## FINAL REPORT

WIND-ENGINEERING STUDY OF EPIA CONTROL TOWER, SAUDI ARABIA
by
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## ()$_{1870}$

## FLUID MECHANICS AND WIND ENGINEERING PROGRAM

## COLLEGE OF ENGINEERING

## COLORADO STATE UNIVERSITY

FORT COLLINS, GOLORADO

# FINAL REPORT <br> WIND-ENGINEERING STUDY OF EPIA CONTROL TOWER, SAUDI ARABIA 

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CSU Project 2-95610
March 1984

[^0]A wind engineering study of the EPIA control tower has been completed. The study consisted of several phases, including boundarylayer wind-tunnel tests on a scale model of the tower to determine the wind loading, a statistical study of on-site wind velocity data to allow proper application of the model test data, and calculation of the response of the tower to these wind loads. The tower's response was expressed as equivalent static wind loads for strength design, and acceleration of the control cab for performance evaluation. Existing literature regarding human response to acceleration in tall structures was reviewed, and various suggested criteria for objectionable motion were adapted to the EPIA control tower. The methodology of the model tests and the application of structural response theory to this data allowed identification of the dominant flow effects around the tower, and various means of reducing the tower's response through changes in mass, stiffness, and size.

The acceleration response of the original tower scheme was quite large in the cross-wind direction, due to a phenomenon known as vortex shedding, prompting concern over human discomfort. A second scheme, made necessary by an extra floor added to the tower cab, caused the vortex shedding to lock-in on the tower's own natural frequency, and drive the response even higher. Several additional schemes were then examined for response, all aimed at reducing the response by avoiding the vortex-shedding frequency. Various measures of response were computed for schemes 3,8 , and 9 . Schemes 8 and 9 were both successful in achieving a reasonable level of response.

The most complete response data is given for scheme 9 , and includes static equivalent design loads, acceleration response, and estimated levels of human perception and objection of motion. All of the tower occupants can be expected to perceive motion every 6 months, on average, and half of them may be able to perceive motion, on average, every 7 days. Evidence suggests that the professional staff holding permanent work positions in the tower should become accustomed to this motion, or can be trained to accept it. Taking this into account, it is predicted that the overall level of objection to motion in the tower will be between 1 and 12 percent of the occupants, depending on the amount of structural damping existing in the tower.
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## NOMENCLATURE

| Symbol | Meaning |
| :---: | :---: |
| A | Reference area, $=\mathrm{DH}$ |
| $\mathrm{a}_{\mathrm{i}}$ | Acceleration at elevation i |
| $\mathrm{C}_{\mathrm{M}}$ | Coefficient of externally-applied base moment, = M/(qAL) |
| $\mathrm{C}_{M}$ | Coefficient of response base moment, $=M /(q A L)$ |
| D | Reference width of structure |
| f | Cyclic frequency in Hz |
| $\mathrm{fS}\left(\mathrm{f}_{\mathrm{r}}\right)$ | Reduced PSD of M |
| $\mathrm{f}_{0}$ | Natural cyclic frequency of structure in fundamental mode of response |
| $\mathrm{fr}_{\mathrm{r}}$ | Reduced frequency, $=\mathrm{fD} / \mathrm{U}$ |
| g | Standard acceleration of gravity ( $32.1740 \mathrm{ft} / \mathrm{sec}^{2}$ ) |
| $\mathrm{g}_{\mathrm{p}}$ | Peak factor |
| H | Reference height of structure |
| $\|\mathrm{H}(\mathrm{f})\|^{2}$ | Mechanical admittance |
| $\left\|H_{a}(\mathrm{f})\right\|^{2}$ | Reduced acceleration admittance |
| $k^{*}$ | Generalized stiffness |
| L | Reference length of structure |
| M | Resultant base moment of externally-applied wind load |
| $\bar{M}, M^{\prime} \hat{M}$ | Mean, fluctuating, and peak values of M |
| M | Response (internal, static equalivalent) base moment |
| $\bar{M}, M^{\prime}, \hat{M}$ | Mean, fluctuating, and peak values of $M$ |
| m* | Generalized mass |
| $\mathrm{m}_{\mathrm{i}}$ | Mass lumped at elevation i |
| P | Effective response force at elevation i |
| $\bar{P}, P^{\prime}, \hat{P}$ | Mean, fluctuating, and peak values of $P$ |
| q | Reference dynamic pressure, $=\rho \mathrm{U}^{2} / 2$ |


| Symbol | Meaning |
| :---: | :---: |
| $S_{M}(f)$ | Power spectral density (PSD) of M |
| S (f) | PSD of $C_{M}$, $=S_{M}(f) /(q A L)^{2}$ |
| U | Reference wind velocity ( $u_{\text {mean }}$ at $z_{g}$ ) |
| u | Wind velocity |
| X | Horizontal coordinate (directed north) |
| y | Horizontal coordinate (directed west) |
| z | Vertical coordinate (directed up) |
| $z_{g}$ | Reference elevation (gradient height in wind boundary layer) |
| $\delta$ | Same as $\mathrm{z}_{\mathrm{g}}$ |
| ${ }^{\circ}()$ | Standard deviation (fluctuating rms) of ( ) |
| $\sigma_{(~)}^{2}$ | Variance of ( ) |
| $\phi_{i}$ | Normalized modal displacement at elevation i |
| $\psi$ | Phase angle |
| $w$ | Circular frequency, $=2 \pi f$ |

# FINAL REPORT <br> WIND-ENGINEERING STUDY OF <br> EPIA CONTROL TOWER, SAUDI ARABIA 

## 1. INTRODUCTION

### 1.1 Background

A $281 \mathrm{ft}(83.53 \mathrm{~m})$ high control tower is currently being engineered by Bechtel Civil \& Minerals, Inc. as part of the Eastern Province International Airport (EPIA) Project for the government of the Kingdom of Saudi Arabia. A model of the proposed structure is shown in Figure 2.5. The occupied portion of the tower, known as a control cab, is supported by a pedestal-type base, which houses an elevator and stairwell, and accounts for $240 \mathrm{ft}(73.15 \mathrm{~m})$ of the total tower height. The pedestal has a hexagonal cross section with six steel columns on a diameter of $22.31 \mathrm{ft}(6.800 \mathrm{~m})$ laced together with steel cross-bracing. All girder to column connections are to be field-bolted. The pedestal is clad with an aluminum skin which forms protruding architectural fins covering the columns. These fins are slightly tapered, and result in a maximum width of $28.2 \mathrm{ft}(8.60 \mathrm{~m})$ at ground level and $25.3 \mathrm{ft}(7.70 \mathrm{~m})$ just below the cab. The average slenderness (height/width) ratio of the pedestal itself, then, is 9.0; accounting for the total height of the tower, the slenderness ratio is 10.5 .

For preliminary design purposes, Bechtel applied the wind loading provisions of the 1982 Uniform Building Code [1], and selected a design wind speed of 80 mph (fastest mile at 33 ft elevation). However, this code contains the following disclaimer:

Structures sensitive to dynamic effects, such as buildings with a height-width ratio greater than five, structures sensitive to wind-excited oscillations, such as vortex shedding or icing, and buildings over 400 ft in height, shall be, and any structure may be, designed in accordance with approved national standards.

The "approved national standard" referred to is ANSI A58.1-1982 [2].
This code describes a procedure which can be used to estimate the alongwind response in the form of the so-called gust response factor--of tall
flexible structures. As the code states, however,

The gust response factor accounts for the additional loading effects due to wind turbulence over the fastest-mile wind speed. It also includes loading effects due to dynamic amplification for flexible buildings and structures, but does not include allowances for the effects of the cross-wind deflection, vortex shedding, or instability due to galloping or flutter. For structures susceptible to loading effects that are not accounted for in the gust response factor, information should be obtained from the recognized references or from wind-tunnel tests.

Due to the slenderness and prismatic nature (i.e., cross section constant with height, offering no interruption to the exposed shape), susceptibility to vortex-shedding must be suspected. There are no known references which address these issues in the context of a geometry similar to the subject tower, and therefore a wind-tunnel study is indicated. In fact the results of the study reported herein show that, for certain wind directions, a high degree of vortex shedding occurs, and greatly magnifies the effective wind load.

In addition to stresses induced by the effective wind load, the importance of the motion itself is widely recognized, as the acceleration associated with the vibratory nature can lead to discomfort of the occupants. This phenomenon is discussed, among other sources, in the National Building Code of Canada [3], wherein it is recognized that such oscillations may be particularly large in the cross-wind direction in
tall slender structures. This code presents a "somewhat tentative" method of estimating the acceleration, briefly discusses the human response to acceleration issue, and suggests a rough criteria to which predicted accelerations should be limited. The acceleration-prediction method is also discussed in reference [4], where it is shown to have been derived from a similar method proposed by Vickery for the building code of Australia [5], which is in turn based on the results of windtunnel studies on a variety of buildings, and carries a warning of extrapolation to structures of significantly different shape or dynamic properties. The Canadian code procedure has, however, been applied to the EPIA tower by Bechtel's structural consultant; it was found that, even at a wind speed of 40 mph (hourly mean at 33 ft ), the acceleration in the across-wind direction alone was far greater than the suggested design criteria. The adequacy of the analytical method was questioned, and a wind-tunnel study was also suggested. Evidently, a further requirement is to more fully investigate available data regarding human response to motion.

The remaining requirement is to establish rational design wind speeds for the EPIA site. The initial selection of 80 mph was arbitrary, and was not based on any available wind statistics. Therefore, as much wind data as possible has been accumulated, and statistical analyses have been used to fit probability distributions to this data.

In view of the preliminary nature of the structural design of the tower, and its anticipated excessive response to wind, design changes where expected as a result of or concurrent with the wind-tunnel study. Therefore, a recently developed type of dynamic model was used in this wind-tunnel study, on which the normalized wind load itself can be
measured. This is distinct from the response of the structure, which is subsequently calculated from the measured wind load; only in this stage it is necessary to take into account the dynamic prototype properties of mass, stiffness, and damping. At the same time, the normalized wind load is scaled to any desired wind speed. In fact, several structural schemes had been proposed by the completion of this study, and the response has been evaluated for these without the need of repeating any of the original wind-tunnel runs. The background, benefits, and limitations of this type of wind-tunnel study are discussed in the remainder of this introductory section.

To summarize, this wind-tunnel study consists of the following tasks:

1. Perform dynamic model tests in the wind tunnel to measure the normalized wind load on the tower. These results are given in Section 3.
2. Statistically analyze available wind data to establish rational design wind speeds; or conversely, the mean recurrence interval of any specified wind.
3. Compute the response of the structure for various structural schemes (i.e., various values of mass, stiffness, and damping). These results are given in Section 4. Wind loads for strength design are in the form of response moments, or equivalent static base moments, for a 50 -year or 100 -year mean recurrence wind. Motion for performance design is in the form of vector resultant rms acceleration of the control cab floor for various recurrence intervals from 0.1 to 10 years.
4. Review all available data concerning human response to motion in a tall building environment. These data are very scarce and not directly applicable. After making rough allowances based on judgement, however, it is possible to estimate the degree of motion perception as well as levels of actual objection, based on acceleration levels experienced and their average recurrence rate. This is the topic of Section 5.

### 1.2 Modeling

The development of boundary-layer wind tunnels has provided a method for determination of wind loads on structures such as the EPIA Control Tower. A boundary-layer wind tunnel differs from other types of wind tunnels in that a thick ( $2-4 \mathrm{ft}$ ) turbulent boundary layer is developed along the floor of a long test section whose characteristics produce an accurate scaled model of the atmospheric boundary layer.

The criteria to be satisfied for accurate modeling of the atmospheric boundary layer and wind loads on a structure have been documented in the literature $[6,7,8]$. In general, the requirements are that the model and prototpye be geometrically similar, that the approach mean velocity at the building site have a vertical profile shape similar to the full-scale flow, that the turbulence characteristics of the flows be similar, and that the Reynolds number for the model and prototype be equal. These criteria are satisfied by constructing a scale model of the structure and its surroundings and performing the wind tests in a wind tunnel specifically designed to model atmospheric boundary-layer flows. The wind tunnel simulation produces both the vertical profile shape in mean velocity and the properly scaled turbulence characteristics.

Reynolds number similarity requires that the quantity $U D / V$ be equal for model and prototype. Since $v$, the kinematic viscosity of air, is identical for both, Reynolds numbers cannot be made equal with reasonable wind velocities. To accomplish this the air velocity in the wind tunnel would have to be as large as the model scale factor times the prototype wind velocity, a velocity which would introduce unacceptable compressibility effects. However, for sufficiently high Reynolds numbers ( $>2 \times 10^{4}$ ) the flow conditions at any location on the structure will be essentially constant for a large range of Reynolds numbers. Typical values encountered are $10^{7}-10^{8}$ for the full-scale and $10^{5}-10^{6}$ for the wind-tunnel model. In this range acceptable flow and wind load similarity is achieved without Reynolds number equality.

Any measurement--velocity, pressure, force, moment, etc.--taken on the model may be extrapolated to the full-size structure (prototype) by expressing it in nondimensional (normalized) or "reduced" form. The reduced value, often referred to as a coefficient, is equally applicable to both model and prototype. For example, the ratio between wind velocities at any two points is a constant applicable at any scale; therefore, the measured velocity at any point, $u$, is reported as a normalized velocity $u / U$, where $U$ is a reference velocity measured at a fixed location. The reference location is arbitrary so long as a corresponding location in the prototype is used. For this reason, the reference location is usually placed where flow conditions a) are not affected by the model structure, and b) can be determined for the prototype site. It is common practice in wind-tunnel work to select the reference location above the test structure at a height equal to the boundary layer thick-ness--the so-called gradient height, denoted $z_{g}$ or $\delta$. The full-scale
(prototype) velocity at this location for a storm of any desired intensity, or mean recurrence interval (e.g., the so-called 50-year wind) is determined from available wind data and an estimated velocity profile.

Similarly, pressures are presented as a pressure coefficient, defined by

$$
C_{p}=\frac{p}{q}
$$

where $p$ is the measured pressure and $q$ is the reference pressure, defined as the dynamic pressure at the reference location:

$$
q=\frac{1}{2} \rho U^{2}
$$

here $\rho$ is the air density. Thus the pressure at any point on the prototype where a measured coefficient exists, for any desired wind storm, can be found by the following steps: 1) determine the reference wind velocity $U, 2$ ) compute the reference pressure $q$, 3 ) multiply this by the pressure coefficient $\quad C_{p}$.

Other measured quantities are treated in the same manner, and require the establishment of a suitable reference quantity. If $\mathrm{D}, \mathrm{H}$, and $L$ are established as the reference width, height, and length, respectively, of the structure, then we can define

```
Reference area \(=\mathrm{A}=\mathrm{DH}\)
Reference force \(=q A\)
Reference moment \(=q A L\)
```

Now if a force $F$ or a moment $M$ is measured in the wind tunnel, the reduced forms are the force coefficient,

$$
c_{F}=\frac{F}{q A}
$$

and the moment coefficient,

$$
C_{M}=\frac{M}{q A L}
$$

When it is required to consider the fluctuation rate, or frequency, of a dynamic quantity, the measured frequency $f$ is expressed as the reduced frequency, defined as

$$
f_{r}=\frac{f D}{U}
$$

where $D$ and $U$ are the reference width and velocity introduced above. As an application of the reduced frequency, a long slender body of width D exposed to a cross flow of velocity $U$ is often observed to shed vortices at a regular frequency, which corresponds to a particular value of $f_{r}$ known as the Strouhal number. This number is a property of the cross-sectional shape of the body, and has been tabulated for a variety of common shapes [9]. Thus, once the Strouhal number for a shape is known, the shedding frequency for any size of the body in any flow velocity may be determined.

The various reference quantities used in this study and their value in the model and prototype are given in Table 1.1.

For many streamlined bodies or those having curved surfaces, any of the various reduced values or coefficients described above may not be absolute constants, but functions of the Reynolds number. Since this number is generally not equal for model and prototype, as discussed above, wind-tunnel studies of such bodies do not always produce valid results. For bluff bodies, or those having sharp edges and corners which determine flow separation, the Reynolds number independence of these values is well established. The EPIA control tower falls into this latter category; therefore, all test results may be safely extrapolated to the prototype structure and any desired wind velocity.

TABLE 1.1
Reference quantities for scaling test data

| Quantity | Symbol | Meaning | Model | Value |
| :--- | :--- | :--- | :--- | :--- |
| Prototype |  |  |  |  |

### 1.3 Measurement Strategy

Two general techniques using a boundary-layer wind tunnel are used to obtain fluctuating loads and responses for structures. In the older method, the structural properties, including stiffness, mass distribution, damping, and natural frequency, are modeled using appropriate scaling criteria. For most cases, a single-mass, rigid model representing the first mode of vibration is used. The model is supported by springs at its base providing up to three degrees of freedom for rotation, and measurement of base moments, about perpendicular axes. The measurements give directly the structure response, as affected by the structure's vibration about each axis at the first mode frequency. Data output can include response base moments, top deflections, and top accelerations. This type of modeling, called aeroelastic, is necessary where motion of the structure is sufficient to modify the wind flow about the structure and hence the wind loading itself. Its chief disadvantage is that the structural properties must be reasonably well established before the model tests can proceed. Significant changes in structural properties usually require retesting.

In the second method, a rigid model of the structure is mounted on a balance (load-measuring device) in such a way that the resulting natural frequency of the balance/model is much higher than the scaled first mode frequencies of the structure. A balance of this type can have up to 6 degrees of freedom for measurement of 3 forces and 3 moments. The measurements give directly the fluctuating wind loads, usually in spectral form, without inclusion of structure inertial loads. The loading data can then be combined analytically with structural properties to obtain the mean and dynamic root-mean-square (rms) structural response. The peak response is estimated by adding the mean value
to a statistically-chosen factor times the rms value. If structural properties change, the analysis can be repeated without having to retest the model.

For the EPIA Control Tower, the second method using a dynamic load balance was selected. The final design structural properties of the tower were sufficiently uncertain at the beginning of the test that structural properties could not be firmly established. The balance was configured to measure 2 components-bending moments about orthogonal $x$ and $y$ axes at the base of the tower. Details of the balance are described in Section 2.2. Load data were obtained in the form of mean values and power spectral densities. The calculation of dynamic response is the subject of Appendix $A$.

Further discussion of the principles of dynamic load model studies is given in Appendix A.

## 2. TEST EQUIPMENT AND INSTRUMENTATION

### 2.1 Wind Tunnel

Three large wind tunnels are available in the Fluid Dynamics and Diffusion Laboratory at Colorado State University for wind engineering investigations. The wind tunnel used for this study was the Industrial Aerodynamics tunnel shown in Figure 2.1. It has a 62 ft long test section with a $6 \times 6 \mathrm{ft}$ cross section. Velocity is continuously variable from 9 to 65 fps using a 75 hp variable pitch fan.

The boundary layer used for this study was obtained using a flow trip--spires and a $7^{\prime \prime}$ barrier wall--at the test section entrance, followed by floor roughness. The roughness was pegboard with 0.5 in . high dowels 0.25 in. in diameter placed 4 in . apart laterally and 3 in . apart in the flow direction. This roughness in combination with the spires and barrier gave a mean velocity profile at the model site given by

$$
\frac{u}{U}=\left(\frac{z}{z_{g}}\right)^{\alpha}
$$

where $u$ is the mean velocity at height $z, U$ is a reference velocity at height $z_{g}$, and $\alpha$ is a power law exponent which varies with floor roughness, and describes the shape of the profile. The parameter values used in this study were $\alpha=0.14$, and $z_{g}=45 \mathrm{in}$. (model scale), or 675 ft (full scale). This profile is shown in Figure 2.2, along with the profile of longitudinal turbulence intensity. These wind-tunnel conditions are a suitable representation of the atmospheric boundary layer with strong winds in flat open country.

### 2.2 Dynamic Load Balance

The balance used in this project is shown in Figures 2.3 and 2.4. Basically it is a strain-sensing apparatus consisting of three



Figure 2.2. Mean Velocity and Turbulence Profiles of Approach Flow in Wind Tunnel


All Dimensions in inches

Figure 2.3. Details of Dynamic Load Balance


Figure 2.4. Photographs of Load Balance and Control Tower Model
main parts: a heavy steel reaction or inertial ring, a steel sprung base plate, and supporting steel cross-beams. The test model is mounted to the base plate, slightly below ground level. The cross-beams allow the base plate--and therefore also the model--to rotate slightly about orthogonal $x$ and $y$ axes, in a horizontal plane at the base of the test model. Strain gages are attached to necked-down segments of these cross-members, and provide an electrical signal proportional to the bending moment about the corresponding axis of rotation. Temperaturecompensating resistors within each gage bridge network are installed within the base of the force balance. All strain gages are p-type silicon semiconductor electrical resistance gages, having a nominal gage factor of about 140. Gage excitation and amplification were provided by Accudata Model 218 gage control/amplifiers, manufactured by Honeywell.

