FLOW CHARACTERISTICS OF LOW WEIR STRUCTURES IN ALLUVIAL CHANNELS

by

J. D. Lawson and D. B. Simons

Experimental program sponsored by
National Science Foundation Grant No. 13391.
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Civil Engineering Department
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I. SYNOPSIS

The flow characteristics associated with low weir structures, with sloping upstream and downstream faces, built across alluvial streams have been investigated in a two feet wide, variable-slope laboratory flume.

Using natural silica sand of specific gravity 2.65 and 0.28 mm mean fall diameter and, later, expanded clay of specific gravity 1.78 and mean fall diameter of 0.37 mm, tests have been conducted to determine the behavior of each material and, associated with this, the variation in flow conditions that could be expected as a result of varying:

(1) flume slope

(2) roughness and shape of structure

(3) submergence

The two sediments have been used in an attempt to approach the problem of sediment selection for a model to simulate a prototype situation.

The effectiveness of three different sizes of triangular block energy dissipators attached to the downstream slope of the structure has been studied qualitatively.

II. INTRODUCTION

Considerable research has been carried out over the years on the discharge characteristics pertaining to a wide variety of weir shapes with crests at varying heights above a fixed upstream base.

For the case of a low weir structure built across a channel or river course in which bed material transport is a feature, less is known of the weir performance. What happens to the sediment on arrival at the structure? What accuracy can be expected in discharge measurement if the structure is to be used as a measuring device and, associated with these questions, what material should be used in a model to reproduce the sediment movements in a natural system?

Recent studies by Simons and Richardson (1962) have shown that adoption of a stage-discharge relationship for a control section in an alluvial river may give significant errors in flow estimation depending on bed configuration i.e., whether dune, ripple or plane bed conditions apply and, in the case of large dunes, whether the stage reading is taken at the crest, trough, or intermediate point of the dune. In the case of a flood passing a gaging station, therefore, it is likely that the stage-discharge relationship may vary throughout the hydrograph, and the final relationship be dependent on bed form which is determined by the intensity and duration of the flood flow.

The present experimental study has been planned having regard to a specific problem concerned with the design and calibration of low measuring weir structures for installation across the Rio Grande, Karaki (1960). In this respect, a broad-crested weir has been adopted with a minimum top width of 10 feet to allow vehicular transport across the structure at times of low river flow. Material availability and methods of construction suggest that sloping up and downstream faces on the structure are likely. Furthermore, a weir crest height of three feet above mean bed level has been suggested for the Rio Grande and this, too, has been adopted in deciding the scale and shape of model for the present study.

III. EXPERIMENTAL EQUIPMENT

The experiments have been conducted in a 60 feet long, 2 feet wide and 2 feet 6 inch deep, plastic-sided flume in the Hydraulic Laboratory at Colorado State University. The flume, pivotted at mid-length, may be tilted from 0 to 0.025 slopes by motorized screw jacks at both ends. Water

(7 c.f.s. maximum) is pumped in a closed circuit from a sump beneath the downstream end of the flume through 12 inch diameter pipe, containing a discharge control gate valve and orifice plate with associated water manometer for flow measurement, into a head box with appropriate wire and wood screens at the upstream end of the flume. A slatted gate across the downstream end of the flume controlled the downstream water level.

The weir structure, made from 5-ply wood and varnished, was placed approximately equidistant from the ends of the flume and sealed against the plastic sides with plasticene. The flume contained a volume of sediment sufficient to provide a 9 inches deep bed for the full length.

Rails along the top of the flume, parallel with the floor of the flume, support a carriage which carries a point gage (reading to 0.001 feet) and the transducer head of a Sonic Depth Sounder.

IV. MEASUREMENTS

Rates of flow in the flume (up to 7 c.f.s.) have been measured by differential water manometer connected to a calibrated orifice plate in the supply pipe.

Flume (and rail) slopes have been set from a pointer on the flume moving over a fixed scale, and checked by surveying methods.

Water and bed surface levels have been measured by point gage on the portable carriage and referred to weir crest level as datum.

Water surface level fluctuations (for constant flow) at the selected gaging point upstream from the structure have been measured by a transistor-operated surface follower in a well connected with the water in the flume at the gaging point. A pen recorded fluctuations of level to 0.005 feet on a 24 hour drum.

Variations in bed surface level at any point have been recorded by Sonic Depth Sounder, Simons, Richardson and Posakony (1961), connected to a Texas Instruments rectilinear strip-chart recorder with chart speeds of 12, 6, 3, 1.5 and 0.75 inches per minute or hour. The same equipment has been used for recording the bed surface configuration on the longitudinal center line of the flume by motorizing the carriage and towing it at constant speed. For convenience, the chart and carriage speeds were related to give a 10:1 distorted chart record. Microswitches on the flume provided reference time markers on the strip chart.

In an effort to obtain a profile flow pattern over the weir, the photographing of immiscible droplets through the side of the flume has been attempted. A mixture of benzene and carbon tetrachloride, adjusted in proportion to give a density the same as water in the flume and suitably colored with Sudan III red dye, has been injected, a 'pea-sized' droplet at a time, from a syringe at one inch increments in water depth. Each immiscible droplet has been photographed with an exposure time sufficient to trace the path over the structure. By double exposing, the complete flow pattern should be available on the one photograph. Insufficient time has prevented the satisfactory development of this flow visualization technique.

Photography has been used extensively in recording flow conditions associated with the various energy dissipating blocks used on the downstream face of the structure.

Although total load and suspended load sampling equipment are available in the laboratory, a study of sediment transport in the vicinity of the model has not been undertaken in the present investigation.

V. SIZE AND SHAPE OF MODEL STRUCTURE

A weir structure with a 10 feet wide flat crest, 2:1 upstream and 3:1 downstream slopes, and 3 feet high above mean bed level has been selected as a possibly suitable structure for the Rio Grande. At a later stage, a similar structure but with 3.1 upstream and 5:1 downstream slopes has been investigated.

To satisfy conditions of dynamic similarity, flume depth and discharge limitations, and yet obtain sediment transporting velocities, an undistorted model of scale 1:5 has been selected. Thus, a prototype discharge of 25 cubic feet per second per foot width of weir (considered maximum for calibration purposes) has been represented by 0.8 cubic feet per second in the model.

It has been convenient to commence tests using a 0.28 mm silica sand (sp.gr. 2.65) that has been studied as an alluvial bed material by Simons and Richardson (1961). They have shown that the bed form for this material varies from plane at Fr = 0.15 through ripples, dunes, transition, plane to antidunes at Fr = 1.0--1.3. For the depths applying in the present weir study, particle movement could be expected to commence at velocities of about 0.9 feet per second and dunes at 1.8 feet per second. The corresponding scaled prototype discharges would be far greater than those at which movement of the natural materials could be expected to occur. In this respect the 0.28 mm sand, though forming ripples and dunes at high model discharges, is not transported at dynamically similar flows. In an attempt to overcome this, a light-weight material has been substituted for the silica sand in later tests.

VI. PROCEDURE

Some 430 runs form the basis of this report. For each run, the general procedure involved recirculating a given discharge of water, with or without sediment (dependent on Q), in the flume at a given slope until equilibrium conditions were established. Slope selection was accomplished in a general sense. In any flow system where discharge, depth and slope can be varied, only two of the three variables can be considered as independent. In the flume study, the discharge was independent, the flume slope preset and the upstream bed screeded parallel to the flume base at the beginning of a run. As the bed configuration upstream from the structure developed, so the slope and depth adjusted to the new condition of bed roughness. Thus, for these experiments the slope and depth are a function of Q and the roughness which develops for that regime of flow. The downstream water surface slopes were controlled by adjusting the tail gate and, except for high degrees of submergence of the structure, had no influence on the upstream flow conditions.

The initial tests were run to determine suitable points for water surface level measurement. Stations 16 and 2, respectively 4 feet upstream and 14 feet downstream from the center line of the weir, were selected as gaging points for head measurement on the weir. These stations, the details of the structure, and the symbols used in the report are shown in Fig. 1. A negative h_2 indicates a free overfall spillway. The dotted lines indicate the modified weir shape and the three inch outside diameter pipe crest installed on the original full-line structure for the final tests.

