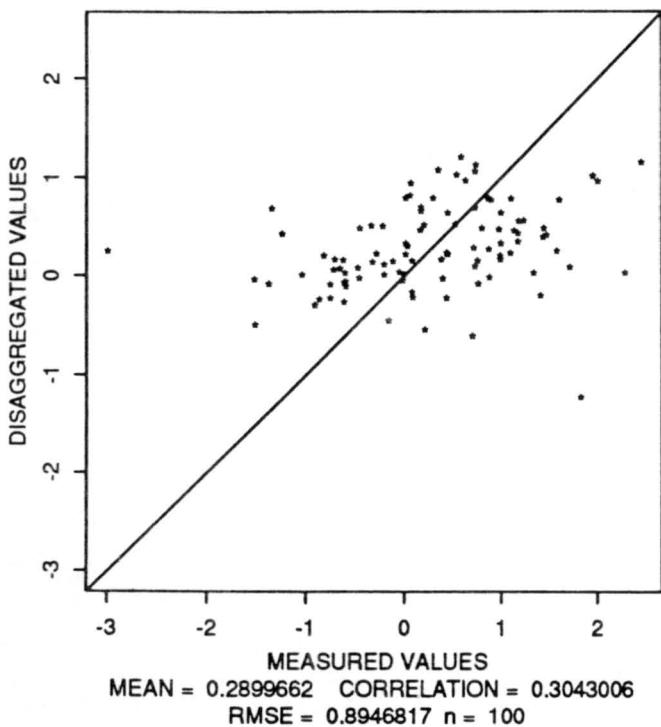
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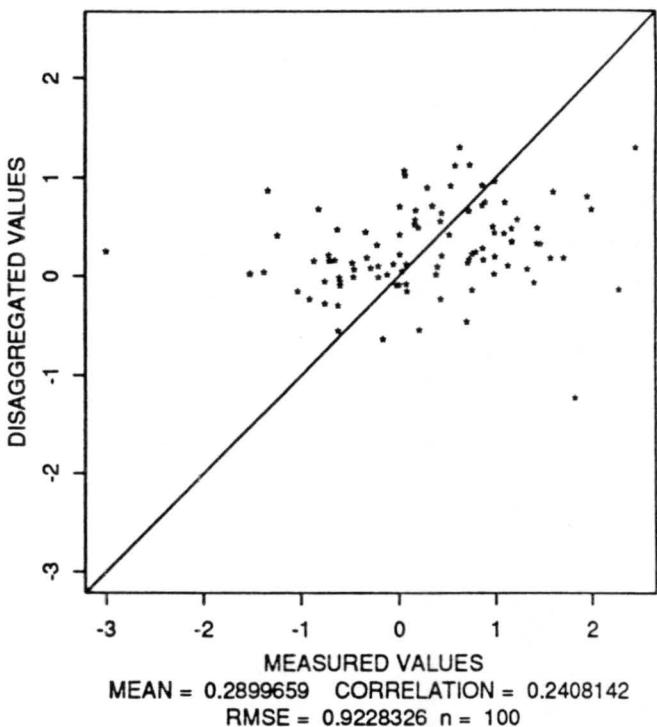
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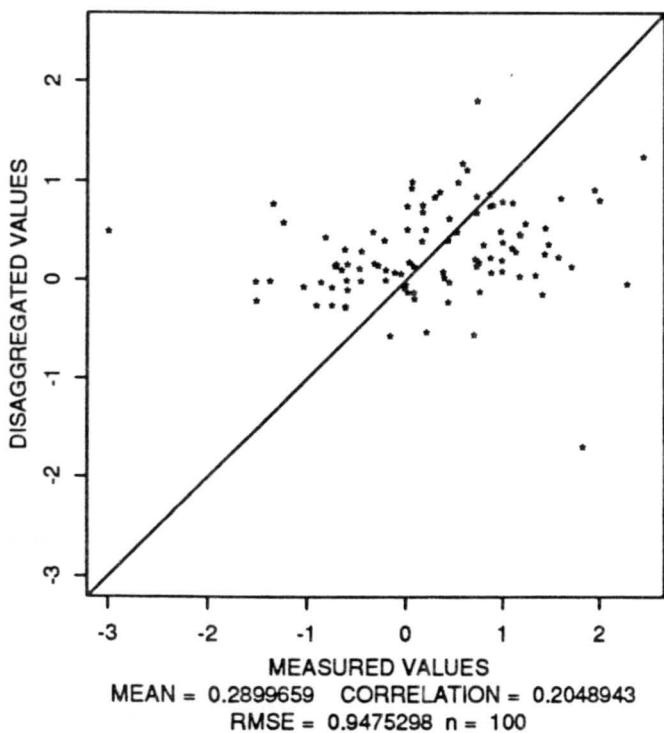
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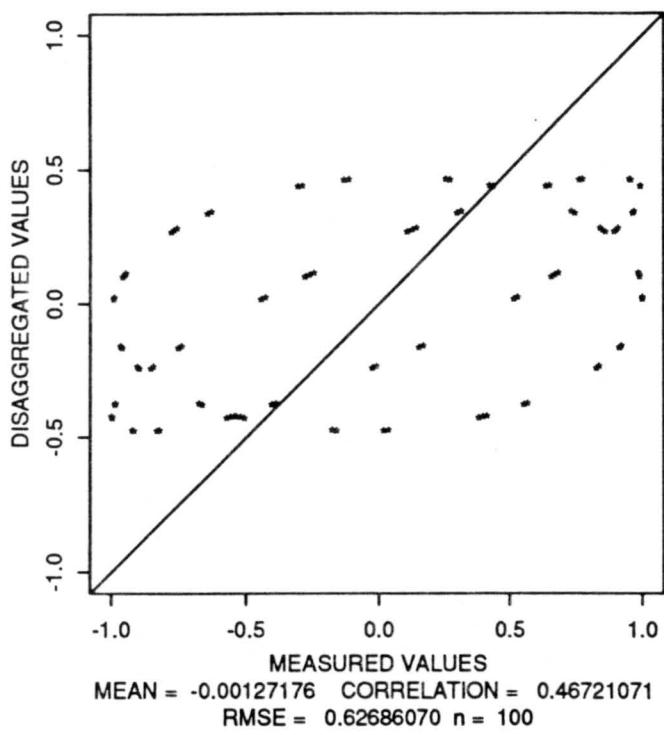
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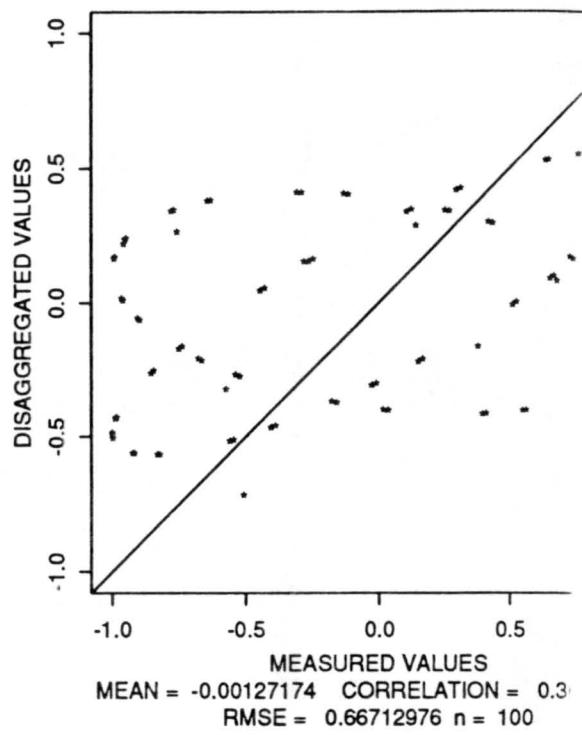
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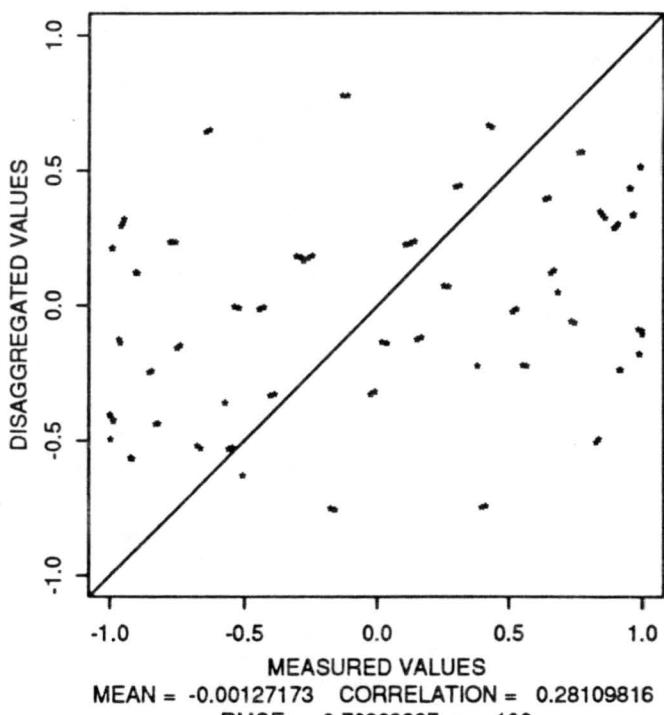
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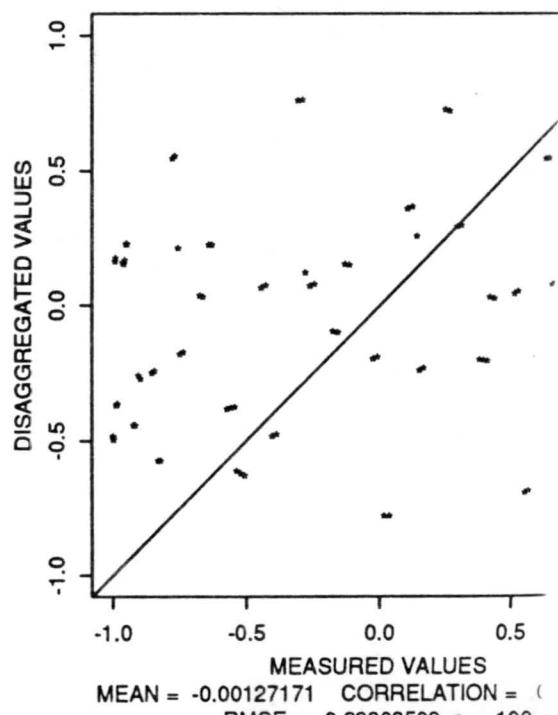
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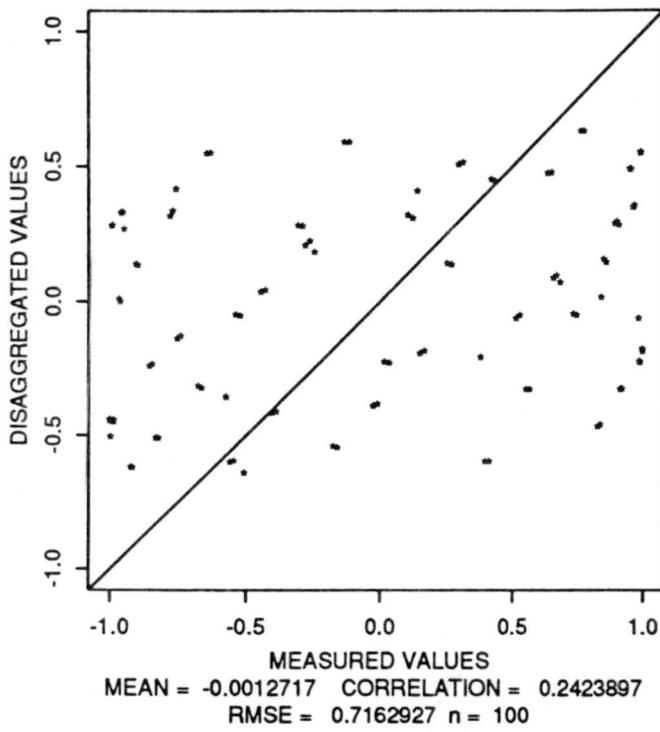
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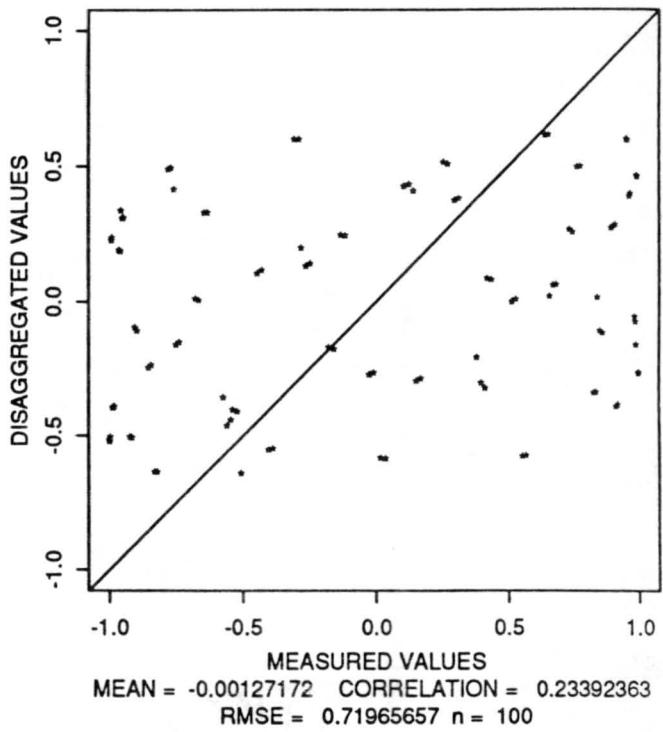
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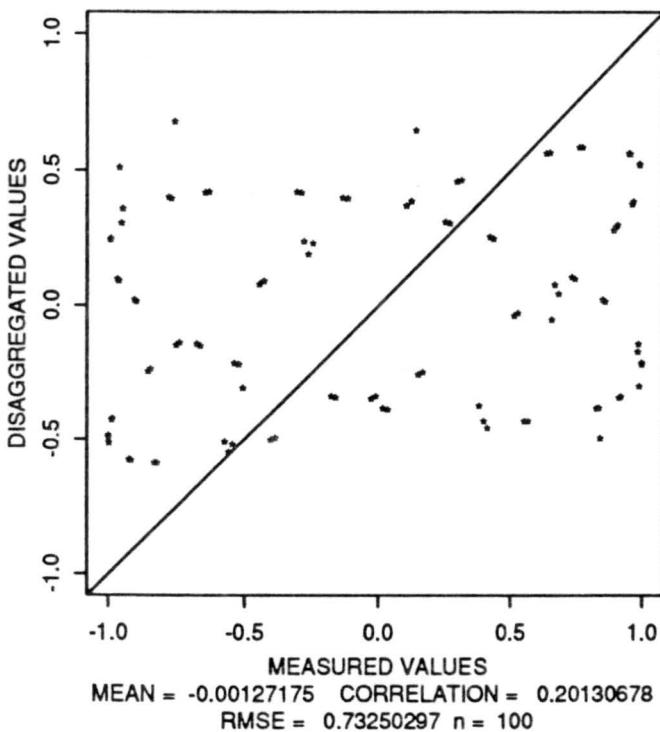
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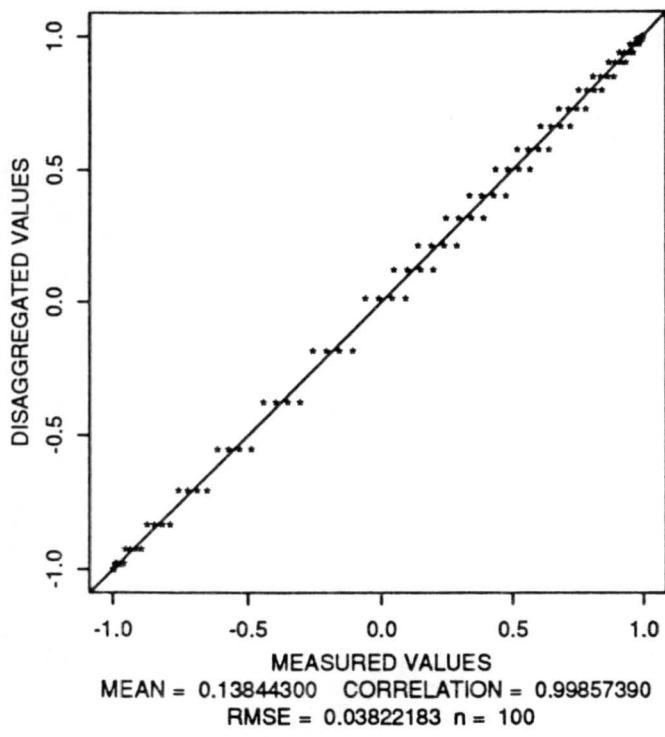
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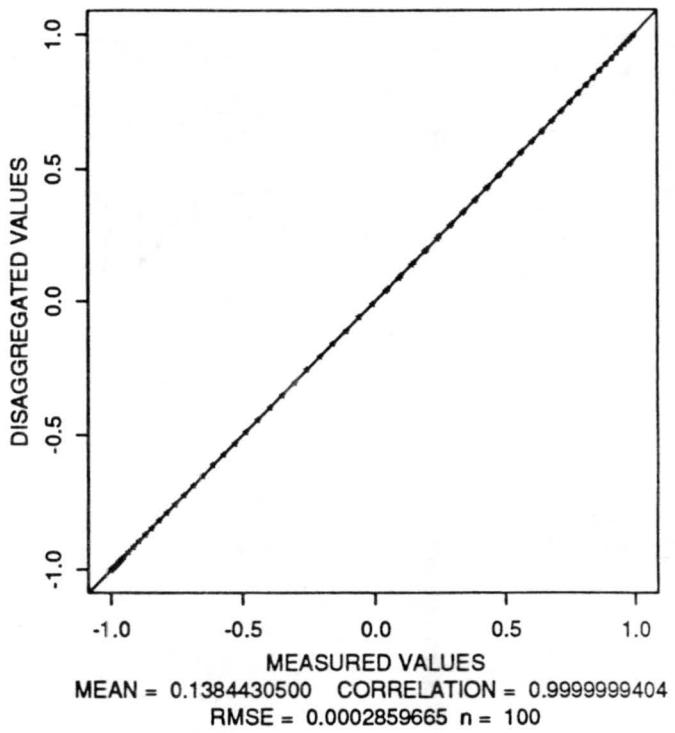
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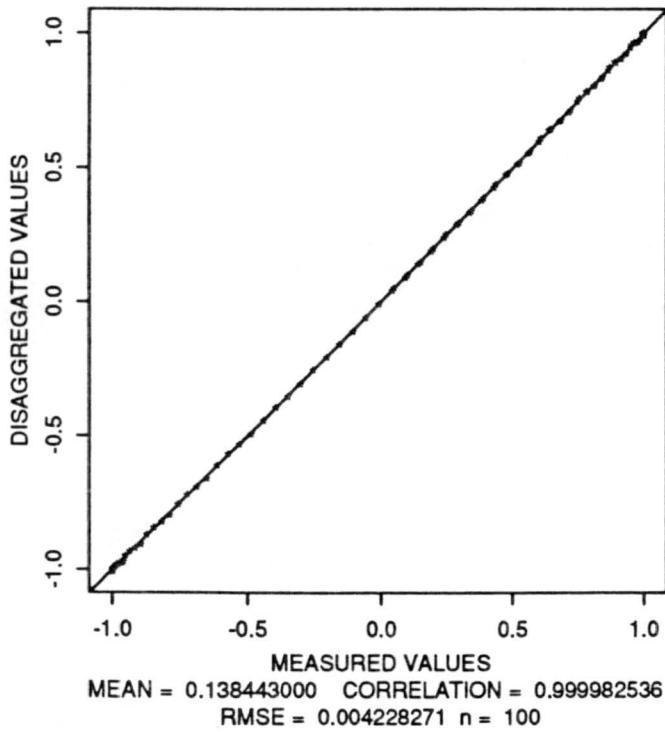
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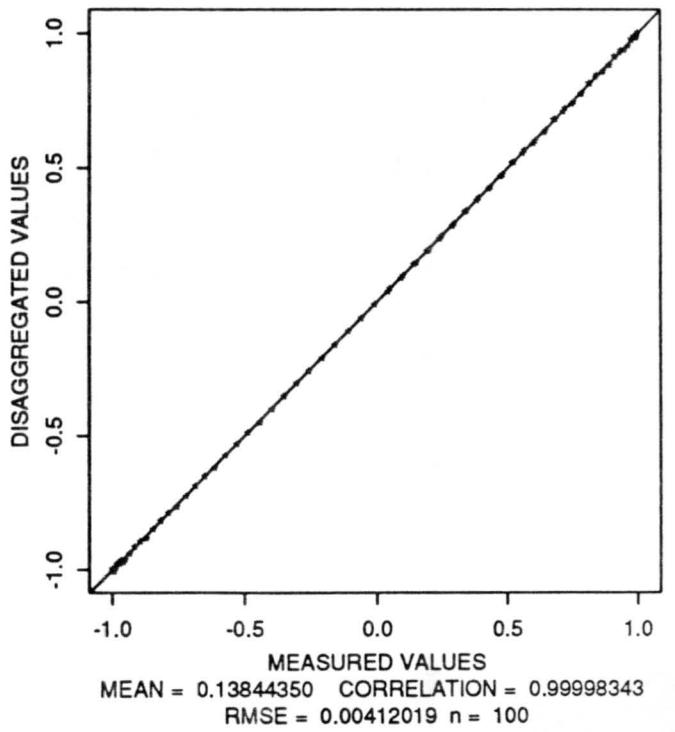
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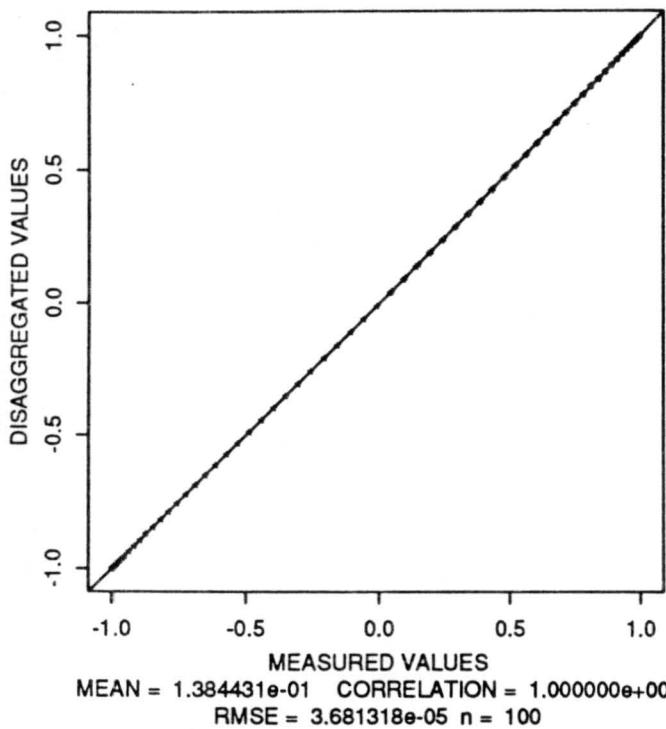
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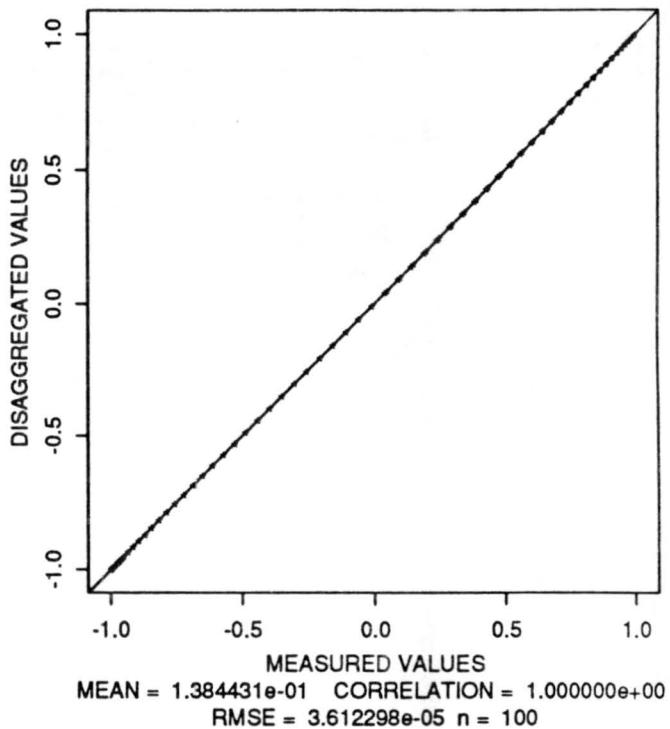
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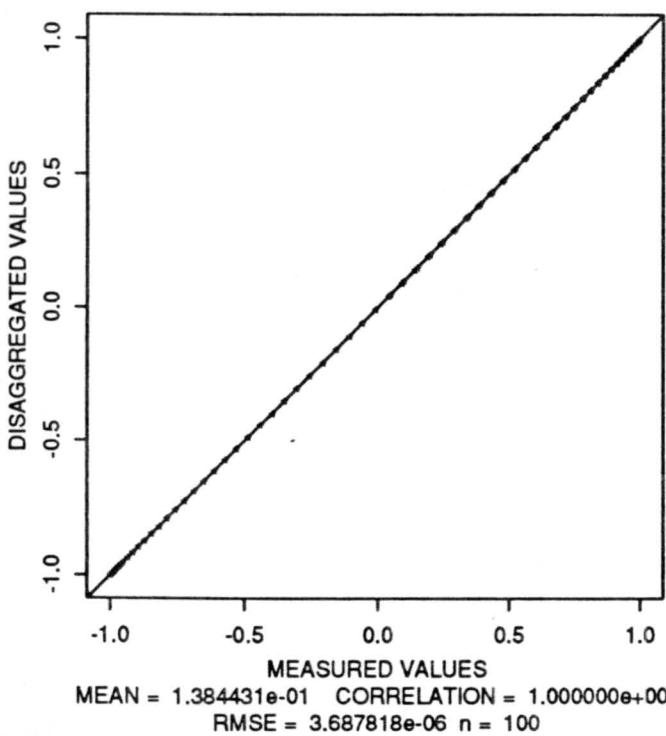
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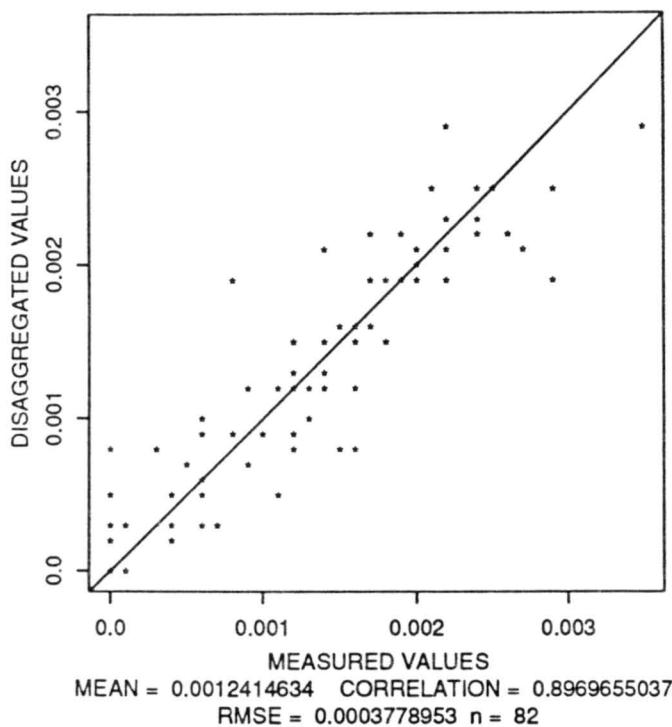
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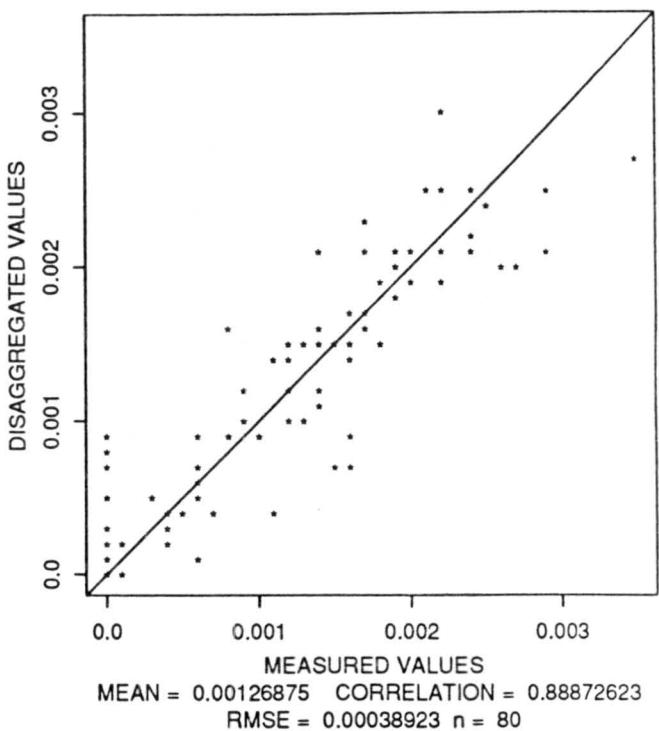
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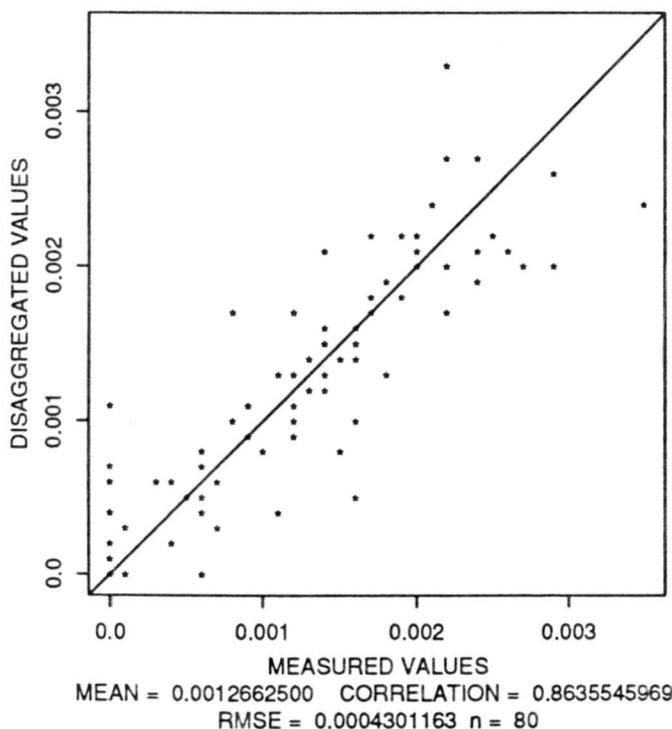
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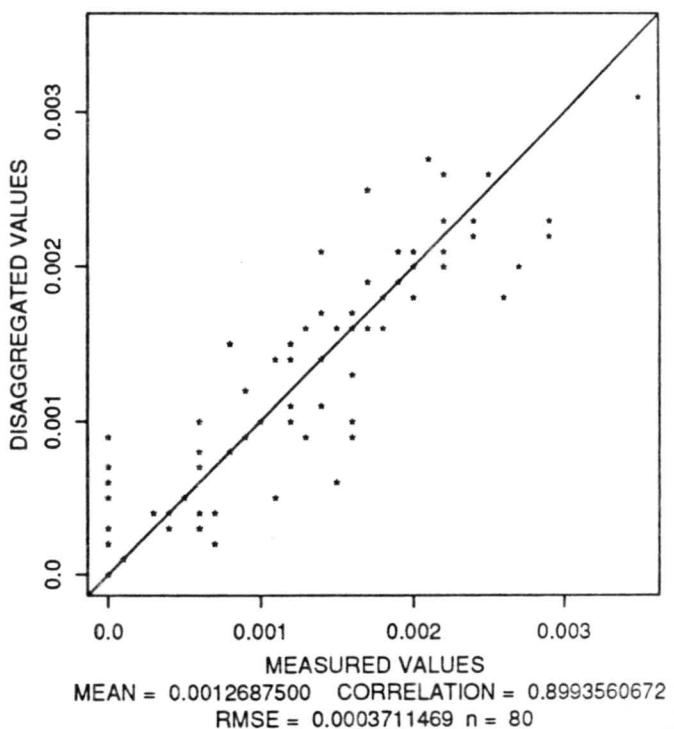
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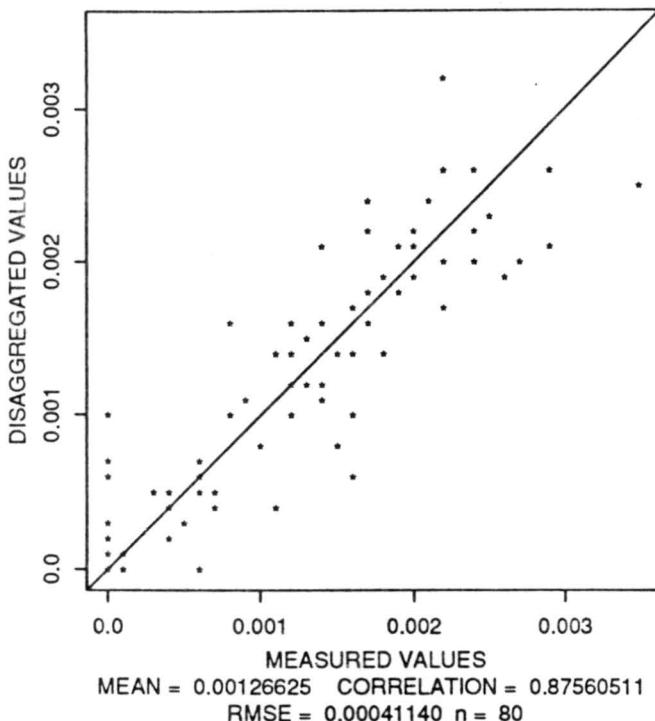
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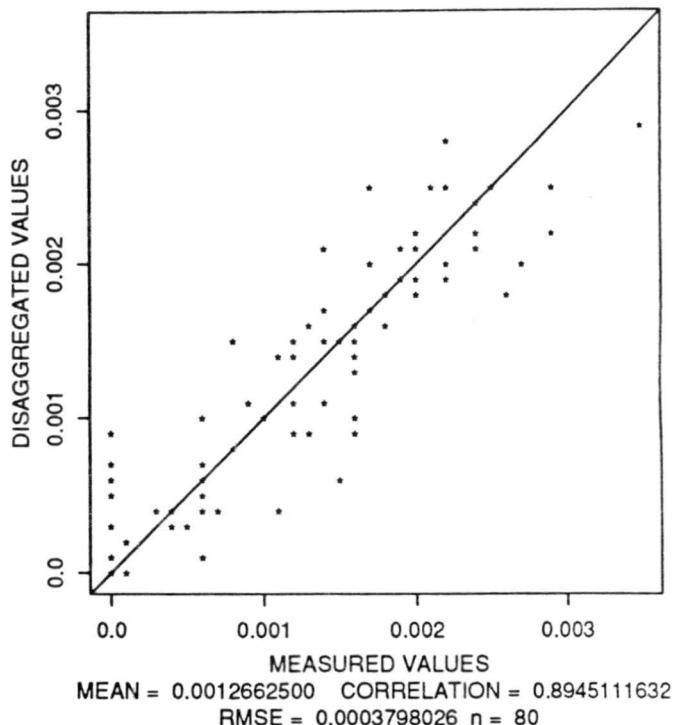
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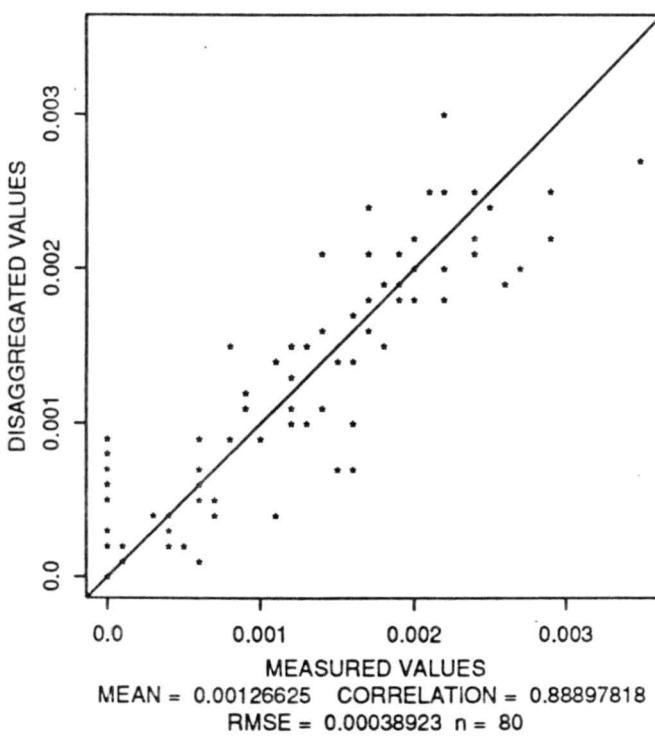
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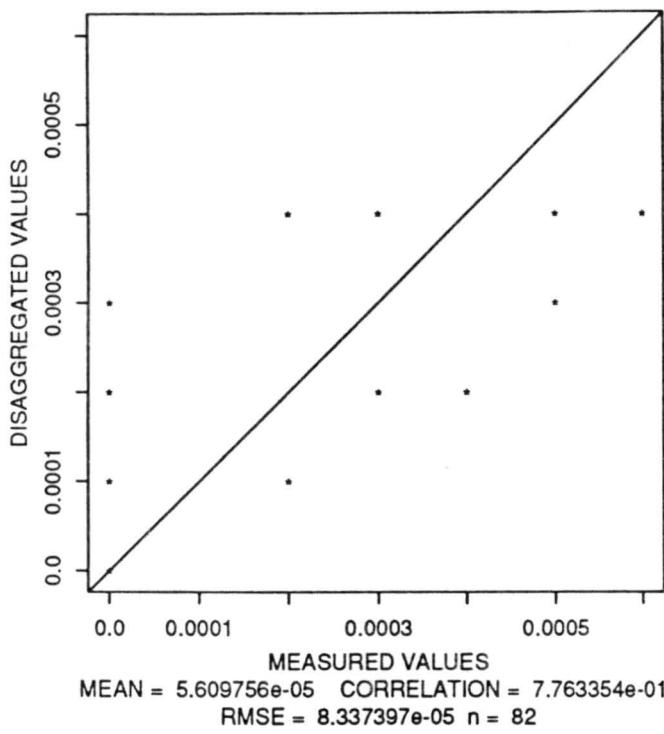
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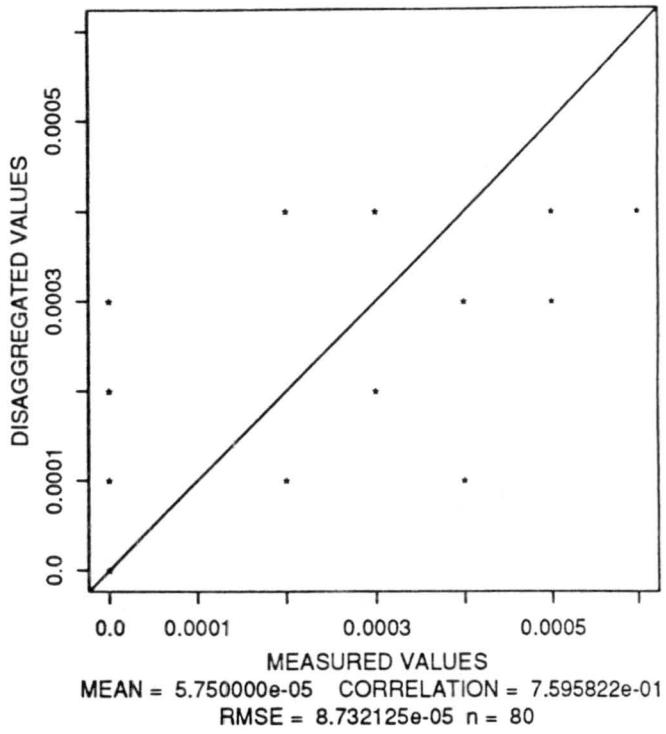
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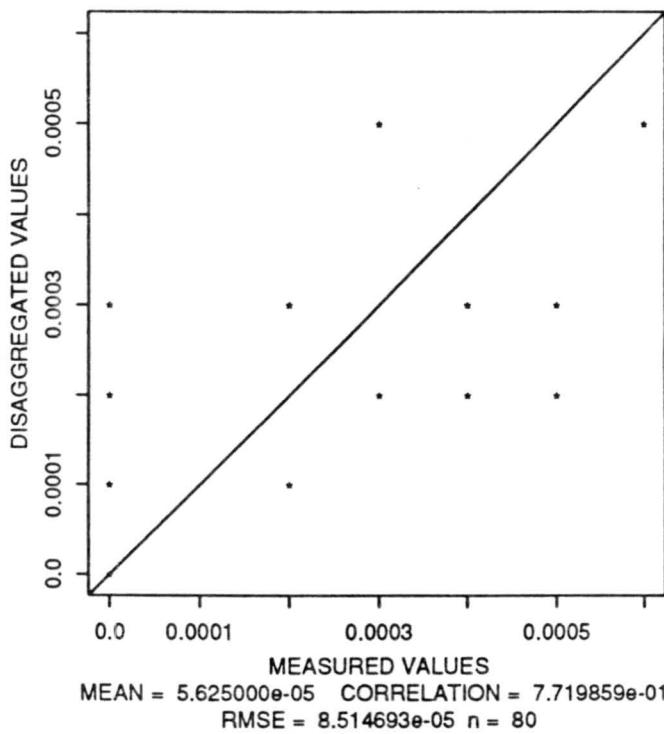
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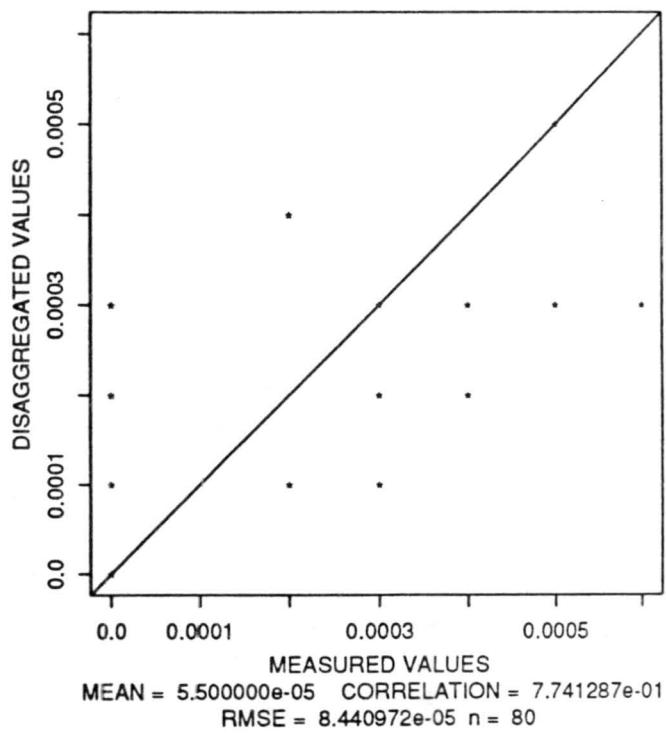
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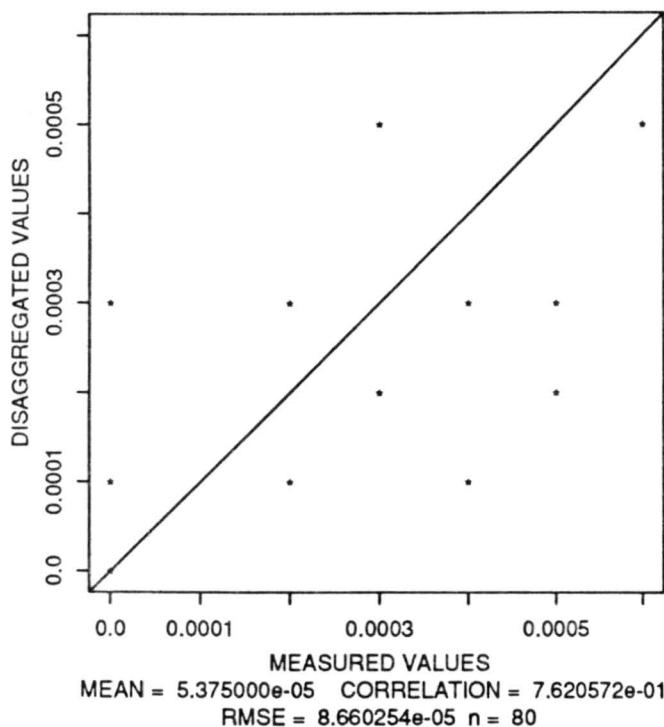
