DISSERTATION

# A DYNAMIC FRAMEWORK FOR EVALUATING TAX POLICIES IN TAIWAN: AN APPLICATION OF OPTIMAL CONTROL THEORY

Submitted by

San-Li Lin

Department of Economics

In partial fulfillment of the requirements

For the Degree of Doctor of Philosophy

Colorado State University

Fort Collins, Colorado

Fall 2006

COLORADO STATE UNIVERSITY

October 19, 2006

WE HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER OUR SUPERVISION BY SAN-LI LIN ENTITLED A DYNAMIC FRAMEWORK FOR EVALUATING TAX POLICIES IN TAIWAN: AN APPLICATION OF OPTIMAL CONTROL THEORY BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

Committee on Graduate Work

Adviser

Department Head/Director

ABSTRACT OF DISSERTATION

A DYNAMIC FRAMEWORK FOR EVALUATING TAX POLICIES IN TAIWAN: AN APPLICATION OF OPTIMAL CONTROL THEORY

In order to eliminate or alleviate the problem of "double taxation" on dividends, the government in Taiwan introduced in 1998 a new tax system with the intent to integrate corporate and personal income taxes. However, some problems, such as the impacts on tax revenues, capital accumulation, and income distribution, may still emerge in this new tax system in the future.

In the presence of heterogeneous income groups with different rates of time preference, industries of different degrees of incorporation with adjustment costs, and government deficits, this study builds a dynamic general equilibrium model to provide a more comprehensive framework capable of capturing a realistic dynamic environment in Taiwan, based on the optimal control theory. This study also displays the concept of net cash flows in accounting, applies it to the model, and sets the objective of industries to pursue the maximization of their net cash flows in the dynamic general equilibrium tax model.

By applying transversality conditions to each sector, this study derives the solution for the theoretical model to ensure that all the equilibrium conditions are satisfied. Then, this study designs several tax policy scenarios, focusing on the corporate

income tax integration plans, to theoretically explore how to apply simulation schemes to the model, to attempt to theoretically investigate the possible effects of these policies on the structure of the model and equilibrium values, and further to check if simulation results are consistent with equilibrium conditions.

In this theoretical model, what we can obtain are the aggregated results for each sector, not the disaggregated ones which can only be done by an empirical study. Therefore, it would be expected that the framework of this study can be used as a blueprint to examine the performance of the new tax system after 1998 if more disaggregated information becomes available in the future, and further evaluate whether the objectives of tax integration are accomplished.

San-Li Lin
Department of Economics
Colorado State University
Fort Collins, CO 80523
Fall 2006

ACKNOWLEDGEMENTS

I would like to express my gratitude to those who provide the support and guidance for the completion of this dissertation.

First of all, I would like to express my deep gratitude to all of my committee members. I thank Dr. Chuen-Mei Fan, my adviser, for encouraging and believing in me to develop my ideas and thoughts, advising me to make this dissertation organized and rational, and letting me look at things seriously and faithfully. Much thanks to Dr. Robert Kling, Dr. John Loomis, and Dr. David Mushinski for their valuable time and constructive comments.

I also would like to express my gratitude to my parents for their encouragement during this period of study. I thank my wife Yuan-Hsin and my children Chun-Chien, Chun-Chiang, and Yu-Fang for their encouragement and support.

My appreciation is extended to my sister, my brothers, and my friends. Finally, I am grateful to the Transworld Institute of Technology, the college I serve in Taiwan, for providing me the chance and supporting me to pursue my doctoral degree.

# DEDICATION

I dedicate this dissertation to my father who is in Heaven.

## TABLE OF CONTENTS

# CHAPTER ONE

# INTRODUCTION

## 1.1 Problem Statement

It has long been well known that the problem of "double taxation" on dividends stems from the tax system in which both corporate and personal income taxes are separately implemented at the same time. Furthermore, under such a tax system, two types of distortions would be introduced. First, from the perspective of corporate income tax, only the interest payments due to the outstanding private bonds, but not the dividends distributed to the shareholders, can be treated as costs and thus be deductible from the corporate income tax base. Accordingly, corporations tend to prefer debt financing to equity financing, which distorts corporate financial rules and leads to high debt/equity ratios and hence high risk of bankruptcy. Second, from the perspective of personal income tax, corporations have incentives to make use of retained earnings to avoid personal income tax for their stockholders, which leads to the so called "lock-in" effect (Fullerton, King, Shoven, and Whalley, 1981; Shoven and Whalley, 1992). The low dividend/retention ratios will impair the efficiency of capital market and block the optimal allocation of resources.

To alleviate these two problems, the government in Taiwan introduced in 1998 a new tax system with the intent to integrate corporate and personal income taxes. In addition to adopting the full imputation system, this new tax system also levies an additional 10% surtax of corporate income tax on retained earnings.

By "the full imputation system", the individual tax liabilities on dividend earnings which have been paid by the corporation on behalf of the individual stockholders should be fully credited against the individual's personal tax liabilities when filing personal income tax (Ministry of Finance, Taiwan, 1998). The Ministry of Finance claims that among the feasible schemes, the full imputation system is most suitable for Taiwan to implement and has been adopted by most of the countries which have implemented corporate income tax integration. One important reason is that all the distortions induced by the separately operated corporate and personal income taxes can be completely eliminated under the scheme of the full imputation system, even though this scheme may generate relatively larger revenue loss than others (Ministry of Finance, Taiwan, 1998).

Under this new system, the previous corporate and personal income tax rates remain unchanged, with the highest marginal rates at 25% and 40%, respectively. Because there is a difference up to 15% between these two top rates, corporations still have an incentive to avoid tax liabilities via retained earnings for their shareholders. Therefore, the authority imposes an additional 10% surtax of corporate income tax on retained earnings to minimize that difference. Another reason for this additional 10% surtax of corporate income tax on retained earnings is to make up for the revenue loss associated with the full imputation system.

However, some problems which the government has to deal with in the future may still emerge in this new tax system. Among them, the effects on tax revenues and income distribution can be easily grasped from the following illustrations.

**The Effects of Integration on Tax Revenues**

Since the implementation of corporate income tax integration in Taiwan in 1998, the corporate income tax revenues have not shown a decreasing tendency. Instead, they vary with the changes in the economic growth rate. On the other hand, because of the time lag when shareholders receive dividends from corporate earnings, the actual impact of corporate income tax integration on the personal income tax revenues will be only effective in 2000. Therefore, from the fact that the average annual rate of increase in the personal income tax revenues for the period 2000 – 2002 is –3.85%, it would be expected that the implementation of corporate income tax integration has brought about a decrease in the personal income tax revenues, and the integration has further contributed to a decrease in the ratio of the personal income tax revenues to the national tax revenues. Nevertheless, because the ratio of the corporate income tax revenues to the national tax revenues has increased year by year, the sum of corporate and personal income tax revenues remained relatively stable, at around 32% of the nation's total tax revenues.

Several figures can be used to further illustrate the finance of Taiwan government. First, the national tax liability rate (National Tax Revenue / GDP) gradually falls from 20.6% in 1990 to 16.2% in 1998, and to 13.6% in 2004 (Ministry of Finance, Taiwan, 2005). This decreasing trend of national tax liability rate mainly is attributed to the seriously eroded tax base. Second, tax revenues have become less than current expenditure since 1995. Third, government savings turn out to be negative since 1999, indicative of the deteriorating trend in the finance of Taiwan government. Finally, the government can not help but significantly increase the issuance of public bonds to finance its various public investments with the annual rate of increase in the value of outstanding government bonds at 10% in 1999 and 25.61% in 2001.

In sum, the implementation of corporate income tax integration in 1998 has adversely affected the personal income tax revenues, with an apparent reduction in the national tax liability rate. Eventually, the government finance falls into deficits and has to rely on financial funds.

## The Effects of Integration on Income Distribution

First, the annual rate of increase in the average disposable income per household has been lower than the economic growth rate since 1994, and has never exceeded 2% since 1998. Furthermore, it becomes less than 1% in 2000, and turns into negative in 2001. Therefore, we can speculate that even with the possibility of an increase in the average disposable income per household after integration, it might be very limited.

The average disposable income per household appears to increase at a decreasing rate for all income groups before 1998. Since 1998, disposable income of some middle and low income groups actually decreased and has not gone back to its pre-corporate income tax integration level. In particular, it appears that the lower the level of the household's average disposable income, the larger the difference between its current and previous levels. In contrast, the average disposable income per household of the highest income group has not decreased in the same period. Therefore, the implementation of corporate income tax integration definitely makes the lower income groups worse off. In fact, income tends to concentrate on the highest income group under corporate income tax integration.

We can also investigate the income inequality indexes to capture the income distribution in more detail. One index is the ratio of income share of the highest 20% to that of lowest 20%, which is less than 5.5 before 1998 and exceeds 6.0 for some periods after 1998. The other is Gini's concentration coefficient, whose highest value is 0.320 prior to 1998 and has gradually increased up to 0.343 in 2003 under corporate income tax integration. Hence, both indexes show that the implementation of corporate tax integration has apparently enlarged the magnitude of income inequality in Taiwan.

This adverse effect of corporate income tax integration on income distribution reflects a shortcoming of corporate tax integration in that it only focuses on the improvement of taxation on dividends. In turn, it makes the high income households better off, because they hold most of the dividends distributed. This is why some researchers argue that corporate income tax integration genuinely benefits neither the low income households nor those who do not invest in stock market (Chuang and Sang, 1998).

**The Functions of the Additional 10% Surtax**

Because the difference between corporate and personal income tax rates can not be completely eliminated under the 1998 tax reform, there still exists the incentive for shareholders to avoid tax liabilities through retained earnings. An official study by the Finance Reform Committee, Executive Yuan (2002) reported that after the implementation of corporate income tax integration, the government has lost tax revenues by 14.9 and 38.5 billion N.T. dollars in 2000 and 2001, respectively.

Therefore, it appears that the additional 10% surtax on retained earnings can not achieve its intended objectives. Instead, it may cause corporations to distribute more dividends to shareholders in order to avoid this additional 10% surtax, and thereby lower the accumulation of corporate capital and the ability to undertake risk. Hence, abolishing the additional 10% surtax on retained earnings has been the most interesting issue being debated since the implementation of corporate income tax integration.

### 1.2 The Importance of the Study

Thus far among the studies on corporate income tax integration in Taiwan using general equilibrium models, most of them are of comparative static analyses in nature, therefore, can not capture the effects of tax integration in a true dynamic sense. This study is one of the few studies that employ a dynamic general equilibrium approach to examine the impacts of corporate income tax integration on income distribution and economic performance in Taiwan economy.

Among the few studies using a dynamic general equilibrium model for tax policy evaluation in Taiwan, some do not analyze the effects of corporate income tax integration on different industries in the production sector, while others do not estimate the effects on different income groups in the household sector. Further, most of the studies adopt the concept of balanced budget incidence in the government sector, which do not allow the government to run deficits and thus can not examine the effects of government deficits induced by corporate income tax integration. Nevertheless, this study attempts to simultaneously simulate all of these three types of effects.

This study encompasses household, production, and government sectors. The household sector consists of m heterogeneous income groups classified by income, with each group possibly responding differently to the corporate income tax integration. The production sector, considering the adjustment costs, is composed of J+1 industry subsectors classified by the degree of incorporation. Furthermore, it allows for the presence of government deficits. Therefore, this study will help policy makers evaluate whether the degree of inequality across income groups is improved and how large the financial crowding-out effects of the new tax system might be, and thereby take appropriate actions to revise or improve upon this new system.

### 1.3 The Objectives of the Study

The purposes of the tax reform implemented in 1998 in Taiwan are to stimulate investment by eliminating double taxation on dividends, to alleviate the distortionary effects of taxation on corporate financial rules, and to promote the efficiency of resource allocation by reducing corporate incentives which make use of retained earnings to avoid personal income tax for their stockholders (Ministry of Finance, Taiwan, 1998). However, the following problems may arise. First, the total government tax revenues may decrease. Second, the distortion of dividend policy may still persist as long as corporate and personal income tax rates are not equal to each other. Third, the additional 10% surtax of corporate income tax levied on retained earnings may impair the accumulation of corporate capital and hence prevent the stock market from developing.

This study will investigate the impacts of corporate income tax integration on income distribution and economic performance by applying a dynamic general equilibrium model in the presence of heterogeneous income groups with different rates of time preferences, industries of different degrees of incorporation with adjustment costs of investment, and government deficits. This study is expected to contribute in the following areas: first, this study will provide a more comprehensive framework capturing a realistic dynamic environment for policy makers to grasp the effects of integrating corporate and personal income taxes. Second, the framework of this study can be used to examine the performance of the new tax system after 1998 if more disaggregated information becomes available in the future, and further evaluate whether the objectives of tax integration are accomplished. Third, this study can be used to inspect the desirability of imposing the additional 10% surtax of corporate income tax on retained earnings.

### 1.4 Data Source for Empirical Studies

For empirical studies, the data used to calibrate the benchmark model and conduct the simulation analysis can be obtained mainly from the following sources:

a. Report on the Survey of Family Income and Expenditure in Taiwan Area of the Republic of China, 2003, Directorate-General of Budget, Accounting and Statistics, Executive Yuan, the Republic of China, October, 2004.
b. National Income in Taiwan Area of the Republic of China, 2004, Directorate-General of Budget, Accounting and Statistics, Executive Yuan, the Republic of China, December, 2004.
c. Monthly Statistics of Finance, Taiwan Area, the Republic of China, June, 2005, Department of Statistics, Ministry of Finance, Executive Yuan, the Republic of China, July, 2005.
d. Business and Industrial Key Financial Ratios, 2004 Edition, Joint Credit Information Center, October, 2004.
e. TEJ Database, Taiwan Economic Journal Co. Ltd.

f. The Report on 2001 Input-Output Tables, The Republic of China, Directorate-General of Budget, Accounting and Statistics, Executive Yuan, the Republic of China, January, 2005.

The major financial and economic indicators of Taiwan economy, the relative data on household income and expenditures, and government revenues and expenditures come from the first three sources described above. The last three sources provide the information on industry sectors. Both sources d and e offer corporate financial data in operation, while the remainder affords input and output data in production. One thing that should be noted is that since data for individual companies are difficult to obtain, especially the details of taxation such as the marginal tax rate levied on individual shareholders and the structure of shareholding, we may have to use proxies, or to set these variables as exogenously given. Therefore, the attempt to examine the impacts of corporate income tax integration on corporate financial decisions may be restricted. It should also be noted that considering the incompleteness of data, we may be constrained to using the data only of listed companies in the stock market.

### 1.5 The Organization of the Study

After the introduction, chapter two is a critical review of the literature. It reviews both the empirical and theoretical studies on corporate income tax integration, focusing on the effects on income distribution and resource allocation. Most of the empirical studies on corporate income tax integration in Taiwan investigate only the resource allocation effects, while only a few studies concentrate on evaluating the impacts of this new tax system on income distribution. Furthermore, among the empirical studies on the effects of integration on income distribution in Taiwan and other countries, most adopt static general equilibrium models. Studies using dynamic general equilibrium models to simulate both income distribution and resource allocation effects are even scarce. The most significant result of these studies is that the size of dynamic efficiency effects of tax integration would differ from that of static ones.

Chapter three first presents the specification of the behavior of each type of agents. In this study, households are heterogeneous in terms of their time preferences. Industries with different degrees of incorporation are considered in the production sector. The adjustment costs of investment are taken into account as well. As to the behavior of the government, it is allowed to run deficits in this study. This chapter will provide specific functional forms to characterize the behavior of each type of agents. Furthermore, this chapter will also identify the equilibrium conditions for each sector and for the closed Taiwan economy as a whole. With respect to the methodology adopted in this study, the temporary Walrasian equilibrium will be used in the scenario for the equilibrium at each period of time. The concept of net cash flows from the perspective of accounting will also be introduced into this study. Most importantly, the optimal control theory will be applied to capture the nature of the optimal dynamic equilibrium.

Chapter four attempts to apply the optimal control theory to the more comprehensive tax model. Then, this chapter derives equilibrium conditions and identifies the solutions for each sector and each market and the whole model by applying transversality conditions to each sector.

Chapter five will explore theoretically how to apply simulation schemes to the model. Then, this chapter will attempt to investigate and compare the difference in the effects of alternative tax policies, focusing on the corporate income tax integration plans, on the model structure and equilibrium values. In the process of simulation analysis, the results derived from alternative corporate income tax integration plans, which are presented as counterfactual cases, will be compared to the solutions derived from the model described in the previous two chapters, which is treated as the benchmark case.

Finally, chapter six will first summarize the simulation results and then offer several conclusions based on the simulation results in this study. The basic contributions of this study will be highlighted. Furthermore, policy implications will be drawn as references for policy makers in pursuing future taxation improvement and reform. This chapter will conclude with some suggestions for future research.

# CHAPTER TWO

# LITERATURE REVIEW

Among the empirical studies, some adopt a general equilibrium model to estimate the effects of corporate income tax integration on income distribution and economic efficiency. Others, without using a general equilibrium model, focus on specific issues in economic efficiency for the production sector, but not on income distribution.

## 2.1 Studies Using a General Equilibrium Model

### 2.1.1 Empirical Studies on the United States

It has long been argued that corporate income tax implemented separately from personal income tax may induce adverse effects on the economy. Following the seminal work on tax incidence by Arnold Harberger (1962), Fullerton, King, Shoven, and Whalley (FKSW) (1981) apply a general equilibrium model, composed of 19 industries, 16 consumer goods, and 12 consumer groups classified by income range, to evaluate the effects of corporate and personal income tax integration on income distribution and resource allocation. They use 1973 data to estimate the static and the dynamic resource allocation effects respectively for four alternative schemes of corporate income tax integration under the assumption of the government's balanced budget in the United States. Further, it is assumed that the U.S. economy lies on a balanced growth path in terms of the 1973 data, only in which savings decisions based on myopic expectations on the rate of return to capital in each period would be correct.

Plan 1 of these four alternatives is a full integration scheme which merges corporate and personal income taxes. In fact, corporate income tax is repealed, and the personal income tax is imposed on the entire corporate earnings including dividends and retained earnings. Other plans are partial integration schemes. Plan 2 is dividend deduction from corporate income tax base with dividends being taxed at the personal level, leaving the corporate income tax on retained earnings only. Therefore, the double taxation of dividends is eliminated. Plan 3 is dividend deduction from personal income tax base, and taxed at the corporate level so as to remove double taxation. FKSW also argue that both of these two dividend deduction plans will give corporations an incentive to pay out more dividends. Plan 4 is dividend "gross up", which gives shareholders a tax credit of 15% of corporate income tax paid by corporations. Compared to dividend deduction plans, Plan 4 only partially reduces the double taxation of dividends, so that there is still partial distortion due to double taxation remaining.

In order to show the impacts of changes in dividend policies in their model, FKSW take into account the extreme behavior assumption that corporations might distribute their entire profits to shareholders, contrast to the other assumption for dividend deduction plans that corporations might keep the existing dividend policies. They set a parameter for each type of capital income to represent the proportion of that type of capital income to be taxed under the personal income tax. They further specify a fraction for each of industries to show the proportion of that industry's capital income to be taxed under the personal income tax. Through the specifications of these parameters and a set of effective tax rates for each integration plan, this study can capture the changes in net of tax return of capital and simulate the impacts of various corporate income tax integration schemes on the economic structure.

Their simulation results show that under the assumption of the government's balanced budget, full integration will produce an annual static efficiency gain of around $6 billion in 1973 prices, while the amount of this efficiency gain derived from partial integration schemes will be lower. With respect to dynamic efficiency, full integration will yield gains around $500 billion in terms of the present value, or "about 1.0 % of the discounted present value of the GNP stream in the U.S. economy after correction for population growth" (FKSW, 1981). Therefore, it appears that corporate income tax integration will bring about significant positive welfare effects to the United States. As to economic efficiency, every income group becomes better off under full integration which makes full integration a Pareto improvement. Finally, the impacts of corporate income tax integration on income distribution vary with integration plans, depending on the choice of replacement taxes used to preserve government revenue neutrality. According to the FKSW study, dividend deduction from personal income tax base will have a slight benefit to higher income groups, while dividend deduction from corporate income tax base will apparently favor the lower income groups relative to their higher income group counterpart.

FKSW adopt the concept of balanced budget incidence and assume that the U.S. economy is located on a balanced growth path, so that the simulation analysis does not examine the impacts of corporate income tax integration on the government revenues and economic growth rate. Furthermore, since FKSW' model does not allow the presence of government deficits, it does not take into account the adverse crowding-out effects of government deficits on private investment (Pereira, 1993). In addition, capital is set to be completely mobile among industries with instantaneous and costless adjustment. Pereira argues that both postulations are unreasonable, and as a consequence, the FKSW model will result in overestimating the benefits expected from corporate income tax integration.

Ballard, Fullerton, Shoven, and Whalley's (BFSW) (1985) model duplicates FKSW' (1981) basic assumptions and parameters, and the four corporate income tax integration schemes except adding the VAT scaling as the fourth tax replacement to preserve government revenue neutrality. Their model includes 12 consumer groups classified by income range, 15 consumer goods, and 19 industries among which 8 industries are chosen to investigate the impacts of corporate income tax integration on the magnitude of changes in capital stock through reduction in capital tax rates. These 8 industries are divided by the differences in the degree of incorporation, four of these industries with low rates of capital tax under the 1973 law, the other four with high ones. They also adopt a dynamic general equilibrium approach to evaluate the effects of corporate income tax integration on industry structure, economic efficiency, and income distribution. To solve for a competitive equilibrium in which profits are zero and supply equals demand for each good and factor in each period, they invoke the Merrill (1972) variant of Scarf's (1973) algorithm (Shoven and Whalley, 1992).

The static effects of corporate income tax integration come mainly from capital stock being more efficiently allocated among the industrial sectors. The effective tax rates on capital become lower after integration. When the corporate income tax is eliminated under full integration, capital tax rates in industries previously more heavily taxed are significantly reduced so that capital used by these industries becomes cheaper, and these industries will gain the most benefits from full integration and tend to employ more capital relative to the rest of industries. With respect to the dynamic effects of corporate income tax integration, since the after-tax rate of return to capital as a whole goes up as caused by the lower capital tax rate under integration, the relative price of capital rises from 1 to 1.208 in the first equilibrium period under full integration. Thereafter, the relative price of capital falls because there are more savings induced by the higher after-tax rate of return to capital, but the size of reduction in the relative price of capital tends to diminish, down to 1.107 by the final period.

In BFSW' model, full integration generally yields more annual static efficiency gains than partial integration, even though the overall difference in annual static efficiency gains derived from various plans of integration is small. On the other hand, dynamic gains are much larger than static ones. Under full integration, the lump-sum replacement might yield dynamic gains as large as $695 billion in 1973 dollars, or about 1.4% of the present discounted value of welfare (consumption plus leisure) in the base sequence, while dynamic gains with the multiplicative and additive replacements are only about 0.62% and 0.84% respectively. Hence, dynamic welfare effects of corporate income tax integration are sensitive to the choice of replacement taxes used to preserve government revenue neutrality, while static ones are relatively stable. Furthermore, for both dividend deduction plans, the static and dynamic gains under the extreme-behavior assumption that corporations distribute their entire profits to shareholders, are greater than those under the assumption of fixed dividend- retention policies.

As to the distributional effect, corporate income tax integration makes every income group better off, with a U-shaped pattern of gains across income groups. That is, corporate income tax integration will lead to a Pareto improvement according to BFSW' (1985) simulation results. However, the effects of dividend deduction from corporate income tax base appear to be more progressive than that of dividend deduction from personal income tax base.

Several conclusions can be derived from the empirical simulation results of FKSW (1981) and BFSW (1985) together. First, dynamic gains are much larger than static ones. Second, partial integration plans yield less welfare gains than the full integration plan, except dividend deduction from corporate income tax base with extreme behavior assumption in FKSW (1981). Third, among all forms of tax replacements, the lump-sum scaling is shown to always have the largest welfare gains, because of its nondistortionary nature. Fourth, under full integration which completely eliminates the corporate income tax, industries with higher degree of incorporation will gain more. Fifth, for dividend deduction plans, the static and dynamic gains under the extreme-behavior assumption are greater than those under the assumption of fixed dividend- retention policies. Sixth, full integration will make every income group better off, or achieve Pareto improvement. Seventh, the impacts of corporate income tax integration on income distribution vary with integration plans, depending on the choice of replacement taxes used to preserve government revenue neutrality. Finally, the effects of dividend deduction from corporate income tax base appear to be more progressive than those of dividend deduction from personal income tax base.

Pereira (1993) constructs a dynamic general equilibrium model of the U.S. economy to investigate the impacts of corporate income tax integration on the intertemporal and intersectoral efficiency and the distribution of wealth and capital. His focuses are on optimal intertemporal investment decisions, optimal intersectoral allocation of investment, optimal intertemporal household consumption/savings and labor/leisure decisions, government deficits, and financial crowding-out effects. The taxation system in his model represents mainly the situation in the United States before the Tax Reform Act of 1986. There are two integration methods simulated in his model: the full integration achieved by repealing the corporate income tax and the partial integration by dividend deduction from the corporate income tax base.

Pereira's study differs from FKSW and BFSW in several respects. First, it allows for government deficits which are optimally determined. Second, it assumes that investment decisions are forward-looking, and are optimally determined in the presence of adjustment costs and imperfect capital mobility. Third, it contains an endogenous sequential equilibrium structure, based on dynamic economic behavior, and in which the link between adjacent periods is provided by the recursive transitions of the stock variables in the economy.

Pereira's model consists of 4 industry sectors classified by the degree of incorporation, 3 consumer groups divided by income range, and the government. Each of these three types of agents faces a dynamic environment in a finite horizon framework. The optimal intertemporal decisions of the different agents are derived from an intertemporal specification of the agents' objectives and constraints. The objective of the producer is to determine endogenously optimal supplies and demands for the different production inputs through maximizing the present value of the intertemporal net cash flows subject to a value added linear homogeneous Cobb-Douglas production technology, a quadratic adjustment costs technology, the equation of motion for the capital stocks, the nonnegativity constraints on the decision variables, and future price expectations (Pereira, 1988, 1993). The optimal government behavior can be derived from the maximization of an intertemporal social welfare function, which is given by a loglinear Cobb-Douglas function (Pereira, 1988), subject to a recursive sequence of budget constraints. The objective of the household is to make the optimal consumption/savings and labor/leisure decisions through maximizing the present value of his utility function characterized by a two-level nested structure with a linearly homogeneous Cobb-Douglas structure, constrained by a recursive sequence of budget constraints, terminal state constraints, and a sequence of future price expectations (Pereira, 1988). The data set and parameter specification in his model are consistent with those reported in BFSW (1985), and are enlarged to cover additional issues not considered in BFSW.

In order to capture both the incomplete and sequential nature of the U.S. economy and the limitations of foresight into the future, Pereira adopts the temporary Walrasian equilibrium in his model. That is, all markets are assumed to clear; equilibrium in the short run is parametric on the expectations of future prices held by the different agents as well as future taxation parameters; and the optimal transitions of the stock variables between adjacent equilibria are determined endogenously, given the equilibrium prices and net demands (Pereira, 1993). In terms of simulation design, Pereira further applies the concept of equal yield to policy evaluation implemented by comparing the differences between equilibria of both the base case that reflects the 1973 U.S. economy before integration and several counterfactual cases that reflect different policy scenarios. The "equal yield" means that "the size of the government is kept constant" in terms of "the same level of social welfare in both the base case and the revised cases" (Pereira, 1993). Three equal-yield alternatives are used in Pereira's model to finance the optimal level of public expenditures necessary to keep the level of social welfare in the counterfactual cases the same as that in the base case. These three schemes differ in the marginal crowding-out effects they induce (Pereira, 1988). Tax-financed equal yield blocks marginal financial crowding-out effects by changing tax revenues with constant base case deficits. Bond-financed equal yield maximizes marginal financial crowding-out effects by changing deficits with constant base case tax revenues. Mixed-financed equal yield generates an intermediate between the former two schemes by changing a mix of deficits and tax revenues (Pereira, 1988, 1993). Pereira further notes that when we are to analyze the efficiency effects of integration with different equal-yield alternatives we require the computation of the changes in other tax rates necessary to neutralize the tax changes under consideration. Accordingly, he uses increased personal income tax rates as the replacement mechanism.

Pereira obtains several simulation results which are not all the same as those in FKSW (1981) and/or BFSW (1985).

First, the elimination of corporate income tax and its replacement by increased personal income tax rates, i.e., the full integration in his study, will yield very modest and often negative long-run efficiency gains at best at 0.17% of the present value of future consumption and leisure, well below the estimates in FKSW and BFSW. Pereira points out two reasons for this difference in efficiency gains. One reason is the presence of adjustment costs, which introduce the rigidities of investment decisions in his model. Therefore, capital is not fully mobile across industries, and adjusting capital towards its optimal level is costly in both time and money. The other reason is that the endogenously recursive nature of equilibrium in his model can better capture the intertemporal distortions caused by higher marginal income tax rates.

Second, the average long-run gains are more than twice as large as the average short-run gains, which implies a sharply increasing intertemporal efficiency pattern. In addition to the same reasons as described in the first result, Pereira reasons that it takes time for the investment efficiency effects of integration to occur.

Third, the efficiency gains from full integration are negatively correlated to the size of change in the personal income tax rates. Numerically, a 1% increase in the tax replacement leads to about 1% decrease in the efficiency gains.

Fourth, even though government deficits generated by both bond-financed and mixed-financed schemes of equal yield under full integration are slightly lower by 0.024% and 0.005% respectively than those generated by tax-financed equal yield, efficiency gains with tax-financed equal yield are always higher than those with the other two schemes of equal yield. There appears to be no significant effects of full integration on the changes in government deficits. Pereira notes that this is because the corporate income tax revenues foregone due to integration are actually shifted to be collected in the form of personal income tax revenues. He further notes that because there is no significant marginal crowding-out effects from integration in the U.S. economy, the comparisons of different scenarios in FKSW and BFSW, in which balanced budget is assumed so that financial crowding-out effects are not considered, may not be seriously biased.

Fifth, partial integration, in the form of dividend deduction from the corporate income tax base, generates systematically negative efficiency effects. Furthermore, long-run losses are greater than short-run losses. Intuitively, this result shows that the elimination of double taxation on dividend income under partial integration can not directly yield efficiency gains, which is different from those in FKSW and BFSW. Pereira presents two reasons for this difference. Though FKSW and BFSW also take into account intertemporal consumption and labor/leisure decisions in their models, the endogenously recursive nature of equilibrium in Pereira's model can better capture the distortions induced by higher marginal income tax rates. The other reason for this difference can be attributed to the different types of replacement tax used in their models. That is, FKSW and BFSW use increased corporate tax rates, not the increased personal income tax rates used by Pereira, as a replacement mechanism. On the other hand, with dividend deduction from corporate income tax base, dividends are treated as interest payments, which theoretically can eliminate the distortion due to corporate tax preference toward debt financing, and then encourage corporations to issue more equity to finance their investment and distributing more dividends. In turn, partial integration should make debt/equity ratios lower, dividend/retention ratios higher, and investment financing more efficient. Hence, Pereira argues that since the effects of integration on corporate financial decisions are not considered in his study and in FKSW and BFSW, the simulation results in all these studies tend to be biased downwards.

Sixth, unlike the result simulated in FKSW and BFSW, full integration in Pereira's study is shown not to be a Pareto improvement in terms of changes in intertemporal utility. Numerically, high-income group benefits the most, more than 11% in terms of utility gain; while the low-income group suffers a utility loss of about 6.5%.

Finally, under full integration, the industries in the sector with highest degree of incorporation gain the most in terms of capital accumulation, which is the same as the result in BFSW. However, changes in capital accumulation are less than 1% under full integration, which are relatively small compared with the result in BFSW. Pereira points out that this difference between the sizes of changes in capital accumulation obtained from both models is generated by the presence of adjustment costs in his model. Accordingly, capital is not perfectly mobile among industries, and it takes time for capital to adjust to its optimal levels.

Theoretically, the integration of corporate and personal income taxes should be expected to affect the optimal household portfolio decisions and the optimal corporate financial rules and dividend/retention policies. That is, from the household perspective, the rates of return on financial assets will vary with integration. From the corporate perspective, the tax preference favoring bonds financing against equity financing and favoring retained earnings against dividends will disappear or diminish after integration. Accordingly, under integration, the corporate debt/equity ratios and dividend/retention ratios will tend to fall and rise, respectively. However, since financial decisions are set to be exogenously given in Pereira's model, the effects of integration on the financial decisions by households and corporations are ignored. Therefore, it is not allowed for households and corporations to adjust their financial decisions to the optimal level after integration, so that the efficiency effects of integration may be underestimated.

### 2.1.2 Empirical Studies on Taiwan

Even though there are so many empirical studies related to the issues of corporate income tax integration in Taiwan, few of them focus on investigating the impacts of this new tax system on income distribution. Further, among these studies even fewer apply dynamic general equilibrium models.

#### 2.1.2.1 Static Studies

Su (1986) uses a comparative static analysis approach, originally developed by Fullerton et al. (1979), to evaluate the economic effects of corporate income tax integration in Taiwan. He also adopts the fixed-point computation algorithm developed by Scarf (1973) to build his empirical computable general equilibrium (CGE) model based on 1981 data. In his analysis of simulations, there are four alternative schemes of corporate income tax integration: the partnership method, the dividend-paid deduction method, the dividend-received exemption method, and the partial imputation method with 15% tax credit. Only the partnership method is of full integration, others are of partial integrations. Among the three partial integration methods, the dividend-paid deduction method is integration at corporate level, while the other two are integration schemes at personal (shareholder) level.

Su's model consists of 15 production sectors and 10 consumer groups. On the production side, the production technology for every sector is characterized by constant returns to scale (CRTS) Leontief production function. On the demand side, the utility function of households is assumed to be characterized by constant elasticity of substitution (CES). With regard to the capital side, it is assumed that production sector's demand for capital input is composed of all households' capital supply and government's capital endowment. As to the income tax structure, it is designed to be a progressive one. Further, the tax the individuals have to pay on their capital income is withheld in advance when corporations pay their corporate income tax. Su's model illustrates the process by which this withholding mechanism affects the household sector.

Su obtains the following results from his study. First, the old tax system prior to corporate income tax integration induces significant distortions to corporate investment decisions. Second, both the partnership method and the dividend-paid deduction method will not only considerably raise the real net national production (NNP), but also slightly lower the magnitude of inequality in income distribution. Third, those alternative schemes of corporate income tax integration at the personal level (especially the dividend-received exemption method) will accentuate the degree of distortions in economic efficiency and inequality in income distribution. Finally, under all of these four alternatives, both the corporate income tax revenues and the total government revenues will decrease. Therefore, Su suggests that the government should avoid choosing the dividend-received exemption method as the corporate income tax integration scheme.

Although Su's model includes progressive personal income tax which is consistent with the reality in Taiwan, data set is confined to only one period of time over 1981 so that his model can only be used to conduct the comparative static analysis of corporate income tax integration, but not to simulate the impacts on prices, capital accumulation, and economic growth.

Ma (2003) simulates the 1996 economy in Taiwan assuming that the new tax system with corporate income tax integration were already implemented. Besides, based on both the input-output table and taxation data in 1996, she formulates a Social Accounting Matrix (SAM) to be used as data base of her CGE model. She further investigates the impacts of corporate income tax integration on households' disposable income, the production value of industries, and economic growth through changes in tax system affecting rate of returns to individual capital, from the short-run and the long-run perspectives. In the short run, capital stock, investment, and the real wage rate are set to be fixed, the impacts of corporate income tax integration are mainly associated with changes in labor. In the long run, the inputs of labor and land are set to be constant, the effects of corporate income tax integration are mainly derived from changes in capital. Hence, Ma's CGE model is static in nature.

Ma builds her empirical single-country CGE model based on the ORANI-G model originally intended for the study of economic efficiency in Australia. Her model includes 39 production sectors and 5 consumer groups classified by average disposable income per household. In order that her study can simulate the impacts of corporate income tax integration on income distribution, she introduces income-expenditure equations into the ORANI-G model. On the other hand, since the tax burdens of various sources of capital could differ with corporate income tax integration, her model also includes the specification of capital structure in the production sector, namely, returns to capital and depreciation. Returns to capital are divided into interest, corporate business profits and non-corporate business profits. Further, corporate profits are composed of dividends and retained earnings.

The relevant assumptions and system design of corporate income tax integration in Ma's model are as follows. First, under the new tax system, corporations distribute dividends with tax credit designated to their shareholders so that households' tax payments can be reduced. Since the data of tax credit received by households is hard to obtain, Ma treats this tax credit as transfer payments from corporations to households accompanied by the dividends distributed. Second, since the tax treatments are different between profits of non-corporate business, corporate dividends, and corporate retained earnings, Ma sets respective tax rates for individual capital in her model so that the effects of corporate income tax integration can be captured through policies influencing these tax rates. Third, both of corporate financial and dividend policies are set to be determined endogenously, and are explained by a CES function. Fourth, the procedure of policy shocks caused by the new tax system is set to include: the removal of the business income taxes on profits of non-corporate business and corporate dividends, and increasing the business income tax on corporate retained earnings. Finally, she sets different substitution elasticities for both dividends with respect to retained earnings and interest with respect to profits. She argues that the magnitude of these two kinds of substitution elasticities is an important factor in deciding the degree of changes in after-tax returns to capital among industries.

The simulation results obtained from Ma's study are as follows. In the long run, corporate income tax integration leads to reduction in aggregate price level, significant increase in real GDP, improvement in competitiveness in foreign trade, and rising investment. With regard to income distribution, when the structural adjustment of the economy is complete in the long run, corporate income tax integration will lower the degree of inequality across income groups, with the effects positively related to the size of substitution elasticity. In the short run, however, corporate income tax integration would worsen macroeconomic performance than what is expected: economic resources can not be allocated efficiently, because of the rigidity in capital, investment, and wage rate. Furthermore, in the short run, income distribution becomes worse as substitution elasticity becomes greater.

Although Ma attempts to investigate the impacts of corporate income tax integration on households' disposable income, industries' production value, and economic growth by introducing the long-run and short-run settings in her model, it still is a static CGE model in nature. On the other hand, since the rate of returns to investment is assumed to be identical for all households in her model, the difference of effects on income distribution can not be captured.

From the long-run setting in Ma's (2003) study along with Su's 1986 simulation results from integration at the corporate level, economic growth and improvement in inequality of income distribution can be achieved by corporate income tax integration. However, the results from integration at the personal level in Su's and from the short-run setting in Ma's show that the degree of inequality in income distribution is heightened. Furthermore, Su argues that integration at the personal level will worsen economic efficiency.

#### 2.1.2.2 Dynamic Studies

Shea (1989) sets up a dynamic general equilibrium model for Taiwan to simulate the impacts of three alternative schemes of corporate income tax integration on investment, GNP, economic growth, prices, corporate savings, tax revenues, and other economic variables over the period 1985 – 1991. His dynamic general equilibrium model contains an input-output submodel comprised of equilibria between demand and supply of 8 industrial sectors, a macro submodel including labor marker, capital market, determinant equation of exchange rate and other equations of definitions, and several dynamic equations. The three alternative schemes of corporate income tax integration in his dynamic general equilibrium model are: full dividend-received credit method, partial dividend-received credit method, and partial dividend-paid credit method. Shea also adopts the concept of absolute tax incidence in his model. However, it should be noted that unlike others he does not divide households into several income groups, thus limiting the possibility to simulate the effects of corporate income tax integration on the degree of inequality in his study.

