GROUND WATER-SURFACE WATER RELATIONSHIPS

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Robert E. Glover

Prepared for
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Introduction

Water resources have commonly been considered as falling within one of two separate and distinct categories. These are surface waters and ground waters. As our water supplies are becoming more heavily encumbered, it is beginning to be realized however, that these two categories are not necessarily distinct; and that in irrigation practice, the operations of the surface diverter and the user of ground water may affect each other in very fundamental and important ways. Such possibilities may be brought to attention, for example, by the advent of pumping in an area previously irrigated by surface diversion alone.

The possibility that an established surface diversion right might be adversely affected by pumping operations has led to demands for legal regulation of the pumps. However efforts to frame an equitable ground water law have run into trouble because of the difficulty of evaluating the nature and magnitude of the supposed interferences. Before these relationships can be clarified it will be necessary to evaluate quantitatively the effects of storage and release of ground water. It is the purpose of this paper to describe an analytical procedure by which these evaluations can be made.

Analytical methods

The procedures to be described are an outgrowth of the mathematical methods discovered by Isaac Newton in about the year 1666. (8) Of particular importance are the additional developments of J.B.J. Fourier about 1812. (1) It was in this year that Fourier's methods were presented to the French Academy and stirred up one of the most violent controversies in the history of science. The importance of this in the present connection is that if the budgetary requirement that the rate of rise of the water table within a narrow zone be compatible

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with the rates of flow of ground water into the zone is expressed in mathematical form it leads to differential equations of the type treated by Fourier. This immediately makes it possible to adapt the brilliant developments of Fourier, and his successors, to the calculation of ground water movements. Because of the interest aroused by Fourier's original paper the mathematical resources in this field are exceptionally good. It will be clear that these methods are not new since they represent some 300 years of development by able mathematicians.

The condition of continuity

The types of differential equation described above are obtained if the flow of ground water is computed by the Dupuit-Forschheimer idealization, which applies the surface gradient of the water table, at any point, to the entire saturated thickness below that point and by computing the flow on the basis that the original saturated thickness of the aquifer remains unchanged.

On this basis the requirement that the difference of flow across two planes a distance dx apart in the direction of flow should be compatible with the rate of rise of the water table between the two planes is

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t} \qquad \ldots (1)$$

Where

h represents the height of the water table measured upward from an assumed original stable water table level.

t time

x a distance measured along the path of flow,

and

$$\alpha = \frac{KD}{V}$$

where

K represents the permeability of the aquifer

D the original saturated depth

V the ratio of drainable or fillable voids to the total volume.

Where radial symmetry prevails, as around a pumped well, the basic differential equation takes the form:

$$\alpha \left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} \right) = \frac{\partial h}{\partial t} \qquad \ldots (2)$$

Where

s represents the drawdown from an assumed original stable water table level and

r represents the radius.

In order to estimate the rate of ground water movement in any given case solutions of the above differential equations are needed which conform to the appropriate initial and boundary conditions. A few examples are the following:

(1) For the case of a well pumped at the rate Q drawing water from storage in an aquifer of unlimited extent which conforms to the conditions

When $t = 0.$ $s = 0.$ for $r > 0$

A solution of equation 2 is: (2) (3)

$$S = \frac{Q}{2\pi KD} \int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} \, du \qquad \ldots (3)$$

The integral which appears here is a form of the exponential integral. A table of values can be found in reference 5.

When a river runs over the surface of an aquifer and is in contact with the ground water in it the stream depletion q_1 due to a well at a distance x_1 from the river when pumped at the rate Q can be obtained from this expression in the form: (2) (9)

$$\frac{q_1}{Q} = 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{\frac{x_1}{\sqrt{4\alpha t}}} e^{-u^2} \, du \qquad \ldots (4)$$

The integral which appears in this expression has been extensively tabulated. (4) (5) (6) It is called the "Probability Integral". Charts of a general nature can be constructed from such expressions. A sample is given in figure 1.

