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STEADY UPWARD FLOW FROM WATER TABLES

by

A. Anat, H. R. Duke and A. T. Corey

June 1965



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This research was conducted by the Agricultural Engineering Department, Colorado State University. Funding for most of the research was from Hatch Act Funds, Western Regional Projects W-32 and W-73. The final experiments were conducted by the senior author who was supported by AID funds as a participant from the SEATO Graduate School of Engineering, Bangkok, Thailand.

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## ABSTRACT

Steady upward flow from water tables was investigated experimentally and theoretically under conditions in which the flow rate is controlled by the capacity of the soil to transmit liquid to a dry surface layer. Under these conditions the flow rate was found to depend upon whether the soil-liquid system was on a drainage (falling water table) or an imbibition (rising water table) cycle. For a given depth of water table, the flow rate on the drainage cycle can be at least 100 times greater than on the imbibition cycle.

It was also found that whenever the rate of removal of liquid from the surface layer exceeds the rate at which it can be supplied from the soil below, the flow rate is reduced substantially compared to the flow rate which would otherwise exist on a drainage cycle.

A differential equation was developed which relates the upward flow rate to the depth of the water table in terms of measureable soil parameters. A Fortran program for a IBM computer was developed to solve the differential equation for a wide range of soil parameters. An algebraic expression was also developed which approximates the computer solution very closely.

The use of either the computer program or the algebraic approximation depends upon an accurate measurement of the soil parameters on the appropriate cycle, i. e., drainage or imbibition. The paper explains how this can be done.

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## NOTATIONS AND DEFINITIONS

<u>Symbol</u>	<u>Term</u>	<u>Definition</u>	<u>Dimensions</u>
	<u>Capillary barrier</u>	- a sheet, strip, cup, or plate of porous material having finer pores than the soil medium in which it is in contact. Such a material will remain fully saturated with liquid and will prevent the entry of air into manometer leads, etc. after the soil has been at least partially desaturated. It will, however, permit the conduction of liquid through it.	
C	<u>Conductivity</u>	- the coefficient in Darcy's equation when the medium is fully saturated. This coefficient is a function of both media and fluid properties. It is sometimes called " <u>hydraulic conductivity</u> ."	Lt <sup>-1</sup>
C <sub>e</sub>	<u>Effective conductivity</u>	- the coefficient in Darcy's equation when the medium is not necessarily fully saturated. It is sometimes called " <u>capillary conductivity</u> ."	Lt <sup>-1</sup>
C <sub>e</sub> /C	<u>Relative conductivity</u>	- a scaled conductivity.	none
d	<u>Distance</u>	from a water table to the bottom of a dried surface layer of soil.	L
d.	<u>Scaled distance</u>	from a water table to the bottom of a dried surface layer of soil. It is the ratio $\frac{d}{p_b/\rho g}$ . It could be called a dimensionless depth.	none
	<u>Drainage cycle</u>	- a process in which the liquid pressure continuously decreases and the liquid saturation of the medium also decreases.	
	<u>Evaporativity</u>	- the capacity of the atmospheric environment to evaporate water from an exposed free-water surface.	Lt <sup>-1</sup>
	<u>Hysteresis</u>	- As used here, the term refers to an unsteady flow process in which a decreasing liquid pressure changes to an increasing liquid pressure at some point in a porous medium. The process is often accompanied by little or no change in medium saturation.	
η		A property of the medium which can be related to the <u>pore-size distribution</u> and is defined in terms of the functional relation between C <sub>e</sub> and p <sub>c</sub> ; i.e., $\eta = - \frac{\Delta(\ln C_e)}{\Delta(\ln p_c)} \text{ when } p_c > p_b .$	
		Large values of η indicate a relatively uniform pore size and smaller values indicate a greater range of pore sizes.	
p	<u>"Apparent" pressure</u>	of liquid in soils. When referred to atmospheric pressure as a datum, it is the quantity measured by a tensiometer. It may sometimes differ from the hydrodynamic concept of pressure because of the adsorptive force fields acting near solid surfaces.	FL <sup>-2</sup>
p <sub>c</sub>	<u>Capillary pressure</u>	- the pressure of the gas phase in soils minus the "apparent" pressure of the liquid. It is often called " <u>suction</u> " or " <u>matric suction</u> ." When the gas pressure is atmospheric, p <sub>c</sub> = - p. More generally, p <sub>c</sub> = p <sub>a</sub> - p where p <sub>a</sub> is the pressure of the air.	FL <sup>-2</sup>
p <sub>b</sub>	<u>Bubbling pressure</u>	- approximately the p <sub>c</sub> at which a soil begins to desaturate rapidly or at which the gas phase becomes continuous. A more rigorous definition is implicit in the method of its determination as described by Brooks and Corey [4].	FL <sup>-2</sup>
p.	<u>Scaled capillary pressure</u>	- the dimensionless ratio $\frac{p_c}{p_b}$ .	none
q	<u>Volume flux</u>	- the volume of flow per unit time per unit bulk area of medium.	Lt <sup>-1</sup>

NOTATIONS AND DEFINITIONS--Continued

<u>Symbol</u>	<u>Term</u>	<u>Definition</u>	<u>Dimensions</u>
q.	<u>Scaled volume flux</u>	- the dimensionless ratio $\frac{q}{C}$ .	none
$q_m$	<u>Scaled volume flux</u>	from a water table when $p_c \rightarrow \infty$ at a locus of points above the water table.	none
$\rho g$	<u>Specific weight</u>	(density x acceleration of gravity). In this paper $\rho g$ refers to the liquid phase only.	FL <sup>-3</sup>
S	<u>Saturation</u>	- the fraction of the total pore volume occupied by liquid in a specific bulk element of soil medium.	none
z	<u>Elevation</u>	above a water table.	L
Z.	<u>Scaled elevation</u>	above water table, the dimensionless ratio $\frac{z}{p_b/\rho g}$ .	none
	<u>Water table</u>	- The locus of points in a soil at which the apparent pressure of the liquid is atmospheric. In this paper the liquid referred to may be either water or a hydrocarbon liquid.	

# STEADY UPWARD FLOW FROM WATER TABLES<sup>1/</sup>

by

A. Anat, H. R. Duke, and A. T. Corey<sup>2/</sup>

## INTRODUCTION

Upward flow from water tables subsequent to evaporation and transpiration from soils is a significant phenomenon, particularly in irrigated areas. In some areas, farmers find it advantageous to cause the water table to rise to within a few feet of the root zone of crops as a method of supplying the root zone with water. In areas where soluble salts may be substantial, however, such practices have often led to an unfavorable accumulation of salts within the root zone.

The hydrologist, also, is concerned with upward flow from water tables since such flow can have a substantial effect on the fluctuation of ground water storage.

Many investigators have studied rates of evaporation from soils in which there is a water table. There is general agreement that the rate of evaporation may be controlled by either the capacity of the atmospheric environment to evaporate water or the capacity of the soil to transmit water to the surface. The maximum rate of upward flow will be the lesser of these two capacities. Another conclusion that can be made from the available literature is that, except for very shallow water-table depths, the capacity of the soil to transmit water is the limiting factor. This paper is concerned with the latter situation.

Since hysteresis in the pressure history of soil-liquid systems plays an important role with respect to upward flow rates [12, 15, 16], this study investigates three cases as follows:

Case 1 - The soil liquid follows a drainage cycle only; i.e., the soil is initially completely saturated and afterwards the pressure of the liquid progres-

sively decreases at all points in the system until steady state is reached. This case occurs in nature only when the maximum rate of evaporation from the surface never exceeds the rate at which water is supplied from the water table to the surface and when the water table is either at a constant elevation or is progressively lowered.

Case 2 - The soil liquid follows an imbibition cycle; i.e., the soil is initially dry and liquid is imbibed from below only.

Case 3 - The soil liquid follows a cycle which is neither completely drainage nor completely imbibition; i.e., there are parts of the soil profile in which the liquid pressure may at times increase and other parts in which the pressure may decrease continuously. Such cases are probably common in nature, especially when the rate of evaporation from the surface temporarily exceeds the upward flow rate.

This study, therefore, attempts to answer the following questions:

1. What is the maximum rate of upward flow (as a function of the depth at which liquid exists at atmospheric pressure) and what are the soil parameters that determine this maximum rate?
2. What is the maximum rate of upward flow when the liquid system follows an imbibition cycle completely?
3. How is the maximum rate of upward flow affected by a situation in which the removal of liquid from the surface exceeds the rate at which it can be transmitted to the surface?

<sup>1/</sup>Contribution from the Agricultural Engineering Department, Colorado State University.

<sup>2/</sup>AID Participant (SEATO Graduate School of Engineering), Junior Civil Engineer and Professor of Agricultural Engineering, Colorado State University, Fort Collins, Colorado, respectively.

## PREVIOUS INVESTIGATIONS

Moore [14] was one of the first to investigate upward flow from water tables as affected by measureable hydraulic properties of soils. He introduced water tables at the bottom of soil columns and allowed the soil to imbibe water. The surface was subjected to evaporation and, by employing tensiometer cups placed at intervals along the length of the column, he observed the relationship between water pressure, moisture content, and rate of water loss from the water table.

From these studies, with materials of widely varying texture, Moore concluded that at complete saturation, permeabilities of soils were arranged in the following order:

sand > fine sandy loam > light clay > clay.

When the capillary potential (defined as the work required to pull a unit mass of water away from a unit mass of soil) was decreased to -100 ergs per gram, he found the order to be:

sand < fine sandy loam < light clay < clay.

This conclusion stemmed from observations that, at lower potentials (i.e., greater depths to the water table), the finer soils supported higher evaporation rates.

Gardner [9] analyzed upward flow rates with the water table at any particular depth and any capillary pressure head\* at the surface. For this analysis Gardner started with the "diffusivity equation," an equation describing unsteady flow in partially saturated porous media. He solved this equation for the special case of steady one-dimensional flow in a vertical (upward) direction.