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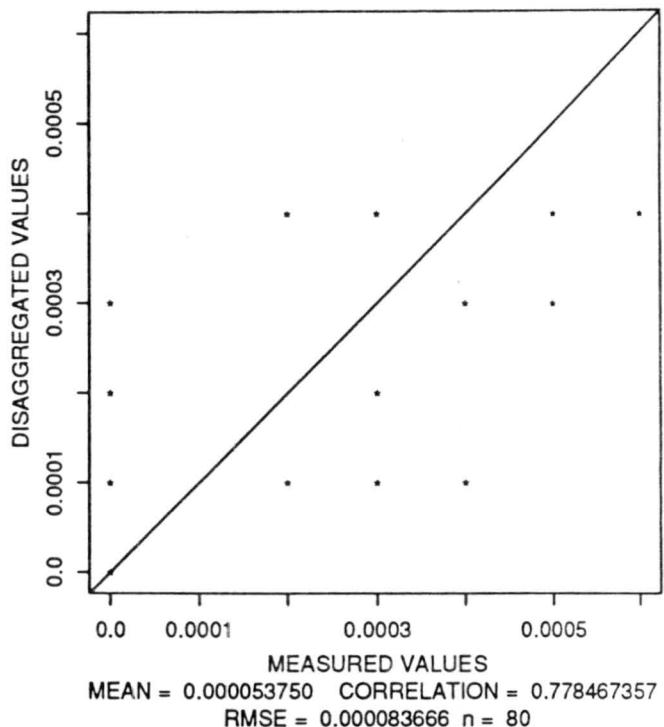
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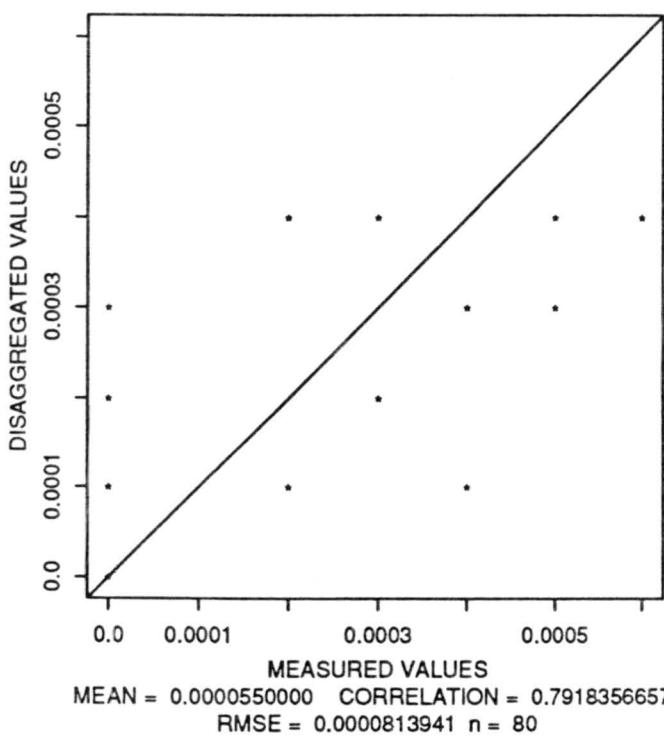
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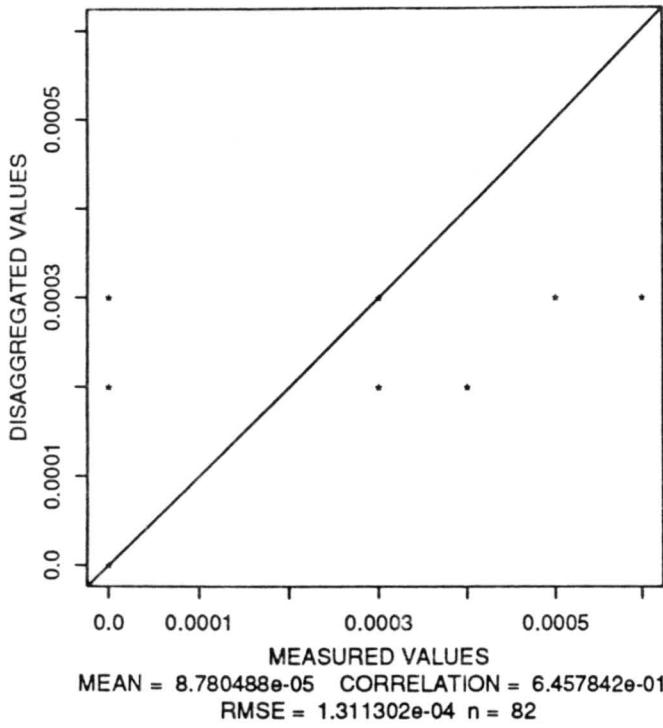
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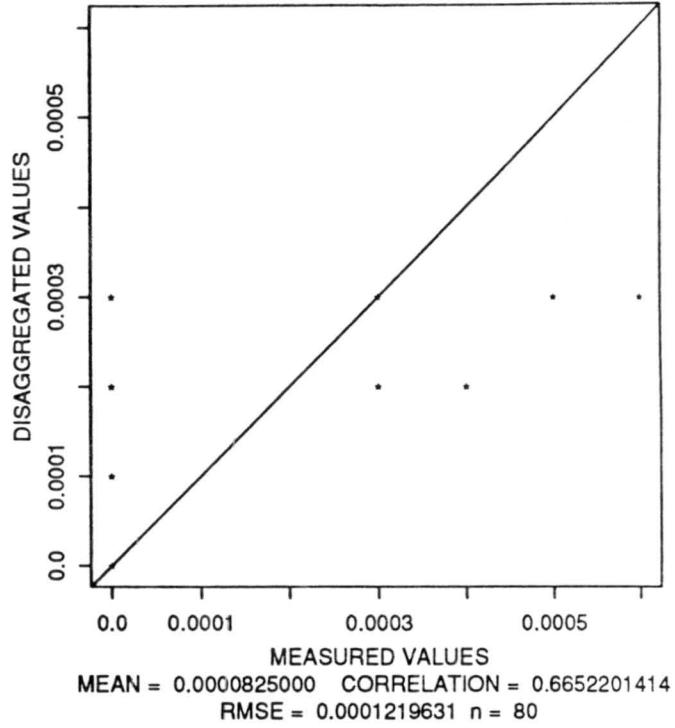
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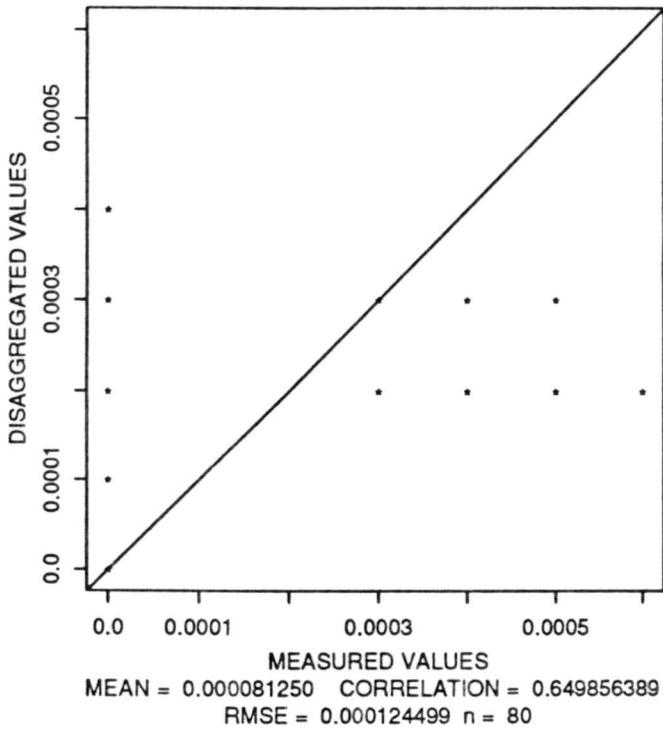
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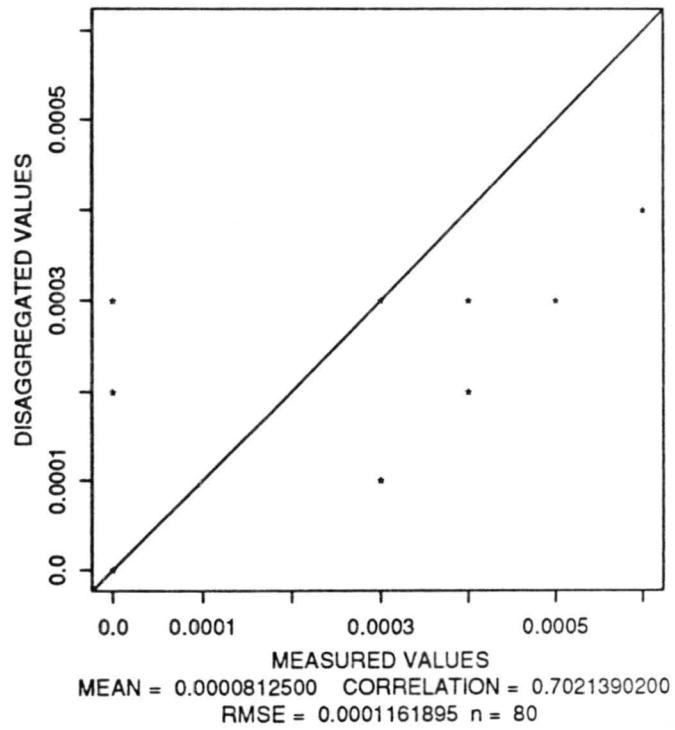
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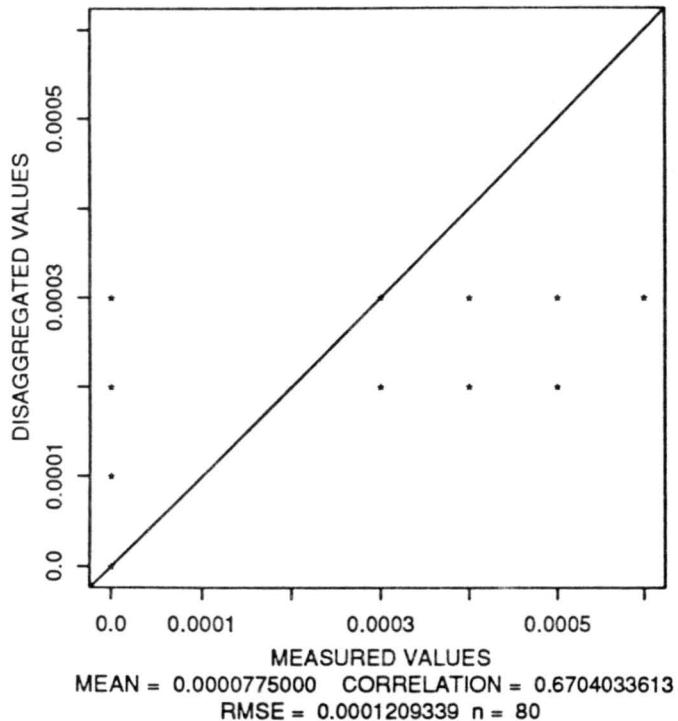
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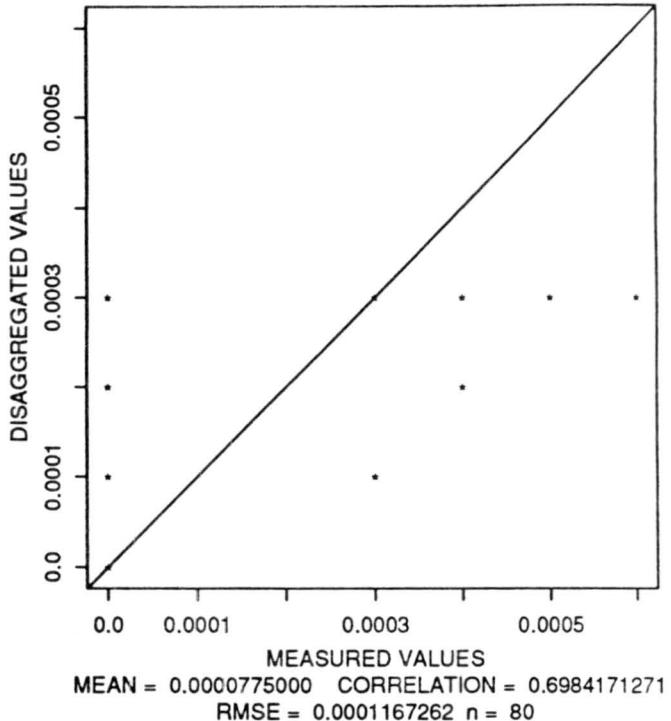
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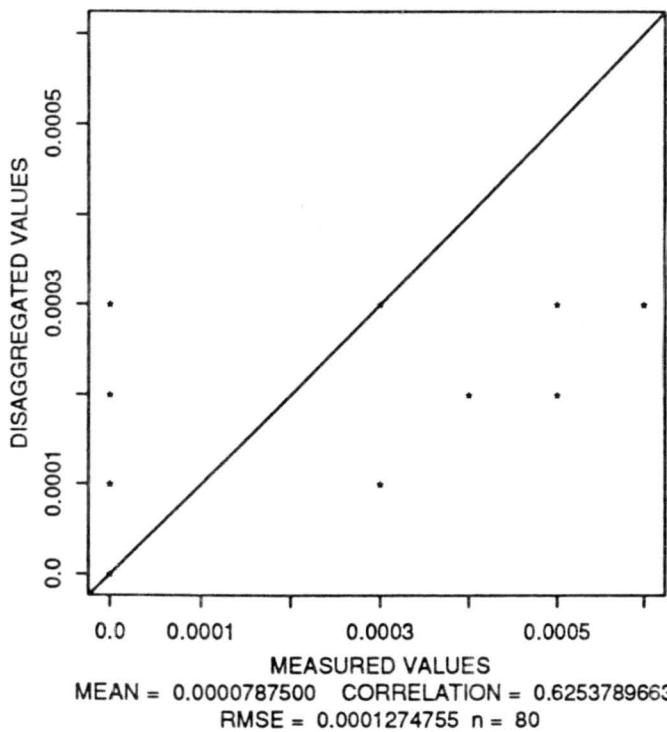
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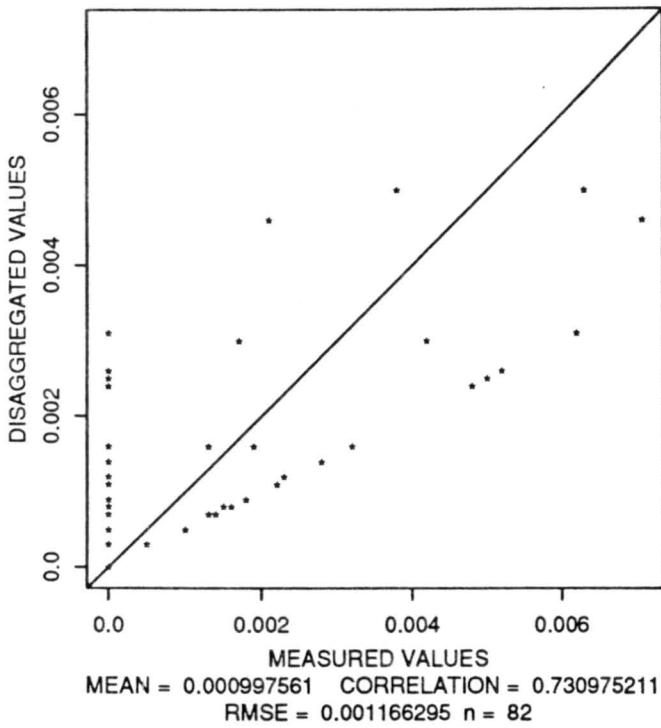
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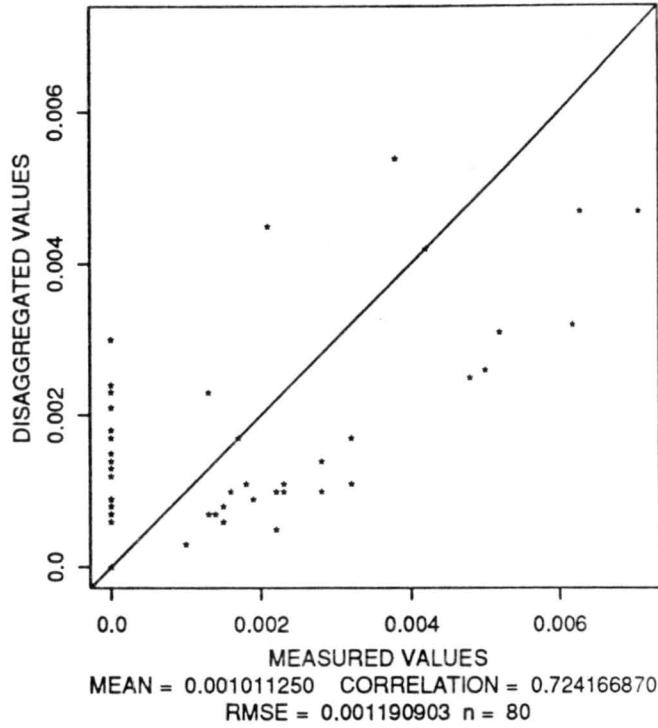
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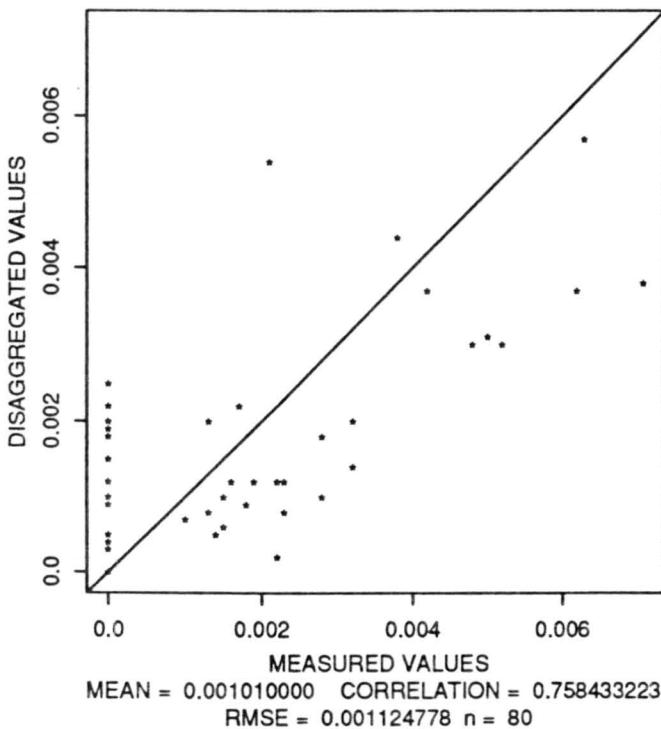
ORDER 0 DISAGGREGATION FOR FPBC



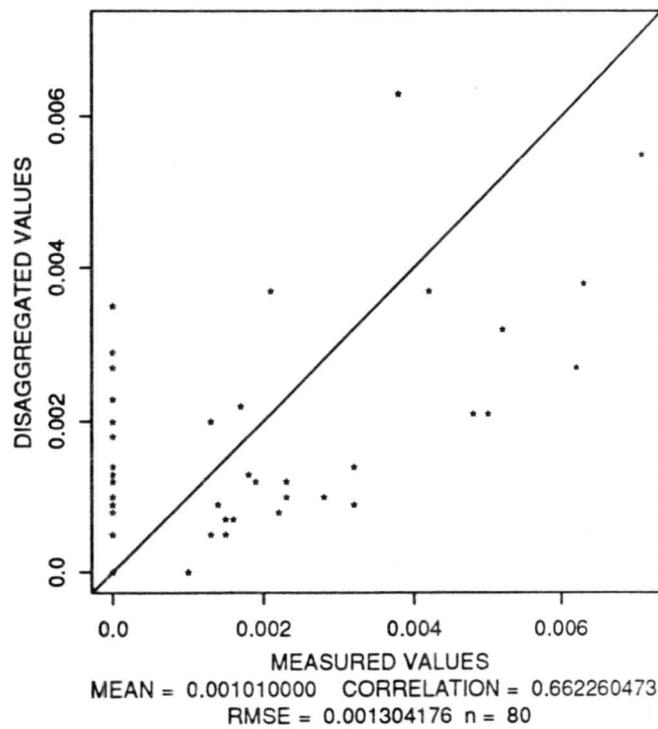
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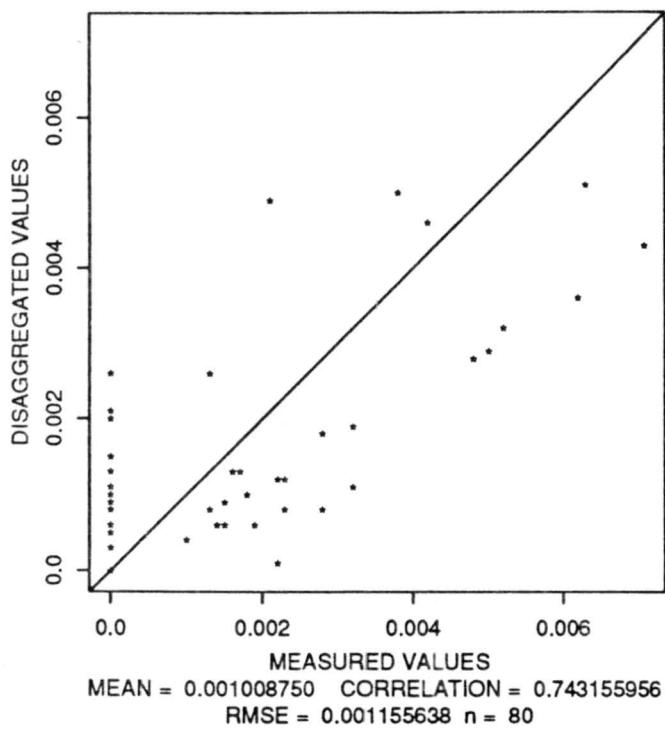
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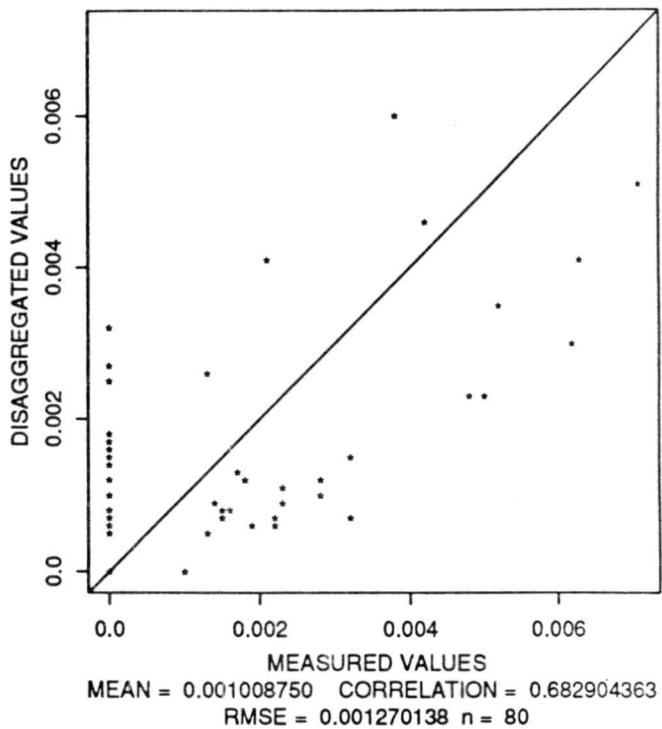
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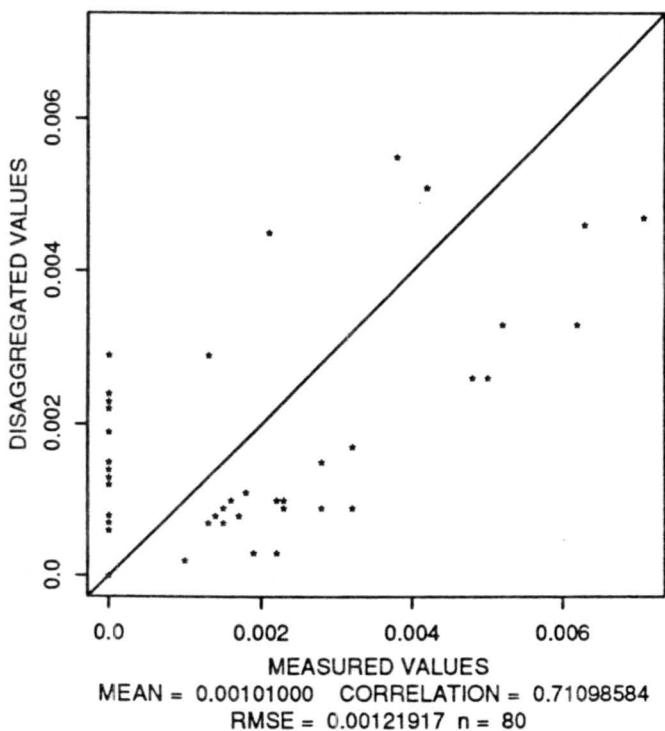
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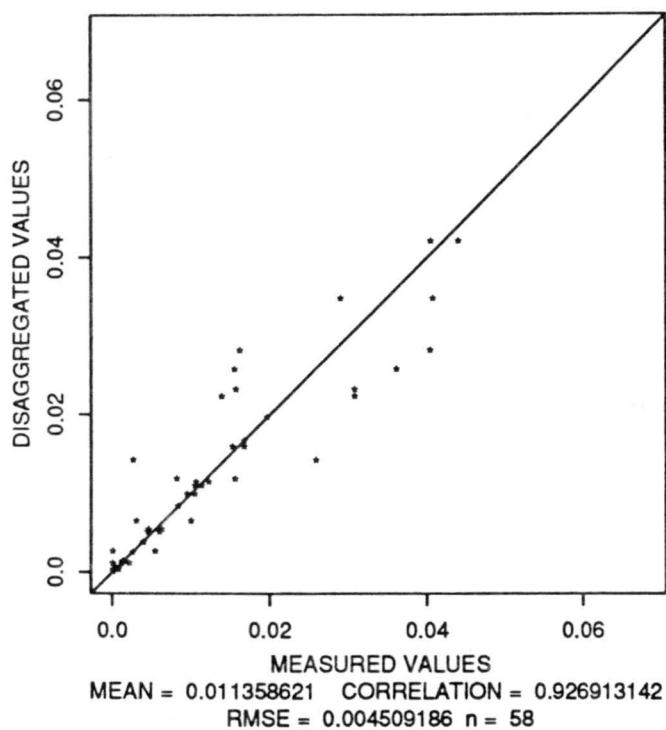
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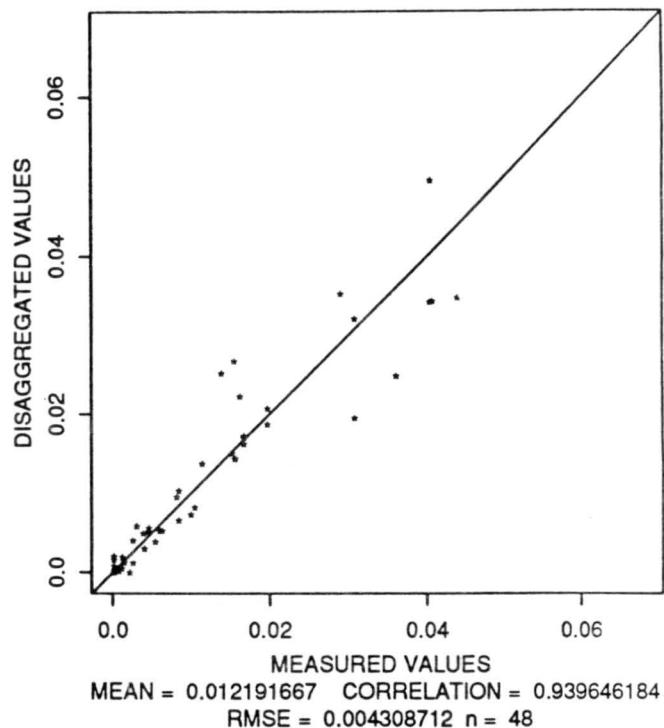
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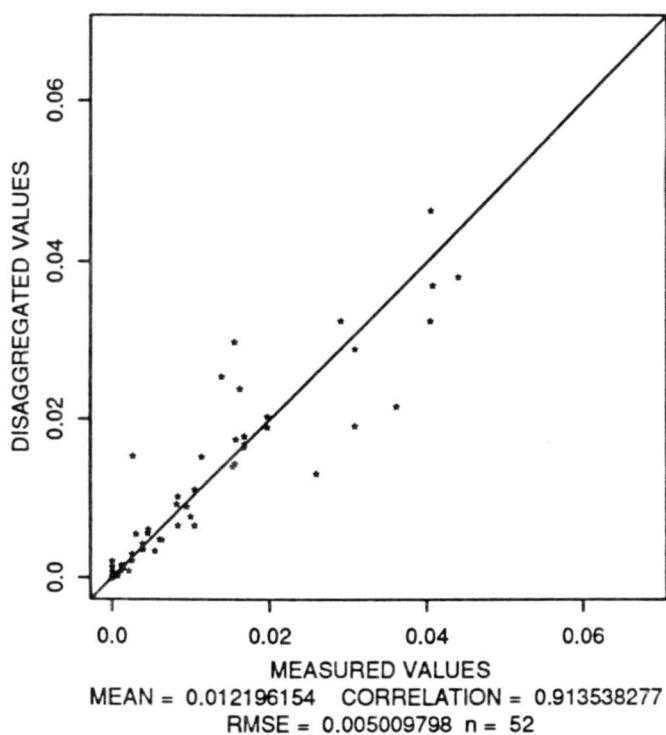
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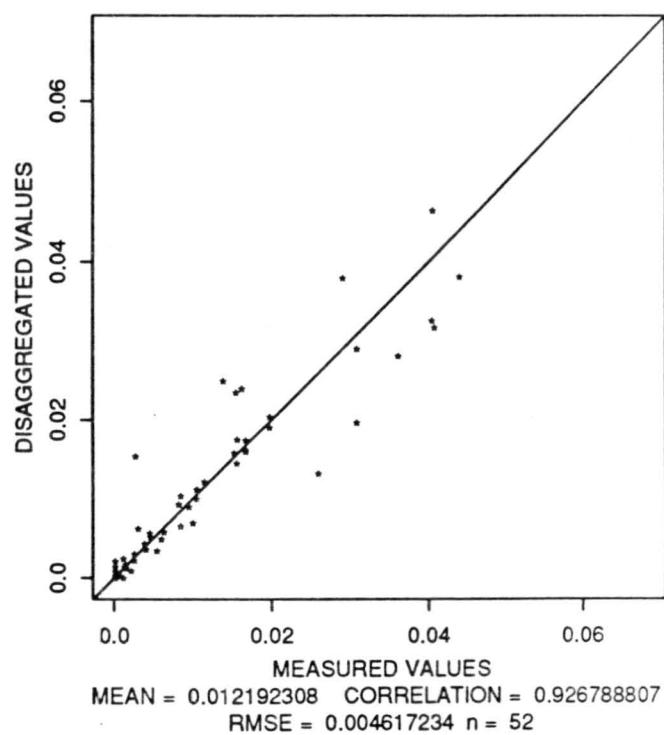
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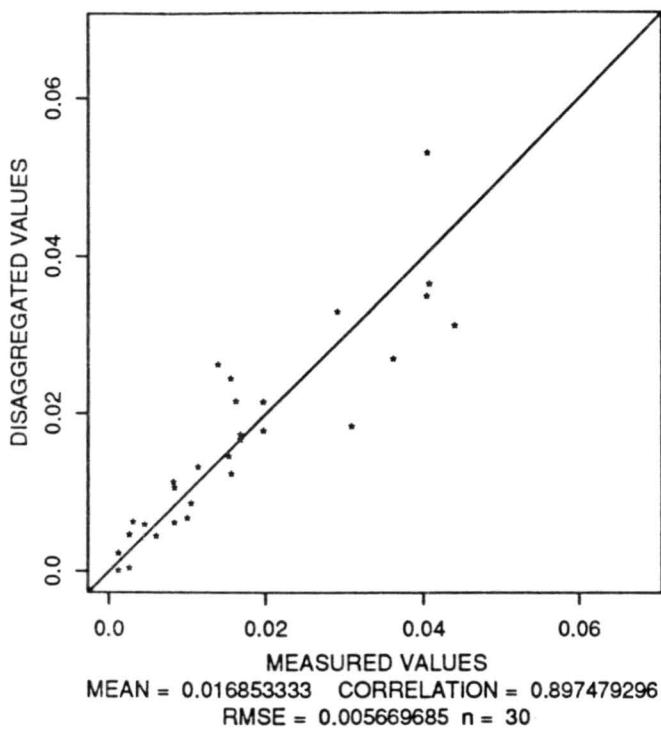
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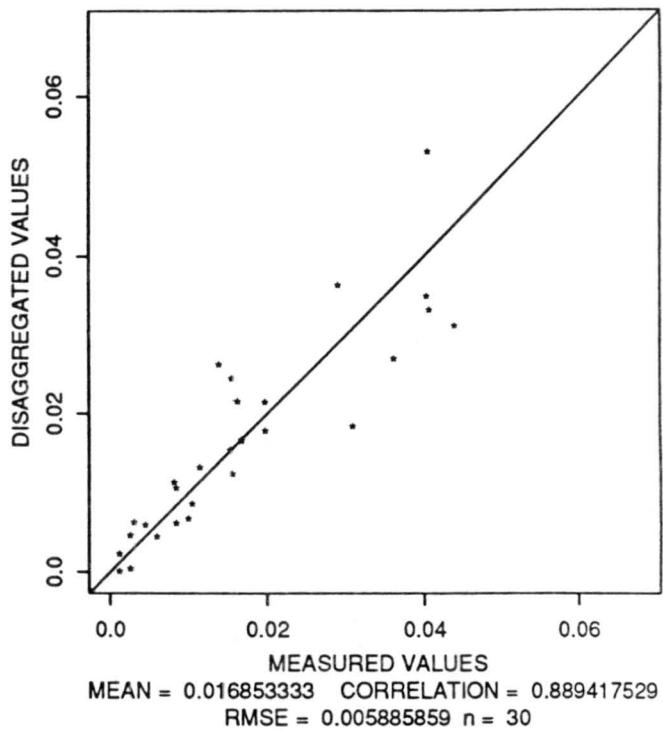
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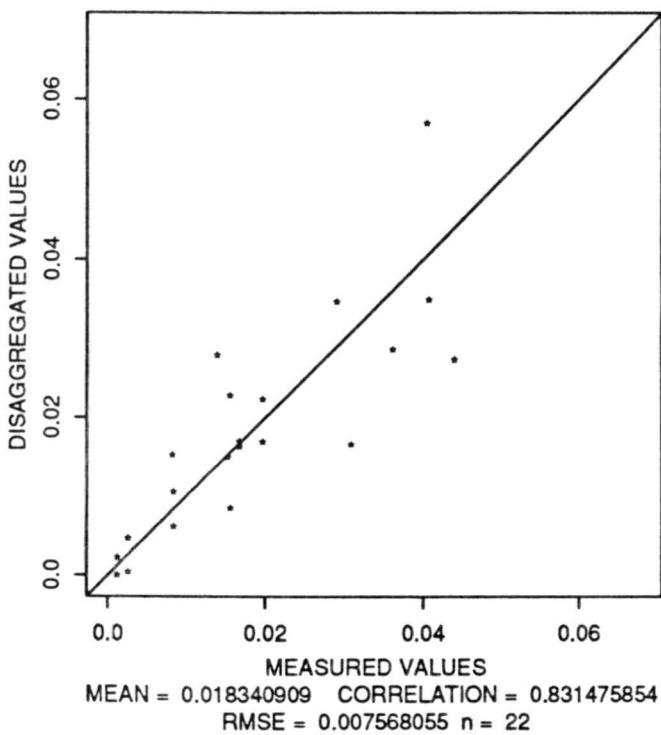
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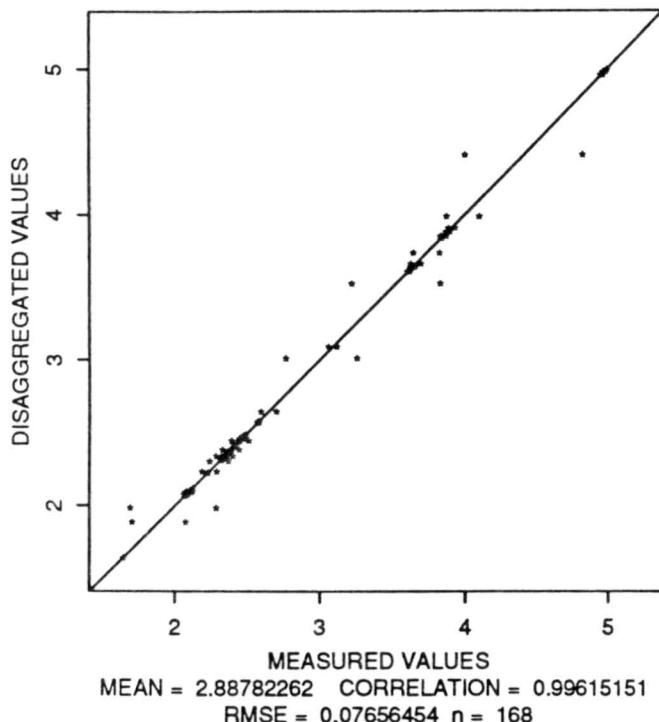
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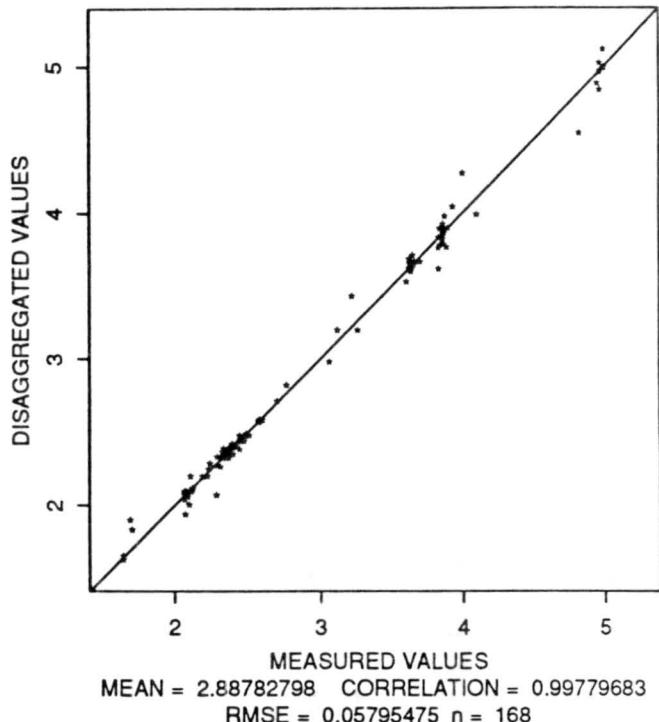
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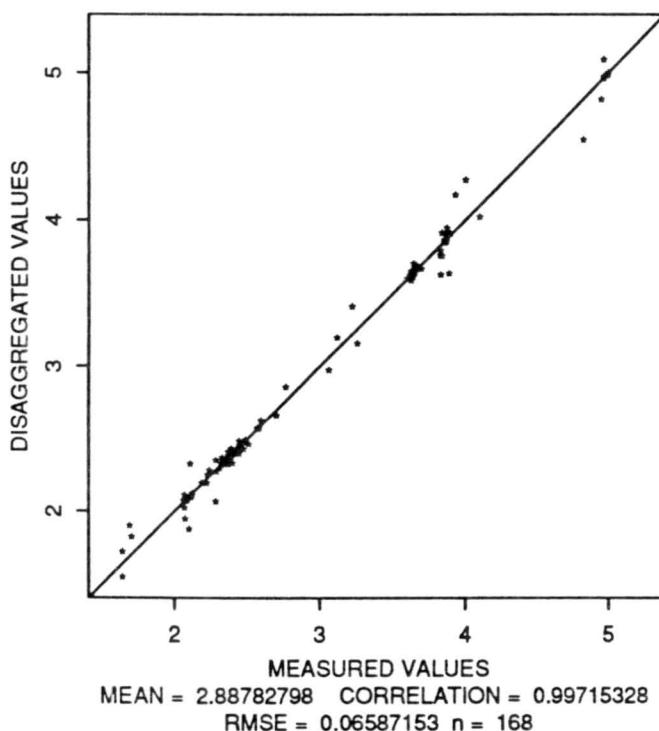
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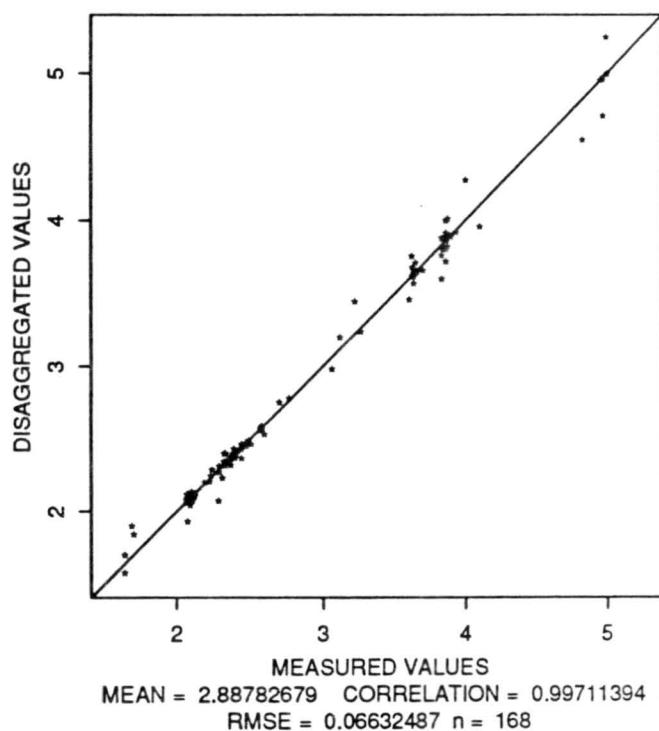
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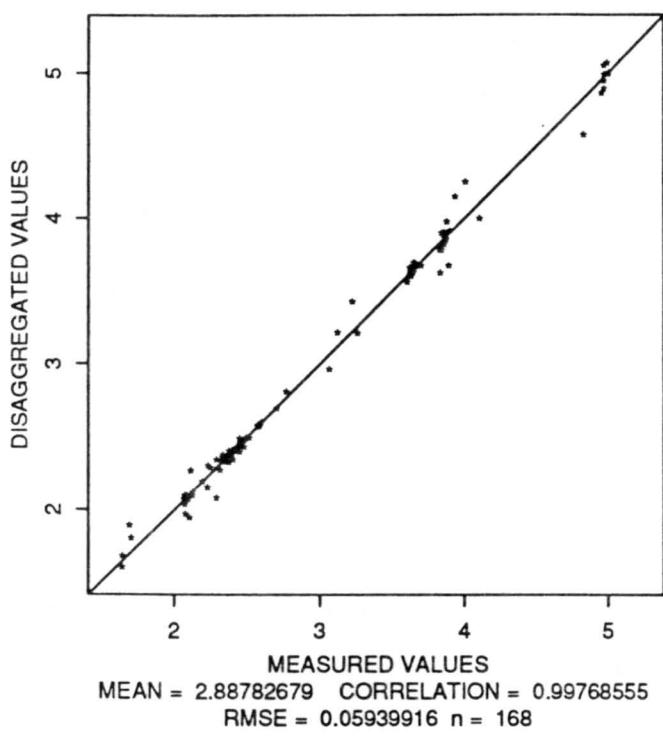
