ROLE OF ANGULAR MOMENTUM TRANPORTS IN TROPICAL STORM DISSIPATION OVER TROPICAL OCEANS

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THESIS

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ABSTRACT

The mean wind field is investigated around eleven tropical storms which dissipated over tropical waters south of 20° north latitude. Wind data at 850 mb and 200 mb are composited in a circular area of 12° latitude radius from the storm center for two periods in the life of these storms: (1) their intensification and mature stages, and (2) their dissipation stage. Vertical shears between these two levels are also composited. Wind data in the composites are converted to radial profiles of mean tangential wind for the two periods of storm life. These profiles are compared to similar profiles found in previous studies for intensifying tropical depressions and for mature tropical storms.

The most significant differences are found in the low level wind field north of the storm centers. The broad easterly flow to the north of the intensifying tropical depressions is also found north of the dying storms during their intensification and mature stages. However, during the dissipation stage the easterly flow has weakened and become westerly over a part of the area. As a result, the mean tangential wind is significantly reduced at all radii thus reducing the import of relative angular momentum into the storm by the mean radial circulation. A deficiency in angular momentum results from a loss to the ocean greater than the import through the boundaries in at least some of the cases investigated.

It is concluded that a lack of angular momentum import through the lateral boundaries may in some cases be an important factor in storm weakening or dissipation. Angular momentum import into storms at low latitudes is performed by the surrounding anticyclones at the outer radii. Maintenance and control of storm intensity is thus directly related to the position and intensity of these anticyclones.

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I. INTRODUCTION

Background

Much theoretical and observational evidence has accumulated to show that a number of physical requirements must be satisfied for development of a tropical storm. Although much emphasis has been placed on the internal workings of the intensification process, evidence is also available to suggest that the process depends on external factors. Similarly there is evidence to indicate that external factors also play a role in storm maintenance and dissipation.

Dunn and Miller (1960) have observed that a strong, broad and persistent trade flow is favorable for storm development. They noted that in many cases development follows a surge in the trade flow, suggesting that intensification may result in some cases from external forcing. Other investigators have related storm genesis to long period general circulation features. Ballenzweig (1958) found that storm genesis was associated with negative monthly mean height anomalies at 700 mb over development regions and positive anomalies to the north and northeast of these regions. This height pattern would lead to easterly anomalous flow at 700 mb. Namias and Dunn (1955) found similar anomalies during August 1955 with the unusually early formation of Hurricanes Connie and Diane in the central Atlantic.

Sartor (1968) has studied variations in relative vorticity patterns from monthly mean wind data at 850 mb in the western Carribean and the Gulf of Mexico. He compared data for months in which storm genesis occurred to months without storm genesis. He found storm genesis was well correlated with high average 850 mb relative vorticity values in the Gulf of Mexico.

Gray (1967) has composited upper wind data at 850 mb and 200 mb relative to a large number of individual disturbances which subsequently intensified. Data was composited to distances of 12^{o} of latitude from the initial disturbance centers during the 36 hour period just prior to reaching tropical storm intensity. Besides showing that disturbances intensify in regions of minimum tropospheric wind shear his data also show that the composite 850 mb wind field has quite strong easterly flow as far as 12^{o} of latitude poleward from the disturbance center.

Riehl and Malkus (1961) have studied the exchange of angular momentum, heat, water vapor, and kinetic energy between Hurricane Daisy, 1958, and its environment. Riehl (1959) did a similar computation for Hurricane Ella, 1958, while Palmén and Riehl (1957) computed the angular momentum and kinetic energy budgets of the mean storm constructed from the data of Hughes (1952) and Jordan (1952). Results of these studies suggest that external factors are important in maintaining the storm and controlling its intensity.

Gray (1967) and Lopéz (1968) have shown that middle level convergence and mass flowing through the storm system as a result of vertical wind shear (ventilation) strongly inhibits the intensification process. Riehl and Malkus (1961) similarly concluded that middle level convergence acted as a constraint on the intensity of Hurricane Daisy, 1958.

Although all of these mechanisms in theory, at least, can act on development, much remains to be learned about their relative importance in actual storm cases. For example many storms fail to reach hurricane intensity when the oceanic heat source is known to be more than adequate. In other cases storms may undergo wide fluctuations in intensity during their life cycle. In yet other cases, storms have completely dissipated over tropical waters. Completely satisfactory reasons for the behavior of these storms has yet to be given.

<u>Purpose of this Study</u>

It is well known that some tropical storms dissipate over tropical waters. Various hypotheses have been given to explain this dissipation but due to a lack of sufficient data, observational substantiation of these hypotheses has been difficult or impossible in individual cases. In the present study, available upper wind data around eleven storms which dissipated over tropical waters are composited, using a procedure similar to that used by Gray

(1967). From the composited data, a picture of the mean wind field surrounding these storms is constructed.

The law of conservation of angular momentum is among the several physical laws which must be satisfied in the tropical storm. The purpose of this study is to compute angular momentum budgets of these dissipating storms from the mean wind fields during two phases of their life cycle, and to compare these budgets with those of the intensifying disturbances studied by Gray (1967). It will be shown that in the dying storms, a deficiency in angular momentum transports from surrounding ciruclations probably contributed to their dissipation.

Selection of Cases for Investigation

All of the particular storm cases investigated in this study occurred in the Western North Atlantic and the Western North Pacific during the period 1958 through 1965. Position and intensity data for the Atlantic cases were obtained from Cry (1964, 1965) and Cry and DeAngelis (1965). Position and intensity data for the cases studied in the Pacific were taken from the Annual Typhoon Reports issued by the Joint Typhoon Warning Center at Guam. All of the cases studied were located with storm centers over tropical water south of 20°N latitude. Some of the storms reached hurricane intensity, while for others, the peak intensity was less than that required for classification as a hurricane. Pertinent data on the eleven cases studied are listed in Table 1.

Table 1. List of storms used in data composites for dying storms.

Year	Date	Name	Location	Maximum Intensity
1958	6-17 Jan	Ophelia	NW Pacific	Trop. Storm
	30 Nov-4 Dec	Pamela	NW Pacific	Trop. Storm
1959	4-12 Mar	Sally	NW Pacific	Trop. Storm
1962	26-30 May	Iris	NW Pacific	Hurricane
	6-12 Nov	Jean	NW Pacific	Hurricane
1963	31 Jul-5 Aug	Arlene	NW Atlantic	Hurricane
	8-12 Sept	Hester	NW Pacific	Trop. Storm
	25-29 Oct	Helena	NW Atlantic	Trop. Storm
1964	13-18 July	Elsie	NW Pacific	Hurricane
	6-10 July	Cora	NW Pacific	Hurricane
1965	12-14 Apr	Wanda	NW Pacific	Hurricane

The data was composited during two periods in the life cycle of these storms. The first composite covered the period during which the storms were intensifying or were at their peak intensity. This is referred to here as the intensification and mature stage. The second composite covers the period extending from the time the storms started to weaken until they were reduced below tropical storm intensity. This period is referred to here as the dissipation stage.

II. COMPOSITING PROCEDURE

Wind data were composited at two levels: 850 mb and 200 mb. The 850 mb level was selected to represent the wind field at the top of the boundary layer surrounding the disturbances. Although winds at 900 mb may have been slightly more representative, the 850 mb level was selected because the wind data were more plentiful there.