### 2.3 Model

The model of the EPIA control tower was constructed as shown in Figures 2.5 and 2.6. The model scale was $1: 180$, which was selected to be consistent with the existing boundary layer and turbulence scales in the wind tunnel, with the measurement capabilities of the balance, and the availability of stock materials from which to build the model.

The pedestal of the model was machined from a stock piece of aluminum tube, as shown in Figure 2.6. This was bolted to the balance using a threaded steel plug inserted into the base of the tube. The tower cab was constructed of thin cardboard and styrofoam for lightness. The combination of a stiff balance, high bearing stresses between the balance platform and the aluminum tube, the rigidity of the aluminum tube, and the lightweight cab, enabled the entire model to react as a single-degree-of-freedom system, in each of the two components.


Figure 2.5. Photographs of Control Tower Model in Wind Tunnel


Figure 2.6. Construction Details of Control Tower Model

Measured natural frequencies of rotation about the $x$ and $y$ axes were 143 Hz and 153 Hz , respectively. The useable frequency bandwidth was modified using electronic filters, as described in Section 2.4.

The entire model tower and balance assembly was mounted on the wind-tunnel turntable (Fig. 2.1) which was rotated to simulate winds from various directions. The tower coordinate system and wind direction convention are shown in Figure 2.7. Note that since the balance rotates with the model, the coordinate system remains aligned with the building, instead of with the flow, as is common in aeronautical wind-tunnel practice.

Due to symmetry of the tower and the flat open country around it, only wind directions from $180^{\circ}$ through $360^{\circ}$ were tested. Nineteen runs were made at $10^{\circ}$ intervals; initial inspection of the results indicated there was no need for finer increments. The first set of runs, referred to as Configuration $A$, did not include any of the nearby terminal buildings in the wind tunnel. Thus, these results may be reflected about the north-south axis to account for wind directions $0^{\circ}$ through $180^{\circ}$. An index of the runs is given in Table 3.1.

A few additional runs were made at selected wind directions from $270^{\circ}$ to $360^{\circ}$ with upwind terminal buildings in place. These are identified as Configuration B.

### 2.4 Instrumentation

As mentioned previously, the wind-tunnel balance is a strain gagebased transducer which, operating in conjunction with a gage controller/ amplifier, produces a signal proportional to the balance moment. The mean balance moment is equal to the mean wind moment applied to the test model; fluctuating loads, however, are somewhat amplified due to resonance of the balance/model system. Because this sytem behaves as a

TERMINAL BUILDINGS


Figure 2.7. Tower Coordinate System and Wind Direction Convention
single-degree-of-freedom system, with a known natural frequency (Section 2.3) and very low damping, the resonant amplification factor may be readily determined as a function of frequency. The amplifier output signal was therefore passed through a low-pass filter, having an attenuation function which very nearly compensates for the resonant amplification, for signal frequencies below the resonant (natural) frequency of the balance. Signal frequencies at and above resonance were attenuated sharply by a second filter, having a steep ( 48 dB / octave) roll-off rate. The primary purpose of this second filter was to prevent digital aliasing in the ensuing power spectral density calculations. The combination of balance resonance, the shaping filter, and the anti-aliasing filter resulted in an essentially flat response (constant gain $\pm 0.5 \mathrm{~dB}$ ) and alias-free system over a bandwidth from 0 Hz to about 70 Hz .

The reference pressure in the wind tunnel--from which the reference velocity is calculated (see Section 1.2 and Table 1.1)--was sensed by a pitot-static tube located above the tower at the refernece height $z_{g}$. The total and static pressure tubes from this sensor were routed to a differential pressure transducer, which provided an output signal proportional to the dynamic pressure $\frac{1}{2} \mathrm{\rho U}^{2}$. The transducer and a dedicated gage control/amplifier were maintained and calibrated together as a unit, and produced a high-level signal precisely related to the reference pressure.

The digital portion of the instrumentation system is centered around an HP 1000 21MX E-Series computer and includes a disc drive, printer, plotter, Digi-Data digital tape drive, and a Preston Scientific 12-bit, 50 KHz , 16 -channel analog-to-digital conversion system (ADC).

All computer software was developed by ERC-FFDL personnel, and performs the following functions: 1) control of ADC channels and sample rates, 2) initiation and transfer of data from ADC to computer memory, 3) conversion of time series from digital to numerical coefficient form using the measured reference pressure, 4) calculation of mean, rms, maximum, and minimum values of all channels, and echoing of this information on user's terminal at wind tunne1, 5) storage of time series data on magnetic tape, 6) calculation of power spectral densities of moment coefficient time series and storage of same on disc, 7) plotting or printing of power spectra, and 8) automatic identification of run number, channel number, channel label, units, wind direction and velocity on all forms of data storage and output. All functions, except for plotting, can be performed by the test conductor operating a remote terminal at the wind tunnel. To decrease the required test time, however, most power spectra were computed after testing by recalling time series data from tape. Power spectra were computed using a standard fast-fourier transform (FFT) algorithm, described in reference [10]. This procedure incorporates a cosine taper data window, and spectral smoothing using both frequency averaging and segment averaging.

## 3. TEST RESULTS

### 3.1 Introduction

This section presents all test data obtained from the EPIA tower model in the wind tunnel. As described previously, all data represents the applied moment due to wind about the $x$ or $y$ axis, referred to an elevation of 13350 mm (just below ground level). These moments have been reduced to coefficient form by dividing by the wind tunnel reference moment, as described in Section 1.2. The coefficients may be scaled to full-scale moments corresponding to any desired wind velocity, by multiplying by a corresponding prototype reference moment (see Table 1.1).

The applied wind moment is divided, for convenience, into its mean and fluctuating components:

$$
M=\bar{M}+M^{\prime}
$$

The fluctuating component, $\mathrm{M}^{\prime}$, is a randomly varying quantity, and is described in terms of its root-mean-square value, denoted ( $\left.M^{\prime}\right)_{r m s}$. This is equivalent to the standard deviation of $M$, and the alternative notation $\sigma_{M}$ is often used.

It is necessary to distinguish the response moment in the structure from the applied moment, and the practice employed herein is to use script letters for response parameters. Thus the response moment is designated $M$, and this is also decomposed into mean and fluctuating parts:

$$
M=M+M^{\prime}
$$

Th mean response $\bar{M}$ is equivalent to the mean load $\bar{M}$. This is not true of the fluctuating response, however, which is equal to the fluctuating applied load, additional load due to inertial acceleration, and
a dissipative load due to damping. The fluctuating rms response, $\sigma$, can be calculated from $\sigma_{M}$, if specific values of the structure's mass, stiffness, and damping are assumed. This is the subject of Section 4.

Applied wind moment data is presented here not only to document the test results, but also because they provide considerable insight regarding the effects of wind direction, wind speed, buildings upwind, tower width, and natural frequency on the response of the tower. Data were obtained in both time domain and frequency domain form. Mean and fluctuating rms values were computed from time domain data. The frequency domain data are in the form of power spectral densities (PSD). This form, which shows the frequency distribution of $M^{\prime}$, is required for the calculation of $\sigma$.

### 3.2 Time Series Evaluation of Loads

The results of all time domain calculations are summarized in Figure 3.1 , in which mean and rms moment coefficients are plotted as a function of wind direction. As described previously, all data were obtained from $180^{\circ}$ to $360^{\circ}$; due to symmetry these may be reflected about $180^{\circ}$ to obtain results for $0^{\circ}$ to $180^{\circ}$ (the mean $x$-moment requires a change in sign). Configuration $B$ data was obtained with terminal buildings upwind at wind directions $270^{\circ}, 280^{\circ}$, and $320^{\circ}-360^{\circ}$ (Table 3.1).

It is of interest to note that the terminal concourse results in a reduction of the mean $x$-moment coefficient at $270^{\circ}$ and $280^{\circ}$. The fluctuating moment, however, is greatly increased; this is due to turbulence in the wake of the concourse. The fluctuating cross-wind moment coefficient, $\left(C_{M}^{\prime}\right)_{\mathrm{rms}}$, is also increased at $270^{\circ}$. From $320^{\circ}-360^{\circ}$, the terminal buildings have very little effect on the mean moment in either direction. The fluctuating moment about either axis is significantly increased at wind direction $330^{\circ}$.



Figure 3.1. Test Results: Externally-Applied Wind Base Moment Coefficient ( $C_{M}$ ) vs Wind Direction

TABLE 3.1
INDEX OF WIND-TUNNEL TEST RUNS

| Run <br> No. | Wind <br> Direction <br> (Deg) | Equiv. Wind <br> Dir. by sym. | Configuration |
| :--- | :--- | :---: | :---: |
|  |  |  |  |
| 7 | $270^{*}$ | 90 | A |
| 8 | $280^{*}$ | 80 | (no adjacent |
| 9 | 290 | 70 | buildings) |
| 11 | 300 | 60 |  |
| 14 | 310 | 50 |  |
| 15 | $320^{*}$ | 40 |  |
| 16 | $340^{*}$ | 30 |  |
| 17 | $350^{*}$ | 20 |  |
| 18 | 260 | 10 |  |
| 22 | 180 | 100 |  |
| 23 | 190 | 170 |  |
| 24 | 200 | 150 |  |
| 25 | 210 | 140 |  |
| 26 | 220 | 120 |  |
| 27 | 230 | 110 |  |
| 28 | 240 |  |  |
| 29 | 250 |  | (upwind terminal |
| 30 | 270 |  | bldgs. included) |
| 37 | 280 |  |  |
| 38 | 320 |  |  |
| 41 | 340 |  |  |
| 42 | 0 |  |  |

*Data obtained only to show effect of removing upwind terminal buildings. True data is Config. B.

### 3.3 Frequency Domain Evaluation of Loads

Power spectral densities (PSDs) of base moments were computed for all of the same run conditions used for time domain data. The PSD of a process, say $x(t)$, is denoted $S_{x}(f)$, and shows the distribution of the mean square value of $x$ with frequency $f$. Its dimension is (dimension of $x)^{2}$ per (unit frequency). The PSD of a moment measured in the wind tunnel, for example, might be measured in (lb-in.) ${ }^{2} / \mathrm{Hz}$. In graphical form it would appear as a curve on the axes $S_{M}(f)$ vs $f$. The area under this curve between two frequencies $f_{1}, f_{2}$ is the contribution to the mean square moment of frequency components between $f_{1}$ and $f_{2}$. The area under the entire spectral curve, excluding 0 Hz , is the total mean square fluctuating moment:

$$
\sigma_{M}^{2}=\int_{0}^{\infty} S_{M}(f) d f=\left(M^{\prime}\right)_{r m s}^{2}
$$

In this case moments have been reduced to coefficient form, and the PSD of moment coefficient has units of coeff ${ }^{2} / \mathrm{Hz}$, or simply $\mathrm{Hz}^{-1}$. If this spectrum were denoted as simply $S(f)$ (PSD of a dimensionless process), then the area under its curve is

$$
\int_{0}^{\infty} S(f) d f=\left(C_{M}^{\prime}\right)_{r m s}^{2}
$$

The square root of this is the same rms value discussed in the preceding section. An example of such a spectrum is given in Figure 3.2(a).

The spectrum $S(f)$ as well as the independent variable $f$ are not dimensionless, and it is desirable to make them so to facilitate scaling to prototype values. Frequency has the dimension $1 /$ time; if a reference frequency is defined as $U / D$ where $U$ is the reference wind-tunnel speed and $D$ the reference width of the model (Table 1.1), the reduced frequency becomes



Figure 3.2. Representative Power Spectral Densities in Dimensional and Reduced Form

$$
f_{r}=\frac{f D}{U}
$$

which was already introduced in Section 1.2. The spectral value $S(f)$ has the dimension of $1 / \mathrm{Hz}$ (time) and can be reduced by multiplying by a reference frequency. The frequency $U / D$ could be used, but it is standard practice to use instead the actual frequency $f$. The reduced spectrum is therefore $f S(f)$, and is generally plotted as a function of fD/U. The spectra of Figure $3.2(\mathrm{a})$, reduced to nondimensional form, are plotted in Figure 3.2(b). There are at least two reasons why it is convenient to reduce the spectrum by multiplying by $f$ : 1) the quantity fS(f) is, directly, the amount of energy which would be transmitted to a mechanical resonator at frequency $f$ (see equation (4.5)); 2) the spectrum may be integrated with respect to $\ell n$ to yield the mean square value, since $d(\ell n f)=d f / f$. For this latter reason the frequency is usually plotted on a logarithmic axis, so that areas under the reduced spectrum curve in various frequency bands can still be geometrically compared for the distribution of energy (a logarithmic y-axis is used simply to conserve space; spatial contributions to energy are therefore distorted in the vertical direction but not the horizontal).

Plots of PSDs for all run cases, as well as numerical tabulations, are given in Appendix D. Characteristics PSDs are shown in Figures 3.2, 3.3 , and 3.4.

### 3.4 Identification of Vortex Shedding

The spectra shown in Figures 3.2-3.4 provide valuable insight to the fluctuating load and its effect on the tower's dynamic response. Regarding Figure 3.2, in particular, note that the wind direction here is $270^{\circ}$, so that $M_{x}$ is an along-wind load, and $M_{y}$ is a cross-wind load. Whether the model scale spectra (Fig. 3.2(a)) or reduced


Figure 3.3. Characteristic PSD at Wind Direction Where Vortex Shedding Occurs


Figure 3.4. Characteristic PSD at Wind Direction Where Vortex Shedding Does Not Occur
nondimensional spectra (Fig. 3.2(b)) are studied, the following features are readily apparent.

At very low frequencies, the along-wind load is much higher than the cross-wind load. This is consistent with the expected behavior of the 0 frequency, or mean, load. As frequency increases, the along-wind loading decreases. This is also consistent with the known behavior of longitudinal wind turbulence. In the cross-wind direction, however, the loading intensity increases with frequency and peaks at a frequency of 25 Hz (at model scale) before decreasing. The model reference width is 0.1239 ft (Table 1.1) and the reference velocity in the wind tunnel for this run was 35.5 fps , so the reduced frequency is

$$
f_{r}=\frac{\mathrm{fD}}{\mathrm{U}}=\frac{(25 / \mathrm{sec})(0.1239 \mathrm{ft})}{35.5 \mathrm{ft} / \mathrm{sec}}=0.087
$$

The actual frequency $f$ of this spectral peak will be different for the prototype structure as well as for various wind velocities, but the reduced frequency $f_{r}$ will remain constant. For the prototype, which has a reference width of $D=22.3 \mathrm{ft}$, the frequency of the spectral peak is

$$
\mathrm{f}=\frac{\mathrm{f}_{\mathrm{r}} \mathrm{U}}{\mathrm{D}}=\frac{(0.087)(\mathrm{U})}{22.3}=0.0039 \mathrm{U} \mathrm{~Hz}
$$

where $U$ is the full-scale reference velocity in fps.
Examination of the load PSDs for other wind directions shows that a similar, though less intense, spectral peak occurs whenever the wind blows parallel to the sides of the tower's hexagonal cross section, i.e. at $210^{\circ}$ (or $150^{\circ}$ by symmetry), $270^{\circ}\left(90^{\circ}\right)$, and $330^{\circ}\left(30^{\circ}\right)$. The loading responsible for the spectral peak is evidently in the cross-wind direction, based on the $270^{\circ}$ data (Fig. 3.2), where the along-wind and crosswind directions are aligned with the $M_{x}$ and $M_{y}$ directions, respectively. At $210^{\circ}$ and $330^{\circ}$ the cross-wind load has a component in both
the $x$ and $y$ directions; indeed a somewhat smaller spectral peak in both the $M_{x}$ and $M_{y}$ data occurs (Fig. 3.3). At wind direction $0^{\circ}$ or $180^{\circ}$, Figure 3.4, the spectral peak is absent from both $M_{x}$ and $M_{y}$. This is also characteristic of wind directions $240^{\circ}\left(120^{\circ}\right)$ and $300^{\circ}$ $\left(60^{\circ}\right)$, where the wind blows across the points of the hexagonal cross section, rather than across the flat sides.

The cause of this peak in some of the loading spectra is evidently vortices being shed from the fins (points of flow separation) of the pedestal section of the tower. Three major observations support this conclusion: 1) the reduced frequency of the spectral peak is very close to the expected value of the Strouhal number for a body such as the pedestal, 2) the cyclical loading is only in the cross-wind direction, and 3) the existence of the phenomenon is dependent on the wind's orientation to the hexagon in a manner which is consistent with the flowrelated mechanism of vortex shedding, wherein a significant afterbody (longitudinal surfaces downwind of the separation point) is required for cross-wind forces to develop.

In Section 4 it will be seen that this vortex-shedding phenomenon has a profound influence on the dynamic response of the tower. It also provides clues as to how the response might be reduced.

## 4. CALCULATION OF STRUCTURAL RESPONSE

### 4.1 Introduction

This section describes the application of the wind-tunnel model test results in determining the implied response of the prototype structure. This consists of two distinct operations; namely, to select a full-scale wind velocity and scale the model loads up to this prototype condition, and to then compute the structure's response to these loads. The detailed steps to be taken are as follows:

1. Select a design wind velocity corresponding to some mean recurrence interval, based on statistical wind data. This velocity is then converted to the reference wind velocity and pressure, as described in Section 1.2.
2. Determine the structure's natural frequency $f_{o}$, and its reduced natural frequency $f_{o} D / U$.
3. Compute the response base moment in the structure, using random vibration techniques.
4. Determine the generalized stiffness $\mathrm{k}^{*}$ and mode shape $\{\phi\}$ of the structure.
5. The rotation of the structure is computed as $\theta=M / k^{*}$. Individual floor displacements are then $\{x\}=\theta\{\phi\}$.
6. Stresses may be computed by imposing the calculated displacements, or alternatively from a static analysis using equivalent loads as in equation 4.8.
7. Compute the acceleration of the control cab floor.

### 4.2 Wind Data

Wind velocity data sets were obtained from two different sources at the EPIA site. These data sets, referred to as EPIA \#1 and EPIA \#2, are
given in Appendix C. They both are based on observations of hourly mean wind speeds in $\mathrm{m} / \mathrm{s}$ at a height of 10 m , and consist of the number of observations at various wind directions falling into various velocity ranges. A probability distribution has been calculated for each direction, as well as for all directions combined, for each of the two data sets.

These distributions were obtained from data sets which, under normal circumstances, would not be considered sufficiently consistent or complete to perform such an analysis. Each data record was only slightly over a year in length. The wind velocities from them should be considered as the best possible estimate of actual wind conditions, as opposed to a conservative estimate, which is normally required for design purposes. This may be acceptable for performance criteria, but would be inappropriate for strength design. The directionality effects are, therefore, not included in high winds such as those occurring every 50 or 100 years; these are therefore assumed equally likely to come from any direction. For lower velocities such as those occurring at least every 10 years, the distribution used was that from whichever data set resulted in the higher velocity for a given probability level.

The results thus obtained are given in Table 4.1 for representative probability levels corresponding to mean recurrence intervals of 1,10 , 50 , and 100 years. These velocities have been converted to mph and to the reference height of 675 ft , using the assumed velocity profile described in Section 2.1. It is of interest also to convert the data to fastest-mile velocities at a height of 33 ft , which is the form specified in U.S. Standards and building codes. Representative values are given in Table 4.2.

TABLE 4.1.
HOURLY MEAN WIND AT 675 FT REFERENCE HEIGHT IN MPH VS WIND DIRECTION FOR VARIOUS RECURRENCE INTERVALS*

| Wind Direction |  | Mean recurrence interval, yrs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Compass Pt | Degrees | , | 10 | 50 | 100 |
| N | 350, 0, 10 | 69.0 | 79.6 | 88.0 | 91.0 |
| NNE | 20, 30 | 64.0 | 72.1 | 88.0 | 91.0 |
| NE | 40, 50 | 39.9 | 44.4 | 88.0 | 91.0 |
| ENE | 60, 70 | 33.2 | 37.0 | 88.0 | 91.0 |
| E | 80, 90, 100 | 33.6 | 37.1 | 88.0 | 91.0 |
| ESE | 110, 120 | 39.9 | 46.7 | 88.0 | 91.0 |
| SE | 130, 140 | 38.6 | 44.0 | 88.0 | 91.0 |
| SSE | 150, 160 | 38.1 | 43.2 | 88.0 | 91.0 |
| S | 170, 180, 190 | 36.1 | 41.7 | 88.0 | 91.0 |
| SSW | 200, 210 | 29.3 | 34.4 | 88.0 | 91.0 |
| SW | 220, 230 | 28.3 | 33.5 | 88.0 | 91.0 |
| WSW | 240, 250 | 24.1 | 29.3 | 88.0 | 91.0 |
| W | 260, 270, 280 | 28.7 | 34.5 | 88.0 | 91.0 |
| WNW | 290, 300 | 30.3 | 35.2 | 88.0 | 91.0 |
| NW | 310, 320 | 38.9 | 44.8 | 88.0 | 91.0 |
| NNW | 330, 340 | 57.6 | 67.8 | 88.0 | 91.0 |
| All | 0-360 | 69.8 | 80.8 | 88.0 | 91.0 |

$\star$ Multiply by 0.655 for velocity at height of 33 ft .
Mulitply by 1.47 for velocity in fps .

