

Modified Use of the “SDF” Semi-Analytical Stream Depletion Model in Bounded Alluvial Aquifers

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Abstract. A widely used semi-analytical model quantifying stream depletions and accretions induced by groundwater pumping and augmented groundwater recharge was reviewed for accuracy when applied in bounded alluvial aquifers. The method is based on the analytical Glover method, but uses a model-derived input parameter to account for mathematically non-ideal conditions such as variable transmissivities and nearby aquifer boundaries. Using this parameter, called the stream depletion factor (SDF) with units of time, often improves estimates and is an approximation that produces only small errors near the time of the SDF. However, recent experience has highlighted the significant errors introduced at larger times, especially in narrow aquifers. This paper reviews the approximations inherent in the method and presents expanded response curves and error curves as functions of well position within the bounded aquifer. It is shown that SDF values mapped in the half of the aquifer nearest the river do not account for aquifer boundaries, and that values in the other half follow a predictable adjustment. By removing the boundary adjustments, the method of images can be legitimately combined with the SDF method, thereby improving estimates in bounded aquifers while still benefiting from the additional information contained in SDF maps and retaining their value as standardized references for water rights administration.

1. Introduction

The “SDF method” refers to a semi-analytical model that is widely used in Colorado for estimating the rate and volume of stream depletion induced by groundwater pumping from alluvial aquifers. It applies equally well to describing stream accretion induced by groundwater recharge operations. The discussions in this paper apply to either of these applications, and for brevity the terms are used interchangeably.

The recent drought in Colorado brought renewed attention to groundwater and surface water management, including the analytical tools used such as the SDF method. At about the same time, numerical modeling performed for a large groundwater recharge project highlighted the significance of the SDF method’s approximations when applied in narrow portions of a bounded aquifer (Halstead and Flory, 2003). To address this observation, this paper

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reviews the approximations inherent in the SDF method and proposes a modification to improve estimates in bounded aquifers.

2. Background

2.1. Analytical Models

Theis (1941) first published a mathematical analysis of transient stream depletion, providing an integral equation to be evaluated graphically or with an infinite series approximation. Glover and Balmer (1954) provided an analysis with a slightly different approach and with results in the form of a probability integral (commonly known as the error function). Hantush (1955) confirmed the Glover and Balmer solution, noted that it was valid in unconfined aquifers for cases in which drawdown was up to 25% of aquifer thickness, and generalized the solution to also apply to a leaky confined aquifer. Glover (1960) presented the integrated form of the rate equation to provide depletion estimates on a volume basis and discussed example applications, including combining it with image wells (Glover 1966, 1968, 1978). Hantush (1965, 1967) provided depletion analyses for cases with a semi-pervious streambed and for right-angle bends in the stream.

These analyses were developed for aquifers that reasonably conform to some combination of mathematically ideal conditions such as straight stream boundaries, fully penetrating streams, permeable streambeds, semi-infinite homogeneous aquifers, etc. In recent years, many investigators have considered and extended stream depletion analysis for a variety of non-ideal cases (e.g., Hunt 1999; Hunt et al. 2001, Fox et al. 2002), including Sophocleous et al. (1995) who found from modeling tests that low streambed conductivity and partial stream penetration affect the accuracy of the Glover analysis more than aquifer heterogeneity. Bouwer and Maddock (2001) describe different types of groundwater-surface water interactions beyond the case described by the Glover and SDF methods.

2.2. Introduction of the SDF Term and Method

Jenkins (1968a) provided a review of analytical stream depletion models with the stated purpose of presenting the computations in a format that was simple and targeted toward the average user, noting that the equations presented in his paper were not new but “seem to have been rather well concealed from most users.” Jenkins also emphasized that residual effects, meaning pumping-induced depletion occurring long after pumping has ceased, are often greater (cumulatively) than the effects that occur during pumping. He provided explicit examples computing intermittent, residual, and cumulative pumping effects, along with providing user-friendly charts for their computation.

Jenkins (1968a) is a widely cited reference for the SDF method since this is where the SDF term is first defined and where its intended use was first mentioned. However, the term’s definition and the utility of the method are better clarified in subsequent publications (Jenkins 1968b, 1970; Jenkins and Taylor 1972, 1974; Moulder and Jenkins 1970). Jenkins (1968b) provided the first details about the determination of SDF values and discusses the extent of

approximation errors he observed when using the method. These details are discussed in Section 3.0.

