DISSERTATION

MODELING OF STREAM/AQUIFER INTERACTION DURING ALLUVIAL WELL DEPLETION

Submitted by

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In partial fulfillment of the requirements

For the Degree of Doctor of Philosophy

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ABSTRACT OF DISSERTATION

MODELING OF STREAM/AQUIFER INTERACTION DURING ALLUVIAL WELL DEPLETION

The conjunctive utilization of surface water and groundwater has become increasingly important as the demand for water intensifies. Water rights decisions for groundwater supplies hydraulically interacting with adjacent surface water bodies require estimation of surface water depletion due to groundwater pumping. However, stream/aquifer exchange due to groundwater extraction of tributary water is commonly analyzed using analytical solutions that oversimplify physical conditions. More complex analytical solutions have recently been developed that include the effects of a streambed layer and stream partial penetration. Such solutions have also been suggested as a way to inversely estimate aquifer parameters and the streambed hydraulic conductivity from field aquifer tests. However, numerous limitations still exist in the application of these solutions.

Research is needed to develop improved analytical solutions and evaluate the significance of the more complex solutions for water rights decisions and for streambed conductivity estimation. The objectives of this research are to develop analytical models of stream/aquifer interaction during alluvial well depletion, evaluate their accuracy in the prediction of aquifer drawdown and stream depletion, and determine their ability to inversely estimate streambed conductivity. This research expands the applicability of recently proposed analytical solutions by considering finite stream width, complex aquifer flow scenarios (i.e., unconfined flow, pumping well partial penetration, delayed drainage from the unsaturated zone, well-bore storage, and well-skin effects), and unsaturated stream/aquifer exchange.

Analytical solutions are derived to account for each of the stream/aquifer scenarios. Differences between the prior analytical solutions, numerical models, and the solutions developed in this research are evaluated to determine conditions when specific stream/aquifer conditions become significant. The theory developed in this research is used to improve the simulation capabilities of a standard, numerical groundwater-flow model, MODFLOW, for simulating stream/aquifer exchange. A stream/aquifer analysis test is performed at the Tamarack State Wildlife Area in eastern Colorado to investigate the inverse estimation of the streambed conductivity using the analytical solutions developed in this research.

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DEDICATION

This dissertation is dedicated to my wife, Amanda, who has supported me tremendously throughout this research. Her support in assisting with general conceptual problems, mathematical derivations and fieldwork has been tremendous. May our lives together continue to be blessed with happiness and success.

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CHAPTER 1

INTRODUCTION

1.1 Background

Quantifying surface water/groundwater interaction in stream/aquifer systems has become an increasingly critical issue for water quantity and quality management and ecosystem habitat conservation. Surface water is commonly hydraulically connected to ground water, but the interactions are difficult to observe and measure. Water in streams and rivers exchange between the active river channel and subsurface pathways during natural flow conditions. Most streams are classified as gaining streams, whereby water in an unconfined aquifer supplements the streamflow. Water in this unconfined aquifer is generally considered a tributary groundwater supply. Water abstraction from wells adjacent to the stream can reduce streamflow by preventing water from recharging the river and/or by inducing a flux from the stream to the underlying aquifer, a result that is known as alluvial well depletion.

Pumping wells located in alluvial aquifers adjacent to a stream commonly provide for the irrigation needs of agricultural producers. The depletive effects on a stream caused by irrigation wells must be accounted for in administering water rights in the western United States. In addition, new water management strategies, such as managed recharge projects, are being utilized to manage hydraulically connected stream and groundwater supplies. Managed groundwater recharge attempts to redirect streamflow by diverting water from the river to recharge ponds during high-flow, low-demand periods. Water is pumped from the alluvial aquifer and generally transported through pipes or ditches to infiltration ponds. The infiltrated water returns to the river as subsurface flow. Recharge ponds are spatially located such that the subsurface flow augments streamflow during critical low-flow, high-demand periods. Quantification of the hydrological impact of such projects requires an in-depth understanding of stream/groundwater interaction.

Bank filtration is recognized as being a possible water treatment process for public water supplies. Bank filtration is defined as the use of systems of groundwater extraction wells to induce infiltration of surface water through the streambed and aquifer materials, taking advantage of the natural filtration processes. The success of bank filtration systems is dependent on the hydrologic exchange of streamflow and also on the biogeochemical transformations enhanced by the streambed (Gollnitz et al., 1997; Wang et al., 1995). Before bank filtration projects can be deemed applicable, an improved understanding of the aquifer conditions and hydraulic controls imposed by different pumping regimes is needed (Hiscock and Grischek, 2002; Schubert, 2002; Gollnitz et al., 1997). Different pumping regimes can significantly influence the patterns and rates of contaminant migration between streams and aquifers (Hiscock and Grischek, 2002; Schubert, 2002).

In summary, understanding the interaction between streams and tributary groundwater supplies and quantification of surface water depletion due to groundwater pumping are becoming increasingly important to understand. However, stream/aquifer exchange due to groundwater extraction of tributary water is commonly analyzed using analytical solutions that oversimplify physical conditions. Existing analytical models typically neglect important surface water/groundwater characteristics such as stream width, the presence of a streambed layer, stream partial penetration, and unsaturated stream/aquifer exchange. The alternatives to the use of such simplified analytical solutions are more complex analytical models that more appropriately represent the physical conditions characteristic of alluvial aquifer systems or sophisticated numerical models that require considerably more time, effort, and data.

Advantages of using analytical solutions include their use to calibrate the more sophisticated numerical solutions and to derive parameters critical to the simulation capability of the numerical model. Recently, analytical solutions have been developed to better represent stream/aquifer interactions by including the effects of both a streambed layer and stream partial penetration. Such solutions have also been suggested as a way to inversely estimate aquifer parameters and the streambed hydraulic conductivity, a parameter that controls the degree of hydraulic interaction between a stream and aquifer. However, numerous limitations still exist in the application of these solutions for deriving streambed conductivity estimates. Furthermore, in-situ measurements of the streambed hydraulic conductivity are possible, but the horizontal and vertical variations in the streambed cross-section create difficulties in deriving a single, representative estimate.

Research is needed to further improve analytical solutions and evaluate the significance of the more complex solutions for water rights decisions and for streambed conductivity estimation. These improved analytical models will better address the question of what percentage of flow at any time comes from the stream versus aquifer storage. Water groups throughout the western United States have recognized this research need. For example, a Technical Scope Committee for Colorado Senate Bill 96-74 (1998) developed a list of tasks for improving groundwater flow modeling. Two of the primary tasks include conducting field tests to obtain reliable estimates of streambed conductivity and improve the modeling of stream/aquifer interaction. The committee notes that sufficient improvement in stream/aquifer modeling requires improved measurements and estimation procedures for stream length and width, vertical streambed hydraulic conductivity, and thickness of the streambed layer.

1.2 Objective and Methodology

The objectives of this research are to develop analytical models of stream/aquifer interaction during alluvial well depletion, evaluate their accuracy in the prediction of aquifer drawdown and stream depletion, and determine their ability to inversely estimate streambed conductivity. This research expands the applicability of recently proposed analytical solutions by considering finite stream width, complex aquifer flow scenarios (i.e., unconfined flow, pumping well partial penetration, delayed drainage from the unsaturated zone, well-bore storage, and well-skin effects), and unsaturated stream/aquifer exchange. Differences between prior analytical solutions, numerical stream/aquifer models, and the analytical solutions developed in this research will be evaluated to determine conditions when specific stream/aquifer conditions become significant in impacting aquifer response and stream depletion. Development of such analytical solutions is expected to identify procedures for improving numerical models capable of simulating stream/aquifer exchange.

The following specific tasks have been identified in order to meet the above objective:

1. Evaluate existing analytical models for simulating drawdown and stream depletion against MODFLOW numerical simulations using hypothetical alluvial aquifer systems.
2. Develop an improved analytical solution that accounts for distributed stream recharge.
3. Investigate the significance of assuming confined flow with fully penetrating wells of insignificant diameter versus accounting for more complex flow scenarios, such as unconfined flow, partially penetrating wells, delayed drainage from the unsaturated zone, well-bore storage and well-skin effects.
4. Evaluate the effect of aquifer parameter uncertainty on analytical streambed conductivity estimations.
5. Expand existing analytical models to account for unsaturated flow between the stream and aquifer.

6. Improve methodologies used by numerical models in accounting for unsaturated stream/aquifer exchange.
7. Perform a field test to investigate the use of complex analytical solutions for inversely estimating streambed conductivity using observed aquifer drawdown during a pumping test next to a stream.

1.3 Organization of this Dissertation

Chapters 2 through 7 of this dissertation are written to be stand-alone technical papers. Each chapter presents an introduction beyond that given in this chapter, literature review, proposed methodology, discussion, and conclusions as they pertain to each topic. Some of the primary figures and equations will be repeated in different chapters to allow each chapter to stand-alone. The individual chapters are derived from the following manuscripts either published or in review in journals or conference proceedings:

- Chapter 2: Fox, G.A. and D.S. Durnford. 2001. Investigation of analytical and numerical models for simulating surface water/groundwater interaction. *Proceedings of the 21st Annual Geophysical Union Hydrology Days,* April 2-5, Fort Collins, CO.
- Chapter 3: Fox, G.A., P. DuChateau, and D.S. Durnford. 2002. Analytical model for aquifer response incorporating distributed pumping-induced stream leakage. *Ground Water* 40(4): 378-384.

- Chapter 4: Fox, G.A. and D.S. Durnford. 2003. STRMAQ: Semi-analytical model for stream/aquifer interaction in confined/unconfined aquifers. *Ground Water* (In Review).
- Chapter 5: Fox, G.A. and D.S. Durnford. 2002. Effect of aquifer parameter uncertainty on analytical estimates of streambed conductance using STRMAQ. *Proceedings of the 22nd Annual Geophysical Union Hydrology Days,* ed. J.A. Ramirez, 86-97.
- Chapter 6: Fox, G.A. and D.S. Durnford. 2003. Unsaturated flow in stream/aquifer conjunctive systems. *Advances in Water Resources* (Accepted).
- Chapter 6: Fox, G.A. 2003. Improving MODFLOW's RIVER package for unsaturated stream/aquifer flow. *Proceedings of the 23rd Annual Geophysical Union Hydrology Days,* ed., J.A. Ramirez. March 31-April 2.
- Chapter 7: Fox, G.A. 2003. Estimating streambed and aquifer parameters from a stream/aquifer analysis test. *Proceedings of the 23rd Annual Geophysical Union Hydrology Days,* ed., J.A. Ramirez. March 31-April 2.

Short but detailed descriptions for chapters 2 through 7 are given below. Chapter 8 summarizes the research work, gives a brief synopsis of the primary conclusions and presents recommendations for future research work.

Chapter 2: Evaluation of Existing Analytical Solutions

Recent analytical solutions in the literature incorporate stream partial penetration and streambed conductivity to more appropriately represent the physical conditions in alluvial aquifer systems during groundwater abstraction. The sensitivity of drawdown and stream leakage to values of streambed conductivity is investigated using a recently proposed analytical solution by Hunt (1999). The analytical solution is then compared to the most widely applied analytical solution for administering tributary groundwater rights. This solution assumes a stream that fully penetrates throughout the saturated thickness of the aquifer without a semipervious streambed layer (Theis, 1941). Hunt's (1999) solution is also compared to MODFLOW numerical groundwater model (McDonald and Harbaugh, 1988). The sensitivity analysis and the model comparisons use characteristic values obtained from aquifer tests at the Tamarack State Wildlife Area in Logan County, Colorado, along the South Platte River. Also, the range of stream partial penetrations over which Hunt's (1999) analytical solution remains valid is determined based on comparisons to the MODFLOW numerical model.

Chapter 3: New Analytical Model Incorporating Distributed Stream Leakage

An analytical solution is developed based on modeling a stream of finite width rather than modeling the stream as a line source. The modified analytical solution more accurately represents the effects of a streambed layer, stream partial penetration, and stream width. The modified analytical solution is evaluated against the Hunt (1999) and

Theis (1941) analytical solutions to determine the impact of finite stream width on stream/aquifer analytical models, utilizing hypothetical case studies for general characteristics of stream/aquifer systems. The proposed solution is also compared to a MODFLOW numerical model simulating stream/aquifer interaction during alluvial well depletion when stream width is significant.

Chapter 4: STRMAQ: A Semi-Analytical Solution for Confined and Unconfined Aquifers

Prior analytical solutions for stream/aquifer interaction during alluvial well depletion assume confined flow with fully penetrating pumping wells of infinitesimal diameter (Hunt, 1999; Hantush, 1965; Theis, 1941). A computer program, called STRMAQ, is developed that incorporates well functions for confined and unconfined aquifer flow under the complexities introduced by pumping well partial penetration, delayed drainage from the unsaturated zone, well-bore storage, and well-skin effects with analytical solutions for stream/aquifer interaction.

Prior analytical models are then compared to STRMAQ based on dimensionless drawdown functions for confined and unconfined aquifers. An analysis is performed for each flow characteristic (i.e., unconfined flow, pumping well partial penetration, delayed drainage from the unsaturated zone, well-skin, and well-bore storage) within STRMAQ. Such comparisons allow a determination of the relevant importance of complex flow parameters on stream/aquifer interaction. STRMAQ is also compared to a MODFLOW numerical model for a hypothetical stream/aquifer system. STRMAQ is used to analyze field data from a stream/aquifer analysis test performed along the Doyleston Drain south of Christchurch, New Zealand (Hunt et al., 2001).

Chapter 5: Sensitivity of Streambed Conductance Estimates to Aquifer Parameter Uncertainty

The development of analytical solutions capable of simulating the presence of a streambed layer suggests that these solutions could be used with a matching-point method to estimate aquifer and streambed parameters from observed aquifer drawdown during alluvial well depletion (Hunt, 1999). However, questions exist about the utility of the proposed estimation procedure and about the possibility of subjective judgment leading to variability in predicted values of aquifer and streambed parameters. In other words, can analytical models that incorporate streambed parameters accurately determine these parameters? What are the expected errors associated with streambed conductivity estimates from analytical solutions versus uncertainty in aquifer parameters?

This research uses an inverse nonlinear routine for estimating streambed conductivity from stream/aquifer analysis tests. A program is developed to automatically estimate streambed hydraulic conductivity from measured drawdown response in defined confined and water table aquifers. STRMAQ is used to generate drawdown profiles for several hypothetical aquifer and streambed scenarios for a stream and aquifer remaining in hydraulic connection, a fully penetrating pumping well, insignificant well-bore storage and no well-skin effects. The program is evaluated based on the ability to predict equivalent values of streambed hydraulic conductivity from the generated drawdown profiles when erroneous trial values of streambed hydraulic conductivity are input. The effect of uncertainty in aquifer parameters, such as aquifer conductivity, specific yield, and storage coefficient, on estimates of the streambed hydraulic conductivity is then investigated.

Chapter 6: Unsaturated Flow in Stream/Aquifer Conjunctive Systems

When the water level in the aquifer is at or above the bottom of the streambed, saturated flow governs the seepage from the stream. However, if the water table in an alluvial aquifer falls a sufficient distance below a semipervious streambed, the head losses in this less conductive layer may cause the region beneath the stream to become unsaturated. Unsaturated flow transforms streams from constant head boundaries to constant flux boundaries, impacting the biogeochemistry in the region beneath the streambed and above the alluvial aquifer. Prior analytical solutions for alluvial well depletions fail to model unsaturated flow between the streambed and water table. Numerical models capable of simulating stream/aquifer interaction, such as MODFLOW, attempt to account for unsaturated stream/aquifer flow but utilize simplifying assumptions about this flow condition.

This research discusses the conditions under which saturated or unsaturated flow occurs and the characteristics of each flow regime. The effect of unsaturated flow is illustrated for the case of stream leakage induced by a well pumping from an aquifer that is hydraulically interacting with a partially penetrating stream. The stream/aquifer interaction routine within MODFLOW is updated based on the theory developed in this research to more appropriately simulate unsaturated stream/aquifer interaction. An approximate semi-analytical solution is proposed for aquifer drawdown and stream depletion that accounts for saturated/unsaturated flow.

Chapter 7: Analysis of Stream/Aquifer Interaction at the Tamarack State Wildlife Area

The Tamarack State Wildlife Area along the South Platte River in eastern Colorado is being investigated as a managed recharge project to redirect streamflow for water quantity management. Two primary surface water/groundwater interactions exist at Tamarack: (1) between the South Platte River and alluvial aquifer and (2) between backwater sloughs, or secondary river channels, and the alluvial aquifer. Knowledge of stream/slough/groundwater interaction could play a vital role in the design and operation of the recharge system.

Quantifying the magnitude of stream/aquifer interactions requires estimates of the streambed hydraulic conductivity. Silt, clay, and organic materials are often deposited in streams resulting in the streambed having a lower hydraulic conductivity than the underlying alluvial aquifer. The interaction of the stream/slough/groundwater at the Tamarack research site is investigated using falling-head permeameter tests to quantify the streambed and sloughbed hydraulic conductivity. Also, a stream/aquifer analysis test is performed at the site to investigate this complex surface water/groundwater interaction during alluvial well depletions. The stream/aquifer analysis test is also used to investigate the use of complex analytical solutions, such as those presented in Chapters 3 and 4, for inversely estimating streambed conductance from observed aquifer drawdown.

Drawdown is measured in observation wells located between a pumping well and the slough channel during groundwater extraction. The measured drawdown response is matched to the analytical solutions developed in this research. Predicted aquifer and streambed parameters from the analytical solutions are compared to reported parameter values from historical site characterizations and measured streambed and sloughbed conductivity from the permeameter tests.

CHAPTER 2

EVALUATION OF EXISTING ANALYTICAL SOLUTIONS

2.1 Introduction

Conjunctive use of groundwater and surface water has become a critical issue in the management of water resources in the western United States. Water rights decisions for tributary groundwater supplies are typically based on analytical solutions that oversimplify physical conditions. These analytical models neglect characteristics such as a streambed layer of lower hydraulic conductivity than the alluvial aquifer, stream partial penetration and aquifer heterogeneity. Hunt (1999) developed an analytical model that incorporates the effects of a streambed layer and stream partial penetration to predict aquifer drawdown and stream depletion during alluvial well depletion.

This chapter presents a comparison of the model by Hunt (1999) that incorporates partial stream penetration and streambed conductivity with the Theis (1941) equation and a numerical groundwater model, MODFLOW (McDonald and Harbaugh, 1988). Conclusions are drawn concerning the importance of incorporating streambed parameters and the usefulness of Hunt's more complex solution in modeling groundwater/surface water interactions.

2.1.1 *Theis and Hantush Analytical Models*

The Theis equation (Theis, 1941) is typically used for water rights administration. The equation assumes an infinitely long, straight, completely penetrating stream in a homogeneous aquifer, as shown in Figure 2.1. Changes in water table elevations are assumed small compared to the saturated thickness of the aquifer, leading to the Dupuit flow assumption (Freeze and Cherry, 1979). No parameters account for a semipervious streambed layer. Applying the principle of superposition, image wells are used to simulate a constant head boundary condition at the stream, and drawdown is given by:

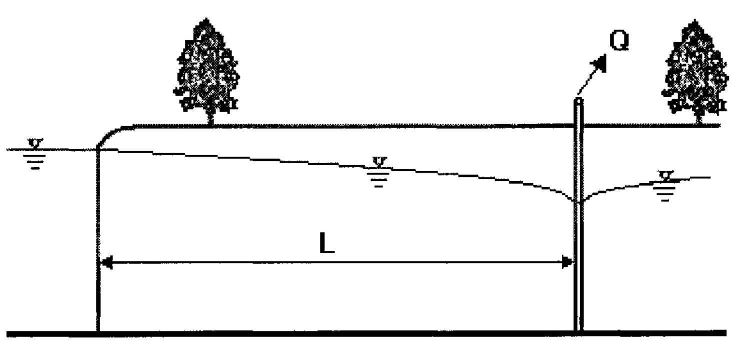


Figure 2.1 - Physical situation simulated by Theis (1941) analytical solution. Modified from Hunt (1999).

$$s_w(u) = \frac{Q}{4\pi T}\{W(u) - W(u_i)\} \quad (2.1)$$

where s_w is the drawdown in the semi-infinite domain [L], Q is the pumping rate [L^3T^{-1}], T is the transmissivity of the aquifer [L^2T^{-1}], and W(u) and W(u_i) are the well functions for the real and image well, respectively.

In addressing limitations of the Theis equation, Hantush (1965) developed an analytical model that considers the effects of a semipervious streambed. The semipervious streambed was represented as a vertical layer of lower conducting material extending throughout the saturated thickness of the aquifer. The Hantush model was based on the principal of additional seepage resistance due to this semipervious layer. Seepage resistance extends the distance between the well and stream by an 'effective distance'. Therefore, the streambed layer of lower hydraulic conductivity creates a flow resistance, R, proportional to

$$R = \frac{K}{(K_{sb} / M)} \quad (2.2)$$

where K and K_{sb} are the hydraulic conductivity of the aquifer and streambed [LT^{-1}], respectively, and M is the streambed thickness [L].

Existing methodologies based on these simplified analytical solutions are widely applied in administering tributary groundwater rights (Spalding and Khaleel, 1991). For example, the USGS has standardized a procedure for analyzing the timing of flows between an aquifer and stream called the stream depletion factor (SDF). Jenkins (1968) originally developed the SDF in studying stream depletion by groundwater pumping. The SDF methodology makes several simplifying assumptions about the flow regime and stream/aquifer characteristics and, in general, makes use of the Theis (1941) solution. SDF is defined as the time when the volume of stream depletion reaches 28% of the total volume pumped. Mathematically, SDF is expressed as

$$SDF = \frac{L^2 S}{T} \tag{2.3}$$

where L is the perpendicular distance from the pumped well to the stream [L], S is the storage coefficient or specific yield, and T is the transmissivity of the aquifer [L^2T^{-1}].

2.1.2 Evaluation of Analytical and Numerical Models

Recently, the limitations of prior analytical models in simulating stream/aquifer interactions have been the subject of considerable research (Conrad and Beljin, 1996; Sophocleous et al., 1995; Spalding and Khaleel, 1991). Spalding and Khaleel (1991) study streambed conductance and stream partial penetration and conclude that analytical models "...often neglect conditions that exist in typical stream/aquifer systems." Sophocleous et al. (1995) further emphasize such points in evaluating the performance of analytical stream/aquifer solutions with MODFLOW for varying stream/aquifer characteristics. Semipervious streambed layers as quantified by streambed/aquifer hydraulic conductivity contrast, stream partial penetration, and aquifer heterogeneity are determined to be the most important assumptions in analytical solutions.

Field studies document that streambed conductivity one to three orders of magnitude lower than aquifer conductivity is common (Calver, 2001; Larkin and Sharp, 1992). Sophocleous et al. (1995) conclude that the streambed layer is the most significant assumption in analytical and numerical modeling of stream/aquifer interaction. Sophocleous et al. (1995) evaluate the effects of the resistance formulation of analytical models (i.e., Hantush model) against MODFLOW, holding the effective distance and streambed thickness constant such that streambed layer effects can be modeled by the hydraulic conductivity ratio (K/K_{sb}). Results indicate that for conductivity contrasts greater than 1/100, analytical solutions over predict stream depletion by 58-71%. Conrad and Beljin (1996) also indicate large deviations (near 20%) between models when streambed conductance (K_{sb}) is two orders of magnitude less than the aquifer hydraulic conductivity (K).

Typical alluvial aquifer systems consist of a shallow, relatively wide, partially penetrating stream. Larkin and Sharp (1992) note that stream penetration ranges from 5-50% for alluvial aquifer systems throughout the United States. Sophocleous et al. (1995) model a hypothetical stream/aquifer system with partial penetration of 10% using MODFLOW and compare the results to analytical solutions assuming insignificant partial penetration. Results indicate that stream partial penetration reduces stream leakage in general, with analytical solutions differing by 10 to 61% from the MODFLOW solutions. These differences are the result of numerical models deriving a significant component of well yield from the side of the river opposite of the well. Accounting for partial penetration is therefore necessary to adequately predict the interaction of surface water and groundwater.

2.1.3 Hunt's Analytical Model

As demonstrated by Conrad and Beljin (1996) and Sophocleous et al. (1995), the Theis (1941) and Hantush (1965) analytical models fail to adequately represent the physical conditions representative of alluvial aquifer systems. Hunt (1999) develops an analytical model that incorporates streambed conductance and stream partial penetration in the simulation of a pumping well located near a stream, as shown in Figure 2.2.

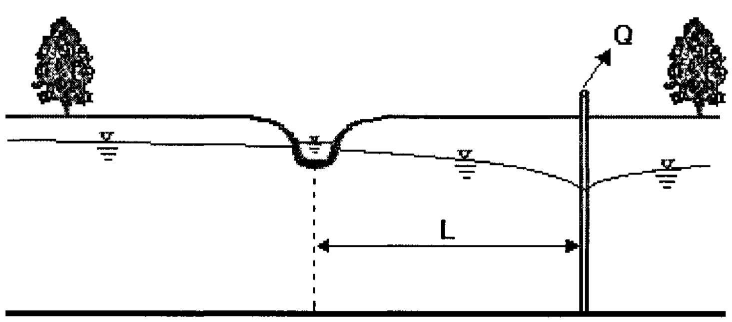


Figure 2.2 - Physical situation simulated by Hunt's (1999) analytical solution. Modified from Hunt (1999).

Hunt's model assumes a homogeneous, isotropic aquifer of infinite extent, with Dupuit flow. The model also assumes that changes in water surface elevation due to pumping are small, and vertical and horizontal streambed cross-sections are small compared to the aquifer saturated thickness. Seepage flow rates from the river into the aquifer are assumed linearly proportional to the head gradient between the aquifer and stream, dependent upon a streambed conductance parameter, λ [LT^{-1}]. Streambed conductance is a function of the streambed hydraulic conductivity. The product of the streambed conductance and the head gradient between the aquifer and river gives the stream leakage per unit length of river. Hunt derived both a streamflow depletion equation (2.4) and drawdown equation (2.5) applicable throughout the infinite domain:

$$\frac{Q_s}{Q} = erfc\left(\sqrt{\frac{SL^2}{4Tt}}\right) - \exp\left(\frac{\lambda^2 t}{4ST} + \frac{\lambda L}{2T}\right) erfc\left(\sqrt{\frac{\lambda^2 t}{4ST}} + \sqrt{\frac{SL^2}{4Tt}}\right) \tag{2.4}$$

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{E_1\left[\frac{(L-x)^2 + y^2}{4Tt/S}\right] - \int_0^\infty e^{-\theta} E_1\left[\frac{(L+|x|+2T\theta/\lambda)^2 + y^2}{4Tt/S}\right] d\theta\right\} \tag{2.5}$$

where Q_s is the stream depletion rate [L^3T^{-1}], L is the distance between the well and the stream [L], E_1 is the well function, S is the aquifer storage coefficient, t is the time since the start of pumping [T], and x and y are the locations within the infinite domain with respect to a datum at the river on a perpendicular line with the well [L].

The streambed conductance in Hunt's solution, λ, is related to the streambed conductance used in MODFLOW (McDonald and Harbaugh, 1988). MODFLOW's streambed conductance, C [L^2T^{-1}], is a function of streambed hydraulic conductivity

(K_{sb}), length (L) and width (W) of the stream in the finite difference cell, and the depth of the streambed (M):

$$C = \frac{K_{sb} L W}{M} \tag{2.6}$$

MODFLOW calculates stream leakage as a product of C and the head gradient between the river and aquifer. The relationship between λ and C is given by

$$\lambda = \frac{C}{L} = \frac{K_{sb} W}{M} \tag{2.7}$$

2.2 Methodology

The sensitivity of Hunt's analytical model to varying streambed conductance is investigated based on predicted drawdown and stream leakage. Hunt's analytical solution is compared to the Theis (1941) analytical solution and MODFLOW numerical groundwater model. MODFLOW, in this comparison, simulates a homogeneous, isotropic aquifer, with a partially penetrating, infinite length stream of insignificant width.

The sensitivity analysis and the model comparisons are based on characteristic values obtained from aquifer tests at the Tamarack Ranch State Wildlife Area in Logan County, Colorado, along the South Platte River. These characteristic values are input into the models to compute drawdown profiles from the river to the well and also stream leakage as a function of well pumping rate versus time. The following parameter values are representative of the stream/aquifer properties at the Tamarack site:

- Aquifer properties: K=300 ft d^{-1} (1.1×10^{-1} cm-s^{-1}), S=0.2, and b=100 ft (30.5 m), where b is the saturated thickness of the aquifer.
- Pumping properties: L=1000 ft (304.8 m) and single well pumping at 3.9 cfs (1.1×10^{-1} m^3-s^{-1}).
- Stream properties: 5% partial penetration of the stream, ratio of stream width (W) to thickness of the streambed (M) of 25 ft-ft^{-1} (m-m^{-1}), and K_{sb}=1 to 10 ft-d^{-1} (3.5×10^{-4} to 3.5×10^{-3} cm-s^{-1}).

The streambed conductivity value is the most uncertain of the input parameters. Therefore, the streambed hydraulic conductivity, K_{sb}, is varied from 1.0 to 100.0 ft-d^{-1} (3.5×10^{-4} to 3.5×10^{-2} cm-s^{-1}).

Hunt's (1999) solution also assumes that the horizontal and vertical dimensions of the streambed cross-section are small compared to the saturated thickness of the aquifer. Therefore, a primary question involves the range of stream partial penetrations that Hunt's solution appropriately simulates. Several MODFLOW simulations were performed for the South Platte case with varying degrees of partial penetration (100%, 25%, 10%, 5%, and 1%).

2.3 Results

The sensitivity of Hunt's analytical model to streambed conductivity is investigated based on predicted drawdown and stream leakage. As noted previously, Hunt's streambed conductance, λ, is a function of the streambed hydraulic conductivity, K_{sb}, as shown in equation (2.7). The sensitivity of predicted stream leakage and aquifer drawdown to variations in the streambed conductance are illustrated in Figures 2.3 and 2.4, respectively.

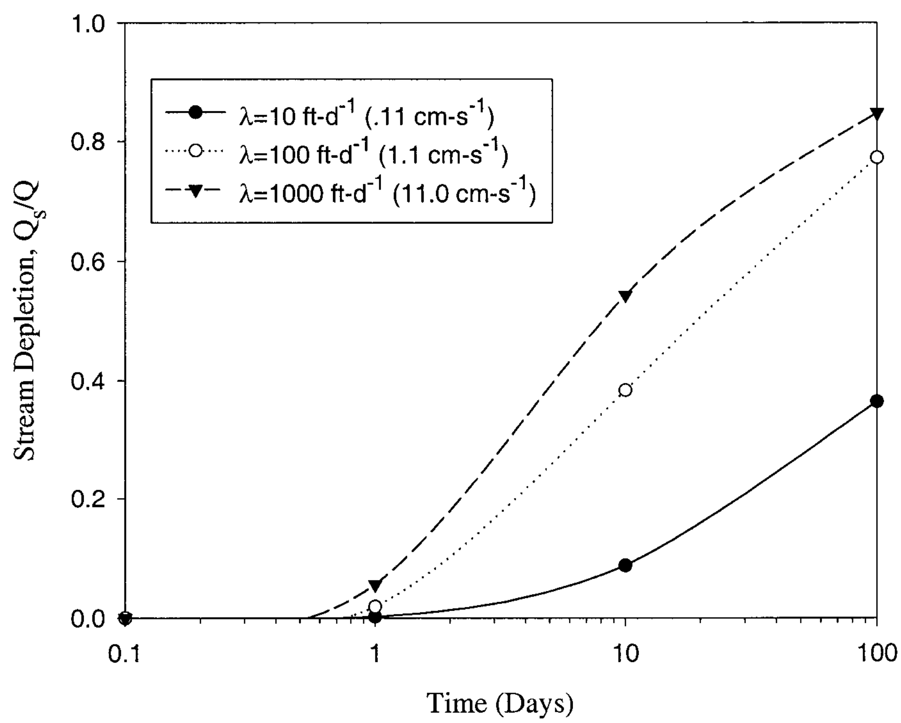


Figure 2.3 - Sensitivity of Hunt's analytical solution to varying leakage coefficient (λ) for stream leakage.

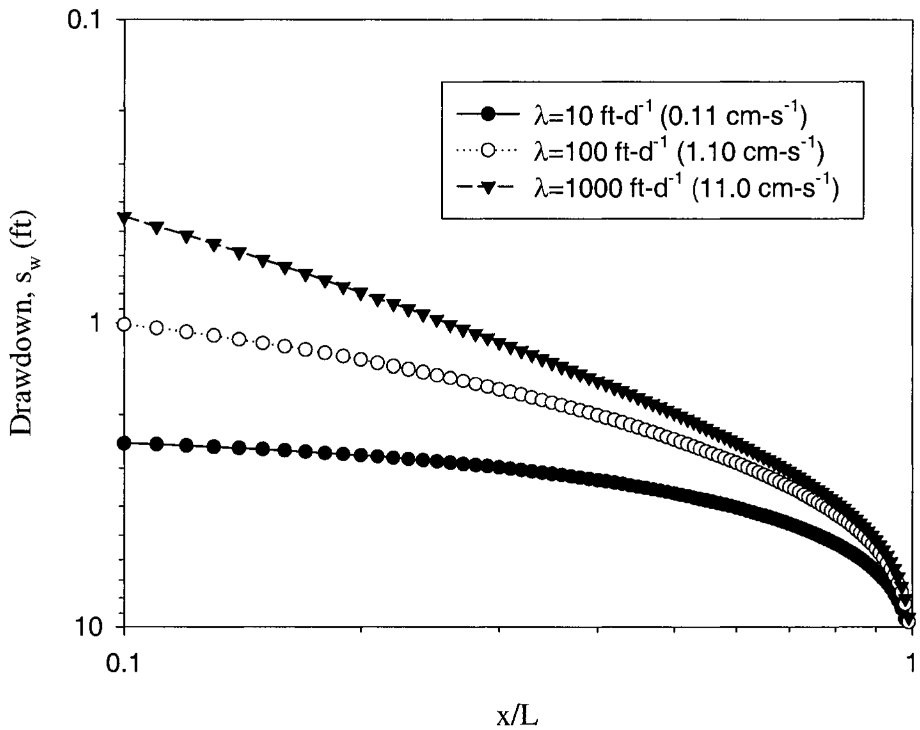


Figure 2.4 - Sensitivity of Hunt's analytical solution to varying leakage coefficient (λ) for drawdown versus x/L at t=100 days of pumping.

As the streambed conductivity is decreased, the fraction of the discharge contributed by leakage (Q_s/Q) decreases due to the lower conductance of the streambed. A decrease in the streambed conductance results in a corresponding increase in the observed drawdown at the river (i.e., x/L=0.1) with drawdowns approaching 2-3 ft (0.6-0.9 m) for a streambed conductance less than 10 ft-d^{-1} (0.11 cm-s^{-1}), as shown in Figure 2.4. MODFLOW numerical simulations demonstrated similar sensitivity to changes in streambed conductance for varying streambed hydraulic conductivity. Therefore, assuming an incorrect value of the streambed conductance could result in considerable error in the predictions generated by both Hunt's solution and using a MODFLOW numerical model.

Hunt's model is then compared with the Theis (1941) solution and a MODFLOW numerical simulation for the South Platte case. MODFLOW is assumed to be the best solution for the conditions being modeled. Comparisons are based on drawdown profiles from the stream to the pumping well and stream leakage as a function of time. The drawdown profile predicted by Theis (1941), Hunt (1999), and MODFLOW numerical model after 100 days of pumping for the Tamarack site using the same hypothetical conditions is shown in Figure 2.5, assuming K_{sb}=10 ft-d^{-1} (3.5E-3 cm-s^{-1}). Note that the Theis (1941) solution models a river that fully penetrates throughout the saturated thickness of the aquifer without a streambed layer. Therefore, the Theis (1941) solution assumes that as much water can be obtained from the river as possible to satisfy the aquifer stress created by the pumping well. Negligible drawdown is predicted at the river throughout the simulation.

Both Hunt's (1999) solution and MODFLOW generate significantly different drawdowns than the Theis (1941) solution. Hunt's (1999) solution appears to better match MODFLOW simulations of drawdown and stream leakage as compared to the more frequently used Theis (1941) solution, especially when the influence of stream partial penetration and the presence of a semipervious streambed layer are physically important. The Theis (1941) solution, and therefore the SDF methodology, accurately simulates aquifer drawdown and stream depletion as long as the stream fully penetrates throughout the saturated thickness of the aquifer and the conductivity of the streambed layer is not significantly different than the aquifer hydraulic conductivity.

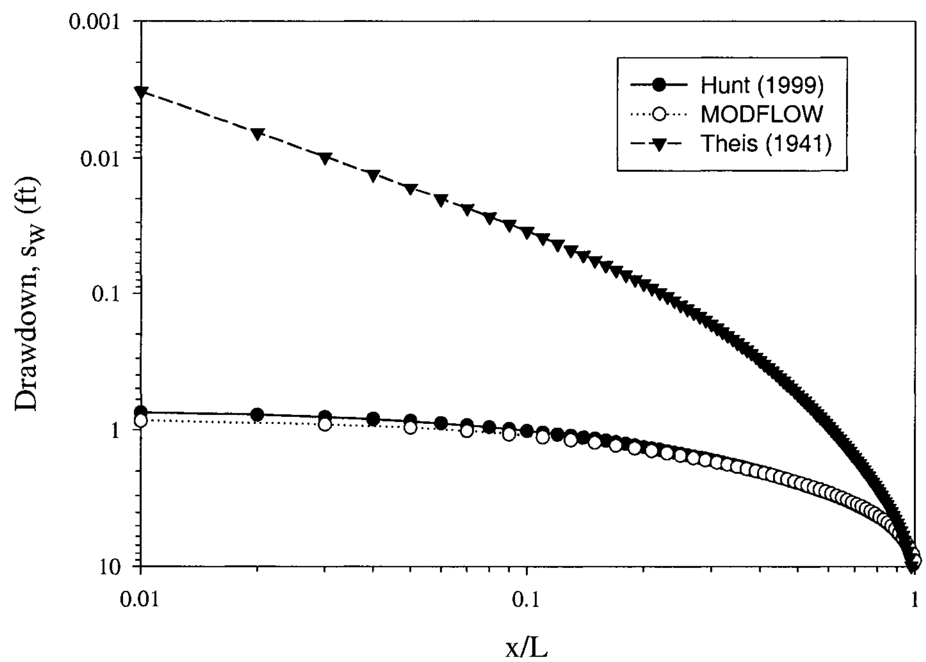


Figure 2.5 - Comparison between Theis, Hunt, and MODFLOW drawdown profiles along a 1000 ft transect from the river (x=0.01) to the well (x=L) for South Platte case at t=100 days of pumping.

Generally, Hunt's analytical solution for drawdown matches the MODFLOW numerical model for all K_{sb} values considered. Therefore, insignificant differences are observed between the two models in the predicted leakage as a function of time. Cases can exist however when predictions of stream leakage can be significantly different. For example, Hunt's model assumes that no matter what the drawdown, the stream remains in hydraulic connection with the aquifer (Rushton, 1999). MODFLOW's RIVER package, which simulates the stream/aquifer interaction, stabilizes leakage from the stream when the water table falls below the bottom of the streambed (Anderson and Woessner, 1992). As a result, Hunt's analytical solution can over-predict stream leakage when the water level in the aquifer falls below the bottom of the streambed. This phenomenon will be discussed in more detail in Chapter 6.

Hunt's analytical solution also accounts for stream partial penetration. The degree of stream partial penetration is varied within the MODFLOW model to determine the range of penetrations over which the analytical solutions are valid. MODFLOW simulations for full stream penetration of the aquifer result in drawdown profiles more similar to the Theis (1941) solution than Hunt's (1999) solution. MODFLOW simulations for less than 10% partial penetration most closely match Hunt's (1999) analytical solution based on comparisons between leakage and drawdown. Therefore, Hunt's analytical solution more appropriately simulated alluvial aquifer conditions when stream partial penetration is less than 10%.

2.4 Summary and Conclusions

Hunt (1999) proposes an analytical solution that more appropriately represents the physical conditions of alluvial aquifer systems under conditions of alluvial well depletions than Theis (1941). This research compares Hunt's (1999) solution for stream/aquifer interactions with the Theis (1941) analytical solution, which is widely applied in administering tributary groundwater rights, and with a MODFLOW numerical groundwater flow model, which is commonly used to simulate stream/aquifer interaction under complex hydraulic conditions. Hunt's (1999) analytical solution considers stream partial penetration and leakage through a streambed layer. The sensitivity of Hunt's (1999) solution to streambed hydraulic conductivity demonstrates that accurate measurements of streambed conductivity are required to accurately simulate effects of pumping on stream depletion. Hunt's solution matches MODFLOW simulations of drawdown and stream leakage better than the Theis (1941) equation. Hunt's solution most closely matches MODFLOW when the degree of partial penetration of the stream is less than 10%. Therefore, Hunt's (1999) solution provides a valuable stream/aquifer analysis tool when the conditions of stream partial penetration and the presence of a streambed layer are important. The Theis (1941) solution, and therefore the SDF methodology, adequately simulates aquifer drawdown and stream depletion as long as the stream fully penetrates throughout the saturated thickness of the aquifer and the conductivity of the streambed layer is not significantly different than the aquifer hydraulic conductivity.

CHAPTER 3

ANALYTICAL MODEL FOR AQUIFER RESPONSE INCORPORATING DISTRIBUTED STREAM LEAKAGE

3.1 Introduction

Pumping from a well next to a stream reduces groundwater levels and withdrawals water from the stream in response to the aquifer stress. Early analytical models of drawdown due to pumping from an aquifer in hydraulic connection with a stream typically assume a completely penetrating stream. The most well known solution is the Theis solution (Theis, 1941). This solution assumes an infinitely long, straight, completely penetrating stream in a homogeneous, isotropic, confined aquifer stressed by a pumping well extracting water at a constant discharge rate, as shown in Figure 3.1 (a). Glover and Balmer (1954) expressed the Theis solution in terms of stream depletion.

Hantush (1965) developed an analytical model that included the effects of a semipervious streambed. The semipervious streambed was represented as a vertical layer of lower conducting material extending throughout the saturated thickness of the aquifer, as shown in Figure 3.1 (b). The Hantush model uses the principal of additional seepage resistance, which is a function of the ratio of the aquifer hydraulic conductivity (K) to the streambed hydraulic conductivity (K_{sb}).

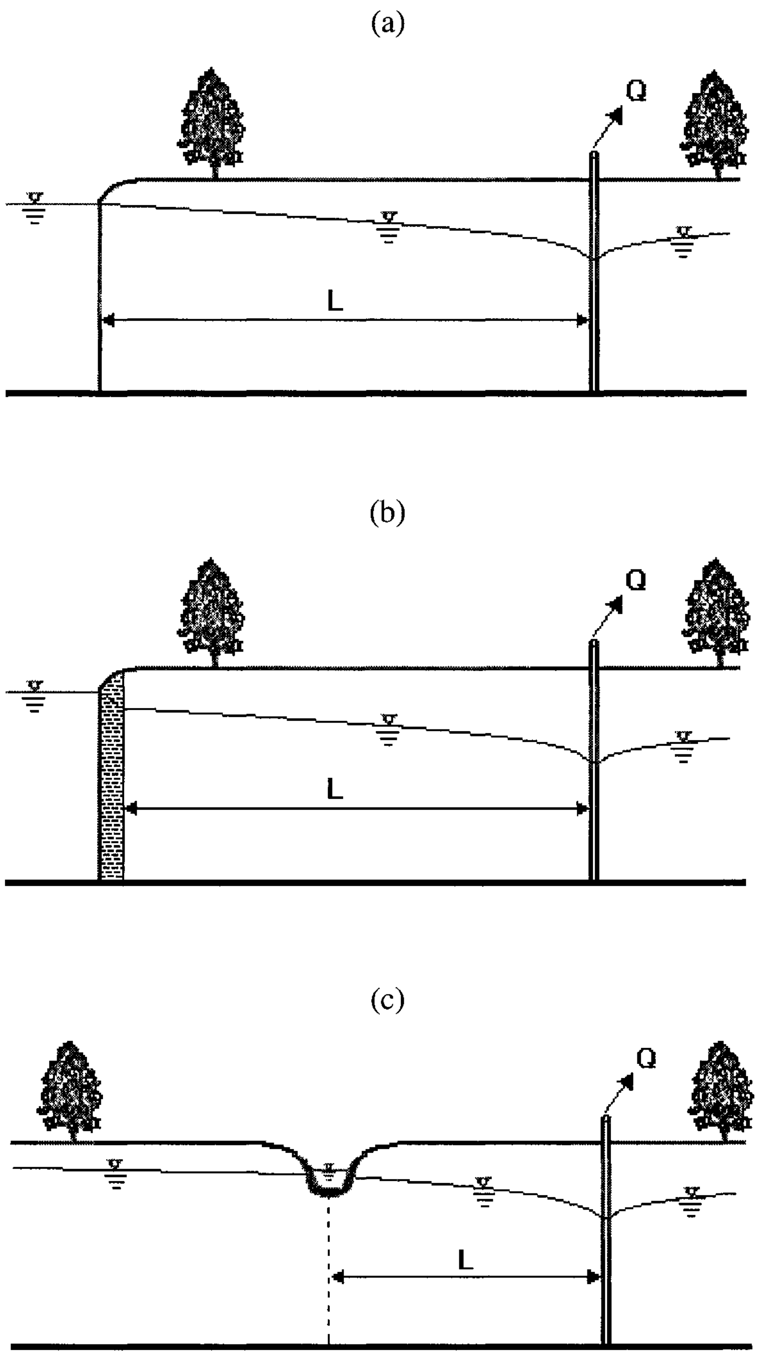


Figure 3.1 - Conceptualization of the problem and notation considered by (a) Theis (1941), (b) Hantush (1965), and (c) Hunt (1999). Modified from Hunt (1999). L is the distance between the stream and pumping well. Q is the discharge rate of the pumping well.

Recently, stream/aquifer interaction has been the subject of considerable research. Streambed conductance, stream partial penetration, and aquifer heterogeneity are the most important assumptions in analytical solutions (Conrad and Beljin, 1996; Sophocleous et al., 1995; Spalding and Khaleel, 1991). Field studies have documented that streambed conductivity is often as much as one to three orders of magnitude lower than aquifer conductivity (Larkin and Sharp, 1992). For conductivity contrasts greater than two orders of magnitude, analytical solutions over predict stream depletion by 58 to 71% (Sophocleous et al., 1995).

Conrad and Beljin (1996) showed large differences (near 20%) between analytical and numerical models when K_{sb} was two orders of magnitude less than K. Stream partial penetration reduced stream leakage in general, with analytical solutions differing by 10 to 61% from the MODFLOW (McDonald and Harbaugh, 1988) simulations. These differences are the result of numerical models deriving a significant component of well yield from the side of the river opposite of the well. Stream partial penetration can range from 5 to 50% for alluvial aquifer systems throughout the United States (Larkin and Sharp, 1992). Zlotnik and Huang (1999) considered the effects of stream partial penetration and low-permeability sediments on aquifer response to stream-stage hydrographs.

Hunt (1999) developed solutions for water table drawdown and stream depletion that incorporated streambed conductance and stream partial penetration. Hunt's model assumes a homogeneous, isotropic aquifer of infinite horizontal extent. The model also assumes that changes in water table elevation due to pumping are small and that vertical and horizontal streambed cross sections are small compared to the aquifer's saturated thickness, as shown in Figure 3.1 (c). Seepage from the river into the aquifer is assumed linearly proportional to the head gradient between the aquifer and stream. This assumption remains valid only when the stream remains in hydraulic connection with the aquifer (Rushton, 1999). Otherwise, the stream becomes perched above the water table and stream depletion becomes constant.