Shea attempts to investigate the impacts of corporate income tax integration on the economy mainly through the exogenous adjustment of income tax. The simulation procedures in his study are as follows. First, the values of all the exogenous variables are set over the period 1986 – 1991, at 1986 constant prices. Second, the simulation values of all the endogenous variables under the old tax system prior to corporate income tax integration over the same period are obtained. Third, the simulation values of all the endogenous variables under the corporate income tax integration over the same period are obtained through the modification of the tax system according to the three alternatives of the new system. Finally, in order to illustrate the impacts of corporate income tax integration on the economy, the simulation values derived under the old tax system are compared with those derived under the new one.

The following simulation results are expected from Shea's dynamic general equilibrium model: the implementation of corporate income tax integration in the future will significantly raise real disposable income, while it will also induce apparent rise to market interest rate and dramatic fall in corporate savings. On the other hand, the new tax system will not only have negligible impacts on real household consumption, prices, and exchange rate, but also slightly lower the total government current revenues, particularly the total direct tax revenues, among which corporate income tax revenues decrease while personal income tax revenues increase. Most importantly, it appears that corporate income tax integration will not effectively stimulate investment and promote economic growth.

According to his simulation results, Shea indicates that the worry of significant losses of government revenues by the opponents of corporate income tax integration is not warranted. Besides, the argument of increase in investment and economic growth by the advocators of integration is not supported empirically. Therefore, Shea concludes that "the benefits of income tax integration…are…mainly in rationalizing income tax system, reducing business income tax evasion, raising firms' equity financing ratio, and lowering personal income tax evasion by reducing corporate retained earnings." Furthermore, he suggests that "when an income tax integration system is implemented, the government should adopt an expansionary monetary policy to prevent the rising of interest rate caused by corporate-saving reductions and government-budget deficit, so that investment can be really stimulated."

Although Shea applies his dynamic general equilibrium model to the open economy (with monetary sector) to investigate the impacts of corporate income tax integration on prices, capital accumulation, and economic growth, his study does not examine the potential impacts of corporate income tax integration on income distribution because the household sector is not divided into several income groups. In addition, he employs a flat average income tax rate instead of a progressive marginal rate structure, which is more in line with the reality in Taiwan.

Jiang (2002) applies a dynamic general equilibrium model characterized by an endogenous formation of income distribution to examine the impacts of corporate income tax integration on income distribution and economic efficiency in Taiwan. She agrees with Sarte (1997) in that households with different income will differ in their rates of time preference and thereby adapt their behaviors variously to the implementation of corporate income tax integration. Therefore, under the progressive income tax structure, households' capital accumulations will vary with their income leading to the formation of a new income distribution.

Jiang's model includes household, production, and government sectors. The household sector consists of 5 income groups divided by average household disposable income. Suppose that the utility functions are identical for each household and for every period, and are characterized by CES. Households' savings can be used to purchase bonds and equity. The former let households earn interest income, while the latter generates dividends or capital gains. Further, assume that the tax rate on interest income and dividends is the same as that on labor income. As a result of arbitrage conducted by households, the after-tax rate of returns of various financial assets will be identical in equilibrium, so that households view these financial assets as completely substitutable to each other. Therefore, the after-tax capital income of households will not vary with their asset structures in equilibrium.

As to the production sector, it is assumed that the aggregate production function form of the whole society is time-invariant for every period and characterized by CRTS in the form of Cobb-Douglas. The production sector can finance its investment by issuing additional bonds, issuing new equity, and employing retained earnings. Among these sources of funds, interest payments to bond holders can be deducted from corporate income tax base to reduce tax liabilities. Besides, the marginal tax rate on corporate income is assumed to be constant, and the ratio of each source of funds to the total funds is exogenously given in the model.

With regard to government revenues, households' labor income, interest, and dividends are put together to be taxed, while capital gains are taxed separately. Assume that the asset structures for all households are identical, so that government can collect personal income tax, capital gains tax, and corporate income tax from private sectors for every period, according to the ratios of bonds and equity to households' assets and the ratios of dividends and retained earnings to households' assets. Further, suppose that the marginal tax rate on personal income which applies to all households is formulated by the government according to the equal sacrifice principle. That is, among households, the difference between utilities obtained from before and after-tax personal income is constant, and thereby the marginal tax rate derived from equal sacrifice principle is a progressive one (Young, 1990).

In Jiang's study, there are two alternative schemes of corporate income tax integration. One is full integration which is divided into two cases: basic model and case with low ratio of earnings retained. The other is partial integration characterized by dividend-paid deduction (or credit) method, which is also divided into two cases: case with an additional 10% surtax of corporate income tax imposed on retained earnings and case without surtax on retained earnings. Full integration is designed to abolish corporate income tax completely by switching the tax liabilities due to corporate earnings onto all shareholders, so that the distortions caused by corporate income tax can be completely eliminated. As a result, government income tax revenues are derived exclusively from personal income tax. Therefore, the cost of capital used by corporations becomes equal to interest rate because there is no more corporate income tax under full integration. Besides, since retained earnings are merged into households' income and are taxed as personal income tax under full integration, in case when the effective personal income tax rate is greater than that of corporate income tax, the tax liabilities of retained earnings will be exaggerated, which discourages capital accumulation by households.

On the other hand, partial integration keeps corporate income tax, while allowing dividends to be entirely deducted from corporate earnings. That is, under the dividend-paid deduction type of partial integration, the corporate income tax base will only includes retained earnings. Thus, Jiang points out that the dividend-paid deduction method will lower the cost of capital used by corporations, which in turn encourages corporations to raise their demand for capital and thereby affects households' intertemporal decisions. Hence, the households with low rate of time preference will tend to hold more capital goods, while households with high rate of time preference will tend to reduce their debts, so that government revenues will also be affected. In case there is a great difference between personal and corporate income tax rates, corporations will have incentives to increase retained earnings to avoid tax liabilities for their shareholders. Therefore, most countries attempting to integrate corporate and personal income taxes will take relevant actions to reduce this kind of distortions (Jiang, 2002). For example, the implementation of corporate income tax in Taiwan is accompanied by an additional 10% surtax of corporate income tax on retained earnings.

As to the simulation analysis, Jiang first sets the given values of all the exogenous variables in her model according to past studies and the secondary data over the period 1987 – 1996 in Taiwan. Second, she obtains the simulation values of all the endogenous variables in the stationary equilibrium under the old tax system prior to corporate income tax integration for 1997. Third, she finds the simulation values of all the endogenous variables under various corporate income tax integration schemes for the same period. Fourth, in order to illustrate the impacts of corporate income tax integration on the economy, all the simulation values derived under the old tax system are compared with those derived under the new one. Finally, sensitivity analysis and dynamic analysis are conducted.

The main simulation results from Jiang's study are as follows. Pareto improvement can be reached in the long run under partial integration, while the economic efficiency caused by the implementation of full integration depends upon both the ratio of corporate earnings retained and the difference between corporate and personal income tax rates. With regard to income distribution, corporate income tax integration can effectively lower the degree of inequality across income groups, and the effects vary positively with the size of integration. In addition, the 10% surtax of corporate income tax on retained earnings will prevent the integration schemes from improving economic efficiency and equalizing income distribution.

Jiang' study is one of the few that apply a dynamic general equilibrium model to investigate the impacts of corporate income tax integration on both income distribution and economic efficiency in Taiwan. However, her study ignores the impacts of corporate income tax integration on the choice of corporate sources of funds (i.e., bonds-equity policy) and the dividends payout policy (i.e., dividend-retention policy) by setting financing decision exogenously. Theoretically, with the distortion of double taxation on dividends, corporations tend to not only raise debt/equity ratio, but also retain more earnings. Once corporate income tax integration has been implemented, corporations should lower their debt/equity ratios and the ratios of earnings retained. Further, her study only emphasizes on the analysis of income distribution for households, but does not take the change of industry structure into account. Finally, her model, for a closed economy, does not include the foreign trade sector.

In sum, Shea (1989) predicts that corporate income tax integration will have no apparent influence on government revenues, the economic growth, and investment. On the other hand, Jiang's (2002) simulation results indicate that partial integration could attain Pareto improvement and equalize income distribution, while the additional 10% surtax of corporate income tax on retained earnings will generate the opposite results.

## 2.2 Studies without Using a General Equilibrium Model in Taiwan

### 2.2.1 Effects of Integration on Corporate Dividend Policies

The change in dividend payout ratio can be used to examine whether corporate dividend policies are significantly affected by corporate income tax integration. Wang (2003) points out that since the objective of the dividend policies set by corporations is to minimize the total tax liabilities of corporations themselves and their shareholders and to maximize shareholders' after-tax returns, corporations usually will take into account the corporate income tax, shareholders' dividend income tax, capital gains tax, and tax on retained earnings when they are setting dividend policies.

Kuo (2000) finds from his empirical study the fact that the additional 10% surtax of corporate income tax on retained earnings significantly influences the listed companies' inclination to induce more dividends against retained earnings. At the same time, Lu (2000) further suggests that with corporate income tax integration corporations tend to raise their cash dividend payout ratio, but decrease stock dividends. On the other hand, he points out that there is significantly positive relationship between the imputation tax credit ratio and the dividend/retention ratio. That is, the higher the imputation tax credit ratio, the higher is the dividend/retention ratio. Chang (2002) also attempts to investigate whether corporations will raise dividend payout ratio to avoid or reduce the additional 10% surtax imposed on retained earnings. Her empirical result shows that the higher the imputation tax credit ratio, the higher the cash dividend payout ratio would be. This finding demonstrates that corporations will make use of dividends to lower retained earnings, thereby to avoid the additional 10% surtax of corporate income tax imposed on retained earnings.

Like Lu (2000), Wang (2003), reaches the same result that there is a significantly positive relationship between the imputation tax credit ratio and the dividend payout ratio. However, one of his study results is opposed to Lu's (2000), displaying that under corporate income tax integration there is no apparent increase in cash dividend payout ratio, and corporations significantly prefer stock dividends to cash dividends. Wang and Chen (2003) in investigating the effects of the new tax system implemented in 1998 on corporate shareholder structures, suggest that corporations tend to increase dividend payout ratio under this new tax system, causing individual shareholders to pay more income tax. They further point out that in order to reduce the tax burdens, individual shareholders may shift their stocks to some holding companies to make dividend income inaccessible for government tax under the new income tax system. Furthermore, Wang and Chen point out the following: first, the percentages of corporate shareholders of listed companies become higher after 1998; second, the higher the corporations' imputation tax credit ratios under the new tax system, the lower the percentages of their corporate shareholders; third, the percentages of overseas shareholders become smaller after 1998.

In sum, all these studies share a common result that the 1998 tax reform significantly affects corporate dividend policies and induces corporations to raise dividend payout ratio.

### 2.2.2 Effects of Integration on Corporate Capital Accumulation

Some researchers have argued that the corporate income tax integration would improve the accumulation of corporate capital. This is because corporate retained earnings are accumulated for their genuine needs, not for the purpose of avoiding tax liabilities. On the other hand, the Ministry of Finance (1998) further notes that under the new tax system, by means of distributing their current earnings in the form of stock dividends, corporations can acquire their internal funds without being levied an additional 10% surtax. Furthermore, the corporate income taxes on stock dividends, paid at the corporate level, can be credited against personal income tax. Therefore, the Ministry of Finance concludes that the corporate income tax integration with an additional 10% surtax of corporate income tax on retained earnings will not induce unfavorable impact on the formation of corporate internal funds.

However, Huang's (2001) empirical study examines the impacts of corporate income tax integration on corporate capital structure, and obtains different results from the official arguments. She points out that the additional 10% surtax of corporate income tax on retained earnings will result in the reduction in corporate internal funds, the increase in corporate capital cost, discouraging corporations from capital accumulation, and impairing their financial decisions.

With regard to the formation of corporate external funds, the Ministry of Finance (1998) argues that the corporate income tax integration will bring positive effects. The reason is that under the full imputation system the top tax rate that shareholders have to pay for their investment decreases from 55% to 40%, which will encourage investors to purchase corporate stocks and thereby be conducive to corporation's equity financing.

Nevertheless, Chang (2002) indicates that those industries which enjoy the tax relief and are relatively capital intensive usually accumulate their funds through retained earnings before 1998. But, the corporate income tax integration with an additional 10%

surtax on retained earnings discourages these industries from accumulating funds by this method. Liang (2003) also presents the same argument as Chang, arguing that imposing an additional 10% surtax of corporate income tax on retained earnings, which are intended to support future operation and investment, will have unfavorable impacts on corporate capital accumulation and reinvestment. Imposing 10% surtax on retained earnings will also aggravate corporate tax liabilities and raise costs of corporate internal funds, contrast to the intent of corporate income tax integration to cut tax. If, as a consequence, corporations shift to external financing, they might encounter higher financial risks in the future. Therefore, Liang suggests that the government should seriously consider abolishing the "additional 10% surtax on retained earnings."

Even though the theoretical argument presented by some researchers and the government authority suggests positive effects of the new tax system on corporate capital accumulation in Taiwan, there is no empirical evidence to support this argument. Rather, most of the empirical results show that the additional 10% surtax of corporate income tax on retained earnings impairs corporate capital accumulation.

# CHAPTER THREE

# METHODOLOGY AND MODEL SPECIFICATION

The literature on corporate income tax integration shows that since it takes time for capital to adjust to the optimal level, the dynamic effects are significantly divergent from the static ones; that the effects of partial integration plans are always different from those of full integration ones; and that integration plans at the personal level benefit mainly the higher income households, thus, less progressive than those at the corporate level. Therefore, it is expected that empirical results vary by research objectives, data sources, analytical approaches, and model specifications including assumptions of variables and functions, choices of parameters, integration alternatives, and simulation schemes used in the study.

This chapter will first present the specification of the behavior of three types of agents: households, producers, and the government. Then, the equilibrium conditions for each sector and for the economy as a whole will be identified. The structure of the model and the behavioral assumptions of producers and the government are essentially adopted from those presented in Pereira (1988), and partly from Romer (2001), while some behavioral assumptions of the household are derived from Sarte (1997) and Jiang (2002). Some modifications will be made according to the objectives of this study to fit the realistic economic situation.

### 3.1 Summary of Behaviors of Agents

#### 3.1.1 Households

One of the objectives of this study is to evaluate the effects of tax reform on income distribution. In order to reach this objective, it is necessary to divide households into several groups.

In this study, there are m income groups classified by income. Adopted from the setting in Sarte (1997) and Jiang (2002), the rates of time preferences are assumed to be invariant in terms of time but different across income groups to show their various behavioral responses to the change in tax policies, so that discount factors may vary with income groups over the model horizon. Further, since the higher income groups have lower rates of time preference than the lower income groups (Jiang, 2002), the former has higher discount factor which inversely varies with rate of time preference.

Following the work of Pereira (1993), the income of each income group at every period consists of labor income, transfer income from the government, and wealth-generated income (i.e., capital income). The wealth-generated income is further divided into dividend income, capital gains, and interest income from public and private bonds.

Based on the principle of equity, personal income tax, a linear progressive income tax scheme with time-invariant marginal tax rate for each income group, is imposed in this study on the income from dividends and interest, as well as on labor income, but not on capital gains.

Income is either saved in the form of the increment in wealth or used for the purchase of consumption goods which is subject to an ad valorem sales tax. Savings are defined as the difference between disposable income and consumption, and can be referred to as intertemporal transfers of wealth to finance future consumption in this study (Pereira, 1993). Further, savings can be used to invest in financial assets: public and private bonds as well as equity. Bonds will yield interest income to households, while equity will generate dividends and capital gains which will be imposed on an ad valorem tax.

Since at equilibrium, different financial assets will yield the same after-tax rate of returns, households will treat them as perfect substitutes. Therefore, the structure of asset portfolio in equilibrium will not make any difference for households in terms of the after-tax rate of returns. For the sake of simplicity, it is assumed in this study that the actual structure of the asset portfolio held by households will be determined by the market equilibrium conditions. That is, all income groups will have the same structure of asset portfolio as that in the market as a whole. Consequently, each income group will own a share of the market portfolio identical to its share of total wealth in the economy (Pereira, 1993). It should be noted that since different marginal tax rates are applied to different income groups, the after-tax rates of returns from the same financial assets will be different across income groups.

Each household's utility function, defined over consumption goods and leisure, is well-behaved and time-invariant for all income groups, and is characterized by a two-level nested structure, as in Pereira (1988). Households will make their consumption/leisure decisions at the top level of utility function, while "their pattern of disaggregated consumption" will be chosen at the lower level (Pereira, 1988). Households pursue the objective to maximize the present value of intertemporal utility, subject to an intertemporal budget constraint and some other conditions. The intertemporal budget constraint implies that the sum of the present value of the current and the future incomes has to be equal to the present value of the expenditures (Pereira, 1993). The optimal household consumption/savings and labor/leisure decisions can be derived from the intertemporal optimization process.

### 3.1.2 Producers

The impacts of corporate income tax integration on resource allocation are also examined in this study. With respect to the production sector, this study will focus on investigating the different allocation effects of integration on industries with different degrees of incorporation.

When discussing the production dynamics, industry subsectors with different degrees of incorporation, as identified by the different corporate income tax rates, will be included in this study. Furthermore, since it takes time for capital to build, install, and adjust to its optimal level and capital is not completely mobile across industries, adjustment costs of investment exist in reality and there are different rates of returns to capital to different industries (Johnson, 1989; Pereira, 1988; Romer, 2001). Hence, adjustment costs will be taken into account in this study and are presumed to affect industries' investment expenditures and thereby investment behavior of producers.

The behavioral assumptions of producers in this study follow those of Pereira's (1988). The revenues of producers come from sales of outputs, while their expenditures consist of labor costs, intermediate input costs, interest payments on outstanding bonds, and investment expenditures. For completeness, the investment tax credit and the taxes on the use of labor services will be included as well. Therefore, the actual investment expenditures in this study encompass three factors: acquisition costs, adjustment costs, and investment tax credit.

Theoretically, there are two types of adjustment costs: internal and external. The former varies with capital stock, while the latter is associated with the change in price of investment goods. For simplicity, only internal adjustment costs are considered in this study.

From the perspective of accounting, profits are not identical to cash (Hand, Issaks, and Sanderson, 2005) and cash flows indicate a corporation's financial health and are "likely to be correlated with future profitability" (Romer, 2001). Further, those corporations without enough cash to support their operations are most likely to fail. Hence, Hand, Issaks, and Sanderson (2005) point out that "many firms have failed, not through the absence of profits, but simply from their inability to pay wages and creditors on time." Accordingly, the key factor for the development and even the survival of a corporation is cash flows, not profits. Therefore, the producer's dynamic behavior in this study is to maximize his intertemporal net cash flows, rather than his intertemporal profits. Hereafter this study will employ the concepts used in a corporation's income statement and cash flow statement to illustrate the structure of producers' revenues and expenditures.

In an income statement (Diamond, Stice, and Stice, 2000; Pyle and Larson, 1981), the gross profits of producers are obtained by subtracting sales costs, represented by the use costs of intermediate inputs, from the sales revenues. Then, the operating income is equal to gross profits minus operating expenses which would include depreciation allowances and the use costs of labor services in this study. An ad valorem labor tax is imposed on the use of labor services. Such a labor tax reflects the fraction that producers pay for their employees including pension program, unemployment insurance, and compensation. By subtracting other expenses such as the interest expenses from operating income, we can further obtain earnings before taxes. Finally, an ad valorem corporate income tax is applied to earnings before taxes, and thereby from which we can derive earnings (also called net income) which can be either distributed as dividends or retained. Hence, both interest payments and depreciation allowances can be deducted from the corporate income tax base.

On the other hand, in a cash flow statement (Atrill and McLaney, 1997; Diamond, Stice, and Stice, 2000; Vickman, 1995), cash flows are generated from three sources: operating activities, investing activities, and financing activities. Since depreciation allowances, already treated as one of the operating expenses in the income statement, is not a real cash expense, it is added up with net income to obtain the net cash provided by operating activities. The net cash used in investing activities in this study is represented by actual investment expenditures. The net cash provided by financing activities increases by issuing new bonds and additional equity, and decreases when dividends are paid to shareholders. Thus, producers' net cash flows during period t are composed of changes in cash flows from operating activities, investing activities, and financing activities.

With respect to the functions associated with the production sector, the value-added production function, defined over the space of capital and labor and assumed to be time-invariant for all producers, is "twice continuously differentiable, strictly increasing in every input, and concave"(Pereira, 1988) ; the adjustment costs are assumed to be a convex function of the gross investment, and to be "twice continuously differentiable, nonnegative, and strictly increasing" (Johnson, 1989; Pereira, 1993); and the evolution of capital stock can be represented by an equation of motion obtained by subtracting the depreciation of capital from the gross investment.

As to the demand of funds, producers' expenditures on operating activities and investing activities can be financed by retained earnings and issuance of new bonds and additional equity. The issuance of new bonds and additional equity is equivalent to an increment in producers' financial liabilities. As in Pereira (1988) and Jiang (2002), for simplicity, the industry-specific dividend/retention ratio and debt/equity ratio are exogenously given in this study, even though both postulations are inconsistent with the reality. Furthermore, capital markets are assumed to be perfect.

The objective of each producer is to maximize the present value of intertemporal net cash flows, subject to a value-added production function, an adjustment costs function, the equation of motion for capital stock, and some boundary conditions. Following the optimal behavior of producers, the optimal production and investment decisions can be derived. That is, producers can determine their optimal plans for output supply and demand for labor, intermediate inputs, investment, and (external) funds.

### 3.1.3 The Government

Even a perfectly competitive market can not solve all agents' economic problems so that the government has a legitimate role to enhance the performance of the market economy (Tresch, 2002). The government participates in four types of economic activities: collecting taxes, making transfer payments to households, providing public goods and services, and selling public bonds as means of government deficit financing (Pereira, 1988, 1993).

By incorporating the tax systems set in Pereira (1988), government revenues in this study are drawn from six different sources: a progressive personal income tax on households' labor income, interest income, and dividend income; an ad valorem sales tax on household consumption goods; an ad valorem capital gains tax on households and industries; an ad valorem labor tax on labor services used by industries and the government; an ad valorem investment tax credit, which actually is a subsidy from the government to industry's acquisition costs; and an ad valorem corporate income tax on industries' earnings before taxes.

Following the work of Pereira (1988), in this study, the government imposes a labor tax on itself for the use of labor services. Therefore, with the disappeared income effect, the price effect of this labor tax on the government can be interpreted as the opportunity cost of the use of labor services by the government (Pereira, 1988). To show that the government provides better pension programs for its employees, the labor tax rate on the government is set to be higher than that on industries.

On the expenditure side, government revenues can be used to make lump-sum transfer payments and interest payments on outstanding public bonds, and to finance the provision of public goods. The costs of the provision of public goods include investment expenditures, the costs of using labor services, and expenditures on consumption goods. For simplicity, adjustment costs will be ignored in the government sector.

The government is allowed to run deficits in this study so that in each period the sum of government expenditures may exceed total government revenues. Then, following the work of Shea (1989), the induced government deficits are assumed to be financed by selling public bonds in the bond market, with the size of bonds issued equal to the increment in government liabilities.

With respect to the functions related to the government sector, public goods are assumed to be provided by using consumption goods, labor and capital as inputs through a well-behaved, time-invariant, and additively separable provision function. Following the assumption in Pereira (1988), the public good, with a shadow price to reflect its social value, is not subject to market pricing. It does not enter the set of household budget constraints, thus is not a control variable in households' maximization problems. The evolution of government liabilities can be expressed by an equation of motion derived by subtracting government revenues from the sum of transfer payments, expenditures on the provision of public goods, and interest payments on outstanding public bonds.

The optimal government behavior can be obtained from the similar maximization process as that used in the household sector. That is, the objective of the government can be achieved by maximizing the present value of the intertemporal provision of public goods, subject to an intertemporal budget constraint and some other conditions. The intertemporal budget constraint implies that the total present value of current and future government revenues has to be equal to the total present value of current and future government expenditures. In turn, the government can determine the optimal quantities of demand for consumption goods, labor, investment goods and funds.

## 3.2 The Dynamic General Equilibrium Model of the Taiwan Economy

### 3.2.1 Households

Households receive their income from labor services and wealth, and from government transfers. Income is spent on consumption and savings. Savings can be invested in bonds and equity. Households maximize their intertemporal utilities represented by an additively time-separable and semi-log linear function, discounted by a time-invariant factor varying with income groups. Households are subject to a recursive sequence of intertemporal budget constraints and some boundary conditions.

The household's utility is a function of consumption and leisure, expressed as:

$$(1)\quad U_h(Y_{hjt}, L_{ht}) = \sum_j \alpha_{hj} \ln Y_{hjt} + (1 - \sum_j \alpha_{hj}) \ln(L_h^* - L_{ht}),\quad h = 1,\ldots, m;\; j = 1,\ldots, J,$$

where $U_h(\bullet)$ is the utility of household h, with $U_1(\bullet)$ and $U_m(\bullet)$ denoting the utilities of the lowest and the highest income groups, respectively; $Y_{hjt}$ is the quantity of consumption good j demanded by household h at period t; $(L_h^* - L_{ht})$ represents the leisure taken by household h, with $L_h^*$ and $L_{ht}$ being the total available time and labor supply of household h, respectively; and $\alpha_{hj}$ is the expenditure share of household h on consumption good j. Here one thing has to be kept in mind, i.e., in order to be able to solve households' and hence the whole model's maximization problems, the number of consumption goods, J, must be no less than that of income groups, m, which is for mathematic reasoning, rather than economic logic.

The household's gross income can be written as:

$$(2)\quad GI_{ht} = w_t L_{ht} + (1 - \sum_i e_{it})\, r_t W_{ht} + \sum_i e_{it} [\, DIV_{it} / (P_{Eit-1} E_{it})\, ]\, W_{ht}$$
$$+ \sum_i (P_{Eit} - P_{Eit-1}) E_{hit} + TR_{ht}, \qquad i = 1,\ldots, J, I,$$

where $GI_{ht}$ is the gross income of household h, $W_{ht}$ is household h's wealth at the beginning of period t, $DIV_{it}$ is dividends distributed by industry i, $E_{it}$ is the quantity of equity of industry i at the beginning of period t, $E_{hit}$ is the quantity of equity of industry i owned by household h, $TR_{ht}$ is the transfers received by household h, $P_{Eit}$ and $P_{Eit-1}$ are prices of equity of industry i at period t and t-1 respectively, $w_t$ is the wage rate, $r_t$ is the interest rate, $e_{it}$ is the share of equity of industry i in household h's wealth (i.e., $P_{Eit-1} E_{hit} / W_{ht}$), and $( 1 - \sum_i e_{it} )$ is the share of public and private bonds in household h's wealth. "I" represents the investment sector.

Hence, $w_t L_{ht}$ represents household h's labor income. Wealth-generated income of household h includes: dividend income $\sum_i e_{it} [ DIV_{it} / ( P_{Eit-1} E_{it} ) ] W_{ht}$, capital gains $\sum_i (P_{Eit} - P_{Eit-1}) E_{hit}$, and interest income $( 1 - \sum_i e_{it} ) r_t W_{ht}$.

The household's disposable income can be written as:

$$(3) \quad DI_{ht} = a_h + ( 1 - pit_h ) \{w_t L_{ht} + ( 1 - \sum_i e_{it} ) r_t W_{ht} + \sum_i e_{it} [ DIV_{it} / ( P_{Eit-1} E_{it} ) ] W_{ht} \} + ( 1 - cgt_{ht} ) \sum_i ( P_{Eit} - P_{Eit-1} ) E_{hit} + TR_{ht}$$

where $DI_{ht}$ is the disposable income of household h, $a_h$ is set to be negative to make sure that marginal tax rates would be greater than average tax rates, $pit_h$ is the marginal income tax rate for household h, and $cgt_{ht}$ is the capital gains tax rate. In this case of linear progressive income tax schedule, the marginal income tax rate for higher income groups is set to be greater than that for lower income groups.

The household's after-tax expenditures can be written as:

$$(4) \quad ATE_{ht} = \sum_j ( 1 + st_{jt} ) P_{jt} Y_{hjt}$$

where $ATE_{ht}$ is the after-tax expenditures of household h, $st_{jt}$ is the ad valorem sales tax rate on consumption good j, $P_{jt}$ is the price of consumption good j. Hence, $P_{jt} Y_{hjt}$ is the pretax expenditures of household h on consumption good j.

The household's savings can be written as:

(5) $SAV_{ht} = DI_{ht} - ATE_{ht}$

$$= a_h + ( 1 - pit_h ) \{w_t L_{ht} + ( 1 - \sum_i e_{it} ) r_t W_{ht} + \sum_i e_{it} [ DIV_{it} / ( P_{Eit-1} E_{it} ) ] W_{ht} \}$$

$$+ ( 1 - cgt_{ht} ) \sum_i ( P_{Eit} - P_{Eit-1} ) E_{hit} + TR_{ht} - \sum_j ( 1 + st_{jt} ) P_{jt} Y_{hjt}$$

where $SAV_{ht}$ is the savings of household h, defined by the difference between disposable income and the after-tax consumption expenditures. On the other hand, since savings are conceived as intertemporal transfers of wealth to finance future consumption, it can also be written as:

(6) $SAV_{ht} = W_{ht+1} - W_{ht}$

where $W_{ht+1}$ is household h's wealth at the beginning of period t+1, i.e., "wealth at the end of period t to be transferred into period t+1 after all expenditures have been incurred" (Pereira, 1988). That is, savings at period t are equivalent to the increment in wealth ($W_{ht+1} - W_{ht}$) and "represent the new funds" supplied by household h at period t to "the rest of the economy" (Pereira, 1988). Furthermore, Savings can be invested in public and private bonds and equity. Therefore, savings can be further written as:

(7) $SAV_{ht} = FD_{ht}$

(8) $SAV_{ht} = \sum_i ( B_{hit+1} - B_{hit} ) + ( B_{hgt+1} - B_{hgt} ) + \sum_i P_{Eit} ( E_{hit+1} - E_{hit} )$

where $FD_{ht}$ is the new funds supplied by household h at period t, $B_{hit}$ and $B_{hit+1}$ are bonds of industry i owned by household h at the beginning of period t and t+1 respectively, $B_{hgt}$ and $B_{hgt+1}$ are government bonds owned by household h at the beginning of period t and t+1, respectively, $E_{hit}$ and $E_{hit+1}$ are the quantities of equity of industry i owned by household h at the beginning of period t and t+1, respectively. Further, $(B_{hit+1} - B_{hit})$ is the increment in industry i's bonds owned by household h, $(B_{hgt+1} - B_{hgt})$ is the increment in government bonds owned by household h, and $(E_{hit+1} - E_{hit})$ is the increment in quantity of industry i's equity owned by household h.

Hence, the savings of household h at period t can be represented by three components: the changes in period t of all private bonds, government bonds, and values of all equity owned by the household.

On the other hand, it is assumed in this study that all households have the same asset portfolio structure as that in the market. Therefore, each household will own a share of the market portfolio equivalent to his share of total wealth in the economy. Accordingly, each household's wealth can be written as:

(9) $W_{ht} = \sum_i B_{hit} + B_{hgt} + \sum_i P_{Eit-1}\, E_{hit}$

(10) $B_{hit} = \xi_{ht}\, B_{it}$

(11) $B_{hgt} = \xi_{ht}\, B_{gt}$

(12) $E_{hit} = \xi_{ht}\, E_{it}$

where $\xi_{ht}$, represented by $W_{ht} / \sum_h W_{ht}$, is household h's share of the market portfolio at the beginning of period t. $B_{it}$ and $B_{gt}$ are outstanding bonds of industry i and the government at the beginning of period t, respectively. Equation (9) represents household h's wealth at the beginning of period t, consisting of private and public bonds and equity owned by the household. Equations (10) – (12) represent the specific asset owned by household h as a share of the corresponding market asset.

The optimal behavior of household h is to maximize the present value of the utility subject to a recursive sequence of budget constraints and some boundary conditions (Leonard and Long, 1992). Formally, the objective of household h is to find $(Y_{hjt}, L_{ht}; W_{ht})$ that maximizes

(13) $\sum_{0 \leq t \leq T} \delta_h^t U_h(Y_{hjt}, L_{ht}) = \sum_{0 \leq t \leq T} \delta_h^t [\sum_j \alpha_{hj} \ln Y_{hjt} + (1 - \sum_j \alpha_{hj}) \ln (L_h^* - L_{ht})]$

subject to the following constraints for each period t

(14) $W_{ht+1} - W_{ht} = \sum_i (B_{hit+1} - B_{hit}) + (B_{hgt+1} - B_{hgt}) + \sum_i P_{Eit} (E_{hit+1} - E_{hit})$

$$= a_h + (1 - pit_h) \{w_t L_{ht} + (1 - \sum_i e_{it}) r_t W_{ht} + \sum_i e_{it} [DIV_{it} / (P_{Eit-1} E_{it})] W_{ht}\}$$

$$+ (1 - cgt_{ht}) \sum_i (P_{Eit} - P_{Eit-1}) E_{hit} + TR_{ht} - \sum_j (1 + st_{jt}) P_{jt} Y_{hjt}$$

with

(15) $Y_{hjt} \geq 0$, and $0 \leq L_{ht} \leq L_h^*$, for all $1 \leq j \leq J$, $1 \leq h \leq m$

(16) $W_{h0} = W_h^* = \sum_i B_{hi}^* + B_{hg}^* + \sum_i P_{Ei,-1} E_{hi}^*$

(17) $W_{hT+1} = 0$; $B_{hiT+1} = 0$; $B_{hgT+1} = 0$; $P_{EiT} E_{hiT+1} = 0$

where $\delta_h$ is the household-specific discount factor for household h with $0 < \delta_1 \leq \delta_2 \leq \ldots \leq \delta_h \leq \ldots \leq \delta_m < 1$, $W_h^*$, $B_{hi}^*$, and $B_{hg}^*$ are respectively initial values of wealth of household h, bonds of industry i owned by household h, and government bonds owned by household h. $P_{Ei,-1}$ is the price of equity of industry i at the period -1 (i.e., one period before the initial period); $E_{hi}^*$ is the initial quantity of equity of industry i owned by household h; thus, $P_{Ei,-1} E_{hi}^*$ is the initial value of equity of industry i owned by household h. $W_{hT+1} = 0$, $B_{hiT+1} = 0$, $B_{hgT+1} = 0$, and $P_{EiT} E_{hiT+1} = 0$ all are terminal values. Equation (14) denotes the budget constraint, and (15) – (17) are boundary conditions. Specifically, $W_{hT+1} = 0$ implies that the present value of current and future disposable income of household h has to be equal to the present value of his after-tax expenditures (Pereira, 1988).

Hereafter this study will apply the maximum principle, the central result of optimal control theory originally developed by Pontryagin et al., to the dynamic general equilibrium model to find the optimal household consumption/savings and labor/leisure decisions.

The maximization problem in discrete time for the behavior of household h in this study can be restated in terms of the optimal control problem. Since the terminal condition of the state variable in the household sector is given in the finite horizon model (i.e., both $W_{hT+1}$ and T are fixed), this kind of optimal control problem is attributed to the case of fixed terminal point with the transversality condition replaced by the terminal condition (Chiang, 1992).

First, the Hamiltonian at period t of this problem can be written as:

$$(18)\quad H_{ht}(Y_{hjt}, L_{ht}; W_{ht}; \lambda_{ht}) = \delta_h^{\,t}\,[\textstyle\sum_j \alpha_{hj}\, ln\, Y_{hjt} + (1 - \sum_j \alpha_{hj})\, ln\,(L_h^{*} - L_{ht})] + \lambda_{ht}\,\{a_h$$

$$+ (1 - pit_h)\,[w_t L_{ht} + (1 - \textstyle\sum_i e_{it})\, r_t\, W_{ht} + \sum_i e_{it}\,[\, DIV_{it} / (P_{Eit-1}\, E_{it})\,]\, W_{ht}\,]$$

$$+ (1 - cgt_{ht}) \textstyle\sum_i (P_{Eit} - P_{Eit-1})\, E_{hit} + TR_{ht} - \sum_j (1 + st_{jt})\, P_{jt}\, Y_{hjt}\}$$

where $H_{ht}(\bullet)$ is the Hamiltonian function of household h at period t. In this optimal control problem, $Y_{hjt}$ and $L_{ht}$ are the control variables which "can be controlled freely", $W_{ht}$ is the state variable "whose value at any time is determined by past decision", and $\lambda_{ht}$ is the costate variable which is "the shadow value of the state variable" (Romer, 2001). The Hamiltonian function is similar to the Lagrangian function. Thus, the dynamic shadow price, $\lambda_{ht}$, is akin to a Lagrangian multiplier and can be interpreted as the marginal utility of household h's wealth at period t.

According to the maximum principle, the necessary conditions are set as follows:

$$(19)\quad \partial H_{ht} / \partial Y_{hjt} = 0, \qquad\qquad t = 0, 1, \ldots, T$$

(20) $\partial H_{ht} / \partial L_{ht} = 0,$ $\quad t = 0, 1, \ldots, T$

(21) $\partial H_{ht} / \partial W_{ht} = -(\lambda_{ht} - \lambda_{ht-1}),$ $\quad t = 1, 2, \ldots, T$

(22) $\partial H_{ht} / \partial \lambda_{ht} = W_{ht+1} - W_{ht},$ $\quad t = 0, 1, \ldots, T$

with

(23) $W_{hT+1} = 0$

Equations (21) and (22) are the equations of motion, respectively for the costate variable, $\lambda_{ht}$, and the state variable, $W_{ht}$ (Chiang, 1992). Equation (23) is the terminal condition for the state variable, and is also the transversality condition of this optimal control problem.

Then, from the derivatives of the Hamiltonian with respect to $Y_{hjt}$, $L_{ht}$, $W_{ht}$, and $\lambda_{ht}$, we can obtain the following first-order conditions for the optimization problem of household h at period t.

(24) $\partial H_{ht} / \partial Y_{hjt} = \delta_h^t [\alpha_{hj} / Y_{hjt}] + \lambda_{ht} [-(1 + st_{jt}) P_{jt}] = 0$

(25) $\partial H_{ht} / \partial L_{ht} = \delta_h^t [-(1 - \sum_j \alpha_{hj}) / (L_h^* - L_{ht})] + \lambda_{ht} [(1 - pit_h) w_t] = 0$

(26) $\partial H_{ht} / \partial W_{ht} = \lambda_{ht} (1 - pit_h) \{ (1 - \sum_i e_{it}) r_t + \sum_i e_{it} [DIV_{it} / (P_{Eit-1} E_{it})] \}$

$$= -(\lambda_{ht} - \lambda_{ht-1})$$

(27) $\partial H_{ht} / \partial \lambda_{ht} = a_h + (1 - pit_h) \{ w_t L_{ht} + (1 - \sum_i e_{it}) r_t W_{ht}$

$$+ \sum_i e_{it} [DIV_{it} / (P_{Eit-1} E_{it})] W_{ht} \} + (1 - cgt_{ht}) \sum_i (P_{Eit} - P_{Eit-1}) E_{hit}$$

$$+ TR_{ht} - \sum_j (1 + st_{jt}) P_{jt} Y_{hjt}$$

$$= W_{ht+1} - W_{ht}$$

### 3.2.2 Producers

Producers maximize the present value of their net cash flows subject to the production technology, an adjustment costs function, the equation of motion for capital stock, and boundary conditions. Net cash flows show that in addition to retained earnings, producers' actual investment expenditures at period t can also be financed by net income and external funds (i.e., issuing new bonds and additional equity) during period t.