2.3. Adoption of the SDF Method

The USGS performed modeling and mapping of SDF values for the alluvial aquifers along the South Platte and Arkansas Rivers in Colorado (Hurr et al. 1972a, 1972b). These maps have been widely used in water rights planning. Limited information about the modeling efforts can be found in Jenkins and Taylor (1972, 1974) and Warner et al. (1994). SDF modeling was also performed in the Platte and Kansas River basins in Wyoming, Nebraska, and Kansas as part of a Missouri River Basin hydrology study (Missouri Basin States Association, 1982).

In 1974, a division water court in Colorado approved the Amended Rules and Regulations for the South Platte which state that stream depletions caused by a well should be calculated using the Glover formula “or by other accepted engineering formulae appropriately modified to reflect the pertinent physical conditions” (Water Division No. 1, 1974). MacDonnell (1988) states that the groundwater augmentation plan developed by the Fort Morgan Reservoir and Irrigation Company used the stream depletion factor to analyze depletions, accretions, and the “net stream effect”, and that “the water court essentially adopted the analytical approach suggested by Fort Morgan.” Warner et al. (1994) noted that the SDF method was the predominant method used to compute groundwater pumping and recharge effects on the South Platte River.

3. Review of the SDF Method

3.1. Definition of the SDF

Glover’s rate equation (Glover and Balmer 1954) can be written as:

$$q/Q = \operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) = \operatorname{erfc}\left(\sqrt{\frac{a^2 S}{4tT}}\right) \quad (1)$$

which relates stream depletion rate (q) to aquifer pumping rate (Q) as a function of aquifer properties and time (t). These aquifer properties are: the perpendicular distance from the well to the stream (a), storativity (S), and aquifer transmissivity (T). Integrating this rate equation gives:

$$\frac{v}{Qt} = \left(\frac{a^2}{2tT/S} + 1\right) \operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) - \left(\frac{a}{\sqrt{4tT/S}}\right) \left(\frac{2}{\sqrt{\pi}}\right) \exp\left(\frac{-a^2}{4tT/S}\right) \quad (2)$$

which relates cumulative stream depletion volume (v) to cumulative pumped volume (Qt). These relationships are plotted on Figure 1. Jenkins (1968b) defines the term “response curve” as “a curve showing the relation between pumping time and volume of stream depletion” (Equation 2).

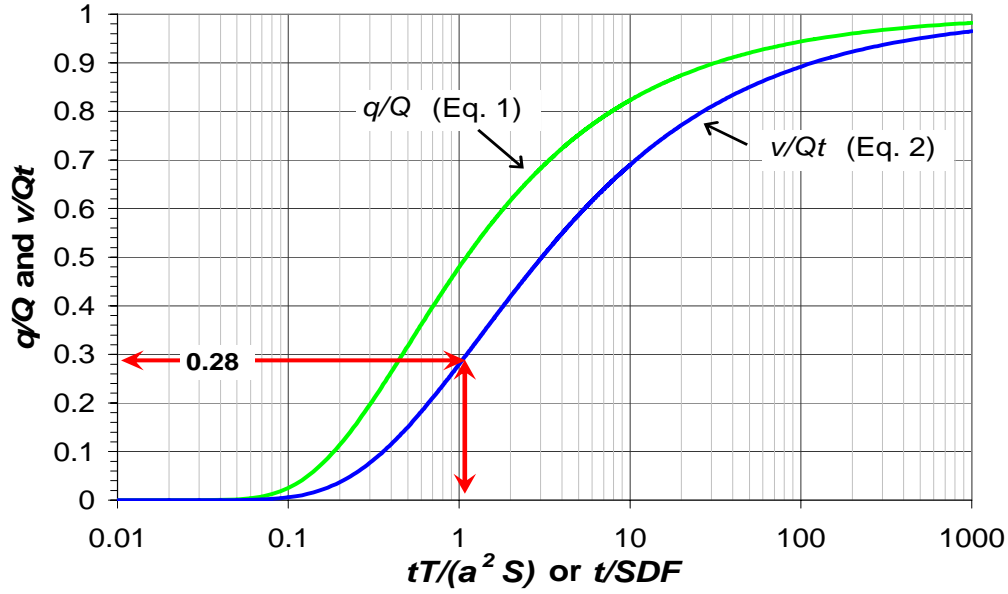


Figure 1. Ideal response curves for depletion rate and volume.

The “Glover method” refers to using Equations (1) or (2) with the inputs a , S , and T as they were previously defined. Jenkins (1968a, 1968b) created the term SDF to serve as a simple but useful input to the Glover equations, the optimal value of which would be determined through numerical modeling of a complex site. The SDF would serve as a model-calibrated replacement to the Glover method’s inputs. When these model-derived SDF values are used, this is referred to as the SDF method.