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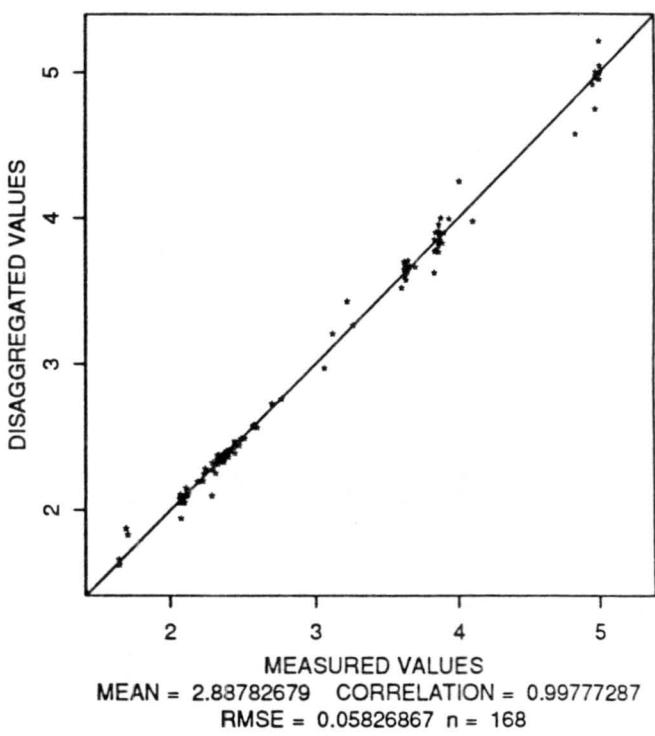
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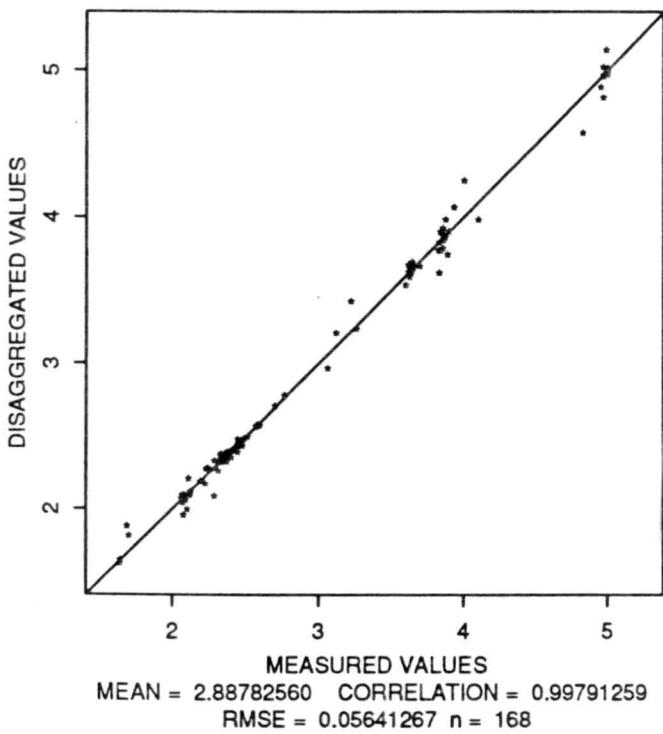
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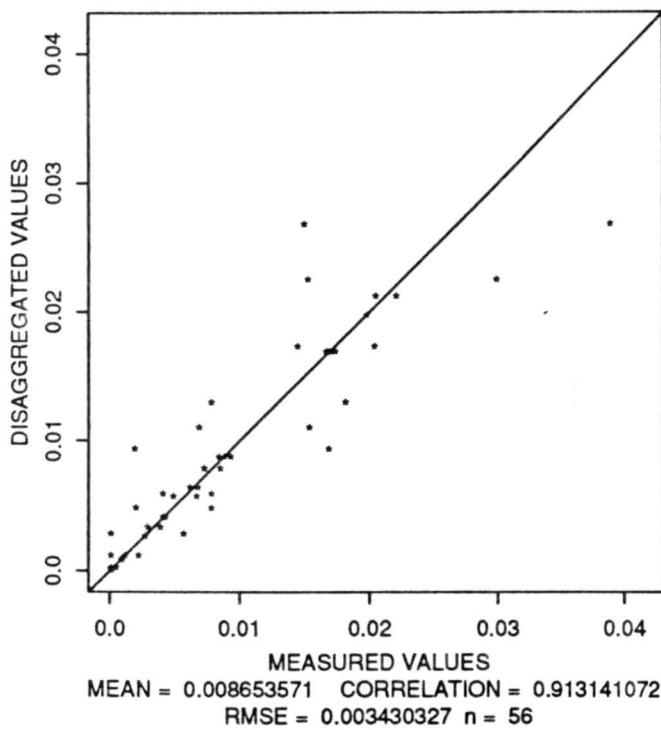
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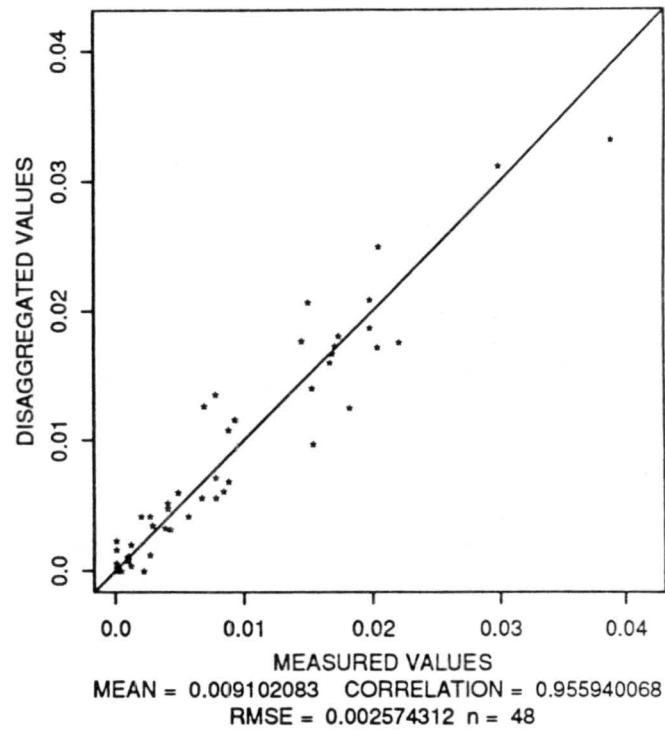
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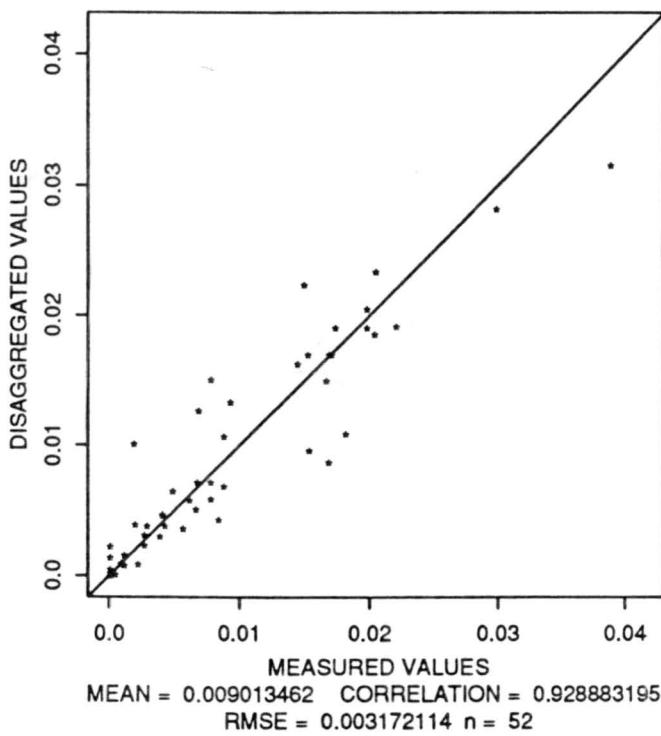
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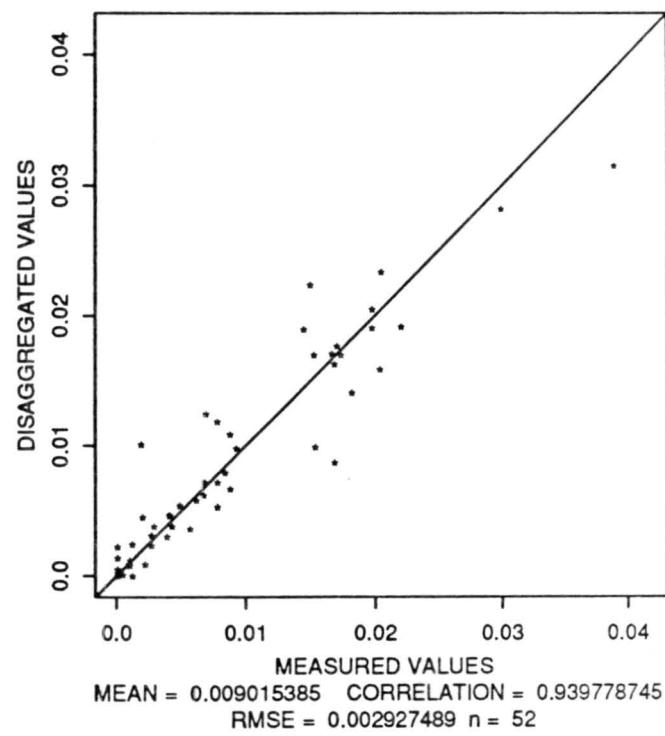
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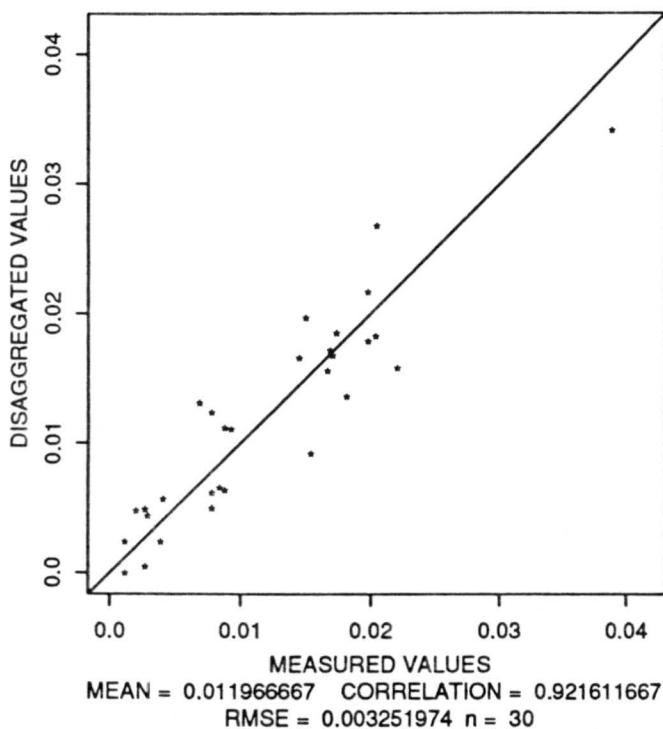
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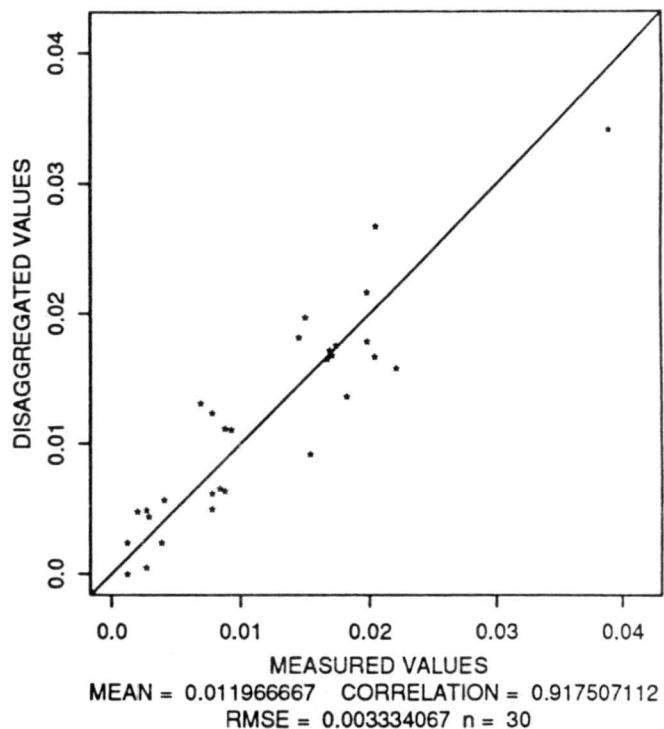
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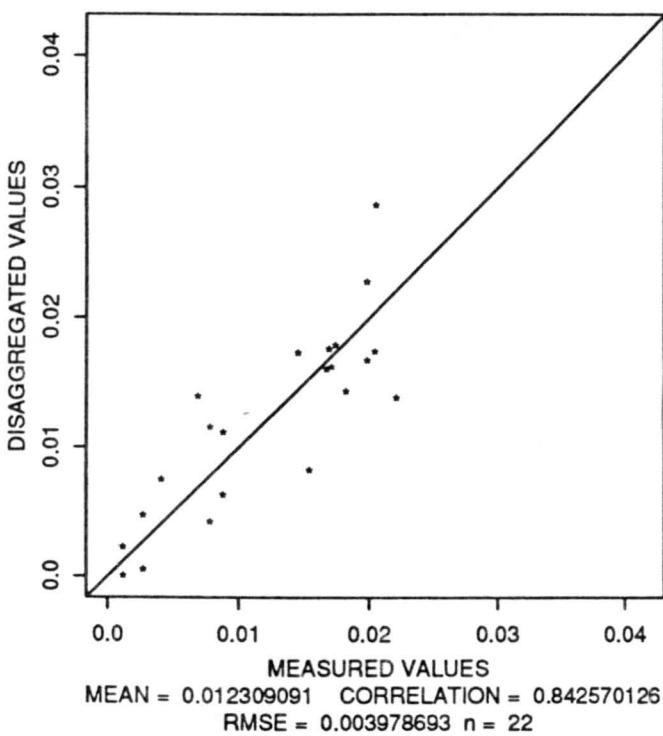
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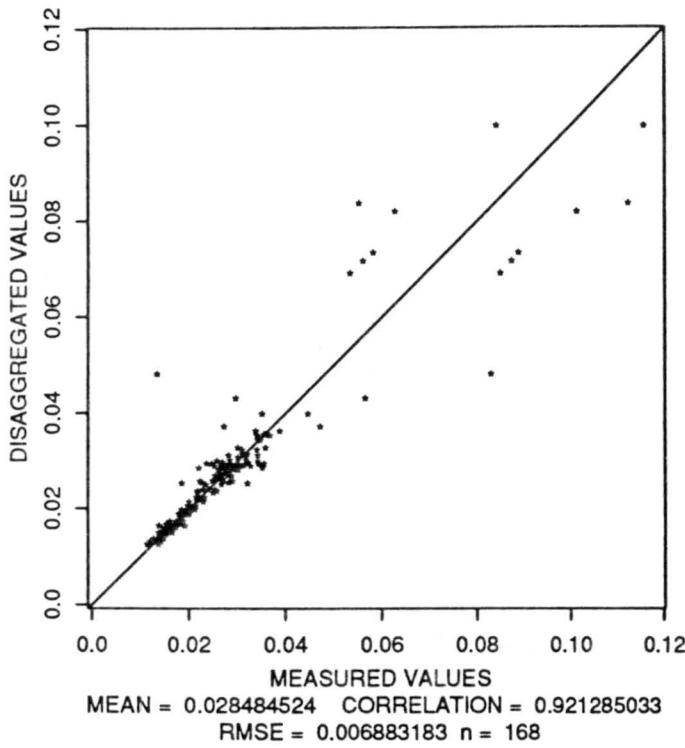
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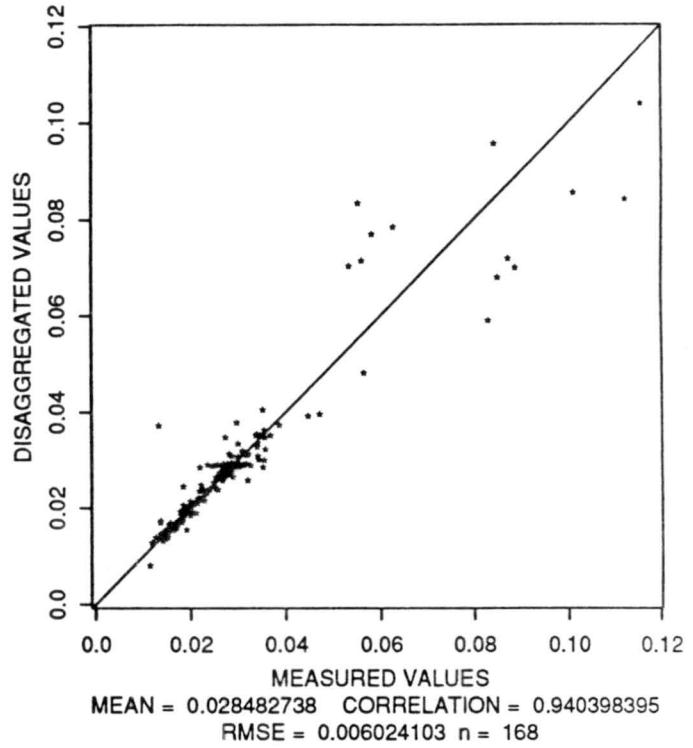
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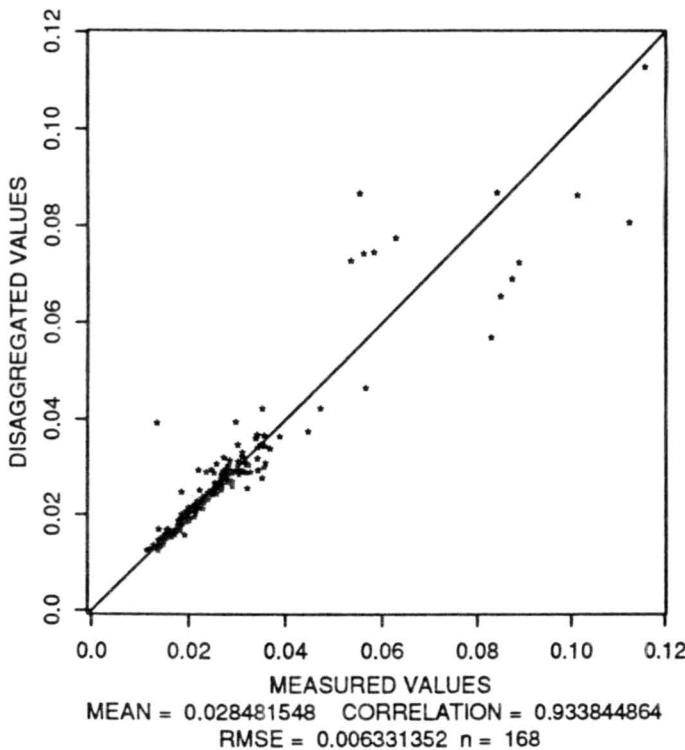
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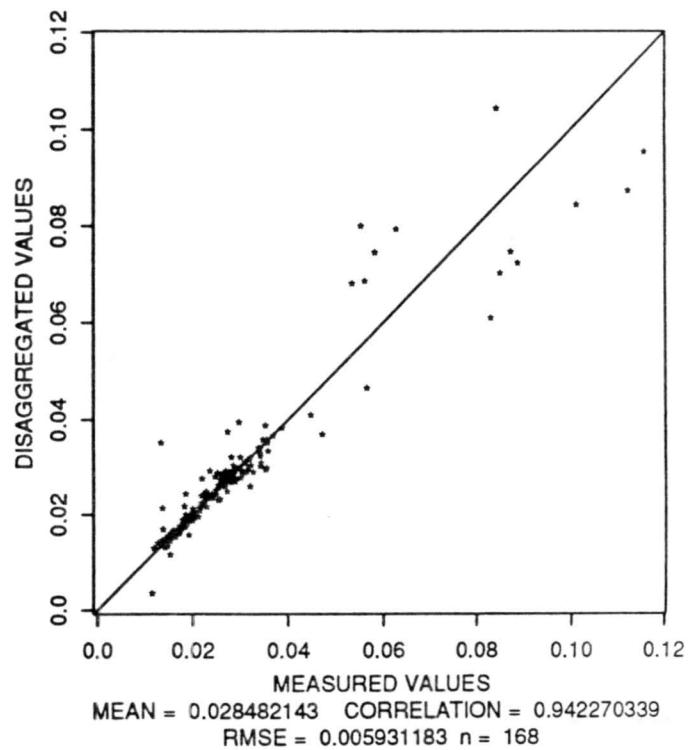
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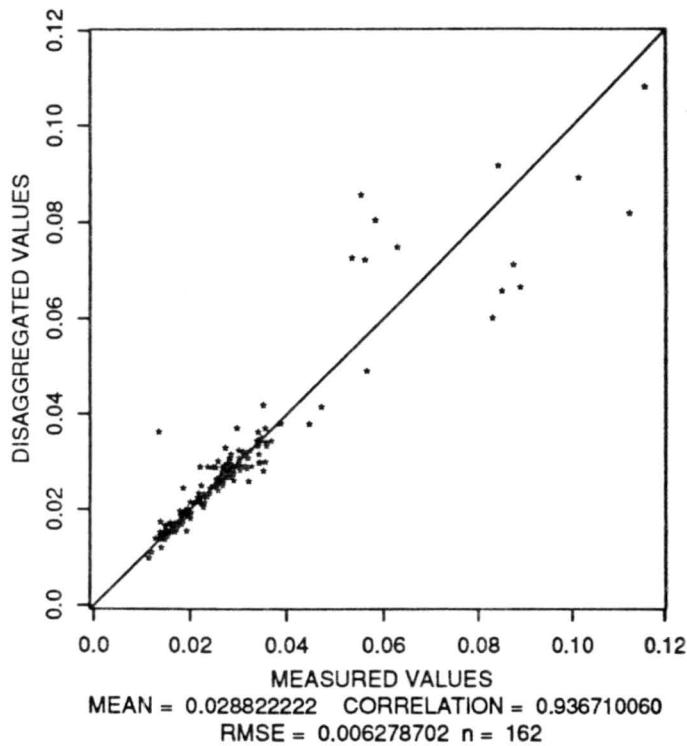
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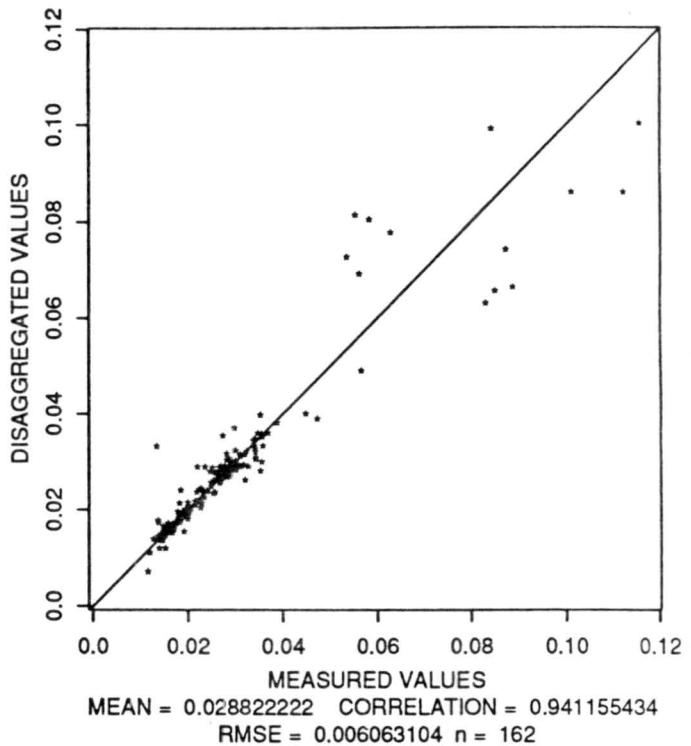
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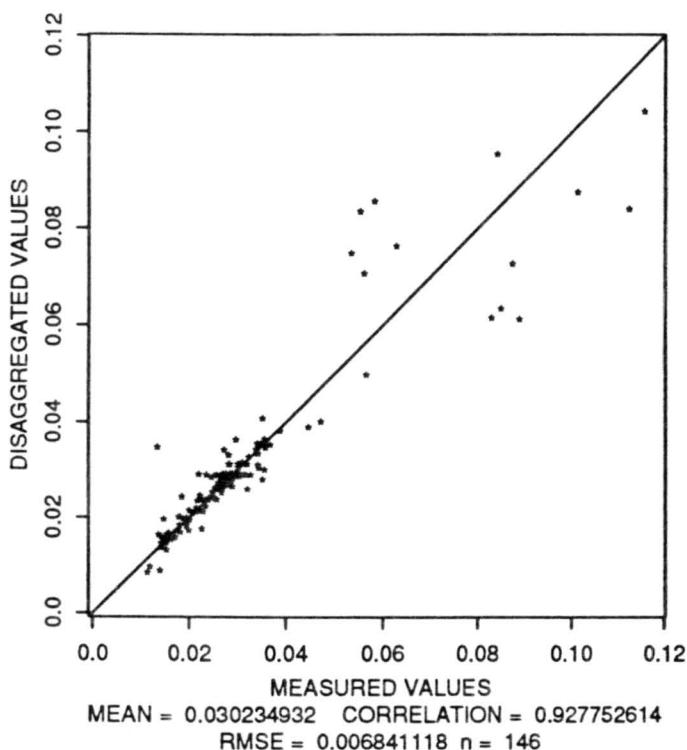
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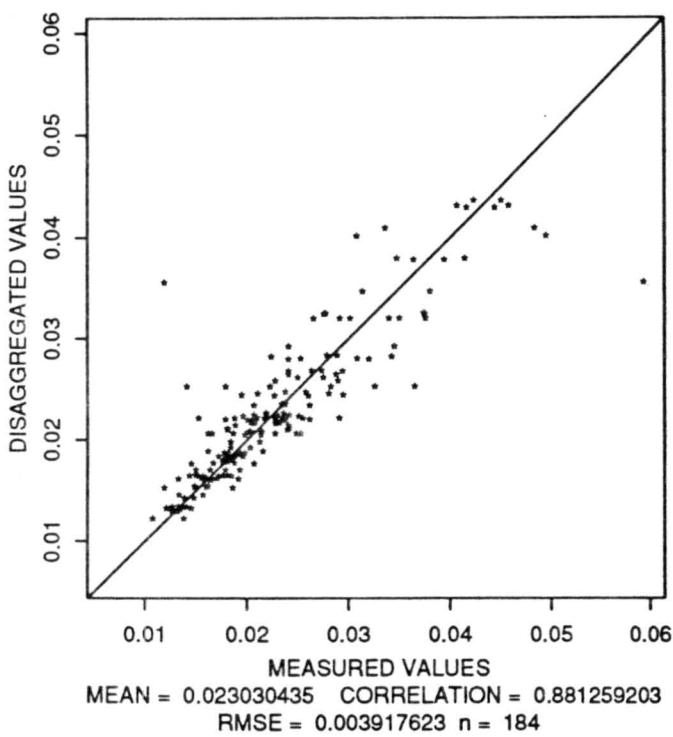
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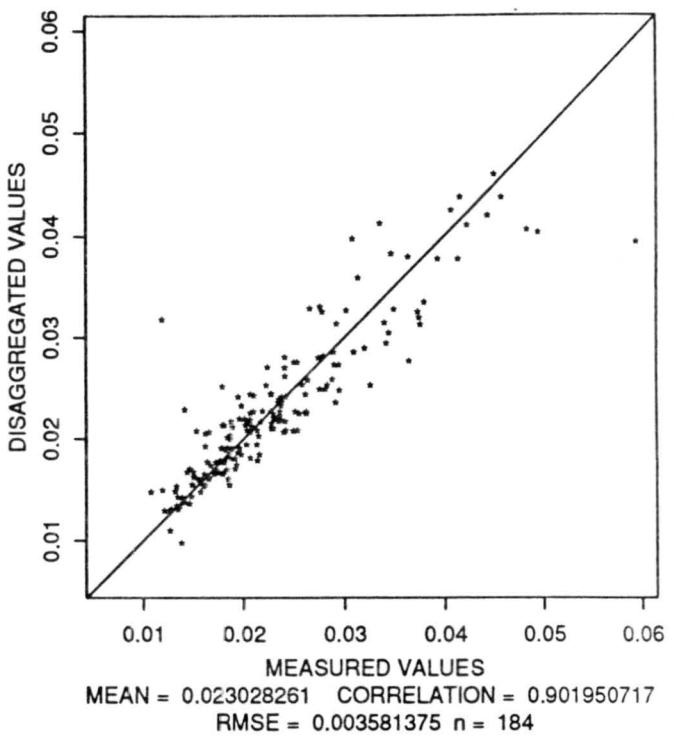
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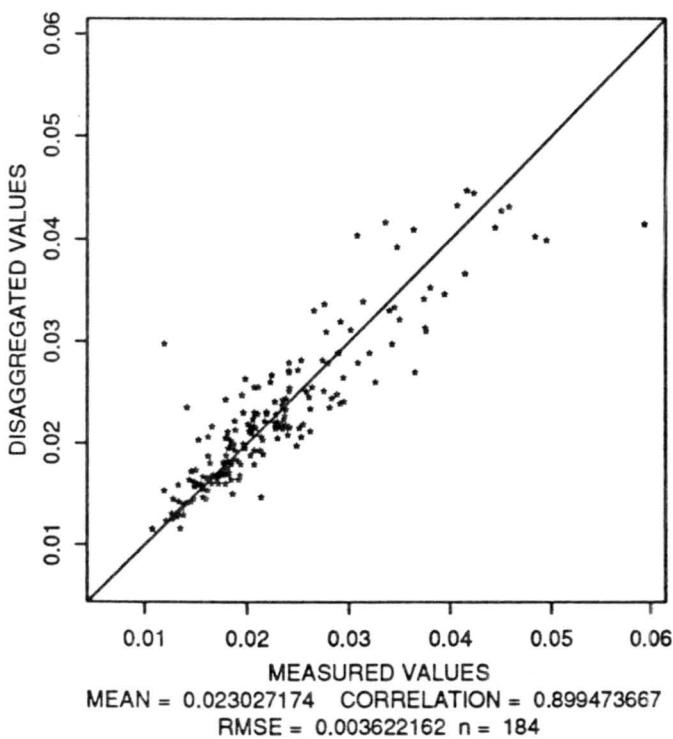
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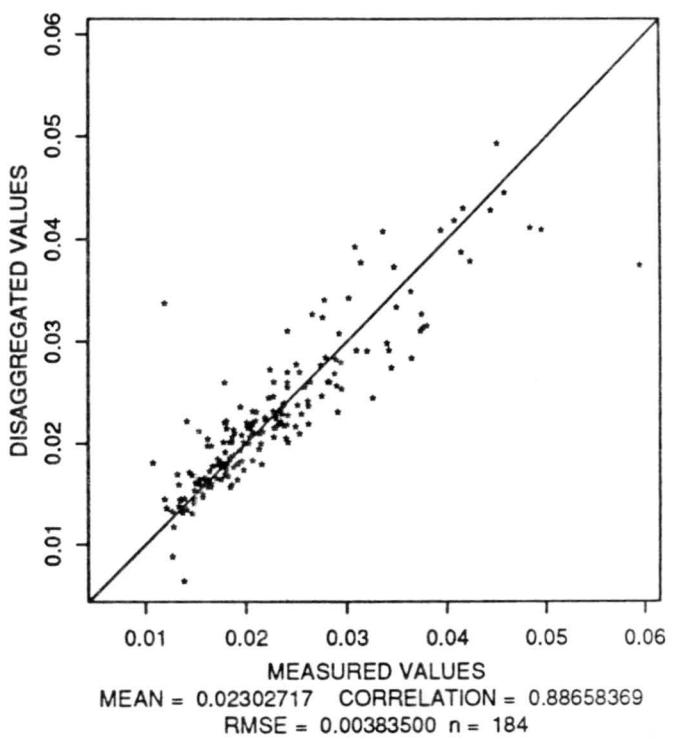
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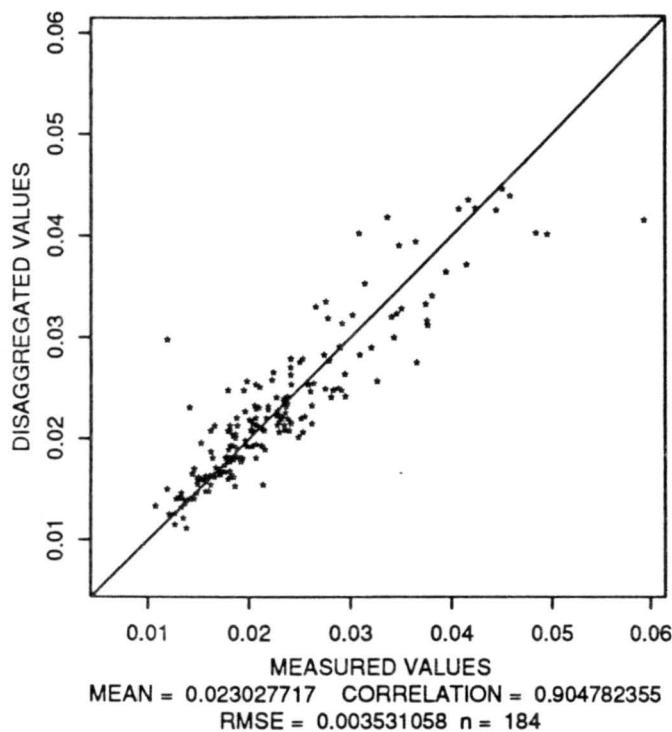
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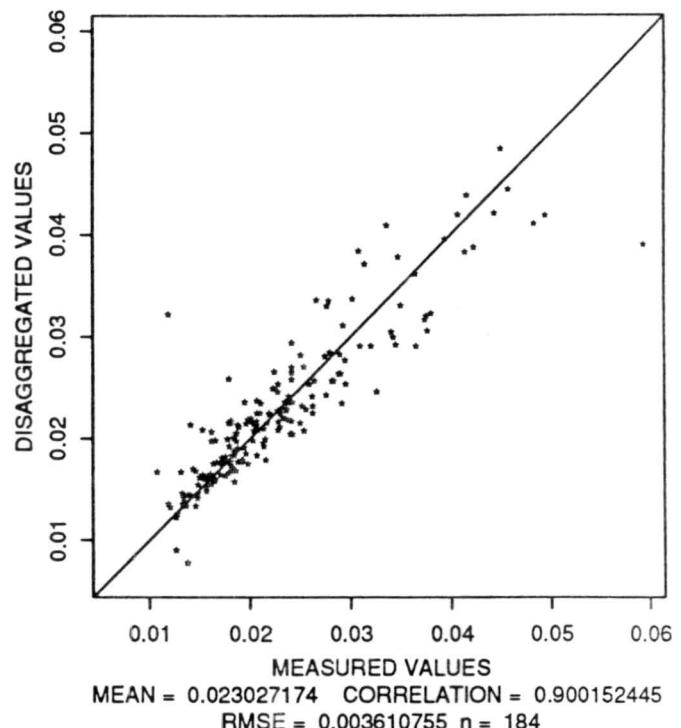
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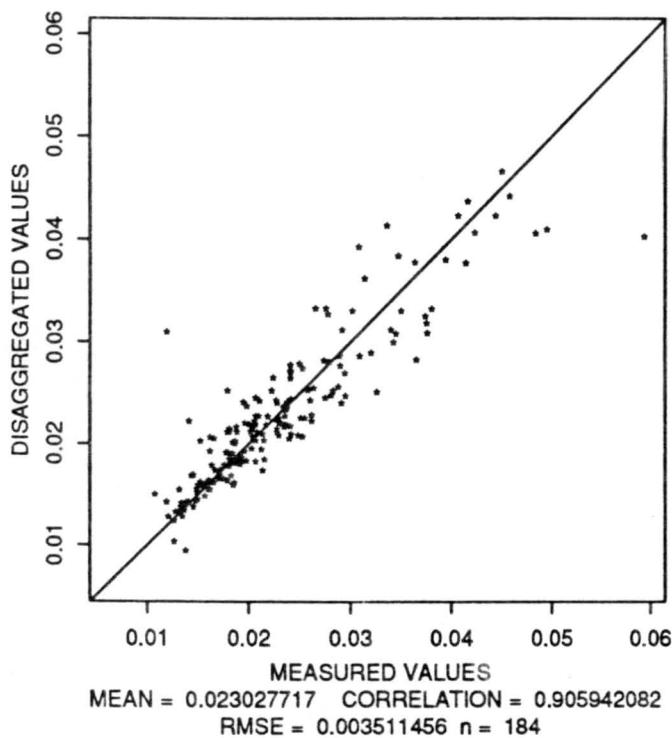
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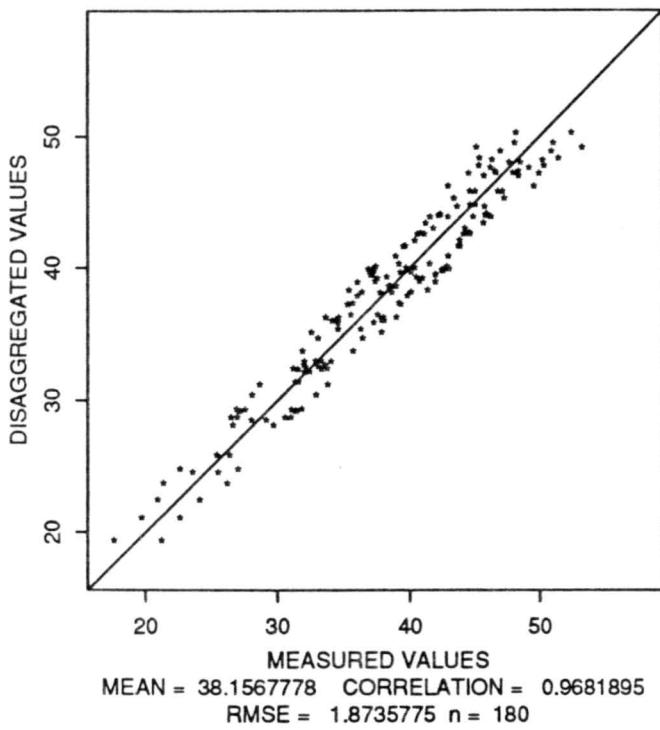
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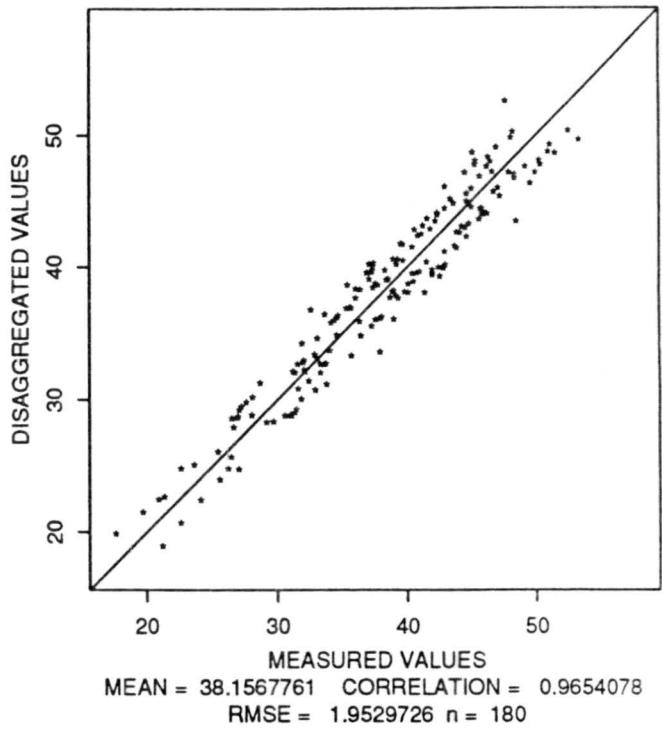
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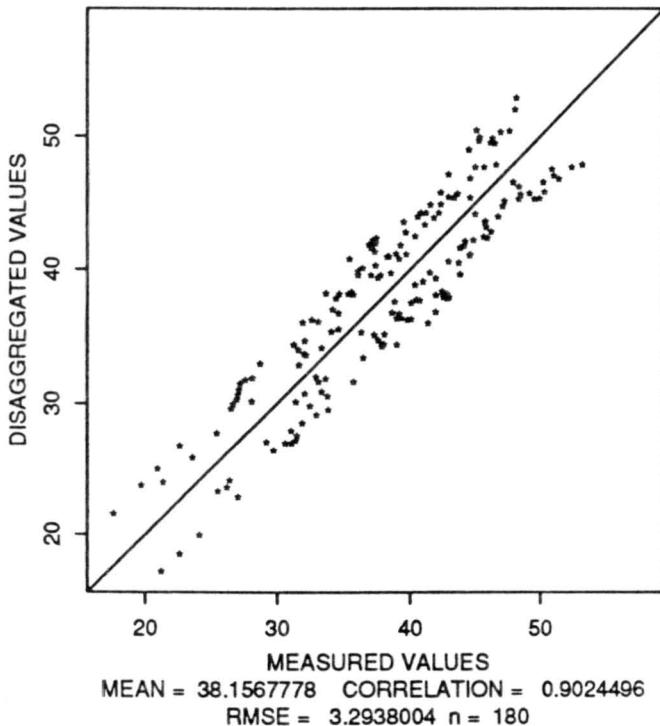
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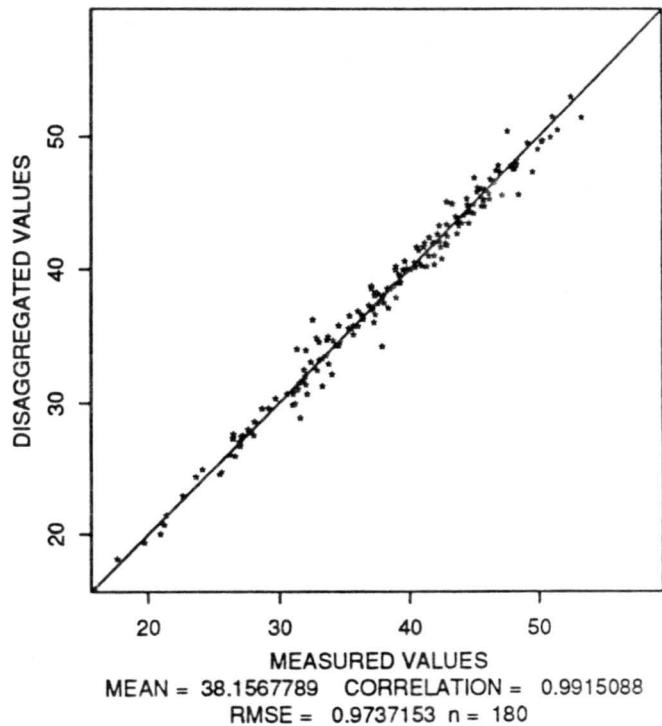
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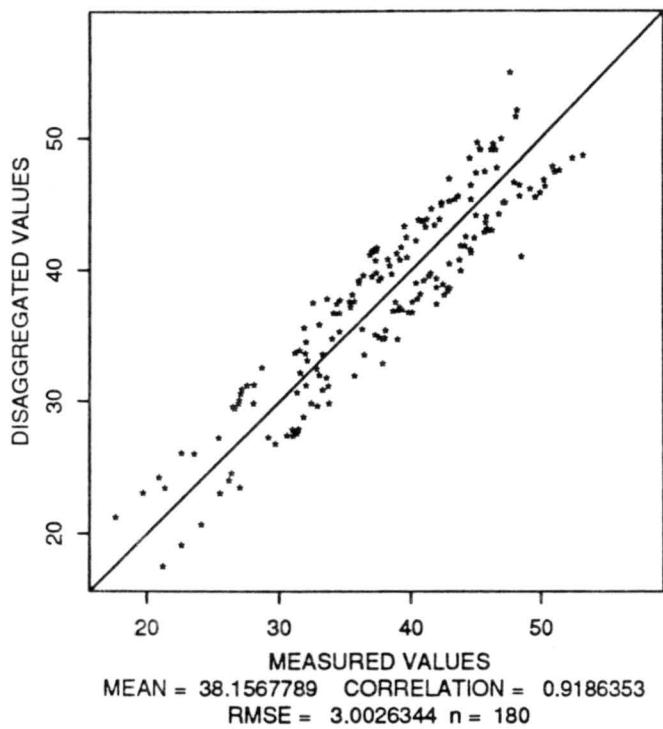