TABLE 4.2
FASTEST-MILE AND HOURLY-MEAN WIND VELOCITIES FOR VARIOUS RECURRENCE INTERVALS, INDEPENDENT OF DIRECTION

| $\begin{gathered} \text { Elev. } \\ \text { ft. } \end{gathered}$ | Type of Meas. | Mean recurrence interval in years |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.1 | 1 | 10 | 50 | 100 | 350 |
| 33 | Fastest mile | 46 | 56 | 65 | 72.2 | 75 | 80 |
| 33 | hourly mean | 37.9 | 45.7 | 52.9 | 57.6 | 59.6 | 63.0 |
| 675 | hourly mean | 57.8 | 69.8 | 80.8 | 88.0 | 91.0 | 96.1 |

### 4.3 Dynamic Properties of Structure

The dynamic structural properties required to determine dynamic response include the generalized mass $m^{*}$, generalized stiffness $k^{*}$, natural frequency $f_{o}$, and critical damping ratio $\zeta$. The damping ratio is assumed to be 1 percent $(\zeta=0.01)$ except where noted (see further discussion in Section 5.4). The other values, referred to as "scheme 1 " properties, were determined from the following information supplied by Bechtel: natural frequency $f_{o}$, mass distribution $\{m\}$, and displacements $\{\delta\}$ computed for a horizontal load distribution as in Equation 12-3 of the Uniform Building Code [1]. This information is given in Table 4.3. Note that $z$ is the vertical axis of the structure so that $m_{i}$ is the mass lumped at height $z_{i}$, etc.

From the displacements $\{\delta\}$ a mode shape $\{\phi\}=\alpha\{\delta\}$ was determined which represents a "best fit" to a straight line mode shape $\{z\}$, according to the criterion

$$
\Sigma(\Delta z)_{i} z_{i}=\alpha \Sigma(\Delta z)_{i} \delta_{i}
$$

from which $\alpha$ may be determined. Using this mode shape, the generalized mass is

$$
\mathrm{m}^{*}=\Sigma \mathrm{m}_{\mathrm{i}} \phi_{\mathrm{i}}
$$

and the generalized stiffness is

$$
k^{*}=\left(2 \pi f_{o}\right)^{2} m^{*}
$$

These values are given in Table 4.3.
As a result of preliminary response calculations using these structural properties, which indicated a very high level of acceleration at the control cab floor level, and based on design charts previously provided (see Section 4.5), Bechtel began a series of modifications to the tower configuration. A total of 9 schemes were distinguished, which investigated various means of adding stiffness to the structure. Scheme 2 , however, involved the addition of one floor; this became an architectural requirement after the original design was formulated. These structural schemes are summarized in Table 4.4 (calculated values of $\mathrm{m}^{*}$, $k^{*}$, and $f_{o}$ provided by Bechtel).

Many of the schemes involve a widening of the tower pedestal, as indicated in the table. In schemes 4 through 7 only the lower half of the pedestal is widened, and this is assumed to have no effect on the applied wind loads. In schemes 8 and 9 the upper portion is also widened. The change is small, however, and it is assumed that the reduced loads--i.e., moment coefficients $C_{M}$ and reduced PSD fS(fre are not altered. The effect is accounted for when the reduced coefficients and frequencies are scaled to prototype values, a process which involves the reference width $D$. A small adjustment in the reference width has therefore been made in schemes 8 and 9 , based on the average width of the top third of the tower pedestal, as indicated in Table 4.4. It is noted that small deviations from the stated widths, should they be ultimately made, would have a negligible effect on the tower's response.

TABLE 4.3
CALCULATION OF MODE SHAPE, GENERALIZED MASS, AND GENERALIZED STIFFNESS (SCHEME 1)

| Elevation <br> $m m$ | $\mathbf{z}$ <br> in. | $\Delta z_{\mathbf{i}}$ <br> in. | $\delta_{\mathbf{i}}$ <br> $\mathbf{i n}$. | $\phi_{\mathbf{i}}=\alpha \delta_{\mathbf{i}}$ | $\mathbf{w}_{\mathbf{i}}$ <br> kips |
| :---: | ---: | :---: | :---: | :---: | ---: |
|  |  |  |  |  |  |
| 103530 | 3543 | 83 | 45 | 4838 | 174 |
| 99130 | 3377 | 103 | 39 | 4193 | 115 |
| 98110 | 3337 | 78.5 | 38 | 4085 | 55 |
| 95140 | 3220 | 78.5 | 36 | 3870 | 127 |
| 94120 | 3180 | 78.5 | 35 | 3762 | 53 |
| 91150 | 3063 | 111 | 33 | 3550 | 59 |
| 88490 | 2958 | 161 | 32 | 3440 | 66 |
| 82980 | 2741 | 213 | 28 | 3010 | 97 |
| 77660 | 2532 | 213 | 25 | 2690 | 106 |
| 72150 | 2315 | 213 | 22 | 2360 | 117 |
| 66830 | 2106 | 213 | 19 | 2040 | 124 |
| 61320 | 1889 | 213.5 | 16 | 1720 | 135 |
| 56000 | 1679 | 213.5 | 13 | 1400 | 142 |
| 50490 | 1462 | 213 | 11 | 1180 | 149 |
| 45170 | 1253 | 213 | 8 | 860 | 152 |
| 3960 | 1036 | 213.5 | 6 | 645 | 153 |
| 34340 | 826 | 213.5 | 4 | 430 | 154 |
| 28830 | 609 | 213 | 2.5 | 270 | 163 |
| 23510 | 400 | 213 | 1.2 | 130 | 169 |
| 18000 | 183 | 200 | 0.25 | 27 | 895 |
| 13350 | 0 | 91.5 | 0 | 0 | $-\ldots-$ |

$\Sigma 3543$
$\Sigma(\Delta z)_{i} z_{i}=6,276,000$ in. ${ }^{2}$
$\Sigma\left(\Delta z_{i}\right) \delta_{i}=58,357$ in. ${ }^{2}$
$\alpha=6,276,000 / 58,357=107.5$
Generalized Weight $\mathrm{w}^{*}=\Sigma \phi_{i}^{2} \mathrm{w}_{\mathrm{i}}=1.5108 \mathrm{E} 13 \mathrm{lb}-\mathrm{in} .{ }^{2}$
Generalized Mass $\mathrm{m}^{*}=\mathrm{W} / \mathrm{g}=1.5108 \mathrm{E} 13 / 386$

$$
=3.913 \mathrm{E} 10 \mathrm{lb}-\mathrm{in} . / \mathrm{sec}^{2}
$$

Natural frequency $f_{0}=0.6135 \mathrm{~Hz}$ (by Bechtel)
$\omega_{o}=2 \pi f_{o}=3.855 \mathrm{sec}^{-1}$
Generalized Stiffness $\mathrm{k}^{*}=w_{0}^{2} \mathrm{M}=5.814 \mathrm{E} 11 \mathrm{lb}-\mathrm{in}$.

TABLE 4.4.
DYNAMIC PROPERTIES OF PROTOTYPE TOWER SCHEMES

| Scheme <br> No. | Description |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Notes: 1. Out-to-out distance of opposite fins.
2. Assumed effective value is average width over top third of pedestal.
3. $D=22.31 \mathrm{ft}$ is diameter of column centers in scheme 1 ; $D$ for other schemes obtained by multiplying by ratio of effective pedestal width.
4. Modal displacement at control cab level. Where unknown, 349 ft is assumed.

### 4.4 Moment and Displacement Response (Equivalent Static Loads)

Analytical Procedure. The concept of load and response in a structure was introduced in Section 3.1, where each of these was expressed as the sum of mean and fluctuating parts:

$$
\begin{aligned}
& M=\bar{M}+M^{\prime} \\
& M=\bar{M}+M^{\prime}
\end{aligned}
$$

where $M$ is the applied base moment due to wind, and $M$ is the resulting internal (response) base moment. Of the mean values it is true that $\mathbb{M}=\bar{M}$, but generally ${ }^{\prime}>M^{\prime}$ due to resonance. The method of relating these, which is based on the principles of modal analysis and random vibration, will now be briefly described. Details of this subject are given in Appendix B.

In Section 4.3 it was seen that the structure's mode shape, $\{\phi\}$, was normalized to approximate the straight line $\{z\}$. The base moment $M$ is then the generalized load of the fundamental mode of response (eq. B.5). Moreover, since nearly all of the dynamic response is in the fundamental mode, $\mathrm{M}^{\dagger}$ is accepted as the generalized load corresponding to the fluctuating response $M^{\prime}$. Therefore, these quantities are linked by results of random vibration theory applicable to a single-degree-offreedom system. In particular, combining eqs. (B.7) and (B.9) provides a means of computing fluctuating rms response:

$$
\begin{equation*}
\sigma^{2}=\int_{0}^{\infty} S_{M}(f) d f=\int_{0}^{\infty}|H(f)|^{2} S_{M}(f) d f \tag{4.1}
\end{equation*}
$$

All of the essential dynamic properties of the structure are incorporated in $|H(f)|^{2}$, the mechanical admittance:

$$
|H(f)|^{2}=\frac{1}{\left[1-\left(f / f_{o}\right)^{2}\right]^{2}+\left(2 \zeta f / f_{o}\right)^{2}}
$$

If the integrand in eq. (4.1) is multiplied by $f$ and integrated with respect to $\ln f$ (which is permissible since $f \cdot d \ell n=d f$ ) and each side is divided by the square of the reference moment $q A L$, there results

$$
\left(\frac{\sigma_{M}}{q A L}\right)^{2}=\int_{0}^{\infty}|H(f)|^{2} \frac{f S_{M}(f)}{(q A L)^{2}} d(\ln f)
$$

or

$$
\left(C_{H}^{\prime}\right)_{r m s}^{2}=\int_{0}^{\infty}|H(f)|^{2} f S(f) \frac{d f}{f}
$$

where $\quad C_{M}$ is a response moment coefficient completely analogous to $C_{M}$, and $f S(f)$ is the reduced load PSD introduced in Section 3.3. If all frequencies $f$ in this equation are now expressed in terms of the reduced frequency $f_{r}=f D / U$, it becomes

$$
\begin{equation*}
\left(C_{M}^{\prime}\right)_{r m s}^{2}=\int_{o}^{\infty}\left|H\left(f_{r}\right)\right|^{2} f S\left(f_{r}\right) \frac{d f_{r}}{f_{r}} \tag{4.2}
\end{equation*}
$$

where

$$
\left|H\left(f_{r}\right)\right|^{2}=\frac{1}{\left[1-\left(f_{r} / f_{r o}\right)^{2}\right]^{2}+\left(2 \zeta f_{r} / f_{r o}\right)^{2}}
$$

and

$$
f_{r o}=\frac{f_{o} D}{U}
$$

is the reduced natural frequency. Equation (4.2) is now completely dimensionless, and specifies the procedure to compute the rms fluctuat-. ing response moment. In steps,

1. Select a reference wind velocity $U$ and compute the reduced natural frequency $f_{r o}=f_{o} D / U$.
2. Compute the mechanical admittance $\left|H\left(f_{r}\right)\right|^{2}$.
3. Multiply this by the reduce load PSD $f S\left(f_{r}\right)$.
4. Integrate the result with respect to $\ln f_{r}$. The square root of this result is $\left(C_{M}^{\prime}\right)_{r m s}$.
5. Multiply by the reference moment to obtain the rms fluctuating response:

$$
\begin{equation*}
\sigma_{M}=(q A L)\left(C_{M}^{\prime}\right)_{\mathrm{rms}} \tag{4.3}
\end{equation*}
$$

The peak response can now be computed using equation (B.10):

$$
\begin{equation*}
\hat{M}=\bar{M}+g_{p} \sigma_{M} \tag{4.4}
\end{equation*}
$$

where $\hat{M}$ is the expected peak moment, and $g_{p}$ is a peak factor defined by equation (B.11).

Approximate Analysis. When a structure is lightly damped and the load PSD is reasonably broad-band, eq. (4.1) may be approximated by the following well-known white-noise approximation:

$$
\sigma_{M}^{2}=S_{M}\left(f_{o}\right) \int_{o}^{\infty}|H(f)|^{2} d f
$$

The integration in this equation can be performed analytically, and results in

$$
\sigma_{M}^{2}=\frac{\pi}{4 \zeta} f_{o} S_{M}\left(f_{o}\right)
$$

Dividing this equation by the square of the reference moment and incorporating the reduced frequency leads to

$$
\begin{equation*}
\left(\mathrm{C}_{M}^{\prime}\right)_{\mathrm{rms}}=\sqrt{\frac{\pi}{4 \xi} \mathrm{fS}\left(\mathrm{f}_{\mathrm{ro}}\right)} \tag{4.5}
\end{equation*}
$$

This is an approximate alternative to eq. (4.2). Although eq. (4.2) was used to compute all of the results to follow, the approximation above provides much valuable insight. The fluctuating response is seen to be inversely proportional to the square root of the damping coefficient:

$$
\sigma_{M} \propto \zeta^{-1 / 2}
$$

Also, the fluctuating response is proportional to the square root of the reduced load spectrum evaluated at the structure's natural reduced frequency:

$$
\sigma_{M} \propto \sqrt{\mathrm{f}_{\mathrm{o}} \mathrm{~S}\left(\mathrm{f}_{\mathrm{ro}}\right)}
$$

For any given reduced load PSD $\mathrm{fS}\left(\mathrm{f}_{\mathrm{r}}\right)$, this relation shows immediately the effect of natural reduced frequency on the fluctuating response. Consider the $M y$ load of Fig. 3.2(b) for example, which shows the reduced load PSD typical of wind directions where vortex shedding occurs. The spectrum peaks at $f_{r} \approx 0.087$ and drops sharply on either side of this reduced frequency. Obviously the natural reduced frequency of the structure should be as different from this value as possible to keep the response low.

For a 100 -year recurrence wind, $\mathrm{U}=91 \mathrm{mph}=133 \mathrm{fps}$, and the natural reduced frequency for structural scheme 1 is

$$
f_{r o}=\frac{f_{o} D}{U}=\frac{(0.613 / \mathrm{sec})(22.31 \mathrm{ft})}{133 \mathrm{ft} / \mathrm{sec}}=0.103
$$

which is slightly higher than the reduced shedding frequency at the PSD peak. Evidently any increase in $f_{0}$, increase in $D$, or reduction in $U$ would be quite beneficial in lowering the response.* For structural scheme 2, the natural reduced frequency for a 100 -year wind is 0.084 . The response will be very high, and will probably peak at a somewhat lower wind velocity.*

[^1]When a large response is indicated due to coincidence or near coincidence of the structure's natural frequency and the vortex-shedding frequency, the motion of the structure is liable to affect the wind loading and thus modify the load PSD. This is known as a "lock-in" phenomenon, and can be described as two effects: 1) the frequency of the load PSD peak is shifted to coincide exactly with the structure's natural frequency, 2) the magnitude of the load PSD in the neighborhood of this peak is increased even further. These are qualitative effects, and no known theory is available to either predict or quantify the phenomenon.

For this study, it was assumed that lock-in can occur when the reduced natural frequency is within 10 percent of the reduced frequency of the load PSD peak. When this occurred, the entire load PSD was shifted in frequency, to account for the first effect described above, before applying eq. (4.2). The second effect cannot be accounted for, and the resulting response can only be interpreted as a lower limit to the actual probable response.

Calculation Results. Response base moments for structural scheme 1 and a 100 -year recurrence wind are given as a function of wind direction in Figure 4.1. The damping ratio in these results--as well as all successive results except where noted--is 1 percent of critical $(\zeta=0.01)$. Peak moments were computed according to eq. (4.4),

$$
\hat{M}=\bar{M}+g_{p} \sigma_{M} .
$$

Peak and mean values, $\hat{M}$ and $\bar{M}$, are shown in the figure. The value of the peak factor $g_{p}$ is identified as "PF".

It is observed that the dynamic response is highly dependent on wind direction and peaks at $60^{\circ}$ intervals centered at $30^{\circ}, 90^{\circ}, \ldots$


Figure 4.1. Response Base Moments as Function of Wind Direction (Scheme 1, 100-Yr Wind)
$270^{\circ}, 330^{\circ}$. Based on the discussion in the preceeding section, this is due to vortex schedding from the tower pedestal. It is also observed that the upwind terminal buildings tend to increase the peak moments somewhat. The highest moment is approximately $68,000 \mathrm{k}-\mathrm{ft}$. The reduced natural frequency for this condition is $f_{o} D / U=(0.613)(22.33) /(133$ $\mathrm{fps})=0.103$. This is sufficiently higher than the reduced frequency of vortex shedding (.087) to prevent lock-in.

Similar results are shown for structural scheme 2 in Figure 4.2. The recurrence interval here is only 50 years, but even at this lower wind velocity, the reduced natural frequency is $f_{o} D / U=(0.498)(22.31) /$ $(129 \mathrm{fps})=0.086$. This very nearly coincides with the shedding frequency, and the peak response is greatly increased. The largest base moment is now approximately $96,000 \mathrm{k}-\mathrm{ft}$, at wind direction $330^{\circ}$. Furthermore, vortex-shedding lock-in must be assumed to occur at all critical wind directions. As discussed in the preceding section, the indicated peaks can only be interpreted as lower limits; the actual peak could be significantly higher.

Figure 4.3 shows the response base moments vs wind direction for scheme 9, for a 50 -year wind. Results for damping ratios of both 0.005 and 0.010 are shown. The natural reduced frequency is $f_{o} D / U=(0.645)$ $(22.97) /(129)=0.115$. This is significantly higher than the shedding frequency, and lock-in will not occur. The highest moment is approximately $50,000 \mathrm{k}-\mathrm{ft}$ (at 0.01 damping). Figure 4.4 shows the same data for a 100 -year wind.

The effect of natural frequency and damping on the moment response is shown in the design charts of Figure 4.5. All of the data in this figure corresponds to a 50 -year recurrence wind velocity and wind



Figure 4.2. Response Base Moments as Function of Wind Direction (Scheme 2, $50-\mathrm{Yr}$ Wind) (Caution: see text before applying data)



Figure 4.3. Response Base Moments as Function of Wind Direction (Scheme 9, 50-Yr Wind)



Figure 4.4. Response Base Moments as Function of Wind Direction (Scheme 9, 100-Yr Wind)


Figure 4.5. Design Charts for Response Base Moments ( $50-\mathrm{Yr}$ Wind at $330^{\circ}$ )
direction $330^{\circ}$, which was selected as the most critical direction.* These results were obtained from the load PSD of run 42 (see Appendix D) using equations (4.2) and (4.4), having incorporated various values of $\zeta$ and $f_{r o}$ in the mechanical admittance $\left|H\left(f_{r}\right)\right|^{2}$.

The abscissae in these plots are actually reduced frequency $f_{o} D / U$, but are stated as a frequency $f_{o}$ by multiplying by the 50 -year reference wind velocity and dividing by an assumed constant reference width $D=22.31 \mathrm{ft}$. Vertical lines have been drawn on the plots at various values of $f_{o}$ corresponding to various structural schemes. For schemes 8 and 9 the actual width $D$ is greater than 22.31 ft (see Table 4.4) so a compensating adjustment is made in the natural frequency by multiplying it by the ratio $C=D /(22.31 \mathrm{ft})$. Similarly the moment (ordinate) $M$ was obtained from $C_{M}$ by assuming a constant reference moment $q D H L$. To compensate for the increase in $D$, the indicated moment must also be multiplied by the ratio $C$.

In scheme 8, for example, the reference width is 25.72 ft , so $C=25.72 / 22.31=1.15$. The effective natural frequency is $(0.654 \mathrm{~Hz})(1.15)=0.754 \mathrm{~Hz}$. At $\zeta=0.01$, the indicated $x$-moment is $38,000 \mathrm{k}$-ft. The actual x -moment is therefore $(38,000)(1.15)=$ $43,700 \mathrm{k}-\mathrm{ft}$. In scheme $9, \quad \mathrm{C}=22.97 / 22.31=1.03 ;$ effective $\mathrm{f}_{\mathrm{o}}=$ $(0.645 \mathrm{~Hz})(1.03)=0.664 \mathrm{~Hz}$; indicated $M_{x}$ at 1 percent damping $=$ $48,000 \mathrm{k}-\mathrm{ft}$; actual x -moment $=49,400 \mathrm{k}-\mathrm{ft}$.

The design charts show clearly that scheme 2 is a poor choice due to its natural frequency. A great improvement is achieved by increasing the natural frequency, and in some cases the width of the tower also.

The above procedure in which loads are adjusted by an effective reference width is approximate, and slightly conservative since some of the wind load is applied to the tower cab, which was not increased in width. This is particularly true for along-wind load, since then the cab accounts for a large part of the load. In the critical across-wind directions, however, nearly all of the load is due to vortex shedding on the tower pedestal, and the approximation is quite good.