VII. THEORY

The discharge coefficient C for the structures has been obtained from the basic formula

$$Q = C.\ L.\ H.^{3/2}$$

where Q = flume discharge in cubic feet per second

L = weir width = 1.978 feet

H = total head on the weir

$= h_{16} + V_{16}^2/2g$

V_{16} = mean velocity at Station 16 in feet per second

$= Q/1.978 \times d_{16}$

VIII. EXPERIMENTAL RESULTS

A. Effect of Surface Roughness of Weir on Discharge Coefficient

The majority of the tests was conducted with a varnished 5 plywood weir surface. To investigate the effect of roughness, however, one set of tests was carried out with a 20 mesh brass screen stretched tightly across the complete Structure A.

With a flume slope of 0.0011 and low tailwater levels throughout, upstream head readings were taken at each discharge with and without the brass screen attached. The additional roughness reduced the value of discharge coefficient by as much as ten percent. Fig. 2. The effectiveness with which large roughness elements (rock) can be used to control flow was demonstrated by Stepanich (1962). His structure consisted of a long flat chute with sufficient roughness to maintain subcritical flow on at least part of the structure for a limited range of discharge. The rock roughened structure creates sufficient turbulence to entrain the total bed material load and can therefore, be used as an effective location for measuring sediment discharge.

B. Weir Performance with Upstream Dunes

To determine the effect of dunes on weir performance, equilibrium conditions at several high rates of flow were established in the flume. As though unaware of the presence of the weir, the dunes advanced to the structure without shape modification and dispersed over it; thus providing, at times, an upstream bed level flush with the weir crest followed by a depression corresponding to the trough of the dune, see Fig. 3.