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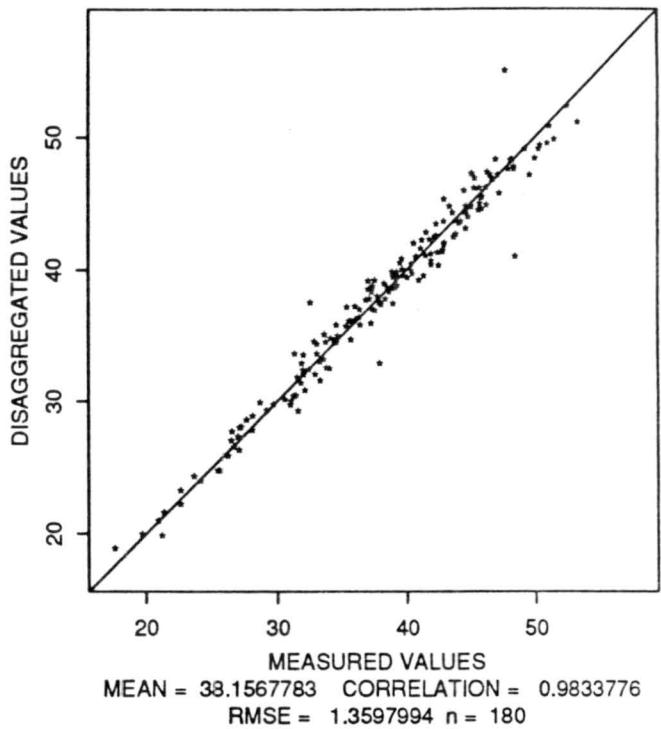
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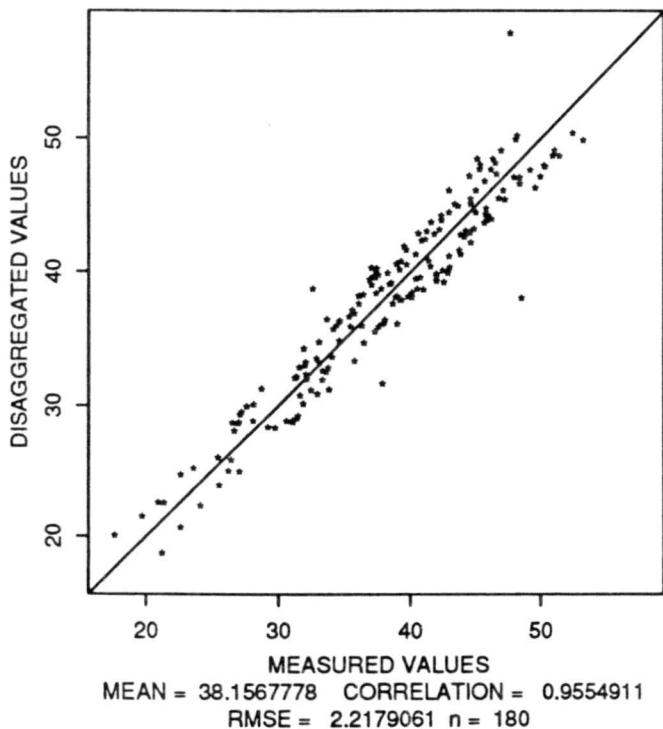
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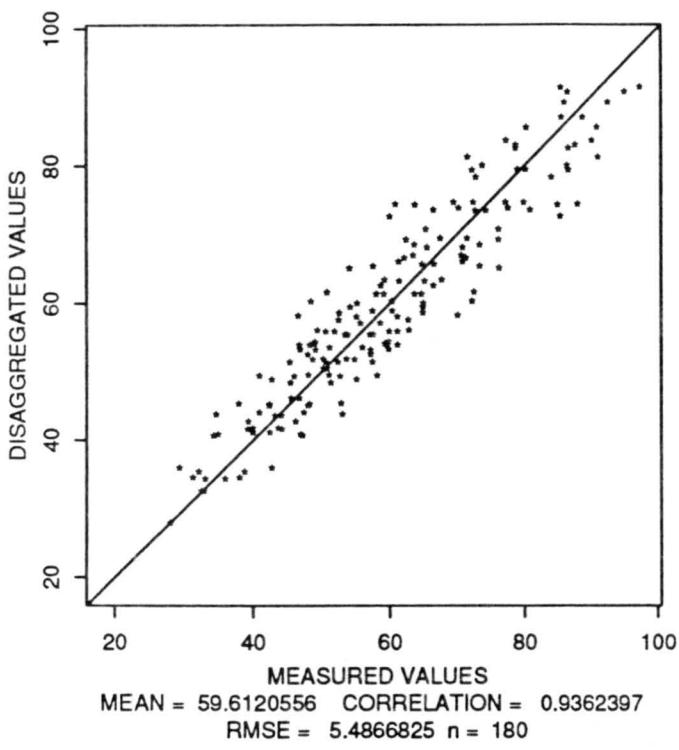
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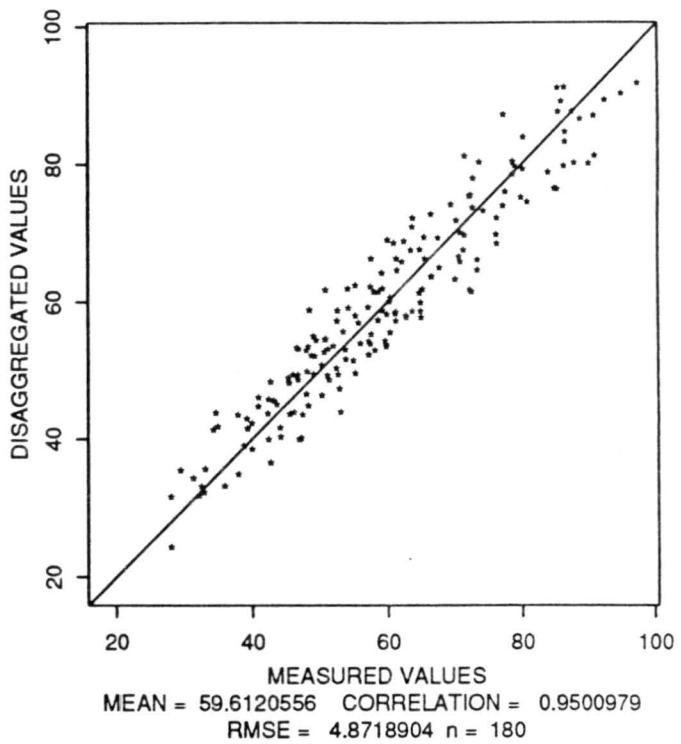
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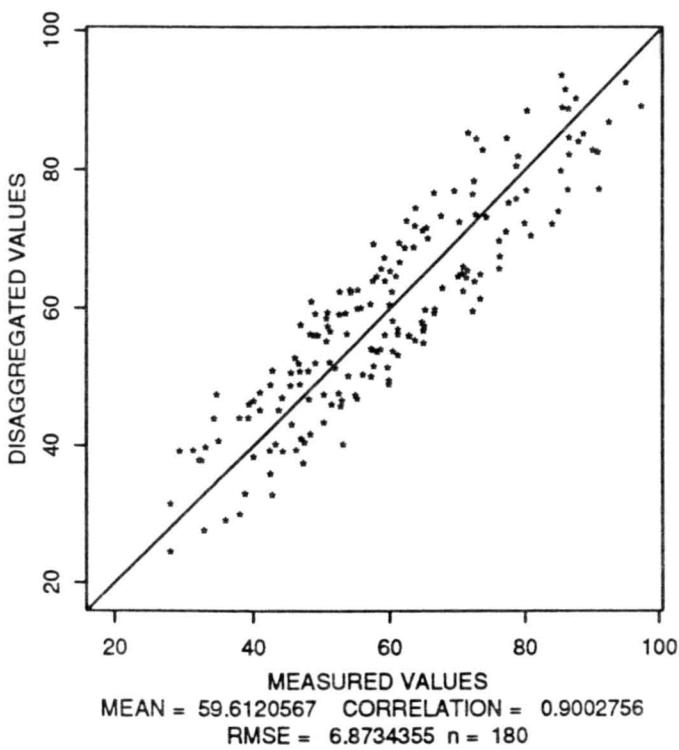
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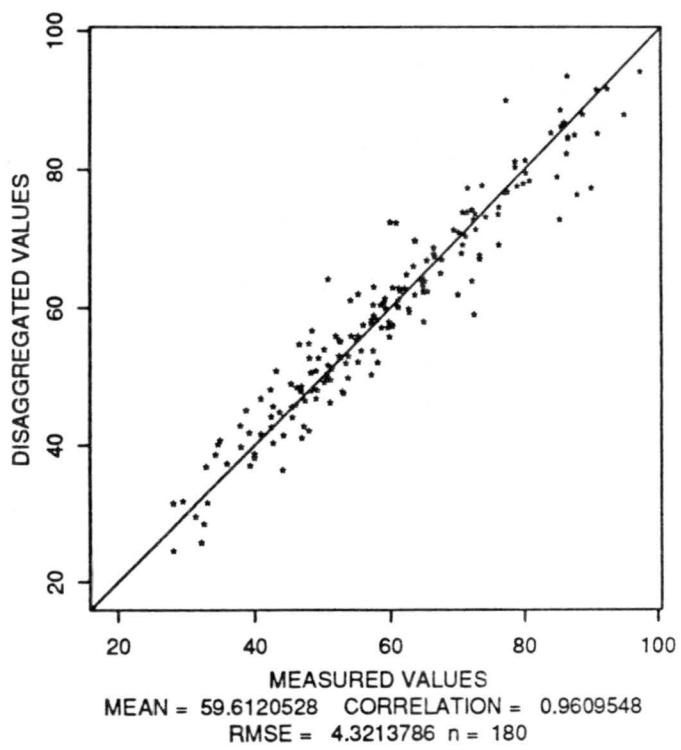
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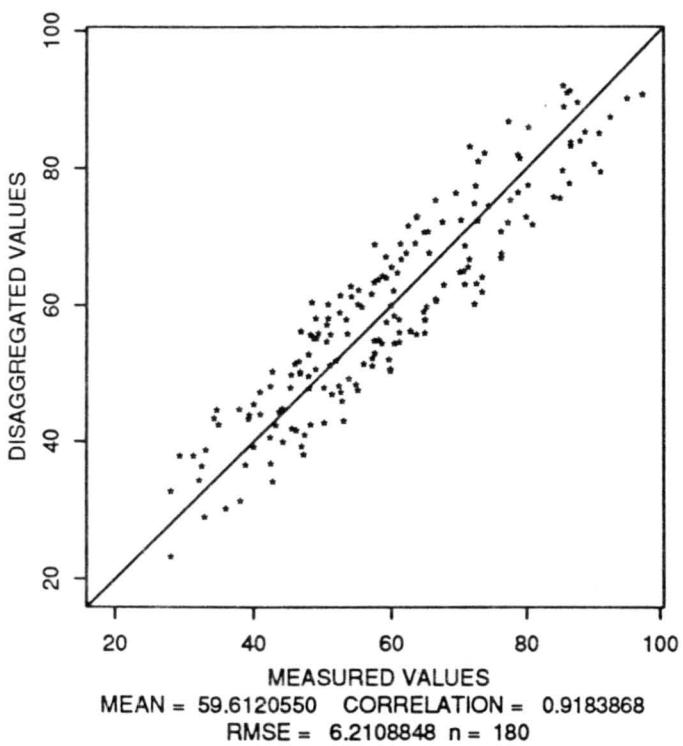
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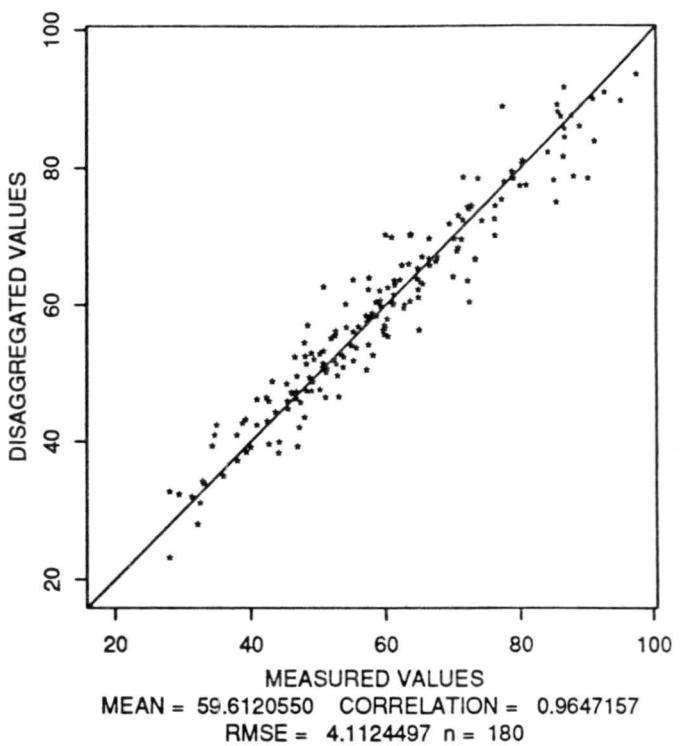
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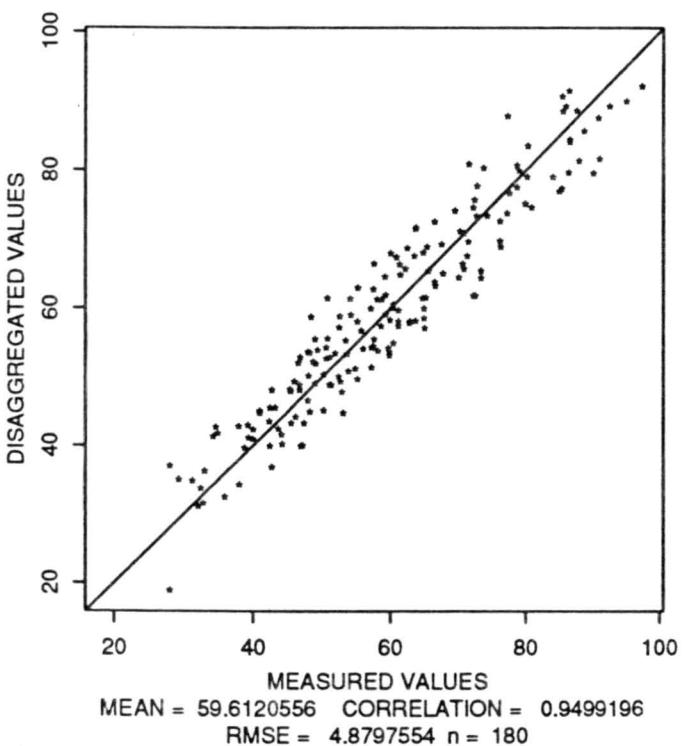
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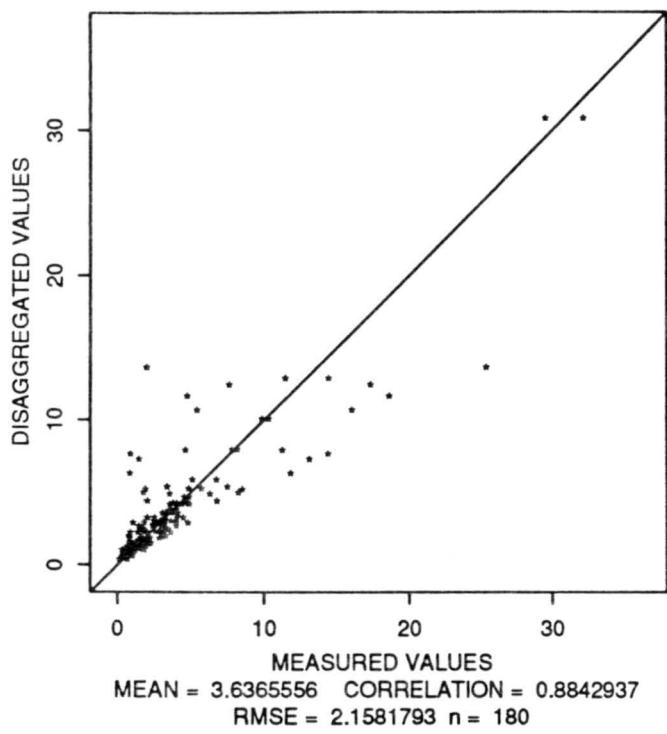
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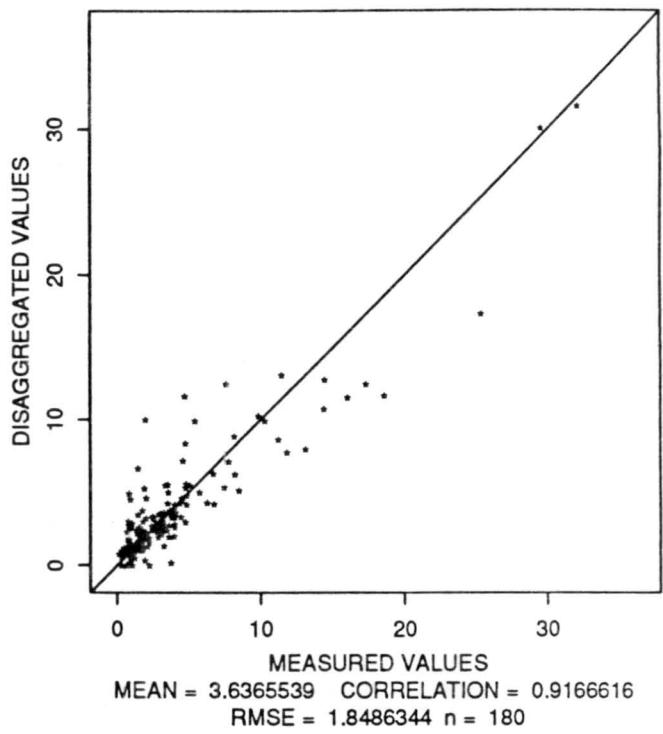
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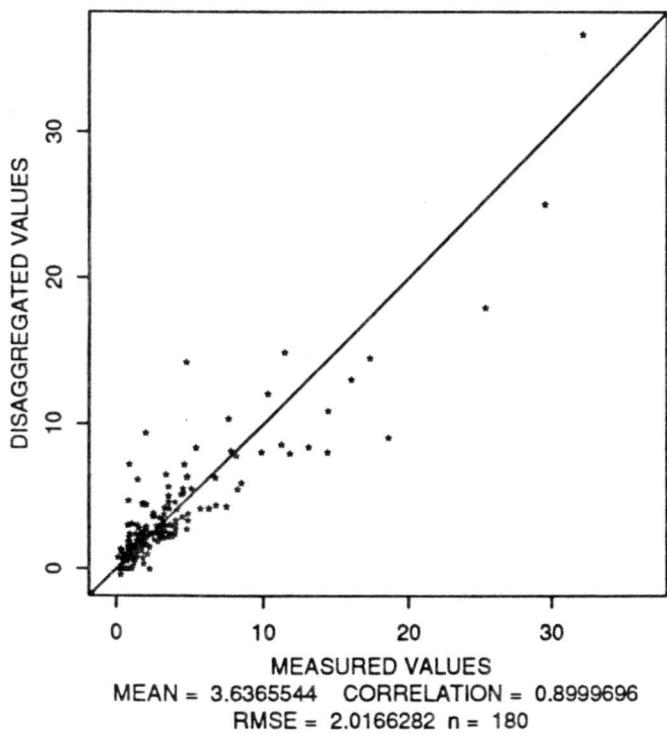
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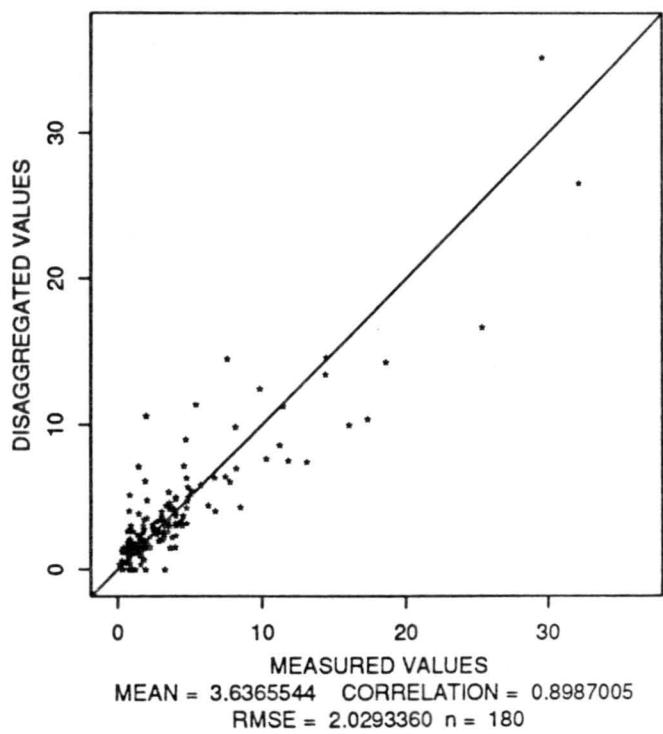
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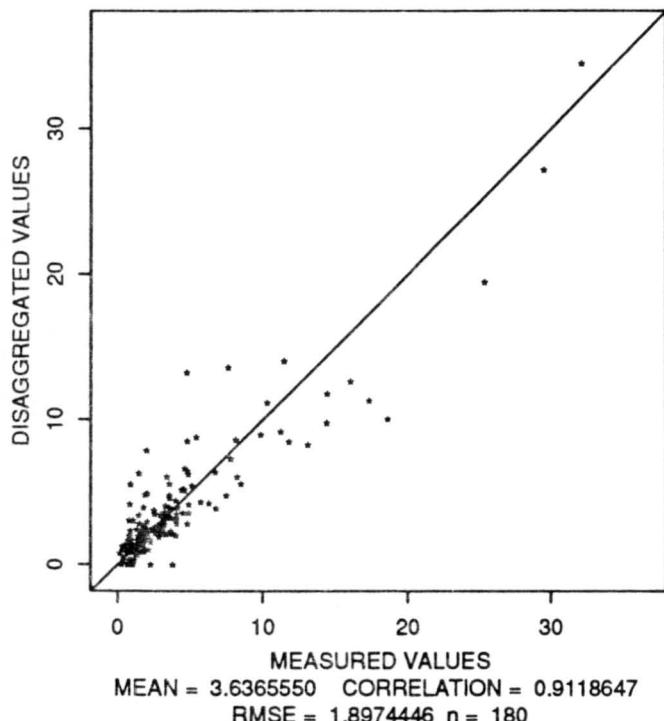
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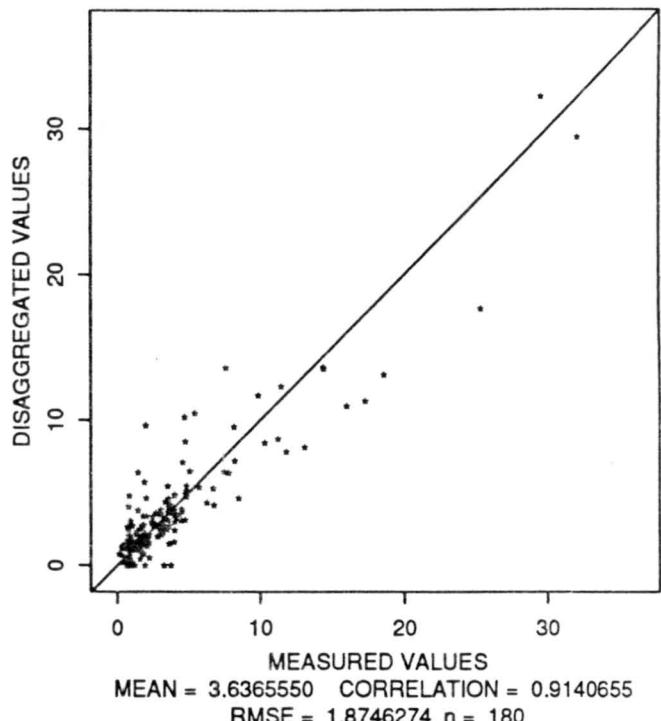
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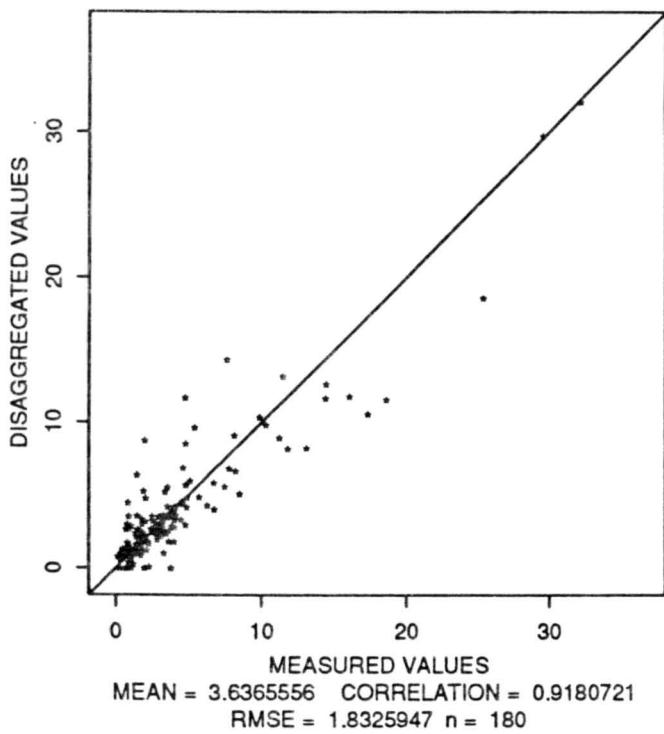
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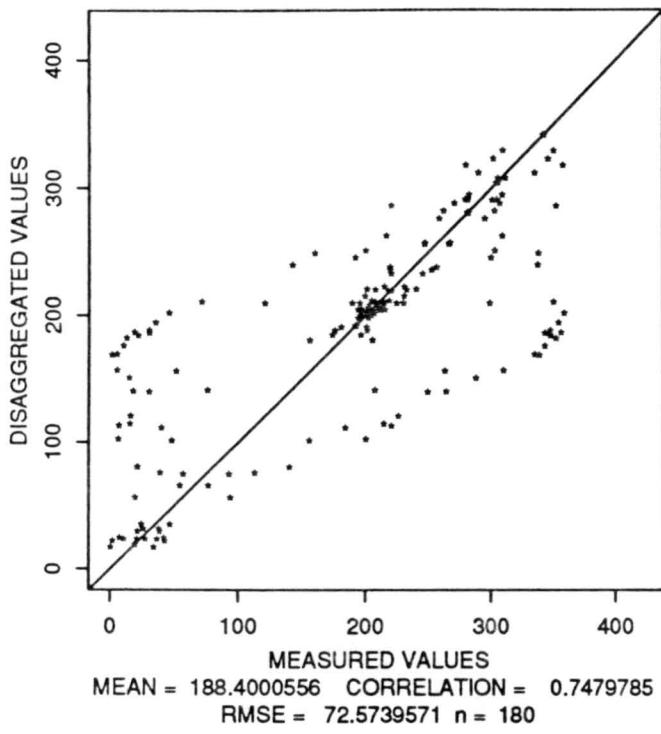
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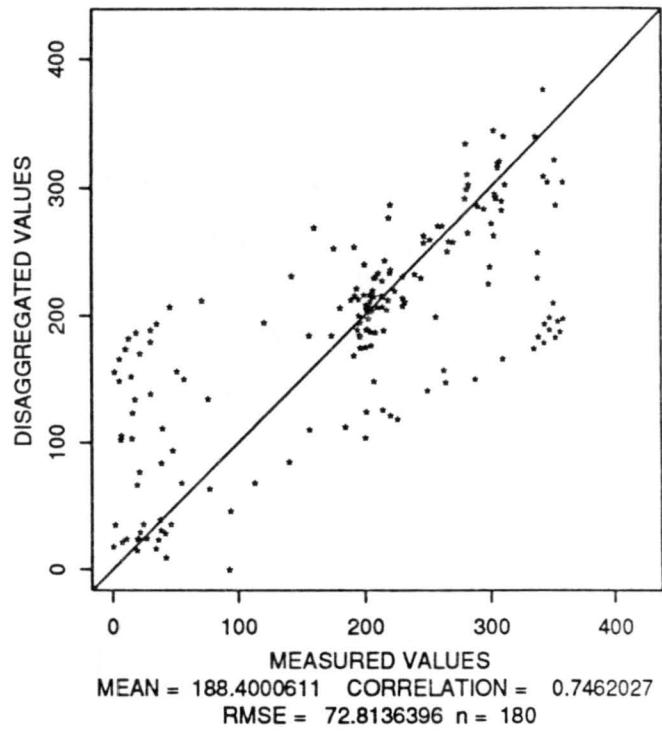
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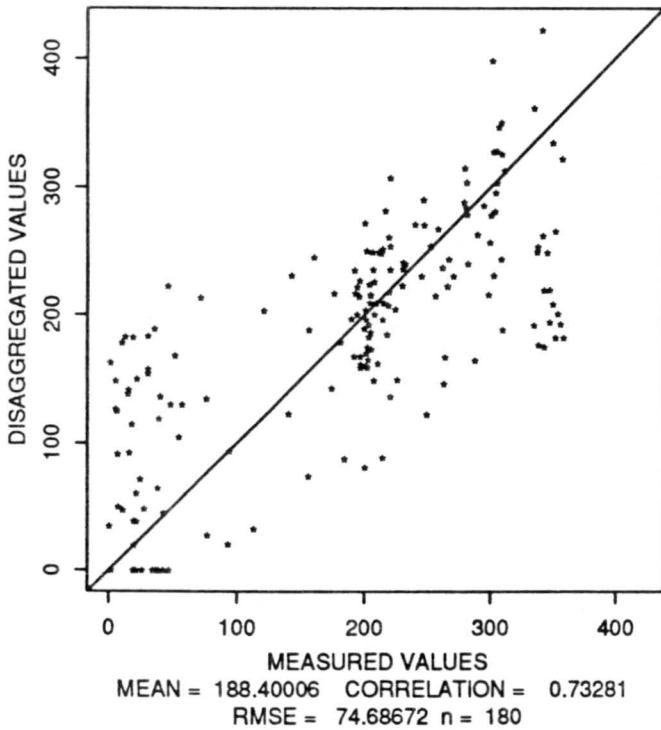
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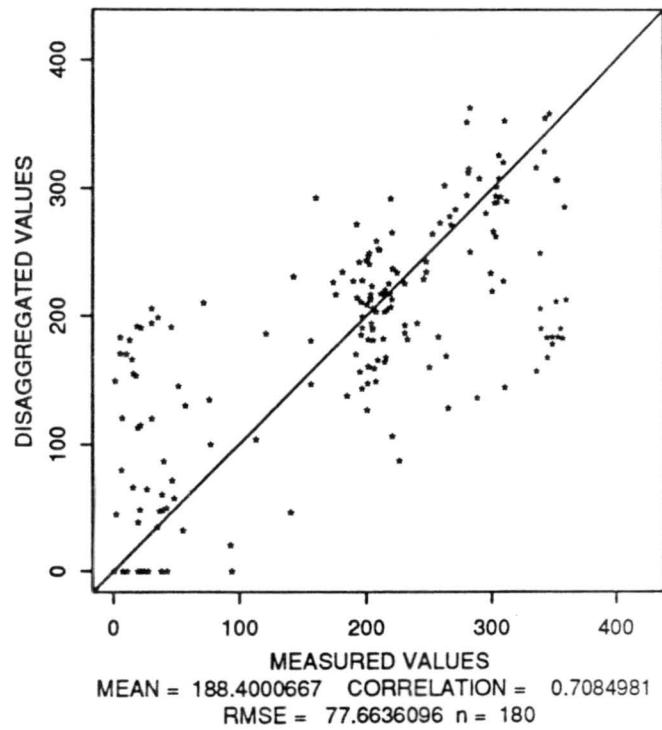
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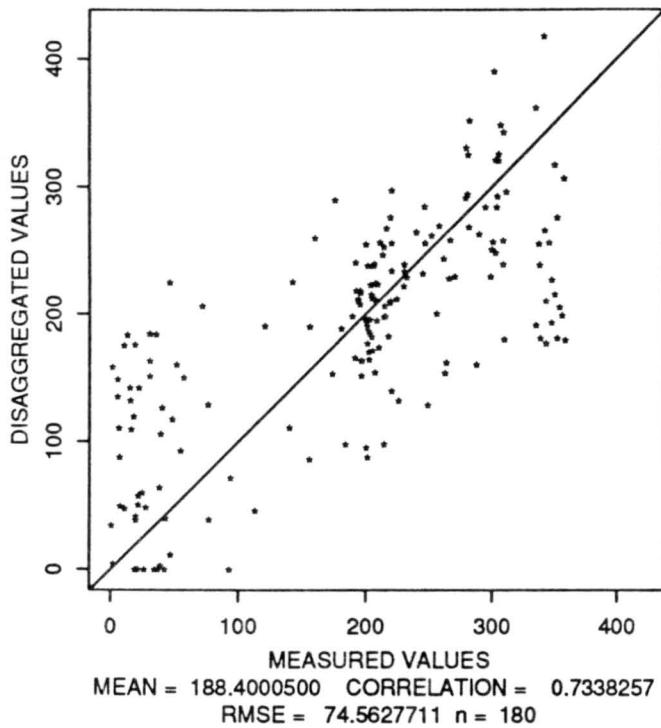
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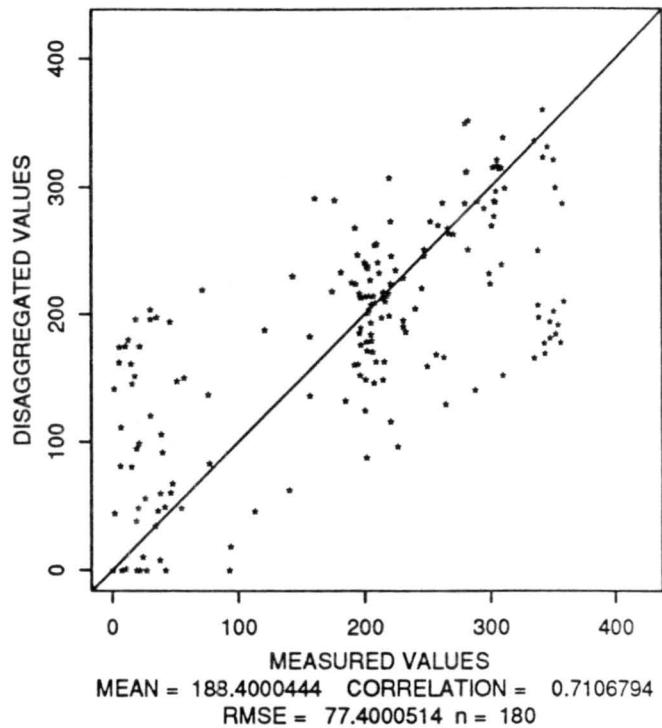
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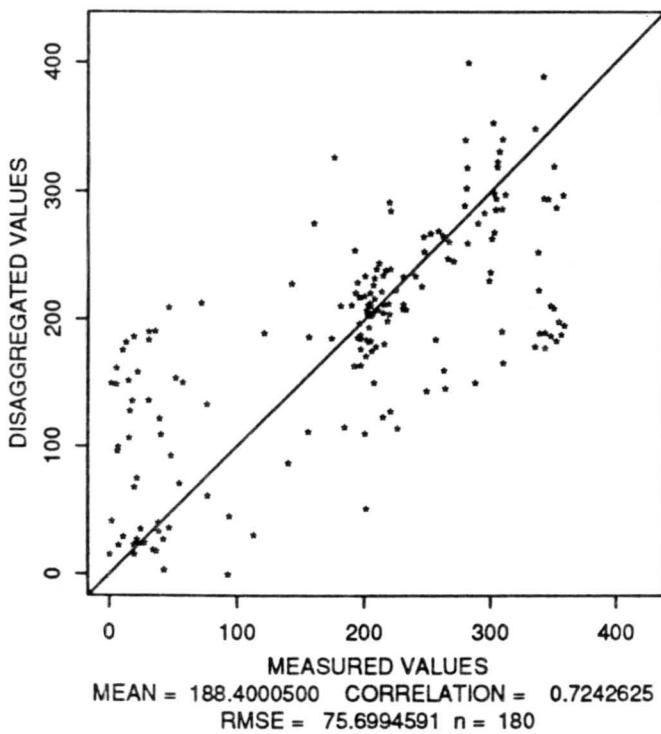
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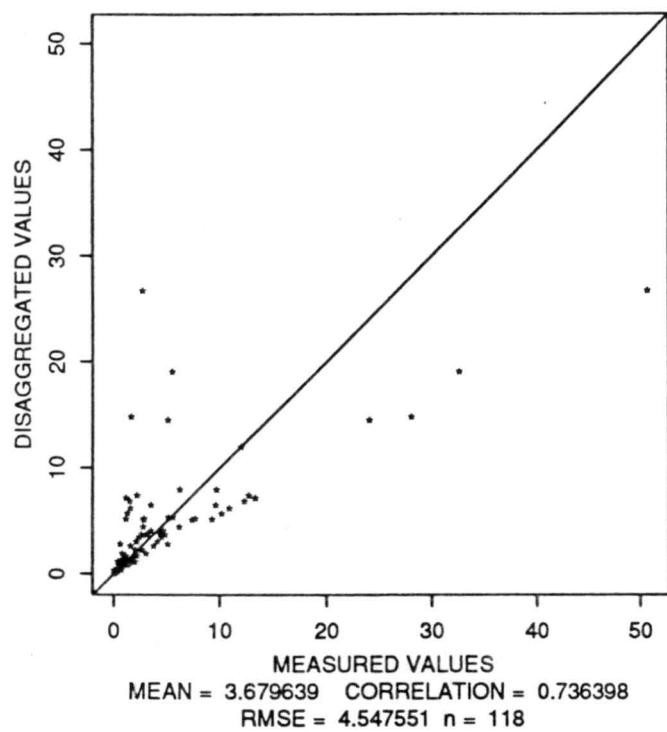
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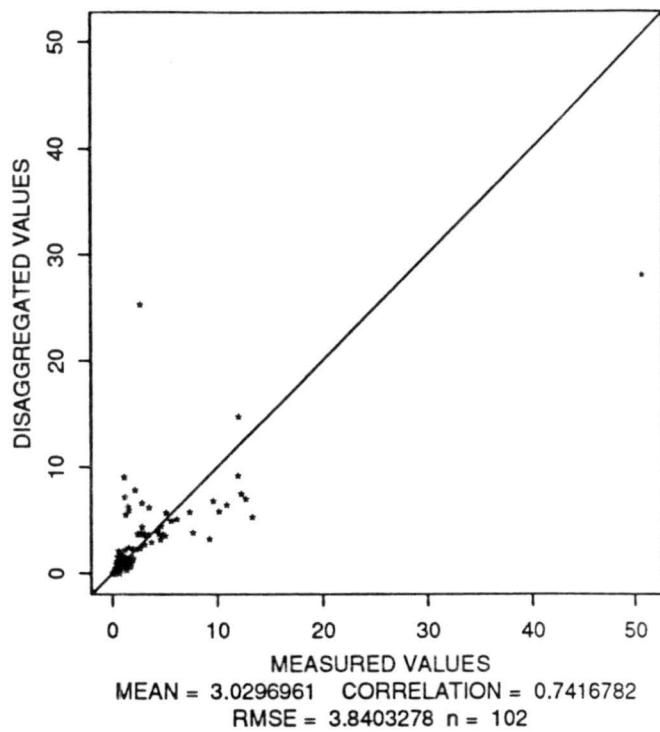
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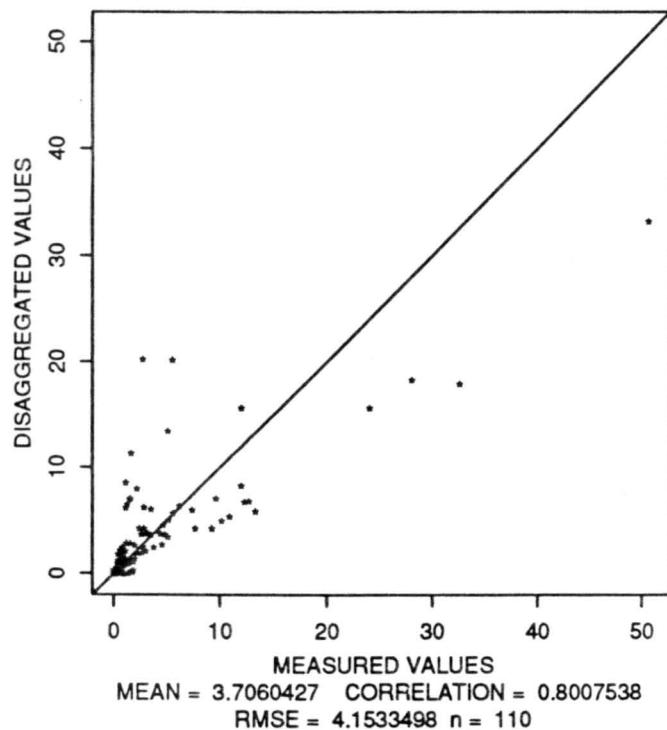
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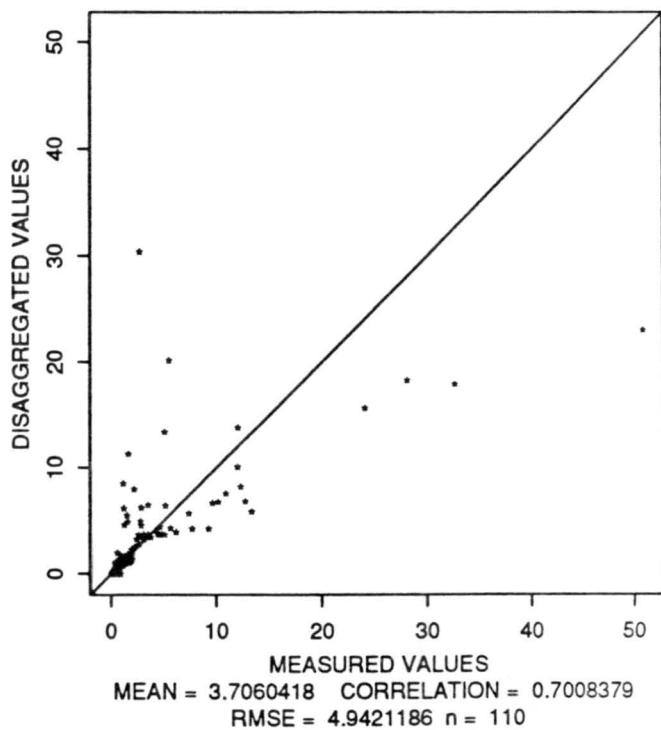
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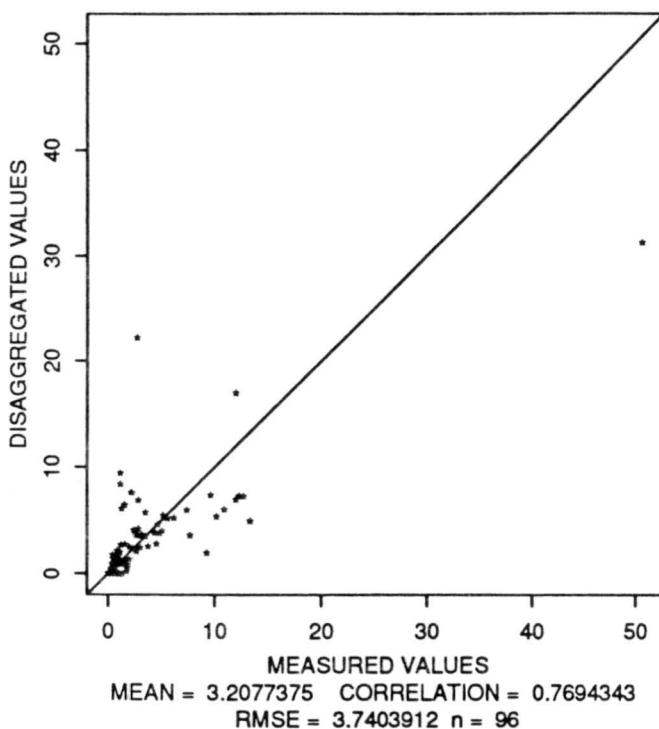
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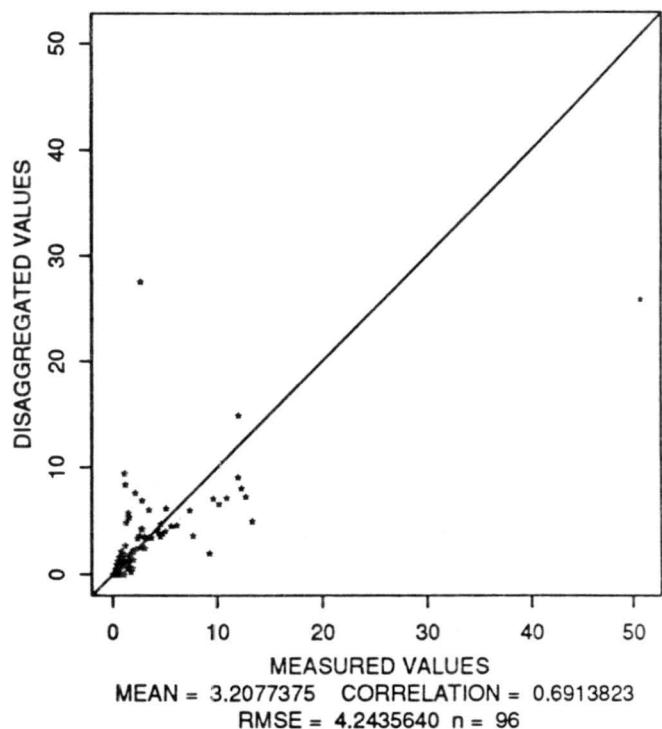