The 200 mb wind field was selected for compositing because this level has been shown to be the level at which outflow is concentrated (Izawa, 1964, Palmén and Riehl, 1957, Jordan, 1952). Wind data at that level are important in determining the angular momentum budget of the storms. In addition the vertical wind shear between 850 mb and 200 mb is a measure of the tropospheric ventilation of heat away from the disturbances. This has been strongly correlated with tropical storm development (Gray, 1967) and may also play a significant role in storm dissipation.

All available wind observations within a radius of 12° latitude from the storm center were converted to u (eastward) and v (northward) components and used in the composites. Data were located in the composite with respect to the geographical distance and direction from the instantaneous storm center. Vertical shears were computed in both components where an observation was available at both the 850 mb and 200 mb levels.

Since storm movement was not subtracted from individual wind data, the composites do not represent the wind field relative to the moving storm centers. However, in this study only mean values of the radial and tangential winds, averaged around a circular boundary at each level, were used in the calculations. These mean values are unaffected by storm motion.

The circular data area was divided into 3 annular rings of 4° of latitude radial increment each. The outer two rings were divided into 16 sectors while the inner ring was divided into eight sectors. All observations of each type within a given sector were averaged. The averaged values for each sector were plotted on concentric circles at 2°, 6°, and 10° of latitude from the storm center.

III. RESULTS OF DATA COMPOSITES

Tropospheric Vertical Wind Shear

Gray (1967) has shown that a minimum of tropospheric (vertical) wind shear is required for intensification of tropical disturbances. Lopéz (1968) has also shown high shear to be a constraint on the intensification of an easterly disturbance. In view of these results, the present author expected to find large values of tropospheric shear, indicative of strong ventilation, in the vertical shear composites for the dying storms during the dissipation stage. The composites, however, did not show this to be true. A minimum of vertical shear in both the u and v component was also found over the dissipating storm centers.

The appearance of minima in vertical wind shear in the surrounding wind field suggests that vertical shear probably plays a small role in contributing to storm dissipation, at least for storms south of 20° latitude. At higher latitudes however, the influence of mid-latitude westerlies may result in a greater influence from vertical shear.

850 mb Zonal Wind Field

Figure 1 shows the composite of the 850 mb zonal wind field found by Gray (1967) from 190 tropical disturbances which later became tropical storms. All of these intensified south of 20° latitude. Figure 2 shows the 850 mb zonal wind field surrounding the dying storms during the intensification and mature stages, while Fig. 3 shows the 850 mb zonal wind field for these same storms during the dissipation stage.

Figures 1 and 2 show that on the average, the easterly flow was markedly stronger north of the storm centers during the intensification and mature stages of the dying storms than it was for the intensifying disturbances. At first this may appear to result from the increased pressure gradient associated with the higher storm intensity. However, if this were the sole reason, the area of strong easterly flow would not have so great an east-west extent, but would be restricted to an area just north of the storm center. Examination of individual surface analyses during the periods in the composite revealed that the band of easterlies in Fig. 2 resulted from the persistent appearance of rather strong trade flow north of the storms during this stage. In many cases, a rather intense high pressure cell was found north of the storm center.

In order to obtain a representative pattern of the 850 mb zonal wind field during the dissipation stage, some smoothing of the analysis was done in Fig. 3. Two of the sectors on the 6° latitude circle had only one observation each (value underscored) which did not appear to be representative of the general pattern shown by the data in the diagram. For this reason, these values were disregarded

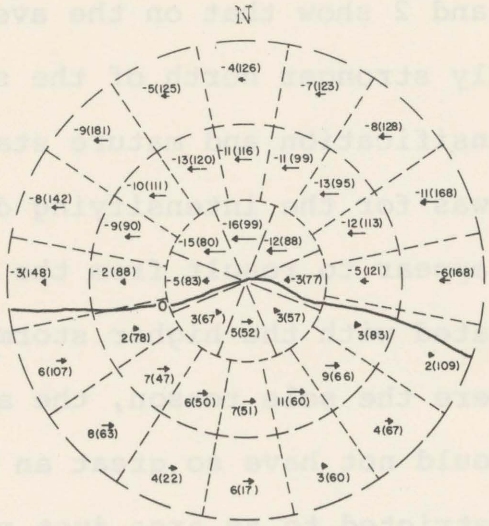

Fig. 1. Composite of the mean 850 mb u component from available rawin data for 190 tropical depressions which later developed into tropical storms in three areas south of 20° latitude (Gray, 1967). Numbers on the left in each sector denote the mean value of the u component in knots; those in parenthesis show the number of observations comprising the mean value. The arrows denote direction and magnitude of the u components. The dashed circles represent distances of 4°, 8°, and 12° lat. from the disturbance center.

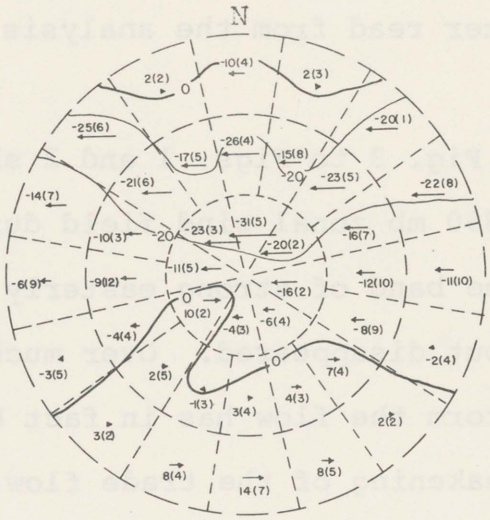

Fig. 2. Composite of the mean 850 mb u component from available rawin data for the 11 dying storms during their intensification and mature stages. See Fig. 1 for further explanation.

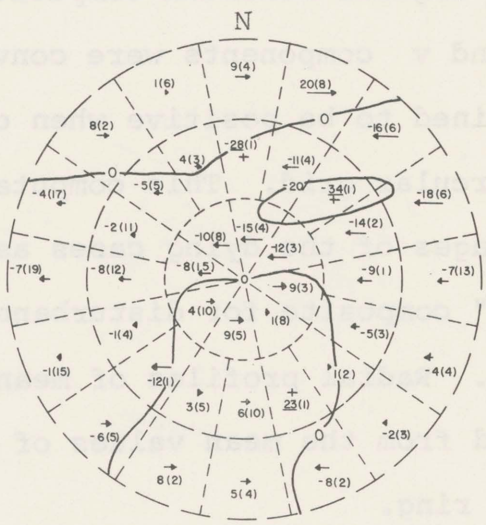

Fig. 3. Composite of the mean 850 mb u component from available rawin data for 11 dying storms during their dissipation stage. Underscored values of the u component denote single observations which were not considered representative and were not drawn for in the isopleth analysis. See Fig. 1 for further explanation.

in the analysis. Interpolated values for these grid points were later read from the analysis for use in computations.

Comparing Fig. 3 to Figs. 1 and 2 shows a noticeable change in the 850 mb zonal wind field during the dissipation stage. The band of strong easterly flow north of the storm has all but disappeared. Over much of the region north of the storm the flow has in fact become westerly. This general weakening of the trade flow during dissipation was also evident in the surface analyses. The area and magnitude of the westerly winds south of the storm center have also been reduced.

850 and 200 mb Mean Tangential Wind Profiles

For use in angular momentum computations, the 850 mb and 200 mb u and v components were converted to tangential winds (defined to be positive when cyclonic) at each point of the circular grid. This computation was carried out for both stages of the dying cases as well as for Gray's "3-area" composite for disturbances developing south of 20° latitude. Radial profiles of mean tangential wind were constructed from the mean values of the tangential wind in each annular ring.