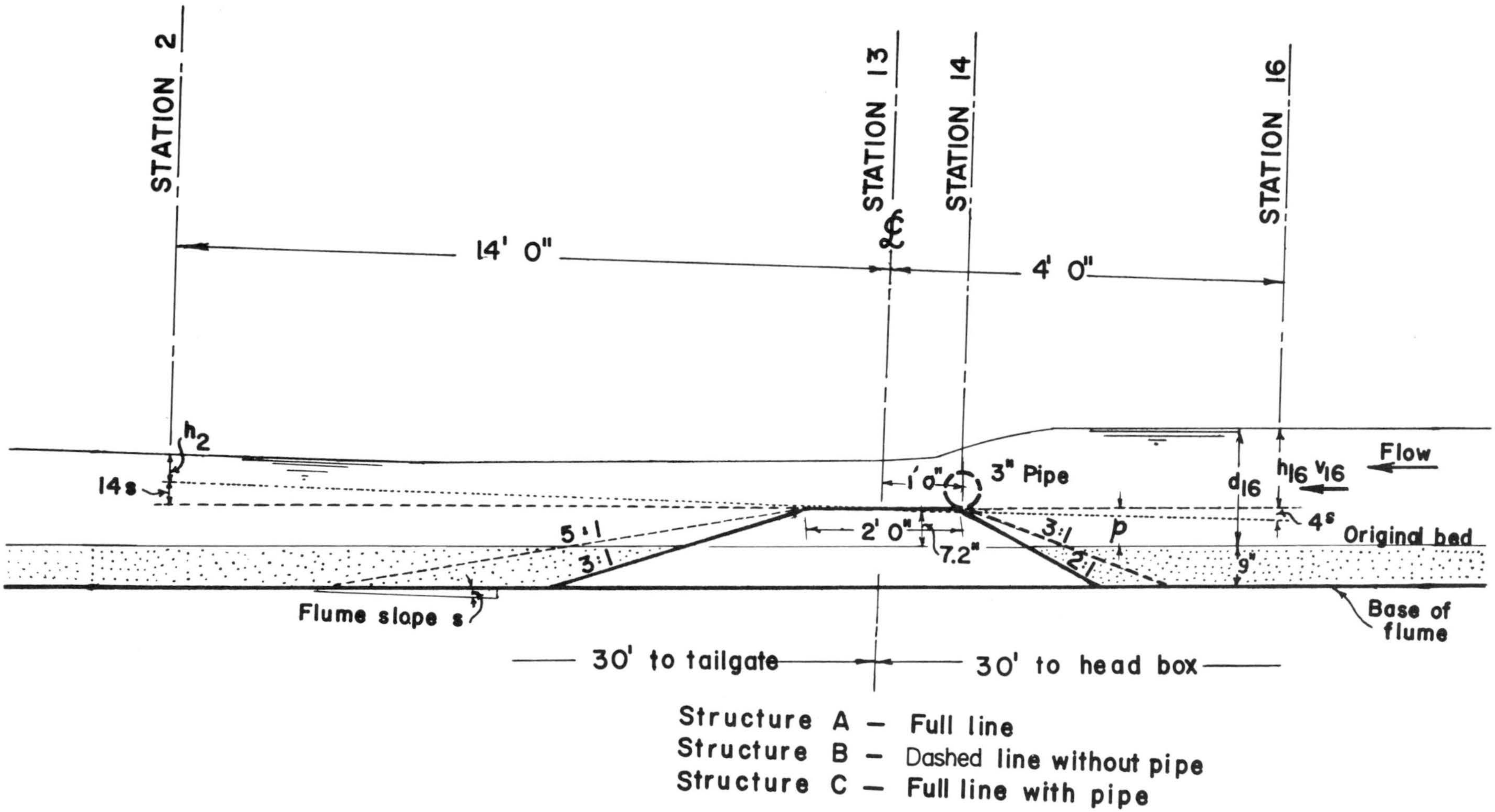


FIGURE 1 GENERAL ARRANGEMENT OF WEIR STRUCTURES

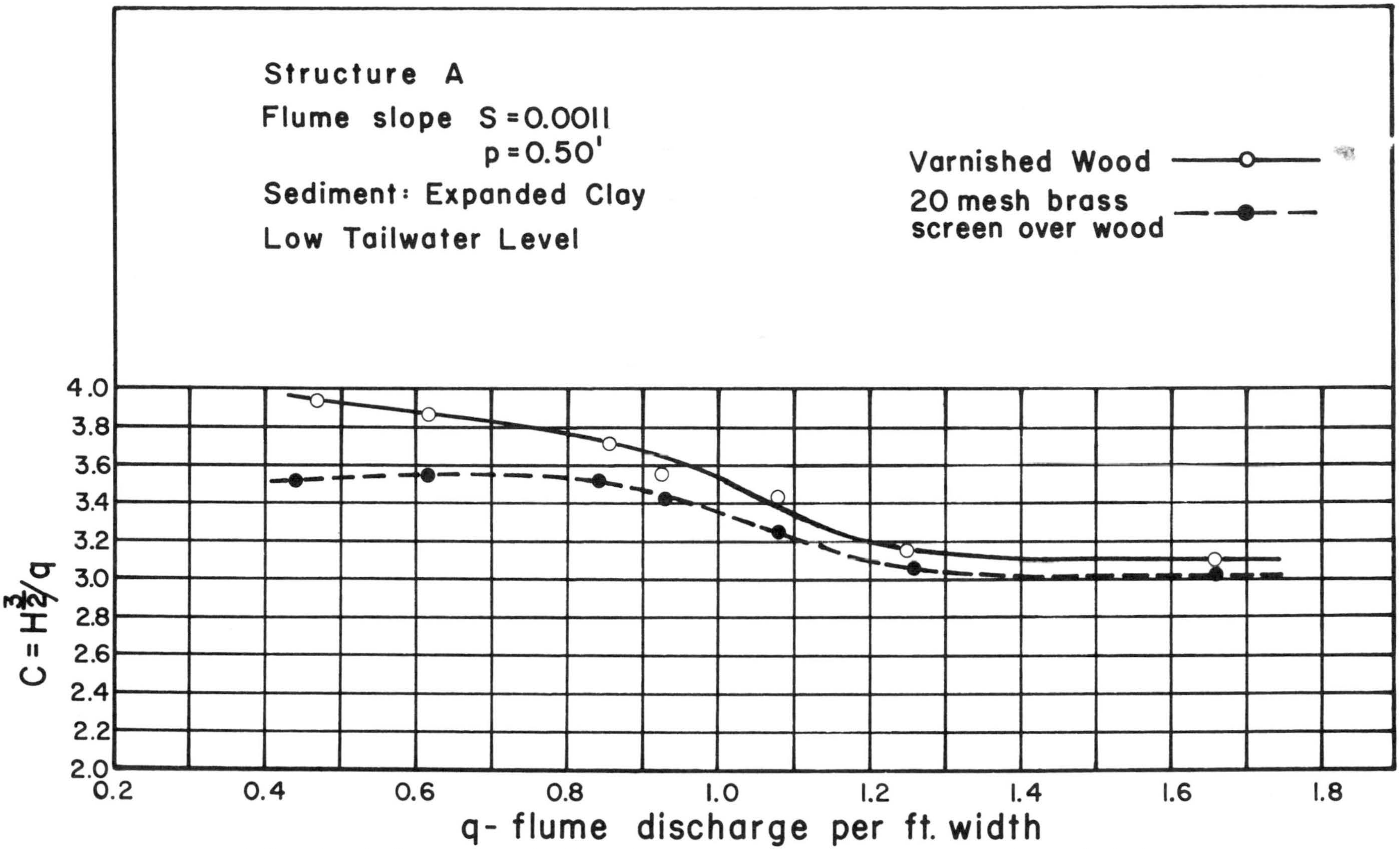


Fig. 2 Effect of Surface Roughness on Coefficient C

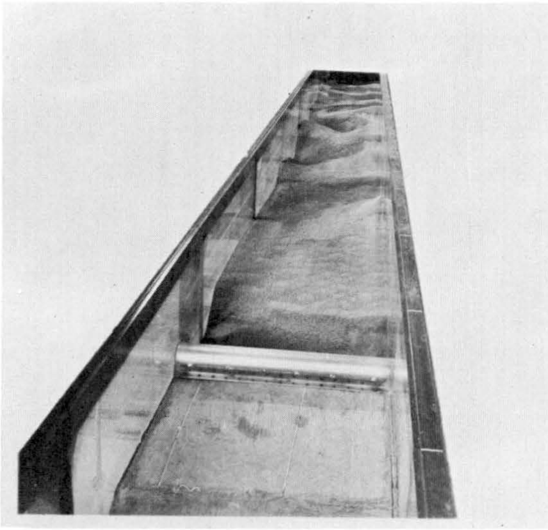


Fig. 3 Dune pattern upstream from Structure C

The water surface level fluctuations at Station 16, associated with the dune movement, were recorded by the surface follower operating over a 15 hour period, Fig. 4. The recorded variations were checked by taking 30 point gage readings at one minute intervals--the mean values being in close agreement by the two methods. It was shown, for the one constant discharge selected, that a $\pm$ 3.8 percent variation in measured head (corresponding to a $\pm$ 5.7 percent error in discharge) could be expected.

The fluctuation in bed level at Station 16 was also recorded by Sonic Depth Sounder, Fig. 5.

By establishing equilibrium conditions at several constant rates of flow under the condition of the dune configuration, it was shown, by calculation of downstream sand volumes, that the mean upstream bed level rose with increase in discharge.

C. Discharge Coefficient for the Structures

Most of the testing program has been devoted to a study of the discharge coefficient for:

(i) three different structure shapes:

Structure A - 2 feet wide broad crest weir with 2:1 and 3:1 slopes.

Structure B - 2 feet wide broad crest weir with 3:1 and 5:1 slopes.

Structure C - 3 inch O.D. pipe attached to the upstream edge of Structure A. (see Fig. 1).

(ii) the full range of discharge (7 c.f.s. maximum).

(iii) a wide range of approach velocities.

(iv) p values (height of structure above bed on upstream side) from 0 to 6 inches.

(v) a wide range of tailwater levels.

(vi) simulated flood hydrographs.

(vii) two different sediments - silica sand and expanded clay.

In general, it became apparent at an early stage that, due to the changing upstream depths resulting from sediment movement, it was important to include velocity of approach in discharge coefficient determination, if any reasonable accuracy in discharge measurement was to be achieved. In this report, the head on the weir is composed of the measured head plus the mean velocity head at Station 16.