The equation obtained by Gardner is

$$z = \int_0^{p_c/\rho g} \frac{d(p_c/\rho g)}{1 + q/C_e} \quad (1)$$

where  $z$  is the elevation (above the water table) at which the capillary pressure  $p_c$  has any particular value;  $q$  is the volume flux (dimensions of velocity); and  $C_e$  is the effective conductivity, a function of  $p_c$  having the dimensions of velocity. The capillary pressure  $p_c$  is defined as the difference in pressure between the air and the "apparent" pressure of the liquid at a point on the interface between these fluid phases. The quantity  $\rho g$  is the specific weight of the liquid. The water table is defined as the locus of points at which the liquid pressure is equal to the air pressure; i.e.,  $p_c = 0$ . In order to obtain equation (1), Gardner used the boundary conditions that  $p_c = 0$  at the water table and some other particular value at the elevation  $z$ .

The integral in equation (1) can be evaluated only when the functional relationship  $C_e(p_c)$  is known from laboratory measurements. After analysing the data of various investigators, Gardner stated that the relation between  $C_e$  and  $p_c$  is expressed very well by

$$\frac{a}{C_e} = \left[ \left( \frac{p_c}{\rho g} \right)^\eta + b \right] \quad (2)$$

where  $a$ ,  $b$ , and  $\eta$  are constants for particular soils. The constants  $a$  and  $b$ , however, are not dimensionless and depend on the units used to express  $C_e$  and  $p_c$ . Gardner stated that, in general,  $\eta$  was larger for coarser textured soils, but that for most soils investigated (at that time) values of  $\eta$  equal to 2 or 3 gave the best fit.

Substituting equation (2) into equation (1), Gardner obtained limiting values of  $q$  as  $p_c$  approached infinity for cases in which  $\eta$  had the values 3/2, 2, 3, and 4. Gardner pointed out that the integral of equation (1) did not have a limit for  $\eta \leq 1$  but stated that no soil had been found with so low an  $\eta$  value.

Gardner and Fireman [10] conducted laboratory studies of evaporation from soil columns packed in lucite cylinders 12 cm in diameter and 60 to 100 cm in length. These columns were initially saturated and allowed to drain, and were saturated again two or three times in order to obtain a structure as near as possible to soils in the field. The water was supplied from a reservoir through porous cups in the lower end of the column. A range of water-table depths was simulated by varying the vertical position of the inflow reservoir.

The investigators also obtained a range of atmospheric evaporative conditions by varying the rate of air circulation above the columns and the incident radiation to the top of the columns. When evaporativity (as measured by evaporation from a free-water surface) was increased to fairly high rates, the evaporation rates from the soil columns appeared to approach a maximum as predicted by theory. The maximum rates obtained for this particular experiment were not compared with theoretical values since  $C_e$  as a function of  $p_c$  was not available for the particular soil used.

For two soils, Chino clay and Pachappa sandy loam, the evaporativity was maintained at a relatively high rate and the water table was lowered in increments. In this case, data were available to check the evaporation rates with values computed using equation (1). The agreement between the measured and theoretical values was satisfactory.

\*The authors have changed Gardner's original nomenclature and symbols to correspond to those used in this paper.

PREVIOUS INVESTIGATIONS AT COLORADO STATE UNIVERSITY

In 1957 Staley [18] submitted a Masters Thesis which attempted to determine (1) how the evaporation rate from a fine sand is affected by independently varying the wind velocity and the depth of the water table, and (2) how the functional relationship between evaporation rates under specified ambient conditions and the depth of the water table can be related to easily measurable properties of the sand.

1. Wind-Tunnel Experiments

Two columns of a fine sand were placed side by side in a test section of a wind tunnel. Each column was 46 inches in length and 12 inches in diameter. The columns were placed so that their surfaces were flush with the floor of the test section. One column, in which the water table was maintained at the surface, was used to evaluate the evaporativity while the water-table depth in the second column was varied.

Both columns were initially fully saturated before the water table was lowered in one of the columns. The columns were supplied by water from a constant head source such that the rate of intake could be measured. Runs were made with the wind velocity varying from 0 to 50 feet per second while the water table was varied independently between the surface and a depth of 30 inches. The runs were conducted so that the column with the variable water table was always on the drainage cycle. Each run was continued until steady state flow was approached.

A critical depth of water table for the fine sand was found at about 24 inches. With the water table above this depth, evaporation rates were of the same order of magnitude as from the free-water surface. When the water table was below 24 inches, the flow rate decreased rapidly with depth and at 30 inches the rate was too small to be measured accurately with the apparatus employed.

The critical water-table depth was related to properties of the capillary pressure - desaturation curve. It was found that the critical depth corresponded to the value of  $p_c/\rho g$  at which the initially saturated sand began to desaturate rapidly with further increases in  $p_c$ . This value of  $p_c$  has been called bubbling pressure  $p_b$  by Brooks and Corey [4].

A. Theoretical analysis - In analysing the problem of upward flow from water tables, Staley expressed the flow rate, the capillary pressure and the elevation above the water table in terms of scaled variables as follows:

- (1) flow rate--the ratio of the flow rate to the hydraulic conductivity  $q/C$ ,
- (2) capillary pressure--the ratio of capillary pressure to the bubbling pressure  $p_c/p_b$ ,

- (3) elevation--the ratio of elevation above the water table to the bubbling pressure head  $\frac{z}{p_b/\rho g}$ .

Staley also derived an equation for the steady upward flow rate  $q$  (identical to equation 1) but used an approach somewhat different from that employed by Gardner. Staley wrote Darcy's law for the case of one-dimensional flow in the vertical direction and re-arranged this equation to give an expression explicitly describing the rate of change of  $p_c$  with  $z$  as a function of  $q$  and  $C_e$ , i.e.,

$$\frac{d\left(\frac{p_c}{\rho g}\right)}{dz} = 1 + \frac{q}{C_e} \tag{3}$$

When this equation is solved for  $z$ , the result is equation (1).

Staley expressed the functional relationship  $C_e(p_c)$  in two parts rather than as the continuous function proposed by Gardner. Staley assumed that  $C_e$  is a constant for  $p_c < p_b$ , and for  $p_c > p_b$ ,  $C_e$  is given by

$$C_e = C \left(\frac{p_b}{p_c}\right)^\eta \tag{4}$$

where  $C$  is the value of  $C_e$  when media are fully saturated.

By a proper choice of the constants  $a$  and  $b$  in equation (2), the relationship proposed by Gardner and that proposed by Staley may result in practically identical functions for  $C_e$  when the value of  $\eta$  is the same.

Staley computed the value of  $\eta$  for his sand using a method proposed by Burdine [5] and arrived at a value of 16. This was surprising in view of the observation of Gardner that  $\eta$  would not be larger than about 4. After reviewing the literature of the petroleum industry, Staley concluded that a value of  $\eta$  equal to 8 should be about average for sands and that the larger value of  $\eta$  for his sand was due to its extremely uniform pore size. Later measurements by Schleusener [15], Scott and Corey [17] and Brooks and Corey [4] (using steady-state methods of measurement) indicate that an  $\eta$  of 16 is not uncommon but is, in fact, close to an average for a tightly packed unconsolidated sand.

Staley solved equation (1) by a numerical method for  $\eta = 8$  and a particular water-table depth to obtain a relationship between  $\frac{p_c}{p_b}$  (at the surface) and  $\frac{q}{C}$ . The water-table depth assumed was such as to give a value of  $\frac{z}{p_b/\rho g}$  at the surface of 2.5. His solution verified the conclusion

of Gardner that  $q$  should approach a limiting value as  $p_c$  (at the surface) becomes large.

B. Experimental results - Because the experiments of Staley were run in a wind tunnel which was not equipped to supply radiant energy at the column surfaces, it was impossible for him to produce evaporativities sufficiently high to cause the evaporation rates to reach limiting values for most of the water-table depths studied. When the water-table was lowered sufficiently so that a limiting value might have been reached, the flow rates were too small to be measured accurately with the equipment employed.

In one case, when the water table was at 26 inches,  $q$  actually decreased when the evaporativity was increased to the maximum extent possible. Staley assumed this to be experimental error. He, therefore, drew his curve of  $q$  as a function of the evaporation rate from the free-water surface to indicate a limiting rate with increasing evaporativity rather than showing the decrease that was measured.

## 2. Experiments in Chamber with Controlled Environment

In later experiments (conducted in a chamber with controlled temperature, humidity, and incident radiation such that evaporativity could be raised to much higher levels), Schleusener [15] found that when evaporativity was increased beyond a critical level, the flow rates decreased. This result was too consistent to be explained as experimental error. Schleusener and Corey [16] attempted to discover the reason for this apparent failure of the theory.

One possibility was that it was caused by a severe temperature gradient as the surface was heated by incident radiation. They tried, therefore, to produce a decrease in upward flow from wet soil to drier soil by artificially producing an enormous temperature gradient. They could not produce a decrease of sufficient magnitude by this method to explain the results observed. Furthermore, they were able to produce similar decreases in flow rates by increasing the wind velocity in a tunnel in which there was no incident radiation and the temperature gradient was in the opposite direction.

A. Hysteresis theory - Schleusener and Corey finally concluded that the reduction in flow rates at

high evaporativity was due to a reversal in the direction of pressure change at a point below the evaporating surface. They reasoned that a high evaporativity initially moves water out of the surface layer faster than it can be supplied from below. This reduces the conductivity of the surface layer to such an extent that finally water can move into the layer below faster than it can be conducted through the dried layer. This results in a reduction of  $p_c$  at some point below the surface so that the hydraulic gradient up to that point is reduced, thus decreasing the flow rate.

Schleusener and Corey also presented a capillary model to describe in detail how this hysteresis probably takes place. Their concept, however, is apparently difficult to visualize and has been severely criticized by many readers of their paper.

In later experiments King and Schleusener [12] showed that cyclic conditions of radiation, temperature, and humidity (simulating the daily fluctuations of these variables in the field) resulted in the same trends as were observed with steady ambient conditions. In particular, the reduction of flow rates at high levels of evaporativity (depending on the water-table depth) were still observed.