The production technology of an industry is represented by a time-invariant Leontief structure expressed as:

(28) $Y_{it} = \text{Min}\{ F_i( K_{it}, L_{it}); Y_{i1t} / c_{i1}, Y_{i2t} / c_{i2}, \ldots, Y_{ijt} / c_{ij}, \ldots, Y_{iJt} / c_{iJ} \}, \quad i = 1,\ldots,J,I,$

where $Y_{it}$ is the output of industry i; $F_i( K_{it}, L_{it})$ is value-added production function of industry i, characterized by Cobb-Douglas structure, defined over the domain of capital $K_{it}$ and labor $L_{it}$; $Y_{ijt}$ is the output of industry j used as intermediate input by industry i, for j = 1,...,J. The Leontief parameter (i.e., input coefficient) for industry i, $c_{ij}$, represents the amount of intermediate input j needed to produce one unit of output i.

The time-invariant Cobb-Douglas production function can be expressed as:

(29) $F_i( K_{it}, L_{it}) = Y_{it} = K_{it}^{\beta i} L_{it}^{1-\beta i}, \quad i = 1,\ldots,J,I,$

where $\beta_i$ is the capital share in industry i, and $1-\beta_i$ is the labor share.

The evolution of capital stock can be written as:

(30) $K_{it+1} - K_{it} = I_{it} - \Phi_{it} K_{it}$

where $K_{it+1}$ and $K_{it}$ are industry i's capital stocks at the beginning of period t + 1 and t, respectively; $I_{it}$ is gross investment in industry i; and $\Phi_{it}$ is the industry-specific depreciation rate of capital stock in industry i. Hence, $( K_{it+1} - K_{it} )$ represents industry i's net investment at period t, and $\Phi_{it} K_{it}$ represents the depreciation in industry i.

The gross profits and the operating income can be written as:

(31) $GP_{it} = P_{it} F_i( K_{it}, L_{it}) - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) Y_{it}$

$= [ P_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) ] K_{it}^{\beta i} L_{it}^{1-\beta i}$

(32) $OI_{it} = GP_{it} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ]$

$$= [ P_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) ] K_{it}^{\beta i} L_{it}^{1-\beta i} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ) ]$$

where $GP_{it}$ and $OI_{it}$ are industry i's gross profits and operating income, respectively. The price of intermediate input j is represented by $P_{jt}$. The labor tax rate, $lt_t$, is assumed to be constant across producers, $lt_t w_t L_{it}$ represents the labor tax paid by industry i, and $( 1+ lt_t ) w_t L_{it}$ represents the total use costs of labor services. $\Phi_{it}'$ is the depreciation rate for tax purpose, $K_{it}'$ is the capital stock that is subject to tax, and $\Phi_{it}' K_{it}'$ represents the depreciation allowances in industry i at period t.

The interest payments due to outstanding bonds can be written as:

(33) $INT_{it} = r_t B_{it}$

where $INT_{it}$ is the interest payments by industry i. Since interest payments can be deducted from corporate income tax base, the actual interest payments can be represented by $( 1- cit_{it} ) r_t B_{it}$. The industry-specific corporate income tax rate on industry i, $cit_{it}$, reflects the different degrees of incorporation across industry (Pereira, 1988). The higher the degree of incorporation is, the higher the corporate income tax rate will be applied to.

The earnings before taxes and the earnings (i.e., net income) can be expressed as:

(34) $EBT_{it} = OI_{it} - INT_{it}$

$$= [ P_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) ] K_{it}^{\beta i} L_{it}^{1-\beta i} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ] - r_t B_{it}$$

(35) $NI_{it} = ( 1- cit_{it} ) EBT_{it}$

$$= ( 1- cit_{it} )\{ [ P_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) ] K_{it}^{\beta i} L_{it}^{1-\beta i} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ] - r_t B_{it} \}$$

(36) $NI_{it} = RE_{it} + DIV_{it}$

where $EBT_{it}$ and $NI_{it}$ are industry i's earnings before taxes and net income, respectively. $RE_{it}$ is industry i's retained earnings.

The adjustment costs function is set to be of a quadratic form in this study and expressed as:

(37) $ADC_i ( I_{it} ) = 0.5\ b_i\ I_{it}^2$

where $ADC_i (\bullet)$ is the adjustment costs function, and $b_i$ (which is positive) is the industry-specific and time-invariant adjustment costs parameter.

The investment tax credit subsidized by the government can be written as:

(38) $ITC_{it} = itc_{it}\ P_{It}\ I_{it}$

where $ITC_{it}$ is the industry-specific investment tax credit which industry i benefits from the purchase of investment goods. $itc_{it}$ is the ad valorem investment tax credit rate on the acquisition costs. $P_{It}\ I_{it}$ represents the acquisition costs, and $P_{It}$ is the price of investment goods.

The actual investment expenditures can be written as:

$$(39)\ AIE_{it} = P_{It}\ I_{it} + ADC_i ( I_{it} ) - ITC_{it} = P_{It}\ I_{it} + 0.5\ b_i\ I_{it}^2 - itc_{it}\ P_{It}\ I_{it}$$
$$= ( 1 - itc_{it} )\ P_{It}\ I_{it} + 0.5\ b_i\ I_{it}^2$$

where $AIE_{it}$ is the actual investment expenditures in industry i.

The net cash flows can be written as:

$$(40)\ NCF_{it} = [NI_{it} + \Phi_{it}'\ K_{it}'] - AIE_{it} + [\ P_{Eit}\ ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]$$
$$= ( 1 - cit_{it} )\ \{ [\ P_{it} - \textstyle\sum_{1 \leq j \leq J} ( c_{ij}\ P_{jt} ) ]\ K_{it}^{\beta i}\ L_{it}^{1-\beta i} - [ ( 1 + lt_t )\ w_t\ L_{it}$$
$$+ \Phi_{it}'\ K_{it}' ) ] - r_t\ B_{it} \} + \Phi_{it}'\ K_{it}' - [ ( 1 - itc_{it} )\ P_{It}\ I_{it} + 0.5\ b_i\ I_{it}^2 ]$$
$$+ [\ P_{Eit}\ ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]$$
$$= ( 1 - cit_{it} )\ \{ [\ P_{it} - \textstyle\sum_{1 \leq j \leq J} ( c_{ij}\ P_{jt} ) ]\ K_{it}^{\beta i}\ L_{it}^{1-\beta i} - ( 1 + lt_t )\ w_t\ L_{it} \}$$

$$- ( 1- cit_{it} ) r_t B_{it} + cit_{it} \Phi_{it}' K_{it}' - [ ( 1- itc_{it} ) P_{It} I_{it} + 0.5 b_i I_{it}^2 ]$$

$$+ [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]$$

where $NCF_{it}$ is industry i's net cash flows at period t, $[NI_{it} + \Phi_{it}' K_{it}']$ is the net cash provided by operating activities, $AIE_{it}$ represents the net cash used in investing activities, and $[ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]$ is the net cash provided by financing activities. Since both interest payments and depreciation allowances can be deducted from the corporate income tax base, the actual interest payments on outstanding bonds are $( 1- cit_{it} ) r_t B_{it}$, and the depreciation allowances benefit industry i's cash flows by $cit_{it} \Phi_{it}' K_{it}'$.

Since producers' expenditures in operating activities and investing activities can be financed by retained earnings and by issuing new bonds and additional equity, the new funds demanded by industry i can be expressed as:

$$(41)\quad FD_{it} = [AIE_{it} + ( 1 - cgt_{it} ) ( P_{Eit} - P_{Eit-1} ) E_{it} + DIV_{it} ] - [ NI_{it} + \Phi_{it}' K_{it}' ]$$

$$= [ ( 1- itc_{it} ) P_{It} I_{it} + 0.5 b_i I_{it}^2 ] + ( 1 - cgt_{it} ) ( P_{Eit} - P_{Eit-1} ) E_{it} + DIV_{it} - [ RE_{it} + DIV_{it} + \Phi_{it}' K_{it}' ]$$

$$= [ (1- itc_{it}) P_{It} I_{it} + 0.5 b_i I_{it}^2 ] + (1 - cgt_{it}) (P_{Eit} - P_{Eit-1}) E_{it} - RE_{it} - \Phi_{it}'K_{it}'$$

$$(42)\quad FD_{it} = P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} )$$

where $FD_{it}$ is the external funds demanded by industry i at period t, and $cgt_{it}$ is the capital gains tax rate on industry i. Equation (42) shows that external funds $FD_{it}$ come from issuing additional equity and new bonds.

In expressions (40) and (41), a new attempt is made in presenting an alternative and more appropriate concept of net cash flows (than what is used in Pereira (1988)), especially the cash flows from financing activities.

The issuance of new bonds and additional equity can be conceived as the increment in producers' financial liabilities which have to be liquidated by the end of the model horizon (Pereira, 1993). Thus, we have:

(43) $FD_{it} = FL_{it+1} - FL_{it}$

(44) $FL_{iT+1} = 0$

where $FL_{it}$ and $FL_{it+1}$ are industry i's financial liabilities at the beginning of period t and t+1, respectively, and $FL_{iT+1} = 0$ represents the terminal value of financial liabilities.

Since the industry-specific dividend/retention ratio and the debt/equity ratio are exogenously given in this study, the net income $NI_{it}$ in (35) and (36) and the external funds $FD_{it}$ in (42) can be rewritten as follows:

(45) $NI_{it} = ( 1 - cit_{it} ) EBT_{it} = \phi_i NI_{it} + ( 1 - \phi_i ) NI_{it} = RE_{it} + DIV_{it}$

(46) $FD_{it} = P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) = \psi_i FD_{it} + ( 1 - \psi_i ) FD_{it}$

where $\phi_i$ is the dividend/retention parameter representing the fraction of corporate net income (i.e., earnings) retained by industry i at period t, and $\psi_i$ is the debt/equity parameter representing the fraction of external funds financed by issuing additional equity by industry i. Thus, $\phi_i NI_{it}$ and $(1 - \phi_i) NI_{it}$ are industry i's retained earnings and dividends, respectively; the additional equity and the new bonds issued by industry i are represented respectively by $\psi_i FD_{it}$ and $(1 - \psi_i) FD_{it}$. Specifically, they can be restated as:

(47) $RE_{it} = \phi_i ( 1 - cit_{it} ) EBT_{it}$

(48) $DIV_{it} = ( 1 - \phi_i ) ( 1 - cit_{it} ) EBT_{it}$

(49) $\psi_i FD_{it} = P_{Eit} ( E_{it+1} - E_{it} )$

(50) $( 1 - \psi_i ) FD_{it} = ( B_{it+1} - B_{it} )$

with

(51) $P_{Ei,-1}\, E_{i0} = P_{Ei,-1}\, E_i^*$

(52) $P_{EiT}\, E_{iT+1} = 0$

(53) $B_{i0} = B_i^*$

(54) $B_{iT+1} = 0$

where $E_i^*$ is the initial quantity of equity of industry i; $B_i^*$ is the initial outstanding bonds of industry i. Expressions (52) and (54) both are terminal values.

It should be noted that even though the corporate dividend policies and financial rules are exogenously given to industries, they are endogenously determined by the model in this study.

Under the assumption of perfect capital market, the price of equity of industry i is the present value of the expected per share future stream of dividends, i.e.,

(55) $P_{Ei0} = \sum_{1 \leq t \leq T} [\, \Pi_{0 \leq s \leq t} (1+ r_s)^{-1} \,]\, DIV_{it}^e / E_{it}^e$

where $P_{Ei0}$ is the initial price of equity of industry i, $DIV_{it}^e$ is the expected dividends distributed by industry i at period t, and $E_{it}^e$ is the expected quantity of equity of industry i at the beginning of period t. Thus, $DIV_{it}^e / E_{it}^e$ is the expected per share dividends distributed by industry i at period t.

The objective of each producer is to maximize the present value of intertemporal net cash flows, subject to a Cobb-Douglas production function, a quadratic adjustment costs function, the equation of motion for capital stock, and some boundary conditions. Accordingly, producers can determine the optimal output supply, and the optimal demand for labor, intermediate inputs, investment, and external funds. Formally, the problem of producer i is to find ( $Y_{ijt}$ , $L_{it}$ , $I_{it}$; $K_{it}$ ) that maximizes

(56) $\sum_{0 \leq t \leq T} [\, \Pi_{0 \leq s \leq t} (1+ r_s)^{-1} \,]\, NCF_{it} =$

$$\sum_{0 \le t \le T} [\Pi_{0 \le s \le t} (1+r_s)^{-1}] \{ (1-cit_{it}) [ (P_{it} - \sum_{1 \le j \le J} c_{ij} P_{jt}) K_{it}^{\beta i} L_{it}^{1-\beta i} - (1+lt_t) w_t L_{it} ] - (1-cit_{it}) r_t B_{it} + cit_{it} \Phi_{it}' K_{it}' - [ (1-itc_{it}) P_{It} I_{it} + 0.5 b_i I_{it}^2 ] + [ P_{Eit} (E_{it+1} - E_{it}) + (B_{it+1} - B_{it}) - DIV_{it} ] \}$$

subject to the following constraints for each period t

(57) $K_{it+1} - K_{it} = I_{it} - \Phi_{it} K_{it}$

with

(58) $Y_{ijt} \ge 0, \;\; L_{it} \ge 0, \;\; I_{it} \ge 0, \;\; K_{it} \ge 0$, for all $1 \le j \le J$, $i = 1, \dots, J, I$

(59) $K_{i0} = K_i^*$

(60) $K_{iT+1} \ge 0$

where $K_i^*$ is the initial value of capital stock of industry i. Equation (57) is the same as (30) and denotes the evolution of capital stock. Expressions (58) – (60) are boundary conditions.

By applying the optimal control theory to the producer's dynamic behavior, the optimal production and investment decisions can be derived. Since the terminal value of the state variable in the production sector is free to vary, only subject to $K_{iT+1} \ge 0$ in the finite horizon model (i.e., T is fixed and $K_{iT+1}$ is free but non-negative), this kind of optimal control problem is attributed to the case of a truncated vertical terminal line (Chiang, 1992).

First, the producer's maximization problem can be expressed in terms of the Hamiltonian function as:

(61) $$H_{it}(L_{it}, I_{it}; K_{it}; \lambda_{it}) = [\Pi_{0 \le s \le t} (1+r_s)^{-1}] \{ (1-cit_{it}) [ (P_{it} - \sum_{1 \le j \le J} c_{ij} P_{jt}) \bullet K_{it}^{\beta i} L_{it}^{1-\beta i} - (1+lt_t) w_t L_{it} ] - (1-cit_{it}) r_t B_{it} + cit_{it} \Phi_{it}' K_{it}' - [ (1-itc_{it}) P_{It} I_{it} + 0.5 b_i I_{it}^2 ] + [ P_{Eit} (E_{it+1} - E_{it}) + (B_{it+1} - B_{it}) - DIV_{it} ] \} + \lambda_{it} [ I_{it} - \Phi_{it} K_{it} ]$$

where $H_{it}(\bullet)$ is the Hamiltonian function of industry i at period t. In this producer's optimal control problem, $L_{it}$ and $I_{it}$ are the control variables, $K_{it}$ is the state variable, and $\lambda_{it}$ is the costate variable. In the production sector, the dynamic shadow price, $\lambda_{it}$, can be interpreted as the marginal net cash flows of capital at period t to be installed at period t+1 (Pereira, 1988).

According to the maximum principle, the necessary conditions can be expressed as follows:

(62) $\partial H_{it} / \partial L_{it} = 0, \quad t = 0, 1, \ldots, T$

(63) $\partial H_{it} / \partial I_{it} = 0, \quad t = 0, 1, \ldots, T$

(64) $\partial H_{it} / \partial K_{it} = -(\lambda_{it} - \lambda_{it-1}), \quad t = 1, 2, \ldots, T$

(65) $\partial H_{it} / \partial \lambda_{it} = K_{it+1} - K_{it}, \quad t = 0, 1, \ldots, T$

with

(66) $\lambda_{iT} \geq 0$, $K_{iT+1} \geq 0$, and $\lambda_{iT} K_{iT+1} = 0$

The expressions (64) and (65) are equations of motion respectively for $\lambda_{it}$ and for capital. Equation (66) is the transversality condition for the case of truncated vertical terminal line in the optimal control problem.

Then, from the derivatives of the Hamiltonian with respect to $L_{it}$, $I_{it}$, $K_{it}$, and $\lambda_{it}$, we can obtain the following first-order conditions for the optimization problem of industry i at period t.

(67) $\partial H_{it} / \partial L_{it} = [\Pi_{0 \leq s \leq t}(1+r_s)^{-1}](1 - cit_{it})[(P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt}) \bullet$

$$(1-\beta_i) K_{it}^{\beta i} L_{it}^{-\beta i} - (1 + lt_t) w_t] = 0$$

(68) $\partial H_{it} / \partial I_{it} = [\Pi_{0 \leq s \leq t}(1+r_s)^{-1}]\{-[(1 - itc_{it}) P_{It} + b_i I_{it}]\} + \lambda_{it} = 0$

(69) $\partial H_{it} / \partial K_{it} = [\Pi_{0 \leq s \leq t}(1+r_s)^{-1}](1 - cit_{it})[(P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt}) \bullet$

$$\beta_i K_{it}^{\beta i-1} L_{it}^{1-\beta i} ] + \lambda_{it} [-\Phi_{it}] = -(\lambda_{it} - \lambda_{it-1})$$

(70) $\partial H_{it} / \partial \lambda_{it} = I_{it} - \Phi_{it} K_{it} = K_{it+1} - K_{it}$

Here, one would note that even though the $NCF_{it}$ set in equation (40) is different from that in Pereira (1988), the first-order conditions are not affected.

### 3.2.3 The Government

The government raises revenues from six different taxes: a progressive personal income tax, an ad valorem sales tax, an ad valorem capital gains tax, an ad valorem labor tax, an ad valorem investment tax credit (i.e., a subsidy to producers), and an ad valorem corporate income tax. Government deficits occur when the sum of its various expenditures and payments exceeds its total tax revenues, and can be financed by selling public bonds. In this study, the optimal behavior of the government is assumed to maximize the present value of the provision of public goods, discounted by a time-invariant factor, subject to a recursive sequence of intertemporal budget constraints, and some other conditions.

The provision of public goods can be represented by a time-invariant and semi-log linear function, expressed as:

(71) $F_g(K_{gt}, L_{gt}, Y_{gjt}) = \gamma_{gk}\, \ln K_{gt} + \gamma_{gL} \ln L_{gt} + \sum_j \gamma_{gj} \ln Y_{gjt}$, $\quad j = 1,\ldots,J$,

where $F_g(\bullet)$ is the provision function of the government, $K_{gt}$ is the government's capital stock at the beginning of period t, $L_{gt}$ is the labor used by the government, and $Y_{gjt}$ is the quantity of good j used by the government as input to provide public goods. The coefficients $\gamma_{gk}$, $\gamma_{gL}$, and $\gamma_{gj}$ are the contributing shares of capital, labor, and good j, respectively, in the provision function. Further, the sum of these coefficients is equal to one.

The evolution of capital stock can be written as:

(72) $K_{gt+1} - K_{gt} = I_{gt} - \Phi_{gt} K_{gt}$

where $K_{gt+1}$ is the government's capital stock at the beginning of period t+1, $I_{gt}$ is the gross investment in the government sector, and $\Phi_{gt}$ is the depreciation rate of capital stock in the government sector. Hence, ( $K_{gt+1} - K_{gt}$ ) represents the government's net investment at period t, and $\Phi_{gt} K_{gt}$ represents the depreciation in the government sector at period t.

The personal income tax revenues can be written as:

(73) $PIT_t = \sum_h \{-a_h + pit_h [ w_t L_{ht} + ( 1 - \sum_i e_{it} ) r_t W_{ht} + \sum_i e_{it} [ DIV_{it} / ( P_{Eit-1} E_{it} ) ] W_{ht} ] \}$

where $PIT_t$ is the personal income tax revenues raised from all income groups at period t.

The sales tax revenues can be written as:

(74) $ST_t = \sum_h ( \sum_j st_{jt} P_{jt} Y_{hjt} )$

where $ST_t$ is the sales tax revenues generated from the purchase of all consumption goods by all households at period t.

The capital gains tax revenues can be written as:

(75) $CGT_t = \sum_h [ cgt_{ht} \sum_i ( P_{Eit} - P_{Eit-1} ) E_{hit} ] + \sum_i cgt_{it} ( P_{Eit} - P_{Eit-1} ) E_{it}$

where $CGT_t$ is the capital gains tax revenues generated from the rising prices of equity of all industries owned by all households and all industries.

The labor tax revenues can be written as:

(76) $LT_t = \sum_i lt_t w_t L_{it} + lt_{gt} w_t L_{gt}$

where $LT_t$ is the labor tax revenues raised from the use of labor services by all industries and the government. The labor tax rate on the government, $lt_{gt}$, is set to be higher than that on industries, $lt_t$.

The investment tax credit can be written as:

$$(77) \quad ITC_t = \sum_i ITC_{it} = \sum_i itc_{it} P_{It} I_{it}$$

where $ITC_t$ is the investment tax credit subsidized by the government to all industries with the purchases of property, plant and equipment (i.e., the acquisition of investment good). Thus, it is a negative tax revenues for the government.

The corporate income tax revenues can be written as:

$$(78) \quad CIT_t = \sum_i cit_{it} \{ [ P_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) ] K_{it}^{\beta i} L_{it}^{1-\beta i} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ] - r_t B_{it} \}$$

where $CIT_t$ is the corporate income tax revenues generated from all industries.

Therefore, the total government tax revenues can be represented by:

$$(79) \quad TGT_t = PIT_t + ST_t + CGT_t + LT_t - ITC_t + CIT_t$$

where $TGT_t$ is the total government tax revenues collected at period t which is the sum of the six different tax revenues described above.

The investment expenditures can be written as:

$$(80) \quad IE_{gt} = P_{It} I_{gt}$$

where $IE_{gt}$ is government's investment expenditures, $P_{It} I_{gt}$ represents the acquisition costs.

The sum of government's various expenditures and payments can be expressed as:

$$(81) \quad TGE_t = [ \sum_j P_{jt} Y_{gjt} + ( 1+ lt_{gt} ) w_t L_{gt} + IE_{gt} ] + TR_{gt} + r_t GL_t$$
$$= [ \sum_j P_{jt} Y_{gjt} + ( 1+ lt_{gt} ) w_t L_{gt} + P_{It} I_{gt} ] + TR_{gt} + r_t GL_t$$

(82) $TR_{gt} = \sum_h TR_{ht}$

where $TGE_t$ is the total government expenditures at period t, $TR_{gt}$ is government's transfer payments to households, and $GL_t$ is government liabilities at the beginning of period t. Hence, $\sum_j P_{jt} Y_{gjt}$, $( 1+ lt_{gt} ) w_t L_{gt}$, and $P_{It} I_{gt}$ are government's expenditures in j goods , labor, and investment good, respectively; $r_t GL_t$ represents government's interest payments on outstanding public bonds. Equation (82) denotes that government's transfer payments are equivalent to the sum of transfers received by individual households.

Government deficits can be financed by issuing public bonds which represent government's demand for funds and the increment in government liabilities. Further, government liabilities have to be liquidated by the end of the model horizon. Thus, we have:

(83) $GL_{t+1} - GL_t = TGE_t - TGT_t$

$$= [ \sum_j P_{jt} Y_{gjt} + ( 1+ lt_{gt} ) w_t L_{gt} + P_{It} I_{gt} + TR_{gt} + r_t GL_t ] - TGT_t$$

(84) $FD_{gt} = GL_{t+1} - GL_t = B_{gt+1} - B_{gt}$

with

(85) $GL_0 = GL^*$

(86) $GL_{T+1} = 0$

where $GL_{t+1}$ is government liabilities at the beginning of period t+1, $GL^*$ is the initial value of government liabilities, $FD_{gt}$ is the funds demanded by the government at period t, and $B_{gt+1}$ and $B_{gt}$ are outstanding public bonds at the beginning of period t+1 and t, respectively. Further, $(GL_{t+1} - GL_t)$ and $(B_{gt+1} - B_{gt})$ represent the increment in government liabilities and public bonds at period t. Therefore, equation (83) denotes that government deficits at period t are the increment in government liabilities at the same time, and equation (86) is the terminal value for government liabilities.

The optimal behavior of the government is assumed to maximize the present value of the provision of public goods in this study, subject to a recursive sequence of intertemporal budget constraints, the equation of motion for capital stock, and boundary conditions. Accordingly, the optimal government expenditure decisions can be derived from the maximization process. Formally, the problem of the government is to find ( $Y_{gjt}$, $L_{gt}$, $I_{gt}$; $GL_t$, $K_{gt}$ ) that maximizes

(87) $\sum_{0 \leq t \leq T} \delta_g^t F_g( K_{gt}, L_{gt}, Y_{gjt} ) = \sum_{0 \leq t \leq T} \delta_g^t [\gamma_{gk} \ln K_{gt} + \gamma_{gL} \ln L_{gt} + \sum_j \gamma_{gj} \ln Y_{gjt} ]$

subject to the following constraints for each period t

(88) $GL_{t+1} - GL_t = [ \sum_j P_{jt} Y_{gjt} + ( 1+ lt_{gt} ) w_t L_{gt} + P_{It} I_{gt} + TR_{gt} + r_t GL_t ] - TGT_t$

(89) $K_{gt+1} - K_{gt} = I_{gt} - \Phi_{gt} K_{gt}$

with

(90) $Y_{gjt} \geq 0$, $L_{gt} \geq 0$, $I_{gt} \geq 0$, $K_{gt} \geq 0$, for all $1 \leq j \leq J$

(91) $GL_0 = GL^*$

(92) $GL_{T+1} = 0$

(93) $K_{g0} = K_g^*$

(94) $K_{gT+1} \geq 0$

where $\delta_g$ is the time-invariant discount factor for the government, and $K_g^*$ is the initial value of government's capital stock. Equation (88) is the same as (83), and denotes the budget constraint. Equation (89) is the same as (72), and denotes the evolution of capital stock in the government sector. The expressions (90) – (94) are boundary conditions, among which equation (92) implies that the present value of current and future government revenues has to be equal to the present value of current and future government expenditures on inputs, interest payments on outstanding public bonds, and transfer payments.

By applying the maximum principle to government's dynamic behavior, the optimal government expenditure decisions can be derived. Since there are two state variables in the government sector, we have to take into account two transversality conditions respectively corresponding to the state variables in this optimal control problem.

First, the Hamiltonian at period t of this problem can be written as:

$$(95)\quad H_{gt}( Y_{gjt}, L_{gt}, I_{gt}; GL_t, K_{gt}; \lambda_{gGLt}, \lambda_{gkt} ) = \delta_g^{\,t} [\gamma_{gk}\, \ln K_{gt} + \gamma_{gL} \ln L_{gt} + \textstyle\sum_j \gamma_{gj} \ln Y_{gjt} ]$$
$$+ \lambda_{gGLt} \{ \textstyle\sum_j P_{jt} Y_{gjt} + ( 1+ lt_{gt} )\, w_t L_{gt} + P_{It} I_{gt} + TR_{gt} + r_t\, GL_t - TGT_t \}$$
$$+ \lambda_{gkt} [ I_{gt} - \Phi_{gt} K_{gt} ]$$

where $H_{gt}( \bullet )$ is the Hamiltonian function of the government at period t. In this optimal control problem, $Y_{gjt}$, $L_{gt}$, and $I_{gt}$ are the control variables, $GL_t$ and $K_{gt}$ are the state variables, and $\lambda_{gGLt}$ and $\lambda_{gkt}$ are the costate variables. In the government sector, the Hamiltonian function represents the total provision of public goods at period t, and the implicit provision of both liabilities and capital stock transferred to next period. The dynamic shadow prices, $\lambda_{gGLt}$ and $\lambda_{gkt}$, can be interpreted as the marginal provision at period t of government liabilities and capital, respectively.

According to the maximum principle, the necessary conditions can be set as follows:

$$(96)\quad \partial H_{gt} / \partial Y_{gjt} = 0, \qquad t = 0, 1, \ldots, T$$

$$(97)\quad \partial H_{gt} / \partial L_{gt} = 0, \qquad t = 0, 1, \ldots, T$$

(98) $\partial H_{gt} / \partial I_{gt} = 0,$ $\quad t = 0, 1, \ldots, T$

(99) $\partial H_{gt} / \partial GL_t = -(\lambda_{gGLt} - \lambda_{gGLt-1}),$ $\quad t = 1, 2, \ldots, T$

(100) $\partial H_{gt} / \partial K_{gt} = -(\lambda_{gkt} - \lambda_{gkt-1}),$ $\quad t = 1, 2, \ldots, T$

(101) $\partial H_{gt} / \partial \lambda_{gGLt} = GL_{t+1} - GL_t,$ $\quad t = 0, 1, \ldots, T$

(102) $\partial H_{gt} / \partial \lambda_{gkt} = K_{gt+1} - K_{gt},$ $\quad t = 0, 1, \ldots, T$

with

(103) $GL_{T+1} = 0$

(104) $\lambda_{gkT} \geq 0, K_{gT+1} \geq 0,$ and $\lambda_{gkT} K_{gT+1} = 0$

Expressions (99) – (102) are equations of motion respectively for the costate variables, $\lambda_{gGLt}$ and $\lambda_{gkt}$, and for the state variables, $GL_t$ and $K_{gt}$. Equations (103) and (104) are the transversality conditions in this optimal control problem.

Then, from the derivatives of the Hamiltonian with respect to $Y_{gjt}$, $L_{gt}$, $I_{gt}$, $GL_t$, $K_{gt}$, $\lambda_{gGLt}$, and $\lambda_{gkt}$, we can obtain the following first-order conditions for the optimization problem of the government at period t.

(105) $\partial H_{gt} / \partial Y_{gjt} = \delta_g^{\,t} [\gamma_{gj} / Y_{gjt}] + \lambda_{gGLt} P_{jt} = 0$

(106) $\partial H_{gt} / \partial L_{gt} = \delta_g^{\,t} [\gamma_{gL} / L_{gt}] + \lambda_{gGLt} [(1 + lt_{gt}) w_t - lt_{gt} w_t] = 0$

(107) $\partial H_{gt} / \partial I_{gt} = \lambda_{gGLt} [P_{It}] + \lambda_{gkt} = 0$

(108) $\partial H_{gt} / \partial GL_t = \lambda_{gGLt} r_t = -(\lambda_{gGLt} - \lambda_{gGLt-1})$

(109) $\partial H_{gt} / \partial K_{gt} = \delta_g^{\,t} [\gamma_{gk} / K_{gt}] + \lambda_{gkt} [-\Phi_{gt}] = -(\lambda_{gkt} - \lambda_{gkt-1})$

(110) $\partial H_{gt} / \partial \lambda_{gGLt} = \{ \sum_j P_{jt} Y_{gjt} + (1 + lt_{gt}) w_t L_{gt} + P_{It} I_{gt} + TR_{gt} + r_t GL_t - TGT_t \}$

$= GL_{t+1} - GL_t$

(111) $\partial H_{gt} / \partial \lambda_{gkt} = I_{gt} - \Phi_{gt} K_{gt} = K_{gt+1} - K_{gt}$

### 3.2.4 Equilibrium Conditions

Following the work of Pereira (1988), temporary Walrasian equilibrium will also be introduced into this study. A temporary Walrasian equilibrium implies that "all current markets are assumed to clear"; "equilibrium in the short run is parametric on the expectations of future prices held by agents as well as the future taxation parameters"; therefore, "the intertemporal equilibrium path is conceived as a sequence of short-run, temporary equilibria parametric on future price expectations" (Pereira, 1988). Further, in each sector, the link between two such consecutive equilibria can be accomplished by the optimal transitions of the state variables in terms of the optimal control theory.

From the first-order conditions described above for agents' dynamic optimization problem, we can derive the closed-form solutions to all control variables, state variables, and costate variables at every period t. Nevertheless, Pereira (1988) suggests that by the definition of temporary Walrasian equilibrium, only the initial values of these solutions matter.

Now, we will roughly list the equilibrium conditions for all markets, including consumption goods markets, the labor market, the investment good market, as well as the financial market.

The equilibrium conditions for consumption goods markets can be represented by:

$$(112)\quad Y_{jt}^{S}(P_t;\ P_t^{*};\ \theta_j) = \sum_{1 \le h \le m} Y_{hjt}^{D}(P_t;\ P_t^{*};\ \theta_h) + \sum_{1 \le f \le J} Y_{fjt}^{D}(P_t;\ P_t^{*};\ \theta_f) + Y_{Ijt}^{D}(P_t;\ P_t^{*};\ \theta_I) + Y_{gjt}^{D}(P_t;\ P_t^{*};\ \theta_g) \qquad j = 1,\ldots,J$$

where $Y_{jt}^{S}(\bullet)$ is the quantity of consumption good j supplied by industry j, $P_t$ is a sequence of price vectors represented by ($P_{1t},\ldots, P_{Jt}; P_{It}; w_t; r_t$), $P_t^{*}$ is a sequence of point expectations about future prices represented by ($P_{1t}^{*},\ldots, P_{Jt}^{*}; P_{It}^{*}; w_t^{*}; r_t^{*}$), and $\theta_j$ is industry j's financing rules. $Y_{hjt}^{D}(\bullet)$ is the quantity of consumption good j demanded by household h, and $\theta_h$ is household h's preference parameters. $Y_{fjt}^{D}(\bullet)$ is the quantity of consumption good j demanded as intermediate inputs by industry f, and $\theta_f$ is technology parameters of industry f. $Y_{Ijt}^{D}(\bullet)$ is the quantity of consumption good j demanded by the investment sector, and $\theta_I$ is technology parameters of the investment sector. $Y_{gjt}^{D}(\bullet)$ is the quantity of consumption good j demanded by the government, and $\theta_g$ is technology parameters of the government sector and current and expected tax policy rules.

The equilibrium conditions for the labor market can be represented by:

$$(113)\quad \sum_{1 \le h \le m} L_{ht}^{S}(P_t;\ P_t^{*};\ \theta_h) = \sum_{1 \le j \le J} L_{jt}^{D}(P_t;\ P_t^{*};\ \theta_j) + L_{It}^{D}(P_t;\ P_t^{*};\ \theta_I) + L_{gt}^{D}(P_t;\ P_t^{*};\ \theta_g)$$

where $L_{ht}^{S}(\bullet)$ is the quantity of labor supplied by household h, and $L_{jt}^{D}(\bullet)$, $L_{It}^{D}(\bullet)$, and $L_{gt}^{D}(\bullet)$ are the quantities of labor demanded by industry j, the investment sector, and the government, respectively.

The equilibrium conditions for the investment good market can be represented by:

$$(114)\quad I_{It}^{S}(P_t;\ P_t^{*};\ \theta_I) = \sum_{1 \le j \le J} I_{jt}^{D}(P_t;\ P_t^{*};\ \theta_j) + I_{It}^{D}(P_t;\ P_t^{*};\ \theta_I) + I_{gt}^{D}(P_t;\ P_t^{*};\ \theta_g)$$

where $I_t^{S}(\bullet)$ is the quantity of investment good supplied by the investment sector, and $I_{jt}^{D}(\bullet)$, $I_{It}^{D}(\bullet)$, and $I_{gt}^{D}(\bullet)$ are the quantities of investment good demanded by industry j, the investment sector, and the government, respectively.

The equilibrium conditions for the financial market can be represented by:

$$(115)\quad \sum_{1 \le h \le m} FD_{ht}^{S}(P_t;\ P_t^{*};\ \theta_h) = \sum_{1 \le j \le J} FD_{jt}^{D}(P_t;\ P_t^{*};\ \theta_j) + FD_{It}^{D}(P_t;\ P_t^{*};\ \theta_I) + FD_{gt}^{D}(P_t;\ P_t^{*};\ \theta_g)$$

where $FD_{ht}^S(\bullet)$ is the new funds supplied by household h, and $FD_{jt}^D(\bullet)$, $FD_{It}^D(\bullet)$, and $FD_{gt}^D(\bullet)$ are the new funds demanded by industry j, the investment sector, and the government, respectively.

The precise equilibrium conditions will be reported after the closed-form solutions to all the control, state, and costate variables in this study are derived in next chapter.

# CHAPTER FOUR

# DERIVATION OF THE SOLUTION FOR THE GENERAL EQUILIBRIUM MODEL

According to first-order conditions, discussed in section 3.2, for the optimization problems of household h, industry i, and the government at period t, we can directly obtain the quasi closed-form solutions for the control variables for each sector which are functions of the unknown initial value(s) of costate variable(s). Further, by expressing the initial value of costate variable as being parametric on the initial value of a predetermined specific state variable, we can indirectly derive the closed-form solution for the control variables. Nevertheless, in the context of temporary Walrasian equilibrium, only the initial values matter for the definition of equilibrium (Pereira, 1988). In turn, we still have to check whether all the solutions in each sector satisfy all the equilibrium conditions described in section 3.2.4.

## 4.1 Solution for Households

Since the terminal condition of the state variable in the household sector is given in the finite horizon model (i.e., both $W_{hT+1}$ and T are fixed), this kind of optimal control problem is attributed to the case of fixed terminal point with the transversality condition replaced by terminal condition (23) $W_{hT+1} = 0$.

### 4.1.1 The Quasi Closed-Form Solution for Control Variables, $Y_{hjt}$ and $L_{ht}$

From the first-order conditions (24) and (25) for the control variables in the optimization problem of household h at period t, we have

(S1) $Y_{hjt} = (\delta_h^t \alpha_{hj}) / [\lambda_{ht} (1 + st_{jt}) P_{jt}]$

(S2) $L_{ht} = L_h^* - [\delta_h^t (1 - \sum_j \alpha_{hj})] / [\lambda_{ht} (1 - pit_h) w_t]$

Then, from expression (26), the equation of motion for the costate variable, we can obtain

(S3) $\lambda_{ht} = B_t^{-1} \lambda_{ht-1}, \qquad t = 1, \ldots, T$

(S4) $\lambda_{ht} = (\Pi_{1 \le s \le t} B_s^{-1}) \lambda_h^* = \lambda_h^* / (\Pi_{1 \le s \le t} B_s)$

where $\lambda_h^* = \lambda_{h0}$ is unknown in this stage, and $B_s = 1 + (1 - pit_h) \{ (1 - \sum_i e_{is}) r_s + \sum_i e_{is} [DIV_{is} / (P_{Eis-1} E_{is})] \}$.