The governing equation for Hunt's (1999) model is the following two-dimensional, partial differential equation:

$$T\left(\frac{\partial^2 s_w}{\partial x^2}+\frac{\partial^2 s_w}{\partial y^2}\right)=S\frac{\partial s_w}{\partial t}-Q\delta(x-L)\delta(y)+\lambda s_w\delta(x) \qquad (3.1)$$

where x and y are the coordinates within the infinite domain with respect to a datum at the river on a perpendicular line with the well [L], t is the time since the start of pumping [T], T is transmissivity or the product of the aquifer hydraulic conductivity (K) and the saturated thickness of the aquifer (b) [L^2T^{-1}], s_w is drawdown [L], S is the storage coefficient, Q is the constant pumping rate [L^3T^{-1}], δ is the Delta Dirac function, L is the distance between the pumping well and the stream [L], and λ is the streambed leakance coefficient [LT^{-1}]. The leakance coefficient (λ) is expressed as a function of streambed hydraulic conductivity (K_{sb}), the stream width (W) and thickness of the streambed layer (M):

$$\lambda=\frac{K_{sb}W}{M} \qquad (3.2)$$

The initial condition is that the system is initially at rest and the boundary conditions are that no drawdown occurs an infinite distance from the pumping well. Hunt (1999) derived an equation for drawdown (s_w) applicable throughout the infinite domain:

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{ E_1\left[\frac{(L-x)^2+y^2}{4Tt/S}\right] - \int_0^\infty e^{-\theta} E_1\left[\frac{\left(L+|x|+2T\theta/\lambda\right)^2+y^2}{4Tt/S}\right] d\theta \right\} \tag{3.3}$$

where E_1 is the exponential integral or well function (Theis, 1935). Tables of values and polynomial approximations to the well function are widely available (e.g., Freeze and Cherry, 1979). Hunt's closed form solution to this problem assumes insignificant stream width.

Hunt's model is shown to be sensitive to K_{sb} in Chapter 2. Therefore, accurate measurements of streambed hydraulic conductivity are required to accurately simulate effects of pumping on aquifer drawdown (Christensen, 2000). In Chapter 2, it is also shown that Hunt's solution was found to match MODFLOW simulations of drawdown significantly better than the Theis (1941) equation. Hunt et al. (2001) evaluated the solution in a field test, in which they suggested that the assumptions on λ require that the ratio of distance between the pumping well and the stream (L) to stream width (W) be much greater than unity (i.e., L/W>>1). Hunt et al. (2001) assume L/W=20 was appropriate, "...in which case a stream of finite width can be expected upon physical grounds to have the same effect upon drawdowns as an infinitely long line source."

Considering a finite width stream should better determine conditions when stream width becomes significant in affecting stream/aquifer interaction. The hypothesis of this research is that finite stream width is significant in predicted aquifer response for pumping wells located adjacent to partially penetrating streams. This research presents an analytical solution for aquifer drawdown due to pumping from a well located adjacent to a finite width stream and evaluates the effect of finite stream width in modeling aquifer drawdown in response to pumping. The analytical solution in this research models recharge from the stream as a distributed source (i.e., across a finite stream width) rather than as a line source.

3.2 Proposed Analytical Model

3.2.1 Derivation

The analytical model for aquifer response to a distributed stream leakage incorporates the finite dimensions of stream width. Instead of using the Delta Dirac function, $\delta(x)$, as Hunt did on the λ term in the governing differential equation as shown in equation (3.1), the solution in this research uses the difference between two Heaviside functions, $H_v(x)$. The modified partial differential equation is

$$T\left(\frac{\partial^2 s_w}{\partial x^2}+\frac{\partial^2 s_w}{\partial y^2}\right)=S\frac{\partial s_w}{\partial t}-Q\delta(x-L)\delta(y)+\lambda s_w\{H_v(x+w)-H_v(x-w)\} \qquad (3.4)$$

where w is the half-width of the stream (i.e., w=W/2). In Hunt's model, $\delta(x)$ is different from zero only at W=0, leading to the restriction of insignificant stream width. The difference between the two $H_v(x)$ functions allows leakage to occur along the entire stream width, and the function equals zero outside of the stream boundaries.

The origin of the coordinate system is at the center of the stream (x=0) and along a perpendicular line from the pumping well to the stream (y=0), as shown in Figure 3.2. Note that this coordinate system is defined to mathematically express the governing differential equations within four separate domains:

- Domain I: pumping well side of the stream past the pumping well ($L<x<\infty$)
- Domain II: pumping well side of the stream between the stream and pumping well ($w<x<L$)
- Domain III: beneath the stream ($-w<x<w$)
- Domain IV: non-pumping well side of the stream ($-\infty<x<-w$)

The boundary conditions are different for each region within the infinite domain. For $L<x<\infty$, the governing partial differential equation can be modified as the two $H_v(x)$ functions cancel one another for all x. Similar reasoning results in the following sets of partial differential equations for their corresponding domains:

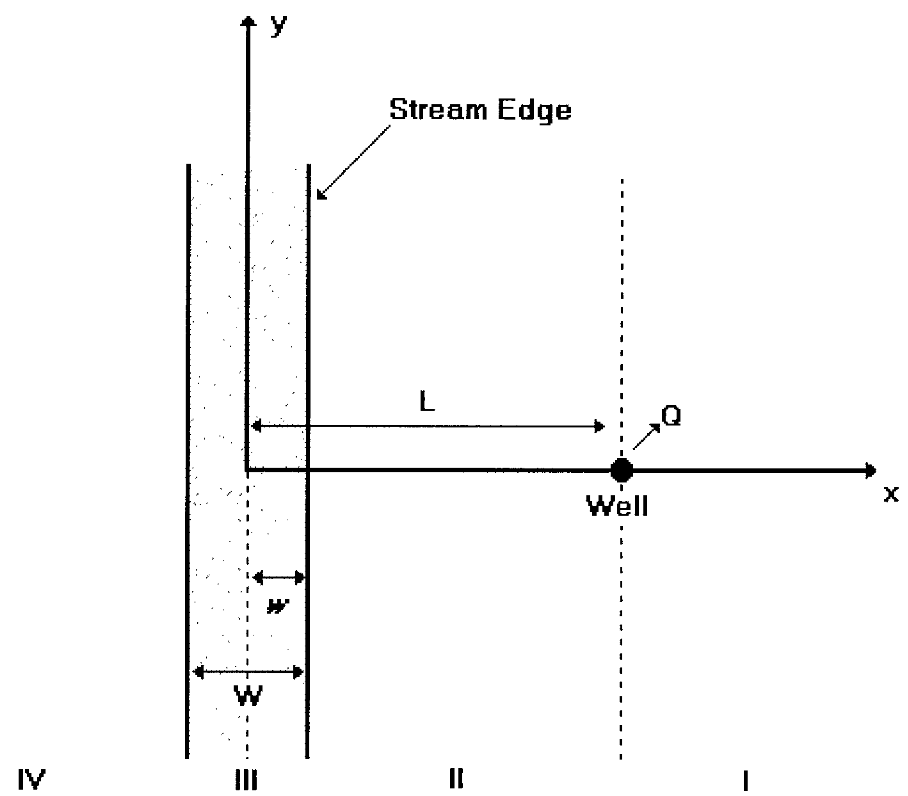


Figure 3.2 - Coordinate system and variable definition for the proposed analytical model. Roman numerals represent solution domains. L is the distance between the stream and pumping well. Q is the discharge rate of the pumping well. W is the width of the stream and w is the half-width of the stream.

Domain I: $$T\left(\frac{\partial^2 s_{w,0}}{\partial x^2}+\frac{\partial^2 s_{w,0}}{\partial y^2}\right)=S\frac{\partial s_{w,0}}{\partial t} \qquad L<x<\infty \quad y\in R \quad t>0 \tag{3.5}$$

Domain II: $$T\left(\frac{\partial^2 s_{w,1}}{\partial x^2}+\frac{\partial^2 s_{w,1}}{\partial y^2}\right)=S\frac{\partial s_{w,1}}{\partial t} \qquad w<x<L \quad y\in R \quad t>0 \tag{3.6}$$

Domain III: $$T\left(\frac{\partial^2 s_{w,2}}{\partial x^2}+\frac{\partial^2 s_{w,2}}{\partial y^2}\right)=S\frac{\partial s_{w,2}}{\partial t}+\lambda s_{w,2} \quad -w<x<w \quad y\in R \quad t>0 \tag{3.7}$$

Domain IV: $$T\left(\frac{\partial^2 s_{w,3}}{\partial x^2}+\frac{\partial^2 s_{w,3}}{\partial y^2}\right)=S\frac{\partial s_{w,3}}{\partial t} \quad -\infty<x<-w \quad y\in R \quad t>0 \tag{3.8}$$

The following condition must hold true, which can be achieved by imposing a discontinuity at the well:

$$T\left(\frac{\partial^2 s_{w,i}}{\partial x^2}+\frac{\partial^2 s_{w,i}}{\partial y^2}\right)=S\frac{\partial s_{w,i}}{\partial t}-Q\delta(x-L)\delta(y) \quad x>w \quad y\in R \quad t>0 \tag{3.9}$$

Fourier and Laplace transforms are used to derive an analytical solution for drawdown at locations between the pumping well and the stream. The analytical solution is derived from the mathematical model outlined in equations (3.4) through (3.9). For i=0...3, letting $u_k(x,y,p)$ be the Laplace transform in t of $s_{w,i}$, and $U_k(x,\alpha,p)$ be the Fourier transform in y of u_k, equations (3.5) through (3.8) are rewritten as:

$$-U_1''(x,\alpha,p)+d_1^2U_1(x,\alpha,p)=\frac{Q}{pT}\delta(x-L) \quad w<x<L \tag{3.10}$$

$$-U_2''(x,\alpha,p)+d_2^2U_2(x,\alpha,p)=0 \quad -w<x<w \tag{3.11}$$

$$-U_3''(x,\alpha,p)+d_1^2U_3(x,\alpha,p)=0 \quad -\infty<x<-w \tag{3.12}$$

where,

$$d_1^2 = \alpha^2 + \frac{Sp}{T}$$
$$d_2^2 = d_1^2 + \frac{\lambda}{T} \qquad (3.13)$$

Integrating equations (3.10) through (3.12) yields

$$U_1(x,\alpha,p) = C_0 e^{-d_1 x} \quad x > L$$
$$U_1(x,\alpha,p) = C_1 e^{-d_1 x} + C_2 e^{d_1 x} \quad w < x < L \qquad (3.14)$$

$$U_2(x,\alpha,p) = C_3 e^{-d_2 x} + C_4 e^{d_2 x} \quad -w < x < w \qquad (3.15)$$

$$U_3(x,\alpha,p) = C_5 e^{d_1 x} \quad x < -w \qquad (3.16)$$

U(x,α,p) and its derivative must be continuous at x=-w and x=w. Also, U(x,α,p) must be continuous at x=L and have a Delta Dirac discontinuity at x=L. The constants C_0 through C_5 can be determined from these continuity relationships.

The domain between the stream and pumping well (i.e., Domain II) is of primary interest to hydrogeologists. The constants C_1 and C_2 are required to derive the drawdown function in this domain. These constants can be expressed in the following form:

$$C_1 = -\frac{Q}{pT}\frac{d_2^2 - d_1^2}{2d_1}\left[\frac{1 - e^{-4wd_2}}{(d_1 + d_2)^2 - e^{-4wd_2}(d_1 - d_2)^2}\right]e^{d_1(2w-l)}$$
$$C_2 = \frac{Q}{pT}\frac{e^{-d_1 l}}{2d_1} \quad (3.17)$$

so that,

$$U_1 = \frac{Q}{pT}\frac{e^{-d_1(L-x)}}{2d_1} - \frac{Q}{pT}\left[\frac{d_2^2 - d_1^2}{2d_1}\left(\frac{1 - e^{-4wd_2}}{(d_1 + d_2)^2 - e^{-4wd_2}(d_1 - d_2)^2}\right)e^{-d_1(x-2w+L)}\right] \quad (3.18)$$

Derivation of the analytical model requires the inversion of the Fourier and Laplace transforms. In order to simplify the inversion process, U_1 is expressed in the following form:

$$U_1 = \frac{Q}{pT}\frac{e^{-d_1(L-x)}}{2d_1} - \frac{Q}{pT}\left[\frac{d_2^2 - d_1^2}{2d_1}\left(\frac{e^{2wd_1} - e^{2wd_1-4wd_2}}{(d_1 + d_2)^2 - e^{-4wd_2}(d_1 - d_2)^2}\right)e^{-d_1(L+x)}\right] \quad (3.19)$$

The exponential terms within the parentheses of equation (3.19) are simplified through the use of a Taylor series expansion about w=0. A three-term Taylor series representation results in the following expression for U_1:

$$U_1 = \frac{Q}{pT}\frac{e^{-d_1(L-x)}}{2d_1} - \frac{Q}{pT}\left[\frac{d_2^2 - d_1^2}{2d_1}\left(\frac{w}{d_1} + \frac{d_1^2 - d_2^2}{d_1^2}w^2\right)e^{-d_1(L+x)}\right] \quad (3.20)$$

Using the definitions of d_1 and d_2 in equation (3.13), several terms within the brackets of equation (3.20) can be simplified. Substituting these simplifications into equation (3.20) and rearranging results in the following expression:

$$U_1 = \frac{Q}{2T}\frac{e^{-d_1(L-x)}}{pd_1} - \frac{Q}{2T}\left[\left(\frac{\lambda w}{T}\frac{1}{pd_1^2} - \frac{\lambda^2 w^2}{T^2}\frac{1}{pd_1^3}\right)e^{-d_1(L+x)}\right] \tag{3.21}$$

The successive Fourier and Laplace transform of the first term in equation (3.21) results in the familiar Theis (1935) solution:

$$F^{-1}\left\{L^{-1}\left(\frac{e^{-d_1(L-x)}}{pd_1}\right)\right\} = \frac{1}{2\pi}E_1\left[\frac{(L-x)^2 + y^2}{4Tt/S}\right] \tag{3.22}$$

An inversion formula for the second term in equation (3.21) for U_1 follows from

$$\frac{d}{dx}\left(\frac{e^{-d_1(L+x)}}{pd_1^2}\right) = \frac{e^{-d_1(L+x)}}{pd_1} \tag{3.23}$$

hence,

$$\frac{e^{-d_1(L+x)}}{pd_1^2} = \int_x^\infty \frac{e^{-d_1(L+z)}}{pd_1}dz \tag{3.24}$$

and,

$$F^{-1}\left\{L^{-1}\left(\frac{e^{-d_1(L+x)}}{pd_1^2}\right)\right\} = \int_x^\infty \frac{1}{2\pi}E_1\left[\frac{(L+z)^2 + y^2}{4Tt/S}\right]dz \tag{3.25}$$

Similar reasoning is used to derive an inversion formula for the third term:

$$F^{-1}\left\{L^{-1}\left(\frac{e^{-d_1(L+x)}}{pd_1^3}\right)\right\} = \int_x^{\infty}\int_z^{\infty}\frac{1}{2\pi}E_1\left[\frac{(L+\zeta)^2+y^2}{4Tt/S}\right]d\zeta\,dz \tag{3.26}$$

Equations (3.22), (3.25), and (3.26) result in the drawdown function for any (x,y,t) where w<x<L, referred to as the Fox-DuChateau-Durnford (FDD) analytical solution:

$$\begin{aligned} s_w(x,y,t) &= \frac{Q}{4\pi T}E_1\left[\frac{(L-x)^2+y^2}{4Tt/S}\right] - \frac{Q}{4\pi T}\left(\frac{\lambda w}{T}\right)\int_x^{\infty}E_1\left[\frac{(L+x')^2+y^2}{4Tt/S}\right]dx' \\ &+ \frac{Q}{4\pi T}\left(\frac{\lambda w}{T}\right)^2\int_x^{\infty}\int_{x'}^{\infty}E_1\left[\frac{(L+\zeta)^2+y^2}{4Tt/S}\right]d\zeta\,dx' \end{aligned} \tag{3.27}$$

Additional terms to the Taylor series expansion can be included to derive more complex solutions. Such higher term expansions will use the same inversion formulas along with the following inversion formula for the n$^{\text{th}}$ power of d_1:

$$F^{-1}\left\{L^{-1}\left(d_1^n\,\frac{e^{-d_1(L+x)}}{pd_1}\right)\right\} = \frac{d^n}{dx^n}\left(\frac{1}{2\pi}E_1\left[\frac{(L+x)^2+y^2}{4Tt/S}\right]\right) \tag{3.28}$$

The mathematical model in equation (3.27) is evaluated numerically. Because the exponential integral (E_1), or well function, is a smooth curve, integration can be performed using straightforward numerical techniques, such as the trapezoidal rule (Cheney and Kincaid, 1994). The analytical solution simplifies to three terms all relating to the well-known Theis well function.

3.2.2 Mathematical Behavior

The distributed stream recharge modifies the Theis (1935) solution through two separate terms when using a three-term Taylor series expansion. The first term is similar to the modification proposed by Hunt (1999) and is referred to as the first order streambed integral (FOSI). The second term represents an additional modification of the Theis (1935) solution, becoming increasingly important as the stream half-width (w) increases. This term is referred to as the second order streambed integral (SOSI). The analytical solution simplifies to the Theis (1935) solution when $w=0$. If $w<1.0$ m, results approach those from Hunt's (1999) solution.

Mathematically, the FOSI and SOSI terms have similar shapes, with the difference between the two functions subtracting from the Theis (1935) contribution to the analytical solution. The contribution of each of these terms to the dimensionless drawdown function is shown in Figure 3.3 for $L/W=100$ and $\lambda L/T=1$. The drawdown profile predicted by the FDD analytical solution has a similar shape to the drawdown profile predicted by Hunt's solution. The analytical solution predicts greater drawdown than the Theis (1941) solution with image well theory. However, the FDD analytical solution with a significant finite width predicts less drawdown than Hunt's solution. This is discussed further in the next section.

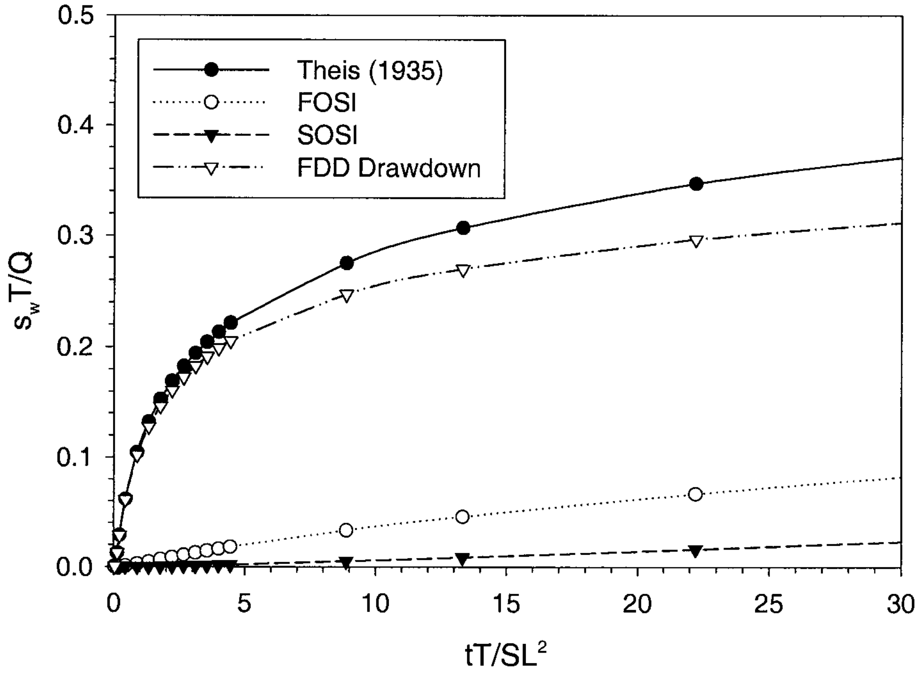


Figure 3.3 - Mathematical behavior of the Theis (1935), First Order Streambed Integral (FOSI), Second Order Streambed Integral (SOSI), and FDD drawdown function for L/W=100 and $\lambda L/T=1$. $s_w T/Q$=dimensionless drawdown. tT/SL^2=dimensionless time.

Utilization of the Taylor series expansion creates a range of times over which the solution remains stable. The stability of the solution is a function of the stream width: instability occurs at earlier times as stream width increases. In general, as the availability of water from the distributed recharge source increases (i.e., greater stream width or streambed hydraulic conductivity), the analytical solution becomes unstable at earlier times. Identification of this instability is not difficult. Over time, the change in drawdown at some (x, y) location should decrease steadily to some "pseudo-steady" drawdown. The solution becomes unstable when the change in drawdown increases between time steps. Even though the solution is temporally limited, the temporal range of validity is within the range of times expected for most stream/aquifer analysis tests, especially for typical values of aquifer and stream parameters. Furthermore, the FDD analytical solution allows the determination of when finite stream width becomes significant and, therefore, when Hunt's (1999) analytical solution can accurately be applied.

3.3 Comparison of Analytical Models

The FDD and Hunt (1999) analytical models are compared using dimensionless drawdown functions defined by Hunt (1999). The dimensionless drawdown functions (s_wT/Q versus tT/SL^2) are dependent upon location within the (x, y) plane. Values are compared in this research for an (x, y) location close to the stream (x/L=0.2) and on a perpendicular line between the stream and pumping well (y=0). Results are compared for L/W ranging from 10 to 1000.

A comparison of dimensionless drawdown for the FDD analytical solution and Hunt's analytical solution is shown in Table 3.1 for L/W=100. Note that when $\lambda L/T=0$ (i.e., no recharge flux from the stream), the two analytical solutions are equivalent to the Theis (1935) equation. However, when $\lambda L/T$ is increased to relatively small values (i.e., 0.1-0.5), deviations between the two analytical solutions become significant (i.e., greater than 5%) as tT/SL^2 increases. Increased stream width results in greater deviations between the two analytical solutions. Figure 3.4 shows dimensionless drawdown for the Theis ($\lambda L/T=0$), Hunt, and FDD analytical solutions for L/W=200 and $\lambda L/T=0.1$.

Table 3.1 - Comparison of dimensionless drawdown at x/L=0.2 and y/L=0.0 between FDD analytical model and Hunt's (1999) analytical solution for L/W=100.

$\lambda L/T$	tT/SL^2	FDD s_wT/Q	Hunt s_wT/Q	% Difference*
0.0	0.1	6.87E-03	6.87E-03	0.00
0.0	0.2	2.47E-02	2.47E-02	0.00
0.0	0.5	6.83E-02	6.83E-02	0.00
0.0	0.7	8.87E-02	8.87E-02	0.00
0.0	0.9	1.05E-01	1.05E-01	0.00
0.0	1.0	1.12E-01	1.12E-01	0.00
0.1	0.1	6.86E-03	6.87E-03	0.10
0.1	0.2	2.45E-02	2.47E-02	0.49
0.1	0.5	6.65E-02	6.77E-02	1.79
0.1	0.7	8.55E-02	8.77E-02	2.52
0.1	0.9	1.00E-01	1.03E-01	3.15
0.1	1.0	1.07E-01	1.10E-01	3.43
0.5	0.1	6.82E-03	6.85E-03	0.45
0.5	0.2	2.40E-02	2.45E-02	2.03
0.5	0.5	6.17E-02	6.57E-02	6.02
0.5	0.7	7.77E-02	8.39E-02	7.39
0.5	0.9	9.01E-02	9.79E-02	8.00
0.5	1.0	9.55E-02	1.04E-01	8.08

* % Difference = {(Hunt-FDD)/Hunt}*100

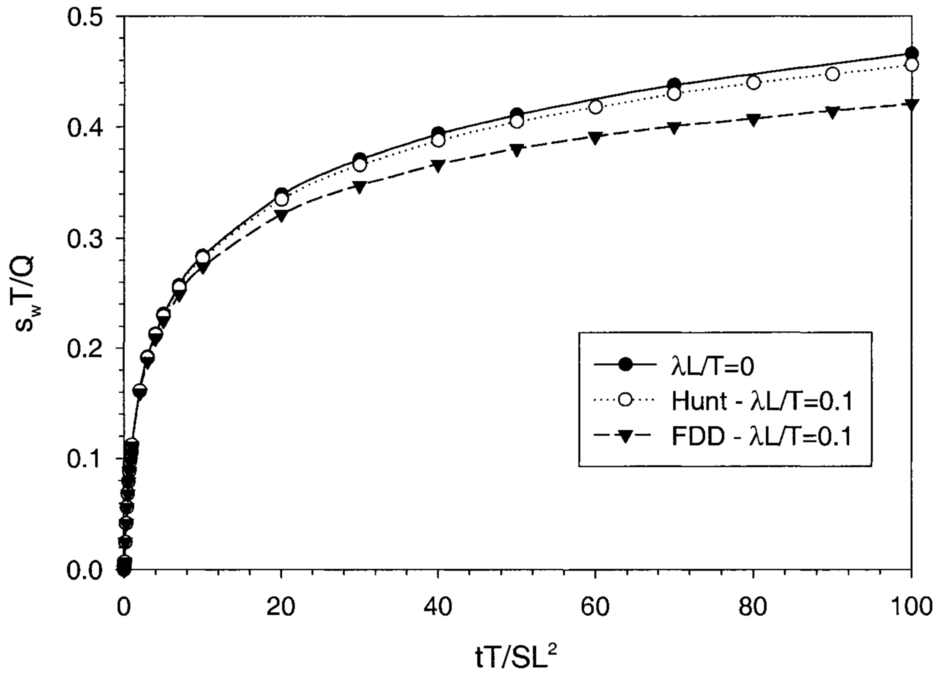


Figure 3.4 - Dimensionless drawdown function (s_wT/Q) for an observation well located at x/L=0.2 and y/L=0.0 for Hunt (1999) and FDD analytical solutions for L/W=200 and λL/T=0.1.

The FDD, Hunt (1999), and Theis (1941) analytical solutions are also compared for a hypothetical alluvial aquifer system along the South Platte River in eastern Colorado. This alluvial aquifer consists of coarse sands and gravels in hydrologic connection with a finite width stream. Representative values of the aquifer and streambed parameters were obtained from aquifer tests and site characterizations at a research site. The case study utilized representative values of streambed parameters (K_{sb}=1.0 x 10^{-4} cm-s^{-1}, M=1.5 m) and a range of L/W from 10-100. Comparisons are based on predicted drawdown profiles after initiation of pumping at x/L=0.2 and y=0.

Figures 3.5, 3.6, and 3.7 compare the Theis (1941) equation, Hunt's solution (1999), and the FDD analytical model for L/W=50, L/W=25, and L/W=15, respectively. The Theis solution with image well theory is dependent on stream width in determination of L and therefore is slightly different in the three figures. When the ratio of distance between the stream and pumping well to the stream width (i.e., L/W) is greater than 50, the percent difference between Hunt's solution and the FDD analytical solution is small (i.e., less than 1% difference). The Hunt (1999) and FDD solutions plot on top of each other in Figure 3.5.

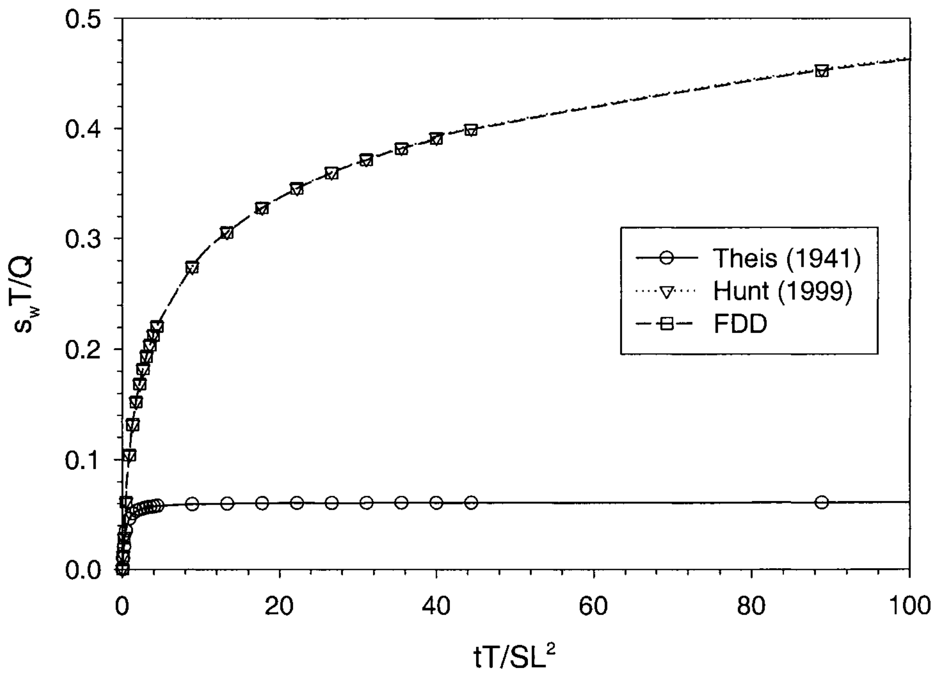


Figure 3.5 - Comparison of Theis (1941), Hunt (1999), and FDD analytical solutions for a hypothetical stream/aquifer system with x/L=0.2, y/L=0.0 and L/W=50.

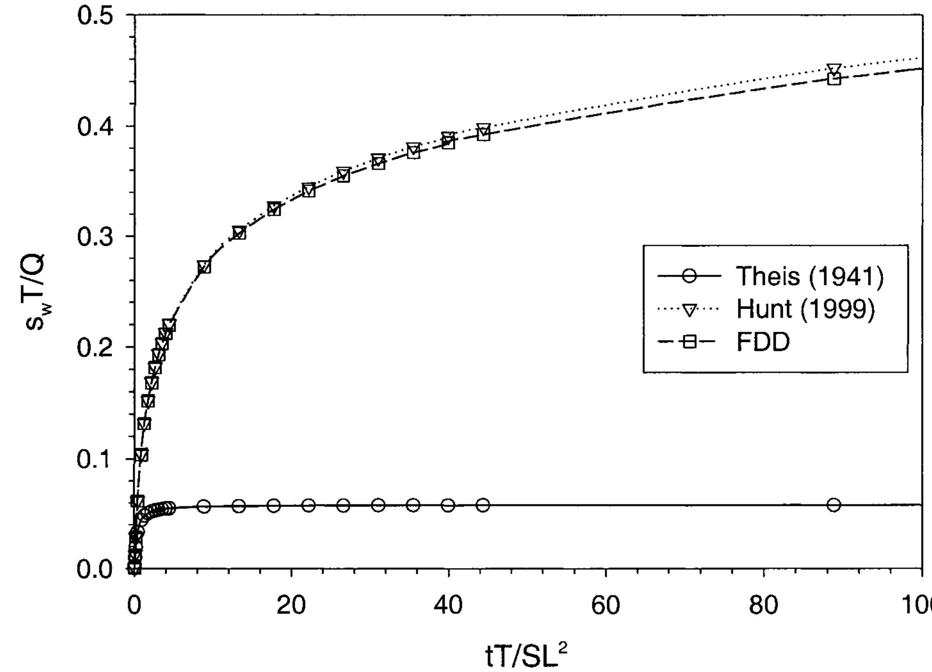


Figure 3.6 - Comparison of Theis (1941), Hunt (1999), and FDD analytical solutions for a hypothetical stream/aquifer system with x/L=0.2, y/L=0.0 and L/W=25.

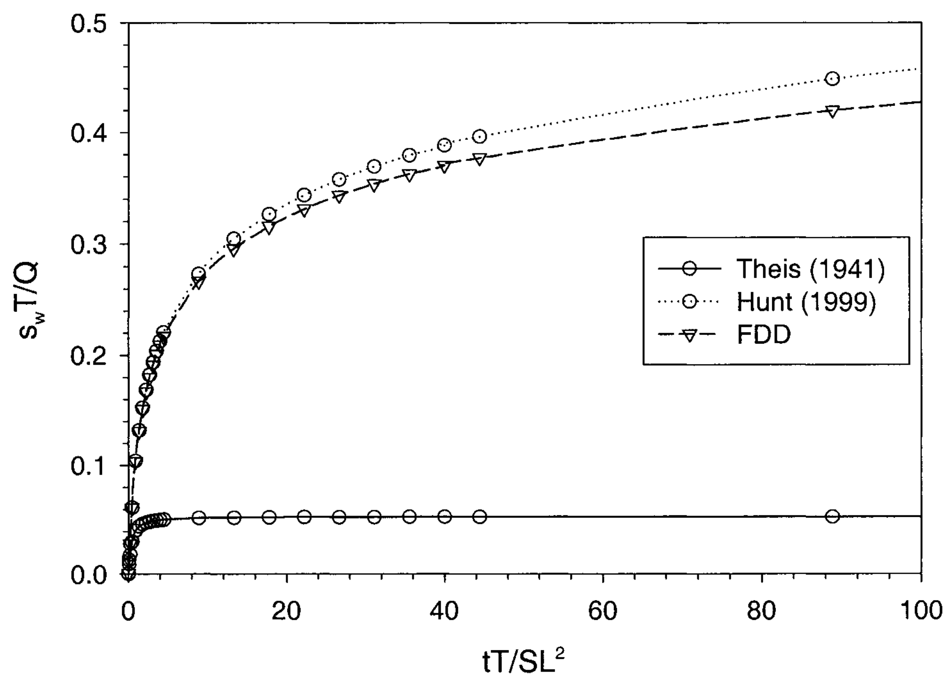


Figure 3.7 - Comparison of Theis (1941), Hunt (1999), and FDD analytical solutions for a hypothetical stream/aquifer system with x/L=0.2, y/L=0.0 and L/W=15.

The critical limit for modeling a finite width stream as a line source appears to be when L/W=25 for this hypothetical stream/aquifer system. The percent difference in dimensionless drawdown between the solutions increases as stream width increases. When L/W=25, Hunt's solution deviates from the FDD analytical model by less than 1% at $tT/SL^2=10$ and less than 5% at $tT/SL^2=100$. When L/W<15, Hunt's model deviates by approximately 5% at $tT/SL^2=10$ and by approximately 10% at $tT/SL^2=100$.

3.4 Comparison to MODFLOW

The finite-difference, numerical groundwater flow model, MODFLOW (McDonald and Harbaugh, 1988), models the interaction of streams and aquifers through the RIVER package. The RIVER package simulates seepage to the groundwater system through the use of a streambed conductance parameter, C:

$$C = \frac{K_{sb} LW}{M} \tag{3.29}$$

where K_{sb} is the hydraulic conductivity of the riverbed material [LT-1], L is the length of stream reach within each finite difference cell [L], W is the width of stream reach within each cell [L], and M is the thickness of the riverbed [L]. The flow between the stream and the groundwater system, Q_s, is given by:

$$Q_s = C(H_{RIV} - h_{i,j,k}) \tag{3.30}$$

where H_{RIV} is the head in the stream and $h_{i,j,k}$ is the head at the node in the cell underlying the stream reach. Equation (3.30) provides an acceptable approximation of the flow as long as water levels in the aquifer do not fall below the bottom of the streambed such that seepage from the stream ceases to depend on the head in the aquifer.

Hunt's (1999) solution is compared to the proposed FDD analytical solution based on drawdown simulated by the numerical groundwater flow model, MODFLOW, for the case where finite stream width is important. A pumping well with a constant discharge rate of 1000.0 m^3-day^{-1} was modeled in an unconfined aquifer (T=1000.0 m^2-day^{-1}, S=0.0001, S_y=0.15), 50 m from a partially penetrating stream with K_{sb}=0.1 m-d^{-1}, W=5 m, and M=1 m, as shown in Figure 3.8. MODFLOW simulated the drawdown response over 10 days in a head observation well located 10 m from the stream on a perpendicular line between the stream and pumping well.

The Hunt (1999) analytical solution and the proposed FDD solution simulated the same physical situation, and the predicted drawdown response in an observation well located 10 m from the stream were compared to the drawdown response simulated by MODFLOW, as shown in Figure 3.9. The Theis (1941) solution, which models a fully penetrating stream with no semipervious streambed, is also shown in Figure 3.9. Hunt's (1999) analytical solution over predicted the drawdown in the observation well, while the FDD most closely matched the numerically simulated drawdown response. These results further emphasize that stream width may be an important consideration in the analytical modeling of pumping wells located close to streams with significant width.

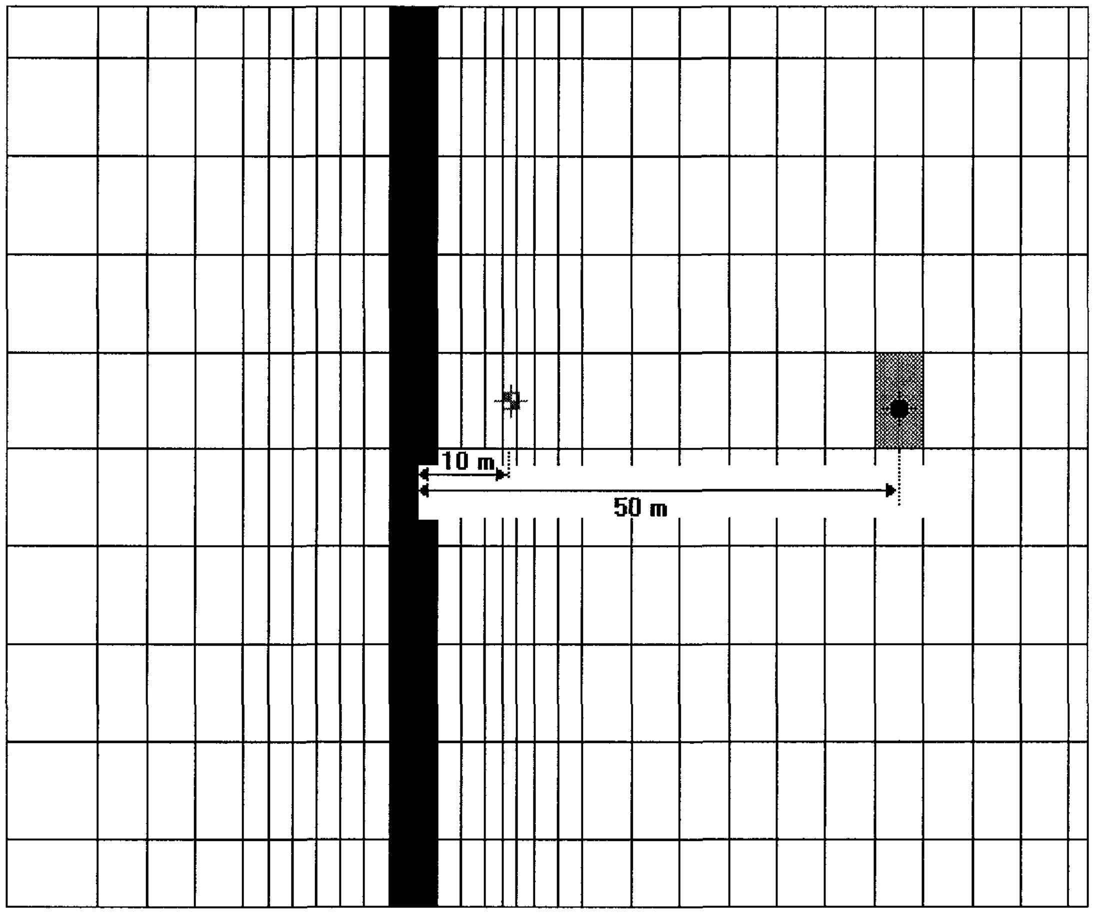


Figure 3.8 - MODFLOW numerical simulation of a pumping well located in an unconfined aquifer and 50 m from a partially penetrating stream of width 5 m. A head observation well is located 10 m from the stream.

3.5 Summary and Conclusions

This research developed an analytical model for drawdown in an aquifer hydraulically connected to a finite width stream. The analytical model modified Hunt's (1999) analytical solution through the use of Heaviside functions to allow a distributed recharge flux through the entire stream width rather than modeling the stream as a line source. Fourier and Laplace transforms along with Taylor series expansions were utilized to derive the analytical model.

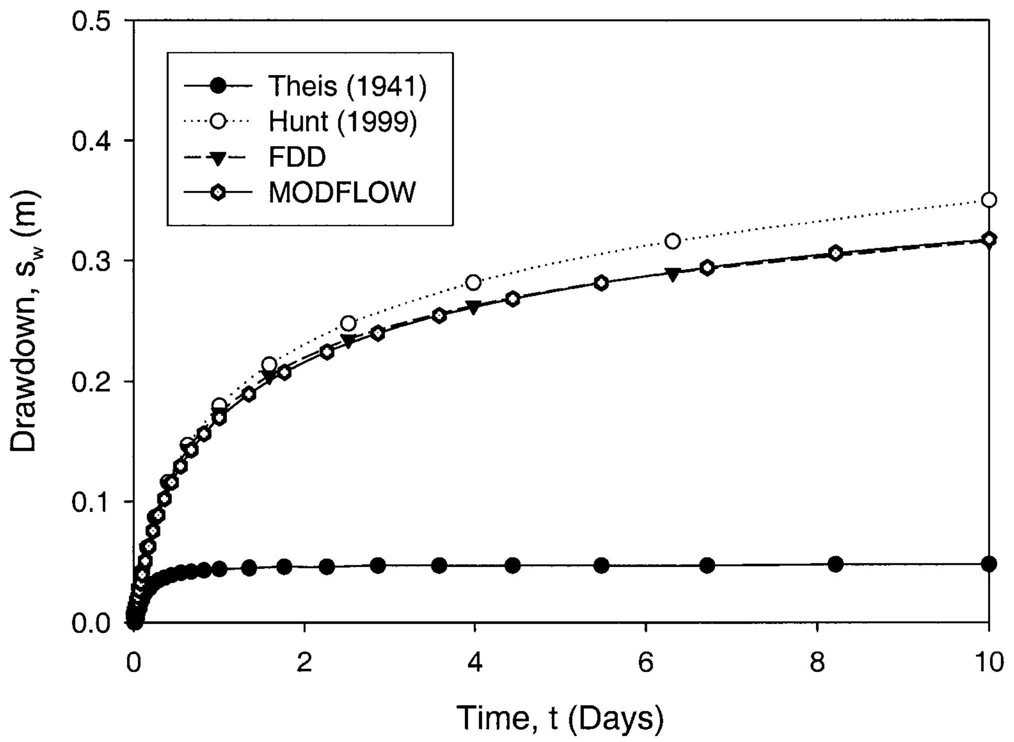


Figure 3.9 - Comparison of the Theis (1941), Hunt (1999), and the FDD analytical solutions against a MODFLOW numerical simulation of drawdown.

One of the primary advantages of the model is its simplicity in consisting of the Theis (1935) solution with integral modifications based on the exponential integral. The analytical solution incorporates parameters for the streambed leakance, a function of the streambed hydraulic conductivity. The proposed analytical solution could be used to derive estimates of streambed parameters. In-stream measurements of streambed hydraulic conductivity provide estimates at a point. However, it is well known that streambed hydraulic conductivity can have both horizontal and vertical variations. The benefit of using the FDD analytical solution is that the solution provides an average estimate more aligned with the data requirements of numerical ground water models such as MODFLOW. Research should be performed in the future to investigate the utilization of the analytical model with stream/aquifer analysis test data, determining the difference in prediction between this analytical model and prior solutions. Specific conclusions of this research include the following:

- The proposed analytical solution approaches the Theis (1935) equation when the width of the stream approaches zero.
- The proposed analytical solution results in equivalent drawdown as predicted by Hunt's analytical solution for insignificant stream widths (i.e., widths less than 1.0 m).
- Utilization of a Taylor series expansion in the derivation of the proposed solution creates a range of times over which the solution remains stable. However, the solution's stable temporal range is well within the time frame of stream/aquifer analysis tests.
- Deviations between Hunt's analytical solution and the FDD analytical model increase as stream width increases.
- Comparison of analytical solutions for a hypothetical stream/aquifer system concluded that Hunt's analytical solution deviates less than 5% from the FDD

analytical model for $tT/SL^2 \leq 100$ as long as L/W>25. When L/W<15, deviations in predicted drawdown are as much as 10-20% at $tT/SL^2=100$.

- The FDD analytical solution most closely matches drawdown response predicted by a MODFLOW numerical model compared to the Theis (1941) and Hunt (1999) solutions when stream width becomes significant.

CHAPTER 4

STRMAQ: A SEMI-ANALYTICAL SOLUTION FOR STREAM/AQUIFER INTERACTION IN CONFINED AND UNCONFINED AQUIFERS

4.1 Introduction

The development of improved analytical models of surface water/groundwater interaction has been the subject of considerable research in the past few years (Hunt et al., 2001; Hunt, 1999; Zlotnik and Huang, 1999). This chapter presents a more comprehensive analytical model for computing aquifer drawdown due to pumping from a confined or unconfined aquifer hydraulically connected to a partially penetrating stream. This model is used to determine the importance of partial penetration of the pumping well, well-bore storage, and well-skin effects in stream/aquifer interaction. Well-skin effects refer to the flow resistance that occurs at the interface between the aquifer and well bore due to restrictions at the well screen and bridging by sand particles across the well-screen openings. Well-bore storage refers to water stored in the well bore of a finite diameter well (Barlow and Moench, 1999).

The model presented in this research removes some of the limitations of Hunt's (1999) analytical solution and, therefore, contributes to the stream/aquifer science by providing a more comprehensive analytical tool. Hunt's (1999) solution for stream/aquifer interaction is limited to confined flow with fully penetrating wells. The analytical solution has been used to inversely derive estimates of the streambed conductance, λ [LT^{-1}], aquifer transmissivity, T [L^2T^{-1}], and the aquifer storage coefficient, S, in an aquifer test performed adjacent to a stream. However, a difficulty in estimating parameter values was due to delayed yield of the unconfined aquifer where the model was applied (Hunt et al., 2001). It is hypothesized that partial penetration of the pumping well, well-skin effects, and well-bore storage may also create difficulties in accurately estimating stream/aquifer parameters. The well function in Hunt's (1999) solution is replaced with modified well functions for complex confined (Dougherty and Babu, 1984) and unconfined (Moench, 1997) aquifer flow that include the effects of partial penetration of the pumping well, delayed drainage from the unsaturated zone, well-skin effects, and well-bore storage. STRMAQ includes the parameters λ, T, and S, as well as specific yield, S_y, and the ratio of vertical to horizontal hydraulic conductivity (K_z/K) to solve for aquifer response under complex flow scenarios.

4.2 Background

The well-known Theis (1935) equation calculates drawdown resulting from a well pumping at a constant rate in an infinite, homogeneous, isotropic aquifer of constant saturated thickness. Theis (1941) presents a solution for a well pumping from an aquifer that is in hydraulic connection with a fully penetrating stream with no semipervious streambed layer. Hantush (1965) modified the Theis (1941) solution to include a semipervious streambed that extended the entire thickness of the fully penetrating stream. Conrad and Beljin (1996), Sophocleous et al. (1995), and Spalding and Khaleel (1991) compare these early analytical models to numerical simulations. Their research concludes that the most critical limitations of analytical models are their failure to adequately account for a semipervious streambed, stream partial penetration, and aquifer heterogeneity.

Hunt (1999) developed an analytical model that accounts for both a semipervious streambed and stream partial penetration. Hunt's equation for aquifer response, or drawdown (s_w), is:

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{ E_1\left[\frac{(L-x)^2+y^2}{4Tt/S}\right] - \int_0^\infty e^{-\theta} E_1\left[\frac{\left(L+|x|+2T\theta/\lambda\right)^2+y^2}{4Tt/S}\right]d\theta \right\} \tag{4.1}$$

where x and y are coordinates in an infinite domain with respect to an origin at the stream on a perpendicular line through the well [L], t is the time since the start of pumping [T], T is transmissivity [L^2T^{-1}], S is the storage coefficient, Q is the pumping rate [L^3T^{-1}], L is the distance between the pumping well and the stream [L], λ is a streambed conductance coefficient [LT^{-1}], θ is the variable of integration, and E_1 is the exponential integral or Theis well function, defined as:

$$E_1(u) = \int_u^\infty \frac{1}{\omega} e^{-\omega} d\omega \tag{4.2}$$

where u is:

$$u = \frac{(x^2 + y^2)}{4Tt / S} \tag{4.3}$$

The streambed conductance, λ, is defined as a function of the hydraulic conductivity of the streambed (K_{sb}), the stream width (W), and the thickness of the semipervious streambed layer (M):

$$\lambda = K_{sb}\frac{W}{M} \tag{4.4}$$

Note that the first term in (4.1) is equivalent to the Theis (1935) solution for drawdown in an infinite aquifer:

$$s_w(x, y, t) = \frac{Q}{4\pi T} E_1\left(\frac{x^2 + y^2}{4Tt / S}\right) \tag{4.5}$$

The Theis well function is well known with readily available tables of values and polynomial approximations (Freeze and Cherry, 1979). While more comprehensive than previous models, Hunt's (1999) model makes several simplifying assumptions:

- drawdown is small compared to saturated thickness of the aquifer,

- the aquifer is of infinite extent and is homogeneous and isotropic,
- horizontal and vertical dimensions of the streambed cross-section are small compared to the saturated thickness of the aquifer,
- changes in the water table elevation are small so that the aquifer remains hydraulically connected to the stream.