Writing Equations (1) and (2) as a function of the SDF gives Equations (3) and (4), respectively:

$$q/Q = \operatorname{erfc}\left(\sqrt{\frac{SDF}{4t}}\right) \quad (3)$$

$$\frac{v}{Qt} = \left(\frac{SDF}{2t} + 1\right) \operatorname{erfc}\left(\sqrt{\frac{SDF}{4t}}\right) - \left(\sqrt{\frac{SDF}{4t}}\right) \left(\frac{2}{\sqrt{\pi}}\right) \exp\left(-\frac{SDF}{4t}\right) \quad (4)$$

The SDF is defined by Jenkins (1970) as “the time coordinate of the point where $v = 28\%$ of Qt on a curve relating v and t .” In a mathematically ideal aquifer, the SDF time can be calculated as a^2S/T since $q/Qt = 0.28$ when $tT/a^2S = 1$ on the non-dimensional response curve (Figure 1), but that term does not define it. In a non-ideal system, the actual time when v/Qt equals 28% is dependant on additional factors not accounted for by the Glover equation parameters.

The SDF was defined this way so it could serve as a reference point “anchoring” actual response curves to the mathematically ideal response

curve. The value of $v/Qt = 0.28$ corresponds to a convenient point since it is the coordinate where $t/SDF = 1$ (Figure 1), but the value of 28% is arbitrary and “has no special significance” in itself (Jenkins 1970; Jenkins and Taylor, 1974).

3.2 The Use and Utility of the SDF

As noted, numerical modeling is used to determine the SDF value that best matches real response curves. The modeling takes into account the effects of boundaries and other non-ideal complexities. The SDF then serves as a single descriptor of complex flow behavior. The basic premise of the SDF method is that the stream response curves in complex stream-aquifer systems are approximated by the mathematically ideal Glover equation. This assumes that shapes of non-ideal response curves are similar to the ideal curve shown in Figure 1. However, this is not always the case. While all response curves do match at the point where $t/SDF = 1$ (by definition), non-ideal curves may have different shapes that deviate from the ideal curve at other points in time.

Jenkins recognized the different shapes of non-ideal curves and accordingly presented the SDF method as approximation. However, in 266 simulations of locations in the Arkansas River valley, he found that most curves matched reasonably well in the range between $t/SDF = \frac{1}{2}$ and $t/SDF = 2$ (Jenkins 1968b). The error due to the differences between ideal and “actual” (modeled) response curves was considered worth the efficiency gained: complex numerical modeling would be required only once to develop an SDF map. The map would then support the evaluation of multiple pumping management plans (Jenkins 1968b, Jenkins and Taylor 1974). The SDF method was more feasible than repeated modeling while still improving the accuracy of computations at locations not easily characterized by analytical methods. This was the real utility of the proposed method.

4.0 Stream Depletion Behavior in Bounded Aquifers

4.1. Image Method

Glover’s analysis considers the aquifer to be semi-infinite, meaning in practice that the aquifer boundary lies far enough away from the well that its effects are negligible. In many sections of alluvial aquifers, however, the boundary has a significant effect. These effects can be accounted for by combining the Glover method with the image well method. The general image well method can be found in most groundwater texts. The infinite series image well pattern required for the case of pumping in a bounded alluvial aquifer is shown by McWhorter and Sunada (1977, p. 127), although in this case the wells “across the river” are already included in the Glover analysis; only the wells across the impermeable boundary need to be added.

Jenkins (1968b) recognized that aquifer boundary effects could be calculated precisely with the image method. However, the image method still assumes other ideal conditions such as straight and parallel boundaries and homogenous aquifer properties. In contrast, the SDF method can account for all non-ideal conditions (albeit approximately), including and in addition to the aquifer boundaries.

4.2. Response Curves for Bounded Aquifers

Using the image method, we constructed response curves for a number of bounded (but otherwise ideal) aquifers with various widths (W), T/S values, and well locations. The response curve shapes are a function only of well position with respect to the stream and aquifer boundaries, i.e., the ratio a/W (Figure 2). These curves are plotted in Figure 3. Note again that by definition all curves match at the point where $t = \text{SDF}$. For wells closer to the river than the boundary ($a/W < 0.5$), the differences in the curve shapes are small and develop well after the SDF time. For wells closer to the boundary, the differences are larger, becoming apparent closer to the SDF time.

