

THESIS

NUMERICAL SIMULATION FOR CONVECTION OF CONTAMINANTS IN GROUNDWATER

Submitted by

Leslie W. Pittman

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ABSTRACT

A numerical model which simulates the convective transport of conservative ions through groundwater aquifers is presented and discussed. The model uses the fully implicit, central finite difference technique to predict transient, two-dimensional areal groundwater level or piezometric head fluctuations, the corresponding flows, and the convective transport of conservative ions. The model uses the fully explicit finite difference technique to calculate the contaminant concentrations. The model neglects the dispersion portion of the convective-dispersion equation. Either square or rectangular grids which remain constant throughout the study period may be used.

Simple longitudinal and radial flow problems are solved. In addition, criteria to assure convergence and stability of the model are developed theoretically and empirically. The sensitivity of the model to variations in grid size, time increment and seepage velocity are presented.

The study was limited to confined aquifers. However, the model has been developed to handle unconfined aquifers also. The study was also limited to homogeneous and isotropic porous media.

Leslie W. Pittman
Civil Engineering Department
Colorado State University
Fort Collins, Colorado 80523
Fall, 1977

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CHAPTER I
INTRODUCTION

Water is the most abundant substance on earth. It is estimated that there is six times as much water on the earth as there is feldspar, the most abundant solid material. Of all the fresh water, approximately 97% -- about 8 trillion acre-feet -- is groundwater (Universal Oil Products, 1974).

Rapid population and industrial growth have led to extensive utilization of groundwater for municipal, agricultural, and industrial purposes. At the same time, improper disposal of human wastes, garbage and toxic chemicals, in addition to agricultural irrigation and fertilization, and a general unfamiliarity with groundwater hydrology have resulted in many instances of groundwater contamination.

These problems have finally begun to make officials aware of the potential for contamination of groundwater aquifers. The need for managing the quality of groundwater has also become apparent. In order to study and predict the areal distribution of contaminants in groundwater aquifers, it is necessary to understand the movement of the contaminants through the groundwater systems.

Groundwater Quality Modeling and its Limitations

As a means to study the movement of contaminants in groundwater aquifers, numerical models using digital computers have been developed.

These models simulate the movement of groundwater with additional provisions to simulate the transport and dispersion of contaminants.

Recent attempts at developing groundwater quality models have been limited primarily to conservative substances, principally the mineral salts. While this simplifies the development process, it is not undesirable since certain conservative parameters (e.g. total dissolved solids, chlorides, etc.) are of significant interest to water quality agencies.

These water quality models generally model only one contaminant at a time. The models are normally two-dimensional simulating areal or vertical distribution of the contaminant. While many models ignore the effects of soil chemistry, some models (Water Resources Engineers, Inc., 1969, and Perez et al, 1972) take the effects of the unsaturated soil zone into account.

Various numerical techniques have been used in the attempt to simulate the convective transport and dispersion of contaminants in groundwater aquifers. The finite difference technique has been adapted using the fully explicit, fully implicit (including forward, centered, and backward in time approaches), and the alternating-direction methods for solution. Previous research has indicated that, while these methods are valid, large amounts of computer time and storage usually limit their application.

The finite element technique has been studied more recently because it supposedly has advantages over the finite difference technique. However, some studies have shown that the Rayleigh-Ritz method exhibits convergence and stability problems and is not applicable to cases where

convection is the dominant transport mode. Another approach, the Galerkin method, requires large amounts of computer storage and time because small time steps and grid sizes must be used during the early calculations.

The method of characteristics is another numerical technique used to simulate convective transport and dispersion. Initial research where concentration values of the stationary grids were plotted indicated a significant amount of numerical dispersion. However, additional studies were made and accurate results obtained when the concentrations of the moving points were plotted.

Objectives of this Study

The previous discussion indicates that serious limitations are inherent in most numerical methods used to simulate convection and dispersion in groundwater aquifers. Most of the models require that the coefficient of dispersion be known and that the users have an extensive understanding of numerical methods, groundwater hydrology and the convection-dispersion process. In addition, these models often require extensive field data and very large amounts of computer time and storage.

Sunada (McWhorter et al, 1977) presented a finite difference numerical model called WTQUAL1 which simulates the convective transport of conservative ions. This model simplifies the simulation of contaminant movement through groundwater aquifers by neglecting the dispersion process. The model ignores the effects of the unsaturated soil

column and handles only one conservative parameter at a time. By using the fully explicit technique to calculate contaminant concentrations, the model minimizes the need for costly computer storage and time.

The objectives of this study are to:

1. Briefly discuss WTQUAL1, paying particular attention to the groundwater flow equation and the mass balance equation used to calculate the contaminant concentrations.
2. Modify WTQUAL1 so that it is applicable to a wider variety of problems.
3. Verify the numerical model by comparing numerical solutions to appropriate analytic solutions.
4. Develop criteria to assure convergence and stability of the model.
5. Perform a sensitivity analysis of the model, specifically evaluating those terms which appear in the stability criteria.

A review of literature dealing with numerical simulation of convection and dispersion in groundwater aquifers will be made. The problems encountered in these approaches will be noted and the effort of this study directed toward minimization or elimination of these problems. The existing numerical model will be modified and a hypothetical aquifer developed for which both numerical and analytic solutions for convection problems will be made. Computer runs will be made primarily on the HP 9830A desk-top computer with verification of these runs being made on the CDC 6400. Theoretical and empirical approaches will be used to develop criteria to assure convergence and stability of the model. Analyses will be performed to determine the sensitivity of the model to

changes in grid size, length of time increment and seepage velocity.

In addition, runs will be made to verify that the model produces accurate results for the two-dimensional convection process resulting from radial flow from a recharge well.

CHAPTER II
CONVECTION AND DISPERSION IN POROUS MEDIA

The first recorded study of dispersion in porous media was done inadvertently by Slichter (1905). In attempting to determine the rate of movement of groundwater, he injected a salt solution into a well and observed the time of arrival at an observation well down-gradient. He noted that the salt did not arrive at the observation well as a slug, but that the salt concentration gradually increased to some maximum value and then decreased. Since that time, much work has been done on the properties of dispersion and molecular diffusion of contaminants in groundwater.

Variation of concentration of contaminants in groundwaters is caused by three processes: convection, dispersion and molecular diffusion. Convection is the transportation of contaminants associated with groundwater flow and is based on the average seepage velocity. Dispersion in a porous media is associated with the convection process and results from a mechanical mixing caused by the individual fluid particles traveling at variable velocities through the pore spaces of the media and along microscopic path lines. Molecular diffusion results directly from the thermal motion of the individual fluid molecules and takes place under the influence of a concentration gradient. A detailed discussion of convection and dispersion is contained in Bear (1972).

Due to the difficulty encountered in trying to describe the boundary conditions on a microscopic scale (i.e. diffusion), the system is usually described on a macroscopic scale (i.e. convection and dispersion). Dispersion is a function of three physical properties: the fluid, the porous media, and fluid flow. Fluid properties of concern are density, viscosity, contaminant concentration, and miscibility of fluids in systems containing two or more fluid types. Media properties affecting dispersion are permeability, pore geometry, and pore space dimensions. Velocity is the major flow property.

According to Scheidegger (1961), and deJosselin deJong and Bossen (1961), the convection and dispersion of a contaminant in fluid flow through a saturated homogeneous porous medium is described by the differential equation:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i} \left[D_{ij} \frac{\partial C}{\partial x_j} - v_i C \right] \quad (2-1)$$

where D_{ij} = coefficient of dispersion

C = the contaminant concentration

t = time

v_i = the component of the seepage velocity vector in a cartesian coordinate system, and

x_i ($i=1,2,3$) = the cartesian space coordinates.

The double subscripting of D_{ij} and x_i, x_j represent the tensorial nature of the dispersion process. The first term in brackets represents the dispersion process while the second term represents convection.

Analytic Solution of Longitudinal Convection and Dispersion

For longitudinal convection and dispersion in a homogeneous and isotropic porous medium with a plane source at $x_3=0$ (see Figure 2-1), Equation 2-1 becomes:

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x_3^2} - v_3 \frac{\partial C}{\partial x_3} \quad (2-2)$$

where D_L is the longitudinal dispersion coefficient neglecting molecular diffusion. The initial and boundary conditions needed for the schematic in Figure 2-1 to apply for steady state flow in the direction of the velocity vector are as follows: the concentration $C=C_0$ of the contaminant source is constant for all times; the concentration in the porous media is initially zero for all values of x_3 ; the concentration at x_3 equal infinity is always zero. Mathematically, these initial and boundary conditions are given by:

$$\begin{aligned} C(0,t) &= C_0 \quad ; \quad t \geq 0 \\ C(x_3,0) &= 0 \quad ; \quad x_3 \geq 0 \\ C(\infty,t) &= 0 \quad ; \quad t \geq 0. \end{aligned} \quad (2-3)$$

Ogata and Banks (1961) used Laplace transforms to obtain the following solution to Equations 2-2 and 2-3:

$$\frac{C}{C_0} = \frac{1}{2} \left[\operatorname{erfc} \left(\frac{x_3 - v_3 t}{2\sqrt{D_L t}} \right) + \exp \left(\frac{v_3 x_3}{D_L} \right) \operatorname{erfc} \left(\frac{x_3 + v_3 t}{2\sqrt{D_L t}} \right) \right] \quad (2-4)$$

where $\operatorname{erfc}=1-\operatorname{erf}$. For areas not close to the source and where $x_3 > v_3 t$, the second term in Equation 2-4 can be omitted and the equation simplified to:

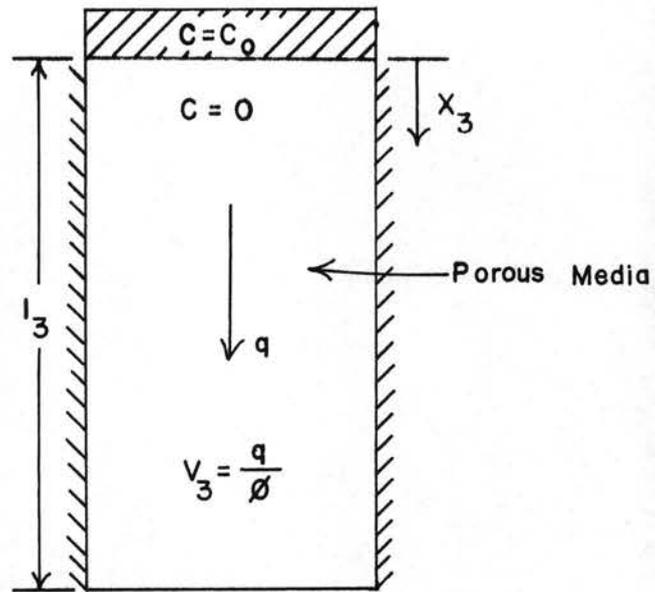


Figure 2-1. Schematic of longitudinal dispersion.

$$\frac{C}{C_0} = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{x_3 - v_3 t}{2\sqrt{D_L t}} \right) \right] \quad (2-5)$$

Where $x_3 < v_3 t$, the applicable equation is:

$$\frac{C}{C_0} = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x_3 - v_3 t}{2\sqrt{D_L t}} \right) \right] \quad (2-6)$$

Equations 2-5 and 2-6 assume that the dispersion is symmetric about the point $C/C_0 = 0.50$ and assume steady state flow but non-steady state dispersion.

Analytic Solution of Longitudinal and Lateral Dispersion and Longitudinal Convection

If the source area is less than the area through which flow is occurring, longitudinal and lateral dispersion and longitudinal convection will occur. A schematic of this condition is shown in Figure 2-2(a).

For a homogeneous and isotropic media with one dimensional flow in the x_3 direction only, the governing equation is:

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x_3^2} + D_T \frac{\partial^2 C}{\partial x_2^2} - v_3 \frac{\partial C}{\partial x_3} \quad (2-7)$$

where D_T is the lateral (transverse) dispersion coefficient. For steady state conditions and with flow in the direction of the velocity vector, the initial and boundary conditions for this equation are:

$$\begin{aligned} C(x_2, 0, t) &= C_0 & ; & \quad 0 \leq x_2 \leq b & ; & \quad t \geq 0 \\ C(x_2, 0, t) &= 0 & ; & \quad b \leq x_2 \leq \ell_2 & ; & \quad t \geq 0 \\ \frac{\partial C(0, x_3, t)}{\partial x_2} &= 0 & ; & \quad t > 0 \\ \frac{\partial C(\ell_2, x_3, t)}{\partial x_2} &= 0 & ; & \quad t > 0 \\ C(x_2, \infty, t) &= \text{bounded} \\ C(x_2, x_3, 0) &= 0 & ; & \quad 0 \leq x_2 \leq \ell_2 & ; & \quad x_3 > 0 \end{aligned} \quad (2-8)$$

The actual dispersion process is graphically shown in Figure 2-2(b). Harleman and Rumer (1963) gave the following approximate steady state

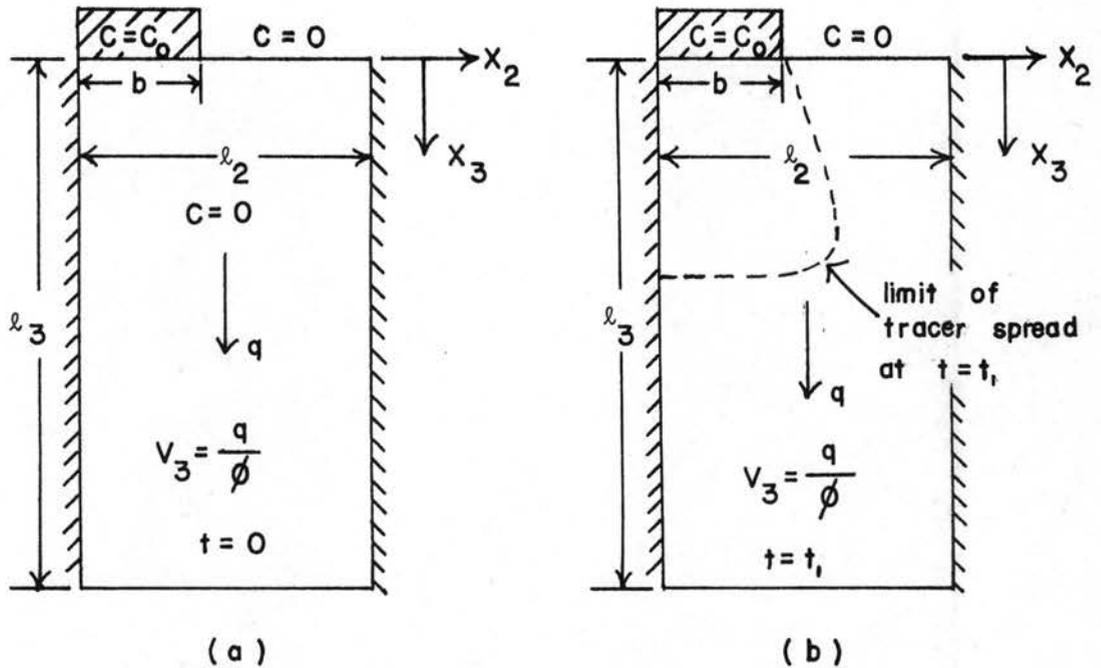


Figure 2-2. Schematic of longitudinal and lateral dispersion.

solution to Equations 2-7 and 2-8:

$$\frac{C}{C_0} = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{x_3 - b}{2 \sqrt{D_T \frac{x_3}{v_3}}} \right) \right] \quad (2-9)$$

Analytic Solution for Radial Convection and Dispersion

The partial differential equation governing radially symmetric diverging flow and the associated convection and dispersion from a well is (Bear, 1972):

$$\frac{\partial C}{\partial t} = v a_I \frac{\partial^2 C}{\partial r^2} - v \frac{\partial C}{\partial r} \quad (2-10)$$

where r is the radial distance from the recharging well and a_I is the longitudinal dispersivity of the porous media expressed as the coefficient of dispersion D_L divided by the seepage velocity v .

For a well of radius r_w injecting a fluid at a constant rate into a confined aquifer, the initial and boundary conditions for Equation 2-10 are:

$$\begin{aligned} C(r,t) &= C_0 \quad ; \quad t > 0 \quad ; \quad r = r_2 \\ C(r,0) &= 0 \quad ; \quad r > r_w \\ C(\infty,t) &= 0 \quad ; \quad t > 0 \end{aligned} \tag{2-11}$$

Because of the nonlinearity of Equation 2-10 (resulting from the fact that v is a function of r), exact analytic solutions are difficult to obtain. However, deJosselin deJong (Lau et al, 1959) obtained the following approximate analytic solution to Equation 2-10

$$\frac{C}{C_0} = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{r - \bar{r}}{\sqrt{\frac{4}{3} a_I \bar{r}}} \right) \right] \tag{2-12}$$

where \bar{r} is the average radius of the body of injected water.

Raimondi et al (1959) suggested an approximate solution to Equation 2-10 based on the assumption that the influence of dispersion becomes small in comparison to the local convective effect as the contaminant moves away from the source. They assumed that the influence of dispersion and diffusion on the concentration distribution as the contaminant moves past any point becomes small as compared to the accumulated effect of dispersion and diffusion that has taken place up to that point. For the case of a well continuously injecting a contaminant of constant concentration they derived the equation:

$$\frac{C}{C_0} = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{r^2/2 - Gt}{\sqrt{\frac{4}{3} a_1 r^3}} \right) \right] \quad (2-13)$$

where G is obtained from the relationship

$$G = \frac{Q}{2\pi h \phi} = vr \quad (2-14)$$

where h is the saturated thickness of a confined aquifer, v is the seepage velocity and ϕ is the porosity.

Dispersion Coefficients

Ebach and White (1958) developed the following empirical relationship for the longitudinal dispersion coefficient based on experiments over a wide range of particle sizes and shapes and where the Reynolds numbers (R) were less than 100:

$$\frac{D_L}{v} = \alpha_1 \left(\frac{vd}{v} \right)^{\beta_1} \quad (2-15)$$

where $\left(\frac{vd}{v} \right) =$ the Reynold's number R

v = the fluid velocity

d = the particle size of the porous media

v = the kinematic viscosity of the fluid

α_1 and β_1 = porous medium coefficients.

They found that α_1 is strongly dependent on the porous medium while β_1 is a function of the flow regime and the porous medium. Various experimental values for α_1 and β_1 have been obtained and are listed in Table 2-1.

TABLE 2-1. Experimentally Determined Values for α_1 and β_1

Reference	α_1	β_1
Harleman and Rumer (1963)	0.66	1.2
Hoopes and Harleman (1965)	1.70	1.2
Ebach and White (1958)	1.92	1.06

Harleman et al (1963) also correlated the longitudinal dispersion coefficient with intrinsic permeability and obtained the following empirical relationship:

$$\frac{D_L}{v} = \alpha_2 \left(\frac{v\sqrt{k}}{v} \right)^{\beta_2} \quad (2-16)$$

where k is the intrinsic permeability. They found $\alpha_2=54$ for spheres and 88 for sand with $\beta_2=1.2$ for both media.

Attempts to fit the lateral dispersion coefficient into a form similar to Equation 2-15 led to the following equation:

$$\frac{D_T}{v} = \alpha_3 \left(\frac{vd}{v} \right)^{\beta_3} \quad (2-17)$$

Experimental values for α_3 range from 0.036 (Harleman and Rumer, 1963) to 0.11 (Hoopes and Harleman, 1965). Both studies estimated β_3 to be 0.7.

Review of Finite Difference Simulation Techniques

Numerical methods approximate the governing differential equations, allowing for high speed solution of a set of equations. While the use of analytic solutions is limited to simplistic problems, the numerical models can be applied to problems involving many source or sink terms and a variety of boundary conditions. To verify the validity of a numerical scheme, it is necessary to compare numerical and analytic solutions for the same problems. A discussion of numerical methods is contained in Conte (1965).

Douglas, Peaceman and Rachford (1959) solved the problem of miscible displacement in a two-dimensional flow field using an alternating-direction implicit scheme. This work was expanded on by Peaceman and Rachford (1962) and by Blair and Peaceman (1963). Two significant problems were often encountered: either the results were effected by numerical dispersion or the solution developed severe oscillations, especially in the regions where the concentration changed rapidly. Numerical dispersion is the error associated with the numerical approximation of the governing differential equation.

In an effort to overcome the problems of instability and numerical dispersion, Stone and Brian (1963) developed a method which consisted of writing a general finite difference equation for the one-dimensional convective-dispersion equation. Their method contained arbitrary weighting coefficients for approximations to the space and time derivatives of the contaminant concentration. With proper choice of weighting coefficients, the model did reduce oscillations and numerical dispersion but could not handle two- and three-dimensional problems.

Shamir and Harleman (1966) took advantage of the Stone and Brian scheme by transforming the two-dimensional dispersion equation into a potential flow coordinate system (i.e., equipotential and streamlines). In this case, the velocity is everywhere tangential to the streamlines and the equation becomes one-dimensional in the convective term. They observed that C/C_0 values greater than 1.0 occurred behind the dispersion front. While the method did exhibit this oscillation, the solution was considered stable since the magnitude of the oscillation was constant with time.

Shamir and Harleman also discussed several basic finite difference approaches of simulating the movement of contaminants in groundwater aquifers. They stated that the fully explicit method of calculating the contaminant concentrations was impractical due to the large amounts of computer time required to solve even very simple problems. They determined that the grid size in the direction of flow must be on the order of a single grain size. They also concluded that the maximum admissible time increment was of the order required for the mean velocity to cover a distance equal to a fraction of the grain size. This particular numerical scheme required that the coefficient of dispersion be known to solve the dispersion equation.

Review of Finite Element Simulation Techniques

The finite element technique supposedly has several advantages over the finite difference technique. Chief among these are the use of smaller amounts of computer time and less storage, additional flexibility available to model irregular shaped basins by using triangular grids as

opposed to square or rectangular grids, and a minimizing of numerical dispersion. Whereas the finite difference technique solves the dispersion equation directly, the finite element technique uses a functional which is minimized for each triangular element. The resulting set of equations is then solved.

Price, Cavendish and Varga (1968) used the Galerkin method for solution of the one-dimensional diffusion-convection equation. They obtained more accurate results while requiring less computer time than central or non-central finite difference approximations and the method of characteristics.

Guyman (1970a) applied the Rayleigh-Ritz finite element technique to the solution of the nonsteady state one-dimensional diffusion-convection equation and later extended it to the two-dimensional case (1970b). The method was not applicable to problems defined by mixed partial differential equations. Later, Guyman (1972) suggested an improvement to his previous solution of the convective dispersion equation but that method displayed numerical dispersion and gave erratic results for small values of the dispersion parameters. It was concluded that the method was not applicable to convection dominated mass transport.

The efforts by Guyman and additional work by Nalluswami (1971) suggested that the method could not be applied to convection dominated transport which is the case for a majority of field situations. In addition, the time domain solution was comprised of inherently explicit schemes and so exhibited convergence and stability problems.

Prakash (1974) applied the Galerkin method to the flow of salt water towards partially penetrating wells located in homogeneous and isotropic aquifers consisting of a fresh water layer overlying a salt water layer. He found that in order to prevent oscillations in the solution of the flow equation, small initial time steps must be followed by gradually increasing time step size. He also found that the number of elements required for reasonably accurate results with a linear interpolation function has to be very large, necessitating huge amounts of computer storage. Due to the combined effect of small initial time steps and the large number of elements, the number of iterations to be performed becomes very large.

Review of Method of Characteristics Simulation Technique

Garder, Peaceman and Pozzi (1964) used the method of characteristics to solve the problem of miscible displacement of an oil-solvent system in order to reduce numerical dispersion. The method gave good results for low oil-solvent viscosity ratios but ignored the tensorial nature of dispersion. However, the method required large amounts of computer time and storage while giving results slightly more accurate than previous, cheaper methods.

Reddell and Sunada (1970) developed flow and convective dispersion equations for non-homogeneous, unsteady flow fields. The flow equation was solved using an implicit numerical technique and the convective dispersion equation was solved using the method of characteristics. This technique required so much excessive storage to catalog information on each moving point that auxiliary storage was required. They observed that the magnitude of error did not converge to a minimum value

regardless of the number of points used per grid in the simple one-dimensional case. They also observed that the concentration profile lagged the actual frontal movement when the moving points remained inside a grid throughout a time step. Use of a weighted-average in the concentration calculations and taking the tensorial nature of dispersion into account allowed for more accurate calculations.

Kraeger (1972) applied the method of characteristics as developed by Reddell and Sunada to a field problem. She observed numerical dispersion inherent in the method of characteristics which did not show up in Reddell and Sunada's study. She attributed this to the grid size used. Reddell and Sunada used grids on the order of fractions of centimeters in size, allowing the actual physical dispersion to partially absorb the numerical smear. Kraeger used grids approximately one-half mile square which clearly indicated the numerical dispersion. She also encountered difficulty in trying a trial-and-error process to arrive at a time increment which produced a concentration distribution resembling the data collected in the field.

Shariatmadar Taleghani (1974) used the method of characteristics to solve the convective-dispersion equation for the case of partially penetrating wells pumping from an aquifer consisting of a fresh water layer underlain by salt water. He obtained accurate results when the concentrations of the moving points were plotted. Kraeger had plotted the concentrations of the stationary grids while neglecting the dispersion process. Therefore, she would have obtained a vertical front regardless of the value of the dispersion coefficient. Shariatmadar Taleghani concluded that the numerical dispersion which Kraeger noted

was a result of the manner in which she plotted the concentration distribution curves and not the grid size used.

Summary of Previous Research

The development of the analytic solutions for dispersion in porous media were major accomplishments. However, their applicability to physical situations is severely restricted. The effect of boundary conditions and inability to handle more than one contaminant source are serious limitations.

The numerical methods which have been developed offer a significant improvement over the analytic solutions but have their own limitations. These methods often require large amounts of costly computer time and storage. Many of the models are affected by instability. In addition, some models require extensive physical data such as longitudinal and lateral dispersion coefficients which are often economically impossible to obtain or which simply are unavailable.

CHAPTER III

THE GROUNDWATER QUALITY MODEL

The groundwater staff of the Civil Engineering Department at Colorado State University recognized the need to develop a numerical approach to simulate two-dimensional flow in aquifers consisting of multiple sources and sinks. The basic philosophy of the model was presented in Bittinger et al (1967). Eckhardt (1976) modified the model to handle confined and unconfined leaky aquifers. Sunada (McWhorter et al, 1977) added provisions to simulate the convection of conservative water quality parameters and called the model WTQUAL1.

Groundwater Flow Equation

The basic non-linear partial differential equation describing two-dimensional transient flow in a saturated porous medium may be derived from the mass continuity equation and Darcy's Law and written as (Jacob, 1950):

$$\frac{\partial}{\partial x} (K_x h \Delta y \frac{\partial H}{\partial x}) \Delta x + \frac{\partial}{\partial y} (K_y h \Delta x \frac{\partial H}{\partial y}) \Delta y = S \frac{\partial h}{\partial t} \Delta x \Delta y + Q \quad (3-1)$$

where h = saturated thickness of aquifer (L)

H = water table elevation above datum (L)

K = hydraulic conductivity (L/T)

S = storage coefficient (dimensionless)

Q = net groundwater withdrawal (L³/T)

x, y = space dimensions (L)

t = time dimension (T).

Equation 3-1 has no general solution. However, by making use of the grid system shown in Figure 3-1, a finite difference approximation of this equation will allow a numerical solution. Equation 3-1 written in implicit, central finite difference form is as follows:

$$\begin{aligned}
 & [AH_{i,j-1} + BH_{i,j+1} + CH_{i-1,j} + DH_{i+1,j} - (A+B+C+D+E)H_{i,j}]^{t+\Delta t} \\
 & = Q - EH_{i,j}^t
 \end{aligned}
 \tag{3-2}$$

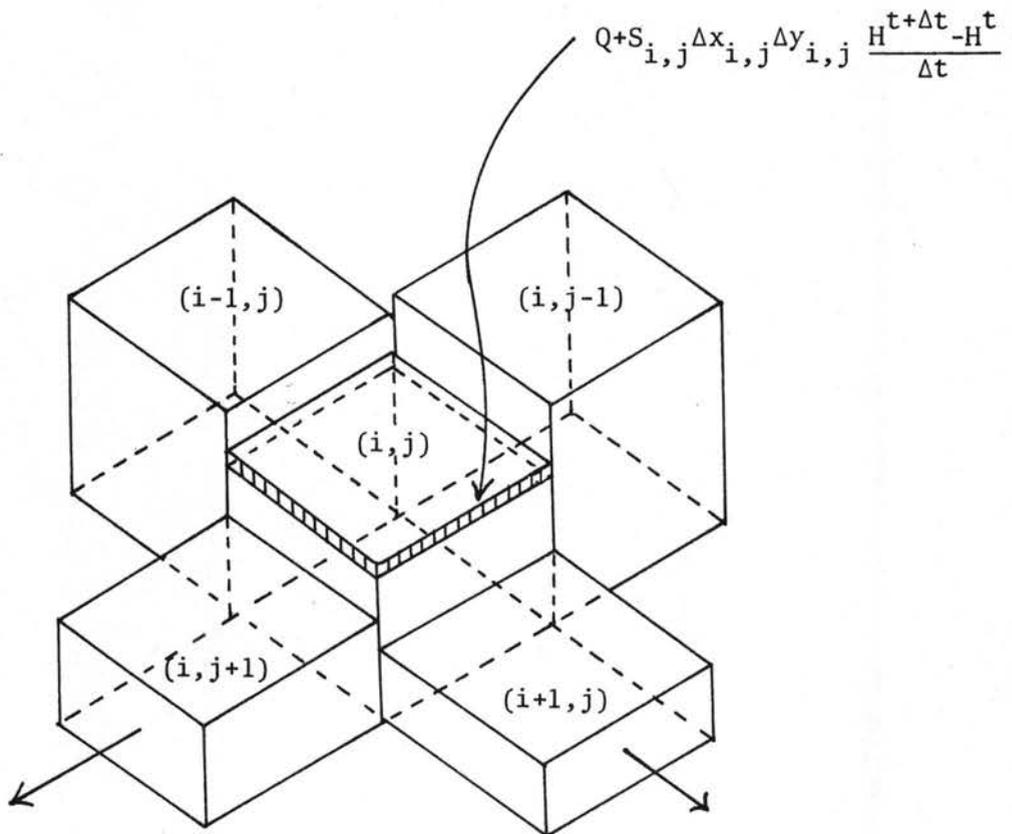


Figure 3-1. Finite difference grid notation (adapted from Bittinger, et al).

where,

$$A = \frac{2K_{i,j} \cdot K_{i,j-1} \cdot \Delta y_{i,j} \cdot \Delta y_{i,j-1} \cdot h_{i,j-1/2}}{\Delta y_{i,j} \cdot K_{i,j} \cdot \Delta x_{i,j-1} + \Delta y_{i,j-1} \cdot K_{i,j-1} \cdot \Delta x_{i,j}}$$

$$B = \frac{2K_{i,j} \cdot K_{i,j+1} \cdot \Delta y_{i,j} \cdot \Delta y_{i,j+1} \cdot h_{i,j+1/2}}{\Delta x_{i,j} \cdot K_{i,j+1} \cdot \Delta y_{i,j+1} + \Delta x_{i,j+1} \cdot K_{i,j} \cdot \Delta y_{i,j}}$$

$$C = \frac{2K_{i,j} \cdot K_{i-1,j} \cdot \Delta x_{i,j} \cdot \Delta x_{i-1,j} \cdot h_{i-1/2,j}}{\Delta y_{i,j} \cdot K_{i-1,j} \cdot \Delta x_{i-1,j} + \Delta y_{i-1,j} \cdot K_{i,j} \cdot \Delta x_{i,j}}$$

$$D = \frac{2K_{i,j} \cdot K_{i+1,j} \cdot \Delta x_{i,j} \cdot \Delta x_{i+1,j} \cdot h_{i+1/2,j}}{\Delta y_{i,j} \cdot K_{i+1,j} \cdot \Delta x_{i+1,j} + \Delta y_{i+1,j} \cdot K_{i,j} \cdot \Delta x_{i,j}}$$

$$E = \frac{S_{i,j} \cdot \Delta x_{i,j} \cdot \Delta y_{i,j}}{\Delta t}$$

Equations 3-1 and 3-2 are subject to the Dupuit-Forchheimer assumptions and also assume that the fluid and porous medium are incompressible.