The results of these calculations are given in Table 2. Tangential wind speeds are given in units of m sec^{-1} for convenience in later calculations. The radial profiles constructed from the data in Table 2 are shown in Figs. 4 and 5. In constructing these profiles, values of

Table 2. Mean values of the tangential wind component for the three annular rings at the 850 mb and 200 mb levels for the two stages of the dying storms and for Gray's developing disturbances.

Class	Inner ring Radius	Inner ring 850mb	Inner ring 200mb	6^{o} lat. 850mb	6^{o} lat. 200mb	10^{o} lat. 850mb	10^{o} lat. 200mb
	Tangential wind speed (m sec^{-1})						
Dying storms (Developing and mature stages)	2.5^{o}	7.4	0.3	5.1	-2.9	3.0	-5.1
Dying storms (Dissipation stage)	2.5^{o}	4.5	-1.9	3.5	-1.4	-0.2	-4.2
Gray's developing disturbance	$2\ ^{o}$	4.7	-0.3	4.5	-3.5	2.9	-5.6

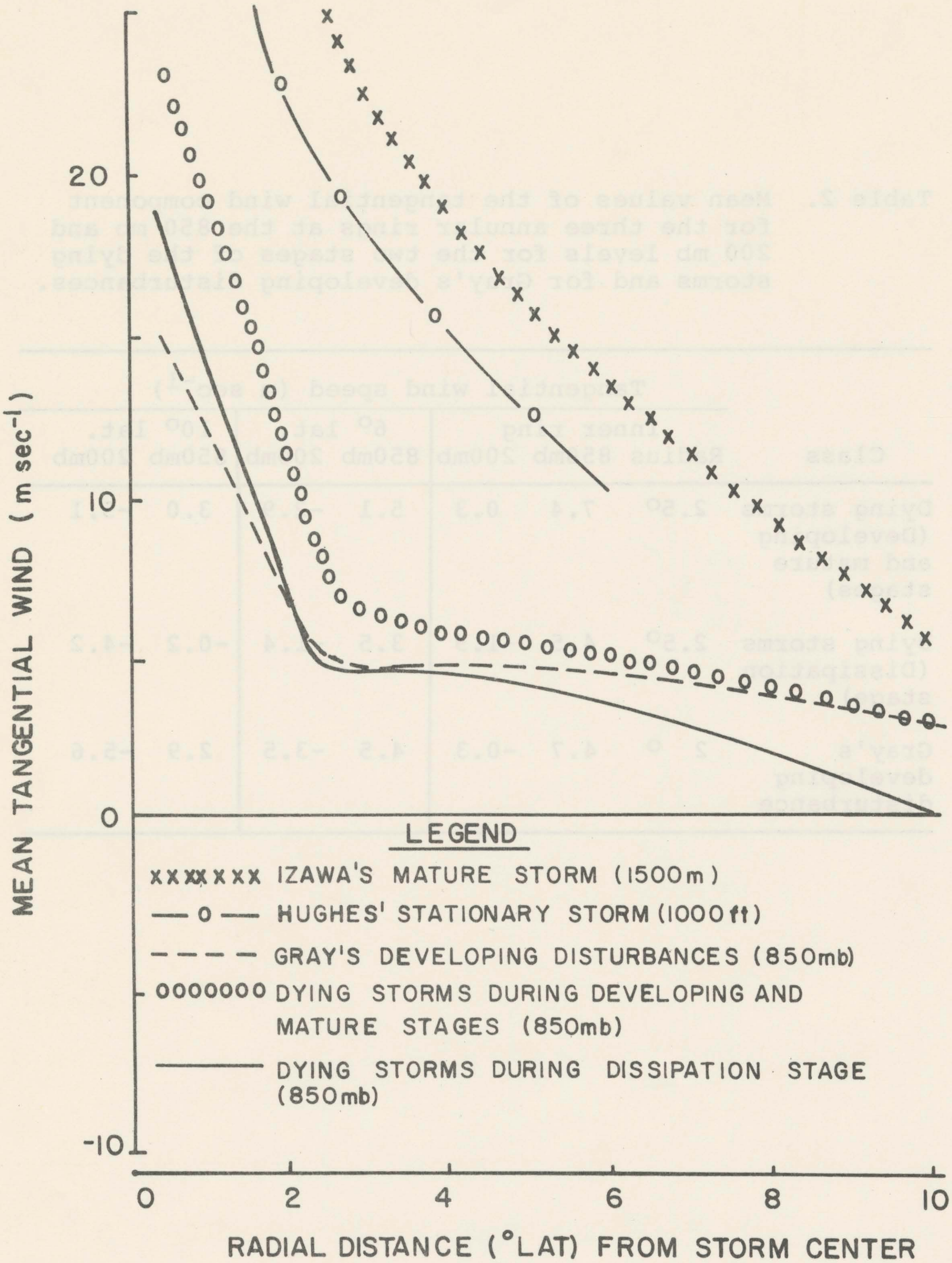

Fig. 4. Radial profiles of mean tangential wind representing the inflow layer for five different classes of storms as shown in the legend. The two upper profiles were taken from Izawa (1964) and Hughes (1952) respectively.