Therefore, $Y_{hjt}$ and $L_{ht}$ can be expressed as a function of $\lambda_h^*$ :

(S5) $Y_{hjt} = (\delta_h^t \alpha_{hj}) (\Pi_{1 \le s \le t} B_s) / [\lambda_h^* (1 + st_{jt}) P_{jt}]$

(S6) $L_{ht} = L_h^* - [\delta_h^t (1 - \sum_j \alpha_{hj})] (\Pi_{1 \le s \le t} B_s) / [\lambda_h^* (1 - pit_h) w_t]$

### 4.1.2 The Closed-Form Solution for Control Variables, $Y_{hjt}$ and $L_{ht}$

From expression (27), the equation of motion for the state variable, we can obtain

$$
\begin{aligned}
\text{(S7)}\; W_{ht+1} = &\, [ a_h + (1 - pit_h) w_t L_{ht} + (1 - cgt_{ht}) \textstyle\sum_i (P_{Eit} - P_{Eit-1}) E_{hit} + TR_{ht} \\
&\quad - \textstyle\sum_j (1 + st_{jt}) P_{jt} Y_{hjt} ] \\
&\quad + \{ 1 + (1 - pit_h) [ (1 - \textstyle\sum_i e_{it}) r_t + \textstyle\sum_i e_{it} [DIV_{it} / (P_{Eit-1} E_{it})] ] \} W_{ht} \\
= &\, A_t + B_t W_{ht}, \qquad t = 0, 1, \ldots, T
\end{aligned}
$$

$$
\begin{aligned}
\text{(S8)}\; W_{ht} &= A_{t-1} + \textstyle\sum_{0 \le s \le t-2} [ A_s (\Pi_{s+1 \le \tau \le t-1} B_\tau ) ] + W_h^* (\Pi_{0 \le s \le t-1} B_s) \\
&= \textstyle\sum_{0 \le s \le t-1} [ A_s (\Pi_{s+1 \le \tau \le t-1} B_\tau ) ] + W_h^* (\Pi_{0 \le s \le t-1} B_s), \qquad t = 2, 3, \ldots, T,
\end{aligned}
$$

where $W_h^* = W_{h0}$ is predetermined in this study,

$A_t = a_h + (1 - pit_h)\, w_t L_{ht} + (1 - cgt_{ht}) \sum_i (P_{Eit} - P_{Eit-1}) E_{hit} + TR_{ht} - \sum_j (1 + st_{jt}) P_{jt} Y_{hjt}$,

and $B_t = 1 + (1 - pit_h) \{ (1 - \sum_i e_{it}) r_t + \sum_i e_{it} [ DIV_{it} / (P_{Eit-1} E_{it}) ] \}$.

At the terminal period (t = T+1), $W_{ht}$ becomes

$$\text{(S9)}\ W_{hT+1} = A_T + \sum_{0 \le t \le T-1} [ A_t ( \Pi_{t+1 \le s \le T} B_s ) ] + W_h^* ( \Pi_{0 \le t \le T} B_t )$$

$$= \sum_{0 \le t \le T} [ A_t ( \Pi_{t+1 \le s \le T} B_s ) ] + W_h^* ( \Pi_{0 \le t \le T} B_t )$$

Then, by applying the terminal condition (23) $W_{hT+1} = 0$, which is also the transversality condition of household h's optimization problem, to equation (S9), we obtain

$$\text{(S10)}\ W_h^* = - ( \Pi_{0 \le t \le T} B_t )^{-1} \sum_{0 \le t \le T} [ A_t ( \Pi_{t+1 \le s \le T} B_s ) ]$$

$$= - \sum_{0 \le t \le T} [ A_t ( \Pi_{0 \le s \le t} B_s )^{-1} ]$$

By setting $C_t = a_h + (1 - cgt_{ht}) \sum_i (P_{Eit} - P_{Eit-1}) E_{hit} + TR_{ht}$, thereby $A_t = C_t + (1 - pit_h)\, w_t L_{ht} - \sum_j (1 + st_{jt}) P_{jt} Y_{hjt}$, then $W_h^*$ can be further expressed as function of $Y_{hjt}$ and $L_{ht}$:

$$\text{(S11)}\ W_h^* = - \sum_{0 \le t \le T} \{ [ C_t + (1 - pit_h)\, w_t L_{ht} - \sum_j (1 + st_{jt}) P_{jt} Y_{hjt} ] / ( \Pi_{0 \le s \le t} B_s ) \}$$

Subsequently, by substituting expressions (S5) and (S6) into (S11), $\lambda_h^*$ can be solved as:

$$\text{(S12)}\ \lambda_h^* = ( \sum_{0 \le t \le T} \delta_h^t ) / [ B_0 \{ W_h^* + \sum_{0 \le t \le T} [ D_t / ( \Pi_{0 \le s \le t} B_s ) ] \} ]$$

where $D_t = C_t + (1 - pit_h)\, w_t L_h^*$

$$= a_h + (1 - cgt_{ht}) \sum_i (P_{Eit} - P_{Eit-1}) E_{hit} + TR_{ht} + (1 - pit_h)\, w_t L_h^*$$

Finally, the closed-form solution for control variables can be derived by substituting (S12) into (S5) and (S6):

$$\text{(S13)}\ Y_{hjt} = ( \delta_h^t \alpha_{hj} ) ( \Pi_{1 \le s \le t} B_s ) B_0 \{ W_h^* + \sum_{0 \le t \le T} [ D_t / ( \Pi_{0 \le s \le t} B_s ) ] \}$$

$$/ [ ( \sum_{0 \le t \le T} \delta_h^{\,t} ) ( 1 + st_{jt} ) P_{jt} ]$$

$$= \{ ( \delta_h^{\,t} \alpha_{hj} ) ( \Pi_{0 \le s \le t} B_s ) W_h^* + ( \delta_h^{\,t} \alpha_{hj} ) ( \Pi_{0 \le s \le t} B_s ) \sum_{0 \le t \le T} [ D_t$$

$$/ ( \Pi_{0 \le s \le t} B_s ) ] \} / [ ( \sum_{0 \le t \le T} \delta_h^{\,t} ) ( 1 + st_{jt} ) P_{jt} ]$$

$$= \{ ( \delta_h^{\,t} \alpha_{hj} ) ( \Pi_{0 \le s \le t} B_s ) W_h^* + ( \delta_h^{\,t} \alpha_{hj} ) \{ \sum_{t+1 \le s \le T} [ D_s / ( \Pi_{t+1 \le \tau \le s} B_\tau ) ]$$

$$+ \sum_{0 \le s \le t} [ D_s ( \Pi_{s+1 \le \tau \le t} B_\tau ) ] \} \} / [ ( \sum_{0 \le t \le T} \delta_h^{\,t} ) ( 1 + st_{jt} ) P_{jt} ]$$

(S14) $$L_{ht} = L_h^* - [ \delta_h^{\,t} ( 1 - \sum_j \alpha_{hj} ) ] ( \Pi_{1 \le s \le t} B_s ) \; B_0 \{ W_h^* + \sum_{0 \le t \le T} [ D_t$$

$$/ ( \Pi_{0 \le s \le t} B_s ) ] \} / [ ( \sum_{0 \le t \le T} \delta_h^{\,t} ) \; ( 1 - pit_h ) w_t ]$$

$$= L_h^* - \{ [ \delta_h^{\,t} ( 1 - \sum_j \alpha_{hj} ) ] ( \Pi_{0 \le s \le t} B_s ) W_h^*$$

$$+ [ \delta_h^{\,t} ( 1 - \sum_j \alpha_{hj} ) ] ( \Pi_{0 \le s \le t} B_s ) \bullet \sum_{0 \le t \le T} [ D_t / ( \Pi_{0 \le s \le t} B_s ) ] \}$$

$$/ [ ( \sum_{0 \le t \le T} \delta_h^{\,t} ) \; ( 1 - pit_h ) w_t ]$$

$$= L_h^* - \{ [ \delta_h^{\,t} ( 1 - \sum_j \alpha_{hj} ) ] ( \Pi_{0 \le s \le t} B_s ) W_h^* + [ \delta_h^{\,t} ( 1 - \sum_j \alpha_{hj} ) ] \bullet$$

$$\{ \sum_{t+1 \le s \le T} [ D_s / ( \Pi_{t+1 \le \tau \le s} B_\tau ) ] + \sum_{0 \le s \le t} [ D_s ( \Pi_{s+1 \le \tau \le t} B_\tau ) ] \} \}$$

$$/ [ ( \sum_{0 \le t \le T} \delta_h^{\,t} ) \; ( 1 - pit_h ) w_t ]$$

### 4.1.3 The Initial Values of Variables Related to Household h

First of all, the initial value of wealth (i.e., the state variable), $W_h^*$, is predetermined. While the initial value of costate variable, $\lambda_h^*$, shown as equation (S12), is actually a function of $D_t$ and $B_t$ over the whole model horizon. Therefore, the value of $\lambda_h^*$ could not be obtained simply from the household sector itself.

According to equations (S1) and (S2), the demand for consumption good j ($Y_{hjt}^D$) and the labor supply ($L_{ht}^S$) at the initial period can be expressed as:

(S15) $Y_{hj0}^D = \alpha_{hj} / [ \lambda_h^* ( 1 + st_{j0} ) P_{j0} ]$, $\qquad j = 1,\ldots,J$

(S16) $L_{h0}^S = L_h^* - ( 1 - \sum_j \alpha_{hj} ) / [ \lambda_h^* ( 1 - pit_h ) w_0 ]$

According to equations (5)-(7), supply of new funds and the equation of motion for wealth ($FD_{ht}^S = W_{ht+1} - W_{ht}$) at the initial period can be expressed as:

$$(S17)\ FD_{h0}^S = W_{h1} - W_h^*$$

$$= a_h + (1 - pit_h)\{w_0 L_{h0}^S + (1 - \sum_i e_{i0})\, r_0 W_h^* + \sum_i e_{i0} [\, DIV_{i0} / (P_{Ei,-1} E_i^*)\,]\, W_h^*\}$$

$$+ (1 - cgt_{h0}) \sum_i (P_{Ei0} - P_{Ei,-1}) E_{hi}^* + TR_{h0} - \sum_j (1 + st_{j0}) P_{j0} Y_{hj0}^D$$

where $Y_{hj0}^D$ and $L_{h0}^S$ can be obtained from expressions (S15) and (S16), respectively.

Hence, the value of wealth at the beginning of period 1 can be represented by:

$$(S18)\ W_{h1} = W_h^* + a_h + (1 - pit_h)\{w_0 L_{h0}^S + (1 - \sum_i e_{i0})\, r_0 W_h^*$$

$$+ \sum_i e_{i0} [\, DIV_{i0} / (P_{Ei,-1} E_i^*)\,]\, W_h^*\} + (1 - cgt_{h0}) \sum_i (P_{Ei0} - P_{Ei,-1}) E_{hi}^*$$

$$+ TR_{h0} - \sum_j (1 + st_{j0}) P_{j0} Y_{hj0}^D$$

which is the same as the result from directly setting t=0 into equation (S7).

On the other hand, since the wealth of household h can be invested in bonds and equity, the value of wealth at the beginning of period t can be represented by :

$$(S19)\ W_{ht} = \sum_i B_{hit} + B_{hgt} + \sum_i P_{Eit-1} E_{hit}$$

which is the same as equation (9).

The components of wealth can be written as:

$$(S20)\quad B_{hit} = \xi_{ht} B_{it}$$

$$(S21)\quad B_{hgt} = \xi_{ht} B_{gt}$$

$$(S22)\quad E_{hit} = \xi_{ht} E_{it}$$

where $\xi_{ht}$, represented by $W_{ht} / \sum_h W_{ht}$, is household h's share of the market portfolio at the beginning of period t. Expressions (S20)-(S22) are the same as (10)-(12), respectively.

Thus, the initial value of wealth can be written as:

$$(S23)\ W_h^* = \sum_i B_{hi}^* + B_{hg}^* + \sum_i P_{Ei,-1} E_{hi}^*$$

which is the same as equation (16). And the composition of wealth at period 1 can be represented by:

(S24) $W_{h1} = \sum_i B_{hi1} + B_{hg1} + \sum_i P_{Ei0} E_{hi1}$

(S25) $B_{hi1} = \xi_{h1} B_{i1}$

(S26) $B_{hg1} = \xi_{h1} B_{g1}$

(S27) $E_{hi1} = \xi_{h1} E_{i1}$

In sum, even though only the initial values matter in solving the optimization problem in the context of temporary Walrasian equilibrium, we can not independently obtain the values of all variables related to the household sector without considering the effects of other sectors. That is, only if $\lambda_h^*$ is solved can we obtain the initial values of all the control variables and hence the supply of new funds, and the values of the composition of wealth at the beginning of period 1. Therefore, we have to depend on the equilibrium conditions for all markets to deal with the solution for $\lambda_h^*$. This will be discussed in detail later in section 4.4.2.

### 4.2 Solution for Producers

The optimal control problem of the production sector is attributed to the case of a truncated vertical terminal line (i.e., T is fixed, and $K_{iT+1}$ is free but non-negative) with transversality conditions in equation (66) $\lambda_{iT} \geq 0$, $K_{iT+1} \geq 0$, and $\lambda_{iT} K_{iT+1} = 0$, which are also called the Kuhn-Tucker conditions. Because of the complexity of computation in solving this kind of problem, this study will follow the suggestion introduced by Chiang (1992) to deal with this problem. That is, in reality we can always try the $\lambda_{iT} = 0$ condition first, and check whether the resulting value of $K_{iT+1}$ satisfies the terminal restriction $K_{iT+1} \geq 0$. Then, if it does, the problem is solved. Otherwise, we have to set $K_{iT+1} = 0$, and treat the problem as the case of a fixed terminal point whose solving process is similar to the one used in the household sector.

### 4.2.1 The Solution for Control Variables, $L_{it}$ and $I_{it}$

From the first-order conditions (67) and (68) for the control variables in the optimization problem of industry i at period t, we have

(S28) $( L_{it} / K_{it} ) = \{ ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) ( 1 - \beta_i ) / [ ( 1 + lt_t ) w_t ] \}^{1/\beta i}$

(S29) $L_{it} = K_{it} \{ ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) ( 1 - \beta_i ) / [ ( 1 + lt_t ) w_t ] \}^{1/\beta i}$

(S30) $I_{it} = [ 1 / b_i ] [ \lambda_{it} \Pi_{0 \leq s \leq t} (1 + r_s) - ( 1 - itc_{it} ) P_{It} ]$

where $L_{it}$ is expressed as a function of $K_{it}$, and $I_{it}$ is a function of $\lambda_{it}$.

From expression (70), the equation of motion for the state variable, we can obtain

(S31) $K_{it+1} = I_{it} + ( 1 - \Phi_{it} ) K_{it}$, $\qquad t = 0,\ldots,T$

(S32) $K_{it} = I_{it-1} + \sum_{0 \leq s \leq t-2} I_{is} [ \Pi_{s+1 \leq \tau \leq t-1} ( 1 - \Phi_{i\tau} ) ] + K_i^* \Pi_{0 \leq s \leq t-1} ( 1 - \Phi_{is} )$

$= \sum_{0 \leq s \leq t-1} I_{is} [ \Pi_{s+1 \leq \tau \leq t-1} ( 1 - \Phi_{i\tau} ) ] + K_i^* \Pi_{0 \leq s \leq t-1} ( 1 - \Phi_{is} )$, $\quad t = 0,\ldots,T$

where $K_{it}$ can be expressed as a function of $I_{it}$ and the predetermined $K_i^*$.

Then, the terminal value of capital stock, $K_{iT+1}$, can be represented by:

(S33) $K_{iT+1} = \sum_{0 \leq t \leq T} I_{it} [ \Pi_{t+1 \leq s \leq T} ( 1 - \Phi_{is} ) ] + K_i^* \Pi_{0 \leq t \leq T} ( 1 - \Phi_{it} )$

Further, by substituting equation (S29) into (29) $F_i ( K_{it}, L_{it} ) = Y_{it} = K_{it}^{\beta i} L_{it}^{1-\beta i}$, $Y_{it}$ can be expressed as a function of $K_{it}$:

(S34) $Y_{it} = K_{it}^{\beta i} \{ K_{it} \{ ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) ( 1 - \beta_i ) / [ ( 1 + lt_t ) w_t ] \}^{1/\beta i} \}^{1-\beta i}$

$= K_{it} \{ ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) ( 1 - \beta_i ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta i) / \beta i}$

Then, by substituting equation (S34) into (28)$Y_{it} = \text{Min}\{ F_i ( K_{it}, L_{it} ); Y_{i1t} / c_{i1}, Y_{i2t} / c_{i2}, \ldots, Y_{ijt} / c_{ij}, \ldots, Y_{iJt} / c_{iJ} \}$, $Y_{ijt}$ can be also expressed as a function of $K_{it}$:

(S35) $Y_{ijt} = c_{ij} Y_{it} = c_{ij} K_{it} \{ ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) ( 1 - \beta_i ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta i) / \beta i}$

Subsequently, from expression (69), the equation of motion for the costate variable, we can obtain

(S36) $\lambda_{it} = ( 1 - \Phi_{it} )^{-1} \{ \lambda_{it-1} - [ \Pi_{0 \leq s \leq t} (1 + r_s)^{-1} ] [ ( 1 - cit_{it} ) ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt}) \bullet ( \beta_i K_{it}^{\beta i - 1} L_{it}^{1-\beta i} ) ] \}, \qquad t = 1, \ldots, T.$

By substituting equation (S28) into (S36), $\lambda_{it}$ can be expressed as a function of $\lambda_i^*$:

(S37) $\lambda_{it} = ( 1 - \Phi_{it} )^{-1} \{ \lambda_{it-1} - [ \Pi_{0 \leq s \leq t} (1 + r_s)^{-1} ] ( 1 - cit_{it} ) ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) \bullet \beta_i \bullet \{ ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) ( 1 - \beta_i ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta i) / \beta i} \}$

$= ( 1 - \Phi_{it} )^{-1} \lambda_{it-1} - ( 1 - \Phi_{it} )^{-1} E_t$

(S38) $\lambda_{it} = \lambda_i^* \Pi_{1 \leq s \leq t} ( 1 - \Phi_{is} )^{-1} - \sum_{1 \leq s \leq t} [ E_s \Pi_{s \leq \tau \leq t} ( 1 - \Phi_{i\tau} )^{-1} ]$

where $\lambda_i^* = \lambda_{i0}$ is unknown in this stage, and $E_t = [ \Pi_{0 \leq s \leq t} (1 + r_s)^{-1} ] ( 1 - cit_{it} ) ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) \bullet \beta_i \bullet \{ ( P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt} ) ( 1 - \beta_i ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta i) / \beta i}$.

At the terminal period ($t = T$), $\lambda_{it}$ becomes

(S39) $\lambda_{iT} = \lambda_i^* \Pi_{1 \leq t \leq T} ( 1 - \Phi_{it} )^{-1} - \sum_{1 \leq t \leq T} [ E_t \Pi_{t \leq s \leq T} ( 1 - \Phi_{is} )^{-1} ]$

**Case 1**: Setting $\lambda_{iT} = 0$ as the Transversality Condition

Then, by applying the transversality condition, set here as $\lambda_{iT} = 0$, to equation (S39), we obtain

(S40) $\lambda_i^* = \sum_{1 \leq t \leq T} [ E_t \Pi_{1 \leq s \leq t-1} ( 1 - \Phi_{is} ) ]$

where $\lambda_i^*$ is parametric on exogenous variables and other parameters.

Therefore, $\lambda_i^*$ is solved by equation (S40), under the given transversality condition $\lambda_{iT} = 0$. Consequently, we can obtain the solution for all control variables, state variable, costate variable, and thereby the relative variables, such as supply of good i and demand for intermediate input j.

By substituting equation (S40) into (S38), $\lambda_{it}$ can be solved as:

$$\text{(S41)}\ \lambda_{it} = \Pi_{1\leq s\leq t}(1-\Phi_{is})^{-1}\{\textstyle\sum_{1\leq t\leq T}[E_t\,\Pi_{1\leq s\leq t-1}(1-\Phi_{is})]\}$$
$$-\textstyle\sum_{1\leq s\leq t}[E_s\,\Pi_{s\leq\tau\leq t}(1-\Phi_{i\tau})^{-1}]$$
$$=\textstyle\sum_{t+1\leq s\leq T}[E_s\,\Pi_{t+1\leq\tau\leq s-1}(1-\Phi_{i\tau})]$$

Then, $I_{it}$ can be solved by substituting equation (S41) into (S30) as:

$$\text{(S42)}\ I_{it} = [1/b_i]\{[\Pi_{0\leq s\leq t}(1+r_s)]\textstyle\sum_{t+1\leq s\leq T}[E_s\,\Pi_{t+1\leq\tau\leq s-1}(1-\Phi_{i\tau})]-(1-itc_{it})P_{It}\}$$

By substituting equation (S42) into (S32), with the predetermined $K_i^*$, we can solve $K_{it}$, which can further be used to solve $L_{it}$, $Y_{it}$, and $Y_{ijt}$. The expressions representing the solution for $K_{it}$, $L_{it}$, $Y_{it}$, and $Y_{ijt}$ are very complicated and are omitted here.

**Case 2**: Setting $K_{iT+1} = 0$ as the Transversality Condition

Then, by applying the transversality condition, set here as $K_{iT+1} = 0$, to equation (S33), we obtain

$$\text{(S40')}\ K_i^* = -[\Pi_{0\leq t\leq T}(1-\Phi_{it})^{-1}]\{\textstyle\sum_{0\leq t\leq T} I_{it}[\Pi_{t+1\leq s\leq T}(1-\Phi_{is})]\}$$
$$= -\{\textstyle\sum_{0\leq t\leq T} I_{it}[\Pi_{0\leq s\leq t}(1-\Phi_{is})^{-1}]\} = -\textstyle\sum_{0\leq t\leq T} I_{it}\,a_t^{-1}$$

where $a_t = \Pi_{0\leq s\leq t}(1-\Phi_{is})$ and $K_i^*$ is a function of $I_{it}$.

By substituting equation (S38) into (S30), $I_{it}$ can be expressed as a function of $\lambda_i^*$:

$$\text{(S41')}\ I_{it} = [1/b_i]\{\Pi_{0\leq s\leq t}(1+r_s)\ \{\lambda_i^*\,\Pi_{1\leq s\leq t}(1-\Phi_{is})^{-1}$$

$$- \sum_{1 \le s \le t} [\, E_s \Pi_{s \le \tau \le t} (1 - \Phi_{i\tau})^{-1} ] \} - (1 - itc_{it}) P_{It} \}$$

$$= b\, [\, \lambda_i^* c_t (1 - \Phi_{i0})\, a_t^{-1} - f_s c_t - (1 - itc_{it})\, P_{It} ]$$

where $b = 1/b_i$, $c_t = \Pi_{0 \le s \le t}(1 + r_s)$, and $f_s = \sum_{1 \le s \le t} [\, E_s \Pi_{s \le \tau \le t} (1 - \Phi_{i\tau})^{-1} ]$.

By substituting equation (S41') into (S40'), we can obtain

$$\text{(S42')}\ \lambda_i^* = \{ - K_i^* + b \sum_{0 \le t \le T} \{ [\, f_s c_t + (1 - itc_{it})\, P_{It} ]\, a_t^{-1} \} \}$$

$$/ \{ b\, (1 - \Phi_{i0}) \sum_{0 \le t \le T} \{ [c_t\, a_t^{-1}]\, a_t^{-1} \} \}$$

$$= \{ - K_i^* + b \sum_{0 \le t \le T} [\, f_s c_t a_t^{-1} + (1 - itc_{it})\, P_{It} a_t^{-1} ] \}$$

$$/ [\, b\, (1 - \Phi_{i0}) \sum_{0 \le t \le T} (c_t\, a_t^{-2}) ]$$

where $\lambda_i^*$ is parametric on the predetermined $K_i^*$, exogenous variables and other parameters.

Therefore, $\lambda_i^*$ is solved by equation (S42'), under the given transversality condition $K_{iT+1} = 0$. Consequently, we can obtain the solution for all control variables, state variable, costate variable, and thereby the relative variables, such as supply of good i and demand for intermediate input j.

By substituting equation (S42') into (S38) and (S41'), we can solve $\lambda_{it}$ and $I_{it}$, and with the predetermined $K_i^*$, they can further be used to solve $K_{it}$. Then, by substituting the solved $K_{it}$ into expressions (S29), (S34), and (S35), we can solve $L_{it}$, $Y_{it}$, and $Y_{ijt}$, respectively. The expressions representing the solution for $K_{it}$, $L_{it}$, $Y_{it}$, and $Y_{ijt}$ are very complicated and are omitted here.

In sum, according to the boundary condition (58), $Y_{ijt}$, $L_{it}$, $I_{it}$, and $K_{it}$ are non-negative. Then, $K_i^*$ in equation (S40') is likely to be negative, which is inconsistent with the boundary condition. Therefore, Case 2 seems to be infeasible. On the other hand, in

Case 1, as long as both ( $P_{it} - \sum_{1 \leq j \leq J} c_{ij} P_{jt}$ ) in equation (S29) and [ $\lambda_{it} \Pi_{0 \leq s \leq t} (1+r_s) - (1 - itc_{it}) P_{It}$ ] in equation (S30) are non-negative, so are $Y_{ijt}$, $L_{it}$, $I_{it}$, and $K_{it}$. Thus, Case 1 is much more likely to work.

### 4.2.2 The Initial Values of Variables Related to Industry i

First of all, the initial value of capital (i.e., the state variable), $K_i^*$, is predetermined. On the other hand, the initial value of costate variable, $\lambda_i^*$, is shown as equation (S40) and is parametric on exogenous variables and other parameters under the setting $\lambda_{iT} = 0$. Therefore, the value of $\lambda_i^*$ can be directly determined inside the production sector.

According to equation (S30), demand for investment good by industry i ($I_{it}^D$) at the initial period can be expressed as:

$$\text{(S43)}\quad I_{i0}^D = [\,1/b_i\,]\,[\,\lambda_i^* \Pi_{0 \leq s \leq 0} (1+r_s) - (1 - itc_{i0})\, P_{I0}\,]$$
$$= [\,1/b_i\,]\,[\,\lambda_i^* (1+r_0) - (1 - itc_{i0})\, P_{I0}\,]$$

According to equation (S29), the labor demanded by industry i ($L_{it}^D$) at the initial period can be expressed as:

$$\text{(S44)}\quad L_{i0}^D = K_i^* \left\{ (P_{i0} - \textstyle\sum_{1 \leq j \leq J} c_{ij} P_{j0})(1-\beta_i) / [\,(1+lt_0)\, w_0\,] \right\}^{1/\beta_i}$$

From expressions (S34) and (S35), the supply of good i ($Y_{it}^S$) and the demand for intermediate input j by industry i ($Y_{ijt}^D$) at the initial period can be expressed as:

$$\text{(S45)}\quad Y_{i0}^S = K_i^* \left\{ (P_{i0} - \textstyle\sum_{1 \leq j \leq J} c_{ij} P_{j0})(1-\beta_i) / [\,(1+lt_0)\, w_0\,] \right\}^{(1-\beta_i)/\beta_i}$$

$$\text{(S46)}\quad Y_{ij0}^D = c_{ij} K_i^* \left\{ (P_{i0} - \textstyle\sum_{1 \leq j \leq J} c_{ij} P_{j0})(1-\beta_i) / [\,(1+lt_0)\, w_0\,] \right\}^{(1-\beta_i)/\beta_i}$$

From expressions (48) and (34), we have

$$(S47)\ DIV_{it} = ( 1-\phi_i ) ( 1-cit_{it} ) \{ ( P_{it} - \sum_{1 \le j \le J} c_{ij} P_{jt} ) Y_{it} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}'] - r_t B_{it} \}$$

Therefore, the dividends distributed from industry i to households ($DIV_{it}$) at the initial period can be expressed as:

$$(S48)\ DIV_{i0} = ( 1-\phi_i ) ( 1-cit_{i0} ) \{ ( P_{i0} - \sum_{1 \le j \le J} c_{ij} P_{j0} ) Y_{i0}^S - [ ( 1+ lt_0 ) w_0 L_{i0}^D + \Phi_{i0}' K_{i0}'] - r_0 B_i^* \}$$

where $Y_{i0}^S$ and $L_{i0}^D$ are from expressions (S45) and (S44), respectively.

From expressions (47) and (34), we have

$$(S49)\ RE_{it} = \phi_i ( 1-cit_{it} ) \{ ( P_{it} - \sum_{1 \le j \le J} c_{ij} P_{jt} ) Y_{it} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}'] - r_t B_{it} \}$$

Therefore, the earnings retained by industry i ($RE_{it}$) at the initial period can be expressed as:

$$(S50)\ RE_{i0} = \phi_i ( 1-cit_{i0} ) \{ ( P_{i0} - \sum_{1 \le j \le J} c_{ij} P_{j0} ) Y_{i0}^S - [ ( 1+ lt_0 ) w_0 L_{i0}^D + \Phi_{i0}' K_{i0}'] - r_0 B_i^* \}$$

From equation (41), the demand for external funds by industry i ($FD_{it}^D$) at the initial period can be expressed as:

$$(S51)\ FD_{i0}^D = ( 1-itc_{i0} ) P_{I0} I_{i0}^D + 0.5\, b_i ( I_{i0}^D )^2 + ( 1 - cgt_{i0} ) ( P_{Ei0} - P_{Ei,-1} ) E_i^* - RE_{i0} - \Phi_{i0}' K_{i0}',$$

where $E_i^* = E_{i0}$ is predetermined. $I_{i0}^D$ and $RE_{i0}$ are from expressions (S43) and (S50).

From equation (S31), the capital stock of industry i ($K_{it}$) at the beginning of period 1 can be expressed as:

$$(S52)\ K_{i1} = I_{i0}^D + ( 1 - \Phi_{i0} ) K_i^*$$

From (49) and (50), we have

(S53) $P_{Eit}\, E_{it+1} - P_{Eit}\, E_{it} = \psi_i\, FD_{it}$

(S54) $B_{it+1} - B_{it} = ( 1- \psi_i )\, FD_{it}$

Therefore, the initial evolution of equity can be shown as:

(S55) $P_{Ei0}\, E_{i1} - P_{Ei0}\, E_i^* = \psi_i\, FD_{i0}^D$, or

(S56) $P_{Ei0}\, E_{i1} = P_{Ei0}\, E_i^* + \psi_i\, FD_{i0}^D$

and the initial evolution of bonds can be expressed as:

(S57) $B_{i1} - B_i^* = ( 1- \psi_i )\, FD_{i0}^D$, or

(S58) $B_{i1} = B_i^* + ( 1- \psi_i )\, FD_{i0}^D$

where $B_i^* = B_{i0}$ is predetermined. $FD_{i0}^D$ is from equation (S51).

From expressions (42) and (43), we have

(S59) $FD_{it} = ( P_{Eit}\, E_{it+1} + B_{it+1}) - ( P_{Eit}\, E_{it} + B_{it} ) = FL_{it+1} - FL_{it}$

Thus, financial liabilities at the beginning of period t+1 can be represented by:

(S60) $FL_{it+1} = P_{Eit}\, E_{it+1} + B_{it+1}$

Further, we can obtain

(S61) $FL_{i1} = P_{Ei0}\, E_{i1} + B_{i1}$

where $P_{Ei0}\, E_{i1}$ and $B_{i1}$ can be obtained from (S56) and (S58), respectively.

In sum, by setting $\lambda_{iT} = 0$ as the transversality condition in the optimization problem of industry i in the context of temporary Walrasian equilibrium, we can obtain the initial values of costate variable, control variables and hence the relative variables, such as supply of good i, demand for intermediate input j, demand for external funds, the distribution of industry i's net income. On the other hand, we can also obtain the values of capital stock, equity, bonds, and financial liabilities at the beginning of period 1. Therefore, we can independently find the solution for all the control variables, state variable, the costate variable, and other relative variables inside the production sector over the model horizon.

### 4.3 Solution for the Government

Since there are two state variables in the government sector, we have to take into account two transversality conditions respectively corresponding to the state variables, represented by equation (103) $GL_{T+1} = 0$ which is similar to (23) $W_{hT+1} = 0$ in the household sector, and equation (104) $\lambda_{gkT} \geq 0$, $K_{gT+1} \geq 0$, and $\lambda_{gkT} K_{gT+1} = 0$ which is akin to equation (66) $\lambda_{iT} \geq 0$, $K_{iT+1} \geq 0$, and $\lambda_{iT} K_{iT+1} = 0$ in the production sector. Therefore, the process to derive the solution for control variables, state variables, and costate variables in the government sector is basically the same as that used in the other two sectors.

For simplicity, $\lambda_{gkT} = 0$ can be directly substituted for the Kuhn-Tucker conditions $\lambda_{gkT} \geq 0$, $K_{gT+1} \geq 0$, and $\lambda_{gkT} K_{gT+1} = 0$ as one of transversality conditions in the government sector, and check if the resulting value of $K_{gT+1}$ satisfies the terminal restriction $K_{gT+1} \geq 0$.

#### 4.3.1 The Quasi Closed-Form Solution for Control Variables, $Y_{gjt}$, $L_{gt}$ and $I_{gt}$

From the first-order conditions (105), (106), and (111) for the control variables in the optimization problem of the government at period t, we have

(S62) $Y_{gjt} = -\,\delta_g{}^t \gamma_{gj} / ( \lambda_{gGLt} P_{jt} )$

(S63) $L_{gt} = -\,\delta_g{}^t \gamma_{gL} / ( \lambda_{gGLt} w_t )$

(S64) $I_{gt} = K_{gt+1} - ( 1 - \Phi_{gt} ) K_{gt}$

In order to express $I_{gt}$ as a function of $\lambda_{gkt}$, we have to proceed with the following steps. First, from the first-order condition (109), we have

(S65) $K_{gt} = \delta_g^{\,t}\, \gamma_{gk} / [\, \lambda_{gkt-1} - (1 - \Phi_{gt})\, \lambda_{gkt}\,]$ for $t = 1, \ldots, T$, and thus

(S65') $K_{gt+1} = \delta_g^{\,t+1}\, \gamma_{gk} / [\, \lambda_{gkt} - (1 - \Phi_{gt+1})\, \lambda_{gkt+1}\,]$

Then, from the first-order conditions (108) and (107), we have

(S66) $\lambda_{gGLt} = (1 + r_t)^{-1} \lambda_{gGLt-1}, \qquad t = 1, \ldots, T$

(S67) $\lambda_{gkt} = - \lambda_{gGLt} P_{It}$ or $\lambda_{gGLt} = - \lambda_{gkt} P_{It}^{-1}$

(S67') $\lambda_{gkt+1} = - \lambda_{gGLt+1} P_{It+1} = - [\, (1 + r_{t+1})^{-1} \lambda_{gGLt}\,]\, P_{It+1}$

$$= - (1 + r_{t+1})^{-1} [- \lambda_{gkt} P_{It}^{-1}]\, P_{It+1} = \lambda_{gkt} (1 + r_{t+1})^{-1} P_{It}^{-1} P_{It+1},$$ and

(S67'') $\lambda_{gkt-1} = - \lambda_{gGLt-1} P_{It-1} = - [\, (1 + r_t)\, \lambda_{gGLt}\,]\, P_{It-1}$

$$= - (1 + r_t) [- \lambda_{gkt} P_{It}^{-1}]\, P_{It-1} = \lambda_{gkt} (1 + r_t) P_{It}^{-1} P_{It-1}$$

Further, by substituting expressions (S67') and (S67''), derived from the connection between $\lambda_{gkt}$ and $\lambda_{gGLt}$, into expressions (S65') and (S65) respectively, we can transform $K_{gt}$ and $K_{gt+1}$ into functions of $\lambda_{gkt}$, represented by:

(S68) $K_{gt} = \delta_g^{\,t}\, \gamma_{gk} / \{\, [\lambda_{gkt} (1 + r_t) P_{It}^{-1} P_{It-1}] - (1 - \Phi_{gt})\, \lambda_{gkt}\,\}$, and

$$= \delta_g^{\,t}\, \gamma_{gk} \lambda_{gkt}^{-1} \{\, [\, (1 + r_t) P_{It}^{-1} P_{It-1}] - (1 - \Phi_{gt})\,\}^{-1}, \qquad t = 1, \ldots, T$$

(S68') $K_{gt+1} = \delta_g^{\,t+1}\, \gamma_{gk} / \{\, \lambda_{gkt} - (1 - \Phi_{gt+1})\, [\, \lambda_{gkt} (1 + r_{t+1})^{-1} P_{It}^{-1} P_{It+1}]\,\}$

$$= \delta_g^{\,t+1}\, \gamma_{gk} \lambda_{gkt}^{-1} [\, 1 - (1 - \Phi_{gt+1})(1 + r_{t+1})^{-1} P_{It}^{-1} P_{It+1}]^{-1}$$

Therefore, $I_{gt}$ can be expressed as a function of $\lambda_{gkt}$:

$$\text{(S69)}\ I_{gt} = \delta_g^{\,t+1}\, \gamma_{gk} \lambda_{gkt}^{-1} [\, 1 - (1 - \Phi_{gt+1})(1 + r_{t+1})^{-1} P_{It}^{-1} P_{It+1}]^{-1}$$
$$- (1 - \Phi_{gt})\, \delta_g^{\,t}\, \gamma_{gk} \lambda_{gkt}^{-1} [\, (1 + r_t) P_{It}^{-1} P_{It-1} - (1 - \Phi_{gt})\,]^{-1}$$
$$= \delta_g^{\,t+1} \gamma_{gk} \lambda_{gkt}^{-1} \{\, [\, 1 - (1 - \Phi_{gt+1})(1 + r_{t+1})^{-1} P_{It}^{-1} P_{It+1}]^{-1}$$
$$- (1 - \Phi_{gt})\, \delta_g^{-1} [\, (1 + r_t) P_{It}^{-1} P_{It-1} - (1 - \Phi_{gt})\,]^{-1} \}$$

where t = 1, …, T. It should be noted that when t = 0, $I_{g0}$ (= $K_{g1} - ( 1 - \Phi_{g0} ) K_g^*$ ) should be written as:

$$(S69') \; I_{g0} = \delta_g \gamma_{gk} \lambda_{gk}^{* \, -1} [ 1 - ( 1 - \Phi_{g1} )( 1 + r_1 )^{-1} P_{I0}^{-1} P_{I1} ]^{-1} - ( 1 - \Phi_{g0} ) K_g^*$$

where $\lambda_{gk}^* = \lambda_{gk0}$, and $K_g^* = K_{g0}$.