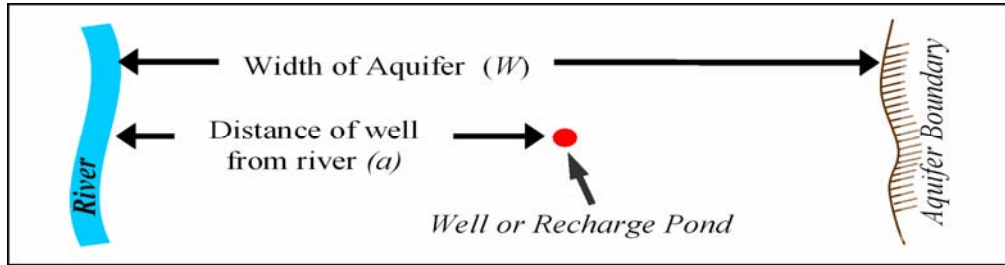


Figure 2. Schematic of a bounded aquifer showing well location notation.

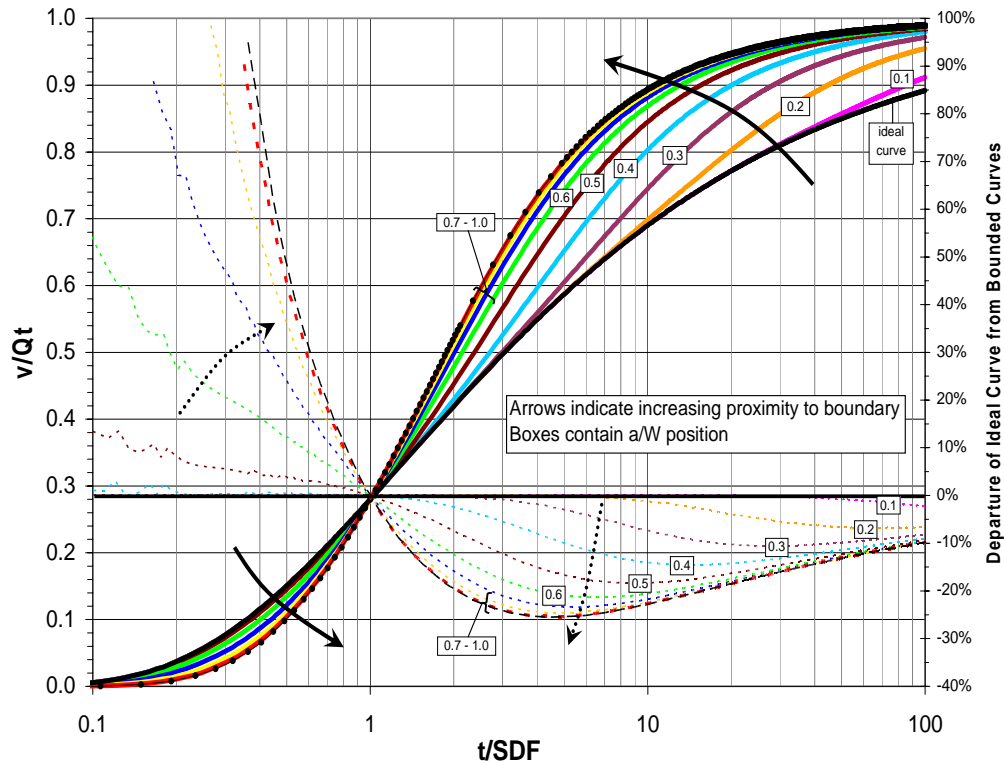


Figure 3. Response curves for bounded (but otherwise ideal) aquifers.

Differences between the ideal curve shape and the various bounded curves are plotted in Figure 3 as percent error. The deviation is less than 30% for all wells for $t > 0.7SDF$ and less than 10 to 20% at all times for the wells further from the boundary. Uncertainty in values of transmissivity (T) or storativity (S) is often much greater than 30%.

These errors were evident in recent modeling evaluations of a large groundwater recharge project in a narrow part of the South Platte aquifer. They were significant in that case because the timescales of interest at the site were much larger than the SDF values of the operating wells and recharge pond locations (i.e., $t/SDF \gg 2$). Figure 3 could be used to help assess when errors are likely to be significant for a given location and time.

5. A Modified Use of SDF Maps

5.1. Difficulty in Characterizing Heterogeneous Aquifers

Awareness that the SDF method can lead to errors in bounded aquifers might prompt water professionals to instead use the Glover method combined with the image method. However, this poses challenges in characterizing other non-ideal conditions, including irregular stream boundaries and heterogeneous aquifers. For example, there are questions about selecting an appropriate transmissivity value from the detailed transmissivity maps provided by the USGS along with the SDF maps.