Christensen (2000) found that Hunt's (1999) model was sensitive to the streambed conductance (λ). A significant benefit of Hunt's (1999) analytical solution is the ability to derive estimates of the streambed conductance (λ) from aquifer tests performed adjacent to a partially penetrating stream (Hunt et al., 2001).

Hunt (1999) assumes, as did the earlier models, that the saturated thickness of the aquifer is constant. Pumping from unconfined aquifers, however, results in changes in the aquifer saturated thickness and both horizontal and vertical components of flow (Charbeneau, 2000). Neuman (1972, 1975) showed that the vertical components of flow could be characterized by a delay index when drawdown is small compared to the initial aquifer thickness. Neuman also assumes that the specific yield, S_y, is much greater than the storage coefficient, S. Neuman (1972, 1975) derived the following equation for drawdown, s_w, which is applicable to unconfined aquifers:

$$s_w(x, y, t) = \frac{Q}{4\pi T} W'(u_A, u_B, \eta) \tag{4.6}$$

where W′ is the Neuman or unconfined well function. The parameters that define the argument of the well function in equation (4.6) are

$$u_A = \frac{(x^2 + y^2)S}{4Tt} \tag{4.7}$$

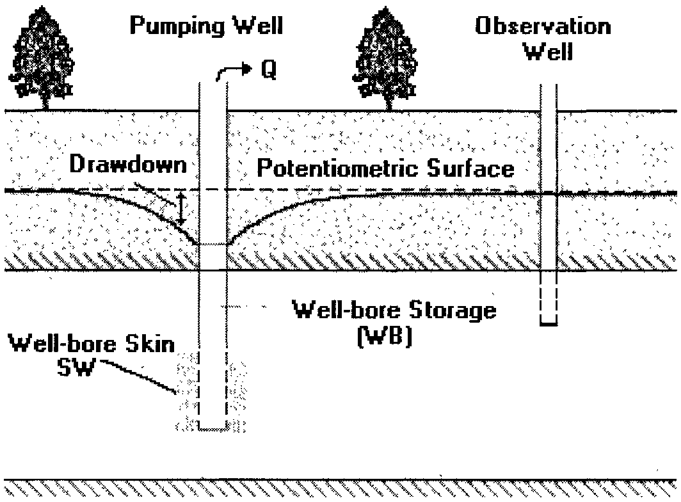
$$u_B = \frac{(x^2 + y^2)S_y}{4Tt} \tag{4.8}$$

$$\eta = \frac{K_z(x^2 + y^2)}{KH^2} \tag{4.9}$$

where K_z is the vertical hydraulic conductivity [LT^{-1}], K is the radial hydraulic conductivity [LT^{-1}], and H is the average saturated thickness of the unconfined aquifer [L]. In general, the unconfined well function characterizes early time behavior using S and late time behavior using S_y.

Moench (1994) demonstrated that Neuman's model estimated K_z, K, and S_y with reasonable accuracy. However, accurate estimates of S were not always possible because of the dependence of these estimates on early time data, which is often complicated by well-bore storage and well-bore skin effects at the pumped well (Moench, 1997). Analytical models have been developed that account for partial penetration of pumping and observation wells, well-bore storage, and well-skin effects in confined (Dougherty and Babu, 1984) and unconfined (Moench, 1997) aquifers. The flow conditions represented by these two analytical solutions are shown in Figure 4.1.

(a)



(b)

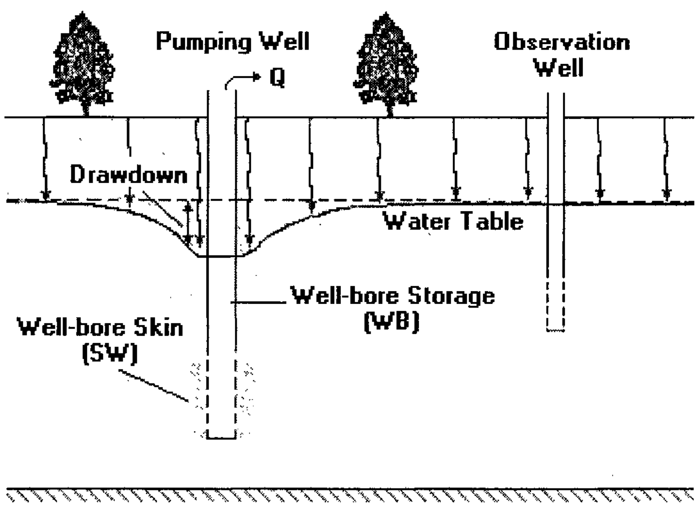


Figure 4.1 - Flow scenarios solved by analytical models of (a) Dougherty and Babu (1984) and (b) Moench (1997). Modified from Barlow and Moench (1999).

Dougherty and Babu (1984) and Moench (1997) calculate a dimensionless drawdown (h_D), where

$$h_D(x, y, t) = \frac{4\pi K_r b}{Q} s_w(x, y, t) \tag{4.10}$$

with the product of K_r and b equal to the transmissivity (T). Comparison of equation (4.10) with the general form of the Theis (1935) solution, as given by equation (4.5), shows that the dimensionless drawdown (h_D) is analogous to the well function (E_1). To generalize, equation (4.10) can be expressed as

$$s_w(x, y, t) = \frac{Q}{4\pi T} W_E \tag{4.11}$$

where W_E is an analogous or modified well function. The analytical solutions of Dougherty and Babu (1984) and Moench (1997) are included in a program called WTAQ (Barlow and Moench, 1999), in which the analytical solution of Moench (1997) is extended to account for delayed drainage into the aquifer from the unsaturated zone. Drainage from the unsaturated zone in a water-table aquifer is represented by a series of exponential terms:

$$K_z \frac{\partial h}{\partial z} = -S_y \int_0^t \frac{\partial h}{\partial t'} \sum_{i=1}^{M} \frac{\alpha_i}{M} \exp[-\alpha_i (t - t')] dt' \tag{4.12}$$

where K_z is the vertical hydraulic conductivity of the aquifer, h is the head in the water table aquifer, α_i is the i^{th} empirical drainage constant, and M is the number of dimensionless drainage constants. The influence of drainage from the unsaturated zone has been reported as being a source of significant discrepancies between simulated and measured drawdown for intermediate time data (Moench et al., 2001).

4.3 Proposed Model

In this research, the Laplace-transform solutions outlined in WTAQ (Barlow and Moench, 1999) are used to evaluate the analogous well functions for the analytical models of Dougherty and Babu (1984) and Moench (1997). These are referred to as W_E^{DB} and W_E^{M}, respectively. The algorithms for the analogous well functions are then imbedded into the Hunt (1999) stream/aquifer analytical model, which is solved for drawdown in complex confined and unconfined aquifer flow in hydraulic connection with a partially penetrating stream. The modified equations for confined and unconfined flow are given by equations (4.13) and (4.14), respectively:

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{W_E^{DB}(t,T,S,r) - \int_0^\infty e^{-\theta} W_E^{DB}(t,T,S,\lambda,r')d\theta\right\} \tag{4.13}$$

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{W_E^{M}(r,t,T,S,S_y) - \int_0^\infty e^{-\theta} W_E^{M}(r',t,T,S,S_y,\lambda)d\theta\right\} \tag{4.14}$$

where

$$r = \left\{(L-x)^2 + y^2\right\}^{0.5} \tag{4.15}$$

$$r' = \left\{\left(L + |x| + \frac{2T\theta}{\lambda}\right)^2 + y^2\right\}^{0.5} \tag{4.16}$$

Similar to WTAQ, STRMAQ solves W_E^{DB} and W_E^{M} using parameters to account for partial penetration and two dimensionless parameters to account for well-bore storage and well-skin effects. STRMAQ requires specification of the depths below the top of the aquifer or water table to the top (z_{PD}) and bottom (z_{PL}) of the screened interval to account for partial penetration of the pumping well. Well-bore skin effects are quantified for either a finite-diameter or infinitesimal-diameter well using a well-bore skin parameter (SW)

$$SW = \frac{K_r d_s}{K_s r_w} \tag{4.17}$$

where K_s is the hydraulic conductivity of the well-bore skin and d_s is the thickness of the skin (Barlow and Moench, 1999). STRMAQ simulates well-bore storage effects only when a finite-diameter pumping well is specified. STRMAQ calculates a dimensionless well-bore storage parameter (WB):

$$WB = \frac{\pi r_c^{\,2}}{\left[2\pi r_w^2 S_S \left(z_{pl} - z_{pd}\right)\right]} \tag{4.18}$$

where r_c is the inside radius of the pumping well in the interval where water levels are changing [L], r_w is the radius of the pumping well screen [L], z_{pl} is the depth from the water table to the bottom of the well screen, and z_{pd} is the depth from the water table to the top of the well screen, and S_S is the specific storage [L^{-1}] (Barlow and Moench, 1999).

Stream depletion is estimated by assuming the discharge, Q', through the stream is linearly proportional to the streambed conductance, λ, and the difference between the water level in the stream, H_w, and the head in the aquifer beneath the stream, h(0,y,t):

$$Q' = \lambda[H_w - h(0,y,t)] = \lambda s_w(0,y,t) \tag{4.19}$$

Integrating along the infinite stream length gives the total stream discharge:

$$Q_s = \int_{-\infty}^{\infty} \lambda s_w(0,y,t)\,dy \tag{4.20}$$

4.4 Limitations of Proposed Model

In order for the form of the recharge integral in Hunt's (1999) solution to remain valid, the assumption of primarily horizontal flow near the stream must be satisfied. Critical considerations in meeting this requirement include the distance between the stream and pumping well (L), anisotropy ratio (K_r/K_z), stream width (W), and the aquifer thickness (b). A partially penetrating pumping well will create significant vertical flow components at the stream if the distance between the stream to pumping well (L) is less than $2b(K_r/K_z)^{1/2}$ (Hantush, 1964). Therefore, STRMAQ is limited to cases where $L>2b(K_r/K_z)^{1/2}$. However, Christensen (2000) suggests the use of multiple observation wells during stream/aquifer analysis tests. Even though partial penetration of the pumping well may not influence drawdown near the stream, it may influence drawdown in observation wells located near the pumping well. STRMAQ can analyze the effects of partial penetration in these observation wells as long as partial penetration of the pumping well does not result in the Dupuit flow assumption being violated at the stream.

Butler et al. (2001) notes that a moderate degree of anisotropy ($K_r/K_z=10$) can be neglected if the ratio of the distance between the pumping well and stream (L) to the stream width (W) is greater than 15. For a high degree of anisotropy ($K_r/K_z=100$), the impact of anisotropy can be neglected when the ratio of distance between the pumping well and stream (L) to the stream width (W) is greater than 75. In Chapter 3, the impact of distributed stream recharge across a finite width stream on stream/aquifer interaction was investigated. As long as the ratio of the distance between the pumping well and the stream (L) to the stream width (W) is greater than 25, stream width effects are negligible in most situations.

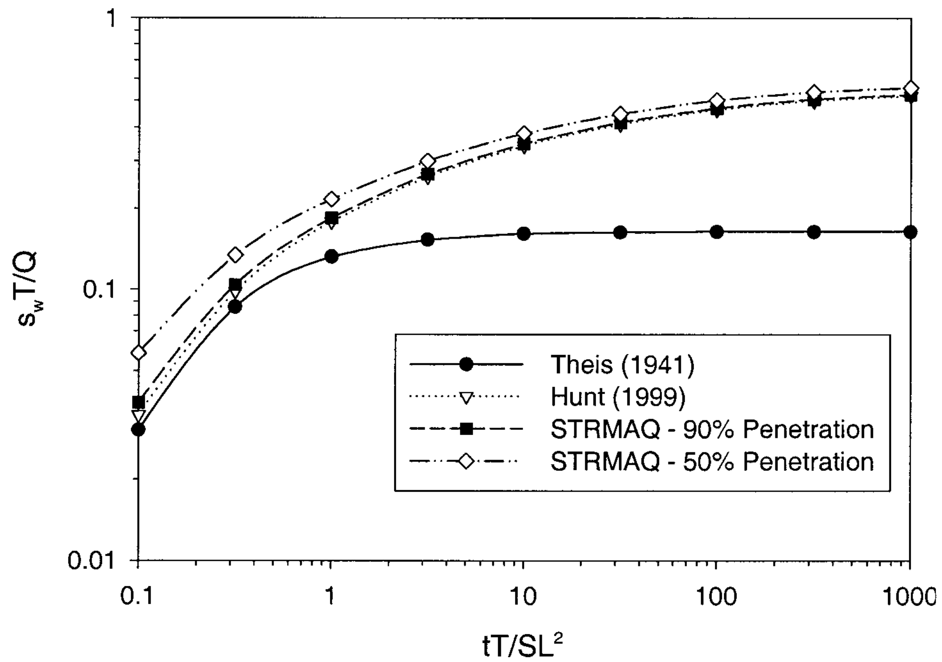
4.5 Results and Discussion

Dimensionless time-drawdown curves are used to compare STRMAQ to Hunt's (1999) analytical solution and the Theis (1941) equation. Parameters for unconfined flow, partial penetration, well-skin effects, and well-bore storage that are included in STRMAQ but not available in Hunt's (1999) solution or the Theis (1941) equation are varied and the sensitivity of drawdown to these variations is evaluated. Dimensionless time (tT/SL^2) versus dimensionless drawdown (s_wT/Q) curves for confined aquifers and modified dimensionless time (tT/L^2) versus dimensionless drawdown (s_wT/Q) curves for unconfined aquifers are compared for a single observation well halfway between the stream and pumping well ($x/L=0.5$) on a direct transect between the two ($y/L=0$). For all analyses, K_z is assumed equivalent to K. The models are compared for three stream systems: $\lambda L/T=0.1$, 1.0, and 10.0, representing low, moderate, and high conducting streambeds.

4.5.1 Stream/Aquifer Interaction in Confined Aquifers

The Theis (1941) and Hunt (1999) equations for aquifer drawdown are compared to STRMAQ for a fully penetrating, 90%, and 50% partially penetrating pumping wells in a confined aquifer with three different streambed conductance values. Results are shown in Figure 4.2 for the low and high conducting streambeds. Well-bore storage and well-bore skin effects are assumed negligible and the well is fully screened. As expected, STRMAQ and Hunt (1999) predict equivalent drawdown when modeling a fully penetrating pumping well. The Theis (1941) equation predicts less drawdown because the stream is assumed to fully penetrate the aquifer without a semipervious streambed layer. Partial penetration of the well increases drawdown (Barlow and Moench, 1999).

(a)



(b)

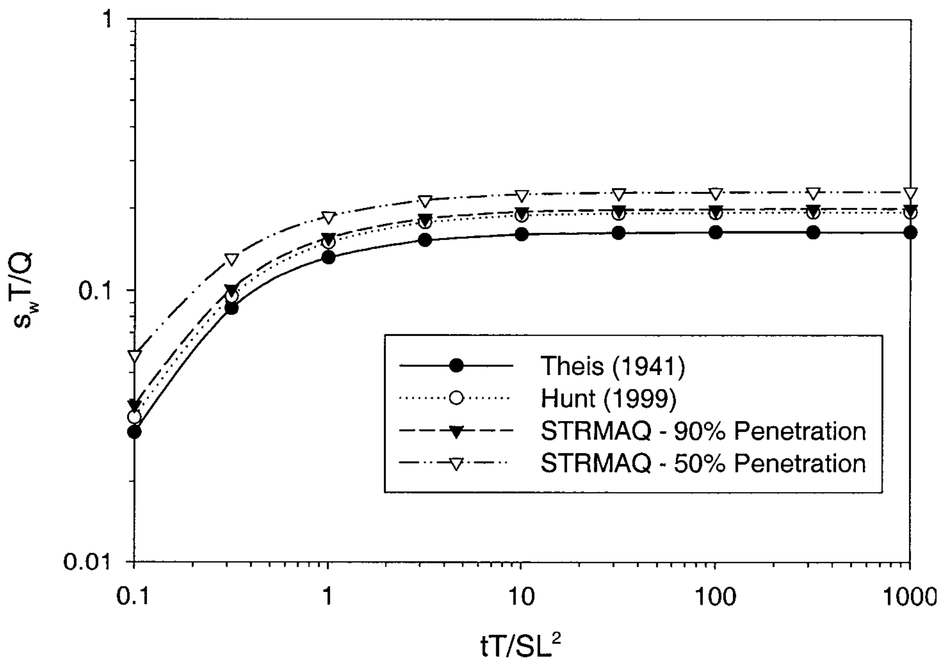


Figure 4.2 - Comparison of Theis (1941) and Hunt (1999) analytical solutions with STRMAQ considering partial penetration effects in a confined aquifer for (a) low ($\lambda L/T=0.1$) and (b) high conducting ($\lambda L/T=10.0$) streambeds.

Differences in predicted drawdown between Hunt's solution and STRMAQ also increase as the streambed becomes more permeable (i.e., as $\lambda L/T$ increases). These results suggest that partial penetration effects on drawdown are most significant for high conducting streambeds. The difference between model predictions is generally higher at low values of non-dimensional time (tT/SL^2) and decreases as non-dimensional time (tT/SL^2) increases. For a low conducting streambed, differences in non-dimensional drawdown (s_wT/Q) are generally less than 10% as long as the degree of partial penetration is greater than 50% and for large non-dimensional time.

For more permeable streambeds, more than 10% difference in non-dimensional drawdown between Hunt's (1999) solution and STRMAQ is observed for all non-dimensional time when partial penetration is less than 50%. Percent differences in drawdown are less than 10% for partial penetrations greater than 90%. Ignoring partial penetration of the pumping well may lead to inaccurate estimates of stream/aquifer parameters, especially λ, in aquifer tests performed adjacent to partially penetrating streams.

STRMAQ is also compared to the Theis (1941) and Hunt (1999) equations with well-bore skin effects but not considering partial penetration. Results for a low conducting streambed are shown in Figure 4.3. Well-skin effects will not affect drawdown in the aquifer without well-bore storage, but when both effects are included, early-time drawdown is less than the drawdown that would occur in the absence of a skin (Barlow and Moench, 1999). As the dimensionless well-skin parameter (SW) is increased, the delayed response for early time drawdown becomes more significant. A significant delay in early-time drawdown data is not observed until the dimensionless well-skin parameter (SW) is greater than 10. Similar results are observed for the moderate and high conducting streambeds, suggesting that well-skin effects on dimensionless drawdown are not dependent on streambed conductance in stream/aquifer interaction. Well-skin effects would not affect estimates of λ in stream/aquifer analysis tests.

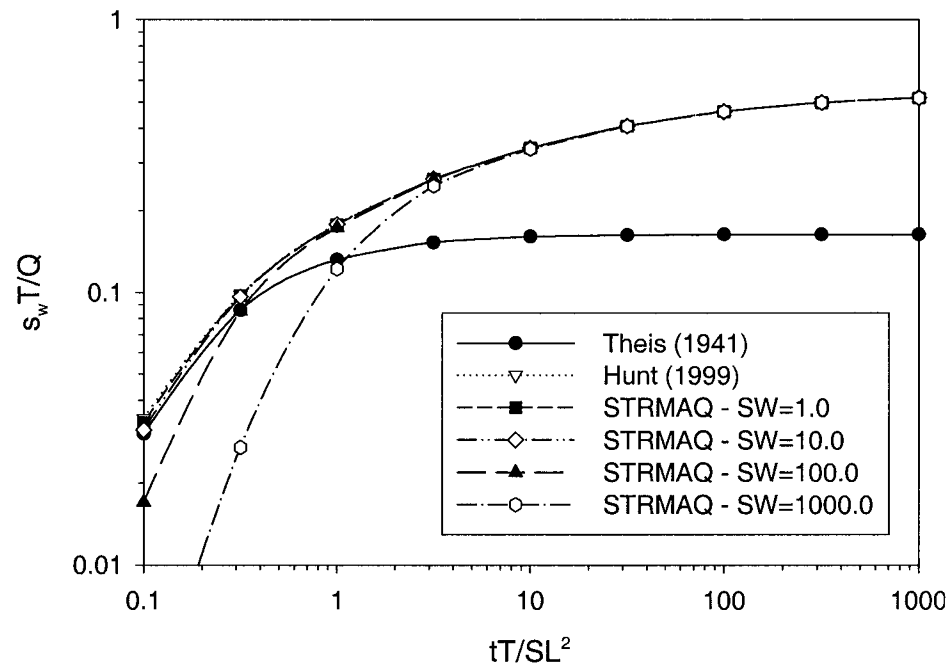


Figure 4.3 - Comparison of Theis (1941) and Hunt (1999) analytical solutions with STRMAQ considering well-skin effects in a confined aquifer. SW is a dimensionless well-skin parameter.

Well-bore storage effects on dimensionless drawdown are investigated by calculating WB for a small well radius (r_w=0.1 m) and for a large well radius (r_w=0.3 m).

Results are shown in Figure 4.4 for a low conducting stream. WB is dependent on several parameters: the ratio of r_c/r_w, specified at r_c/r_w=0.1, 0.5, and 1 for each well radius; specific storage (S_S), assumed 10^{-6} m^{-1}; and the difference between z_{pl} and z_{pd}, assumed equal to a saturated thickness of 30 m. When WB approaches 0, STRMAQ is equivalent to Hunt's solution for both the small and large well radii and for all streambed conductance values.

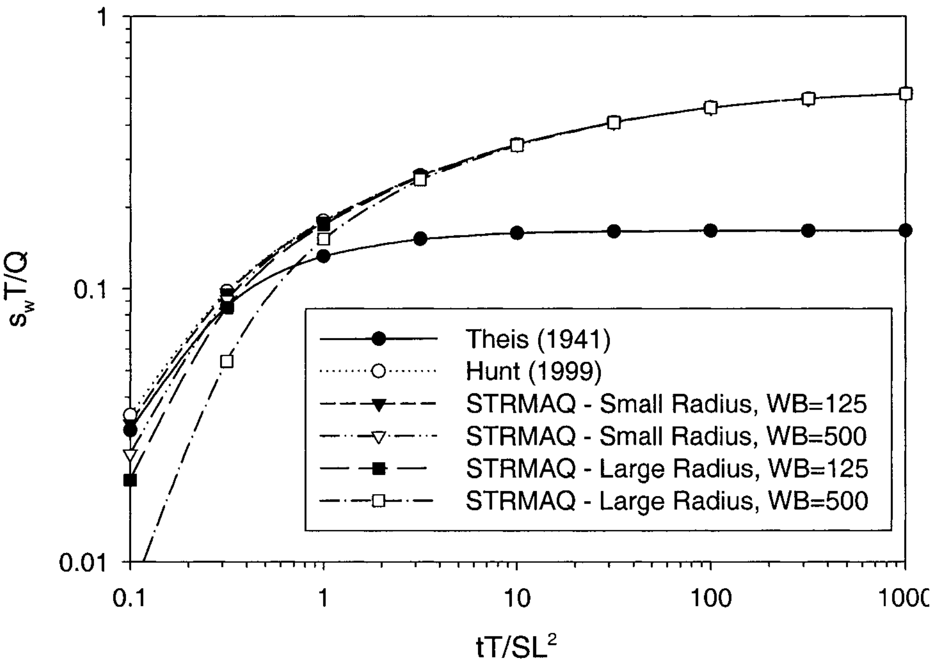


Figure 4.4 - Comparison of Theis (1941) and Hunt (1999) analytical solutions with STRMAQ considering well-bore storage in a confined aquifer. WB is a dimensionless well-bore storage parameter.

For a small radius well, the impact of well-bore storage is significant when WB>125. However, for the large well radius, well-bore storage is significant when WB>5. Also, similar to the results for well-bore skin effects, well-bore storage only impacts early-time drawdown when compared to Hunt's (1999) solution. However, well-bore storage effects can impact late-time drawdown if the transmissivity (T) is very low. Similar results are observed for moderate and high conducting streambeds, suggesting that well-bore storage effects on dimensionless drawdown are not dependent on streambed conductance in stream/aquifer interaction. Unless the effects of well-bore storage impact late-time drawdown, well-bore storage effects would not impact the accuracy of λ estimates in stream/aquifer analysis tests.

4.5.2 Stream/Aquifer Interaction in Unconfined Aquifers

STRMAQ can be used to solve for drawdown in an unconfined aquifer hydraulically interacting with a partially penetrating stream. This more realistically models stream/aquifer interaction. Most tributary groundwater systems consist of an unconfined aquifer in direct hydraulic connection to a partially penetrating stream. The drawdown profile in an unconfined aquifer consists of two regions, with early-time drawdown governed by S and late-time drawdown by S_y.

Partial penetration of the pumping well in an unconfined aquifer is varied from 50 to 100%, assuming that well-skin and well-bore storage effects are negligible. The effect of partial penetration of the pumping well on stream/aquifer dimensionless drawdown for a medium conducting streambed is shown in Figure 4.5. In this figure, the Hunt (1999) solution is based on confined flow. The full penetration case and 90% and 50% partial penetration cases are based on unconfined flow. The unconfined aquifer is assumed to have S=0.001 and S_y=0.1. Note that a modified non-dimensional time (tT/L^2) is used as the dependent variable rather than the earlier version of non-dimensional time (tT/SL^2).

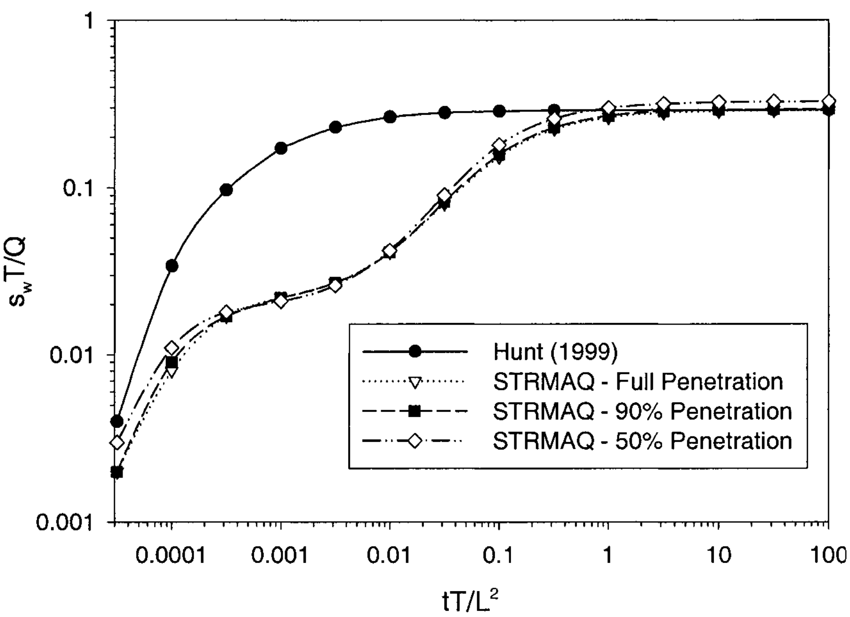


Figure 4.5 - Comparison of Hunt's (1999) analytical solution and STRMAQ considering partial penetration effects in an unconfined aquifer for a moderate ($\lambda L/T$=1.0) conducting streambed.

Differences in predicted dimensionless drawdown (s_wT/Q) between a fully penetrating well and partial penetrating wells are similar for all streambed conductance values at early times. However, throughout the range of non-dimensional time (tT/L^2), differences in predicted drawdown between a fully penetrating well and partially penetrating wells increase as the streambed conductance increases. Partial penetration effects are most significant on early-time drawdown when partial penetration is less than 50%. Late-time drawdown is significantly affected by partial penetrations of less than 90% with the effects increasing as the streambed conductance increases. In summary, partial penetration effects on drawdown are most significant for high conducting streambeds, suggesting that partial penetration of the pumping well should be considered in predicting drawdown in stream/aquifer systems. Ignoring partial penetration of the pumping well may lead to inaccurate estimates of λ in aquifer tests performed adjacent to partially penetrating streams.

The dimensionless well-skin parameter (SW) is varied to investigate well-skin effects on unconfined aquifer drawdown in a stream/aquifer system (Figure 4.6). Well-bore storage is assumed negligible and the pumping well is fully penetrating. SW=0 simulates an unconfined aquifer with no well-skin effects. Similar to the confined case, SW affects early-time drawdown but not late-time drawdown. Differences in predicted drawdown from the case of SW=0 occur as SW approaches 100. Well-skin effects on drawdown show no dependency on streambed conductance in an unconfined stream/aquifer system.

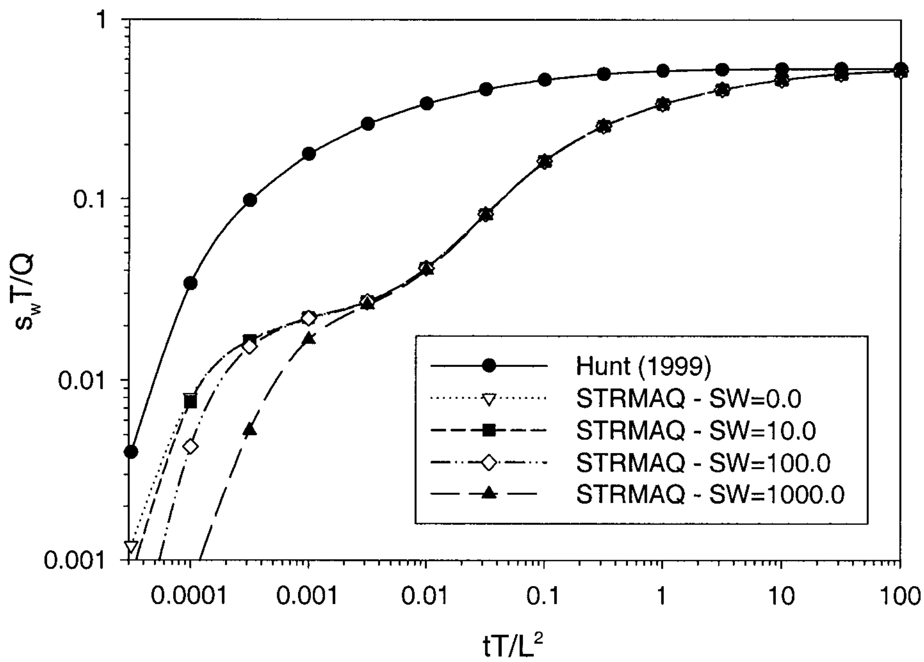


Figure 4.6 - Comparison of Hunt's (1999) analytical solution and STRMAQ considering well-skin effects in an unconfined aquifer. SW is a dimensionless well-skin parameter.

Well-bore storage effects are included independently of well-skin and partial penetration (Figure 4.7). As in the confined case, the dimensionless well-bore storage parameter (WB) is varied for three streambed conductance values and for two well radii: small well radius (r_w=0.1 m) and large well radius (r_w=0.3 m). WB=0 represents an unconfined aquifer with no well-bore storage effects. Again, well-bore storage only affects early-time drawdown, with significant effects occurring when WB>125 for the small well radius and WB>5 for the large well radius. Similar to well-skin effects, well-bore storage shows no dependency on streambed conductance in an unconfined stream/aquifer system.

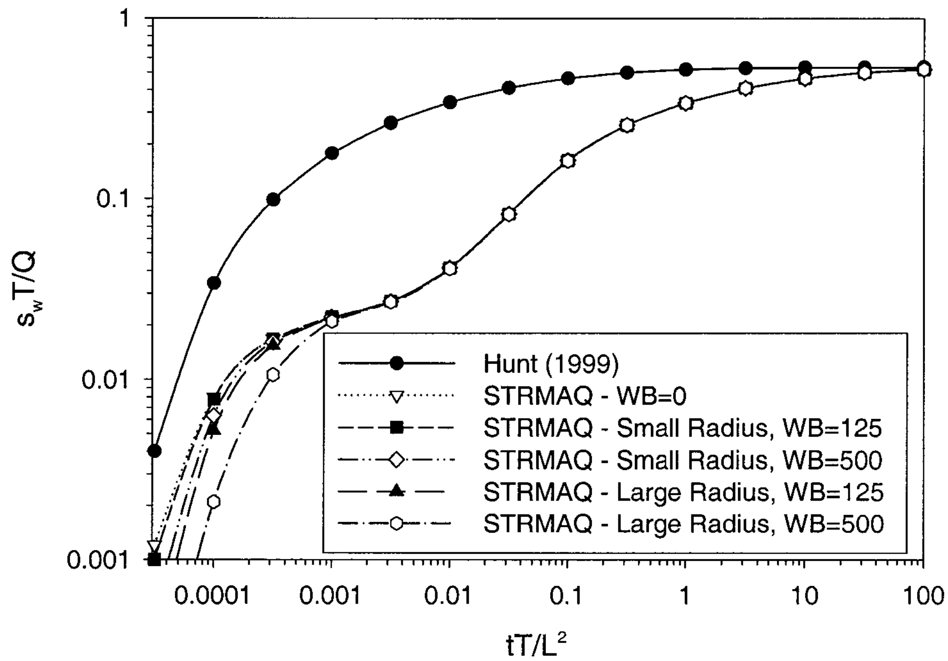


Figure 4.7 - Comparison of Hunt's (1999) analytical solution and STRMAQ considering well-bore storage in an unconfined aquifer. WB is a dimensionless well-bore storage parameter.

A comparison of STRMAQ with the numerical ground water flow model, MODFLOW (McDonald and Harbaugh, 1988), is shown in Figure 4.8. The MODFLOW model simulated a fully penetrating pumping well with a constant discharge of 0.058 m^3-sec^{-1} located 200 m from a 10-m wide stream in a water-table aquifer with T=0.012 m^2-sec^{-1} (K_r=K_z=5.78 x 10^{-4} m-sec^{-1}, b=20 m), S=0.001, and S_y=0.15. Drawdown profiles are compared for two observations wells located between the pumping well and stream at (x/L=0.20, y/L=0) and (x/L=0.50, y/L=0).

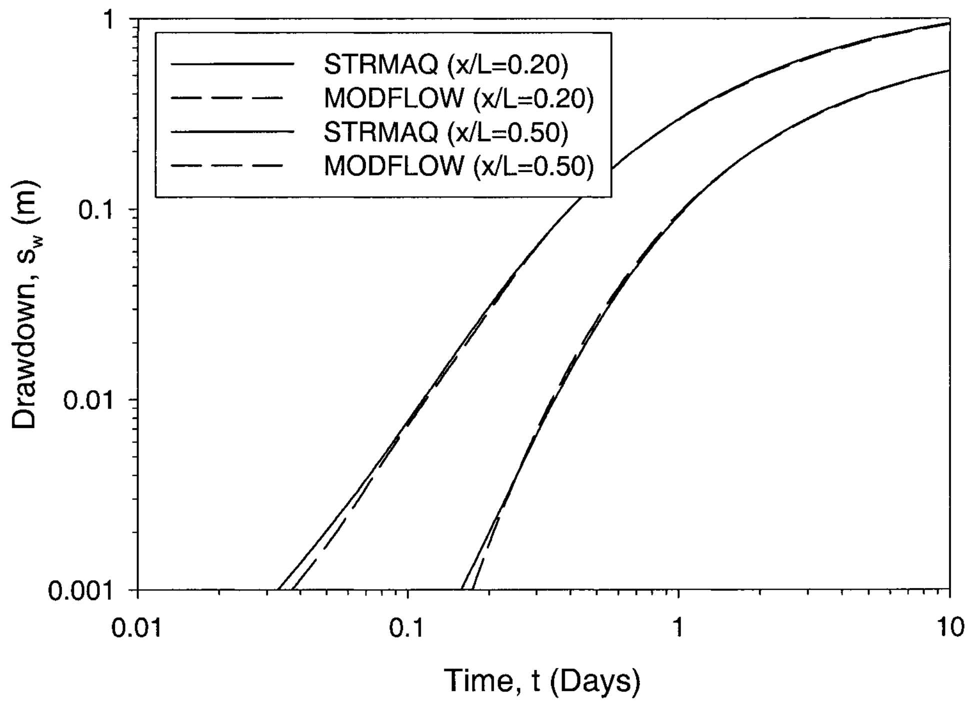


Figure 4.8 - Comparison of STRMAQ versus MODFLOW numerical flow model simulating a fully penetrating pumping well in an alluvial aquifer and hydraulically interacting with a partially penetrating stream.

4.6 Field Application of STRMAQ

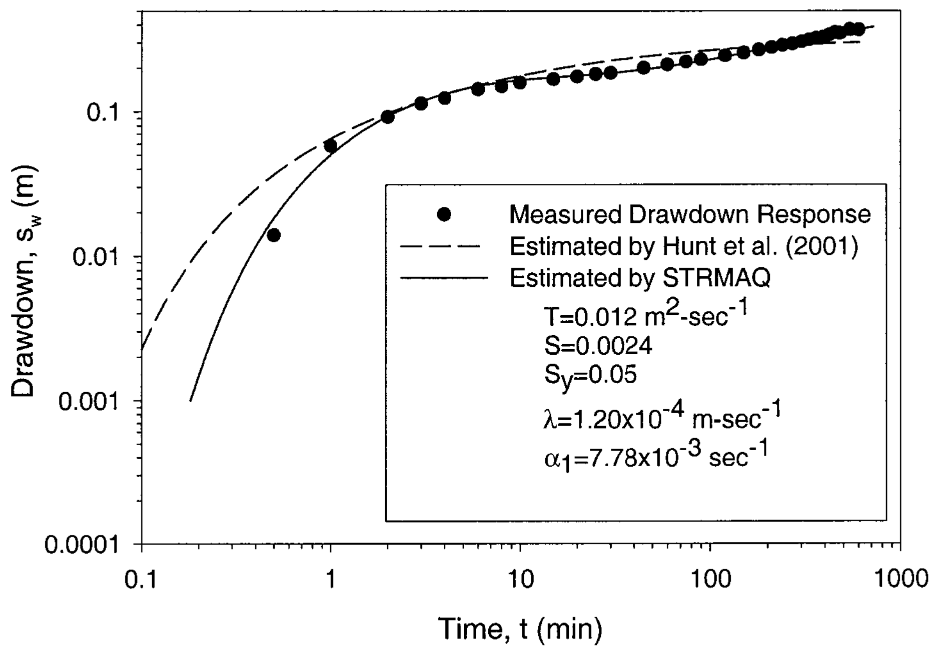
Hunt et al. (2001) report results from a field experiment near the Doyleston Drain south of Christchurch, New Zealand. A pumping well was located approximately 55 m from the center of a 2.5-m wide drain. The aquifer was 20-m thick, unconsolidated, and composed of primarily sand and gravel. A 2.8-m thick layer of less permeable material, with a hydraulic conductivity on the order of 10^{-9} m-sec^{-1}, overlaid the aquifer.

Drawdown was measured in two observation wells located between the stream and pumping well and two observation wells located on the side of the pumping well away from the stream. Weirs upstream and downstream of the pumping well were used to measure changes in streamflow, or stream depletion. The aquifer test was performed with a constant discharge rate of 0.0175 m^3-sec^{-1}, and drawdown measurements were made for 10 hours.

Hunt et al. (2001) state that drawdown measured at later times lacked sufficient precision to determine a value of λ because the geology and observation well drawdowns were characteristic of drawdowns in an unconfined, delayed yield aquifer. A matching point process was used to estimate T and S from observation well drawdowns at early times and measurements of stream depletion flows were used to estimate λ at later times. Results from this procedure resulted in the following ranges for the aquifer and streambed parameters: T=0.016 to 0.025 m^2-sec^{-1}, S=0.0015 to 0.0022, and λ=0.63x10^{-4} to 0.92x10^{-4} m-sec^{-1}.

STRMAQ was used to match the observation well drawdown response from Hunt et al. (2001). Only the two observation wells located closer to the stream (observation wells 4 and 5) were used in this analysis because fitting the drawdown response measured in these wells allow an estimate of the streambed conductivity. Figure 4.9 compares the measured drawdowns, the calculated drawdowns of Hunt et al. (2001), and the calculated drawdowns from STRMAQ for observation wells 4 and 5. STRMAQ was able to match the delayed yield response of the aquifer, and estimates of T, S, and λ were similar to the range of estimates of Hunt et al. (2001) for both observation wells.

(a)



(b)

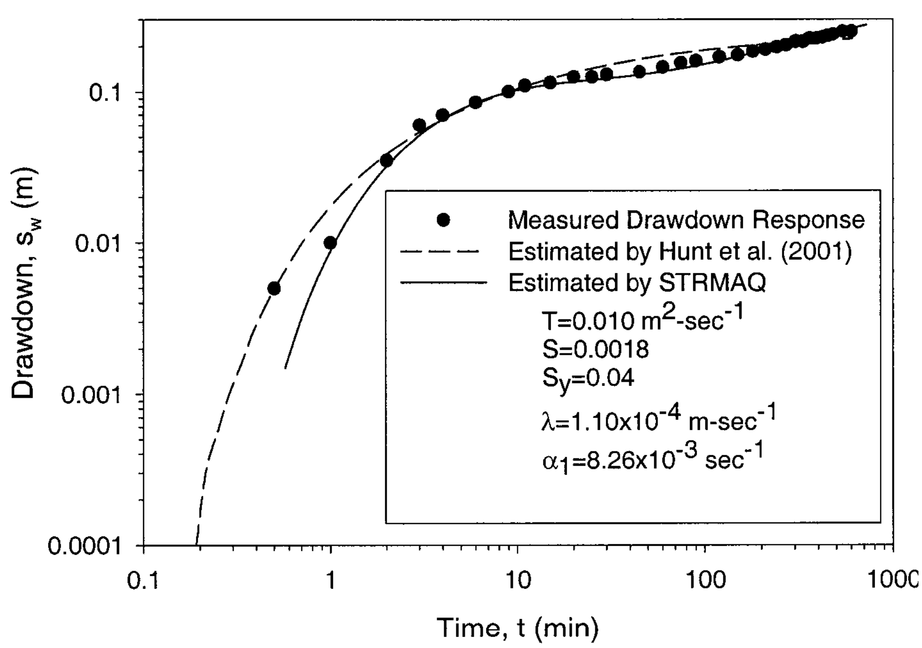


Figure 4.9 - Comparison of measured, Hunt et al. (2001), and STRMAQ drawdown versus time for (a) observation well 4 (x/L=0.53, y/L=0) and (b) observation well 5 (x/L=0.11, y/L=0). Data from field experiment near the Doyleston Drain south of Christchurch, New Zealand by Hunt et al. (2001).

STRMAQ was used to estimate a specific yield of 0.04-0.05. The overlying layer has a conductivity characteristic of a silt or clay subsoil (Bear, 1972). For such materials, the specific yield, S_y, is reported to range from 0.01-0.10. Delayed drainage was estimated using a single exponential relation, i.e., M=1 in equation (4.12). A single exponential relation was used due to the lack of specific information about the drainage in the unsaturated zone at the site.

4.7 Summary and Conclusions

This research presents an analytical model for drawdown in a stream/aquifer system for confined and unconfined aquifers with partial penetration of the pumping well, delayed drainage, well-skin effects, and well-bore storage. The analytical model, referred to as STRMAQ, combines Hunt's (1999) solution for stream/aquifer interaction with analytical solutions of Dougherty and Babu (1984) and Moench (1997). Combination of the solutions is performed through the use of analogous well functions. The computer program makes use of complex flow analytical solutions provided in the computer program WTAQ, developed by Barlow and Moench (1999). STRMAQ is used to determine the influence of the complex aquifer flow parameters on stream/aquifer interaction and on the ability to derive estimates of the streambed conductance from aquifer tests adjacent to partially penetrating streams.

Drawdown computed using STRMAQ is equivalent to Hunt's analytical solution when modeling a fully penetrating well in a confined aquifer with no well-skin or well-bore storage effects. The delayed yield of an unconfined aquifer could significantly impact predicted drawdown and the accuracy of streambed conductance estimates in stream/aquifer analysis tests. Partial penetration of the pumping well was determined important in stream/aquifer analytical models, with the effects more significant for high conducting streambeds. Well-bore storage and well-skin effects delayed early-time drawdown response when the dimensionless well-skin parameter exceeded 100 and the dimensionless well-bore storage parameter exceeded 5 for larger well radii and 125 for smaller well radii.

STRMAQ is shown to compare well to the numerical groundwater flow model, MODFLOW, based on predicted aquifer response for a hypothetical stream/aquifer system. Field data from a stream/aquifer analysis test performed by Hunt et al. (2001) is analyzed using STRMAQ and compared to an equivalent analysis using the analytical solution of Hunt (1999). STRMAQ was able to match the delayed yield response of the water-table aquifer, and estimates of aquifer transmissivity, storage coefficient, and streambed conductance were similar to the range of estimates of Hunt et al. (2001) for both observation wells analyzed.

STRMAQ greatly extends the applicability of stream/aquifer analytical solutions. This is especially important considering that most stream/aquifer interaction problems deal with unconfined aquifers in direct hydrologic connection with partially penetrating streams. STRMAQ provides a mechanism for relating the hydraulic parameters of a confined or unconfined aquifer (radial hydraulic conductivity, vertical hydraulic conductivity, storage coefficient, and specific yield) with streambed conductance. The

FORTRAN code for the STRMAQ program is included in Appendix A. Also, sample input and output files and the program's README file are included in Appendix B.

CHAPTER 5

SENSITIVITY OF STREAMBED CONDUCTANCE ESTIMATES TO AQUIFER PARAMETER UNCERTAINTY

5.1 Introduction

Recently, analytical solutions for stream/aquifer interaction due to pumping wells hydraulically interacting with partially penetrating streams have been the subject of considerable research (see Chapters 3 and 4). Analysis of stream/aquifer interaction is generally based on the Theis (1941) solution. The Theis (1941) solution models a well pumping from an aquifer that is in hydraulic connection with a fully penetrating stream with no semipervious streambed layer. Hantush (1965) modified the Theis (1941) solution to include a semipervious streambed that extended the entire thickness of the fully penetrating stream. The most critical limitations of these analytical models are their failure to adequately account for a semipervious streambed, stream partial penetration, and aquifer heterogeneity. Hunt (1999) developed an improved analytical model that accounted for both a semipervious streambed and stream partial penetration. Hunt's (1999) solution for stream/aquifer interaction is limited to confined flow with fully penetrating wells.

STRMAQ (Chapter 4) further removes some of the limitations of prior analytical solutions by accounting for unconfined flow, pumping well partial penetration, delayed drainage from the unsaturated zone, well-skin effects, and well-bore storage (Figure 5.1).

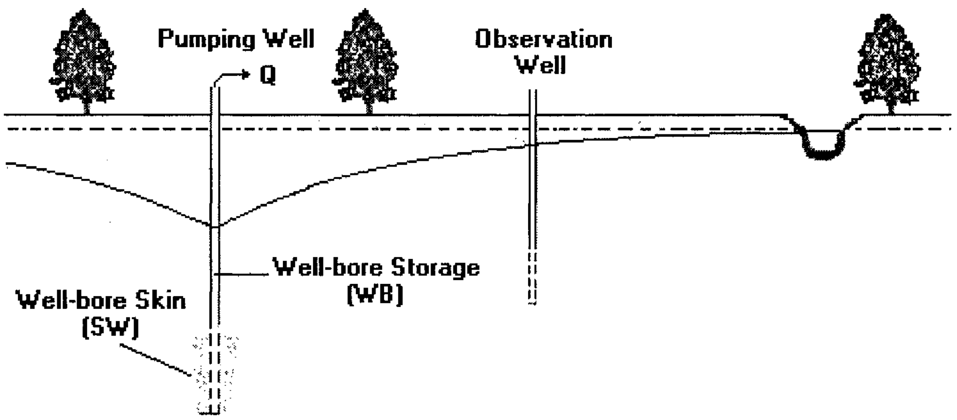


Figure 5.1 - Physical condition solved by the analytical model, STRMAQ.