The subscript notation refers to particular grid blocks in a five-grid system as indicated in Figure 3-1. The superscript t refers to the starting time or previous time level, Δt is the time increment and $t+\Delta t$ is the current time level. Equation 3-2 is written for each grid in the study area for each designated time increment. The system of equations for the first time increment is solved simultaneously for the values of $H_{i,j}$ at the end of the time increment. These computed values of $H_{i,j}$ are then used as initial values in the system of equations representing the next time increment.

The coefficients A, B, C, and D are computed for each grid at the beginning of each time increment and are held constant during the time increment. The term $(h_{i,j-1/2})$ in the equation for coefficient A is the effective saturated thickness between grids $(i,j-1)$ and (i,j) calculated by the following approximation:

$$h_{i,j-1/2} = \text{MAX}(H_{i,j}, H_{i,j-1}) - \text{MAX}(Z_{i,j}, Z_{i,j-1}) \quad (3-3)$$

where Z equals the bedrock elevation above a datum. A similar expression may be written for the h term in the equations for the coefficients B, C, and D. The value for the storage coefficient, S , included in coefficient E, varies spatially but remains constant in time. If the time increment is also constant, coefficient E will remain constant in time for each grid.

The rate of net groundwater withdrawal, Q , represents the deep percolation of precipitation and applied surface water, and the rate of net withdrawal by pumping. The extraction of water by phreatophytes or the addition of water by artificial recharge could also be included in the value of Q . It is necessary to calculate an average value of Q for each grid for each time increment.

Water Quality Aspects of WTQUAL1

WTQUAL1 was developed primarily to model the convection of contaminants which are picked up when water flows through strip mine tailings. The model allows for a continuous contaminant source in those cases where the contaminants go into solution over a long period of time. The model considers as slug sources those cases where the contaminants go into solution in a relatively short period of time.

WTQUAL1 uses the fully explicit method to determine relative contaminant concentrations during each time increment. However, the model only allows consideration of water quality parameters within the aquifer itself. Other contaminant sources such as rivers, lakes, irrigated land and artificial recharge areas cannot be considered when using WTQUAL1.

The fully explicit method, which assumes complete and instantaneous mixing of the waters in the aquifer, satisfies the objective of a simplistic convective transport simulator. In addition, the fully explicit method as utilized in WTQUAL1 provides results of sufficient accuracy to render the numerical simulator a valid approach.

Since it was desirable to develop a simplistic simulator of convection in groundwater aquifers, it was felt that WTQUAL1 offered an excellent starting point. Because WTQUAL1 was based on a groundwater flow model, all the required inputs and outputs of the groundwater system, the geologic parameters and a provision for either slug or continuous contaminant injection were available. The primary modification required was to make provisions for including and varying input contaminant concentrations for each source variable for each desired time increment to be studied. This involves estimating relative concentrations of contaminants in rainfall, water applied as irrigation, constant head sources (e.g., lakes, rivers, ponds), artificial recharge areas, and the underflow into the aquifer from outside the area being studied.

Modifications to WTQUAL1

The first modification to WTQUAL1 involved providing for the input of initial contaminant concentrations for all source waters. This was accomplished by expanding Subroutine READPH (see Appendix B for a flowchart of the computer program and Appendix C for a description of all subroutines). Provisions were made for reading in, as either slug or continuous sources, initial contaminant concentrations for precipitation, water applied as irrigation, water

artificially recharged to the aquifer, and constant head sources such as rivers, lakes and ponds. These initial concentrations can be input as a constant throughout the study area or can be varied from grid to grid.

The second modification, which is actually an extension of the previous one, involved providing for the change of the contaminant concentrations of all the source waters at each time increment of analysis. This was accomplished by adding Subroutine READC to the program. A controlling variable, AGGIE, is included in the initial data input to the numerical model. Depending on the value of AGGIE, the model either uses the initial contaminant concentrations for each time increment of analysis or control is transferred to READC at the beginning of each time increment and new relative concentration values are read in. Concentrations for flows through boundary grids into the study area are included in this data input.

The next modification occurred in Subroutine QFIX. In order for the relative contaminant concentration calculation to be made later in the program, it was necessary to convert all flows to a consistent volumetric unit and to identify and store this volume for each source and sink for each grid in the study area. These terms are then retrieved by Subroutine BYFLOW where the actual concentration calculations are made.

Due to the structure of Subroutine BYFLOW, the concentration calculation process was broken into two steps. First, an intermediate change in contaminant mass for the current time level was computed based on all source and sink terms except constant head grids. This step includes the actual flow of groundwater within the aquifer. Then, all constant head

contributions to each grid were calculated. A new total change in contaminant mass was computed, converted to a relative concentration value and combined with the relative concentration value at the previous time step to give the new relative concentration value for each grid.

For confined aquifers, the model automatically excludes contaminants resulting from precipitation, water applied as irrigation, water artificially recharged to the aquifer, and phreatophyte consumption. However, contaminants removed by pumping and added by constant head sources in direct contact with the aquifer are included.

These modifications make the numerical model suitable for application to many problems involving either confined or unconfined aquifers. This modified version of WTQUAL1 is called WTQUAL2.

Explicit Contaminant Mass Balance Equation

The fully explicit mass balance equation used in calculating the contaminant concentrations is very similar to that used in the development of the basic groundwater flow equation. In general terms, the mass balance equation can be written as:

$$\left[\begin{array}{l} \text{RATE OF CHANGE} \\ \text{OF CONTAMINANT} \\ \text{IN THE AQUIFER} \end{array} \right] = \left[\begin{array}{l} \text{RATE OF CONTAMINANT} \\ \text{INFLOW TO THE} \\ \text{AQUIFER} \end{array} \right] - \left[\begin{array}{l} \text{RATE OF CONTAMINANT} \\ \text{OUTFLOW FROM THE} \\ \text{AQUIFER} \end{array} \right] \quad (3-4)$$

where the assumption is made that the rate of contaminant removal within the aquifer is zero (i.e. we are modeling a conservative substance).

The grid system used is identical to that shown in Figure 3-1. However, for more detailed illustration, a typical grid is shown in Figure 3-2 indicating the various parameters which effect the mass

balance calculation. Neglecting the dispersion process, the applicable explicit mass balance equation in finite difference form is:

$$C_{i,j}^{t+\Delta t} = C_{i,j}^t + [(V^{t+\Delta t} \cdot C^t)_{i,j-1} + (V^{t+\Delta t} \cdot C^t)_{i-1,j} - V_{i+1,j}^{t+\Delta t} \cdot C_{i,j}^t - V_{i,j+1}^{t+\Delta t} \cdot C_{i,j}^t + (W \cdot C)_{i,j}^{t+\Delta t} - VLEAK_{i,j}^{t+\Delta t} \cdot C_{i,j}^t] / [U_{i,j}^{t+\Delta t}] \quad (3-5)$$

where $C_{i,j}$ = relative contaminant concentration in the grid i,j
 $V_{i,j-1}$ = volume of flow from grid $i,j-1$ to grid i,j
 $C_{i,j-1}$ = relative contaminant concentration corresponding to $V_{i,j-1}$
 $V_{i-1,j}$ = volume of flow from grid $i-1,j$ to grid i,j

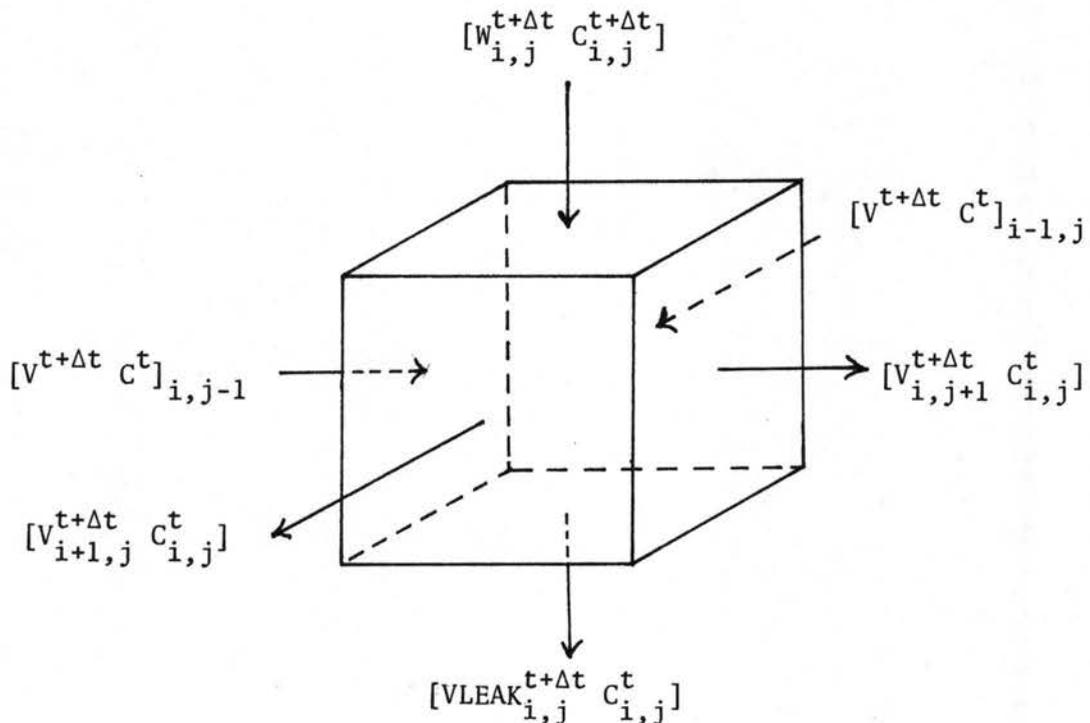


Figure 3-2. Typical aquifer grid illustrating parameters which influence a change in concentration.

$C_{i-1,j}$ = relative contaminant concentration corresponding to
 $V_{i-1,j}$
 $V_{i+1,j}$ = volume of flow from grid i,j to grid $i+1,j$
 $V_{i,j+1}$ = volume of flow from grid i,j to grid $i,j+1$
 $VLEAK_{i,j}$ = volume of flow from grid i,j through the leaky layer
 beneath the grid (applicable only if leaky aquifer
 conditions exist)

$U_{i,j}$ = total volume of water stored in grid i,j .

The term $(W \cdot C)_{i,j}^{t+\Delta t}$ is determined from the following relationship:

$$\begin{aligned}
 (W \cdot C)_{i,j}^{t+\Delta t} = & [(V \cdot C)_{PPT} + (V \cdot C)_{RCHR} + (V \cdot C)_{APW} + (V \cdot C)_{SQR} \\
 & - (V \cdot C)_{PHR} - (V \cdot C)_{PUM}]_{i,j}^{t+\Delta t} \quad (3-6)
 \end{aligned}$$

where the volumes, V , and relative contaminant concentrations, C , apply to precipitation (PPT), artificial recharge (RCHR), water applied as irrigation (APW), recharge from constant head sources such as rivers, lakes and ponds (SQR), phreatophyte consumption (PHR), and pumping from wells (PUM). This equation encompasses the modifications made to WTQUAL1 allowing the contaminant concentrations of all source and sink waters to be taken into account.

Description of the Numerical Model

WTQUAL2 uses the fully implicit, central difference technique to predict transient, two-dimensional areal groundwater level (or piezometric head) fluctuations and the corresponding flows. Based upon these flows, the model uses the fully explicit mass balance technique to simulate the convection of contaminants through the aquifer.

The study area is overlain with a grid system. The selection of grid dimensions are dependent upon the stability criteria of the concentration calculation (see Chapter IV). The rectangular grid system is oriented to allow for easy boundary approximation, provide for easy adaption of hydrologic and geologic data, and to meet the required stability criteria.

The program reads in the number of rows and columns for the entire grid system, including those of the buffer zones which are built into the program (Olson, 1973). The desired time increment of analysis, total time of analysis, and time increment printout are also input to the program.

The dimensions of each grid and values for hydraulic conductivity, bedrock elevation, ground surface elevation, storage coefficient or specific yield, coefficient for the fraction of each grid that is irrigated, initial relative concentrations for all source waters, and initial relative concentrations for each of the grids in the aquifer are read as input data. All values are held constant throughout the time of analysis except for source water concentrations which may be changed at the beginning of each time step. New values of the aquifer concentrations are calculated for each time increment based on the previous concentration of each grid and the addition or loss of contaminant during the time period Δt .

Contaminant concentrations are read in and calculated as relative concentrations ranging from 0.0 to 1.0. Normally, a source concentration is considered to have a value of 1.0. However, if concentrations are anticipated which might exceed a source concentration, then an arbitrary

value can be assigned to the relative concentration value of 1.0 and all other values will be referenced to it.

The initial water table (or piezometric head) elevations are also read in for each grid. Impermeable boundary grids, constant head boundary grids, and grids with horizontal underflow are identified by coding the initial water table elevations. For boundary grids having underflow, the difference in water elevation between the outermost boundary grid and the next inner grid is held constant throughout the total time of the analysis (i.e., a constant hydraulic gradient is maintained).

The program also reads in hydrologic data for annual precipitation (the model assumes a uniform depth of precipitation over the entire study area), annual water applied as irrigation (the model assumes a uniform application of the water over the irrigated portion of the grid in question), annual phreatophyte extraction, gross annual pumping withdrawal, and annual application of water to recharge pits. The annual precipitation, irrigation, phreatophyte, pumping, and recharge values are read in for each grid of the study area for the year to be analyzed. One set of annual distribution coefficients is read in for each of the five types of hydrologic data. The coefficients represent the percentage of annual precipitation, irrigation, phreatophyte consumption, pumping, and recharge that occurs during each of the time increments. The coefficients are read in initially and remain constant throughout one year of analysis but may be changed at the beginning of each additional year of analysis.

The program also reads in coefficients that represent the percentage of precipitation, applied water, and recharge water that percolates to the water table. Another coefficient is read in to represent the percentage of the gross pumping withdrawal that does not return to the water table.

The program uses the Gauss elimination method to solve the system of equations for each time step. The program output at desired time steps includes the following:

1. Matrix of net vertical withdrawal of water from each grid including precipitation, applied water, pumping, artificial recharge, phreatophyte consumption and leakage.
2. List of overdrawn or flooded grids.
3. List of grids, if any, which change from confined to unconfined or unconfined to confined.
4. Matrix of discharge between grids in the *i*-directions. Flow down is considered positive and flow upward is negative. Discharge in the first row of the matrix is the flow between grids in row 1 and 2, and so on for the remainder of the grids. Therefore, the value in the last row is always zero.
5. Matrix of discharge between grids in the *j*-direction. Flow right is positive and flow left is negative. Discharge in the first column of the matrix is the flow between grids in column 1 and 2 and so on for the remainder of the grids. Therefore, the value in the last column is always zero.
6. Matrix of net flow from constant head grids.

7. Table of water balance computations.
8. Matrix of water table or piezometric head elevations.
9. Matrix of relative contaminant concentrations.

CHAPTER IV

NUMERICAL SIMULATION OF CONVECTIVE TRANSPORT

As mentioned in the introduction, a major purpose of this research was to study a simplistic numerical technique to simulate convection of contaminants in groundwater aquifers. This attempt at a simplistic model is based on the fully explicit method of calculating contaminant concentrations. In order to compare numerical and analytic results, the aquifer studied must have both numerical and analytic solutions.

A hypothetical one-dimensional, homogeneous and isotropic, steady state situation was developed using representative values of aquifer properties for a coarse sand commonly encountered in actual physical situations (McWhorter and Sunada, 1977). The assumptions and calculations associated with the development of the hypothetical situation are discussed in Appendix A and a schematic of the layout is shown in Figure 4-1. In order to assure that flow was steady state, the saturated thickness of the aquifer was held constant and a constant hydraulic gradient was maintained.

Verification of the Longitudinal Convection Case

The longitudinal convection case was verified using a one-dimensional, steady state flow situation with a constant contaminant source located along the inflow boundary of the model. Hydraulic conductivity was uniform throughout the model and the piezometric head was oriented to provide a constant gradient in the direction of flow

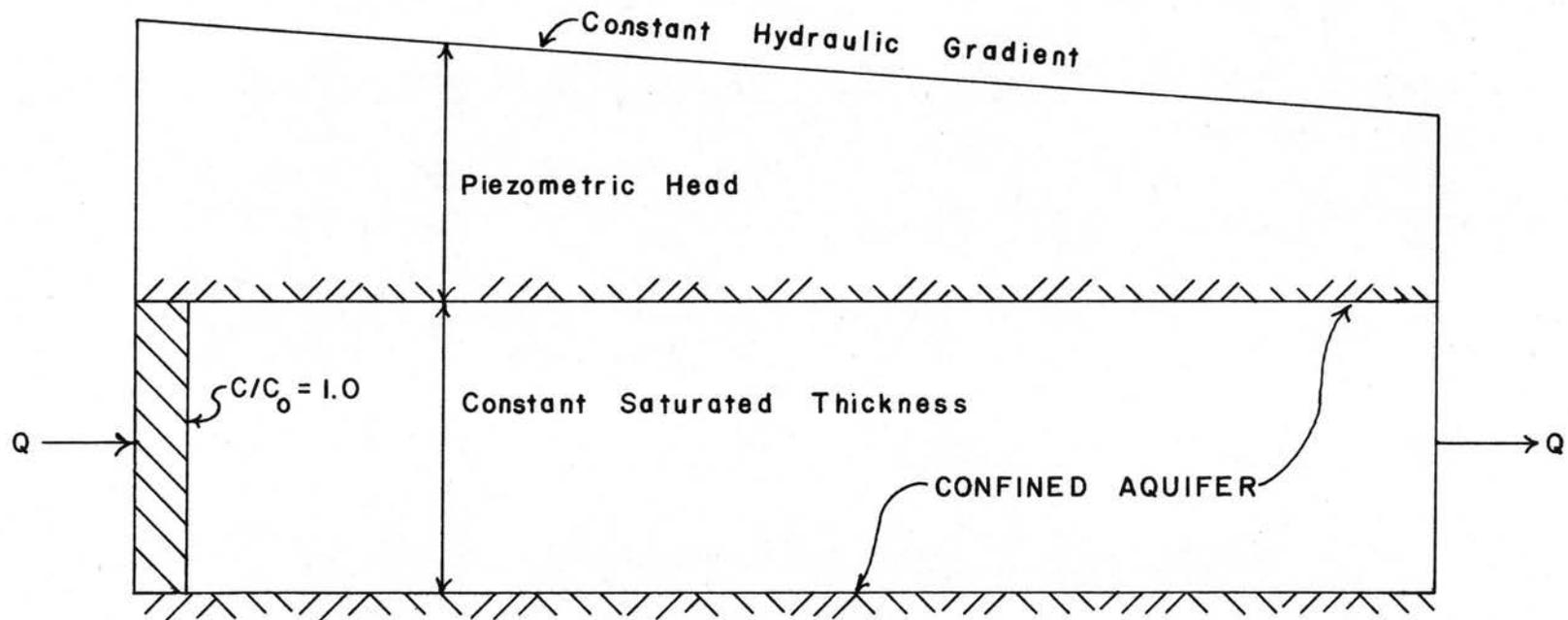


Figure 4-1. Schematic of the hypothetical aquifer with constant contaminant source.

and zero gradient perpendicular to the direction of flow. Boundaries parallel to the direction of flow were considered impermeable. This results in a constant seepage velocity v being maintained throughout the aquifer in the direction of flow.

The numerical results were compared with those derived from Equations 2-5 and 2-6. The results for time equal 90, 180, 270, and 360 days and grid size equals 115 feet for a time increment of 30 days and grid size equals 40 feet for a time increment of 10 days are shown graphically in Figures 4-2 and 4-3, respectively.

The analytic solution is at all times located a distance equal to the product of the seepage velocity and the elapsed time from the contaminant source. The numerical solution satisfies this condition for the case where C/C_0 has a value of approximately 0.5.

The numerical model yielded relative concentration values which decreased gradually with increasing distance from the contaminant source. The shape of the numerically determined curve can be attributed to the numerical dispersion inherent in the model. This numerical dispersion is a result of the error which occurs from numerically approximating the governing differential equation. It is a function of the numerical model and is independent of the aquifer properties.

The abrupt change of the analytic solutions shown in Figures 4-2 and 4-3 is a result of neglecting the dispersion process. Therefore, the curves shown are actually vertical lines. The analytic solutions including the dispersion process were calculated but not plotted. The shape of these curves could not be distinguished from the curves neglecting the dispersion process.

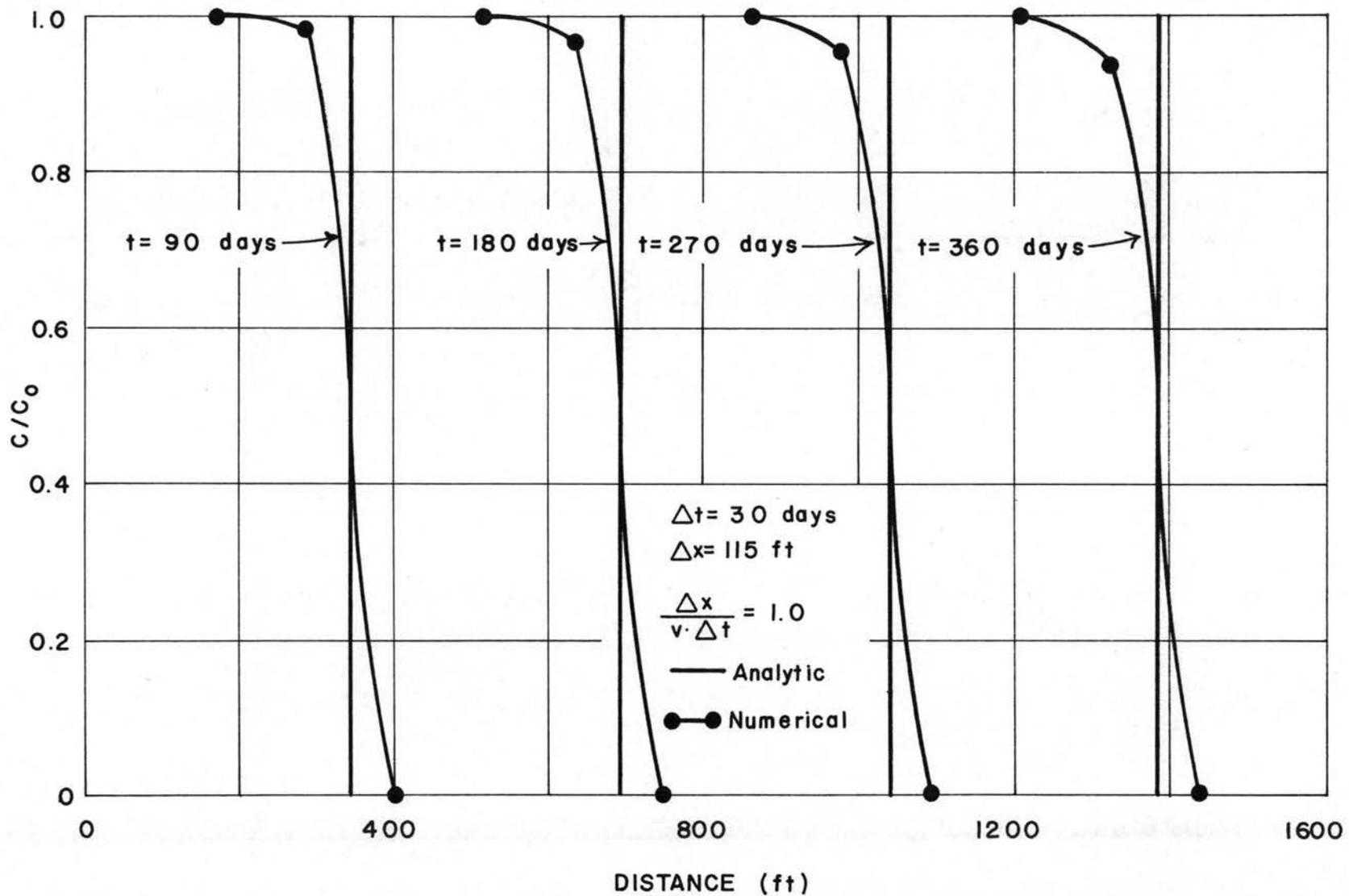


Figure 4-2. Numerical versus analytic solutions for $\Delta x/v\Delta t=1$ and $\Delta t=30$ days.

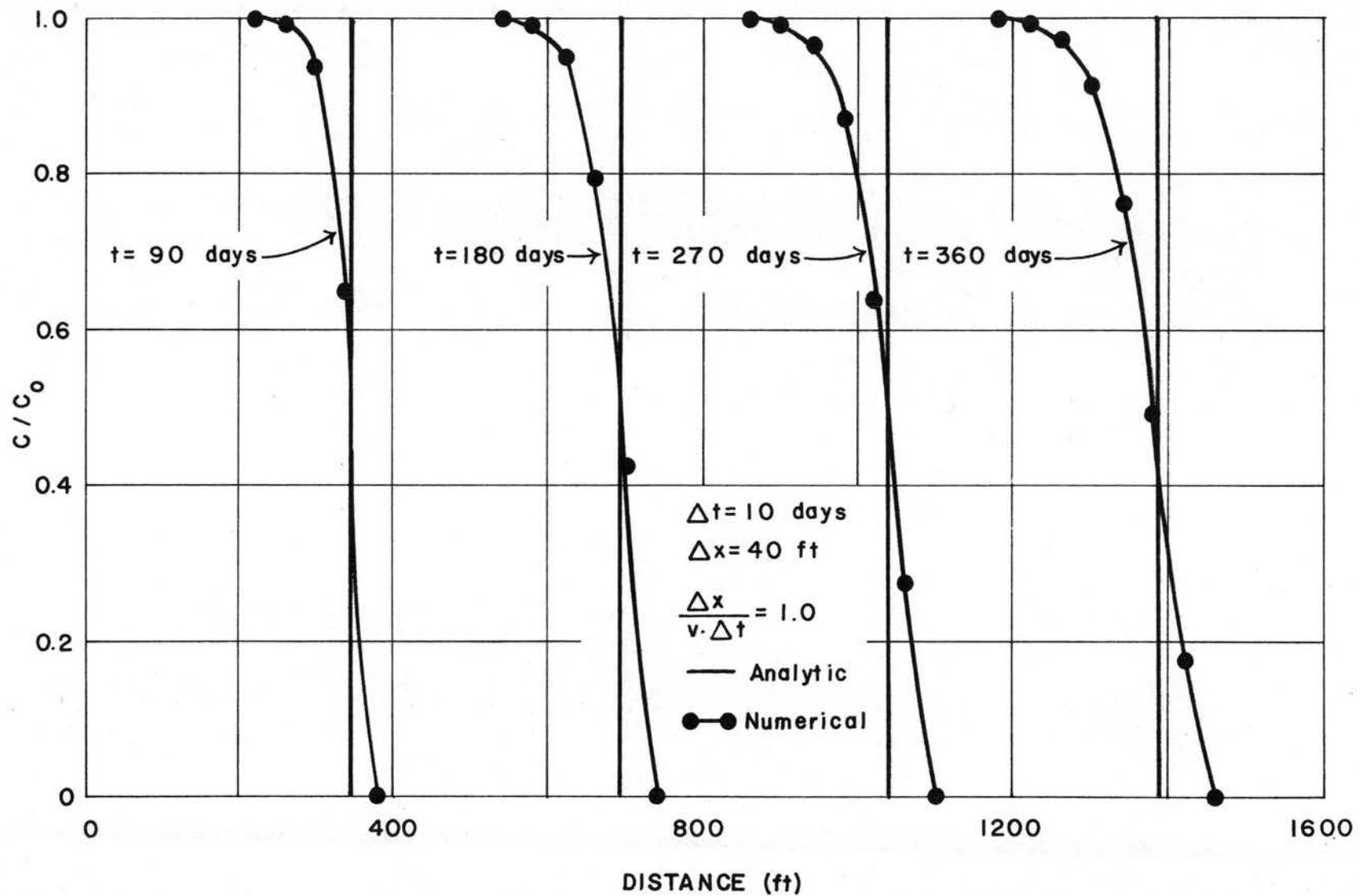


Figure 4-3. Numerical versus analytic solution for $\Delta x/v\Delta t=1$ and $\Delta t=10$ days.

Previous research indicates that analytic solutions are usually S-shaped curves similar to the numerical solutions shown. The seepage velocities and the associated dispersion coefficients for the analytic solutions used in this study are typical for actual physical situations and are low relative to those used by many previous researchers. As a result, the analytic solutions take on essentially vertical profiles.

The results shown in Figures 4-2 and 4-3 indicate that the method used to calculate the relative concentrations of the contaminant is a valid method, especially in the region near the point where $C/C_0=0.5$. For the particular cases illustrated, the method gives results to within approximately ± 10 percent for all relative concentration values $0 \leq C/C_0 \leq 1.0$ at time equal 360 days.