This challenge is better understood by reviewing the mathematical derivation of the Glover analysis (Theis 1941; Glover and Balmer 1954; McWhorter and Sunada 1977). The first step uses the Theis equation which describes drawdown in space and time as a function of transmissivity (and storativity) of the entire aquifer, particularly T within the “radial cone of depression.” The space derivative of this drawdown, taken perpendicular to the stream and evaluated at the stream, gives the hydraulic gradient at the stream boundary. The second step of the analysis multiplies this gradient by the transmissivity along the stream and integrates this product along the entire reach (mathematically from $-\infty$ to ∞) to get total stream depletion.

Glover (1978) also provides an analysis of the distribution of depletion along the stream as a function of both time and aquifer properties. It can be shown that at later times (when q/Q approaches one), half of the accretion occurs within a stream reach of length $2a$ (centered on the well), and that 87% of the accretion occurs along a reach length equal to $10a$. Considering this multidimensional nature of stream depletion and the wide area influencing it, any method of characterizing transmissivity clearly needs to account for variable transmissivity over the entire aquifer. Averaging values over a one-dimensional flow path would be theoretically invalid.

5.2. Complexities Integrated into Available SDF Maps

In the concept of the SDF, complex behavior can be represented by Glover’s equations and “a value of time that reflects the integrated effects of the following: irregular impermeable boundaries; stream meanders; aquifer properties and their areal variations; distance to the point from the stream; and imperfect hydraulic connection between the stream and the aquifer” (Jenkins and Taylor 1974). Jenkins noted that these factors could be taken into account

in the SDF modeling, with the level of detail for any particular modeling project chosen to suit the project’s particular needs and available computing capacity.

Based on the documentation we reviewed, it appears that the SDF maps produced by the USGS (e.g., Hurr et al. 1972a,b) account for the areal variation of aquifer properties, the presence of aquifer boundaries and their irregular shapes (e.g., meandering streams and the presence of tributaries), and the well or pond location in the aquifer with respect to these features. It is not apparent whether the modeling performed for these maps accounted for any semi-permeable stream boundaries.

5.3. Combining the SDF Method with the Image Method

Jenkins (1968b) observed in a series of model tests that the SDF equaled a^2S/T in the half of the aquifer nearest the stream, and that the SDF was progressively less than a^2S/T as the boundary was approached. Figure 4 shows the expected difference (i.e., the model-derived adjustment) between SDF values and a^2S/T as a function of well position (a/W) and caused by the boundary effects only. This is similar to the plot obtained by Jenkins (1968b), although the grid in his model introduced some scatter for the points near the stream.

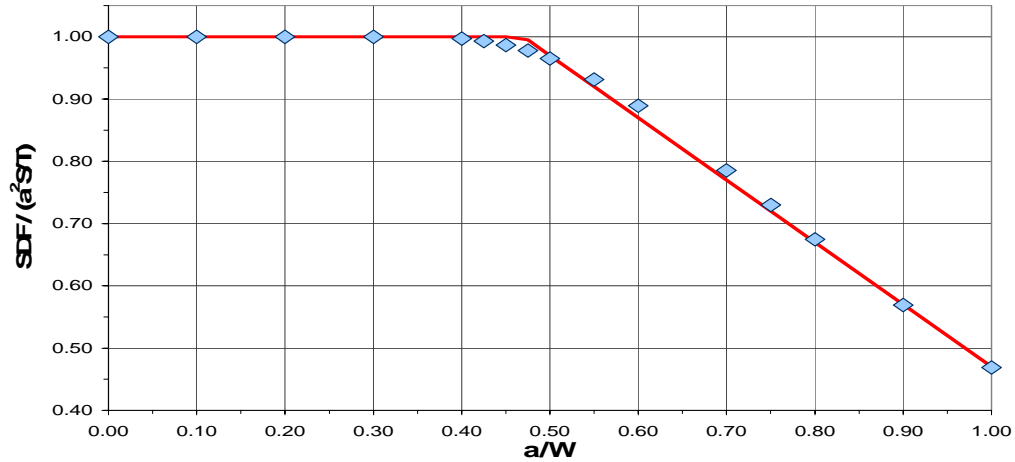


Figure 4. SDF adjustment as a function of aquifer position.

The lack of adjustment near the stream can be verified by considering the influence of the boundary to be significant when $q/Q = 0.01$ for the closest boundary image well. This corresponds to $v/Qt = 0.002$. Setting Equation (1) equal to 0.01, the argument of the complimentary error function for the image well, $[r_i^2S/(4Tt)]^{1/2}$, is equal to 1.821. And, at the time of the SDF of the real well, $t = a^2S/T$, so substitution yields $r_i/a = 3.642$. Accounting for the boundary location with respect to r_i yields $a/W = 0.43$. Thus, for wells located at $a/W < 0.43$, the boundary will have a negligible effect (0.2% of Qt) on depletion through the SDF time.