STRMAQ was derived by replacing the well function, E_1, in Hunt's (1999) solution with modified well functions for complex confined, represented as W_E^{DB}, (Dougherty and Babu, 1984) and unconfined, represented as W_E^{M}, (Moench, 1997) aquifers, such that the drawdown, $s_w(x,y,t)$, is:

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{W_E^{DB}(r,t,K,K_z,S) - \int_0^{\infty} e^{-\theta} W_E^{DB}(r',t,K,K_z,S,\lambda)d\theta\right\} \quad (5.1)$$

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{W_E^{M}(r,t,K,K_z,S,S_y) - \int_0^{\infty} e^{-\theta} W_E^{M}(r',t,K,K_z,S,S_{y,}\lambda)d\theta\right\} \quad (5.2)$$

where

$$r = \left\{ (L - x)^2 + y^2 \right\}^{1/2} \tag{5.3}$$

$$r' = \left\{ \left(L + |x| + \frac{2T\theta}{\lambda} \right)^2 + y^2 \right\}^{1/2} \tag{5.4}$$

and where x and y are coordinates in an infinite domain with respect to an origin at the stream on a perpendicular line through the well [L], t is the time since the start of pumping [T], T is transmissivity [L^2T^{-1}], S is the storage coefficient, S_y is the specific yield, K is the radial hydraulic conductivity [LT^{-1}], K_z is the vertical hydraulic conductivity [LT^{-1}], Q is the constant pumping rate [L^3T^{-1}], L is the distance between the pumping well and the stream [L], λ is a streambed conductance coefficient [LT^{-1}], and θ is the variable of integration. STRMAQ uses the streambed conductance parameter, λ, for specifying the degree of hydraulic connection between the stream and aquifer:

$$\lambda = \frac{K_{sb} W}{M} \tag{5.5}$$

where K_{sb} is the streambed hydraulic conductivity [LT^{-1}], W is the width of the stream [L], and M is the thickness of semipervious streambed layer [L].

Hunt (1999) suggests the use of a matching-point method for estimating aquifer and streambed parameters from observed aquifer drawdown data. Hunt's (1999) analytical solution has been used to inversely derive estimates of the streambed conductance, λ, aquifer transmissivity, T, and the aquifer storage coefficient, S, in an aquifer test performed adjacent to a stream (Hunt et al., 2001). However, a difficulty in estimating parameter values was due to delayed yield of the semi-confined aquifer where the model was applied. Furthermore, this matching point procedure requires subjective judgment that can lead to variability in predicted values of aquifer and streambed parameters.

The alternative to a matching point process is the use of inverse parameter estimation routines. Several programs have been developed to automatically estimate aquifer hydraulic properties, i.e., aquifer hydraulic conductivity (K), specific storage (S_S), and specific yield (S_y). Such automated techniques overcome the sometimes tedious, trial and error approach of type-curve analysis. Early routines fit observed data to the Theis (1935) solution (Das Gupta and Joshi, 1984). Advances were then made for estimating aquifer parameters from aquifer test analysis of leaky aquifers (Kashyap et al., 1988). Heidari and Moench (1997) used nonlinear least-squares techniques to estimate water-table aquifer parameters. The computer program UCODE (Poeter and Hill, 1998) estimates parameters for numerical or analytical models.

Most parameter estimation techniques for aquifer test analysis are based on nonlinear regression modeling. Nonlinear least-squares techniques have been found to complement but not replace graphical interpretation methods (Heidari and Moench, 1997; Chandler et al., 1981). The Levenberg-Marquardt method is a nonlinear least-squares routine based on the following chi-square (χ^2) merit function that measures the agreement between measured or observed data with model predictions:

$$\chi^2 = \sum_{i=1}^{N}\left(\frac{y_i - y(x_i; a_1 \ldots a_M)}{\sigma_i}\right)^2 \tag{5.6}$$

where it is assumed that a model with M adjustable parameters a_j, $j=1....M$ is being fit to N data points (x_i, y_i) $i=1...N$, each with a known standard deviation σ_i (Press et al., 1992). Small values of the chi-square merit function represent close agreement between observed and predicted data. The parameters of the model can be adjusted to minimize the value of the chi-square merit function, resulting in best-fit parameters for the observed data. The nonlinear dependencies of model parameters require the problem to be solved iteratively (Press et al., 1992). Adjustment of parameters continues until the change in the merit function decreases below a specific convergence level. The recommended Marquardt recipe includes the following steps in iterating to convergence of χ^2:

1. Assign initial values to the set of fitted parameters **a** and assign $\lambda_{LM}=0.001$, where λ_{LM} is a nondimensional step size factor used in locating the minimum χ^2
2. Solve for the increment ($\delta\mathbf{a}$) and evaluate $\chi^2(\mathbf{a}+\delta\mathbf{a})$
3. If $|\chi^2(\mathbf{a}+\delta\mathbf{a})-\chi^2(\mathbf{a})|$>convergence level then:
   a. If $\chi^2(\mathbf{a}+\delta\mathbf{a})\geq\chi^2(\mathbf{a})$, increase λ_{LM} by a factor of 10 and repeat step 2
   b. If $\chi^2(\mathbf{a}+\delta\mathbf{a})<\chi^2(\mathbf{a})$, decrease λ_{LM} by a factor of 10, update the trial solution and repeat step 2

This research performs a sensitivity analysis based on the effect of uncertainty in confined and water-table aquifer parameters (i.e., radial hydraulic conductivity, specific yield, specific storage, and the ratio of vertical to radial hydraulic conductivity) on streambed conductance estimation. Such an investigation will determine the accuracy of analytical streambed conductance estimation techniques relative to aquifer parameter uncertainty.

5.2 Methodology

A program is developed to automatically estimate streambed conductance, λ, from observed drawdown response in defined confined and water-table aquifers (i.e., true values of K, K_z/K, S_S, and S_y are known). The program makes use of the Levenberg-Marquardt method for nonlinear least squares modeling, allowing the input of known values of aquifer parameters and trial values of streambed conductance, and then fits a model to the observed drawdown response by updating streambed conductance until the χ^2 merit function is minimized. A "shotgun approach" is used to adjust initial streambed conductance values (Cheney and Kincaid, 1994). Characteristic aquifer, stream, and pumping well characteristics for both a confined and water-table aquifer are outlined in Table 5.1.

STRMAQ is used to generate drawdown profiles for two hypothetical aquifer and streambed scenarios, as shown in Figures 5.2 and 5.3 for an observation well at x/L=0.2. The effect of uncertainty in aquifer parameters (K, K_z/K, S_S, and S_y) on estimates of streambed conductance, λ, is investigated. The observed drawdown profiles generated by STRMAQ for the defined aquifers are input into the estimation program. The program is used to determine the effect of uncertainty in K, K_z/K, S_S, and S_y on the nonlinear least-squares estimation of streambed conductance.

Table 5.1 - Aquifer and pumping well parameters for hypothetical confined and water-table aquifers.

Aquifer Parameters	*Water-Table*	*Confined*
Hydraulic Conductivity, K (m-day^{-1})	100	100
Anisotropy Ratio, K_z/K*	0.1	0.1
Aquifer Thickness, b (m)	30	30
Specific Storage, S_S (m^{-1})	0.0001	0.0001
Storage Coefficient, S	0.003	0.003
Specific Yield, S_y	0.2	
Pumping Well Parameters*		
% Penetration	100	
Radius of Well (m)	0.1	
Pumping Rate, Q (m^3-day^{-1})	10000	
Distance between Well and Stream, L (m)	50	
Well-Skin Parameter, SW	0	

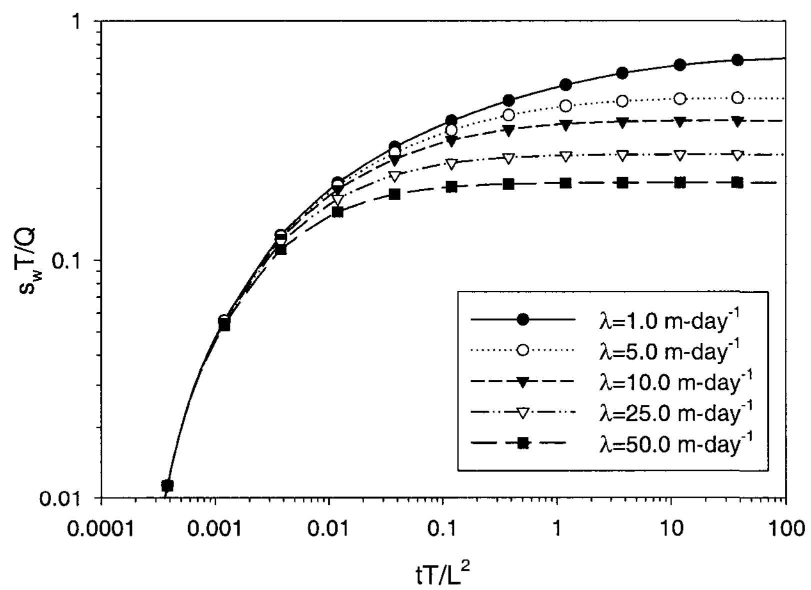


Figure 5.2 - Drawdown response for variable streambed conductance (λ) in a confined aquifer using STRMAQ for an observation well at x/L=0.2.

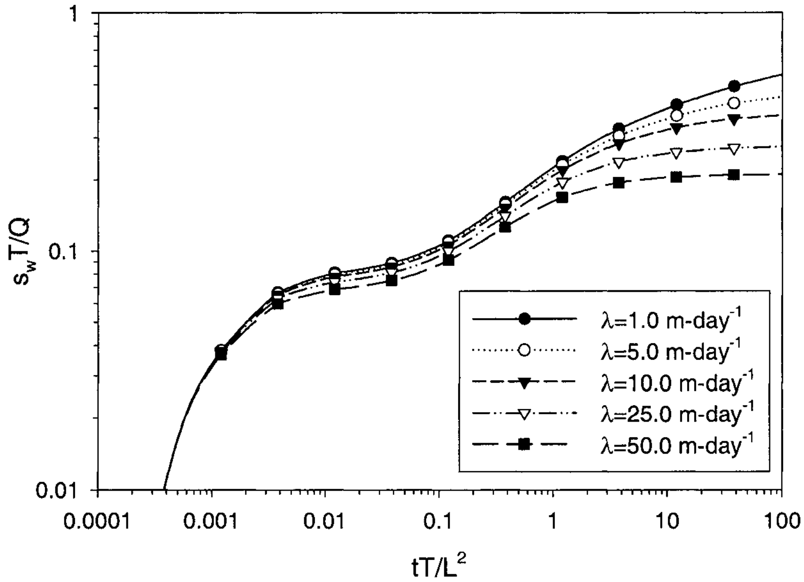


Figure 5.3 - Drawdown response for variable streambed conductance (λ) in a water-table aquifer using STRMAQ for an observation well at x/L=0.2.

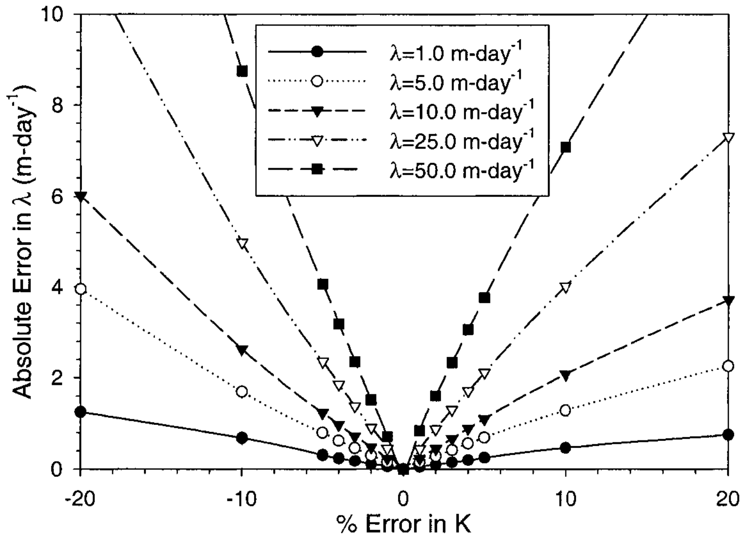
Sensitivity analyses for the effect of aquifer parameter uncertainty on streambed conductance estimations are performed by individually varying each parameter within a 20% range above and below the true parameter value, as given in Table 5.1. The effect of uncertainty is quantified by plotting percent errors in aquifer parameters versus absolute error in predicted streambed conductance. This procedure is performed for streambed conductance ranging between 1.0 and 50.0 m-day^{-1} and for different observation well locations (i.e., x/L=0.2, y/L=0 and x/L=0.5, y/L=0.0).

5.3 Results and Discussion

The absolute error in predicted streambed conductance versus percent error in confined aquifer parameters is shown in Figures 5.4 and 5.5. Initial investigations are based on the confined aquifer, varying the radial hydraulic conductivity, as shown in Figure 5.4, specific storage, as shown in Figure 5.5, and ratio of vertical to radial hydraulic conductivity. Absolute errors in predicted streambed conductance increase with observation well distance from the stream for uncertainty in aquifer parameters. More accurate estimates of streambed conductance are obtained when observation wells are located closer to the stream. Uncertainty in radial hydraulic conductivity results in approximately 10 times greater absolute deviations in predicted conductance than those corresponding to uncertainty in specific storage. Absolute errors in streambed conductance due to ±20% error in specific storage and the ratio of vertical to radial hydraulic conductivity are less than 1.0 m-day^{-1} and 0.1 m-day^{-1}, respectively.

The relationship between the absolute error in streambed conductance and percent error in aquifer parameters is not linear. When percent errors in aquifer parameters are positive, the aquifer is assumed to have a greater storage capacity. This increased storage results in less demand for water from the stream to satisfy the pumping well discharge requirements, resulting in a lower predicted streambed conductance. The predicted streambed conductance gets smaller as the percent error in aquifer parameters increases, limited by a zero streambed conductance that simulates a non-conductive streambed. In other words, the rate of change in the absolute error in the streambed conductance is small compared to changes in the percent error in aquifer parameters, resulting in a nonlinear relationship.

(a)



(b)

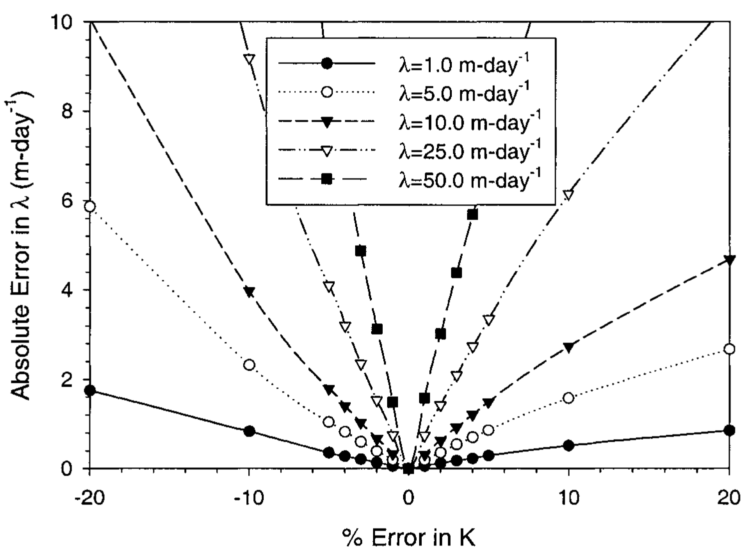


Figure 5.4 - Absolute error in predicted streambed conductance (λ) versus percent error in radial hydraulic conductivity (K) for a confined aquifer and an observation well at (a) x/L=0.2 and (b) x/L=0.5.

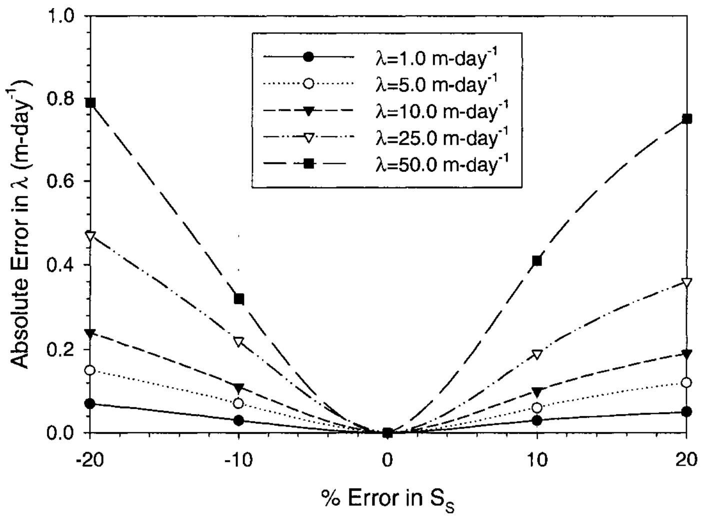


Figure 5.5 - Absolute error in predicted streambed conductance (λ) versus percent error in aquifer specific storage (S_S) for a confined aquifer and observation well at x/L=0.2.

As the percent errors in aquifer parameters become negative, the aquifer is assumed to have less storage capacity, creating a greater demand for streamflow to satisfy the pumping well discharge requirements. This greater demand for streamflow increases the predicted streambed conductance compared to the true conductance and causes the rate of change in the absolute error in streambed conductance to increase much more rapidly with increases in percent errors in aquifer parameters.

The effect of uncertainty in water-table aquifer parameters is then investigated, varying the radial hydraulic conductivity, as shown in Figure 5.6, specific yield, as shown in Figure 5.7, specific storage, and ratio of vertical to radial hydraulic conductivity. For the water-table aquifer, specific storage and the ratio of vertical to radial hydraulic conductivity does not result in significant absolute errors (i.e., less than 0.1 m-day^{-1}) in predicted streambed conductance for either location of observation well. Specific storage only affects early-time drawdown in water-table aquifers, during which there is insignificant stream depletion.

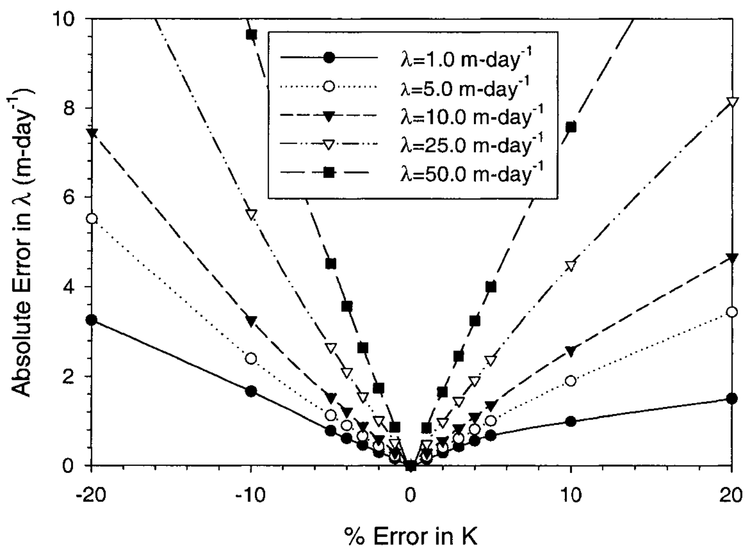


Figure 5.6 - Absolute error in predicted streambed conductance (λ) versus percent error in radial hydraulic conductivity (K) for a water-table aquifer and observation well at x/L=0.2.

For the water-table aquifer, absolute errors in predicted streambed conductance again increase with observation well distance from the stream and percent errors associated with radial hydraulic conductivity are larger than those associated with specific yield, specific storage, or the ratio of vertical to radial hydraulic conductivity. Absolute errors due to percent errors in specific yield for the water-table aquifer are greater than the absolute errors due to uncertainty in specific storage in the confined aquifer.

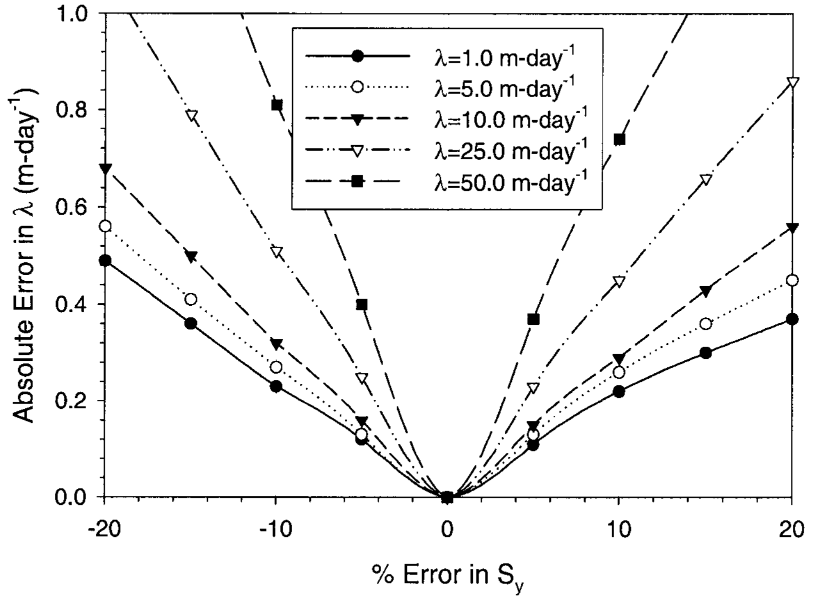


Figure 5.7 - Absolute error in predicted streambed conductance (λ) versus percent error in aquifer specific yield (S_y) for a water-table aquifer and observation well at x/L=0.2.

Absolute errors in streambed conductance due to uncertainty in radial hydraulic conductivity are also greater in the water-table aquifer compared to the confined aquifer for less conductive streambeds. For example, for a streambed conductance, λ, of 10.0 m-day^{-1}, an estimated aquifer hydraulic conductivity, K, of 110 m-day^{-1} (i.e., +10% error), and an observation well at x/L=0.2, the absolute error in predicted streambed conductance is 2.07 m-day^{-1} and 2.58 m-day^{-1} in the confined and water-table aquifers, respectively. The difference in absolute errors in predicted streambed conductance between the confined and water-table aquifers is greater for the observation well at x/L=0.5. However, as streams become more conductive, the absolute errors in predicted streambed conductance for the confined and water-table aquifers converge.

5.4 Summary and Conclusions

A non-linear least squares program based on the Levenberg-Marquardt method of nonlinear modeling is developed to investigate the prediction of streambed conductance in defined confined and water-table aquifers. Analytical solutions have recently been developed for the interaction of aquifers with partially penetrating streams. These solutions are suggested as a mechanism for estimating the streambed conductance parameter to quantify the degree of interaction between the groundwater and surface water resources. Questions exist about subjective judgment in the use of such estimation techniques and the accuracy of estimates with uncertain aquifer parameters.

The effect of aquifer parameter uncertainty on analytical streambed conductance estimates is evaluated based on observed drawdown response of hypothetical confined and water-table aquifers using STRMAQ. Aquifer parameters are varied within 20% of their assumed true value and the streambed conductance is predicted. Comparisons are made between the absolute errors in predicted streambed conductance with the percent error in aquifer parameters for varying degrees of streambed conductance.

In general, greater absolute errors occur due to uncertainty in radial hydraulic conductivity rather than specific storage, specific yield, or the ratio of vertical to radial hydraulic conductivity. Absolute errors in streambed conductance for uncertainty in radial hydraulic conductivity are greater in the water-table aquifer compared to the confined aquifer for less conductive streambeds and then the absolute errors converge as the streambed conductivity increases. Absolute errors due to percent errors in specific yield for the water-table aquifer are also greater than the absolute errors due to specific storage in the confined aquifer. The effect of aquifer parameter uncertainty on estimated streambed conductance becomes greater when observation wells are located further from the stream. When aquifer parameters, especially radial hydraulic conductivity, are not known with less than 10% error, analysts must be aware that derived estimates of streambed conductance may have significant uncertainty.

CHAPTER 6

UNSATURATED FLOW IN STREAM/AQUIFER CONJUNCTIVE SYSTEMS

6.1 Introduction

The hydraulic connection between surface water and groundwater resources has long been recognized as a crucial factor in the management of water resources. However, new questions in water supply, water quality, and stream ecology are emerging as water supplies become over appropriated, water quality degradation becomes more widespread, and water management must satisfy the dual objectives of water supply and ecosystem habitat conservation (Allan, 1995; Winter et al., 1988). These new concerns are causing researchers and managers to take a new look at the complex hydrology at the interface between surface water and groundwater.

An important groundwater/surface water interface is stream leakage to an underlying aquifer. Unsaturated flow can occur within this interface under a number of different scenarios. Ephemeral streams are commonly perched above the groundwater table. In cases where the stream and aquifer are initially connected, groundwater extraction from wells can initiate unsaturated flow depending on the streambed hydrologic properties. If the streambed is thin with approximately the same physical and hydraulic properties as the underlying soil, flow through the region beneath the streambed is saturated, and the discharge will be a function of the location of the water table. However, the streambed often has a conductivity that is orders of magnitude less than the underlying aquifer and is sufficiently thick so that most of the hydraulic head loss occurs in the less conductive layer (Calver, 2001; Larkin and Sharp, 1992; Rosenshein, 1988). This conductivity contrast is more realistically analyzed as a layered system rather than a homogeneous profile. In general, seepage from a stream with a less permeable streambed can be characterized by three flow regimes: saturated flow, a transition zone, and unsaturated flow. The location of the water table, soil properties, and accessibility of an air phase determine the regime.

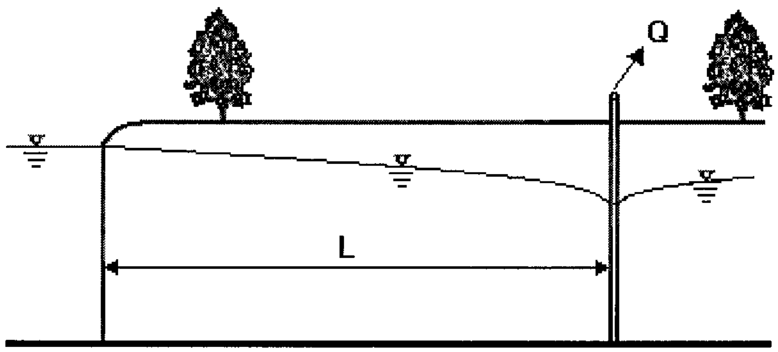
The objective of this research is to discuss the development and implications of unsaturated stream/aquifer flow. First, the conditions that determine the flow regime and the characteristics of each regime are discussed. Then, the effect of an unsaturated flow is illustrated for the case of stream leakage induced by a well pumping from an aquifer that is hydraulically interacting with a partially penetrating stream. A pumping well will intercept groundwater, reducing recharge to the stream and/or extracting water from the stream. While this problem has been analyzed extensively for decades (Butler et al., 2001; Hunt et al., 2001; Hunt, 1999; Conrad and Beljin, 1996; Sophocleous et al, 1995; Hantush, 1965; Theis, 1941), it is always assumed that the stream is a constant head boundary and flow between the stream and aquifer is saturated.

The most well known equation for drawdown due to a pumping well in an infinite aquifer is the Theis (1935) solution. Theis expanded this equation to simulate a pumping well adjacent to a stream through image well theory (Theis, 1941). This solution assumes an infinitely long, straight, completely penetrating stream in a homogeneous, isotropic, confined aquifer stressed by a well pumping at a constant discharge rate, as shown in Figure 6.1(a). Glover and Balmer (1954) expressed the Theis (1941) solution in terms of stream depletion. Hantush (1965) developed an analytical model that considers the effects of a semipervious streambed. The semipervious streambed is represented as a vertical layer of lower conducting material extending throughout the saturated thickness of the aquifer, as shown in Figure 6.1(b). Hunt (1999) developed an analytical solution for drawdown and stream depletion that accounts for a semipervious streambed and stream partial penetration, as shown in Figure 6.1(c).

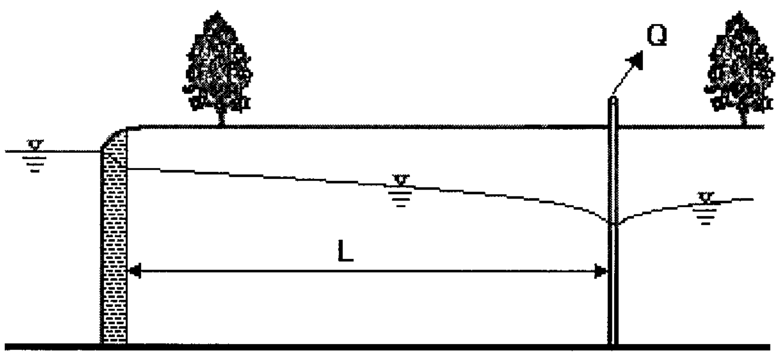
Butler et al. (2001) developed an analytical solution that considers finite stream width and an aquifer of limited lateral extent. However, these analytical solutions fail to model the case where the region between the streambed and water table becomes unsaturated. If the water table falls a sufficient distance below the bottom of the streambed, the higher head loss in the less conductive streambed will cause the region beneath the streambed to become unsaturated. When this occurs, the stream and aquifer are often said to hydraulically "disconnect" (Rushton, 1999; Bouwer, 1978). The stream and underlying aquifer remain hydraulically connected, albeit through unsaturated flow. A more appropriate term is that the stream is "perched" above the water table.

Unsaturated flow is important mathematically because it transforms streams from constant head boundaries, where the discharge from the stream is dependent on the drawdown in the aquifer, to constant flux boundaries, where the discharge is independent of the position of the water table. Unsaturated flow is also important environmentally because it impacts biogeochemical reactions in the region beneath the streambed.

(a)



(b)



(c)

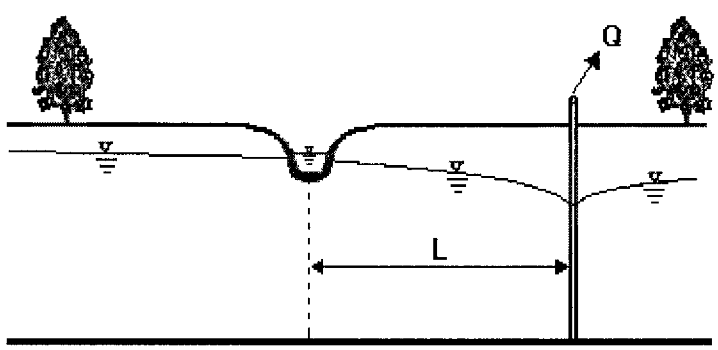


Figure 6.1 - Outline of the problem considered by (a) Theis (1941), (b) Hantush (1965), and (c) Hunt (1999). L=distance between the stream and pumping well and Q=discharge rate of the pumping well. Modified from Hunt (1999).

6.2 Stream/Aquifer Interaction

A homogeneous and isotropic streambed is assumed to exist between the stream and underlying aquifer. The streambed is assumed to have a lower hydraulic conductivity than the underlying soil. It is also assumed that leakage from the stream to the aquifer is steady, vertical, one-dimensional flow. For these conditions, it can be shown that the streambed will remain saturated, as discussed by Corey (1994). The following discussion follows an analysis similar to that presented by Corey (1994) for flow through layered profiles and addresses three stream/aquifer hydrologic states or regimes:

- Regime A – Saturated Flow: Stream leakage rate depends on the location of the water table.
- Regime B – Transition zone: Stream leakage rate governed by unsaturated flow hydraulic conditions.
- Regime C – Unsaturated Gravity-Driven Flow: Stream leakage rate is not a function of the location of the water table.

6.2.1 Regime A: Saturated Flow

Saturated flow occurs in the region beneath the streambed when the water table is located within the streambed, when the water table is slightly below the bottom of the streambed but water pressures are not sufficiently negative to desaturate the subsoil, or if there are no pathways for the air phase to enter the soil beneath the streambed. When the water table is located within the streambed, the specific discharge, q [LT^{-1}], through that region of the streambed between the bottom of the stream and the water table (assuming one-dimensional, vertical flow, where a downward flux is assumed to be negative) is:

$$q = -K_{sb} \frac{s_w}{s_w - H_w} \tag{6.1}$$

where K_{sb} is the saturated hydraulic conductivity of the streambed [LT^{-1}] and s_w is drawdown [L], defined as the distance between the water level in the stream, H_w [L], and the water table. If the water table is located just at the bottom of the streambed, equation (6.1) reduces to

$$q = -K_{sb} \frac{H_w + M}{M} \tag{6.2}$$

where M is the thickness of the streambed [L]. In both of these simple cases, the hydraulic head in the stream is driving the flow and viscous resistance is proportional to the hydraulic conductivity of the streambed. If the water table declines to an elevation below the bottom of the streambed, the specific discharge is given by:

$$q = -K_{sb} \frac{H_w + M - h_w}{M} \tag{6.3}$$

where h_w is the water pressure head at the bottom of the streambed [L]. Because the water table is below the streambed, the water pressure head, h_w, at the streambed/aquifer interface is negative. It is useful to consider water pressure head, h_w, in terms of capillary pressure head, h_c [L]. If pore fluid pressures are measured with respect to ambient air pressures, assumed zero, then

$$h_c = -h_w \tag{6.4}$$

Rewriting equation (6.3) in terms of capillary pressure head, h_c, gives:

$$q = -K_{sb} \frac{H_w + M + h_c}{M} \tag{6.5}$$

The region below the streambed may or may not desaturate, depending on the magnitude of the water pressure head at the interface between the streambed and aquifer and if air has access to enter the zone beneath the streambed. Desaturation of the underlying soil will occur if the capillary pressure head is greater (i.e., the water pressure head is more negative) than an air entry capillary pressure head, h_e [L]. The air entry pressure head is that capillary pressure head below which the nonwetting phase (in this case air) becomes discontinuous and is not capable of flowing through the material (Corey, 1994). The entry pressure head loosely represents the height of the capillary fringe in a uniform soil.

The value of an entry pressure head can be estimated by curve fitting capillary pressure-saturation data (Corey, 1994; Corey, 1992). There is considerable ambiguity regarding this critical capillary pressure head and it is variously referred to as a displacement pressure head, a bubbling pressure head, and an entry pressure head (Corey, 1994; White et al., 1972). If there are no pathways for air, the region below the streambed will also remain saturated no matter how far the water table drops below the streambed. This research will assume that pathways exist for air to enter the zone beneath the streambed.

6.2.2 Regime B: Transition Flow

Regime B is a transition zone between saturated flow and gravity-driven, unsaturated flow. Unsaturated flow occurs when the water table falls a sufficient distance below the bottom of the streambed so that some of the pores desaturate, but the water table is not sufficiently far below the streambed for a unit hydraulic gradient to exist. It can be shown that the streambed will remain saturated (Corey, 1994). Figure 6.2 shows the distribution of capillary pressure head in this region for this regime.

The specific discharge of water through the unsaturated zone is given by:

$$q = -K(h_c)\left(1 - \frac{\partial h_c}{\partial z}\right) \tag{6.6}$$

where h_c is the capillary pressure head, $K(h_c)$ is a constitutive relation between hydraulic conductivity and capillary pressure, and z is the depth within the unsaturated zone.

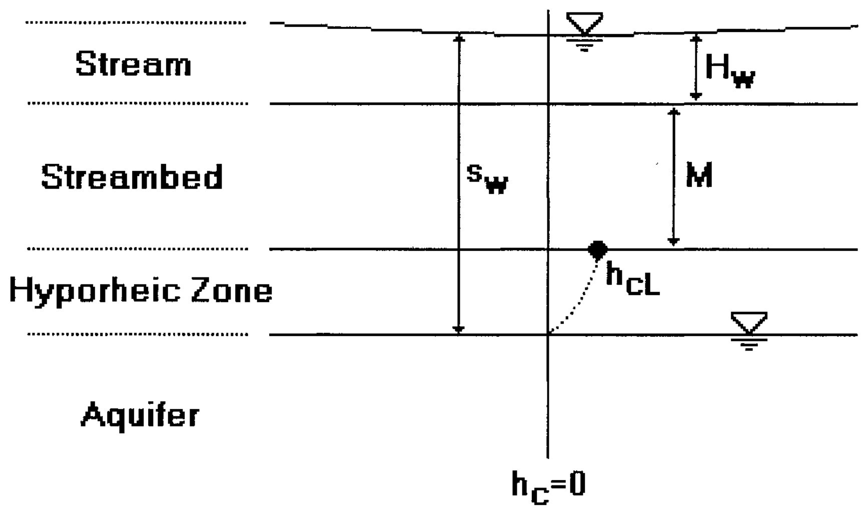


Figure 6.2 – Capillary pressure head (h_c) distribution in the region beneath the streambed during regime B, prior to the formation of a unit hydraulic gradient beneath the streambed. h_{cL}=interface capillary pressure head.

It is convenient to fit algebraic expressions to the capillary pressure head-hydraulic conductivity relationship. Among the most popular of these algebraic expressions are the Gardner (1958), Brooks and Corey (1964), and van Genuchten (1980) equations. The Brooks-Corey equations are

$$
\begin{aligned}
K(h_c) &= K_s & h_c &\le h_e \\
K(h_c) &= K_s\left(\frac{h_e}{h_c}\right)^{\eta} & h_c &> h_e
\end{aligned}
\tag{6.7}
$$

where K_s is the saturated hydraulic conductivity, h_c is capillary pressure head, h_e is the entry capillary pressure head, and η is a parameter dependent on the pore-size distribution index (Corey, 1994; Brooks and Corey, 1964). Rearranging equation (6.6) and replacing $K(h_c)$ with the Brooks-Corey equation results in:

$$\frac{\partial h_c}{\partial z} = 1 - \frac{|q|}{K_s}\left(\frac{h_c}{h_e}\right)^{\eta} \qquad h_c > h_e \tag{6.8}$$

Other constitutive relationships for $K(h_c)$ can be used instead of the Brooks-Corey equations. Equation (6.8) can be integrated from the water table to the bottom of the streambed to relate the drawdown, s_w, and the capillary pressure head at the bottom of the streambed, h_{cL}, in terms of the hydraulic conductivity of the subsoil and soil parameters, h_e and η:

$$s_w - M - H_w = \int_0^{h_{cL}} \frac{\partial h_c}{1 - \frac{|q|}{K_s}\left(\frac{h_c}{h_e}\right)^{\eta}} \tag{6.9}$$

Equation (6.5) with $h_c = h_{cL}$ and equation (6.9) can be solved simultaneously for the specific discharge, q, and the interface capillary pressure head, h_{cL}.

The specific discharge in regime B has a magnitude between those in regimes A and C, but under most circumstances, the length of time that this regime applies is small. Representative values of the entry pressure head, h_e, and Brooks-Corey parameter, η, for different subsoils are shown in Table 6.1. Using these representative values, equation (6.8) is solved for the change in the capillary pressure head gradient with elevation ($\partial h_c/\partial z$) as a function of the capillary pressure head (h_c).

Table 6.1 - Typical values of entry pressure head, h_e, and Brooks-Corey parameter, η, on a drainage curve. Values are obtained from Bear (1972) and Carsel and Parrish (1988).

Soil Type	**h_e (cm of water)**	η
Coarse Sand	2-10	8.0
Medium Sand	10-35	6.5
Fine Sand	35-70	5.9
Silt	70-150	3.1
Clay	>200-400	2.6

Results are shown in Figure 6.3 for fine, medium and coarse subsoils with two different ratios of the specific discharge to saturated hydraulic conductivity, $|q|/K_s$. When $\partial h_c/\partial z=0$, a unit hydraulic gradient exists in the region beneath the streambed. The formation of this unit hydraulic gradient signals the end of regime B. This figure demonstrates that the capillary pressure gradient $\partial h_c/\partial z$ goes to zero quickly, especially for medium and coarse sand subsoils.

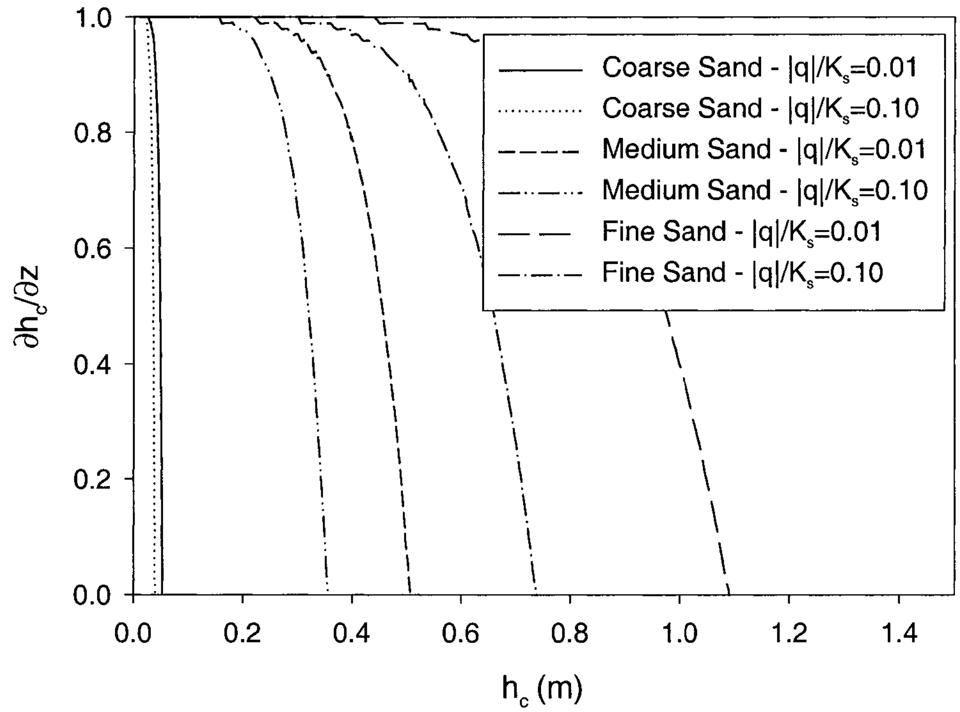


Figure 6.3 – Gradient of capillary pressure head with elevation ($\partial h_c/\partial z$) as a function of the interface capillary pressure head (h_c) for different soil types and the ratio of specific discharge to saturated hydraulic conductivity, $|q|/K_s$.

6.2.3 Regime C: Unsaturated Gravity-Driven Flow

Regime C is defined by the presence of only gravity-driven flow ($\partial h_c/\partial z=0$) in contrast to the other regimes where there is also a pressure gradient in the unsaturated zone beneath the streambed (Figure 6.4). In regime C, stream leakage reaches a maximum, as the interface capillary pressure head, h_{cL}, becomes a maximum. This maximum interface capillary pressure head will be referred to as the ultimate interface capillary pressure head, h_{cu}. This ultimate interface capillary pressure head is a function of the air entry pressure head, h_e, and saturated hydraulic conductivity, K_s, of the aquifer:

$$h_{cu} = h_e \left(\frac{K_s}{|q|} \right)^{1/\eta} \tag{6.10}$$

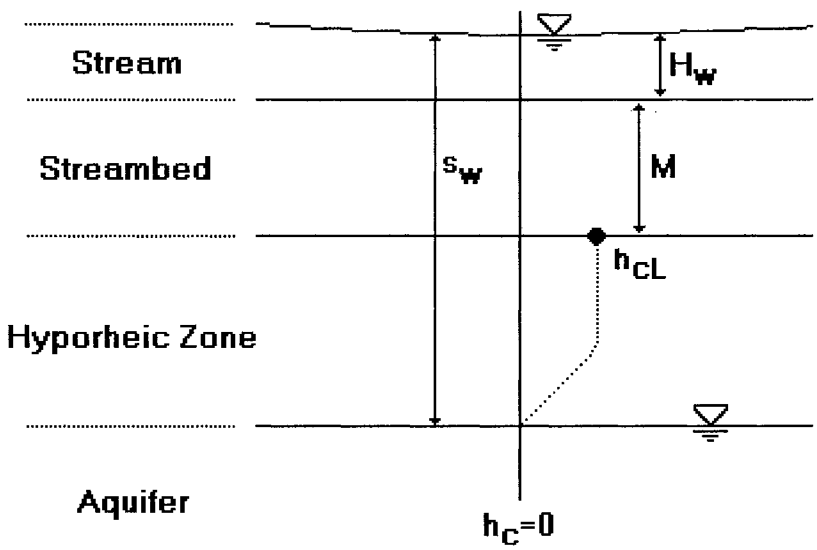


Figure 6.4 – Capillary pressure head (h_c) distribution in the unsaturated zone during regime C, after formation of a unit hydraulic gradient. h_{cL}=interface capillary pressure head.

The specific discharge, q_{max}, is given by:

$$q_{max} = -K_{sb}\left[1 + \frac{(h_{cu} + H_w)}{M}\right] \tag{6.11}$$

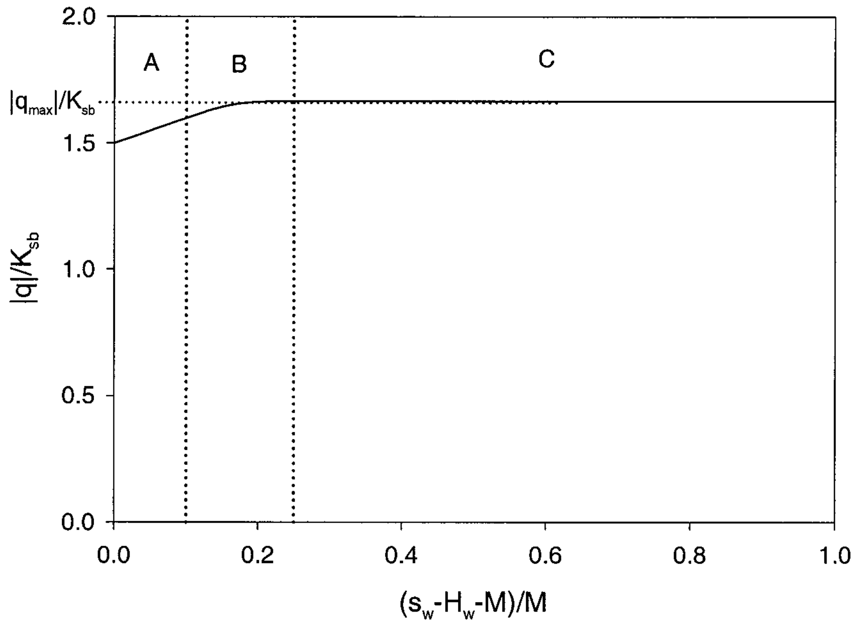
and simplifying equation (6.6) as:

$$q_{\max} = -K_s \left(\frac{h_e}{h_{cu}} \right)^{\eta} \qquad h_{cu} > h_e \tag{6.12}$$

Equations (6.11) and (6.12) can be solved simultaneously for q_{max} and h_{cu}.

In regime C, the specific discharge and ultimate capillary pressure head are independent of the position of the water table. Thus, the stream changes from a constant head boundary to a constant flux boundary. Figure 6.5 shows an example assuming a stream with flow depth, H_w=0.5 m, a 1-m thick streambed, and hydraulic conductivity, K_{sb}, of 0.1 m/day, overlays a subsoil with a saturated hydraulic conductivity, K_s, of 10 m/day, η=6.5, and h_e=0.1 m. These values are characteristic of a silt streambed overlying a medium, sandy aquifer (Carsel and Parrish, 1988; Rawls and Brakensiek, 1982). Figure 6.5(a) plots the specific discharge, nondimensionalized by K_{sb}, as a function of position of the water table below the streambed, nondimensionalized by the streambed thickness, M. This figure shows that (a) stream leakage varies with water table position in regimes A and B, (b) the transition from Regime A to Regime C occurs over a small range of drawdowns, (c) a constant maximum leakage rate, $|q_{max}|$, is obtained when a unit hydraulic gradient occurs in regime C, and (d) the stream leakage does not change with any additional decline in water table position in regime C.

(a)



(b)

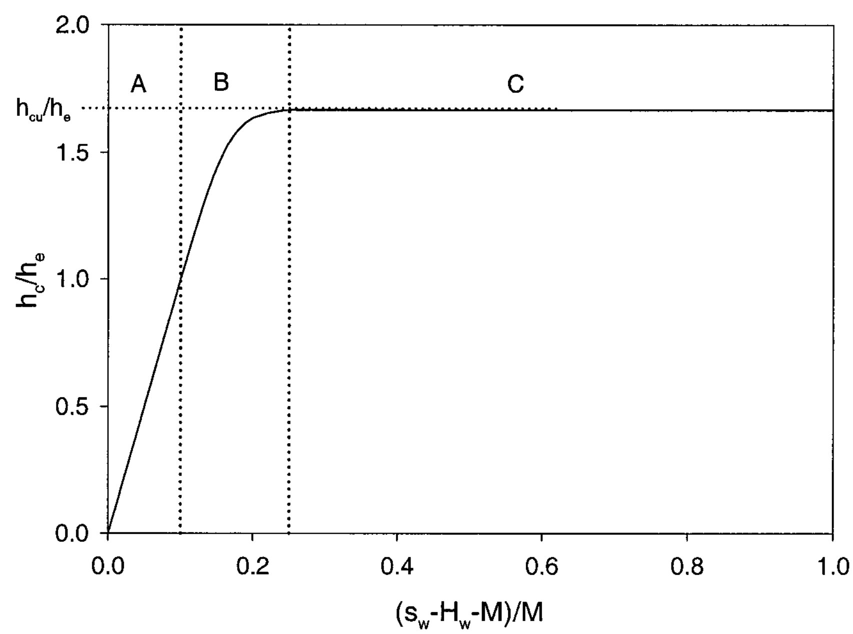


Figure 6.5 – (a) Specific discharge from the stream, $|q|$, divided by the streambed hydraulic conductivity, K_{sb}, and (b) interface capillary pressure head, h_{cL}, divided by the entry pressure head of the aquifer, h_e, as a function of water table position for regimes A (saturated flow), B (transition), and C (unsaturated flow).