There is more uncertainty in the procedure when applied to scheme 9, in which the pedestal fins are significantly tapered. There is some evidence that such tapering reduces the intensity of vortex-shedding, but the change in geometry is too severe for this to be quantitatively deduced from the wind-tunnel test data. It is believed that the application of an effective reference width as indicated in Table 4.4 is the most accurate means of assessing the response of scheme 9 available, while maintaining a degree of conservatism, without retesting the model.

Force Distribution with Height. If the total response (or static equivalent) force acting at the ith floor is expressed as the sum of a mean and a fluctuating component, i.e.,

$$
P_{i}=\bar{P}_{i}+P_{i}^{\prime}
$$

then, in a manner analogous to that used for the base moment in the previous section, the peak expected force may be written

$$
\begin{equation*}
\hat{P}_{i}=\bar{P}_{i}+g_{p} \sigma_{P_{i}} \tag{4.6}
\end{equation*}
$$

where $g_{p}$ is the same peak factor which was determined for the base moment. The distribution of the mean forces $\bar{P}_{i}$ cannot be determined exactly, but an effective means of estimating it will be described below. A means of estimating the rms fluctuating forces $\sigma_{P_{i}}$ will
first be described. Reference is made to Appendix B concerning modal analysis concepts.

Since the structure's motion is essentially in a normal mode, the fluctuating equivalent static load at floor $i$ is proportional to the mass and the modal deflection at that floor:

$$
P_{i}^{\prime}=\alpha m_{i} \phi_{i}
$$

These forces can be related to the base moment $M$, since

$$
M^{\prime}=\Sigma P_{i}^{\prime} z_{i}
$$

Substituting for $P_{i}^{\prime}$ leads to

$$
M^{\prime}=\alpha \Sigma m_{i} \phi_{i} z_{i} \approx \alpha m^{*}
$$

where $\mathrm{m}^{*}$ is the generalized mass (see Tables 4.3, 4.4). This allows the proportionality constant $\alpha$ to be evaluated, and the equation above for $P_{i}^{\prime}$ becomes

$$
P_{i}^{\prime}=\frac{M^{\prime}}{m^{\mp}} m_{i} z_{i}
$$

This equation shows that the individual fluctuating floor loads, $P_{i}^{\prime}$, may be determined from the fluctuating base moment, $M^{\prime}$. The rms forces can now be expressed as

$$
\begin{equation*}
\sigma_{\mathcal{P}_{i}}=\frac{\mathrm{m}_{\mathrm{i}} \mathrm{z}_{\mathrm{i}}}{\mathrm{~m}^{*}} \sigma_{M} \tag{4.7}
\end{equation*}
$$

A simple approximation for the peak forces $\hat{P}$ may be obtained by assuming that the mode shape $\phi_{i}$ approximates the static deflected shape. In this case the above analysis applies to the mean response in addition to the fluctuating part, and therefore also to the total response. Under this assumption $P_{i}$ and $M$ can be substituted for $P_{i}^{\prime}$ and $M^{\prime}$. The expected peak forces then become

$$
\begin{equation*}
\hat{P}_{i} \cong \frac{m_{i} z_{i}}{m^{*}} \hat{M} \tag{4.8}
\end{equation*}
$$

The expected peak base moment $\hat{M}$ may be read directly from Figures 4.1-4.5.

This approximation may not be good for wind directions where the mean response is a large fraction of the peak response, depending on how much the static deflected shape deviates from a straight line. Note, however, that in many cases the largest response occurs in the crosswind direction, where the mean response is very small; in these cases the approximation is excellent.

Displacements of the tower may be treated in a manner analogous to the analysis of forces in the preceding section. Thus the peak expected lateral deflection at floor $i$ is

$$
\hat{\Delta}_{i}=\bar{\Delta}_{i}+g_{p} \sigma_{\Delta}
$$

where $\bar{\Delta}$ is the mean static deflection, $\sigma_{\Delta}$ is the fluctuating rms deflection, and $g_{p}$ is the peak factor introduced above. The dynamic displacements are obtained from equation (B.2),

$$
\Delta_{i}^{\prime}=\theta^{\prime} \phi_{i}
$$

where $\theta^{\prime}$ is the fluctuating rotation of the approximate straight-line mode shape of the structure (thus $\theta^{\prime}$ approximates the generalized coordinate $\xi$ ). By Appendix B $\theta^{\prime}=M^{\prime} / k^{*}$, so

$$
\begin{equation*}
\Delta_{i}^{\prime}=\frac{M^{\prime}}{k^{*}} \phi_{i} \tag{4.9}
\end{equation*}
$$

Multiplying by the peak factor and taking the rms value results in

$$
g_{p} \sigma_{\Delta}=\frac{1}{k^{*}} g_{p} \sigma \phi_{i}
$$

The generalized stiffness $\mathrm{k}^{*}$ is given in Table 4.4, and the value $g_{p} \sigma$ may be read from Figures 4.1-4.5.

In the preceding section regarding force distributions, a simplifying assumption was made that the static deflected shape can be approximated by a straight line. This lead to the simple equation (4.8) for the peak expected force. Parallel treatment of the displacements leads to

$$
\begin{equation*}
\hat{\Delta}_{i}=\frac{\phi_{i}}{k^{\hbar}} \hat{M} \tag{4.10}
\end{equation*}
$$

The value $\hat{M}$ may be read directly from Figures 4.1-4.5. Note again that the accuracy of this approximation depends on how well the deflected shape can be fit by a straight line, and on the relative contributions of mean and fluctuating response to the total response. The equation is quite accurate for the cross-wind response, for example, where the mean response is zero.

The above procedure will be illustrated by the following example, applicable to scheme 9. The 50 -year wind from direction $330^{\circ}$, and a damping ratio of .01 , are considered. From either Figure 4.3 or 4.5, the peak $x$-moment is $\hat{M}_{x}=49,400 \mathrm{k}-\mathrm{ft}$. From Table 4.4, the generalized stiffness is $k^{*}=8.97 \mathrm{E} 10 \mathrm{lb}-\mathrm{ft}=8.97 \mathrm{E} 7 \mathrm{k}-\mathrm{ft}$, and the modal deflection at the control cab level is $\phi=349 \mathrm{ft}$. Substituting these values into eq. (4.10), the expected peak displacement of the control cab is

$$
\begin{aligned}
\hat{\Delta}_{\text {c.c. }} & =\frac{349 \mathrm{ft}}{8.97 \mathrm{E} 7 \mathrm{k}-\mathrm{ft}}(49,400 \mathrm{k}-\mathrm{ft}) \\
& =.192 \mathrm{ft}
\end{aligned}
$$

This displacement is due to rotation about the x-axis; i.e., it is in the $y$-direction. Simultaneously, $*$ the peak $y$-moment is $\hat{M}_{y}=47,000 \mathrm{k}-\mathrm{ft}$ and will result in a peak displacement in the $x$-direction of .183 ft .

FThe two moment (as well as displacement) components are essentially independent random variables, and to assume that each may reach its peak value simultaneously is slightly conservative.

### 4.5 Acceleration Response

Analytical Procedure. The theory by which the response base moment and deflection are calculated from a load PSD can also be used, with slight modification, to calculate the acceleration response. If the fluctuating displacement at any floor level, $\Delta_{i}^{\prime}$, is viewed in the frequency domain, then the component at frequency $f$ can be expressed as

$$
\Delta_{i}^{\prime}=\Delta_{i o} \sin (2 \pi f t+\psi)
$$

and the acceleration, obtained by differentiating twice with respect to time, is

$$
a_{i}=\ddot{\Delta}_{i}=-(2 \pi f)^{2} \Delta_{i}^{\prime}
$$

Using equation (4.9), this may be written

$$
\begin{equation*}
a_{i}=-(2 \pi f)^{2} \frac{\phi_{i}}{k^{*}} M \tag{4.11}
\end{equation*}
$$

This can be adapted to equation (4.1), with the result

$$
\begin{equation*}
\sigma_{a i}^{2}=\left(\frac{\phi_{i}}{k^{\frac{1}{*}}}\right)^{2} \int_{0}^{\infty}(2 \pi f)^{4}|H(f)|^{2} S_{M}(f) d f \tag{4.12}
\end{equation*}
$$

which is the general result required to compute the rms acceleration from the load PSD $S_{M}(f)$.

For application to model test results a nondimensional form of this equation is needed, which can be obtained with the following sequence of operations:

$$
\begin{aligned}
& \sigma_{a i}^{2}\left(\frac{k^{*}}{\phi_{i}}\right)^{2}\left(\frac{D}{U}\right)^{4}=\int_{0}^{\infty}\left(2 \pi \frac{\mathrm{fD}}{U}\right)^{4}|H(f)|^{2} \mathrm{fS}_{M}(f) \frac{d f}{f} \\
& \sigma_{a i}^{2} \frac{k^{*}{ }^{2} D^{4}}{\phi_{i}^{2} U^{4}(q A L)^{2}}=\int_{0}^{\infty}\left(2 \pi f_{r}\right)^{4}\left|H\left(f_{r}\right)\right|^{2} \frac{f_{M}(f)}{(q A L)^{2}} \frac{d f_{r}}{f_{r}}
\end{aligned}
$$

Finally,

$$
\begin{equation*}
\left(C_{a}\right)_{r m s}^{2}=\left(\frac{\sigma_{a i}}{a_{\text {iref }}}\right)^{2}=\int_{o}^{\infty}\left|H_{a}\left(f_{r}\right)\right|^{2} f S\left(f_{r}\right) \frac{d f_{r}}{f_{r}} \tag{4.12}
\end{equation*}
$$

where $C_{a}$ is the acceleration coefficient, the reference acceleration is

$$
a_{i r e f}=\left(\frac{q A L}{k^{*}}\right)\left(\frac{U}{D}\right)^{2} \phi_{i}
$$

and the reduced acceleration admittance is

$$
\left|H_{a}\left(f_{r}\right)\right|^{2}=\left(2 \pi f_{r}\right)^{4}\left|H\left(f_{r}\right)\right|^{2}
$$

Equation (4.12) is completely nondimensional and was used to obtain acceleration coefficients from the reduced load PSDs $f S\left(f_{r}\right)$, using various values of $\zeta$ and $f_{r o}$ in the acceleration admittance function. The acceleration at the control cab level was then obtained by multiplying by the reference acceleration, with $\phi_{i}$ evaluated at this level (Table 4.4). RMS acceleration components $\sigma_{a x}$ and $\sigma_{a y}$, due to rotation about the $x$ and $y$ axes, respectively, were computed separately. These were then vectorially combined to obtain the total resultant rms acceleration:

$$
\sigma_{a}=\sqrt{\sigma_{a x}^{2}+\sigma_{a y}^{2}}
$$

Approximate Analysis. For an intuitive understanding of the effects of load PSD, natural frequency, and damping on acceleration, it is desirable to develop an approximate relationship as was done for base moments. From equations (4.5) and (4.9),

$$
\begin{aligned}
\sigma_{\Delta_{i}} & =\frac{\phi_{i}}{k^{*}} \sigma_{M}=(q A L)\left(\frac{\phi_{i}}{k^{*}}\right)\left(C_{M}^{\prime}\right)_{r m s} \\
& =\left(\frac{q A L}{k^{*}}\right) \phi_{i} \sqrt{\frac{\pi}{4 \zeta} f_{o} S\left(f_{r o}\right)}
\end{aligned}
$$

If all of the motion is assumed to be at resonance, then $a_{i}=\left(2 \pi f_{0}\right)^{2} \Delta_{i}$, and

$$
\begin{aligned}
\sigma_{a i} & \cong\left(\frac{q A L}{k^{*}}\right) \phi_{i} f_{o}^{2} \sqrt{\frac{\pi}{4 \zeta} f_{o} S\left(f_{r o}\right)} \\
& \propto \frac{f_{o}^{2}}{k^{*}} \sigma_{M}
\end{aligned}
$$

This is the approximate alternative to equation (4.12). The acceleration response is thus seen to behave just as the base moment response, with two exceptions: 1) the base moment response must be multiplied by $f_{o}^{2}$, which tends to offset the benefit achieved by raising the natural frequency well above that of vortex shedding; 2) stiffness affects the acceleration directly in addition to its influence on the natural frequency.

Calculation Results. The response acceleration at the control cab level in structural scheme 1 , for a constant reference velocity of 69.8 mph is given as a function of wind direction in Figure 4.6. This velocity would represent a 1 -year mean recurrence interval if wind were assumed equally likely to come from any direction (Table 4.1). The acceleration is the rms value of the vector sum of the accelerations due to motion about both the $x$ and $y$ axes, expressed as a ratio of the gravitational acceleration $g$. The periodic increase in response at $60^{\circ}$ intervals in wind direction are even more apparent in acceleration than in base moment. This is because the individual components have been combined into a vector resultant, and because the mean acceleration is zero. An increase in response due to upwind terminal buildings is also readily apparent.

Based on this data, the following three runs have been selected for further investigation:


Figure 4.6. Acceleration vs Wind Direction, Scheme 1 (Approx. 1-Yr Wind Assumed Constant with Direction)

1. Wind direction $270^{\circ}$, Configuration $B$, with the terminal concourse upwind, which will be referred to as simply $270^{\circ}$. This appears to be the most critical wind direction.
2. Wind direction $270^{\circ}$, Configuration A, which can be interpreted as wind direction $30^{\circ}, 90^{\circ}, 120^{\circ}, \ldots 210^{\circ}$, and will hereafter be referred to as wind direction $90^{\circ}$. This is the most common situation which results in large accelerations.
3. Wind direction $360^{\circ}$, Configuration B. The presence of the upwind building increases the acceleration slightly. This data is referred to below as wind direction $0^{\circ}$, and is conservatively representative of wind directions $60^{\circ}, 120^{\circ}$, etc. Note that $0^{\circ}$ and $180^{\circ}$ are the most frequently occurring wind directions, and also result in minimum acceleration.

Additional results in the form of design charts for these three cases, and wind velocities of 69.8 and 80.8 mph (corresponding to 1 - and 10 -year recurrence intervals, if velocities are assumed independent of direction) are given in Figures 4.7, 4.8, and 4.9. These charts show the vector resultant rms acceleration plotted against the natural frequency of the structure $f_{o}$, for various values of generalized stiffness K. Each curve corresponds to a constant value of this stiffness, expressed in terms of the stiffness of scheme 1 , $K \emptyset$. The natural frequency $\mathrm{F} \emptyset$ on the abscissa is expressed as a ratio of the scheme 1 natural frequency of 0.613 Hz .

Note that a change in mass, with stiffness held fixed, is equivalent to a change in natural frequency by the ratio

$$
\frac{F \phi}{0.613}=\sqrt{\frac{M \phi}{M}}
$$



Figure 4.7. Design Chart for Acceleration, Wind Direction $270^{\circ}$ (Velocity Assumed Constant with Direction)


Figure 4.8. Design Chart for Acceleration, Wind Direction $90^{\circ}$ (Velocity Assumed Constant with Direction)


Figure 4.9. Design Chart for Acceleration, Wind Direction $0^{\circ}$ (Velocity Assumed Constant with Direction)
where $F \emptyset$ is the new natural frequency, $M$ the new generalized mass, and $M \emptyset$ the scheme 1 generalized mass. Thus the effect of changing the mass can be seen by tracing along a curve corresponding to the stiffness.

Changing the generalized stiffness and mass in the same proportion leaves the natural frequency unchanged; the effect of such a change can be seen by jumping vertically among the constant stiffness curves.

Finally, the effect of changing both stiffness and mass but not in the same proportion can be seen by jumping to the appropriate stiffness curve, and tracing along this curve to the new natural frequency.

EXAMPLE: The 10 -year recurrence acceleration at wind direction $0^{\circ}$ is .012 G (point 1 in Figure 6b). If $M$ is increased 25 percent, the frequency will change by $\sqrt{1 / 1.25}=.89$ and the acceleration becomes .0095 G (point 2). If instead K were increased by 25 percent, the frequency change is $\sqrt{1.25}=1.12$ and the new acceleration is .010 G (point 3). If both mass and stiffness were increased 25 percent, the frequency will not change, and the acceleration would be . 0095 G (point 4).

All of the preceding data assumes that wind of a given velocity is equally likely to come from any direction. For example, the interpretation of Figure 4.6 is that, once every year on the average, an acceleration of approximately 0.012 g will occur from 6 different wind directions (the acceleration at $270^{\circ}$ and $330^{\circ}$, as indicated by Configuration $B$, is higher due to the terminal buildings upwind).

Table 4.1 indicates that nearly all high winds occur within a narrow band of wind directions centered at $0^{\circ}$. The above data,
therefore, are overly conservative since the only response-sensitive wind directions likely to be excited are $30^{\circ}$ and $330^{\circ}$. This data has, therefore, been recomputed, taking into account the wind velocities of Table 4.1 and the natural frequency and stiffness corresponding to scheme 3. These results are shown in Figure 4.10. The interpretation of this plot is best illustrated by an example. Referring to the points plotted as squares, once every year on the average the following will occur: a north wind will produce an acceleration of from 0.0043 g to 0.0054 g (of which the larger should be assumed), an NNE wind will produce an acceleration of 0.0069 g , a NE wind will produce an acceleration of 0.0015 g , etc.

To arrive at a single recurrence interval for various acceleration levels, taking into account all wind directions, these data must be computed and combined in a somewhat different manner. For example, the recurrence interval of a 0.006 g acceleration is found as follows. First, wind-tunnel data from all significant wind directions is analyzed to determine the lowest wind velocity at each direction which would cause an acceleration of at least 0.006 g . Figure 4.10 indicates that only $10^{\circ}$ (representing N ), $20^{\circ}$ (representing NNE), and $330^{\circ}$ (representing NNW) need to be considered. Second, the wind velocity at each direction is substituted into the probability distribution of winds at that direction (see Appendix C), to find the probability of at least that high a wind occurring at that direction. Finally, these probabilities for all significant wind directions are numerically added to obtain the probability of the given acceleration at any wind direction. The reciprocal of this probability is the recurrence interval.


Figure 4.10. Acceleration vs Wind Direction (Scheme 3, Damping Ratio - . 01)

These calculations have been completed for a range of acceleration levels, critical damping ratios of 0.005 and 0.01 , and the structural properties of scheme 3. Results are shown in Figure 4.11(a). Similar results for structural scheme 9 are given in (b). As explained earlier, there is no conservatism in these results, as there is in the normal design sense, due to uncertainty in the wind velocity probability distributions. The actual accelerations are as likely to be higher than the indicated values as they are to be lower. A limited amount of these calculations have also been made for scheme 8. It was found that the mean recurrence interval of a 0.010 g rms acceleration is 19 years, at a damping ratio of 0.01 , or 2.25 years, if the damping ratio is 0.005 .

Key results of the above figures are summarized in Table 4.5.

TABLE 4.5
MEAN RECURRENCE INTERVAL OF VARIOUS ACCELERATION LEVELS FOR STRUCTURAL SCHEMES 3, 8, AND 9

| Damping <br> Coefficient | Acceleration | Scheme 3 | Scheme 8 | Scheme 9 |
| :---: | :---: | :--- | :---: | :--- |
| 0.005 | 0.004 g | 0.03 years | -- | 0.025 years |
|  | 0.010 | 0.6 | 2.25 | 0.9 |
| 0.01 | 0.004 | 0.07 | -- | 0.15 |
|  | 0.010 | 3.5 | 19 | 13 |


a) Scheme 3

b) Scheme 9

Figure 4.11. Acceleration vs Mean Recurrence Interval Including All Wind Directions (Schemes 3, 9)
5. HUMAN RESPONSE TO ACCELERATION

### 5.1 Literature Review

It is generally agreed that acceleration provides the best measure of possible human discomfort due to motion in tall structures; however, there is very little data available by which this issue can be judged quantitatively. Approximately five sources are available in the literature which address this problem: Chang [11], Chen and Robertson [12], Hansen et al. [13], Irwin [14], and the Canadian Building Code [3].

The paper by Chang is the earliest of these, and does not have the benefit of some later research. His suggestions, which are based on data extrapolated from aircraft industry tests conducted at a much higher frequency range than is directly applicable to tall structure vibration, may be summarized as follows: rms acceleration levels less than 0.0035 g are not perceptible, and levels from 0.0035 g to 0.011 g are at the threshold of perceptibility. The effects of the frequency and duration of the vibration are not identified, and no specific recommendation regarding recurrence intervals or the degree of perception or objection by the occupants is made.

Chang also quotes the results of experiments by Feld, who studied human response to vibration for application to the design of the World Trade Center. The design criteria adapted in that case was a limiting acceleration of 0.007 g not to exceed an occurrence rate of 12 times a year. Such a high recurrence rate implies that 0.007 g is being interpreted as the threshold of perception. These test data were probably obtained at vibration frequencies substantially lower than what will be experienced in the EPIA tower, and more recent research indicates that a lower threshold may be applicable to the EPIA tower.

Chang also reports some interesting observations on the Chicago John Hancock Tower: that common motion is probably at the threshold of perception, that 75 percent of the occupants have been aware of motion, and "almost all persons had adapted to and come to accept sway."