The boundary still has an effect on wells on the stream-side half of the aquifer; but the effects are significant only at times greater than the SDF. Also note that even if a well located at $a/W < 0.43$ is pumped for a time less than the SDF, the residual effects are still computed for times much longer than the pumping time and boundary effects may still be significant.

Figure 5 shows an example where $SDF = 100$ days with no boundary present and plots response curves for progressively closer boundaries (for a/W from 0.1 to 1.0). For all locations of $a/W \leq 0.40$, v/Qt reaches 28% at 100 days, and for $a/W \approx 0.5$ the change is negligible. These wells exhibit boundary effects after the SDF time. In contrast, the wells with $a/W > 0.5$ are affected earlier: lowering the to $SDF = 80$ days for $a/W = 0.7$, for example.

Since they were not affected, SDF values in the stream half of the aquifer can be used directly with the method of images without any over-accounting for boundary effects. In the other half of the aquifer, the adjustment follows a predictable pattern (Figure 4), allowing this boundary-induced adjustment to be easily removed. The potential benefit of this approach is that the SDF values incorporate other non-ideal effects in addition to the boundaries.

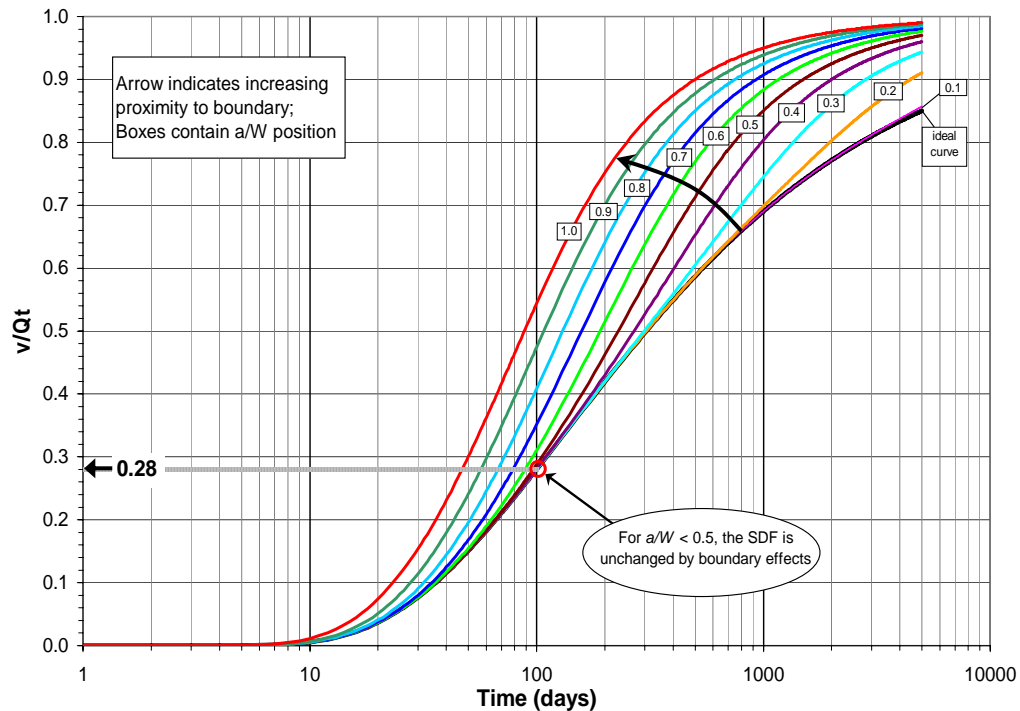


Figure 5. Boundary effects develop after the SDF time for wells far from the boundary (i.e., does not affect SDF time for locations $a/W < 0.5$).

5.4. Test of the Proposed Modified SDF Method

By removing the boundary-induced portion of the SDF adjustment, all mapped SDF values can be combined with the image method. This allows the user to better account for boundaries while retaining other aquifer information integrated in SDF values (e.g., spatial variations in T). This approach was

tested against a detailed numerical model (MODFLOW) constructed for a large-scale groundwater recharge project at the Tamarack State Wildlife Area along the South Platte River (Halstead and Flory, 2003). The model has been calibrated against transient field conditions and considered to simulate relevant site behavior successfully.