A discussion of the factors listed above follows

1. Velocity of approach

Runs were carried out with approach velocities at station 16 ranging from about 1 to 4 ft/sec. The water surface slopes associated with these velocities ranged from 0.011 to 0.015. The flatter slope corresponded approximately to the bed slope at one of the proposed weir sites on the Rio Grande, Karaki (1960). Both sediments were used and free overfall conditions maintained at the weir in these experiments.

The upstream bed configuration varied from plane bed with no sediment movement at large p values, (see Fig. 1) small flow and flat slope, through the ripple and dune zone, to plane bed flush with weir crest (p= 0), high sediment discharge at large flow (including supercritical) and steep slopes.

Except at certain low flows, the discharge coefficient was constant for the range of conditions tested, indicating that slope is not an important variable, providing velocity of approach is considered, see Figs. 6 and 7.

2. p value

At the commencement of most tests a value of p was selected and the upstream bed of sediment screeded parallel to the base of the flume. As the sediment began to move so, in general, p varied.

However, in one set of tests using Structure A, silica sand and a flume slope of 0.011, values of p for every inch from 0 to 6 inches were set and maintained (if necessary by smoothing the bed before reading the water surface) and the discharge coefficient determined up to discharges as high as feasible without excess movement, see Figs. 8 and 9. In the case of p= 6", flume discharges up to 2.50 c.f.s. were possible without sediment movement.

The logarithmic plot (Fig. 8) shows relatively constant C at high Q , but great variability in

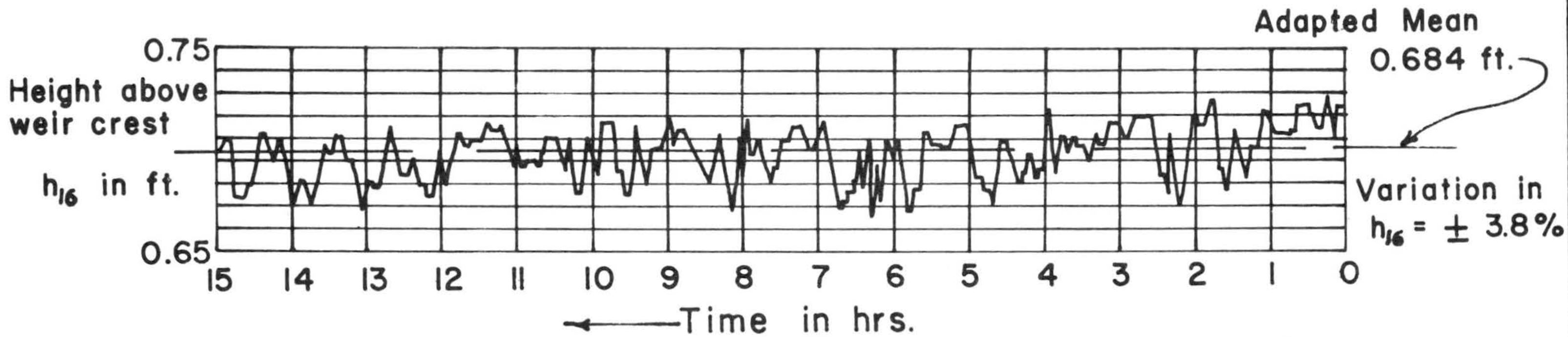


Fig. 4 Water Surface Level Fluctuations at Station 16 for Q = 4.10 cfs as Recorded by Surface Follower

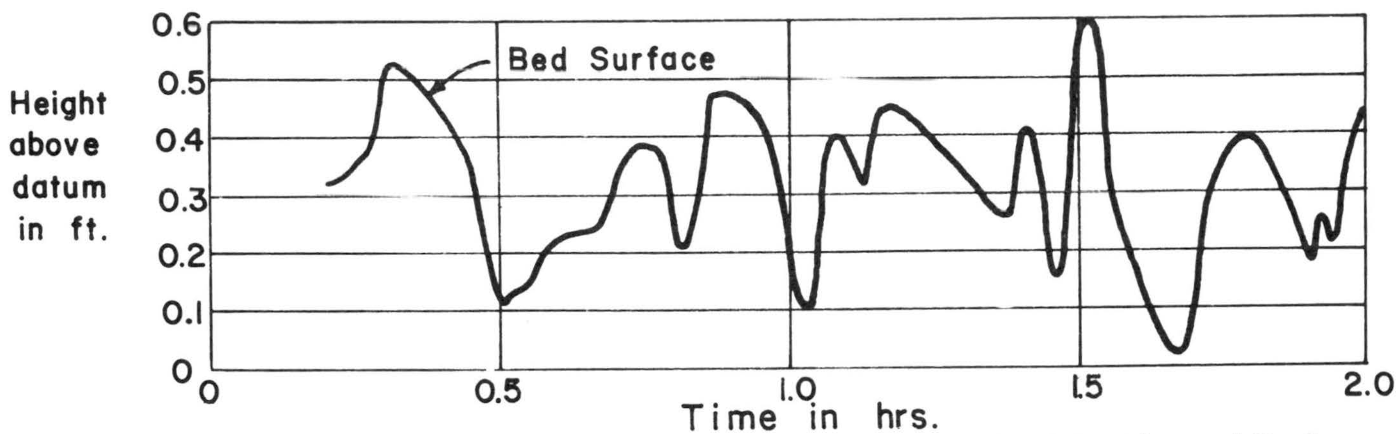


Fig. 5 Bed Level Fluctuations at Station 16 for Q = 4.10 cfs as Recorded by Sonic Depth Sounder