Chen and Robertson reported on a sophisticated series of tests designed to determine the threshold of perception of motion of persons in a realistic office tower environment. Their results are summarized in Table 5.1. It is cautioned, however, that all of Chen and Robertson's data were collected at frequencies ranging from 0.05 to 0.2 Hz ; the above extrapolation to 0.6 Hz may not be valid but is probably more realistic than Chang's results.

TABLE 5.1
ROOT-MEAN-SQUARE ACCELERATION LIMITS OF PERCEPTION AS A RATIO OF GRAVITATIONAL ACCELERATION [12]

| Frequency | 98th \%ile | 90th \%ile | 50th \%ile | 10th \%ile | 2nd \%ile |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.05 Hz | 0.0120 g | 0.0088 g | 0.0052 g | 0.0031 g | 0.0022 g |
| 0.10 | 0.0155 | 0.0113 | 0.0067 | 0.0039 | 0.0029 |
| 0.15 | 0.0118 | 0.0086 | 0.0051 | 0.0030 | 0.0022 |
| 0.20 | 0.0096 | 0.0070 | 0.0041 | 0.0024 | 0.0018 |
| 0.30 | 0.0075 | 0.0055 | 0.0032 | 0.0019 | 0.0014 |
| 0.40 | 0.0065 | 0.0048 | 0.0028 | 0.0017 | 0.0012 |
| 0.50 | 0.0060 | 0.0044 | 0.0026 | 0.0015 | 0.0011 |
| 0.60 | 0.0056 | 0.0041 | 0.0024 | 0.0014 | 0.0010 |

Note: nth percentile is that level of acceleration which can be sensed by $\mathrm{n} \%$ of the population.

Hansen et al. conducted a research program involving acceleration measurements on two tall buildings during one windstorm each, and subsequent interviews of the occupants of the top one-third of each building. It was then determined what percentage of the occupants would "object to" the observed level of motion if it occurred at various recurrence intervals. The acceleration levels involved were 0.002 g and 0.005 g
(rms), and only these two motion intensities could be analyzed. Their results are summarized in Figure 5.1.

A key feature of Hansen's interpretation is that a person's objection depends not only on the level of acceleration, but also on the recurrence interval of that acceleration. For any given acceleration level, there is a continuous function associating levels of objection with recurrence intervals. Figure 5.1 is two such functions Hansen found for acceleration levels of 0.002 g and 0.005 g . Note that under this interpretation a question such as "How many times a year will $\mathrm{X} \%$ of the people object?" is irrational; the appropriate question is simply "What percentage of the people will object?"

Their suggested performance criteria is that no more than 2 percent of the (top one-third of the) building occupants should object to the motion. This criteria may be met by various combinations of acceleration/recurrence interval combinations, one of which is an acceleration of 0.005 g rms every six years. Another possible combination would be 0.002 g rms every (approximately) 1.5 years.

Irwin has reviewed a large amount of data regarding many aspects of human response to vibration, and included frequency of motion as a variable which strongly affects this response. Concerning normal activities in tall buildings, his suggested criteria (and interpretation thereof) is the same as that proposed by Hansen et al., except that allowances are made for a much broader frequency range. For example, his suggested maximum rms acceleration for a 5-year recurrence interval--that which will produce a 2 percent objection level--is 0.005 g at a frequency of 0.2 Hz , and decreases to 0.0033 at 0.6 Hz and 0.0028 g at frequencies 1 Hz and higher. He also presents an application to offshore drilling platforms which suggests that trained personnel can perform routine


Figure 5.1. Human Objection vs Mean Recurrence Interval for Two Levels of Acceleration Based on Actual Field Studies (From Hansen, Reed, and Van Marcke, "Human Response to Wind-Induced Motion of Buildings," Journal of the Structural Division, ASCE, July 1978, pp. 1589-1605)
skilled tasks adequately in such an environment at much higher motion levels. Irwin's suggested design acceleration for this case is in fact six times higher than that for ordinary tall buildings. This increased allowance follows the notion that a person's tolerance for motion can be increased if he is trained to expect it. This allowance is perhaps not applied to normal tall buildings since the lay person who inhabits it normally believes that buildings should not move at all.

Finally, the National Building Code of Canada (NBC) suggests that the perception threshold of rms acceleration is from 0.0035 g to 0.011 g , and that suitable design limits for a 10 -year recurrence interval are about this same amount*. The lower limit is suggested to apply to residential buildings and the upper limit to office buildings. This seems to be in agreement with the previously-discussed notion that trained personnel performing skilled tasks have a higher tolerance level.

This NBC criterion appears to be more permissive than others described above for building application; however, the following should be noted: the NBC provisions used to estimate acceleration in structures assumes that wind comes from the least favorable (most critical) direction, and so the limiting criteria must also be applied at that wind direction.

All of the human-response data found in the literature, discussed above, is conveniently summarized in Figure 5.2.

### 5.2 Adaptation of Criteria to EPIA Control Tower

In contrast to the rest of this program, acceleration limits are not a straightforward engineering problem for which a right or wrong

FActual values stated in the code are higher, and represent peak accelerations. Suitable allowance has been made to convert them to rms values.


Figure 5.2. Summary of Data Describing Human Response to Acceleration and Proposed Performance Standards (see text for complete references)
solution exists. Criteria suggested in the literature range from a "low" of 0.002 g every 1.5 years, or 0.005 g every 6 years (Hansen et al., and agreed with by Irwin for buildings), to a "high" of 0.011 g every 10 years (NBC Canada). Adoption of any of these proposals is an issue of subjective judgement, and may also be subject to the economics and feasibility of the particular situation. Thus, an ideal structure should meet Hansen's criteria, but if this cannot be met or can be met only with extreme costs, then higher motion levels can be accepted. This latitude exists because the issue is one of performance, and not safety.

Without knowledge of these issues regarding the control tower, it is suggested that a subjective decision be made as follows. Hansen et al. have developed the most rational method of specifying a limit, i.e., the concept of allowing many different acceleration level/recurrence interval combinations. However, their adaption of a 2 percent objection level is arbitrary and quite possibly too low, at least for other than residential buildings. Irwin is in agreement with them for residential buildings, but suggests allowable accelerations six times higher for active working environments (e.g., off-shore drilling platforms or bridge maintenance). This is probably too high for the control tower environment. The Canadian Building Code suggests that office buildings can accept three times the acceleration of a residential building. Therefore, a suitable limit might be three times that suggested by Hansen. His data, however, was obtained from buildings with a substantially lower frequency than the control tower, and both Irwin and Chen and Robertson indicated that an adjustment to a lower acceleration limit be made to account for the higher frequency. This
can be done based on Irwin's data, which implies a reduction factor of 0.67 , (assuming a natural frequency of 0.6 Hz ). The net result of these two factors (3 for a more liberal motion tolerance, and $2 / 3$ for the increased frequency) is that Hansen's acceleration data can be numerically doubled, while maintaining the same recurrence rates and objection levels.

For convenience, Hansen's results of Figure 5.1 are repeated as Figure 5.3, with the acceleration levels modified as indicated above.

It is suggested that the upper curve, corresponding to an acceleration of .010 g , is a better measure of motion objection in the EPIA tower than is the lower curve. The lower curve, representing .004 g acceleration, is based on data originally related to . 002 g acceleration, which is essentially the threshold of perception. The objection levels as determined by Hansen were based on the concept that many persons will object to motion if they can perceive it at all. After becoming accustomed to motion, however, it is likely that objection to motion will not occur until accelerations are well above the perception level.

### 5.3 Evaluation of Human Response in EPIA Control Tower

Scheme 1. Although acceleration levels vs recurrence interval for all wind directions have not been computed, an adequate estimate can be obtained from Figure 4.6. This indicates that, at a reference wind velocity of 69.8 mph , the acceleration would be 0.012 to 0.013 g at wind directions $30^{\circ}, 90^{\circ}, 150^{\circ}$, and $210^{\circ}$; it would be 0.015 to 0.017 g at directions $270^{\circ}$ and $330^{\circ}$. If wind directionality statistics were not taken into account, the mean recurrence interval of each of these events would be 1 year (Table 4.1). In other words, the acceleration would
exceed 0.012 g on the average of 6 times per year. This is far above any of the proposed criteria summarized in Figure 5.2. According to Hansen's modified data of Figure 5.3, an objection level of at least 50 percent of the occupants would be expected.

When the wind direction is considered as indicated in Table 4.1, the high accelerations indicated above would be likely to occur only at wind directions $30^{\circ}$ and $330^{\circ}$ every year, and even at these wind directions the intensity would be reduced due to slightly lower wind velocities.

Scheme 2. Due to an architectural requirement significant mass was added to the tower cab. The resulting scheme 2 had such a low natural frequency that vortex-shedding interacted severly with the dynamic response of the tower and increased it dramatically, as described in Section 4.4. The lock-in phenomenon is assumed to exist, and the response cannot be quantitatively evaluated using the methods of this program.

Scheme 3. This scheme was an attempt to maximize the natural frequency within the architectural limits of the original pedestal design. The frequency achieved, 0.546 Hz , is just high enough to prevent vortex-shedding lock-in. The acceleration response was calculated, taking into account wind directional effects, in Figure 4.11(a). Using this data in conjucntion with Figure 5.3 allows the level of objection to be estimated, based on an rms acceleration of 0.010 g . From Figure $4.11(a)$, with 1 percent damping, this will occur every 3.5 years. Referring now to Figure 5.3, the level of objection would be 3.5 percent. If the damping ratio were more conservatively taken as 0.005 , the recurrence interval would be 1 year, and about 11 percent of the


Figure 5.3. Hansen's Acceleration Response Data Adapted for Application to the EPIA Control Tower
population would object. These objection levels may not seem abnormally high; however, it should be remembered that, by multiplying Hansen's acceleration levels by a factor of 3 , it is assumed that the tower occupants are accustomed to working in that environment, they understand that such motion levels are a normal occurrence, and have come to expect such motion.

The data of Figure $4.11(\mathrm{a})$ is repeated as Figure $5.4(\mathrm{a})$, in which acceleration criteria of Sections 5.1 and 5.3 have been superimposed. Some response measures are summarized in Table 5.2 for comparison with schemes 8 and 9.

Scheme 8. Limited analyses have been performed on scheme 8, using the method described above. At a damping ratio of 0.01 , the recurrence interval of 19 years ( 0.010 g rms acceleration) falls outside the range of Hansen's data; extrapolation leads to an objection level of 0.5 percent. At a damping ratio of 0.005 , the recurrence interval is 2.25 years, and the level of objection is about 5 percent. The high natural frequency, high stiffness, and increased reference width all contribute to the reduction in acceleration.

Scheme 9. Complete acceleration analysis has been performed on scheme 9, as was described previously for scheme 3. The results were in Figure 4.11(b), which is repeated here as Figure $5.4(\mathrm{~b})$ with acceleration criteria superimposed. Some results are also summarized in Table 5.2.

### 5.4 Modifications to Reduce Response

Structural Properties. The approximate functional relationship between rms acceleration, natural frequency, stiffness, and loading PSD was derived in equation (4.13):


b) Scheme 9

Figure 5.4. Control Tower Acceleration Compared to Levels of Motion Perception and Objection (Schemes 3, 9)

TABLE 5.2
SUMMARY OF HUMAN RESPONSE MEASURES IN SCHEMES 3, 8, AND 9

| Measure of human response |  | Damping ratio 0.005 |  |  | Damping ratio 0.010 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \text { Scheme } \\ 3 \end{gathered}$ | $\begin{gathered} \text { Scheme } \\ 8 \end{gathered}$ | $\begin{aligned} & \text { Scheme } \\ & 9 \end{aligned}$ | Scheme 3 | $\begin{aligned} & \text { Scheme } \\ & 8 \end{aligned}$ | Scheme |
| Mean recurrence interval of motion perceivable by various \% of occupants | 50\% | always | - | 3 days | 1 wk | - | 1 wk |
|  | 90\% | 11 days | - | 11 days | 1 mo | - | 2 mo |
|  | 98\% | 18 days | - | 1 mo | 3 mo | - | 6 mo |
| Percent of occupants objecting based on 0.004 g acceleration (Figure 5.3)* |  | 80\%+ | - | 80\%+ | 60\% | - | 30\% |
| Percent of occupants objecting based on 0.010 g acceleration (Figure 5.3)* |  | 17\% | 5\% | 12\% | 3.5\% | 0.5\% | 1\% |

$$
\sigma_{\mathrm{a}} \propto \frac{\mathrm{f}_{\mathrm{o}}^{2}}{\mathrm{k}^{*}} \sqrt{\frac{\pi}{4 \zeta} \mathrm{f}_{\mathrm{o}} \mathrm{~S}\left(\mathrm{f}_{\mathrm{ro}}\right)}
$$

Based on this equation, the following would have a beneficial effect on the acceleration:

1. Increase the natural reduced frequency $f_{r o}=f_{o} D / U$, by increasing $f_{o}$ or $D$ (see Section 4.4, Approximate Analysis).
2. Reduce the natural frequency $f_{0}$.
3. Increase the generalized stiffness $k^{*}$.
4. Increase the damping ratio $\zeta$.

Items 1 and 2 are contradictory in the requirement on $f_{0}$; the actual influence must be determined by calculation. Representative results were shown in the design charts of Figures 4.7-4.9. Figures 4.7 and 4.8 , at wind directions $270^{\circ}$ and $90^{\circ}$, are also representative of $330^{\circ}$ and $30^{\circ}$, and therefore also of the total acceleration. These figures indicate the optimum natural frequency to be about 0.61 to 0.70 Hz .

If $f_{r o}$ is to be increased, it is evidently more beneficial to do so by increasing $D$ rather than $f_{o}$. This is in part why scheme 8 shows the lowest acceleration response.

Increasing the stiffness is clearly the most effective means of reducing acceleration. This will act directly in inverse proportion, and also indirectly through its effect on the natural frequency. The generalized mass $m^{*}$ can be varied to help optimize $f_{o}$ independently of $\mathrm{k}^{*}$. Further explanation can be found in Section 4.5.

Apparently scheme 8 or 9 represents the highest stiffness available within acceptable architectural limits and the structural system of a braced frame. Additional stiffness could possibly be achieved by using
a shell structure, or one having a structural skin of steel or concrete in place of the present architectural aluminum.

Structural damping was discussed in Section 4.3. It would be beneficial to use bolted connections in highly stressed areas, rather than welded connections. The benefit achieved is difficult to quantify, but it might be appropriate to assume a damping ratio of $\zeta=0.01$, rather than 0.005 . An even higher value could be assumed if concrete (not prestressed or post-tensioned) were used along with the steel frame.

Mechanical Hardware. Damping can probably be increased substantially by adding energy-dissipating devices within the structure. This type of scheme was incorporated in the World Trade Center Towers [15].

Tuned-mass dampers are a recent innovative approach to motion reduction in tall structures. Such devices have been installed in the John Hancock Tower in Boston [16] and the Citicorp building in New York City [17]. Technical information can be found in references 16-21.

The analysis or design of these devices is beyond the scope of the present project.

Architectural Modifications. Much of the response motion in the EPIA control tower is induced by an abnormally high loading PSD, which is the result of vortex shedding. This shedding reaches a high intensity because of the long uninterrupted architectural lines of the slender pedestal. The loading could probably be reduced by suitable architectural modifications of the nature of those shown schematically in Figure 5.5. Interruption of vortex formation is the principle of schemes a), b), and c) in this figure. Similar devices are widely used


Figure 5.5(a). Reduction of Dynamic Response by Architectural Modification


Figure 5.5(b). Reduction of Dynamic Response by Architectural Modification


Figure 5.5(c). Reduction of Dynamic Response by Architectural Modification


Figure 5.5(d). Reduction of Dynamic Response by Architectural Modification


Figure 5.5(e). Reduction of Dynamic Response by Architectural Modification
on tall stacks for this same purpose [22]. Such devices will reduce the fluctuating loads (e.g., acceleration, cross-wind moment) but increase the mean load. Perforating the cladding, as in d), should reduce all wind loads. Unfortunately, these schemes cannot be quantitatively evaluated from the test data obtained on the original tower geometry.

Increasing the width of the pedestal, as in Figure 5.5(e), is an effective means of increasing the natural reduced frequency $f_{\text {ro }}$ of the tower, as described above. This method was incorporated in structural scheme 8. Vortex shedding will still occur on this tower, but at lower frequency than on other schemes. Since the shedding frequency is always less than the natural frequency, lowering the shedding frequency results in less resonant amplification in the response.

## 6. SUMMARY

A wind-tunnel study has been performed on a model of the EPIA control tower, for the purpose of evaluating its dynamic response and effective wind load. Because the response was expected to be excessive and structural modifications were anticipated, a dynamic study was conducted in which the power spectral density of the externally-applied wind load could be determined. The tower response was then calculated from the load data, using various values of mass, stiffness, and damping. A two-degree-of-freedom system was assumed, incorporating the fundamental modes of response about orthogonal axes at the base of the tower. Lumped-mass properties were converted to generalized modal properties for this purpose using modal analysis.

Wind records were obtained from the EPIA site and statistically analyzed to obtain rational design wind speeds. The dependency of wind speed with direction was considered for the evaluation of the tower performance, but not for strength design.

The tower response was computed in the form of equivalent static loads, and rms acceleration at the control cab level. Design charts were prepared to show the effect of changes in structural properties on the response.

From the wind loading power spectral densities, vortex shedding was identified as a major contributor to the high response of the tower. Guidelines were established to keep the tower's natural frequency high to prevent lock-in from occurring.

A significant weight was subsequently added to the tower control cab, resulting in the Scheme 2 structure. The natural frequency of this scheme was such that vortex shedding could lock in, significantly
increasing response. Only a non-conservative estimate of the lower bound of acceleration could be made, which showed excessive response. Additional structural schemes explored various methods of increasing the natural frequency to prevent lock-in and reduce the level of response. Schemes 3,8 , and 9 were evaluated in terms of the design base moment for strength, and acceleration response for performance. The equivalent static base moment for a 50 -year wind in these schemes is $110,000 \mathrm{k}-\mathrm{ft}, 43,700 \mathrm{k}-\mathrm{ft}$, and $49,400 \mathrm{k}-\mathrm{ft}$, respectively (1 percent damping). The mean recurrence interval of a .010 g rms acceleration level is 3.5 years, 19 years, and 13 years, respectively ( 1 percent damping).

Existing literature was reviewed to obtain data regarding human response to acceleration in a tall building environment. These data were investigated for their consistency and adapted for application to the EPIA tower. Comparison of these results to the calculated response acceleration enabled estimates to be made of a) recurrence intervals at which motion will be perceptible to the tower occupants, and b) overall levels of objection to motion.

Fifty percent of the occupants should be able to perceive motion, on the average, every 7 days in schemes 3 and 9 . Virtually all of the occupants will perceive motion every 3 months in scheme 3, and every 6 months in scheme 9. Overall levels of objection, based on an acceleration level of .010 g and a reasonable interpretation of published criteria, are 3.5 percent, 0.5 percent, and 1 percent of the occupants in chemes 3, 8, and 9, respectively. On this basis, schemes 8 and 9 appear to be satisfactory in terms of performance.

The acceleration response of all 9 structural schemes would be considered excessive or marginal at best, if a strict application of published human response criteria were made. The criteria are generally not directly applicable, however, since they pertain to frequency ranges related to taller structures, or untrained personnel unaccustomed to motion. Modifications were made to the published criteria to make them applcable to the EPIA control tower.

Suggestions and guidelines were established by which the tower's response could be further reduced, if desired.