On the other hand, from the first-order condition (109), we have

$$(S70) \; \lambda_{gkt} = ( 1 - \Phi_{gt} )^{-1} [ \lambda_{gkt-1} - \delta_g^{\,t} ( \gamma_{gk} / K_{gt} ) ].$$

By setting $N_t = \delta_g^{\,t} ( \gamma_{gk} / K_{gt} )$, the solution for $\lambda_{gkt}$ and $\lambda_{gkT}$ can be derived as:

$$(S71) \; \lambda_{gkt} = \lambda_{gk}^* \Pi_{1 \le s \le t} ( 1 - \Phi_{gs} )^{-1} - \Sigma_{1 \le s \le t} [ N_s \Pi_{s \le \tau \le t} ( 1 - \Phi_{g\tau} )^{-1} ]$$

$$= \lambda_{gk}^* \Pi_{1 \le s \le t} ( 1 - \Phi_{gs} )^{-1} - F_t, \qquad t = 1, \ldots, T$$

$$(S72) \; \lambda_{gkT} = \lambda_{gk}^* \Pi_{1 \le t \le T} ( 1 - \Phi_{gt} )^{-1} - \Sigma_{1 \le t \le T} [ N_t \Pi_{t \le s \le T} ( 1 - \Phi_{gs} )^{-1} ]$$

$$= \lambda_{gk}^* \Pi_{1 \le t \le T} ( 1 - \Phi_{gt} )^{-1} - F_T$$

where $F_t = \Sigma_{1 \le s \le t} [ N_s \Pi_{s \le \tau \le t} ( 1 - \Phi_{g\tau} )^{-1} ]$ and $\lambda_{gkt}$ and $\lambda_{gkT}$ actually are functions of $K_{gt}$ and their initial value ($\lambda_{gk}^*$) which is unknown in this stage.

If the terminal value of $\lambda_{gkt}$ is set to be equal to zero (i.e., $\lambda_{gkT} = 0$) and as the transversality condition, then from equation (S72), we have

$$(S73) \; \lambda_{gk}^* = \Sigma_{1 \le t \le T} [ N_t \Pi_{1 \le s \le t-1} ( 1 - \Phi_{gs} ) ]$$

$$= \Sigma_{1 \le t \le T} \{ (\delta_g^{\,t} \gamma_{gk} / K_{gt}) [ \Pi_{1 \le s \le t-1} ( 1 - \Phi_{gs} ) ] \}$$

where $\lambda_{gk}^*$ can be expressed as a function of $K_{gt}$.

Therefore, by substituting equation (S73) into (S71), $\lambda_{gkt}$ can further be expressed as:

$$(S74) \; \lambda_{gkt} = N_{t+1} + \Sigma_{t+2 \le s \le T} [ N_s \Pi_{t+1 \le \tau \le s-1} ( 1 - \Phi_{g\tau} ) ]$$

$$= \Sigma_{t+1 \le s \le T} \{ (\delta_g^{\,s} \gamma_{gk} / K_{gs}) [ \Pi_{t+1 \le \tau \le s-1} ( 1 - \Phi_{g\tau} ) ] \}, \qquad t = 1, \ldots, T-1,$$

where $\lambda_{gkt}$ can be expressed as a function of $K_{gt}$.

Consequently, from expressions (S67) and (S71), the solution for $\lambda_{gGLt}$ and $\lambda_{gGL}{}^*$ can be derived as:

$$(S75)\ \lambda_{gGLt} = -\lambda_{gkt} P_{It}{}^{-1}$$

$$= -P_{It}{}^{-1} \{ \lambda_{gk}{}^* \Pi_{1 \le s \le t} (1 - \Phi_{gs})^{-1} - F_t \}, \qquad t = 1, \ldots, T$$

$$(S76)\ \lambda_{gGL}{}^* = -\lambda_{gk}{}^* P_{I0}{}^{-1}$$

Eventually, the quasi closed-form solution for $Y_{gjt}$ and $L_{gt}$ can be obtained by substituting equation (S75) into (S62) and (S63), respectively, and expressed as functions of $\lambda_{gk}{}^*$ (and $K_{gt}$):

$$(S77)\ Y_{gjt} = \delta_g{}^t \gamma_{gj} P_{jt}{}^{-1} P_{It} \{ \lambda_{gk}{}^* \Pi_{1 \le s \le t} (1 - \Phi_{gs})^{-1} - F_t \}^{-1}, \qquad t = 1, \ldots, T$$

$$= \gamma_{gj} P_{j0}{}^{-1} P_{I0} \lambda_{gk}{}^{*-1}, \qquad t = 0$$

$$(S78)\ L_{gt} = \delta_g{}^t \gamma_{gL} w_t{}^{-1} P_{It} \{ \lambda_{gk}{}^* \Pi_{1 \le s \le t} (1 - \Phi_{gs})^{-1} - F_t \}^{-1}, \qquad t = 1, \ldots, T$$

$$= \gamma_{gL} w_0{}^{-1} P_{I0} \lambda_{gk}{}^{*-1}, \qquad t = 0$$

In addition, the quasi closed-form solution for $I_{gt}$ can be obtained by substituting equation (S71) into (S69), and expressed as functions of $\lambda_{gk}{}^*$ (and $K_{gt}$):

$$(S79)\ I_{gt} = \delta_g{}^{t+1} \gamma_{gk} \{ [ 1 - (1 - \Phi_{gt+1})(1 + r_{t+1})^{-1} P_{It}{}^{-1} P_{It+1}]^{-1}$$

$$- (1 - \Phi_{gt}) \delta_g{}^{-1} [ (1 + r_t) P_{It}{}^{-1} P_{It-1} - (1 - \Phi_{gt}) ]^{-1} \}$$

$$\cdot \{ \lambda_{gk}{}^* \Pi_{1 \le s \le t} (1 - \Phi_{gs})^{-1} - F_t \}^{-1}, \qquad t = 1, \ldots, T$$

$$= \delta_g \gamma_{gk} \lambda_{gk}{}^{*-1} [ 1 - (1 - \Phi_{g1})(1 + r_1)^{-1} P_{I0}{}^{-1} P_{I1}]^{-1} - (1 - \Phi_{g0}) K_g{}^*, \quad t = 0$$

#### 4.3.2 The Closed-Form Solution for Control Variables, $Y_{gjt}$, $L_{gt}$ and $I_{gt}$

From expression (111), the equation of motion for the state variable $K_{it}$, we can obtain

$$(S80)\ K_{gt+1} = I_{gt} + (1 - \Phi_{gt}) K_{gt}, \qquad t = 0, \ldots, T$$

(S81) $K_{gt} = \sum_{0 \le s \le t-1} I_{gs} [\, \Pi_{s+1 \le \tau \le t-1}(1 - \Phi_{g\tau}) \,] + K_g^* \, \Pi_{0 \le s \le t-1} (1 - \Phi_{gs}), \qquad t = 2, \ldots, T+1,$

where $K_{gt}$ can be expressed as a function of $I_{gt}$ and the predetermined $K_g^*$.

Then, the terminal value of capital stock, $K_{gT+1}$, can be represented by:

(S82) $K_{gT+1} = \sum_{0 \le t \le T} I_{gt} [\, \Pi_{t+1 \le s \le T} ( 1 - \Phi_{gs} ) \,] + K_g^* \, \Pi_{0 \le t \le T} ( 1 - \Phi_{gt} )$

On the other hand, from the first-order condition (110), the equation of motion for the other state variable $GL_t$, we can obtain

(S83) $GL_{t+1} = ( 1 + r_t ) GL_t + ( \sum_j P_{jt} Y_{gjt} + ( 1 + lt_{gt} ) w_t L_{gt} + P_{It} I_{gt} + TR_{gt} - TGT_t )$

$= M_t + ( 1 + r_t ) GL_t, \qquad t = 0, 1, \ldots, T$

(S84) $GL_t = M_{t-1} + \sum_{0 \le s \le t-2} M_s [\, \Pi_{s+1 \le \tau \le t-1} ( 1 + r_\tau ) \,] + GL^* \, \Pi_{0 \le s \le t-1} ( 1 + r_s )$

$= \sum_{0 \le s \le t-1} M_s [\, \Pi_{s+1 \le \tau \le t-1} ( 1 + r_\tau ) \,] + GL^* \, \Pi_{0 \le s \le t-1} ( 1 + r_s ) \quad t = 2, 3, \ldots, T+1,$

where $GL_t$ is a function of $Y_{gjt}$, $L_{gt}$, $I_{gt}$, and the predetermined $GL^*$.

$TGT_t = PIT_t + ST_t + CGT_t + LT_t - ITC_t + CIT_t,$

$TGL_t = TGT_t - lt_{gt} w_t L_{gt} = PIT_t + ST_t + CGT_t + \sum_i lt_t w_t L_{it} - ITC_t + CIT_t,$ and

$M_t = \sum_j P_{jt} Y_{gjt} + ( 1 + lt_{gt} ) w_t L_{gt} + P_{It} I_{gt} + TR_{gt} - TGT_t$

$= \sum_j P_{jt} Y_{gjt} + w_t L_{gt} + P_{It} I_{gt} + TR_{gt} - TGL_t.$

Then, from equation (S84), the terminal value of government liabilities, $GL_{T+1}$, can be represented by:

(S85) $GL_{T+1} = \sum_{0 \le t \le T} M_t [\, \Pi_{t+1 \le s \le T} ( 1 + r_s ) \,] + GL^* \, \Pi_{0 \le t \le T} ( 1 + r_t )$

Further, by combining with the transversality condition (103) $GL_{T+1} = 0$, the initial value of government liabilities, $GL^*$, can be written as:

(S86) $GL^* = - \{ \sum_{0 \le t \le T} M_t [\, \Pi_{t+1 \le s \le T} ( 1 + r_s ) \,] \} / [\, \Pi_{0 \le t \le T} ( 1 + r_t ) \,]$

$= - \sum_{0 \le t \le T} \{ M_t / [\, \Pi_{0 \le s \le t} ( 1 + r_s ) \,] \}$

$= - \sum_{0 \le t \le T} \{ M_t [\, \Pi_{0 \le s \le t} ( 1 + r_s )^{-1} \,] \}$

$$= - \sum_{0 \le t \le T} \{ [ \sum_j P_{jt} Y_{gjt} + w_t L_{gt} + P_{It} I_{gt} + TR_{gt} - TGL_t ]$$

$$\cdot [ \Pi_{0 \le s \le t} ( 1 + r_s )^{-1} ] \}$$

where $GL^*$ is expressed as a function of $Y_{gjt}$, $L_{gt}$, and $I_{gt}$.

Subsequently, by subsituting expressions (S77), (S78), and (S79) into (S86), $\lambda_{gk}^*$ can be expressed as a function of the predetermined $GL^*$ and the unknown $K_{gt}$ through the following equation.

$$\text{(S87)}\ GL^* = - \sum_{1 \le t \le T} \{ [ \sum_j P_{jt} Y_{gjt} + w_t L_{gt} + P_{It} I_{gt} ] [ \Pi_{0 \le s \le t} ( 1 + r_s )^{-1} ] \}$$

$$- [ \sum_j P_{j0} Y_{gj0} + w_0 L_{g0} + P_{I0} I_{g0} ] ( 1 + r_0)^{-1}$$

$$- \sum_{0 \le t \le T} \{ [ TR_{gt} - TGL_t ] [ \Pi_{0 \le s \le t} ( 1 + r_s )^{-1} ] \}$$

$$= - \sum_{1 \le t \le T} \{ \delta_g^t P_{It} [ \lambda_{gk}^* \Pi_{1 \le s \le t} ( 1 - \Phi_{gs} )^{-1} - F_t ]^{-1} [ \Pi_{0 \le s \le t} ( 1 + r_s )^{-1} ]$$

$$\cdot [ ( 1 - \gamma_{gk} ) + \delta_g \gamma_{gk} \{ [ 1 - ( 1 - \Phi_{gt+1} ) ( 1 + r_{t+1} )^{-1} P_{It}^{-1} P_{It+1} ]^{-1}$$

$$- ( 1 - \Phi_{gt} ) \delta_g^{-1} [ ( 1 + r_t ) P_{It}^{-1} P_{It-1} - ( 1 - \Phi_{gt} ) ]^{-1} \} ] \}$$

$$- \lambda_{gk}^{*-1} P_{I0} ( 1 + r_0)^{-1} \{ ( 1 - \gamma_{gk} ) + \delta_g \gamma_{gk} [ 1 - ( 1 - \Phi_{g1} )( 1 + r_1 )^{-1} P_{I0}^{-1} P_{I1} ]^{-1} \}$$

$$+ P_{I0} ( 1 - \Phi_{g0} ) ( 1 + r_0)^{-1} K_g^* - \sum_{0 \le t \le T} \{ [ TR_{gt} - TGL_t ] [ \Pi_{0 \le s \le t} ( 1 + r_s )^{-1} ] \}$$

where $\sum_j \gamma_{gj} + \gamma_{gL} + \gamma_{gk} = 1$, $F_t = \sum_{1 \le s \le t} [ N_s \Pi_{s \le \tau \le t} ( 1 - \Phi_{g\tau} )^{-1} ]$

Since we can not obtain the exact solution for $\lambda_{gk}^*$ here, the closed-form solution of control variables in the government sector can not be derived.

### 4.3.3 The Initial Values of Variables Related to the Government

First of all, the initial values of state variables (i.e., government liabilities and capital sotck), $GL^*$ and $K_g^*$, are predetermined. While the initial values of costate variables, $\lambda_{gk}^*$, shown as equation (S87), and hence $\lambda_{gGL}^*$ are actually functions of $TGL_t$

(i.e., $PIT_t + ST_t + CGT_t + \sum_i lt_t\, w_t L_{it} - ITC_t + CIT_t$) over the whole model horizon. In addition, the values of variables associated with the household sector, such as $PIT_t$, $ST_t$, and $CGT_t$ here, are unknown because the value of $\lambda_h^*$ could not be obtained simply from the household sector. Therefore, the values of $\lambda_{gk}^*$ and $\lambda_h^*$ are to be solved by employing equilibrium conditions in section 4.4.

According to expressions (S77), (S78), and (S79), the demand for consumption good j ($Y_{gjt}^D$), labor ($L_{gt}^D$), and investment good ($I_{gt}^D$) at the initial period can be respectively expressed as:

(S88) $Y_{gj0}^D = \gamma_{gj}\, P_{j0}^{-1}\, P_{I0}\, \lambda_{gk}^{*-1}$

(S89) $L_{g0}^D = \gamma_{gL}\, w_0^{-1}\, P_{I0}\, \lambda_{gk}^{*-1}$

(S90) $I_{g0}^D = \delta_g\, \gamma_{gk}\, \lambda_{gk}^{*-1} [\, 1 - (\, 1 - \Phi_{g1}\,)(\, 1 + r_1\,)^{-1} P_{I0}^{-1} P_{I1}]^{-1} - (\, 1 - \Phi_{g0}\,)\, K_g^*$

From equations (83) and (84), the demand for new funds ($FD_{gt}^D$) at the initial period can be expressed as:

(S91) $FD_{g0}^D = GL_1 - GL_0 = GL_1 - GL^*$

$$= r_0\, GL^* + \sum_j P_{j0}\, Y_{gj0}^D + w_0\, L_{g0}^D + P_{I0}\, I_{g0}^D + TR_{g0} - TGL_0$$

where $Y_{gj0}^D$, $L_{g0}^D$, and $I_{g0}^D$ can be obtained from expressions (S88), (S89), and (S90), respectively.

From equation (S80), the evolution of capital stock at the initial period can be represented by:

(S92) $K_{g1} = I_{g0}^D + (\, 1 - \Phi_{g0}\,)\, K_g^*$

or

(S93) $K_{g1} - K_g^* = I_{g0}^D - \Phi_{g0}\, K_g^*$

From equation (83), the evolution of government liabilities at the initial period can be represented by:

(S94) $GL_1 - GL^* = r_0\, GL^* + \sum_j P_{j0}\, Y_{gj0}{}^D + w_0\, L_{g0}{}^D + P_{I0}\, I_{g0}{}^D + TR_{g0} - TGL_0$

which is the same as (S91), or

(S95) $GL_1 = (\, 1 + r_0 \,)\, GL^* + \sum_j P_{j0}\, Y_{gj0}{}^D + w_0\, L_{g0}{}^D + P_{I0}\, I_{g0}{}^D + TR_{g0} - TGL_0$

From expressions (73)-(79), the personal income tax revenues ($PIT_t$), the sales tax revenues ($ST_t$), the capital gains tax revenues ($CGT_t$), the labor tax revenues ($LT_t$), the investment tax credit ($ITC_t$), the corporate income tax revenues ($CIT_t$), and the total government tax revenues ($TGT_t$) at the initial period can be respectively expressed as:

(S96) $PIT_0 = \sum_h \{ - a_h + pit_h\, [\, w_0\, L_{h0}{}^S + (\, 1 - \sum_i e_{i0} \,)\, r_0\, W_h{}^*$

$+ \sum_i e_{i0}\, [\, DIV_{i0} \,/\, (\, P_{Ei,-1}\, E_i{}^* \,)\,]\, W_h{}^* \,] \}$

(S97) $ST_0 = \sum_h (\, \sum_j st_{j0}\, P_{j0}\, Y_{hj0}{}^D \,)$

(S98) $CGT_0 = \sum_h [\, cgt_{h0} \sum_i (\, P_{Ei0} - P_{Ei,-1} \,)\, E_{hi}{}^* \,] + \sum_i cgt_{i0}\, (\, P_{Ei0} - P_{Ei,-1} \,)\, E_i{}^*$

(S99) $LT_0 = \sum_i lt_0\, w_0 L_{i0}{}^D + lt_{g0} w_0 L_{g0}{}^D$

(S100) $ITC_0 = \sum_i ITC_{i0} = \sum_i itc_{i0}\, P_{I0}\, I_{i0}{}^D$

(S101) $CIT_0 = \sum_i cit_{i0}\, \{\, [\, P_{i0} - \sum_{1 \le j \le J} (\, c_{ij}\, P_{j0} \,)\,]\, Y_{i0}{}^S - [\, (\, 1 + lt_0 \,)\, w_0\, L_{i0}{}^D + \Phi_{i0}'\, K_{i0}' \,]$

$- r_0\, B_i{}^* \}$

(S102) $TGT_0 = PIT_0 + ST_0 + CGT_0 + LT_0 - ITC_0 + CIT_0$

From equation (82), the government's transfer payments to households ($TR_{gt}$) at the initial period can be written as:

(S103) $TR_{g0} = \sum_h TR_{h0}$

In sum, since we can not independently obtain the values of all variables related to the government sector without considering the effects of other sectors, we may have to rely on the equilibrium conditions of all markets together to solve for $\lambda_{gk}^*$. That is, only if $\lambda_h^*$ is solved, can we obtain the values of $TGL_t$ over the whole model horizon, and hence find the value of $GL_t$ which can further be used to solve other variables associated with the government sector. This will be discussed in detail later.

### 4.4 Solution for Equilibrium Conditions

Thus far we derived the closed-form solutions for all control variables, state variables, and costate variables at every period t for each sector, respectively. All these solutions have to satisfy the equilibrium conditions of all markets described in section 3.2.4. Hence, by using the solutions derived above, we will separately find the precise equilibrium conditions for each market, which can further be applied to obtain the initial values of those variables previously unsolved in each sector, especially the costate variables $\lambda_h^*$ and $\lambda_{gk}^*$.

#### 4.4.1 The Solution for All Markets

##### 4.4.1.1 The Solution for Consumption Goods Markets

From equation (112), the equilibrium conditions for the market of consumption good j at period t can be expressed in its simple form:

(S104) $Y_{jt}^S = \sum_{1 \le h \le m} Y_{hjt}^D + \sum_{1 \le f \le J} Y_{fjt}^D + Y_{Ijt}^D + Y_{gjt}^D, \qquad j = 1,\ldots,J, \quad t = 0,\ldots,T$

From equation (S34), the quantity of consumption good j supplied by industry j can be expressed as:

(S105) $Y_{jt}^S = K_{jt} \{ ( P_{jt} - \sum_{1 \le f \le J} c_{jf} P_{ft} ) ( 1- \beta_j ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta j) / \beta j}$

From equation (S1), the quantity of consumption good j demanded by individual household h can be expressed as:

(S106) $Y_{hjt}^{D} = \delta_h^{t} \alpha_{hj} / [\lambda_{ht} (1 + st_{jt}) P_{jt}]$, and then the demand by the whole household sector is

(S107) $\sum_{1 \leq h \leq m} Y_{hjt}^{D} = \sum_{1 \leq h \leq m} \{ \delta_h^{t} \alpha_{hj} / [\lambda_{ht} (1 + st_{jt}) P_{jt}] \}$

From equation (S34), the quantity of consumption good f supplied by industry f can be expressed as:

(S108) $Y_{ft}^{S} = K_{ft} \{ (P_{ft} - \sum_{1 \leq j \leq J} c_{fj} P_{jt}) (1 - \beta_f) / [(1 + lt_t) w_t] \}^{(1-\beta f) / \beta f}$

Accordingly, from equation (S35), the quantity of consumption good j demanded as intermediate inputs by industry f is

(S109) $Y_{fjt}^{D} = c_{fj} K_{ft} \{ (P_{ft} - \sum_{1 \leq j \leq J} c_{fj} P_{jt}) (1 - \beta_f) / [(1 + lt_t) w_t] \}^{(1-\beta f) / \beta f}$, and the intermediate demand, for consumption good j, by all producers of consumption goods is

(S110) $\sum_{1 \leq f \leq J} Y_{fjt}^{D}$

$$= \sum_{1 \leq f \leq J} \{ c_{fj} K_{ft} \{ (P_{ft} - \sum_{1 \leq j \leq J} c_{fj} P_{jt}) (1 - \beta_f) / [(1 + lt_t) w_t] \}^{(1-\beta f) / \beta f} \}$$

From equation (S34), the quantity of investment good supplied by industry I (i.e., the investment sector) can be expressed as:

(S111) $I_{It}^{S} = K_{It} \{ (P_{It} - \sum_{1 \leq j \leq J} c_{Ij} P_{jt}) (1 - \beta_I) / [(1 + lt_t) w_t] \}^{(1-\beta_I) / \beta_I}$

Accordingly, from equation (S35), the quantity of consumption good j demanded as intermediate inputs by industry I is

(S112) $Y_{Ijt}^{D} = c_{Ij} K_{It} \{ (P_{It} - \sum_{1 \leq j \leq J} c_{Ij} P_{jt}) (1 - \beta_I) / [(1 + lt_t) w_t] \}^{(1-\beta_I) / \beta_I}$

From equations (S62) and (S67), the quantity of consumption good j demanded by the government is

(S113) $Y_{gjt}^{D} = - \delta_g^{t} \gamma_{gj} / (\lambda_{gGLt} P_{jt}) = \delta_g^{t} \gamma_{gj} P_{jt}^{-1} P_{It} \lambda_{gkt}^{-1}$

Therefore, by substituting expressions (S105), (S107), (S110), (S112), and (S113) into (S104), the equilibrium conditions for the market of consumption good j at period t can be rewritten as:

$$\text{(S114)}\ K_{jt} \{ ( P_{jt} - \sum_{1 \le f \le J} c_{jf} P_{ft} ) ( 1- \beta_j ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta j) / \beta j}$$

$$= \sum_{1 \le h \le m} \{ \delta_h^{\ t} \alpha_{hj} / [ \lambda_{ht} ( 1 + st_{jt} ) P_{jt} ] \}$$

$$+ \sum_{1 \le f \le J} \{ c_{fj} K_{ft} \{ ( P_{ft} - \sum_{1 \le j \le J} c_{fj} P_{jt} ) (1- \beta_f ) / [ (1 + lt_t ) w_t ] \}^{(1-\beta f) / \beta f} \}$$

$$+ c_{Ij} K_{It} \{ ( P_{It} - \sum_{1 \le j \le J} c_{Ij} P_{jt} ) ( 1- \beta_I ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta_I) / \beta_I}$$

$$+ \delta_g^{\ t} \gamma_{gj} P_{jt}^{-1} P_{It} \lambda_{gkt}^{-1}, \qquad t = 0,\ldots,T$$

Thus, the equilibrium conditions for the market of consumption good j at the initial period can be written as:

$$\text{(S115)}\ K_j^* \{ ( P_{j0} - \sum_{1 \le f \le J} c_{jf} P_{f0} ) ( 1- \beta_j ) / [ ( 1 + lt_0 ) w_0 ] \}^{(1-\beta j) / \beta j}$$

$$= \sum_{1 \le h \le m} \{ \alpha_{hj} / [ \lambda_h^* ( 1 + st_{j0} ) P_{j0} ] \}$$

$$+ \sum_{1 \le f \le J} \{ c_{fj} K_f^* \{ ( P_{f0} - \sum_{1 \le j \le J} c_{fj} P_{j0} ) (1- \beta_f ) / [ (1 + lt_0 ) w_0 ] \}^{(1-\beta f) / \beta f} \}$$

$$+ c_{Ij} K_I^* \{ ( P_{I0} - \sum_{1 \le j \le J} c_{Ij} P_{j0} ) ( 1- \beta_I ) / [ ( 1 + lt_0 ) w_0 ] \}^{(1-\beta_I) / \beta_I}$$

$$+ \gamma_{gj} P_{j0}^{-1} P_{I0} \lambda_{gk}^{*-1}$$

#### 4.4.1.2 The Solution for the Labor Market

From equation (113), the equilibrium conditions for the labor market at period t can be expressed in its simple form:

$$\text{(S116)}\ \sum_{1 \le h \le m} L_{ht}^{\ S} = \sum_{1 \le j \le J} L_{jt}^{\ D} + L_{It}^{\ D} + L_{gt}^{\ D}, \qquad j = 1,\ldots,J;\ \ t = 0,\ldots,T$$

From equation (S2), the quantity of labor supplied by household h can be expressed as:

(S117) $L_{ht}^{S} = L_h^{*} - [\ \delta_h^{t}\ (\ 1 - \sum_{1 \le j \le J} \alpha_{hj}\ )\ ] \,/\, [\ \lambda_{ht}\ (\ 1 - pit_h\ )\ w_t\ ]$, and then the labor supply of the entire household sector is

(S118) $\sum_{1 \le h \le m} L_{ht}^{S} = \sum_{1 \le h \le m} \{\ L_h^{*} - [\ \delta_h^{t}\ (\ 1 - \sum_{1 \le j \le J} \alpha_{hj}\ )\ ] \,/\, [\ \lambda_{ht}\ (\ 1 - pit_h\ )\ w_t\ ]\ \}$

From equation (S29), the quantity of labor demanded by industry j is

(S119) $L_{jt}^{D} = K_{jt}\ \{\ (\ P_{jt} - \sum_{1 \le f \le J} c_{jf} P_{ft}\ )\ (\ 1 - \beta_j\ ) \,/\, [\ (\ 1 + lt_t\ )\ w_t\ ]\ \}^{1/\beta_j}$, and then the demand for labor by all producers of consumption goods can be expressed as:

(S120) $\sum_{1 \le j \le J} L_{jt}^{D} = \sum_{1 \le j \le J} \{\ K_{jt}\ \{\ (\ P_{jt} - \sum_{1 \le f \le J} c_{jf} P_{ft}\ )\ (\ 1 - \beta_j\ ) \,/\, [\ (\ 1 + lt_t\ )\ w_t\ ]\ \}^{1/\beta_j}\ \}$

From equation (S29), the quantity of labor demanded by the investment sector is

(S121) $L_{It}^{D} = K_{It}\ \{\ (\ P_{It} - \sum_{1 \le j \le J} c_{Ij} P_{jt}\ )\ (\ 1 - \beta_I\ ) \,/\, [\ (\ 1 + lt_t\ )\ w_t\ ]\ \}^{1/\beta_I}$

From equations (S63) and (S67), the quantity of labor demanded by the government is

(S122) $L_{gt}^{D} = -\ \delta_g^{t}\, \gamma_{gL} \,/\, (\ \lambda_{gGLt}\ w_t\ ) = \delta_g^{t}\ \gamma_{gL}\, w_t^{-1}\ P_{It}\, \lambda_{gkt}^{-1}$

Therefore, by substituting expressions (S118), (S120), (S121), and (S122) into (S116), the equilibrium conditions for the labor market at period t can be rewritten as:

$$
\begin{aligned}
\text{(S123)}\ & \textstyle\sum_{1 \le h \le m} \{\ L_h^{*} - [\ \delta_h^{t}\ (\ 1 - \sum_{1 \le j \le J} \alpha_{hj}\ )\ ] \,/\, [\ \lambda_{ht}\ (\ 1 - pit_h\ )\ w_t\ ]\ \} \\
& = \textstyle\sum_{1 \le j \le J} \{\ K_{jt}\ \{\ (\ P_{jt} - \sum_{1 \le f \le J} c_{jf} P_{ft}\ )\ (\ 1 - \beta_j\ ) \,/\, [\ (\ 1 + lt_t\ )\ w_t\ ]\ \}^{1/\beta_j}\ \} \\
& \quad + K_{It}\ \{\ (\ P_{It} - \textstyle\sum_{1 \le j \le J} c_{Ij} P_{jt}\ )\ (\ 1 - \beta_I\ ) \,/\, [\ (\ 1 + lt_t\ )\ w_t\ ]\ \}^{1/\beta_I} \\
& \quad + \delta_g^{t}\ \gamma_{gL}\, w_t^{-1}\ P_{It}\, \lambda_{gkt}^{-1}, \qquad\qquad t = 0,\ldots,T
\end{aligned}
$$

Thus, the equilibrium conditions for the labor market at the initial period can be written as:

$$
\begin{aligned}
\text{(S124)}\ & \textstyle\sum_{1 \le h \le m} \{\ L_h^{*} - [\ (\ 1 - \sum_{1 \le j \le J} \alpha_{hj}\ )\ ] \,/\, [\ \lambda_h^{*}\ (\ 1 - pit_h\ )\ w_0\ ]\ \} \\
& = \textstyle\sum_{1 \le j \le J} \{\ K_j^{*}\ \{\ (\ P_{j0} - \sum_{1 \le f \le J} c_{jf} P_{f0}\ )\ (\ 1 - \beta_j\ ) \,/\, [\ (\ 1 + lt_0\ )\ w_0\ ]\ \}^{1/\beta_j}\ \}
\end{aligned}
$$

$$+ K_I^* \{ ( P_{I0} - \sum_{1 \le j \le J} c_{Ij} P_{j0} ) ( 1 - \beta_I ) / [ ( 1 + lt_0 ) w_0 ] \}^{1/\beta_I}$$

$$+ \gamma_{gL} w_0^{-1} P_{I0} \lambda_{gk}^{*-1}$$

#### 4.4.1.3 The Solution for the Investment Good Market

From equation (114), the equilibrium conditions for the investment good market at period t can be expressed in its simple form:

(S125) $I_{It}^S = \sum_{1 \le j \le J} I_{jt}^D + I_{It}^D + I_{gt}^D$, $\qquad j = 1,\ldots,J; \quad t = 0,\ldots,T$

From equation (S34), the quantity of investment good supplied by the investment sector can be expressed as:

(S126) $I_{It}^S = K_{It} \{ ( P_{It} - \sum_{1 \le j \le J} c_{Ij} P_{jt} ) ( 1 - \beta_I ) / [ ( 1 + lt_t ) w_t ] \}^{(1-\beta_I)/\beta_I}$

which is the same as (S112).

From equation (S30), the quantity of investment good demanded by industry j is

(S127) $I_{jt}^D = [ 1/ b_j ] [ \lambda_{jt} \Pi_{0 \le s \le t} (1+ r_s) - ( 1 - itc_{jt} ) P_{It} ]$, and then the demand by all producers of consumption goods can be represented by:

(S128) $\sum_{1 \le j \le J} I_{jt}^D = \sum_{1 \le j \le J} [ 1/ b_j ] [ \lambda_{jt} \Pi_{0 \le s \le t} (1+ r_s) - ( 1 - itc_{jt} ) P_{It} ]$

From equation (S30), the quantity of investment good demanded by the investment sector is

(S129) $I_{It}^D = [ 1/ b_I ] [ \lambda_{It} \Pi_{0 \le s \le t} (1+ r_s) - ( 1 - itc_{It} ) P_{It} ]$

From expressions (S69) and (S69'), the quantity of investment good demanded by the government for t = 1,…,T is

(S130) $I_{gt}^D = \delta_g^{t+1} \gamma_{gk} \lambda_{gkt}^{-1} \{ [ 1 - ( 1 - \Phi_{gt+1} ) ( 1 + r_{t+1} )^{-1} P_{It}^{-1} P_{It+1} ]^{-1}$

$$- ( 1 - \Phi_{gt} ) \delta_g^{-1} [ ( 1 + r_t ) P_{It}^{-1} P_{It-1} - ( 1 - \Phi_{gt} ) ]^{-1} \},$$

while that for t = 0 (the initial period) should be

(S131) $I_{g0}^{D} = \delta_g \gamma_{gk} \lambda_{gk}^{* \, -1} [\, 1 - (\, 1 - \Phi_{g1}\,)(\, 1 + r_1\,)^{-1} P_{I0}^{-1} P_{I1}]^{-1} - (\, 1 - \Phi_{g0}\,) K_g^{*}$

Therefore, by substituting expressions (S126), (S128), (S129), and (S130)/(S131) into (S125), the equilibrium conditions for the investment good market for t = 1,…,T and for t = 0 can be respectively rewritten as

(S132) $K_{It} \{ (\, P_{It} - \sum_{1 \le j \le J} c_{Ij} P_{jt}\,)(\, 1- \beta_I\,) / [\, (\, 1 + lt_t\,) w_t\,] \}^{(1-\beta_I)/\beta_I}$

$= \sum_{1 \le j \le J} [\, 1/ b_j\,] [\, \lambda_{jt} \Pi_{0 \le s \le t} (1+ r_s) - (\, 1- itc_{jt}\,) P_{It}\,]$

$+ [\, 1/ b_I\,] [\, \lambda_{It} \Pi_{0 \le s \le t} (1+ r_s) - (\, 1- itc_{It}\,) P_{It}\,]$

$+ \delta_g^{t+1} \gamma_{gk} \lambda_{gkt}^{-1} \{ [\, 1 - (\, 1 - \Phi_{gt+1}\,)(\, 1 + r_{t+1}\,)^{-1} P_{It}^{-1} P_{It+1}]^{-1}$

$- (\, 1 - \Phi_{gt}\,) \delta_g^{-1} [\, (\, 1 + r_t\,) P_{It}^{-1} P_{It-1} - (\, 1 - \Phi_{gt}\,)\,]^{-1} \}, \quad t = 1,\ldots,T$

(S133) $K_I^{*} \{ (\, P_{I0} - \sum_{1 \le j \le J} c_{Ij} P_{j0}\,)(\, 1- \beta_I\,) / [\, (\, 1 + lt_0\,) w_0\,] \}^{(1-\beta_I)/\beta_I}$

$= \sum_{1 \le j \le J} [\, 1/ b_j\,] [\, \lambda_j^{*} (1+ r_0\,) - (\, 1- itc_{j0}\,) P_{I0}\,]$

$+ [\, 1/ b_I\,] [\, \lambda_I^{*} (1+ r_0\,) - (\, 1- itc_{I0}\,) P_{I0}\,]$

$+ \delta_g \gamma_{gk} \lambda_{gk}^{* \, -1} [\, 1 - (\, 1 - \Phi_{g1}\,)(\, 1 + r_1\,)^{-1} P_{I0}^{-1} P_{I1}]^{-1} - (\, 1 - \Phi_{g0}\,) K_g^{*}, \qquad t = 0.$

#### 4.4.1.4 The Solution for the Financial Market

From equation (115), the equilibrium conditions for the financial market at period t can be expressed in its simple form:

(S134) $\sum_{1 \le h \le m} FD_{ht}^{S} = \sum_{1 \le j \le J} FD_{jt}^{D} + FD_{It}^{D} + FD_{gt}^{D}, \qquad j = 1,\ldots,J; \quad t = 0,\ldots,T$

From expressions (5) and (7), the new funds supplied by household h is

(S135) $FD_{ht}^{S} = a_h + (\, 1 - pit_h\,) \{ w_t L_{ht}^{S} + (\, 1 - \sum_i e_{it}\,) r_t W_{ht} + \sum_i e_{it} [\, DIV_{it} / (\, P_{Eit-1} E_{it}\,)\,]$

$\bullet W_{ht} \} + (\, 1 - cgt_{ht}\,) \sum_i (\, P_{Eit} - P_{Eit-1}\,) E_{hit} + TR_{ht} - \sum_j (\, 1 + st_{jt}\,) P_{jt} Y_{hjt}^{D},$

and then the supply of all households can be represented by:

(S136) $\sum_{1 \leq h \leq m} FD_{ht}^{S}$

$$= \sum_{1 \leq h \leq m} \{ a_h + ( 1 - pit_h ) \{ w_t L_{ht}^{S} + ( 1 - \sum_i e_{it} ) r_t W_{ht} + \sum_i e_{it} [ DIV_{it} / ( P_{Eit-1} E_{it} ) ] \bullet W_{ht} \} + ( 1 - cgt_{ht} ) \sum_i ( P_{Eit} - P_{Eit-1} ) E_{hit} + TR_{ht} - \sum_j ( 1 + st_{jt} ) P_{jt} Y_{hjt}^{D} \},$$

$i = 1,\ldots,J,I.$

From equation (41), the new funds demanded by industry j is

(S137) $$FD_{jt}^{D} = (1 - itc_{jt}) P_{It} I_{jt}^{D} + 0.5 b_j (I_{jt}^{D})^2 + (1 - cgt_{jt}) ( P_{Ejt} - P_{Ejt-1} ) E_{jt} - RE_{jt} - \Phi_{jt}' K_{jt}',$$

and then the demand by all producers of consumption goods can be represented by:

(S138) $$\sum_{1 \leq j \leq J} FD_{jt}^{D} = \sum_{1 \leq j \leq J} \{ ( 1 - itc_{jt} ) P_{It} I_{jt}^{D} + 0.5 b_j (I_{jt}^{D})^2 + ( 1 - cgt_{jt} ) ( P_{Ejt} - P_{Ejt-1} ) E_{jt} - RE_{jt} - \Phi_{jt}' K_{jt}' \}$$

From equation (41), the new funds demanded by the investment sector is

(S139) $$FD_{It}^{D} = ( 1 - itc_{It} ) P_{It} I_{It}^{D} + 0.5 b_I (I_{It}^{D})^2 + ( 1 - cgt_{It} ) ( P_{EIt} - P_{EIt-1} ) E_{It} - RE_{It} - \Phi_{It}' K_{It}'$$

From expressions (83) and (84), the new funds demanded by the government is

(S140) $$FD_{gt}^{D} = [ \sum_j P_{jt} Y_{gjt}^{D} + ( 1 + lt_{gt} ) w_t L_{gt}^{D} + P_{It} I_{gt}^{D} + TR_{gt} + r_t GL_t ] - TGT_t$$

Therefore, by substituting (S136), (S138), (S139), and (S140) into (S134), the equilibrium conditions for the financial market at period t can be rewritten as:

(S141) $$\sum_{1 \leq h \leq m} \{ a_h + ( 1 - pit_h ) \{ w_t L_{ht}^{S} + ( 1 - \sum_i e_{it} ) r_t W_{ht} + \sum_i e_{it} [ DIV_{it} / ( P_{Eit-1} E_{it} ) ] \bullet W_{ht} \} + ( 1 - cgt_{ht} ) \sum_i ( P_{Eit} - P_{Eit-1} ) E_{hit} + TR_{ht} - \sum_j ( 1 + st_{jt} ) P_{jt} Y_{hjt}^{D} \}$$

$$= \sum_{1 \leq j \leq J} \{ ( 1 - itc_{jt} ) P_{It} I_{jt}^{D} + 0.5 b_j (I_{jt}^{D})^2 + ( 1 - cgt_{jt} ) ( P_{Ejt} - P_{Ejt-1} ) E_{jt} - RE_{jt} - \Phi_{jt}' K_{jt}' \}$$

$$+ ( 1 - itc_{It} ) P_{It} I_{It}^{D} + 0.5 b_I (I_{It}^{D})^2 + ( 1 - cgt_{It} ) ( P_{EIt} - P_{EIt-1} ) E_{It} - RE_{It} - \Phi_{It}' K_{It}'$$

$$+ [ \sum_j P_{jt} Y_{gjt}^{D} + ( 1 + lt_{gt} ) w_t L_{gt}^{D} + P_{It} I_{gt}^{D} + TR_{gt} + r_t GL_t ] - TGT_t, \qquad t = 0,\ldots,T$$

Thus, the equilibrium conditions for the financial market at the initial period can be written as:

$$(S142)\ \sum_{1 \le h \le m} \{ a_h + (1 - pit_h) \{w_0 L_{h0}^S + (1 - \sum_i e_{i0}) r_0 W_h^* + \sum_i e_{i0} [ DIV_{i0} / ( P_{Ei,-1} E_i^* ) ]$$

$$\bullet W_h^*\} + ( 1 - cgt_{h0} ) \sum_i ( P_{Ei0} - P_{Ei,-1} ) E_{hi}^* + TR_{h0} - \sum_j (1 + st_{j0}) P_{j0} Y_{hj0}^D \}$$

$$= \sum_{1 \le j \le J} \{ ( 1 - itc_{j0} ) P_{I0} I_{j0}^D + 0.5\, b_j (I_{j0}^D)^2 + ( 1 - cgt_{j0} ) ( P_{Ej0} - P_{Ej,-1} ) E_j^*$$

$$- RE_{j0} - \Phi_{j0}' K_{j0}' \}$$

$$+ ( 1 - itc_{I0} ) P_{I0} I_{I0}^D + 0.5\, b_I (I_{I0}^D)^2 + (1 - cgt_{I0}) (P_{EI0} - P_{EI,-1}) E_I^* - RE_{I0} - \Phi_{I0}' K_{I0}'$$

$$+ [ \sum_j P_{j0} Y_{gj0}^D + ( 1 + lt_{g0} ) w_0 L_{g0}^D + P_{I0} I_{g0}^D + TR_{g0} + r_0 GL^* ] - TGT_0$$

### 4.4.2 The Solution for $\lambda_h^*$ and $\lambda_{gk}^*$

From the analysis discussed prior to this section, once the initial values of the costate variables, $\lambda_h^*$ and $\lambda_{gk}^*$, are solved, we can obtain values of all the variables related to the household sector for every period recursively, and with the solved values of all the variables, the values of $TGL_t$ and hence $GL_t$ over the whole model horizon can be obtained.