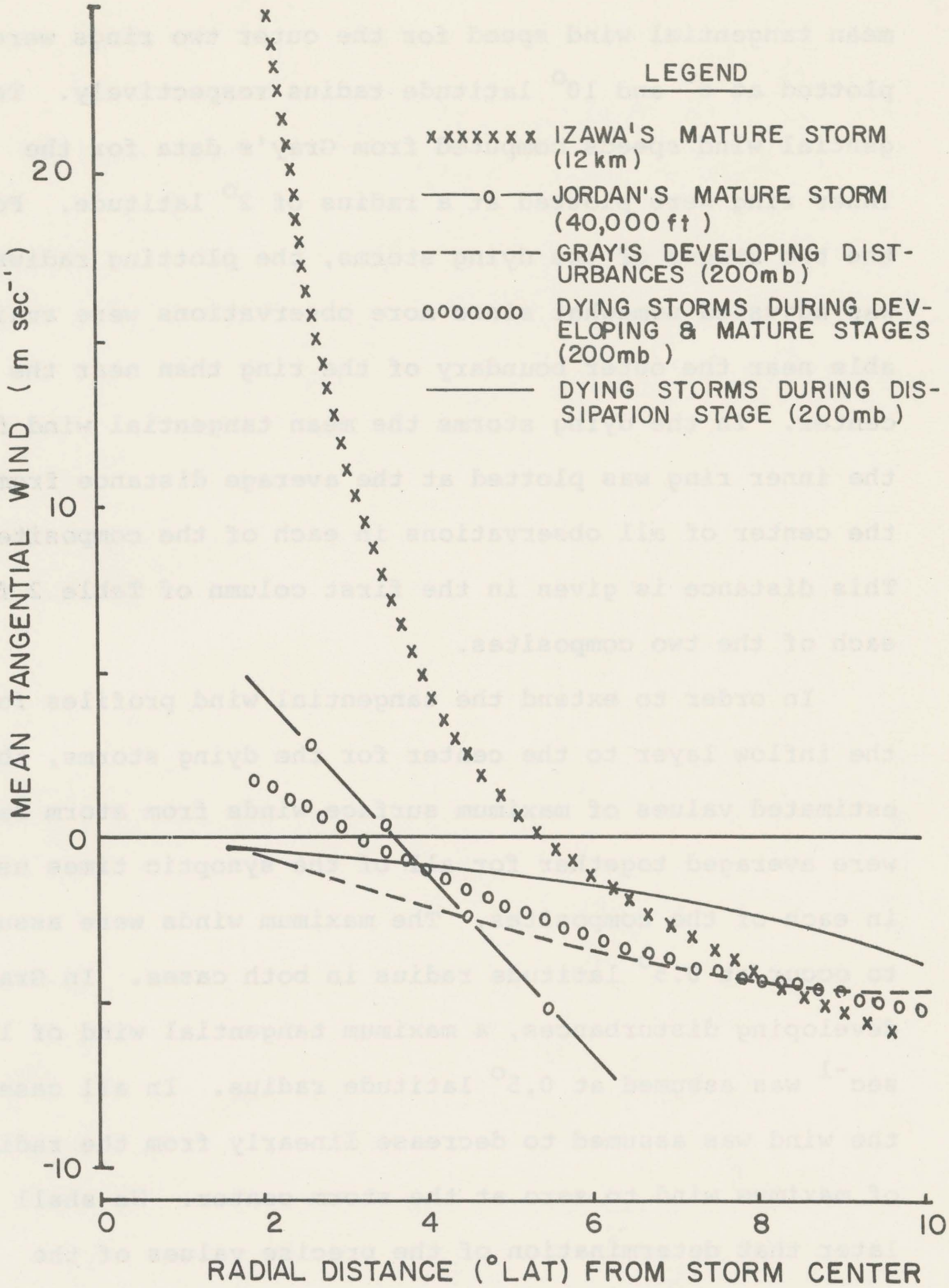

Fig. 5. Radial profiles of mean tangential wind representing the outflow layer for five different classes of storms as shown in the legend. Two of these profiles were taken from Izawa (1964) and Jordan (1952).

mean tangential wind speed for the outer two rings were plotted at 6° and 10° latitude radius respectively. Tangential wind speeds computed from Gray's data for the inner ring were plotted at a radius of 2° latitude. For the two stages of the dying storms, the plotting radius was adjusted somewhat since more observations were available near the outer boundary of the ring than near the center. In the dying storms the mean tangential wind for the inner ring was plotted at the average distance from the center of all observations in each of the composites. This distance is given in the first column of Table 2 for each of the two composites.

In order to extend the tangential wind profiles for the inflow layer to the center for the dying storms, the estimated values of maximum surface winds from storm data were averaged together for all of the synoptic times used in each of the composites. The maximum winds were assumed to occur at 0.5° latitude radius in both cases. In Gray's developing disturbances, a maximum tangential wind of 15 m sec^{-1} was assumed at 0.5° latitude radius. In all cases the wind was assumed to decrease linearly from the radius of maximum wind to zero at the storm center. We shall see later that determination of the precise values of the distribution of tangential wind near the storm core is not critical to the computations to be performed here.

In addition to the tangential wind profiles computed in this study, comparison profiles for a mean stationary hurricane as found by Hughes (1952) and Jordan (1952) and a mean profile for a number of Pacific typhoons as found by Izawa (1964) are also shown in Figs. 4 and 5.

Figure 4 shows that the mature storms of Izawa and Hughes show a strong cyclonic circulation to great distances from the storm center. In the case of Gray's developing disturbances and in the dying storms during developing and mature stages, the cyclonic circulation is also present at all radii, although it is weaker than in Hughes' and Izawa's storms. In the dissipation stage a marked reduction in the mean cyclonic circulation occurs at all radii with the largest reduction between 6^o and 10^o latitude radius.

IV. ESTIMATES OF THE ANGULAR MOMENTUM BUDGET

The tangential wind profiles which were constructed from the data in Section III for the two stages of the dying storms were used to quantitatively investigate the angular momentum budgets of these storms. Similar calculations were made from the tangential wind profiles for Gray's (1967) developing disturbances. This section describes the method used and presents the results of computations based on the observed tangential wind profiles. Wherever possible, standard notation is used. (See Appendix I for a complete list of symbols.) The equations are developed in a cylindrical (r, θ, z) coordinate system. Volume integrations are performed over a cylinder whose axis coincides with the vertical storm axis.

Theory of the Angular Momentum Budget

The time rate of change of the absolute angular momentum, M, of a parcel about any axis is given by:

$$\frac{dM}{dt} = \Sigma T_i \qquad (1)$$

where M denotes the absolute angular momentum and the T_i represents the individual torques acting on the parcel about the same axis.

The absolute angular momentum, M_z, per unit mass of a parcel in a tropical storm about the vertical storm axis is given by:

$$M_z = v_\theta \; r + \frac{fr^2}{2} \tag{2}$$

Here v_θ represents the tangential component of the total wind vector and r is the radial distance from the storm axis. The horizontal component of the wind at any radius is used. The effect of the earth's curvature, which is not taken into account in computing the tangential wind relative to the vertical storm axis, may be neglected within a distance of 12° latitude from the storm center.

Two forces exert a torque on the air particles about the vertical storm axis: the tangential pressure gradient $(- \frac{1}{\rho r} \frac{\partial p}{\partial \theta})$ and the tangential frictional acceleration (F_θ). Expanding the total derivative of Eq. (1) and introducing the above two torques, we arrive at the result:

$$\frac{\partial M_z}{\partial t} + \mathbf{V}_3 \cdot \nabla_3 M_z = (- \frac{1}{\rho} \frac{\partial p}{r \partial \theta} + F_\theta) r \tag{3}$$

If we integrate Eq. (3) over the mass inside some cylinder of constant radius moving with the storm and apply the continuity equation and the divergence theorem we obtain:

$$\int_V \frac{\partial}{\partial t} (\rho M_z) dV = - \int_S \rho M_z v_n dS + \int_V \rho F_\theta r dV \tag{4}$$

where the local time derivative is taken with respect to the moving storm system, v_n is the normal wind component relative to the moving boundary, positive when directed outward, dS is an element of area on the boundary, and V denotes volume. In the integration over the volume, the net torque resulting from the tangential pressure gradient vanishes. This leads to the result that only friction and boundary fluxes can appreciably alter the total absolute angular momentum of a storm about its vertical axis.

It should be noted that for a storm moving northward, there is a slight increase in the vertical component of absolute angular momentum which results from the tilting of the storm's vertical axis relative to the earth's axis of rotation. However, this effect can be shown to be several orders of magnitude smaller than the frictional and advective terms and may be neglected.

In the boundary flux calculations we may assume that no advective momentum transports occur through the bottom (ocean surface) or top (100 mb surface) boundaries. The flux term in Eq. (4) may then be expressed by:

$$- \int_S \rho M_z v_n dS = - \int_0^{z_T} \int_0^{2\pi} \rho M_z v_r r d\theta dz \tag{5}$$

where z_T is the height of the top of the cylindrical volume. The net transport of earth's angular momentum ($fr^2/2$) into a cylinder will vanish except where there is a net transport of mass across the boundary. Since net mass fluxes are negligible,[1] the advective boundary flux

[1]It can be shown that even for rapidly deepening or filling tropical cyclones, the net mass flux across the boundary is very small compared to the mass circulation through the storm.

term will involve only the relative angular momentum (i.e., $v_\theta r$). Finally if we express the transport in any incremental layer in terms of a mean and eddy transport, Eq. (5) may be written in the form:

$$\int_S \rho v_\theta v_r r dS \approx \sum_{\Delta z} 2\pi\rho(\overline{v}_\theta\overline{v}_r + \overline{v_\theta' v_r'})r^2\Delta z \tag{6}$$

Here $\overline{v}_\theta$ and $\overline{v}_r$ represent average values of these velocity components around the circular boundary at a given level. v_θ' represents the deviation of the tangential component of the total wind from the mean tangential component, while v_r' represents the deviation of the radial wind relative to the moving storm from the mean radial wind at that level. The first term in parentheses on the right of Eq. (6) represents the momentum transport by the mean mass flow across the vertical walls of a given height increment, while the second term represents the eddy transports of angular momentum resulting from the correlation between deviations of the radial and tangential components from their respective mean values.