In addition, Figures 4-2 and 4-3 indicate that the results are stable for all times. Thus, the accuracy of the solution does not decrease with time. If the initial error which is introduced during the very early time steps when large concentration gradients are present can be minimized, then a high degree of accuracy can be maintained throughout the period of study.

Verification of the Radial Convection Case

To show that the numerical model is applicable to problems other than simple one-dimensional flow, the model was run for a simplistic two-dimensional case. This involved simulating the injection of a contaminant into a confined aquifer through a recharge well. A constant rate of flow containing a conservative contaminant was injected into a homogeneous and isotropic confined aquifer and the convection of the contaminant was radially symmetric about the location of the well.

The model was first run for a square grid network where all grids were of a uniform size. The center grid of the system was used to simulate a recharge well by maintaining a constant head throughout the period of analysis. This resulted in radial, diverging flow into the aquifer from the recharge grid. The flow rate from the recharge grid into the aquifer remained constant for all time. The piezometric head of all grids surrounding the recharge grid were initially level and a uniform constant head was maintained on all boundary grids.

The concentration distribution curves for various times and the associated analytic solutions for Equation 2-13 neglecting dispersion are shown in Figure 4-4. It can be seen that the numerical solutions lag the analytic solutions by a large but relatively constant value. This can be attributed to the fact that the numerical model (using a rectangular coordinate system) is trying to simulate purely radial, diverging flow. The fact that the model only approximates this condition results in the errors shown.

In order to minimize this problem, a run was made for an almost identical hypothetical situation except the grid sizes were varied radially, from small dimensions near the recharge grid to larger grids on the edge of the grid network. The results for this condition are plotted in Figure 4-5. These results are better with regard to the location of the point where $C/C_0=0.5$, improving on the results shown in Figure 4-4.

Semi-logarithmic plots were made of piezometric head versus radial distance at time equal 270 days for both the uniform and variable grid size problems. While the results showed that neither solution was

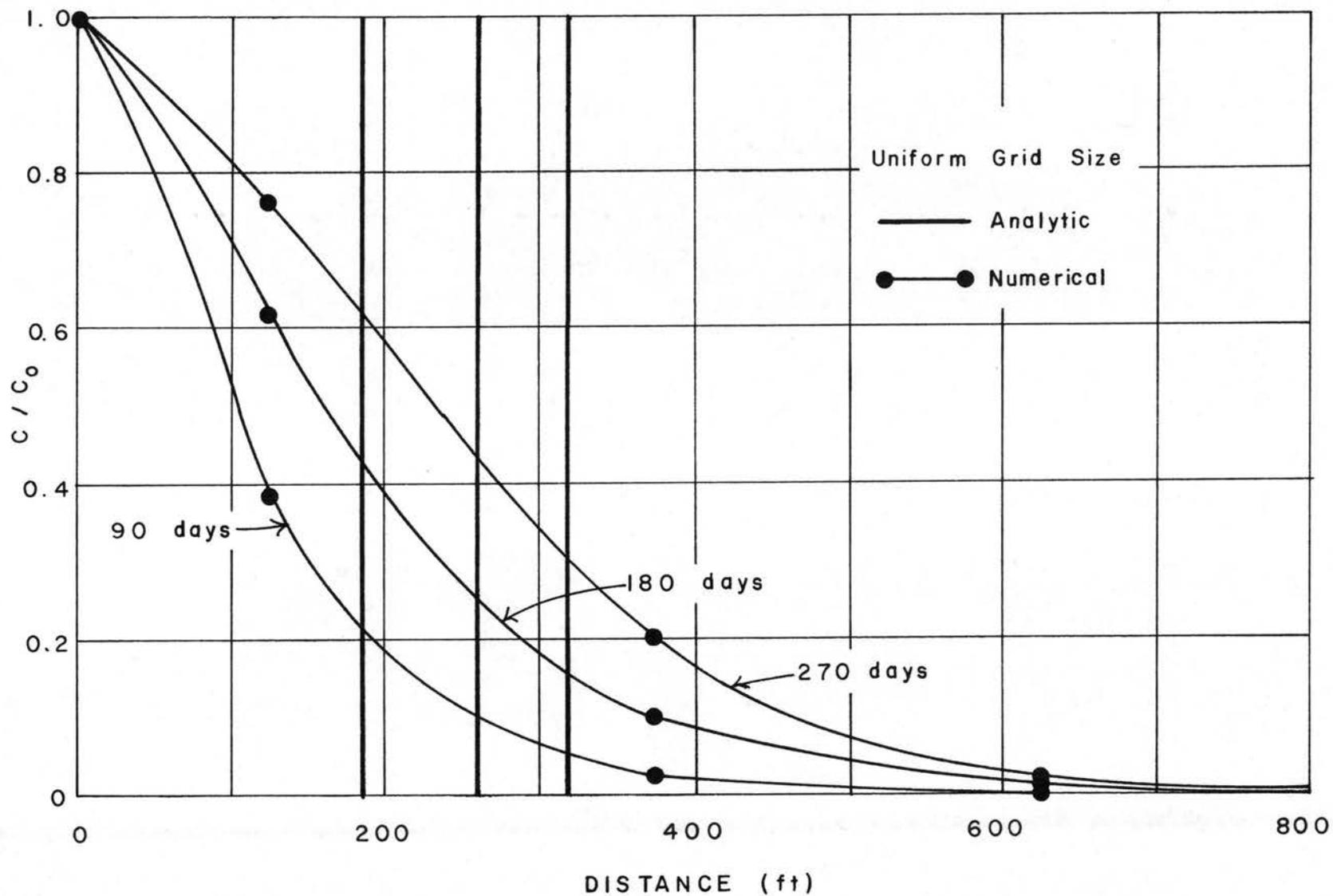


Figure 4-4. Concentration distribution curves for radial flow in a uniform size grid network.

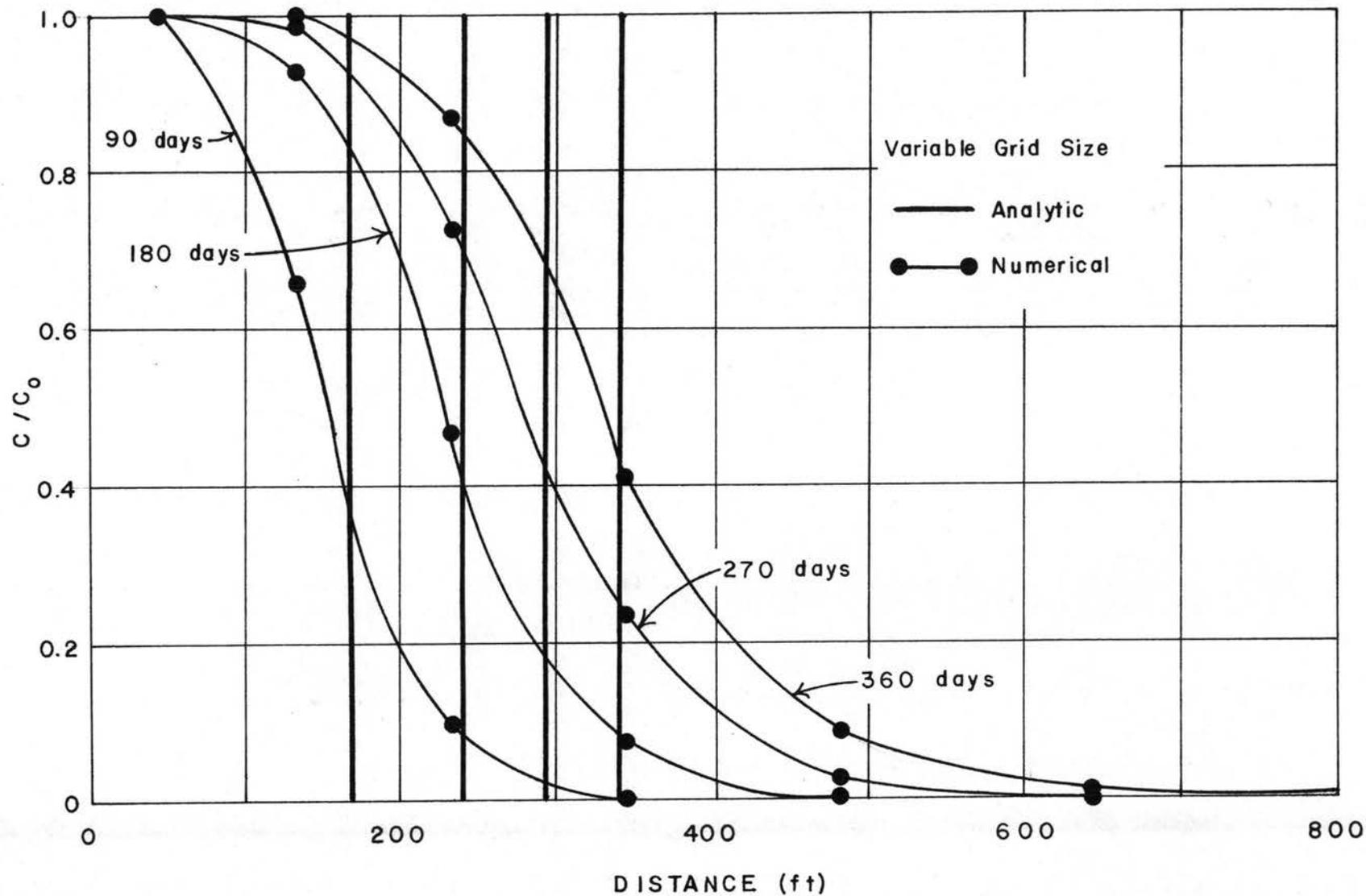


Figure 4-5. Concentration distribution curves for radial flow in a variable size grid network.

exact with regard to the radial flow case, both numerical solutions gave good approximations at points located away from the recharge grid. The curve for the variable grid size problem became linear at a radial distance of approximately 200 feet while the uniform grid size problem did not become linear until a radial distance of approximately 500 feet had been reached. The fact that the variable grid size problem gives a better approximation of the radial flow case nearer the recharge grid explains the reason for the improved accuracy over the uniform grid size problem.

The concentration distribution curves in Figures 4-4 and 4-5 exhibit significant amounts of numerical dispersion. As time increases the magnitude of the numerical dispersion increases. However, the error in distance between the numerical and analytic solutions at the point where $C/C_0=0.5$ remains constant with increasing time.

For radially symmetric, diverging flow with a constant flow rate, the velocity of the fluid decreases with increasing distance from the recharge grid. It will be shown in Chapter V that the degree of numerical dispersion is a function of the grid size, time increment and seepage velocity.

Stability Criteria

The general equation governing longitudinal convection and dispersion, which was discussed previously, is

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - v \frac{\partial C}{\partial x} \quad (2-2)$$

For the purpose of developing the simplistic model, the term $D_L \frac{\partial^2 C}{\partial x^2}$ was assumed to be zero. This reduced Equation 2-2 to

$$\frac{\partial C}{\partial t} = -v \frac{\partial C}{\partial x} \quad (4-1)$$

Using the backward finite difference expansion, Equation 4-1 becomes

$$\frac{C_i^t - C_i^{t+\Delta t}}{\Delta t} = -v \frac{C_i^t - C_{i-1}^t}{\Delta x} \quad (4-2)$$

Rearranging Equation 4-2 and solving for $C_i^{t+\Delta t}$ yields

$$C_i^{t+\Delta t} = \frac{v \cdot \Delta t}{\Delta x} [C_{i-1}^t - C_i^t] + C_i^t \quad (4-3)$$

Noting that contaminant concentrations are at all times between zero and one, and that the worst condition is given by $C_i^t=0$, $C_{i-1}^t=1.0$ and $C_{i-1}^{t+\Delta t}=1.0$, Equation 4-3 reduces to

$$1 = \frac{v \cdot \Delta t}{\Delta x} [1 - 0] + 0 \quad (4-4)$$

Simplifying this equation leads to the stability criteria

$$\frac{\Delta x}{v \cdot \Delta t} \geq 1.0 \quad (4-5)$$

An identical criteria was developed empirically and is discussed below.

Upon examining the runs made using the numerical model, it was noted that the volume of water flowing through a finite difference grid during each time increment must not exceed the volume of water stored in the grid during the time increment under study or severe oscillation and instability of the solution would occur. Mathematically, this necessary condition for the one-dimensional flow case may be expressed as

$$Q \leq \Delta x \cdot \Delta y \cdot h \cdot \phi \quad (4-6)$$

where Q = volume of flow through the grid

$\Delta x, \Delta y$ = grid dimensions

h = saturated thickness

ϕ = porosity.

Noting that the volume of flow through the grid may be expressed using Darcy's law, the grid dimensions and the time increment, Equation 4-6 may also be written as

$$v \cdot \Delta y \cdot \Delta t \cdot h \cdot \phi \leq \Delta x \cdot \Delta y \cdot h \cdot \phi \quad (4-7)$$

where v is the seepage velocity and Δt is the time increment.

Cancelling like terms and rearranging this equation, we get

$$\frac{\Delta x}{v \cdot \Delta t} \geq 1.0 \quad (4-8)$$

This relationship (which is identical to that developed by expanding the governing partial differential equation) indicates that the grid dimension in the direction of flow must at all times be greater than or equal to the seepage velocity times the time increment. If this criteria is not met, severe oscillations and instability, as shown in Figures 4-6 and 4-7, will occur. It can also be shown that the greatest accuracy of the model occurs when $\frac{\Delta x}{v \cdot \Delta t} = 1.0$. Figures 4-2 and 4-3, discussed earlier, are plots of two specific instances where this stability criteria has a value of unity.

Conservation of Contaminant Mass

Contaminant mass within the groundwater system is conserved at all times. This is a result of using the fully explicit method to calculate relative contaminant concentrations (i.e. mass of contaminant within each

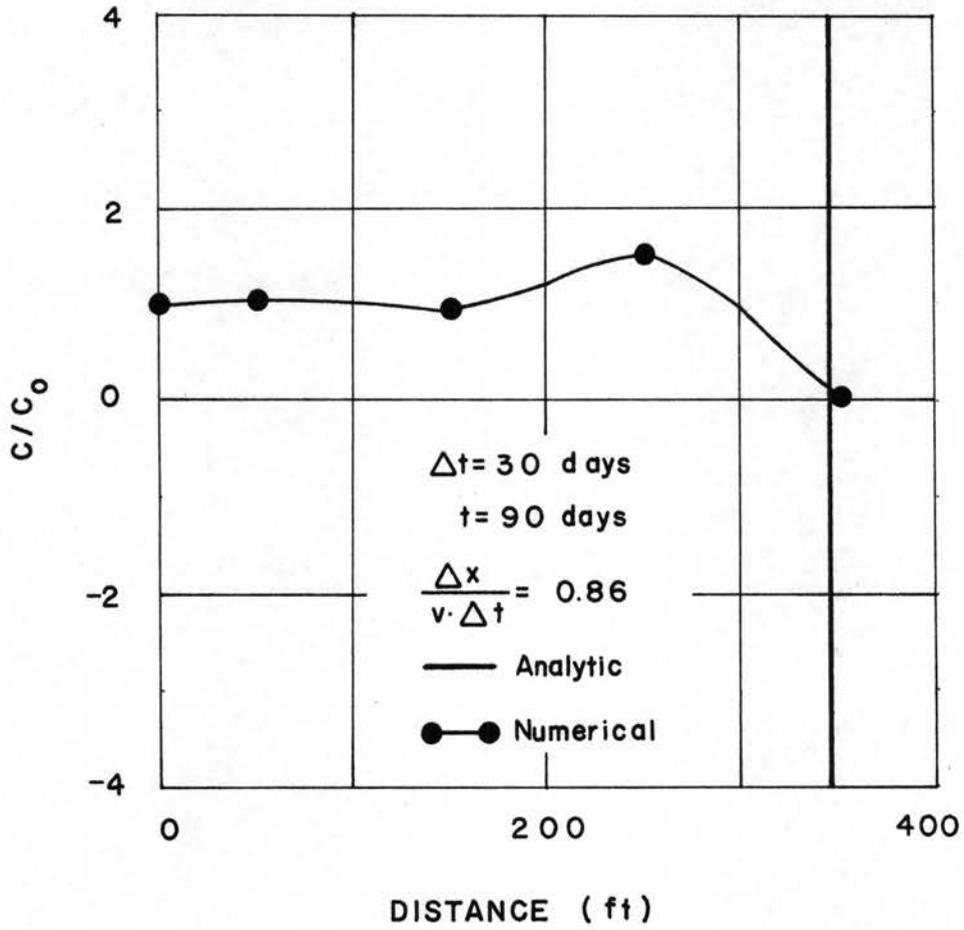


Figure 4-6. Oscillation of numerical solution for $\frac{\Delta x}{v \cdot \Delta t} = 0.86$ after 90 days.

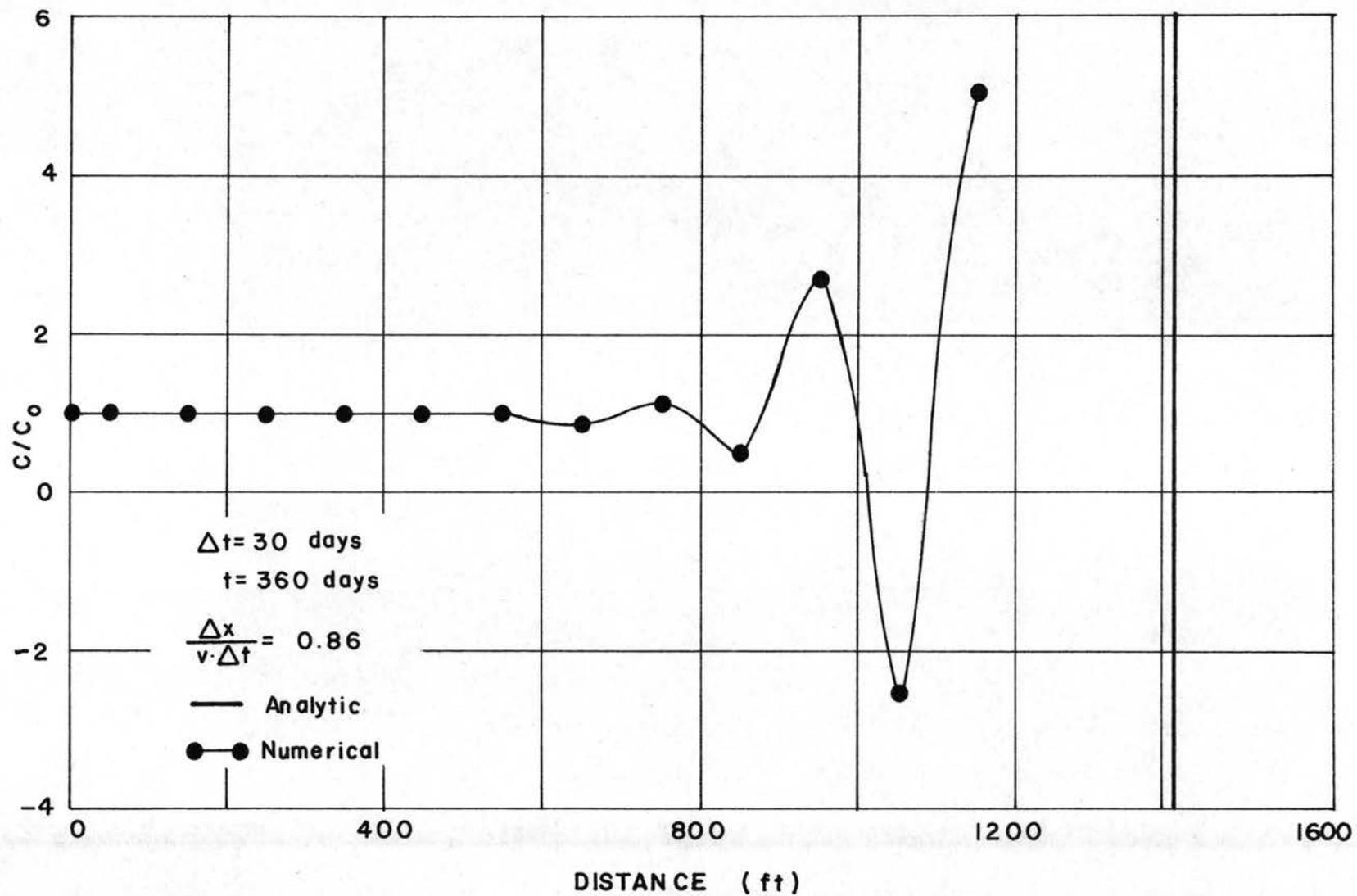


Figure 4-7. Oscillation of numerical solution for $\frac{\Delta x}{v \cdot \Delta t} = 0.86$ after 360 days.

grid). Chapter III contains a detailed discussion of the concentration calculation process.

All C/C_0 values range between 0.0 and 1.0. Since concentrations beyond the third significant figure are not normally of interest, the model only prints out concentrations to three decimal places. Many computers carry numbers to 10 or 15 significant figures. Therefore, grids with a printed concentration of 0.000 often have small contaminant concentrations. In order for a total conservation of contaminant mass to occur, these small concentrations must always be taken into account.

CHAPTER V
SENSITIVITY ANALYSIS OF THE NUMERICAL MODEL

The response behavior of the groundwater system to the convection of a contaminant may be influenced by many input variables and the interaction of many system parameters. The number of possible combinations of these factors is infinite -- not only in terms of magnitude of the factors, but also variations in time and space. It is seldom economically feasible to quantitatively evaluate all or most of the input variables and system parameters with precision. It is, however, very important that the effect of these variables on the accuracy of the model be known.

While most aquifers are not homogeneous and isotropic, these basic assumptions were made in order to simplify the development of the numerical model. As such, sensitivity of the numerical model to variations in the aquifer properties (i.e., permeability, stratification, porosity, storage coefficient, etc.) will not be discussed here. The parameters grid size, time increment and seepage velocity are the primary components of the stability criteria. The effects of these properties on the accuracy of the numerical model should be studied prior to the effects of the aquifer properties.

Grid Size

For the purpose of analyzing the model's sensitivity to grid size, time increments of 10, 30 and 45 days were chosen. Previous work with the basic groundwater flow model indicated that these time increments yield results of sufficient accuracy for the groundwater flow. Also, a total model time of 360 days was chosen to compare the results of the various grid sizes.

Figure 5-1 shows the concentration distribution curves for grid sizes 115, 300, 500 and 1000 feet when the time increment is 30 days. It can be seen that the numerical dispersion increases with increasing grid size. As grid size increases, the numerical solution, as evidenced by the point where $C/C_0=0.5$, lags the analytic solution by an increasing amount. However, the calculated distance where $C/C_0=0.5$ for the 1000 foot grid size only lags the analytic solution by approximately 10 percent. Table 5-1 illustrates the percent error of distance for each grid size at various C/C_0 values.

TABLE 5-1. Value of C/C_0 Versus Percent Error in Calculated Distance

C/C_0	PERCENT DISTANCE ERROR			
	115'	300'	500'	1000
0	3.6	82.9		
0.2	1.4	30.2		
0.4	0	7.2		
0.5	0	- 1.4	-3.6	-11.0
0.6	0	- 9.3		
0.8	- 1.4	- 31.0		
1.0	-11.5	-100.0		

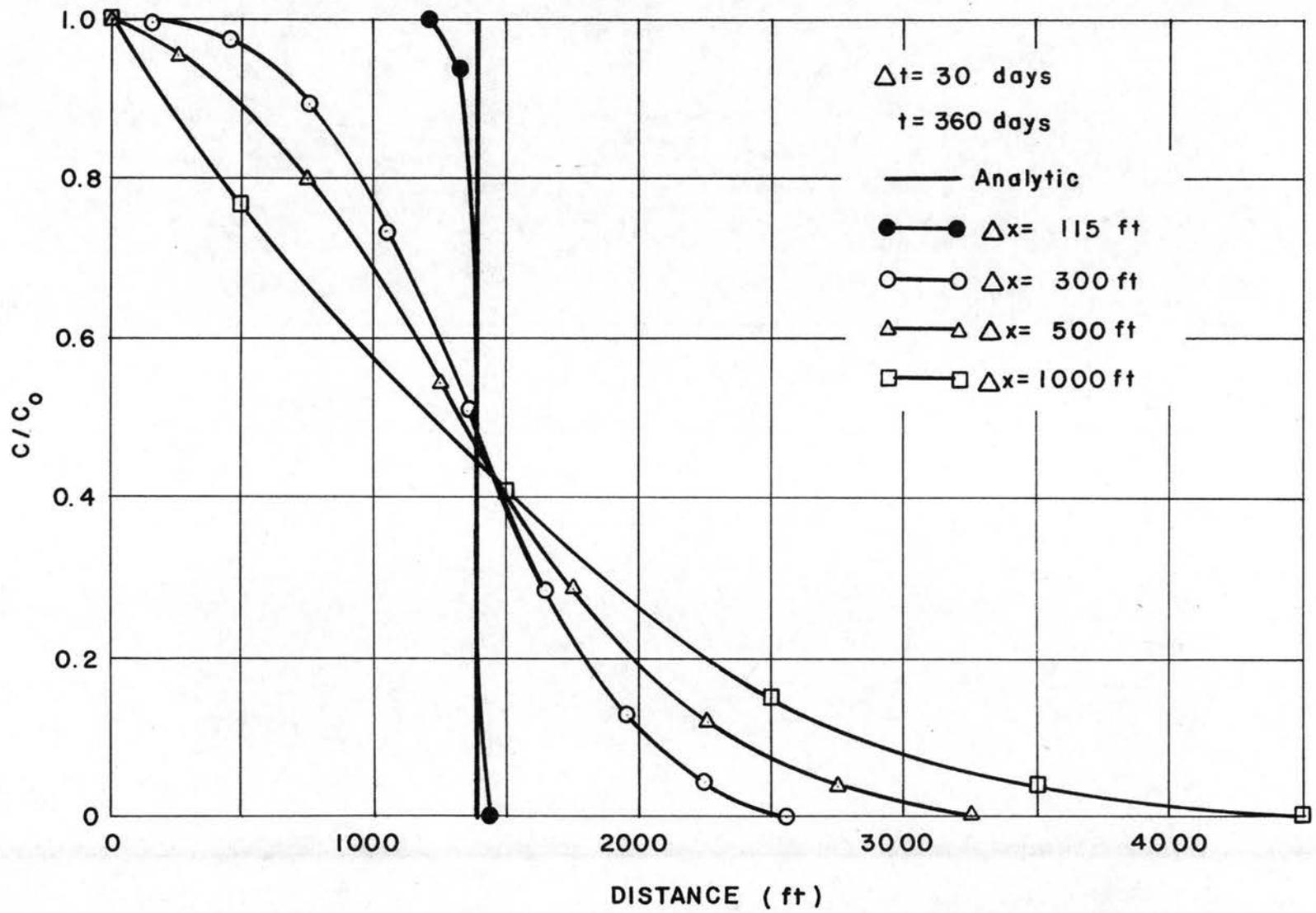


Figure 5-1. Concentration distribution curves for various grid sizes with $\Delta t = 30$ days.

The data in Table 5-1 indicates that if a value of $\frac{\Delta x}{v \cdot \Delta t} = 1$ is chosen (in this instance this corresponds to a grid size of 115 feet), then accuracy within ± 10 percent can be obtained for the entire concentration distribution curve while accuracy to within $\pm 2\%$ can be obtained for all values $0.2 \leq C/C_0 \leq 0.8$. Thus, the model has the capability of giving very accurate results. For $\frac{\Delta x}{v \cdot \Delta t} = 2.6$ (a grid size of 300 feet), accuracy to less than ± 10 percent can be obtained for all values $0.4 \leq C/C_0 \leq 0.6$. As grid size increases, the model does lose accuracy. Yet, very good results are obtained for the location of the point where $C/C_0 = 0.5$ for all grid sizes. Figures 5-2 and 5-3 show the concentration distribution curves for various grid sizes when the time increment equals 10 days and 45 days, respectively. These figures confirm the conclusions drawn with respect to the 30 day time increment.

The data for grid sizes 115 feet and 300 feet in Table 5-1 also indicate that the numerical model does not produce a symmetric numerical dispersion pattern. This can be attributed to the fully explicit mass balance technique which is used to calculate the relative contaminant concentrations. Regardless of the speed at which the contaminant front moves, the numerical model advances the contaminant concentrations one grid with each calculation. As the time of analysis increases, so does the numerical dispersion and non-symmetry of the concentration distribution curve. Figures 5-1, 5-2, and 5-3 all indicate this pattern. However, by keeping the value of $\frac{\Delta x}{v \cdot \Delta t}$ as close to 1.0 as possible, this numerical dispersion and non-symmetric pattern is kept at a minimum, and in the case where $\frac{\Delta x}{v \cdot \Delta t} = 1$ actually stabilizes.