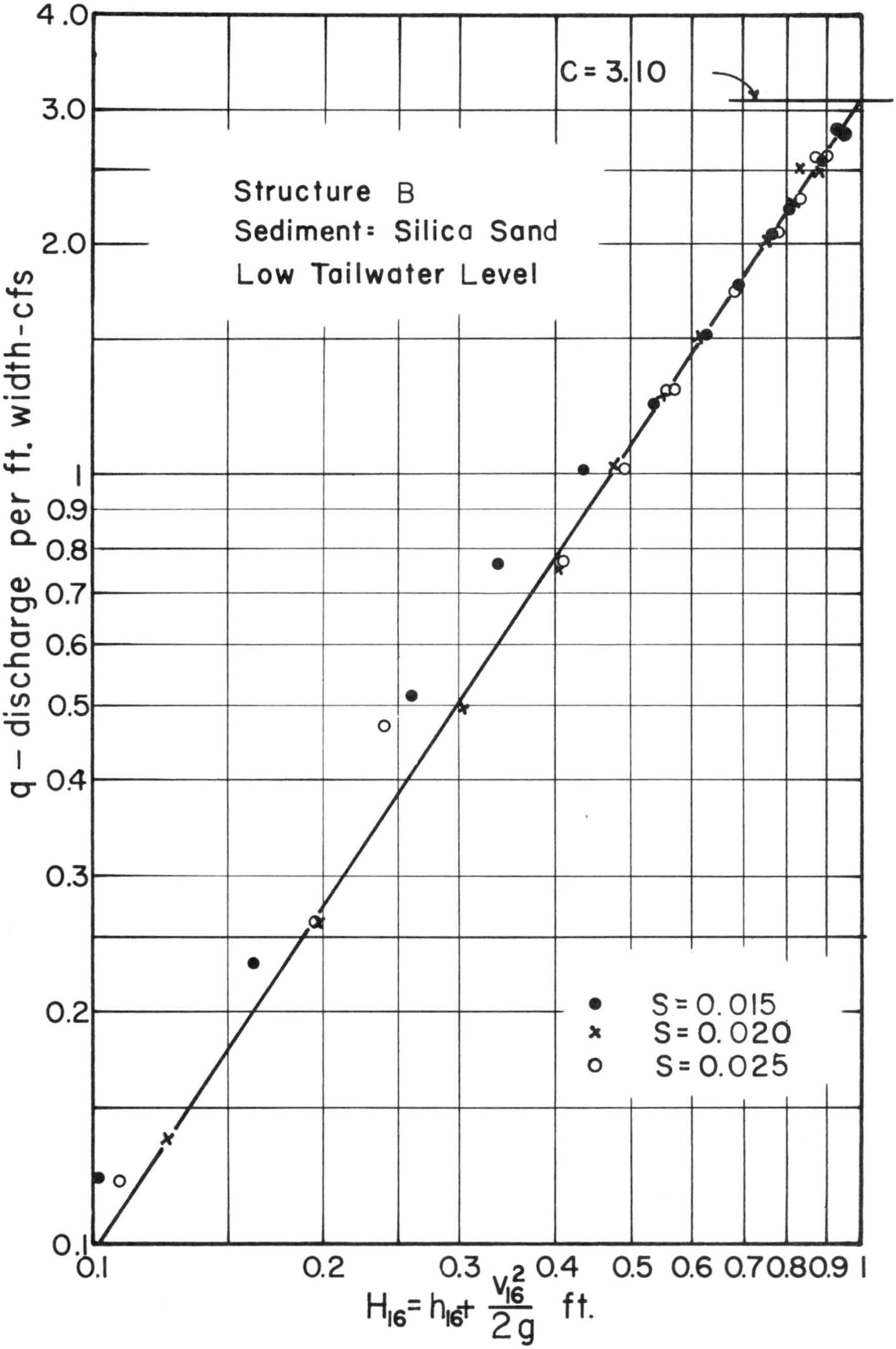


Fig. 6 Effect of Flume Slope S on Coefficient C

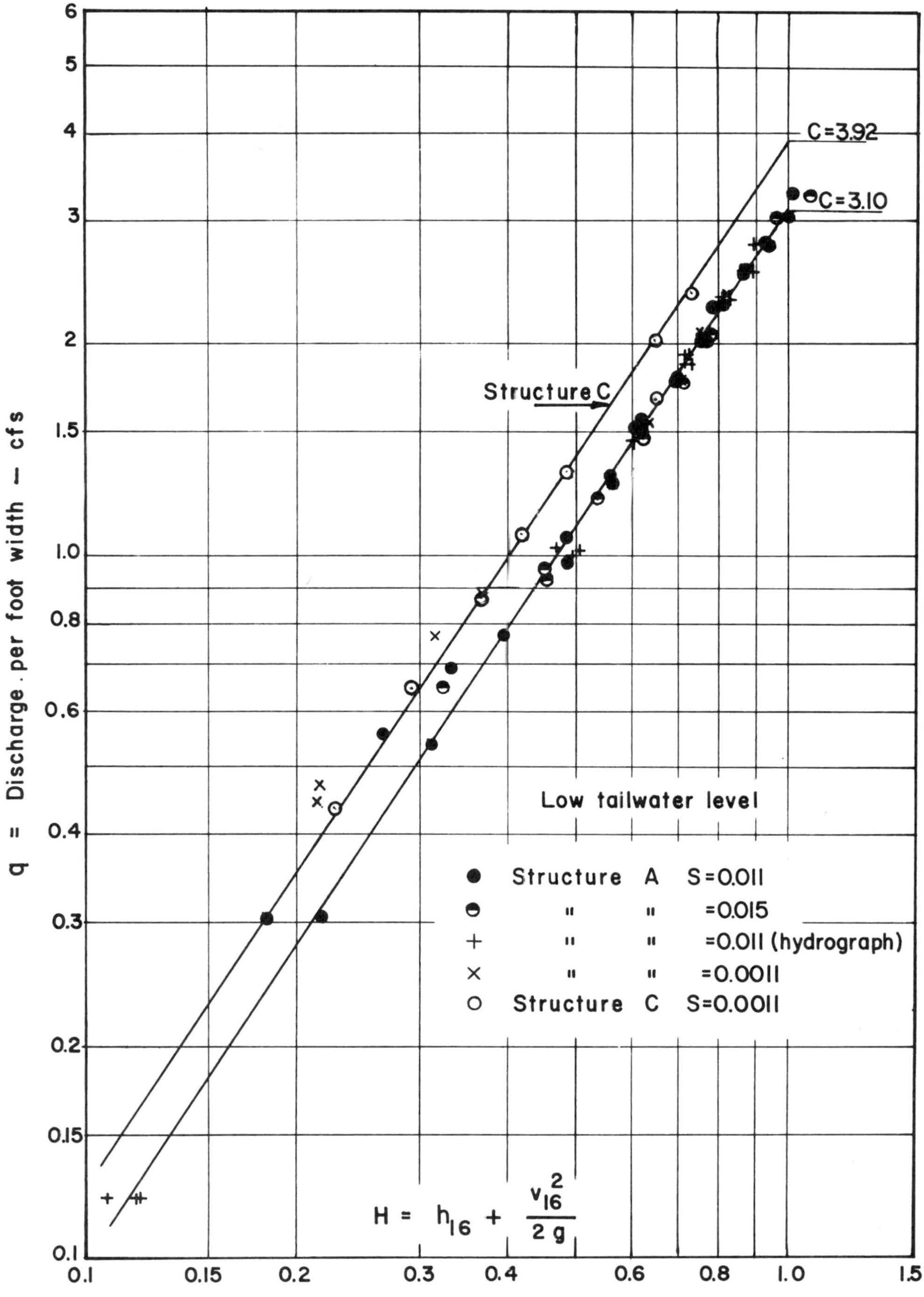


FIGURE 7 VALUES OF C USING EXPANDED CLAY SEDIMENT

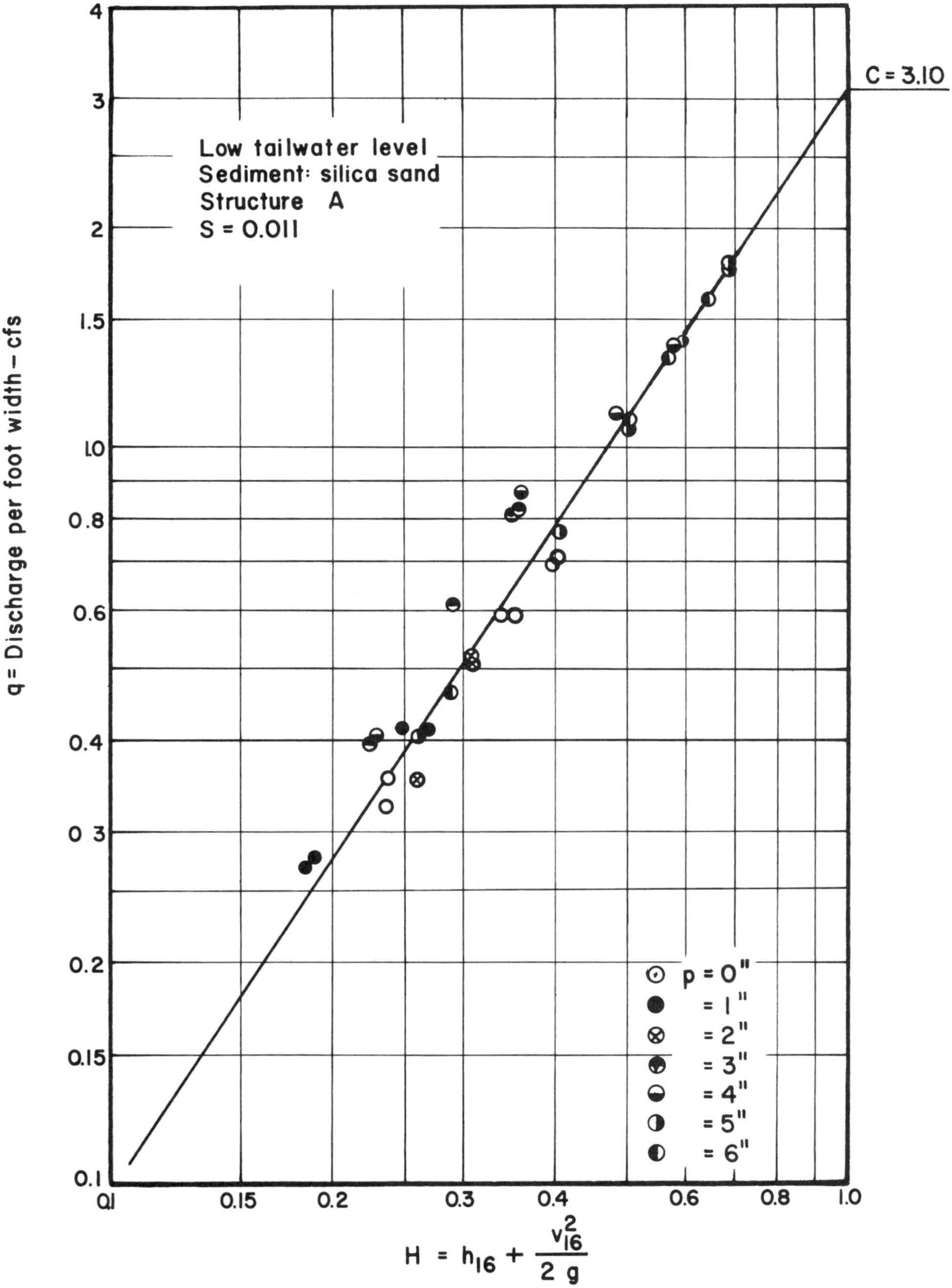


FIGURE 8 EFFECT OF P-VALUE ON COEFFICIENT C

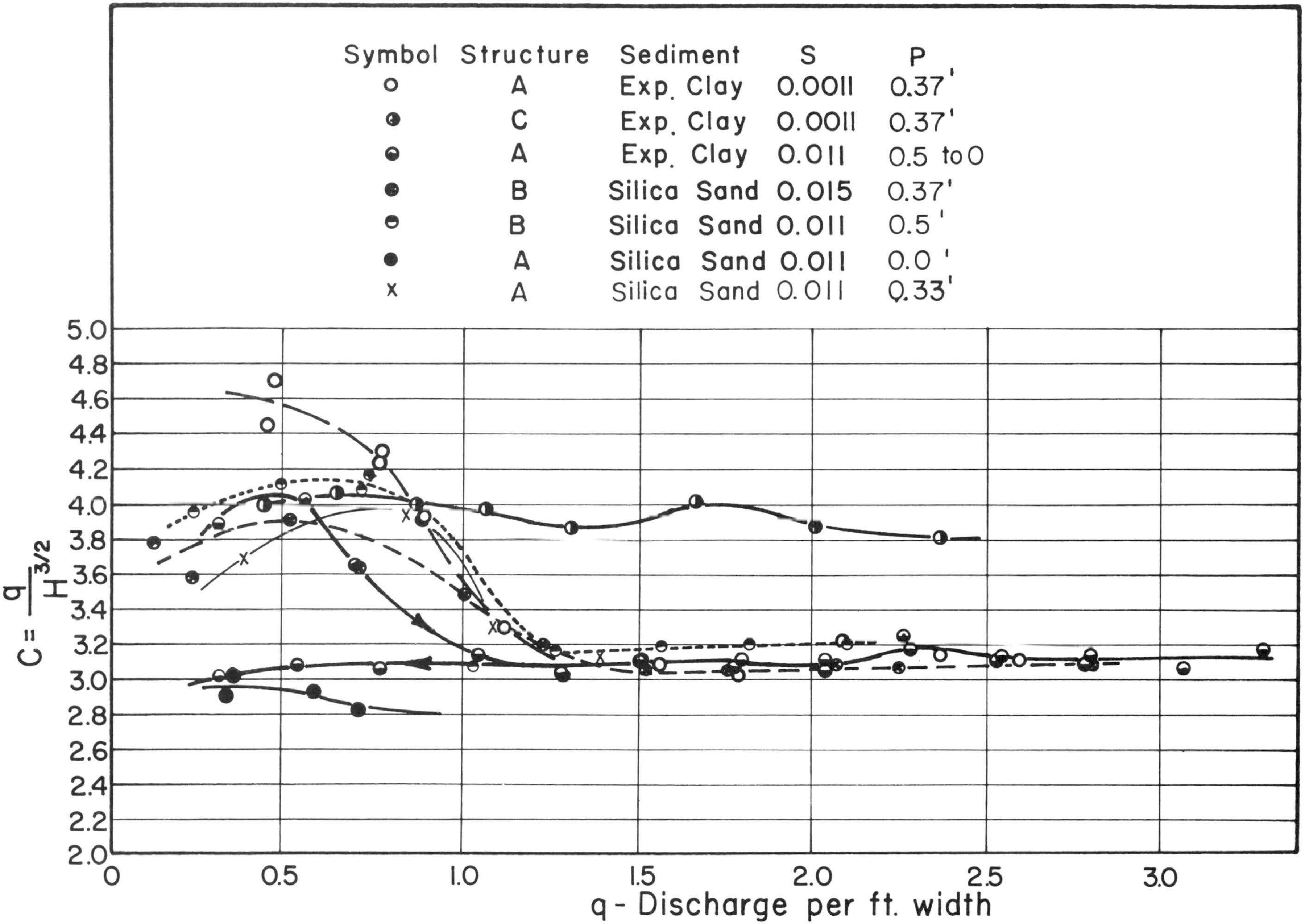


Fig. 9 Variation of Coefficient C with Discharge

the lower range of Q . To better illustrate the latter, C values for p = 4" and p = 0" have been plotted on rectilinear paper, see Fig. 9. The wide variation in C is discussed later in the report.