The frictional losses of angular momentum from within a volume to the surroundings may be expressed in terms of the torque exerted by the tangential shearing stress acting on the boundaries. If we neglect losses of angular momentum through small scale eddy stresses acting at the vertical walls, then the frictional losses will occur primarily at the ocean surface and may be expressed in the form:

$$\int_V \rho F_\theta \, r dV = \int_{S_O} \tau_{\theta z_O} \, r dS_O \tag{7}$$

where S_O denotes the ocean surface, and $\tau_{\theta z_O}$ is the tangential shearing stress at the ocean surface. Similar computations on losses of angular momentum to the surroundings have been made by Pfeffer (1958) and Riehl and Malkus (1961).

The above equations will be used to calculate momentum transport and frictional angular momentum losses for the storm data presented in this paper.

Previous Investigations

Riehl (1961) has calculated the transport of angular momentum in the surface inflow layer for several mature hurricanes. He found that nearly all of the relative angular momentum transport in the inflow layer is performed by the mean radial circulation. His computations were carried out at radii of 200, 400, and 600 km (1.8°, 3.6°, and 5.4° latitude) from the storm center. Palmén and Riehl (1957) performed angular momentum calcualtions using a composite of the mean wind distribution around tropical storms obtained from the data of Hughes (1952) and Jordan (1952). Relative angular momentum transports by the mean circulation as well as the eddies were calculated at all levels from 1000 mb to 100 mb. Their results showed that, although the majority of the inward momentum transport was performed by the mean radial circulation, the magnitude of the eddy transport relative to that by the mean radial circulation increased with increasing distance from the storm center. At 6^{o} latitude radius, the eddies were found to transport about 37% of the total relative angular momentum. Pfeffer (1958) arrived at slightly higher values for the eddy transports on the basis of computations using the same data. All studies indicated, however, that within 3^{o} latitude of the storm center, nearly all of the relative angular momentum is transported by the mean radial circulation.

Computations from Data in This Study

The following calcualtions were performed from the profile of mean tangential wind for Gray's developing disturbances, and from similar profiles for the dying storms during the developing and mature stages and during the dissipation stage. These profiles were taken to represent tangential wind profiles around three classes of storms, namely, intensifying, steady state, and dissipating storms respectively. These profiles are represented in Figs. 4 and 5.

1. Estimate of the frictional loss of angular momentum to the ocean surface. The tangential shearing stress was computed from the equation:

$$\tau_{\theta z_o} = -\rho C_D v_{\theta_o} |\mathbb{V}_o| \simeq -\rho C_D v_{\theta_o} |v_{\theta_o}| \qquad (8)$$

where C_D is the surface drag coefficient. The zero subscript denotes the wind at anemometer level. $\tau_{\theta z_o}$ is defined to be positive when the ocean surface exerts a counterclockwise torque on the atmosphere. In this computation, it was assumed that the total wind speed is approximated by the tangential wind. It was also assumed that the tangential winds at anemometer level were approximated in the different cases by the curves depicted in Fig. 4. A constant value of $C_D = 1.4 \times 10^{-3}$ was used on the basis of results obtained by Palmén and Riehl (1957). The density was taken to be 1.1×10^{-3} gm cm^{-3} for the inflow calculations.

Substituting Eq. (8) into Eq. (7), we obtain an expression for the frictional loss, L_o, of angular momentum to the ocean surface inside a given cylinder:

$$L_o = - \int_{S_o} \tau_{\theta z_o} r dS_o \approx 2\pi \sum_{\Delta r} r^2 \rho C_D \overline{v}_{\theta_o} |\overline{v}_{\theta_o}| \Delta r$$

$$\approx \frac{2\pi}{3} \sum_{\Delta r} \rho C_D \overline{v}_{\theta_o} |\overline{v}_{\theta_o}| (r_2{}^3 - r_1{}^3) \qquad (9)$$

Here r_2 and r_1 denote the outer and inner radius respectively of an annular ring. $\overline{v}_{\theta_o}$ denotes the mean value of the tangential wind in each ring. Frictional losses of angular momentum were computed inside a radius of 10^o latitude. From Eq. (9) we see that cyclonic winds transfer angular momentum to the ocean surface, while anticyclonic winds extract angular momentum from the ocean surface.

2. An estimate of the transport of relative angular momentum inward by the mean radial wind in the inflow layer. Observational studies (Jordan, 1952, Izawa, 1964) show that the mass transport inward toward the storm center is confined largely to the surface inflow layer and is largely due to frictional turning of the surface wind toward lower pressure. Furthermore, the turning angle has in general been found to be about 20-22° at the surface (Hughes, 1952, Krueger, 1957, Gray, 1967, Ausman, 1959, Mendenhall, 1967). On the basis of these results, the radial wind used for computing the transports was assumed to be given at the surface by:

$$\bar{v}_{r_o} = - \bar{v}_{\theta_o} \tan 22^{\circ}$$

and assumed to decrease linearly to zero at the top of the inflow layer. The inflow layer was taken to be 1500 meters thick at all radii on the basis of radial wind profiles as published in Palmén and Riehl (1957) and Pfeffer (1958). Although some inflow occurs above this layer, it is reasonable to assume that the bulk of the mass flow into the tropical storm occurs in the lowest 1500 meters. From the above assumptions the first term on the right side of Eq. (6) was evaluated at radii of 3°, 6°, and 10° latitude.

3. <u>An estimate of the transport of relative angular momentum by the mean radial circulation in the upper tropospheric outflow layer</u>. This was estimated by assuming (a) mass balance so that the mass outflow rate at any radius for each type of storm would equal the mass inflow rate in the surface inflow layer, (b) that the outflow is concentrated in a thin layer (about 100 mb thick) at 200 mb, and (c) that the mean tangential wind at a given radius at the 200 mb level is representative of the mean tangential wind in the outflow layer. All of these assumptions are justified by observational evidence. Using the mass transports computed in the inflow layer and tangential winds from the curves in Fig. 5, the relative angular momentum transports were evaluated in the outflow layer at radii of 3^{o}, 6^{o}, and 10^{o} latitude. These transports, combined with those in the inflow layer, represent a first approximation to the transport of relative angular momentum by the mean radial circulation.

No attempt was made to evaluate eddy transports of relative angular momentum from the composite data. The method of compositing in this study was such that asymmetries in the storm are not well represented in the composites.

The computed frictional losses of angular momentum are presented in Table 3. The losses are tabulated to show their relative distribution as a function of radius.

Table 3. Frictional losses of angular momentum to the ocean surface for the three classes of storms investigated. The second column shows the frictional loss within each annular ring, while the third column shows the total loss inside the cylinders of 3^{o}, 6^{o}, and 10^{o} radius.

Class	Radial increment	Incremental momentum loss (10^{22}gm·cm^2·sec^{-2})	Cumulative total from storm center (10^{22}gm·cm^2·sec^{-2})
Dying storms	0-1.5	4.2	
(developing	1.5-3	6.5	10.7
and mature	3-6	25.9	36.6
stage)	6-10	51.3	87.9
Dying storms	0-1.5	2.4	
(dissipation	1.5-3	4.4	6.8
stage)	3-6	12.1	18.9
	6-10	11.2	30.1
Gray's	0-1.5	2.1	
developing	1.5-3	4.0	6.1
disturbance	3-6	16.4	22.5
	6-10	49.5	72.0

It is apparent that frictional losses near the storm center are only a small part of the total. For purposes of the total angular momentum budget calculations, the precise determination of maximum winds near the center is not critical. Although the winds are light in the outer bands of the storm, the angular momentum losses there are relatively high because of the greater area and torque arm.