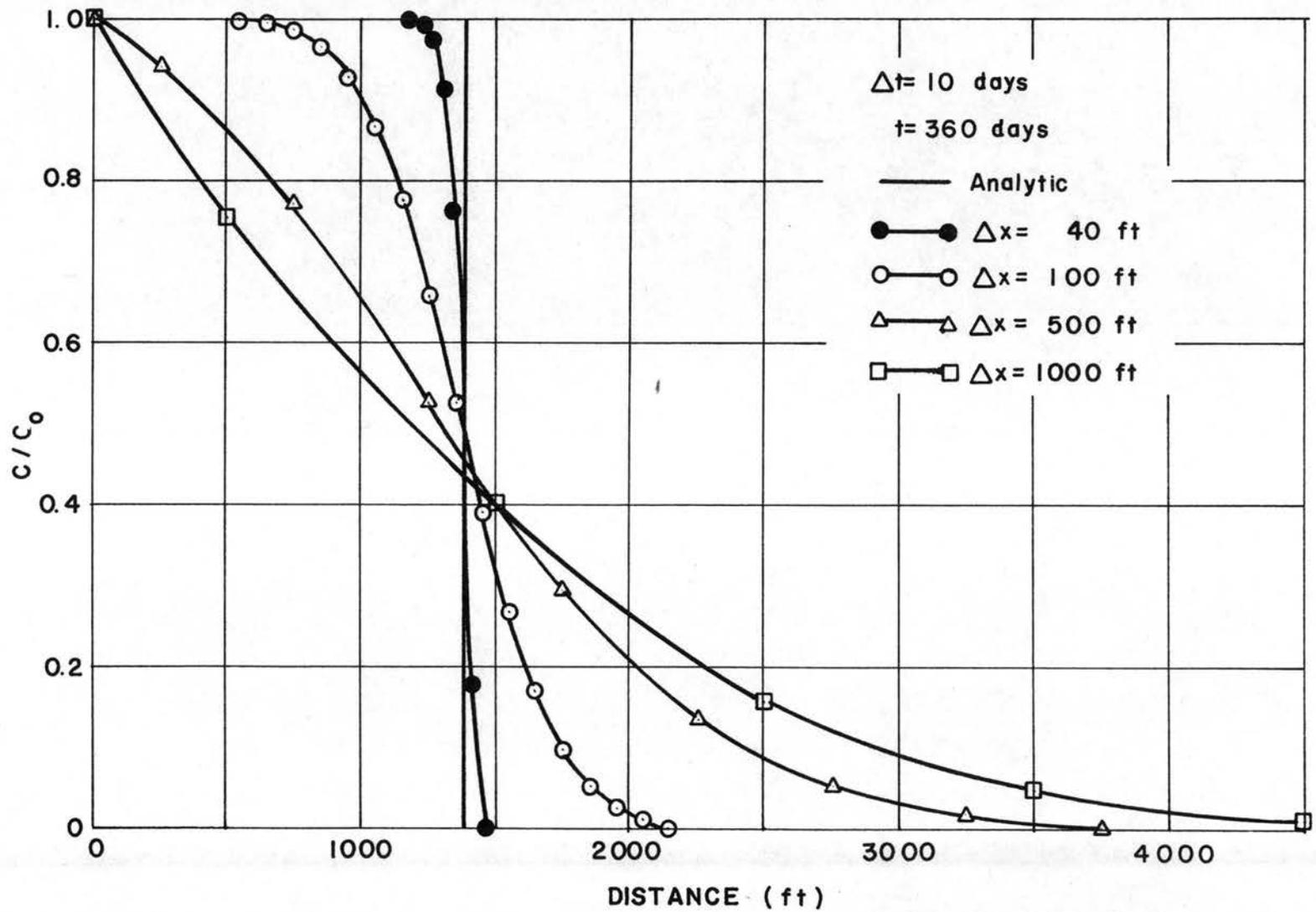


Figure 5-2. Concentration distribution curves for various grid sizes with $\Delta t=10$ days.

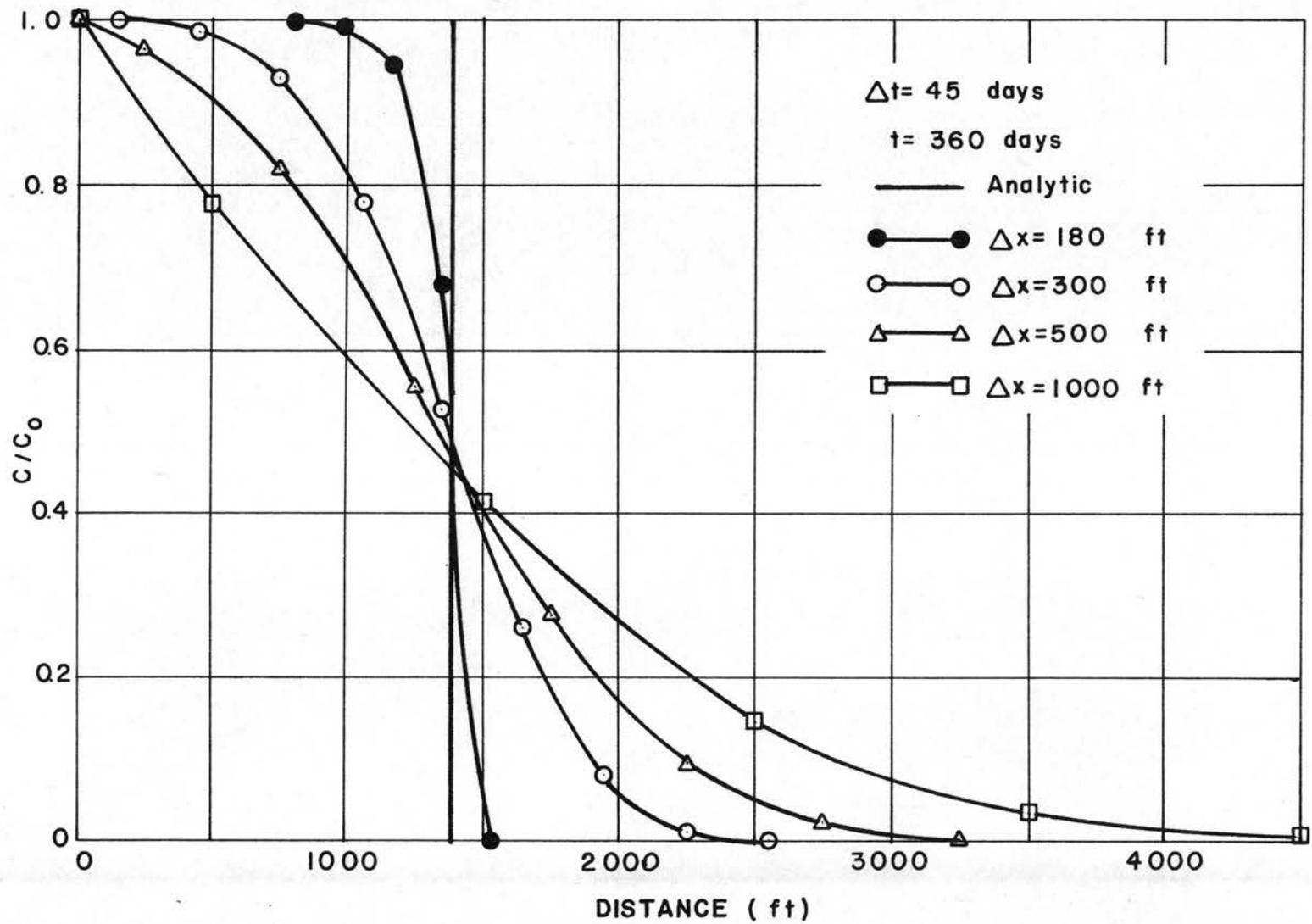


Figure 5-3. Concentration distribution curves for various grid sizes with $\Delta t = 45$ days.

Time Increment

Figure 5-4 shows the relationship of the concentration distribution curves for time increments of 10, 30 and 45 days with a constant grid size of 300 feet at time equal 360 days. This illustrates that the use of a small time increment actually leads to an increase in numerical dispersion. This is due to the use of the fully explicit mass balance technique to calculate the relative concentration values. If a time increment of 10 days is used, three calculations are made over a thirty-day period as opposed to one for a thirty-day time increment. Since each calculation moves the contaminant down-gradient one grid, the model has a tendency to smear the front with each mass balance calculation. The use of as large a time increment as possible while meeting the stability criteria will keep this numerical smear to a minimum.

From Figure 5-4, it can be seen that each time increment gives approximately the same distance for a C/C_0 value of 0.5. Therefore, it can be concluded that while the time increment does have some effect on the accuracy of the numerical model, the effect is not as severe as that caused by variation in grid size.

The maximum value which can be chosen will be dictated by the hydrologic accuracy of the numerical model and the stability criteria. Figures 5-5 and 5-6 show the concentration distribution curves for various times when the grid size equals 500 feet and 115 feet, respectively. These figures confirm the conclusions drawn with respect to the 300 foot grid size. It should be noted that the larger variation

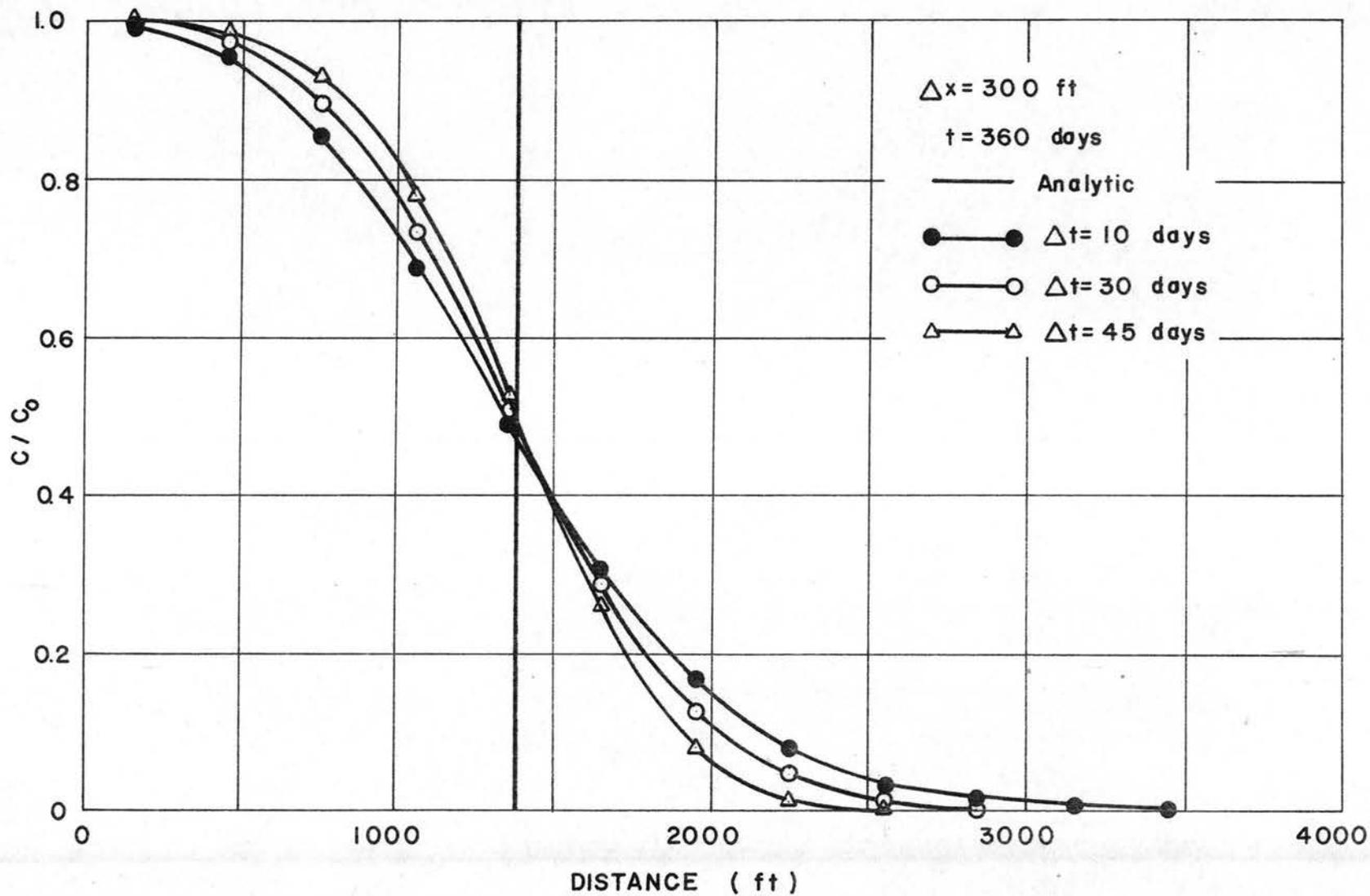


Figure 5-4. Concentration distribution curves for various time increments with $\Delta x=300$ feet.

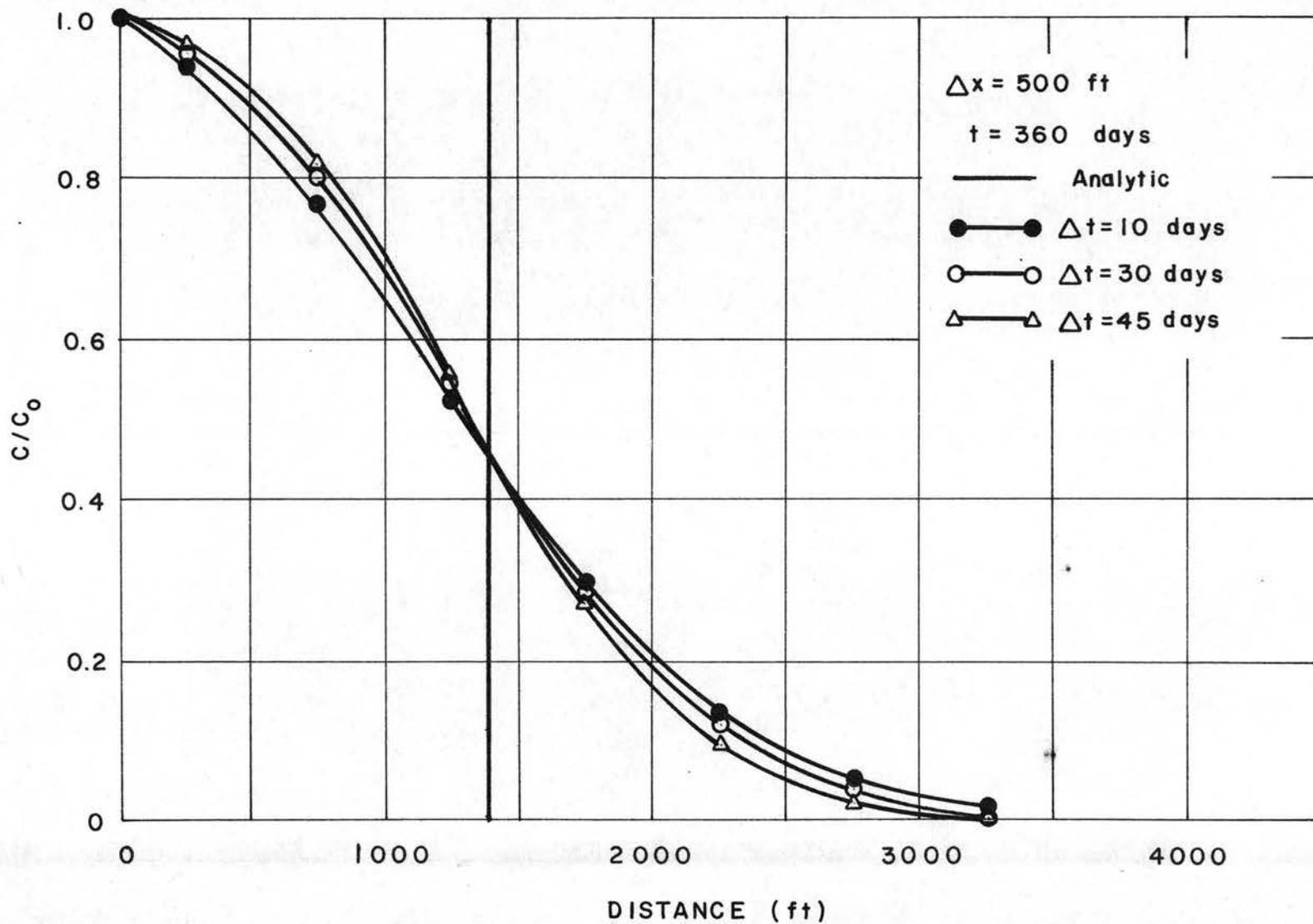


Figure 5-5. Concentration distribution curves for various time increments with $\Delta x = 500$ feet.

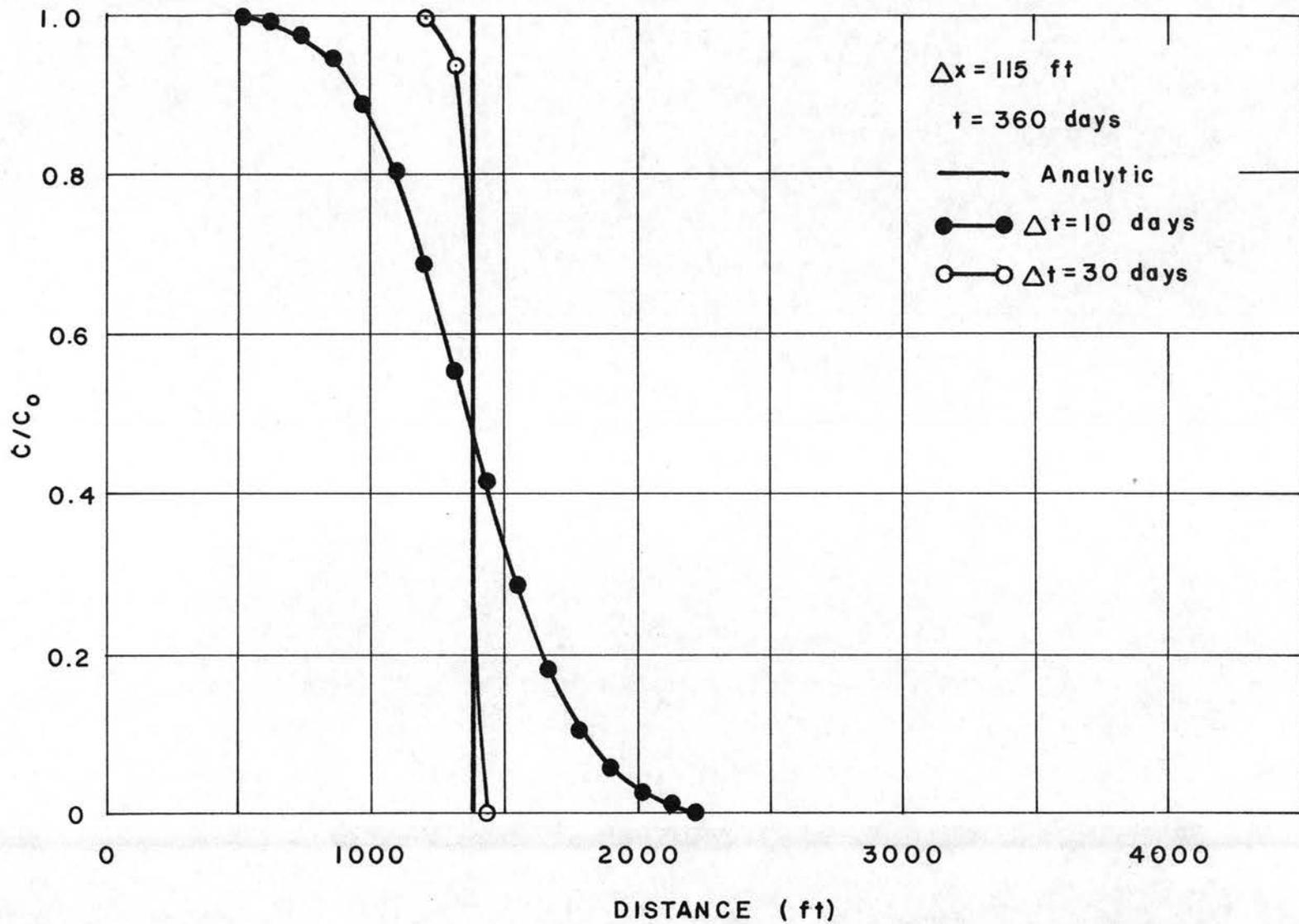


Figure 5-6. Concentration distribution curves for various time increments with $\Delta x=115$ feet.

indicated by Figure 5-6 is due to the fact that the $\frac{\Delta x}{v \cdot \Delta t}$ value for the 30 day time increment is 1.0, resulting in a very accurate approximation to the analytic solution which somewhat distorts the comparison with the 10 day time increment.

Seepage Velocity

As indicated by Equation 2-16, the dispersion coefficient is directly related to the seepage velocity. The analytic results presented previously are all related to one hypothetical case where the coefficient of longitudinal dispersion was calculated to be 1.44×10^{-3} ft²/day. It should be noted that the coefficient of dispersion was estimated based on Equation 2-16 (Harleman, et al, 1963). While there is a small loss of accuracy in estimating the coefficient in this manner, the research by Harleman, et al, indicated that this is a valid relationship which gives accurate results. The results of this study tend to indicate that this estimation is very accurate and that confidence can be expressed in the results obtained based upon this empirical relationship.

To verify the validity of the numerical model for general use, the model was run for hypothetical cases using various values of seepage velocity. Table 5-2 lists the particular cases studied. As with the original hypothetical case, McWhorter and Sunada (1977) was used as a reference to obtain typical values of porosity and hydraulic conductivity for groundwater aquifers.

The concentration distribution curves for the medium sand and medium gravel are shown in Figures 5-7 and 5-8, respectively. It is apparent that the numerical model is valid for these different values of the seepage velocity and the associated dispersion coefficients. As discussed in the previous section, when $\frac{\Delta x}{v \cdot \Delta t} = 1$ the numerical model produces a close approximation to the analytic solution. As the value of $\frac{\Delta x}{v \cdot \Delta t}$ increases, numerical dispersion increases. However, the model continues to produce accurate results with respect to the location of the point where $C/C_0 = 0.5$ for all values of the seepage velocity. From this analysis, it can be concluded that while the numerical model neglects the dispersion process, the effect of dispersion for typical aquifer properties is very small relative to convection over the time periods used and thus the model is valid for the range of seepage velocities studied.

TABLE 5-2. Data for Cases Tested

CASE	TYPE OF MATERIAL	POROSITY ϕ	HYDRAULIC CONDUCTIVITY K ft/day	SEEPAGE VELOCITY v ft/day	DISPERSION COEFFICIENT D_L ft ² /day
1	Medium Sand	0.41	40	0.975	1.26×10^{-4}
2	Coarse Sand	0.39	150	3.846	1.44×10^{-3}
3	Medium Gravel	0.31	1140	36.84	7.32×10^{-2}

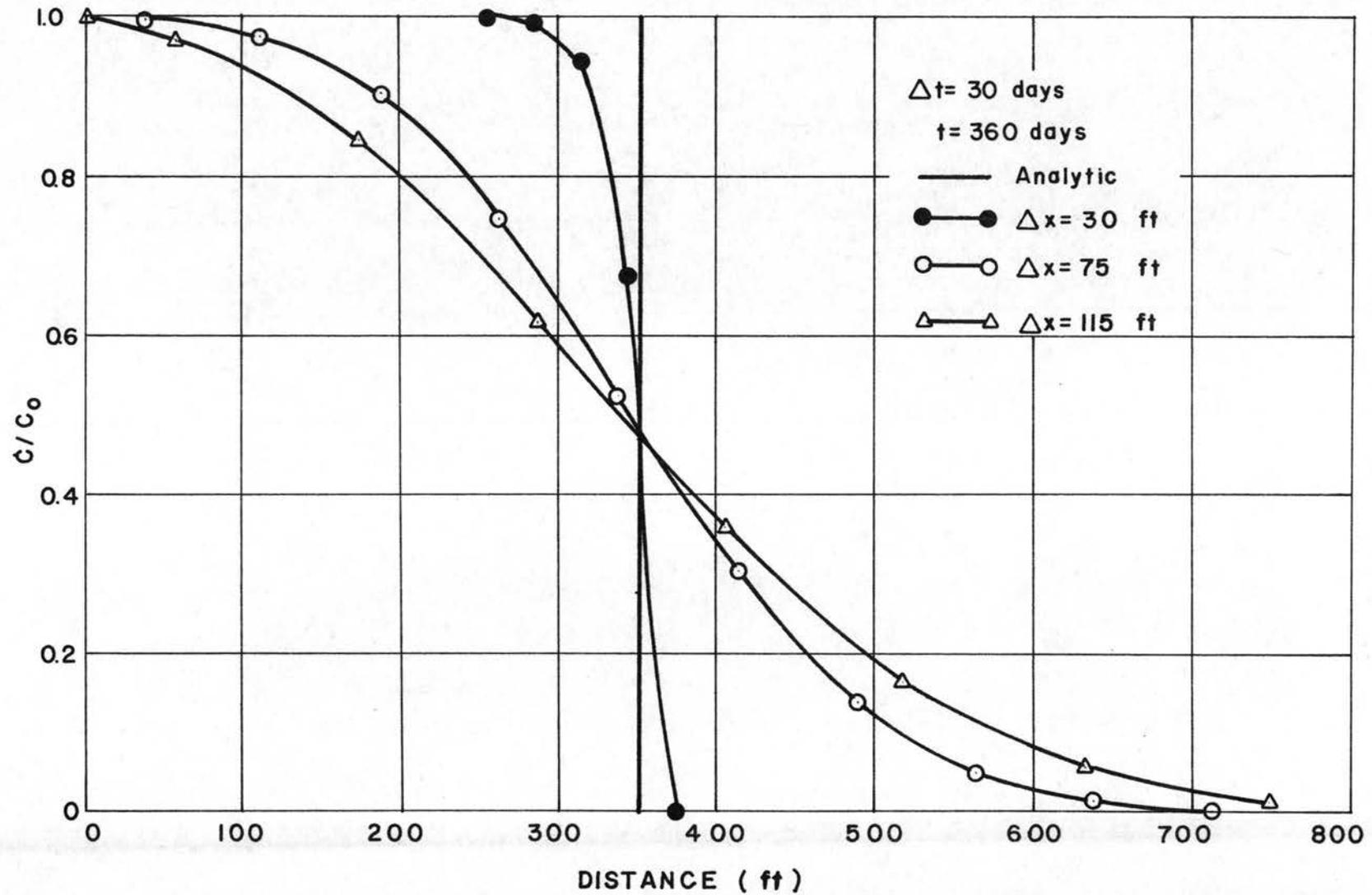


Figure 5-7. Concentration distribution curves for a medium sand.

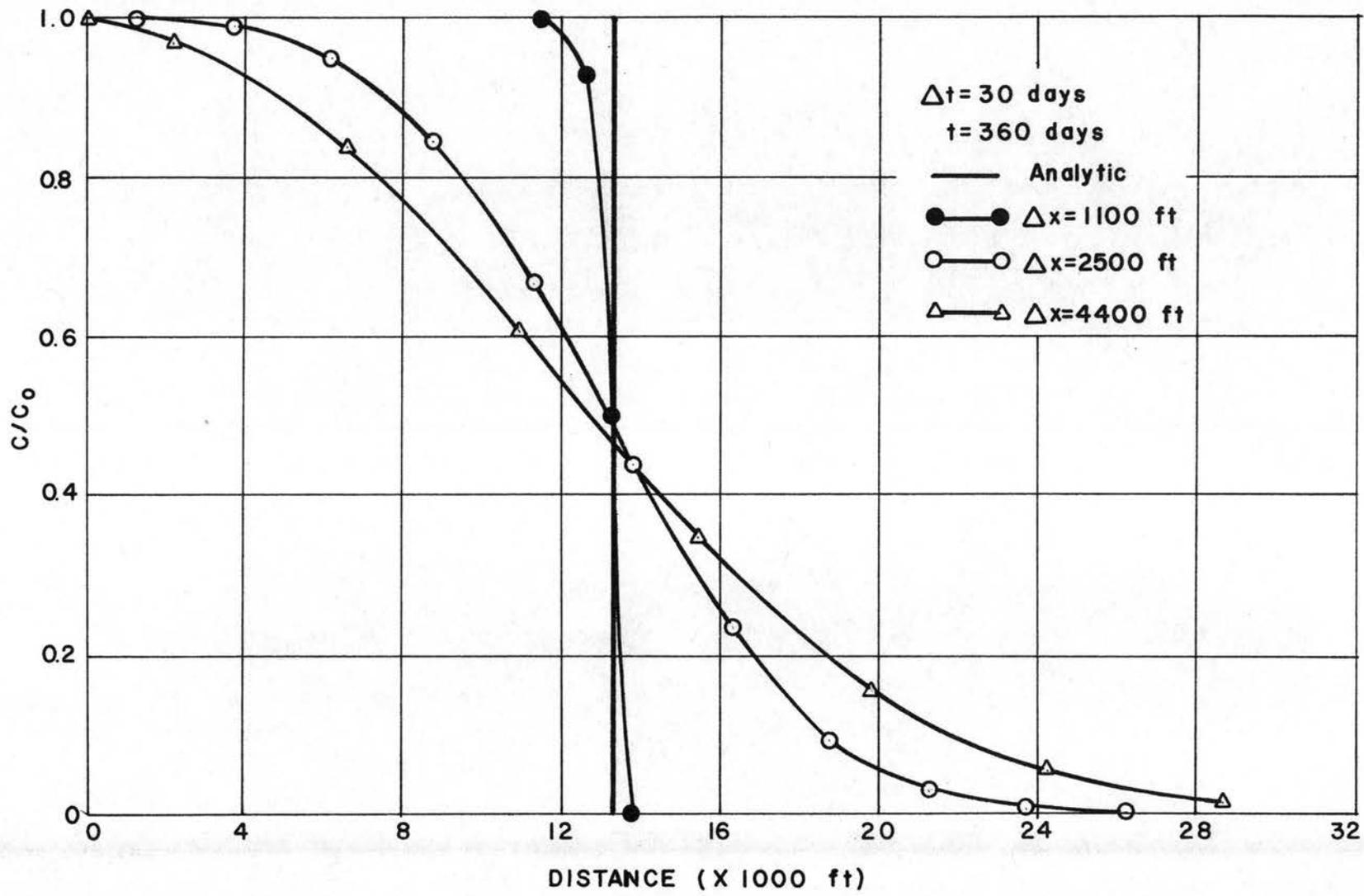


Figure 5-8. Concentration distribution curves for a medium gravel.

Summary of the Sensitivity Analysis

Results of the sensitivity analysis produce three independent conclusions. First, the grid size has a very large effect on the accuracy of the numerical model. For a constant time increment, the larger the grid size, the larger the numerical dispersion. Second, the value of the time increment chosen has little effect on the accuracy of the model. However, the larger the time increment, the greater the accuracy. Finally, the model is valid for a relatively wide range of seepage velocities commonly encountered in groundwater systems.

Collectively, additional conclusions can be drawn. Since the best numerical solutions occur when $\frac{\Delta x}{v \cdot \Delta t} = 1$ and when large time increments are used, this forces the use of large grid sizes which reduce computer time and storage requirements. So long as the value of $\frac{\Delta x}{v \cdot \Delta t}$ remains close to 1.0, the use of larger grid sizes does not significantly effect the accuracy of the model. In addition, the accuracy of the model for cases where $\frac{\Delta x}{v \cdot \Delta t} = 1$ is so good that results for all values $0 < C/C_0 < 1$ can be used with confidence. However, as the value of $\frac{\Delta x}{v \cdot \Delta t}$ increases, the range over which the C/C_0 values are acceptable decreases. Regardless, for all grid sizes the model produces very accurate locations of the point where $C/C_0 = 0.5$.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

From the results of this study, it can be concluded that the numerical model is a valid numerical approach for simulating convection in confined groundwater aquifers. The accuracy of the model is strongly dependent on the value of the function $\frac{\Delta x}{v \cdot \Delta t}$. If the value of this function can be maintained near unity, very good accuracy of the dispersion process can be obtained. While larger values of the function result in increasing amounts of numerical dispersion, the model at all times locates the point where $C/C_0=0.5$ with good accuracy.