Figure 6.5(b) plots the interface capillary pressure head (h_{cL}), nondimensionalized by the entry pressure head of the subsoil (h_e), as a function of the position of the water table below the streambed. During saturated flow (regime A), pore spaces are completely filled with water because the water pressure heads are not sufficiently negative to desaturate the pores. The interface capillary pressure head increases with increasing drawdown in regime B, reaching a constant, maximum capillary pressure head. This maximum capillary pressure head, or ultimate interface capillary pressure head, h_{cu}, indicates gravity-driven flow is occurring in the unsaturated zone. Once h_{cu} is reached, the capillary pressure head does not increase with additional declines in the water table.

6.3 Improved Stream Leakage Package for MODFLOW

MODFLOW is a widely used, finite-difference flow model for simulating saturated groundwater flow and is capable of simulating the interaction between streams and underlying alluvial aquifers. MODFLOW models stream/aquifer interaction using the RIVER or STREAM packages (Harbaugh and McDonald, 1996). The RIVER package assumes that the stream stage remains constant throughout a given stress period within the model. This constant stream stage is then utilized to calculate the flux of water between the stream and aquifer system, proportional to the head gradient between the river and aquifer and a streambed conductance parameter. The limitations in modeling the stream with a constant head in a given stress period led to the development of the

STREAM package. The STREAM package is a streamflow routing model limited to steady flow through a rectangular, prismatic channel (Prudic, 1989).

MODFLOW's RIVER and STREAM packages accurately account for saturated flow and account for unsaturated flow by making simplifying assumptions. When the aquifer head is above the bottom of the streambed (i.e., saturated flow), the specific discharge through the streambed is assumed proportional to the difference in hydraulic head between the stream and aquifer:

$$q = -\frac{K_{sb}}{M} s_w \tag{6.13}$$

where q is specific discharge [LT^{-1}] with a downward flux assumed negative, K_{sb} is the streambed hydraulic conductivity [LT^{-1}], M is the streambed thickness [L], and s_w is the drawdown defined as the difference between the water level in the stream, h_w [L], and the hydraulic head in the aquifer, h [L]. If the aquifer head drops below the bottom of the streambed, MODFLOW's packages assume that the seepage is no longer proportional to the aquifer head and becomes dependent on the water level in the stream and the streambed thickness, M [L]:

$$q = -\frac{K_{sb}}{M}\left(H_w + M\right) \tag{6.14}$$

Osman and Bruen (2002) presented an improved method for incorporating unsaturated flow into the RIVER package. Their research made use of the van

Genuchten (1980) relationship for the capillary pressure head-hydraulic conductivity relation. Additionally, Osman and Bruen make the simplifying assumption that the seepage during the transition regime (Regime B) is proportional to the difference in the water level in the stream and the aquifer head.

The proposed improvement to the RIVER package in this research is based on the use of the Brooks-Corey (1964) relationship. Similar to the current version of the RIVER package, saturated flow occurs as long as the hydraulic head in the aquifer is above the bottom of the streambed (i.e., $s_w \leq H_w + M$). Seepage between the stream and aquifer is governed by equation (6.13). Saturated flow also occurs as long as the capillary pressure head does not exceed the air entry capillary pressure head (i.e., $s_w \leq H_w + M + h_e$).

Regime C occurs when the aquifer head falls a sufficient distance below the bottom of the streambed such that a maximum capillary pressure head, h_{cu}, will form at the bottom of the streambed. Specific discharge through the streambed and unsaturated zone will be governed by equations (6.11) and (6.12), which can be solved simultaneously for q_{max} and h_{cu}.

The transition regime occurs when the hydraulic head is sufficiently far below the bottom of the streambed (i.e., $s_w > H_w + M + h_e$) but not sufficiently far enough below to create a unit hydraulic gradient (i.e., $s_w < H_w + M + h_{cu}$). A capillary pressure head is created at the bottom of the streambed that increases the force to induce water through the streambed, such that seepage through the streambed is given by:

$$q = -\frac{K_{sb}}{M}\left(H_w + M + h_{cl}\right) \tag{6.15}$$

where h_{cl} is the capillary pressure head at the bottom of the streambed. Seepage through the unsaturated zone during Regime B is given by equation (6.9). The integral in equation (6.9) can be expressed as a Lerch Phi function (Erdelyi, 1953):

$$s_w - M - H_w = \frac{h_{cl}}{\eta} LerchPhi\left(\frac{q}{K_s}\left(\frac{h_{cl}}{h_e}\right)^{\eta}, 1, \frac{1}{\eta}\right) \tag{6.16}$$

where LerchPhi(z,a,v) is given by the following infinite series as long as $|z|<1$ (Erdelyi, 1953):

$$LerchPhi(z,a,v) = \sum_{n=0}^{\infty} \frac{z^n}{(v+n)^a} \tag{6.17}$$

Equations (6.15) and (6.16) can be solved simultaneously for q and h_{cl}.

Three different modifications of the current version of the MODFLOW RIVER package, RIV_M, are created for purposes of determining the influence of each regime on unsaturated stream/aquifer exchange. The three packages are summarized in Table 6.2. The first modified version, RIV_N, removes any consideration of unsaturated flow from the RIVER package. This version simulates the assumptions commonly addressed in analytical stream/aquifer solutions by assuming the stream remains a constant head boundary condition no matter how far the aquifer head drops below the bottom of the streambed.

Table 6.2 - Summary of four MODFLOW RIVER packages used to investigate importance of unsaturated flow in stream/aquifer interaction.

RIV_M	Standard Version of MODFLOW RIVER Package
	Saturated Flow: $s_w \leq H_w+M$, $q \propto s_w$
	Unsaturated Flow: $s_w > H_w+M$, $q \propto H_w+M$
RIV_N	Only Saturated Flow
	Saturated Flow: $s_w > 0$, $q \propto s_w$ No Unsaturated Flow
RIV_AC	Accounts for Regimes A, B and C (Similar to Osman and Bruen, 2002)
	Saturated Flow: $s_w \leq H_w+M+h_{cu}$, $q \propto s_w$
	Unsaturated Flow: $s_w > H_w+M+h_{cu}$, $q \propto H_w+M+h_{cu}$
RIV_U	Accounts for Regimes A, B and C
	Saturated Flow: $s_w \leq H_w+M+h_e$, $q \propto s_w$ Transition Zone: $H_w+M+h_e > s_w > H_w+M+h_{cu}$, $q \propto H_w+M+h_{cl}$
	Unsaturated Flow: $s_w \geq H_w+M+h_{cu}$, $q \propto H_w+M+h_{cu}$

The next modified version is a RIVER package that simulates only Regimes A (saturated flow), uses Regime A to simulate the transition zone (Regime B), and Regime C (unsaturated flow), RIV_AC. In this scenario, saturated flow is assumed to occur until the drawdown falls a sufficient distance below the bottom of the streambed to form a maximum capillary pressure head (i.e., $s_w < H_w+M+h_{cu}$). This scenario is similar to the proposed modification of Osman and Bruen (2002). The final modified version of the RIVER package, RIV_U, incorporates flow conditions governed by all three regimes. The FORTRAN code for the updated MODFLOW RIVER packages are provided in Appendix C.

The four MODFLOW RIVER packages listed in Table 6.2 are compared based on predicted drawdown and stream depletion for a hypothetical stream/aquifer system. A MODFLOW numerical model is constructed to simulate a pumping well located in alluvial aquifer next to a partially penetrating stream. The alluvial aquifer is assumed homogenous, isotropic, and consisting of coarse sands with an air entry capillary pressure head, h_e, of 5 cm and a pore size distribution index of 2 (i.e., η=8). The stream is assumed to have a width of 5 m and a constant stream stage of 0.5 m. The streambed is assumed to be homogeneous and isotropic, consisting of fine sands and having a hydraulic conductivity, K_{sb}, equal to 0.5 m-d^{-1} and thickness of 0.5 m.

The modified RIVER packages are independently integrated into the MODFLOW numerical model. Aquifer drawdown as predicted by the four different RIVER packages is compared in Figure 6.6 based on dimensionless drawdown, s_wT/Q, versus dimensionless time, tTS_y/SL^2, where T is the aquifer transmissivity [L^2T^{-1}], S_y is the specific yield, S is the storage coefficient, L is the distance between the stream and pumping well [L], and Q is the constant discharge rate of the pumping well [L^3T^{-1}]. Drawdown is measured at the location on the stream that is nearest to the pumping well.

For this scenario, unsaturated flow occurs when $tTS_y/SL^2>100$. The maximum capillary pressure head is 8 cm and the maximum specific discharge equals 1.08 m-d^{-1}. RIV_N simulates a continually saturated stream/aquifer exchange. RIV_N is equivalent to recently proposed analytical solutions, such as Hunt's (1999) solution, suggested to improve administering water rights in the western United States. Assuming flow stays saturated beneath the streambed causes the specific discharge to continue to increase with drawdown no matter how far the aquifer head falls below the bottom of the streambed.

Therefore, this RIVER package predicts less aquifer drawdown compared to the other packages that account for unsaturated flow.

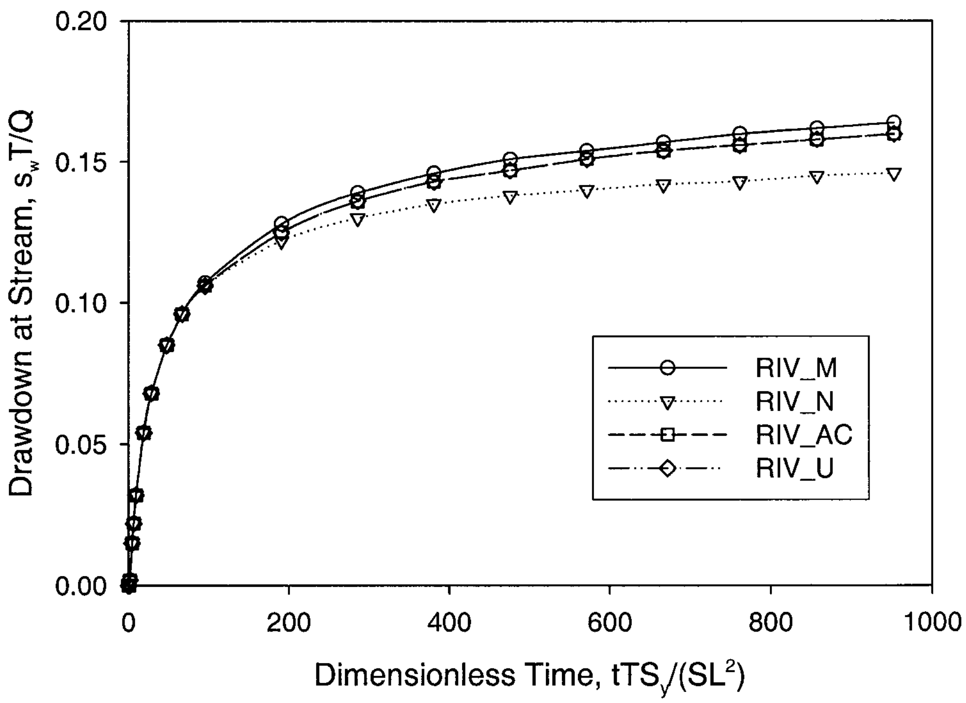


Figure 6.6 – Comparison of predicted dimensionless drawdown at the stream by four different MODFLOW RIVER packages.

RIV_M is the current RIVER package included in MODFLOW. RIV_M limits the amount of specific discharge to be proportional to the sum of the river stage and the streambed thickness. Therefore, RIV_M predicts a greater amount of drawdown compared to RIV_AC and RIV_U. An important result from this analysis is that RIV_AC and RIV_U predict minimal differences in aquifer drawdown. Stream depletion as predicted by the four different RIVER packages is compared in Figure 6.7 based on dimensionless depletion, Q_s/Q, versus dimensionless time, tTS_y/SL^2, where Q_s is the volumetric depletion rate from the stream $[L^3T^{-1}]$.

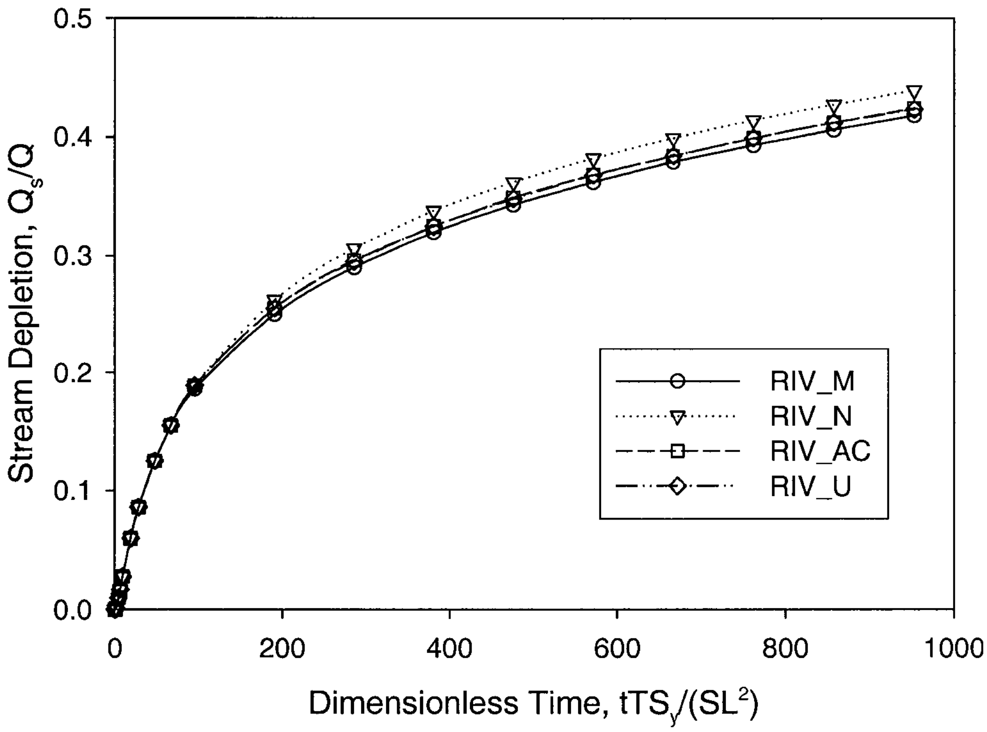


Figure 6.7 – Comparison of dimensionless stream depletion versus dimensionless time as predicted by four different MODFLOW RIVER packages.

In cases where the difference between the entry and maximum capillary pressure heads is more significant, the transition regime could become important and therefore should be included in an updated RIVER package within MODFLOW. However, modeling Regime B using the flow conditions of Regime A may be appropriate for many stream/aquifer interaction scenarios where the underlying alluvial material is represented by medium to coarse sands.

6.4 Analytical Approximation for Saturated/Unsaturated Flow

Stream depletion and aquifer drawdown in response to a pumping well hydraulically interacting with a stream has been the subject of intensive research for decades. Recently, Hunt (1999) developed analytical solutions for drawdown and stream depletion from a partially penetrating stream with a semipervious streambed. However, this solution is limited to saturated flow between the stream and aquifer (i.e., regime A). In the following sections, Hunt's solution is introduced and then equations are presented that modify his solution to analyze drawdown and stream depletion for both saturated and gravity-driven, unsaturated flow (regime C).

6.4.1 Hunt's (1999) Solution for Saturated Flow

The governing equation for Hunt's solution is a two-dimensional, partial differential equation with a sink due to pumping, Q, and recharge source from the stream, λs_w:

$$T\left(\frac{\partial^2 s_w}{\partial x^2}+\frac{\partial^2 s_w}{\partial y^2}\right)=S\frac{\partial s_w}{\partial t}-Q\delta(x-L)\delta(y)+\lambda s_w\delta(x) \qquad (6.18)$$

where x and y are horizontal coordinates with respect to a datum at the center of the river on a perpendicular line with the well [L], t is the time since the start of pumping [T], T is transmissivity [L^2T^{-1}], S is storage coefficient, Q is a constant well discharge rate [L^3T^{-1}],

δ is the Delta Dirac function, L is the distance between the pumping well and the center of the stream, and λ is a streambed conductance coefficient [LT^{-1}] given by:

$$\lambda = \frac{K_{sb}W}{M} \tag{6.19}$$

where K_{sb} is the streambed hydraulic conductivity [LT^{-1}], W is stream width [L], and M is streambed thickness [L]. The third term on the right side of equation (6.18) accounts for stream leakage and explicitly shows a dependency on drawdown. The variable λ is derived from the total discharge per unit stream length (Q'), which Hunt (1999) assumes as

$$Q' = -K_{sb}W\left(\frac{s_w}{M}\right) = -\lambda s_w \tag{6.20}$$

where s_w is the drawdown in the aquifer [L]. The stream is assumed partially penetrating with a semipervious streambed and the aquifer is homogeneous, isotropic and of infinite horizontal extent. The stream is modeled as a constant head boundary with vertical and horizontal streambed cross-sections small compared to the aquifer's saturated thickness. Hunt derived an equation for drawdown, s_w, in terms of the Theis well function, E_1 (1935):

$$s_w(x,y,t) = \frac{Q}{4\pi T}\left\{E_1\left[\frac{(L-x)^2+y^2}{4Tt/S}\right] - \int_0^{\infty} e^{-\theta}E_1\left[\frac{(L+|x|+2T\theta/\lambda)^2+y^2}{4Tt/S}\right]d\theta\right\} \tag{6.21}$$

where

$$E_1(u) = \int_u^{\infty} \frac{1}{\omega} e^{-\omega} d\omega \tag{6.22}$$

and u is the argument of the well function. Hunt also derived an equation for stream depletion, Q_S, as a function of the pumping well discharge, Q:

$$\frac{Q_S}{Q} = erfc(\sqrt{\frac{SL^2}{4Tt}}) - \exp(\frac{\lambda^2 t}{4ST} + \frac{\lambda L}{2T}) erfc(\sqrt{\frac{\lambda^2 t}{4ST}} + \sqrt{\frac{SL^2}{4Tt}}) \tag{6.23}$$

Hunt's model assumes that changes in water table elevation are small, and the stream/aquifer connection is saturated (Hunt, 1999; Rushton, 1999).

6.4.2 Proposed Solution for Saturated/Unsaturated Flow

When a pumping well is located sufficiently close to a stream and/or the groundwater extraction rate causes significant depletion, part of the stream may become perched above the water table (Rushton, 1999). The stream will initially become perched nearest the well and then the length of stream undergoing unsaturated flow increases over time as pumping continues. Therefore, a more realistic model is one in which segments of the stream closest to the pumping well become perched while other contributing stream segments remain connected to the aquifer by saturated flow, as shown in Figure

6.8, where y_{hd} is the time-varying half-length of stream with unsaturated flow. This research proposes an approximating semi-analytical solution to model the scenario presented in Figure 6.8, making the following assumptions: (1) changes in the water surface elevation due to pumping are small, (2) vertical and horizontal streambed cross-sections are small compared to the aquifer's saturated thickness, (3) ratio of vertical to horizontal flow components in the aquifer is small (Dupuit flow assumption), (4) drawdown is small compared to the saturated thickness of the aquifer, and (5) the transition flow regime can be assumed to have a negligible impact on stream/aquifer exchange.

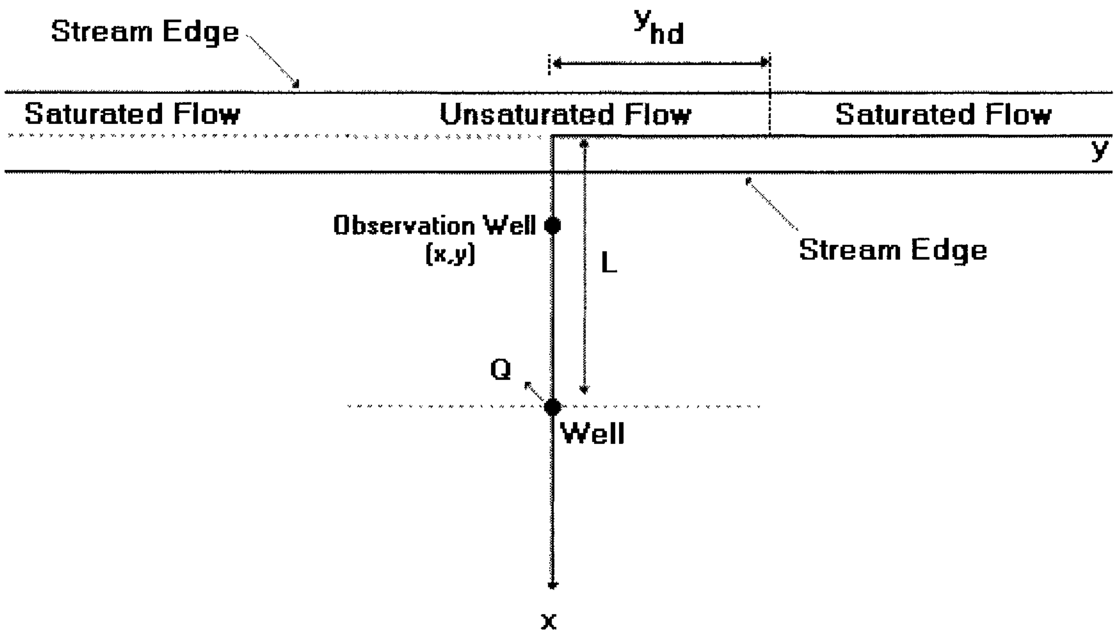


Figure 6.8 - Coordinate system and variable definition for the proposed saturated/unsaturated flow model. y_{hd}= time-varying length of stream for which unsaturated flow occurs.

The drawdown contributions of flow from the unsaturated flow stream segments and the stream segments that remain saturated are linearly superimposed. Superposition is appropriate since the assumptions result in linear governing partial differential equations. The influence of the groundwater extraction well is simulated using the Theis equation (1935):

$$s_T = \frac{Q}{4\pi T} E_1 \left[\frac{(L-x)^2 + y^2}{4Tt/S} \right] \tag{6.24}$$

Hunt (1999) gives the equation governing the recharge contribution by an infinite stream undergoing saturated flow as

$$\frac{Q}{4\pi T} \int_0^{\infty} e^{-\theta} E_1 \left[\frac{(L+|x|+2T\theta/\lambda)^2 + y^2}{4Tt/S} \right] d\theta \tag{6.25}$$

where Q is the constant discharge rate of the pumping well. However, during unsaturated flow, segments of the stream closest to the pumping well become perched while other contributing stream segments remain connected through saturated flow. The recharge contribution to drawdown by these saturated flow stream segments can be approximated using a modification of Hunt's integral expression given by equation (6.25). The modification of Hunt's integral expression is based on integrating only along the stream reach that remains connected to the aquifer through saturated flow rather than over an infinite stream length. This is accomplished by modifying the lower limit of the integration in equation (6.25), such that:

$$\frac{Q}{4\pi T}\int_{\theta'}^{\infty} e^{-\theta} E_1\left[\frac{(L+|x|+2T\theta/\lambda)^2+y^2}{4Tt/S}\right]d\theta \tag{6.26a}$$

where

$$\theta' = \frac{f(T,L,\lambda)y_{hd}\lambda}{2T} \tag{6.26b}$$

The function, f(T,L,λ), in θ' is derived empirically based on comparisons to the updated MODFLOW RIVER package, RIV_AC:

$$f(T,L,\lambda) = \sqrt{\frac{T}{L\lambda}} \tag{6.26c}$$

The recharge contribution to drawdown from the unsaturated stream segment, s_U, is modeled as an infinite line of recharge wells supplying water to the alluvial aquifer at the rate q_{max}:

$$s_U = \int_{-y_{hd}}^{y_{hd}} \frac{q_{max}W}{4\pi T} E_1\left[\frac{x^2+(y-l)^2}{4Tt'(l)/S}\right]dl \tag{6.27}$$

where W is the stream width, l is the variable of integration relating to the length along the stream segment with unsaturated flow, and $t'(l)$ is the time period that unsaturated flow has occurred. This equation is developed assuming unit gradient conditions (i.e., regime C) throughout the unsaturated zone. Equation (6.27) is numerically equivalent to Glover's (1960) analytical solution for the rise of a groundwater mound due to continuous recharge over a rectangular strip.

The time period that unsaturated flow has occurred at a particular stream location within y_{hd} depends on l. Unsaturated flow initiates at the stream location nearest the pumping well and then the length of stream with unsaturated flow increases over time. At the stream location nearest to the pumping well, recharge to the alluvial aquifer occurs at the constant rate q_{max} for the entire time period of unsaturated flow. However, at the stream location dividing saturated and unsaturated flow, recharge is just beginning to occur at the rate q_{max}. As a first approximation, the semi-analytical solution assumes a squared distribution, as shown in Figure 6.9, for the relationship between the time period that unsaturated flow has occurred and the location, l, along the stream reach with unsaturated flow, $t'(l)$. This distribution is selected based on results from the RIV_AC package of MODFLOW.

Linear superposition results in an equation for aquifer drawdown when the stream has both saturated and unsaturated flow:

$$s_w(x, y, t) = s_T - s_S - s_U \tag{6.28}$$

The first term on the right-hand side of equation (6.28) is equivalent to the Theis (1935) equation. The second term is the drawdown contribution due to recharge from stream segments with regions beneath the streambed undergoing saturated flow, as given by equation (6.26). The last term is the drawdown contribution due to recharge from stream segments with zones undergoing unsaturated flow, given by equation (6.27).

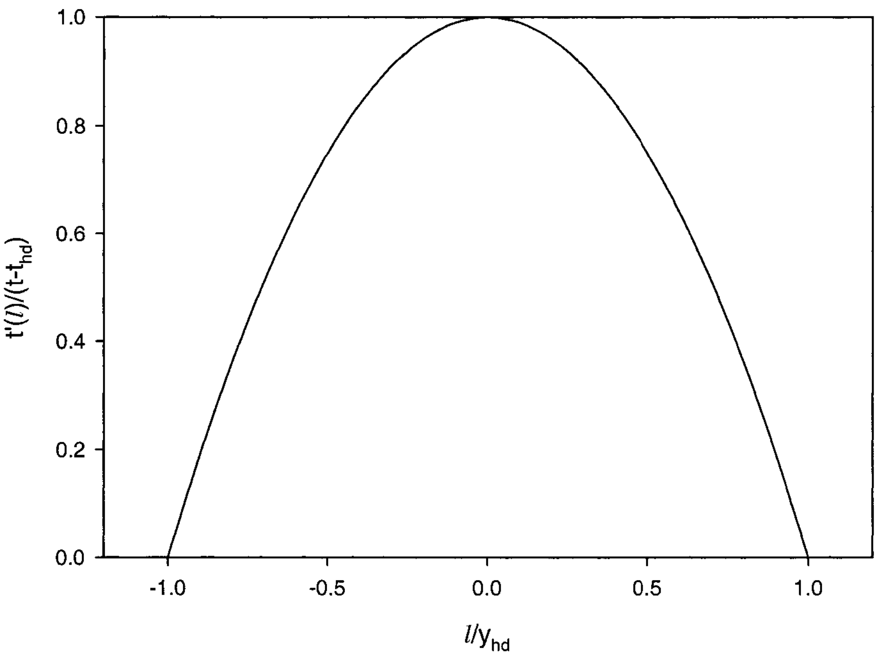


Figure 6.9 – Approximating relationship for the time period that unsaturated flow occurs, $t'(l)$, versus location along the stream with unsaturated flow, l.

An approximation for the stream depletion occurring during saturated/unsaturated flow is derived based on the streamflow contribution from the perched stream segment and the streamflow contribution from the saturated flow stream segment. The stream depletion at the moment that the stream begins to perch is

$$Q_S(t_{hd}) = Q\left(erfc(\sqrt{\frac{SL^2}{4Tt_{hd}}}) - \exp(\frac{\lambda^2 t_{hd}}{4ST} + \frac{\lambda L}{2T}) erfc(\sqrt{\frac{\lambda^2 t_{hd}}{4ST}} + \sqrt{\frac{SL^2}{4Tt_{hd}}}) \right) \tag{6.29}$$

Once the stream begins to perch (i.e., $t > t_{hd}$), additional stream depletion is derived from the stream segments with unsaturated flow. If the recharge wells representing this unsaturated stream segment are assumed to operate for an equivalent time period, the volumetric discharge from this part of the stream (Q_{UNSAT}) is given by

$$Q_{UNSAT}(t) = |q_{\max}| W y_{hd} = K_{sb}\left[1 + \frac{(h_{cu} + H_w)}{M}\right] W y_{hd} \tag{6.30}$$

However, since the unsaturated flow initiates at the stream location nearest the pumping well and then the length of stream with unsaturated flow increases over time, Q_{UNSAT} is given by

$$Q_{UNSAT}(t) = \frac{\int_{-y_{hd}}^{y_{hd}} q_{\max} W t'(l) dl}{t - t_{hd}} \tag{6.31}$$

where $t'(l)$ is assumed to be governed by a squared distribution as shown in Figure 6.9. The stream depletion from saturated flow stream segments (Q_{SAT}) is dependent on the drawdown:

$$Q_{SAT}(t) = 2\lambda \int_{y_{hd}}^{\infty} s_w(0, y', t)\, dy' \tag{6.32}$$

Note that this equation makes use of the symmetry relationship along the saturated stream segments. The stream depletion can then be expressed as a function of the stream depletion at the onset of unsaturated flow, the stream depletion from the unsaturated stream segment, and the stream depletion from the saturated flow stream segment using numerical techniques:

$$\begin{gathered} Q_S(t_o) = Q_S(t_{hd}) \\ Q_S(t_i) = Q_S(t_{i-1}) + \{Q_{UNSAT}(t_i) - Q_{UNSAT}(t_{i-1})\} + \{Q_{SAT}(t_i) - Q_{SAT}(t_{i-1})\} \end{gathered} \tag{6.33}$$

The following steps summarize the solution methodology:

- Solve (6.11) and (6.12) simultaneously for q_{max} and h_{cu}.
- Solve for the time, t_{hd}, at which unsaturated flow initiates at the stream location nearest the well, i.e., (x,y)=(0,0), using (6.21). The time the stream begins to perch is when $s_w(0,0,t_{hd})=H_w+M+h_{cu}$.
- If $t<t_{hd}$, then drawdown and stream depletion are given by (6.21) and (6.23).
- If $t>t_{hd}$, then
    1. Solve for y_{hd} iteratively using (6.28), where $s_w(0,y_{hd},t)=H_w+M+h_{cu}$.
    2. Solve equations (6.29), (6.31) and (6.32) for the components of stream depletion.
    3. Evaluate $s_w(x,y,t)$ and Q_S using equations (6.28) and (6.33).

Since the semi-analytical solution is based on the methodology of Hunt (1999), the solution is limited to certain stream/aquifer conditions. The primary limitation is that horizontal flow must predominate in the aquifer. Critical considerations for meeting this criterion include the distance between the pumping well and stream, anisotropy ratio, stream width, and aquifer thickness, as discussed by Butler et al. (2001). A partially penetrating pumping well will create significant vertical flow components at the stream if the distance between the stream and pumping well (L) is less than $2b(K_h/K_v)^{1/2}$, where b is the aquifer thickness, K_v is the vertical aquifer hydraulic conductivity, and K_h is the horizontal aquifer hydraulic conductivity. A moderate degree of anisotropy ($K_h/K_v=10$) can be neglected if the ratio between the distance between the pumping well and stream (L) to the stream width (W) is greater than 15. For a high degree of anisotropy ($K_h/K_v=100$), the impact of anisotropy can be neglected when the ratio of distance between the pumping well and stream (L) to the stream width (W) is greater than 75 (Butler et al., 2001). Because the solution superimposes contributions from unsaturated and saturated flow stream segments that are represented by line sources, stream width effects can influence the validity of the semi-analytical solution. Research has indicated that stream width effects become significant when the ratio of the distance between the stream and pumping well (L) to stream width (W) is less than 25 (see Chapter 3).

6.4.3 Comparison of Analytical Solutions

The significance of unsaturated flow is investigated using a hypothetical stream/aquifer system whose properties are given in Table 6.3. The aquifer is coarse sand with a hydraulic conductivity 100 times greater than the overlying semipervious streambed layer. Specific yield, S_y, is used in place of the storage coefficient, S, to simulate the storage capabilities of an unconfined aquifer. The proposed saturated/unsaturated semi-analytical solution is compared to the analytical solution of Hunt (1999), which assumes a saturated stream/aquifer connection, based on estimated drawdown and stream depletion. The drawdown and stream depletion curves for Hunt's solution are not strictly valid after the time when the stream becomes perched above the water table but are shown to illustrate the differences in the two solutions.

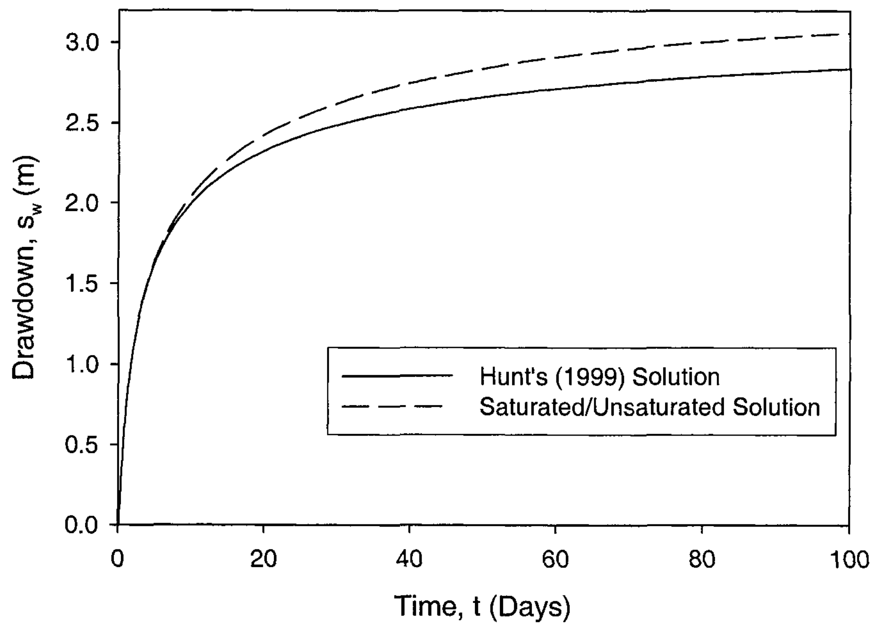
Table 6.3 - Characteristic aquifer, stream, and streambed properties for a hypothetical stream/aquifer system.

Aquifer Parameters	
Pumping Rate, Q (m^3-day^{-1})	10000.0
Saturated Hydraulic Conductivity, K_s (m-day^{-1})	50.0
Transmissivity, T (m^2-day^{-1})	1000.0
Specific Yield, S_y	0.1
Entry Pressure Head, h_e (m)	0.05
Brooks-Corey Parameter, η	8
Pumping Well Location	
Distance Between Stream and Pumping Well, L (m)	100.0
Observation Well Location	
x-Coordinate Location, x (m)	10.0
y-Coordinate Location, y (m)	0.0
Stream and Streambed Parameters	
Streambed Hydraulic Conductivity, K_{sb} (m-day^{-1})	0.5
Stream Width, W (m)	2.5
Streambed Thickness, M (m)	0.5
Water Level in the Stream, H_w (m)	0.5

Figure 6.10 shows that drawdown at a location 10 m from the stream is greater when accounting for unsaturated flow compared to Hunt's (1999) equation. Stream depletion is shown to be less. Unsaturated gravity-driven flow begins at a location on the stream nearest the pumping well at approximately 2.5 days after the initiation of pumping. The maximum specific discharge from the unsaturated segments, $|q_{max}|$, is 1.08 m/day and the ultimate interface capillary pressure head, h_{cu}, is 0.08 m. In Hunt's solution, the specific discharge is 1.0 m/day when the water table is just at the bottom of the streambed. However, assuming flow stays saturated beneath the streambed, specific discharge continues to increase with drawdown.

The stream length with unsaturated flow, y_{hd}, is shown in Figure 6.11. Figure 6.11 also shows that the perched stream length increases rapidly after unsaturated flow begins at 2.5 days. This curve can be explained because the total contributing length, as defined by an effective radius of influence along the stream, increases with time. As the stream becomes perched, stream discharge in the perched length becomes limited to the maximum discharge, q_{max}. Because the well cannot draw more water from the stream in the nearest reach, it draws more water from aquifer storage, corresponding to larger drawdown compared to the saturated flow case where specific discharge is not limiting. As drawdown in the aquifer increases, the radius of influence increases. Thus, a limited constant specific discharge in the perched reach results in more of the stream contributing to well discharge compared to the saturated case.

(a)



(b)

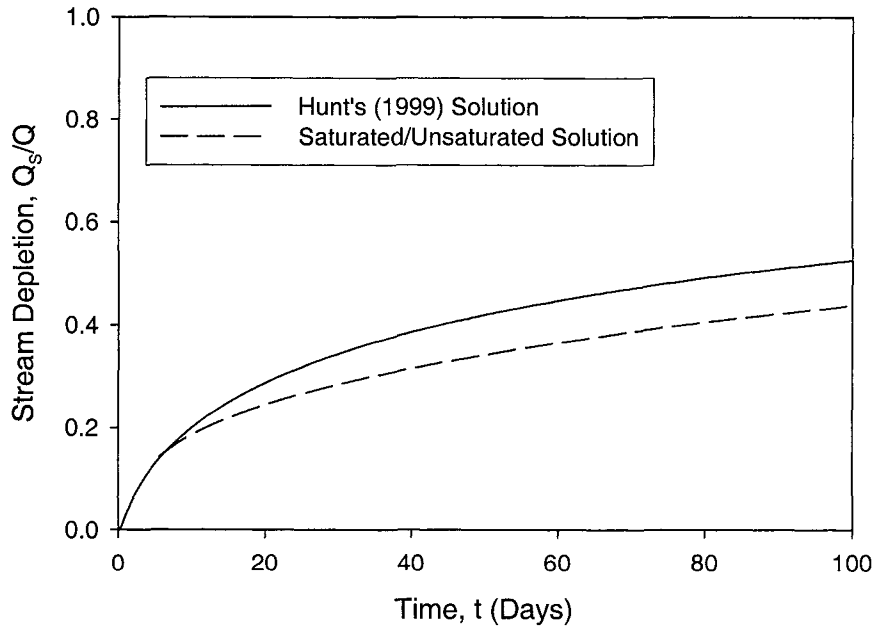


Figure 6.10 – Comparison between the proposed solution and Hunt (1999) for (a) drawdown, s_w, at (x/L, y/L)=(0.1, 0) and (b) stream depletion as a function of discharge rate of the pumping well, Q_S/Q.

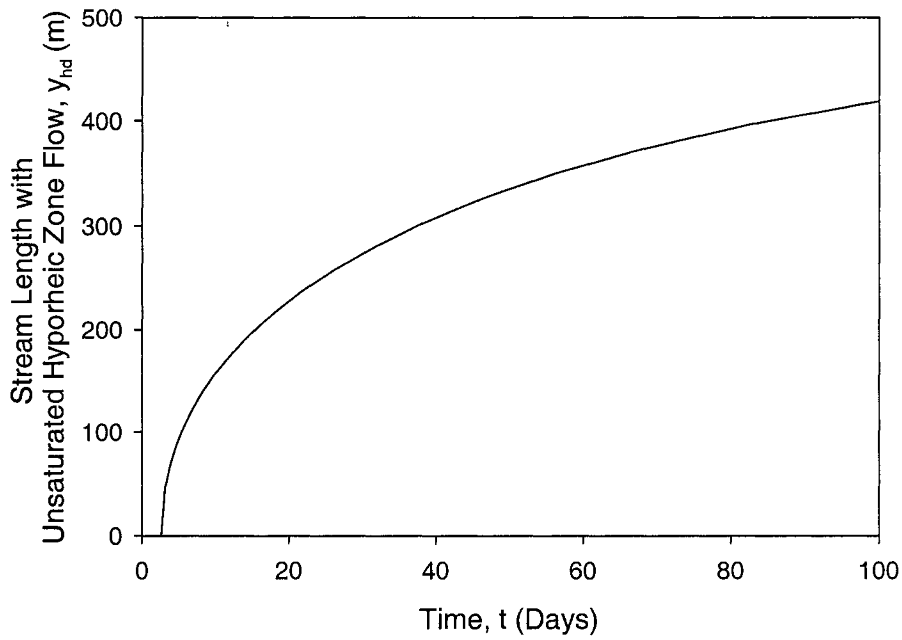


Figure 6.11 – Half-length of stream with unsaturated flow, y_{hd}, versus time for hypothetical stream/aquifer system.

An intriguing question related to Figures 6.10 and 6.11 is what happens when other wells are pumping from the same aquifer. Additional pumping can create interference with the expansion of the radius of influence. If only a finite stream reach is available to meet the pumped well discharge requirements, the length of stream with an unsaturated zone will increase until the entire finite stream is perched. At that time, stream depletion would reach a limit and additional well discharge would need to be provided by release of aquifer storage only, resulting in even larger drawdowns compared to those in Figure 6.10. These larger drawdowns have implications for pump and well efficiencies.

Cases can exist where the increase in specific discharge between the saturated and unsaturated flow scenarios can result in stream leakage satisfying the flow demand created by pumping. The capillary pressure forces that are created during unsaturated flow will create a greater specific discharge compared to the specific discharge during saturated flow. Therefore, a range of physical conditions exists such that unsaturated flow could satisfy the flow demand created by the groundwater extraction well with less drawdown.

The sensitivity of drawdown and stream depletion to streambed hydraulic conductivity, K_{sb}, streambed thickness, M, stream width, W, entry pressure head of the aquifer, h_e, Brooks-Corey parameter, η, and the water level in the stream, H_w, is evaluated using the proposed semi-analytical solution. The parameter values in Table 6.3 are varied by ±50% of their initial values. Lower streambed conductivity results in greater drawdown and earlier initiation of unsaturated flow. Higher streambed conductivity results in larger maximum specific discharge, as can be observed by inspection of equation (6.11), supplying more water to satisfy the aquifer stress created by pumping, and delaying the time at which unsaturated flow begins. Stream depletion is also sensitive to variations in the streambed hydraulic conductivity.

The maximum specific discharge is inversely related to the streambed thickness. Therefore, as thickness increases, the specific discharge decreases and drawdown in the aquifer increases, indicating that more well discharge is supplied by aquifer storage. Streambed thickness also affects the time at which unsaturated flow begins. Since drawdown is measured with respect to the water level in the stream, a larger drawdown is needed to induce unsaturated flow for a thicker streambed.

The stream width also influences the total amount of water available to recharge the aquifer. Total stream depletion increases proportionately to stream width, all other factors being the same, resulting in less aquifer drawdown needed to satisfy the well discharge requirements. This, in turn, delays the onset of unsaturated flow below the stream. The water level in the stream influences the time when unsaturated flow begins since drawdown is measured with respect to the water level in the stream. A larger drawdown is needed to induce unsaturated flow for deeper streams. The water level in the stream also affects the specific discharge, as shown by equation (6.11). Drawdown and specific discharge were found to be insensitive to the entry pressure head and the Brooks-Corey parameter, η, for the range of parameter values investigated.

6.4.4 Comparison of Proposed Analytical Solution and MODFLOW

A comparison based on predicted drawdown is shown in Figure 6.12 between the MODFLOW numerical model with the RIV_U package and the proposed analytical solution for saturated/unsaturated flow. Predictions are for the hypothetical stream/aquifer system discussed in the previous section on modification of the MODFLOW RIVER package and the drawdown shown is for an observation well directly beneath the stream.

6.5 Implications for Biogeochemistry

When streams transition between gaining and losing and/or where episodic pumping leads to frequent transient unsaturated conditions, aeration of the region beneath the streambed can occur and influence biogeochemical processes. Dissolved oxygen concentrations will increase, influencing chemical transformations. For example, if pathways exist for air to enter the zone below a streambed that has become unsaturated, dissolved oxygen concentrations increase, preventing the development of anaerobic conditions that are beneficial for many nutrient transformations. Anaerobic processes have been shown to be important for organic matter oxidation in some streams (Baker et al., 2000). Anaerobic conditions are also important to promote nitrate as an electron acceptor instead of oxygen (Duff and Triska, 2000).

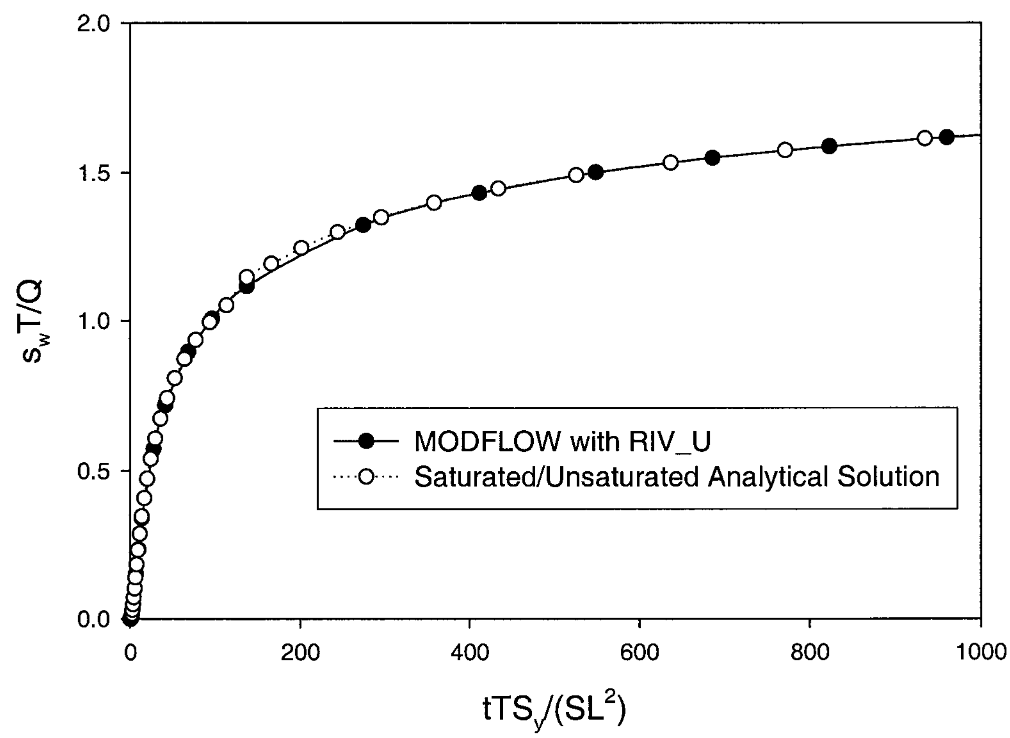


Figure 6.12 – Comparison of dimensionless drawdown at the stream, (x,y)=(0,0), versus dimensionless time as predicted by the MODFLOW model with the RIV_U package and the proposed analytical solution.

Aeration of the region beneath the streambed could significantly influence the effectiveness of bank filtration systems for nutrient transformations. Small municipalities across the United States are experiencing increased pressures to improve filtration of surface water resources and further reduce nitrate concentrations in drinking water sources. Instead of constructing improved water quality treatment facilities, many municipalities have investigated the possibility of modifying the treatment process by obtaining source water through groundwater extraction wells located in the alluvium, i.e. a bank filtration process, rather than directly from the surface water supply. In bank filtration, groundwater extraction wells cause water to be interchanged between a stream and alluvial aquifer through a streambed of contrasting hydraulic conductivity. Bank filtration attempts to take advantage of the natural filtration as water flows through the porous material and possible anaerobic conditions that develop in this region beneath the streambed (Doussan et al., 1998). However, if groundwater extraction leads to unsaturated flow conditions, aeration could potentially prolong the aerobic conditions and prevent nutrient transformations.