The imports of relative angular momentum at the three radii are shown in Table 4 for each class of storm considered. Positive values denote inward transport of momentum. Note that in the outflow layer a positive contribution to the relative angular momentum budget occurs everywhere except near the center because of the negative values of the tangential wind in that layer.

Table 5 is a summary of the mean radial transports and frictional losses for the three classes. The column labeled "Res" is given by the import by the mean radial circulation minus the frictional loss to the ocean surface. Negative values indicate that more momentum is lost to the ocean surface than is imported by the mean radial circulation. This term is a measure of the angular momentum which must be imported by the eddy transport mechanism if the circulation is to remain in balance.

The mass flow rates in Table 4 are observed to vary somewhat with radius in the three classes. In particular the zero mass flux at 10° latitude radius is probably

Table 4. Inward transports of relative angular momentum by the mean radial circulation (including inflow and outflow layers) for the three classes of storms.

Class	Radius	Mass flow (10^{11}gm·sec^{-1})	Inward angular Momentum transport (10^{22}gm·cm^{2}·sec^{-2})		
			Inflow layer	Outflow layer	Total
Dying storms (developing and mature stage)	3^o	43	9.4	0.6	10.0
	6^o	67	22.6	12.6	35.2
	10^o	68	22.6	34.0	56.6
Dying storms (dissipation stage)	3^o	29	4.4	0.6	5.0
	6^o	45	10.0	0.6	10.6
	10^o	0	0	0	0
Gray's developing disturbance	3^o	31	5.0	1.2	6.2
	6^o	60	17.6	13.2	30.8
	10^o	68	22.6	34.0	56.6

unrepresentative of actual conditions. This is an inevitable result of the method used to compute mass fluxes in this study. It should be noted, however, that even with a non-zero mass flux, the import of angular momentum by the inflow layer will vanish at 10^{o} latitude radius in the dissipation stage because of a zero tangential wind.

Table 5. Summary of angular momentum budget estimates for the three classes of storms. $\overline{v}_{\theta}\overline{v}_{r}$ represents the transport of angular momentum inward by the mean radial circulation and "L_{o}" denotes the loss of angular momentum to the ocean surface inside the given radius. The column labeled "Res" is given by the import by the mean radial circulation minus the frictional loss to the ocean surface. Units are 10^{22}gm·cm^{2}·sec^{-2}.

Case	3^{o}			6^{o}			10^{o}		
	$\overline{v}_{\theta}\overline{v}_{r}$	L_{o}	Res	$\overline{v}_{\theta}\overline{v}_{r}$	L_{o}	Res	$\overline{v}_{\theta}\overline{v}_{r}$	L_{o}	Res
Dying storms (developing and mature stage)	10	11	-1	35	37	-2	57	88	-31
Dying storms (dissipation stage)	5	7	-2	11	19	-8	0	30	-30
Gray's developing disturbance	6	6	0	31	23	8	57	72	-15

V. INTERPRETATION OF RESULTS

Maintenance of the Tropical Storm

Figure 6a compares the ratios of imports by the mean radial circulation to frictional losses inside the cylinder for the three storm classes at three different radii. It is apparent that this ratio is markedly decreased in the dying storms during their dissipation stage when compared to the same ratio during earlier stages of the storm. The somewhat larger values of this ratio for Gray's developing disturbances suggest that a surplus of angular momentum (at least inside 6° radius) was available for disturbance intensification. The lack of angular momentum import during the dissipation stage indicates that a deficiency in angular momentum was probably a contributing factor in storm dissipation.

Figure 6a shows a decrease with increasing radius in the ratio of radial imports to frictional losses for all three classes. Even in the developing disturbances the import by the mean radial circulation is less than the frictional loss inside 10° latitude radius. This suggests, in agreement with findings by Palmén and Riehl (1957), that the eddy transports must increase in importance with distance from the storm center in order to maintain balance against frictional losses. The author has found in preliminary calculations with tangential wind profiles from

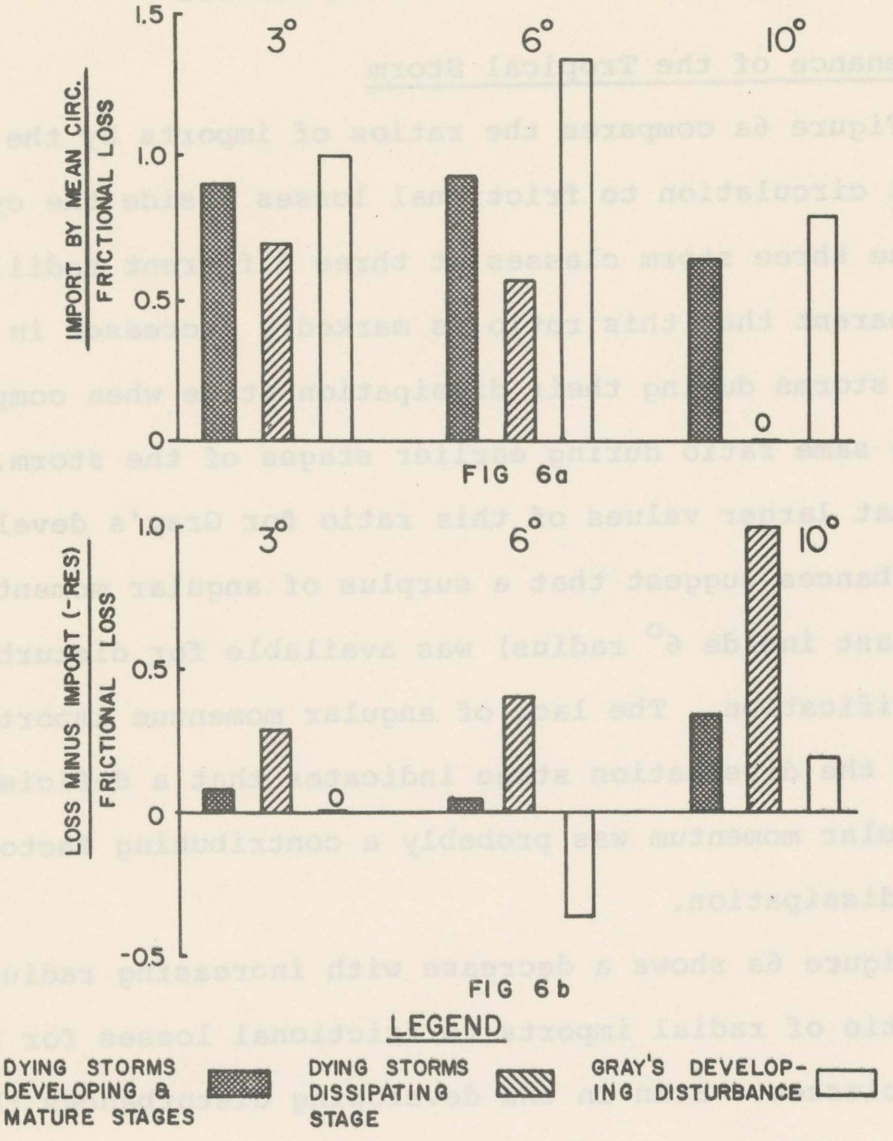

Fig. 6. Top diagram (Fig. 6a) is a comparison of the ratio of import by the mean radial circulation to surface frictional loss inside the given radius for three classes of storms. The lower diagram (Fig. 6b) compares the ratio of loss minus import to surface frictional loss inside a given radius for the same three classes. "Loss minus import" is the negative of the residual (Res) in Table 5. Values are given in both diagrams at 3^{o}, 6^{o}, and 10^{o} lat. radius.