3. Type of sediment

The original silica sand (0.28 mm mean fall diameter) was replaced by expanded clay sediment (0.37 mm mean fall diameter) for the later tests. This substitution was intended to provide a lighter weight, smaller fall diameter sediment that would move at lower velocities, more nearly corresponding to the prototype situation. In fact, as indicated by the fall diameters, the behavior of the two materials was practically the same except that, resulting from the wider range of particle sizes for the clay, ripples of the fine fraction formed at lower velocities than with the silica sand. Had time permitted the removal of the large particle size fraction from the 5 tons of expanded clay, the original objective would have been more nearly attained.

4. Submergence

The effect of submergence on each of the three structures was investigated by raising the tailwater level in increments above weir crest level and recording water surface level at Station 16, maintaining constant discharge. A submergence ratio, defined as the ratio of downstream head to upstream head above weir crest as datum, of 0.85 could be tolerated on all structures without affecting the discharge coefficient, Fig. 10.

5. Hydrographs

To illustrate more graphically the variations in C, p and velocity that may be expected at a low weir structure subjected to a flood, two simulated flood hydrographs were run in the flume. The first hydrograph had equal rising and falling stages and was applied to structure B , with silica sand and the steep flume slope of 0.025, Fig. 11. The second hydrograph had a falling stage equal to twice the rising stage and was applied to Structure A, with expanded clay material, and a flume slope of 0.011, Fig. 12.

In both cases the peak flow was selected large enough that, given sufficient time, it would cause sediment to build up as a plane bed flush with the weir crest. The initial bed surface was smooth and at flume slope at the beginning of each test.

The variation in C in Fig. 11 is worthy of comment. At low discharge and large p value at the beginning of the flood, C has a high value, as reported previously. At high discharge, C has a normal, relatively constant value. As flow decreases in the falling stage, C again rises but to a smaller value than at the beginning of the flood. This is thought to be due to the change in p value and the change in bed roughness--originally plane bed and finally dunes left over from the previous higher flows. Unfortunately, the limited upstream length of flume prevented the evaluation of bed roughness.

The above variations in C are not apparent in Fig. 12. This may be due to the flatter flume slope and the different shape of hydrograph i.e. the longer time in the falling stage to restore the bed to a smooth condition.

The change in bed form, from rising to falling stage, at approximately the same discharge is shown in Fig. 13. These are bed conditions from the hydrograph of Fig. 12.

6. Structure shape

No measurable difference in flow behavior could be discerned between broad-crested Structures A and B. The discharge coefficient at high flows was the same for both, see Figs. 6 and 7, and the same variability in C at low flows was apparent. So far as energy dissipation and anti-scour devices are concerned, the downstream slope of the structure requires consideration. This aspect was not seriously considered in the present study.

Structure C was investigated as a possibly suitable alternative prototype structure requiring less volume of rock or other fill material for the same weir height, as well as affording the opportunity of installing sections of smaller diameter pipe on the same invert to provide any required width of rectangular notch for more accurate measurement of low flows.

The flume tests on Structure C produced some interesting results. In the first place, the p value and flume slope were set the same as for a previous test on Structure A, and expanded clay was used in both cases. Apart from the higher C value for Structure C (to be expected with the better shape of crest), the value of C remained practically constant over the full range of discharge whereas, for Structure A, high values of C occurred at low flows, see appropriate curves of Figs. 7 and 9. This variation is discussed in the next section. It was noted, too, that the flat apron immediately downstream from the pipe crest assisted in creating a hydraulic jump for a wide range of tailwater levels nearer to the structure than in the case of Structures A and B. This could result in a saving in anti-scour devices.

7. Variability in value of discharge coefficient at low flow

A possible explanation of this phenomenon is that as the discharge and approach velocity increase, so separation and increased curvature of flow at the junction of the upstream face and the horizontal crest of the structure cause a reduction in C.