6.6 Summary and Conclusions

The objective of this chapter is to discuss the development and implications of unsaturated flow beneath a stream. Saturated flow is typically assumed for seepage between a stream and an underlying aquifer. However, this flow can become unsaturated if the water table falls a sufficient distance below a less permeable streambed. Flow from a stream to an underlying aquifer can be characterized by three hydrologic states or regimes: saturated flow, a transition regime, and unsaturated, gravity-driven flow below the streambed. The development of a unit hydraulic gradient in the unsaturated zone is rapid and the transition regime can be ignored in analyzing stream/aquifer interaction. One of the most important differences between saturated and unsaturated flow beneath a streambed is that unsaturated flow transforms streams from constant head boundaries to constant flux boundaries. Equations are developed to calculate the maximum, limiting flux that can occur under unsaturated conditions.

The effects of an unsaturated zone are illustrated for the case of stream leakage induced by a well pumping from an aquifer that is hydraulically interacting with a partially penetrating stream. It is often assumed that the stream is a constant head boundary and flow between the stream and aquifer is saturated. However, when pumping wells are located sufficiently close to a stream, gravity-driven unsaturated flow can occur and the stream depletion rate is no longer a function of the drawdown.

The stream/aquifer interaction package, RIVER, within the numerical groundwater flow model, MODFLOW, is improved based on the theory developed in this research. Four different RIVER packages are created to further investigate the influence of the transition regime and unsaturated flow regime on stream/aquifer interaction during alluvial well depletions. Accounting for the numerically complex transition regime may not be critical in many stream/aquifer interaction scenarios where the underlying alluvial material is represented by medium to coarse sands.

A semi-analytical solution for drawdown and stream depletion is presented for transient, unsaturated flow. Linear superposition is used to derive equations for aquifer drawdown and stream depletion when portions of the stream are perched. The proposed semi-analytical equations for aquifer drawdown and stream depletion are illustrated for a hypothetical stream/aquifer system and results are compared to an analytical solution that assumes saturated flow. Unsaturated flow limits the specific discharge of water from the stream to satisfy aquifer stress. Perhaps more importantly, the length of stream that becomes perched can be predicted. It is shown that unsaturated flow in response to a pumping well results in a larger contributing stream length and, in most cases, more aquifer drawdown when compared to a solution that assumes the stream/aquifer connection remains saturated. The proposed analytical solution is also shown to compare favorably based on predicted drawdown to the updated RIVER package that simulates all three flow regimes.

CHAPTER 7

ANALYSIS OF STREAM/AQUIFER INTERACTION AT THE TAMARACK STATE WILDLIFE AREA

7.1 Introduction

The Tamarack State Wildlife Area in eastern Colorado is a managed recharge project to redirect flows in the South Platte River for water quantity management. The managed recharge project is Colorado's contribution to the *Cooperative Agreement for Platte River Research and Other Efforts Relating to Endangered Species Habitat along the Central Platte River Nebraska* (July 1, 1997). Colorado has agreed to supplement flows in the South Platte River at the Nebraska state border during critical low-flow periods.

Two primary surface water/groundwater interactions exist at Tamarack: (1) between the South Platte River and alluvial aquifer and (2) between backwater sloughs, or secondary river channels, and the alluvial aquifer. Knowledge of stream/slough/groundwater interaction could play a vital role in the design and operation of the recharge system. Quantifying the magnitude of stream/aquifer interaction requires estimates of the streambed hydraulic conductivity. Silt, clay, and organic materials are often deposited in streams resulting in a lower hydraulic conductivity in the streambed compared to the underlying alluvial aquifer. Also, streambed conductivity can vary significantly between different stream reaches due to the different flow and depositional environments. The interaction of the stream/slough/groundwater at the Tamarack research site is investigated using falling-head permeameter tests to quantify the streambed and sloughbed hydraulic conductivity. Also, a stream/aquifer analysis test is performed at the site to investigate the surface water/groundwater interaction during alluvial well depletion. Water levels are measured in the alluvial aquifer, slough, and South Platte River to determine the depletive effects of pumping. The stream/aquifer analysis test also investigates the use of complex analytical solutions, such as those presented in Chapters 3 and 4, for inversely estimating streambed conductivity from observed aquifer drawdown.

7.2 Description of Tamarack

The Tamarack State Wildlife Area in Logan County, Colorado, has been identified by the state as a source of redirecting flows in the South Platte River during critical low flow periods. The primary recharge components at the Tamarack Managed Recharge Project are shown in Figure 7.1. Under conditions of unappropriated flow in the river, pumping wells in the floodplain (R1, R2, R3, R5, R6, R7 and R8) extract water from the alluvial aquifer and pump the water through pipes to off-channel recharge ponds (Figure 7.1). Diverted surface water is allowed to infiltrate and return to the river as subsurface flow. The recharge ponds are spatially located so that the subsurface flow augments streamflow during critical low-flow, high-demand periods.

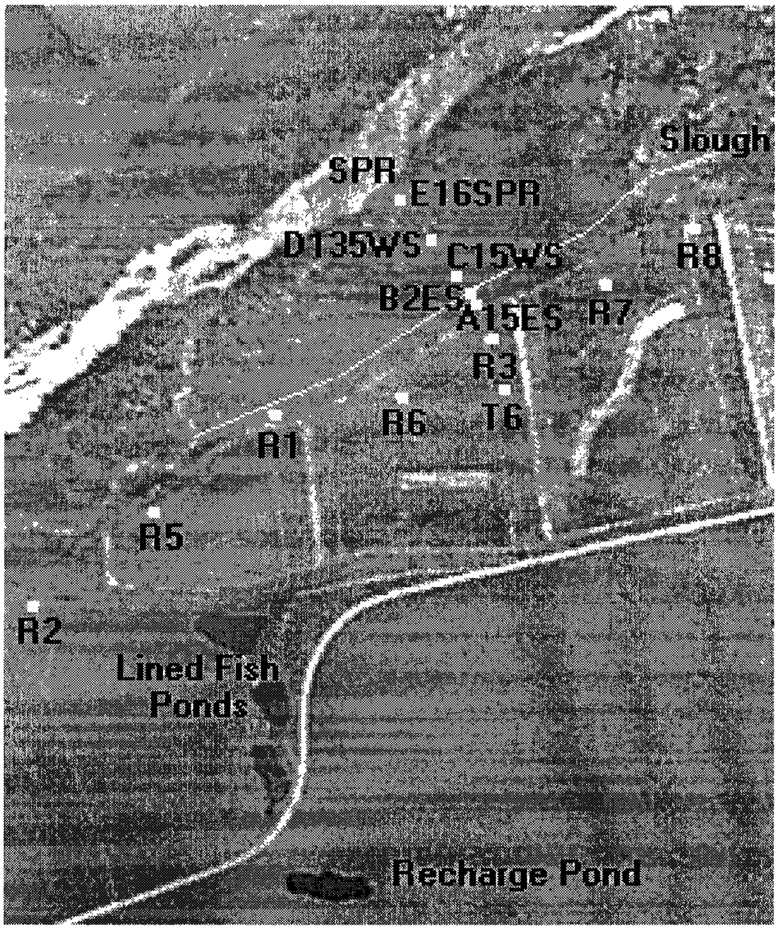


Figure 7.1 - Location of existing pumping wells, slough channels, and South Platte River (SPR). R1, R2, R3, R5, R6, R7, and R8 are pumping wells.

The alluvial aquifer consists of an underlying layer of impermeable shale overlain by alluvial deposits of high permeability (Burns, 1985). The alluvial aquifer is characterized as braided stream deposits with interbedded clay, silt, sand, and gravel. Depths to impermeable shale vary from approximately 20 to 60 m. Burns (1985) characterizes the alluvial aquifer as high conducting sand with a hydraulic conductivity of 60 m-d^{-1}. Recent aquifer tests at the site have estimated the aquifer hydraulic conductivity from 60 to 200 m-d^{-1} (Halstead, 2002).

The South Platte River is a shallow, braided river with considerable variation in width seasonally. Several secondary channels, or backwater sloughs, are present within the floodplain during periods of moderate to high flow in the South Platte River. These sloughs are expected to influence both the depletion of the river by the groundwater wells and the return flows toward the river. The slough channels are estimated to have an average width, W, of approximately 7 m.

7.3 Investigation of Stream/Slough/Aquifer Interaction

Two types of tests were used to estimate streambed hydraulic conductivity at the Tamarack site: falling head permeameter and a stream/aquifer analysis test. Recently proposed stream/aquifer analytical solutions, such as those in Chapters 3 and 4, suggest that streambed conductivity can be inversely estimated by matching the observed drawdown response in one or more observation wells to analytical predictions of drawdown. The objective of this stream/aquifer analysis test is to investigate the use of complex analytical solutions for estimating streambed conductivity by comparing streambed conductivity estimates to measured conductivity from the permeameter tests.

7.3.1 Permeameter Tests to Quantify Streambed Conductivity

Grain-size analyses of streambed samples, slug tests, in situ permeameter tests, and seepage flux measurements with seepage meters are used to estimate streambed hydraulic conductivity. Landon et al. (2001) compare these techniques for estimating the streambed hydraulic conductivity in sandy streambeds in the Platte River in Nebraska. They conclude that field permeameter tests are adequate for determining the streambed conductivity in the upper 0.25 m of the streambed. They also note that the spatial variability between stream transects is greater than variability in measured streambed conductivity between different techniques.

Permeameter tests involve pushing a pipe partially into the streambed. Water is added to this pipe to induce a hydraulic gradient through the sediments inside the pipe. In a falling head permeameter test (Figure 7.2), the water level inside the pipe is allowed to fall while the water level and time are measured. Vertical streambed hydraulic conductivity can be calculated for the falling-head permeameter tests using an application of Darcy's equation (Landon et al., 2001) or the Hvorslev (1951) equation. Application of Darcy's equation for a falling-head permeameter yields

$$K_{sb} = \frac{L}{t_1 - t_0} \ln\left(\frac{H_0}{H_1}\right) \qquad (7.1)$$

where K_{sb} is the vertical, streambed hydraulic conductivity, L is the sediment interval being tested, t_1-t_0 is the elapsed time, H_0 is the initial displacement, and H_1 is the displacement at time t_1. The Hvorslev (1951) solution is similar to the Darcy's equation in that it assumes uniform sediment within and below the permeameter, but also accounts for anisotropic conditions:

$$K_{sb} = \frac{\frac{\pi D}{11\,m} + L}{t_1 - t_0} \ln\left(\frac{H_0}{H_1}\right) \tag{7.2}$$

where D is the diameter of the permeameter and m is the isotropic transformation ratio, $\sqrt{K_h / K_{sb}}$, where K_h is the horizontal, streambed hydraulic conductivity.

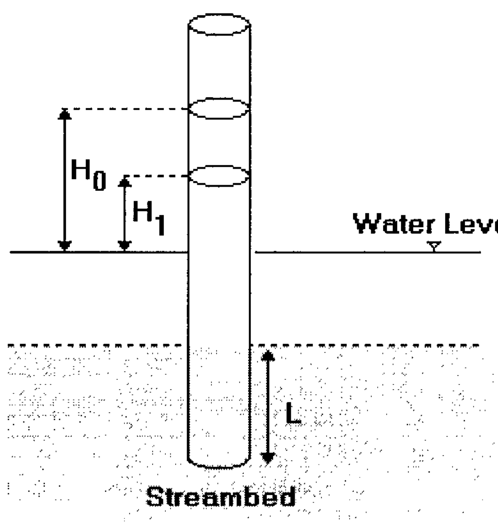


Figure 7.2 - Schematic diagram of falling head permeameter used to measure vertical streambed hydraulic conductivity.

The streambed hydraulic conductivity of the South Platte River and backwater slough channels at the Tamarack State Ranch Wildlife Area was measured using falling-head permeameter tests. The permeameter was a 100 cm long, polyvinyl chloride (PVC) pipe with an inside diameter of 15.24 cm. The permeameter was installed into the streambed with minimum disturbance to depths of 25 and 40 cm. Water was added to the PVC pipe and water level measurements were taken every ten to twenty seconds until the total time for the test was approximately three minutes or until measurable changes in the water level were not observed. The data obtained from these tests were analyzed using both Darcy's law and the Hvorslev (1951) equation for streambed hydraulic conductivity, K_{sb}.

Results of the falling-head permeameter tests for five sampling locations in the South Platte River are shown in Table 7.1. For the case where the permeameter was installed to a depth of 25 cm, the Darcy and Hvorslev equations predicted an average streambed hydraulic conductivity of 143.7 and 168.7 m-d^{-1}, respectively. For an installation depth of 40 cm, the Darcy and Hvorslev equations predicted an average streambed hydraulic conductivity of 64.3 and 71.3 m-d^{-1}, respectively. These results suggest that the deposits deeper than 25 cm below the surface of the streambed limit groundwater/surface water fluxes in the South Platte River. Landon et al. (2001) document similar results for the streambed of the Platte River in eastern Nebraska.

Table 7.1 - Results from falling-head permeameter tests to quantify the streambed hydraulic conductivity, K_{sb} (m-d^{-1}), of the South Platte River at the Tamarack State Wildlife Area.

	K_{sb} Darcy's Equation		K_{sb} Hvorslev (1951) Equation	
Site Location	L=25 cm	L=40 cm	L=25 cm	L=40 cm
1	142.8	83.2	167.6	92.3
2	138.1	68.0	162.1	75.4
3	143.6	47.8	168.5	53.1
4	132.4	47.7	155.5	52.9
5	161.6	74.7	189.8	82.8
Average	143.7	64.2	168.7	71.3
Standard Deviation	11.0	16.0	12.9	17.7

The hydraulic conductivity of the sloughbed was also measured using a falling-head permeameter test at three locations adjacent to pumping well R3 (see Figure 7.1). Data and results for the three backwater slough measurement locations are shown in Table 7.2 and suggest that a considerable permeability difference (e.g., K_{sb}<1.0 m-d^{-1}) is present. Streambed materials in the slough consist of coarse sands and gravels similar to the South Platte River, but are much more inundated by organic materials that significantly limit the exchange of water between the slough channel and alluvial aquifer. The thickness of this organically restricted layer is approximated to be 50 to 75 cm. A picture of the conditions representative of the slough channel at the Tamarack research site is shown in Figure 7.3.

Table 7.2 – Falling-head permeameter test for determining the hydraulic conductivity, K_{sb} (m-d^{-1}), of the sloughbed at the Tamarack State Wildlife Area.

	K_{sb} Darcy's Equation	K_{sb} Hvorslev (1951) Equation
Site Location	L=25 cm	L=25 cm
1	0.1	0.1
2	0.1	0.2
3	0.8	0.9
Average	0.3	0.4
Standard Deviation	0.4	0.4

Figure 7.3 – Slough channel at the Tamarack State Wildlife Area.

7.3.2 Stream/Aquifer Analysis Test

Field measurement of the streambed hydraulic conductivity using falling-head permeameters gives a point measurement. Research has indicated that streambed conductivity can vary significantly in both the horizontal and vertical directions within a streambed (Landon et al., 2001). The use of analytical solutions provides a mechanism for aggregating point measurement variations into a single effective conductivity more aligned with the input requirements of numerical ground water flow models. Hunt (1999) was the first to suggest that drawdown in one or more observation wells located between a pumping well and stream can be used with analytical solutions that account for streambed conductivity to inversely estimate both aquifer and streambed hydraulic properties. Only three such field tests have been documented to the author's knowledge: one presented by Hunt et al. (2001) as analyzed in Chapter 4, another by Nyholm and Christensen (2000), and one presented by Nyholm et al. (2002).

The location and setup for the stream/aquifer analysis test at the Tamarack site is shown in Figure 7.1. Well R3 was pumped at a constant discharge rate, Q=10,900 m^3-d^{-1} (2000 gpm), for 24 hours on December 6th-7th, 2002. Water levels were measured periodically in the five observation wells depicted in Figure 7.1: one located 15 m from the slough on the pumping well side, A15ES; one located 2 m from the slough on the pumping well side, B2ES; one located 15 m from the slough on the side of the South Platte River, C15WS; one located 135 m from the slough on the side of the South Platte River, D135WS; and one located 16 m from the South Platte River, E16SPR. Water levels were also measured in the pumping well, in an existing observation well (T6) on the side of the pumping well opposite from the slough channel, in the slough channel, and in the South Platte River. Water level measurements were made using manual water level recorders. Figure 7.4 shows the initial water levels along a transect from the pumping well, through the slough channel, and to the South Platte River.

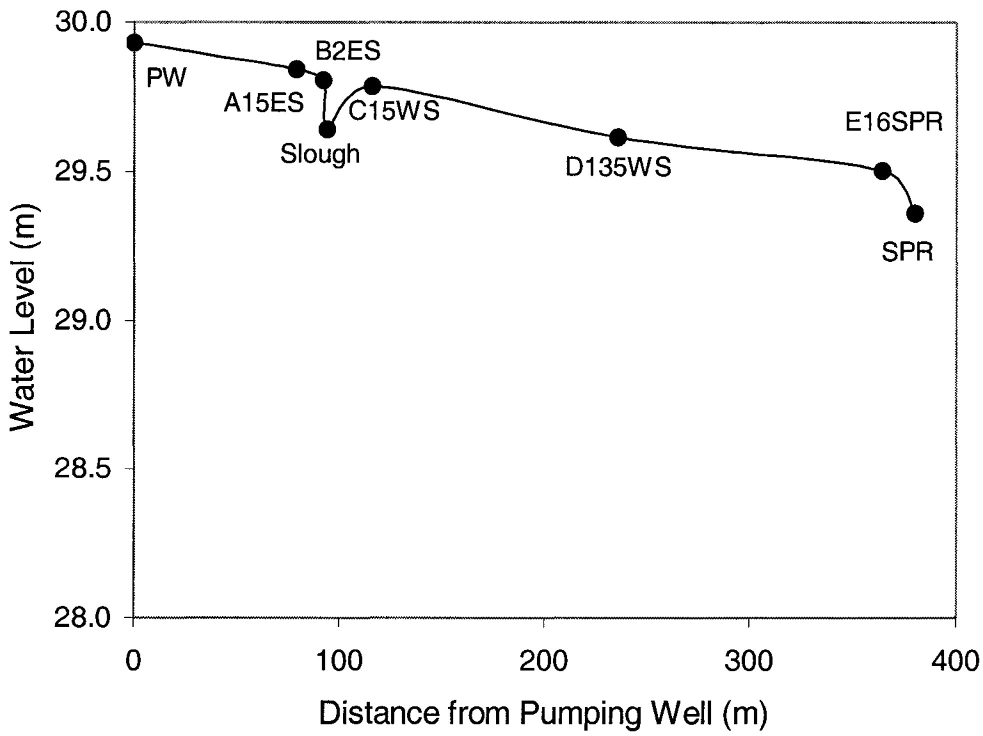


Figure 7.4 – Transect of initial water level before stream/aquifer analysis test. PW=pumping well and SPR=South Platte River.

The measured drawdown response in observation wells A15ES and B2ES are shown in Table 7.3. In general, water levels in the observation wells declined rapidly during the first ten to thirty minutes after initiation of groundwater abstraction. Analytical solutions such as those developed in this research assume that the head in the surface water body does not change during the pumping test. During the 24-hour pumping period, the water level in the slough channel declined by less than 0.1 m. Water levels were also measured in the South Platte River. Throughout the duration of the pumping test, the water level in the river declined by less than 0.01 m.

Table 7.3 – Measured drawdown response in observation wells A15ES and B2ES, located between the pumping well and slough channel, during stream/aquifer analysis test.

Observation Well A15ES		Observation Well B2ES	
Elapsed Time (min)	**Drawdown (m)**	**Elapsed Time (min)**	**Drawdown (m)**
0.0	0.000	0.0	0.000
11.4	0.049	12.9	0.034
13.3	0.052	14.0	0.034
18.6	0.058	19.7	0.040
39.7	0.079	40.6	0.052
48.6	0.085	49.4	0.055
62.9	0.088	63.7	0.061
83.3	0.101	84.1	0.070
114.7	0.119	113.7	0.076
144.8	0.128	143.3	0.085
192.0	0.143	191.2	0.098
347.2	0.192	349.5	0.131
599.4	0.262	600.5	0.177
1090.7	0.375	1091.7	0.247
1249.8	0.378	1250.6	0.262
1274.5	0.393	1275.9	0.268
1356.9	0.399	1357.5	0.277
1456.9	0.405	1456.3	0.287

Since observation wells A15ES and B2ES are located between the slough and pumping well, these drawdown measurements are used with an analytical solution to estimate the sloughbed hydraulic conductivity. Note that the finite slough width becomes critical in influencing the interaction between the slough channel and the alluvial aquifer since the distance between the slough and pumping well, L, is 94 m, resulting in an L/W ratio of less than 15. As such, the analytical solutions of Hunt (1999) and the proposed analytical model, STRMAQ (Chapter 4), are not valid. In order to analyze this data, this research combines the analysis presented in Chapter 3 on finite stream width effects and the STRMAQ package to develop a solution capable of addressing the critical slough/aquifer interaction conditions. Predicted drawdown is fit to the observed drawdown curve in observation wells A15ES and B2ES using a trial and error approach.

For observation well A15ES, the following parameter values result in a least squares best match: transmissivity, T, of 3750 m^2-d^{-1}; aquifer hydraulic conductivity, K, of 125 m-d^{-1}; anisotropy ratio, K/K_z, of 7.8, where K_z is the vertical hydraulic conductivity; specific storage, S_s, of 2.5 x 10^{-4} m^{-1}; specific yield, S_y, of 0.20; and a streambed hydraulic conductivity of 0.1 m-d^{-1}. Observed drawdowns and the model-predicted drawdowns are shown in Figure 7.5. For observation well B2ES, the following parameter values result in a least squares best match: transmissivity, T, of 3900 m^2-d^{-1}; aquifer hydraulic conductivity, K, of 130 m-d^{-1}; anisotropy ratio, K/K_z, of 8.1; specific storage, S_s, of 2.7 x 10^{-4} m^{-1}; specific yield, S_y, of 0.22; and a streambed hydraulic conductivity of 0.5 m-d^{-1}. Observed and model predicted drawdowns for B2ES are shown in Figure 7.6.

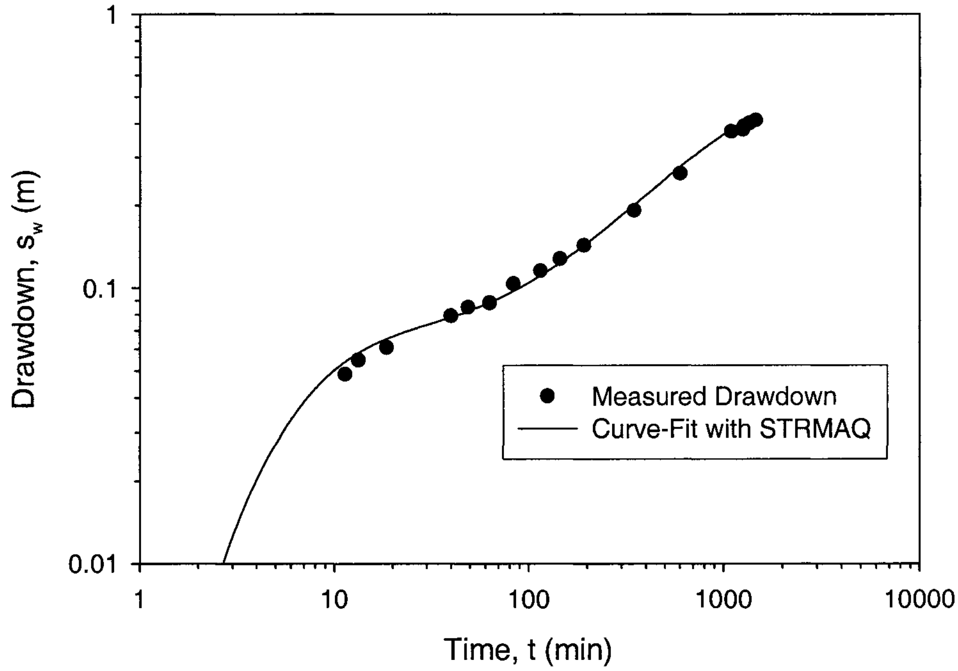


Figure 7.5 – Comparison of measured and predicted drawdown at observation well A15ES.

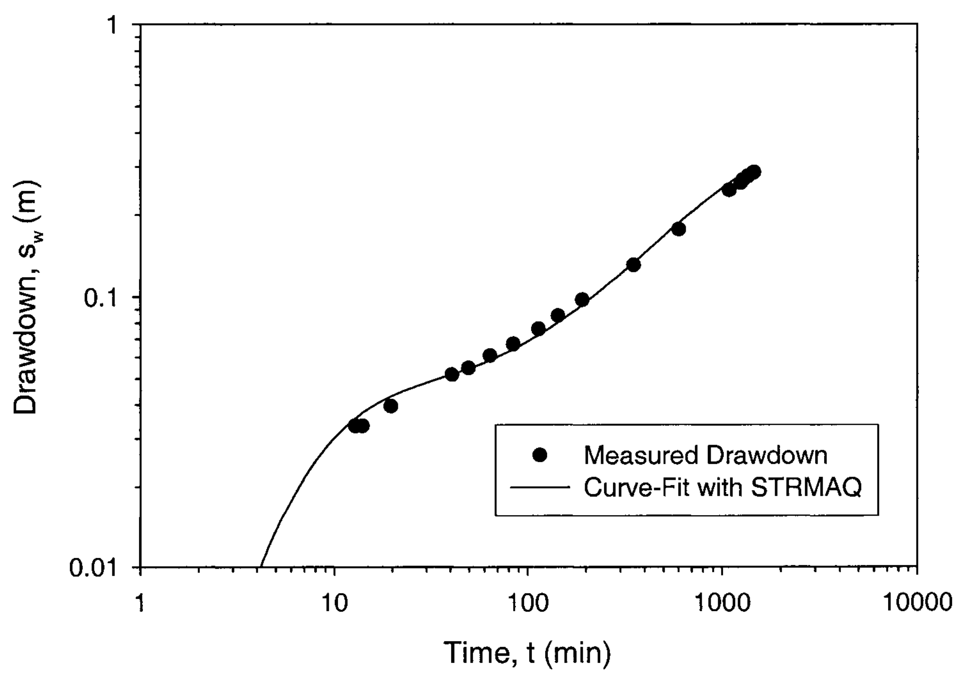


Figure 7.6 – Comparison of measured and predicted drawdown at observation well B2ES.

Parameter values for both observation wells agree with reported values of aquifer parameters at the site (Halstead, 2002; Burns, 1985; Hurr and Schneider, 1973) and also with measured streambed conductivity in the slough channel from the falling-head permeameter tests. Several recent aquifer tests have been performed at Tamarack by researchers with the Northern Colorado Water Conservancy District and the Colorado Division of Wildlife (Halstead, 2001). Based on these data sources, Halstead (2001) reports reasonable values for aquifer parameters that match the parameter values obtained from the stream/aquifer analysis tests:

- Transmissivity, T: approximately 1000 to 4500 m^2-d^{-1} with an average of 2800 m^2-d^{-1}
- Aquifer Conductivity, K: approximately 60 to 200 m-d^{-1} with an average of 130 m-d^{-1}
- Specific Yield, S_y: 0.12-0.30 with an average of 0.15

Even though not used to estimate the conductivity of the sloughbed, the observation wells located between the slough and South Platte River provide information about the degree of surface water/groundwater interaction at Tamarack. The drawdown response in observation well C15WS, located 15 m from the slough on the opposite side from the pumping well, is shown in Table 7.4. The water level response in this well indicates that some percentage of the aquifer stress is being satisfied by flow from the aquifer on the non-pumping side of the slough channel. Water levels in observation well D135WS, located 135 m from the slough on the side opposite of the pumping well, decline by approximately 0.1 m during the 24-hour period. Water levels in observation well E16SPR decline by less than 0.01 m suggesting that the pumping well does not influence the South Platte River during the first 24-hours of pumping. However, continued and/or more intense pumping could deplete both the slough channel and South Platte River.

Table 7.4 – Measured drawdown response in observation well C15WS, located on the non-pumping well side of the slough channel, during stream/aquifer analysis test.

Elapsed Time (min)	Drawdown (m)
0.0	0.000
24.7	0.037
35.8	0.043
53.9	0.046
58.4	0.049
90.4	0.058
105.3	0.064
138.7	0.067
185.8	0.075
1096.9	0.183
1258.5	0.201
1333.5	0.207
1446.7	0.216

The water level transect from the pumping well, through the slough channel, and to the South Platte River at the end of the stream/aquifer analysis test is shown in Figure 7.7. The initial water level along this same transect (as presented in Figure 7.4) is also shown to depict the influence of the pumping well on the slough/river/aquifer interaction.

Groundwater abstraction creates a significantly different water level profile, converting the slough channel from a groundwater discharge zone to a recharge zone during pumping. The influence of the pumping well extends beyond the slough. At the end of the test, the influence of the pumping well is beginning to reach the South Platte River and it is expected that continued pumping would eventually result in significant depletion at the river.

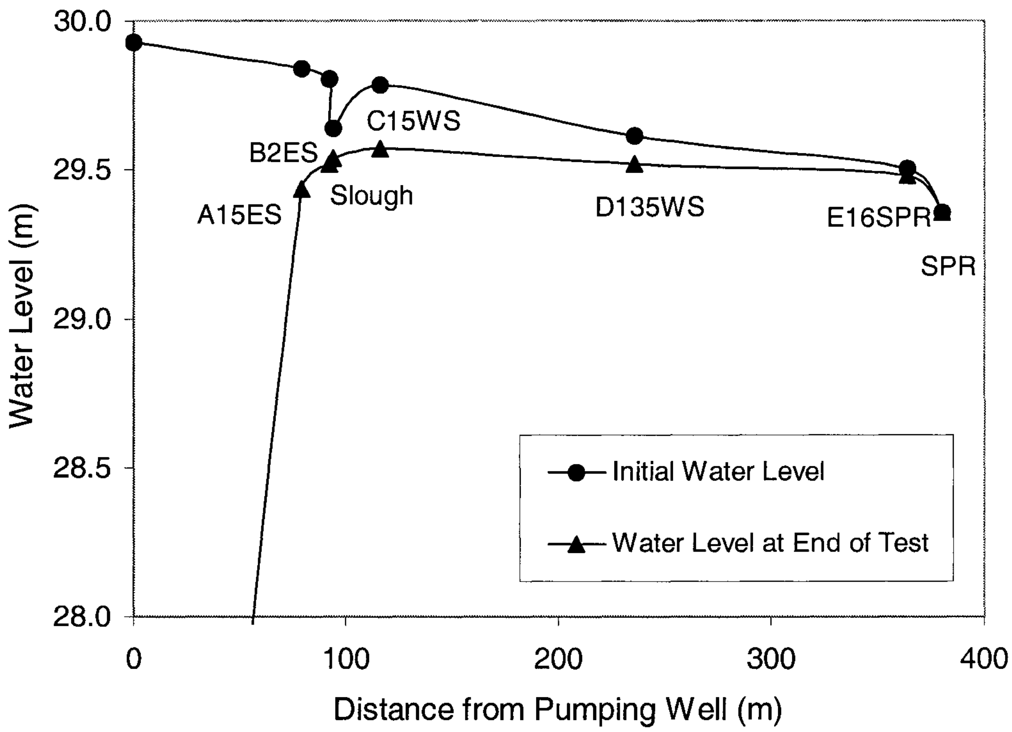


Figure 7.7 – Transects of the initial and final water levels near pumping well R3. SPR=South Platte River.

An interesting consideration is the stream/aquifer interaction that occurs during full operation of the managed recharge project, such that all the pumping wells shown in

Figure 7.1 will be pumping and recharge will be occurring from the infiltration ponds. The combined influence of all the pumping wells is expected to create a stream/aquifer interaction scenario where flow is being depleted from both the slough and South Platte River simultaneously. Also, with the restrictive layer in the sloughbed 50 to 75 cm thick, drawdown due to several pumping wells could perch the slough above the alluvial aquifer, converting the slough from a constant head boundary condition to a specified flux boundary condition, as discussed in Chapter 6. More research is needed to determine whether this perching of the slough channel occurs.

7.4 Summary and Conclusions

The interaction between the South Platte River and backwater slough channels with the underlying alluvial aquifer at the Tamarack State Wildlife Area in eastern Colorado is investigated. The Tamarack site is being suggested as a possible managed recharge project to supplement flows in the South Platte River during critical low-flow, high-demand periods. Pumping wells located next to the backwater sloughs and the South Platte River extract water from the alluvial aquifer. The interaction of the surface water resources with groundwater in the alluvial aquifer is investigated using field tests to determine the degree of hydraulic interaction between the slough and aquifer and the South Platte River and aquifer.

Falling-head permeameter tests are performed to estimate the hydraulic conductivity of the streambed and sloughbed. Permeameter tests are performed using a

PVC-pipe pushed partially into the streambed. Water is added to this pipe to induce a hydraulic head on the sediments inside the pipe. The water level inside the pipe is allowed to fall while the displacement is measured. Falling-head permeameter tests estimate the hydraulic conductivity of the riverbed to be approximately 150 m-d^{-1} in the upper 25 cm and approximately 70 m-d^{-1} in the upper 40 cm. The hydraulic conductivity of the bed materials in the slough is much less due to the presence of organic materials that clog the pore spaces of the coarse sand grains. Falling-head permeameter tests estimate the sloughbed conductivity to range between 0.1 to 1.0 m-d^{-1}.

Also, a stream/aquifer analysis test (i.e., pumping test next to the slough and stream) is performed at the Tamarack site. Recently proposed analytical solutions, such as those in Chapters 3 and 4, suggest that streambed conductivity can be inversely estimated by matching the observed drawdown response in one or more observation wells to analytical predictions of drawdown. This objective of this stream/aquifer analysis test is not only to investigate surface water/groundwater interaction scenarios at the Tamarack site, but also investigate the use of complex analytical solutions for estimating streambed conductivity. The stream/aquifer analysis test makes use of a single pumping well located 94 m from the slough channel and 380 m from the South Platte River. Drawdown is measured over a 24-hour period in five observation wells: two observation wells are located within the 94 m between the slough and pumping well and three observation wells are located between the slough and South Platte River. Finite stream width is noted to be critical in influencing the interaction between the slough channel and the alluvial aquifer since the distance between the slough and pumping well is relatively small. In order to analyze this data, this research combines the analysis presented in Chapter 3 on finite stream width effects and the STRMAQ package to develop a solution capable of addressing all critical slough/aquifer interaction conditions at the site. Predicted drawdown is individually matched to the observed drawdown in the two observation wells between the pumping well and sloughs channel. Estimates of critical aquifer and streambed parameters are obtained, and these estimates match reported parameter values from other aquifer tests performed at the site.

CHAPTER 8

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

8.1 Summary and Conclusions

The objectives of this research were to develop analytical models of stream/aquifer interaction during alluvial well depletion, evaluate their accuracy in the prediction of aquifer drawdown and stream depletion, and determine their ability to inversely estimate streambed conductivity. This research expands the applicability of recently proposed analytical solutions by considering finite stream width, complex aquifer flow scenarios (i.e., unconfined flow, pumping well partial penetration, delayed drainage from the unsaturated zone, well-bore storage, and well-skin effects), and unsaturated stream/aquifer exchange. Differences between prior analytical solutions, numerical stream/aquifer models, and the analytical solutions developed in this research were evaluated to determine conditions when specific stream/aquifer conditions become significant in impacting aquifer response and stream depletion.

Existing analytical models, including the Theis (1941) and Hunt (1999) solutions, for simulating drawdown were evaluated against MODFLOW numerical simulations using hypothetical alluvial aquifer systems. Recently proposed analytical solutions were shown to more appropriately simulate aquifer drawdown and stream depletion compared to more simplified solutions. Next, an analytical solution referred to as the FDD analytical solution was developed. The analytical model allowed a distributed recharge flux through the entire stream width rather than modeling the stream as a line source. Results indicate that when the distance between the pumping well and stream is small compared to the width of the stream, finite width effects could significantly influence predicted drawdown. As long as the ratio of the distance between the pumping well and the stream with respect to the stream width is greater than 25, stream width effects are negligible in most alluvial aquifer scenarios.

The effects of unconfined flow, partially penetrating wells, delayed drainage from the unsaturated zone, well-bore storage and well-skin effects were incorporated into stream/aquifer analytical models. The resulting model was referred to as STRMAQ. Partial penetration of the pumping well was determined to be important in stream/aquifer analytical models, with the effects more significant for highly conducting streambeds. Delayed drainage of the unsaturated zone influenced intermediate time drawdown data. Well-bore storage and well-skin effects delayed early-time drawdown response and were not as important as the other parameters in efforts to estimate the streambed hydraulic conductivity. Data from a pumping test performed adjacent to a stream near the Doyleston Drain south of Christchurch, New Zealand was analyzed using STRMAQ. STRMAQ was able to match the delayed yield response of the aquifer significantly better than the Hunt (1999) solution.

Inverse estimation of the streambed hydraulic conductivity using analytical models was shown to require accurate estimates of aquifer parameters, especially aquifer hydraulic conductivity. Greater errors in estimating streambed conductivity occurred due to uncertainty in aquifer hydraulic conductivity compared to specific storage, specific yield, or the ratio of vertical to horizontal hydraulic conductivity. The effect of aquifer parameter uncertainty on estimated streambed conductance became greater when observation wells were located further from the stream.

Methodologies used by numerical models, such as MODFLOW, in accounting for unsaturated stream/aquifer exchange were evaluated. Unsaturated stream/aquifer exchange was determined to have a significant influence on stream/aquifer interaction by transforming streams from constant head boundaries to constant flux boundaries. Unsaturated stream/aquifer interaction in response to a pumping well results in a larger contributing stream length and, in most cases, more aquifer drawdown when compared to a solution that assumed the stream/aquifer connection remains saturated.

Finally, stream/aquifer interaction at the Tamarack State Wildlife Area in eastern Colorado was investigated. Falling-head permeameter tests were used to quantify the streambed and sloughbed hydraulic conductivity, measured to be between 70 and 150 m-d^{-1} for the South Platte River and between 0.1 and 1.0 m-d^{-1} for the backwater slough channel. A stream/aquifer analysis test was performed, and observed drawdowns in observation wells were matched to predictions of aquifer drawdown by the analytical solutions developed in this research. Aquifer and streambed parameter estimates from the analytical solutions matched parameter values obtained from historical pumping tests and the falling-head permeameter tests.

8.2 Recommendations for Future Research

This research has investigated the impact of a number of different stream/aquifer conditions on the analytical modeling of stream/aquifer interaction. The goal of this research has been to address the necessity of more complex solutions for water rights decisions and also begin to answer the question of how much pump discharge comes from the stream versus aquifer storage. Also, this research investigated the use of these improved analytical solutions with data obtained from aquifer tests performed next to a stream as a methodology for estimating the streambed conductivity and therefore, quantifying the hydraulic interaction between a stream and aquifer. Even though significant advances have been made in this field, the following activities are recommended for continuing the improvement of modeling stream/aquifer interaction:

- Only four known field tests, including the one presented in this research, of the influence of a pumping well next to a partially penetrating stream have been documented in the literature. Additional field tests need to be performed to determine under what hydrogeologic conditions inverse estimation of the streambed conductivity is viable.
- The analytical solutions developed in this research need to be further tested by comparison to numerical models under a number of different hydrologic scenarios and also with data obtained from additional field tests.
- The capability of the analytical models in simulating the hydrologic conditions in the stream needs to be improved. For example, methodologies need to be

developed that account for transient streamflow conditions, especially during significant depletion.

- The theory on unsaturated flow needs to be rigorously tested. Questions exist as to whether air will enter the zone underneath the streambed. Aeration of the unsaturated stream/aquifer exchange will impact contaminant transport between the stream and aquifer.
- A field project needs to be performed to investigate unsaturated stream/aquifer exchange. Such a field test would require an intensively instrumented site not impacted by the influence of adjacent pumping wells or recharge. Critical measurements during the field test would include aquifer drawdown in observation wells, the amount of stream depletion, and the length of stream experiencing unsaturated flow.
- Existing analytical and numerical models of stream/aquifer interaction neglect the dynamics of sediment/particle transport and streambed clogging by using a constant streambed leakage factor. Thus, modeling efforts aimed at predicting the complexities in stream/aquifer interaction fail when streambed clogging becomes significant or when the river morphology changes rapidly. Improved modeling tools capable of accounting for streambed clogging need to be developed to improve the analysis of water and geochemical constituent cycling during stream/aquifer interaction.
- An improved, dynamic numerical modeling tool for analyzing stream/aquifer interaction over large spatial scales also needs to be developed. A suggested modeling framework could consist of the numerical flow model capable of

simulating stream/aquifer interaction. This model could be combined with a particle transport model to account for streambed clogging. Also, the flow model could be linked with a sediment transport model to predict changes in streambed morphology over time. Such integrated models could predict changes in the streambed permeability during both saturated and unsaturated hydrologic exchange. The integrated modeling package should be demonstrated for several alluvial aquifer systems.