By integrating this expression with respect to time the total depletion of the river can be obtained in the form:

$$\frac{\int_0^t q_1\,dt}{Qt} = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{x_1}{\sqrt{4\alpha t}}} e^{-u^2}du - \frac{2}{\pi}\left(\frac{x_1{}^2}{4\alpha t}\right)\sqrt{\pi}\int_{\frac{x_1}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du \quad \ldots (5)$$

A plot of this expression is shown in figure 2. The integral

$$\sqrt{\pi}\int_{\frac{x_1}{\sqrt{4\alpha t}}}^{\infty}\frac{e^{-u^2}}{u^2}\,du$$

has been tabulated by Mr. M. W. Bittinger. This table may be found in reference 3. It may be noted that these charts are of a very general nature and can be applied to a wide range of conditions if only the aquifer properties are known.

A solution of equation 1 subject to the conditions

$h = H$ for $0 < x < L$ when $t = 0$

$h = 0$ when $x = 0$ for $t > 0$

$h = 0$ when $x = L$ for $t > 0$

is: (2) (3)

$$h = H \frac{4}{\pi} \sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-\frac{n^2\pi^2 \alpha t}{L^2}}}{n} \operatorname{Sin}\frac{n\pi}{L}x \qquad \ldots (6)$$

The part of the drainable volume remaining can be obtained from this expression in the form:

$$\frac{\int_0^L hdx}{H_o L} = \frac{8}{\pi^2} \sum_{n=1,3,5\ldots}^{n=\infty} \frac{e^{-\frac{n^2\pi^2 \alpha t}{L^2}}}{n^2} \qquad \ldots (7)$$

A plot of this expression is shown on figure 3. These expressions may be used to estimate drainage rates to parallel drains a distance L apart, or, since there is no flow across the plane at $x = L/2$ they may be used to estimate the return flow due to deep percolation losses from irrigation in an irrigated river valley of width L with the river in the middle of the valley.

Nature of the solutions

The condition of continuity described above is of a budgetary nature and imposes the requirement that the total flow across the boundaries and the water remaining in storage must equal the volume originally present in the aquifer. This condition is imposed upon the solutions and it follows that the amounts of water involved are exactly accounted for even though the differential equation is an exact expression of the physical conditions only if the rise or fall of the water table is infinitesimally small when compared with the original saturated depth. Whatever inaccuracies there may be will then appear only in the estimate of the time required for the changes to take place. The movements of ground water are treated as transient phenomena by these methods. This idealization will be found to accord well with the conditions to be found in the field.

Comments:

Plans for engineering works to promote the conservation of water and to provide for its effective use have generally been concerned with surface waters only. Such plans are usually based upon studies utilizing the records of runoff for a series of years. These studies are also of a budgetary nature but generally require no mathematical operations beyond arithmetic. Even though it is recognized that the storage and release of ground water may be an important factor it has heretofore been difficult to include this factor because there has been no means available for accounting for the time delays inherent in ground water movements.* By utilizing the analytical methods described herein this difficulty can be overcome, the factors introduced by ground water movements can be evaluated and the studies can be completed.

Even though specialized mathematical skills are required in this procedure the results of the mathematical work can be incorporated into simple charts, such as the ones shown, and thereafter the inclusion of the ground water factors can become a routine matter requiring no familiarity with advanced mathematical procedures. The charts will not have to be remade for each new case. They are sufficiently general to require only a knowledge of the aquifer characteristics, and its geometry. They can be prepared on either a flow or volume basis. The developments described are adequate to treat the factors of pumpage, return flow from irrigations, canal leakage and seepage from reservoirs. The cases described illustrate a method which can be extended to many other cases. The spreading of water from recharge areas, for example, can also be treated in this way.

Summary

The analytical methods described herein will permit the storage and return of ground water to be included in water budget studies required for the planning of engineering works for water conservation purposes. Charts can be prepared which will permit these factors to be included on a routine basis. Ground water-surface water relationships in existing developments can also be clarified and the effect of future developments can be assessed.