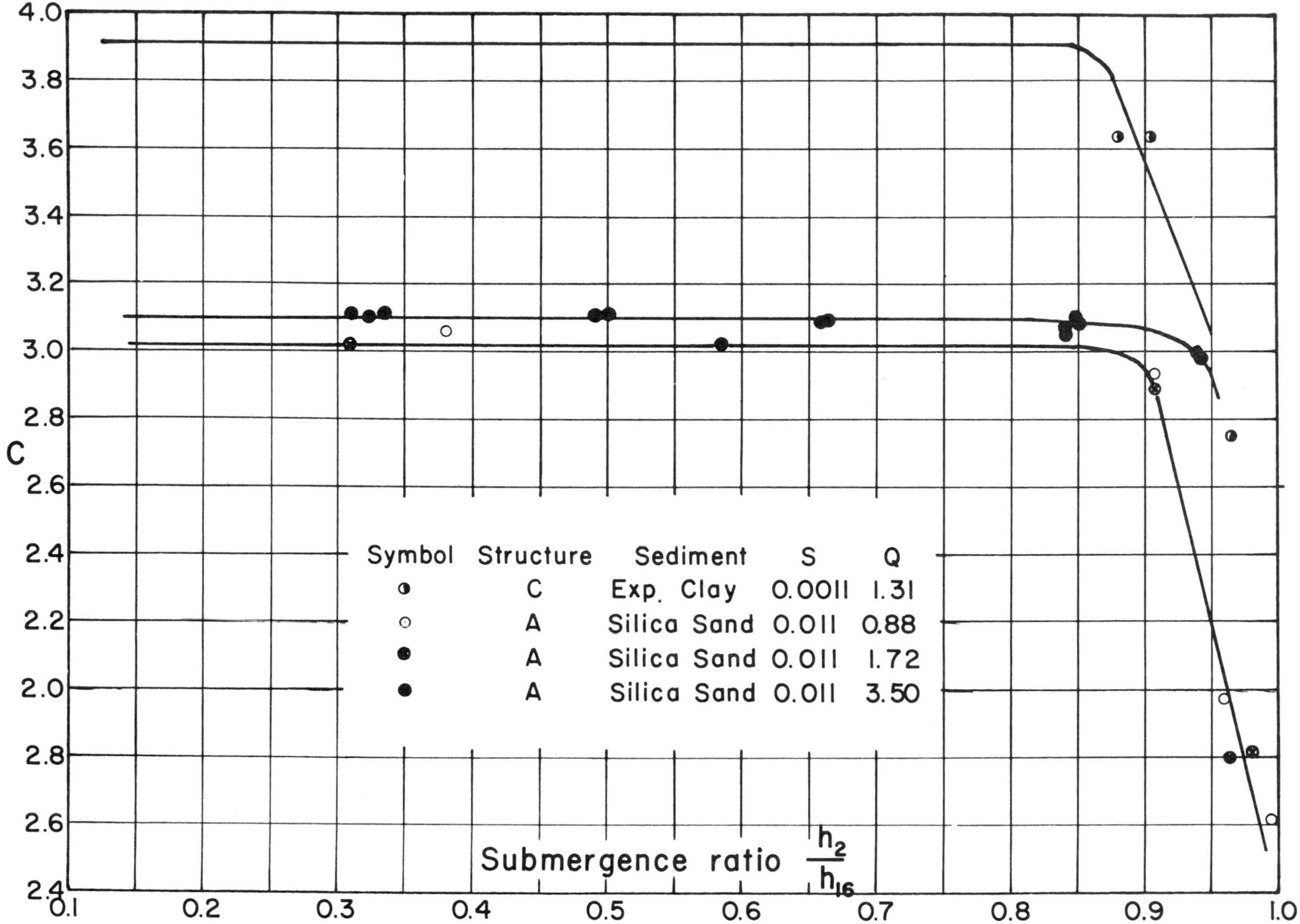


Fig. 10 Effect of Submergence on Coefficient C

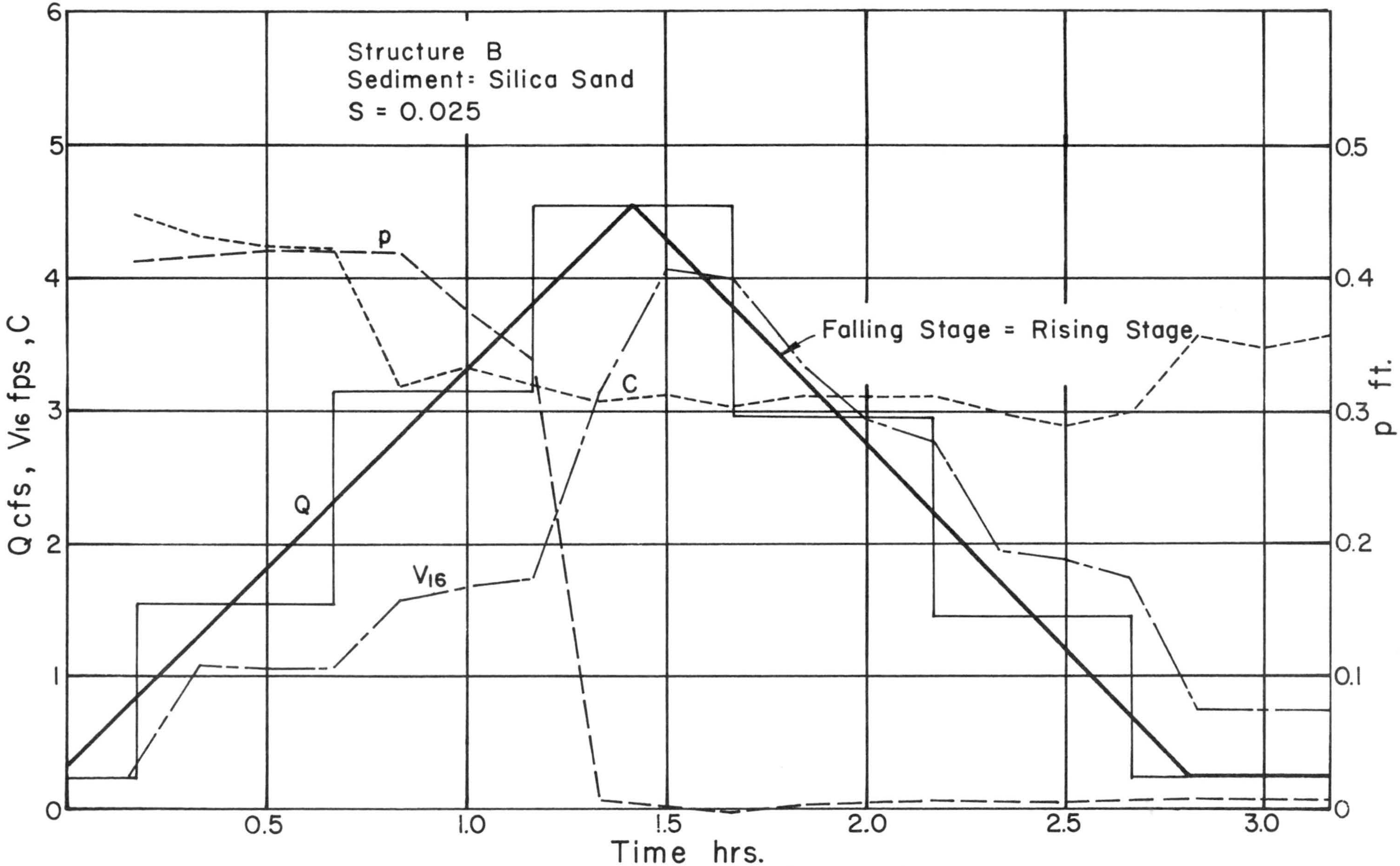


Fig. 11 Simulated Hydrograph

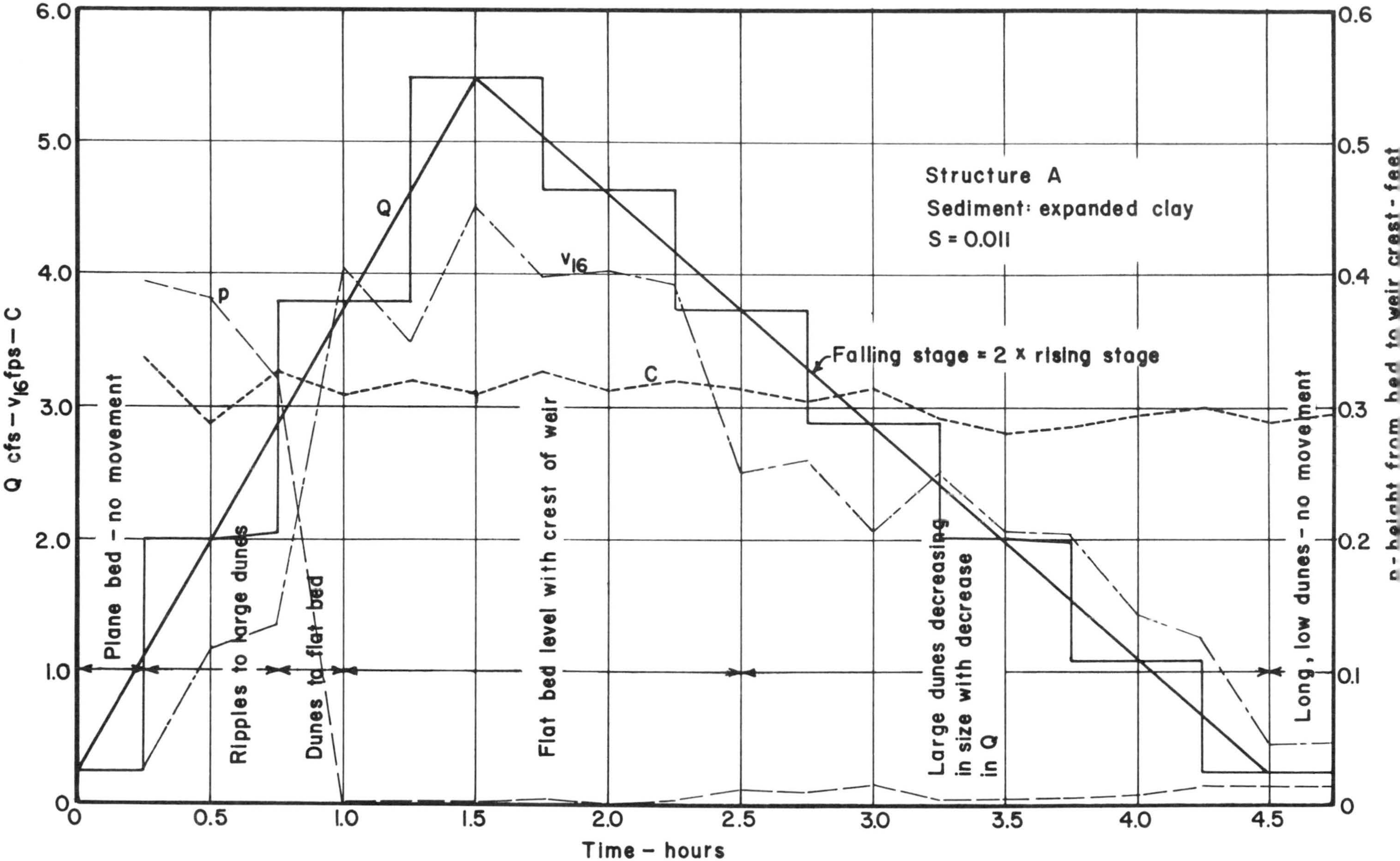
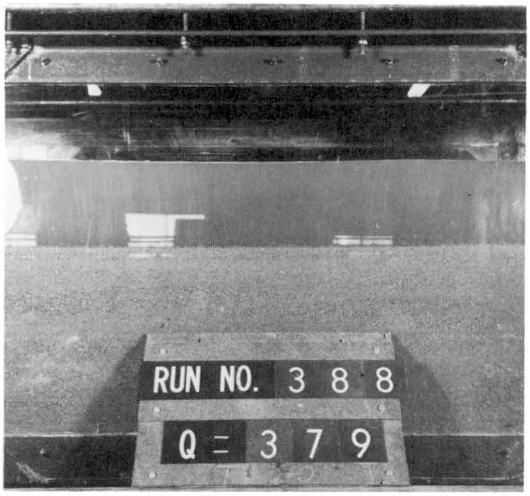
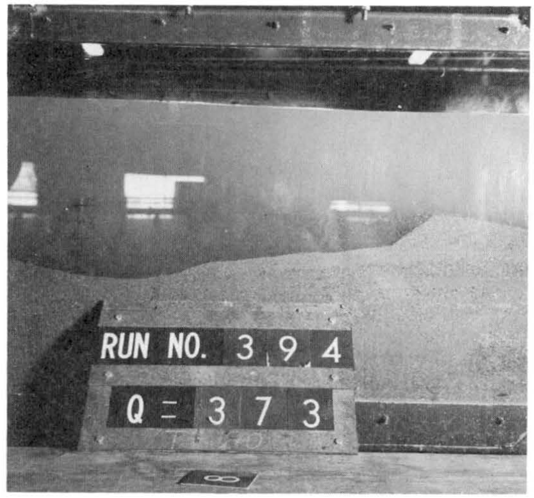