First of all, the value of $\lambda_{gk}^*$ can be obtained by solving equation (S133) as:

$$(S143)\ \lambda_{gk}^* = \delta_g \gamma_{gk} [ 1 - ( 1 - \Phi_{g1} ) ( 1 + r_1 )^{-1} P_{I0}^{-1} P_{I1} ]^{-1}$$

$$\cdot \{ K_I^* \{ ( P_{I0} - \sum_{1 \le j \le J} c_{Ij} P_{j0} ) ( 1 - \beta_I ) / [ ( 1 + lt_0 ) w_0 ] \}^{(1-\beta_I) / \beta_I}$$

$$+ ( 1 - \Phi_{g0} ) K_g^* - \sum_{1 \le j \le J} [ 1 / b_j ] [ \lambda_j^* (1 + r_0 ) - ( 1 - itc_{j0} ) P_{I0} ]$$

$$- [ 1 / b_I ] [ \lambda_I^* (1 + r_0 ) - ( 1 - itc_{I0} ) P_{I0} ] \}^{-1}$$

where $K_I^*$ and $K_g^*$ are predetermined, and $\lambda_j^*$ and $\lambda_I^*$ have already been solved by equation (S40) with the transversality condition $\lambda_{iT} = 0$ for i = 1,…,J, I.

Then, by substituting the solved $\lambda_{gk}^*$ into equation (S115), the value of $\lambda_h^*$ can be obtained theoretically. However, there is a mathematical problem we have to deal with in solving $\lambda_h^*$ in the empirical work. That is, in this study, there are J consumption goods represented by j in equation (S115) and m income groups represented by h. In order to obtain m values of $\lambda_h^*$ in this problem, J has to be equal to or greater than m mathematically.

In sum, by further employing equilibrium conditions, we can eventually solve all the initial values of control, state, and costate variables and other relative variables. In addition, once all the initial values (equilibria) in this study, characterized by temporary Walrasian equilibrium, are obtained, we can thereby recursively derive the equilibrium values of all variables for every period t for the system as a whole. Nevertheless, in order to be able to solve the whole model's maximization problems, the number of consumption goods, J, must be no less than that of income groups, m, which is for mathematic reasoning, rather than based on economic logic.

# CHAPTER FIVE

# SIMULATION DESIGN AND RESULTS

Generally, in a dynamic general equilibrium model characterized by a finite horizon, the decision time frame is set at one hundred years. Therefore, this kind of empirical model can be used to evaluate the short-run and long-run impacts of various policy scenarios just by setting the desirable values of time period t into the model. However, with respect to simulation analysis, this study focuses on theoretically exploring how to apply simulation schemes to the model and further investigating the possible effects of alternative price expectations, labor tax, investment tax credit, and alternative corporate income tax integration plans on the model structure and equilibrium values. Especially, this study will compare the difference in the effects of alternative simulations of corporate income tax integration on the agents and on control variables, state variables, and other financial variables, respectively. There are various corporate income tax integration schemes. In order to evaluate the effects of different corporate income tax integration schemes on the model structure and equilibrium values, this study will compare three integration plans: one full and two partial integration plans.

## 5.1 Price Expectations

In a dynamic general equilibrium model, the formation of price expectations has to be made and has significant impacts on simulation results. For simplicity, this study will merely show three types of expectations in explaining the difference of these three approaches in simulation design, i.e., myopic expectations, constant rate of growth, and perfect foresight.

There are several types of prices in this model: prices of consumption goods and investment good, wage rate, interest rate, and the price of equity. Myopic expectations indicate that the initial values of prices are expected to prevail into the terminal period. That is, $P_t^e = P_0$ for $t = 1,\ldots, T$. On the other hand, "constant rate of growth" type of price expectations refers to the one with that the growth rate of initial prices is expected to prevail into the terminal period. That is, $P_t^e = P_0 (P_0 / P_{-1})^t$ for $t = 1,\ldots, T$. Finally, "perfect foresight" indicates that prices over the model horizon can be perfectly anticipated. That is, $P_t^e = P_t$ for $t = 1,\ldots, T$. Therefore, externally, the theoretical model with the assumption of perfect foresight is exactly identical to the one shown in this study.

Thus, in order to obtain the equilibrium value of one variable for a specific future period t, we just have to substitute the expected price of the specific period for that in equilibrium conditions shown in chapter three and four. For example, based on perfect foresight, myopic expectations and constant rate of growth, the equilibrium value of the total government expenditures at period t ($TGE_t$) in equation (81) can be expressed as (R1), (R2), and (R3) respectively:

(R1) $TGE_t = \sum_j P_{jt} Y_{gjt} + (1 + lt_{gt}) w_t L_{gt} + P_{It} I_{gt} + TR_{gt} + r_t GL_t$

(R2) $TGE_t = \sum_j P_{j0} Y_{gjt} + (1 + lt_{gt}) w_0 L_{gt} + P_{I0} I_{gt} + TR_{gt} + r_0 GL_t$

(R3) $TGE_t = \sum_j P_{j0} (P_{j0} / P_{j,-1})^t Y_{gjt} + (1 + lt_{gt}) w_0 (w_0 / w_{-1})^t L_{gt} + P_{I0} (P_{I0} / P_{I,-1})^t I_{gt}$

$+ TR_{gt} + r_0 (r_0 / r_{-1})^t GL_t$

In addition, the model such as that specified in chapter three with myopic expectations usually can be treated as the base case for the comparison with alternative policy scenarios in an empirical study.

### 5.2 Labor Tax

Any difference in equilibria with and without labor tax will also be investigated in this study. Labor tax affects the costs of labor services used by industries and the government. Since labor tax is imposed only on industries and the government, there seems no direct effect on variables attributed to the household sector in case labor tax is abolished from the model.

For the production sector, abolishing labor tax at period t=0 (i.e., $lt_t=0$, and $lt_{gt}=0$) will influence industries' expenditures and thus their operating income, earnings, demand for external funds ($FD_{i0}^D$), net cash flows ($NCF_{i0}$), and the dividends ($DIV_{i0}$) and retained earnings ($RE_{i0}$) based on the assumption of a constant dividend/retention ratio at the same period in the model.

Among the first-order conditions for the optimization problem of industries, the change in labor tax solely has something to do with $\partial H_{it} / \partial L_{it}$ in equation (67), i.e., the derivative of the Hamiltonian with respect to labor. Nevertheless, from equations (S44)-(S46) and (S40), abolishing labor tax at t=0 will directly raise the equilibrium values of labor demand by industries ($L_{i0}^D$), intermediate input demanded by industries ($Y_{ij0}^D$), supply of consumption goods and investment good ($Y_{i0}^S$ , i.e., $Y_{j0}^S$ and $I_{I0}^S$), and the marginal net cash flows of capital ($\lambda_i^*$) respectively at the same period. Through the increased $\lambda_i^*$, this policy also raises the equilibrium value of investment good demanded by industries ($I_{i0}^{D}$) which further increases the value of capital stock at next period ($K_{i1}$). Consequently, the enlarged $K_{i1}$ will make the equilibrium values in period t+1 of labor demand by industries ($L_{i1}^{D}$), intermediate input demanded by industries ($Y_{ij1}^{D}$), and supply of consumption goods and investment good ($Y_{j1}^{S}$ and $I_{I1}^{S}$) greater than their original values.

In addition, even with the higher $L_{i0}^{D}$ and $Y_{i0}^{S}$, we can not be sure about whether the equilibrium values of earnings before taxes ($EBT_{i0}$), net income ($NI_{i0}$), $DIV_{i0}$, and $RE_{i0}$ increase or not simply from equations (34)-(36) in this theoretical model. But we can almost presume that they would be greater under this policy. Furthermore, we can not ensure the change in the values of $NCF_{i0}$ and $FD_{i0}^{D}$ and hence the quantity of industries' equity ($E_{i1}$) and the outstanding bonds of industries ($B_{i1}$) from equations (40), (S51), (S56), and (S58), respectively.

According to equation (S37), $\lambda_{it}$ will be continuously greater under this policy than keeping the old labor tax rate. Then, from equations (S30), (S31), (S29), (S34), and (S35) in order, $I_{it}^{D}$, $K_{it+1}$, $L_{it+1}^{D}$, $Y_{jt+1}^{S}$, $I_{It+1}^{S}$, and $Y_{ijt+1}^{D}$ all are greater than their original values in the same period.

On the other hand, the change in labor tax instantaneously affects the total government expenditures ($TGE_0$) and tax revenues ($TGT_0$) focusing on labor tax revenues ($LT_0$) and corporate income tax revenues ($CIT_0$). Futhermore, this policy will affect the funds demanded by the government ($FD_{g0}^{D}$) apparently through the deviation between $TGE_0$ and $TGT_0$.

Among the first-order conditions for the optimization problem of the government, the change in labor tax has connections to $\partial H_{gt} / \partial L_{gt}$ and $\partial H_{gt} / \partial \lambda_{gGLt}$ in equations

(106) and (110), i.e., the derivatives of the Hamiltonian with respect to labor and the marginal provision of government liabilities, respectively.

From equation (S143), abolishing labor tax also has an impact on the initial value of marginal provision of government capital ($\lambda_{gk}^*$), but the exact impact can not be identified in this study. Then, no matter how $\lambda_{gk}^*$ is affected, the absolute value of $\lambda_{gGL}^*$ will vary positively with $\lambda_{gk}^*$ (because $\lambda_{gGLt}$ is less than zero) by the connection between them, while the equilibrium values of labor, consumption goods, and investment good demanded by the government at the same period ($L_{g0}^D$, $Y_{gj0}^D$, and $I_{g0}^D$) and thereby the capital stock at next period ($K_{g1}$) will vary with $\lambda_{gk}^*$ in the opposite direction. Subsequently, from equation (S70), the change in the equilibrium values of $\lambda_{gk1}$ and hence $\lambda_{gGL1}$ is uncertain and depends on the relative effects of $K_{g1}$ and $\lambda_{gk}^*$ on $\lambda_{gk1}$. Furthermore, from equations (S113), (S122), and (S130), the equilibrium values of consumption goods, labor, and investment good demanded by the government at period t ($Y_{gjt}^D$, $L_{gt}^D$, and $I_{gt}^D$) will always be inversely affected by $\lambda_{gkt}$.

On the other hand, even though there is no direct impact of abolishing labor tax on the household sector, we can discuss it indirectly from equation (S115) from which the initial marginal utility of household's wealth ($\lambda_h^*$) can be derived as long as $J \geqq m$, but the effect of this policy on $\lambda_h^*$ is still ambiguous in this theoretical model. Anyway, we still find the relationship between $\lambda_h^*$ and the initial equilibrium values of consumption goods demanded and labor supplied by households ($Y_{hj0}^D$ and $L_{h0}^S$) from equations (S15) and (S16). That is, $Y_{hj0}^D$ varies negatively with $\lambda_h^*$, while $L_{h0}^S$ positively. Similarly, equations (S1) and (S2) also show that the relationship between $\lambda_{ht}$ and $Y_{hjt}^D$ and $L_{ht}^S$ are the same as that at the initial period.

As to the impact of abolishing labor tax at the initial period on government tax revenues, the only one affected is the investment tax credit, the government's subsidies given to industries, which becomes greater due to the higher $I_{i0}^D$. Of course, the labor tax revenues become zero.

In turn, no matter whether labor tax is eliminated or lowered at period t=0, theoretically we can easily derive again all the equilibrium values of the control, state, costate, and other financial variables for each sector over the whole model horizon, while we can not be sure about its impacts on some variables even for the initial period (t=0) in this theoretical simulation analysis. That is, we can not directly and completely explain the effect of a change in labor tax on equilibrium values simply from this theoretical model. Rather, through an empirical study, we can clearly explore the global influence of such a policy on all variables for all agents at each period.

### 5.3 Investment Tax Credit

Here, this study attempts to show the effects of abolishing the investment tax credit (ITC) on the model structure and equilibrium values. This indicates that the value of the investment tax credit rate ($itc_{it}$) has to be set at zero into the theoretical model. Since ITC is only given to industries from the government, there seems no direct effect on variables attributed to the household sector under this policy.

For the production sector, abolishing ITC at period t=0 will directly influence industries' actual investment expenditures ($AIE_{i0}$) and thus their $NCF_{i0}$ and $FD_{i0}^D$ at the same period in the model. Therefore, it will directly affect the equilibrium conditions for the investment good market and the financial market.

Among the first-order conditions for the optimization problem of industries, the change in $itc_{it}$ only affects $\partial H_{it} / \partial I_{it}$ in equation (68), i.e., the derivative of the Hamiltonian with respect to investment. Therefore, from equations (S38) and (S40), $\lambda_i^*$ and $\lambda_{it}$ have nothing to do with this policy. Consequently, equations (S30), (S42), and (S43) apparently show that this policy will make equilibrium values of $I_{i0}^D$ and $I_{it}^D$ less than those without this policy. On the other hand, equations (S44)-(S46) indicate that $L_{i0}^D$, $Y_{j0}^S$, $I_{I0}^S$, and $Y_{ij0}^D$ all are not affected by this policy instantaneously, while equations (S31), (S29), (S34), and (S35) show that $I_{it}^D$ will make equilibrium values of $K_{it+1}$ and hence $L_{it+1}^D$, $Y_{jt+1}^S$, $I_{It+1}^S$, and $Y_{ijt+1}^D$ all less than their original values. In addition, from equations (34)-(36), there is no change in equilibrium values of $EBT_{i0}$, $NI_{i0}$, $DIV_{i0}$, and $RE_{i0}$ due to $L_{i0}^D$, $Y_{j0}^S$, and $I_{I0}^S$ remaining the same under this policy. But we can not ensure the change in the values of $FD_{i0}^D$ and hence $E_{i1}$, $B_{i1}$, and $NCF_{i0}$ from equations (S51), (S56), (S58), and (40), respectively.

With respect to the government sector, the abolishment of ITC instantaneously affects the total government tax revenues ($TGT_0$) and further the funds demanded by the government ($FD_{g0}^D$). Among the first-order conditions for the optimization problem of the government, the change in ITC only affects $\partial H_{gt} / \partial \lambda_{gGLt}$ in equation (110), i.e., the derivatives of the Hamiltonian with respect to the marginal provision of government liabilities.

Eequation (S143) shows that abolishing ITC makes the initial value of marginal provision of government capital ($\lambda_{gk}^*$) and hence the absolute value of $\lambda_{gGL}^*$ lower. Then, from equations (S77)-(S80), the equilibrium values of consumption goods, labor, and investment good demanded by the government at the same period ($Y_{gj0}^D$, $L_{g0}^D$, and $I_{g0}^D$)

and thereby the capital stock at next period ($K_{g1}$) will vary with $\lambda_{gk}^*$ in the opposite direction. Therefore, from equation (81), the value of $TGE_0$ increases. Subsequently, from equation (S70), the change in the equilibrium values of $\lambda_{gk1}$ and hence $\lambda_{gGL1}$ is uncertain and depends on the relative effect of $K_{g1}$ and $\lambda_{gk}^*$ on $\lambda_{gk1}$ simultaneously. Furthermore, from equations (S113), (S122), and (S130), the equilibrium values of labor, investment good, and consumption goods demanded by the government at period t ($L_{gt}^D$, $I_{gt}^D$, and $Y_{gjt}^D$) will always be inversely affected by $\lambda_{gkt}$.

Even though there is no direct impact of abolishing ITC on the household sector, we can discuss it from equation (S115) from which the initial marginal utility of household's wealth ($\lambda_h^*$) can be derived, and this policy will indirectly raise the equilibrium value of $\lambda_h^*$ through $\lambda_{gk}^*$. Then, equations (S15) and (S16) show that the abolishment of ITC lowers $Y_{hj0}^D$ and raises $L_{h0}^S$ through $\lambda_h^*$, respectively. Besides, the lower $Y_{hj0}^D$ and the higher $L_{h0}^S$ with the unchanged $DIV_{i0}$ make the equilibrium values of $DI_{h0}$ and hence $SAV_{h0}$ and households' wealth ($W_{h1}$) greater through equations (3)-(6). Since we can not be sure about the value of $DIV_{i1}$ from equations (34) and (48), so can't be sure about the equilibrium value of $\lambda_{h1}$. From equations (S1) and (S2), the relationship between $\lambda_{ht}$ and $L_{ht}^S$ and $Y_{hjt}^D$ are the same as that at the initial period.

As to the impact of abolishing ITC on other individual tax revenues collected by the government at the initial period, capital gains tax revenues ($CGT_0$) and corporate income tax revenues ($CIT_0$) remain the same; personal income tax revenues ($PIT_0$) increase due to the higher $L_{h0}^S$; sales tax revenues ($ST_0$) decrease due to the lower $Y_{hj0}^D$; and labor tax revenues ($LT_0$) increase due to the higher $L_{g0}^D$ which simultaneously induces the government to pay more for the labor. Therefore, we can not be sure about the change in the value of $TGT_0$ and hence $GL_1$ and $FD_{g0}{}^D$ from equations (79), (83), and (84), respectively.

In sum, the abolishment of ITC does not have any impact on $\lambda_i{}^*$ and $\lambda_{it}$ at all. Similarly, this policy has no instantaneous effect on the initial equilibrium values of $L_{i0}{}^D$, $Y_{j0}{}^S$, $I_{I0}{}^S$, and $Y_{ij0}{}^D$, and hence $EBT_{i0}$, $NI_{i0}$, $DIV_{i0}$, and $RE_{i0}$. Besides, the equilibrium values of all the control variables and the state variable in the production sector are lower than their original values. The inverse relationship between costate variables and control variables in the government sector under this policy can be established as that under the abolishment of labor tax. The difference is that the change in the initial values of costate variables and hence control variables and the value of government capital at the beginning of period 1 is able to be determined. Under the policy of abolishing ITC, we have higher values of $\lambda_h{}^*$, $L_{h0}{}^S$, $DI_{h0}$, $SAV_{h0}$, and $W_{h1}$, and lower $Y_{hj0}{}^D$. The relationship between $\lambda_{ht}$, $Y_{hjt}{}^D$, and $L_{ht}{}^S$ is akin to that under the policy of abolishing labor tax.

### 5.4 Full Integration

In this study, full integration is designed to completely abolish the corporate income tax (CIT) and then households would have to pay personal income taxes (PIT) on income from both dividends ($DIV_{it}$) and retained earnings ($RE_{it}$). That is, the corporate income tax rate ($cit_{it}$) is set to be zero and therefore industries' earnings before and after taxes are identical to each other ($EBT_{it}=NI_{it}$) in this study. Besides, households' tax burdens intuitively become heavier due to the additional retained earnings which they do not really receive at the time they report their personal income taxes. Of course, this will certainly affect the total government tax revenues ($TGT_t$).

For the production sector, abolishing CIT at period t=0 will directly eliminate industries' tax burdens on income, intuitively increase the values of $NI_{i0}$, $DIV_{i0}$, and $RE_{i0}$, and thus affect their $NCF_{i0}$ and $FD_{i0}^{D}$ at the same period in the model. Therefore, it will directly affect the equilibrium conditions for the financial market.

Among the first-order conditions for the optimization problem of industries, the change in $cit_{it}$ simultaneously affects $\partial H_{it} / \partial L_{it}$ and $\partial H_{it} / \partial K_{it}$ in equations (67) and (69), i.e., the derivatives of the Hamiltonian with respect to labor and capital, respectively. From equations (S44)-(S46), $L_{i0}^{D}$, $Y_{j0}^{S}$, $I_{I0}^{S}$, and $Y_{ij0}^{D}$ all are not instantaneously affected by this policy. Therefore, from equations (35), (36), (S48), and (S50), under full integration, $NI_{i0}$, $DIV_{i0}$, and $RE_{i0}$ will have greater equilibrium values than before the integration. But we can not ensure the change in the values of $FD_{i0}^{D}$ and hence $E_{i1}$, $B_{i1}$, and $NCF_{i0}$ from equations (S51), (S56), (S58), and (40), respectively.

However, from equations (S40)-(S43), the equilibrium values of $\lambda_i^{*}$, $\lambda_{it}$, $I_{i0}^{D}$, and $I_{it}^{D}$ become greater under this integration plan. Consequently, equations (S31), (S29), (S34), and (S35) show that $I_{it}^{D}$ will make the equilibrium values of $K_{it+1}$ and hence $L_{it+1}^{D}$, $Y_{jt+1}^{S}$, $I_{It+1}^{S}$, and $Y_{ijt+1}^{D}$ all greater than their original values before this policy. However, we can not be sure about the values of $NI_{i1}$ and hence $DIV_{i1}$ and $RE_{i1}$ from equations (35), (47), and (48).

With respect to the government sector, full integration instantaneously affects the total government tax revenues ($TGT_0$) and further the funds demanded by the government ($FD_{g0}^{D}$). Among the first-order conditions for the optimization problem of the government, the change in $cit_{it}$ only affects $\partial H_{gt} / \partial \lambda_{gGLt}$ in equation (110).

From equation (S143) and (S75), full integration makes the initial value of marginal provision of government capital ($\lambda_{gk}^*$) and hence the absolute value of $\lambda_{gGL}^*$ greater through the higher $\lambda_i^*$. Then, from equations (S77)-(S80), the equilibrium values of consumption goods, labor, and investment good demanded by the government at the same period ($Y_{gj0}^D$, $L_{g0}^D$, and $I_{g0}^D$) and thereby the capital stock at next period ($K_{g1}$) will vary with $\lambda_{gk}^*$ in the opposite direction. Therefore, the value of $TGE_0$ in equation (81) decreases. Subsequently, from equation (S70), the change in the equilibrium values of $\lambda_{gk1}$ and hence $\lambda_{gGL1}$ is uncertain and depends on the relative effect of $K_{g1}$ and $\lambda_{gk}^*$ on $\lambda_{gk1}$ simultaneously. Furthermore, equations (S113), (S122), and (S130) show that the equilibrium values of labor, investment good, and consumption goods demanded by the government at period t ($L_{gt}^D$, $I_{gt}^D$, and $Y_{gjt}^D$) will always be inversely affected by $\lambda_{gkt}$.

As to the household sector, full integration will directly influence households' disposable income ($DI_{ht}$) and savings ($SAV_{ht}$). Among the first-order conditions for the optimization problem of households, this policy simultaneously affects $\partial H_{ht} / \partial W_{ht}$ and $\partial H_{ht} / \partial \lambda_{ht}$ in equations (26) and (27), i.e., the derivatives of the Hamiltonian with respect to wealth and the marginal utility of households' wealth, respectively. From equation (S115), this policy will lower the equilibrium value of $\lambda_h^*$ through the higher $\lambda_{gk}^*$. Then, equations (S15) and (S16) show that full integration raises $Y_{hj0}^D$ and lowers $L_{h0}^S$ through $\lambda_h^*$, respectively. Even though we have the higher $DIV_{i0}$ and $RE_{i0}$ with the lower $L_{h0}^S$ and the higher $Y_{hj0}^D$, we can not exactly ensure the change in the values of $DI_{h0}$ and hence $SAV_{h0}$ from equations (3)- (5), so can't be sure about the equilibrium values of $\lambda_{h1}$ and $W_{h1}$ derived from equation (S3) and (6), respectively. Nevertheless, $DI_{h0}$,

$SAV_{h0}$, and $W_{h1}$ tend to become smaller under full integration. From equations (S1) and (S2), the relationship between $\lambda_{ht}$ and $Y_{hjt}^D$ and $L_{ht}^S$ is the same as that at the initial period.

As to the impact of full integration on other individual tax revenues collected by the government at the initial period, sales tax revenues ($ST_0$) increase due to the higher $Y_{hj0}^D$; capital gains tax revenues ($CGT_0$) remain the same; investment tax credit ($ITC_0$) increases due to the higher $I_{i0}^D$; and labor tax revenues ($LT_0$) decrease due to the lower $L_{g0}^D$ which simultaneously induces the government to pay less for the labor. In particular, even though in the literature it is often argued that full integration will make personal income tax revenues (PIT) greater, we can not exactly point out whether $PIT_0$ becomes greater or not due to the lower $L_{h0}^S$ in this theoretical model. Therefore, we can not be sure about the change in the values of $TGT_0$ and hence the government liabilities ($GL_1$) and the funds demanded by the government ($FD_{g0}^D$) from equations (79), (83), and (84), respectively.

In sum, full integration apparently raises all the equilibrium values of all control, state, and costate variables in the production sector over the model horizon, except the initial values of $K_i^*$ which is predetermined, and $L_{i0}^D$, $Y_{j0}^S$, $I_{I0}^S$, and $Y_{ij0}^D$ which are not affected by this policy. Therefore, $NI_{i0}$, $DIV_{i0}$, and $RE_{i0}$ also increase under this policy. It should be noted that compared to the policy of abolishing ITC, this policy has the completely opposite impact on control, state, and costate variables in the production sector. The inverse relationship between costate variables and control variables in the government sector under full integration is the same as those described in section 5.2 and 5.3. The difference is that the initial values of costate variables increase and hence control variables decrease and the value of government capital at the beginning of period

1 also decrease. Under this policy, we have lower values of $\lambda_h^*$ and $L_{h0}^S$ and higher $Y_{hj0}^D$, which is opposite to those in the policy of abolishing ITC. The relationship between $\lambda_{ht}$, $Y_{hjt}^D$, and $L_{ht}^S$ is identical to that under the former two policies.

### 5.5 Partial Integration A: Dividends Deducted from the Corporate Income Tax Base

In this study, partial integration A is designed to partially eliminate the problem of double taxation on dividends[1] by deducting dividends from the corporate income tax base. Therefore, dividends ($DIV_{it}$) are taxed only at the personal level, and the corporate income tax on retained earnings ($RE_{it}$) remains at the corporate level. With respect to the structure of the model, the main effect of this policy is on the production sector, especially on net income ($NI_{it}$); the government sector is affected only in its tax revenues ($TGT_t$) and liabilities ($GL_{t+1}$); there is almost no impact on the household sector.[2]

For the production sector, partial integration A at period t=0 will directly influence industries' $NI_{i0}$ and thus their $NCF_{i0}$, $FD_{i0}^D$, $DIV_{i0}$, and $RE_{i0}$ in the model. Therefore, it will directly affect the equilibrium conditions for the financial market.

All the first-order conditions for the optimization problem of industries stay the same under partial integration A. Therefore, the equilibrium values of all control, state, and costate variables in the production sector are completely not affected by this policy over the whole model horizon. However, at each period t, partial integration A raises equilibrium values of $NI_{it}$, $DIV_{it}$, $RE_{it}$, and $NCF_{it}$, but lowers the equilibrium value of

[1] Stockholders would actually be better off if corporate income taxes were partially shifted forward to consumers or backward to other factors of production in the first place. Therefore, the double taxation thesis is based on the assumption that corporations are unable to shift their taxes.

[2] Ibid.

$FD_{it}^{D}$ which further makes the equilibrium values of $E_{it+1}$ and $B_{it+1}$ smaller through equations (49) and (50), respectively.

With respect to the government sector, partial integration A instantaneously affects the total government tax revenues ($TGT_0$) and the funds demanded by the government ($FD_{g0}^{D}$). Among the first-order conditions for the optimization problem of the government, this policy only affects the value of $TGT_t$ in $\partial H_{gt} / \partial \lambda_{gGLt}$ in equation (110).

Nevertheless, the equilibrium values of all control and state variables and one of costate variables, $K_{gt}$, in the government sector are completely not affected by this policy over the whole model horizon. Therefore, equation (81) indicates that the initial equilibrium value of total government expenditures ($TGE_0$) will remain the same.

Even though there is no direct impact of partial integration A on the household sector, the indirect impact still comes into effect. From equation (S115), this policy will not affect the equilibrium value of $\lambda_h^*$ through the unchanged $\lambda_{gk}^*$. Therefore, equations (S15) and (S16) show that partial integration also has no impact on the equilibrium value of $Y_{hj0}^{D}$ and $L_{h0}^{S}$ through $\lambda_h^*$. Besides, the unchanged $Y_{hj0}^{D}$ and $L_{h0}^{S}$ with the greater $DIV_{i0}$ make the equilibrium values of $DI_{h0}$, $SAV_{h0}$, and $W_{h1}$ greater through equations (3)-(6). With the higher $DIV_{i1}$ and $W_{h1}$ and the lower $E_{i1}$, we still can not exactly ensure the change in the value of $\lambda_{h1}$, since we are not sure about the change in the value of $B_s$ in equation (S3). Therefore, values of $Y_{hj1}^{D}$ and $L_{h1}^{S}$ also can not be determined through equations (S1) and (S2), respectively. For the similar reason, the change in values of $DI_{h1}$, $SAV_{h1}$, and $W_{h2}$ can not be determined from equations (3)-(6).

As to the impact of partial integration A on government tax revenues, the initial sales tax revenues ($ST_0$) remain the same, while the values at following periods ($ST_t$) are uncertain due to the uncertain $Y_{hjt}^D$; the initial capital gains tax revenues ($CGT_0$) remain the same, while the values at following periods ($CGT_t$) may tend to decrease mainly due to the lower $E_{it}$ ; both the labor tax revenues ($LT_0$ and $LT_t$) and the investment tax credit ($ITC_0$ and $ITC_t$) are not affected by partial integration A; the initial corporate income tax revenues ($CIT_0$) decrease due to the deducted dividends, and thus the values at following periods ($CIT_t$) may also tend to decrease; the initial personal income tax revenues ($PIT_0$) increase due to the higher dividends. Since the magnitude of increase in $PIT_0$ is apparently less than that of decrease in $CIT_0$, the total government tax revenues ($TGT_0$) would inevitably decrease. In addition, from equations (83) and (84), with the unchanged $TGE_0$ and the lower $TGT_0$, $FD_{g0}^D$ tends to increase, which would consequently raise the government liabilities and the outstanding public bonds at the beginning of period 1 ($GL_1$ and $B_{g1}$).

### 5.6 Partial Integration B: Dividends Deducted from the Personal Income Tax Base

In this study, partial integration B is designed to partially eliminate the problem of double taxation on dividends by exempting dividends from the personal income tax. Therefore, dividends ($DIV_{it}$) are taxed only at the corporate level and there is no tax on households' dividend income ($\sum_i e_{it} [\, DIV_{it} / ( P_{Eit-1} E_{it} ) \,] W_{ht}$) at the personal level. That is, the personal income tax revenues ($PIT_t$) merely consist of households' labor income ($w_t L_{ht}$) and interest income ($( 1 - \sum_i e_{it} ) r_t W_{ht}$). With respect to the structure of the model, the main effect of this policy is on the household sector, especially on disposable income ($DI_{ht}$) and hence savings ($SAV_{ht}$) and wealth ($W_{ht+1}$) ; the government sector is affected only on $PIT_t$ and hence total tax revenues ($TGT_t$) and liabilities ($GL_{t+1}$); there is almost no impact on the production sector.[3]

Intuitively, there is no direct impact of partial integration A on the production sector. All the first-order conditions for the optimization problem of industries remain the same under partial integration B. Therefore, the equilibrium values of all control, state, and costate variables in the production sector are completely not affected by this policy over the whole model horizon. Furthermore, at each period t, partial integration B maintains all equilibrium values of $NI_{it}$, $DIV_{it}$, $RE_{it}$, $NCF_i$, $FD_{it}^D$, $E_{it+1}$ and $B_{it+1}$ at their original levels.

With respect to the government sector, partial integration B instantaneously affects $PIT_0$ and further $TGT_0$ and $FD_{g0}^D$. Among the first-order conditions for the optimization problem of the government, this policy only affects the value of $TGT_t$ in $\partial H_{gt} / \partial \lambda_{gGLt}$ in equation (110). Nevertheless, the equilibrium values of all control and state variables and one of costate variables, $K_{gt}$, in the government sector are completely not affected by this policy over the whole model horizon. Therefore, equation (81) indicates that the initial equilibrium value of total government expenditures ($TGE_0$) will remain the same.

For the household sector, partial integration B at period t=0 will directly influence households' $DI_{h0}$ and hence $SAV_{h0}$ and $W_{h1}$ in the model. Therefore, it will directly affect the equilibrium conditions for the financial market.

[3] Same assumption as footnote 1 applies.

Among the first-order conditions for the optimization problem of households, this policy simultaneously affects $\partial H_{ht} / \partial W_{ht}$ and $\partial H_{ht} / \partial \lambda_{ht}$ in equations (26) and (27), respectively. From equation (S115), this policy will not affect the equilibrium value of $\lambda_h^*$ through the unchanged $\lambda_{gk}^*$. Therefore, equations (S15) and (S16) show that partial integration B also has no impact on the equilibrium values of $Y_{hj0}^D$ and $L_{h0}^S$ through $\lambda_h^*$. Besides, with the unchanged $Y_{hj0}^D$, $L_{h0}^S$ and $DIV_{i0}$, this policy raises the equilibrium values of $DI_{h0}$, $SAV_{h0}$, and $W_{h1}$ through equations (3)-(6). With the unchanged $DIV_{i1}$ and $E_{i1}$ and the higher $W_{h1}$, we still can not exactly ensure the change in the value of $\lambda_{h1}$, since the change in the value of $B_s$ in equation (S3) can not be determined. That is, with the unchanged $DIV_{i1}$ and $E_{i1}$ and the higher $W_{h1}$, the share of equity of industry i in household h's wealth ($e_{it}$, i.e., $P_{Eit-1}\, E_{hit} / W_{ht}$) decreases and thus the share of public and private bonds in household h's wealth ($1 - \sum_i e_{it}$) increases. This makes the change in the value of $B_s$ uncertain. Therefore, values of $Y_{hj1}^D$ and $L_{h1}^S$ also can not be determined through equations (S1) and (S2), respectively. For the similar reason, the change in values of $DI_{h1}$, $SAV_{h1}$, and $W_{h2}$ can not be determined from equations (3)-(6).

As to the impact of partial integration B on government tax revenues, the initial sales tax revenues ($ST_0$) remain the same, while the values at following periods ($ST_t$) are uncertain due to the uncertain $Y_{hjt}^D$; the initial capital gains tax revenues ($CGT_0$) remain the same, while the values at following periods ($CGT_t$) are uncertain mainly due to the uncertain $E_{hit}$; all of the labor tax revenues ($LT_0$ and $LT_t$), investment tax credit ($ITC_0$ and $ITC_t$), and corporate income tax revenues ($CIT_0$ and $CIT_t$) are not affected by partial integration B; the initial personal income tax revenues ($PIT_0$) decrease due to the deducted dividends, and thus the values at following periods ($PIT_t$) may also tend to decrease. Therefore, the total government tax revenues ($TGT_0$) would inevitably decrease. In addition, from equations (83) and (84), with the unchanged $TGE_0$ and the lower $TGT_0$, $FD_{g0}^D$ will increase, which in turn raises $GL_1$ and $B_{g1}$.

# CHAPTER SIX

# CONCLUSIONS AND POLICY IMPLICATIONS

In order to provide a more comprehensive framework capable of capturing a realistic dynamic environment in Taiwan, this study builds a dynamic general equilibrium model in the presence of heterogeneous income groups with different rates of time preferences, industries of different degrees of incorporation with adjustment costs of investment, and government deficits, based on the optimal control theory. Therefore, it would be expected that this model can be used to investigate the impacts of corporate income tax integration on income distribution and economic performance. This study also derives the solution for the theoretical model to ensure that all the equilibrium conditions are satisfied. Then, focusing on the corporate income tax integration plans, several tax policy scenarios are designed to theoretically explore how to apply simulation schemes to the model, to attempt to theoretically investigate the possible effects of these policies on the structure of the model and equilibrium values, and further to check if simulation results are consistent with equilibrium conditions.

## 6.1 Summary of Simulation Results

In such a dynamic theoretical model, the simulation results can not completely show the process of evolution of equilibrium values derived from changes in the tax policy. Nevertheless, we still can obtain some important outcome by summarizing the simulation results for all tax policies from the perspective of equilibrium conditions. We can explain the impacts of a specific tax policy on variables in equilibrium conditions through the change in their equilibrium values. We can also approximate the extreme short-run (with t=0) and various long-run (with different values of t) effects of this tax policy on equilibrium values of these variables. Furthermore, comparison of simulation results of different policies bears interesting implications. Even though some of the changes in equilibrium values of variables are uncertain, they will not affect our conclusions as long as other variables in the same market at the same period together do not violate the equilibrium conditions. In other words, the study framework ensures that equilibrium conditions remain consistent with different sets of relevant parameters and variables.

## 6.2 Conclusions

According to the model we built, the corresponding solutions derived, and the simulation results analyzed in this study, the following can be concluded:

### The Model

1. The existence of solutions for the theoretical model can be established and all the equilibrium conditions are satisfied under alternative tax policies.