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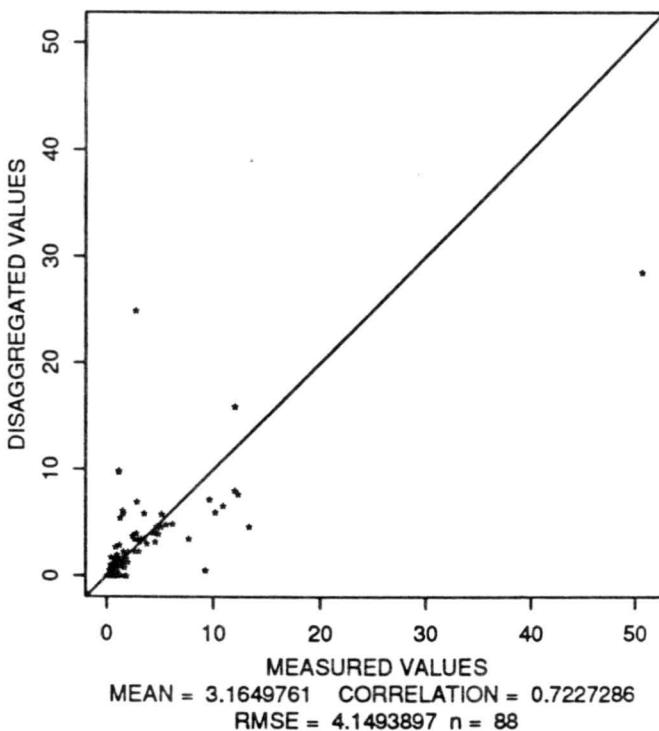
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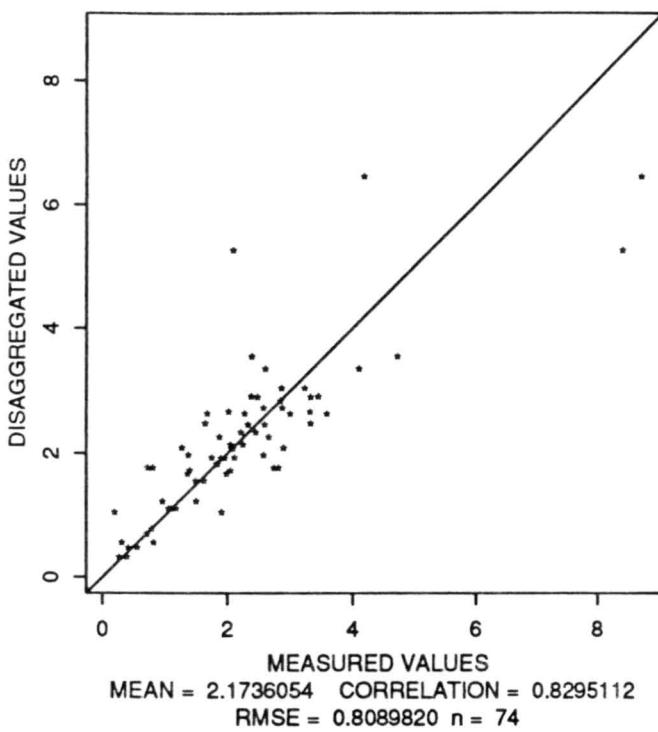
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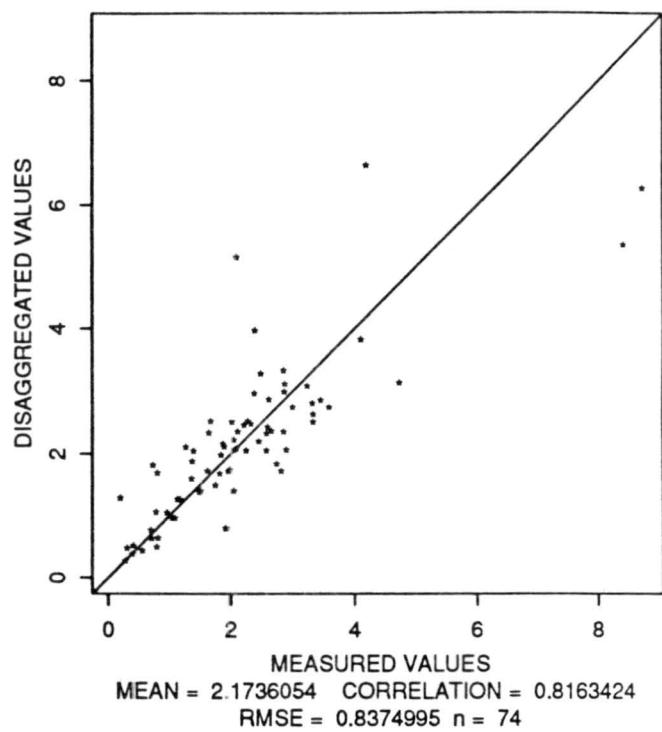
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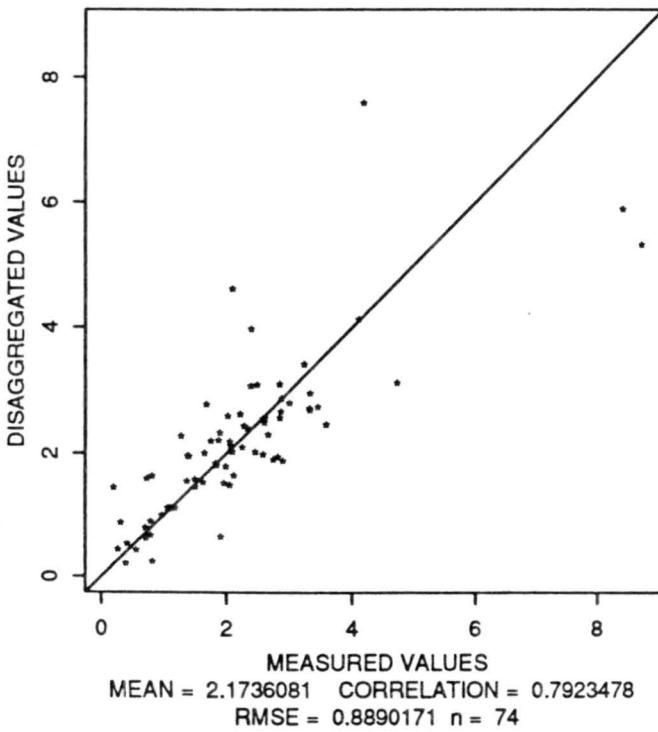
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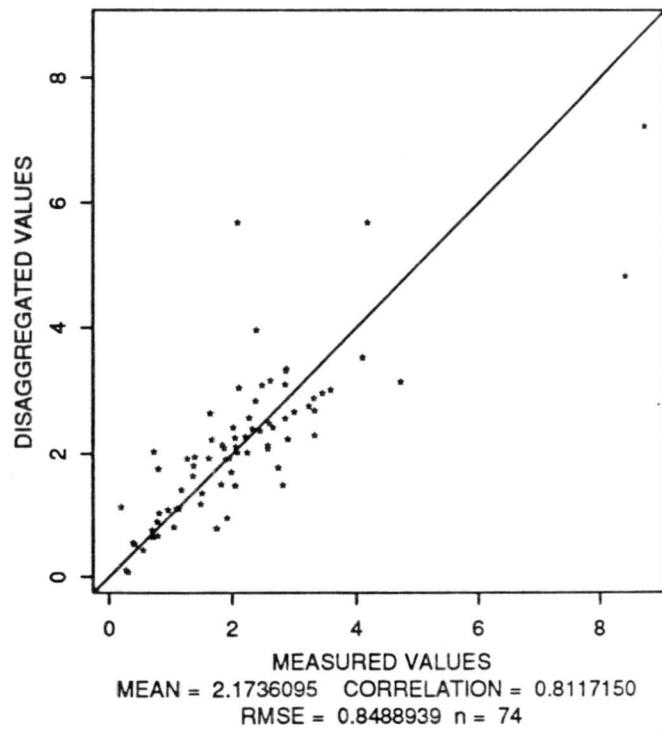
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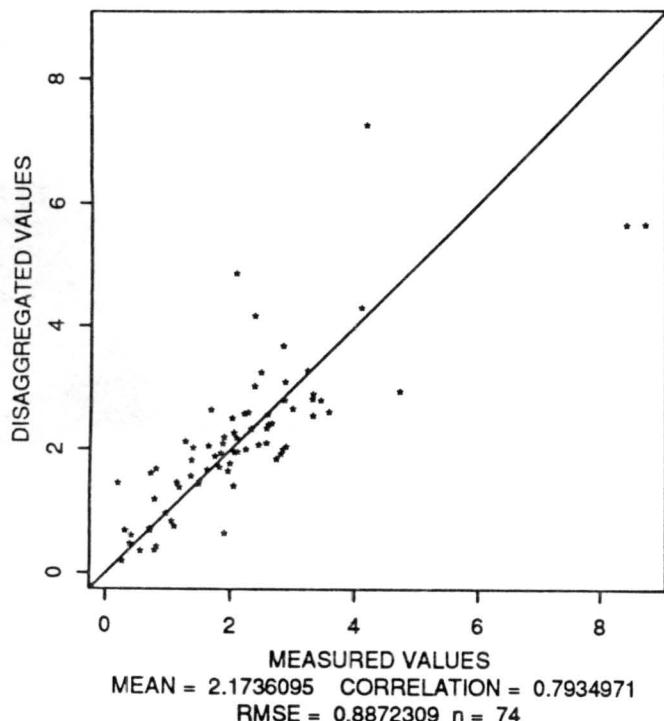
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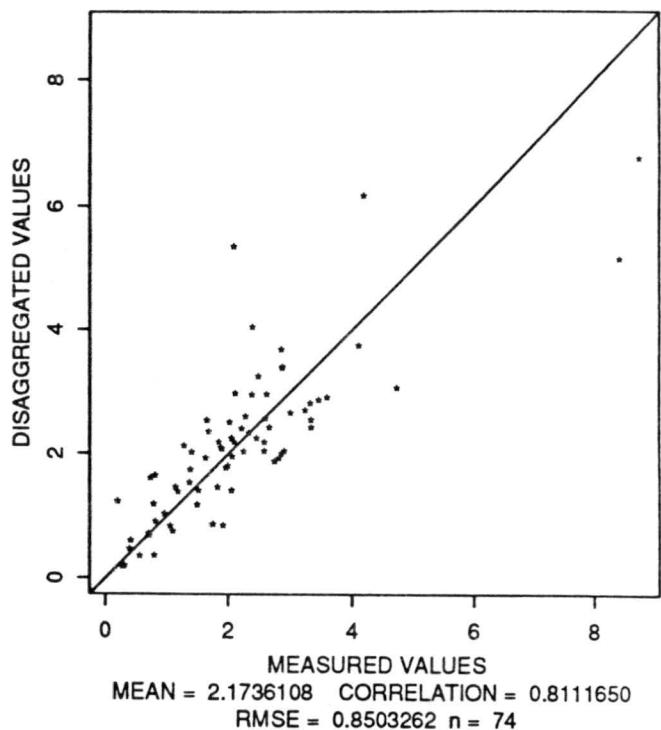
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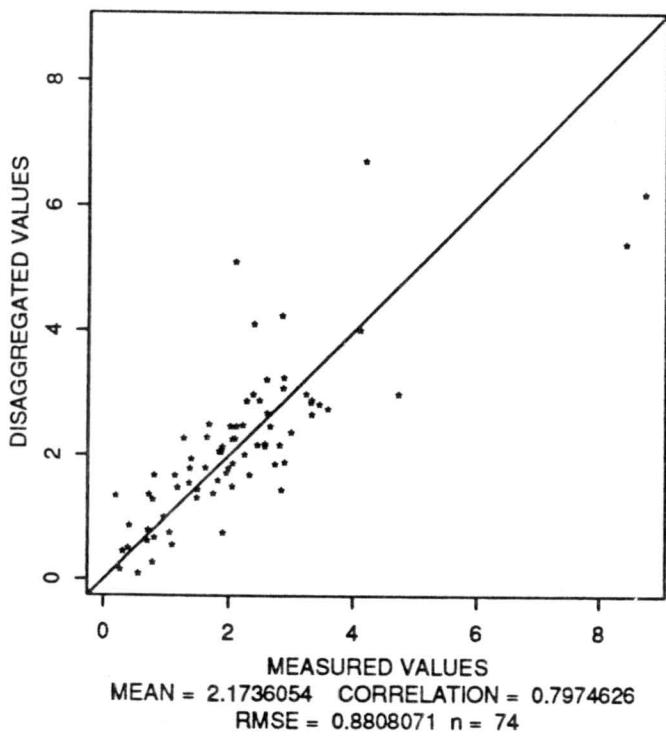
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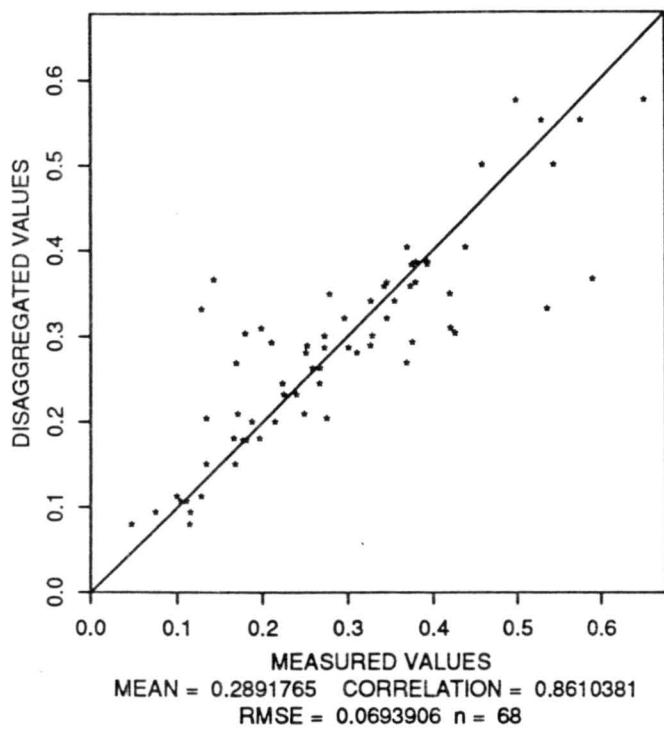
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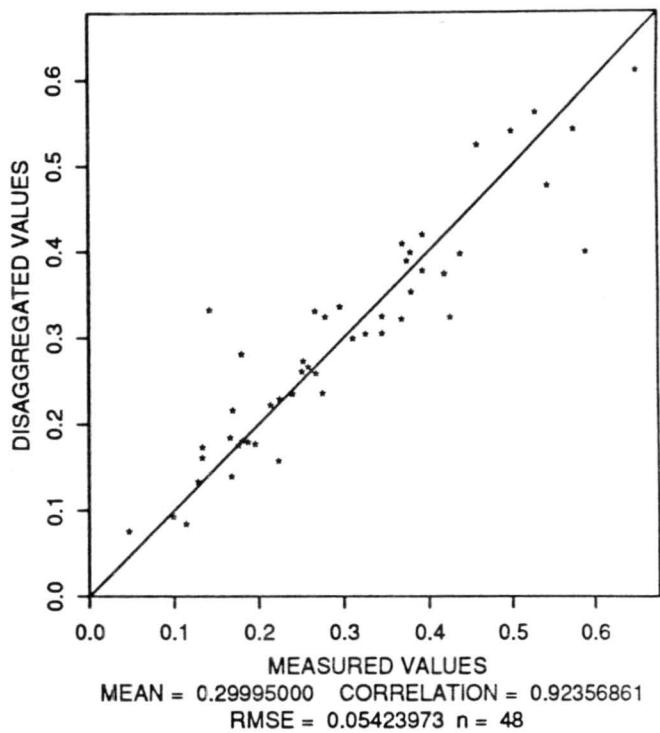
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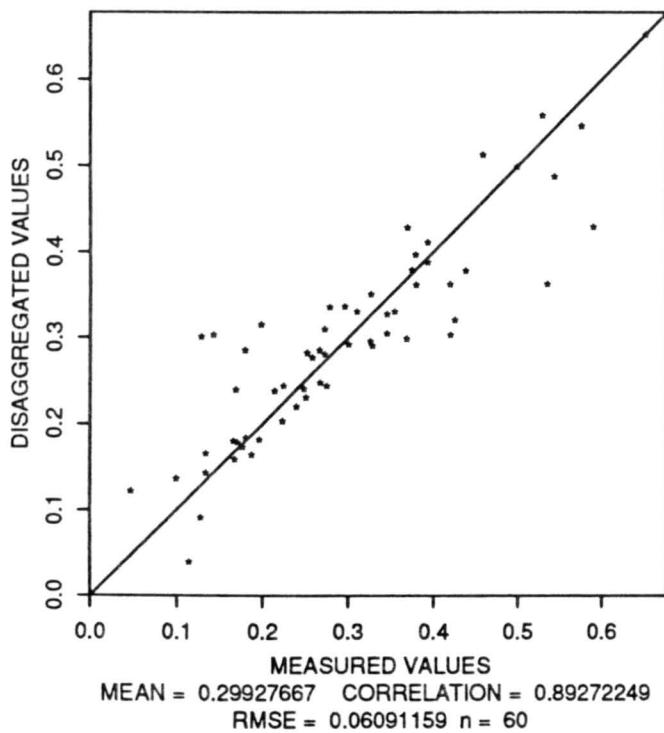
ORDER 0 DISAGGREGATION FOR NSOILC



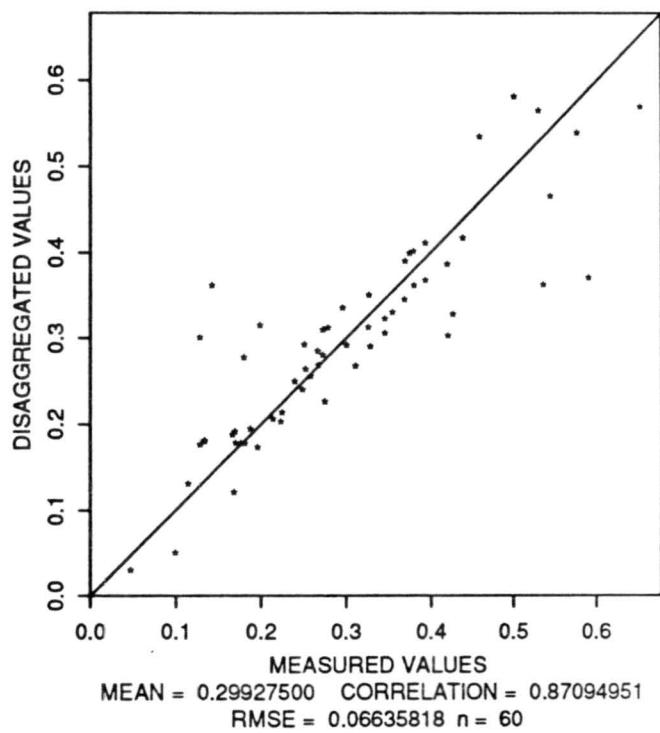
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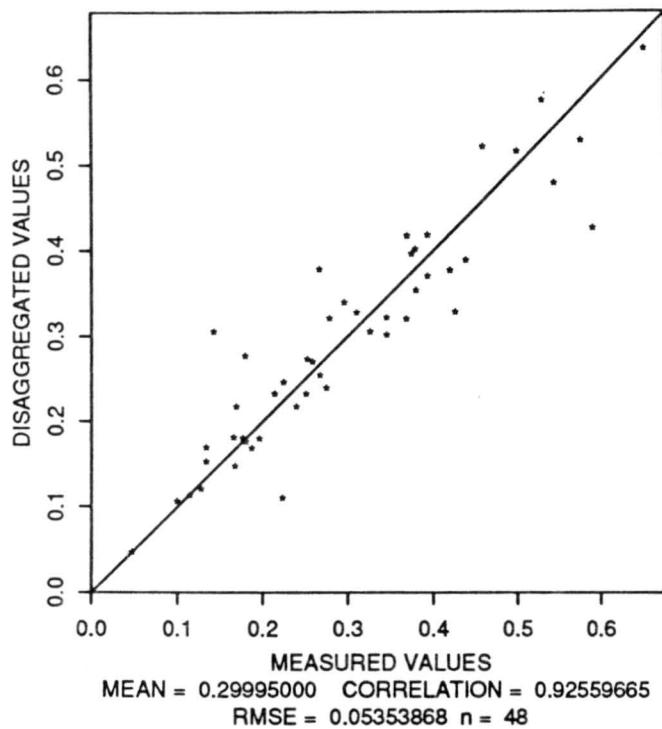
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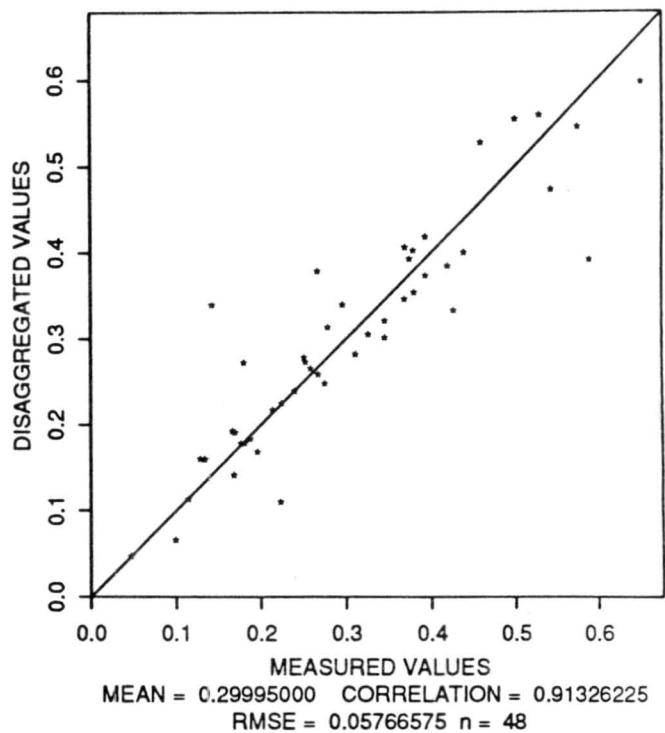
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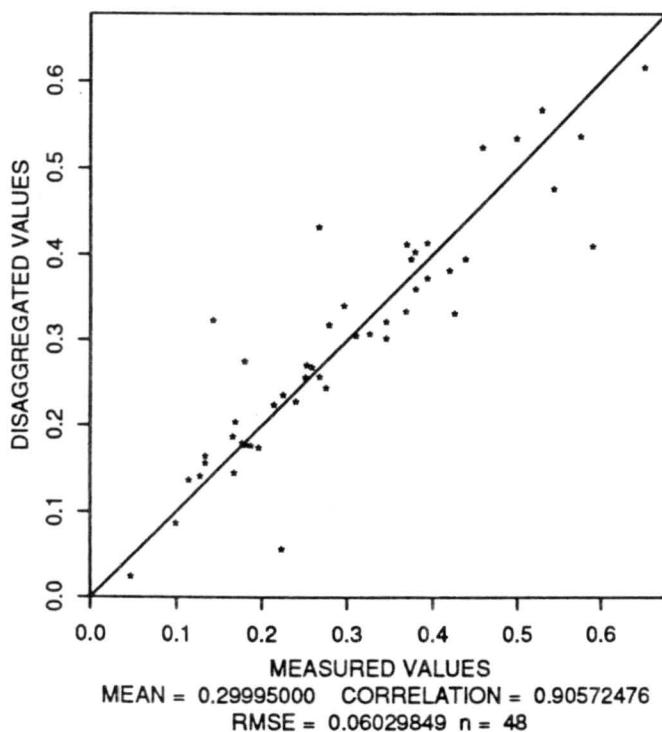
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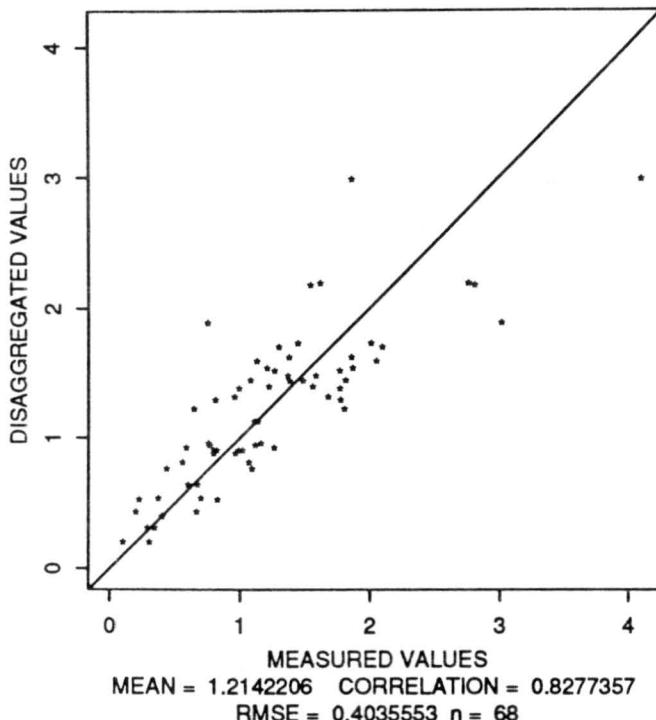
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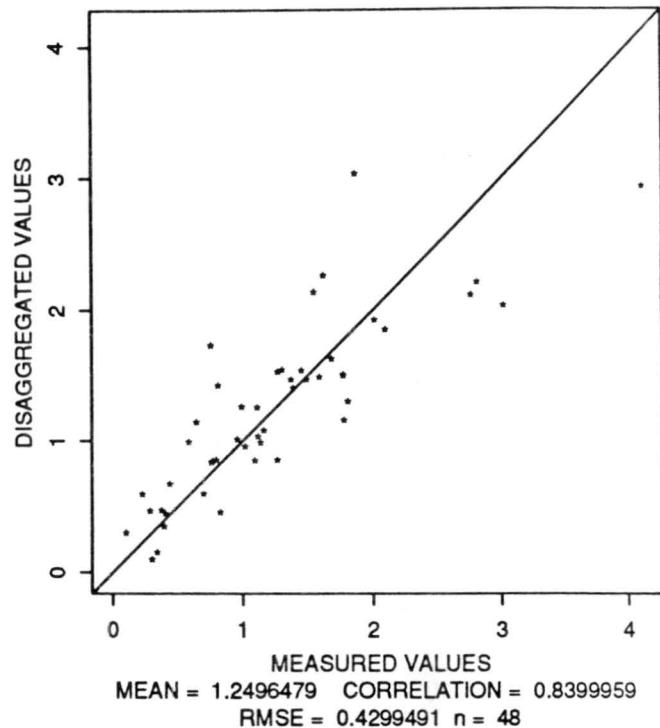
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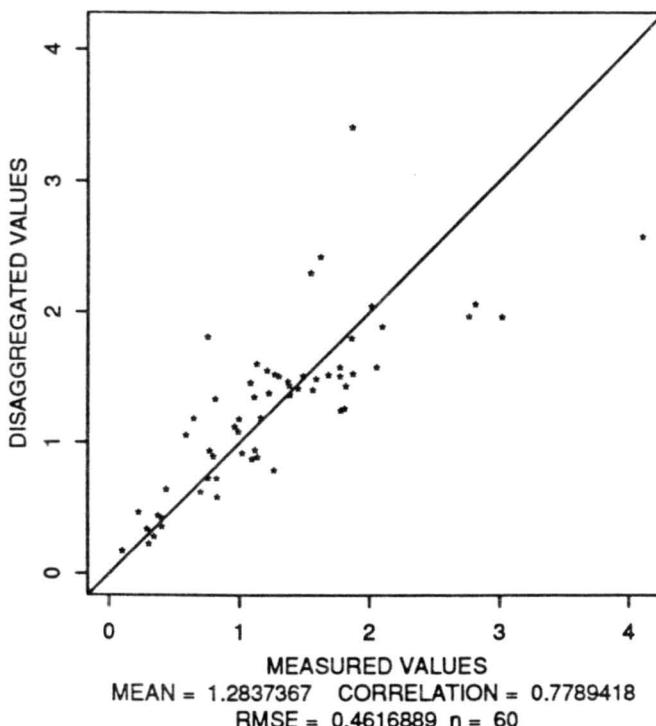
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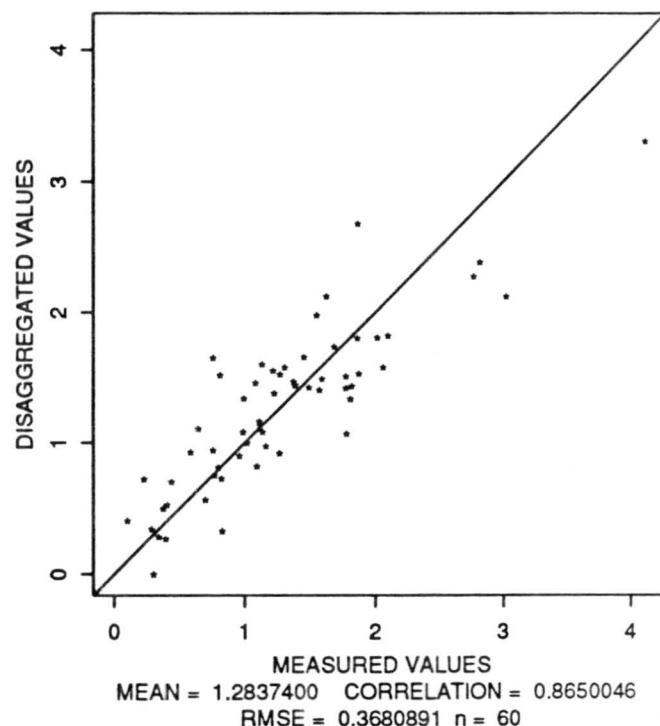
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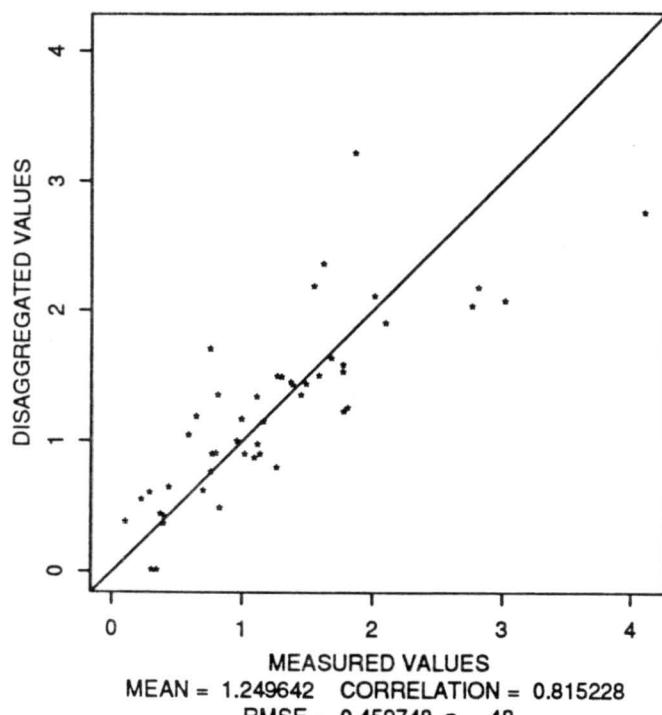
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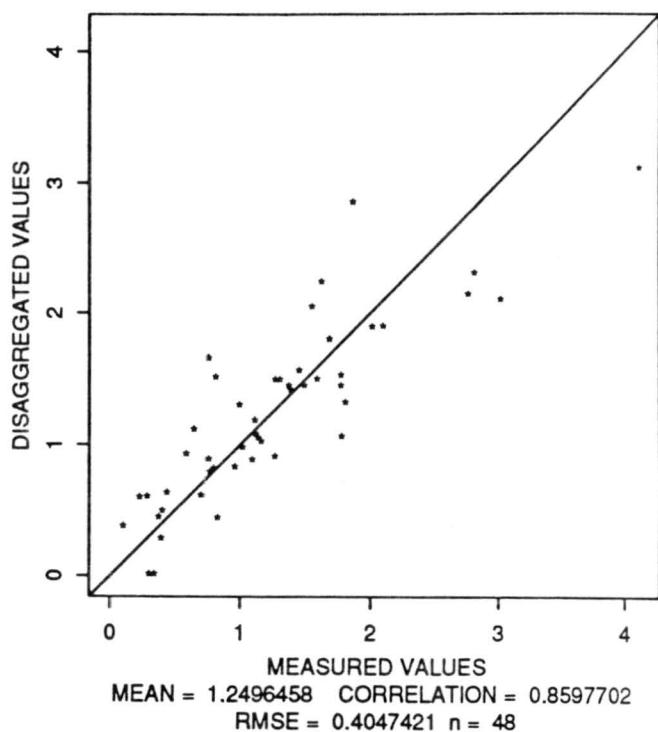
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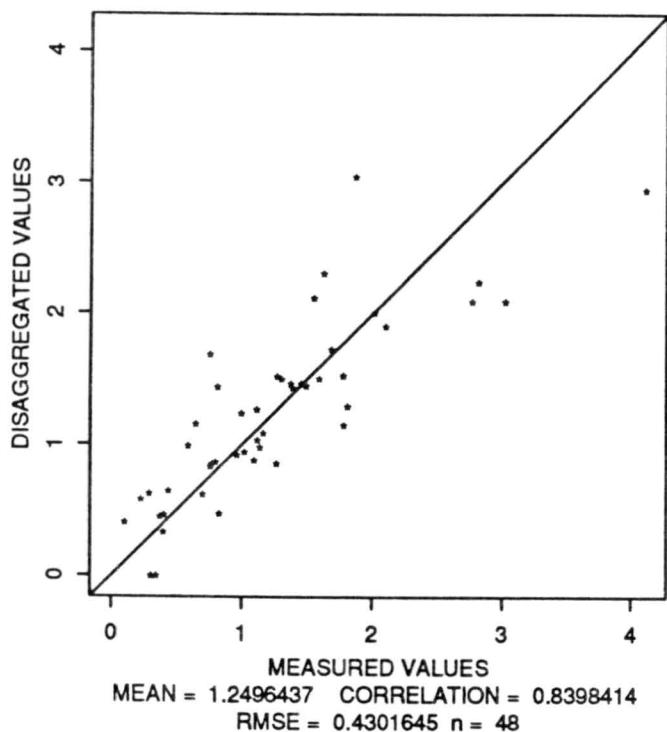
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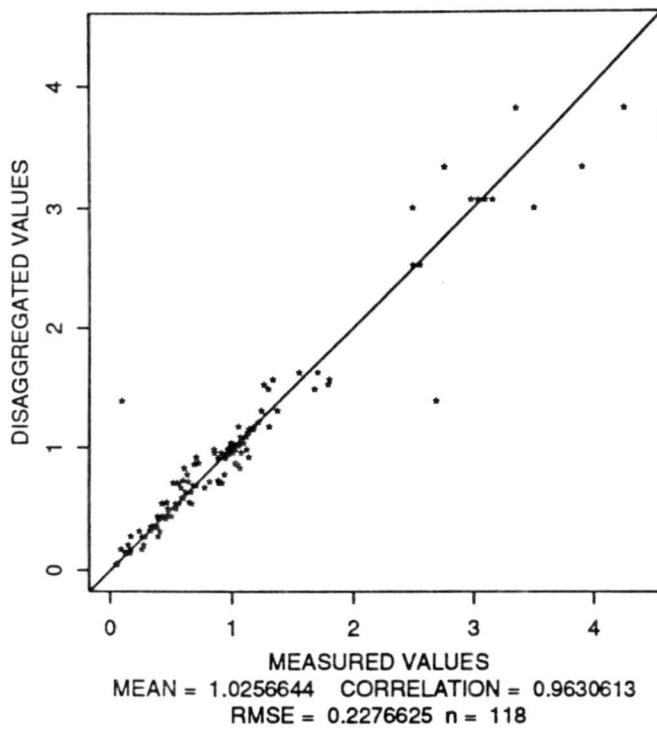
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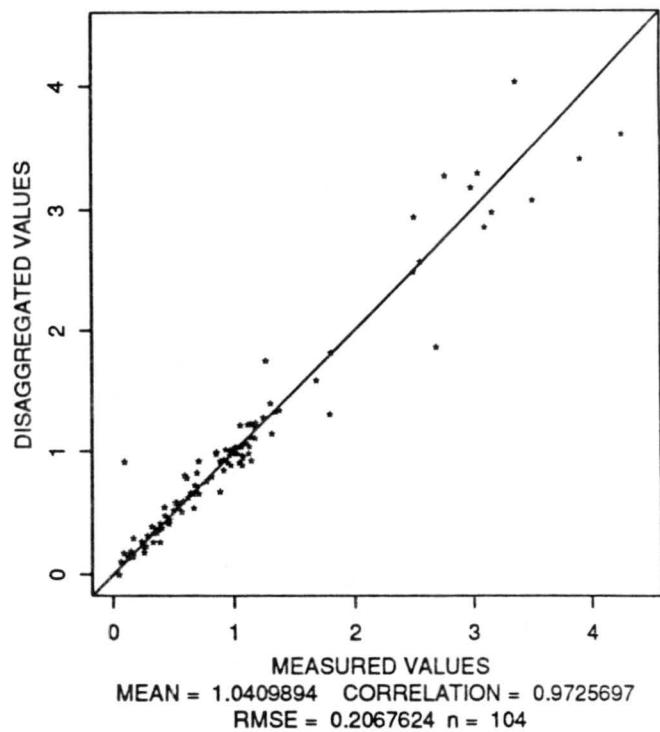
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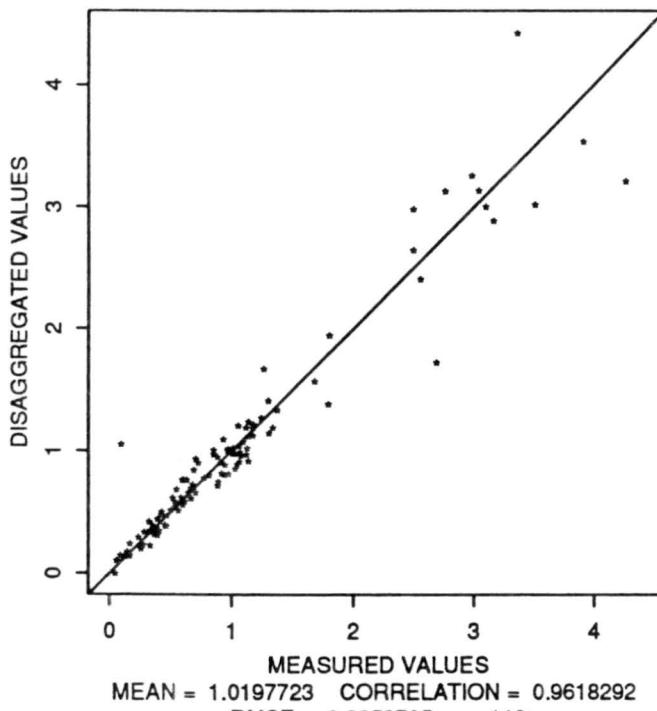
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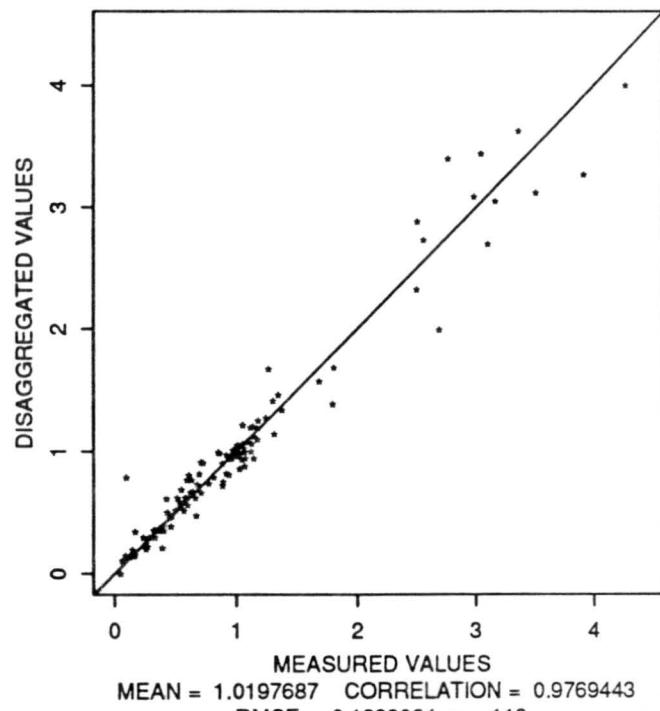
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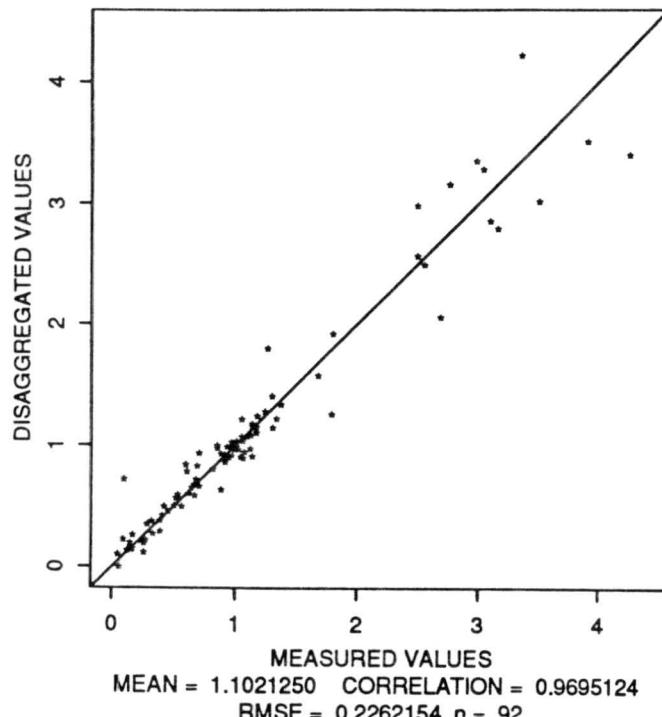
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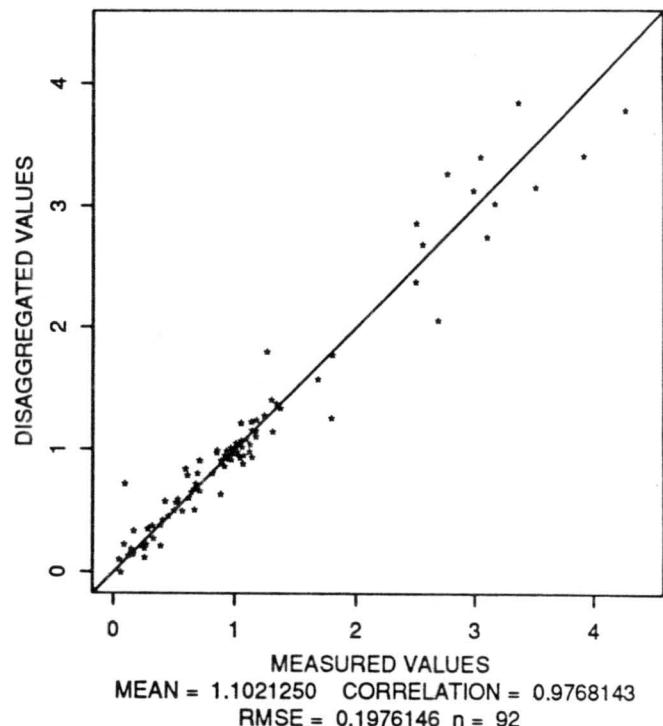