Acknowledgments

The procedures described herein include developments made at the U. S. Bureau of Reclamation and at Colorado State University.

* Return flows have sometimes been estimated from experience in other areas.

References

(1) The Analytical Theory of Heat by Joseph Fourier. Translation by Alexander Freeman, M.A., G. E. Stechert Co.

(2) Studies of Ground Water Movement. Technical Memorandum No. 657. U. S. Bureau of Reclamation. March 1960.

(3) Source Material for a course in Transient Ground Water Hydraulics, compiled by R. E. Glover and M. W. Bittinger, Colorado State University. April 1959.

(4) A Short Table of Integrals by B. O. Peirce. Ginn and Co. (Contains values of the integral

$$\frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx).$$

(5) Heat Conduction by L. K. Ingersoll, O. J. Zobel, and A. C. Ingersoll. McGraw-Hill 1948. (Contains tables of the integrals

$$\frac{2}{\sqrt{\pi}}\int_0^x e^{-\beta^2}d\beta \quad \text{and} \quad \int_x^\infty \frac{e^{-\beta^2}}{\beta}d\beta).$$

(6) Tables of the Error Function and its Derivatives. U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series No. 41. (Contains a 15 place table of the integral

$$\frac{2}{\sqrt{\pi}}\int_0^x e^{-\alpha^2}d\alpha).$$

(7) Tables of Functions and of Zeros of Functions. U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series No. 37. (Contains a table of

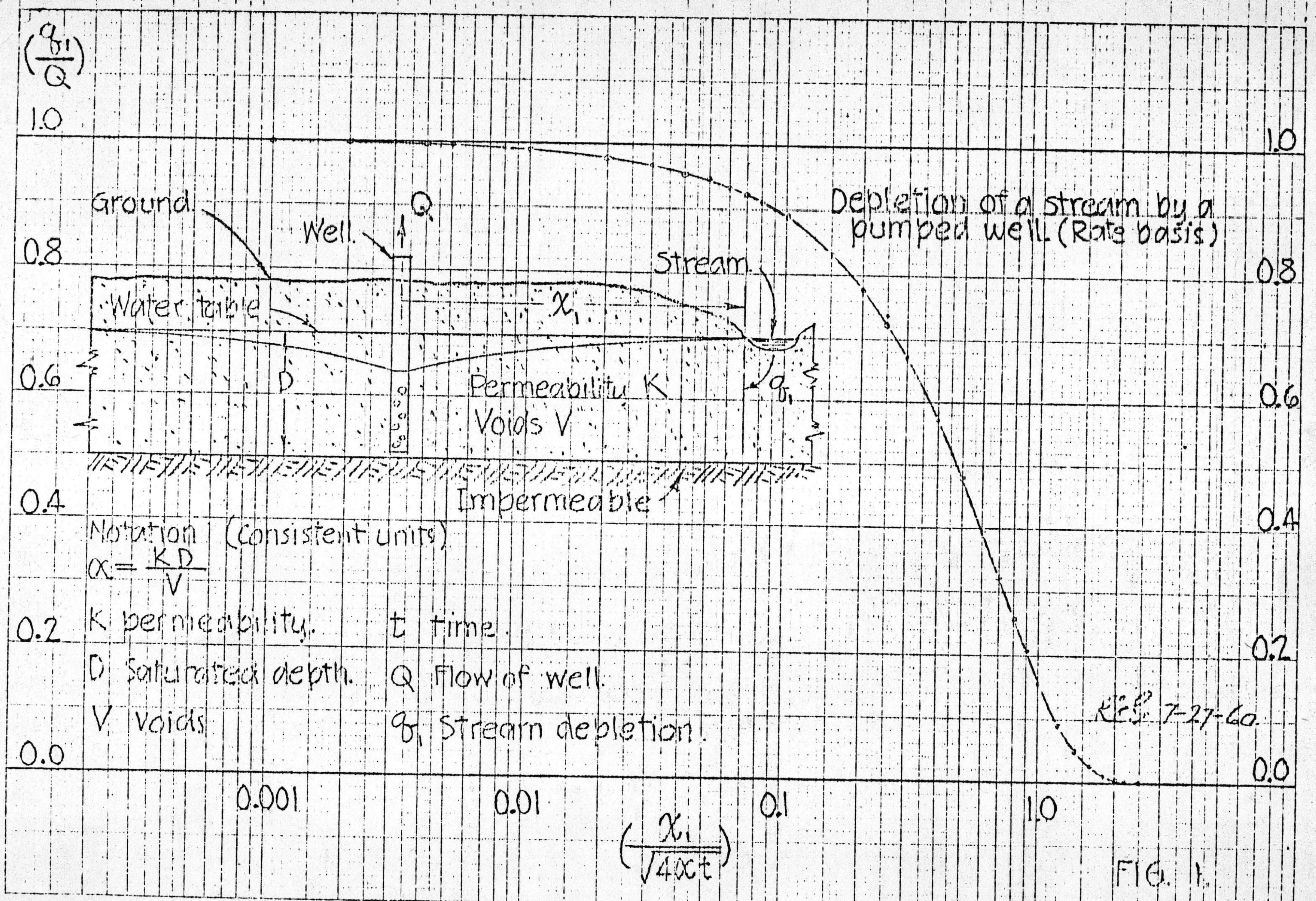
$$E_1(x) = \int_x^\infty \frac{e^{-u}}{u}du).$$