With regard to the sensitivity of the model, conclusions can be drawn about each of the terms which appear in the function $\frac{\Delta x}{v \cdot \Delta t}$. With v and Δt held constant, increased numerical dispersion results with larger grid sizes. With Δx and v held constant, varying Δt has a much smaller effect on model accuracy than varying Δx . However, as Δt is increased, numerical dispersion is minimized and the accuracy of the model increases. The model is valid for a wide range of seepage velocities subject to the limitations imposed by varying Δx and Δt . It is shown that, for a given seepage velocity, the best results are obtained by maximizing Δt within the limits of the accuracy of the groundwater flow portion of the model and minimizing Δx so that the

value of $\frac{\Delta t}{v \cdot \Delta x}$ approaches 1.0. While this criteria does somewhat limit grid size, grid sizes on the order of hundreds of feet should be possible for most cases.

While the majority of work done as a part of this study was based on the one-dimensional, steady state flow condition, the model was developed to simulate two-dimensional, areal distribution of contaminants. For the radially symmetric, diverging flow situation, the model produced relatively good results as the numerical approximation of the groundwater flow equation approached the solution for pure radial flow. The numerical dispersion increased with increasing distance from the contaminant source and is attributable to the increase in the value of the function $\frac{\Delta x}{v \cdot \Delta t}$ as the radial distance increases.

Several runs were made in which various numbers of recharge pits, pumping wells, constant head sources and phreatophyte sources were used. These indicated that the programming modifications made are correct insofar as computer language is concerned. However, no hypothetical cases were run to determine the accuracy and sensitivity of these contaminant sources.

In general, the numerical model minimizes many of the problems encountered with the development of previous numerical models. The model does not require prohibitive amounts of computer time or storage. It has the capability of handling impermeable, constant head and constant gradient boundaries. When the stability criteria is followed, the model is stable and converges to a reasonably accurate solution.

It is recommended that the following areas be studied with regard to the fully explicit mass balance approach of simulating convection in groundwater aquifers:

1. The validity of the model should be verified for non-homogeneous confined aquifers.
2. The validity of the model for both homogeneous and non-homogeneous unconfined aquifers should be established.
3. The model should be applied to an actual field problem.

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APPENDIX A

DETERMINATION OF HYPOTHETICAL AQUIFER PROPERTIES

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In order to show that WTQUAL2 is a valid dispersion simulator, it was felt that the data used in the comparison of the analytic and numerical solutions should be representative of values commonly encountered in actual physical situations. Therefore, typical values of hydraulic conductivity and porosity were chosen from a table in McWhorter and Sunada (1977) containing maximum, minimum and arithmetic mean values for soils ranging from the finest silts and clays to coarse gravels. The values $K=150$ ft/day and $\phi=0.39$ were chosen for the sensitivity analysis of grid size and time increment and are representative of a typical coarse sand.

Once these values were chosen, it was then necessary to choose a value for the hydraulic gradient. Review of several actual physical situations indicated that a gradient of 0.01 (10 feet change in vertical elevation per 1000 feet change in horizontal distance) is typical and so this value was chosen.

Assuming that the flow regime would be laminar (this will be checked later), Darcy's law was applied. The version used in this situation was

$$q = K \cdot \frac{dh}{dl} \quad (A-1)$$

where q is the Darcy velocity, K is hydraulic conductivity, and $\frac{dh}{dl}$ is the hydraulic gradient. Using the previously chosen values for K and $\frac{dh}{dl}$, a Darcy velocity q of 1.50 ft/day was obtained. Dividing this value by the porosity gave a seepage velocity v of 3.846 ft/day.

At this point in the analysis an additional assumption had to be made. For the purpose of this study, a groundwater temperature of 50°F was chosen. The groundwater system was then assumed to be isothermal throughout the period of study. The appropriate fluid properties of water at this temperature are

dynamic viscosity	$\mu = 2.735 \times 10^{-5} \text{ lb-sec/ft}^2$
kinematic viscosity	$\nu = 1.410 \times 10^{-5} \text{ ft}^2/\text{sec}$
density	$\rho = 1.94 \text{ slugs/ft}^3$
acceleration of gravity	$g = 32.2 \text{ ft/sec}^2$
specific weight	$\gamma = 62.4 \text{ lb/ft}^3$

Applying the relationship between hydraulic conductivity and intrinsic permeability k

$$k = \frac{K \cdot \mu}{\rho \cdot g} = \frac{K \cdot \mu}{\gamma} \quad (\text{A-2})$$

yielded a permeability of $7.609 \times 10^{-10} \text{ ft}^2$.

At this point, Equation 2-16 as developed by Harleman et al (1963) was applied and a longitudinal dispersion coefficient value $D_L = 1.439 \times 10^{-3} \text{ ft}^2/\text{day}$ was obtained. This value was then used in Equation 2-5 to obtain the analytic solution values of the concentration distribution curve.

To verify that the flow regime was laminar, the Reynold's number R was calculated from the following equation:

$$R = \frac{v\sqrt{k}}{\nu} \quad (A-3)$$

This calculation yielded $R=8.708 \times 10^{-5}$. Since this value is less than 0.01, the flow regime can be characterized as laminar and Darcy's law applies.

To determine the storage coefficient, it was necessary to estimate values for the pore volume compressibility and the compressibility of water due to the formation lying above the confined aquifer. Pore volume compressibility α_p was assumed to be $3 \times 10^{-5} \text{ psi}^{-1}$ and compressibility of water β was assumed to be $3.3 \times 10^{-6} \text{ psi}^{-1}$. These values are relatively constant for most problems commonly encountered in groundwater hydrology and are therefore assumed to be representative for the condition being studied.

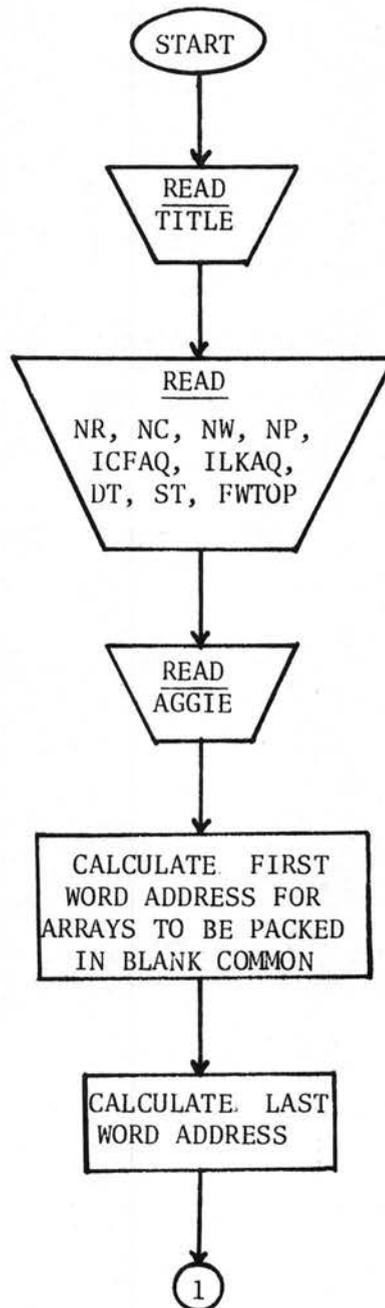
A specific storage S_s of $5.68 \times 10^{-5} \text{ ft}^{-1}$ was obtained by applying the formula

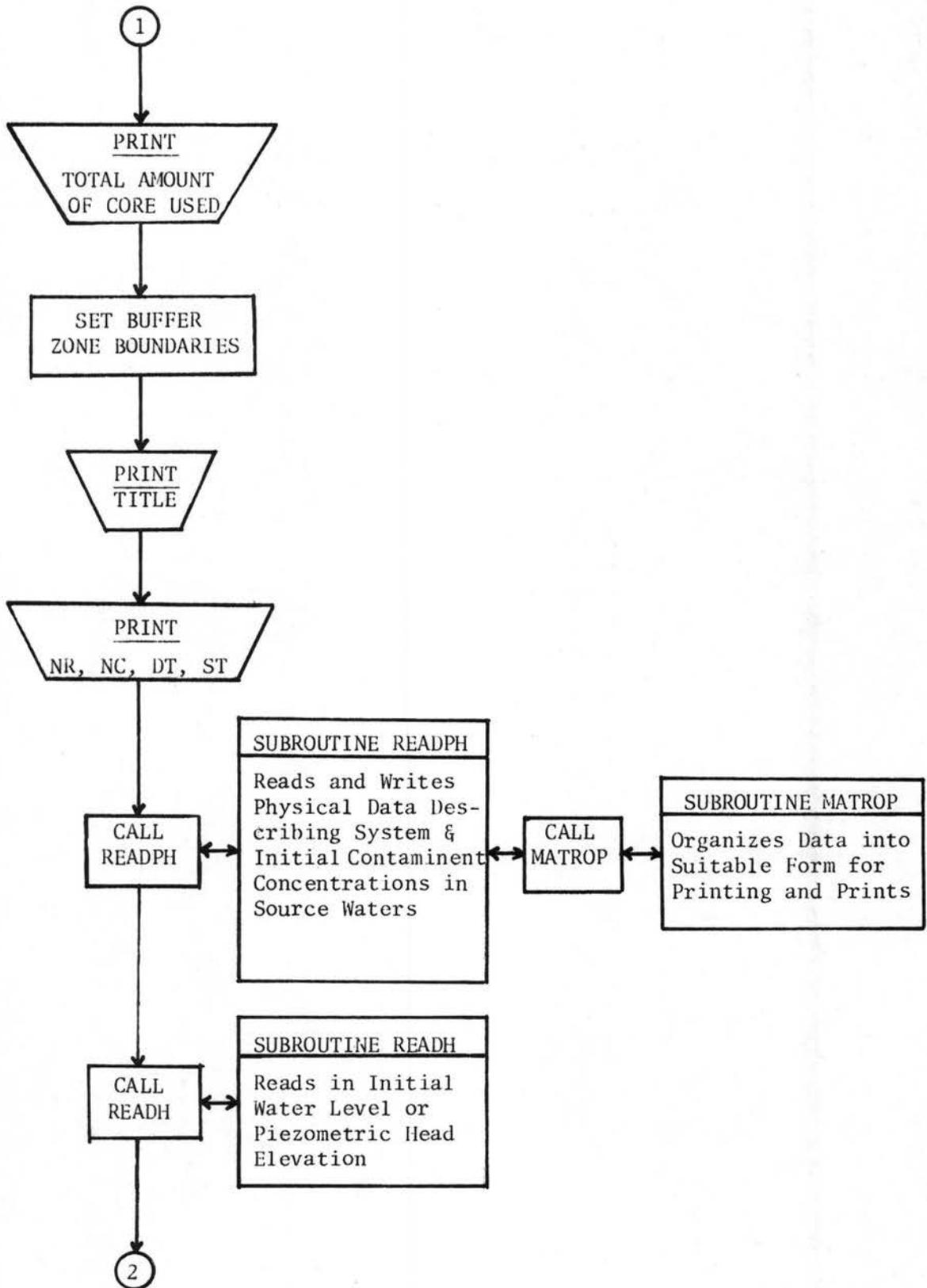
$$S_s = \rho \cdot g \cdot \phi \cdot (\alpha_p + \beta) \quad (A-4)$$

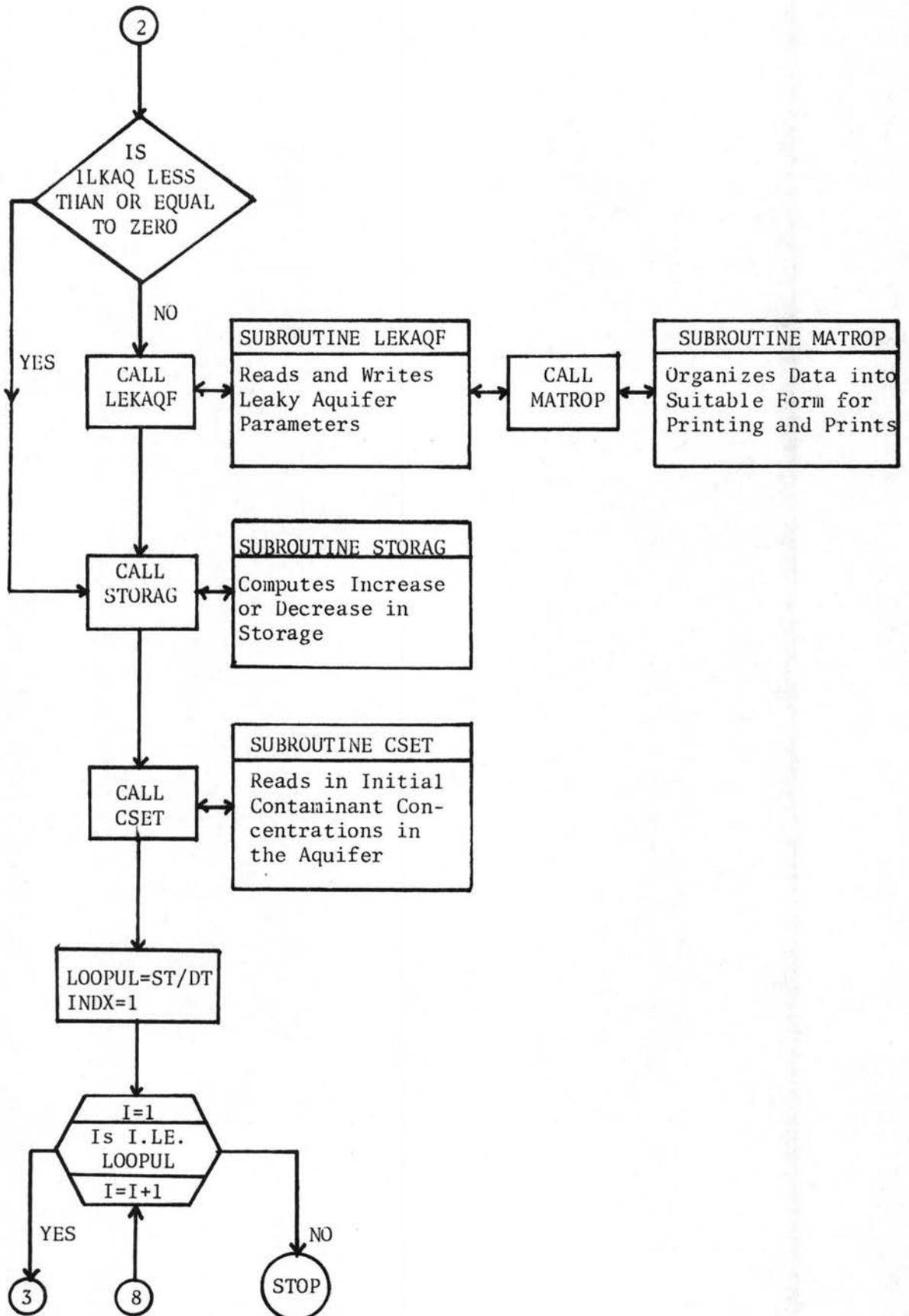
With a confined aquifer thickness set at 60 feet, a storage coefficient S of 0.0034 was obtained.

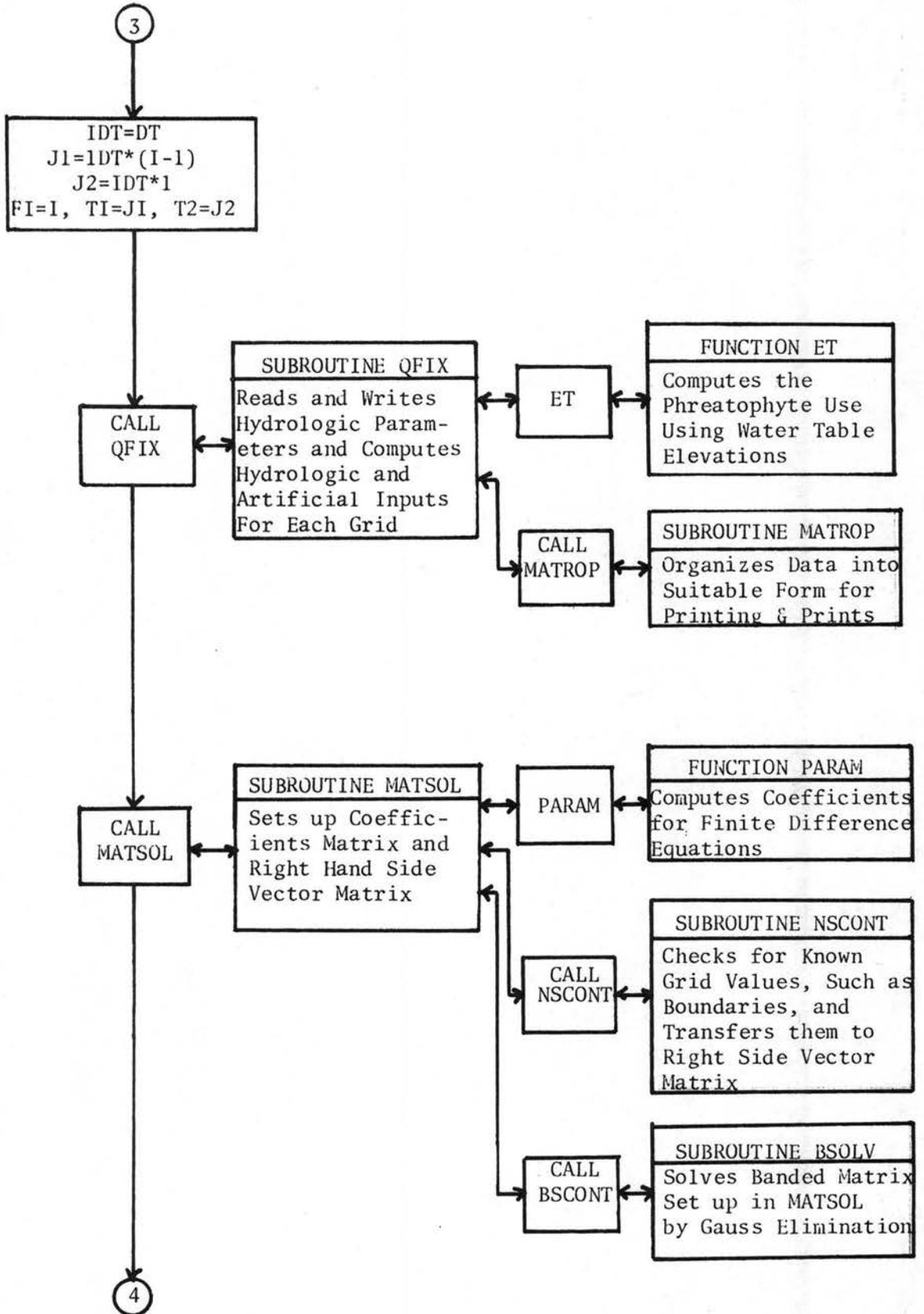
APPENDIX B
PROGRAM FLOW CHART

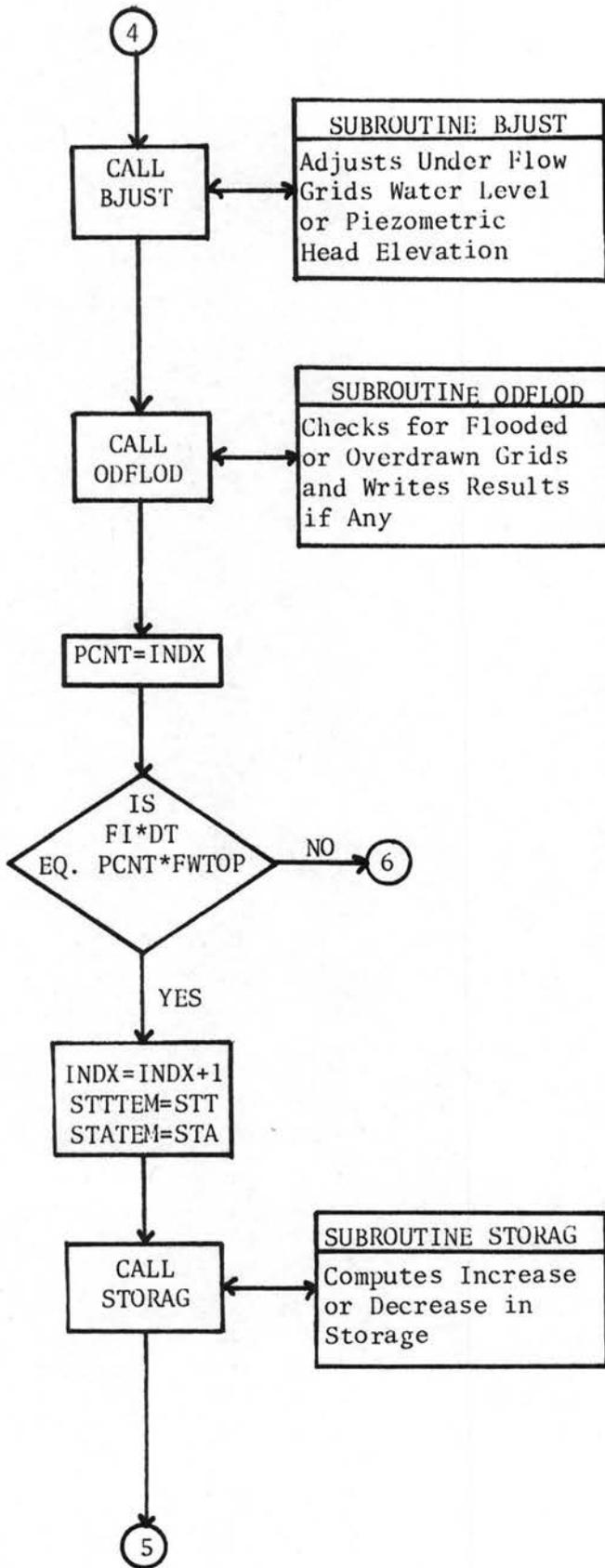
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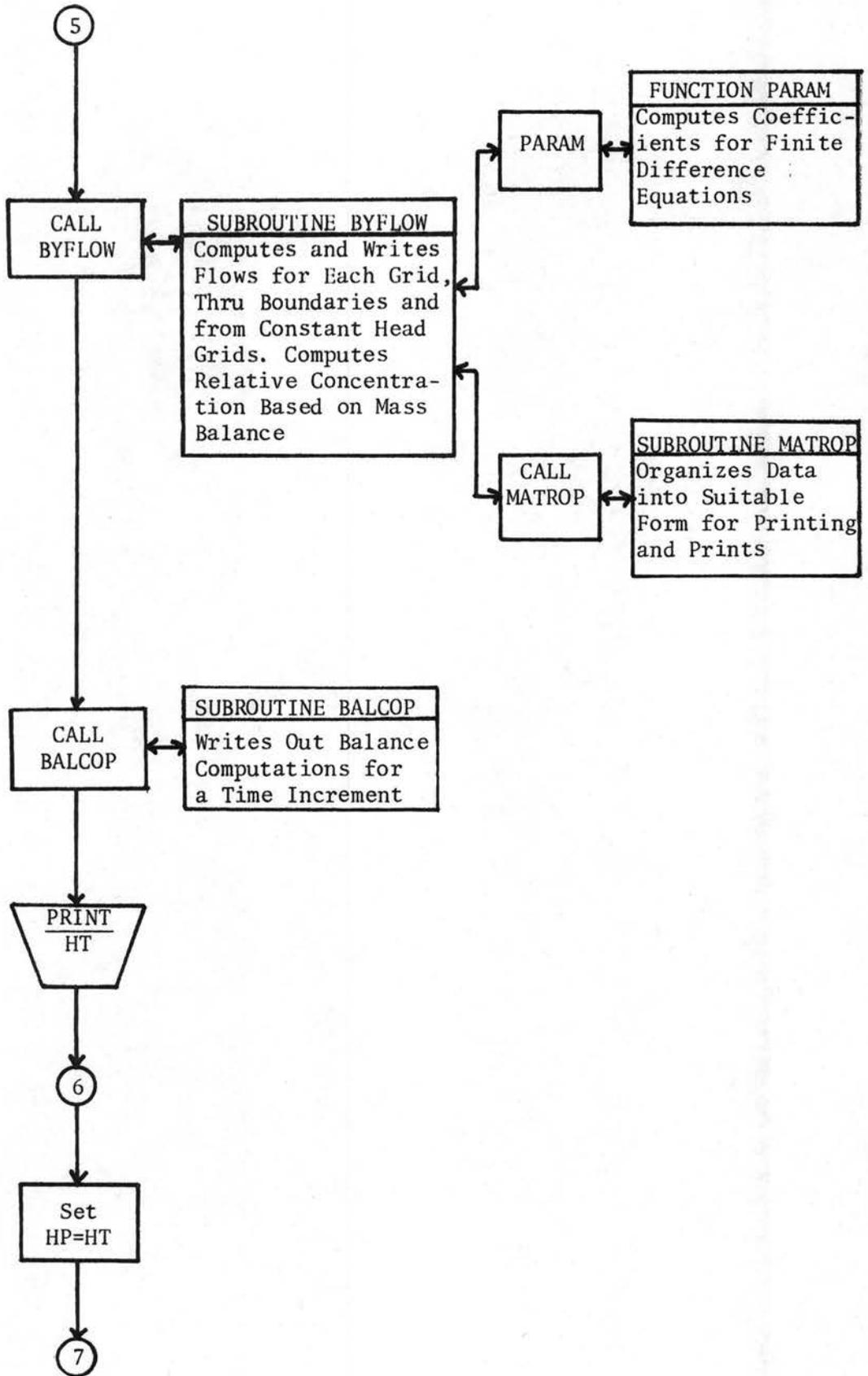


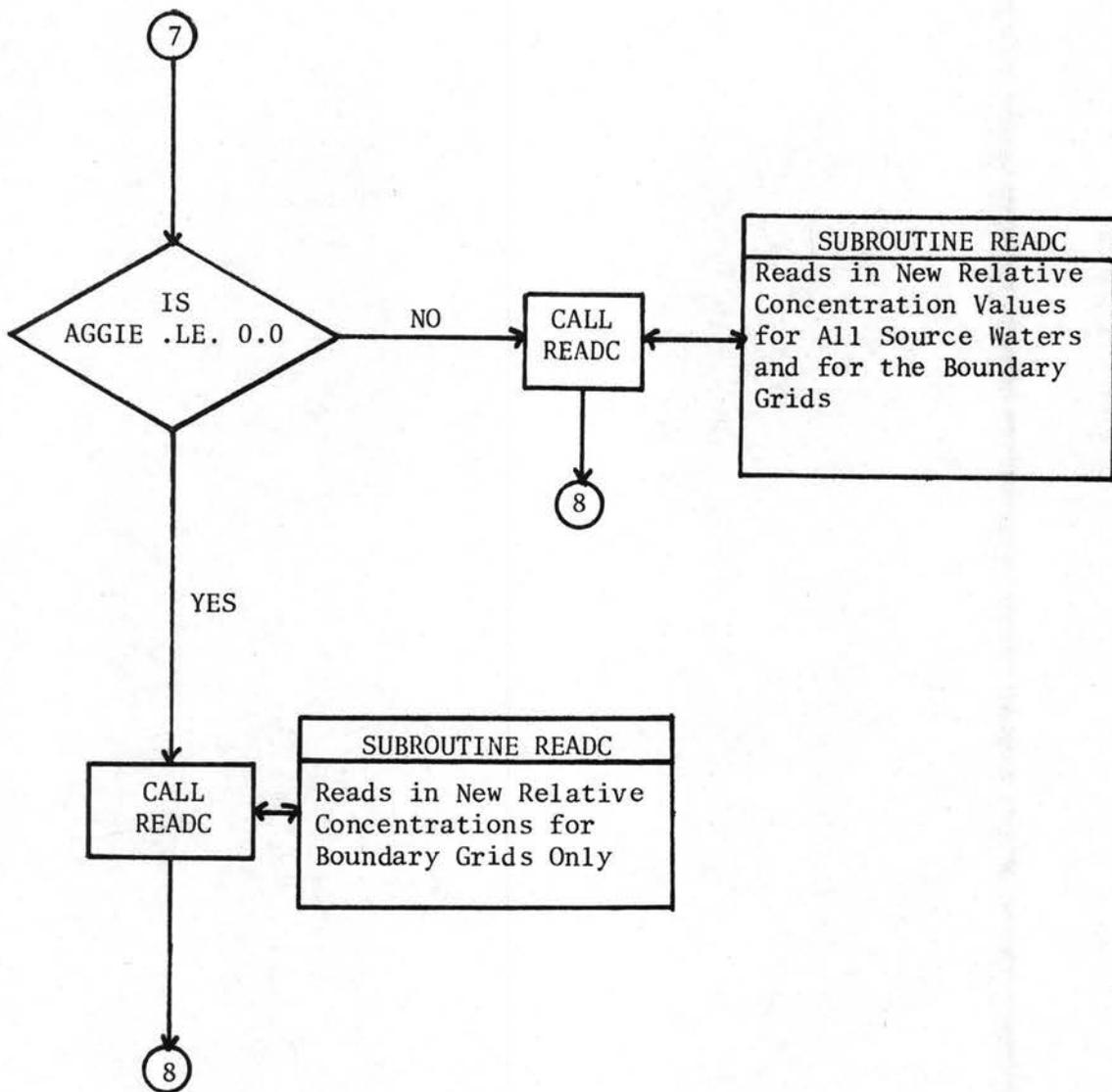












APPENDIX C
DESCRIPTION OF SUBPROGRAMS

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Subroutine READPH

This subroutine reads and writes the physical data describing the study area. The following variables are read and printed: DX, DY, FK, Z, CS, CPPT, CAPW, CRCHR, CSQR, G, PHI, and PHIC. CA is also read but printed later. Coded values of CS are printed. Only one data card is required if all variables are uniform for each grid, otherwise each parameter that is variable must be read in matrix form. Variables DX and DY require only NC and NR values, respectively.

CALLED FROM: Main Program

SUBPROGRAMS USED: MATRØP

IMPORTANT VARIABLES: DX, DY, FK, Z, G, PHI, PHIC, CA, CS, CPPT, CAPW,
 CRCHR, CSQR

Subroutine READH

This subroutine reads the initial coded water level or piezometric head elevations. H is decoded and set equal to HT and HP. One data card is required if the initial water level is horizontal, otherwise the entire H-matrix must be read.