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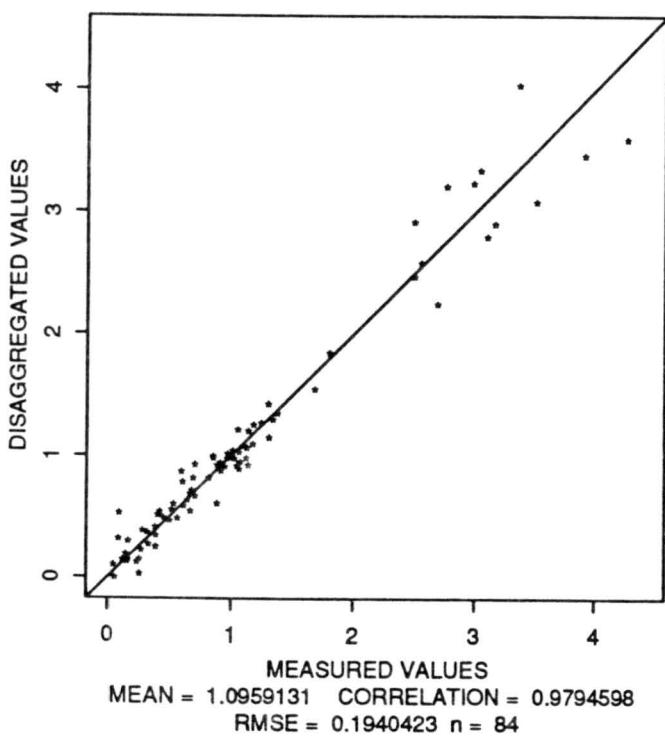
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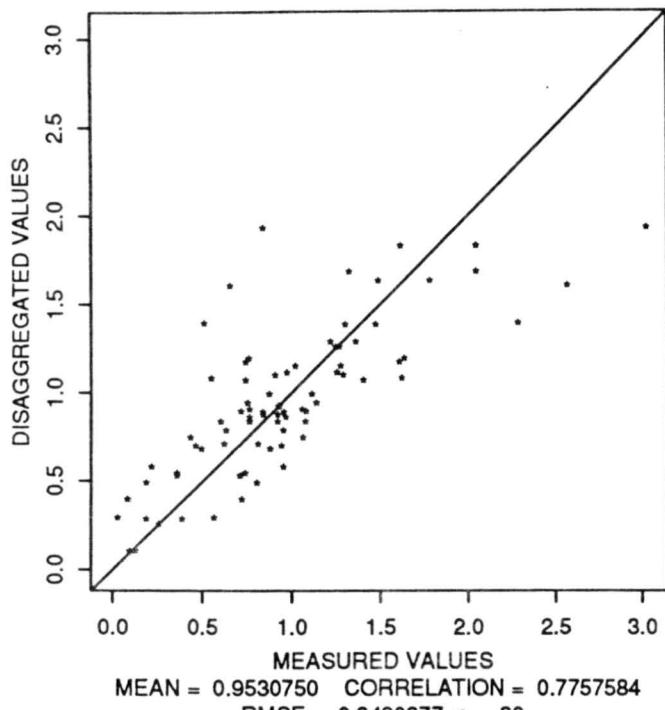
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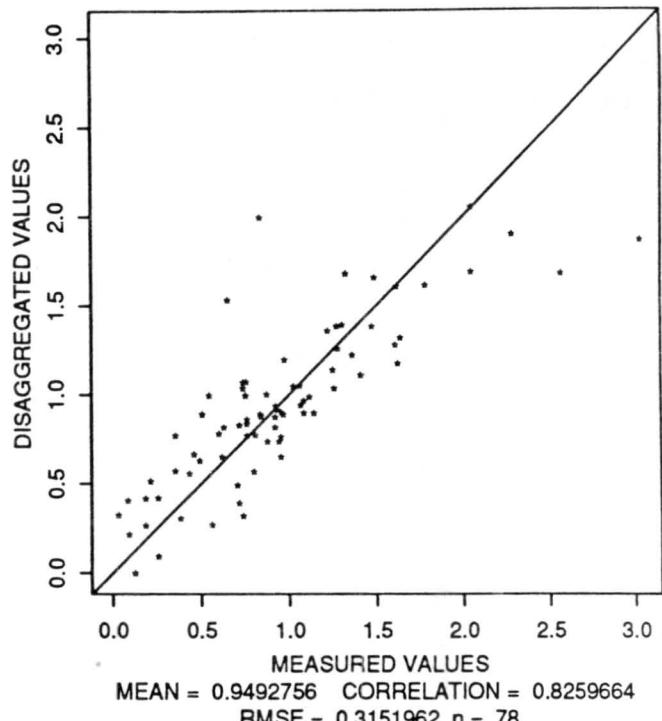
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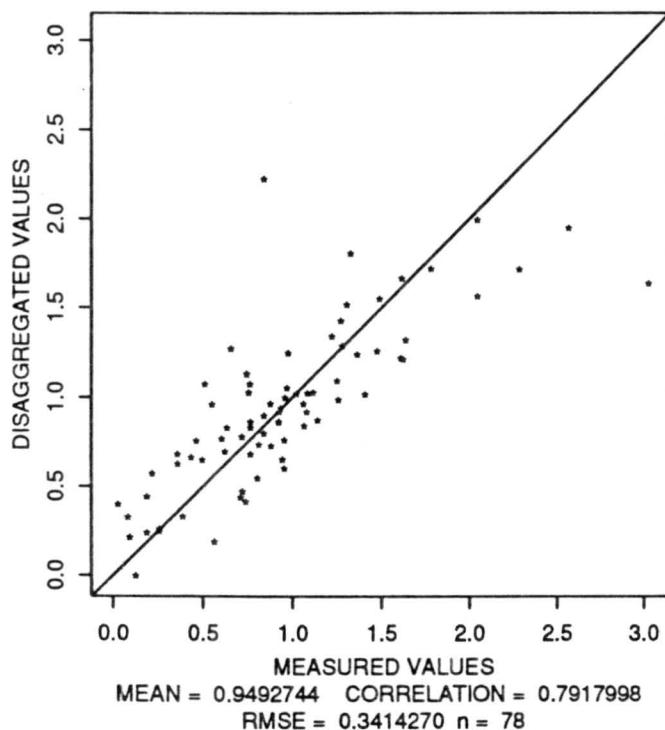
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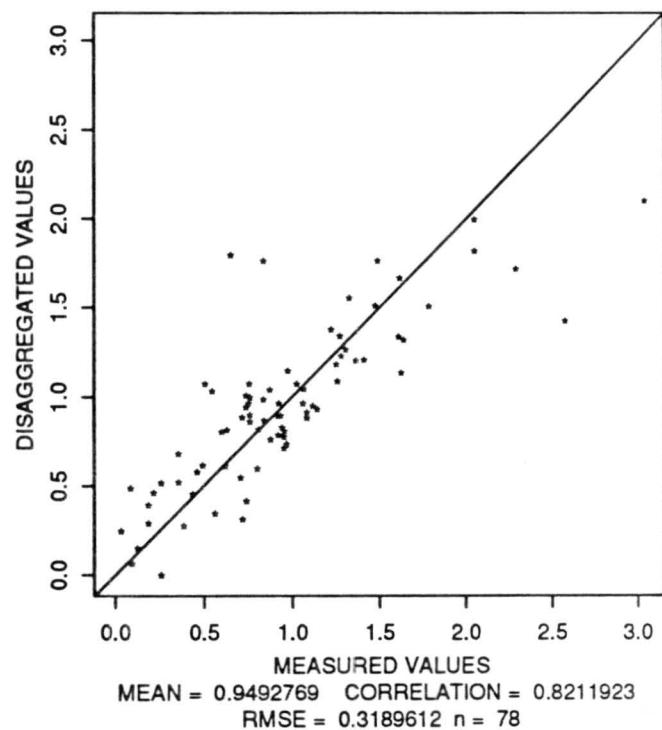
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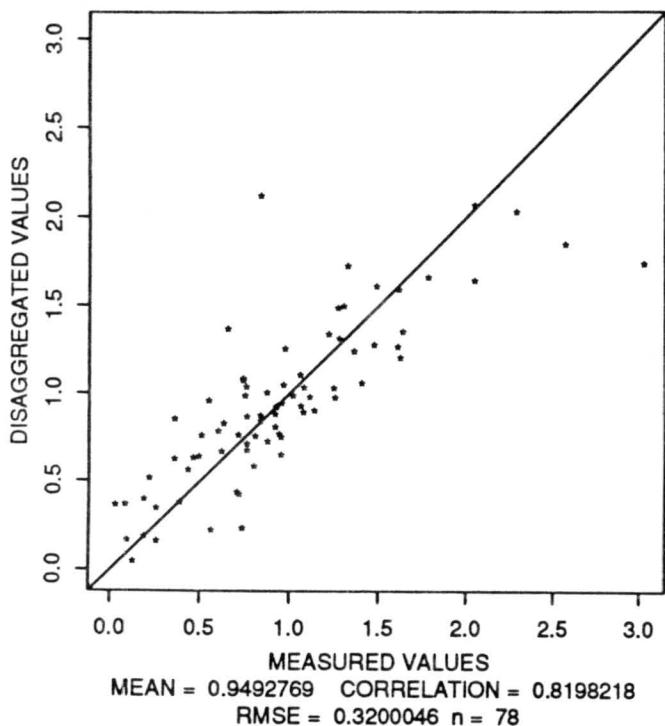
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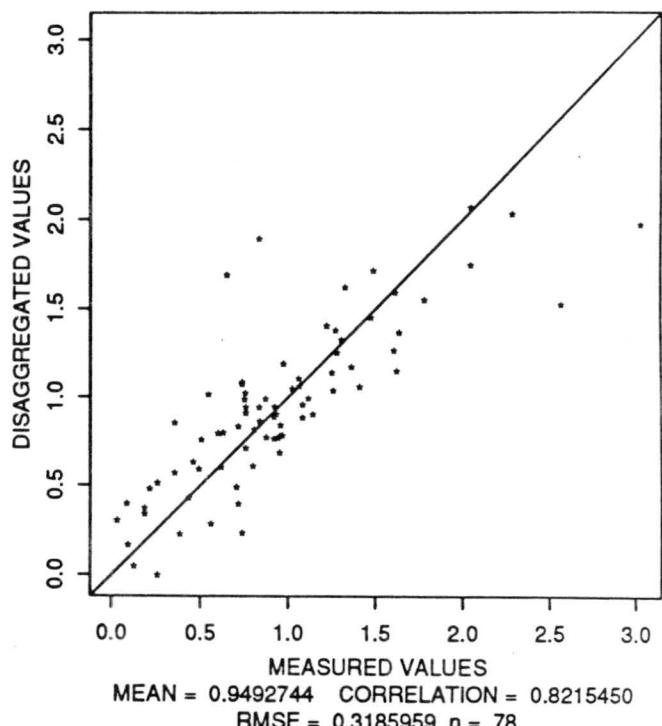
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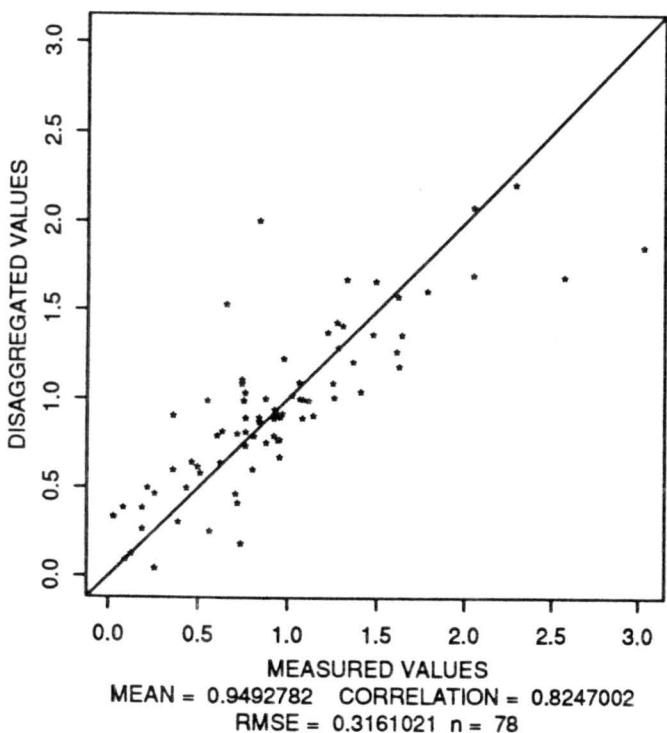
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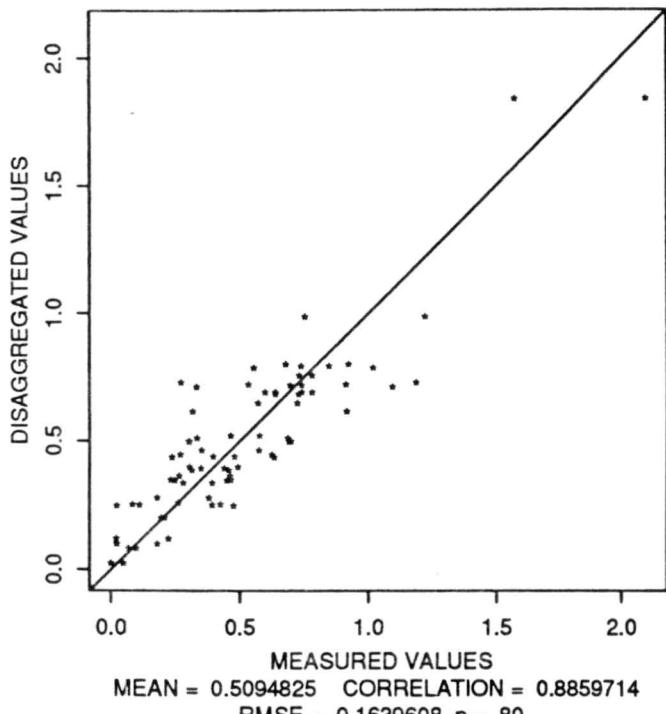
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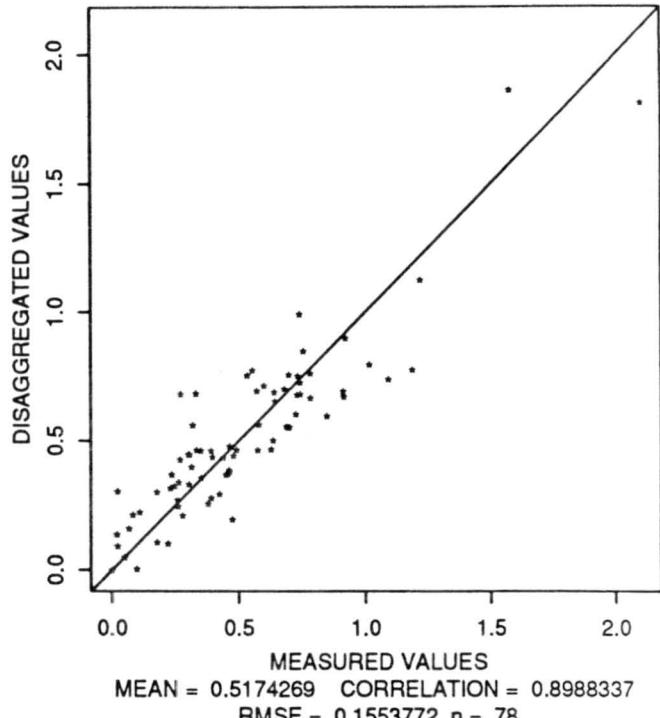
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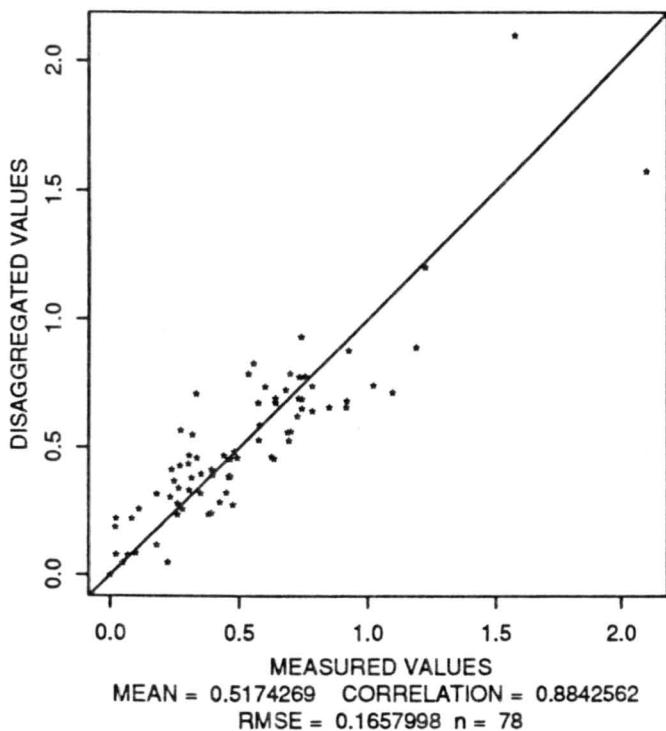
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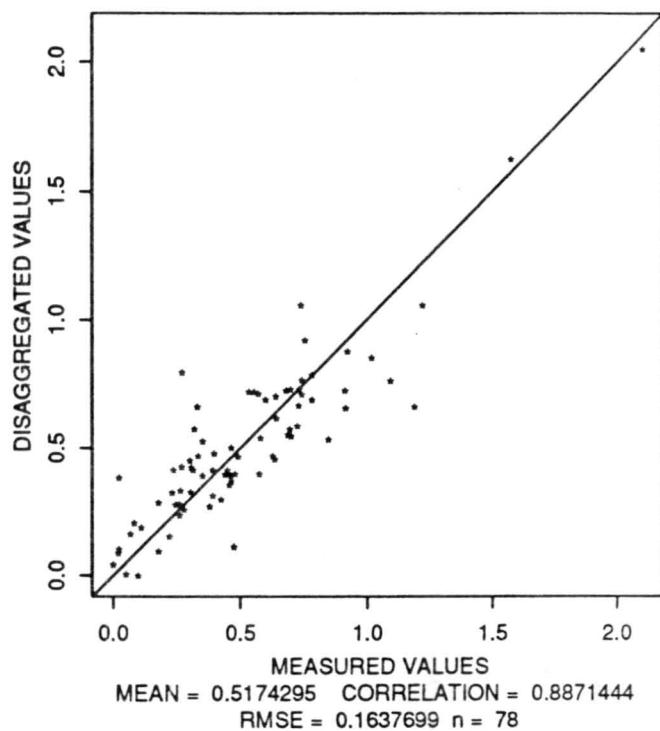
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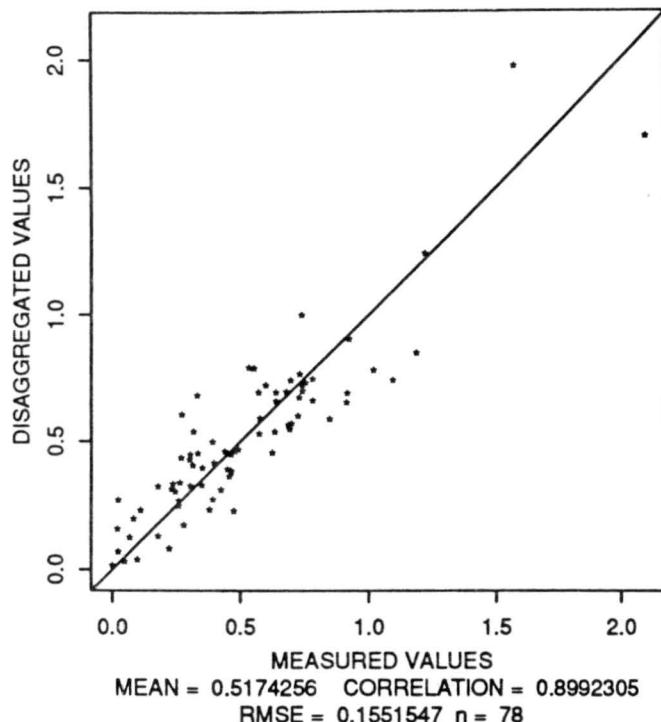
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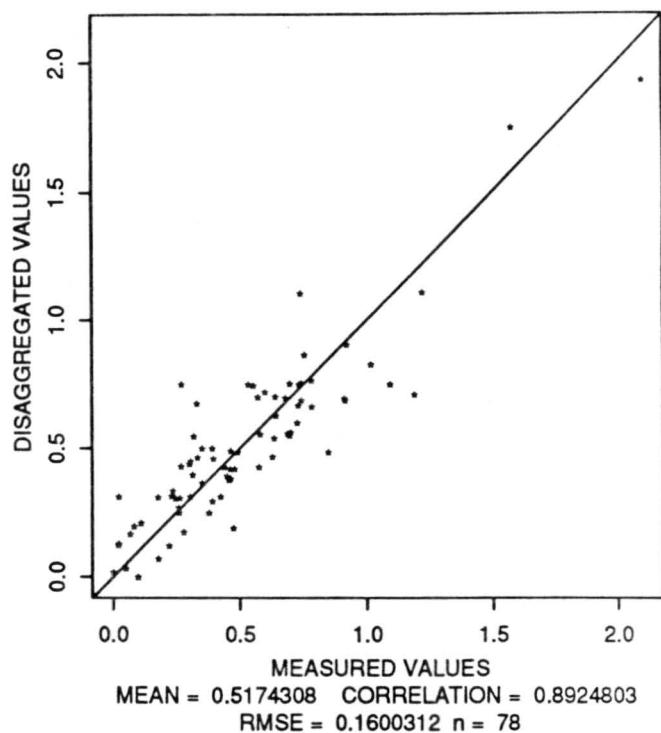
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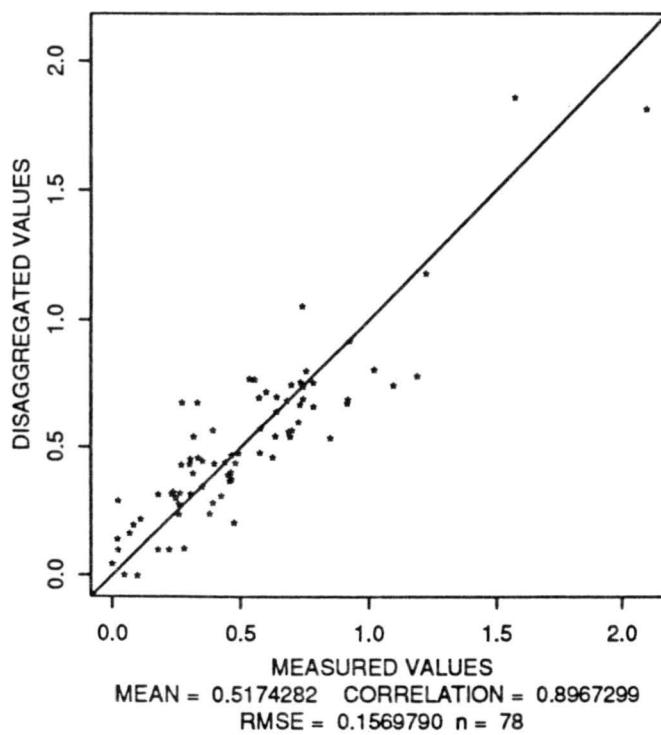
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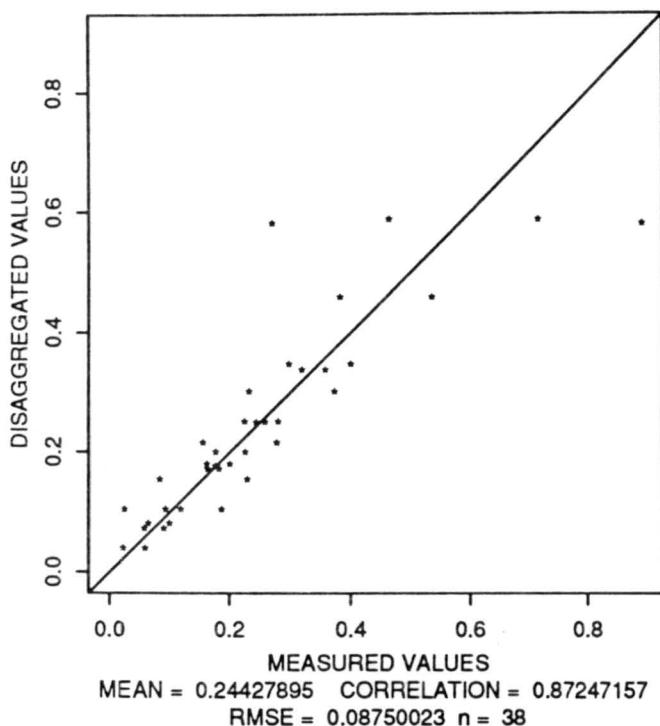
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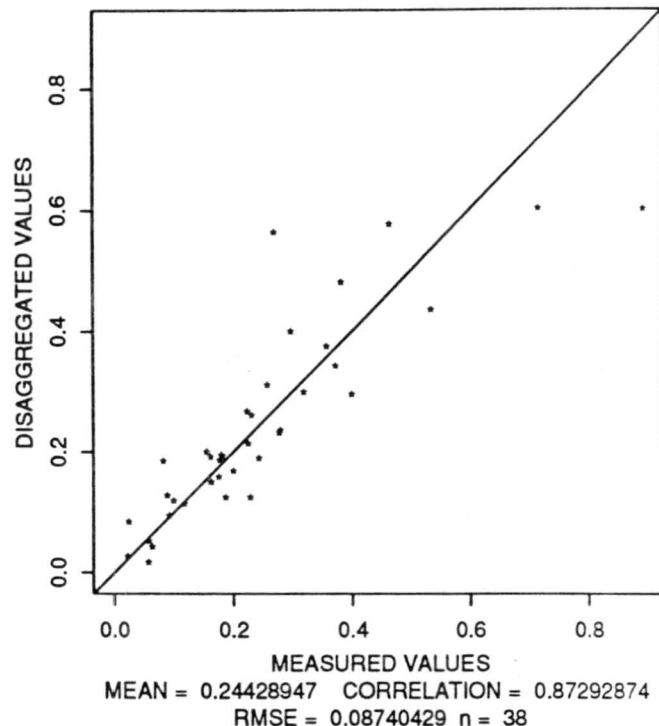
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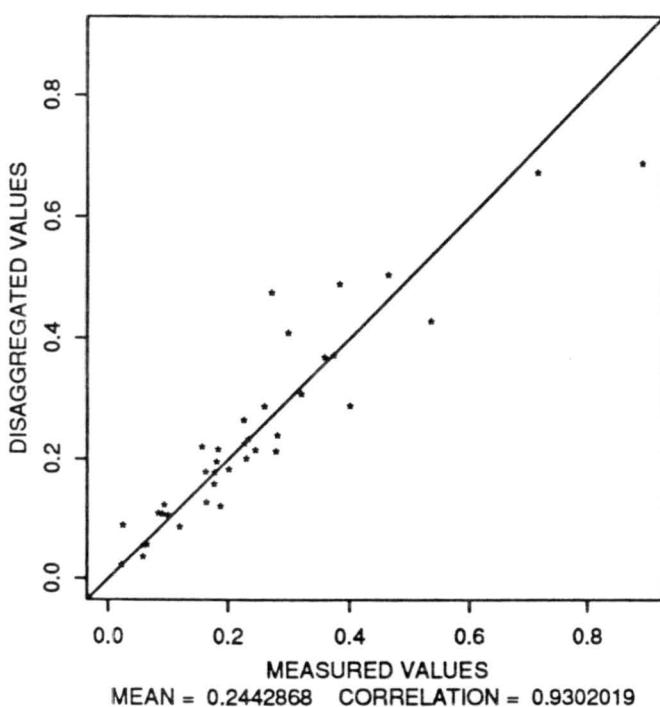
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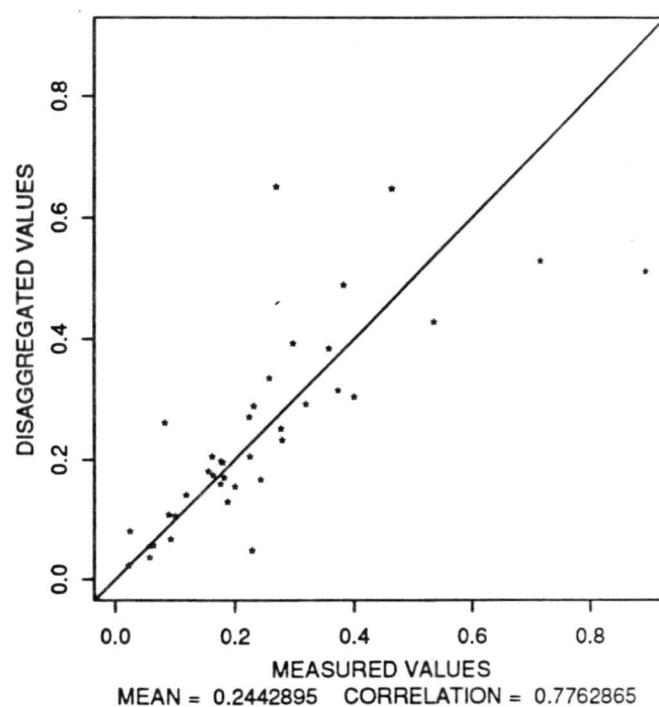
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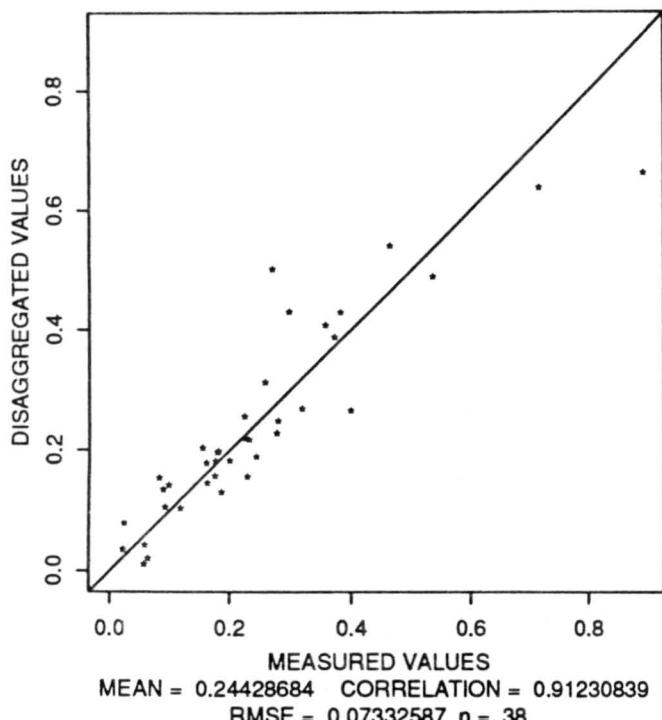
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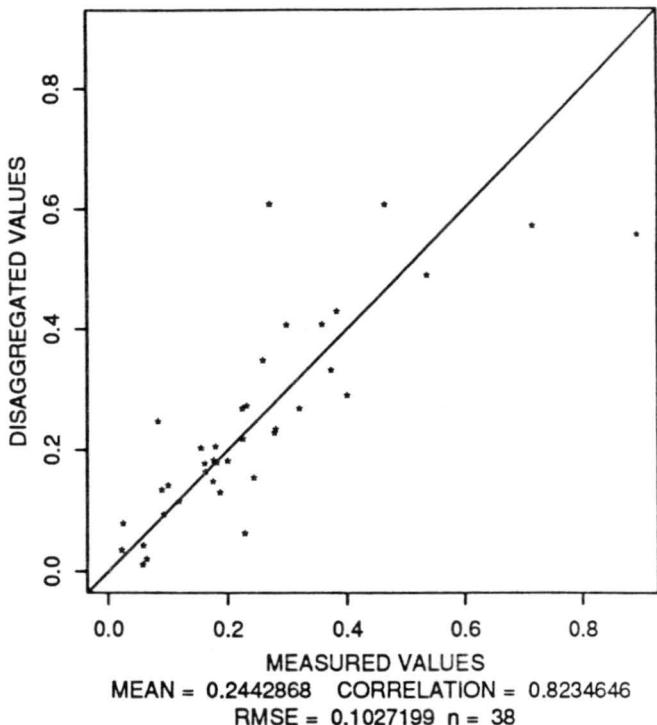
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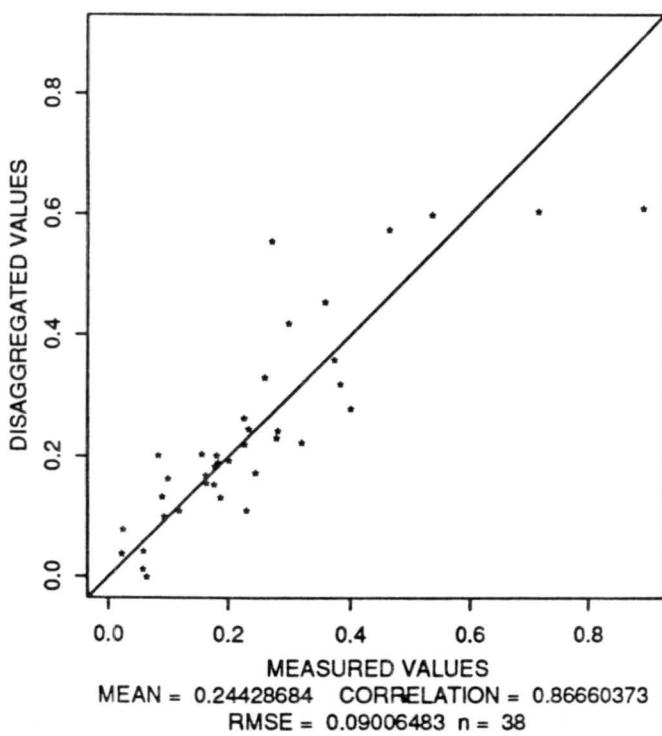
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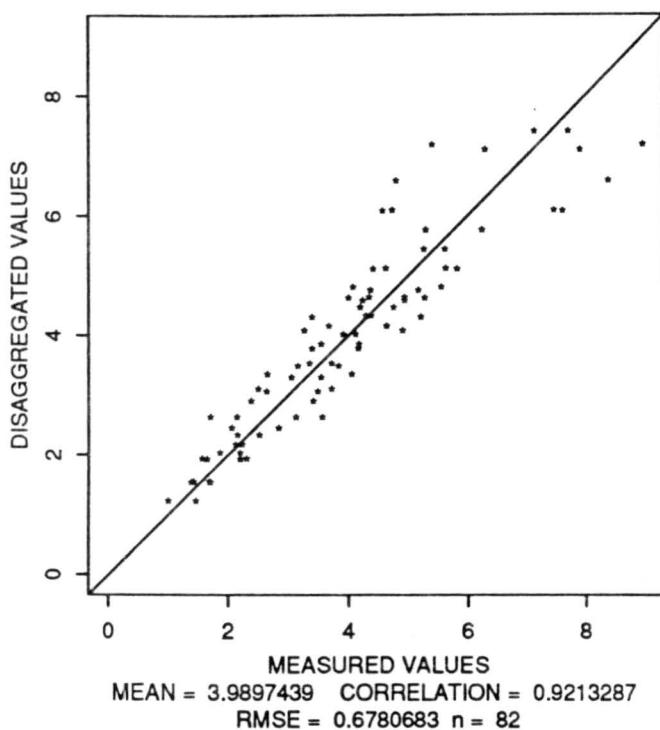
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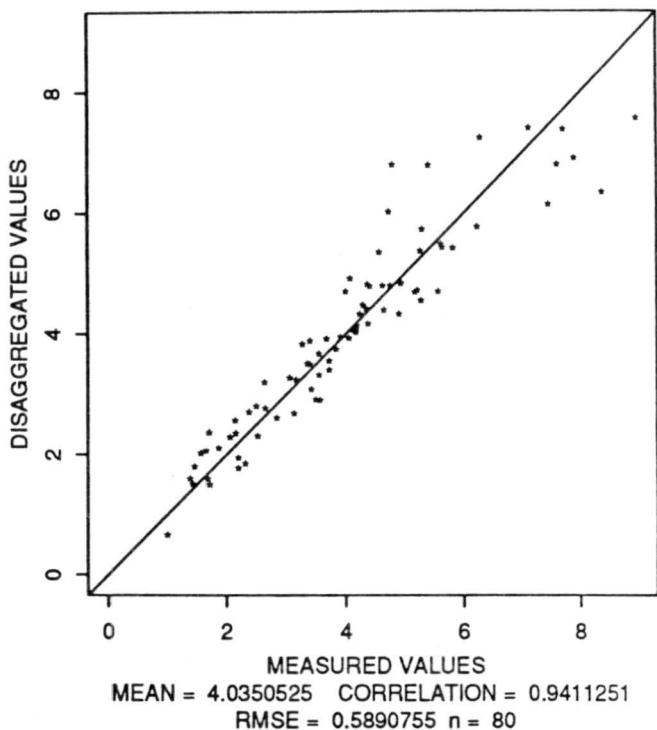
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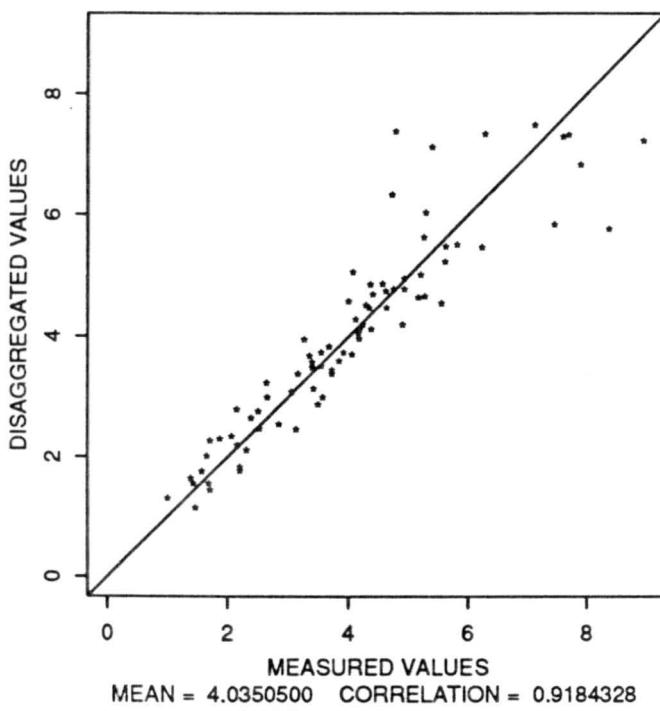
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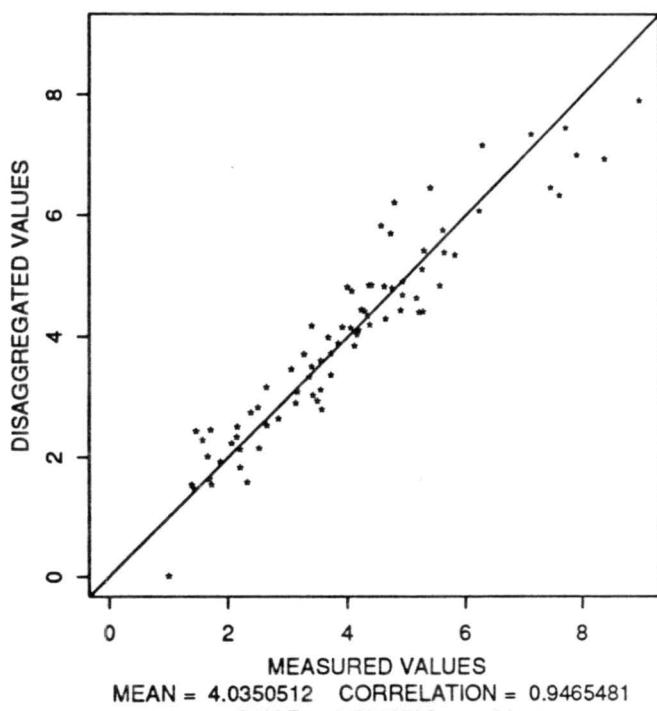
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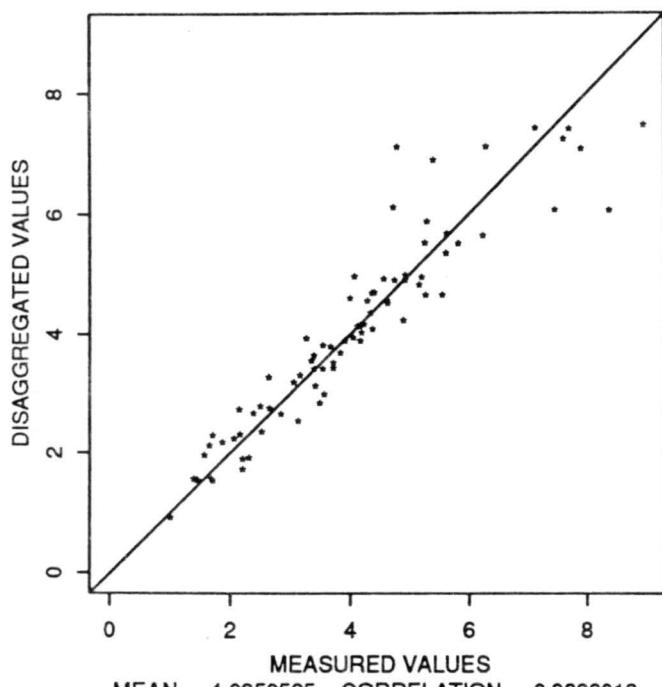
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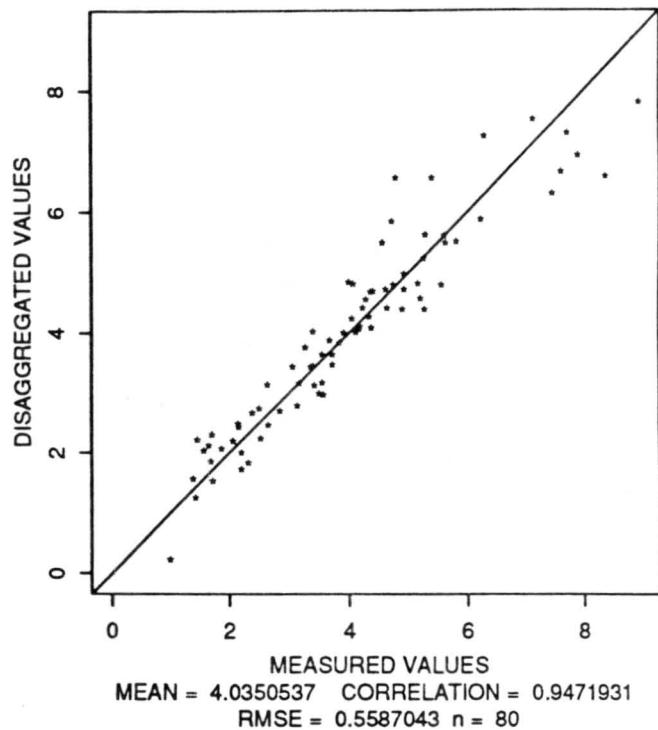