## REFERENCES

1. Uniform Building Code, Part V, Chapter 23, International Conference of Building Officials, Whittier, CA, 1982.
2. American National Standards Institute, "American National Standard Building Code Requirements for Minimum Design Loads in Buildings and Other Structures," ANSI Standard A58.1, 1982.
3. National Building Code of Canada, Part 4 - Design, National Research Council of Canada, Ot tawa, 1980.
4. Simiu, Emil, "Modern Developments in Wind Engineering: Part 4," Engineering Structures, V. 5, October, 1983, pp. 273-281.
5. Vickery, B. J., "Notes on Wind Forces on Tall Building," Annex to Australiani Standard 1170, Part 2-1973, SAA Loading Code, Part 2 Wind Forces, Standards Association of Australia, Sydney, $19 \overline{7} 3$.
6. Chermak, J. E., "Laboratory Simulation of the Atmospheric Boundary Layer," AIAA Jl., Vol. 9, September 1971.
7. Cermak, J. E., "Applications of Fluid Mechanics to Wind Engineering," A Freeman Scholar Lecture, Journal of Fluid Engineering, ASME, Vol. 97, No. 1, March 1975.
8. Cermak, J. E., "Aerodynamics of Buildings," Annual Review of Fluid Mechanics, Vol. 8, 1976, pp. 75-106.
9. "Wind Forces on Structures," Trans. ASCE, V. 126, 1961, pp. 11241198.
10. Bendat, J. S., and Piersol, A. G., Random Data: Analysis and Measurement Procedures, Wiley-Intersciece, New York, 1971, pp. 314329.
11. Chang, F. K., "Human Response to Motions in Tall Buildings," Journal of the Structural Division, ASCE, Vol. 99, No. ST6, June, 1973, pp. 1259-1272.
12. Chem, P. W., and Robertson, L. E., "Human Perception Thresholds of Horizontal Motion," Journal of the Structural Division, ASCE, Vol. 98, No. ST8, Proc. Paper 9142, Aug., 1972, pp. 1681-1695.
13. Hansen, R. J., Reed, J. W., and Van Marcke, E. H., "Human Response to Wind-Induced Motion of Buildings," Journal of the Structural Division, Vol. 99, No. ST7, July, 1973, pp. 1589-1605.
14. Irwin, A. W., "Human Response to Dynamic Motion of Structures," The Structural Engineer, Vol. 56A, No. 9, Sept., 1978, pp. 237-244.
15. Feld, L. S., "Superstructure for $1350-\mathrm{ft}$ World Trade Center," Civil Engineering, ASCE, June, 1971, pp. 66-70.
16. "Hancock Tower Now to Get Dampers," Engng News Record, 30 October 1975, 11.
17. Isyumov, N., Holmes, J., and Davenport, A. G. "A Study of Wind Effects for the First National City Corporation Project - New York, USA, University of Western Ontario Research Report BLWT-551-75, London, Ontario, Canada, 1975.
18. Wiesner, K. B., "Tuned Mass Dampers to Reduce Building Wind Motion," Preprint 3510, ASCE Convention and Exposition, Boston, 2-6 April 1979.
19. McNamara, R. J., "Tuned Mass Dampers for Buildings, J. Struct. Div., ASCE, 1977, Vol. 13, No. ST9, Proc. Paper No. 13200, pp. 1785-1798.
20. Luft, R. W., "Optimal Tuned Mass Dampers for Buildings," J. Struct. Div., ASCE, Proc. Paper 15017, December 1979, pp. 2766-2772.
21. "Tuned Mass Dampers Sway Skyscrpaers in Wind," Engng. New Record, 18 August 1977, pp. 28-29.
22. Simiu, E., and R. H. Scanlan, Wind Effects on Structures: An Introduction to Wind Engineering, John Wiley $\overline{\&}$ Sons, New York, 1978.

## APPENDIX A

## THEORY OF THE DYNAMIC WIND TUNNEL MODEL

The traditional method of evaluating a tall building's response to wind loading is to use an aeroelastic model in a boundary-layer wind tunnel capable of modeling the atmospheric boundary layer. Such a test may be viewed as a physical analog of the governing equation, which in simplified form is

$$
m \ddot{y}+c \dot{y}+k y=f(t)
$$

The aeroelastic model is scaled to the prototype, built, and "tuned" to proper values of the mass $m$, damping coefficient $c$, and stiffness $k$. The wind tunnel environment is configured to provide a properly scaled external loading $f(t)$. This loading is a combination of the turbulent wind velocity and its interaction with the model. Only the approaching velocity and tubulence distribution is measured and known: the proper external model load $f(t)$ exists but cannot be determined directly. The resulting motion of the model corresponds to $y$, the solution of the differential equation.

It is important to understand that the test result is the solution to a complete differential equation which corresponds to particular values of mass, stiffness, and damping. If any of these parameters are changed, the model must be adjusted accordingly and the test repeated.

The governing differential equation would be simple to solve analytically if the external loading $f(t)$ were known. For a large class of structures this loading will be independent of the parameters on the left-hand-side of the equation and will be determined by the wind velocity field in conjunction with the building's geometry--i.e., it
will be independent of the building's motion. Thus one is led to seek a method of measuring the external loading term $f(t)$ directly on a suitable model in the wind tunnel, after which the response $y$ may be calculated for any desired combination of mass, stiffness, or damping.

This type of model will be referred to as a dynamic--as opposed to aeroelastic--model. The model itself is rigid and geometrically similar to the prototype. Unlike the aeroelastic model, however, it is mounted on a very stiff base moment balance which allows negligibly small rotations about the base to occur. Stated another way, the balance/model system has a very high natural frequency compared to that of aeroelastic balance/model system (which is intentionally made equal to the scaled natural frequency of the prototype). The balance is instrumented to measure the base moment on the model, and because of its high natural frequency, this may be considered to be the dynamic loading due to wind. It also turns out to be, very nearly, the proper form for use in the governing equation (for a structure with a linear mode shape, it is exactly the required form). The bandwidth of the system is from 0 Hz up to about one-half of its natural frequency.

A method of solving the governing differential equation is available by considering the system in the frequency, rather than time, domain, as in the attached figure. This solution technique also provides valuable insight into loading and response mechanisms. Part a) shows an abstract representation of a general linear system as a "black box" having an input $x(t)$ and output $y(t)$. The box is completely characterized by its complex frequency response $H(f)$. When the input (and therefore the output also) are random functions, a simple input/ output relationship is


Schematic Representation of Wind Loading on a Tall Structure as a Linear System in the Frequency Domain
a) General linear system
b) Structure/wind interaction as a simple system
c) Separation into two systems; one empirical and one analytical

$$
S_{y}(f)=|H(f)|^{2} \cdot S_{x}(f)
$$

where $S_{x}$ and $S_{y}$ are the power spectral densities of the input and output, and $|H(f)|^{2}$ is a real function known as a "transfer" or "admittance" function. A statistical description of the output $y(t)$ is available through its variance, $\sigma_{y}^{2}$, which can be easily computed from the spectrum $S_{y}(f)$.

A similar system in the context of wind loading on a tall building is shown in b). The input is the wind velocity $u(z, t)$, now a function of height as well as time. The output $y(z, t)$ is also a function of height; various response parameters such as moment, displacement, or acceleration may be interpreted as the response. An appropriate form of $H(f)$ corresponds to the desired response parameter. This function is very complicated and unknown in general, as it represents effects distributed over the height of the structure, and the (poorly understood) mechanism by which fluid exerts pressure on a body of arbitrary shape as it flows around it.

It is convenient, however, to separate this function into two parts, one of which contains the complicated or unknown processes, and another which is amenable to analytical treatment. This is shown schematically in part c) of the figure where we now have two general linear systems. The first is characterized by an "aerodynamic admittance," which transforms the wind velocity field into an equivalent scalar load $F(t)$. This is the "generalized load" as used in modal analysis, defined as

$$
F(t)=\int_{0}^{h} q(z, t) \phi(a) d z
$$

where $q(z, t)$ is the pressure distribution due to wind, and $\phi(z)$ is the mode shape. The fundamental mode shape is nearly linear, i.e.,
$\phi(z) \approx z$, so the generalized load is approximately the same as the base moment. This is directly measurable in the dynamic model wind tunnel test.

The second linear system, for which the generalized load is the input, is called "mechanical admittance." Its governing equation, in the time domain, is

$$
M \ddot{\xi}+C \xi+K \xi=F(t)
$$

where now $M, C$, and $K$ are the generalized mass, damping, and stiffness, respectively, of the fundamental mode. These are all known ( $C$ must be assumed). Since $F(t)$ has been measured in the dynamic model test, this equation can be solved analytically for the "generalized response" $\xi$.

The solution is most conveniently performed in the frequency domain, where $F$ and $\xi$ are in the form of power spectral densities, and

$$
S_{\xi}(f)=\left|H_{m}(f)\right|^{2} \cdot S_{F}(f)
$$

According to the governing equation as formulated above, $\xi$ is equivalent to the base rotation of the structure. Alternate forms of the equation may be written, however, such that $\xi$ is some other response parameter. In the frequency domain solution, this corresponds to using various forms of the mechanical admittance. Some examples are:

$$
\begin{array}{ll}
\xi= & \text { base moment: } \\
& \left|H_{m}(f)\right|^{2}=\frac{1}{\left[1-\left(f / f_{o}\right)^{2}\right]^{2}+\left[2 \zeta f / f_{o}\right]^{2}} \\
& \text { rotation about base: }
\end{array} \quad\left|H_{m}(f)\right|^{2}=\frac{1 / K^{2}}{\left[1-\left(f / f_{o}\right)^{2}\right]^{2}+\left[2 \zeta f / f_{o}\right]^{2}}
$$

$$
\text { rotational acceleration: } \quad\left|H_{m}(f)\right|^{2}=\frac{(2 \pi f)^{4} / K^{2}}{\left[1-\left(f / f_{o}\right)^{2}\right]^{2}+\left[2 \zeta f / f_{o}\right]^{2}}
$$

In the above $f_{o}$ is the natural frequency of the fundamental mode,

$$
f_{o}=\frac{1}{2 \pi} \sqrt{K / M}
$$

and $\zeta$ is the critical damping ratio,

$$
\zeta=\frac{C}{2 \sqrt{K M}}
$$

The total procedure may now be summarized as follows. A rigid, geometrically similar scale model of the prototype is built and mounted on a stiff base moment balance, then placed in a properly scaled wind tunnel environment. The balance has a suitable dynamic bandwidth due to its high natural frequency, and thus generates a signal corresponding to the externally applied dynamic base moment due to wind. The base moment is interpreted as the generalized load for a single-degree-of-freedom system equivalent to the fundamental mode of the real structure. The power spectral density of this load is computed from the measured time series. This load spectrum will be valid as long as the geometry of the structure is unchanged; mass, stiffness and damping do not affect it. A mechanical admittance function is then formulated, depending on mass, stiffness, damping and the desired response parameter. This function is multiplied by the load spectrum, which results in a response spectrum. The response spectrum may be numerically integrated to obtain the variance or root-mean-square value of the response. If results are then desired for other values of mass, stiffness, or damping, it is only necessary to repeat numerical computations, without additional wind tunnel testing.
APPENDIX B
MODAL ANALYSIS AND RANDOM VIBRATION
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Reduction to SDOF System Using Modal Analysis
A common analytical model of a tall building, suitable for the analysis of dynamic horizontal loading, is to consider it a chain of lumped masses, connected in series by beam elements. Displacements in the $x$-direction, $y$-direction, and rotations about the (vertical) z-axis are considered independently. For each of these components, the structural "frame" model will have $n$ degrees of freedom, corresponding to the side-sway (or rotation) of each floor or lumped mass point. The system may then be described by an $n \times n$ stiffness matrix [k], with an "input" n-component load vector $\{P\}$, and an "output" n-component displacement vector $\{x\}$.

In a static system, these are related by the familiar equation

$$
\{P\}=[k]\{x\}
$$

When the loading varies with time and height, however, a complete description would be

$$
[m]\{\ddot{x}\}+[c]\{\dot{x}\}+[k]\{x\}=\{P\}
$$

where [m] is the matrix of lumped masses, [c] is a matrix of damping coefficients, and $\{x\}$ and $\{P\}$ are now functions of time. This represents a system of $n$ simultaneous equations, which would be difficult to solve even if $\{P\}$ could be determined. If a transformation to a system of "generalized coordinates" is applied, these simultaneous equations are simplified to $n$ uncoupled equations. Each equation corresponds to one of the generalized coordinates, $\xi_{i}$, which also has an associated natural frequency $f_{i}$ and mode shape $\{\phi\}_{i}$. Furthermore, due to the frequency distribution of wind energy, almost all of the excitation occurs in the fundamental mode associated with the lowest
natural frequency. It is then only necessary to consider the first of the generalized coordinate equations, which appears as

$$
\begin{equation*}
\mathrm{m}^{*} \ddot{\xi}+\mathrm{c}^{*} \dot{\xi}+\mathrm{k} * \xi=\mathrm{p} * \tag{B.1}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{m}^{*} & =\{\phi\}^{\mathrm{T}}[\mathrm{~m}]\{\phi\} \\
\mathrm{c}^{*} & =\{\phi\}^{\mathrm{T}}[\mathrm{c}]\{\phi\} \\
\mathrm{k}^{*} & =\{\phi\}^{\mathrm{T}}[\mathrm{k}]\{\phi\} \\
\mathrm{p}^{*} & =\{\phi\}^{\mathrm{T}}\{\mathrm{P}\}
\end{aligned}
$$

These are referred to as the generalized mass, damping, stiffness, and load, respectively. This governing equation is that of a conventional single-degree of freedom system. The solution of the system, $\xi$, is related to the actual system by

$$
\begin{equation*}
\{x\}=\xi\{\phi\} . \tag{B.2}
\end{equation*}
$$

It can be shown that the natural frequency of the system is

$$
\begin{equation*}
f_{o}=\frac{1}{2 \pi} \sqrt{k^{\star} / m^{*}} . \tag{B.3}
\end{equation*}
$$

A further key property of tall structures is that the mode shape may be approximated by a straight line, that is $\phi_{i}=\alpha z_{i}$ or $\{\phi\}=$ $\alpha\{z\}$. Since the magnitude of a mode shape is arbitrary, $\alpha$ may be taken as unity. The generalized mass then becomes

$$
\begin{equation*}
m^{*}=\{z\}^{T}[m]\{z\}=\Sigma m_{i} z_{i}^{2} \tag{B.4}
\end{equation*}
$$

which is approximately the mass moment of inertia, $I$, about the base. The generalized load is

$$
\begin{equation*}
\mathrm{P}^{*}=\{z\}^{\mathrm{T}}\{\mathrm{P}\}=\Sigma \mathrm{P}_{\mathrm{i}} \mathrm{z}_{\mathrm{i}} \tag{B.5}
\end{equation*}
$$

which is the moment about the base, M. The displacement vector is

$$
\{x\}=\xi\{z\}, \text { or } x(z)=\xi z
$$

which shows that $\xi$ is the rotation of the structure (which remains a straight line by assumption) about its base, $\theta$. By analogy to the SDOF system the generalized stiffness $k^{*}$ is equivalent to a simple rotational stiffness $k_{\theta}$. Introducing the critical damping ratio,

$$
\begin{aligned}
\zeta & =\frac{c}{c_{c r}}=\frac{c^{*}}{c_{c}^{\star}}=\frac{c^{*}}{2 \sqrt{k^{*} m^{*}}} \\
& =2 m^{*} \omega_{0}
\end{aligned}
$$

where $\omega_{0}=2 \pi f_{0}$ is the natural circular frequency, the governing equation (B.1) may be rewritten as

$$
\begin{equation*}
I \ddot{\theta}+2 I \zeta w_{o} \dot{\theta}+k_{\theta} \theta=M(t) \tag{B.6}
\end{equation*}
$$

## Solution of the Governing Equation Using Random Vibration Theory

Equation (B.6) is most easily solved in the frequency domain when the loading $M(t)$ is random in time, since an arbitrary function of time can be described by a superposition of sinusoidal functions. For such a harmonic function at frequency $f$,

$$
M(t)=M_{o} \sin (2 \pi f t+\psi)
$$

the solution is (in magnitude, ignoring phase)

$$
\theta(t)=\frac{1}{k_{\theta}}|H(f)| \cdot M(t)
$$

where the frequency response function $H(f)$ is defined as

$$
|H(f)|=\frac{1}{\sqrt{\left[1-\left(f / f_{o}\right)^{2}\right]^{2}+\left(2 \zeta f / f_{o}\right)^{2}}}
$$

In terms of the spring load $M$, which is equal to

$$
M=k_{\theta} \theta,
$$

the solution is simply

$$
M(t)=|H(f)| \cdot M(t) .
$$

Note that $M(t)$ and $M(t)$ are both moments, and that $|H(f)|$ is dimensionless. If the driving frequency $f$ is near the structure's natural frequency $f_{o}$, and the damping ratio $\zeta$ is low, this function is an amplification factor describing a condition known as resonance. $M(t)$ is properly described as an externally applied base moment, whereas $M(t)$ is an internal or "response" base moment. Note that if $M(t)$ were externally applied to the structure and a static analysis performed, the calculated response (displacement or internal forces) would be the same as the actual response due to the real fluctuating load $M(t)$. Thus $M(t)$ is also referred to as a "static equivalent load."

When the loading $M(t)$ is random in time, its statistical description as a superposition of harmonics is its "power spectral density," denoted $S_{M}(f)$. The response moment is then also described as a power spectral density, $S_{M}(f)$. The general result of random vibration theory is the relationship between these two, which is simply

$$
\begin{equation*}
S_{M}(f)=|H(f)|^{2} \cdot S_{M}(f) \tag{B.7}
\end{equation*}
$$

The relating function $|H(f)|^{2}$ is just the square of the frequency response function, and is referred to in general as the transfer function:

$$
\begin{equation*}
|H(f)|^{2}=\frac{1}{\sqrt{\left[1-\left(f / f_{o}\right)^{2}\right]^{2}+\left(2 \zeta f / f_{o}\right)^{2}}} \tag{B.8}
\end{equation*}
$$

This particular transfer function is also referred to as the "mechanical admittance."

The principal usefulness of the power spectral density of a function is that its variance may be computed as the area under the spectrum:

$$
\begin{equation*}
\sigma_{M}^{2}=\int_{0}^{\infty} S_{M}(f) d f \tag{B.9}
\end{equation*}
$$

The response is expected to be normally distributed (Gaussian) statistically, and thus can be completely described by its variance $\sigma_{M}^{2}$ and mean value $\bar{M}$ (note that the mean response $\bar{M}$ is equal to the mean load $\bar{M})$. Since the structural damping is very low, the response is also "narrow band," i.e., it can be loosely described as vibration at a single frequency $f_{o}$ with randomly varying amplitude. Each cycle of vibration has a maximum and negative "peak" value associated with it. It is this series of peaks which are of interest insofar as structural design for strength or stiffness is concerned. It can be shown that these peak values obey a Rayleigh probability distribution, which is easily obtained from $\sigma_{\mathbb{M}}^{2}$. The expected value, variance, etc. of these peaks could be easily found from the Rayleigh distribution. What is more desirable, however, are statistics describing the largest peak which is likely to occur. Such an analysis is beyond the scope of this review, and the following result is simply stated:

$$
\begin{equation*}
\hat{M}=\bar{M}+g_{p} \sigma_{M} \tag{B.10}
\end{equation*}
$$

Here $\hat{M}$ is the expected value of the largest peak $M$ occurring during a duration $T$ of the loading $M$. The so-called "peak factor" $g_{p}$ is calculated as follows [B3]

$$
\text { where } \quad \begin{align*}
g_{p} & =\mu+\frac{.5772 \mu}{\mu^{2}-1}  \tag{B.11}\\
\mu & =\sqrt{2 \ln V T}+\frac{\ln \sqrt{2 \ln V T}}{\sqrt{2 \ln V T}} \\
v & =f_{o} \sqrt{8 \zeta}
\end{align*}
$$

## Nomenclature

[c] Damping matrix
c* Generalized damping
$\mathrm{f}_{\mathrm{o}} \quad$ Natural cyclic frequency of fundamental mode
g Peak factor
$|\mathrm{H}(\mathrm{f})|^{2}$ Mechanical admittance (transfer function)
I Mass moment of inertia of fundamental mode approximated by straight-line shape
[k] Stiffness matrix
k* Generalized stiffness
$k_{\theta} \quad$ Rotational stiffness of fundamental mode approximated by straight-line shape

Resultant base moment of externally-applied wind load
Response (internal, static equivalent) base moment
[m] Mass matrix
m* Generalized mass
$\mathrm{m}_{\mathrm{i}}$
$\{P\} \quad$ Vector of $P_{i}$
$\mathrm{P}_{\mathrm{i}} \quad$ Resultant force of wind load acting at floor i
P* Generalized load
$\left.S_{( }\right)^{(f)}$ Power spectral density of ()
\{x\} Vector of floor displacements
$\{\mathrm{z}\} \quad$ Vector of $\mathrm{z}_{\mathrm{i}}$
$z_{i} \quad$ Height of floor i
$\zeta \quad$ Critical damping ratio
$\theta \quad$ Rotation of fundamental mode approximated by straight-line shape
$\xi \quad$ Generalized coordinate
${ }^{\sigma}() \quad$ Standard deviation (fluctuating rms) of ( )
$\sigma_{( }^{2} \quad$ Variance of ( )
$\{\phi\} \quad$ Mode shape
$\psi \quad$ Phase angle
$\omega_{0} \quad$ Natural circular frequency
( ${ }^{-}$Temporal mean of ( )
( ) Expected peak value of ( )

## APPENDIX B--REFERENCES

B1. Clough, Ray W., and J. Penzien, Dynamics of Structures, McGraw-Hill Book Company, New York, 1975.

B2. Crandall, Stephen H., and William D. Mark, Random Vibration in Mechanical Systems, Academic Press, New York, 1963.

B3. Davenport, A. G., N. Isyumov, H. Rothman, and H. Tanaka, "WindInduced Response of Suspension Bridges - Wind Tunnel Model and Full Scale Observations," Wind Engineering, Proceedings of the Fifth International Conference, Cermak, J. E. (ed.), Fort Collins, Colorado, July 1979, Pergamon Press, Oxford, 1980, Vol. 2, pp. 807-824.