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APPENDIX A

FORTRAN Code for STRMAQ

```
!    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
!    ~                                                                  ~
!    ~                        ~~~~ STRMAQ ~~~~                          ~
!    ~                                                                  ~
!    ~    COMPUTER PROGRAM THAT COMBINES HUNT'S (1999)                  ~
!    ~                                                                  ~
!    ~    ANALYTICAL SOLUTION FOR STREAM/AQUIFER                        ~
!    ~                                                                  ~
!    ~    INTERACTION WITH ANALYTICAL MODELS FOR                        ~
!    ~                                                                  ~
!    ~    FLOW IN A CONFINED OR WATER-TABLE AQUIFER WITH                ~
!    ~                                                                  ~
!    ~    AXIAL-SYMMETRIC FLOW TO A FINITE- OR                          ~
!    ~                                                                  ~
!    ~    INFINITESIMAL-DIAMETER PUMPED WELL                            ~
!    ~                                                                  ~
!    ~                        VERSION 1.01                              ~
!    ~            Garey A. Fox, Colorado State University               ~
!    ~               Department of Civil Engineering                    ~
!    ~                                                                  ~
!    ~    NOTE: PROGRAM MODIFIED DIRECTLY FROM WTAQ                     ~
!    ~                   Barlow and Moench (1999)                       ~
!    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

      PARAMETER (IMAXX1=25,IMAXX2=200)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DIMENSION ALPHA(5), GAMMA(5)
      DIMENSION TDPLOT(IMAXX1,IMAXX2),HDPLOT(IMAXX1,IMAXX2)
      DIMENSION TIMEPW(IMAXX2),XMEASPW(IMAXX2),TIMEOB(IMAXX2),
     2    XMEASOB(IMAXX2)
      DIMENSION TDRDSQT(IMAXX2),TDRDSQTY(IMAXX2),HDTPLOT(IMAXX2),
     2    HDTYPLOT(IMAXX2)
      INTEGER IN, IO, IP, N, IPLOT
      CHARACTER*50 IFNAME, OFNAME, PFNAME, TEMP
       CHARACTER STRING*80,TITLE*80,WELLTXT(IMAXX1)*6, AQTYPE*11
      DATA WELLTXT/'HDPW','HDOB1','HDOB2','HDOB3','HDOB4',
     2    'HDOB5','HDOB6','HDOB7','HDOB8','HDOB9','HDOB10','HDOB11',
     3    'HDOB12','HDOB13','HDOB14','HDOB15','HDOB16','HDOB17',
     4    'HDOB18','HDOB19','HDOB20','HDOB21','HDOB22','HDOB23',
     5    'HDOB24'/

      COMMON /IOUNIT/ IN,IO,IP
      COMMON /PAR1/ IPWD,IRUN,IPWS,NOBWC,IOWS,IDPR
      COMMON /PAR2/ NGAMMA,IDRA,NS,KK,NMAX,NTMS
      COMMON /PAR3/ IFORMAT,IAQ,NLC,NOX
      COMMON /PAR4/ ITS,IMEAS,NTSPW
      COMMON /PAR5/ BB,HKR,XKD,SS,SY,RW,QQ
      COMMON /PAR6/ BETAW,SIGMA,GAMMA
      COMMON /PAR7/ RERRNR,RERRSUM,TDLAST,TLAST
      COMMON /PAR8/ R,ZP,Z1,Z2,WDP
      COMMON /PAR9/ V(20),XLN2,EXPMAX
      COMMON /PAR10/ RD,ZD,ZD1,ZD2
      COMMON /PAR11/ XLD,XDD,WD,SW
      COMMON /PAR12/ TIMEPW, XMEASPW
```

```
!A      OPEN INPUT AND OUTPUT FILES
        OPEN(UNIT=4,FILE='Files.txt',STATUS='OLD')
        OPEN(UNIT=5,FILE='Test.txt',STATUS='UNKNOWN')
        IN = 15
        IO = 16
        IP = 17
!       READ INPUT AND OUTPUT FILE NAMES
        READ(4,*) TEMP
        READ(4,5005) IFNAME
        N=LENCHR(IFNAME)
        IF(N.LT.1) THEN
        WRITE(*,*) 'ERROR OPENING INPUT FILE'
        STOP
        ENDIF
        OPEN(IN,FILE=IFNAME,STATUS='OLD')
        READ(4,5005) OFNAME
        N=LENCHR(OFNAME)
        IF(N.LT.1) THEN
        WRITE(*,*) 'ERROR OPENING OUTPUT FILE'
        STOP
        ENDIF
        OPEN(IO,FILE=OFNAME)
        WRITE(*,8767)
8767    FORMAT(/,'~~~~~~~~~~~STREAM AQUIFER~~~~~~~~~~~~~~~~',/,/
   &      /,'~~~~~      VERSION 1.01          ~~~~~',/,
   &      /,'~~~~~   CREATED BY: Garey Fox      ~~~~~',/,
   &       '~~~~~          Colorado State University  ~~~~~',/,/)
        WRITE(*,*) ' INPUT FILE NAME:  ',IFNAME
        WRITE(*,*) ' OUTPUT FILE NAME:  ',OFNAME

!       FORMAT STATEMENTS
 5005   FORMAT(A50)
!       WRITE PROGRAM BANNER TO RESULT FILE
        CALL BANNER(IO)
!       READ AND PREPARE INPUT DATA
        READ(IN,'(A70)') STRING
        TITLE=STRING
        IFORMAT=1
!       READ AND TEST AQUIFER TYPE
!       AQTYPE=Aquifer Type (CONFINED or WATER TABLE)
        IAQ=2
        READ(IN,'(A11)') STRING
        AQTYPE=STRING
        IF(AQTYPE.EQ.'CONFINED')IAQ=0
        IF(AQTYPE.EQ.'WATER TABLE')IAQ=1
        WRITE(*,*) ' SPECIFIED AQUIFER TYPE:  ',AQTYPE
!       IF IAQ STILL EQUALS 2, UNRECOGNIZED AQUIFER TYPE; STOP
        IF(IAQ.EQ.2)THEN
        WRITE(IO,104)
        STOP
        ENDIF
!       READ AND TEST AQUIFER PROPERTIES
!       BB=Saturated Thickness
!       HKR=Kr (Horizontal Hydraulic Conductivity)
!       HKZ=Kz (Vertical Hydraulic Conductivity)
!       SS=Specific Storage
```

```
!       AT=Aquifer Transmissivity
!       XKD=Ratio of Vertical to Horizontal Hydraulic Conductivity
!       ASC=Aquifer Storage Coefficient
!       SIGMA=Ratio of Storativity to Specific Yield
        READ(IN,*,ERR=1000) BB, HKR, HKZ, SS, SY
        IF(BB.LE.0.0D0)THEN
        WRITE(IO,105)
        STOP
        ENDIF
        IF(HKR.LE.0.0D0.OR.HKZ.LE.0.0D0)THEN
        WRITE(IO,105)
        STOP
        ENDIF
        IF(SS.LE.0.0D0)THEN
        WRITE(IO,105)
        STOP
        ENDIF
        AT=HKR*BB
        XKD=HKZ/HKR
        ASC=SS*BB
        IF(IAQ.EQ.1)THEN
        IF(SY.LE.0.0D0)THEN
        WRITE(IO,105)
        STOP
        ENDIF
        SIGMA=ASC/SY
        ENDIF
!       SIGMA = 0 FOR CONFINED AQUIFERS
        IF(IAQ.EQ.0)SIGMA=0.0D0
!       READ AND TEST DRAINAGE PROPERTIES
!       IDRA=Type of Drainage at Water Table (0-Instantaneous, 1-Delayed)
!       NALPHA=Number of Drainage Constants (Must be Less Than or Equal to 5)
!       ALPHA(I)=NALPHA Drainage Constants
        READ(IN,*,ERR=1000) IDRA, NALPHA
        IF(IDRA.LT.0)THEN
        WRITE(IO,106)
        STOP
        ENDIF
        IF(NGAMMA.LT.0.OR.NALPHA.LT.0)THEN
        WRITE(IO,106)
        STOP
        ENDIF
        IF(NGAMMA.GT.5.OR.NALPHA.GT.5)THEN
        WRITE(IO,106)
        STOP
        ENDIF
        IF(IDRA.EQ.0)NALPHA=0
        IF(IAQ.EQ.0)IDRA=0
        IF(IAQ.EQ.0)NALPHA=0
        IF(IDRA.EQ.1)THEN
        READ(IN,*,ERR=1000)(ALPHA(I),I=1,NALPHA)
!       CONVERT DIMENSIONAL ALPHAS TO DIMENSIONLESS GAMMAS
        DO 5 I=1,NALPHA
        GAMMA(I)=ALPHA(I)*BB*SY/HKZ
 5      CONTINUE
        NGAMMA=NALPHA
```

```
      DO 10 I=1,NGAMMA
      IF(GAMMA(I).LT.0.0D0)THEN
      WRITE(IO,106)
      STOP
      ENDIF
10    CONTINUE
      ELSE
      READ(IN,*,ERR=1000)ALPHA(1)
      ENDIF
!     READ AND TEST TIME-STEP INFORMATION
!     TLAST=Largest Value of Time (on Log Scale)
!     NLC=Number of Logarithmic Cycles
!     NOX=Number of Equally Spaced Times Per Logarithmic Cycle
      ITS=0
      IMEAS=0
      ILINE=8
      READ(IN,*,ERR=1000) TLAST, NLC, NOX
      IF(TDLAST.LT.0.0D0.OR.TLAST.LT.0.0D0)THEN
      WRITE(IO,107)
      STOP
      ENDIF
      IF(NLC.LT.0.OR.NOX.LT.0)THEN
      WRITE(IO,107)
      STOP
      ENDIF
!     READ AND TEST PROGRAM-SOLUTION VARIABLES
!     RERRNR=Relative Error for Newton-Rhapson Iteration (Not For Confined)
!     RERRSUM=Relative Error for Finite Summations (Confined, Suggested=1.0D-07)
!     NMAX=Maximum Number of Terms Permitted in Finite Summations (Suggested=200)
!     NTMS=Factor Used to Determine Number of Terms in Finite Summations (WT, Confined=0.0)
!     NS=Number of Terms Used in Stehfest Algorithm (Suggested, NS=8-12)
      READ(IN,*,ERR=1000) RERRNR, RERRSUM, NMAX, NTMS, NS
      IF(RERRNR.LT.0.0D0.OR.RERRSUM.LT.0.0D0)THEN
      WRITE(IO,108)
      STOP
      ENDIF
      IF(NMAX.LT.0.OR.NTMS.LT.0)THEN
      WRITE(IO,108)
      STOP
      ENDIF
      IF(NS.LT.0)THEN
      WRITE(IO,108)
      STOP
      ENDIF
!     READ AND TEST PUMPED-WELL INFORMATION
!     IPWS=Type of Pumped Well (0-Partially, 1-Full)
!     IPWD=Type of Diameter of Pumped Well (0-Infinitesimal, 1-Finite)
!     PWSTRM=Distance From Stream to Pumping Well
!     QQ=Pumping rate of well (L^3/T)
!     RW=Radius of Pumped Well Screen (L)
!     RC=Inside Radius of Pumped Well
!     ZPD=Depth Below Top of Aquifer or Initial Water Table to Top of Screened Interval (L)
!     ZPL=Depth Below Top of Aquifer or Initial Water Table to Bottom of Screen (L)
!     SW=Well-bore Skin Parameter
      READ(IN,*,ERR=1000) IPWS, IPWD
      IF(IPWS.LT.0.OR.IPWS.GT.1)THEN
```

```
      WRITE(IO,109)
      STOP
      ENDIF
      IF(IPWD.LT.0.OR.IPWD.GT.1)THEN
      WRITE(IO,109)
      STOP
      ENDIF
      READ(IN,*,ERR=1000) QQ, RW, RC, ZPD, ZPL, SW
      IF(RW.LT.0.0D0.OR.RC.LT.0.0D0)THEN
      WRITE(IO,110)
      STOP
      ENDIF
      IF(ZPD.LT.0.0D0.OR.ZPL.LT.0.0D0)THEN
      WRITE(IO,110)
      STOP
      ENDIF
      IF(WD.LT.0.0D0.OR.SW.LT.0.0D0)THEN
      WRITE(IO,110)
      STOP
      ENDIF
      IF(QQ.LT.0.0D0)THEN
      WRITE(IO,110)
      STOP
      ENDIF
!     CALCULATE PUMPING-WELL DIMENSIONLESS PARAMETERS
!     RWD=Ratio of Radius of Well to Saturated Thickness
!     BETAW=Dimensionless Parameter
!     WD=Well-Bore Storage Parameter
!     XDD=Dimensionless Distance of Top Screen to Saturated Thickness
!     XLD=Dimensionless Distance of Bottom Screen to Saturated Thickness
      RWD=RW/BB
      BETAW=XKD*(RWD*RWD)
      IF(IPWD.EQ.0)WD=0.0D0
      IF(IPWD.EQ.1)THEN
      WD=((RC/RW)**2)/(2.0D0*SS*(ZPL-ZPD))
      ENDIF
      IF(IPWS.EQ.0)THEN
      XDD=ZPD/BB
      XLD=ZPL/BB
      ENDIF
      IF(IPWS.EQ.1)THEN
      BCALC=ZPL-ZPD
      IF(BCALC.NE.BB)THEN
      WRITE(IO,111)
      STOP
      ENDIF
      ENDIF
      IF(IPWS.EQ.1)THEN
      XDD=0.0D0
      XLD=1.0D0
      ENDIF
      IRUN=1
!     READ AND TEST STREAM/STREAMBED INFORMATION
!     SBKSB=Streambed Hydraulic Conductivity
!     STRMW=Stream Width
!     SBM=Streambed Thickness
```

```
      READ(IN,*,ERR=1000) PWSTRM, SBKSB, SBM, STRMW
      IF(SBKSB.LT.0.D0.OR.STRMW.LT.0.D0) THEN
      WRITE(*,*) 'ERROR IN STREAM/STREAMBED PARAMETERS'
      STOP
      ENDIF
      IF (SBM.EQ.0.D0) THEN
      WRITE(*,*) 'ERROR IN STREAM/STREAMBED PARAMETERS'
      STOP
      ENDIF
      SLAMBDA=SBKSB*STRMW/SBM
!     READ AND TEST OBSERVATION-WELL INFORMATION
!     NOBWC=Number of Observation Wells or Piezometers
 35   READ(IN,*,ERR=1000) NOBWC
      IF(NOBWC.LT.0.OR.NOBWC.GT.(IMAXX1-1))THEN
      WRITE(IO,113)(IMAXX1-1)
      STOP
      ENDIF
!     WRITE INPUT DATA TO RESULT FILE
!     TITLE
      WRITE(IO,114)TITLE
!     AQUIFER TYPE
      IF(IAQ.EQ.0)AQTYPETXT='CONFINED AQUIFER'
      IF(IAQ.EQ.1)AQTYPETXT='WATER-TABLE AQUIFER'
      WRITE(IO,115) AQTYPETXT
!     HYDRAULIC PROPERTIES
      WRITE(IO,116)
      WRITE(IO,117)BB
      WRITE(IO,120)HKR
      WRITE(IO,121)HKZ
      WRITE(IO,118)XKD
      WRITE(IO,122)AT
      WRITE(IO,123)SS
      IF(IAQ.EQ.1)WRITE(IO,124)SY
      WRITE(IO,125)ASC
      IF(IAQ.EQ.1)WRITE(IO,119)SIGMA
!     DRAINAGE PROPERTIES
      IF(IAQ.EQ.1)THEN
      IF(IDRA.EQ.0)WRITE(IO,126)IDRA
      IF(IDRA.EQ.1)THEN
      WRITE(IO,127)IDRA
      WRITE(IO,129)(ALPHA(I),I=1,NALPHA)
      WRITE(IO,128)(GAMMA(I),I=1,NGAMMA)
      ENDIF
      ENDIF
!     TIME-STEP AND PROGRAM-SOLUTION VARIABLES
      WRITE(IO,132)
      WRITE(IO,131)TLAST,NLC,NOX
      WRITE(IO,135)
      WRITE(IO,136)RERRNR,RERRSUM,NMAX,NTMS,NS
!     PUMPING-WELL INFORMATION
      WRITE(IO,137)
      IF(IPWD.EQ.0)WRITE(IO,138)IPWD
      IF(IPWD.EQ.1)WRITE(IO,139)IPWD
      IF(IPWS.EQ.0)WRITE(IO,140)IPWS
      IF(IPWS.EQ.1)WRITE(IO,141)IPWS
      WRITE(IO,142)QQ
```

```
      WRITE(IO,143)
      WRITE(IO,144)RW,ZPD,ZPL,WD,SW
      WRITE(IO,145)BETAW
!     FORMAT STATEMENTS
 100  FORMAT(A70)
 101  FORMAT(A25)
 102  FORMAT(/,/,3X,'ANALYSIS FORMAT (LINE 2) IS INVALID.',
  2   ' PROGRAM STOPPED.')
 103  FORMAT(A11)
 104  FORMAT(/,/,3X,'AQUIFER TYPE IS INVALID. PROGRAM STOPPED.')
 105  FORMAT(/,3X,'A HYDRAULIC PROPERTY OF THE AQUIFER (INPUT LINE',
  2   ' 4) IS LESS THAN OR',/,3X,' EQUAL TO ZERO.',
  3    ' PROGRAM STOPPED.')
 106  FORMAT(/,3X,'A DRAINAGE VARIABLE OR PROPERTY (IDRA, NGAMMA,',
  2   ' NALPHA, GAMMA(I), OR',/,3X,'ALPHA(I)) IS NOT CORRECT.',
  3   '  PROGRAM STOPPED.')
 107  FORMAT(/,3X,'A TIME-STEP VARIABLE (TDLAST, TLAST, NLC, NOX,',
  2   '  ITS, OR IMEAS) IS LESS THAN',/,3X,' ZERO.  PROGRAM STOPPED.')
 108  FORMAT(/,3X,'A PROGRAM-SOLUTION VARIABLE (RERRNR, RERRSUM, NMAX,',
  2   ' NTMS, OR NS) IS LESS',/,3X,'THAN ZERO.  PROGRAM STOPPED.')
 109  FORMAT(/,3X,'EITHER IPWS OR IPWD IS NOT EQUAL TO 0 OR 1',
  2   '  PROGRAM STOPPED.')
 110  FORMAT(/,3X,'A PUMPING-WELL VARIABLE (RW, RC, ZPD, ZPL, QQ, WD,',
  2   ' OR SW) IS LESS',/,3X,'THAN ZERO.  PROGRAM STOPPED.')
 111  FORMAT(/,2X,'ZPL-ZPD DOES NOT EQUAL INITIAL SATURATED THICKNESS',
  2   ' SPECIFIED FOR',/,2X,'AQUIFER. MODIFY ZPD, ZPL, OR BB.',
  3   ' PROGRAM STOPPED.')
 112  FORMAT(/,3X,'INFORMATION ON PUMPED-WELL TIME-DRAWDOWN DATA',
  2   ' (NTSPW, IRUN, OR TIMEPW(I))',/,3X,'IS NOT CORRECT.  PROGRAM',
  3   ' STOPPED.')
 113  FORMAT(/,2X,'SPECIFIED VALUE OF NOBWC IS LESS THAN ZERO OR ',
  2    'GREATER THAN',I3,'. NOBWC ',/,2X,'MUST BE LESS THAN OR EQUAL',
  3   ' TO (IMAXX1-1). IMAXX1 IS SPECIFIED IN THE',/,2X,'COMPUTER',
  4   ' CODE. REDUCE THE NUMBER OF OBSERVATION WELLS OR PIEZOMETERS',
  5   /,2X,'OR INCREASE THE VALUE OF IMAXX1 IN THE CODE.  PROGRAM',
  6    ' STOPPED.')
 114   FORMAT(3X,A70)
 115  FORMAT(/,/,5X,A20)
 116  FORMAT(/,/,/,15X,'*** AQUIFER HYDRAULIC PROPERTIES ***',/)
 117  FORMAT(2X,' SATURATED THICKNESS (BB): ',8X,D12.3,
  2   ' (units of length)')
 118  FORMAT(2X,' RATIO OF VERTICAL TO HORIZONTAL',/,3X,
  2   ' HYDRAULIC CONDUCTIVITY (XKD): ',3X,D12.3,' (dimensionless)')
 119  FORMAT(2X,' RATIO OF STORATIVITY TO',/,3X,
  2   ' SPECIFIC YIELD (SIGMA): ',9X,D12.3,' (dimensionless)')
 120   FORMAT(2X,' HORIZONTAL HYDRAULIC',/,3X,' CONDUCTIVITY (HKR): ',
  2   13X,D12.3,' (units of length per time)')
 121  FORMAT(2X,' VERTICAL HYDRAULIC',/,3x,' CONDUCTIVITY (HKZ): '
  2   13x,D12.3,' (units of length per time)')
 122  FORMAT(2X,' CALCULATED TRANSMISSIVITY: ',7X,D12.3,
  2   ' (units of length squared',/,51X,'per time)')
 123  FORMAT(2X,' SPECIFIC STORAGE (SS): ',11X,D12.3,
  2   ' (units of inverse length)')
 124  FORMAT(2X,' SPECIFIC YIELD (SY): ',13X,D12.3,
  2   ' (dimensionless)')
 125  FORMAT(2X,' CALCULATED STORATIVITY: ',10X,D12.3,
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  2     ' (dimensionless)')
126     FORMAT(2X,' DRAINAGE AT WATER TABLE (IDRA): ',3X,I3,
  2     ' (instantaneous)')
127     FORMAT(2X,' DRAINAGE AT WATER TABLE (IDRA): ',3X,I3,
  2     ' (delayed)')
128     FORMAT(2X,' DIMENSIONLESS DRAINAGE PARAMETER(S) (GAMMAS): ',/,
  2     3X,5(1PD11.2))
129     FORMAT(2X,' DRAINAGE PARAMETER(S) (ALPHAS) (units of inverse',
  2     ' length):',/,3X,5(1PD11.2))
130     FORMAT(/,/,/,16X,'*** PROGRAM SOLUTION VARIABLES ',
  2     ' ***',/,/,4X,'LARGEST VALUE',7X,'NUMBER OF',7X,'DRAWDOWN',
  3     ' CALCULATIONS',/,3X,'OF TIME (TDLAST)',3X,'LOG CYCLES',
  4     ' (NLC)',4X,'PER LOG CYCLE (NOX)',/,3X,'----------------',
  5     '-------------------------------------------')
131     FORMAT(7X,D9.3,11X,I2,18X,I2)
132     FORMAT(/,/,/,16X,'*** PROGRAM SOLUTION VARIABLES ',
  2     ' ***',/,/,4X,'LARGEST VALUE',7X,'NUMBER OF',7X,'DRAWDOWN',
  3     ' CALCULATIONS',/,3X,'OF TIME (TLAST)',4X,'LOG CYCLES',
  4     ' (NLC)',4X,'PER LOG CYCLE (NOX)',/,3X,'----------------',
  5     '-------------------------------------------')
133     FORMAT(/,/,/,16X,'*** PROGRAM SOLUTION VARIABLES ',
  2     ' ***',/,/,3X,'USER-SPECIFIED TIMES; MEASURED DRAWDOWN',
  3     ' DATA NOT SPECIFIED')
134     FORMAT(/,/,/,16X,'*** PROGRAM SOLUTION VARIABLES ',
  2     ' ***',/,/,3X,'USER-SPECIFIED TIMES; MEASURED DRAWDOWN',
  3     ' DATA SPECIFIED')
135     FORMAT(/,/,5X,'RERRNR',6X,'RERRSUM',7X,'NMAX',7X,'NTMS',
  2     5X,'NS',/,3X,'------------------------------------------',
  3     '--------')
136     FORMAT(4X,D9.3,3X,D9.3,4X,I5,6X,I5,6X,I2)
137     FORMAT(/,/,/,3X,'*** PUMPED-WELL CHARACTERISTICS AND',
  2     ' CALCULATED DRAWDOWN ***',/)
138     FORMAT(3X,'WELL-DIAMETER TYPE (IPWD):',11X,I1,
  2     ' (infinitesimal diameter)')
139     FORMAT(3X,'WELL-DIAMETER TYPE (IPWD):',11X,I1,
  2     ' (finite diameter)')
140     FORMAT(3X,'SCREENED INTERVAL (IPWS):',12X,I1,
  2     ' (partially penetrating)')
141     FORMAT(3X,'SCREENED INTERVAL (IPWS):',12X,I1,
  2     ' (fully penetrating)')
142     FORMAT(3X,'PUMPING RATE OF WELL (QQ):',9X,D11.3,
  2     ' (cubic length per time)')
143     FORMAT(/,19X,'SCREENED INTERVAL',4X,'WELL BORE',3X,'WELL BORE',
  2     /,4X,'WELL RADIUS',6X,'ZPD',6X,'ZPL',8X,'STORAGE',6X,'SKIN',
  3     /,3X,'-----------------------------------------------------------')
144     FORMAT(5X,D8.2,5X,D8.2,2X,D8.2,4X,D8.2,4X,D8.2)
145     FORMAT(/,/,3X,'BETAW = ',D12.3)

!C      DEFINE SELECTED PROGRAM PARAMETERS
        PI=3.141592653589793D0
        F1=(SS*RW*RW)/HKR
        F2=QQ/(4.0D0*PI*HKR*BB)
!D      EXPMAX IS THE MAXIMUM ALLOWABLE ABSOLUTE VALUE OF EXPONENTIAL
        ARGUMENTS
        EXPMAX=708.D0
!E      CALL LINVST TO CALCULATE COEFFICIENTS USED FOR THE STEHFEST
```

```
      CALL LINVST(V,NS)
      XLN2=DLOG(2.D0)
!F    CALCULATE TIME-STEP PARAMETERS FOR LOG-CYCLE FORMATS
      NTS=NOX*NLC + 1
      DELTD=10.D0**(1.0D0/NOX)
!G    CALCULATE REQUIRED NUMBER OF TIME-DRAWDOWN CURVES
      KKTOT=NOBWC+1
!H    CALCULATE TIME-DRAWDOWN CURVES FOR OBSERVATION WELL
      KK=2
      KI=1
      IF(KI.EQ.1)WRITE(IO,8106)
      WRITE(IO,8112)KI
      READ(IN,*,ERR=1000) IOWS, IDPR
      IF(IOWS.LT.0.OR.IOWS.GT.2)THEN
      WRITE(IO,8107)
      STOP
      ENDIF
      IF(IDPR.LT.0.OR.IDPR.GT.1)THEN
      WRITE(IO,8107)
      STOP
      ENDIF
      IF(IDPR.EQ.1)THEN
      IF(IOWS.EQ.2)THEN
      WRITE(IO,8107)
      STOP
      ENDIF
      ENDIF
      READ(IN,*,ERR=1000) OBWX,OBWY,Z1,Z2,ZP,RP,XLL
!     CALCULATE RADIAL DISTANCE FROM OBSERVATION TO PUMPING WELL BASED
      ON DISTANCE
!     BETWEEN PUMPING WELL AND STREAM
      R=DSQRT((PWSTRM-OBWX)**2+OBWY**2)
      IF(R.LT.0.0D0.OR.ZP.LT.0.0D0)THEN
      WRITE(IO,8108)
      STOP
      ENDIF
      IF(Z1.LT.0.0D0.OR.Z2.LT.0.0D0)THEN
      WRITE(IO,8108)
      STOP
      ENDIF
      IF(WDP.LT.0.0D0.OR.RP.LT.0.0D0)THEN
      WRITE(IO,8108)
      STOP
      ENDIF
      IF(XLL.LT.0.0D0)THEN
      WRITE(IO,8108)
      STOP
      ENDIF
      RD=R/RW
      RDSQ=RD*RD
      BETA=BETAW*RDSQ
!H4   CALCULATE DIMENSIONLESS POSITION OF OBSERVATION WELL OR
!     OBSERVATION
!     PIEZOMETER. NOTE SWITCH IN DEFINITION OF WELL-SCREEN POSITION FROM
!     WATER-TABLE REFERENCE TO BOTTOM-OF-AQUIFER REFERENCE
!H5   IF IOWS = 0, PARTIALLY PENETRATING OBSERVATION WELL
```

```
      IF(IOWS.EQ.0)THEN
      ZD=0.0D0
      ZD1=1.0D0 - (Z2/BB)
      ZD2=1.0D0 - (Z1/BB)
      ENDIF
!H6   IF IOWS = 1, FULLY PENETRATING OBSERVATION WELL
      IF(IOWS.EQ.1)THEN
      BCALC=Z2-Z1
      IF(BCALC.NE.BB)THEN
      WRITE(IO,8109)
      STOP
      ENDIF
      ZD=0.0D0
      ZD1=0.0D0
      ZD2=1.0D0
      ENDIF
!H7   IF IOWS = 2, OBSERVATION PIEZOMETER
      IF(IOWS.EQ.2)THEN
      ZD=1.0D0 - (ZP/BB)
      ZD1=0.0D0
      ZD2=0.0D0
      ENDIF
      IF(IDPR.EQ.0)WDP=0.0D0
      IF(IDPR.EQ.1)THEN
      XM=DSQRT(XKD)
      XX=XLL/(2.0D0*XM*RP)
      XXX=XX+DSQRT(1.0D0+XX*XX)
      FPRIME=XLL/DLOG(XXX)
      WDP=((RP/RW)**2)/(2.0D0*SS*FPRIME)
      ENDIF
      IRUN=1
!H8   WRITE OBSERVATION-WELL INFORMATION TO OUTPUT FILE
      IF(IOWS.EQ.0)WRITE(IO,8113)
      IF(IOWS.EQ.1)WRITE(IO,8114)
      IF(IOWS.EQ.2)WRITE(IO,8115)
      IF(IOWS.EQ.0.OR.IOWS.EQ.1)WRITE(IO,8116)R,Z1,Z2,WDP
      IF(IOWS.EQ.2)WRITE(IO,8117)R,ZP,WDP
      WRITE(IO,8118)BETA
!H9   FINISHED CALCULATING DIMENSIONLESS VARIABLES
!H10  DEFINE DIMENSIONLESS TIME (TD) AND ITERATION VARIABLES
      RDSQ=1.0D0
      TD=TLAST*DELTD/F1
!H11  NOW, ENTER TIME-STEP LOOP
!     SET INITIAL VALUES OF SWITCHES FOR MINIMUM DRAWDOWN
!     CALCULATIONS
      IHD=1
      IHDH=1
      DO 30 NT=1,NTS
      HDBDRAW=0.0D0
!H12  DETERMINE DIMENSIONLESS TIME OF CURRENT TIME STEP.
!     DRAWDOWNS ARE CALCULATED FROM LARGEST TO SMALLEST TIME.
!     THUS, DIVIDE CURRENT TIME BY DELTD TO REDUCE TIME BY DELTD:
      TD=TD/DELTD
!H13  IF IHDH OR IHD HAVE BEEN SET TO 0, THEN PREVIOUS DRAWDOWN
!     WAS LESS THAN 1.0D-2; THUS, NO NEED TO CALCULATE DRAWDOWNS
!     FOR SMALLER TIMES (THAT IS, MINIMUM DRAWDOWN ALLOWED IS
```

```
!         1.0D-2)
!H14      IF CONFINED AQUIFER (IAQ=0), CALL LTST2
          IF(IAQ.EQ.0)THEN
          IF(IHDH.EQ.0)GOTO 2235
          R=DSQRT((PWSTRM-OBWX)**2+OBWY**2)
          RD=R/RW
          RDSQ=RD*RD
          BETA=BETAW*RDSQ
          CALL LTST2(TD,HD)
          IF(DABS(HD).LT.1.0D-3)IHDH=0
          FOI=HD
          WRITE(*,3998) F1*TD
3998      FORMAT(/,' CALCULATING HUNT INTEGRAL FOR TIME: ',F9.4)
          DELTP=1.0D-04
          SOISUM=0.0D0
          SOIR1=0.0D0
          SOI=0.0D0
          SOITEMP=1.0D12
          SOIPP=0.0D0
          DO WHILE (SOITEMP>1.0D-04)
          SOIPP=SOIPP+DELTP
          SOIR1=DEXP(-SOIPP)
          RI=DSQRT((PWSTRM+ABS(OBWX)+2*HKR*BB*SOIPP/
&         SLAMBDA)**2+OBWY**2)
          RD=RI/RW
          BETA=BETAW*RDSQ
          CALL LTST2(TD,HD)
          IF(HD.LT.1.0D-10) HD=0.0D0
          SOITEMP=SOIR1*HD
          SOISUM=SOISUM+HD*SOIR1*DELTP
          ENDDO
          SOI=SOISUM
          HDBDRAW=F2*(FOI-SOI)
          ENDIF
!H15      IF WATER-TABLE AQUIFER (IAQ=1), CALL LTST3
          IF(IAQ.EQ.1)THEN
          IF(IHD.EQ.0)GOTO 2235
          R=DSQRT((PWSTRM-OBWX)**2+OBWY**2)
          RD=R/RW
          RDSQ=RD*RD
          BETA=BETAW*RDSQ
          CALL LTST3(TD,HD)
          IF(DABS(HD).LT.1.0D-2)IHD=0
          FOI=HD
          WRITE(*,1998) TD*F1
1998      FORMAT(/,'  CALCULATING HUNT INTEGRAL FOR TIME: ',F9.4)
          DELTP=1.0D-04
          SOISUM=0.0D0
          SOIR1=0.0D0
          SOI=0.0D0
          SOITEMP=1.0D12
          SOIPP=0.0D0
          DO WHILE (SOITEMP>1.0D-04)
          SOIPP=SOIPP+DELTP
          SOIR1=DEXP(-SOIPP)
          RTEST=((PWSTRM+ABS(OBWX)+2*HKR*BB*SOIPP/
```

```
&       SLAMBDA)**2+OBWY**2)
        RI=DSQRT(RTEST)
        RD=RI/RW
        BETA=BETAW*RDSQ
        CALL LTST3(TD,HD)
        IF(HD.LT.1.0D-10) HD=0.0D0
        SOITEMP=SOIR1*HD
        SOISUM=SOISUM+HD*SOIR1*DELTP
        ENDDO
        SOI=SOISUM
        HDBDRAW=F2*(FOI-SOI)
        ENDIF
 2235   IF(HDBDRAW.LT.1.0D-04) HDBDRAW=0.0D0
        RTD=F1*TD
        RHD=HDBDRAW
        TDPLOT(1,NT)=RTD
        HDPLOT(1,NT)=RHD
!H16    END TIME LOOP FOR DRAWDOWN CURVE
 30     CONTINUE
!I      WRITE TIME-DRAWDOWN TO OUTPUT FILE
        DO 40 NT=1,NTS
        NTT=NTS-NT+1
        WRITE(IO,8120)TDPLOT(1,NTT),HDPLOT(1,NTT)
 40     CONTINUE
!K      FORMAT STATEMENTS
 8100   FORMAT(/,3X,'DIMENSIONLESS TIME   DIMENSIONLESS DRAWDOWN',
   2    /,7X,'(TDRDSQ)',18X,'(HD)',/,3X,'------------------',
   3    '   ----------------------')
 8101   FORMAT(/,17X,'CALCULATED',/,6X,'TIME       DRAWDOWN',
   2    /,6X,'----       ----------')
 8102   FORMAT(/,48X,'RELATIVE',/,18X,'MEASURED',6X,'CALCULATED',7X,
   2    'ERROR',/,6X,'TIME       DRAWDOWN      DRAWDOWN    ',
   3    '  (PERCENT)'/,6X,'----       ----------    ----------    ',
   4    ' ---------- ')
 8103   FORMAT(/,/,9X,'****   PUMPED WELL   ****')
 8104   FORMAT(/,5X,'TIME         CALCDD')
 8105   FORMAT(/,5X,'TIME         MEASDD       CALCDD       ',
   2    'RELERR')
 8106   FORMAT(/,/,/,1X,'*** OBSERVATION-WELL CHARACTERISTICS AND',
   2    ' CALCULATED DRAWDOWN ***',/)
 8107   FORMAT(/,3X,'EITHER IOWS OR IDPR IS INCORRECT.  PROGRAM STOPPED.')
 8108   FORMAT(/,3X,'AN OBSERVATION-WELL VARIABLE (R, Z1, Z2, ZP, WDP,',
   2    ' RP, OR XLL) IS LESS',/,3X,'THAN ZERO.  PROGRAM STOPPED.')
 8109   FORMAT(/,2X,'Z2(I)-Z1(I) DOES NOT EQUAL SATURATED',
   2    ' THICKNESS SPECIFIED FOR AQUIFER.',/,2X,'MODIFY Z2(I),',
   3    ' Z1(I), OR BB. PROGRAM STOPPED.')
 8110   FORMAT(/,3X,'INFORMATION ON OBSERVATION-WELL TIME-DRAWDOWN DATA',
   2    ' (NTSOB, IRUN,',/,3X,'OR TIMEOB(I)) IS NOT CORRECT.  PROGRAM',
   3    ' STOPPED.')
 8111   FORMAT(/,/,13X,'****   DRAWDOWN NOT CALCULATED   ****')
 8112   FORMAT(/,/,9X,'****   OBSERVATION WELL OR PIEZOMETER',I3,
   2    2X,'****')
 8113   FORMAT(/,/,3X,'PARTIALLY PENETRATING OBSERVATION WELL')
 8114   FORMAT(/,/,3X,'FULLY PENETRATING OBSERVATION WELL')
 8115   FORMAT(/,/,3X,'OBSERVATION PIEZOMETER')
 8116   FORMAT(/,/,3X,'DISTANCE FROM',/,5X,'CENTER OF',29X,
```

```
     2     'DELAYED RESPONSE',/,4X,'PUMPED WELL',10X,'Z1',10X,'Z2',
     3     9X,'FACTOR',/,3X,'--------------------',
     4     '-----------------------------------',/,5X,D9.3,7X,D9.3,
     5     3X,D9.3,4X,D9.3)
 8117      FORMAT(/,/,3X,'DISTANCE FROM',/,5X,'CENTER OF',16X,
     2     'DELAYED RESPONSE',/,4X,'PUMPED WELL',8X,'ZP',10X,'FACTOR',/,
     3     3X,'------------------------------------------',/,5X,D9.3,
     4     5X,D9.3,5X,D9.3)
 8118      FORMAT(/,/,3X,'DRAWDOWN CALCULATED FOR BETA = ',D12.3)
 8119      FORMAT(5X,D11.4,12X,D11.4)
 8120      FORMAT(1X,F15.7,4X,F11.4)
 8121      FORMAT(1X,D11.4,4X,D11.4,4X,D11.4,4X,D11.4)
 8122      FORMAT(1X,E11.4,4X,E11.4)
 8123      FORMAT(1X,E11.4,4X,E11.4,4X,E11.4,4X,E11.4)
 8124      FORMAT(/,'   TDRDSQ     HDT     TDYRDSQ     HDTY',2X,
     2     5(4X,A6))
 8125      FORMAT(1X,9(1X,E9.3))
 8126      FORMAT(/,'   TIME',2X,5(4X,A6))
 8127      FORMAT(1X,6(1X,E9.3))
 8128      FORMAT(1X,D11.4,4X,D11.4,4X,D11.4,8X,'****')
 8129      FORMAT(1X,E11.4,4X,E11.4,4X,E11.4,8X,'****')
!          CLOSE INPUT AND OUTPUT FILES, STOP, AND END
           CLOSE(UNIT=IN)
           CLOSE(UNIT=IO)
           STOP
 1000      CALL RDERR(IO,STRING,ILINE)
           END


!     **************************************************
!     *                                                *
!     *           SUBROUTINE OFILE                     *
!     *                                                *
!     *        FROM BARLOW AND MOENCH (1999)           *
!     *                                                *
!     **************************************************
!
!          SUBROUTINE OFILE OPENS INPUT, RESULT, AND PLOT FILES
           SUBROUTINE OFILE(IPLOT)
           CHARACTER*50 IFNAME, OFNAME, PFNAME, TEMP
           INTEGER IN, IO, IP, N, IPLOT
           COMMON /IOUNIT/ IN,IO,IP
           IN = 15
           IO = 16
           IP = 17
!          READ INPUT AND OUTPUT FILE NAMES
           READ(4,*) TEMP
           READ(4,5) IFNAME
           N=LENCHR(IFNAME)
           IF(N.LT.1) THEN
           WRITE(*,*) 'ERROR OPENING FILE'
           STOP
           ENDIF
           OPEN(IN,FILE=IFNAME,STATUS='OLD')
           READ(4,5) OFNAME
           N=LENCHR(OFNAME)
```

```
      IF(N.LT.1) THEN
      WRITE(*,*) 'ERROR OPENING FILE'
      STOP
      ENDIF
      OPEN(IO,FILE=OFNAME)
!     FORMAT STATEMENTS
5     FORMAT(A50)
      RETURN
      END


!   ****************************************************
!   *                                                  *
!   *              FUNCTION LENCHR                     *
!   *                                                  *
!   *         FROM BARLOW AND MOENCH (1999)            *
!   *                                                  *
!   ****************************************************
!
!     FUNCTION LENCHR CALCULATES THE LENGTH OF A CHARACTER STRING
!     (CODE FROM R. STEVE REGAN, USGS)
!
      INTEGER FUNCTION LENCHR
  I           ( STRING )
      CHARACTER*(*) STRING
!     + + + ARGUMENT DEFINITIONS + + +
!     STRING - Input string to determine length of
!     + + + LOCAL VARIABLES + + +
      INTEGER MAX, I
!     + + + INTRINSICS + + +
      INTEGER LEN
      INTRINSIC LEN
!     + + + END SPECIFICATIONS + + +
      MAX = LEN(STRING)
      DO 10 I = MAX, 1, -1
      IF (STRING(I:I).NE.' ') THEN
      LENCHR = I
      GOTO 20
      ENDIF
 10   CONTINUE
      LENCHR = 0
 20   RETURN
      END


!   ****************************************************
!   *                                                  *
!   *            SUBROUTINE BANNER                     *
!   *      MODIFIED FROM BARLOW AND MOENCH (1999)      *
!   *                                                  *
!   *                                                  *
!   ****************************************************
!
      SUBROUTINE BANNER(IO)
      WRITE(IO,100)
```

```
100     FORMAT(/,10X,
  2     '   ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~',/,
  3     10X,
  4     '   ~                                        ~',/,
  5     10X,
  6     '   ~            ~~~~~ STRMAQ ~~~~~              ~',/,
  7     10X,
  8     '   ~                                        ~',/,
  9     10X,
  4     '   ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~',
  5     /,/)
        RETURN
        END

!    ****************************************************
!    *                                                  *
!    *            SUBROUTINE RDERR                      *
!    *                                                  *
!    *       FROM BARLOW AND MOENCH (1999)              *
!    *                                                  *
!     ****************************************************
!
!       SUBROUTINE RDERR WRITES AN ERROR MESSAGE TO THE RESULT FILE
!       AND STOPS PROGRAM EXECUTION
!
        SUBROUTINE RDERR(IO,STRING,ILINE)
        CHARACTER STRING*80,VAR(19)*50
        DATA VAR/' TITLE',
   2     ' ANALYSIS FORMAT',
   3     ' AQUIFER TYPE',
   4     ' AQUIFER PROPERTIES',
   5     ' DRAINAGE TYPE AND NUMBER OF DRAINAGE PARAMETERS',
   6     ' DRAINAGE PARAMETERS',
   7     ' TDLAST NLC NOX',
   8     ' ITS IMEAS',
   9     ' TLAST NLC NOX',
   &      ' RERRNR RERRSUM NMAX NTMS NS',
   1     ' IPWS IPWD',
   2     ' PUMPED-WELL CHARACTERISTICS',
   3     ' NTSPW IRUN',
   4     ' PUMPED-WELL MEASURED TIME-DRAWDOWN DATA',
   5     ' NOBWC',
   6     ' IOWS IDPR IRUN',
   7     ' OBSERVATION-WELL CHARACTERISTICS',
   8     ' NTSOB',
   9     ' OBSERVATION-WELL MEASURED TIME-DRAWDOWN DATA'/
        WRITE(IO,1)ILINE
        WRITE(IO,'(A80)')STRING
        WRITE(IO,2) VAR(ILINE)
        WRITE(IO,3)
 1      FORMAT(' Aborting due to error in line from input file:',
  2     2X,I5)
 2      FORMAT(' While attempting to read variables',A40)
 3      FORMAT(' Check for incorrect positions or data types...')
        STOP
        END
```

```
!     ****************************************************
!    *                                                  *
!    *          SUBROUTINE LINVST                       *
!    *                                                  *
!    *       FROM BARLOW AND MOENCH (1999)              *
!    *                                                  *
!     ****************************************************
!
!       SUBROUTINE LINVST CALCULATES COEFFICIENTS USED FOR THE
!       STEHFEST ALGORITHM
        SUBROUTINE LINVST(V,NS)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        DIMENSION G(20),V(20),HS(20)
        G(1)=1.D0
        NH=NS/2
        DO 1 IS=2,NS
1       G(IS)=G(IS-1)*IS
        HS(1)=2.D0/G(NH-1)
        DO 3 IS=2,NH
        FI=IS
        IF(IS.EQ.NH) GOTO 2
        HS(IS)=FI**(NH)*G(2*IS)/(G(NH-IS)*G(IS)*G(IS-1))
        GOTO 3
2       HS(IS)=FI**(NH)*G(2*IS)/(G(IS)*G(IS-1))
3       CONTINUE
        SN=2*(NH-NH/2*2)-1
        DO 4 IS=1,NS
        V(IS)=0.D0
        K1=(IS+1)/2
        K2=IS
        IF(K2.GT.NH)K2=NH
        DO 5 KS=K1,K2
        IF(2*KS-IS.EQ.0) GOTO 6
        IF(IS.EQ.KS) GOTO 7
        V(IS)=V(IS)+HS(KS)/(G(IS-KS)*G(2*KS-IS))
        GOTO 5
6       V(IS)=V(IS)+HS(KS)/(G(IS-KS))
        GOTO 5
7       V(IS)=V(IS)+HS(KS)/G(2*KS-IS)
5       CONTINUE
        V(IS)=SN*V(IS)
        SN=-SN
4       CONTINUE
        RETURN
        END

!     ****************************************************
!    *                                                  *
!    *          SUBROUTINE LTST1                        *
!    *                                                  *
!    *       FROM BARLOW AND MOENCH (1999)              *
!    *                                                  *
!     ****************************************************
!
```

```
!       SUBROUTINE LTST1 CALCULATES THE LAPLACE TRANSFORM SOLUTION FOR
!       DRAWDOWN FOR FLOW TO A FULLY PENETRATING WELL OF INFINITESIMAL
!       DIAMETER IN A CONFINED AQUIFER (THEIS SOLUTION).
        SUBROUTINE LTST1(TD,HDT)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        COMMON /PAR2/ NGAMMA,IDRA,NS,KK,NMAX,NTMS
        COMMON /PAR9/ V(20),XLN2,EXPMAX
        COMMON /PAR10/ RD,ZD,ZD1,ZD2
        XP=0.D0
        DO 1 I=1,NS
        PP=XLN2*I/TD
        CA=RD*DSQRT(PP)
        IF(CA.GT.EXPMAX) CA=EXPMAX
        RE0=BESSK0(CA)
        PDL=RE0/PP
1       XP=XP+V(I)*PDL
        HDT=2.D0*XP*XLN2/TD
        RETURN
        END

!    ****************************************************
!    *                                                  *
!    *            SUBROUTINE LTST2                      *
!    *                                                  *
!    *    MODIFIED FROM BARLOW AND MOENCH (1999)        *
!    *                                                  *
!    ****************************************************
!
!       SUBROUTINE LTST2 CALCULATES THE LAPLACE TRANSFORM SOLUTION FOR
!       DRAWDOWN FOR FLOW TO A FINITE DIAMETER, PARTIALLY PENETRATING
!       WELL IN A CONFINED AQUIFER (MODIFIED SOLUTION OF DOUGHERTY AND
!       BABU, 1984). DELAYED DRAWDOWN RESPONSE AT OBSERVATION WELLS
!       IS INCLUDED.

        SUBROUTINE LTST2(TD,HD)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        COMMON STATEMENTS
        COMMON /IOUNIT/ IN,IO,IP
        COMMON /PAR1/ IPWD,IRUN,IPWS,NOBWC,IOWS,IDPR
        COMMON /PAR2/ NGAMMA,IDRA,NS,KK,NMAX,NTMS
        COMMON /PAR6/ BETAW,SIGMA,GAMMA
        COMMON /PAR7/ RERRNR,RERRSUM,TDLAST,TLAST
        COMMON /PAR8/ R,ZP,Z1,Z2,WDP
        COMMON /PAR9/ V(20),XLN2,EXPMAX
        COMMON /PAR10/ RD,ZD,ZD1,ZD2
        COMMON /PAR11/ XLD,XDD,WD,SW
        HD=0.D0
        IF(IRUN.EQ.0.AND.KK.EQ.1) RETURN
        PI=3.141592653589793D0
        IF(IPWS.EQ.1) THEN
        XDD=0.D0
        XLD=1.D0
        ENDIF
        XP=0.0D0
        DO 1 I=1,NS
        PP=XLN2*I/TD
```

```
      Q0=DSQRT(PP)
      Q0RD=Q0*RD
      IF(Q0.GT.EXPMAX) Q0=EXPMAX
      IF(Q0RD.GT.EXPMAX) Q0RD=EXPMAX
      RE0=BESSK0(Q0)
      RE1=BESSK1(Q0)
      RE0X=BESSK0(Q0RD)
      A0=RE0*(XLD-XDD)/(Q0*RE1)
      E0=RE0X*(XLD-XDD)/(Q0*RE1)
      A=0.D0
      E=0.D0
      IF(IPWS.EQ.1) GOTO 30
      IF(IOWS.EQ.1) GOTO 30
      SUMA=0.D0
      SUME=0.D0
      NNN=0
10    NNN=NNN+1
      IF(NNN.GE.NMAX) GOTO 40
      SUMTA=SUMA
      SUMTE=SUME
      XNPI=NNN*PI
      QN=DSQRT(BETAW*XNPI*XNPI+PP)
      IF(QN.GT.EXPMAX) QN=EXPMAX
      DB=DSIN(XNPI*(1.0D0-XDD))
      DA=DSIN(XNPI*(1.0D0-XLD))
      IF(IPWS.EQ.1) DA=0.D0
      SINES=DB-DA
      RE0=BESSK0(QN)
      RE1=BESSK1(QN)
      XNUM=RE0*SINES*SINES/(XNPI*(XLD-XDD))
      XDEN=0.5D0*QN*RE1*XNPI
      A=XNUM/XDEN
      SUMA=SUMTA+A
      QNRD=QN*RD
      IF(QNRD.GT.EXPMAX) QNRD=EXPMAX
      RE0X=BESSK0(QNRD)
      IF(IOWS.EQ.0)
     1 XNUM=RE0X*SINES*(DSIN(XNPI*ZD2)
     2 -DSIN(XNPI*ZD1))/(XNPI*(ZD2-ZD1))
      IF(IOWS.EQ.2)
     1 XNUM=RE0X*SINES*DCOS(XNPI*ZD)
      E=XNUM/XDEN
      SUME=SUMTE+E
      IF(IPWS.EQ.0.AND.NNN.LT.25) GOTO 10
      ERRA=DABS(SUMTA-SUMA)
      IF(ERRA.LT.RERRSUM*SUMA) GOTO 40
      ERRE=DABS(SUMTE-SUME)
      IF(ERRA.LT.RERRSUM*SUMA.AND.ERRE.LT.RERRSUM*SUME) GOTO 40
      GOTO 10
40    CONTINUE
      A=SUMA
30    DENOM=(1.D0+WD*PP*(A0+A+SW))
      E=SUME
      IF(IDPR.EQ.0) PDL=(E0+E)/(PP*DENOM)
      IF(IDPR.EQ.1) THEN
      SLUGF=1.D0/(1.D0+WDP*PP)
```

```
      PDL=SLUGF*(E0+E)/(PP*DENOM)
      ENDIF
      XP=XP+V(I)*PDL
1     CONTINUE
      HD=2.D0*XP*XLN2/(TD*(XLD-XDD))
      IF(NNN.GE.NMAX) WRITE(IO,100)
100   FORMAT('PROGRAM CONTINUES TO NEXT TIME STEP BECAUSE NNN',
    2 ' EXCEEDS NMAX.')
      RETURN
      END


!    ************************************************
!    *                                              *
!    *              SUBROUTINE LTST3                *
!    *                                              *
!    *      MODIFIED FROM BARLOW AND MOENCH (1999)  *
!    *                                              *
!    ************************************************
!
!     SUBROUTINE LTST3 CALCULATES THE LAPLACE TRANSFORM SOLUTION FOR
!     DRAWDOWN FOR FLOW TO A FINITE DIAMETER, PARTIALLY PENETRATING
!     WELL IN A WATER-TABLE AQUIFER WITH DELAYED DRAWDOWN RESPONSE AT
!     OBSERVATION WELLS.
      SUBROUTINE LTST3(TD,HD)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION GAMMA(5)
      COMMON /IOUNIT/ IN,IO,IP
      COMMON /PAR1/ IPWD,IRUN,IPWS,NOBWC,IOWS,IDPR
      COMMON /PAR2/ NGAMMA,IDRA,NS,KK,NMAX,NTMS
      COMMON /PAR6/ BETAW,SIGMA,GAMMA
      COMMON /PAR7/ RERRNR,RERRSUM,TDLAST,TLAST
      COMMON /PAR8/ R,ZP,Z1,Z2,WDP
      COMMON /PAR9/ V(20),XLN2,EXPMAX
      COMMON /PAR10/ RD,ZD,ZD1,ZD2
      COMMON /PAR11/ XLD,XDD,WD,SW
      SLUGF=0.0D0
      HD=0.D0
      IF(IRUN.EQ.0.AND.KK.EQ.1) RETURN
!     EMPIRICAL ESTIMATE OF NECESSARY NUMBER OF NNN TERMS NEEDED FOR
!     THE FINITE SUMS. (SMALL VALUES OF BETA AND TD REQUIRE MORE
!     TERMS THAN DO LARGE VALUES.)
!     EMPIRICAL ESTIMATE OF NECESSARY NUMBER OF NNN TERMS NEEDED FOR THE
!     FINITE SUM. (SMALL VALUES OF BETA REQUIRE MORE TERMS THAN LARGE
!     VALUES.)
      BETA=BETAW*RD*RD
      NX=NTMS*2.**((-DLOG10(BETA)-2.0D0))
      IF(NX.LT.4) NX=4
      PI=3.141592653589793D0
      IF(IPWS.EQ.1) THEN
      XDD=0.D0
      XLD=1.D0
      ENDIF
      IF(IOWS.EQ.1) THEN
      ZD1=0.D0
      ZD2=1.D0
```

```
      ENDIF
      XP=0.0D0
      DO 1 I=1,NS
      PP=XLN2*I/TD
      IF(IDRA.EQ.1)THEN
      RHS=0.0D0
      DO 5 J=1,NGAMMA
      RHS1=1.0D0/(SIGMA*BETAW+PP/GAMMA(J))
      RHS=RHS+RHS1
 5    CONTINUE
      XNGAMMA=NGAMMA
      RHS=PP*RHS/XNGAMMA
      ELSE
      RHS=PP/(SIGMA*BETAW)
      ENDIF
      SUMA=0.D0
      SUME=0.D0
      NNN=0
10    NNN=NNN+1
      IF(NNN.EQ.NX) GOTO 40
!     ASSIGN A STARTING POINT FOR EPS
      IF(NNN.GT.1) GOTO 15
      IF(RHS.LT.1.D0) EPS=DSQRT(RHS)
      IF(RHS.GE.1.D0) EPS=DATAN(RHS)
15    IF(NNN.GT.1) EPS=EPS0+PI
      SUMTA=SUMA
      SUMTE=SUME
!     CALCULATE EPSILON FOR CURRENT VALUE OF PP USING NEWTON-RAPHSON
!     TECHNIQUE
      N=0
30    EPS0=EPS
      N=N+1
      IF(N.GT.100)WRITE(IO,100)
      IF(N.GT.100)STOP
      A1=DSIN(EPS0)
      A2=DCOS(EPS0)
      F=EPS0*A1-RHS*A2
      FP=EPS0*A2+A1+RHS*A1
      EPS=EPS0-F/FP
      IF(DABS(EPS-EPS0).GT.RERRNR*EPS) GOTO 30
      QN=DSQRT(BETAW*EPS*EPS+PP)
      IF(QN.GT.EXPMAX) QN=EXPMAX
      DB=DSIN(EPS*(1.0D0-XDD))
      DA=DSIN(EPS*(1.0D0-XLD))
      IF(IPWS.EQ.1) DA=0.D0
      SINES=DB-DA
      RE0=BESSK0(QN)
      RE1=BESSK1(QN)
      XNUM=RE0*SINES*SINES/(EPS*(XLD-XDD))
      XDEN=0.5D0*QN*RE1*(EPS+0.5D0*DSIN(2.D0*EPS))
      A=XNUM/XDEN
      SUMA=SUMTA+A
      QNRD=QN*RD
      IF(QNRD.GT.EXPMAX) QNRD=EXPMAX
      RE0X=BESSK0(QNRD)
!     IF IOWS.EQ.0, THEN PARTIALLY PENETRATING OBSERVATION WELL
```

```
      IF(IOWS.EQ.0)
    1 XNUM=RE0X*SINES*(DSIN(EPS*ZD2)
    2 -DSIN(EPS*ZD1))/(EPS*(ZD2-ZD1))
!     IF IOWS.EQ.1, THEN FULLY PENETRATING OBSERVATION WELL
      IF(IOWS.EQ.1)
    1 XNUM=RE0X*SINES*DSIN(EPS)/EPS
!     IF IOWS.EQ.2, THEN OBSERVATION PIEZOMETER
      IF(IOWS.EQ.2)
    1 XNUM=RE0X*SINES*DCOS(EPS*ZD)
      E=XNUM/XDEN
      SUME=SUMTE+E
      GOTO 10
 40   CONTINUE
      A=SUMA
      DENOM=(1.D0+WD*PP*(A+SW))
      E=SUME
      IF(IDPR.EQ.0) PDL=E/(PP*DENOM)
      IF(IDPR.EQ.1) THEN
      SLUGF=1.D0/(1.D0+WDP*PP)
      PDL=SLUGF*E/(PP*DENOM)
      ENDIF
      XP=XP+V(I)*PDL
1     CONTINUE
      HD=2.D0*XP*XLN2/(TD*(XLD-XDD))
 100  FORMAT(' PROGRAM STOPPED BECAUSE NEWTON-RAPHSON ITERATIONS',
    2 ' EXCEED 100')
      RETURN
      END


!    **************************************************
!    *                                                *
!    *             FUNCTION BESSK0                    *
!    *                                                *
!    *        FROM BARLOW AND MOENCH (1999)           *
!    *                                                *
!    **************************************************
!
!     FUNCTION BESSK0 CALCULATES THE ZERO-ORDER MODIFIED BESSEL FUNCTION
!     OF THE SECOND KIND. SOURCE: PRESS AND OTHERS (1992).
      DOUBLE PRECISION FUNCTION BESSK0(X)
      DOUBLE PRECISION X,Y,P1,P2,P3,P4,P5,P6,P7,
     * Q1,Q2,Q3,Q4,Q5,Q6,Q7,BESSI0
      DATA P1,P2,P3,P4,P5,P6,P7/-0.57721566D0,0.42278420D0,0.23069756D0,
     * 0.3488590D-1,0.262698D-2,0.10750D-3,0.74D-5/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7/1.25331414D0,-0.7832358D-1,0.2189568D-1,
     * -0.1062446D-1,0.587872D-2,-0.251540D-2,0.53208D-3/
      IF (X.LE.2.D0) THEN
      Y=X*X/4.D0
      BESSK0=(-DLOG(X/2.D0)*BESSI0(X))+(P1+Y*(P2+Y*(P3+
     * Y*(P4+Y*(P5+Y*(P6+Y*P7))))))
      ELSE
      Y=(2.D0/X)
      BESSK0=(DEXP(-X)/DSQRT(X))*(Q1+Y*(Q2+Y*(Q3+
     *  Y*(Q4+Y*(Q5+Y*(Q6+Y*Q7))))))
      ENDIF
```

```
      RETURN
      END

!    ****************************************************
!    *                                                  *
!    *            FUNCTION BESSI0                       *
!    *                                                  *
!    *       FROM BARLOW AND MOENCH (1999)              *
!    *                                                  *
!    ****************************************************
!
!     FUNCTION BESSI0 CALCULATES THE ZERO-ORDER MODIFIED BESSEL FUNCTION
!     OF THE FIRST KIND. SOURCE: PRESS AND OTHERS (1992).
      DOUBLE PRECISION FUNCTION BESSI0(X)
      DOUBLE PRECISION X,Y,AX,P1,P2,P3,P4,P5,P6,P7,
     *    Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9
      DATA P1,P2,P3,P4,P5,P6,P7/1.0D0,3.5156229D0,3.0899424D0,1.2067492D
     *    0,
     *    0.2659732D0,0.360768D-1,0.45813D-2/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9/0.39894228D0,0.1328592D-1,
     *    0.225319D-2,-0.157565D-2,0.916281D-2,-0.2057706D-1,
     *    0.2635537D-1,-0.1647633D-1,0.392377D-2/
      IF (DABS(X).LT.3.75D0) THEN
      Y=(X/3.75D0)**2
      BESSI0=P1+Y*(P2+Y*(P3+Y*(P4+Y*(P5+Y*(P6+Y*P7)))))
      ELSE
      AX=DABS(X)
      Y=3.75D0/AX
      BESSI0=(DEXP(AX)/DSQRT(AX))*(Q1+Y*(Q2+Y*(Q3+Y*(Q4
     *    +Y*(Q5+Y*(Q6+Y*(Q7+Y*(Q8+Y*Q9))))))))
      ENDIF
      RETURN
      END

!    ****************************************************
!    *                                                  *
!    *            FUNCTION BESSK1                       *
!    *                                                  *
!    *       FROM BARLOW AND MOENCH (1999)              *
!    *                                                  *
!    ****************************************************
!
!     FUNCTION BESSK1 CALCULATES THE FIRST-ORDER MODIFIED BESSEL FUNCTION
!     OF THE SECOND KIND. SOURCE: PRESS AND OTHERS (1992).
      DOUBLE PRECISION FUNCTION BESSK1(X)
      DOUBLE PRECISION X,Y,P1,P2,P3,P4,P5,P6,P7,
     *    Q1,Q2,Q3,Q4,Q5,Q6,Q7,BESSI1
      DATA P1,P2,P3,P4,P5,P6,P7/1.0D0,0.15443144D0,-0.67278579D0,
     *    -0.18156897D0,-0.1919402D-1,-0.110404D-2,-0.4686D-4/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7/1.25331414D0,0.23498619D0,-0.3655620D-1,
     *    0.1504268D-1,-0.780353D-2,0.325614D-2,-0.68245D-3/
      IF (X.LE.2.0) THEN
      Y=X*X/4.0
      BESSK1=(LOG(X/2.0)*BESSI1(X))+(1.0/X)*(P1+Y*(P2+
```

```
*     Y*(P3+Y*(P4+Y*(P5+Y*(P6+Y*P7))))))
      ELSE
      Y=2.0/X
      BESSK1=(EXP(-X)/SQRT(X))*(Q1+Y*(Q2+Y*(Q3+
*     Y*(Q4+Y*(Q5+Y*(Q6+Y*Q7))))))
      ENDIF
      RETURN
      END


!    **************************************************
!    *                                                *
!    *               FUNCTION BESSI1                  *
!    *                                                *
!    *         FROM BARLOW AND MOENCH (1999)          *
!    *                                                *
!    **************************************************
!
!     FUNCTION BESSI1 CALCULATES THE FIRST-ORDER MODIFIED BESSEL FUNCTION
!     OF THE FIRST KIND. SOURCE: PRESS AND OTHERS (1992).
      DOUBLE PRECISION FUNCTION BESSI1(X)
      DOUBLE PRECISION X,Y,AX,P1,P2,P3,P4,P5,P6,P7,
*     Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9
      DATA P1,P2,P3,P4,P5,P6,P7/0.5D0,0.87890594D0,0.51498869D0,
*     0.15084934D0,0.2658733D-1,0.301532D-2,0.32411D-3/
      DATA Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9/0.39894228D0,-0.3988024D-1,
*     -0.362018D-2,0.163801D-2,-0.1031555D-1,0.2282967D-1,
*     -0.2895312D-1,0.1787654D-1,-0.420059D-2/
      IF (ABS(X).LT.3.75) THEN
      Y=(X/3.75)**2
      BESSI1=X*(P1+Y*(P2+Y*(P3+Y*(P4+Y*(P5+Y*(P6+Y*P7))))))
      ELSE
      AX=ABS(X)
      Y=3.75/AX
      BESSI1=(EXP(AX)/SQRT(AX))*(Q1+Y*(Q2+Y*(Q3+Y*(Q4+
*     Y*(Q5+Y*(Q6+Y*(Q7+Y*(Q8+Y*Q9))))))))
      ENDIF
      RETURN
      END
```