CALLED FROM: Main Program

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: H, HT, HP

Subroutine LEKAQF

This subroutine reads and writes the leaky aquifer parameters. The following variables are read and printed: HL, TL, and FKL. One data card is required if these variables are uniform, otherwise each matrix that is variable must be read.

CALLED FROM: Main Program

SUBPROGRAMS USED: MATRØP

IMPORTANT VARIABLES: HL, TL, FKL

Subroutine CSET

This subroutine initializes the relative concentration throughout the aquifer.

CALLED FROM: Main Program

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: CØ, CT, H, G, CS

Subroutine STØRAG

This subroutine computes the initial storage and increase or decrease of storage. Total area and between station (between buffer zone boundaries) storage is calculated. Also storage of overlap areas is computed.

CALLED FROM: Main Program

SUBROUTINES USED: None

IMPORTANT VARIABLES: STA, STT, STØL, H, HT, Z

Subroutine QFIX

This subroutine reads and writes the hydrologic parameters. The hydrologic and artificial inputs are then calculated for each grid. A value of zero on the input card indicates a particular parameter is not used. The exception to this is the number of grids with phreatophyte use, NGPU. If NGPU is blank, the entire PHR matrix must be read, otherwise the number of grids specified is read. NGPU equal to zero indicates no phreatophyte use.

Coding PHR less than one indicates that phreatophyte use should be calculated every time increment from the previous time step water level elevation. The ET subprogram is used for this.

The factors considered in QFIX are (1) precipitation, (2) applied water as irrigation, (3) phreatophyte use, (4) wells, (5) recharge areas or lines, and (6) leaky aquifer conditions.

CALLED FROM: Main Program

SUBPROGRAMS USED: ET

IMPORTANT VARIABLES: PPT, CPT, YPT, APW, CAW, YAW, NGPU, PHR, YPR,
 WELL, RPUM, YPM, PIT, RCHR, YRC, Q, SQT, SQA,
 REPEAT, CPM

Function ET

This subprogram computes the phreatophyte use for each grid using the water level elevations from the previous time step. If the depth of water table DTWT is negative, an error message is printed. It is anticipated this program, if used, will change with each study area.

CALLED FROM: QFIX
SUBPROGRAMS USED: None
IMPORTANT VARIABLES: ET, DTWT

Subroutine MATSØL

This subroutine sets up the coefficient matrix, CMATRX, and the right hand side vector matrix, CR. CMATRX is a reduced matrix containing only the band of known values in the left side of the difference equations and is written vertically rather than diagonally. Its dimensions are $(NR-2)*(NC-2)$ by $2*NR-3$. The coefficients are computed using Function PARAM and checked for adjacent boundary values of H in subroutine NSCØNT. MATSØL treats known grid values of H. BSØLVE is used to solve the matrix equation set up.

CALLED FROM: Main Program
SUBPROGRAMS USED: PARAM, NSCØNT, BSØLVE
IMPORTANT VARIABLES: CMATRX, CR

Function PARAM

This subprogram computes the coefficients in the left side of the finite difference equation. For confined aquifer analysis, saturated thickness is compared to aquifer thickness and the smallest of the two is used to calculate the coefficient.

CALLED FROM: MATSØL, BYFLØW
SUBPROGRAMS USED: None
IMPORTANT VARIABLES: PARAM

Subroutine NSCØNT

This subroutine transfers the coefficients, in CMATRX, multiplied by their respective H-value, to the right hand side vector matrix in case of adjacent head or known boundary conditions. It also sets coefficients equal to zero in case of adjacent impermeable grids.

CALLED FROM: MATSØL

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: None

Subroutine BSØLVE

This subroutine solves the matrix equation set up in MATSØL by Gauss Elimination. BSØLVE is designed specifically for a diagonal matrix that results from analysis of groundwater systems.

CALLED FROM: MATSØL

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: None

Subroutine BJUST

This subroutine adjusts the underflow boundary water level elevations. Gradients are calculated three grids in from the exterior boundary grids and the gradients are projected back to the exterior boundary grids to obtain new water level elevations. This calculation is performed at even time steps. At odd time steps the water level elevations are held constant and the exterior boundary grids are treated as constant head grids.

CALLED FROM: Main Program

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: H, HT

Subroutine ØDFLØD

This subroutine checks for overdrawn or flooded grids. If either should occur, a message is printed indicating such. For confined aquifer analysis the flooded grid computations are bypassed. Total flooded and overdraw amounts are computed for the total area and between stations.

CALLED FROM: Main Program

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: ØACFTT=ØVT, ØACFTA=ØVA, FACFTT=FVT, FACFTA=FVA

Subroutine BYFLØW

This subroutine computes flows for each grid. Total flow through model boundaries and buffer zone boundaries is calculated as well as flow into the system from constant head grids. The flow equation used is developed from the finite difference equations and uses particular values of the CMATRX. These values are transferred from MATSØL except for boundary values which are calculated in BYFLØW using Function PARAM. Flow is not allowed to or from an impermeable grid and between any two adjacent underflow grids. I-direction and J-direction flows are printed and flows from constant head grids are interpreted and printed as flow from river grids. Relative concentration calculations are made using the flow between grids.

CALLED FROM: Main Program

SUBPROGRAMS USED: PARAM, MATRØP

IMPORTANT VARIABLES: SQGGI, SQGGJ, SQBT, SQBA, SQR, SQRT, SQRA, CS, CPPT, CAPW, CRCHR, CSQR

Subroutine BALCØP

This subroutine writes the balance computations at the desired time steps specified by FWTØP. Mass balance for the entire area cannot always be obtained, due to accounting procedures used to compute mass flow at exterior boundary grids. However, for between stations, which refers to the area between the buffer zone boundaries, mass balance must always be satisfied except for the case when a confined grid becomes unconfined. This error should be small and is indicated by the "TOTALS" in the mass balance output being different than zero. To reduce this error, decrease the value of Δt . For confined aquifer analysis, a message is printed indicating if a grid becomes unconfined.

CALLED FROM: Main Program

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: SQA, SQT, SQRA, SQRT, SQBA, SQBT, STT, STTEM, STA,
STATEM, STØL, ØVA, ØVT

Subroutine MATRØP

This subroutine organizes data or results into a suitable form for printing and then prints.

CALLED FROM: READPH, LEKAQF, QFIX, BYFLØW

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: NR=NØRØW, NC=NØCØL

Subroutine READC

This subroutine reads in new relative concentration values for all source waters and the boundary grids for each time increment of analysis. Execution of this subroutine is controlled by the value AGGIE. If AGGIE is less than or equal to zero, only new concentrations for the boundary grids will be read in. If AGGIE is greater than zero, new values for each source water throughout the grid network can be read in as a single value. Variable concentrations must be read in matrix form. Boundary grid concentrations are read in one value per card for each grid other than impermeable boundaries.

CALLED FROM: Main Program

SUBPROGRAMS USED: None

IMPORTANT VARIABLES: CPPT, CAPW, CRCHR, CSQR, CØ, AGGIE

APPENDIX D
PROGRAM LISTING

C		520
C	NA=(NP-2)*(NC-2)	530
	NB=2*NR-3	540
C		550
	INZR=360./DT	560
C		570
	IDC=NR*NC	580
	IFK=1	590
	IPHI=IDC+1	600
	I7=2*IDC+1	610
	IG=3*IDC+1	620
	IDX=4*IDC+1	630
	IDY=5*IDC+1	640
	ICA=6*IDC+1	650
	IH=7*IDC+1	660
	IHT=8*IDC+1	670
	IHP=9*IDC+1	680
	IQ=10*IDC+1	690
	IWFLL=11*IDC+1	700
	IPIT=12*IDC+1	710
	IPHR=13*IDC+1	720
	ISGGI=14*IDC+1	730
	ISGGJ=15*IDC+1	740
	ISQR=16*IDC+1	750
	IA=17*IDC+1	760
	IB=18*IDC+1	770
	IPHRTMP=19*IDC+1	780
	IHF=20*IDC+1	790
	ICO=21*IDC+1	
	ICT=22*IDC+1	
	ICS=23*IDC+1	
	IAREA=24*IDC+1	
	IOPPT=25*IDC+1	
	IOPPW=26*IDC+1	
	ICRCHR=27*IDC+1	
	IOPHR=28*IDC+1	
	IOPUM=29*IDC+1	
	IOLAK=30*IDC+1	
	ICPPT=31*IDC+1	
	ICAPW=32*IDC+1	
	ICRCHR=33*IDC+1	
	ICPHR=34*IDC+1	
	ICPUM=35*IDC+1	
	ICLEAK=36*IDC+1	
	ICSQR=37*IDC+1	
	IPHIC=ICS+ICFAQ*IDC	
	IHL=IPHIC+ILKAQ*IDC	810
	ITL=IHL+ILKAQ*IDC	820
	IFKL=ITL+ILKAQ*IDC	830
	IEND1=38*IDC+ICFAQ*IDC+ILKAQ*3*IDC	
C		850
	IYPT=IEND1+1	860
	IYPR=IYPT+INZR	870
	IYAH=IYPR+INZR	880
	IYPM=IYAH+INZR	890
	ICPM=IYPM+NW*INZR	900
	IRPUM=ICPM+NW	910
	IYRC=IRPUM+NW	920
	IRCHR=IYRC+NP*INZR	930
	IENC2=IEND1+3*INZR+NW*INZR+2*NW+NP*INZR+NP*1	940
C		950
	ICMATRX=IEND2+1	960
	ICR=IEND2+(NA*NR)+1	970
	IEND3=IEND2+NA+(NA*NB)	980
C		990
	LWA=LOCFC(C(IEND3))	1000
	WRITE(6,230)LWA	1010
	CALL CORF(LWA)	1020
	DO 100 LI=1,IEND3	
	C(LI)=UNDEF	
100	CONTINUE	
C		1030
	LCIW=3	1040
	LCIE=NC-2	1050
	LCJW=3	1060
	LCJE=NR-2	1070
C		1080
	WRITE(6,210)TITLE	1090
	IF(ILKAQ.LE.0)GO TO 110	1100
	WRITE(6,250)	1110

	GO TO 130	1120
110	IF (ICFAQ.LE.0) GO TO 120	1130
	WRITE (6,240)	1140
	GO TO 130	1150
120	WRITE (6,260)	1160
130	WRITE (6,190) NR,NC,DT,ST	1170
C		1180
	CALL READPH (NR,NC,C(IFK),C(IFHI),C(IZ),C(IG),C(IDX),C(IDY),C(ICA)	1190
	1,C(IPHIC),C(ICPPT),C(ICAPW),C(ICPCHR),C(ICSGR),C(ICS),C(IAREA),TAR	
	2EA)	
C		1210
	CALL READH (NR,NC,C(IH),C(IHP),C(IHT),C(IHF),LBC,RBC,TBC,BBC)	
C		1230
	IF (ILKAQ.LE.0) GO TO 140	1240
	CALL LEKAQF (NR,NC,C(IHL),C(ITL),C(IFKL))	1250
C		1260
140	CALL STOPAG (NR,NC,C(IH),C(IHT),C(IZ),C(IDX),C(IDY),C(IPHI),C(IG),	1270
	1,C(IPHIC))	1280
	CALL CSET (NR,NC,C(ICO), C(ICT), C(IH), C(IG),C(ICS))	
C		1290
	LOOPUL=ST/DT	1300
	INDX=1	1310
C		1320
	DO 170 I=1,LOOPUL	1330
C		1340
	IDT=DT	1350
	J1=IDT*(I-1)	1360
	J2=IDT*I	1370
	FI=I	1380
	T1=J1	1390
	T2=J2	1400
C		1410
	CALL OFIX (NF,NC,C(IDX),C(IDY),C(ICA),C(IH),C(IZ),C(IHT),I,C(IG)	1420
1),NW,NP,C(IPHR),C(IWELL),C(IPI T),C(IYFT),C(IYAW),C(IYPR),C(IYPM)	1430
2),C(IRPUM),C(ICPM),C(IYRC),C(IRCHR),INR,C(IHL),C(ITL),C(IFKL),	1440
3	C(IG),C(IPRTP),C(IQPPT),C(IQAPW),C(IQRCHR),C(IQPHR),C(IQPUM),	
4	C(IQLEAK),C(IAREA),TAREA)	
C		1460
	CALL MATSOL (NR,NC,NA,NB,C(IFK),C(IPHI),C(IH),C(IHT),C(IZ),C(ID	1470
1	X),C(IDY),C(IQ),C(ICMATRX),C(ICR),C(IA),C(IA),C(IG),C(IPHIC),C(I	1480
2	IHP),C(IHF))	1490
C		1500
	CALL BJUST (NR,NC,C(IH),C(IHT),C(IHP),C(IDX),C(IDY),I)	1510
C		1520
	CALL ODFLOD (NR,NC,C(IH),C(IHT),C(I7),C(IG),C(IPHI),C(IDX),C(ID	1530
1	Y),C(IPHIC))	1540
C		1550
	SYSTEM=STT	1560
	STATEM=STA	1570
1	CALL STORAG (NR,NC,C(IH),C(IHT),C(IZ),C(IDX),C(IDY),C(IPHI),C(I	1580
	G),C(IPHIC))	1590
C		1600
	PCNT=INDX	1610
	IF ((FI*DT).NE.(PCNT*FWTOP)) GO TO 150	1620
	INDX=INDX+1	1630
C		1640
	CALL BYFLOW (NR,NC,NA,NB,C(IFK),C(IH),C(IHF),C(IZ),C(IDX),C(IDY)	1650
1),C(ISQGGI),C(ISQGGJ),C(ISQR),C(ICMATRX),C(IA),C(IA),C(IHF),C(I	1660
2	G),I,C(IG),C(ICT),C(IPHI),C(IPHIC),C(ICS),C(IQPPT),C(IQAPW),C(I	
3	QRCHR),C(IQPHR),C(IQPUM),C(IQLEAK),C(ICPPT),C(ICAPW),C(ICRCHR)	
4	,C(ICSQR),C(IQ),C(ICPUM),C(ICLEAK),C(ICPHR))	
C		1680
	CALL BALCOP (J1,J2,I,STTEM,STATEM)	1690
C		1700
	WRITE (6,180) T2	1710
	CALL MATROP (NR,NC,C(IHT))	1720
C		1730
150	NCT=0	1740
	DO 160 L=1,NC	1750
	DO 160 K=1,NR	1760
	C(IHP+NCT)=C(IHT+NCT)	1770
	NCT=NCT+1	1780
160	CONTINUE	1790
	IF (I.EQ. LOOPUL) GO TO 170	
	CALL PFADC (NR,NC,C(ICAPW),C(ICRCHR),C(ICSGR),C(ICC),LBC,RBC,TBC,	
	1ARR,AGGIE)	
170	CONTINUE	1800
	STOP	1810
C		1820
180	FORMAT (1H1,44X, 22HHEAD MAP AT TIME LEVEL,F10.2,/1H ,52X, 18H(FEE	
	1T ABOVE DATUM))	

```

190 FORMAT (15H=ROW DIMENSION=I4,21H COLUMN DIMENSION=I4,24H TIM 1850
1E INCREMENTED BY G9.2,5H DAYS,27H TOTAL TIME OF ANALYSIS G9.2,5 18
2H DAYS) 1870
200 FORMAT (8A10) 1880
210 FORMAT (1H1,////,27X,8A10) 1890
220 FORMAT (6I5,5F10.1) 1900
230 FORMAT (1H,///, 20H FIELD LENGTH = ,020) 1910
240 FORMAT (1H=,47X, 25HUNCONFINED AQUIFER ANALYSIS) 1920
250 FORMAT (1H=,48X, 22HLEAKY AQUIFER ANALYSIS) 1930
260 FORMAT (1H=,46X, 27HUNCONFINED AQUIFER ANALYSIS) 1940
270 FORMAT (1F10.1)
C 195
END 196

```

SUBROUTINE READPH

```

SUBROUTINE READPH (NR,NC,FK,PHI,Z,G,DX,DY,CA,PHIC,CPPT,CAPW,CRCHR,
CSQR,CS,AREA,TAREA)
C RP 20
C RP 30
C THIS SUBROUTINE READS AND WRITES THE PHYSICAL DATA DESCRIBING
C THE SYSTEM. RP 40
C RP 50
C FK=PERMEABILITY (FEET/DAY) RP 60
C PHI=EFFECTIVE POROSITY RP 70
C Z=BEDROCK ELEVATION (FEET) RP 80
C G=GROUND SURFACE ELEVATION, OR TOP OF CONFINED AQUIFER (FEET) RP 90
C DX=X-DIMENSION OF GRID (FEET) RP 100
C DY=Y-DIMENSION OF GRID (FEET) RP 110
C CA=FRACTION OF GRID THAT WATER IS APPLIED (DECIMAL) RP 120
C PHIC=CONFINED AQUIFER STORAGE COEFFICIENT RP 130
C DLX=UNIFORM DX RP 140
C DLY=UNIFORM DY RP 150
C FFK=UNIFORM FK RP 160
C ZZ=UNIFORM Z RP 170
C GG=UNIFORM G RP 180
C PPHI=UNIFORM PHI RP 190
C CCA=UNIFORM CA RP 200
C PPHIC=UNIFORM PHIC RP 210
C CS=RELATIVE CONCENTRATION OF CONTAMINANT IN AQUIFER (DECIMAL)
C CPPT=RELATIVE CONCENTRATION OF CONTAMINANT IN PRECIPITATION
C CAPW=RELATIVE CONCENTRATION OF CONTAMINANT IN WATER APPLIED AS
C IRRIGATION
C CRCHR=RELATIVE CONCENTRATION OF CONTAMINANT IN RECHARGE WATERS
C CSQR=RELATIVE CONCENTRATION OF CONTAMINANT FROM CONSTANT HEAD
C SOURCES
C AREA=AREA OF EACH GRID
C TAREA=TOTAL AREA COVERED BY THE GRID NETWORK RP 220
C DIMENSION FK(NR,NC), PHI(NR,NC), Z(NR,NC), G(NR,NC), DX(NR,NC), DY
1(NR,NC), CA(NR,NC), PHIC(NR,NC), CS(NR,NC), AREA(NR,NC), CPPT(NR,
2NC), CAPW(NR,NC), CRCHR(NR,NC), CSQR(NR,NC) RP 230
C COMMON /BLK1/ DT,S7,ICFAC,ILKAQ,LCIF,LCIW,LCJF,LCJW,FWTCP RP 250
C RP 260
C RP 270
C DO 110 J=1,NR RP 280
C DO 110 K=1,NC RP 290
C FK(J,K)=0.0 RP 300
C Z(J,K)=0.0 RP 310
C G(J,K)=0.0 RP 320
C PHI(J,K)=0.0 RP 330
C CA(J,K)=0.0 RP 340
C PHIC(J,K)=0.0 RP 350
C CS(J,K)=0.0
C CPPT(J,K)=0.0
C CAPW(J,K)=0.0
C CRCHR(J,K)=0.0
C CSQR(J,K)=0.0
110 CONTINUE RP 360
C RP 370
C READ (5,440) DLX,DLY,FFK,ZZ,GG,PPHI,CCA,PPHIC RP 380
C RP 390
C IF (DLX.LE.0.0) GO TO 130 RP 400

```

DO 120 J=1,NC	RP 410
DO 120 I=1,NR	RP 420
120 DX(I,J)=DLX	RP 430
GO TO 150	RP 440
130 READ (5,440) (DX(1,J),J=1,NC)	RP 450
DO 140 I=2,NR	RP 460
DO 140 J=1,NC	RP 470
140 DX(I,J)=DX(1,J)	RP 480
150 CONTINUE	RP 490
C	RP 500
IF (OLY.LE.0.0) GO TO 170	RP 510
DO 160 J=1,NC	RP 520
DO 160 I=1,NR	RP 530
160 DY(I,J)=DLY	RP 540
GO TO 190	RP 550
170 READ (5,440) (DY(I,1),I=1,NR)	RP 560
DO 180 J=2,NC	RP 570
DO 180 I=1,NR	RP 580
180 DY(I,J)=DY(I,1)	RP 590
190 CONTINUE	RP 600
TAREA=0.0	
DO 195 J=1,NC	
DO 195 I=1,NR	
AREA(I,J)=DX(I,J)*DY(I,J)	
TAREA=TAREA+AREA(I,J)	
195 CONTINUE	
C	RP 610
IF (FFK.LE.0.0) GO TO 210	RP 620
DO 200 J=1,NR	RP 630
DO 200 K=1,NC	RP 640
200 FK(J,K)=FFK	RP 650
GO TO 220	RP 660
210 READ (5,440) FK	RP 670
C	RP 680
220 IF (ZZ.NE.0.0) GO TO 230	RP 690
IF (SIGN(1.0,ZZ).LT.0.0) GO TO 250	RP 700
230 DO 240 J=1,NR	RP 710
DO 240 K=1,NC	RP 720
240 Z(J,K)=ZZ	RP 730
GO TO 260	RP 740
250 READ (5,440) Z	RP 750
C	RP 760
260 IF (GG.NE.0.0) GO TO 270	RP 770
IF (SIGN(1.0,GG).LT.0.0) GO TO 290	RP 780
270 DO 280 J=1,NR	RP 790
DO 280 K=1,NC	RP 800
280 G(J,K)=GG	RP 810
GO TO 300	RP 820
290 READ (5,440) G	RP 830
C	RP 840
300 IF (PPHI.LE.0.0) GO TO 320	RP 850
DO 310 J=1,NR	RP 860
DO 310 K=1,NC	RP 870
310 PHI(J,K)=PPHI	RP 880
GO TO 330	RP 890
320 READ (5,440) PHI	RP 900
C	RP 910
330 IF (CCA.NE.0.0) GO TO 340	RP 920
IF (SIGN(1.0,CCA).LT.0.0) GO TO 360	RP 930
340 DO 350 J=1,NC	RP 940
DO 350 I=1,NR	RP 950
350 CA(I,J)=CCA	RP 960
GO TO 370	RP 970
360 READ (5,440) CA	RP 980
C	RP 990
370 IF (ICFAO.LE.0) GO TO 400	RP 1000
IF (PPHIC.LE.0.0) GO TO 390	RP 1010
DO 380 I=1,NR	RP 1020
DO 380 J=1,NC	RP 1030
380 PHIC(I,J)=PPHIC	RP 1040
GO TO 400	RP 1050
390 READ (5,440) PHIC	RP 1060
400 READ (5,445) CSS	
IF (CSS.LE.0.0) GO TO 610	
DO 600 J=1,NC	
DO 600 I=1,NR	
600 CS(I,J)=CSS	
GO TO 615	
610 READ (5,440) CS	
615 READ (5,445) CCPPT	
IF (CCPPT.LT.0.0) GO TO 630	

```

DO 620 J=1,NC
DO 620 I=1,NR
620 CPPT(I,J)=CCPPT
GO TO 640
630 READ (5,440) CPPT
C
640 READ (5,445) CCAPW
IF (CCAPW.LT.0.0) GO TO 650
DO 660 J=1,NC
DO 660 I=1,NR
660 CAPW(I,J)=CCAPW
GO TO 670
650 READ (5,440) CAPW
C
670 READ (5,445) CCRCHR
IF (CCRCHR.LT.0.0) GO TO 680
DO 690 J=1,NC
DO 690 I=1,NR
690 CRCHR(I,J)=CCRCHR
GO TO 700
680 READ (5,440) CRCHR
C
700 READ (5,445) CCSQR
IF (CCSQR.LT.0.0) GO TO 710
DO 720 J=1,NC
DO 720 I=1,NR
720 CSQR(I,J)=CCSQR
GO TO 730
710 READ (5,440) CSQR
730 CONTINUE
C
WRITE (6,530)
CALL MATROP (NR,NC,CS)
WRITE (6,540)
CALL MATPOP (NR,NC,CPPT)
WRITE (6,550)
CALL MATROP (NR,NC,CAPW)
WRITE (6,560)
CALL MATPOP (NR,NC,CRCHR)
WRITE (6,570)
CALL MATROP (NR,NC,CSQR)
WRITE (6,450)
CALL MATROP (NR,NC,DX)
WRITE (6,460)
CALL MATROP (NR,NC,DY)
IF (ICFAO.GT.0) GO TO 410
WRITE (6,470)
GO TO 420
410 WRITE (6,510)
420 CALL MATROP (NR,NC,G)
WRITE (6,480)
CALL MATROP (NR,NC,Z)
WRITE (6,490)
CALL MATROP (NR,NC,PHI)
IF (ICFAO.LE.0) GO TO 430
WRITE (6,520)
CALL MATROP (NR,NC,PHIC)
430 WRITE (6,500)
CALL MATROP (NR,NC,FK)
RETURN
C
440 FORMAT (8F10.1)
445 FORMAT (1F10.1)
450 FORMAT (1H1,40X, 50HDELTA=X MAP, SFACING ACROSS IN J-DIRECTION (FEET
1ET) ,/)
460 FORMAT (1H1,41X, 47HDELT=Y MAP, SPACING DOWN IN I-DIRECTION (FEET
1),/)
470 FORMAT (1H1,44X, 41HSURFACE ELEVATION MAP (FEET ABOVE DATUM),/)
480 FORMAT (1H1,44X, 41HREDROCK ELEVATION MAP (FEET ABOVE DATUM),/)
490 FORMAT (1H1,55X, 18HSPECIFIC YIELD MAP,/)
500 FORMAT (1H1,46X, 28HPERMEABILITY MAP (FEET/DAY),/)
510 FORMAT (1H1,30X, 57HTOP OF CONFINED AQUIFER ELEVATION MAP (FEET A
1BOVE DATUM),/)
520 FORMAT (1H1,39X, 40HCONFINED AQUIFER STORAGE COEFFICIENT MAP,/)
530 FORMAT (1H1,51X, 33HINITIAL AQUIFER CONCENTRATION MAP,/)
540 FORMAT (1H1,49X, 39HINITIAL PRECIPITATION CONCENTRATION MAP,/)
550 FORMAT (1H1,49X, 35HINITIAL APPLIED WATER CONCENTRATION MAP,/)
560 FORMAT (1H1,52X, 34HINITIAL RECHARGE CONCENTRATION MAP,/)
570 FORMAT (1H1,48X, 36HCONSTANT HEAD GRID CONCENTRATION MAP,/)
C
END

```

RP 1080

RP 1090
RP 1100
RP 1110
RP 1120
RP 1130
RP 1140
RP 1150
RP 1160
RP 1170
RP 1180
RP 1190
RP 1200
RP 1210
RP 1220
RP 1230
RP 1240
RP 1250
RP 1260
RP 1270
RP 1280
RP 1290

RP 1300
RP 1310
RP 1320
RP 1330
RP 1340
RP 1350
RP 1360
RP 1370
RP 1380
RP 1390
RP 1400

RP 1410
RP 1420

SUBROUTINE READH

```

SUBROUTINE READH (NR,NC,H,HP,HT,HF,LBC,RBC,TBC,BBC)
C
C
C THIS SUPROUTINE READS IN AN INITIAL WATER TABLE ELEVATION OR
C HEAD FOR COMPARING WATER LEVEL CHANGES.
C H=INITAL WATER TABLE ELEVATION OR HEAD (FEET)
C HT=PRESENT WATER TABLE ELEVATION OR HEAD (FEET)
C HP=WATER TABLE ELEVATION OR HEAD AT PREVICUS TIME LEVEL (FEET)
C HW=HORIZONTAL WATER LEVEL
C LBC=LEFT BOUNDARY CODE
C RBC=RIGHT BOUNDARY CODE
C TBC=TOP BOUNDARY CODE
C BBC=BOTTOM BOUNDARY CODE
C IDENTIFICATION OF BOUNDARY VALUES OF H.
C H(I,J) LESS THAN 10,000 = WATER TABLE ELEVATION (NO BOUNDARY)
C H(I,J) GREATER THAN 10000 BUT LESS THAN 20000 = IMPERMEABLE
C H(I,J) GREATER THAN 20000 BUT LESS THAN 30000 = UNDERFLOW
C H(I,J) GREATER THAN 30000 BUT LESS THAN 40000 = CCNSTANT HEAD
C
C DIMENSION H(NR,NC), HT(NR,NC), HP(NR,NC), HF(NR,NC)
C REAL LBC
C
C READ (5,220) HW,LBC,RBC,TBC,BBC
C
C IF (HW.LE.0.0) GO TO 140
C DO 110 I=1,NR
C DO 110 J=1,NC
110 H(I,J)=HW
C DO 120 I=1,NR
C H(I,1)=LBC+HW
C H(I,NC)=RBC+HW
120 CONTINUE
C DO 130 J=1,NC
C H(1,J)=TBC+HW
C H(NR,J)=BBC+HW
130 CONTINUE
C GO TO 150
C
140 READ (5,220) H
150 DO 210 J=1,NC
C DO 210 I=1,NR
C KK=H(I,J)/10000.+1
C GO TO (160,170,180,190), KK
160 HT(I,J)=H(I,J)
C GO TO 200
170 HT(I,J)=H(I,J)-10000.
C GO TO 200
180 HT(I,J)=H(I,J)-20000.
C GO TO 200
190 HT(I,J)=H(I,J)-30000.
200 HP(I,J)=HT(I,J)
C HF(I,J)=HT(I,J)
210 CONTINUE
C RETURN
C
220 FORMAT (8F10.1)
C
C END

```

RH 25
 RH 30
 RH 40
 RH 50
 RH 60
 RH 70
 RH 80
 RH 90
 RH 100
 RH 110
 RH 120
 RH 130
 RH 140
 RH 150
 RH 160
 RH 170
 RH 180
 RH 190
 RH 200
 RH 210
 RH 220
 RH 230
 RH 240
 RH 250
 RH 260
 RH 270
 RH 280
 RH 290
 RH 300
 RH 310
 RH 320
 RH 330
 RH 340
 RH 350
 RH 360
 RH 370
 RH 380
 RH 390
 RH 400
 RH 410
 RH 420
 RH 430
 RH 440
 RH 450
 RH 460
 RH 470
 RH 480
 RH 490
 RH 500
 RH 510
 RH 520
 RH 530
 RH 540
 RH 550
 RH 560
 RH 570
 RH 580

SUBROUTINE LEKAQF

```

SUBROUTINE LEKAQF (NR,NC,HL,TL,FKL)
C
C
C THIS SUPROUTINE READS IN LEAKY AQUIFER PARAMETERS.
C HL=CONSTANT HEAD VALUE CAUSING LEAK (FEET)
C TL=THICKNESS OF LEAKY LAYER (FEET)
C FKL=PERMEABILITY OF LEAKY LAYER (FEET/DAY)
C
C DIMENSION HL(NR,NC), TL(NR,NC), FKL(NR,NC)
C
C