APPENDIX C

WIND VELOCITY DATA AND PROBABILITY DISTRIBUTIONS

WEIBUL DISTRIBUTION FIT $P( \rangle U\rangle=\exp (-\langle U / C) * * K)$

STATION: EPIA \#1, SAUDIA ARABIA


| SE | $P(>U)$ | 0.64 | 0.28 | 0.06 | 0.00 | 0.00 | Sumy , xy |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Y=\ln (-1 n P)$ ) | -0.81 | 0.24 | 1.06 | 2.49 | 2.55 | 5.53 |
|  | $X * Y$ | -0.75 | 0.37 | 2.13 | 5.86 | 6.63 | 14.25 |
|  | slope-K | 2.11 | inter-b | $-2.85$ | $C=e-b / K$ | 3.87 |  |
| SSE | $P() U$ ) | 0.51 | 0.23 | 0.02 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P))$ | -0.41 | 0.37 | 1.32 | 2.51 | 2.56 | 6.36 |
|  | $X * Y$ | -0.37 | 0.56 | 2.67 | 5.90 | 6.67 | 15.42 |
|  | slope-K | 1.90 | inter-b | $-2.29$ | $C=e-b / K$ | 3.34 |  |
| 5 | $P(>)$ | 0.50 | 0.24 | 0.08 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P)$ ) | -0.37 | 0.36 | 0.93 | 2.50 | 2.55 | 5.97 |
|  | $X * Y$ | -0.34 | 0.54 | 1.87 | 5.87 | 6.64 | 14.59 |
|  | slope-K | 1.85 | inter-b | $-2.27$ | $C=e-b / K$ | 3.43 |  |
| SSW | $P() U$ ) | 0.44 | 0.18 | 0.03 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P)$ ) | -0.21 | 0.55 | 1.26 | 2.49 | 2.54 | 6.63 |
|  | $X * Y$ | -0.19 | 0.82 | 2.54 | 5.85 | 6.62 | 15.64 |
|  | slope-K | 1.74 | inter-b | -1.94 | $C=e-b / K$ | 3.05 |  |
| SW | $P(>U)$ | 0.27 | 0.11 | 0.00 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P))$ | 0.28 | 0.81 | 2.40 | 2.44 | 2.50 | 8.43 |
|  | $X * Y$ | 0.26 | 1.21 | 4.84 | 5.73 | 6.50 | 18.54 |
|  | slope-K | 1.48 | inter-b | $-1.10$ | $c=e-b / K$ | 2.10 |  |
| WSW | $P(>U)$ | 0.41 | 0.16 | 0.00 | 0.00 | 0.00 | Sumy, xy |
|  | $Y=\ln (-1 n P))$ | -0.11 | 0.62 | 2.45 | 2.48 | 2.54 | 7.98 |
|  | $X * Y$ | -0.10 | 0.93 | 4.93 | 5.84 | 6.61 | 18.20 |
|  | Slope-K | 1.76 | inter-b | -1.71 | $\mathrm{C}=e-\mathrm{b} / \mathrm{K}$ | 2.64 |  |
| W | $P(>)$ | 0.42 | 0.17 | 0.02 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P))$ | -0.13 | 0.57 | 1.38 | 2.52 | 2.58 | 6.92 |
|  | $X * Y$ | -0.12 | 0.86 | 2.78 | $5.94$ | $6.71$ | 16.16 |
|  | slope-K | 1.73 | inter-b | -1.86 | $C=e-b / K$ | 2.94 |  |
| WNW | $P(>U)$ | 0.49 | 0.17 | 0.04 | 0.00 | 0.00 | Sumy, xy |
|  | $Y=\ln (-\ln P))$ | $-0.34$ | 0.57 | 1.15 | 2.51 | 2.57 | 6.46 |
|  | $X * Y$ | $-0.31$ | 0.86 | 2.31 | 5.91 | 6.69 | 15.46 |
|  | slope-K | 1.81 | inter-b | -2.12 | $C=e-b / K$ | 3.21 |  |
| NW | $P(>)$ | 0.72 | 0.30 | 0.07 | 0.02 | 0.00 | Sumy, xy |
|  | $Y=\ln (-\ln P))$ | $-1.11$ | 0.19 | 0.96 | 1.42 | 2.64 | 4.10 |
|  | $X * Y$ | -1.02 | 0.28 | 1.94 | 3.33 | 6.87 | 11.42 |
|  | slope-K | 2.03 | inter-b | $-2.98$ | $C=s-b / K$ | 4.36 |  |
| NNW | $P(>)$ | 0.79 | 0.53 | 0.23 | 0.07 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P)$ ) | -1.46 | -0.46 | 0.37 | 0.95 | 1.67 | 1.07 |
|  | $X * Y$ | -1.34 | -0.70 | 0.75 | 2.24 | 4.34 | 5.30 |
|  | slope-K | 1.79 | inter-b | $-3.15$ | $c=e-b / K$ | 5.80 |  |
| ALL | $\mathrm{P}(>\mathrm{U})$ | 0.68 | 0.40 | 0.16 | 0.04 | 0.01 | Sumy, xy |
|  | $Y=\ln (-\ln P)$ ) | -0.97 | -0.10 | 0.62 | 1.18 | 1.67 | 2.40 |
|  | X*Y | -0.89 | -0.14 | 1.24 | 2.77 | 4.34 | 7.32 |
|  | slope-K | 1.54 | inter-b | $-2.41$ | $C=e-b / K$ | 4.79 |  |

STATION: EPIA \#1, SAUDIA ARABIA

| T yrs | 0.10 | 0.50 | 1.00 | 2.00 | 5.00 | 10.00 | 50.00 | 100.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 16.5 | 19.2 | 20.2 | 21.2 | 22.4 | 23.3 | 25.3 |
| N | 13.7 | 16.1 | 17.1 | 18.0 | 19.1 | 20.0 | 21.8 | 22.5 |
| NNE | 13.7 | 10.4 | 11.1 | 11.7 | 12.4 | 13.0 | 14.1 | 14.6 |
| NE | 8.7 | 8.6 | 9.1 | 9.5 | 10.0 | 10.4 | 11.2 | 11.6 |
| ENE | 7.4 | 7.7 | 8.9 | 9.4 | 9.8 | 10.4 | 10.8 | 11.6 |
| E | 7.9 | 11.9 |  |  |  |  |  |  |
| ESE | 9.2 | 11.0 | 11.7 | 12.3 | 13.1 | 13.7 | 14.9 | 15.4 |
| SE | 7.0 | 8.3 | 8.9 | 9.4 | 10.0 | 10.4 | 11.3 | 11.7 |
| SSE | 6.6 | 8.0 | 8.5 | 9.1 | 9.7 | 10.2 | 11.2 | 11.6 |
| S | 6.8 | 8.3 | 8.9 | 9.4 | 10.1 | 10.6 | 11.7 | 12.2 |
| SSW | 6.2 | 7.7 | 8.3 | 8.9 | 9.6 | 10.1 | 11.2 | 11.7 |
| SW | 4.2 | 5.7 | 6.3 | 6.9 | 7.6 | 8.1 | 9.2 | 9.7 |
| WSW | 5.2 | 6.5 | 7.1 | 7.5 | 8.1 | 8.6 | 9.5 | 9.9 |
| W | 6.4 | 7.8 | 8.4 | 8.9 | 9.6 | 10.1 | 11.2 | 11.6 |
| WNW | 6.6 | 8.1 | 8.6 | 9.2 | 9.8 | 10.3 | 11.4 | 11.8 |
| NW | 9.3 | 10.8 | 11.4 | 11.9 | 12.6 | 13.1 | 14.2 | 14.6 |
| NNW | 13.4 | 15.9 | 16.9 | 17.8 | 19.0 | 19.9 | 21.7 | 22.5 |
| ALL | 16.6 | 19.1 | 20.1 | 21.1 | 22.4 | 23.3 | 25.4 | 26.3 |

STATION: EPIA \#I, SAUDIA ARABIA Speeds in mph at Elev $675 \mathrm{ft}, 0.14$ power 1 aw

| T yrs | 0.10 | 0.50 | 1.00 | 2.00 | 5.00 | 10.00 | 50.00 | 100.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |
| N | 56.5 | 65.4 | 69.0 | 72.3 | 76.6 | 79.6 | 86.4 | 89.2 |
| NNE | 46.8 | 55.1 | 58.4 | 61.5 | 65.4 | 68.2 | 74.3 | 76.9 |
| NE | 29.8 | 35.6 | 37.8 | 39.8 | 42.4 | 44.3 | 48.3 | 49.9 |
| ENE | 25.3 | 29.4 | 31.0 | 32.5 | 34.3 | 35.6 | 38.4 | 39.5 |
| E | 26.3 | 30.5 | 32.1 | 33.6 | 35.4 | 36.7 | 39.6 | 40.7 |
| ESE | 31.5 | 37.5 | 39.9 | 42.0 | 44.8 | 46.7 | 50.9 | 52.6 |
| SE | 23.8 | 28.5 | 30.3 | 32.0 | 34.1 | 35.6 | 38.8 | 40.1 |
| SSE | 22.5 | 27.3 | 29.2 | 30.9 | 33.1 | 34.7 | 38.1 | 39.5 |
| S | 23.1 | 28.3 | 30.3 | 32.2 | 34.6 | 36.3 | 40.1 | 41.6 |
| SSW | 21.0 | 26.3 | 28.3 | 30.3 | 32.7 | 34.4 | 38.2 | 39.8 |
| SW | 14.4 | 19.5 | 21.5 | 23.5 | 25.9 | 27.7 | 31.6 | 33.2 |
| WSW | 17.9 | 22.4 | 24.1 | 25.7 | 27.8 | 29.3 | 32.5 | 33.8 |
| W | 21.8 | 26.7 | 28.7 | 30.5 | 32.8 | 34.5 | 38.1 | 39.6 |
| WNW | 22.6 | 27.5 | 29.5 | 31.3 | 33.6 | 35.2 | 38.8 | 40.3 |
| NW | 31.9 | 36.9 | 38.9 | 40.8 | 43.1 | 44.8 | 48.5 | 49.9 |
| NWW | 45.7 | 54.3 | 57.6 | 60.8 | 64.9 | 67.8 | 74.3 | 76.9 |
| ALL | 56.8 | 65.2 | 68.7 | 72.1 | 76.4 | 79.6 | 86.7 | 89.7 |

WEIBUL DISTRIBUTION FIT $P( \rangle U)=\exp (-(U / C) * * K)$

STATION: EPIA \#2, SAUDIA ARABIA

C-6

| SE | P(>U) | 0.88 | 0.56 | 0.19 | 0.03 | 0.00 | Sumy , xy |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Y=\ln (-\ln P)$ ) | -2.06 | -0.55 | 0.52 | 1.29 | 2.55 | 1.76 |
|  | X*Y | -1.88 | -0.82 | 1.04 | 3.03 | 6.64 | 8.01 |
|  | slope-K | 2.57 | inter-b | -4.48 | $c=e-b / K$ | 5.70 |  |
| SSE | $\mathrm{P}(>\mathrm{U})$ | 0.89 | 0.51 | 0.17 | 0.02 | 0.00 | Sumy , xy |
|  | $Y=1 n(-1 n P)$ ) | -2.16 | -0.38 | 0.58 | 1.32 | 2.56 | 1.90 |
|  | $X * Y$ | -1.98 | -0.58 | 1.16 | 3.09 | 6.66 | 8.36 |
|  | Slope-k | 2.61 | inter-b | -4.52 | $C=e-b / K$ | 5.65 |  |
| S | P (>U) | 0.79 | 0.25 | 0.05 | 0.02 | 0.00 | Sumy, xy |
|  | $Y=\ln (-1 n P)$ ) | -1.46 | 0.33 | 1.10 | 1.41 | 2.59 | 3.97 |
|  | X*Y | -1.34 | 0.50 | 2.22 | 3.32 | 6.74 | 11.43 |
|  | slope-K | 2.17 | inter-b | -3.28 | $C=e-b / K$ | 4.54 |  |
| SSW | P(>U) | 0.75 | 0.17 | 0.03 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P)$ ) | -1.26 | 0.56 | 1.26 | 2.49 | 2.55 | 5.60 |
|  | X*Y | -1.16 | 0.84 | 2.55 | 5.85 | 6.63 | 14.71 |
|  | Slope-K | 2.29 | inter-b | -3.19 | $c=e-b / K$ | 4.01 |  |
| Sw | $\mathrm{P}(>\mathrm{C})$ | 0.76 | 0.11 | 0.05 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P)$ ) | -1.28 | 0.80 | 1.07 | 2.44 | 2.50 | 5.52 |
|  | X*Y | -1.17 | 1.20 | 2.16 | 5.73 | 6.50 | 14.41 |
|  | Slope-k | 2.20 | inter-b | -3.03 | $C=e-b / K$ | 3.96 |  |
| WSW | P( $>$ U) | 0.60 | 0.04 | 0.00 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P)$ ) | -0.66 | 1.15 | 2.42 | 2.46 | 2.52 | $7.89$ |
|  | $X * Y$ | -0.60 | 1.73 | 4.88 | $5.78$ | 6.55 | 18.33 |
|  | slope-K | 1.92 | inter-b | -2.03 | $C=e-b / K$ | 2.88 |  |
| $\omega$ | P(>U) | 0.78 | 0.14 | 0.00 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-1 n P)$ ) | -1.37 | 0.67 | 2.47 | 2.51 | 2.56 | 6.84 |
|  | $X * Y$ | -1.26 | 1.01 | 4.98 | 5.89 | 6.67 | 17.30 |
|  | slope-K | 2.43 | inter-b | -3.20 | $c=e-b / K$ | 3.72 |  |
| WNW | $\mathrm{P}(>\mathrm{l})$ | 0.85 | 0.27 | 0.02 | 0.00 | 0.00 | Sumy , xy |
|  | $Y=\ln (-\ln P)$ ) | -1.85 | 0.27 | 1.33 | 2.51 | 2.57 | 4.83 |
|  | X*Y | -1.69 | 0.41 | 2.69 | 5.90 | 6.68 | 13.98 |
|  | slope-K | 2.68 | inter-b | -4.06 | $C=e-b / K$ | 4.56 |  |
| NW | P ( $>\mathrm{U})$ | 0.94 | 0.46 | 0.10 | 0.02 | 0.00 | Sumy , xy |
|  | $Y=\ln (-1 n P)$ ) | -2.78 | -0.26 | 0.83 | 1.38 | 2.63 | 1.80 |
|  | X*Y | -2.55 | -0.38 | 1.67 | 3.26 | 6.85 | 8.84 |
|  | slope-K | 2.97 | inter-b | -5.22 | $C=e-b / K$ | 5.79 |  |
| NNW | P ( $>\mathrm{U}$ ) | 0.93 | 0.63 | 0.25 | 0.08 | 0.01 | Sumy, xy |
|  | $Y=\ln (-\ln P)$ ) | -2.64 | -0.79 | 0.32 | 0.90 | 1.51 | -0.70 |
|  | X*Y | -2.42 | -1.19 | 0.64 | 2.13 | 3.94 | 3.10 |
|  | Slope-K | 2.40 | inter-b | -4.65 | $C=e-b / K$ | 6.93 |  |
| ALL | $\mathrm{P}(>\mathrm{l})$ | 0.90 | 0.52 | 0.22 | 0.07 | 0.01 | Sumy, xy |
|  | $Y=\ln (-1 n P)$ ) | -2.20 | -0.43 | 0.40 | 0.98 | 1.53 | 0.28 |
|  | $X * Y$ | -2.01 | -0.65 | 0.81 | 2.31 | 3.97 | 4.43 |
|  | slope-k | 2.13 | inter-b | -3.94 | $c=e-b / K$ | 6.37 |  |

STATION: EPIA \#2, SAUDIA ARABIA

| T yrs | 0.10 | 0.50 | 1.00 | 2.00 | 5.00 | 10.00 | 50.00 | 100.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| N | 15.8 | 17.8 | 18.6 | 19.3 | 20.2 | 20.9 | 22.3 | 22.9 |
| NNE | 15.8 | 17.9 | 18.7 | 19.5 | 20.4 | 21.1 | 22.5 | 23.1 |
| NE | 10.0 | 11.2 | 11.7 | 12.1 | 12.6 | 13.0 | 13.8 | 14.1 |
| ENE | 8.3 | 9.3 | 9.7 | 10.1 | 10.5 | 10.8 | 11.5 | 11.7 |
| E | 8.5 | 9.5 | 9.8 | 10.2 | 10.6 | 10.9 | 11.5 | 11.7 |
| ESE | 9.6 | 10.9 | 11.4 | 11.8 | 12.3 | 12.7 | 13.5 | 13.8 |
| SE | 9.3 | 10.8 | 11.3 | 11.8 | 12.4 | 12.9 | 13.8 | 14.2 |
| SSE | 9.2 | 10.6 | 11.2 | 11.6 | 12.2 | 12.7 | 13.6 | 13.9 |
| S | 8.6 | 10.0 | 10.6 | 11.1 | 11.7 | 12.2 | 13.2 | 13.6 |
| SSW | 6.9 | 8.1 | 8.6 | 9.0 | 9.6 | 9.9 | 10.8 | 11.1 |
| SW | 6.3 | 7.8 | 8.3 | 8.8 | 9.4 | 9.8 | 10.7 | 11.1 |
| WSW | 5.1 | 6.4 | 6.9 | 7.3 | 7.9 | 8.3 | 9.2 | 9.5 |
| W | 6.3 | 7.4 | 7.7 | 8.1 | 8.5 | 8.9 | 9.5 | 9.8 |
| WNW | 7.4 | 8.5 | 8.9 | 9.3 | 9.7 | 10.0 | 10.7 | 11.0 |
| NW | 9.6 | 10.7 | 11.1 | 11.4 | 11.9 | 12.2 | 12.9 | 13.2 |
| NNW | 12.9 | 14.6 | 15.3 | 15.9 | 16.7 | 17.3 | 18.5 | 19.0 |
| ALL | 15.6 | 17.3 | 17.9 | 18.6 | 19.4 | 19.9 | 21.2 | 21.7 |

STATION: EPIA \#2, SAULIA ARABIA
Speeds in mph at Eleu $675 \mathrm{ft}, 0.14$ power law

| T yrs | 0.10 | 0.50 | 1.00 | 2.00 | 5.00 | 10.00 | 50.00 | 100.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |
| N | 54.0 | 60.8 | 63.5 | 66.0 | 69.1 | 71.3 | 76.2 | 78.2 |
| NNE | 54.1 | 61.2 | 64.0 | 66.6 | 69.8 | 72.1 | 77.0 | 79.0 |
| NE | 34.2 | 38.4 | 39.9 | 41.4 | 43.2 | 44.4 | 47.1 | 48.2 |
| ENE | 28.3 | 31.9 | 33.2 | 34.4 | 35.9 | 37.0 | 39.2 | 40.1 |
| E | 29.0 | 32.3 | 33.6 | 34.7 | 36.1 | 37.1 | 39.3 | 40.1 |
| ESE | 32.9 | 37.2 | 38.8 | 40.3 | 42.1 | 43.4 | 46.1 | 47.2 |
| SE | 31.7 | 36.7 | 38.6 | 40.3 | 42.5 | 44.0 | 47.2 | 48.5 |
| SSE | 31.5 | 36.3 | 38.1 | 39.8 | 41.8 | 43.2 | 46.3 | 47.5 |
| S | 29.2 | 34.2 | 36.1 | 37.9 | 40.1 | 41.7 | 45.1 | 46.5 |
| SSW | 23.4 | 27.7 | 29.3 | 30.8 | 32.7 | 34.0 | 36.8 | 37.9 |
| SW | 21.5 | 26.5 | 28.3 | 30.0 | 32.1 | 33.5 | 36.7 | 37.9 |
| WSW | 17.5 | 21.8 | 23.5 | 25.0 | 26.9 | 28.3 | 31.3 | 32.5 |
| W | 21.6 | 25.1 | 26.4 | 27.7 | 29.2 | 30.3 | 32.6 | 33.5 |
| WNW | 25.3 | 29.0 | 30.3 | 31.6 | 33.2 | 34.3 | 36.6 | 37.6 |
| NW | 32.9 | 36.5 | 37.8 | 39.0 | 40.6 | 41.7 | 44.0 | 44.9 |
| NNW | 43.9 | 50.0 | 52.3 | 54.5 | 57.2 | 59.1 | 63.3 | 65.0 |
| ALL | 53.4 | 59.0 | 61.3 | 63.4 | 66.2 | 68.1 | 72.5 | 74.3 |

## APPENDIX D

TEST RESULTS: REDUCED POWER SPECTRAL DENSITIES OF GENERALIZED LOAD

RUN INDEX

| Run <br> No. | Wind Direction (Deg) | Equiv. Wind Dir. by Sym. | Configuration | Page Nos. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Plot | Tabulation |
| 7 | 270* | 90 | A | D-3 | D-28 |
| 8 | 280* | 80 | (no adjacent | D-4 | D-30 |
| 9 | 290 | 70 | buildings) | D-5 | D-32 |
| 11 | 300 | 60 |  | D-6 | D-34 |
| 14 | 310 | 50 |  | D-7 | D-36 |
| 15 | 320** | 40 |  | D-8 | D-38 |
| 16 | 330* | 30 |  | D-9 | D-40 |
| 17 | 340** | 20 |  | D-10 | D-42 |
| 18 | 350** | 10 |  | D-11 | D-44 |
| 22 | 260 | 100 |  | D-12 | D-46 |
| 23 | 180 |  |  | D-13 | D-48 |
| 24 | 190 | 170 |  | D-14 | D-50 |
| 25 | 200 | 160 |  | D-15 | D-52 |
| 26 | 210 | 150 |  | D-16 | D-54 |
| 27 | 220 | 140 |  | D-17 | D-56 |
| 28 | 230 | 130 |  | D-18 | D-58 |
| 29 | 240 | 120 |  | D-19 | D-60 |
| 30 | 250 | 110 |  | D-20 | D-62 |
| 37 | 270 |  | B | D-21 | D-64 |
| 38 | 280 |  | (upwind | D-22 | D-66 |
| 41 | 320 |  | terminal | D-23 | D-68 |
| 42 | 330 |  | buildings | D-24 | D-70 |
| 48 | 340 |  | included) | D-25 | D-72 |
| 49 | 350 |  |  | D-26 | D-74 |
| 50 | 0 |  |  | D-27 | D-76 |

\#Data obtained only to show effect of removing upwind terminal
buildings. True data is Config. B. buildings. True data is Config. B.