B. Other studies - Staley, Schleusener, and King and Schleusener measured the temperature gradients existing in soil profiles during evaporation. They observed, however, that large temperature gradients were confined to the upper surface layer which usually becomes dry. In this region, movement of water is mostly in the vapor phase. They concluded that the temperature gradient could have little effect on the liquid flow rates up to the dried layer and that the rate of liquid flow up to the dried layer usually controlled the evaporation rate. They did not, therefore, relate the evaporation rate to the temperature gradient.

Schleusener [15], Corey and Filmer [6], and Corey and King [7] also studied other aspects of the evaporation process such as the effect of surface treatments, e. g., gravel mulches and also unsteady evaporation in the absence of water tables. These studies are not described here because they are not directly related to the subject of this paper.

Recent investigations at Colorado State University were specifically designed to answer the questions posed in the introduction.

1. Flow Controlled by Siphons

The first of these investigations has been described by Duke [8] in a Masters Thesis submitted in 1965. Duke attempted (by entirely different methods from those employed by previous investigators) to determine how the steady flow rates from water tables can be predicted for any soil with any water-table depth. He assembled and presented data with the purpose of making them usable for engineering estimations applicable to any field situations.

The use of these data, however, depends on the accurate determination of hydraulic conductivities, bubbling pressures, and the values of  $\eta$  for the soils under investigation. The significance and determination of these soil parameters have been discussed in detail by Brooks and Corey [4] and by Brooks [3]. The  $\eta$  referred to is the dimensionless exponent appearing in equation (4) which Brooks and Corey have related to the pore-size distribution of porous media.

A. Solution of flow equation - Duke analysed his data in terms of dimensionless variables introduced by Staley, i.e.,  $q/C$ ,  $p_c/p_b$ ,  $\frac{z}{p_b/\rho g}$ , and  $\eta$ . He designated these as  $q$ ,  $P$ ,  $Z$ , and  $\eta$  respectively, following the precedent established by Miller and Miller [13] and Brooks and Corey [4].

Duke adopted the empirical equation for the functional relationship  $C(P)$  introduced by Staley, i.e., equation (4). He, therefore, wrote equation (1) in the form

$$d = \frac{1}{1+q_m} + \int_1^{\infty} \frac{dP}{1+q_m P^\eta} \quad (5)$$

where  $d$  is the scaled elevation from the water table to the bottom of a layer of dried surface soil, which is the particular value of  $Z$  at the elevation of the bottom of the dried soil layer. The subscript  $m$  implies that  $q_m$  is the maximum value of  $q$ , occurring when  $P \rightarrow \infty$  at the scaled elevation  $d$ .

Duke devised a graphical solution for equation (5) involving the determination of the area under the curves shown in figure 1. He also devised a computer program which determined the integral in equation (5) very quickly for integral values of  $\eta$  from 2 through 20. This range of  $\eta$  includes all values observed for porous media to date. This program was written to continue the solution until the area under the next increment (i.e.,  $\Delta Z$  of

the next successive increment) was less than  $1 \times 10^{-6}$ . That this cutoff point reduced the "tail" (see figure 1) to a negligible amount was proven by comparing the computer solutions (at  $\eta$  values of 2 and 3) with closed solutions of equation (5), similar to those presented by Gardner [9]. These two solutions were consistently alike to 6 significant figures. Closed solutions for  $\eta > 4$  are not practical due to the complexity of the solutions.

A copy of the FORTRAN program (for an IBM 1401 computer) used for this solution is included in Appendix A. The solutions were compiled into a nomograph relating  $q_m$  to  $d$  for each value of  $\eta$ . The complete nomograph is presented in figure 2 and the computed values are given in table 2 in Appendix A.

Although most of Duke's theoretical data were obtained using the computer program, the graph shown in figure 1 serves to illustrate a significant principle. The first term on the right of equation (5) is a constant for a particular value of  $q_m$ .

Consequently, the plot maintains a constant value of  $1/(1+q_m)$  up to  $P = 1$ , then drops more or less rapidly (depending on the value of  $\eta$ ) as  $P$  increases toward infinity. The value of  $1/(1+q_m P^\eta)$  rapidly approaches zero as  $P$  is increased. This means that the increment of  $Z$  over which  $P$  changes from a relatively small value to a very high value is not large. It is possible, therefore, to establish the functional relationship between  $q_m$  and  $d$  (with only small error) by ignoring the mechanism of transport through the drier soil near the surface. This is particularly true for materials having a relatively high value of  $\eta$ .

This is not to say that the mechanism of transport through the drier surface soil has little effect on the evaporation rate. It means, in stead, that whatever factors affect the rate of transport through the surface layer also affect the thickness of the dry surface layer and, therefore, affect the distance from the water table to the dried layer. Knowing the distance from the water table to the dried layer and the necessary soil parameters (also whether the soil liquid has followed a drainage or imbibition cycle), it is possible to compute the flow rate. For definitions of drainage and imbibition cycles, see the section on notations and definitions.

B. Experimental methods - Duke devised a novel method of measuring flow rates for columns in which the capillary pressure was maintained at a zero value at a fixed elevation near the bottom of the column. Rather than depending on evaporation from the surface to produce upward flow, the liquid was removed from the surface through a capillary barrier attached to a siphon. Flow rates were

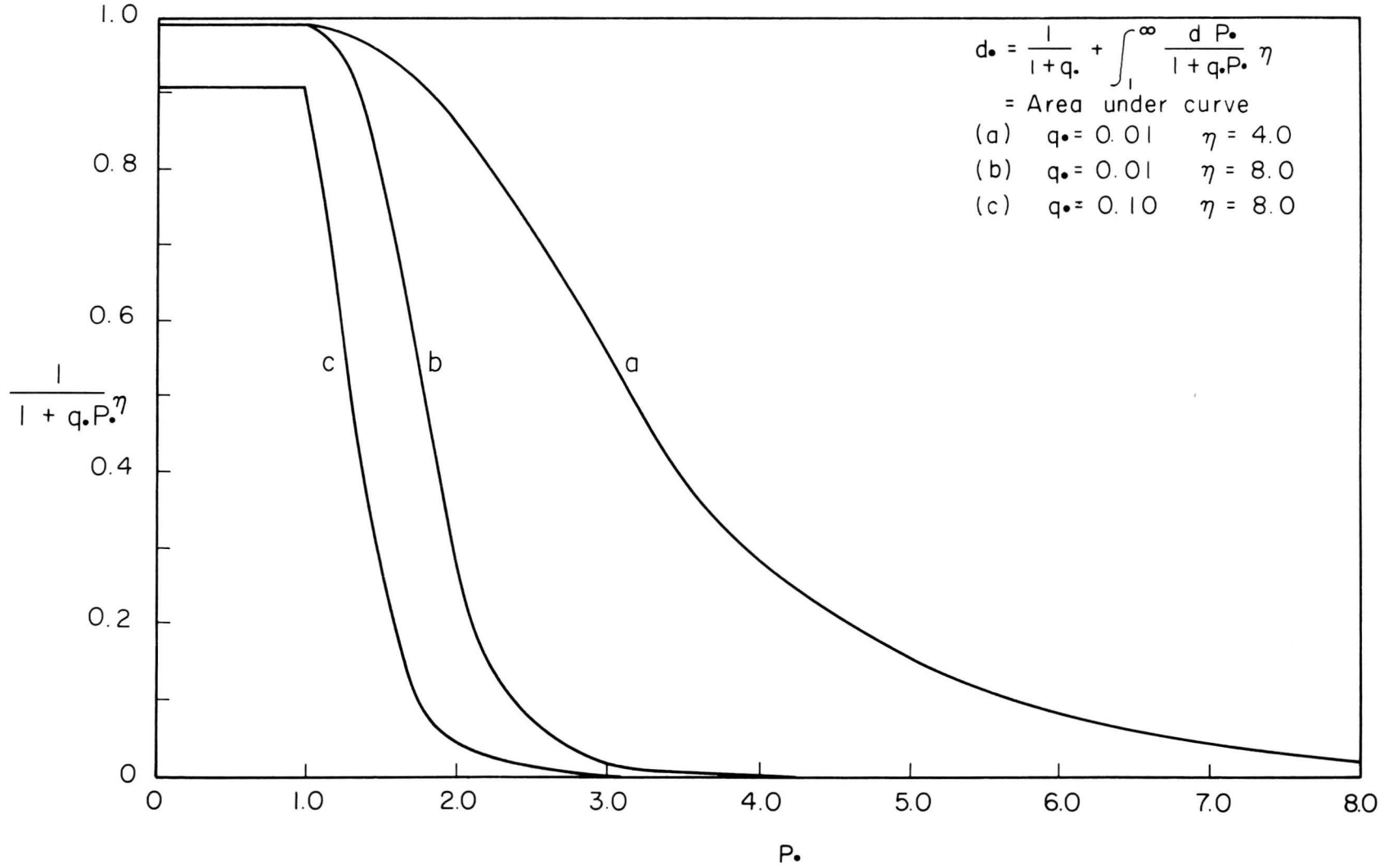


Figure 1. Plot for graphical solution of equation (5).