Izawa (1964) that at 10^{o} latitude from the storm center the eddies must assume a major role in angular momentum transports in order to balance frictional losses in a steady state hurricane.

Figure 6b shows the ratios of "loss minus import" by the mean radial circulation" to "frictional loss" computed from the data in Table 5 for each intensity class. The large values of this ratio for the dying storms during the dissipation stage illustrate further how the angular momentum deficiency contributed to storm dissipation. The negative value of this ratio at 6^{o} radius in Gray's developing disturbance, on the other hand, demonstrates a surplus of angular momentum.

A decrease in the mean transport during dissipation does not necessarily imply a simultaneous reduction in the eddy transport. A quantitative assessment of the variation of eddy momentum transports in such dying storms awaits the availability of more adequate data. However, a qualitative assessment may be obtained from the zonal wind field changes shown between Figs. 2 and 3. The most marked reduction in the easterly trade flow on the 6^{o} and 10^{o} latitude circles is seen to occur to the north of the storm center. Since the predominant direction of movement of these storms was WNW, this represents a reduction in the tangential wind in the right front quadrant of the moving storms. Krueger (1959) has shown that surface inflow at 5° latitude radius is strongest in the right front quadrant of hurricanes. When the greatest reduction in tangential wind occurs in this quadrant, the result is a reduction in the inward eddy momentum transport. Qualitatively, then, the observed changes in the zonal wind would tend to decrease the eddy momentum import as well as the import by the mean radial circulation.

Importance of Trade Flow and the Subtropical Anticyclones

We have shown a relation between strong trade flow and the development and maintenance of tropical storms which is apparently due to the angular momentum balance requirement. Palmén (1956) has shown that at low latitudes storm development is limited by the maximum tangential wind which can be attained by inflowing air under conservation of angular momentum but modified by frictional losses. At low latitudes, the low values of the Coriolis parameter must be compensated by higher imports of relative angular momentum at the storm periphery (radii greater than 5-6° latitude). The stronger trade flow contributes to a high value of the mean tangential wind and thus favors high imports of relative angular momentum.

The subtropical anticyclones must now assume a dual role in maintaining the angular momentum budgets of low latitude tropical cyclones. The first function involves the maintenance of the trades north of the storms. The strength of the trades is dependent on the strength of the subtropical high to the north. The trade flow may weaken or disappear as a result of a decrease in the intensity of the anticyclone. This phenomenon was readily apparent in three of the eleven dying cases studied here. Typhoons Iris, 1962, Cora, 1964, and Elsie, 1964, dissipated after they moved westward from a region of easterly trade flow between the storm and a rather intense anticyclone to a region of westerly flow north of the storm. Such a sequence of events was not so evident in the other eight cases.

The second function of the anticyclones involves the eddy import of relative angular momentum into the storm circulation. Where the flow is anticyclonic relative to the storm axis, friction acts to extract angular momentum from the ocean surface (Ref. Eq. 9). This occurs in the westerly flow north of the subtropical anticyclones. The flow around these anticyclones must then contribute to the transport of relative angular momentum from the westerlies into the trades. Such a mechanism has been described by Pfeffer (1958) in a study of actual storm cases.

Storm Activity in the Eastern North Pacific

Tropical storms in the eastern North Pacific form off the coast of Central America and almost always dissipate over water while moving on a west to northwest course. Figure 7 shows the genesis region with some typical storm tracks. In some cases storm dissipation is due to movement of the storm over cold ocean currents west of 130° W longitude or north of 20° latitude as shown by two of the storm tracks in Fig. 7 (Sadler, 1963). Gray (1967) has shown that large vertical shear and tropospheric ventilation west of 130° W longitude also contribute to storm dissipation. Many storms, however, dissipate between 110° and 120° W longitude as shown by yet another two storm tracks in Fig. 7. This is a region with more than adequate sea surface temperatures and climatologically low vertical wind shears. Since sea surface temperatures and vertical shear cannot always fully explain dissipation so far east, it is possible that the surrounding ciruclations in this region may often be unfavorable for supplying angular momentum to the tropical storms.

In order to investigate this possibility, monthly climatological 1000 mb winds (U.S. Navy, 1966) were used to compute tangential wind profiles relative to hypothetical positions on typical storm tracks. Tangential wind profiles relative to four center positions were computed from August data and are shown in Fig. 8. Because of the nature of the data used the profiles must converge toward a zero value at zero radius. At large distances from the center, however, the profiles represent a rough estimate of the mean tangential wind profile surrounding these center positions for actual storms.

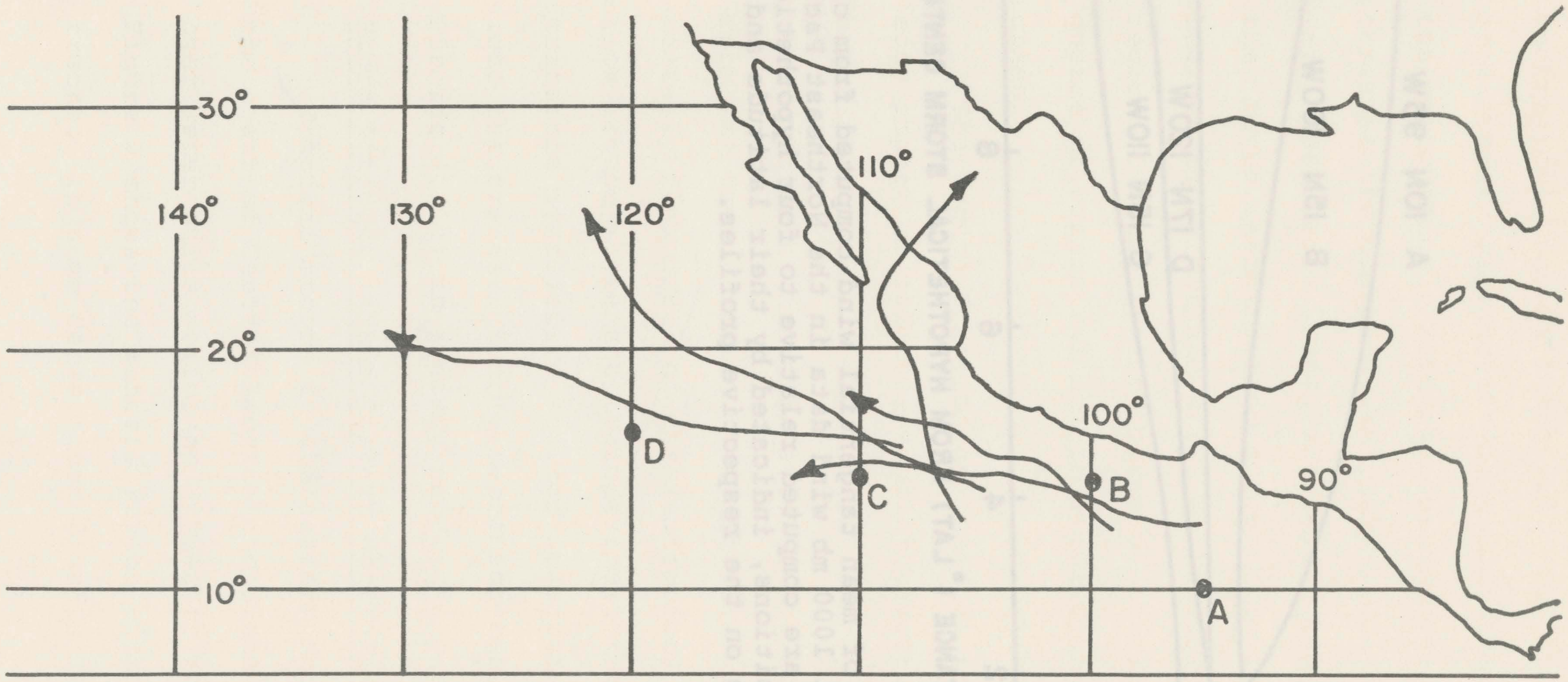

Fig. 7. A map of the Northeast Pacific depicting the region of storm development and typical storm tracks. The four positions (A,B,C, and D) represent the four hypothetical storm positions relative to which the four tangential wind profiles in Fig. 8 were constructed.