References - continued.

Note that $$\int_x^\infty \frac{e^{-u^2}}{u} du = \frac{1}{2} E_1 (x^2).$$

(8) Men of Mathematics by E. T. Bell. Dover

(9) River Depletion Resulting from Pumping a Well Near a River by R. E. Glover and G. G. Balmer. Trans. AGU., Vol. 35 No. 3. June 1954.



5 CYCLES X 70 DIVISIONS
$\left(\frac{q_1}{Q}\right)$
1.0
0.8
0.6
0.4
0.2
0.0
Depletion of a stream by a pumped well (Rate basis)
Ground
Well
Q
Stream
Water table
x_1
D
Permeability K
Voids V
q_1
Impermeable
Notation (Consistent units)
$\alpha = \frac{KD}{V}$
K permeability.
D Saturated depth.
V voids
t time
Q Flow of well
q_1 Stream depletion
0.001
0.01
0.1
1.0
$\left(\frac{x_1}{\sqrt{4\alpha t}}\right)$
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FIG. 1.

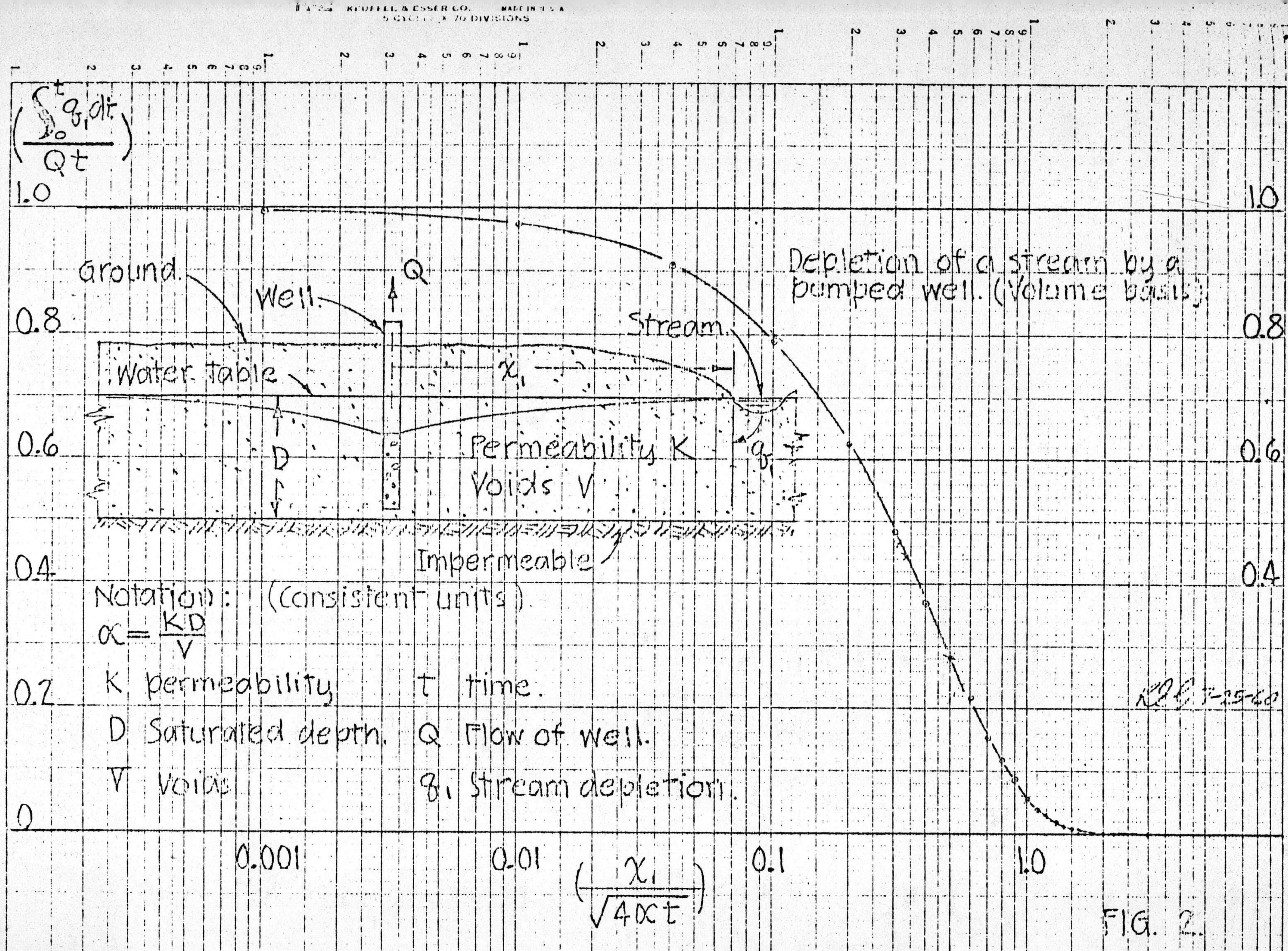
KEUFFEL & ESSER CO. MADE IN U.S.A.
5 CYCLES X 70 DIVISIONS
$\left(\frac{\int_0^t q_1 dt}{Qt}\right)$
1.0
0.8
0.6
0.4
0.2
0
Depletion of a stream by a pumped well. (Volume basis)
Ground.
Well.
Q
Stream.
Water Table
x_1
D
Permeability K
Voids V
q_1
Impermeable
Notation: (Consistent units)
$\alpha = \frac{KD}{V}$
K permeability
D Saturated depth.
V Voids
t time.
Q Flow of well.
q_1 Stream depletion.
0.001
0.01
0.1
1.0
$\left(\frac{x_1}{\sqrt{4\alpha t}}\right)$

FIG. 2.