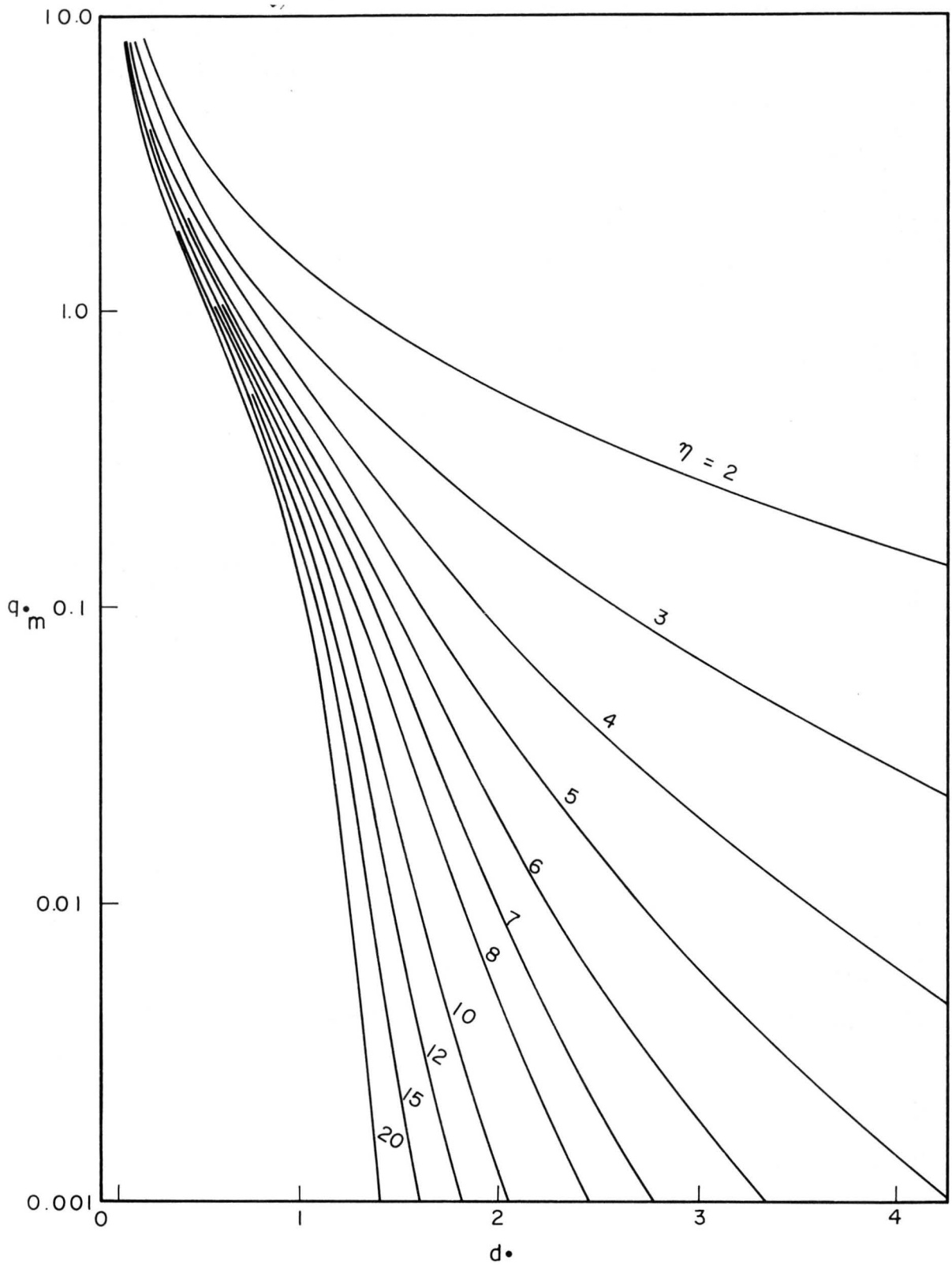


Figure 2. Nomograph of relation among  $d$ ,  $q_m$ , and  $\eta$ .

determined by measuring the outflow from the siphon after the outflow rates became steady with a particular capillary pressure on the top barrier. A temperature control system held the temperature within reasonable limits.

Duke obtained the maximum flow rates by lowering the outflow siphon in increments until the flow rates appeared to approach a limiting value, or as happened in most cases, actually decreased. The water-table depth was varied by cutting the column after the maximum rate had been established for one particular length of column. In all cases flow rates were measured on what was intended to be a drainage cycle.

A hydrocarbon liquid\* with a surface tension much lower than that of water was employed. The result was that a column having a particular scaled length was only half as long as would have been necessary if water had been used. The hydrocarbon has the additional advantage that the soil structure remains stable during the period required for measurements. The columns were repeatedly drained and resaturated without changing the conductivity or other properties.

A wide variety of soil types was used ranging in texture from heavy clay to sands. The heavy clay

soils were highly aggregated. With the method of packing used by Duke, those soils which contained large aggregates did not provide homogeneous columns. This fact made it difficult for Duke to compare his measured flow rates with theoretical values for aggregated soils, because for such soils he could not determine unique values for the parameters  $p_b$  and  $\eta$ .

The details of the equipment and procedures employed by Duke are presented in Appendix A.

C. Comparison of measured flow rates with theoretical values - Duke was able to compare measured values of flow rates with values computed from equation (5) for soils with relatively uniform textures. In such cases the measured values were less than the computed values by an amount ranging from 20 to 50 percent. This discrepancy was attributed to a pressure reversal observed at a tensiometer (near the top of the column) when the outflow siphon was lowered below a critical elevation. Duke considered his results to be further evidence that the hysteresis theory originally proposed by Schleusener and Corey [16] actually applies to soils during upward flow even when the soils undergo progressive desaturation.

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\*Soltrol "C" core test fluid. Phillips Petroleum Company, Special Products Division, Bartlesville, Oklahoma.

## FURTHER INVESTIGATIONS OF THE EFFECT OF HYSTERESIS

In June 1965, A. Anat completed an investigation [1] to determine more precisely the effect of hysteresis on upward flow from water tables. In his experiments, the equipment employed was the same as that used by Duke (see Appendix A). His procedures, in most cases, differed significantly from those followed by Duke.

### 1. Experimental Methods and Results

Anat packed his columns by first filling them with soil and then vibrating them with an electric powered vibrating device until they reached a relatively stable degree of compaction. The filling was accomplished by inserting a small tube (filled with soil and supplied by a funnel) and gradually withdrawing the tube as the column filled. This method avoided the necessity of dropping soil from the top of the column which might have resulted in segregation of particles according to size. It also avoided the segregation obtained by Duke from simultaneous filling and vibrating.

Using this packing technique, curves of relative conductivity as a function of capillary pressure were obtained which could be represented very well by equation (4). Both  $\eta$  and  $p_b$  were determined with precision as shown in figures 3 and 4.

A. Conductivity as a function of capillary pressure on the imbibition cycle - In some cases (see figure 4) curves of relative conductivity as a function of capillary pressure were also obtained on the imbibition cycle. This was accomplished by first draining the columns to a very low saturation (high  $p_c$ ) and then raising the inflow reservoir and outflow siphon in increments. The conductivity was determined after steady state was obtained for each increment.

From these measurements, it was found that the conductivity at particular values of capillary pressure on the imbibition cycle is one or two orders of magnitude less than on the drainage cycle. The exception is for capillary pressures less than the bubbling pressure, in which case, the conductivity on the imbibition cycle is about half that on the drainage cycle, and the values of  $p_b$  were about 0.6 those obtained on the drainage cycle.

It has been pointed out by Bloomsburg and Corey [2] that for capillary pressures less than the bubbling pressure on the imbibition cycle, the conductivity is a function of time since the entrapped air will eventually diffuse from the system and the medium will become fully saturated.

B. Measurement of upward flow rates - The procedure used by Anat for determining maximum upward flow rates differed from that employed by Duke in several ways. The columns used were usually only slightly longer than necessary for determining conductivity as a function of capillary

pressure. Instead of shortening the column after a series of runs, Anat simulated greater depths to a water table by lowering the inflow siphon. He was careful, however, not to desaturate the column below the lower tensiometer ring.

When the system reached a steady state, the lower tensiometer was read and the outflow rate was measured. Knowing the fully-saturated conductivity of the soil, it was possible to compute an equivalent depth to a zone of zero capillary pressure by using Darcy's law.

The first runs were made with the zone of zero capillary pressure at the elevation of the lower tensiometer. The capillary pressure at the surface was gradually increased in very small increments until the outflow appeared to approach a maximum, but the outflow siphon was not lowered more than this.

After this, the inflow siphon was lowered to simulate a greater depth to the water table, but the elevation of the outflow siphon was not changed. This procedure resulted in no pressure reversal at the upper tensiometer. Furthermore, the flow rates agreed very closely with those predicted from equation (5). This fact is shown in figure 5.

It will be noted that Anat plotted  $q_m$  as a function of  $d$  on log-log paper. His reason for doing this was that, theoretically, this curve should approach a slope of  $\eta$  at low values of  $q_m$ , thus indicating a relationship to the relative conductivity curves shown in figures 3 and 4. This result was experimentally confirmed as figure 5 shows.

C. Upward flow on the imbibition cycle - One series of runs was also made by starting with completely dry soil and allowing the soil to imbibe liquid from the lower inflow barrier. The initial imbibition was produced with the inflow siphon at a low elevation. The imbibition took place at an extremely slow rate and a considerable time passed before any flow from the outflow siphon was observed. Readings of the outflow rate were finally made, but it is extremely unlikely that sufficient time was allowed for the flow rate to reach a steady state (or a maximum rate for the particular elevation of the inflow siphon.)

When the inflow siphon was raised, the system reached steady state at increasingly shorter times, and it is probable that the data obtained represented maximum upward flow rates. At any rate, the data for the higher elevation of the inflow siphon agreed very closely with theoretical rates computed using conductivity data obtained on an imbibition cycle as figure 6 shows.