APPENDIX B

Sample Input and Output Files for STRMAQ

Appendix B.1 – Sample Input File

```
SUBMISSION TO GROUND WATER (September 2001)
HUNT
WATER TABLE                                          AQTYPE
 30.0E0 100.0E0  10.0E0  1.0E-4  0.2E0               BB HKR HKZ SS SY
 0    0                                              IDRA  NALPHA
 1.0E9                                               ALPHA(I)
 3.0E2  5      3                                     TLAST  NLC  NOX
 1.0E-10  0.0E0  0     30     8                      RERRNR RERRSUM NMAX NTMS NS
 1  0                                                IPWS IPWD
 1.0E4  0.10E0   0.10E0  0.0E0  30.0E0  0.0E0        QQ RW RC ZPD ZPL SW
 50.0E0  1.0E0  1.0E0  10.0E0                        PWSTRM SBKSB SBM STRMW
 1                                                   NOBWC
 2  0                                                IOWS IDPR
 10.0E0 0.0E0 0.0E0  0.0E0  10.0E0  0.0E0  0.0E0     OBSWX OBSWY Z1 Z2 ZP RP XLL
```

Appendix B.2 – Sample Output File

```
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
~                                                             ~
~                    ~~~~~ STRMAQ ~~~~~                       ~
~                        VERSION 1.01                         ~
~                  G.A. Fox and D.S. Durnford                 ~
~               Department of Civil Engineering               ~
~                  Colorado State University                  ~
~                                                             ~
~     For Further Information: gfox@engr.colostate.edu        ~
~                 Program Developed From WTAQ                 ~
~                   Barlow and Moench (1999)                  ~
~                                                             ~
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~


DATA SET:          SUBMISSION TO GROUND WATER (September 2001)
SOLVER:            HUNT (1999)
AQUIFER TYPE:      WATER-TABLE AQUIFER


~~~~~~~~ AQUIFER PROPERTIES ~~~~~~~~

 SATURATED THICKNESS (BB):                          30.000
 HORIZONTAL HYDRAULIC CONDUCTIVITY (HKR):           100.000
 VERTICAL HYDRAULIC CONDUCTIVITY (HKZ):             10.000
 HKZ/HKR (XKD):                                     .100
 TRANSMISSIVITY:                                    .300D+04
 SPECIFIC STORAGE (SS):                             .100D-03
 SPECIFIC YIELD (SY):                               .200D+00
 CALCULATED STORATIVITY:                            .300D-02
 SIGMA:                                             .015
 DRAINAGE AT WATER TABLE (IDRA):                    0 (Instantaneous)


~~~~~~~~ SOLUTION VARIABLES ~~~~~~~~

LARGEST VALUE      NUMBER OF       DRAWDOWN CALCULATIONS
OF TIME (TLAST)  LOG CYCLES (NLC)  PER LOG CYCLE (NOX)
-------------------------------------------------------------------------------
 300.000                5                   3


RELATIVE ERROR FOR NEWTON-RHAPSON ITERATION:                  .100E-09
RELATIVE ERROR FOR FINITE SUMMATIONS:                         .000E+00
MAXIMUM NUMBER OF TERMS FOR FINITE SUMMATIONS (CONFINED):   0
MAXIMUM NUMBER OF TERMS FOR FINITE SUMMATIONS (UNCONFINED): 30
NUMBER OF STEHFEST TERMS:                                     8
```

~~~~~~~~ PUMPED-WELL CHARACTERISTICS ~~~~~~~~

WELL-DIAMETER TYPE (IPWD):	0 (infinitesimal diameter)
SCREENED INTERVAL (IPWS):	1 (fully penetrating)
PUMPING RATE OF WELL (QQ):	10000.000

WELL RADIUS	SCREENED INTERVAL ZPD	ZPL	WELL BORE STORAGE	WELL BORE SKIN
.10	.00	30.00	.00	.00

~~~~~~~~ STREAM INFORMATION ~~~~~~~~

DISTANCE FROM PUMPED-WELL TO STREAM (PWSTRM):	50.000
STREAMBED HYDRAULIC CONDUCTIVITY (SBKSB):	1.00000
STREAM WIDTH (STRMW):	10.000
STREAMBED THICKNESS (SBM):	1.000
CALCULATED STREAMBED CONDUCTANCE (SLAMBDA):	10.00000

~~~~~~~~ OBSERVATION-WELL INFORMATION ~~~~~~~~

OBSERVATION PIEZOMETER

DISTANCE FROM CENTER OF PUMPED WELL	ZP	DELAYED RESPONSE FACTOR
40.000	10.000	.000

~~~~~~~~ CALCULATED DRAWDOWN ~~~~~~~~

TIME	DRAWDOWN
.0139248	.2661
.0300000	.2841
.0646330	.3197
.1392477	.3870
.3000000	.4976
.6463304	.6440
1.3924767	.7989
3.0000000	.9387
6.4633041	1.0533
13.9247665	1.1402
30.0000000	1.2007
64.6330407	1.2390
139.2476650	1.2611
300.0	1.2728

Appendix B.3 – STRMAQ Readme.TXT File

~~~~~~~~~~ STRMAQ ~~~~~~~~~~

Version 1.01
G.A. Fox and D.S. Durnford
Department of Civil Engineering
Colorado State University

1. Description of STRMAQ

STRMAQ is an analytical model for stream/aquifer interaction with complex flow scenarios in confined and unconfined aquifers. The analytical model incorporates Hunt's (1999) solution for stream/aquifer interaction with analytical solutions of Dougherty and Babu (1984) and Moench (1997). STRMAQ accounts for pumping well partial penetration, well-skin effects, and well-bore storage. Incorporation of the solutions is performed through the use of analogous well functions. The computer program makes use of the Laplace-transform solutions of the complex flow models provided in the computer program WTAQ, developed by Barlow and Moench (1999). STRMAQ is written in FORTRAN and is constructed around the foundation of WTAQ (Barlow and Moench, 1999).

2. Required Files for STRMAQ

Simulation Title, Input File name, and Output File name are specified in the primary data file (Files.txt). Input File name and Output File name should begin with the "STRMAQ" designation:

Input File: STRMAQ*****.txt
Output File: STRMAQ*****.txt

This file should remain as "Files.txt" in the same directory as the executable file (STRMAQ.exe). STRMAQ reads "Files.txt" and opens the specified input and output files. Input File and Output File names must be less than 70 characters in length. A sample input file is included (STRMAQIN.txt). Input files must be organized in the same manner as this sample file (i.e., space delimited). The program writes input data, calculates parameters, and simulates drawdown at a single observation well/piezometer to the Output File.

* NOTE: If the program fails to execute, check the Output File for possible errors in data input.
~~~~~~~~~~

Appendix B.3 - Continued

3. Definition of Variables

STRMAQ uses equivalent variables as used in WTAQ, whenever possible (Barlow and Moench, 1999). All caps are required for the specification of SOLTYPE (HUNT) and AQTYPE (CONFINED or WATER TABLE). Note that solution parameters (RERRNR, RERRSUM, NMAX, NTMS, and NS) are dependent on the type of aquifer. A consistent set of units must be used for all variables.

IFNAME	Input File Name (Less than 70 Characters)
OFNAME	Output File Name (Less than 70 Characters)
SOLTYPE	Type of Solution Methodology (In current version, solver type must be set to HUNT)
AQTYPE	Type of Aquifer (CONFINED or WATER TABLE)
BB	Aquifer Saturated Thickness (Real)
HKR	Horizontal Hydraulic Conductivity (Real)
HKZ	Vertical Hydraulic Conductivity (Real)
SS	Specific Storage (Real)
AT	Aquifer Transmissivity (Real)
XKD	Ratio of Vertical to Horizontal Hydraulic Conductivity (Real)
ASC	Aquifer Storage Coefficient (Real)
SIGMA	Ratio of Storativity to Specific Yield (Real)
IDRA	Type of Drainage at Water Table (Integer: 0-Instantaneous, 1-Delayed)
NALPHA	Number of Drainage Constants (Integer: If IDRA=0 Use 0, If IDRA=1 Must be Less Than or Equal to 5)
ALPHA(I)	NALPHA Empirical Drainage Constants (Real: Use 1.0E9 if NALPHA =0) - See Barlow and Moench (1999)
TLAST	Largest Value of Time (Real)
NLC	Number of Logarithmic Cycles (Integer)
NOX	Number of Equally Spaced Times Per Logarithmic Cycle (Integer)
RERRNR	Relative Error for Newton-Rhapson Iteration (Real: If CONFINED Use 0.0E0, If WATER TABLE SUGGESTED 1.0E-10)
RERRSUM	Relative Error for Finite Summations (Real: If CONFINED Suggested is 1.0D-07, If WATER TABLE Use 0.0E0)
NMAX	Maximum Number of Terms Permitted in Finite Summations (Real: If CONFINED Suggested is 200, If WATER TABLE Use 0)
NTMS	Factor Used to Determine Number of Terms in Finite Summations (If CONFINED Use 0, If WATER TABLE Suggested is 30)
NS	Number of Terms Used in Stehfest Algorithm (Integer: Suggested is 8-12)
IPWS	Penentration of Pumped Well (Integer: 0-Partial, 1-Full)
IPWD	Model Well as Line Source or Finite Diameter (Integer: 0-Infinitesimal, 1-Finite)
PWSTRM	Distance From Stream to Pumping Well (Real)
QQ	Pumping Rate of Well (Real)

RW	Radius of Pumped Well Screen (Real)
RC	Inside Radius of Pumped Well (Real)
ZPD	Depth Below Top of Aquifer or Initial Water Table to Top of Screened Interval (Real)
ZPL	Depth Below Top of Aquifer or Initial Water Table to Bottom of Screened Interval (Real)
SW	Well-bore Skin Parameter (Real)
RWD	Ratio of Radius of Well to Saturated Thickness (Real)
BETAW	Dimensionless Parameter (Real)
WD	Well-Bore Storage Parameter (Real)
XDD	Dimensionless Distance of Top Screen to Saturated Thickness (Real)
XLD	Dimensionless Distance of Bottom Screen to Saturated Thickness (Real)
SBKSB	Streambed Hydraulic Conductivity (Real)
STRMW	Stream Width (Real)
SBM	Streambed Thickness (Real)
SLAMBDA	Streambed Conductance (Real)
NOBWC	Number of Observation Wells or Piezometers (Integer: In Current Version, NOBWC must be set equal to 1)
IOWS	Type of Observation Well/Piezometer (Integer: IOWS=0 for Partially Penetrating Observation Well, IOWS= 1 for Fully Penetrating Observation Well, IOWS=2 for Observation Piezometer)
IDPR	Type of Drainage at Observation Well/Piezometer (Integer: 0-Instantaneous, 1-Delayed)
OBWX	X-Coordinate Distance Between Pumped-Well and Observation Well/Piezometer (Real)
OBWY	Y-Coordinate Distance Between Pumped-Well and Observation Well/Piezometer (Real)
Z1	Depth Below Top of Aquifer or Water-Table to Top of Screened Interval of Observation Well (Real: Use Only if IOWS=0 or 1, Enter 0.0E0 if IOWS=1)
Z2	Depth Below Top of Aquifer or Water-Table to Bottom of Screened Interval of Observation Well (Real: Use Only if IOWS=0 or 1, Enter 0.0E0 if IOWS=1)
ZP	Depth Below Top of Aquifer or Water-Table to Center of Piezometer (Real: Use Only if IOWS=2, Enter 0.0E0 if IOWS=0 or 1)
RP	Inside Radius of Observation Well (Real: Use 0.0E0 if IDPR=0)
XLL	Length of Screened Interval of Observation Well (Real: Enter 0.0E0 if IDPR=0)

4. Contact Information

No warranty, expressed or implied, is made by the developers as to the accuracy of the program. Please report problems and direct suggestions and/or comments to G.A. Fox, A205B Engineering, Department of Civil Engineering, Colorado State University (gfox@engr.colostate.edu).

5. References

Barlow, P.M. and A.F. Moench. 1999. WTAQ: A computer program for calculating drawdowns and estimating hydraulic properties for confined and water-table aquifers. U.S. Geological Survey Water-Resources Investigations Report 99-4225, 74 p.

Dougherty, D.E. and D.K. Babu. 1984. Flow to a partially penetrating well in a double-porosity reservoir. Water Resources Research 20(8): 1116-1122.

Hunt, B. 1999. Unsteady stream depletion from ground water pumping. Ground Water 37(1): 98-102.

Moench, A.F. 1997. Flow to a well of finite diameter in a homogeneous, anisotropic water table aquifer. Water Resources Research 33(6): 1397-1407.

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APPENDIX C

Modified RIVER Packages for MODFLOW

Appendix C.1 – RIV_AC (Regimes A & C)

```
**** UPDATED RIVER PACKAGE
**** RIV5FM and RIV5BD
**** NOVEMBER 2002
**** UNSATURATED FLOW: ONLY REGIMES A AND C

      SUBROUTINE RIV5FM(NRIVER,MXRIVR,RIVR,HNEW,HCOF,RHS,IBOUND,
     1               NCOL,NROW,NLAY,NRIVVL)
C
C-----VERSION 0950 16JULY1992 RIV5FM
C     ******************************************************************
C     ADD RIVER TERMS TO RHS AND HCOF
C     ******************************************************************
C
C     SPECIFICATIONS:
C     ------------------------------------------------------------------
      DOUBLE PRECISION HNEW,RRBOT
      DIMENSION RIVR(NRIVVL,MXRIVR),HNEW(NCOL,NROW,NLAY),
     1        HCOF(NCOL,NROW,NLAY),RHS(NCOL,NROW,NLAY),
     2        IBOUND(NCOL,NROW,NLAY)
C     ------------------------------------------------------------------
C
C
C1------IF NRIVER<=0 THERE ARE NO RIVERS. RETURN.
      IF(NRIVER.LE.0)RETURN
C
C2------PROCESS EACH CELL IN THE RIVER LIST.
      DO 100 L=1,NRIVER
C
C3------GET COLUMN, ROW, AND LAYER OF CELL CONTAINING REACH.
      IL=RIVR(1,L)
      IR=RIVR(2,L)
      IC=RIVR(3,L)
C
C4------IF THE CELL IS EXTERNAL SKIP IT.
      IF(IBOUND(IC,IR,IL).LE.0)GO TO 100
C
C5------SINCE THE CELL IS INTERNAL GET THE RIVER DATA.
      HRIV=RIVR(4,L)
      CRIV=RIVR(5,L)
      RBOT=RIVR(6,L)
            sbksb=rivr(7,l)
            sbm=rivr(8,l)
            hypk=rivr(9,l)
            eph=rivr(10,l)
            psdi=rivr(11,l)
            eta=nint(2.0+3.0*psdi)
      RRBOT=RBOT

C
C6------COMPARE AQUIFER HEAD TO BOTTOM OF STREAM BED.
      IF(HNEW(IC,IR,IL).LE.RRBOT) GOTO 96
```

```
C
C7------SINCE HEAD>BOTTOM ADD TERMS TO RHS AND HCOF.
    RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*HRIV
    HCOF(IC,IR,IL)=HCOF(IC,IR,IL)-CRIV
    GOTO 100
C
C8------SINCE HEAD<BOTTOM ADD TERM ONLY TO RHS.

****************************************************************************
C8GAF - account for unsaturated flow
! 1:  Solve for qmax and hcu
  96 conv=1.0E10
        convold=1.0E10
        hcu=eph
        do
                hcu=hcu+1.0D-3
                conv=abs(sbksb*(1+(hcu+abs(HRIV-RRBOT)-sbm)/sbm)-
        1                   hypk*(eph/hcu)**eta)
                if (conv>convold) then
                        hcu=hcu-2.0D-3
                        convold2=1.0D10
                        do
                                hcu=hcu+1.0D-5
                                conv=abs(sbksb*(1+(hcu+(abs(HRIV-RRBOT)-sbm))/sbm)-
        1                                   hypk*(eph/hcu)**eta)
                                if (conv>convold2) then
                                        hcu=hcu-1.0D-5
                                        GOTO 8291
                                endif
                                convold2=conv
                        enddo
                endif
                convold=conv
        enddo
! 2: Check to see if head is above (rrbot-hcu)
8291    if(hnew(ic,ir,il).ge.(rrbot-hcu)) then
!       FLOW REMAINS SATURATED...
                RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*HRIV
                HCOF(IC,IR,IL)=HCOF(IC,IR,IL)-CRIV
        else if (hnew(ic,ir,il)<(rrbot-eph)) then
!       Regime C - Gravity-Driven Unsaturated Flow
                HHDIFF=(RBOT-hcu)-hnew(ic,ir,il)
                RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*(HRIV-HHDIFF)
                HCOF(IC,IR,IL)=HCOF(IC,IR,IL)-CRIV
        endif

 100 CONTINUE
C
C9------RETURN
    RETURN
    END
    SUBROUTINE RIV5BD(NRIVER,MXRIVR,RIVR,IBOUND,HNEW,
   1    NCOL,NROW,NLAY,DELT,VBVL,VBNM,MSUM,KSTP,KPER,IRIVCB,
   2    ICBCFL,BUFF,IOUT,PERTIM,TOTIM,NRIVVL,IRIVAL)
C-----VERSION 1422 05APRIL1993 RIV5BD
C    *****************************************************************
```

```
C     CALCULATE VOLUMETRIC BUDGET FOR RIVERS
C     ******************************************************************
C
C     SPECIFICATIONS:
C     ------------------------------------------------------------------
      CHARACTER*16 VBNM(MSUM),TEXT
      DOUBLE PRECISION HNEW,HHNEW,CHRIV,RRBOT,CCRIV,RATIN,RATOUT,RRATE
      DIMENSION RIVR(NRIVVL,MXRIVR),IBOUND(NCOL,NROW,NLAY),
     1          HNEW(NCOL,NROW,NLAY),VBVL(4,MSUM),BUFF(NCOL,NROW,NLAY)
C
      DATA TEXT /'   RIVER LEAKAGE'/
C     ------------------------------------------------------------------
C
C1------INITIALIZE CELL-BY-CELL FLOW TERM FLAG (IBD) AND
C1------ACCUMULATORS (RATIN AND RATOUT).
      ZERO=0.
      RATIN=ZERO
      RATOUT=ZERO
      IBD=0
      IF(IRIVCB.LT.0 .AND. ICBCFL.NE.0) IBD=-1
      IF(IRIVCB.GT.0) IBD=ICBCFL
      IBDLBL=0
C
C2------IF CELL-BY-CELL FLOWS WILL BE SAVED AS A LIST, WRITE HEADER.
      IF(IBD.EQ.2) CALL UBDSV2(KSTP,KPER,TEXT,IRIVCB,NCOL,NROW,NLAY,
     1          NRIVER,IOUT,DELT,PERTIM,TOTIM,IBOUND)
C
C3------CLEAR THE BUFFER.
      DO 50 IL=1,NLAY
      DO 50 IR=1,NROW
      DO 50 IC=1,NCOL
      BUFF(IC,IR,IL)=ZERO
50    CONTINUE
C
C4------IF NO REACHES, SKIP FLOW CALCULATIONS.
      IF(NRIVER.EQ.0)GO TO 200
C
C5------LOOP THROUGH EACH RIVER REACH CALCULATING FLOW.
      DO 100 L=1,NRIVER
C
C5A-----GET LAYER, ROW & COLUMN OF CELL CONTAINING REACH.
      IL=RIVR(1,L)
      IR=RIVR(2,L)
      IC=RIVR(3,L)
      RATE=ZERO
C
C5B-----IF CELL IS NO-FLOW OR CONSTANT-HEAD MOVE ON TO NEXT REACH.
      IF(IBOUND(IC,IR,IL).LE.0)GO TO 99
C
C5C-----GET RIVER PARAMETERS FROM RIVER LIST.
      HRIV=RIVR(4,L)
      CRIV=RIVR(5,L)
      RBOT=RIVR(6,L)
            sbksb=rivr(7,l)
            sbm=rivr(8,l)
            hypk=rivr(9,l)
```

```
          eph=rivr(10,l)
          psdi=rivr(11,l)
          eta=nint(2.0+3.0*psdi)
      RRBOT=RBOT
      HHNEW=HNEW(IC,IR,IL)
C
C5D-----COMPARE HEAD IN AQUIFER TO BOTTOM OF RIVERBED.
      IF(HHNEW.GT.(RRBOT-eph)) THEN
C
C5E-----AQUIFER HEAD > BOTTOM THEN RATE=CRIV*(HRIV-HNEW).
        CCRIV=CRIV
        CHRIV=CRIV*HRIV
        RRATE=CHRIV - CCRIV*HHNEW
        RATE=RRATE
      ELSE
C8GAF - account for unsaturated flow
!  1:Solve for qmax and hcu
                    conv=1.0E10
                    convold=1.0E10
                    hcu=eph
                    do
                              hcu=hcu+1.0D-3
                              conv=abs(sbksb*(1+(hcu+abs(HRIV-RRBOT)-sbm)/sbm)-
          1                             hypk*(eph/hcu)**eta)
                              if (conv>convold) then
                                        hcu=hcu-2.0D-3
                                        convold2=1.0D10
                                        do
                                                  hcu=hcu+1.0D-5
                                                  conv=abs(sbksb*(1+(hcu+(abs(HRIV-RRBOT)-sbm))/sbm)-
          1                                                 hypk*(eph/hcu)**eta)
                                                  if (conv>convold2) then
                                                            hcu=hcu-1.0D-5
                                                            GOTO 8292
                                                  endif
                                                  convold2=conv
                                        enddo
                              endif
                              convold=conv
                    enddo
8292                if (KPER.ne.KPEROLD) then
                                        write(*,*) '    Unsaturated Flow: hcu =',hcu
                              KPEROLD=KPER
                    endif
!         Regime C - Gravity-Driven Unsaturated Flow
                              CCRIV=CRIV
                              CHRIV=CRIV*HRIV
                              CHRIV2=CRIV*RRBOT
                              RATE=CHRIV+CRIV*hcu-CHRIV2
                              RRATE=RATE
      END IF
C
C5G-----PRINT THE INDIVIDUAL RATES IF REQUESTED(IRIVCB<0).
      IF(IBD.LT.0) THEN
        IF(IBDLBL.EQ.0) WRITE(IOUT,61) TEXT,KPER,KSTP
   61   FORMAT(1X,/1X,A,'   PERIOD',I3,'   STEP',I3)
```

```
      WRITE(IOUT,62) L,IL,IR,IC,RATE
   62  FORMAT(1X,'REACH',I4,'   LAYER',I3,'   ROW',I4,'   COL',I4,
    1      '  RATE',1PG15.6)
      IBDLBL=1
    END IF
C
C5H------ADD RATE TO BUFFER.
    BUFF(IC,IR,IL)=BUFF(IC,IR,IL)+RATE
C
C5I-----SEE IF FLOW IS INTO AQUIFER OR INTO RIVER.
    IF(RATE)94,99,96
C
C5J-----AQUIFER IS DISCHARGING TO RIVER SUBTRACT RATE FROM RATOUT.
  94 RATOUT=RATOUT-RRATE
    GO TO 99
C
C5K-----AQUIFER IS RECHARGED FROM RIVER; ADD RATE TO RATIN.
  96 RATIN=RATIN+RRATE
C
C5L-----IF SAVING CELL-BY-CELL FLOWS IN LIST, WRITE FLOW.  OR IF
C5L-----RETURNING THE FLOW IN THE RIVR ARRAY, COPY FLOW TO RIVR.
  99 IF(IBD.EQ.2) CALL UBDSVA(IRIVCB,NCOL,NROW,IC,IR,IL,RATE,IBOUND,
   1                NLAY)
    IF(IRIVAL.NE.0) RIVR(NRIVVL,L)=RATE
 100 CONTINUE
C
C6------IF CELL-BY-CELL FLOW WILL BE SAVED AS A 3-D ARRAY,
C6------CALL UBUDSV TO SAVE THEM.
    IF(IBD.EQ.1) CALL UBUDSV(KSTP,KPER,TEXT,IRIVCB,BUFF,NCOL,NROW,
   1                  NLAY,IOUT)
C
C7------MOVE RATES,VOLUMES & LABELS INTO ARRAYS FOR PRINTING.
 200 RIN=RATIN
    ROUT=RATOUT
    VBVL(3,MSUM)=RIN
    VBVL(4,MSUM)=ROUT
    VBVL(1,MSUM)=VBVL(1,MSUM)+RIN*DELT
    VBVL(2,MSUM)=VBVL(2,MSUM)+ROUT*DELT
    VBNM(MSUM)=TEXT
C
C8------INCREMENT BUDGET TERM COUNTER.
    MSUM=MSUM+1
C
C9------RETURN.
    RETURN
    END
```

Appendix C.2 – RIV_U (Regimes A, B & C)

```
**** UPDATED RIVER PACKAGE
**** RIV5FM and RIV5BD
**** NOVEMBER 2002
**** UNSATURATED FLOW: REGIMES A, B AND C

      SUBROUTINE RIV5FM(NRIVER,MXRIVR,RIVR,HNEW,HCOF,RHS,IBOUND,
     1            NCOL,NROW,NLAY,NRIVVL)
C
C-----VERSION 0950 16JULY1992 RIV5FM
C     ****************************************************************
C     ADD RIVER TERMS TO RHS AND HCOF
C     ****************************************************************
C
C     SPECIFICATIONS:
C     ----------------------------------------------------------------
      DOUBLE PRECISION HNEW,RRBOT
      DIMENSION RIVR(NRIVVL,MXRIVR),HNEW(NCOL,NROW,NLAY),
     1        HCOF(NCOL,NROW,NLAY),RHS(NCOL,NROW,NLAY),
     2        IBOUND(NCOL,NROW,NLAY)
C     ----------------------------------------------------------------
C
C
C1------IF NRIVER<=0 THERE ARE NO RIVERS. RETURN.
      IF(NRIVER.LE.0)RETURN
C
C2------PROCESS EACH CELL IN THE RIVER LIST.
      DO 100 L=1,NRIVER
C
C3------GET COLUMN, ROW, AND LAYER OF CELL CONTAINING REACH.
      IL=RIVR(1,L)
      IR=RIVR(2,L)
      IC=RIVR(3,L)
C
C4------IF THE CELL IS EXTERNAL SKIP IT.
      IF(IBOUND(IC,IR,IL).LE.0)GO TO 100
C
C5------SINCE THE CELL IS INTERNAL GET THE RIVER DATA.
      HRIV=RIVR(4,L)
      CRIV=RIVR(5,L)
      RBOT=RIVR(6,L)
           sbksb=rivr(7,l)
           sbm=rivr(8,l)
           hypk=rivr(9,l)
           eph=rivr(10,l)
           psdi=rivr(11,l)
           eta=nint(2.0+3.0*psdi)
      RRBOT=RBOT

C
C6------COMPARE AQUIFER HEAD TO BOTTOM OF STREAM BED.
      IF(HNEW(IC,IR,IL).LE.RRBOT) GOTO 96
C
```

```
C7------SINCE HEAD>BOTTOM ADD TERMS TO RHS AND HCOF.
    RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*HRIV
    HCOF(IC,IR,IL)=HCOF(IC,IR,IL)-CRIV
    GO TO 100
C
C8------SINCE HEAD<BOTTOM ADD TERM ONLY TO RHS.

*****************************************************************************
C8GAF - account for unsaturated flow
! 1:  Solve for qmax and hcu
  96 conv=1.0E10
        convold=1.0E10
        hcu=eph
        do
                hcu=hcu+1.0D-3
                conv=abs(sbksb*(1+(hcu+abs(HRIV-RRBOT)-sbm)/sbm)-
        1                       hypk*(eph/hcu)**eta)
                if (conv>convold) then
                        hcu=hcu-2.0D-3
                        convold2=1.0D10
                        do
                                hcu=hcu+1.0D-5
                                conv=abs(sbksb*(1+(hcu+(abs(HRIV-RRBOT)-sbm))/sbm)-
        1                                       hypk*(eph/hcu)**eta)
                                if (conv>convold2) then
                                        hcu=hcu-1.0D-5
                                        GOTO 8291
                                endif
                                convold2=conv
                        enddo
                endif
                convold=conv
        enddo
! 2: Check to see if head is above hcu
8291    if(hnew(ic,ir,il).ge.(rrbot-hcu)) then
!       Regime B - Unsaturated Flow
                hcl=0.0D0
                convold=1.0D10
                do
                        hcl=hcl+1.0D-5
                        if (hcl>hcu) then
                                hcl=hcu
                                EXIT
                        endif
                        FLHS=(HRIV-hnew(ic,ir,il))-abs(HRIV-RRBOT)
                        cont=1.0D10
                        FLP=0.0D0
                        n=-1
                        do while (cont>1.0D-5)
                                n=n+1
                                temp1=sbksb*(HRIV+SBM+hcl)/SBM
                                temp2=(hcl/eph)**eta
                                cont=((temp1*temp2/HypK)**n)/(1/eta+n)
                                FLP=FLP+cont
                        enddo
                        FRHS=hcl/eta*FLP
```

```
                    convergence=abs(FRHS-FLHS)
                    if (convergence<1.0D-4) EXIT
                    if (convergence>convold) EXIT
                    convold=convergence
                enddo
                HHCLD=(RBOT-hcl)-hnew(ic,ir,il)
                RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*HRIV-CRIV*HHCLD
                HCOF(IC,IR,IL)=HCOF(IC,IR,IL)-CRIV

!               RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*(HRIV)
!               HCOF(IC,IR,IL)=HCOF(IC,IR,IL)-CRIV
!               write(*,*) hnew(ic,ir,il), hcl
        else if (hnew(ic,ir,il)<(rrbot-hcu)) then
!       Regime C - Gravity-Driven Unsaturated Flow
                RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*(HRIV-RBOT+hcu)
!               write(*,*) hnew(ic,ir,il), hcu
        endif
!   RHS(IC,IR,IL)=RHS(IC,IR,IL)-CRIV*(HRIV-RBOT)
 100 CONTINUE
C
C9------RETURN
    RETURN
    END
    SUBROUTINE RIV5BD(NRIVER,MXRIVR,RIVR,IBOUND,HNEW,
   1    NCOL,NROW,NLAY,DELT,VBVL,VBNM,MSUM,KSTP,KPER,IRIVCB,
   2    ICBCFL,BUFF,IOUT,PERTIM,TOTIM,NRIVVL,IRIVAL)
C-----VERSION 1422 05APRIL1993 RIV5BD
C    ******************************************************************
C    CALCULATE VOLUMETRIC BUDGET FOR RIVERS
C    ******************************************************************
C
C    SPECIFICATIONS:
C    ------------------------------------------------------------------
    CHARACTER*16 VBNM(MSUM),TEXT
    DOUBLE PRECISION HNEW,HHNEW,CHRIV,RRBOT,CCRIV,RATIN,RATOUT,RRATE
    DIMENSION RIVR(NRIVVL,MXRIVR),IBOUND(NCOL,NROW,NLAY),
   1      HNEW(NCOL,NROW,NLAY),VBVL(4,MSUM),BUFF(NCOL,NROW,NLAY)
C
    DATA TEXT /'   RIVER LEAKAGE'/
C    ------------------------------------------------------------------
C
C1------INITIALIZE CELL-BY-CELL FLOW TERM FLAG (IBD) AND
C1------ACCUMULATORS (RATIN AND RATOUT).
    ZERO=0.
    RATIN=ZERO
    RATOUT=ZERO
    IBD=0
    IF(IRIVCB.LT.0 .AND. ICBCFL.NE.0) IBD=-1
    IF(IRIVCB.GT.0) IBD=ICBCFL
    IBDLBL=0
C
C2------IF CELL-BY-CELL FLOWS WILL BE SAVED AS A LIST, WRITE HEADER.
    IF(IBD.EQ.2) CALL UBDSV2(KSTP,KPER,TEXT,IRIVCB,NCOL,NROW,NLAY,
   1      NRIVER,IOUT,DELT,PERTIM,TOTIM,IBOUND)
C
C3------CLEAR THE BUFFER.
```

```
      DO 50 IL=1,NLAY
      DO 50 IR=1,NROW
      DO 50 IC=1,NCOL
      BUFF(IC,IR,IL)=ZERO
50    CONTINUE
C
C4------IF NO REACHES, SKIP FLOW CALCULATIONS.
      IF(NRIVER.EQ.0)GO TO 200
C
C5------LOOP THROUGH EACH RIVER REACH CALCULATING FLOW.
      DO 100 L=1,NRIVER
C
C5A-----GET LAYER, ROW & COLUMN OF CELL CONTAINING REACH.
      IL=RIVR(1,L)
      IR=RIVR(2,L)
      IC=RIVR(3,L)
      RATE=ZERO
C
C5B-----IF CELL IS NO-FLOW OR CONSTANT-HEAD MOVE ON TO NEXT REACH.
      IF(IBOUND(IC,IR,IL).LE.0)GO TO 99
C
C5C-----GET RIVER PARAMETERS FROM RIVER LIST.
      HRIV=RIVR(4,L)
      CRIV=RIVR(5,L)
      RBOT=RIVR(6,L)
            sbksb=rivr(7,l)
            sbm=rivr(8,l)
            hypk=rivr(9,l)
            eph=rivr(10,l)
            psdi=rivr(11,l)
            eta=nint(2.0+3.0*psdi)
      RRBOT=RBOT
      HHNEW=HNEW(IC,IR,IL)
C
C5D-----COMPARE HEAD IN AQUIFER TO BOTTOM OF RIVERBED.
      IF(HHNEW.GT.RRBOT) THEN
C
C5E-----AQUIFER HEAD > BOTTOM THEN RATE=CRIV*(HRIV-HNEW).
         CCRIV=CRIV
         CHRIV=CRIV*HRIV
         RRATE=CHRIV - CCRIV*HHNEW
         RATE=RRATE
C5F-----AQUIFER HEAD < BOTTOM THEN RATE=CRIV*(HRIV-RBOT).
      ELSE

C8GAF - account for unsaturated flow
!  1:Solve for qmax and hcu
                  conv=1.0E10
                  convold=1.0E10
                  hcu=eph
                  do
                        hcu=hcu+1.0D-3
                        conv=abs(sbksb*(1+(hcu+abs(HRIV-RRBOT)-sbm)/sbm)-
     1                        hypk*(eph/hcu)**eta)
                        if (conv>convold) then
                              hcu=hcu-2.0D-3
```

```
                              convold2=1.0D10
                              do
                                    hcu=hcu+1.0D-5
                                    conv=abs(sbksb*(1+(hcu+(abs(HRIV-RRBOT)-sbm))/sbm)-
     1                                    hypk*(eph/hcu)**eta)
                                    if (conv>convold2) then
                                          hcu=hcu-1.0D-5
                                          GOTO 8292
                                    endif
                                    convold2=conv
                              enddo
                        endif
                        convold=conv
                  enddo
! 2: Check to see if head is above hcu
8292              if (KPER.ne.KPEROLD) then
                        write(*,*) '    Unsaturated Flow: hcu =',hcu
                        KPEROLD=KPER
                  else

                  endif
                  if(hnew(ic,ir,il).ge.(rrbot-hcu)) then
!           Regime B - Unsaturated Flow
                        hcl=0.0D0
                        convold=1.0D10
                        do
                              hcl=hcl+1.0D-5
                              if (hcl>hcu) then
                                    hcl=hcu
                                    EXIT
                              endif
                              FLHS=(HRIV-hnew(ic,ir,il))-abs(HRIV-RRBOT)
                              cont=1.0D10
                              FLP=0.0D0
                              n=-1
                              do while (cont>1.0D-5)
                                    n=n+1
                                    temp1=sbksb*(HRIV+SBM+hcl)/SBM
                                    temp2=(hcl/eph)**eta
                                    cont=((temp1*temp2/HypK)**n)/(1/eta+n)
                                    FLP=FLP+cont
                              enddo
                              FRHS=hcl/eta*FLP
                              convergence=abs(FRHS-FLHS)
                              if (convergence<1.0D-4) EXIT
                              if (convergence>convold) EXIT
                              convold=convergence
                        enddo
                        CCRIV=CRIV
                        CHRIV=CRIV*HRIV
                        CHRIV2=CRIV*RRBOT
                        RATE=CHRIV+CCRIV*hcl-CHRIV2
                        RRATE=RATE
                  else if (hnew(ic,ir,il)<(rrbot-hcu)) then
!           Regime C - Gravity-Driven Unsaturated Flow
                        CCRIV=CRIV
```

```
                    CHRIV=CRIV*HRIV
                    CHRIV2=CRIV*RRBOT
                    RATE=CHRIV+CRIV*hcu-CHRIV2
                    RRATE=RATE
               endif
      END IF
C
C5G-----PRINT THE INDIVIDUAL RATES IF REQUESTED(IRIVCB<0).
      IF(IBD.LT.0) THEN
         IF(IBDLBL.EQ.0) WRITE(IOUT,61) TEXT,KPER,KSTP
   61    FORMAT(1X,/1X,A,'   PERIOD',I3,'   STEP',I3)
         WRITE(IOUT,62) L,IL,IR,IC,RATE
   62    FORMAT(1X,'REACH',I4,'   LAYER',I3,'   ROW',I4,'   COL',I4,
     1       '   RATE',1PG15.6)
         IBDLBL=1
      END IF
C
C5H------ADD RATE TO BUFFER.
      BUFF(IC,IR,IL)=BUFF(IC,IR,IL)+RATE
C
C5I-----SEE IF FLOW IS INTO AQUIFER OR INTO RIVER.
      IF(RATE)94,99,96
C
C5J-----AQUIFER IS DISCHARGING TO RIVER SUBTRACT RATE FROM RATOUT.
   94 RATOUT=RATOUT-RRATE
      GO TO 99
C
C5K-----AQUIFER IS RECHARGED FROM RIVER; ADD RATE TO RATIN.
   96 RATIN=RATIN+RRATE
C
C5L-----IF SAVING CELL-BY-CELL FLOWS IN LIST, WRITE FLOW.  OR IF
C5L-----RETURNING THE FLOW IN THE RIVR ARRAY, COPY FLOW TO RIVR.
   99 IF(IBD.EQ.2) CALL UBDSVA(IRIVCB,NCOL,NROW,IC,IR,IL,RATE,IBOUND,
     1                    NLAY)
      IF(IRIVAL.NE.0) RIVR(NRIVVL,L)=RATE
  100 CONTINUE
C
C6------IF CELL-BY-CELL FLOW WILL BE SAVED AS A 3-D ARRAY,
C6------CALL UBUDSV TO SAVE THEM.
      IF(IBD.EQ.1) CALL UBUDSV(KSTP,KPER,TEXT,IRIVCB,BUFF,NCOL,NROW,
     1                     NLAY,IOUT)
C
C7------MOVE RATES,VOLUMES & LABELS INTO ARRAYS FOR PRINTING.
  200 RIN=RATIN
      ROUT=RATOUT
      VBVL(3,MSUM)=RIN
      VBVL(4,MSUM)=ROUT
      VBVL(1,MSUM)=VBVL(1,MSUM)+RIN*DELT
      VBVL(2,MSUM)=VBVL(2,MSUM)+ROUT*DELT
      VBNM(MSUM)=TEXT
C
C8------INCREMENT BUDGET TERM COUNTER.
      MSUM=MSUM+1
C
C9------RETURN.
      RETURN
```