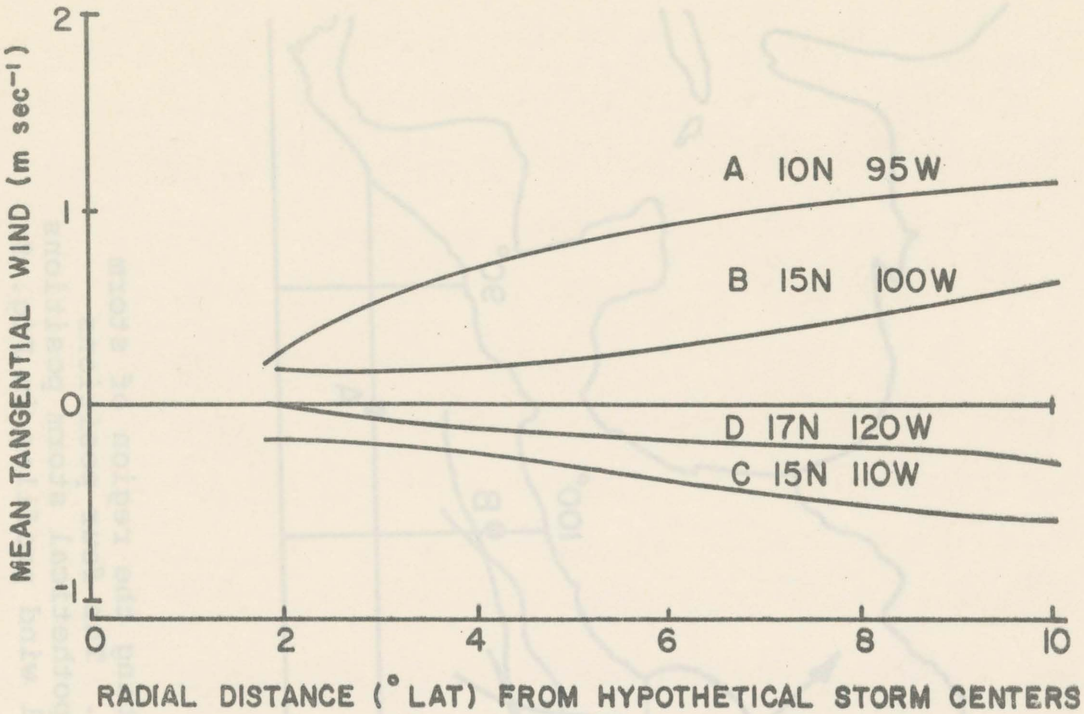

Fig. 8. Profiles of mean tangential wind computed from climatological 1000 mb wind data in the Northeast Pacific. Profiles are computed relative to four hypothetical storm positions, indicated by their latitude and longitude on the respective profiles.

Examination of the curves in Fig. 8 reveals a small but significant change in the tangential wind values between 5^{o} and 10^{o} radius. In the vicinity of 10N-95W where the storms typically form, the monthly mean tangential wind at 10^{o} radius relative to the center is about 1 $m \cdot sec^{-1}$. At positions 15N-110W and 17N-120W (where storms often decrease in intensity) the tangential wind at 10^{o} radius is about -0.5 $m \cdot sec^{-1}$. Profiles for July and September showed the same pattern. These monthly profiles further indicate that angular momentum transports may be an important factor in dissipation of storms over tropical waters.

VI. CONCLUSION

We have shown by physical and mathematical arguments that the inward transport of angular momentum is another of a number of requirements for development and maintenance of tropical storms. Observationally it has also been shown that a deficiency in this transport of angular momentum can be a primary reason for storm dissipation. From this result, it follows that boundary transports of angular momentum may serve as a fundamental constraint on storm intensity at least for low latitude storms. This possibility is certainly suggested by the small values of tangential wind at outer radii in the dying storms compared with the tangential winds at the same radii in mature storms. A fuller understanding of this constraint might increase present capability for forecasting intensity changes.

The results of this study have shown that for low latitude tropical cyclones, angular momentum balance depends not only on the storm circulation itself, but on surrounding circulations as well. The angular momentum import by the mean radial circulation was found to depend on the strength of the trades north of the storm, which is in turn related to the intensity of the subtropical anticyclones north of the storms. Simultaneously the anticyclones contribute to the eddy transport of angular momentum from regions where friction transfers angular momentum from the ocean to the atmosphere.

This investigation of angular momentum transports is far from complete. Variations in time of the eddy transports of angular momentum around the storm periphery have not been observationally investigated. Additional studies of the influence of variations in surrounding circulation patterns would appear to be a profitable future research endeavor. Studies of the angular momentum budget in relation to heat, moisture, and kinetic energy budgets must also be undertaken.

If the importance of angular momentum transports in controlling storm intensity can be verified observationally then a program for systematic aircraft data collection around the periphery of storms should be instituted. Data gained in such a program would be useful in more detailed observational studies of angular momentum transports at the storm periphery. Finally, such data may find its greatest practical value in forecasting storm intensity changes.

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APPENDIX I.

LIST OF SYMBOLS AND THEIR DEFINITIONS

Coordinates

r	radial coordinate in the cylindrical coordinate system. In a storm it represents the radial distance from the storm axis.
θ	angular coordinate in the cylindrical coordinate system
z	the vertical coordinate, taken to be zero at sea level

Geometry

S	surface area of the cylindrical volume
S_o	that portion of the surface area of the cylindrical volume consisting of the air-sea interface
V	volume
Z_T	height of the top of the storm circulation

Forces

F_θ	tangential component of the friction force
p	pressure
T	torque acting on a parcel, measured with respect to the vertical axis of the tropical storm
$\tau_{\theta z}$	tangential component of the surface shearing stress, defined to be positive when the ocean exerts a counterclockwise torque on the atmosphere relative to the storm axis

Motion

f	Coriolis parameter at the latitude of the storm axis
M	absolute angular momentum

Motion

M_z	absolute angular momentum relative to the vertical storm axis
u	eastward component of the horizontal wind
v	northward component of the horizontal wind
v_n	component of the wind normal to the surface of the volume of integration
v_θ	tangential component of the wind, defined to be positive when directed counterclockwise with respect to the instantaneous fixed position of the storm center
v_r	radial component of the wind, defined to be positive in the direction of increasing radius, measured with respect to the moving storm axis
$\mathbb{V}_3$	the three dimensional wind vector

Miscellaneous

∇_3	three dimensional gradient operator
L_o	total frictional loss of angular momentum to the ocean inside a given cylindrical volume
ρ	density
Σ	summation over finite intervals
t	time