Another series of runs was made in which the imbibition cycle was started after draining the soil

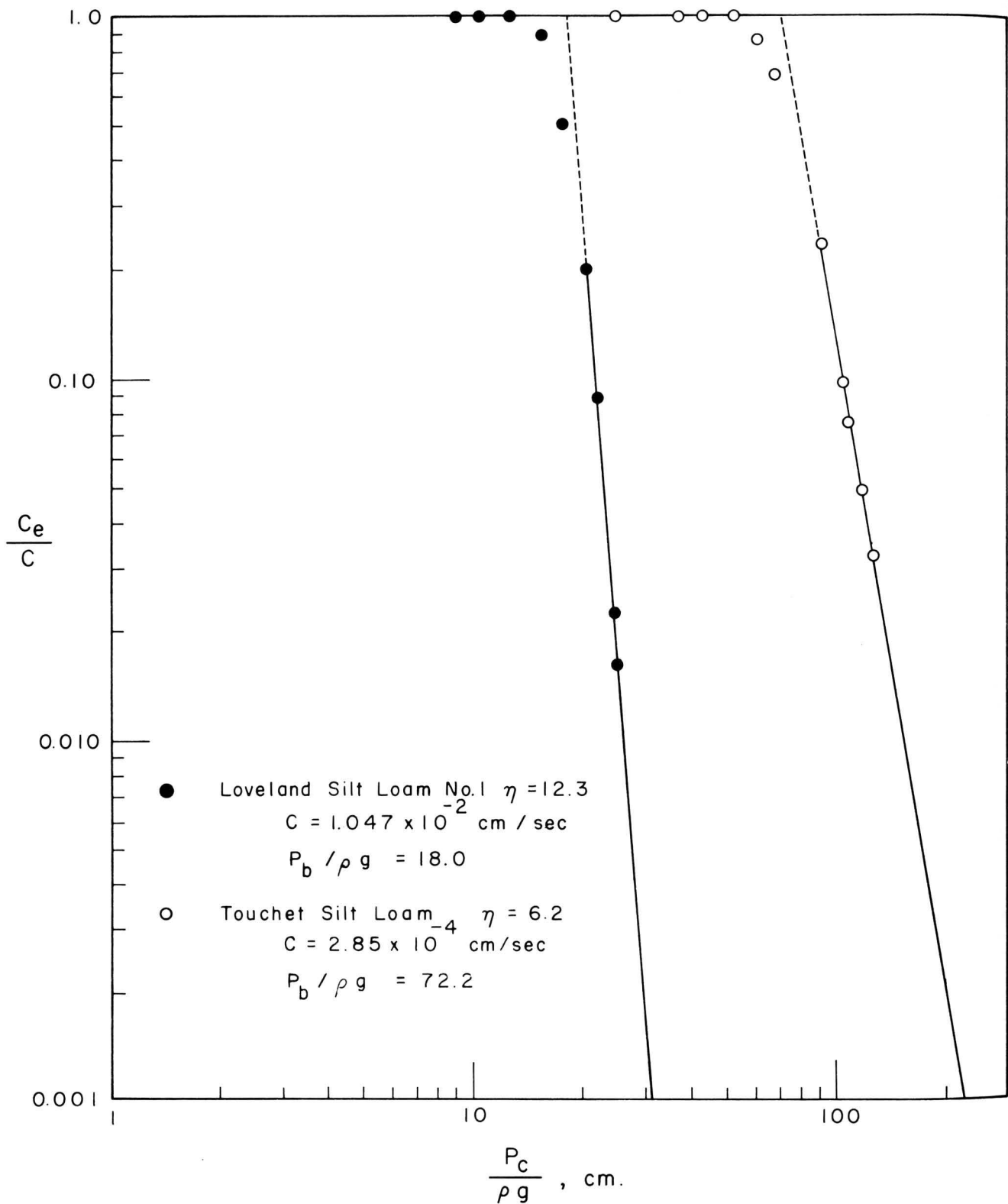


Figure 3. Relative conductivity - capillary pressure curves for drainage cycle.

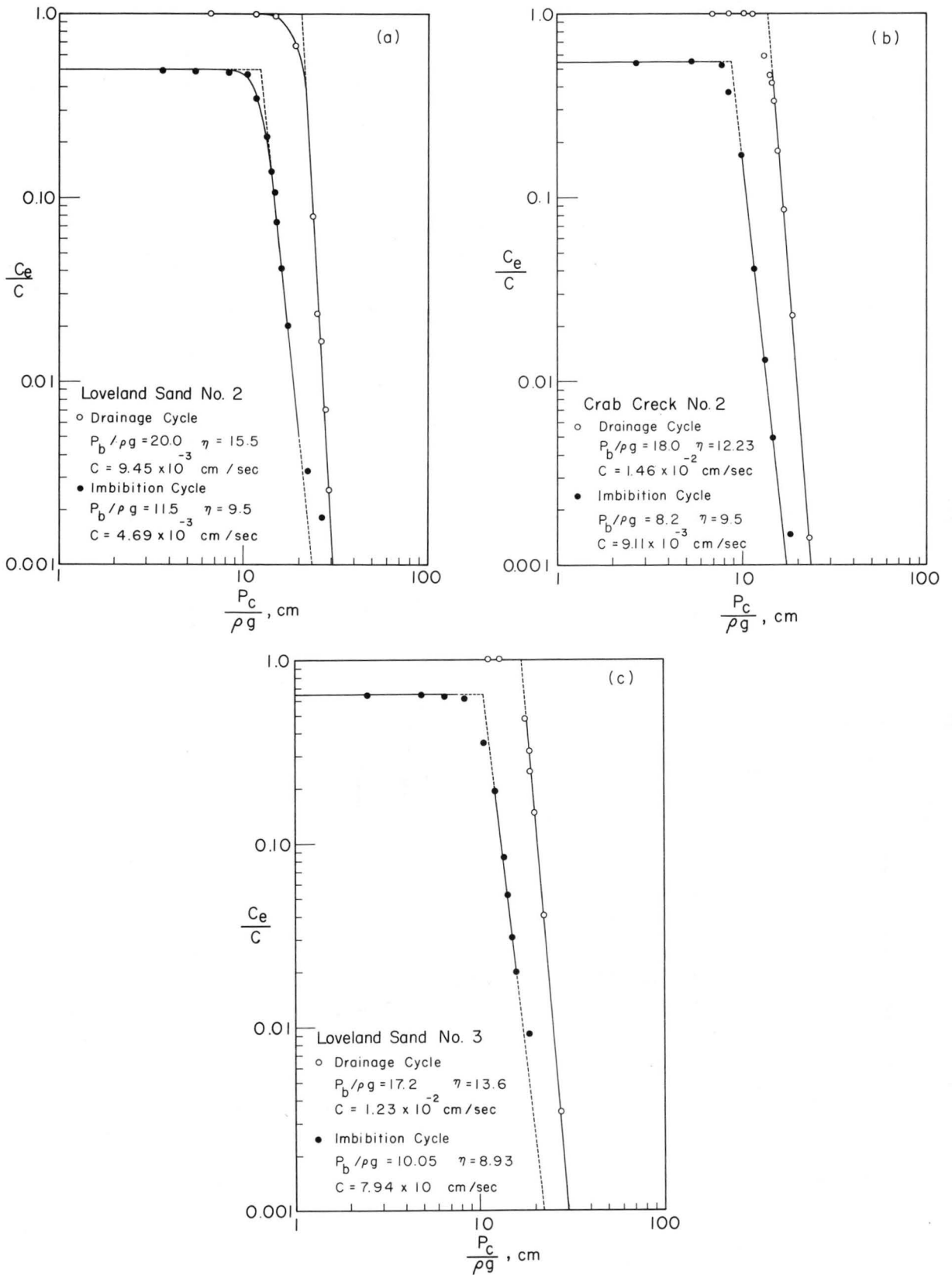


Figure 4. Relative conductivity - capillary pressure curves for drainage and imbibition cycles.

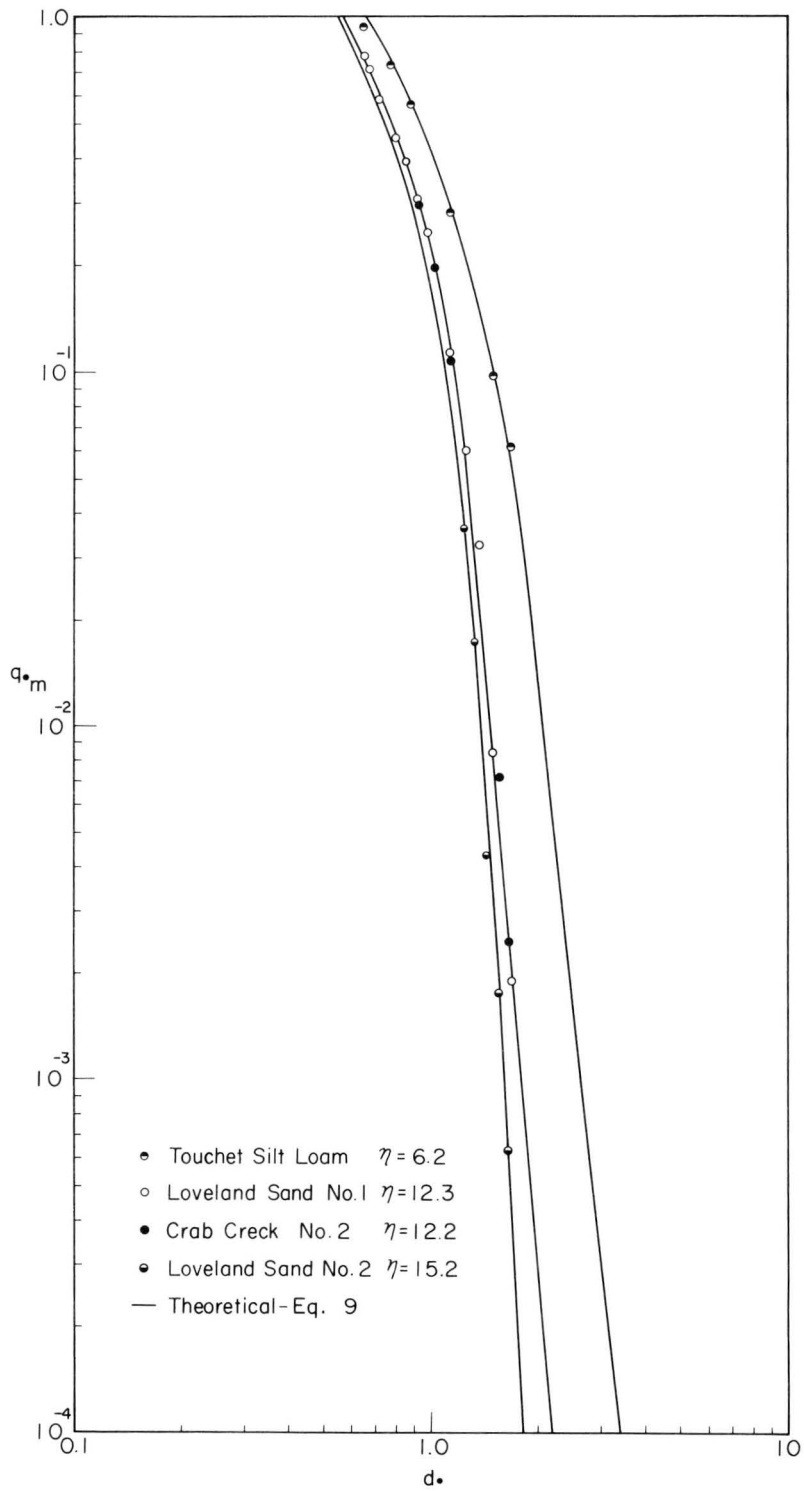


Figure 5. Comparison of experimental values of  $q_m$  as a function of  $d$ , with computed values for drainage cycles.