Two locations were selected to demonstrate the differences in each half of the aquifer: Recharge Pond A, which has a mapped SDF of approximately 60 days and an aquifer position of $a/W = 0.31$, and Recharge Pond B, which has a mapped SDF near 200 days and an aquifer position of $a/W = 0.79$. These SDF values and boundary distances were obtained from the local USGS map (Hurr et al. 1972a). Interpolation between SDF isopleths was done by the square root of SDF values since SDF varies by a^2 (Jenkins, 1968b). The mapped values were modified according to the simple linear fit shown in Figure 4:

$$\begin{aligned} \text{SDF}_{\text{mod}} &= \text{SDF}_{\text{USGS}} \quad (\text{no change}) & \text{for } a/W < 0.47 \\ \text{SDF}_{\text{mod}} &= (1.47 - a/W)^{-1}(\text{SDF}_{\text{USGS}}) & \text{for } a/W > 0.47 \end{aligned} \quad (6).$$

The comparisons for Recharge Pond A and B are shown in Figures 6 and 7, respectively. Figure 6 shows that for Pond A the traditional SDF method (without images) compares well with the model until times much larger than the SDF. Adding image wells provided a good fit at all times.

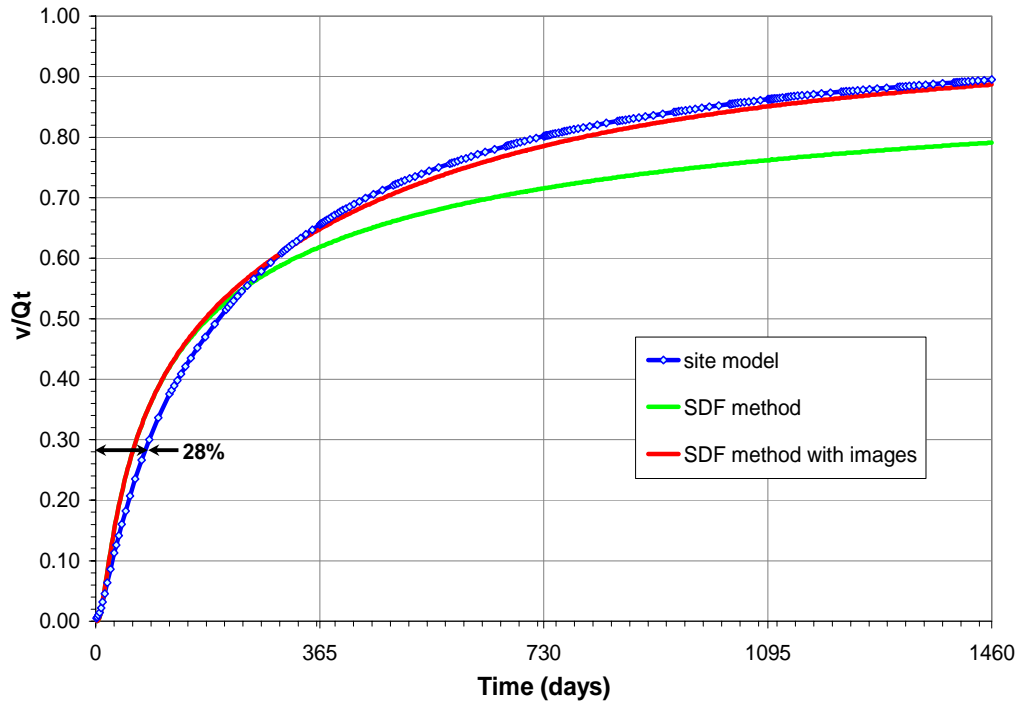


Figure 6. Comparison of analytical response curves to modeled results for a recharge site along the South Platte river. This recharge site was in the half of the aquifer closest to the river ($a/W < 0.5$) where mapped SDF values do not account for boundary effects.

Figure 7 compares the same response curves as Figure 6 plus response curves based on our modified SDF. As expected for this location closer to the boundary, the SDF method without image wells departs from the model significantly, starting immediately after the SDF. Adding image wells to the unadjusted SDF provides a significantly better fit, but as expected, it overcompensates and predicts a greater response. Finally, using the modified SDF value with image wells removed the overcompensation, providing a slightly better fit.