RUN NO. 9 WIND DIRECTION 290 Deg. VEL. $U=35.4$ fps



て**('ffooj) '(f)St






Z**(•ffeoj) '(f)St

RUN NO. 22 WIND DIRECTION 268 Deg. VEL. $U=35.6$ fps




RUN NO. 25 WIND DIRECTION 200 Deg. VEL. $U=35.5 \mathrm{fps}$









RUN NO. 41 WIND DIRECTION 320 Deg. VEL. $U=35.6 \mathrm{fps}$




RUN NO. 48 WIND DIRECTION 340 Deg. VEL. $U=34.5$ fps








FOUER SFECTRAL FILE EPOOg1

| PROJECT HO 5510 COMFICUATION G | UINO YEL : CIRECTIGN: | $\begin{array}{r} 35.43 \\ 290 \end{array}$ |
| :---: | :---: | :---: |
| COHFIGUATION | OIRESTIGT. |  |

MOH-GIMENSIGMAL SPETTRUM N*SCN; OF MXVE.N*OイL


```
```

MEAH=.5240 RAS=.5SOGE-01 RODT(AKEA)= 5G42E-01

```
```

```
```

MEAH=.5240 RAS=.5SOGE-01 RODT(AKEA)= 5G42E-01

```
```



| $21$ |  |
| :---: | :---: |
| E-03 | $599 E$ |
| 564E-63 | 327E-63 |
| $546 E-03$ | $1395-43$ |
| 799E-93 | $151 E-03$ |
| 10ZE-02 | $344 E-43$ |
| 127E-02 | 340E-63 |
| 152E-42 | $564 E-63$ |
| 76E-42 | 224E-03 |
| 212E-02 | 810E-03 |
| $61 E-62$ | 1295-62 |
| O9E-02 | $633 E-93$ |
| 55E-02 | 803E-63 |
| 431E-02 | 159E-42 |
| 28E-02 | 803E-63 |
| 25E-02 | 121E-02 |
| 2E-02 | $824 E-63$ |
| 2 | 107E-42 |


| 485E-01 | . 178E-03 | . 118 | 628E-04 |
| :---: | :---: | :---: | :---: |
| 52+E- 1 | $.1405-03$ | .122 | 423E-64 |
| 562E-01 | . $147 E-03$ | .128 | $571 E-04$ |
| $6015-61$ | . $1185-03$ | .132 | $374 E-04$ |
| $640 E-01$ | . 905E-64 | .144 | 342E-04 |
| 679E- 01 | . $9335-04$ | . 147 | $331 E-04$ |
| $7185-01$ | . $126 E-03$ | . 159 | $265 E-04$ |
| $756 E-71$ | . 940 E-04 | . 163 | $298 E-04$ |
| 795E-01 | . 935E-04 | .175 | $274 E-04$ |
| 834E- 1 | . 937E-04 | .179 | 233E-04 |
| 273E-01 | . $882 \mathrm{E}-04$ | .190 | $219 E-04$ |
| $912 E-01$ | . $649 E-64$ | . 134 | $2115-04$ |
| 951E-01 | . $776 E-04$ | .206 | 379E-04 |
| 989E-01 | . 830 E-04 | . 210 | 250E-14 |
| 143 | . 856E-04 | . 221 | 198E-04 |
| 107 | . $8335-04$ | . 225 | 276E-04 |
| 111 | . 840 OE-64 | . 237 | $217 E-04$ |
| 114 | . $6775-64$ | 241 | 224E-64 |

TIME 12:15 DAY 269 DF 1383
RUN NO
CHANNEL MX IN COEFF. UNITS



# POMER SPECTRAL FILE EPG112 

TIME 12:3
OIRGYEL : 35.63 FPS
RUN HOG
CHANHEL MY IH COEFF. UHITS
PROJECT NO 5610
CGNFIGUATION A


| 0 | $=1.437$ | IN |
| :---: | :---: | :---: |
| U | $=35.63$ | FPS |
| Q*A | $=.2382$ | 185 |
| Q*A*L | $=4.698$ | LB*It |




| 911E-02 | 108E-02 |
| :---: | :---: |
| 101E-01 | 931E-*3 |
| $110 E-61$ | $678 \mathrm{E}-3$ |
| 120E- 01 | 717E-才3 |
| 130E-01 | $741 E-03$ |
| 139E-01 | 913E-03 |
| $1495-01$ | 554E-63 |
| 16.3E-01 | 853E-03 |
| 183E-01 | 668E-43 |
| 202E-01 | 858E-03 |
| $2215-01$ | $7115-63$ |
| 241E-01 | $575 E-63$ |
| 260E- 1 | $463 E-63$ |
| 279E-01 | 438E-63 |
| $299 E-61$ | $564 E-3$ |
| 328E-01 | 370E-03 |
| $366 E-01$ | $372 E-63$ |
| 405E-01 | . $312 \mathrm{E}-63$ |
| 443E-01 | . $235 E-03$ |


| 482E-01 | $3$ |
| :---: | :---: |
| 521E-01 | 247E-03 |
| 55 ¢E- 1 | $1925-43$ |
| 598E-01 | $163 E-03$ |
| $636 E-61$ | $1735-03$ |
| $675 E-61$ | 137E-03 |
| $714 E-01$ | $1495-03$ |
| $752 E-61$ | 159E-03 |
| 791E-01 | $1265-03$ |
| $829 E-01$ | 122E-03 |
| 868E-31 | 106E-03 |
| 907E-61 | $136 E-03$ |
| 345E- 1 | $1265-63$ |
| 984E-61 | 166E-03 |
| 102 | $1315-93$ |
| 106 | $153 E-03$ |
| 110 | . $1045-03$ |
| 114 | $.109 E-03$ |


| .118 | 767E-04 |
| :---: | :---: |
| .122 | . $855 \mathrm{E}-04$ |
| .127 | . 739 E -04 |
| .131 | . $567 \mathrm{E}-04$ |
| .143 | . $586 \mathrm{E}-04$ |
| .147 | . $4915-04$ |
| .158 | $437 E-04$ |
| .162 | 372E-04 |
| .174 | . $2665-04$ |
| .178 | . $2575-04$ |
| .189 | . $2575-04$ |
| .193 | . 225 E-04 |
| .265 | . 3 94E-04 |
| . 208 | . $187 \mathrm{E}-04$ |
| .220 | . 229 E- 04 |
| . 224 | . 228 E-04 |
| . 235 | . 24 ¢E-04 |
| . 239 | .219E-04 |



















## PROJECT NO 5610 <br> CONFIGUATIOA A

OINO YEL: 35 24EFPS
RUN NO
CHANAE
25
IN COEFF. UNITS
HOH-OIKENSIGNAL SPECTRUM N*S《N OF MY VS. N*Q:U:

| D | 1.48 ? | IN |
| :---: | :---: | :---: |
| U | 35.46 | FPS |
| Q* ${ }^{\text {a }}$ | . 2360 | LBS |
| Q* $\mathbf{R}^{*}$ L | 4.653 | LEMI |




PQUER SPECTRRL FILE EPO262
TIME 16: 3 DAY 269 OF 1983



## PROJECT KO 5610 COHFIGUATION A


RUN HO 27
CHANHEL MY IN COEFF. UNITS


| 0 | $=$ | 1.467 | I |
| :---: | :---: | :---: | :---: |
| U | = | 35.43 | FPS |
| 日* ${ }^{\text {a }}$ | = | . 2355 | EBS |
|  | $=$ | 4.845 | LE*IN |










POUER SPECTRAL FILE EPO372


2 SEGMENTS OF 16384 SAMPLES AT 142.05 S SS
HEAN = .3654E-03 RMS = .5616E-01 ROOTGAREA $=.5626 E-01$
N*O/U N* S(H)

|  |  |
| :---: | :---: |
| $\begin{aligned} & 301 E-04 \\ & 602 E-84 \end{aligned}$ | $\begin{array}{r} 924 E-06 \\ 990 E-6 E \end{array}$ |
| 903E-04 | .i3iE-05 |
| $136 E-03$ | 941E-G6 |
| $196 E-03$ | 892E-06 |
| 256E-03 | 332E-95 |
| $316 E-03$ | 165E-65 |
| 376E-03 | 239 |
| 437E-03 | 449E-05 |
| 527E-03 | 369 |
| $647 E-03$ | 750 E |
| 768E-03 | 815 E -05 |
| 888E-03 | 545E-05 |
| 107E-02 | 754 E |
| $131 \mathrm{E}-02$ | 663E-05 |
| $155 \mathrm{E}-02$ | 140E-94 |
| $179 \mathrm{E}-02$ | $116 \mathrm{E}-04$ |
| 203E-02 | 241E-04 |
| 227E-02 | 182E-04 |
| 251E-02 | 29 |
| E-02 | 320 |
| $300 \mathrm{E}-02$ | 378 |
| 324E-02 | 401 |
| $348 \mathrm{E}-02$ | $435 \mathrm{E}-04$ |
| 372E-02 | 575E-64 |
| $408 \mathrm{E}-02$ |  |
| 456E-02 | 560E-04 |
| (1)-02 | 702E-94 |
| 33-02 | 710E-04 |
| 1E-02 | 834 |

N* OJU N * SiN:

| 649E-02 | $114 E-03$ | $395 \mathrm{E}-01$ | $663 E-03$ |
| :---: | :---: | :---: | :---: |
| 697E-02 | . $959 \mathrm{E}-34$ | . 4 可E-01 | $836 E-03$ |
| 745E-02 | . $811 \mathrm{E}-44$ | . $433 \mathrm{SW-61}$ | $681 E-63$ |
| $8185-02$ | .122E-33 |  | 839E-63 |
| 914E-92 | . $824 E-04$ | . $472 \mathrm{E}-01$ | $821 E-03$ |
| 101E-01 | $115 E-03$ | . $482 \mathrm{E}-1$ | $104 E-02$ |
| $1115-01$ | . $1315-03$ | $511 E-41$ | $165 E-62$ |
| 120E-01 | . 12 9E-03 | . $520 \mathrm{E}-01$ | . 18 ¢E-82 |
| 130E-01 | $194 E-93$ | 549E-01 | $125 E-62$ |
| 140E-01 | $.148 E-03$ | $559 E-01$ | . $118 \mathrm{E}-02$ |
| 149E-01 | $.152 E-03$ | 588E-01 | . 170 -02 |
| 159E-61 | . $216 E-33$ | . $597 \mathrm{E}-61$ | . $173 \mathrm{E}-62$ |
| 168E-01 | . $1715-03$ | 636E-61 | .197E-92 |
| 178E-01 | . $2615-33$ | 674E-61 | 209E-62 |
| $188 E-01$ | . $242 E-63$ | 713E-61 | . $246 \mathrm{E}-62$ |
| 197E-01 | . $2245-33$ | . $7515-61$ | . $23+E-02$ |
| 207E-01 | . $386 \mathrm{E}-03$ | . $790 \mathrm{OE}-01$ | . $410 \mathrm{E}-62$ |
| 21\%E-01 | . 368 E-03 | . $8295-61$ | . 468 E -02 |
| 226E-01 | . $306 \mathrm{E}-03$ | 867E-01 | 479E-02 |
| $336 E-01$ | . $3845-03$ | 306E-01 | $477 E-02$ |
| 246E-01 | . $335 \mathrm{E}-63$ | $944 E-61$ | . $361 \mathrm{E}-62$ |
| $255 E-01$ | . $353 \mathrm{E}-33$ | . $983 \mathrm{E}-61$ | . $325 \mathrm{E}-02$ |
| 265E-01 | . 371 E-03 | .102 | . $236 E-42$ |
| 274E-01 | . $475 E-03$ | .106 | . $1665-02$ |
| 284E-01 | .417E-63 | .110 | . $1515-92$ |
| 294E-01 | . $358 \mathrm{E}-3$ | .114 | .117E-02 |
| $303 E-4 i$ | . 365 - 63 | 118 | $104 E-42$ |
| $3185-01$ | . $553 \mathrm{E}-3$ | .121 | . 837E-03 |
| 327E-01 | . $606 E-43$ | .125 | $.753 E-93$ |
| 356E-01 | . 593E-03 | 129 | . $662 \mathrm{E}-03$ |


| 133 | . 571E-03 |
| :---: | :---: |
| 137 | . $453 \mathrm{E}-03$ |
| 141 | . $467 \mathrm{E}-03$ |
| 145 | . $426 \mathrm{E}-03$ |
| 148 | . $345 \mathrm{E}-03$ |
| 152 | . $330 \mathrm{E}-03$ |
| 156 | . $317 \mathrm{E}-03$ |
| 150 | . $245 \mathrm{E}-03$ |
| 164 | . $246 \mathrm{E}-03$ |
| 168 | . $229 \mathrm{E}-03$ |
| 172 | . $233 \mathrm{E}-03$ |
| 175 | . $230 \mathrm{E}-03$ |
| 179 | . $169 \mathrm{E}-63$ |
| 183 | .185E-03 |
| 187 | . $167 \mathrm{E}-93$ |
| 191 | -154E-03 |
| 195 | . 152 E -03 |
| 196 | 137E-03 |
| 202 | . 120 E -0 |
| 200 | . $155 \mathrm{E}-03$ |
| 210 | $108 \mathrm{E}-03$ |
| 214 | -116E-03 |
| 218 | -117E-G3 |
| 222 | -105E-03 |
| 225 | .976E-04 |
| 229 | .947E-04 |
| 233 | . $971 \mathrm{E}-04$ |
| 237 | -907E-04 |
| 241 | . $974 \mathrm{E}-04$ |
| 245 | 878 E |




POUER SPECTRAL FILE EPOHII
PROJECTHO 5610
CONFIGUATION B



602E-02 499E-03




POUER SPECTRAL FILE EPO481
TIME 12:48 BAY 271 OF 1983



POUER SPECTRAL FILE EPO491


2 SEGMEMTE OF 16384 SGMPLES AT 142.05 S/S TOTAL SAMPLE TIME $=230.7$ SEC
$N * O / H \quad N * S(N)$
$\mathrm{N} * \mathrm{O} / \mathrm{U} \quad \mathrm{H} * \mathrm{SCN}$
$H * 0 . U \quad H * S(N)$


| 00 |  |
| :---: | :---: |
| 313E-04 | 746 E |
| 626E-04 | 650 E - |
| 940E-04 | 962E-05 |
| $141 E-03$ | 438E-34 |
| 204E-03 | 960E-45 |
| 266E-03 | 288E-04 |
| 329E-03 | 229E-04 |
| 392E-03 | $132 E-04$ |
| 454E-03 | 306E-64 |
| 548E-03 | 374E-04 |
| 673E-03 | $159 \mathrm{E}-3$ |
| 799E-03 | $1375-03$ |
| 924E-03 | 249E-03 |
| $1115-02$ | 138E-03 |
| 136E-02 | 160E-G3 |
| 161E-02 | $138 E-03$ |
| 186E-92 | $121 E-03$ |
| 211E-02 | 264E-03 |
| 236E-02 | 334E-03 |
| 262E-02 | 199E-03 |
| 287E-02 | 271E-03 |
| 312E-02 | $275 \mathrm{E}-03$ |
| 337E-92 | 448E-43 |
| 362E-02 | $430 \mathrm{E}-3$ |
| 387E-02 | 430E-03 |
| 424E-02 | $236 \mathrm{E}-43$ |
| 475E-02 | 389E-43 |
| 25E-02 | 444E-03 |
| 75E-02 | 367E-03 |
| 25E-02 | 461E-03 |


| $\begin{aligned} & 675 E-02 \\ & 725 E-02 \end{aligned}$ | $\begin{aligned} & 597 E-63 \\ & .512 E-63 \end{aligned}$ |
| :---: | :---: |
| 725E-02 | $.755 E-03$ |
| $8505-02$ | 676E-03 |
| -51E-C2 | 978E-03 |
| $105 E-01$ | 680E-03 |
| 115E-01 | 517 E -03 |
| 125E-01 | 668E-03 |
| $135 E-01$ | S41E-03 |
| 145E-01 | 812E-63 |
| $155 E-1$ | 634E-03 |
| 165E-01 | $455 E-6$ |
| 175 -01 | 592 E - 3 |
| $185 \mathrm{E}-01$ | 518E-63 |
| $195 E-01$ | $6 G E E-03$ |
| 205E-01 | 538E-0.3 |
| $215 E-01$ | 60OE-03 |
| 225E-01 | 571E-03 |
| $235 E-01$ | 467E-03 |
| 245E-01 | $517 \mathrm{E}-63$ |
| 255E-01 | 507E-63 |
| 265E-01 | 485E-03 |
| 275E-01 | 577E-83 |
| 286E-01 | 432 E - 03 |
| 296E-01 | 427E-03 |
| $306 E-01$ | 391E-63 |
| $316 E-01$ | $4815-63$ |
| 331 E-01 | $465 E-03$ |
| $3415-61$ | $445 E-03$ |
| 371E-01 | $369 E-03$ |
| 381E-01 | 345E-03 |


| 4iEE-01 | 402E-03 |
| :---: | :---: |
| $421 \mathrm{E}-01$ | 310E-03 |
| $451 E-01$ | . 375 -03 |
| $461 E-61$ | 370E-03 |
| 491E- ${ }^{\text {c }}$ | . $372 \mathrm{E}-03$ |
| 541E-01 | 346E-03 |
| $531 \mathrm{E}-01$ | 292E-03 |
| 541E-01 | 268E-03 |
| 571E-01 | 262E-03 |
| 581E-01 | 264E-03 |
| $611 E-01$ | 2695-03 |
| $621 E-61$ | 282E-03 |
| 661E-01 | 260E-03 |
| 701E-01 | 322E-93 |
| ? $42 E-01$ | 294E-03 |
| 782E-61 | 290E-03 |
| 322E-01 | 319E-03 |
| 862E-01 | 354E-03 |
| 902E-01 | 401E-03 |
| $942 \mathrm{E}-01$ | 404E-03 |
| 982E-61 | 368E-03 |
| 102 | 398E-03 |
| 106 | 345E-03 |
| 110 | 282E-43 |
| 114 | 226E-03 |
| 118 | 195E-03 |
| 122 | 173E-03 |
| 126 | 153E-03 |
| 130 | 148 E -03 |
| 134 | 123E-03 |


| 138 | 119E-03 |
| :---: | :---: |
| 142 | . $108 \mathrm{E}-03$ |
| 146 | . $103 \mathrm{E}-03$ |
| 154 | . 108 E-93 |
| 154 | .931E-04 |
| 158 | 830E-04 |
| 162 | .802E-04 |
| 166 | . 722 E -04 |
| 170 | . $773 \mathrm{E}-64$ |
| 174 | .642E-64 |
| 178 | 614E-04 |
| 182 | 604E-04 |
| 186 | 585E-04 |
| 190 | 482E-04 |
| 194 | 512E-04 |
| 198 | $425 \mathrm{E}-04$ |
| 202 | $435 \mathrm{E}-04$ |
| 206 | 444E-04 |
| 210 | . 376 E - 4 |
| 214 | . $738 \mathrm{E}-04$ |
| 218 | . $336 \mathrm{E}-04$ |
| 223 | . $327 \mathrm{E}-04$ |
| 227 | - 379 -04 |
| 231 | - $370 \mathrm{E}-64$ |
| 235 | . 3 35E-04 |
| 239 | - 3 34E-04 |
| 243 | . $354 \mathrm{E}-04$ |
| 247 | . $295 \mathrm{E}-04$ |
| 251 | . $362 \mathrm{E}-04$ |
| 255 | . $329 \mathrm{E}-04$ |






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[^1]:    FAt a larger width the response coefficient will decrease, but the reference moment $q A L$ will increase. The net effect on the product of these two depends on the slope of the load spectrum.