2. Since there are interactions between variables inside a single sector and/or intersectors simultaneously in such a dynamic general equilibrium model, the effects of tax policies on equilibrium values can not be exactly identified from a single variable or a

single sector as what would be obtained in a static model. Rather, the connections between variables and sectors should be taken into account globally.

3. This study can not obtain any unambiguous result, except some at the initial period, for the household sector from all simulation analyses on these policies. This is because in this theoretical model household h's share of the market portfolio ($\xi_{ht}$, i.e., $W_{ht} / \sum_h W_{ht}$) is undetermined. This will further affect the quantity of equity of industry i owned by household h ($E_{hit}$, i.e., $\xi_{ht} E_{it}$) and hence the share of equity of industry i in household h's wealth ($e_{it}$, i.e., $P_{Eit-1} E_{hit} / W_{ht}$).

4. Even for the initial period, only some of the changes in equilibrium values can be identitied in this theoretical study, but not all of them.

5. In this theoretical model, what we can obtain are the aggregated results for each sector, not the disaggregated ones which can only be done by an empirical study. Therefore, this study can not derive any exact result on income distribution among income groups and resource allocation among industries from the effects of these tax policies on the change in equilibrium values.

6. Since the dividend/retention ratio ($\phi_i$) and the debt/equity ratio ($\psi_i$) are set to be invariant in this study, the biased tax preference toward retained earnings and debts financing can not be really eliminated or alleviated in this model.

7. Even if these two ratios ($\phi_i$ and $\psi_i$) are set to be free, they can not make the simulation results more apparent. Instead, some results, which are originally unambiguous with these invariant ratios, may become undetermined with the variant ratios.

8. Generally, there is a positive relationship between control variables and costate variable in the production sector, while there is an inverse relationship between control variables and absolute values of costate variables in the government sector. As to the household sector, one of the control variables (demand for consumption goods) varies inversely with the costate variable at each period t, while supply of labor varies positively.

**Price Expectations**

Prices keep unchanged over the finite model horizon under the assumption of myopic expectations, while they are continuous to change at an invariant rate under the assumption of constant rate of growth which makes prices either extremely large or extremely small at the terminal period. This indicates that the result (and/or equilibrium values) of an empirical study will be different depending on the different assumption of price expectations.

**Full Integration**

In the long run, the full integration plan and the abolishment of labor tax will induce the increment in the outputs supplied, inputs demanded, and capital accumulated by industries, while the abolishment of investment tax credit will obtain the opposite results.

**Partial Integration**

1. Both partial integration plans will not affect the inputs demanded by the government at all. Therefore, the public goods provided by the government will not be influenced by both plans.

2. With both partial integration plans, government liabilities will be aggravated at the initial period due to the reduction in total government revenues.

3. The partial integration plan with dividends deducted from the corporate income tax base will only induce the reduction in the funds demanded by industries, but will not affect the outputs supplied, inputs demanded, and capital accumulated by industries at all.

4. The partial integration plan with dividends deducted from the personal income tax base will not affect the outputs supplied, inputs demanded, capital accumulated, and even the funds demanded by industries at all.

## 6.3 Policy Implications

**The Model**

1. According to simulation analyses on all the corporate income tax integration plans in this study, the results indicate that the problem of "double" taxation on dividends actually can be eliminated from the theoretical model.

2. With the full integration and the partial integration with dividends deducted from the corporate income tax base, dividends are treated as interest payment and thus the tax preference toward debts should be eliminated from the theoretical model. However, due to both the dividend/retention ratio and the debt/equity ratio are set to be

constant in this study, corporations are not allowed to adjust their financial decisions to the optimal level after corporate income tax integration.

### Households

In the short run, partial integration plans may not affect households' consumption and leisure activities and hence their utilities, but may raise their savings and wealth.

### Producers

1. The abolishment of labor tax and the full integration plan will promote industries' supply of outputs, demand for inputs, and capital accumulation, while the abolishment of investment tax credit will discourage them.

2. The implementation of full integration, whose impact on industries is the same as the abolishment of labor tax, will alleviate industries' tax liabilities and encourage their production activities.

3. Partial integration plans have no impact on the real production activities and capital accumulation of industries. Between two partial integration plans, the only effect on industries is derived from the one with dividends deducted from the corporate income tax base and will make their liabilities lower.

### The Government

1. The government may reduce the provision of public goods due to the reduction in its demand for inputs resulted from the decrease in total tax revenues under the full integration.

2. Partial integration plans have no impact on government's provision of public goods, while may aggravate its deficits due to the reduction in total tax revenues.

**6.4 Contributions**

The main contributions of this study are as follows:

1. This study provides a more comprehensive framework capturing a realistic dynamic environment in Taiwan. Therefore, it can be appropriately modified to be used as the basic model in an empirical study.

2. This study applies the optimal control theory to a more comprehensive tax model. Furthermore, it derives equilibrium conditions for each sector and for each market in the theoretical model by means of applying transversality conditions to each sector. At the same time, this study identifies the solutions in detail for the model.

3. The objective of maximizing industries' profits is generally set in a traditional general equilibrium model, while net cash flows are most likely to be associated with industries' future profitabilities, and are the base for their perpetuation. Therefore, this study displays the concept of net cash flows in accounting, applies it to the model, and sets the objective of industries to pursue the maximization of their net cash flows in the dynamic general equilibrium tax model.

4. Results stemmed from simulation analyses in this theoretical study can be used as references for future empirical studies.

5. The model built in this study is one of the few dynamic general equilibrium tax models. It can be used for empirical studies to simultaneously evaluate the effects of

corporate income tax integration on both income distribution among households and economic performance among industries in Taiwan economy.

### 6.5 Future Research

Finally, this study suggests several possible directions for future research.

1. Even though this study encompasses the concept of net cash flows from the perspective of accounting to make the tax model closer to the reality, we look forward to engaging more studies applying concepts of accounting to economic analysis, especially for the studies on tax policies.

2. This study implicitly assumes, as in other studies in the literature, that there are interior solutions and then directly applies both the optimal control theory to the dynamic general equilibrium model and the transversality conditions to derive solutions for the model. Future studies may attempt to inspect first if there are interior solutions for the model, then identify if we can directly apply the maximum principle and further transversality conditions to solve the optimization problem for each sector.

3. This study focuses on investigating the effects of changing tax policies on the model structure and solutions for the model. Future studies can further evaluate the effects of directly changing the structure of the model on equilibrium values, such as eliminating the adjustment costs of investment and depreciation of capital from the model.

4. The framework of this tax model can be extended to an open economy in which international trade and finance can become one of the focal points of analysis.

## REFERENCES


Allard, Marie; Bronsard, Camille and Richelle, Yves. "Temporary Pareto Optimum Theory." *Journal of Public Economics*, 1989, 38, pp. 343-68.

Atrill, Peter and Mclaney, Eddie. *Accounting and Finance for Non-specialists*, 2nd. London: Prentice Hall, 1997.

Ballard, Charles L.; Fullerton, Don.; Shoven, John B. and Whalley, John. *A General Equilibrium Model for Tax Policy Evaluation*. Chicago: University of Chicago Press, 1985.

Ballard, Charles L.; Shoven, John B. and Whalley, John. "General Equilibrium Computations of the Marginal Welfare Costs of Taxes in the United States." *American Economic Review*, 1985, 75(1), pp. 128-38.

Bhattarai, Keshab and Whalley, John. "General Equilibrium Modelling of UK Tax Policy," in Sean Holly and Martin Weale, eds., *Econometric Modelling: Techniques and Applications*. Cambridge: Cambridge University Press, 2000, pp. 69-93.

Bovenberg, A. Lans. "Capital Income Taxation in Growing Open Economies." *Journal of Public Economics*, 1986, 31, pp. 347-76.

Branson, William H. *Macroeconomic Theory and Policy*. New York: Addison Wesley, 1989.

Chang, Hsiu-Yu. *Additional 10% Surtax on Undistributed Retained Earnings: Managerial and Market Responses*. Thesis, Chung Yuan Christian University, 2002.

Chiang, Alpha C. *Elements of Dynamic Optimization*. New York: McGraw-Hill, 1992.

Chuang, Shi-Feng and Sang, Tern-Sheng. "The Impacts of Corporate Tax Integration on the Taiwan Economy." *Taiwan Economics and Finance Monthly*, 1998, 34(2), pp.1-9

Diamond, Michael A.; Stice Earl K. and Stice, James D. *Financial Accounting: Reporting and Analysis*, 5th ed. South-Western College Publishing, 2000.

Dixon, Peter B.; Parmenter, B.R.; Sutton, John and Vincent, D.P. *ORANI: A Multisectoral Model of the Australian Economy*. Amsterdam: North-Holland, 1982.

Dorfman, Robert. "An Economic Interpretation of Optimal Control Theory." *American Economic Review*, December 1969, 59(5), pp. 817-31.

Feltenstein, Andrew. "An Intertemporal General Equilibrium Analysis of Financial Crowding Out." *Journal of Public Economics*, 1986, 31, pp. 79-104.

Fullerton, Don; King, A. Thomas; Shoven, John B. and Whalley, John. " Corporate Tax Integration in the United States: A General Equilibrium Approach." *American Economic Review*, September 1981, 71(4), pp. 677-91.

Goulder, Lawrence H. and Summers, Lawrence H. "Tax Policy, Asset Prices, and Growth: A General Equilibrium Analysis." *Journal of Public Economics*, 1989, 38, pp. 265-96

Grandmont, J.-M. "Temporary Equilibrium," in John Eatwell; Murray Milgate and Peter Newman, eds., *General Equilibrium*. New York: The Macmillan Press, 1989, pp. 297-304.

Hand, Len; Issaks, Carolyn and Sanderson, Peter. *Introduction to Accounting for Non-specialists*. London: Thomson, 2005.

Hall, Robert E. and Jorgenson, Dale W. "Tax Policy and Investment Behavior." *American Economic Review*, 1967, 57(3), pp. 391-414.

Harberger, Arnold C. "The Incidence of the Corporation Income Tax." *Journal of Political Economy*, 1962, 70, pp. 215-40.

Hofkes, M.W. *Modelling and Computation of General Equilibrium*. Amsterdam: Thesis Publishers, 1991.

Huang, Hsiao-Ling. *The Impacts of Income Tax Integration on Corporate Capital Structure*. Thesis, National Sun Yat-Sen University, 2001.

Jehle, Geoffrey A. and Reny, Philip J. *Advanced Microeconomic Theory.* New York: Addison Wesley, 2001.

Jiang, Lily. "Corporate tax Integration and Income Distribution in Dynamic Equilibrium." *Taiwan Economic Review*, 2002, 30(2), pp. 205-27.

Johnson, Paul A. *Capital Utilization and Investment When Capital Depreciates in Use.* Unpublished PhD Dissertation, Stanford University, 1989.

King, Robert G. and Plosser, Charles I. "Real Business Cycles I: Introduction." *Journal of Monetary Economics*, 1988, 21, pp. 191-93.

King, Robert G.; Plosser, Charles I. and Rebelo, Sergio T. "Production, Growth and Business Cycles I: the Basic Neoclassical Model." *Journal of Monetary Economics*, 1988, 21, pp. 195-232.

------. "Production, Growth and Business Cycles II: New Directions." *Journal of Monetary Economics*, 1988, 21, pp. 309-41.

Kuo, Lopin. The Effects of Corporate Tax Integration on Dividend Policy. *Public Finance Review*, 2000, 32(3), pp. 28-34.

Leonard, Daniel and Long, Ngo Van. *Optimal Control Theory and Static Optimization in Economics*. Cambridge: Cambridge University Press, 1992.

Liang, Su-Mei. *Effect of Retained Earning Tax on Capital Valuation*. Thesis, National Sun Yat-Sen University, 2003.

Lu, Yu-Shi. *The Impact of Imputation Tax System on Corporate Dividend Policy: An Empirical Study*. Thesis, Feng Chia University, 2000.

Ma, Hui-Chuan. *The Effects of Taiwan's Business and Individual Income Tax Integration on Industry Development and Income Distribution*. Thesis, Taipei University, 2003.

McLure, C. E. and Zodrow, G. R. "The Study and Practice of Income Tax Policy," in J. Quigley and E. Smolensky, eds., *Modern Public Finance*. Cambridge: Harvard University Press, 1994.

Ministry of Finance, Taiwan. *Alternatives of Corporate Tax Integration: Introduction*. 1998.

------. *The Prospect and Project of Tax Reform in Taiwan*. 2005.

Pereira, Alfredo M. "A Dynamic General Equilibrium Analysis of Corporate Tax Integration." *Journal of Policy Modeling*, 1993, 15(1), pp. 63-89.

------. *Corporate Tax Integration in the U.S.: A Dynamic General Equilibrium Analysis*. Unpublished PhD Dissertation, Stanford University, 1988.

Powell, Alan A. and Snape, Richard H. "The Contribution of Applied General Equilibrium Analysis to Policy Reform in Australia." *Journal of Policy Modeling*, 1993, 15(4), pp. 393-414.

Pyle, William W. and Larson, Kermit D. *Fundamental Accounting Principles*, $9^{th}$ ed. Richard D. Irwin, 1981.

Ratti, Ronald A. and Shome, Parthasarathi. "On a General Equilibrium Model of the Incidence of the Corporation Tax under Uncertainty." *Journal of Public Economics*, 1977, 8, pp. 233-38.

Romer, David. *Advanced Macroeconomics*. Singapore: McGraw-Hill, 2001.

Sarte, Pierre-Daniel G. "Progressive Taxation and Income Inequality in Dynamic Competitive Equilibrium." *Journal of Public Economics*, 1997, 66, pp. 145-71.

Scarf, Herbert E. (with Hansen, Terje.) *The computation of Economic Equilibria*. Cowles Foundation Monograph 24, New Haven and London: Yale University Press, 1973.

Scarf, Herbert E. "Computation of General Equilibria," in John Eatwell; Murray Milgate and Peter Newman, eds., *General Equilibrium*. New York: The Macmillan Press, 1989, pp. 84-97.

Schworm, William E. "Financial Constraints and Capital Accumulation." *International Economic Review*, October 1980, 21(3), pp. 643-60.

Seung, C.K. and Kraybill, D.S. "Tax Incentives in an Economy with Public Goods." *Growth and Change*, 1999, 30, pp. 128-47.

Shea, Jia-Dong. "A Dynamic General-Equilibrium Simulation on Tax Integration in Taiwan." *Academia Economic Papers*, 1989, 17(2), pp. 1-53.

Shoven, John B. and Whalley, John. "A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S." *Journal of Public Economics*, 1972, 1, pp. 281-321.

--------. "Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey." *Journal of Economic Literature*, September 1984, 22(3), pp. 1007-51.

--------. *Applying General Equilibrium*. New York: Cambridge University Press, 1992.

--------. "Equal Yield Tax Alternatives: General Equilibrium Computational Techniques." *Journal of Public Economics*, 1977, 8, pp. 211-24.

Su, Dao-Min. A General Equilibrium Analysis of Corporate Tax Integration in Taiwan. Thesis, National Chengchi University, 1986.

Tresch, Rechard. "Introduction to Normative Public Sector Theory," in *Public Finance – a Normative Theory*. Academic Press, 2002.

Vickman, Thomas M. *Handbook of Model Accounting Reports and Formats*, 2nd ed. Englewood Cliffs: Prentice Hall, 1995.

Viscusi, W. Kip and Moore, Michael J. "Rates of Time Preference and Valuations of the Duration of Life." *Journal of Public Economics*, 1989, 38, 297-317.

Wang, Jui-Chih and Chen Ming-Chin. "An Empirical Investigation of Impacts of the Imputation Tax System on Corporate Shareholder Structures." *Journal of Contemporary Accounting*, November 2003, 4(2), pp. 169-90.

Wang, Zong-Siang. *The Impacts of Imputation Tax System on Corporate Dividend Policy: An Empirical Study and Survey*. Thesis, National Sun Yat-Sen University, 2003.

Young, H. Peyton. "Progressive Taxation and Equal Sacrifice." *American Economic Review*, March 1990, 80(1), pp. 253-66.

# APPENDIX

The purpose of this appendix is to illustrate the simulation results, i.e., to show the impacts of tax policies on the model structure for each sector and equilibrium values for each market.

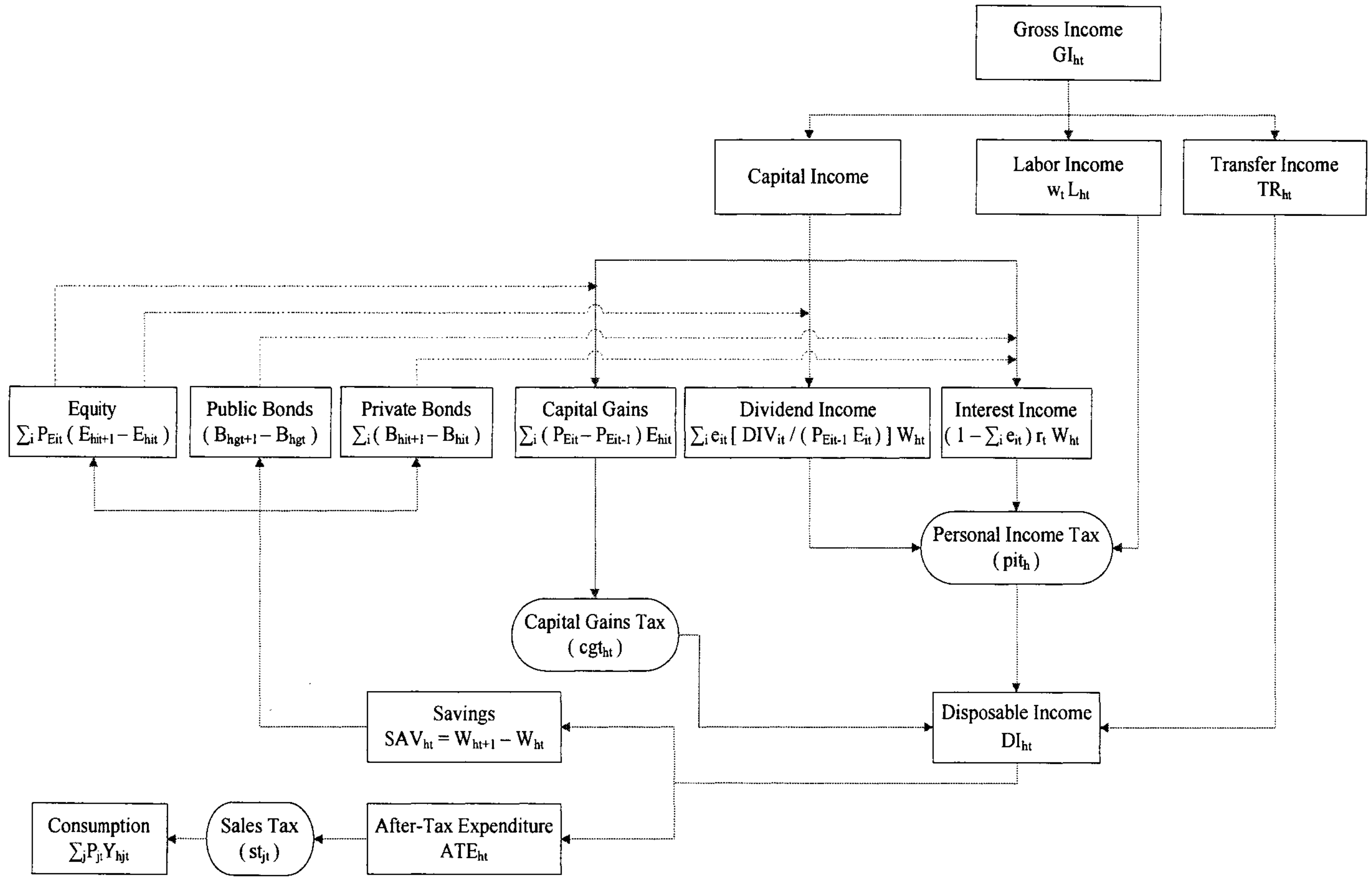


**Figure A1.1 Structure of Households' Income and Expenditures**

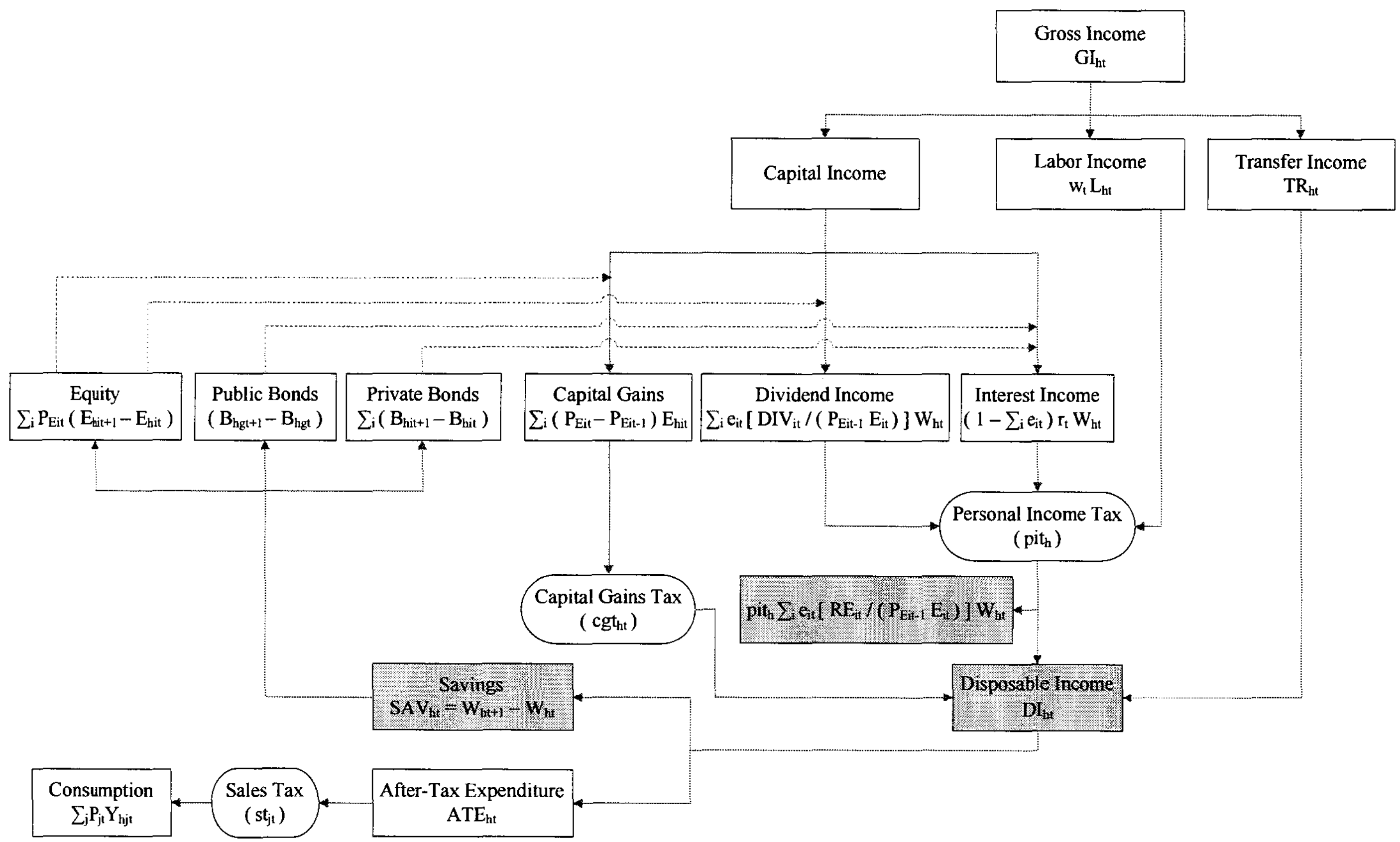


**Figure A1.2 The Effects of Full Integration on the Structure of the Household Sector**

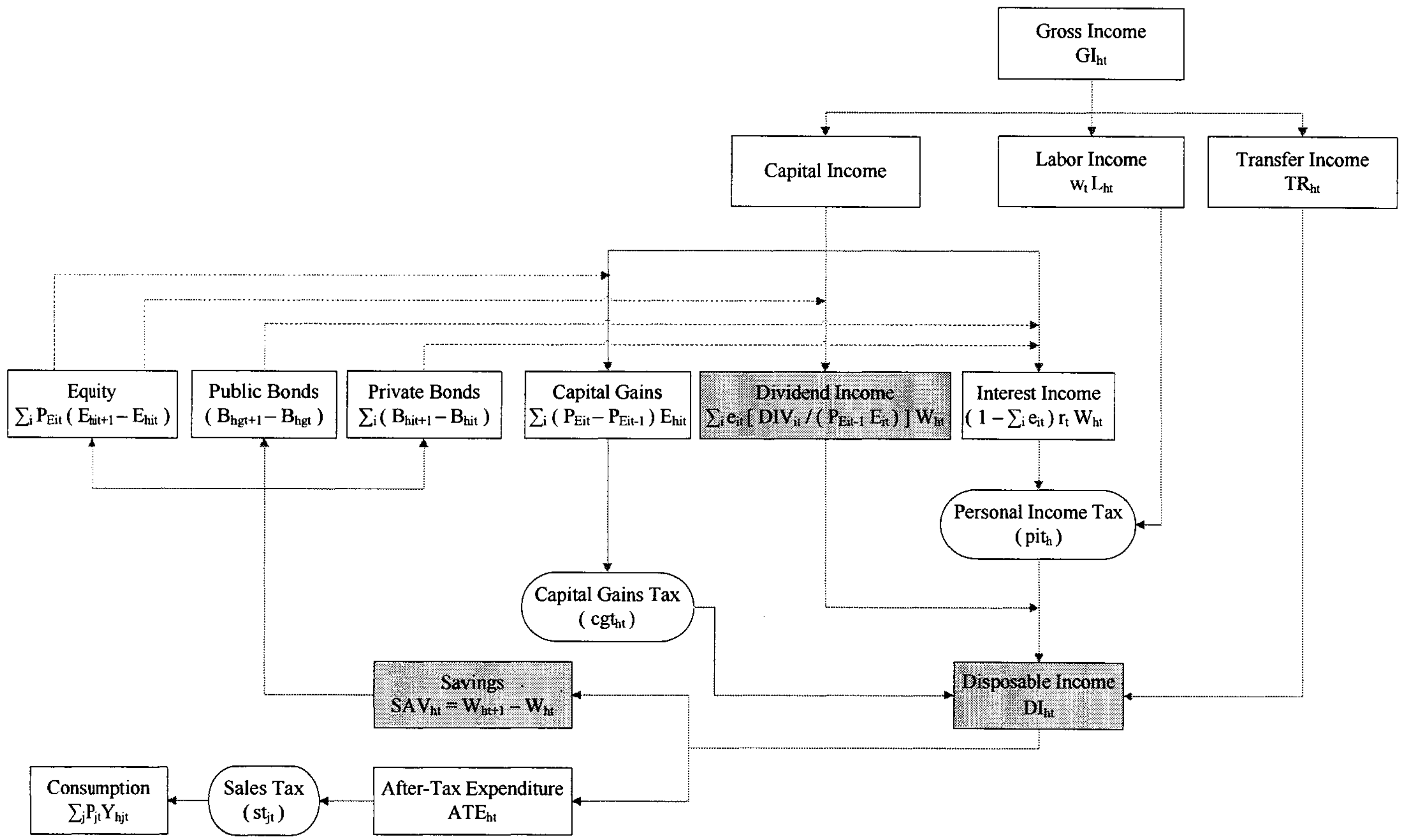


**Figure A1.3 The Effects of Partial Integration B on the Structure of the Household Sector**

**Table A2.1 Structure of Producers' Revenues and Expenditures**

**a. Income Statement (Profit and Loss Statement)**

| | | |
|---|---|---|
| Sales Revenues : | $P_{it} Y_{it}$ | |
| − Sales Costs : | $\sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) Y_{it}$ | (Intermediate Input Costs) |
| Gross Profits : | $GP_{it} = P_{it} Y_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) Y_{it}$ | |
| − Operating Expenses : | $( 1 + lt_t ) w_t L_{it}$ | (Labor Costs) |
| | $\Phi_{it}' K_{it}'$ | (Depreciation Allowances) |
| Operating Income : | $OI_{it} = GP_{it} - [ ( 1 + lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ) ]$ | |
| − Other Expenses : | $r_t B_{it}$ | (Interest Payments) |
| Earnings before Taxes : | $EBT_{it} = OI_{it} - r_t B_{it}$ | |
| − Taxes : | $cit_{it} EBT_{it}$ | (Corporate Income Tax) |
| Earnings (i.e., Net Income) : | $NI_{it} = ( 1 - cit_{it} ) EBT_{it}$ | |
| − Dividends Paid to Shareholders : | $DIV_{it} = ( 1 - \phi_i ) NI_{it}$ | |
| Retained Earnings : | $RE_{it} = \phi_i NI_{it}$ | |

**Table A2.1 Structure of Producers' Revenues and Expenditures** (continue)

**b. Cash Flow Statement**

**Cash Flow from Operating Activities**

| | | |
|---|---|---|
| Earnings (Net Income) : | $NI_{it}$ | |
| + Non-Cash Adjustment : | $\Phi_{it}' K_{it}'$ | (Depreciation Allowances) |

Net Cash Provided by Operating Activities : $NI_{it} + \Phi_{it}' K_{it}'$

**Cash Flow from Investing Activities**

| | | |
|---|---|---|
| Actual Investment Expenditure : | $P_{It} I_{it}$ | (Acquisition Costs) |
| | $+ \ 0.5\, b_i I_{it}^2$ | (Adjustment Costs) |
| | $- \ itc_{it} P_{It} I_{it}$ | (Investment Tax Credit) |

− Net Cash Used in Investing Activities : $(1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2$

**Cash Flow from Financing Activities**

| | |
|---|---|
| Issuance of Additional Equity : | $P_{Eit} ( E_{it+1} - E_{it} )$ |
| + Issuance of New Bonds : | $( B_{it+1} - B_{it} )$ |
| − Dividends Paid : | $DIV_{it}$ |

+ Net Cash Provided by Financing Activities : $P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it}$

Net Increase in Cash and Equivalents (i.e., Net Cash Flows) : $NCF_{it}$

$$
\begin{aligned}
NCF_{it} &= [NI_{it} + \Phi_{it}' K_{it}'] - [ (1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ] \\
&= (1 - cit_{it}) \{ [ P_{it} - \textstyle\sum_{1 \le j \le J} ( c_{ij} P_{jt} ) ] Y_{it} - (1 + lt_t) w_t L_{it} \} \\
&\quad - (1 - cit_{it}) r_t B_{it} + cit_{it} \Phi_{it}' K_{it}' - [ (1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]
\end{aligned}
$$

**Table A2.2 The Effects of Abolishing Labor Tax on the Structure of the Production Sector**

**a. Income Statement (Profit and Loss Statement)**

| | | |
|---|---|---|
| Sales Revenues : | $P_{it} Y_{it}$ | |
| − Sales Costs : | $\sum_{1 \le j \le J} ( c_{ij} P_{jt} ) Y_{it}$ | (Intermediate Input Costs) |
| Gross Profits : | $GP_{it} = P_{it} Y_{it} - \sum_{1 \le j \le J} ( c_{ij} P_{jt} ) Y_{it}$ | |
| − Operating Expenses : | $w_t L_{it}$ | (Labor Costs) |
| | $\Phi_{it}' K_{it}'$ | (Depreciation Allowances) |
| Operating Income : | $OI_{it} = GP_{it} - [ w_t L_{it} + \Phi_{it}' K_{it}' ) ]$ | |
| − Other Expenses : | $r_t B_{it}$ | (Interest Payments) |
| Earnings before Taxes : | $EBT_{it} = OI_{it} - r_t B_{it}$ | |
| − Taxes : | $cit_{it} EBT_{it}$ | (Corporate Income Tax) |
| Earnings (i.e., Net Income) : | $NI_{it} = ( 1 - cit_{it} ) EBT_{it}$ | |
| − Dividends Paid to Shareholders : | $DIV_{it} = ( 1 - \phi_i ) NI_{it}$ | |
| Retained Earnings : | $RE_{it} = \phi_i NI_{it}$ | |

**Table A2.2 The Effects of Abolishing Labor Tax on the Structure of the Production Sector** (continue)

**b. Cash Flow Statement**

**Cash Flow from Operating Activities**

| | | |
|---|---|---|
| Earnings (Net Income) : | $NI_{it}$ | |
| + Non-Cash Adjustment : | $\Phi_{it}' K_{it}'$ | (Depreciation Allowances) |

Net Cash Provided by Operating Activities : $NI_{it} + \Phi_{it}' K_{it}'$

**Cash Flow from Investing Activities**

| | | |
|---|---|---|
| Actual Investment Expenditure : | $P_{It} I_{it}$ | (Acquisition Costs) |
| + | $0.5\, b_i I_{it}^2$ | (Adjustment Costs) |
| – | $itc_{it} P_{It} I_{it}$ | (Investment Tax Credit) |

– Net Cash Used in Investing Activities : $(1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2$

**Cash Flow from Financing Activities**

| | |
|---|---|
| Issuance of Additional Equity : | $P_{Eit} ( E_{it+1} - E_{it} )$ |
| + Issuance of New Bonds : | $( B_{it+1} - B_{it} )$ |
| – Dividends Paid : | $DIV_{it}$ |

+ Net Cash Provided by Financing Activities : $P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it}$

Net Increase in Cash and Equivalents (i.e., Net Cash Flows) : $NCF_{it}$

$$
\begin{aligned}
NCF_{it} &= [NI_{it} + \Phi_{it}' K_{it}'] - [ (1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ] \\
&= (1 - cit_{it}) \{ [ P_{it} - \textstyle\sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) ] Y_{it} - w_t L_{it} \} \\
&\quad - (1 - cit_{it}) r_t B_{it} + cit_{it} \Phi_{it}' K_{it}' - [ (1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]
\end{aligned}
$$

**Table A2.3 The Effects of Abolishing Investment Tax Credit on the Structure of the Production Sector**

**a. Income Statement (Profit and Loss Statement)**

| | | |
|---|---|---|
| Sales Revenues : | $P_{it} Y_{it}$ | |
| − Sales Costs : | $\sum_{1 \le j \le J} ( c_{ij} P_{jt} ) Y_{it}$ | (Intermediate Input Costs) |
| Gross Profits : | $GP_{it} = P_{it} Y_{it} - \sum_{1 \le j \le J} ( c_{ij} P_{jt} ) Y_{it}$ | |
| − Operating Expenses : | $( 1 + lt_t ) w_t L_{it}$ | (Labor Costs) |
| | $\Phi_{it}' K_{it}'$ | (Depreciation Allowances) |
| Operating Income : | $OI_{it} = GP_{it} - [ ( 1 + lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ) ]$ | |
| − Other Expenses : | $r_t B_{it}$ | (Interest Payments) |
| Earnings before Taxes : | $EBT_{it} = OI_{it} - r_t B_{it}$ | |
| − Taxes : | $cit_{it} EBT_{it}$ | (Corporate Income Tax) |
| Earnings (i.e., Net Income) : | $NI_{it} = ( 1 - cit_{it} ) EBT_{it}$ | |
| − Dividends Paid to Shareholders : | $DIV_{it} = ( 1 - \phi_i ) NI_{it}$ | |
| Retained Earnings : | $RE_{it} = \phi_i NI_{it}$ | |

**Table A2.3 The Effects of Abolishing Investment Tax Credit on the Structure of the Production Sector** (continue)

**b. Cash Flow Statement**

**<u>Cash Flow from Operating Activities</u>**

Earnings (Net Income) : $NI_{it}$

+ Non-Cash Adjustment : $\Phi_{it}' K_{it}'$ (Depreciation Allowances)

---

Net Cash Provided by Operating Activities : $NI_{it} + \Phi_{it}' K_{it}'$

**<u>Cash Flow from Investing Activities</u>**

Actual Investment Expenditure : $P_{It} I_{it}$ (Acquisition Costs)

$+ \; 0.5 \, b_i \, I_{it}^2$ (Adjustment Costs)

$- \; 0$ (Investment Tax Credit)

---

− Net Cash Used in Investing Activities : $P_{It} I_{it} + 0.5 \, b_i \, I_{it}^2$

**<u>Cash Flow from Financing Activities</u>**

Issuance of Additional Equity : $P_{Eit} ( E_{it+1} - E_{it} )$

+ Issuance of New Bonds : $( B_{it+1} - B_{it} )$

− Dividends Paid : $DIV_{it}$

---

+ Net Cash Provided by Financing Activities : $P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it}$

---

Net Increase in Cash and Equivalents (i.e., Net Cash Flow) : $NCF_{it}$

$$
\begin{aligned}
NCF_{it} &= [NI_{it} + \Phi_{it}' K_{it}'] - [ ( 1 - 0 ) P_{It} I_{it} + 0.5 \, b_i \, I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ] \\
&= ( 1 - cit_{it} ) \{ [ P_{it} - \textstyle\sum_{1 \le j \le J} ( c_{ij} P_{jt} ) ] Y_{it} - ( 1 + lt_t ) w_t L_{it} \} \\
&\quad - ( 1 - cit_{it} ) r_t B_{it} + cit_{it} \Phi_{it}' K_{it}' - [ P_{It} I_{it} + 0.5 \, b_i \, I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]
\end{aligned}
$$

## Table A2.4 The Effects of Full Integration on the Structure of the Production Sector

### a. Income Statement (Profit and Loss Statement)

| | | |
|---|---|---|
| Sales Revenues : | $P_{it} Y_{it}$ | |
| − Sales Costs : | $\sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) Y_{it}$ | (Intermediate Input Costs) |
| Gross Profits : | $GP_{it} = P_{it} Y_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) Y_{it}$ | |
| − Operating Expenses : | $( 1 + lt_t ) w_t L_{it}$ | (Labor Costs) |
| | $\Phi_{it}' K_{it}'$ | (Depreciation Allowances) |
| Operating Income : | $OI_{it} = GP_{it} - [ ( 1 + lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ) ]$ | |
| − Other Expenses : | $r_t B_{it}$ | (Interest Payments) |
| Earnings before Taxes : | $EBT_{it} = OI_{it} - r_t B_{it}$ | |
| − Taxes : | $0$ | (Corporate Income Tax) |
| Earnings (i.e., Net Income) : | $NI_{it} = EBT_{it}$ | |
| − Dividends Paid to Shareholders : | $DIV_{it} = ( 1 - \phi_i ) NI_{it}$ | |
| Retained Earnings : | $RE_{it} = \phi_i NI_{it}$ | |

**Table A2.4 The Effects of Full Integration on the Structure of the Production Sector** (continue)

**b. Cash Flow Statement**

**Cash Flow from Operating Activities**

Earnings (Net Income) : $NI_{it}$

+ Non-Cash Adjustment : $\Phi_{it}' K_{it}'$ (Depreciation Allowances)

---

Net Cash Provided by Operating Activities : $NI_{it} + \Phi_{it}' K_{it}'$

**Cash Flow from Investing Activities**

Actual Investment Expenditure : $P_{It} I_{it}$ (Acquisition Costs)

$+ \; 0.5\, b_i I_{it}^2$ (Adjustment Costs)

$- \; itc_{it} P_{It} I_{it}$ (Investment Tax Credit)

---

− Net Cash Used in Investing Activities : $(1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2$