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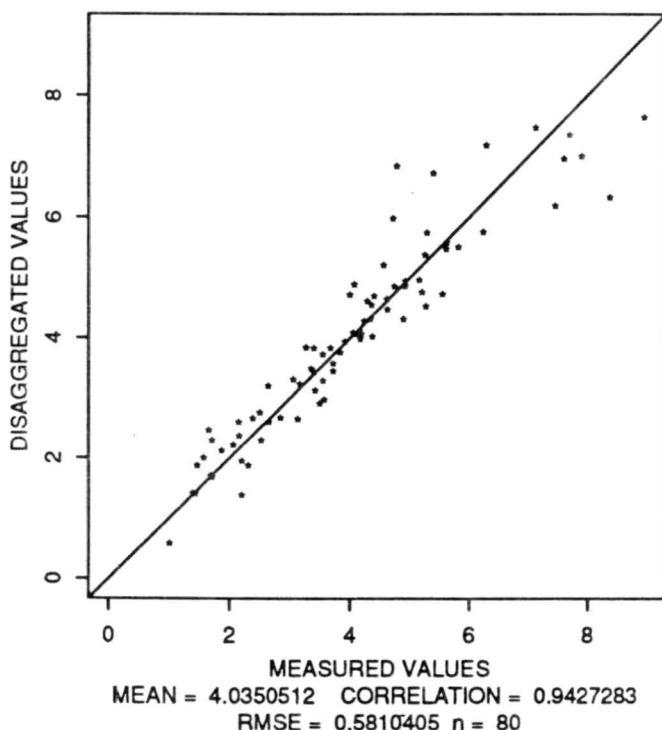
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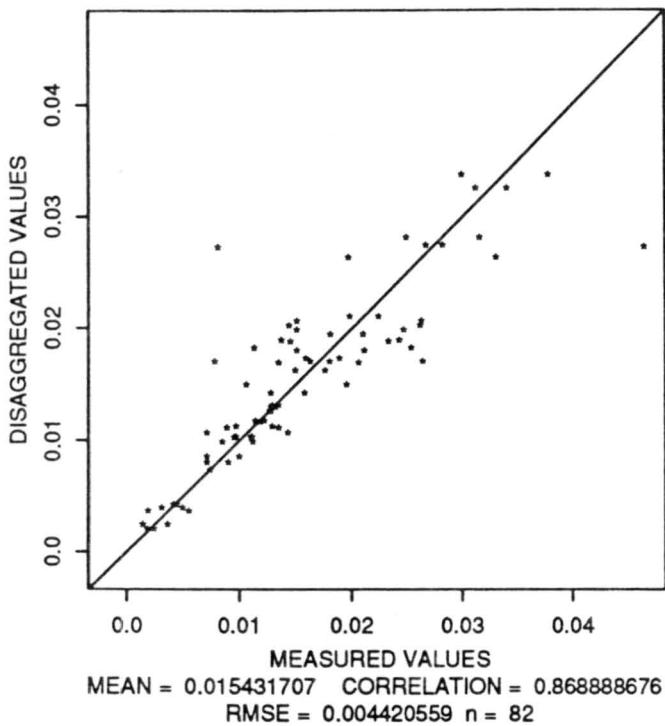
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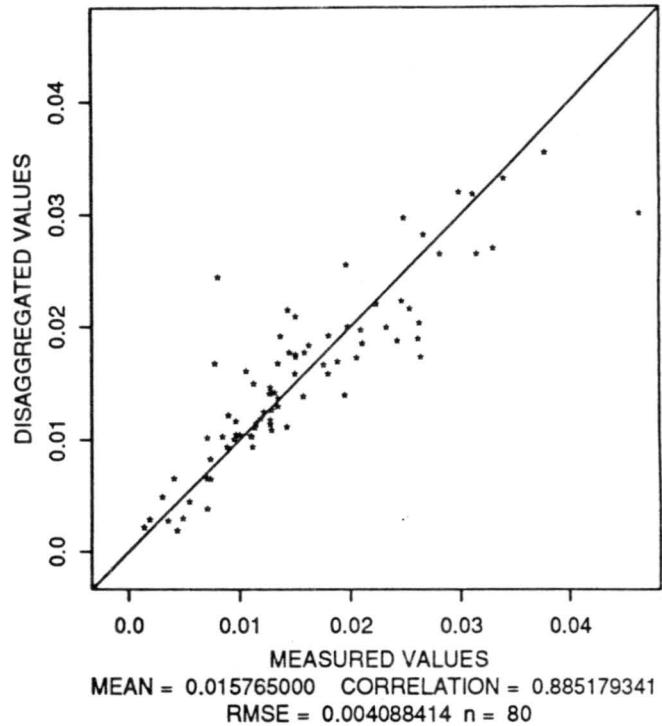
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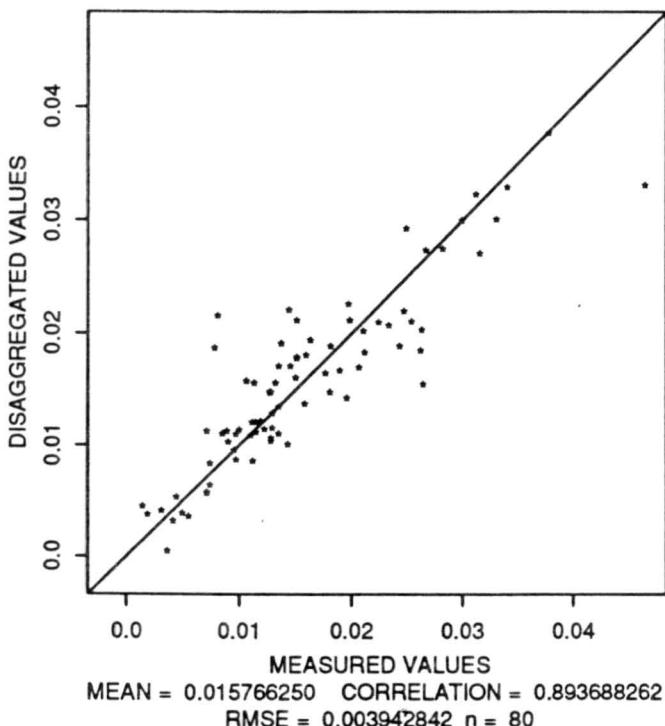
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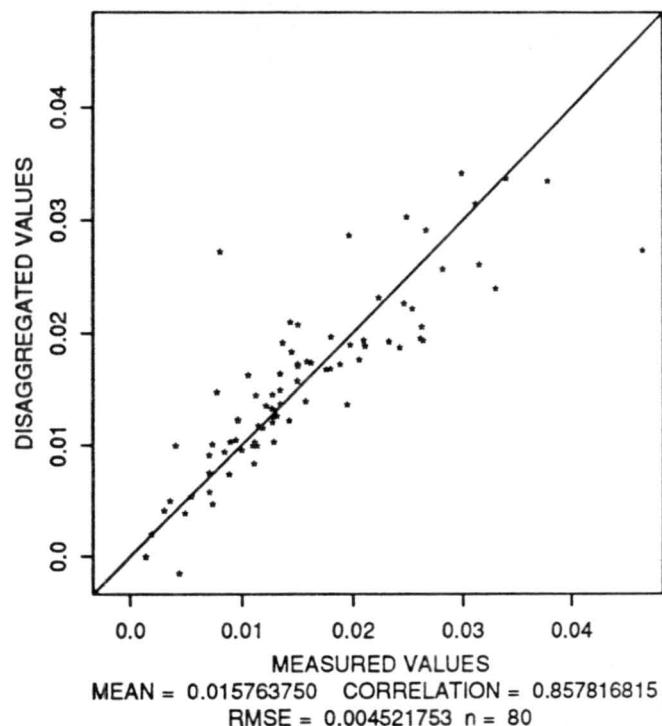
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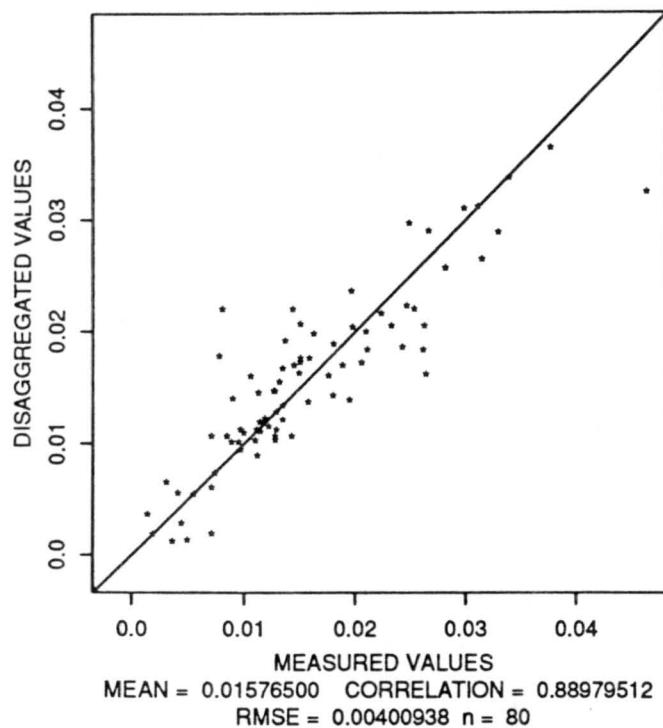
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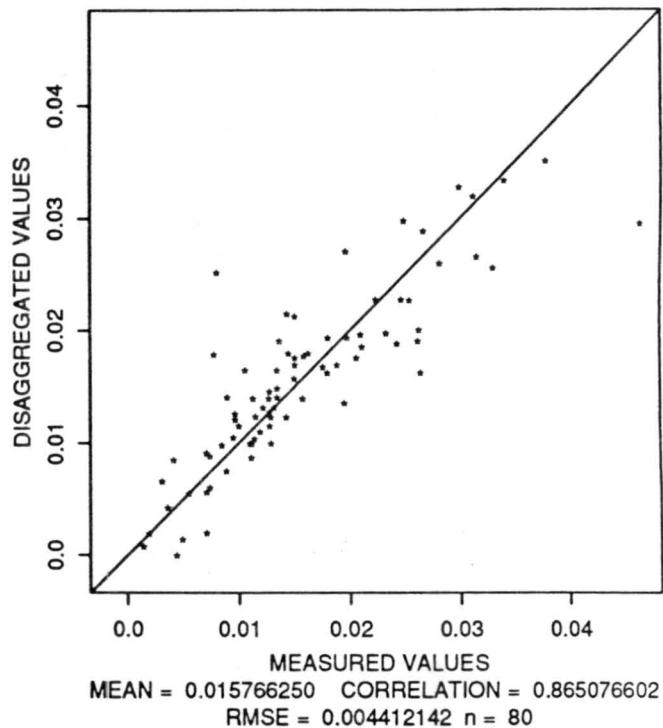
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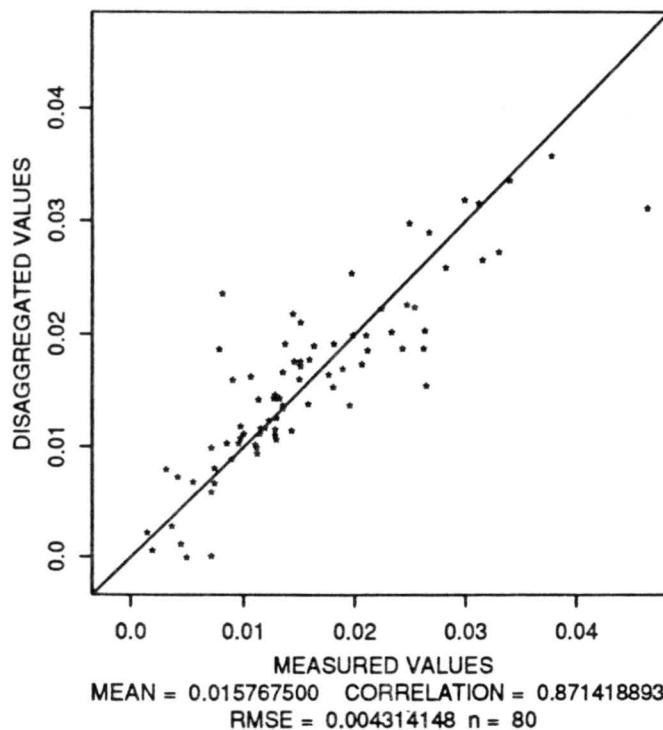
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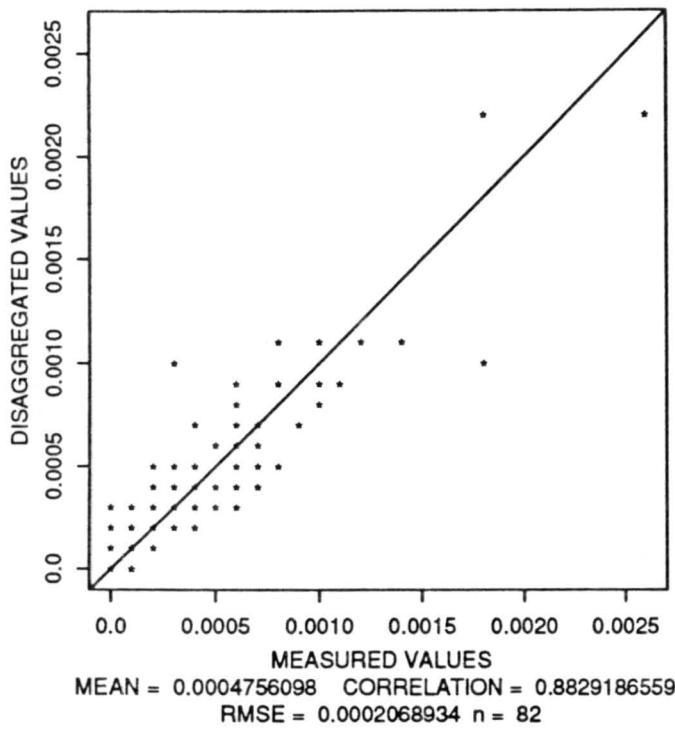
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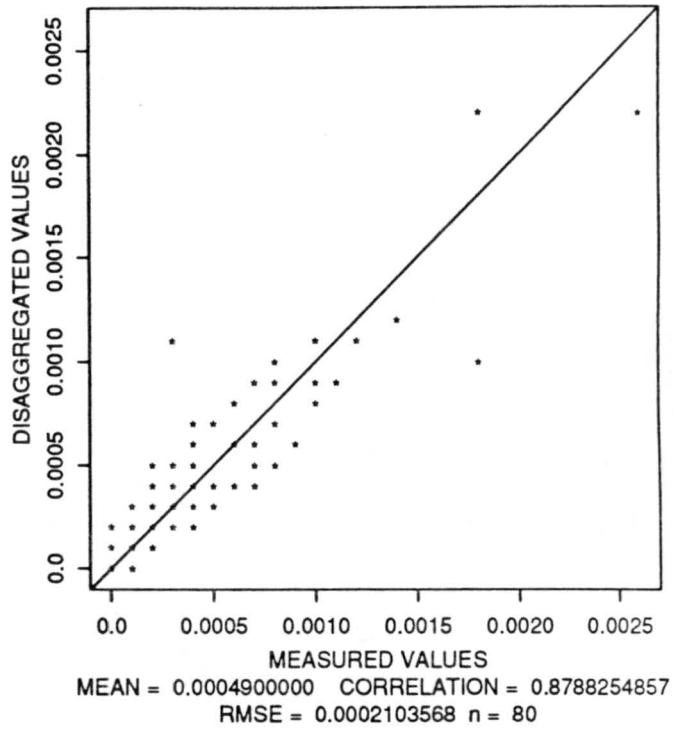
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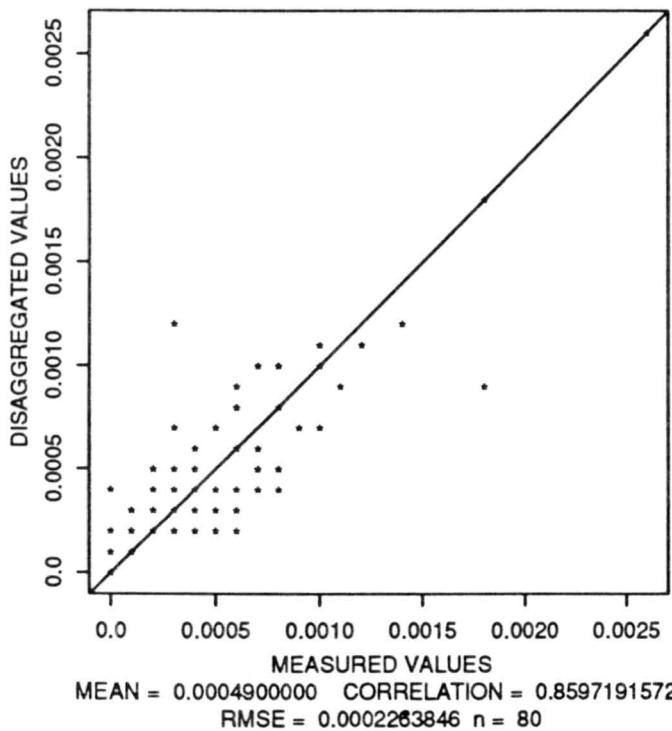
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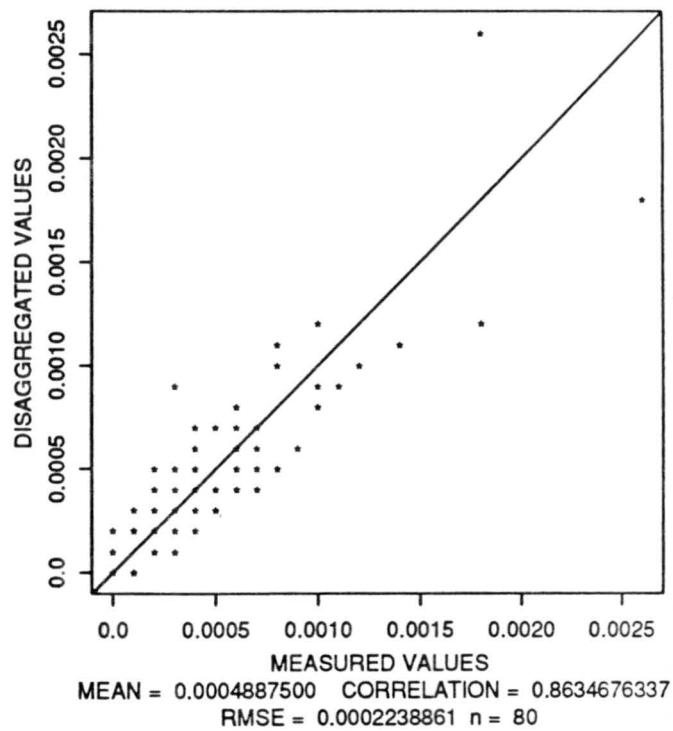
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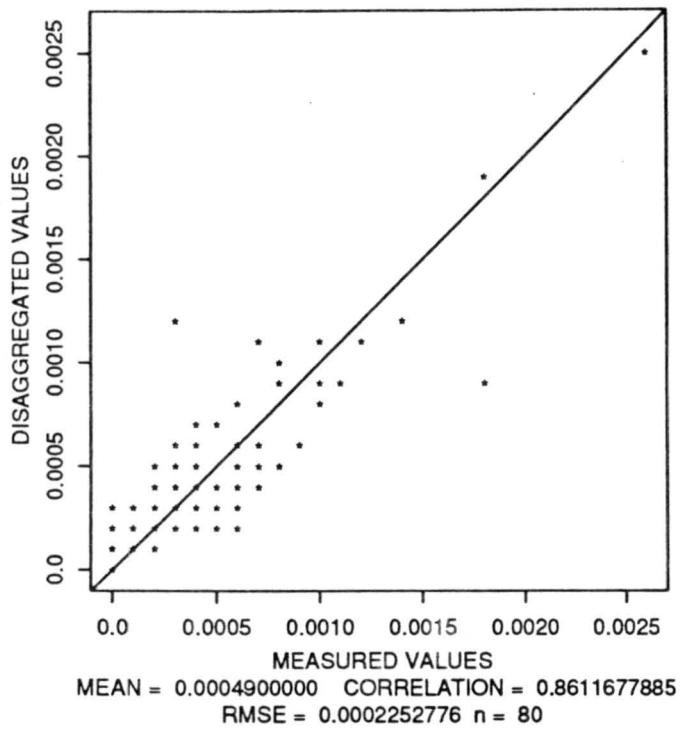
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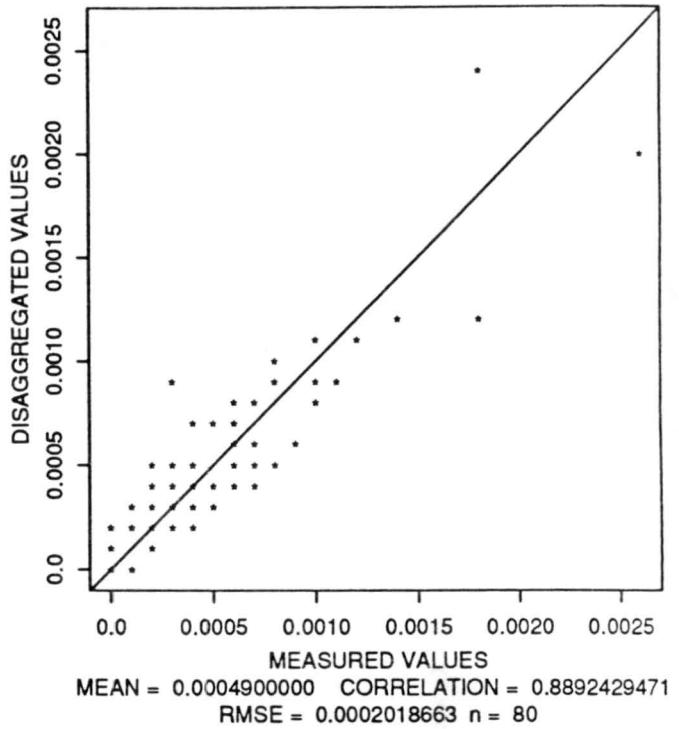
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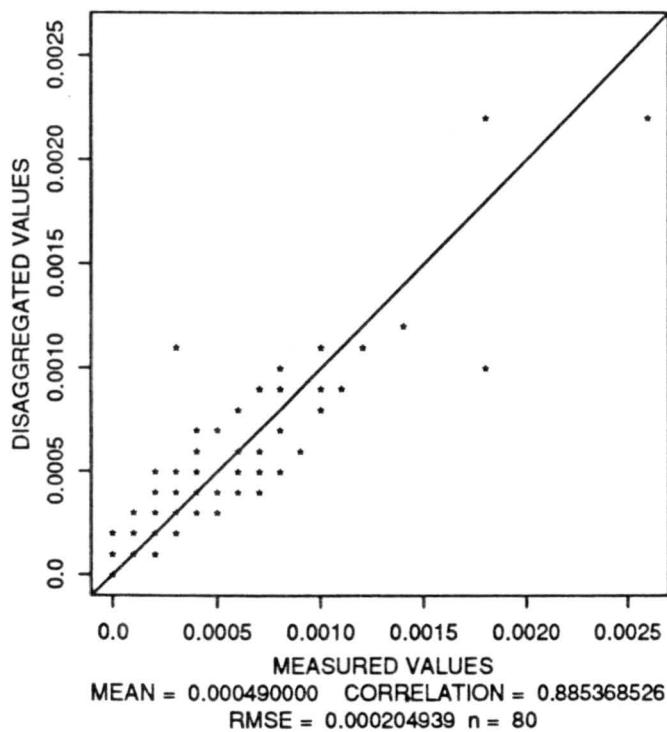
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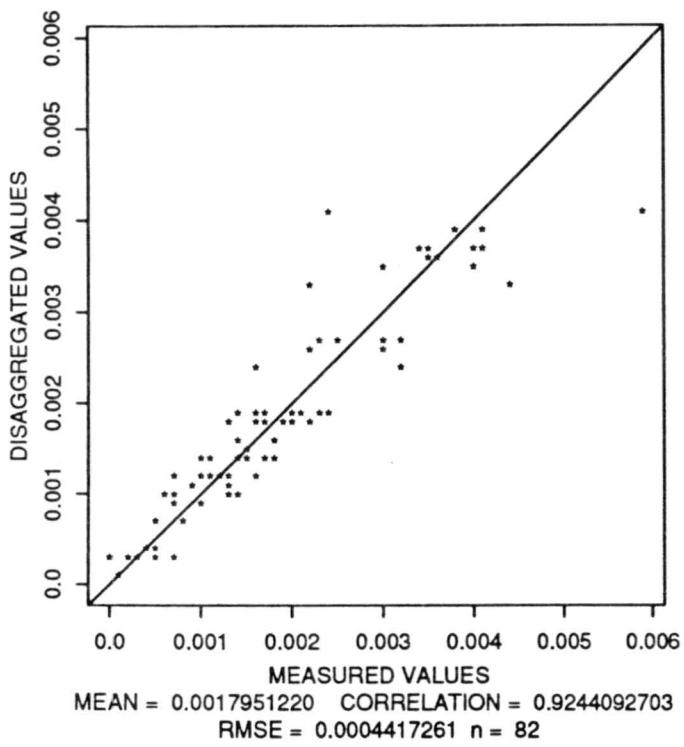
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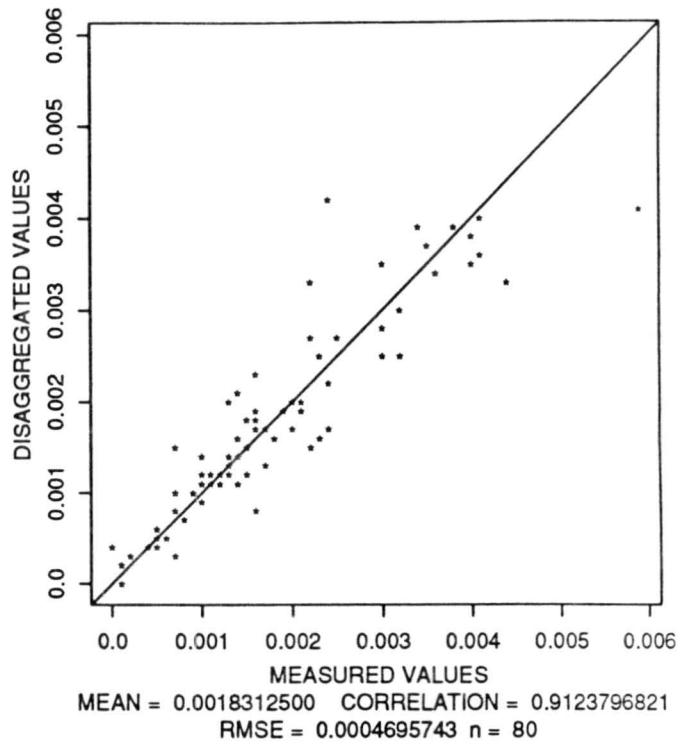
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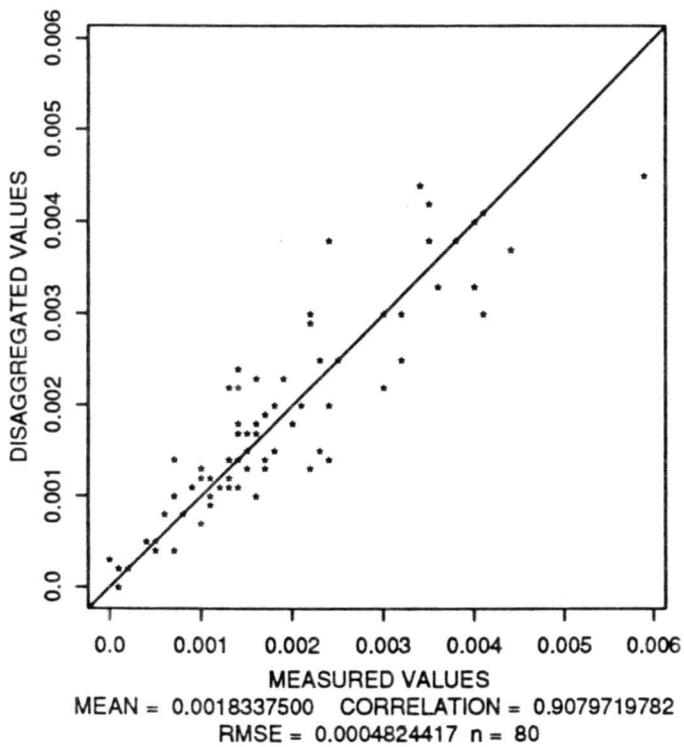
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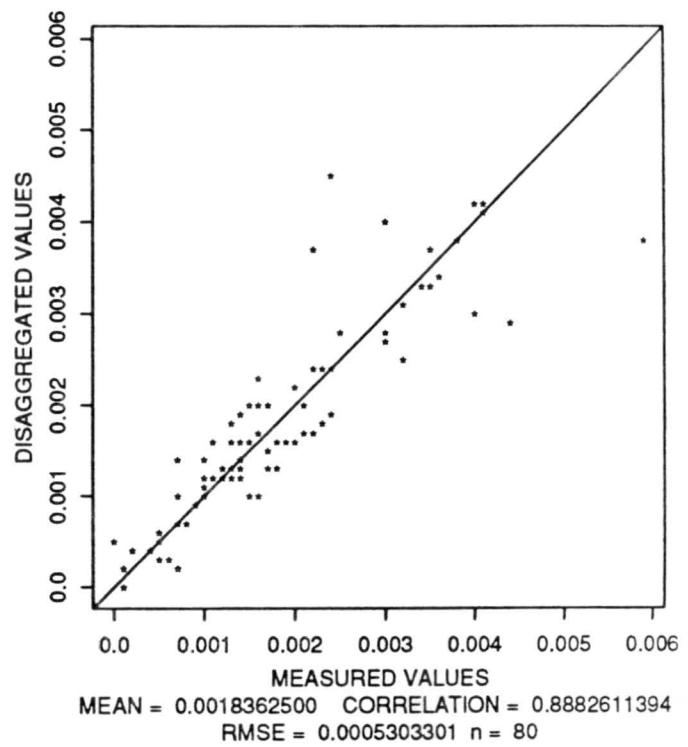
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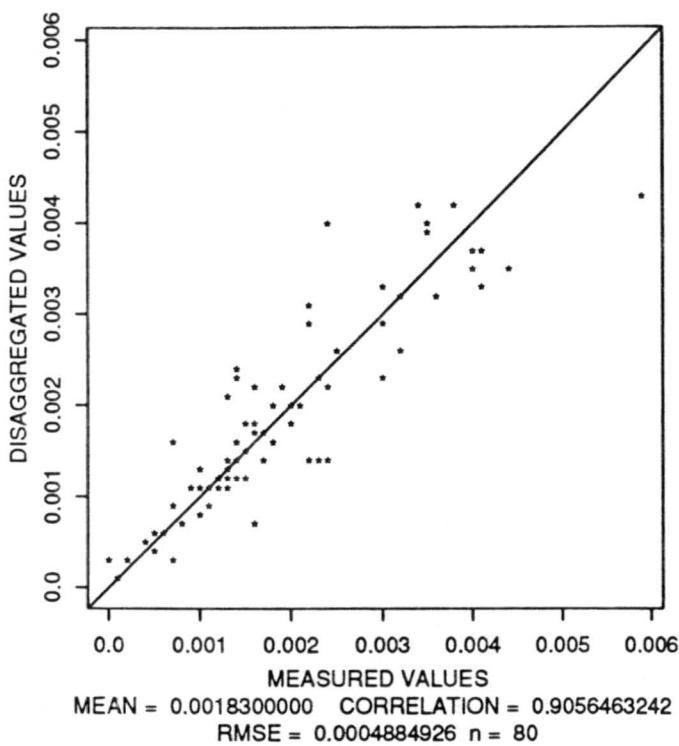
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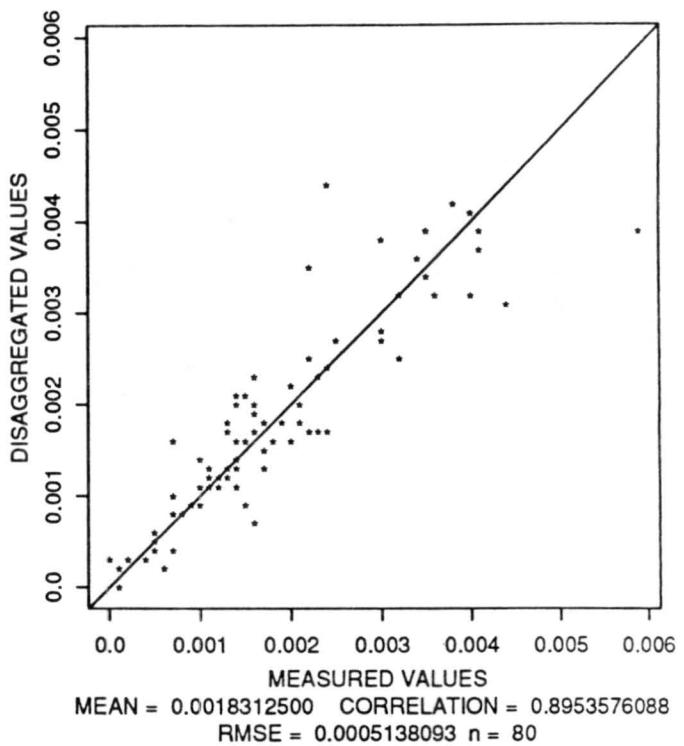
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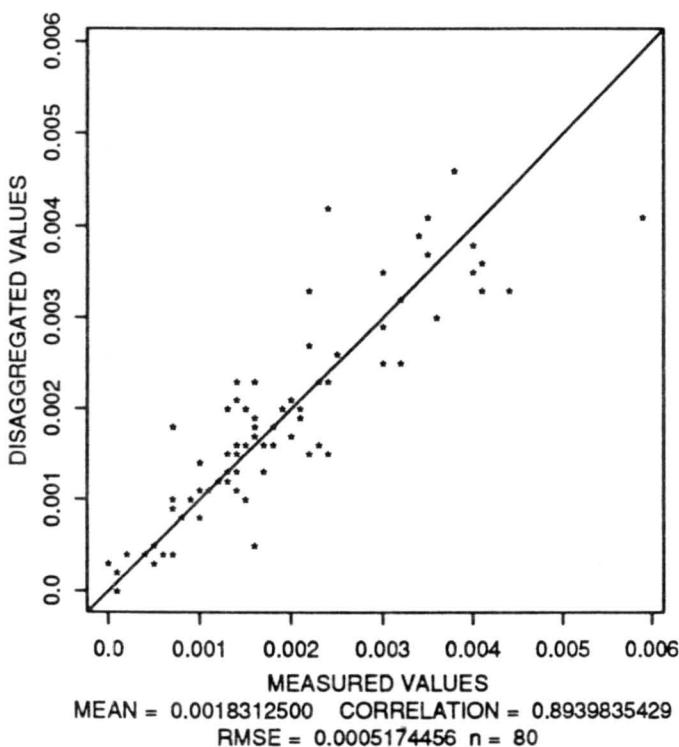
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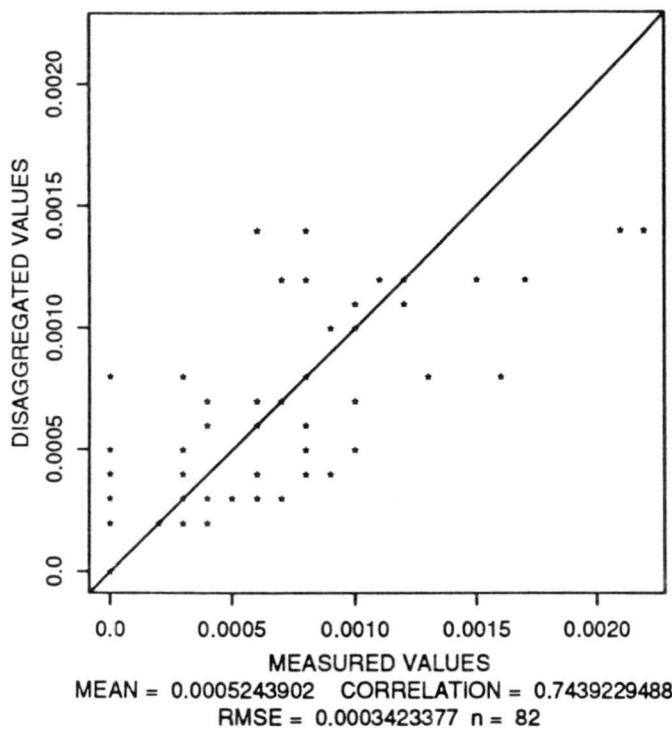
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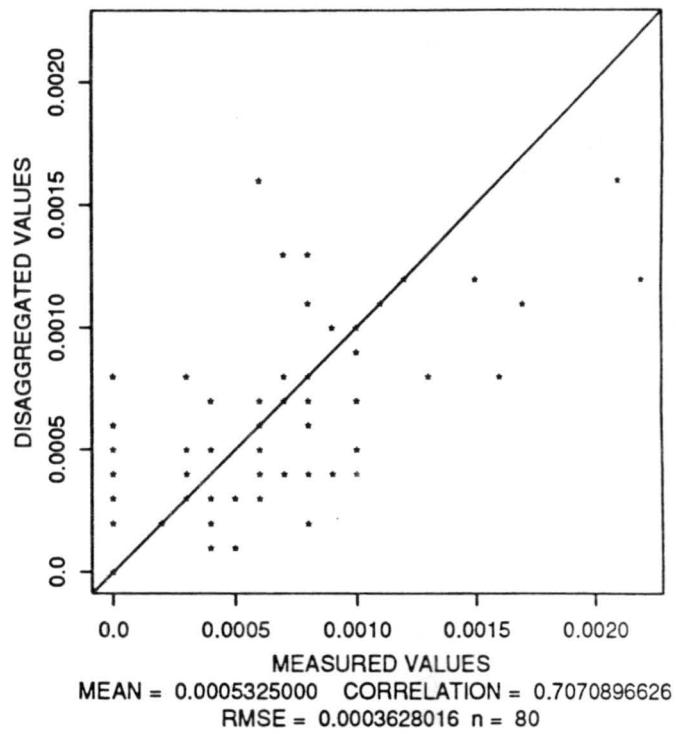
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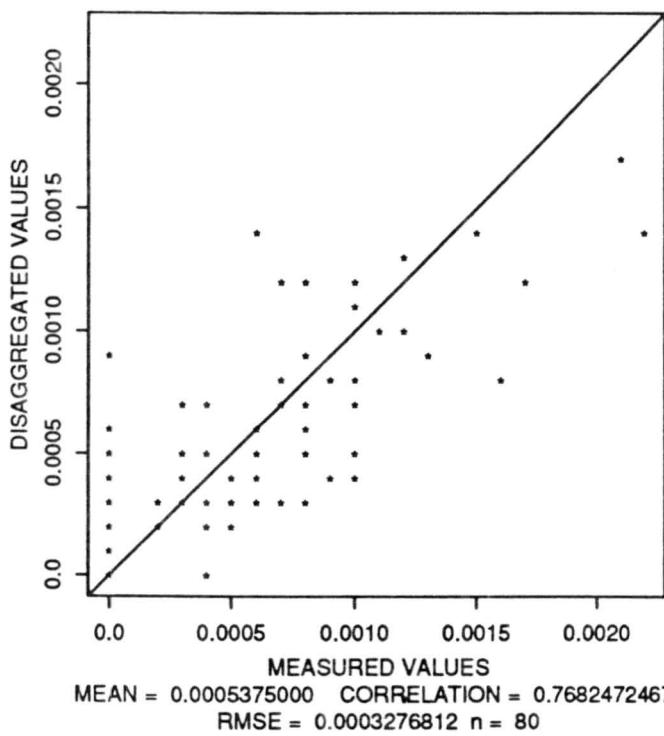
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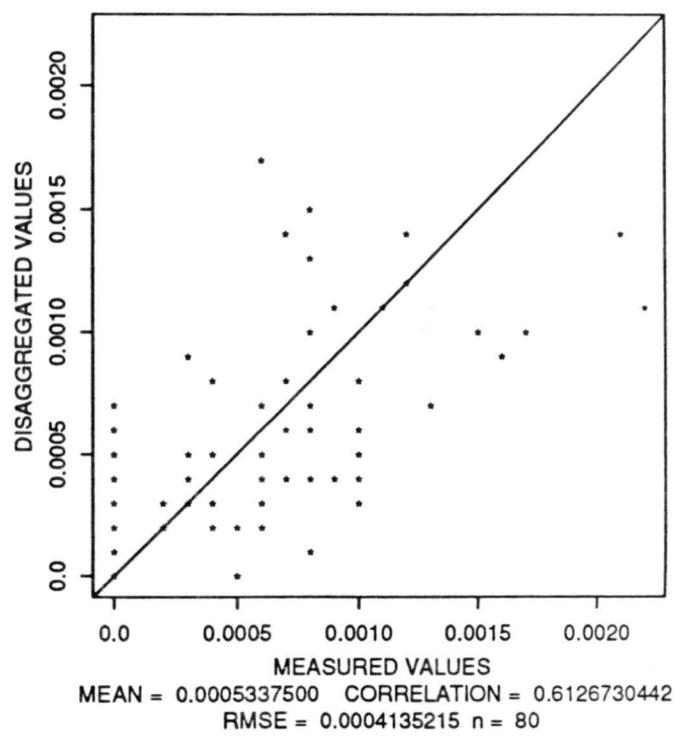
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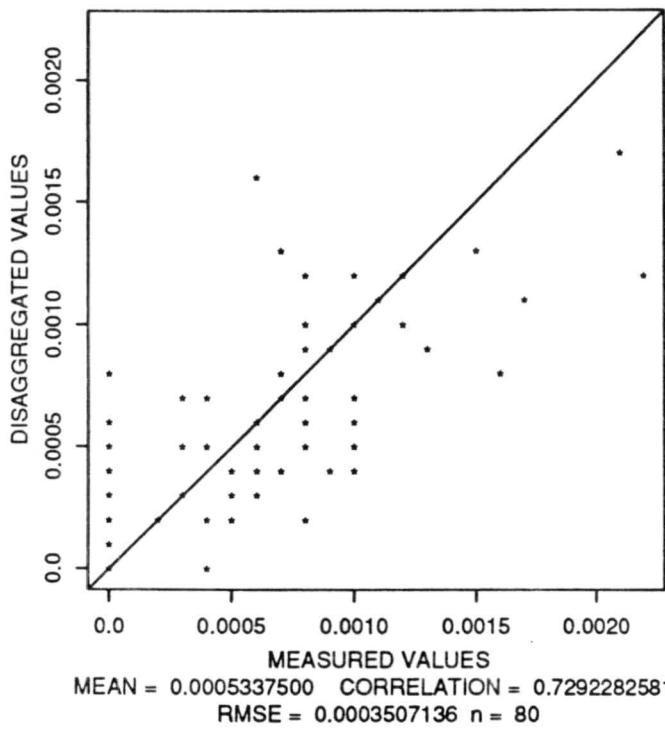
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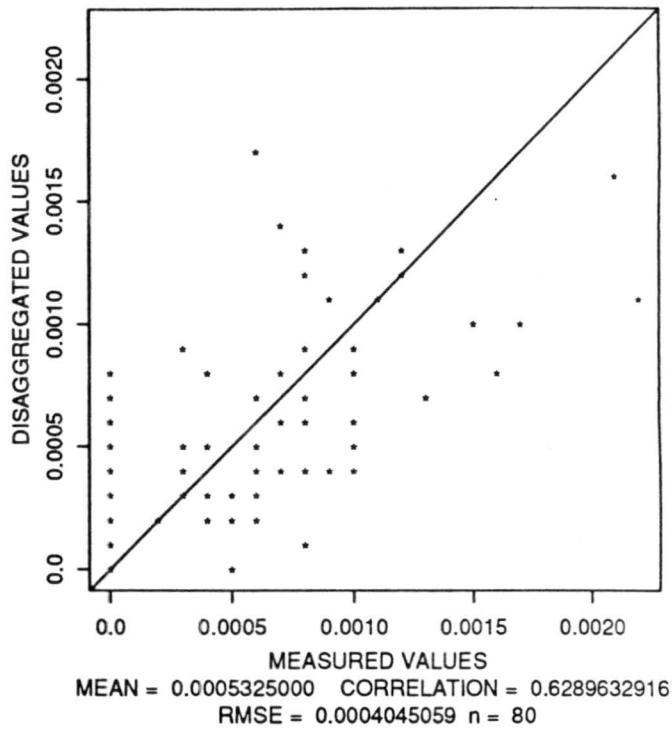