```

LA 10
 LA 20
 LA 30
 LA 40
 LA 50
 LA 60
 LA 70
 LA 80
 LA 90
 LA 100

	DO 110 I=1,NR	LA 110
	DO 110 J=1,NC	LA 120
	HL(I,J)=0.0	LA 130
	TL(I,J)=0.0	LA 140
	FKL(I,J)=0.0	LA 150
110	CONTINUE	LA 160
	READ (5,210) HHL,TTL,FFKL	LA 170
	IF (HHL.LE.0.0) GO TO 130	LA 180
	DO 120 I=1,NR	LA 190
	DO 120 J=1,NC	LA 200
120	HL(I,J)=HHL	LA 210
	GO TO 140	LA 220
130	RFAD (5,210) HL	LA 230
140	IF (TTL.LE.0.0) GO TO 160	LA 240
	DO 150 I=1,NR	LA 250
	DO 150 J=1,NC	LA 260
150	TL(I,J)=TTL	LA 270
	GO TO 170	LA 280
160	READ (5,210) TL	LA 290
170	IF (FFKL.LE.0.0) GO TO 190	LA 300
	DO 180 I=1,NR	LA 310
	DO 180 J=1,NC	LA 320
180	FKL(I,J)=FFKL	LA 330
	GO TO 200	LA 340
190	RFAD (5,210) FKL	LA 350
200	CONTINUE	LA 360
	WRITE (6,220)	LA 370
	CALL MATROP (NR,NC,HL)	LA 380
	WRITE (6,230)	LA 390
	CALL MATROP (NR,NC,TL)	LA 400
	WRITE (6,240)	LA 410
	CALL MATROP (NR,NC,FKL)	LA 420
	RETURN	LA 430
C		LA 440
210	FORMAT (8F10.1)	LA 450
220	FORMAT (1H1,////,38X, 44HHEAD MATRIX CAUSING LEAK (FEET ABOVE DA	LA 460
	1TUM),//)	LA 470
230	FORMAT (1H1,////,44X, 32HTHICKNESS OF LEAKY LAYER (FEET),//)	LA 480
240	FORMAT (1H1,////,36X, 48HVERTICAL PERMEABILITY OF LEAKY LAYER (F	LA 490
	1EET/DAY),//)	LA 500
C		LA 510
	END	LA 520

SUBROUTINE STORAGE

	SUBROUTINE STORAGE (NR,NC,H,HT,Z,DX,DY,S,G,SC)	ST 10
C		ST 20
C		ST 30
C	THIS SUBROUTINE COMPUTES THE INCREASE OR DECREASE IN STORAGE.	ST 40
C	STA=BETWEEN STATIONS STORAGE (AF)	ST 50
C	STT=TOTAL AREA STORAGE (AF)	ST 60
C	STOL=OVERLAP AREA STORAGE (AF)	ST 70
C		ST 80
C	DIMENSION H(NR,NC), HT(NR,NC), Z(NR,NC), DX(NR,NC), DY(NR,NC), S(N	ST 90
	1R,NC), G(NR,NC), SC(NR,NC)	ST 100
C		ST 110
	COMMON /BLK1/ DT,ST,ICFAQ,ILKAO,LCIE,LCIW,LCJE,LCJW,FKTOP	ST 120
	COMMON /BLK2/ STA,STOL,STT,SOA,SOT,SORA,SOBA,SORT,SQBT,OVA,OVT	ST 130
C		ST 140
	NC1=NC-1	ST 150
	NP1=NR-1	ST 160
	STT=0.0	ST 170
	STA=0.0	ST 180
	DO 130 L=2,NC1	ST 190
	DO 130 K=2,NR1	ST 200
	IF (H(K,L).GT.10000.) GO TO 130	ST 210
	IF (ICFAQ.LE.G) GO TO 110	ST 220
	IF (HT(K,L).LE.G(K,L)) GO TO 130	ST 230
	STP=((G(K,L)-Z(K,L))*S(K,L)+(HT(K,L)-G(K,L))*SC(K,L))*DX(K,L)*D	ST 240
1	Y(K,L)/43560.	ST 250
	GO TO 120	ST 260
110	STP=(HT(K,L)-Z(K,L))*DX(K,L)*DY(K,L)*S(K,L)/43560.	ST 270
120	STT=STT+STP	ST 280

```

      IF (L.LT.LCIW) GO TO 130
      IF (L.GT.LCIE) GO TO 130
      IF (K.LT.LCJW) GO TO 130
      IF (K.GT.LCJE) GO TO 130
      STA=STA+STP
130 CONTINUE
      STOL=STT-STA
C      RETURN
C      END

```

ST 290
 ST 300
 ST 310
 ST 320
 ST 330
 ST 340
 ST 350
 ST 360
 ST 370
 ST 380
 ST 390

SUBROUTINE CSET

```

SUBROUTINE CSET (NR,NC,CO,CT,H,G,CS)
C
C THIS SUBROUTINE SETS THE INITIAL RELATIVE CONCENTRATION IN EACH
C GRID OF THE AQUIFER.
C
C CS=INITIAL CODED AQUIFER CONCENTRATION
C CO=INITIAL (OR CURRENT) UNCODED AQUIFER CONCENTRATION
C CT=NEW UNCODED AQUIFER CONCENTRATION
C
C DIMENSION CO(NR,NC), CT(NR,NC),H(NR,NC),G(NR,NC),CS(NR,NC)
C DO 100 J=1,NC
C DO 100 I=1,NR
C CT(I,J)=0.0
C IF (CS(I,J).GT.2.0) GO TO 10
C CO(I,J)=CS(I,J)
C GO TO 100
10 CO(I,J)=CS(I,J)=2.0
100 CONTINUE
C RETURN
C END

```

SUBROUTINE QFIX

```

SUBROUTINE QFIX (NR,NC,DX,DY,CA,H,Z,HT,I,Q,NW,NP,PHR,WELL,PIT,YPT, QF 10
1YAW,YPR,YPM,RPUM,CPM,YFC,RCHR,INYR,HL,TL,FKL,G,PHRMP,CPPT,QAPW,QR
2CHR,QPHR,QPUM,QLEAK,AREA,TAREA)
C
C THIS SUBROUTINE COMPUTES THE HYDROLOGIC AND ARTIFICIAL INPUTS.
C PPT=PRECIPITATION (INCHES/YEAR) QF 30
C CPY=COEF. OF EFFECTIVE PRECIPITATION TO GROUNDWATER (DECIMAL) QF 40
C YPT=DISTRIBUTION OF PRECIPITATION FOR EACH DT OF ONE YEAR QF 50
C (DECIMAL) QF 60
C APW=APPLIED WATER AS A RESULT OF SURFACE IRRIGATION (FEET/YEAR) QF 70
C CAW=COEF. OF DEEP PERCOLATION OF APPLIED WATER (DECIMAL) QF 80
C YAW=DISTRIBUTION OF APPLIED WATER FOR EACH DT OF ONE YEAR QF 90
C (DECIMAL) QF 100
C NGPU=NUMBER OF GRIDS WITH PHREATOPHYTE USE QF 110
C PHR=WATER USED BY PHREATOPHYTES (AF/YEAR). THIS MAY BE QF 120
C CALCULATED FROM THE ET SUBPROGRAM BY CODING PHR VALUES QF 130
C LESS THAN ZERO. QF 140
C YPR=DISTRIBUTION OF PHREATOPHYTE USE FOR EACH DT OF ONE YEAR QF 150
C (DECIMAL) QF 160
C WELL=WELL NUMBER CODE QF 170
C RPUM=AMOUNT EACH WELL PUMPS PER YEAR (AF/YEAR) QF 180
C CPM=COEF. OF GROUNDWATER REMOVED BY PUMPING (DECIMAL) QF 190
C YPM=DISTRIBUTION OF PUMPING FOR EACH DT OF ONE YEAR (DECIMAL) QF 200
C PIT=RECHARGE PIT NUMBER CODE QF 210
C RCHR=AMOUNT EACH PIT RECHARGES PER YEAR (FEET) QF 220
C YRC=DISTRIBUTION OF PIT RECHARGE FOR EACH DT OF ONE YEAR QF 230
C (DECIMAL) QF 240
C Q=NET VALUE OF HYDROLOGIC AND ARTIFICIAL INPUT PER GRID (AF/DAY) QF 250

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C	SQT=TOTAL Q PER DT (AF)	QF 290
C	SQA=TOTAL Q PER DT BETWEEN STATIONS (AF)	QF 300
C	REPEAT=DATA INPUT CODE FOR MULTIPLE YEAR ANALYSIS	QF 310
C		QF 320
	DIMENSION DX(NR,NC), DY(NR,NC), CA(NR,NC), H(NR,NC), Z(NR,NC), HT(QF 330
	1NR,NC), O(NR,NC), PHR(NR,NC), WELL(NR,NC), FIT(NR,NC), YPT(INYR),	QF 340
	2YAW(INYR), YPR(INYR), YPM(INW,INYR), RFUM(INW), CFM(INW), YRC(INP,INYR	QF 350
	3), RCHR(INP), HL(NR,NC), TL(NR,NC), FKL(NR,NC), G(NR,NC), PHRTMP(NR	QF 360
	4,NC), QPPT(NR,NC), QAPW(NR,NC), QPCHR(KR,NC), QPHR(NR,NC), QLFAN(NR	
	5,NC), QPUM(NR,NC), AREA(NR,NC)	
C		QF 380
	COMMON /BLK1/ DT,ST,ICFAQ,ILKAQ,LCIE,LCIW,LCJF,LCJW,FHTOP	QF 390
	COMMON /BLK2/ STA,STOL,STT,SQA,SQT,SQFA,SQBA,SQRT,SQBT,CVA,OVT	QF 400
C		QF 410
	IF (I.NE.1) GO TO 320	QF 420
	ICT=0	QF 430
	INDX=1	QF 440
110	DO 120 K=1,NR	QF 450
	DO 120 L=1,NC	QF 460
	PHR(K,L)=0.0	QF 470
	Q(K,L)=0.0	QF 480
	PHRTMP(K,L)=0.0	QF 490
	WELL(K,L)=0.0	QF 500
	PIT(K,L)=0.0	QF 510
120	CONTINUE	QF 520
	ETCNT=0.0	QF 530
	DO 130 K=1,INYR	QF 540
	YPT(K)=0.0	QF 550
	YAW(K)=0.0	QF 560
	YPR(K)=0.0	QF 570
130	CONTINUE	QF 580
	FFT=HT(I,J)-G(I,J)	QF 590
	READ (5,470) PPT,CPT	QF 600
	IF (PPT.LE.0.0) GO TO 140	QF 610
	READ (5,480) (YPT(K),K=1,INYR)	QF 620
140	READ (5,470) APW,CAW	QF 630
	IF (APW.LE.0.0) GO TO 150	QF 640
	READ (5,480) (YAW(K),K=1,INYR)	QF 650
150	CONTINUE	QF 660
	READ (5,490) NGPU	QF 670
	IF (NGPU.EQ.0) GO TO 170	QF 680
	DO 160 NPR=1,NGPU	QF 690
	READ (5,490) J,K,P	QF 700
	PHR(J,K)=P	QF 710
160	CONTINUE	QF 720
	GO TO 180	QF 730
170	IF (SIGN(1.0,NGPU).GT.0.0) GO TO 210	QF 740
	READ (5,470) PHF	QF 750
180	DO 190 KK=1,NR	QF 760
	DO 190 LL=1,NC	QF 770
	IF (PHR(KK,LL).LT.0.0) GO TO 210	QF 780
	IF (PHR(KK,LL).GT.0.0) GO TO 200	QF 790
190	CONTINUE	QF 800
200	READ (5,480) (YPR(K),K=1,INYR)	QF 810
210	IF (NW.LE.0) GO TO 250	QF 820
	READ (5,490) NW	QF 830
	DO 220 K=1,NW	QF 840
	DO 220 L=1,INYR	QF 850
	YPM(K,L)=0.0	QF 860
220	CONTINUE	QF 870
	DO 240 J=1,NW	QF 880
	READ (5,500) IWNO,K,L,RPUM(J),CPM(J)	QF 890
	WELL(K,L)=IWNO	QF 900
	READ (5,480) (YPM(IWNO,K),K=1,INYR)	
	IF (YPM(J,1).NE.0.0) GO TO 240	QF 920
	IF (SIGN(1.0,YPM(J,1)).GE.0.0) GO TO 240	QF 930
	DO 230 K=1,INYR	QF 940
230	YPM(J,K)=YPM(J-1,K)	QF 950
240	CONTINUE	QF 960
250	IF (NP.LE.0) GO TO 290	QF 970
	READ (5,490) NP	QF 980
	DO 260 K=1,NP	QF 990
	DO 260 L=1,INYR	QF 1000
	YRC(K,L)=0.0	QF 1010
260	CONTINUE	QF 1020
	DO 280 J=1,NP	QF 1030
	READ (5,500) IPNO,K,L,RCHR(J)	QF 1040
	PIT(K,L)=IPNO	QF 1050
	READ (5,480) (YRC(IPNO,K),K=1,INYR)	
	IF (YRC(J,1).NE.0.0) GO TO 280	QF 1070
	IF (SIGN(1.0,YRC(J,1)).GE.0.0) GO TO 280	QF 1080

	DO 270 K=1,INYP	QF 1090
270	YRC(J,K)=YRC(J-1,K)	QF 1100
280	CONTINUE	QF 1110
290	CONTINUE	QF 1120
	WRITF (6,510)	QF 1130
	CALL MATROP (NR,NC,CA)	QF 1140
	WRITE (6,620) APW	QF 1150
	WRITE (6,630) (YAW(K),K=1,INYP)	QF 1160
	WRITF (6,560)	QF 1170
	CALL MATROP (NR,NC,PHF)	QF 1180
	WRITE (6,630) (YPC(K),K=1,INYP)	QF 1190
	WRITE (6,640) PPT	QF 1200
	WRITE (6,630) (YPT(K),K=1,INYP)	QF 1210
	IF (NH.LE.0) GO TO 300	QF 1220
	WRITE (6,520)	QF 1230
	CALL MATROP (NR,NC,WELL)	QF 1240
	WRITF (6,650)	QF 1250
	DO 295 J=1,NH	
295	WRITE(6,660)J,RPUM(J),(YPM(J,K),K=1,INYP)	
300	IF (NF.LE.0) GO TO 310	QF 1270
	WRITE (6,530)	QF 1280
	CALL MATROP (NR,NC,PIT)	QF 1290
	WRITE (6,670)	QF 1300
	DO 305 J=1,NP	
305	WRITE(6,680)J,RCHR(J),(YRC(J,K),K=1,INYP)	
C		QF 1320
310	IF (I.NE.1) GO TO 320	QF 1330
	WRITE (6,590)	QF 1340
	CALL MATROP (NR,NC,H)	QF 1350
	WRITF (6,600) STA,STOL,STT	QF 1360
C		QF 1370
320	ICT=ICT+1	QF 1380
	IF (ICT.LE.INYP) GO TO 330	QF 1390
	ICT=ICT-INYP-1	QF 1400
	READ (5,470) REPEAT	QF 1410
	IF (REPEAT.NE.0.0) GO TO 320	QF 1420
	KYEAR=I/INYP	QF 1430
	WRITE (6,610) KYEAR	QF 1440
	GO TO 110	QF 1450
330	PCNT=INDX	QF 1460
	FI=I	QF 1470
	SQA=0.0	QF 1480
	SOT=0.0	QF 1490
	KCT=0	QF 1510
	DO 440 K=1,NR	QF 1520
	DO 440 L=1,NC	QF 1530
	IF (H(K,L).GT.1.E4) GO TO 440	QF 1540
	IF (HT(K,L).GT.Z(K,L)) GO TO 350	QF 1550
	A=0.0	QF 1560
	IF (KCT.GT.0) GO TO 340	QF 1570
	WRITE (6,580) I	QF 1580
	KCT=1	QF 1590
340	WRITE (6,540) K,L	QF 1600
	GO TO 360	QF 1610
350	A=1.0	QF 1620
360	CONTINUE	QF 1630
C		QF 1640
	QPPT(K,L)=(PPT*CPT*YPT(ICT)*AREA(K,L)/TAREA)/(12.*43560.)	
	QAPH(K,L)=(APH*CAW*CA(K,L)*YAW(ICT)*AREA(K,L)/TAREA)/43560.	QF 1670
	JJJ=PIT(K,L)	QF 1680
	IF (JJJ.LE.0) GO TO 370	
	QRCHR(K,L)=RCHR(JJJ)*YRC(JJJ,ICT)*AREA(K,L)/43560.	QF 1700
	GO TO 380	
370	QRCHR(K,L)=0.0	
380	CONTINUE	QF 1720
	IF (PHR(K,L).GE.0.0) GO TO 390	QF 1730
	QPHR(K,L)=ET(HT(K,L),G(K,L),K,L)*AREA(K,L)/43560.	
	PHRTMP(K,L)=QPHR(K,L)	
	ETCNT=1.	QF 1760
	GO TO 400	QF 1770
390	QPHR(K,L)=PHR(K,L)*YPR(ICT)	
400	CONTINUE	QF 1790
	JJ=WELL(K,L)	QF 1800
	IF (JJ.LE.0) GO TO 410	QF 1810
	QPUM(K,L)=RPUM(JJ)*YPM(JJ,ICT)*CPM(JJ)	
	GO TO 420	QF 1830
410	QPUM(K,L)=0.0	
420	CONTINUE	QF 1850
	IF (ILKAQ.LE.0) GO TO 430	QF 1860
	QLFAK(K,L)=FKL(K,L)*(HT(K,L)-HL(K,L))*AREA(K,L)*DT/(TL(K,L)*435	
	160.)	
	GO TO 431	

SUBROUTINE MATSOL

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SUBROUTINE MATSOL (NOROW,NOCOL,IP,IR,FK,FHI,H,HT,Z,DELY,DFLY,Q,CMA MS 10
1T=X,CR,A,R,G,PHIC,HP,HF) MS 20
C MS 30
C MS 40
C THIS SUBROUTINE SETS UP THE COEFFICIENT MATRIX AND RIGHT HAND MS 50
C SIDE VECTOR MATRIX. MS 60
C CMATRX=COEFFICIENT MATRIX MS 70
C CR=RIGHT HAND SIDE VECTOR MATRIX MS 80
C MS 90
C DIMENSION FK(NOROW,NOCOL), PHI(NOROW,NOCOL), H(NCROW,NCCOL), HT(NC MS 100
1ROW,NOCOL), Z(NOROW,NOCOL), DELX(NORCW,NCCOL), DELY(NCROW,NOCOL), MS 110
2Q(NOROW,NOCOL), CMATRX(IP,IR), CR(IP), A(NOROW,NOCOL), B(NORCW,NOC MS 120
3OL), G(NOROW,NOCOL), PHIC(NORCW,NCCOL), HF(NCROW,NCCOL), HF(NCROW, MS 130
4NOCOL) MS 140
C MS 150
C COMMON /BLK1/ DT,ST,ICFAO,ILKAO,LCIE,LCIW,LCJE,LCJW,FWTOP MS 160
C MS 170
C DELT=DT MS 180
DO 110 J=1,IR MS 190
DO 110 I=1,IP MS 200
CMATRX(I,J)=0.0 MS 210
110 CONTINUE MS 220
DO 120 I=1,NOROW MS 230
DO 120 J=1,NOCOL MS 240
A(I,J)=0.0 MS 250
B(I,J)=0.0 MS 260
120 CONTINUE MS 270
NT=J MS 280
NC1=NOCOL-1 MS 290
NR1=NOROW-1 MS 300
IB=NOROW-2 MS 310
IM=IB+1 MS 320
IC=IM+1 MS 330
ID=2*IB+1 MS 340
DO 160 J=2,NC1 MS 350
DO 160 I=2,NR1 MS 360
NT=NT+1 MS 370
CR(NT)=0.0 MS 380
IF (H(I,J).GE.10000.0) GO TO 150 MS 390
JA=I MS 400
JD=I MS 410
C MS 420
C LEFT (A) MS 430
C MS 440
CMATRX(NT,1)=PARAM(FK(JA,J-1),FK(I,J),HT(JA,J-1),HT(I,J),Z(JA,J MS 450
1 -1),Z(I,J),DELY(JA,J-1),DELY(I,J),DELY(JA,J-1),DELY(I,J),G(JA,J MS 460
2 -1),G(I,J)) MS 470
C MS 480
C TOP (B) MS 490
C MS 500
CMATRX(NT,IB)=PARAM(FK(I-1,J),FK(I,J),HT(I-1,J),HT(I,J),Z(I-1,J MS 510
1 ),Z(I,J),DELY(I-1,J),DELY(I,J),DELY(I-1,J),DELY(I,J),G(I-1,J),G MS 520
2 (I,J)) MS 530
C MS 540
C BOTTOM (C) MS 550
C MS 560
CMATRX(NT,IC)=PARAM(FK(I+1,J),FK(I,J),HT(I+1,J),HT(I,J),Z(I+1,J MS 570
1 ),Z(I,J),DELY(I+1,J),DELY(I,J),DELY(I+1,J),DELY(I,J),G(I+1,J),G MS 580
2 (I,J)) MS 590
A(I,J)=CMATRX(NT,IC) MS 600
C MS 610
C RIGHT (D) MS 620
C MS 630
CMATRX(NT,ID)=PARAM(FK(JD,J+1),FK(I,J),HT(JD,J+1),HT(I,J),Z(JD, MS 640
1 J+1),Z(I,J),DELY(JD,J+1),DELY(I,J),DELY(JD,J+1),DELY(I,J),G(JD, MS 650
2 J+1),G(I,J)) MS 660
B(I,J)=CMATRX(NT,ID) MS 670
C MS 680
C CALL NSCONT (H(JA,J-1),HT(JA,J-1),HT(I,J),Z(JA,J-1),Z(I,J),CMAT MS 690
1 RX(NT,1),CMATRX(NT,IM),CR(NT)) MS 700
CALL NSCONT (H(I-1,J),HT(I-1,J),HT(I,J),Z(I-1,J),Z(I,J),CMATRX( MS 710
1 NT,IB),CMATRX(NT,IM),CR(NT)) MS 720
CALL NSCONT (H(I+1,J),HT(I+1,J),HT(I,J),Z(I+1,J),Z(I,J),CMATRX( MS 730
1 NT,IC),CMATRX(NT,IM),CR(NT)) MS 740
CALL NSCONT (H(JD,J-1),HT(JD,J-1),HT(I,J),Z(JD,J-1),Z(I,J),CMAT MS 750
1 RX(NT,ID),CMATRX(NT,IM),CR(NT)) MS 760
C MS 770
C (E) MS 780
C MS 790

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      IF (ICFAQ.LE.0) GO TO 130
      IF (HT(I,J).LE.G(I,J)) GO TO 130
      STCOEF=PHIC(I,J)
      GO TO 140
130  STCOEF=PHI(I,J)
140  CONTINUE
      CMATRX(NT,IM)=CMATRX(NT,IP)-(CMATRX(NT,1)+CMATRX(NT,IB)+CMATRX(
1    NT,IC)+CMATRX(NT,IO)+(STCOEF*DELX(I,J)*DFLY(I,J))/DELTA)
      CR(NT)=CR(NT)-(HT(I,J)*STCOEF*CELX(I,J)*CELY(I,J))/DELTA=Q(I,J)*
1    43560.0
C
      GO TO 160
150  CMATRX(NT,IM)=1.0
      CR(NT)=HT(I,J)
160  CONTINUE
      REWIND 7
      WRITE (7) CMATRX,CR
      CALL BSOLVE (CMATRX,IP,IR,CR)
      NT=0
      DO 170 J=2,NC1
      DO 170 I=2,NR1
          NT=NT+1
          HT(I,J)=CR(NT)
          HF(I,J)=CR(NT)
170  CONTINUE
      IF (ICFAQ.LE.0) GO TO 230
      REWIND 7
      READ (7) CMATRX,CR
      ICAC=0
      NT=0
      DO 210 J=2,NC1
      DO 210 I=2,NR1
          NT=NT+1
          IF (HT(I,J).LE.G(I,J)) GO TO 180
          IF (HP(I,J)=G(I,J)) 190,190,210
          IF (HP(I,J)=G(I,J)) 210,210,200
180  ERROR=(G(I,J)*(PHI(I,J)-PHIC(I,J)))*DELX(I,J)*DELY(I,J)/DELTA
190  CR(NT)=CR(NT)+ERROR
      CMATRX(NT,NR1)=CMATRX(NT,NR1)+(PHI(I,J)-PHIC(I,J))*DELX(I,J)*DE
1    LY(I,J)/DELTA
      WRITE (6,240) I,J
      ICAC=1
      GO TO 210
200  ERROR=(G(I,J)*(PHI(I,J)-PHIC(I,J)))*DELX(I,J)*DELY(I,J)/DELTA
      CR(NT)=CR(NT)-ERROR
      CMATRX(NT,NR1)=CMATRX(NT,NR1)+(PHIC(I,J)-PHI(I,J))*DELX(I,J)*DE
1    LY(I,J)/DELTA
      WRITE (6,250) I,J
      ICAC=1
210  CONTINUE
      IF (ICAC.EQ.0) GO TO 230
      CALL BSOLVE (CMATRX,IP,IR,CR)
      NT=0
      DO 220 J=2,NC1
      DO 220 I=2,NR1
          NT=NT+1
          HT(I,J)=CR(NT)
          HF(I,J)=CR(NT)
220  CONTINUE
230  RETURN
C
240  FORMAT (1H ,43X, 4HGRID,2I5,5X, 22HUNCONFINED TO CONFINED)
250  FORMAT (1H ,43X, 4HGRID,2I5,5X, 22HCCNFIED TO UNCONFINED)
C
      FND

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FUNCTION PARAM

```

C      FUNCTION PARAM(AK1,AK2,AHT1,AHT2,AZ1,AZ2,AX1,AX2,AY1,AY2,AG1,AG2) PR 30
C      PR 20
C      PR 30
C      THIS SUBPROGRAM COMPUTES THE COEFFICIENTS USED IN MATSOL AND PR 40
C      BYFLOW. IT IS APPLICABLE TO CASES OF VARIABLE OX, CY, FK PR 50
C      AND SATURATED THICKNESS. PR 60
C      PR 70

```

```

COMMON /BLK1/ DT,ST,ICFAQ,ILKAQ,L(IE,LCIN,LCJE,LCJW,FATOP
C
IF (ICFAQ.LE.0) GO TO 110
A=AMIN1(AHT1,AG1)
B=AMIN1(AHT2,AG2)
SATHCK=AMAX1(A,B)-AMAX1(AZ1,AZ2)
GO TO 120
110 SATHCK=AMAX1(AHT1,AHT2)-AMAX1(AZ1,AZ2)
120 PARAM=(2.*AK1*AK2*AY1*AY2*SATHCK)/((AX1*AK2*AY2)+(AX2*AK1*AY1))
C
RETURN
C
END

```

PR 8
PR 9
PR 10
PR 11
PR 12
PR 13
PR 14
PR 15
PR 16
PR 17
PR 18
PR 19
PR 20

SUBROUTINE NSCONT

```

SUBROUTINE NSCONT (HA,HTA,HTM,ZA,ZH,CRXA,CRXM,CRL)
C
C
C THIS SUBROUTINE TRANSFERS THE COEFFICIENTS, MULTIPLIED BY THEIR
C RESPECTIVE H-VALUE, TO THE RIGHT HAND SIDE VECTOR MATRIX IN
C CASE OF ADJACENT CONSTANT HEAD OR KNOWN BOUNDARY CONDITIONS.
C IT ALSO SETS COEFFICIENTS EQUAL TO ZERO IN CASE OF ADJACENT
C IMPERMEABLE BOUNDARIES.
C
IF (HA.LT.20000.0) GO TO 110
CRL=CRL-CRXA*HTA
CRXM=CRXM-CRXA
CRXA=0.0
GO TO 120
110 IF (HA.GE.10000.0) GO TO 130
120 IF ((HTM-ZH).LE.1.0.AND.HTM.GT.HTA) GO TO 130
IF ((HTA-ZA).GT.1.0.OR.HTA.LE.HTM) GO TO 140
130 CRXA=0.0
C
140 RETURN
C
END

```

NC 10
NC 20
NC 30
NC 40
NC 50
NC 60
NC 70
NC 80
NC 90
NC 100
NC 110
NC 120
NC 130
NC 140
NC 150
NC 160
NC 170
NC 180
NC 190
NC 200
NC 210
NC 220

SUBROUTINE BSOLVE

```

SUBROUTINE BSOLVE (D,N,M,V)
C
C
C THIS SUBROUTINE SOLVES THE MATRIX, SET UP IN MATSOL, BY GAUSS
C ELIMINATION.
C
DIMENSION D(N,M), V(N)
C
LR=(M-1)/2
DO 120 L=1,LR
IM=LR-L+1
DO 120 I=1,IM
DO 110 J=2,M
110 D(L,J-1)=D(L,J)
KN=N-L
KM=M-I
D(L,M)=0.0
120 D(KN+1,KM+1)=0.0
LF=LR+1
IM=N-1
DO 130 I=1,IM
NPIV=I
LS=I+1
DO 130 L=LS,LR
IF (ABS(D(L,1)).GT.ABS(D(NPIV,1))) NPIV=L
130 CONTINUE

```

BS 10
BS 20
BS 30
BS 40
BS 50
BS 60
BS 70
BS 80
BS 90
BS 100
BS 110
BS 120
BS 130
BS 140
BS 150
BS 160
BS 170
BS 180
BS 190
BS 200
BS 210
BS 220
BS 230
BS 240
BS 250
BS 260

```

      IF (NPIV.LE.I) GO TO 150
      DO 140 J=1,M
        TEMP=D(I,J)
        D(I,J)=D(NPIV,J)
140    D(NPIV,J)=TEMP
        TEMP=V(I)
        V(I)=V(NPIV)
        V(NPIV)=TEMP
150    V(I)=V(I)/D(I,1)
        DO 160 J=2,M
160      D(I,J)=D(I,J)/D(I,1)
        DO 180 L=LS,LR
          TEMP=D(L,1)
          V(L)=V(L)-TEMP*V(I)
          DO 170 J=2,M
170            D(L,J)=D(L,J)-TEMP*D(I,J)
180          D(L,M)=0.0
          IF (LR.LT.N) LP=LR+1
190    CONTINUE
        V(N)=V(N)/D(N,1)
        JM=2
        DO 210 I=1,IM
          L=N-I
          DO 200 J=2,JM
            KM=L+J
200          V(L)=V(L)-D(L,J)*V(KM-1)
            IF (JM.LT.M) JM=JM+1
210    CONTINUE
C      RETURN
C      END

```