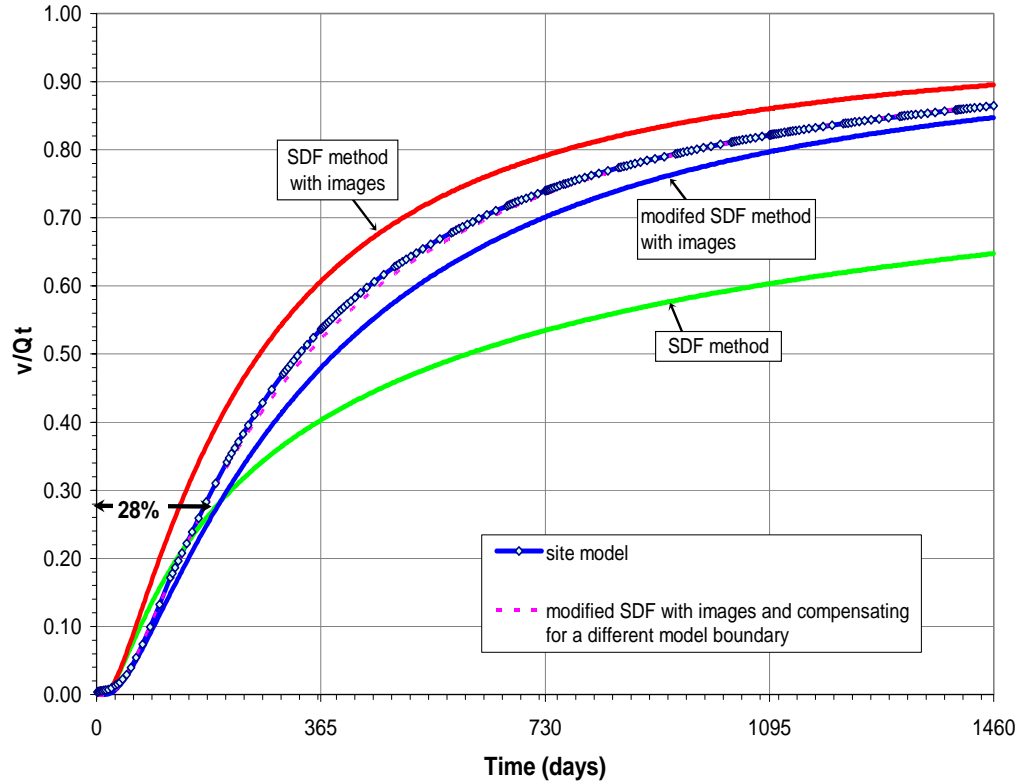


Figure 7. Comparison of analytical response curves to modeled results for a recharge site along the South Platte River. This recharge site was in the half of the aquifer closest to the boundary ($a/W > 0.5$) where mapped SDF values partially account for boundary effects and need to be adjusted before combining them with the image method.

Though slightly better than the unmodified result and much better than the traditional SDF method without image wells, the modified method in this case does not fit well. This appears to be a result of the numerical model having a slightly closer boundary than the USGS maps, causing the model to exhibit an earlier response curve. This difference is partly an artifact of MODFLOW having difficulty simulating a rising water table along the steep bedrock near the boundary. Compensating for this different boundary location, by adjusting the distance in the image method by the difference between the model and the SDF map, gave the modified method a very close fit (Figure 7). Nevertheless,

either method with image wells provided a much closer fit than the traditional SDF approach, and the degree of fit was within the range of uncertainty for many of the input variables.

5.5. Conclusions about the Modified SDF Method

The response curves modeled for these two comparisons are site-specific and may not extend to other sites with equal success even though the approach is conceptually valid. This modified SDF method is still an approximation since, like boundary effects, other non-ideal complexities may have varying degrees of adjustment manifested in the mapped SDF. Still, this initial test produced a good match and showed that the modified method produced SDF values and response curves that compare well to those obtained with site-specific numerical modeling. This modified use of SDF values may be a valuable approach since numerical modeling is not feasible in most cases. The advantage of combining image wells with the SDF method was demonstrated both conceptually and with model results.

6. Summary

This review highlighted how the SDF method approximates effects of aquifer boundaries and provided response curves that can be used to assess potential errors. The need to combine the image method with stream depletion analysis in bounded alluvial aquifers can be significant, and the potential difficulty in selecting appropriate transmissivity values in lieu of using SDF values was discussed. As a solution, this review demonstrated the conceptual validity of using the image method with SDF values without over-accounting for boundary effects: mapped SDF values in the half of the aquifer closest to the stream can be used without modification since they do not include boundary effects, and values in the other half of the aquifer can be used by removing a predictable adjustment. This allows the user to better account for boundary effects while still retaining other non-ideal aquifer effects that are integrated into mapped SDF values. This proposed modification to the SDF method compared well to results obtained with a numerical model of a groundwater recharge site along the South Platte River. The modified method can provide improved stream depletion estimates while retaining the value of SDF maps as consistent and user-friendly references.

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