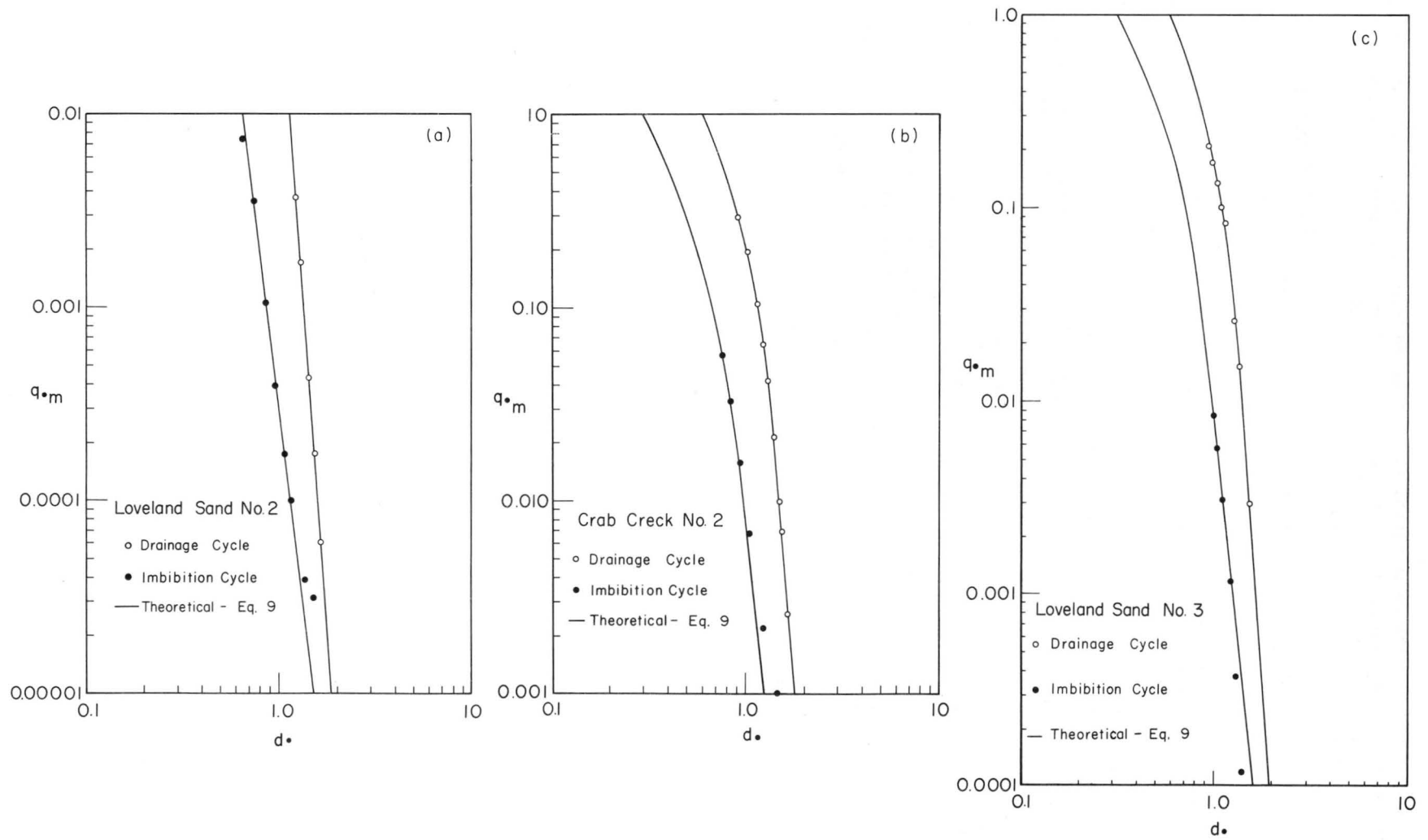


Figure 6. Comparison of experimental values of  $q_m$  as a function of  $d$ , with computed values for drainage and imbibition cycles.

to a high capillary pressure. The inflow siphon was then gradually raised in increments. The flow rates agreed very closely with the theoretical curves obtained by using the soil parameters obtained in an analogous manner, i.e., on the imbibition cycle starting with the soil moist but at a high capillary pressure. These results are shown in figure 6a and 6b.

D. Effect of hysteresis - Anat also performed experiments to determine whether or not the flow rates determined by Duke were affected by hysteresis as Duke had suspected. Figure 7 presents the results of one experiment in which the column was first saturated and the outflow siphon was lowered in increments while the zone of zero pressure was maintained at the lower tensiometer. Afterwards the outflow siphon was raised in increments while the zone of zero pressure was maintained at the same elevation. The broken line represents the theoretical curve for the drainage cycle. It will be noted that the measured flow rates obtained for the imbibition cycle follow a somewhat similar curve but at lower flow rates. The flow rates obtained by lowering the outflow siphon start to follow the theoretical curve, but at some point the liquid is evidently re-

moved from the surface faster than it can be supplied from below, thus producing the hysteresis previously discussed.

E. Effect of rate of removal of surface liquid - Another experiment was performed to determine whether or not the increments by which the outflow siphon was lowered might affect the magnitude of the hysteresis. Runs were made in which the outflow siphon was lowered by increments of 5, 10, and 20 cms respectively. The results are shown in figure 8 in which  $h_p$  represents the negative head on the outflow barrier.

It is evident from these results that the flow rates obtained by Duke were not unique but were affected by the particular increments by which he lowered his outflow siphon. Evidently, increasing the capillary pressure at the surface quickly so that the surface layer is dried quickly results in a more abrupt pressure reversal in the soil below the surface. This apparently increases the hysteresis effect that was first reported by Schleusener and Corey [16]. In any case, flow rates found in this way were 20 to 50 percent lower than theoretical values as was also found by Duke.

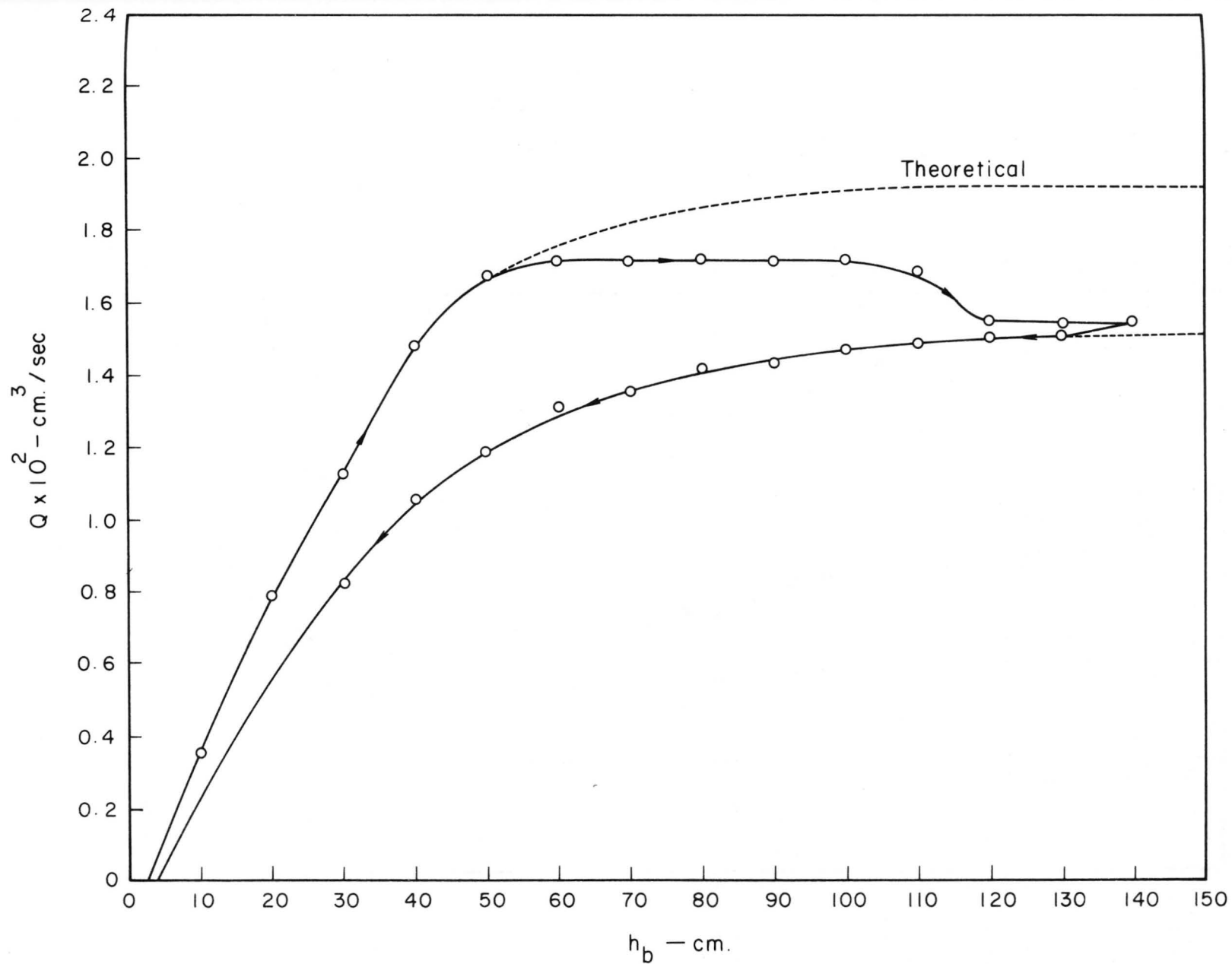


Figure 7. Effect of hysteresis on flow rate as a function of negative head at outflow barrier.