```

BS 270
BS 280
BS 290
BS 300
BS 310
BS 320
BS 330
BS 340
BS 350
BS 360
BS 370
BS 380
BS 390
BS 400
BS 410
BS 420
BS 430
BS 440
BS 450
BS 460
BS 470
BS 480
BS 490
BS 500
BS 510
BS 520
BS 530
BS 540
BS 550
BS 560
BS 570
BS 580

```

SUBROUTINE BJUST

```

      SUBROUTINE BJUST (NR,NC,H,HT,HP,DX,DY,I)
C
C
C      THIS SUBROUTINE ADJUSTS THE UNDERFLOW BOUNDARY WATER TABLE OR
C      HEAD ELEVATIONS. BOUNDARY ELEVATIONS ARE HELD CONSTANT FOR
C      ODD TIME STEPS. GRADIENTS ARE COMPUTED THREE GRIDS IN AND
C      PROJECTED BACK TO OBTAIN NEW WATER LEVEL ELEVATIONS OR
C      HEAD ELEVATIONS AT BOUNDARIES.
C
C      DIMENSION H(NR,NC), HT(NR,NC), HP(NR,NC), DX(NR,NC), DY(NR,NC)
C
C      IF ((I/2)*2).NE.I) RETURN
C
C      NR1=NR-1
C      NC1=NC-1
C
C      DO 120 I2=2,NC1
        IF (H(1,I2).GE.30000.0) GO TO 110
        IF (H(1,I2).LT.20000.0) GO TO 110
        DITP=(HT(2,I2)-HT(3,I2))*(DY(1,I2)+DY(2,I2))/(DY(2,I2)+DY(3,I2))
1      HT(1,I2)=HT(2,I2)+DITP
110      IF (H(NR,I2).GE.30000.0) GO TO 120
        IF (H(NR,I2).LT.20000.0) GO TO 120
        DIRT=(HT(NR-2,I2)-HT(NR-1,I2))*(DY(NR,I2)+DY(NR-1,I2))/(DY(NR-2
1      ,I2)+DY(NR-1,I2))
        HT(NR,I2)=HT(NR1,I2)-DIRT
120    CONTINUE
C
C      DO 140 I1=2,NR1
        IF (H(I1,1).GE.30000.0) GO TO 130
        IF (H(I1,1).LT.20000.0) GO TO 130
        DILT=(HT(I1,2)-HT(I1,3))*(DX(I1,1)+DX(I1,2))/(DX(I1,2)+DX(I1,3))
1      HT(I1,1)=HT(I1,2)+DILT
130      IF (H(I1,NC).GE.30000.0) GO TO 140
        IF (H(I1,NC).LT.20000.0) GO TO 140
        DIRT=(HT(I1,NC-2)-HT(I1,NC-1))*(DX(I1,NC)+DX(I1,NC-1))/(DX(I1,N
1      C-2)+DX(I1,NC-1))
        HT(I1,NC)=HT(I1,NC1)-DIRT
140    CONTINUE
C      RETURN
C      END

```

```

BJ 10
BJ 20
BJ 30
BJ 40
BJ 50
BJ 60
BJ 70
BJ 80
BJ 90
BJ 100
BJ 110
BJ 120
BJ 130
BJ 140
BJ 150
BJ 160
BJ 170
BJ 180
BJ 190
BJ 200
BJ 210
BJ 220
BJ 230
BJ 240
BJ 250
BJ 260
BJ 270
BJ 280
BJ 290
BJ 300
BJ 310
BJ 320
BJ 330
BJ 340
BJ 350
BJ 360
BJ 370
BJ 380
BJ 390
BJ 400
BJ 410
BJ 420
BJ 430
BJ 440
BJ 450

```


SUBROUTINE BYFLOW

```

SUBROUTINE BYFLOW (NR,NC,NA,NB,FK,H,HF,Z,DX,DY,SQGGI,SQGGJ,SQR,CMA BF 10
1TRX,A,B,HP,G,ITIME,CO,CT,PHI,PHIC,CS,CPPT,QAPW,CRCHR,CPHR,OPUM,CLE
2AK,CPPT,CAPW,CRCHR,CSQR,Q,CPUM,CLEAK,CPHR) BF 30
C BF 4
C THIS SUBROUTINE COMPUTES FLOWS FOR EACH GRID.. FLOW THROUGH BF 5
C BOUNDARIES AND FROM CONSTANT HEAD GRIDS IS CALCULATED. BF 60
C THIS SUBROUTINE COMPUTES NEW RELATIVE CONCENTRATIONS FOR EACH GRID
C BASED ON ALL INPUT AND OUTPUT VARIABLES TO EACH GRID.
C
C SQGGI=FLOW BETWEEN GRIDS IN I-DIRECTION (AF) BF 70
C SQGGJ=FLOW BETWEEN GRIDS IN J-DIRECTION (AF) BF 80
C SQBT=TOTAL INFLOW THROUGH BOUNDARIES (AF) BF 90
C SQBA=INFLOW THROUGH BUFFER ZONE BOUNDARIES (AF) BF 100
C SQR=INFLOW FROM CONSTANT HEAD GRIDS BF 110
C SQRT=TOTAL INFLOW FROM RIVER OR CONSTANT HEAD GRIDS (AF) BF 120
C SORA=INFLOW FROM RIVER OR CONSTANT HEAD GRIDS WITHIN BUFFER BF 130
C ZONE BOUNDARIES (AF) BF 14
C BF 150
C DIMENSION FK(NR,NC), H(NR,NC), HF(NR,NC), Z(NR,NC), DX(NR,NC), DY( BF 160
1NR,NC), SQGGI(NR,NC), SQGGJ(NR,NC), SQR(NR,NC), CMATRX(NA,NB), I(N BF 170
2R,NC), G(NR,NC), HP(NR,NC), G(NR,NC), CO(NR,NC), CT(NR,NC), PHI(NR,NC) BF 180
3,PHIC(NR,NC), CS(NR,NC), CPPT(NR,NC), QAFW(NR,NC), CRCHR(NR,NC), CPHR(
4NF,NC), OPUM(NR,NC), QLEAK(NR,NC), Q(NR,NC), CFUM(NR,NC), CLEAK(NR,NC),
5CPHR(NR,NC), CPPT(NR,NC), CAPW(NR,NC), CRCHR(NR,NC), CSQR(NR,NC)
C BF 190
C COMMON /BLK1/ DT,ST,ICFA0,ILKAO,LCIE,LCIW,LCJE,LCJW,FWTOP BF 200
COMMON /BLK2/ STA,STOL,STT,SOA,SOT,SQFA,SQBA,SQRT,SQBT,OVA,OVT BF 210
C BF 220
C LS=LCIE BF 230
C LN=LCIW-1 BF 240
C NS=LCJE BF 250
C NN=LCJW-1 BF 260
C NR1=NR-1 BF 27
C NC1=NC-1 BF 28
C SQBIA=0 BF 290
C SQBJA=0 BF 300
C SQRIA=0.0 BF 31
C SQRJA=0.0 BF 320
C BF 330
C DO 20 LOOP COMPUTES FLOW IN THE I DIRECTION BF 340
C STATEMENTS 10 AND 12 FORCES DISCHARGES AT THE FIRST AND LAST BF 350
C GRIDS TO BE COUNTED AS BOUNDARY GRIDS BF 360
C STATEMENT 14 AND 16 COMPUTES RIVER FLOWS. RIVER MUST BE INTERIOR BF 370
C TO GRIDS 3 AND NR1 OR INTERIOR TO LCIF AND LCIW. BF 380
C BF 390
C DO 210 J=1,NC BF 400
C SQGGI(NR,J)=0.0 BF 410
C DO 210 I=1,NR1 BF 420
C BF 430
C CHECK FOR IMPERMEABLE BOUNDARY BF 44
C CHECK FOR ADJACENT CONSTANT HEAD OR CONSTANT GRADIENT CONDITIONS BF 450
C BF 460
C KK=(H(I,J)/10000.)*1 BF 470
C KL=(H(I+1,J)/10000.)*1 BF 48
C GO TO (110,160,120,130), KK BF 490
110 GO TO (140,160,140,140), KL BF 500
120 GO TO (140,160,160,160), KL BF 510
130 GO TO (140,160,160,140), KL BF 520
C BF 530
C 140 IF (A(I,J).NE.0.0) GO TO 150 BF 540
C SQGGI(I,J)=PARAM(FK(I+1,J),FK(I,J),HP(I+1,J),HP(I,J),Z(I+1,J),Z BF 550
1 (I,J),DY(I+1,J),DY(I,J),DX(I+1,J),DX(I,J),G(I+1,J),G(I,J))*(HF( BF 560
2 I,J)-HF(I+1,J))*DT/43560. BF 570
C GO TO 170 BF 580
150 SQGGI(I,J)=A(I,J)*(HF(I,J)-HF(I+1,J))*DT/43560. BF 590
C GO TO 170 BF 600
160 SQGGI(I,J)=0.0 BF 610
170 IF (I.NE.1) GO TO 180 BF 620
C IF (KK.EQ.4) GO TO 190 BF 630
C SQRI=SQGGI(I,J) BF 640
C BF 650
C SUM OF I=FLOW THROUGH BOUNDARIES BF 660
C BF 670
C SORIA=SQBIA+SQRI BF 680
C GO TO 190 BF 690
180 IF (I.NE.NR1) GO TO 190 BF 700
C IF (KK.EQ.4) GO TO 190 BF 710
C SQRI=-SQGGI(I,J) BF 720

```

C		BF 730
C	SUM OF I=FLOW THROUGH BOUNDARIES	BF 740
C		BF 750
	SQRJA=SQRJA+SQRJ	BF 760
190	IF (I.NE.NN) GO TO 200	BF 770
	IF (J.LT.LCIW) GO TO 210	BF 780
	IF (J.GT.LCIE) GO TO 210	BF 790
	IF (KK.EQ.4) GO TO 210	BF 800
	TQBI=SQGGI(I,J)	BF 810
		BF 820
C	SUM OF I=FLOW THROUGH BUFFER ZONE BOUNDARIES	BF 830
C		BF 840
	TQBIA=TQBIA+TQBI	BF 850
	GO TO 210	BF 860
200	IF (I.NE.NS) GO TO 210	BF 870
	IF (J.LT.LCIW) GO TO 210	BF 880
	IF (J.GT.LCIE) GO TO 210	BF 890
	IF (KK.EQ.4) GO TO 210	BF 900
	TQBI=-SQGGI(I,J)	BF 910
		BF 920
C	SUM OF I=FLOW THROUGH BUFFER ZONE BOUNDARIES	BF 930
C		BF 940
	TQBIA=TQBIA+TQBI	BF 950
210	CONTINUE	BF 960
	WRITE (6,400) ITIME	BF 970
	CALL MATROP (NR,NC,SQGGI)	BF 980
		BF 990
C	DO 40 LOOP COMPUTES FLOWS IN THE J-DIRECTION.	BF 1000
C		BF 1010
	DO 320 I=1, NR	BF 1020
	SQGGJ(I,NC)=0.0	BF 1030
	DO 320 J=1, NC1	BF 1040
	KK=(H(I,J)/10000.)*1	BF 1050
	KL=(H(I,J+1)/10000.)*1	BF 1060
	GO TO (220,270,230,240), KK	BF 1070
220	GO TO (250,270,250,250), KL	BF 1080
230	GO TO (250,270,270,270), KL	BF 1090
240	GO TO (250,270,270,250), KL	BF 1100
		BF 1110
C	IF (B(I,J).NE.0.0) GO TO 260	BF 1120
	SQGGJ(I,J)=PARAM(FK(I,J+1),FK(I,J),HP(I,J+1),HP(I,J),Z(I,J+1),Z	BF 1130
1	(I,J),DX(I,J+1),DX(I,J),CY(I,J+1),CY(I,J),G(I,J+1),G(I,J))*HF(I	BF 1140
2	I,J)=HF(I,J+1)*DT/43560.	BF 1150
	GO TO 280	BF 1160
260	SQGGJ(I,J)=B(I,J)*(HF(I,J)-HF(I,J+1))*DT/43560.	BF 1170
	GO TO 280	BF 1180
270	SQGGJ(I,J)=0.0	BF 1190
280	IF (J.NE.1) GO TO 290	BF 1200
	IF (KK.EQ.4) GO TO 300	BF 1210
	SQBJ=SQGGJ(I,J)	BF 1220
		BF 1230
C	SUM OF J=FLOW THROUGH BOUNDARIES	BF 1240
C		BF 1250
	SQRJA=SQRJA+SQRJ	BF 1260
	GO TO 300	BF 1270
290	IF (J.NE.NC1) GO TO 300	BF 1280
	IF (KK.EQ.4) GO TO 300	BF 1290
	SQRJ=-SQGGJ(I,J)	BF 1300
		BF 1310
C	SUM OF J=FLOW THROUGH BOUNDARIES	BF 1320
C		BF 1330
	SQRJA=SQRJA+SQRJ	BF 1340
300	IF (J.NE.LN) GO TO 310	BF 1350
	IF (I.LT.LCJW) GO TO 320	BF 1360
	IF (I.GT.LCJE) GO TO 320	BF 1370
	IF (KK.EQ.4) GO TO 320	BF 1380
	TQBJ=SQGGJ(I,J)	BF 1390
		BF 1400
C	SUM OF J=FLOW THROUGH BUFFER ZONE BOUNDARIES	BF 1410
C		BF 1420
	TQBJA=TQBJA+TQBJ	BF 1430
	GO TO 320	BF 1440
310	IF (J.NE.LS) GO TO 320	BF 1450
	IF (I.LT.LCJW) GO TO 320	BF 1460
	IF (I.GT.LCJE) GO TO 320	BF 1470
	IF (KK.EQ.4) GO TO 320	BF 1480
	TQBJ=-SQGGJ(I,J)	BF 1490
		BF 1500
C	SUM OF J=FLOW THROUGH BUFFER ZONE BOUNDARIES	BF 1510
C		BF 1520
	TQBJA=TQBJA+TQBJ	BF 1530
320	CONTINUE	BF 1540

```

WRITE (6,410) ITIME
CALL MATROP (NR,NC,SQGGJ)
C
C RELATIVE CONCENTRATION CALCULATIONS
C CALCULATE CHANGE IN RELATIVE CONCENTRATIONS DUE TO ALL VARIABLES
C EXCEPT CONSTANT HEAD SOURCES INSIDE THE BOUNDARY GRIDS.
C SOURCE GRIDS ARE TAKEN AS C=1.0
C DO 10 J=1,NC
  DO 10 I=1,NR
    CPUM(I,J)=CO(I,J)
    CLEAK(I,J)=CO(I,J)
    CPHR(I,J)=0.0
  10 CONTINUE
    DO 530 J=2,NC1
      DO 530 I=2,NR1
        IF (CS(I,J) .GT. 2.0) GO TO 530
        IF (HP(I,J) .GT. 6(I,J)) GO TO 480
        CT(I,J)=CO(I,J-1)*SQGGJ(I,J-1)+CO(I-1,J)*SQGGI(I-1,J)-CO(I,J)*(
        1SQGGI(I,J)+SQGGJ(I,J))+QPPT(I,J)*CPPT(I,J)+CRCHR(I,J)*CRCHR(I,J)+
        2QAPW(I,J)*CAPW(I,J)-QPUM(I,J)*CPUM(I,J)-CPHR(I,J)*CPHR(I,J)-CLEAK(
        3I,J)*CLEAK(I,J)
        GO TO 490
      480 CT(I,J)=CO(I,J-1)*SQGGJ(I,J-1)+CO(I-1,J)*SQGGI(I-1,J)-CO(I,J)*(
        1SQGGI(I,J)+SQGGJ(I,J))-QPUM(I,J)*CPUM(I,J)
      490 IF (SQGGI(I-1,J) .GE. 0.0) GO TO 500
        CT(I,J)=CT(I,J)+(CO(I,J)-CO(I-1,J))*SQGGI(I-1,J)
      500 IF (SQGGJ(I,J) .GE. 0.0) GO TO 510
        CT(I,J)=CT(I,J)-(CO(I+1,J)-CO(I,J))*SQGGI(I,J)
      510 IF (SQGGJ(I,J-1) .GE. 0.0) GO TO 520
        CT(I,J)=CT(I,J)+(CO(I,J)-CO(I,J-1))*SQGGJ(I,J-1)
      520 IF (SQGGJ(I,J) .GE. 0.0) GO TO 530
        CT(I,J)=CT(I,J)-(CO(I,J+1)-CO(I,J))*SQGGJ(I,J)
      530 CONTINUE
C
C SORT=0.0
C DO 330 J=1,NC
  SQGGI(1,J)=0.0
  SQGGI(NR1,J)=0.0
  330 CONTINUE
    DO 340 I=1,NR
      SQGGJ(I,1)=0.0
      SQGGJ(I,NC1)=0.0
    340 CONTINUE
      DO 350 I=2,NR1
        DO 350 J=2,NC1
          SQR(I,J)=0.0
          KK=(H(I,J)/10000.)*1
          IF (KK.NE.4) GO TO 350
          SQR(I,J)=-SQGGI(I-1,J)+SQGGI(I,J)-SQGGJ(I,J-1)+SQGGJ(I,J)
          SORL=SQR(I,J)
C
C TOTAL INFLOW FROM CONSTANT HEAD GRIDS
C
C SORT=SORT+SORL
  350 CONTINUE
C
C TOTAL INFLOW THROUGH BOUNDARIES
C
C SGBT=SQBJA+SQBIA
C IF (SORT.LE.0.0) GO TO 360
C WRITE (6,420) ITIME
C CALL MATROP (NR,NC,SQR)
C
C
C 360 SORA=0.0
  DO 370 J=LCIW,LCIE
    SQGGI(NN,J)=0.0
    SQGGI(NS,J)=0.0
  370 CONTINUE
    DO 380 I=LCJW,LCJE
      SQGGJ(I,LN)=0.0
      SQGGJ(I,LS)=0.0
    380 CONTINUE
      DO 390 I=LCJW,LCJE
        DO 390 J=LCIW,LCIE
          SQR(I,J)=0.0
          KK=(H(I,J)/10000.)*1
          IF (KK.NE.4) GO TO 390
          SQR(I,J)=-SQGGI(I-1,J)+SQGGI(I,J)-SQGGJ(I,J-1)+SQGGJ(I,J)
          SORL=SQR(I,J)
C
C TOTAL INFLOW FROM CONSTANT GRIDS WITHIN BUFFER ZONE BOUNDARIES

```

BF 155
BF 156
BF 157

BF 158
BF 159
BF 160
BF 161
BF 162
BF 163
BF 164
BF 165
BF 166
BF 167
BF 168
BF 169
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BF 171
BF 172
BF 173
BF 174
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BF 193
BF 194
BF 195
BF 196
BF 197
BF 198
BF 199
BF 200
BF 201
BF 202
BF 203
BF 204
BF 205
BF 206
BF 207

```

      SQA=SQFA+SQA*L
390 CONTINUE
C
C      TCTAL INFLOW THROUGH BUFFER ZONE BOUNDARIES
C
      SORA=TORJA+TORIA
C
C      CALCULATE NEW RELATIVE CONCENTRATIONS FOR EACH GRID.
C
      DO 570 J=2,NC1
      DO 570 I=2,NR1
      IF (CS(I,J) .GT. 2.0) GO TO 590
      IF (HP(I,J) .GT. G(I,J)) GO TO 590
      CT(I,J)=CO(I,J)+(CT(I,J)+SQR(I,J)*CSQR(I,J))*(43560./ (DX(I,J)*DY(I
1,J)*PHI(I,J)*(HP(I,J)-Z(I,J)) +Q(I,J)*DT + SQR(I,J)))
      GO TO 570
580 CT(I,J)=CO(I,J)+(CT(I,J)+SQR(I,J)*CSQR(I,J))*(43560./ (DX(I,J)*DY(I
1,J)*(PHI(I,J)*(G(I,J)-Z(I,J))+PHIC(I,J)*(HP(I,J)-G(I,J))) -OPUM(I,
2J)+SQR(I,J)))
      GO TO 570
590 CT(I,J)=CS(I,J)-2.
570 CONTINUE
      TIME=ITIME*DT
      WRITE(6,430) TIME
      CALL MATROP (NR,NC,CT)
      DO 397 J=1,NC
      DO 397 I=1,NR
      CO(I,J)=CT(I,J)
397 CONTINUE
      RETURN
C
C      400 FORMAT (1H1,29X, 56HOISCHARGE IN I-DIRECTION (AC=FT/DT) FOR INCREM
1ENT NUMBER,I6)
C      410 FORMAT (1H1,29X, 56HOISCHARGE IN J-DIRECTION (AC=FT/CT) FOR INCREM
1ENT NUMBER,I6)
C      420 FORMAT (1H1,22X, 74HRIVER FLCW IN EACH GRID MINLS MEANS FLOW FRM
1AQUIFER (AC=FT/DT),/ ,1H ,49X, 16HINCREMENT NUMBER,I6)
C
C      430 FORMAT (1H1,30X, 27HRELATIVE CONCENTRATICN (CA),G10.2,4HODAYS)
      END

```

SUBROUTINE BALCOP

```

      SUBROUTINE BALCOP (J1,J2,I,STTTEM,STATEM)
C
C      THIS SUBROUTINE WRITES OUT THE BALANCE COMPUTATIONS FOR EACH
C      TIME INCREMENT. ALL UNITS ARE IN AC-FT PFR TIME INCREMENT.
C
C      SQA,SQT = APPLIED WATER, BETWEEN STATICS, TOTAL AREA.
C      SORA,SORT = INFLOW FROM RIVER, BETWEEN STATICS, TOTAL AREA.
C      SOBA,SOBT = BOUNDARY INFLOW, BETWEEN STATICS, TOTAL AREA.
C      STT,STTTEM = TOTAL AREA STORAGE AND DECREASE CF STORAGE.
C      STA,STATEM = BETWEEN STATICS STORAGE AND DECREASE CF STORAGE.
C      STOL = STORAGE OF OVERLAP AREAS.
C      OVA,OVT = ILLEGALLY WITHDRAWN.
C      ASTA,ASTT = TOTALS, BETWEEN STATIONS, TOTAL AREA.
C
C      COMMON /BLK2/ STA,STOL,STT,SQA,SQT,SORA,SOBA,SORT,SOBT,OVA,OVT
C
      WRITE (6,110) J1,J2,I
      WRITE (6,120) SQA,SQT
      WRITE (6,130) SORA,SORT
      WRITE (6,140) SOBA,SOBT
      STTTEM=STTTEM-STT
      STATEM=STATEM-STA
      WRITE (6,150) STT,STTTEM
      WRITE (6,160) STA,STATEM
      WRITE (6,170) STOL
      WRITE (6,180) OVA,OVT
      ASTA=SQA+SORA+SOBA+STATEM+OVA
      ASTT=SQT+SORT+SOBT+STTTEM+OVT
      WRITE (6,190) ASTA,ASTT
      RETURN
C

```

110	FORMAT (1H1,13X, 55HMASS BALANCE COMPUTATIONS (AC=FT/DT) FOR SIMUL	BC	330
	LATED TIME,I9, 1H=,I9, 26H AS AT THE END OF PERIOD,I3,///)		
120	FORMAT (50H0 APPLIED WATER (BETWEEN STATIONS - TOTAL AREA),20X,	BC	350
	12F15.2)	BC	360
130	FORMAT (54H0 INFLOW FROM RIVER (BETWEEN STATIONS - TOTAL AREA),	BC	370
	116X,2F15.2)	BC	380
140	FORMAT (52H0 BOUNDARY INFLCW (BETWEEN STATIONS - TOTAL AREA),18	BC	390
	1X,2F15.2)	BC	400
150	FORMAT (47H0 TOTAL AREA STORAGE AND DECREASE OF STORAGE,8X,F15.	BC	410
	12,15X,F15.2)	BC	420
160	FORMAT (53H0 BETWEEN STATIONS STORAGE AND DECREASE OF STORAGE,2	BC	430
	1X,2F15.2)	BC	440
170	FORMAT (29H0 STORAGE OF OVERLAP AREAS,26X,F15.2//)	BC	450
180	FORMAT (56H0 ILLEGALLY WITHDRAWN (BETWEEN STATIONS - TOTAL AREA	BC	460
	1),14X,2F15.2)	BC	470
190	FORMAT (49H= T O T A L S (BETWEEN STATIONS - TOTAL AREA),21X,2	BC	480
	1F15.2)	BC	490
C	END	BC	500
		BC	510

SUBROUTINE MATROP

	SUBROUTINE MATROP (NOROW,NOCOL,B)	MP	10
C		MP	20
C		MP	30
C	THIS SUBROUTINE ORGANIZES DATA OF RESULTS INTO A SUITABLE FORM	MP	40
C	FOR PRINTING AND PRINTS.	MP	50
C		MP	60
C	DIMENSION B(NOROW,NOCOL)	MP	70
C		MP	80
	NOCOLM=NOCOL	MP	90
	ICONT=1	MP	100
	NO1=NOCOLM	MP	110
	IF (NOCOLM.GT.12) NO1=12	MP	120
110	NO2=NOCOLM-12	MP	130
	WRITE (6,140) (JJ,JJ=ICONT,NO1)	MP	140
	DO 120 I=1,NOROW	MP	150
120	WRITE (6,150) I,(B(I,J),J=ICONT,NO1)	MP	160
	IF (NO2.LE.0) RETURN	MP	170
	NOCOLM=NOCOLM-12	MP	180
	ICONT=ICONT+12	MP	190
	IF (NOCOLM.LE.12) GO TO 130	MP	200
	NO1=ICONT+11	MP	210
	GO TO 110	MP	220
130	NO1=ICONT-1+NOCOLM	MP	230
	GO TO 110	MP	240
C		MP	250
	140 FORMAT (1H ,//,3X,12(7X, 1HX,I2)//)	MP	260
	150 FORMAT (1H , 1HY,I2,12F10.3)	MP	270
C		MP	280
C	END	MP	290

SUBROUTINE READC

	SUBROUTINE READC (NR,NC,CAPW,CRCHP,CSCR,CO,LBC,FBC,TBC,EBC,AGGIE)
C	
C	
C	THIS SUBROUTINE READS IN RELATIVE CONCENTRATIONS OF SOURCE WATERS
C	WHICH MAY CHANGE WITH EACH TIME INTERVAL.
C	
C	CAPW=CONCENTRATION IN APPLIED WATER
C	CRCHP=CONCENTRATION IN ARTIFICIALLY RECHARGED WATERS
C	CSCR=CONCENTRATION IN WATERS FROM CONSTANT HEAD GRIDS
C	CO=CONCENTRATION IN BOUNDARY GRIDS WHERE WATER FLOES INTO THE GRID
C	SYSTEM
C	

```

DIMENSION CAPW(NR,NC), CFCCHR(NF,NC), CSOR(NF,NC), CO(NR,NC)
IF (AGGIE .EQ. 0.0) GO TO 120
IF (AGGIE .LT. 0.0) GO TO 230

C
NR1=NR-1
READ (5,500) CCAPW
IF (CCAPW .LT. 0.0) GO TO 130
DO 110 J=1,NC
DO 110 I=1,NR
110 CAPW(I,J)=CCAPW
GO TO 120
100 READ (5,510) CAPW

C
120 READ (5,500) CFCCHR
IF (CFCCHR .LT. 0.0) GO TO 130
DO 140 J=1,NC
DO 140 I=1,NR
140 CFCCHR(I,J)=CFCCHR
GO TO 150
130 READ (5,510) CFCCHR

C
150 READ (5,500) CCSQR
IF (CCSQR .LT. 0.0) GO TO 160
DO 170 J=1,NC
DO 170 I=1,NR
170 CSOR(I,J)=CCSQR
GO TO 180
160 READ (5,510) CSQR

C
180 IF (LBC .NE. 10000.) GO TO 190
280 IF (RBC .NE. 10000.) GO TO 200
290 IF (TBC .NE. 10000.) GO TO 210
300 IF (BBC .NE. 10000.) GO TO 220
GO TO 230
190 READ (5,510) (CO(I,1), I=2,NR1)
GO TO 280
200 READ (5,510) (CO(I,NC), I=2,NR1)
GO TO 290
210 READ (5,510) (CO(1,J), J=1,NC)
GO TO 300
220 READ (5,510) (CO(NR,J), J=1,NC)
500 FORMAT (1F10.1)
510 FORMAT (8F10.1)
230 CONTINUE
END

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APPENDIX E
DATA CODING FORMAT

IBM

FORTRAN Coding Form

GX28-7327-6 U/M 050**
Printed in U.S.A.

PROGRAM	WTQUAL 2	PUNCHING INSTRUCTIONS	GRAPHIC	PAGE 6 OF 6
PROGRAMMER	L.W. PITTMAN	DATE	PUNCH	CARD ELECTED NUMBER
		DECEMBER '77		

LINE	STATEMENT NUMBER	CONT.	FORTRAN STATEMENT																																	IDENTIFICATION SEQUENCE																																											
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33		34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76
8	CCAPW F10.1		UNIFORM VALUE OF NEW CONCENTRATION OF WATER APPLIED AS IRRIGATION, OTHERWISE BLANK CARD																																																																												
8A	CAPW		CAPW	→	(NR * NC) * (F10.1)	READ IN BY COLUMNS																																																																									
9	CCRCHR F10.1		UNIFORM VALUE OF NEW CONCENTRATION OF WATER RECHARGED THRU PITS AND LINE SOURCES, OTHERWISE BLANK CARD																																	NOT NEEDED IF AGGIE .LT. 0.0																																											
9A	CRCHR		CRCHR	→	(NR * NC) * (F10.1)	READ IN BY COLUMNS	IF USED, MUST BE READ IN FOR EACH DT																																																																								
10	CCSOR F10.1		UNIFORM VALUE OF NEW CONCENTRATION OF WATER ORIGINATING FROM CONSTANT HEAD SOURCES, OTHERWISE BLANK CARD																																																																												
10A	CSQR		CSQR	→	(NR * NC) * (F10.1)	READ IN BY COLUMNS																																																																									
11	CB		FOR LEFT BOUNDARY	→	(NR-2) * (F10.1)																																																																										
12	CB		FOR RIGHT BOUNDARY	→	(NR-2) * (F10.1)																																																																										
13	CB		FOR TOP BOUNDARY	→	NC * (F10.1)		ONLY NEEDED IF AGGIE .LT. 0.0																																																																								
14	CB		FOR BOTTOM BOUNDARY	→	NC * (F10.1)																																																																										

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms per pad may vary slightly