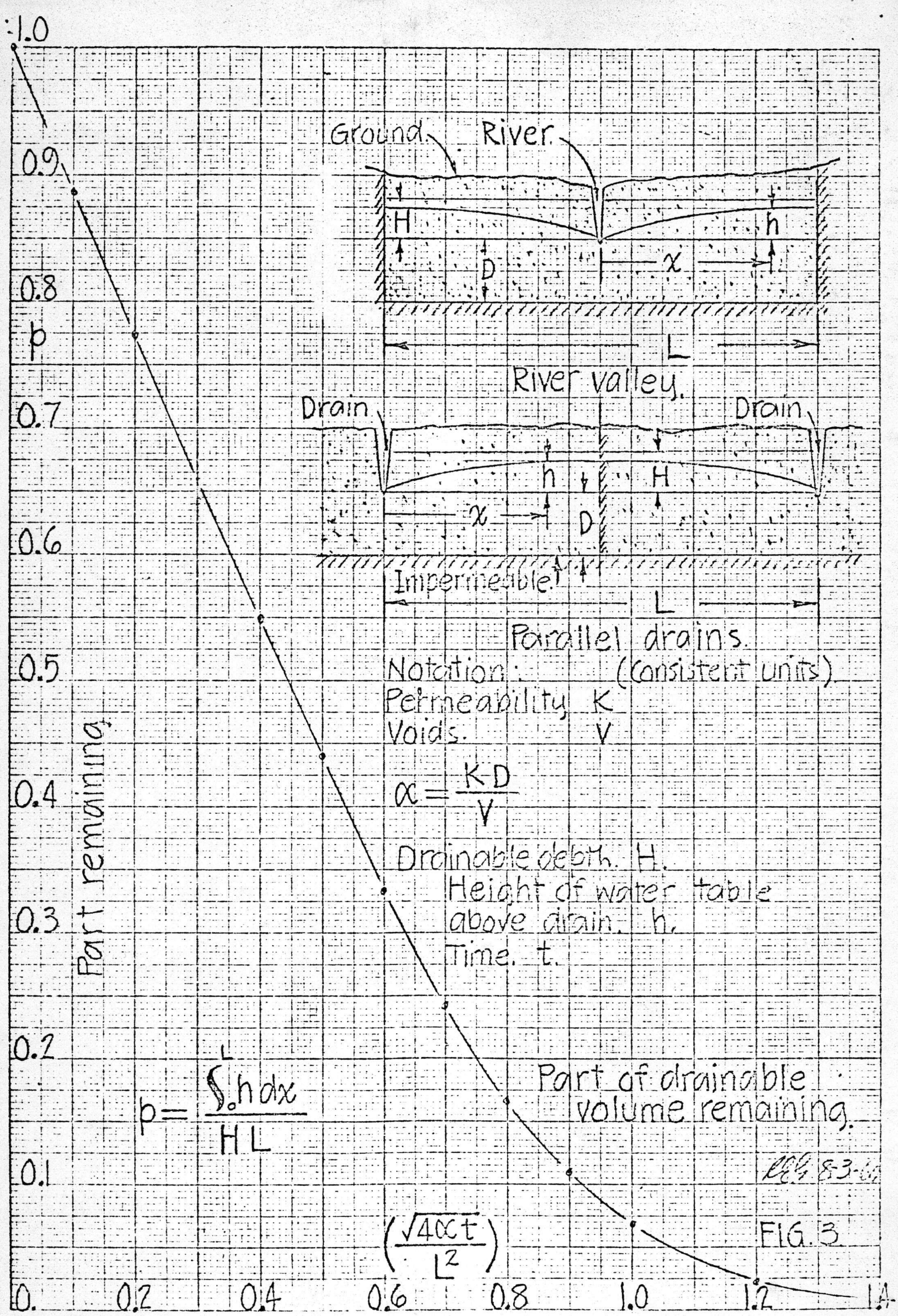
Ground
River
H
h
D
x
L
River valley.
Drain
Drain
h
H
x
D
Impermeable
L
Parallel drains.
Notation: (Consistent units)
Permeability K
Voids. V
$\alpha = \frac{KD}{V}$
Drainable depth. H.
Height of water table above drain. h.
Time. t.
Part remaining
p
Part of drainable volume remaining.
$p = \frac{\int_0^L h\,dx}{HL}$
$\left(\frac{\sqrt{4\alpha t}}{L^2}\right)$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
0.2
0.4
0.6
0.8
1.0
1.2
1.4

FIG. 3