**Cash Flow from Financing Activities**

Issuance of Additional Equity : $P_{Eit} ( E_{it+1} - E_{it} )$

+ Issuance of New Bonds : $( B_{it+1} - B_{it} )$

− Dividends Paid : $DIV_{it}$

---

+ Net Cash Provided by Financing Activities : $P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it}$

---

Net Increase in Cash and Equivalents (i.e., Net Cash Flow) : $NCF_{it}$

$$
\begin{aligned}
NCF_{it} &= [NI_{it} + \Phi_{it}' K_{it}'] - [ (1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ] \\
&= \{ [ P_{it} - \textstyle\sum_{1 \le j \le J} ( c_{ij} P_{jt} ) ] Y_{it} - ( 1 + lt_t ) w_t L_{it} \} - r_t B_{it} + 0 \\
&\quad - [ (1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2 ] \\
&\quad + [ P_{Eit} ( E_{it+1} - E_{it} ) + ( B_{it+1} - B_{it} ) - DIV_{it} ]
\end{aligned}
$$

**Table A2.5 The Effects of Partial Integration A on the Structure of the Production Sector**

**a. Income Statement (Profit and Loss Statement)**

| | | |
|---|---|---|
| Sales Revenues : | $P_{it} Y_{it}$ | |
| − Sales Costs : | $\sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) Y_{it}$ | (Intermediate Input Costs) |
| Gross Profits : | $GP_{it} = P_{it} Y_{it} - \sum_{1 \leq j \leq J} ( c_{ij} P_{jt} ) Y_{it}$ | |
| − Operating Expenses : | $( 1+ lt_t ) w_t L_{it}$ | (Labor Costs) |
| | $\Phi_{it}' K_{it}'$ | (Depreciation Allowances) |
| Operating Income : | $OI_{it} = GP_{it} - [ ( 1+ lt_t ) w_t L_{it} + \Phi_{it}' K_{it}' ) ]$ | |
| − Other Expenses : | $r_t B_{it}$ | (Interest Payments) |
| Earnings before Taxes : | $EBT_{it} = OI_{it} - r_t B_{it}$ | |
| − Taxes : | $cit_{it} ( EBT_{it} - DIV_{it} )$ | (Corporate Income Tax) |
| Earnings (i.e., Net Income) : | $NI_{it} = ( 1 - cit_{it} ) EBT_{it} + cit_{it} DIV_{it}$ | |
| − Dividends Paid to Shareholders : | $DIV_{it} = ( 1 - \phi_i ) ( 1 - cit_{it} ) EBT_{it} / [1 - ( 1 - \phi_i ) cit_{it}]$ | |
| Retained Earnings : | $RE_{it} = \phi_i [ ( 1 - cit_{it} ) EBT_{it} + cit_{it} DIV_{it} ]$ | |

**Table A2.5 The Effects of Partial Integration A on the Structure of the Production Sector** (continue)

**b. Cash Flow Statement**

**Cash Flow from Operating Activities**

Earnings (Net Income) : $NI_{it}$

$+$ Non-Cash Adjustment : $\Phi_{it}' K_{it}'$ (Depreciation Allowances)

---

Net Cash Provided by Operating Activities : $NI_{it} + \Phi_{it}' K_{it}'$

**Cash Flow from Investing Activities**

Actual Investment Expenditure : $P_{It} I_{it}$ (Acquisition Costs)

$+ \; 0.5\, b_i I_{it}^2$ (Adjustment Costs)

$- \; itc_{it} P_{It} I_{it}$ (Investment Tax Credit)

---

$-$ Net Cash Used in Investing Activities : $(1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2$

**Cash Flow from Financing Activities**

Issuance of Additional Equity : $P_{Eit} (E_{it+1} - E_{it})$

$+$ Issuance of New Bonds : $(B_{it+1} - B_{it})$

$-$ Dividends Paid : $DIV_{it}$

---

$+$ Net Cash Provided by Financing Activities : $P_{Eit} (E_{it+1} - E_{it}) + (B_{it+1} - B_{it}) - DIV_{it}$

---

Net Increase in Cash and Equivalents (i.e., Net Cash Flow) : $NCF_{it}$

$$
\begin{aligned}
NCF_{it} &= [NI_{it} + \Phi_{it}' K_{it}'] - [(1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2] \\
&\quad + [P_{Eit} (E_{it+1} - E_{it}) + (B_{it+1} - B_{it}) - DIV_{it}] \\
&= (1 - cit_{it}) \{ [P_{it} - \textstyle\sum_{1 \le j \le J} (c_{ij} P_{jt})] Y_{it} - (1 + lt_t) w_t L_{it} \} \\
&\quad - (1 - cit_{it}) r_t B_{it} + cit_{it} \Phi_{it}' K_{it}' - [(1 - itc_{it}) P_{It} I_{it} + 0.5\, b_i I_{it}^2] \\
&\quad - (1 - cit_{it}) DIV_{it} + [P_{Eit} (E_{it+1} - E_{it}) + (B_{it+1} - B_{it})]
\end{aligned}
$$

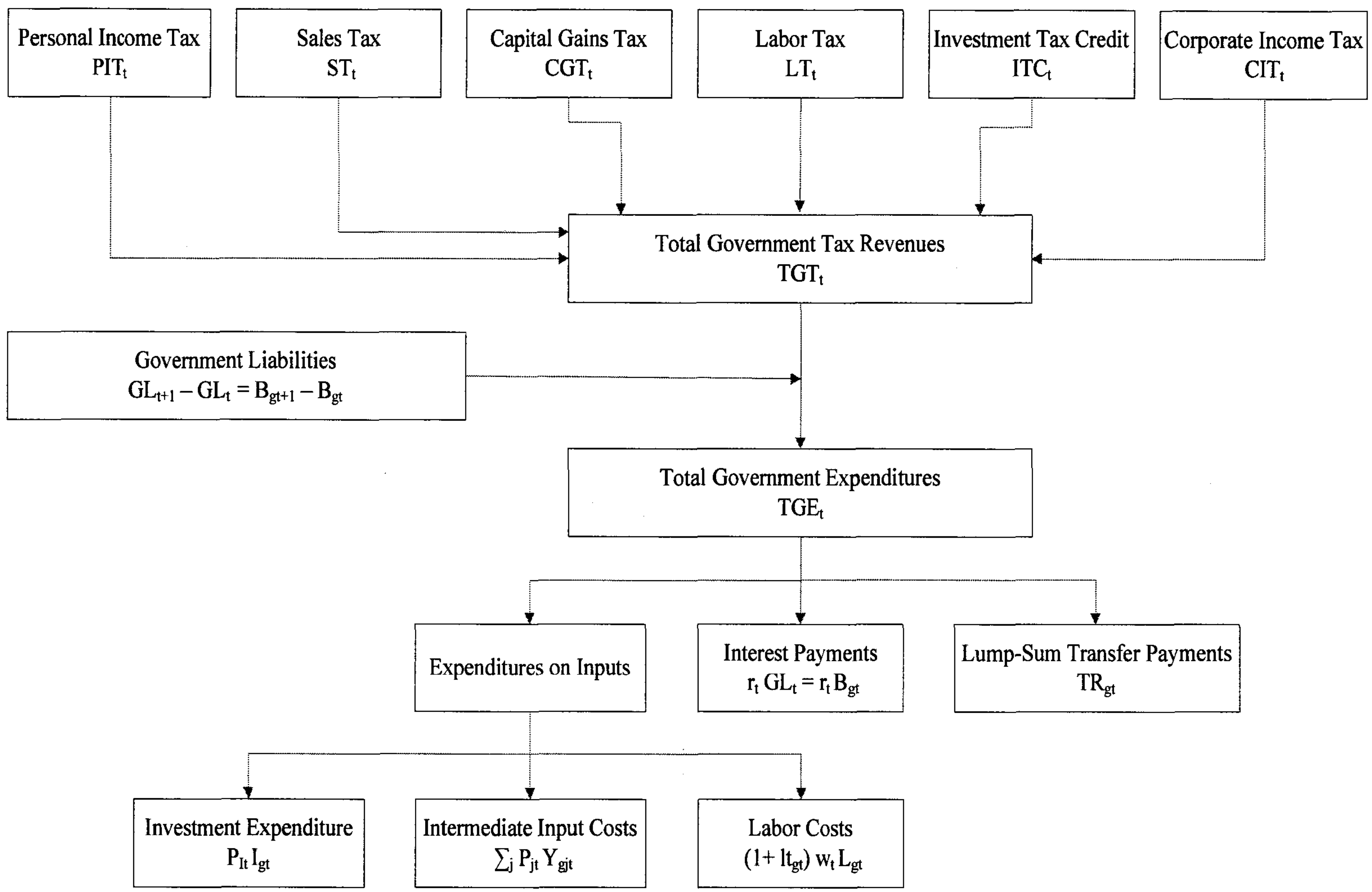


**Figure A3.1 Structure of the Government's Revenues and Expenditures**

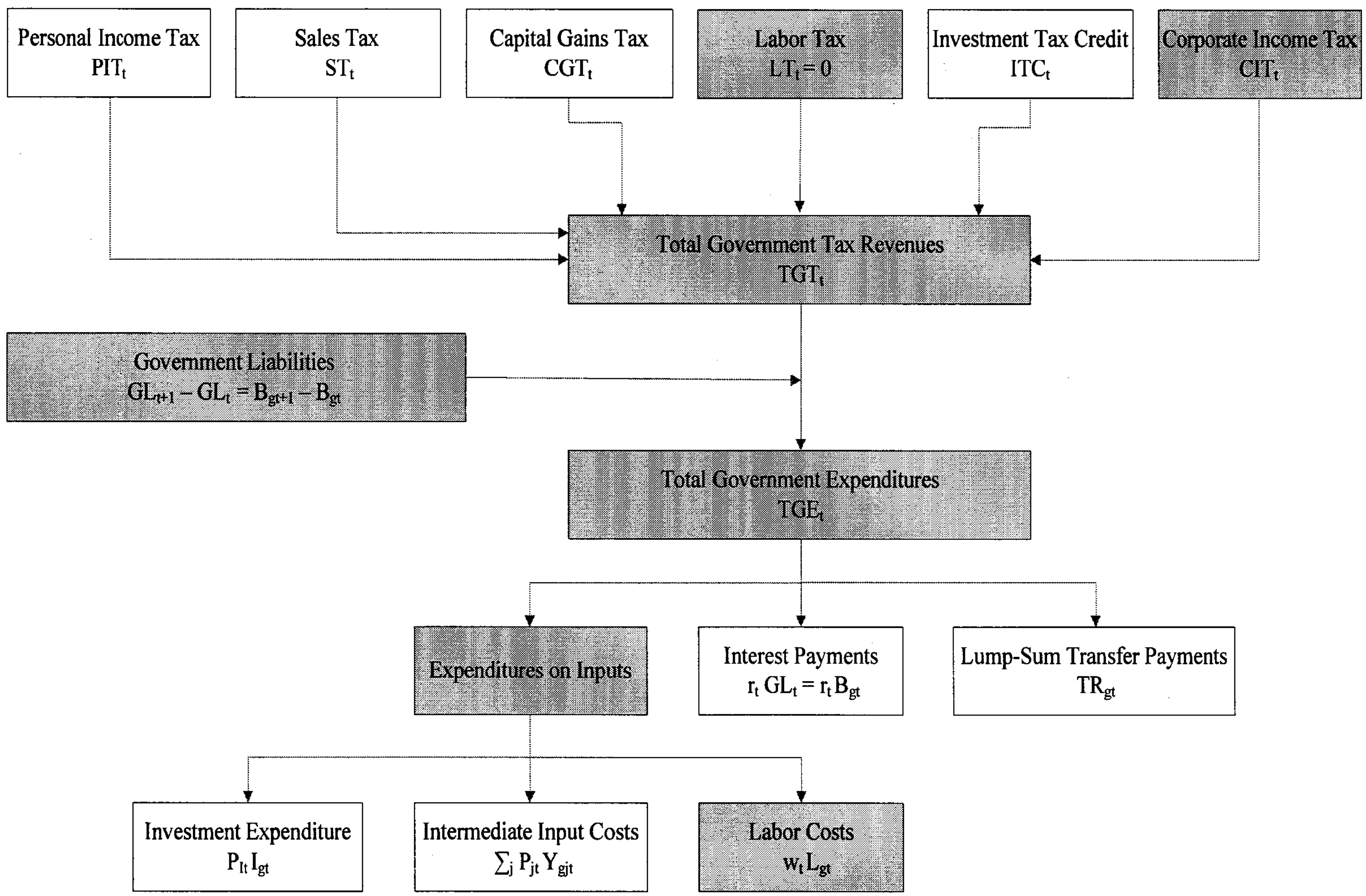


**Figure A3.2 The Effects of Abolishing Labor Tax on the Structure of the Government Sector**

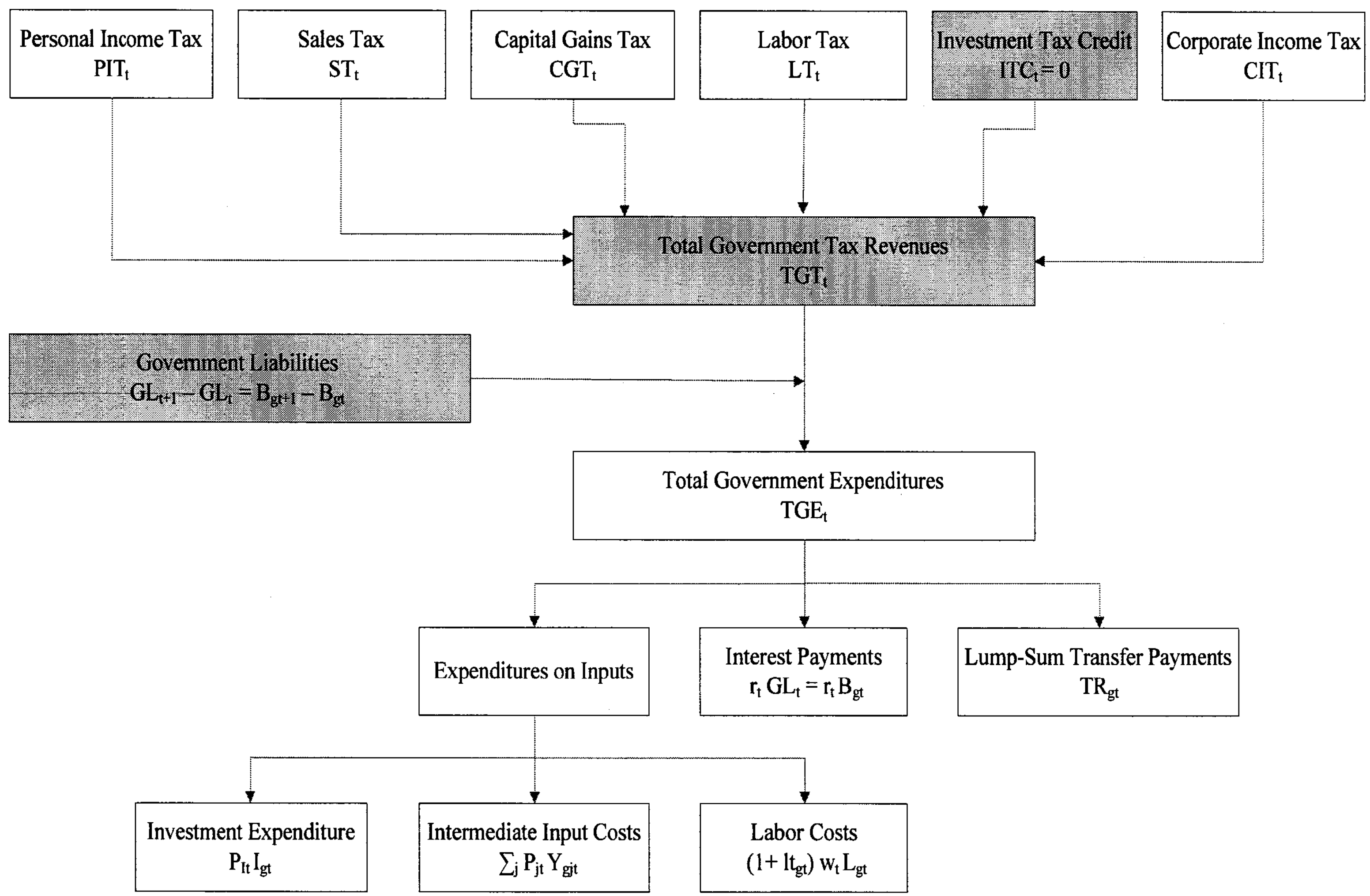


**Figure A3.3 The Effects of Abolishing Investment Tax Credit on the Structure of the Government Sector**

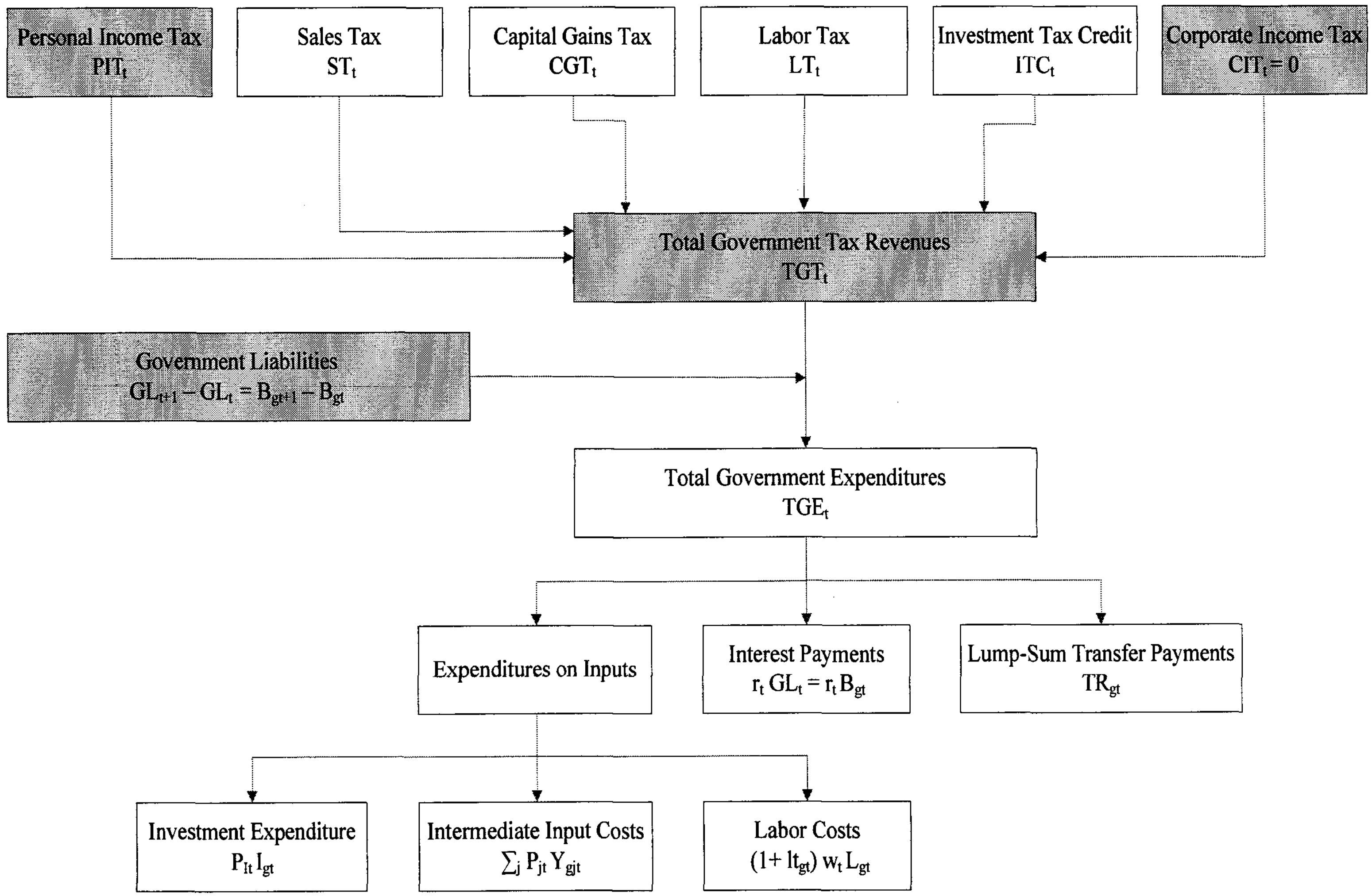


**Figure A3.4 The Effects of Full Integration on the Structure of the Government Sector**

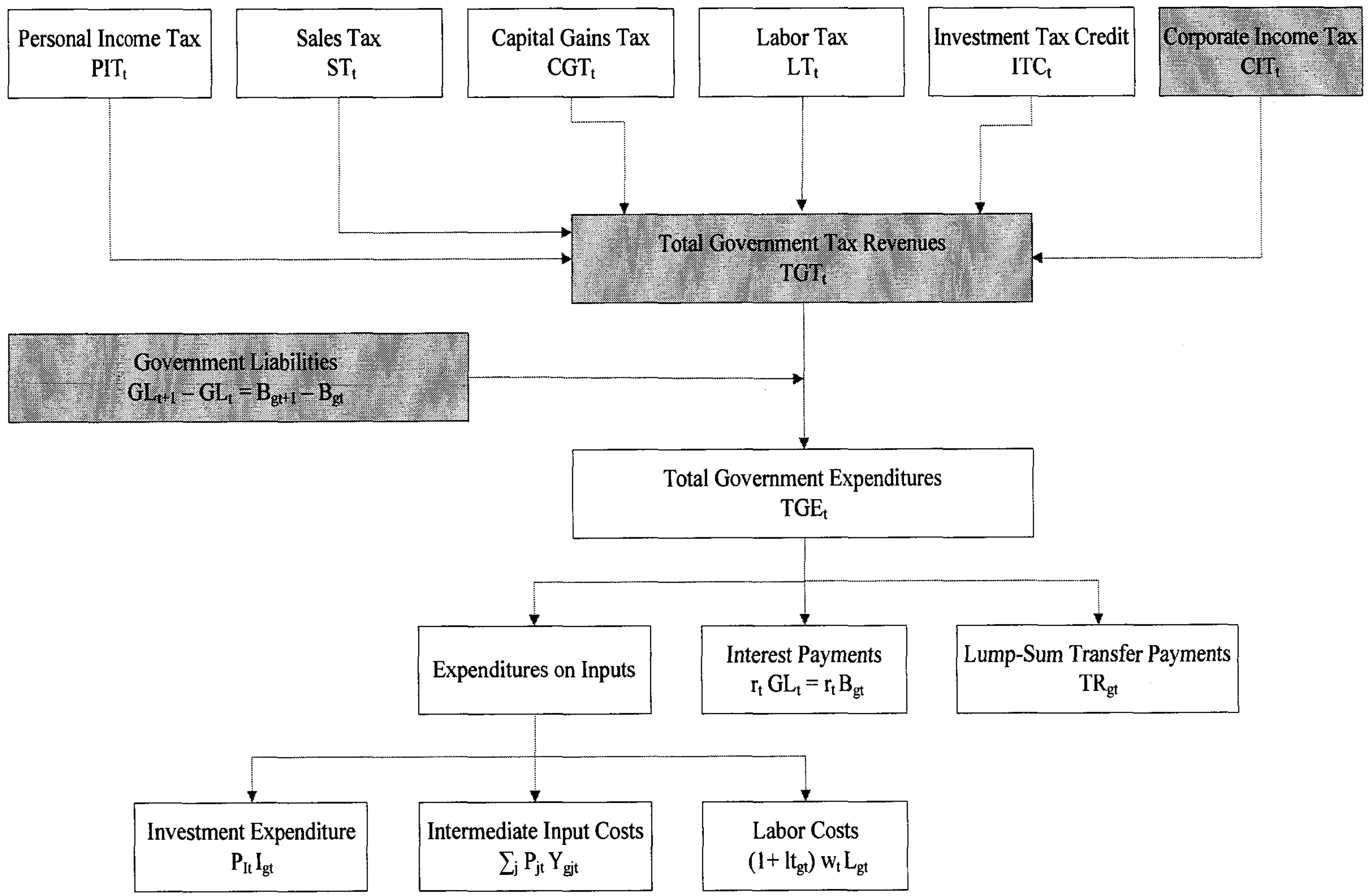


**Figure A3.5 The Effects of Partial Integration A on the Structure of the Government Sector**

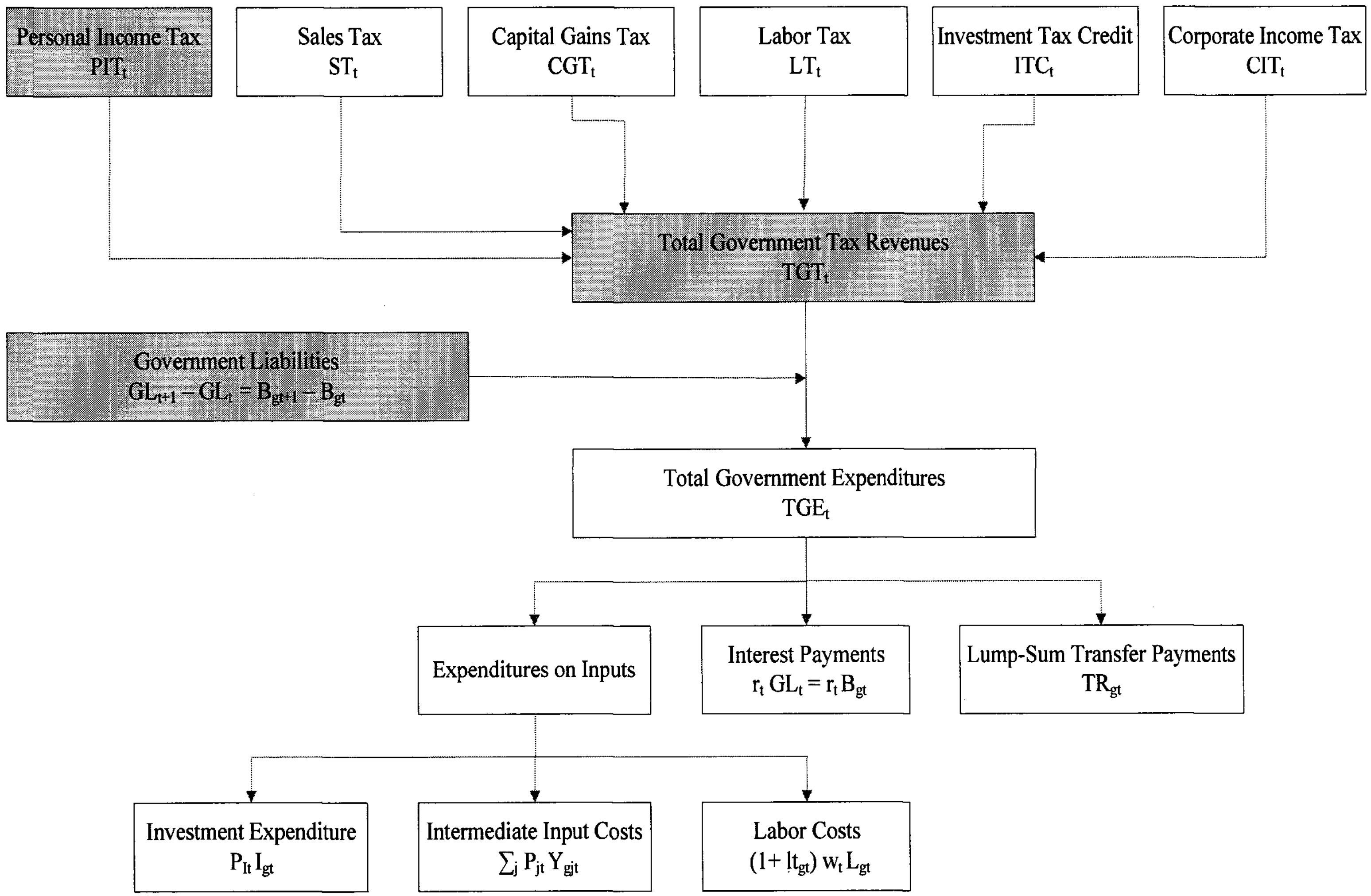


**Figure A3.6 The Effects of Partial Integration B on the Structure of the Government Sector**

**Table A4.1 The Effects of Abolishing Labor Tax on Equilibrium Values**

**1. Solution for Consumption Goods Markets**

(S104) $$Y_{jt}^{S} = \sum_{1 \le h \le m} Y_{hjt}^{D} + \sum_{1 \le f \le J} Y_{fjt}^{D} + Y_{Ijt}^{D} + Y_{gjt}^{D} \qquad j = 1,\ldots,J,$$

| | $Y_{jt}^{S}$ | $\sum_{1 \le h \le m} Y_{hjt}^{D}$ | $\sum_{1 \le f \le J} Y_{fjt}^{D}$ | $Y_{Ijt}^{D}$ | $Y_{gjt}^{D}$ |
|---|---|---|---|---|---|
| t = 0 | ↑ | ? | ↑ | ↑ | ? |
| t | ↑ | ? | ↑ | ↑ | ? |

**2. Solution for the Labor Market**

(S116) $$\sum_{1 \le h \le m} L_{ht}^{S} = \sum_{1 \le j \le J} L_{jt}^{D} + L_{It}^{D} + L_{gt}^{D}$$

| | $\sum_{1 \le h \le m} L_{ht}^{S}$ | $\sum_{1 \le j \le J} L_{jt}^{D}$ | $L_{It}^{D}$ | $L_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | ? | ↑ | ↑ | ? |
| t | ? | ↑ | ↑ | ? |

**3. Solution for the Investment Good Market**

(S125) $$I_{It}^{S} = \sum_{1 \le j \le J} I_{jt}^{D} + I_{It}^{D} + I_{gt}^{D}$$

| | $I_{It}^{S}$ | $\sum_{1 \le j \le J} I_{jt}^{D}$ | $I_{It}^{D}$ | $I_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | ↑ | ↑ | ↑ | ? |
| t | ↑ | ↑ | ↑ | ? |

**4. Solution for the Financial Market**

(S134) $$\sum_{1 \le h \le m} FD_{ht}^{S} = \sum_{1 \le j \le J} FD_{jt}^{D} + FD_{It}^{D} + FD_{gt}^{D}$$

| | $\sum_{1 \le h \le m} FD_{ht}^{S}$ | $\sum_{1 \le j \le J} FD_{jt}^{D}$ | $FD_{It}^{D}$ | $FD_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | ? | ? | ? | ? |
| t | ? | ? | ? | ? |

**Table A4.2 The Effects of Abolishing Investment Tax Credit on Equilibrium Values**

**1. Solution for Consumption Goods Markets**

(S104) $Y_{jt}^S = \sum_{1 \le h \le m} Y_{hjt}^D + \sum_{1 \le f \le J} Y_{fjt}^D + Y_{Ijt}^D + Y_{gjt}^D$ $\quad j = 1,\ldots,J,$

| | $Y_{jt}^S$ | $\sum_{1 \le h \le m} Y_{hjt}^D$ | $\sum_{1 \le f \le J} Y_{fjt}^D$ | $Y_{Ijt}^D$ | $Y_{gjt}^D$ |
|---|---|---|---|---|---|
| t = 0 | – | ↓ | – | – | ↑ |
| t | ↓ | ? | ↓ | ↓ | ? |

**2. Solution for the Labor Market**

(S116) $\sum_{1 \le h \le m} L_{ht}^S = \sum_{1 \le j \le J} L_{jt}^D + L_{It}^D + L_{gt}^D$

| | $\sum_{1 \le h \le m} L_{ht}^S$ | $\sum_{1 \le j \le J} L_{jt}^D$ | $L_{It}^D$ | $L_{gt}^D$ |
|---|---|---|---|---|
| t = 0 | ↑ | – | – | ↑ |
| t | ? | ↓ | ↓ | ? |

**3. Solution for the Investment Good Market**

(S125) $I_{It}^S = \sum_{1 \le j \le J} I_{jt}^D + I_{It}^D + I_{gt}^D$

| | $I_{It}^S$ | $\sum_{1 \le j \le J} I_{jt}^D$ | $I_{It}^D$ | $I_{gt}^D$ |
|---|---|---|---|---|
| t = 0 | – | ↓ | ↓ | ↑ |
| t | ↓ | ↓ | ↓ | ? |

**4. Solution for the Financial Market**

(S134) $\sum_{1 \le h \le m} FD_{ht}^S = \sum_{1 \le j \le J} FD_{jt}^D + FD_{It}^D + FD_{gt}^D$

| | $\sum_{1 \le h \le m} FD_{ht}^S$ | $\sum_{1 \le j \le J} FD_{jt}^D$ | $FD_{It}^D$ | $FD_{gt}^D$ |
|---|---|---|---|---|
| t = 0 | ↑ | ? | ? | ? |
| t | ? | ? | ? | ? |

**Table A4.3 The Effects of Full Integration on Equilibrium Values**

**1. Solution for Consumption Goods Markets**

(S104) $$Y_{jt}^{S} = \sum_{1 \leq h \leq m} Y_{hjt}^{D} + \sum_{1 \leq f \leq J} Y_{fjt}^{D} + Y_{Ijt}^{D} + Y_{gjt}^{D} \qquad j = 1,\ldots,J,$$

| | $Y_{jt}^{S}$ | $\sum_{1 \leq h \leq m} Y_{hjt}^{D}$ | $\sum_{1 \leq f \leq J} Y_{fjt}^{D}$ | $Y_{Ijt}^{D}$ | $Y_{gjt}^{D}$ |
|---|---|---|---|---|---|
| t = 0 | – | ↑ | – | – | ↓ |
| t | ↑ | ? | ↑ | ↑ | ? |

**2. Solution for the Labor Market**

(S116) $$\sum_{1 \leq h \leq m} L_{ht}^{S} = \sum_{1 \leq j \leq J} L_{jt}^{D} + L_{It}^{D} + L_{gt}^{D}$$

| | $\sum_{1 \leq h \leq m} L_{ht}^{S}$ | $\sum_{1 \leq j \leq J} L_{jt}^{D}$ | $L_{It}^{D}$ | $L_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | ↓ | – | – | ↓ |
| t | ? | ↑ | ↑ | ? |

**3. Solution for the Investment Good Market**

(S125) $$I_{It}^{S} = \sum_{1 \leq j \leq J} I_{jt}^{D} + I_{It}^{D} + I_{gt}^{D}$$

| | $I_{It}^{S}$ | $\sum_{1 \leq j \leq J} I_{jt}^{D}$ | $I_{It}^{D}$ | $I_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | – | ↑ | ↑ | ↓ |
| t | ↑ | ↑ | ↑ | ? |

**4. Solution for the Financial Market**

(S134) $$\sum_{1 \leq h \leq m} FD_{ht}^{S} = \sum_{1 \leq j \leq J} FD_{jt}^{D} + FD_{It}^{D} + FD_{gt}^{D}$$

| | $\sum_{1 \leq h \leq m} FD_{ht}^{S}$ | $\sum_{1 \leq j \leq J} FD_{jt}^{D}$ | $FD_{It}^{D}$ | $FD_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | ? | ? | ? | ? |
| t | ? | ? | ? | ? |

**Table A4.4 The Effects of Partial Integration A on Equilibrium Values**

**1. Solution for Consumption Goods Markets**

$$(S104)\quad Y_{jt}^{S} = \sum_{1 \le h \le m} Y_{hjt}^{D} + \sum_{1 \le f \le J} Y_{fjt}^{D} + Y_{Ijt}^{D} + Y_{gjt}^{D} \qquad j = 1,\ldots,J,$$

| | $Y_{jt}^{S}$ | $\sum_{1 \le h \le m} Y_{hjt}^{D}$ | $\sum_{1 \le f \le J} Y_{fjt}^{D}$ | $Y_{Ijt}^{D}$ | $Y_{gjt}^{D}$ |
|---|---|---|---|---|---|
| t = 0 | – | – | – | – | – |
| t | – | ? | – | – | – |

**2. Solution for the Labor Market**

$$(S116)\quad \sum_{1 \le h \le m} L_{ht}^{S} = \sum_{1 \le j \le J} L_{jt}^{D} + L_{It}^{D} + L_{gt}^{D}$$

| | $\sum_{1 \le h \le m} L_{ht}^{S}$ | $\sum_{1 \le j \le J} L_{jt}^{D}$ | $L_{It}^{D}$ | $L_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | – | – | – | – |
| t | ? | – | – | – |

**3. Solution for the Investment Good Market**

$$(S125)\quad I_{It}^{S} = \sum_{1 \le j \le J} I_{jt}^{D} + I_{It}^{D} + I_{gt}^{D}$$

| | $I_{It}^{S}$ | $\sum_{1 \le j \le J} I_{jt}^{D}$ | $I_{It}^{D}$ | $I_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | – | – | – | – |
| t | – | – | – | – |

**4. Solution for the Financial Market**

$$(S134)\quad \sum_{1 \le h \le m} FD_{ht}^{S} = \sum_{1 \le j \le J} FD_{jt}^{D} + FD_{It}^{D} + FD_{gt}^{D}$$

| | $\sum_{1 \le h \le m} FD_{ht}^{S}$ | $\sum_{1 \le j \le J} FD_{jt}^{D}$ | $FD_{It}^{D}$ | $FD_{gt}^{D}$ |
|---|---|---|---|---|
| t = 0 | ↑ | ↓ | ↓ | ↑ |
| t | ? | ↓ | ↓ | ? |

**Table A4.5 The Effects of Partial Integration B on Equilibrium Values**

**1. Solution for Consumption Goods Markets**

| | | | | | | |
|---|---|---|---|---|---|---|
| (S104) | $Y_{jt}^{S} =$ | $\sum_{1 \le h \le m} Y_{hjt}^{D} +$ | $\sum_{1 \le f \le J} Y_{fjt}^{D} +$ | $Y_{Ijt}^{D} +$ | $Y_{gjt}^{D}$ | $j = 1,\ldots,J,$ |
| $t = 0$ | – | – | – | – | – | |
| $t$ | – | ? | – | – | – | |

**2. Solution for the Labor Market**

| | | | | |
|---|---|---|---|---|
| (S116) | $\sum_{1 \le h \le m} L_{ht}^{S} =$ | $\sum_{1 \le j \le J} L_{jt}^{D} +$ | $L_{It}^{D} +$ | $L_{gt}^{D}$ |
| $t = 0$ | – | – | – | – |
| $t$ | ? | – | – | – |

**3. Solution for the Investment Good Market**

| | | | | |
|---|---|---|---|---|
| (S125) | $I_{It}^{S} =$ | $\sum_{1 \le j \le J} I_{jt}^{D} +$ | $I_{It}^{D} +$ | $I_{gt}^{D}$ |
| $t = 0$ | – | – | – | – |
| $t$ | – | – | – | – |

**4. Solution for the Financial Market**

| | | | | |
|---|---|---|---|---|
| (S134) | $\sum_{1 \le h \le m} FD_{ht}^{S} =$ | $\sum_{1 \le j \le J} FD_{jt}^{D} +$ | $FD_{It}^{D} +$ | $FD_{gt}^{D}$ |
| $t = 0$ | ↑ | – | – | ↑ |
| $t$ | ? | – | – | ? |