FIGURE 12 SIMULATED HYDROGRAPH



a. Rising stage



b. Falling stage

Fig. 13 Bed configurations for hydrograph - S = 0.011 - expanded clay

With trapezoidal weirs which have upstream faces that are flatter than those tested the magnitude of C is more nearly constant over the complete range of discharge. This was indicated by the studies of Bazin and the U. S. Deep Waterways Board, King (1954).

The effect of the slope of the upstream face of such a structure on the magnitude of C can be largely eliminated by constructing a vertical sill at the junction of the top of the trapezoidal weir and its downstream face, Karaki (1960). The sill reduces the velocity of approach which in turn reduces the opportunity for adverse flow conditions to develop.

Another interesting point is the constancy in C value for structure C with the pipe crest. With this arrangement the nappe springs free at all but very small discharges and consequently the structure behaves more like a sharp crested weir.

D. Effectiveness of Various Types of Triangular Block Energy Dissipators

The most effective arrangement and size of triangular block dissipators, attached to the downstream face of the structure, were investigated by comparing profile photographs of the downstream region for a range of discharges and tailwater levels, using Structure A and a flume slope of 0.011.

Each type of block had a width equal to the height and a length, from square end to the point of triangle in profile, equal to 3 times the height. The following sizes and arrangements were used:

(i) 3 x 1 x 1 inch blocks close together across the full width of flume with the square end alternatively upstream and downstream.

(ii) 4 1/2 x 1 1/2 x 1 1/2 inch blocks as in (i).

(iii) 6 x 2 x 2 inch blocks as in (i).

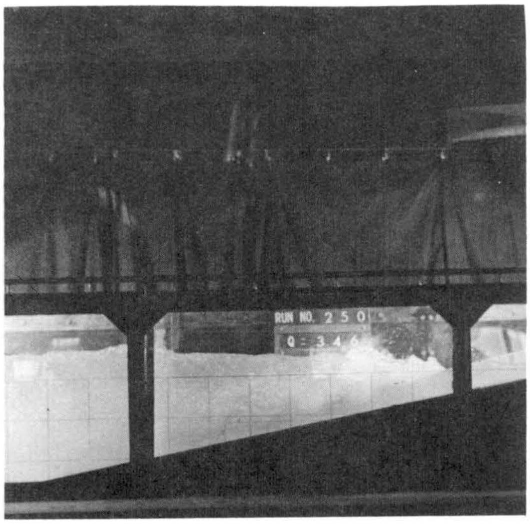
(iv) 4 1/2 x 1 1/2 x 1 1/2 inch blocks with the square end downstream and 1 1/2 inch spaces between blocks.

(v) 6 x 2 x 2 inch blocks as in (iv) with 2 inch spaces.

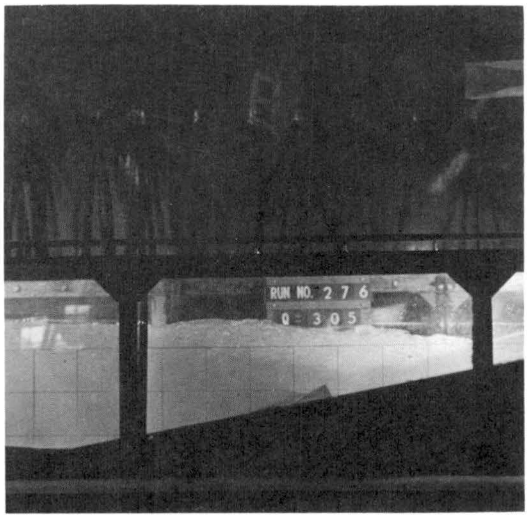
Comparison of flow profile photographs of the above arrangements and the plane structure indicated that the largest blocks spaced close together provided the most effective energy dissipator. The 6 x 2 x 2 inch blocks were later attached to Structure B and tested, with satisfactory result, at flume slopes of 0.011, 0.015, 0.020 and 0.025.

The 6 x 2 x 2 inch blocks were later attached to Structure B and tested, with satisfactory result, at flume slopes of 0.011, 0.015, 0.020 and 0.025.

The high velocity 'diving jet' with the plane structure was dispersed and displaced upward by the blocks as in Fig. 14.



a. Plane downstream face
Q = 3.46 c.f.s.



b. 6 x 2 x 2 inch blocks on downstream face
Q = 3.05 c.f.s.

Fig. 14 Effect of energy dissipator blocks on 5:1 downstream face of Structure B - flume slope = 0.011

IX. CONCLUSIONS

In this exploratory experimental program using a two-foot wide laboratory flume, the following conclusions were reached concerning the flow behavior associated with low weir structures in alluvial channels:

1. The surface roughness of the model structure affects the discharge coefficient, and at some flow conditions, the location of the downstream hydraulic jump. These factors should be considered in scaling up to a prototype weir.

2. Surface level fluctuations, associated with dune movement, at an upstream gaging station indicate the desirability of continuous recording of water surface level if accuracy in discharge measurement is important. Fluctuations in level, corresponding to $\pm$ 6 percent variation in Q , were recorded in the flume study.

3. Upstream dune and ripple patterns are unaffected by the presence of a structure. Upstream bed configuration is a function of depth of flow, rate of flow, slope and duration of flow.

4. Velocity of approach must be considered in discharge coefficient determination due to the changing upstream alluvial bed level.

5. Changes in laboratory flume slope from 0.0011 to 0.025 had negligible effect on discharge coefficient.

6. The depth p of sediment bed surface below weir crest level had negligible effect on C at high flows, but large and uncertain effect at low flows.

7. The two sediments used in the flume experiments behaved in similar manner because their full diameters were nearly the same.

8. All structures could tolerate 85 percent submergence without effect on discharge coefficient.

9. Two simulated hydrographs showed up the variation in C , v and p that could be expected. These experiments show that the bed configuration at any time is a function of the magnitude, shape and duration of the flood and whether considered on the rising or falling section.

10. The upstream slope of a broad crested weir has negligible influence on the discharge performance.

11. Large variations in C value at low flows have been recorded for both broad crested weirs studied. An explanation of these variations at present escapes the author.

12. A structure with pipe crest possessed several advantages over the broad crested weir, including a high discharge coefficient that was constant and stable over the full

range of flume discharge.

13. A comparative study of several sizes of triangular block energy dissipators on the downstream face of the structures showed the largest size (6 x 2 x 2 inch blocks) to be the most effective.

In summary, for accurate flow measurement at a low weir in an alluvial channel it would appear that both water surface and bed surface levels should be recorded, preferably as a continuous record. The velocity of approach should be considered in flow estimation.

Weirs with a pipe crest rather than a broad crest, are favored as having a higher and more stable coefficient of discharge at low rates of flow.

ACKNOWLEDGEMENTS

The experimental program was carried out by Dr. Lawson in the Hydraulic Laboratory at Colorado State University, Fort Collins, Colorado. His activities were financed by a National Science Foundation Grant which was administered by Dr. D. B. Simons, U. S. Geological Survey.

A Fulbright Travel Grant was awarded to Dr. Lawson for the four-month period.

The authors express their thanks to the National Science Foundation, Mr. E. V. Richardson and Mr. H. P. Guy of the U. S. Geological Survey, Mr. S. S. Karaki (responsible for the investigation of structures for the Rio Grande) of Colorado State University, and the staff associated with the Civil Engineering Department for their assistance.

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