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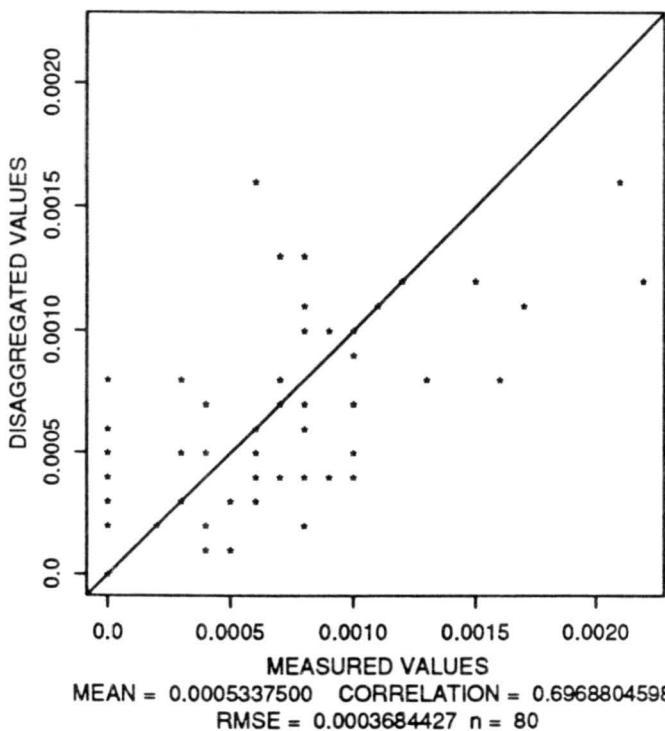
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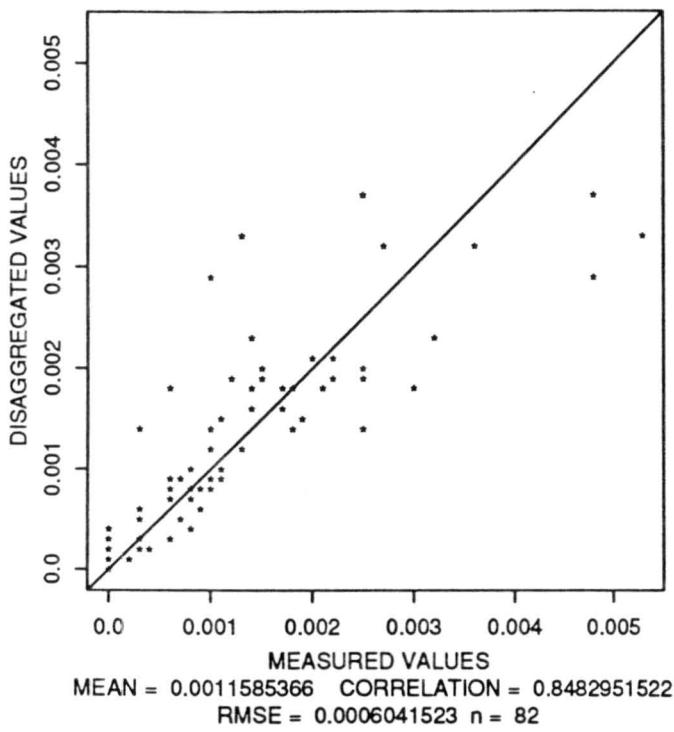
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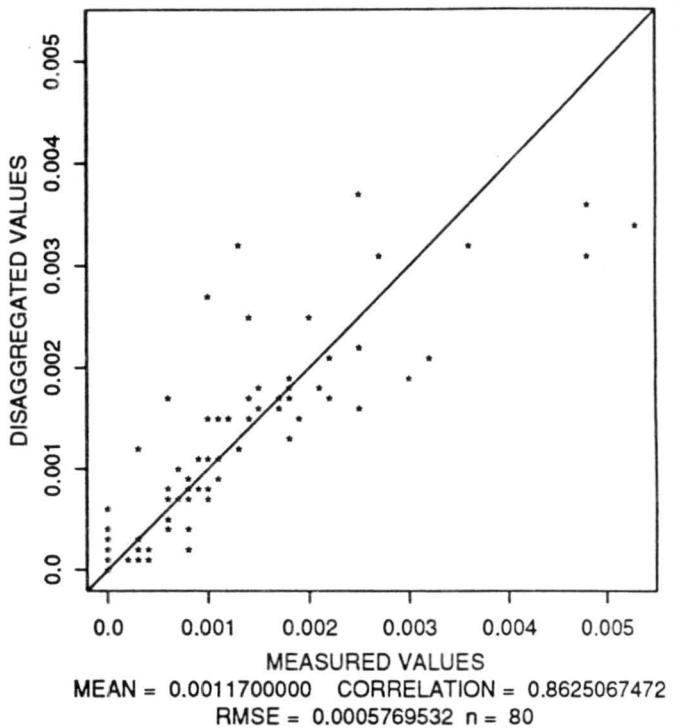
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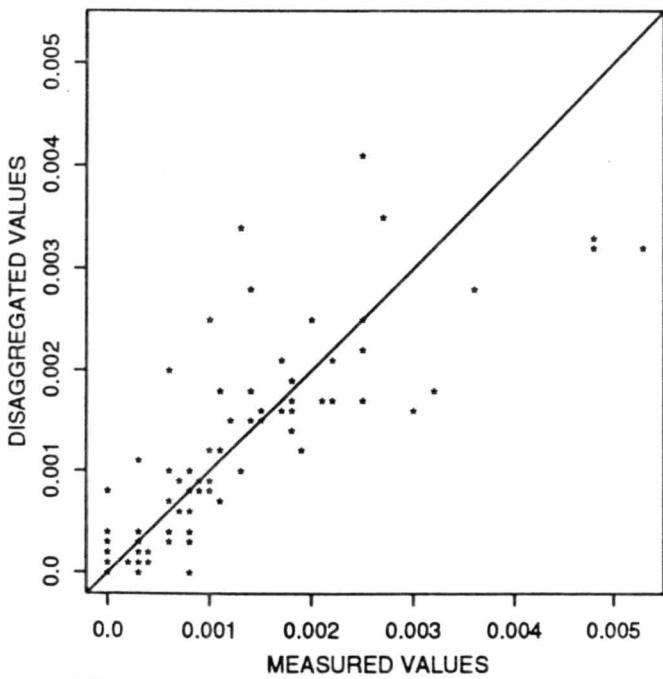
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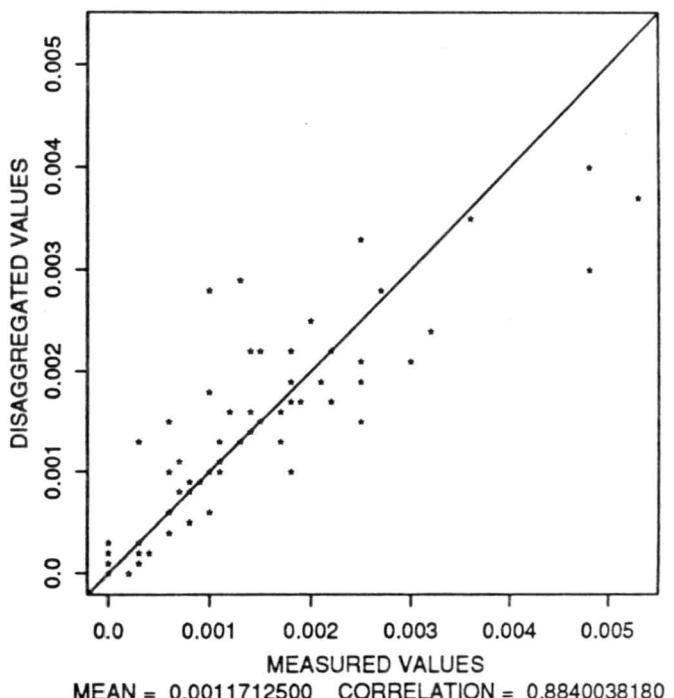
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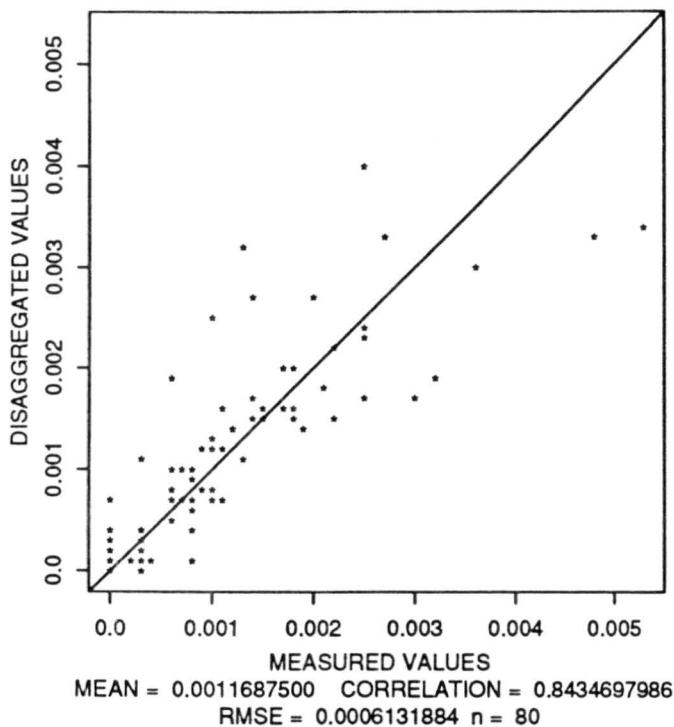
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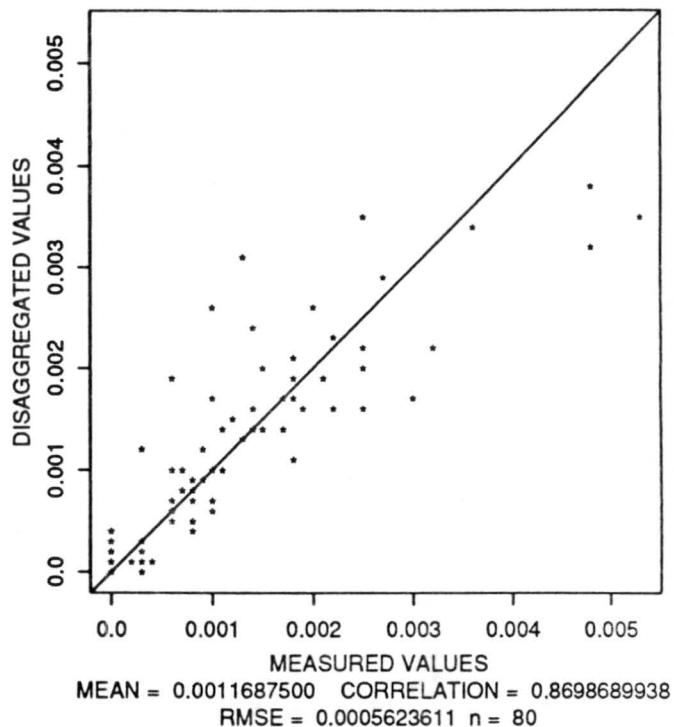
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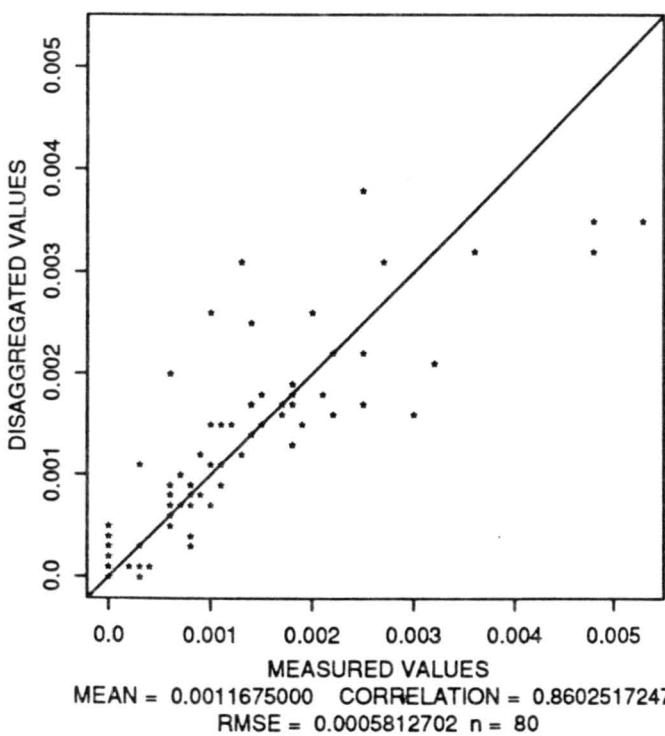
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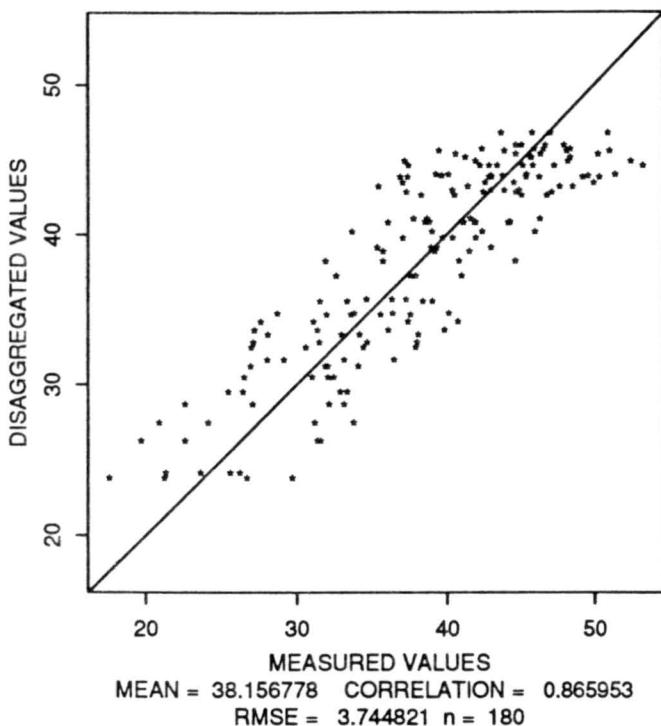
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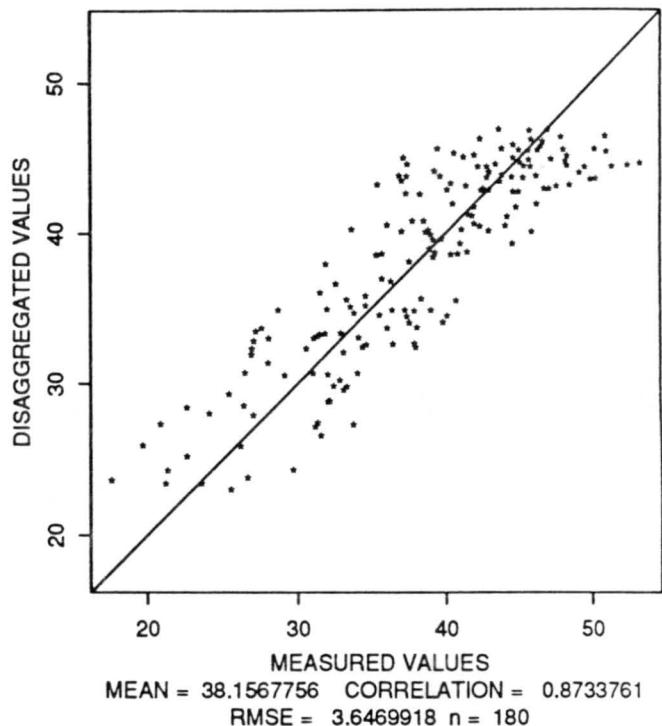
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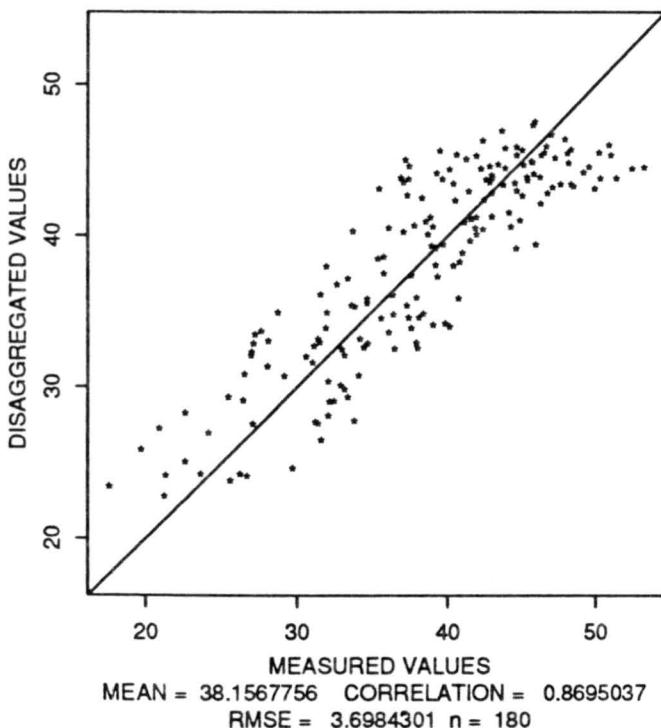
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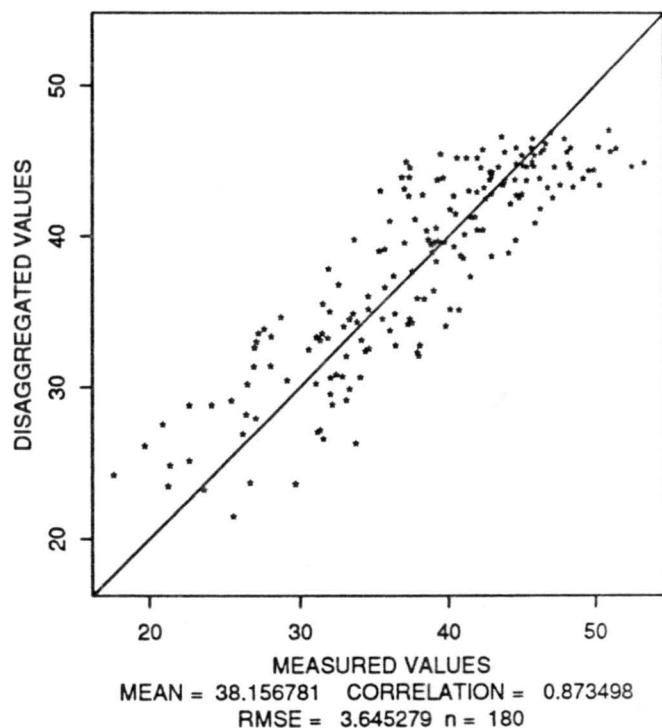
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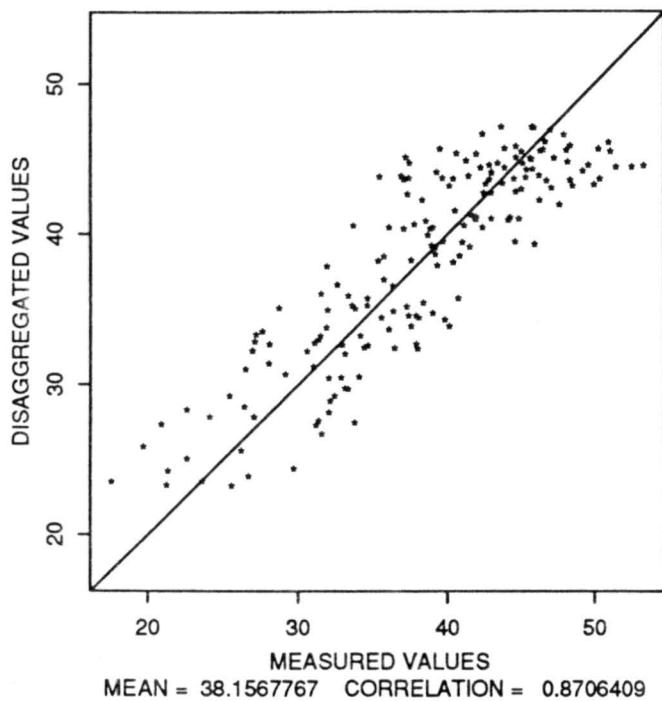
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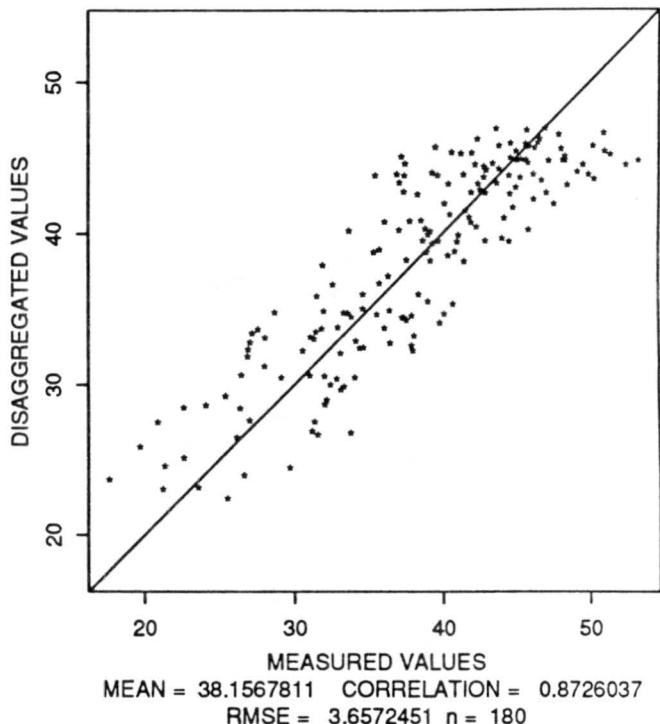
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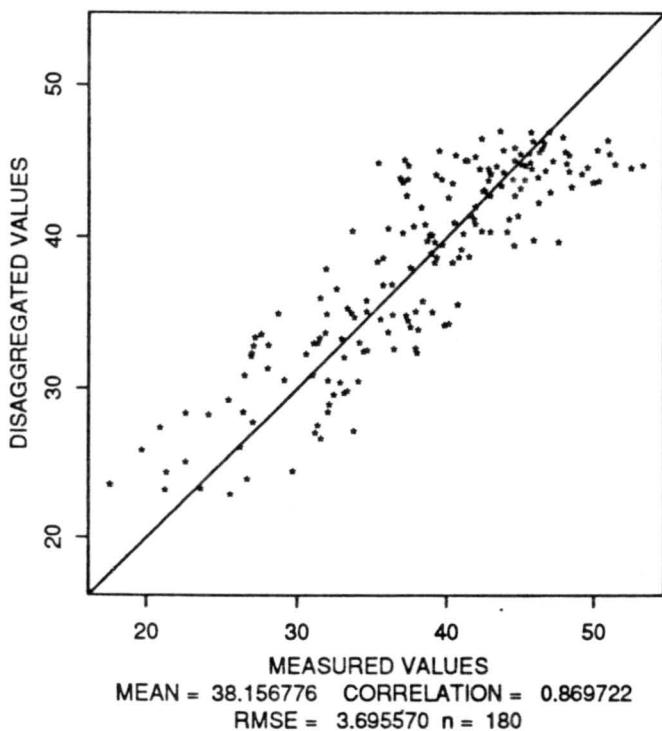
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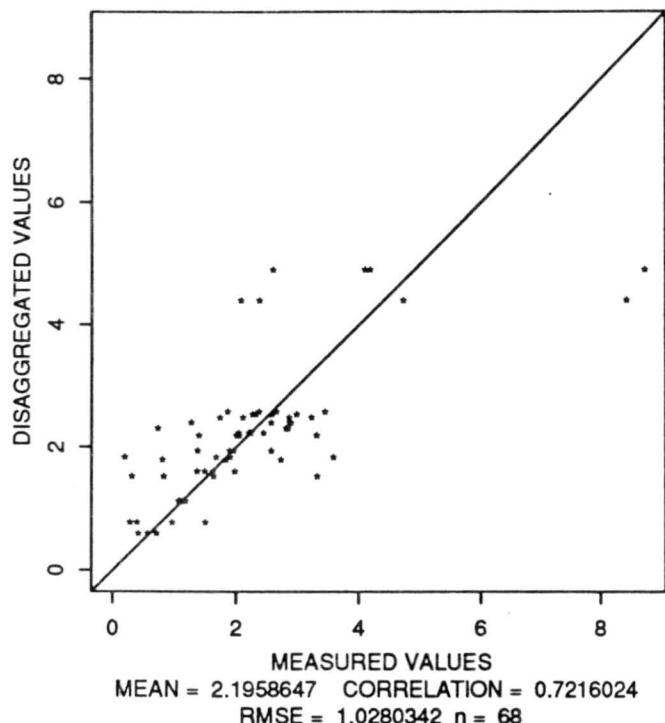
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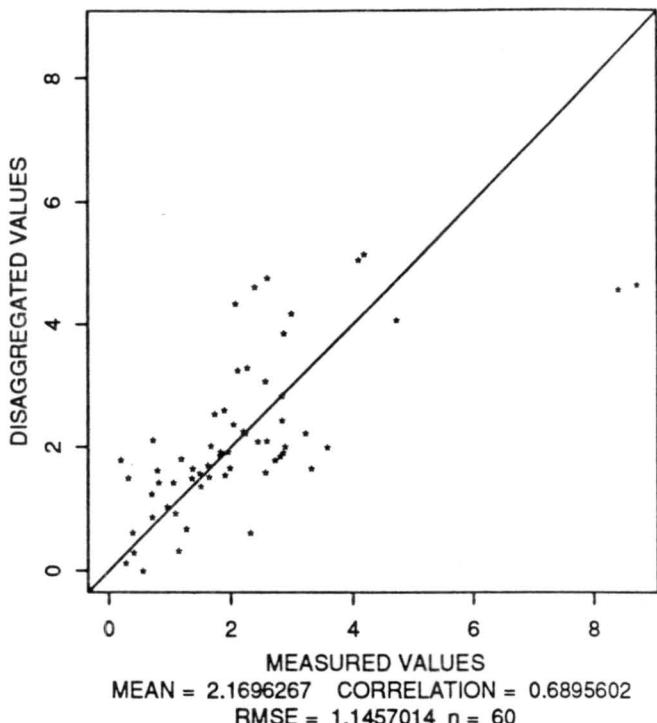
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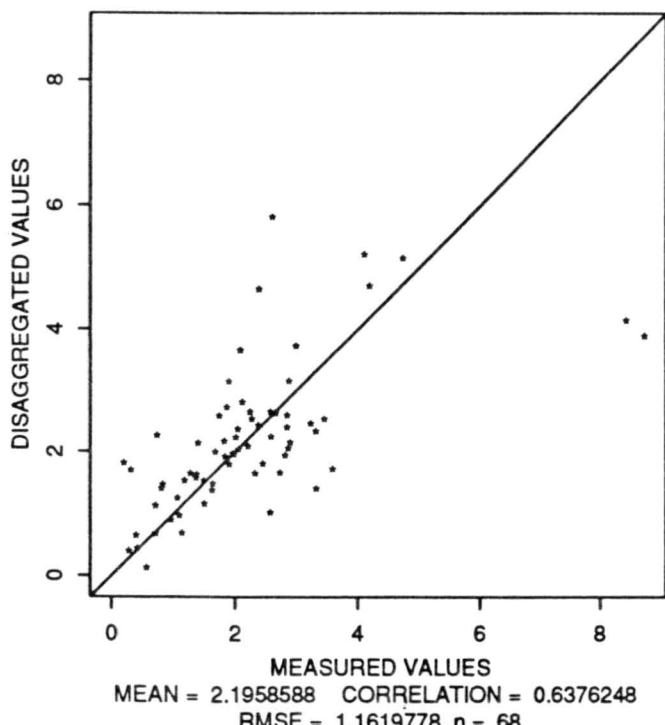
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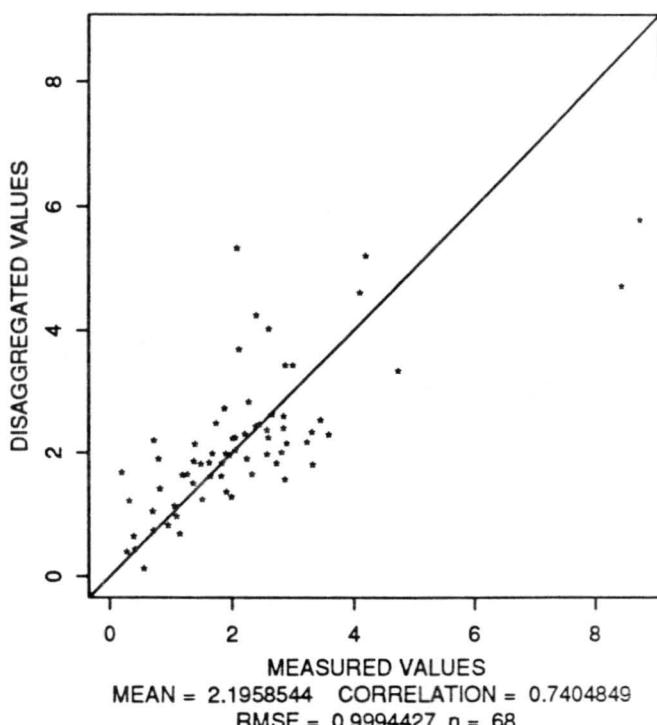
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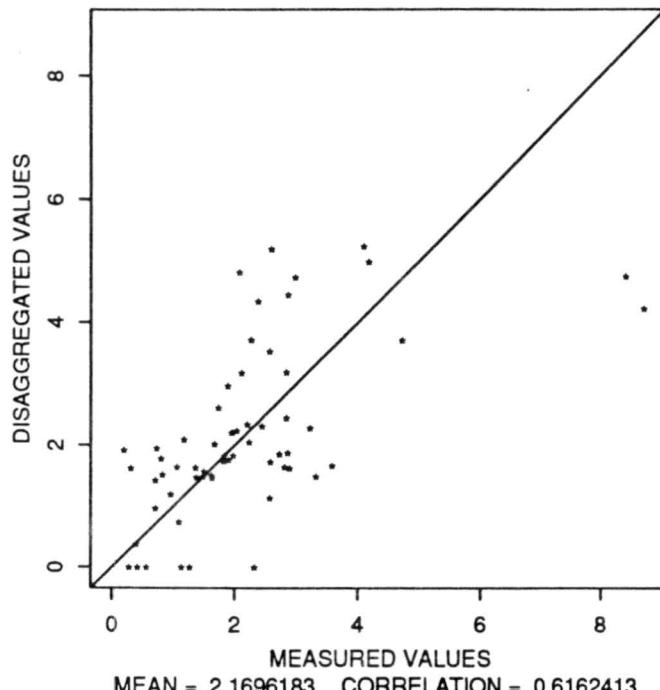
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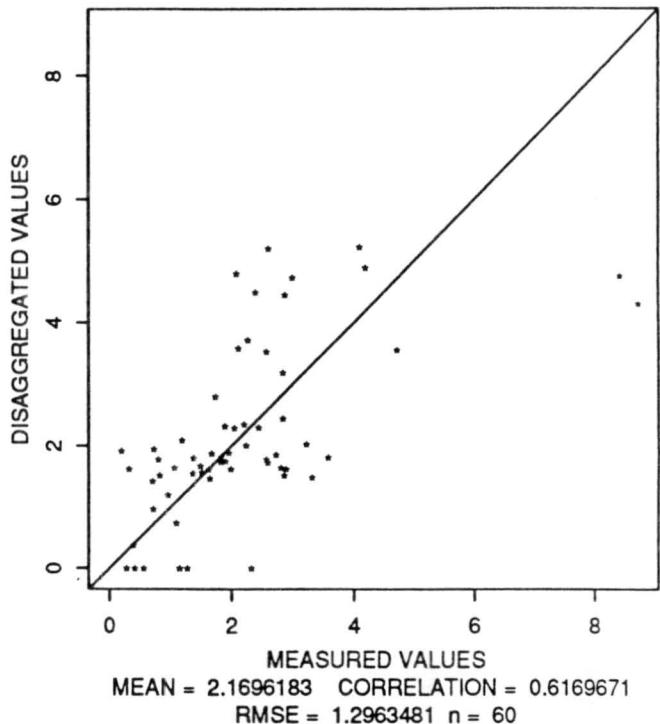
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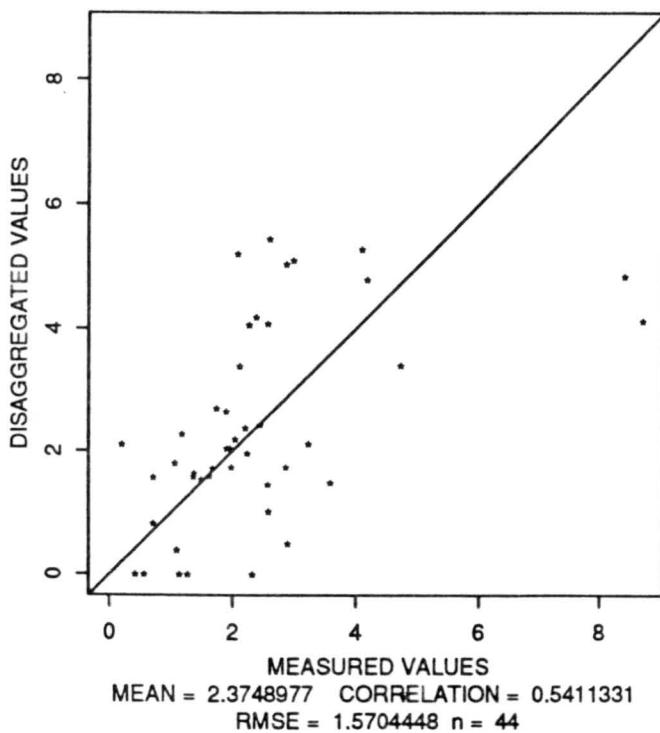
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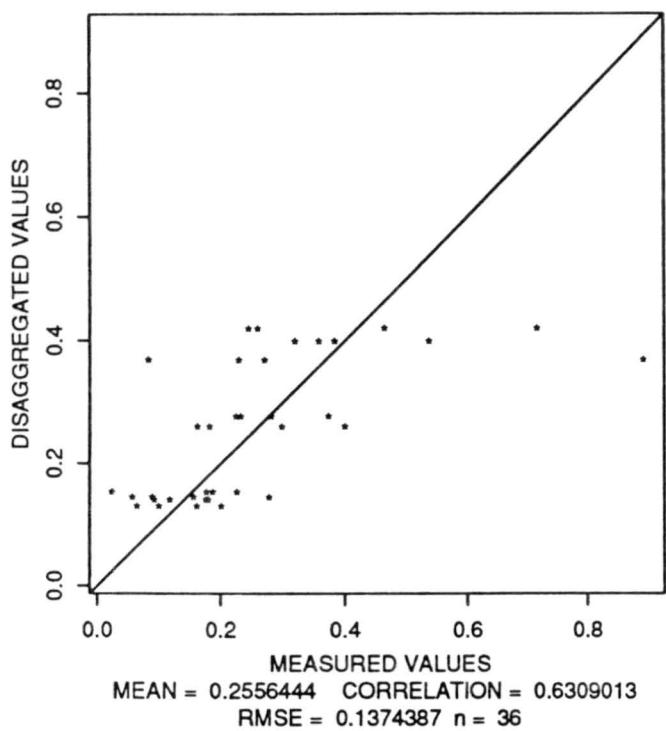
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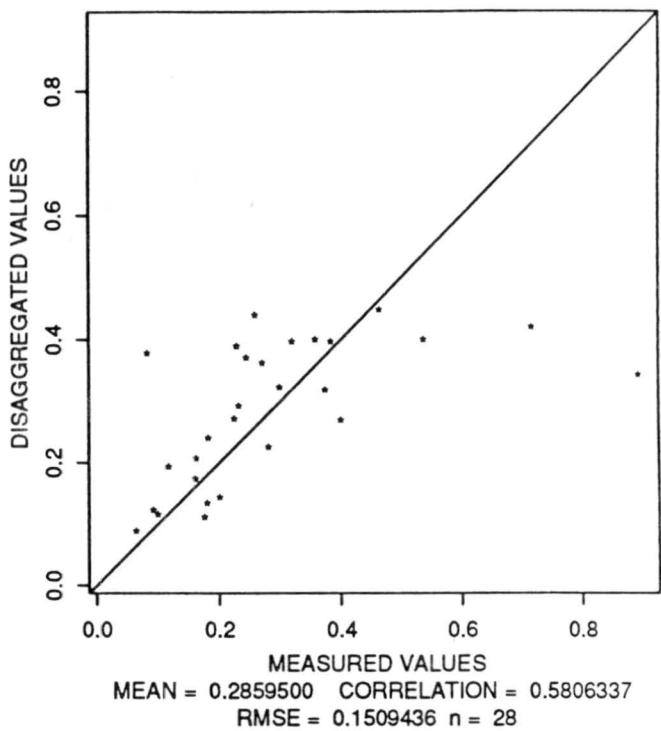
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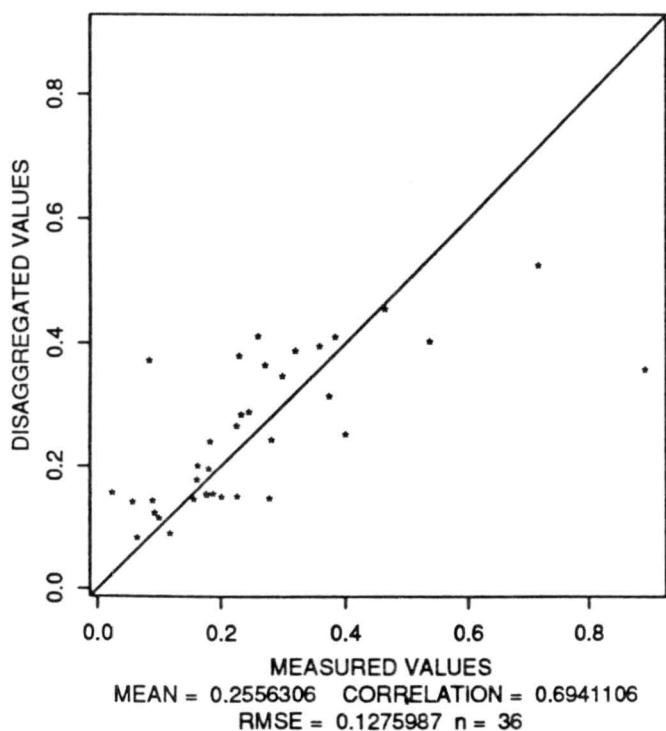
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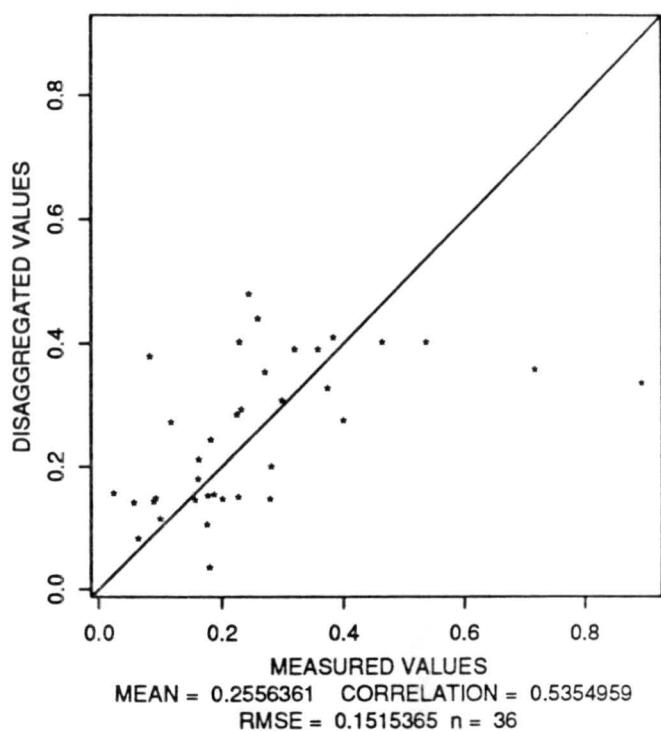
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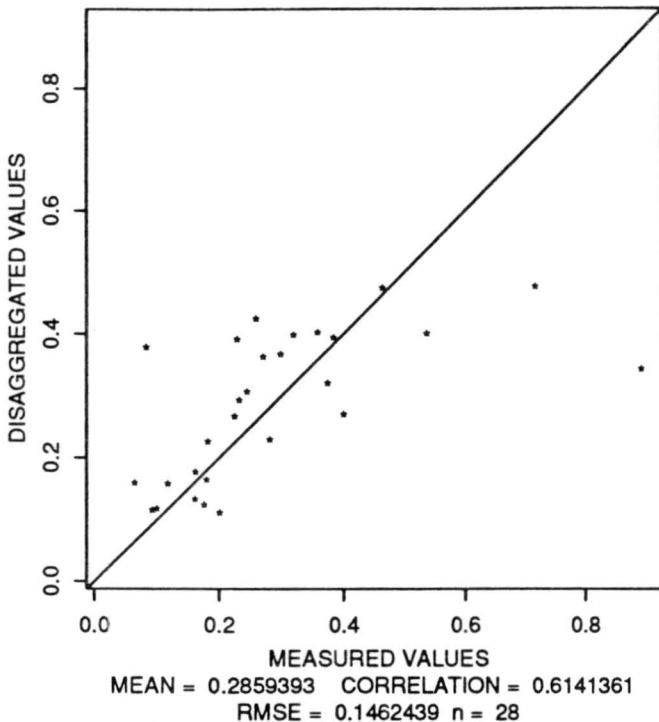
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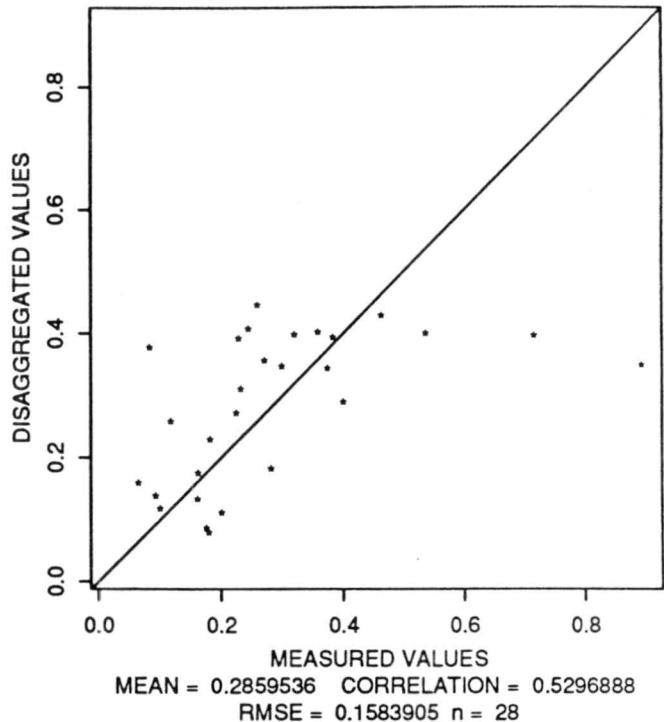
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