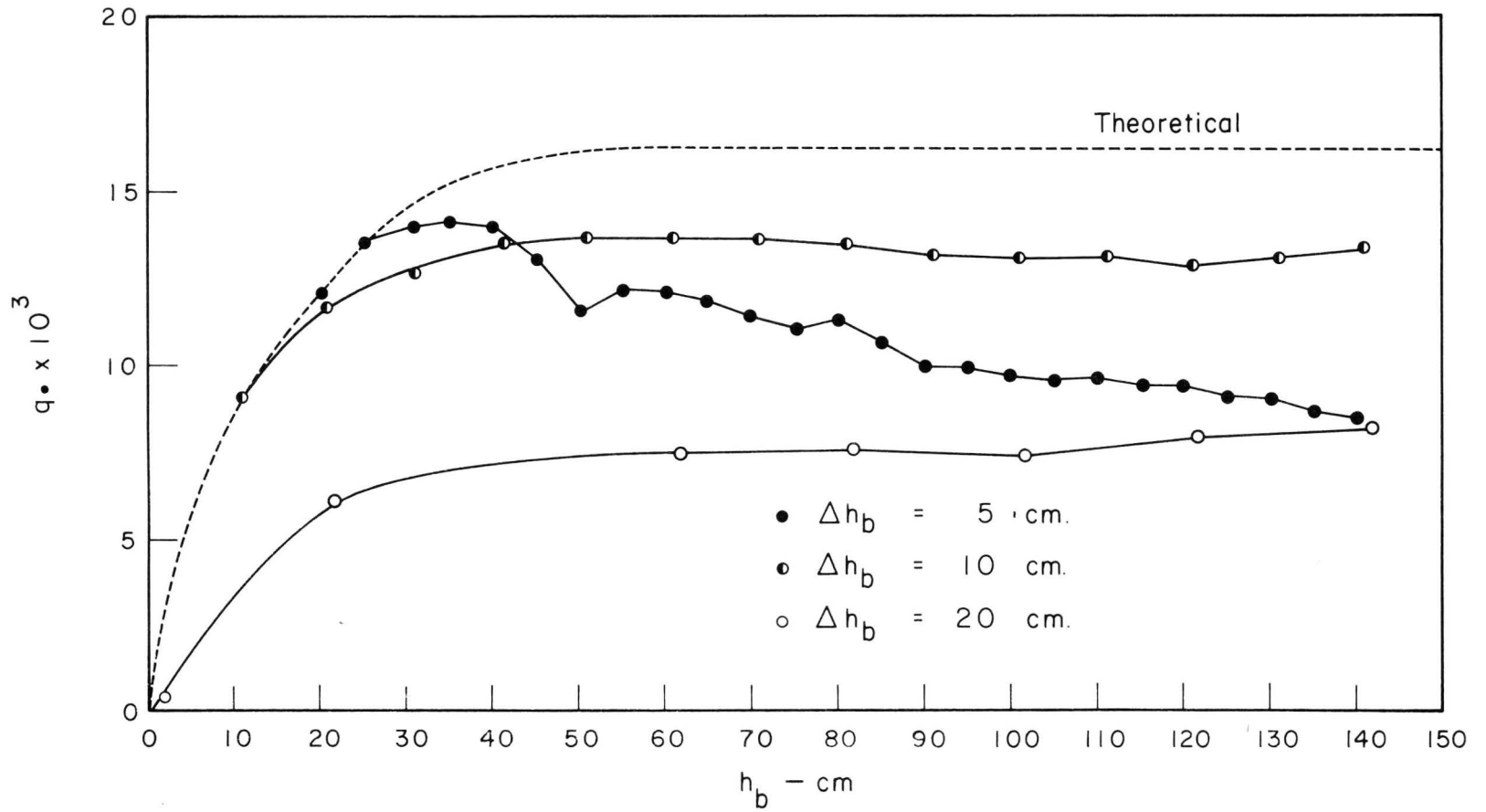


Figure 8. Effect on flow rate of increment of increase of negative head at outflow barrier.

EQUATION FOR APPROXIMATING MAXIMUM UPWARD FLOW RATES

Anat also developed an algebraic equation which approximates very closely the values determined from equation (5) using a computer. His derivation is summarized as follows:

Equation (5) is first written as

$$d. = \frac{1}{1+q_m} + \frac{1}{q_m} \frac{1}{\eta} \int_{q_m}^{\infty} \frac{dx}{1+x} \frac{1}{\eta} \quad (6)$$

in which x is a new variable representing  $P \cdot q_m^{1/\eta}$ . To solve equation (6), the value of the integral

$$\int_0^x \frac{dx}{1+x} \frac{1}{\eta}$$

must be known. The exact value of this integral when  $\eta > 4$  is extremely complex. The technique employed is to expand  $\frac{1}{1+x} \frac{1}{\eta}$  into a convergent series and integrate it term by term.

The convergent series of  $\frac{1}{1+x} \frac{1}{\eta}$  is not the same when  $x < 1$  as when  $x > 1$ . The values of

$$\int_0^x \frac{dx}{1+x} \frac{1}{\eta},$$

therefore, are separated into two cases, i.e.,  $x < 1$  and  $x > 1$ .

By the foregoing techniques, approximate values of the integral are as follows:

Case 1 --  $x < 1$

$$\int_0^x \frac{dx}{1+x} \frac{1}{\eta} \approx x - \frac{x}{\eta+1} \ln(1+x) + \frac{x^2}{4(\eta+1)^2} \quad (7)$$

Case 2 --  $x > 1$

$$\int_0^x \frac{dx}{1+x} \frac{1}{\eta} \approx 1 + \frac{1.886}{\eta^2+1} - \frac{x}{\eta-1} \ln(1+x^{-\eta}) - \frac{x^{-\eta}}{(2\eta-1)^2} \quad (8)$$

Substituting equations (7) and (8) into equation (6), the values of d. corresponding to maximum values of  $p_b/\rho g$ , C, and  $\eta$  are as follows:

Case 1 --  $q_m < 1$

$$d. \approx \frac{1}{q_m} \frac{1}{\eta} \left( 1 + \frac{1.886}{\eta^2+1} \right) - \frac{q_m}{1+q_m} + \frac{\ln(1+q_m)}{\eta+1} \quad (9)$$

Case 2 --  $q_m > 1$

$$d. \approx \frac{1}{1+q_m} + \frac{1}{\eta-1} \ln \frac{(1+q_m)}{q_m} \quad (10)$$

The subscript m in equations (9) and (10) indicates that  $q_m$  represents the maximum value of q., obtained when  $p \rightarrow \infty$  at the scaled elevation d. The equations (9) and (10) are sufficiently accurate when  $\eta \geq 4$ . This is not, however, a serious disadvantage since the values of I(x) can be obtained easily by direct integration when  $\eta = 2$  or 3.

The degree of accuracy of equations (9) and (10) is shown by comparing the values computed from them with the values computed by Duke which are shown in table 2 in Appendix A. A comparison of values (for  $\eta = 4, 8,$  and  $12$ ) computed from the approximate equations with those obtained by Duke are shown in table 1 on the next page. A detailed discussion of the derivation is given in the Ph.D. Dissertation by Anat [1].

For shallow water-table depths or for large values of q., the flow rate is controlled by external evaporative conditions. The authors suspect that equation (10) is of academic interest only. For most actual problems, the value of  $q_m$  is small and if the value of  $q_m < 0.01$ , equation 9 can be approximated by

$$d. \approx \frac{1}{q_m} \frac{1}{\eta} \left( 1 + \frac{1.886}{\eta^2+1} \right) \quad (11)$$

or

$$q_m \approx \left( 1 + \frac{1.886}{\eta^2+1} \right)^\eta d.^{-\eta} \quad (12)$$

Equation (12) is somewhat comparable in form to those which appear in Gardner's paper [9] as equations (18, 19, 20, and 21). The latter, however, were for integral values of  $\eta$  from 1 to 4 only. In deriving these equations, Gardner also assumed that q was small so that q/a approached zero, a being a soil parameter.

An important advantage of equations (9, 10, 11, and 12) over the particular computer program employed by Duke is that the former can be solved for non-integral values of  $\eta$ . Furthermore, these equations will greatly facilitate computation of flow rates through stratified soil even though a computer may be necessary for such computations. The theory of how such computations might be made is presented in the Thesis by Anat [1]. It has not been verified experimentally to date.

TABLE 1

COMPARISON OF VALUES OF  $d$ . AS A FUNCTION OF  $q_m$   
 FROM EQUATION (9) WITH THE VALUES PRESENTED  
 BY DUKE

$q_m$	$\eta = 4$		$\eta = 8$		$\eta = 12$	
	Eq. 9	Duke	Eq. 9	Duke	Eq. 9	Duke
0.001	6.253	6.24	2.439	2.43	1.801	1.80
0.002	5.253	5.25	2.236	2.23	1.70	1.70
0.004	4.419	4.41	2.046	2.04	1.601	1.60
0.008	3.708	3.71	1.874	1.87	1.507	1.50
0.016	3.111	3.11	1.712	1.71	1.415	1.41
0.032	2.601	2.60	1.555	1.55	1.312	1.32
0.064	2.161	2.16	1.398	1.39	1.22	1.22
0.128	1.768	1.77	1.230	1.23	1.10	1.10
0.256	1.403	1.40	1.042	1.04	0.948	0.947
0.512	1.057	1.06	0.826	0.822	0.764	0.762

The data measured by Anat are presented in detail in Appendix B, the conductivity values

being given in table 3 and the upward flow rates in table 4.

## SUMMARY AND CONCLUSIONS

Experiments were conducted at Colorado State University to investigate steady upward flow from water tables. The first investigations began in 1956 (as part of the Western Regional Project W-32) some aspects of which have been reported in previously published literature [12, 16]. The principal purpose of this paper is to report the findings of studies which were completed in 1965 concerning upward flow from water tables.

The objective of the latter studies was to answer the questions:

1. What is the maximum rate of upward flow (as a function of the depth at which liquid exists at atmospheric pressure) and what are the soil parameters that determine this maximum rate?
2. What is the maximum rate of upward flow when the liquid system follows an imbibition cycle completely?
3. How is the maximum rate of upward flow affected by a situation in which the removal of liquid from the surface exceeds the rate at which it can be transmitted to the surface?

A differential equation was developed which relates the maximum scaled flow rate  $q_m$  to the scaled distance  $d$ , from the zone of zero capillary pressure to the bottom of a dry surface layer. The differential equation contains the soil parameter  $\eta$  which is a dimensionless number evaluating the pore-size distribution.

The flow rate  $q$  is scaled by dividing it by the hydraulic conductivity of the fully saturated soil and the distance  $d$  is scaled by dividing by the quantity  $p_b/\rho g$ , in which  $p_b$  is the capillary pressure at which the soil begins to desaturate rapidly.

The differential equation relating  $q_m$  to  $d$  has been solved using a computer program for a range of  $\eta$  from 2 to 20, which includes all values of  $\eta$  found to date. This program, however, is suitable for values of  $\eta$  that are integers only. Algebraic expressions were also developed which approximate the exact solution of the differential equation. The approximation is close enough for any conceivable practical purpose and, furthermore, the algebraic expressions can be evaluated for values of  $\eta$  that are non-integers as well as integers.

It was found that the theoretical equations gave functional relations between  $q_m$  and  $d$ , which agreed extremely well with measured functional

relations when the soil parameters  $C$ ,  $p_b$ , and  $\eta$  were measured with sufficient accuracy. The agreement was good for systems in which the soil liquid followed both an imbibition (rising water table) and a drainage cycle (falling water table), provided the soil parameters were obtained for the appropriate cycle.

Unfortunately, however, the values of  $q_m$  corresponding to a particular value of  $d$ , when the system follows a drainage cycle can be 100 times greater than when the system follows an imbibition cycle. Furthermore, whenever the rate of removal of liquid from a soil surface substantially exceeds the rate at which the liquid can be replaced from the interior of the soil, hysteresis takes place.

The hysteresis process probably takes place as follows:

A large potential gradient at the surface at first removes liquid from the surface layer faster than the liquid can be replaced from the interior. The desaturation of the surface layer results in a large reduction in its conductivity so that a stage is reached when the rate at which the desaturated layer can imbibe water from the interior is somewhat greater than the rate at which it can be conducted through the layer. This causes the pressure of water entering the desaturated layer to increase, thus decreasing the potential gradient producing flow from the interior by the time steady state flow is established. Because the functional relations among degree of saturation, pressure and conductivity contain hysteresis loops, the increased pressure is not accompanied by a material increase in saturation or conductivity in the surface layer.

The result is that when flow takes place from a water table to a soil surface (under conditions that would otherwise produce a drainage cycle) quick drying of the surface will produce flow rates less than when hysteresis does not occur.

The reduction in flow rate, furthermore, depends on the rate at which the surface layer is desaturated; the faster it is desaturated, the greater the reduction in flow rate. Reduction in flow rates of from 20 to 50 percent resulting from hysteresis have been observed. It is possible, of course, that if the surface layer had been desaturated even more quickly than was possible with the apparatus employed, reduction in flow rates greater than 50 percent might have occurred.

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## APPENDIX A

(From Thesis by Duke [8])

## Fortran Computer Program for Solution of Equation (5)

SEQ	STMNT	FORTRAN STATEMENT
	C	PROGRAM FOR HAROLD DUKE
1		H=.05
2		PI=3.1415927
3		N=1
4		DO 20 LLL=1,19
5		N=N-1
6		EN=N
7		Q=.0005
8		DO 20 LL=1,14
9		ZZ=0.0
10		Q=Q*2
11		Z=PI/(EN*SINF(PI/EN)*Q**(1./EN))
12		DO 10 L=1,10
13		R=FLOATF(L)-1.
14		TERM=(H/3.)*((1./(1.+Q*((2.*R*H)**N))) +
	1	(4./(1.+Q*((2.*R*H+H)**N))) +
		(1./(1.+Q*((2.*R*H+H+H)**N))))
15	00010	ZZ=ZZ+TERM
16		Z=Z+ZZ
17		ZZZ=1./(1.+Q)
18		Z=Z+ZZZ
19		PUNCH 1020, N,Q,Z
20	00020	PRINT 1010,N,Q,Z
21		STOP
22	01020	FORMAT (3H N=12,5X,3H Q=P6.3,5X,3H Z=E18.8)
23		END

TABLE 2

VALUES OF  $d$ . FROM COMPUTER SOLUTION OF EQUATION (5)

$q_m$	$\eta$						
	2	3	4	5	6	7	8
0.001	49.67	12.09	6.24	4.25	3.31	2.77	2.43
.002	35.12	9.60	5.25	3.70	2.95	2.51	2.23
.004	24.83	7.61	4.41	3.22	2.62	2.27	2.04
.008	17.56	6.04	3.71	2.80	2.33	2.05	1.87
.016	12.41	4.79	3.11	2.43	2.07	1.85	1.71
.032	8.76	3.79	2.60	2.10	1.83	1.16	1.55
.064	6.17	2.98	2.16	1.80	1.60	1.48	1.39
.128	4.32	2.32	1.77	1.52	1.38	1.29	1.23
.256	2.98	1.76	1.40	1.24	1.14	1.08	1.04
0.512	1.99	1.27	1.06	0.951	0.890	0.850	0.822
1.024	1.26	0.861	0.733	.672	.635	.611	.594
2.048	0.754	.535	.464	.430	.409	.395	.386
4.096	.423	.308	.270	.252	.240	.233	.228
8.192	0.226	0.167	0.148	0.138	0.132	0.128	0.125
$q_m$	$\eta$						
	9	10	11	12	13	14	15
0.001	2.20	2.03	1.90	1.80	1.72	1.65	1.60
.002	2.03	1.89	1.78	1.70	1.63	1.57	1.52
.004	1.88	1.76	1.67	1.60	1.54	1.49	1.45
.008	1.74	1.64	1.56	1.50	1.46	1.42	1.38
.016	1.60	1.52	1.46	1.41	1.37	1.34	1.31
.032	1.47	1.41	1.36	1.32	1.29	1.26	1.24
.064	1.33	1.28	1.25	1.22	1.19	1.17	1.15
.128	1.18	1.15	1.12	1.10	1.08	1.06	1.05
.256	1.01	0.982	0.947	0.947	0.934	0.923	0.914
0.512	0.802	.786	.773	.762	.654	.747	.740
1.024	.581	.571	.563	.557	.552	.547	.543
2.048	.378	.373	.368	.365	.361	.359	.357
4.096	.224	.221	.218	.216	.214	.213	.212
8.192	0.123	0.122	0.120	0.119	0.118	0.118	0.117
$q_m$	$\eta$						
	16	17	18	19	20		
0.001	1.55	1.51	1.47	1.44	1.42		
.002	1.48	1.45	1.42	1.39	1.37		
.004	1.42	1.39	1.36	1.34	1.32		
.008	1.35	1.33	1.31	1.29	1.27		
.016	1.29	1.27	1.25	1.23	1.22		
.032	1.22	1.20	1.19	1.17	1.16		
.064	1.14	1.13	1.11	1.10	1.09		
.128	1.04	1.03	1.02	1.01	1.00		
.256	0.905	0.898	0.892	0.887	0.882		
0.512	.735	.730	.726	.722	.719		
1.024	.540	.537	.534	.532	.530		
2.048	.355	.353	.352	.350	.349		
4.096	.211	.210	.209	.208	.208		
8.192	0.116	0.116	0.116	0.115	0.115		

APPENDIX B

(From Thesis by A. Anat [1])

TABLE 3

RELATIVE PERMEABILITY-CAPILLARY PRESSURE DATA

Soil	$p_c/\rho g(\text{cm.})$	$C_e/C$
<u>Loveland Sand No. 1</u>		
Drainage cycle	10.70	1.0000
	13.00	1.0000
$C = 1.047 \times 10^{-2}$ cm/sec	15.45	0.8960
	15.80	0.8100
$p_b/\rho g = 18.00$ cm	17.80	0.5020
	20.30	0.2000
$\eta = 12.3$	22.00	0.0874
	24.80	0.0226
	25.0	0.016
<u>Touchet Silt Loam</u>		
Drainage cycle	9.65	1.0000
	24.80	1.0000
$C = 2.85 \times 10^{-4}$ cm/sec	37.05	1.0000
	43.50	0.9770
$p_b/\rho g = 72.2$ cm	53.10	0.9090
	60.30	0.8600
$\eta = 6.2$	68.30	0.6800
	71.50	0.6600
	91.35	0.2330
	106.90	0.09680
	107.50	0.07550
	117.50	0.04870
	127.60	0.03200
<u>Loveland Sand No. 2.</u>		
Drainage cycle	6.60	1.00000
	11.30	1.00000
$C = 9.45 \times 10^{-3}$ cm/sec	14.50	1.00000
	18.50	0.69200
$p_b/\rho g = 20.0$ cm	23.50	0.10200
	25.10	0.00234
$\eta = 15.2$	26.10	0.00166
	27.80	0.00071
	29.30	0.000262
Imbibition cycle	26.20	0.00018
	22.25	0.00033
$C = 4.685 \times 10^{-3}$ cm/sec	17.50	0.02050
	16.00	0.04150
$p_b/\rho g = 12.2$ cm	15.00	0.07514
	14.60	0.10920
$\eta = 9.5$	13.20	0.21100
	11.80	0.34870
	10.30	0.46950
	8.20	0.47700
	5.40	0.48500
	3.60	0.48960

TABLE 3--Continued

Soil	$p_c/\rho g(\text{cm.})$	$C_e/C$
<u>Crab Creek No. 2</u>		
Drainage cycle	7.00	1.00000
	8.55	1.00000
$C = 1.462 \times 10^{-2}$ cm/sec	10.33	1.00000
	13.35	0.59000
$p_b/\rho g = 13.5$ cm	14.25	0.45500
	14.70	0.31200
$\eta = 12.2$	15.80	0.17757
	16.70	0.08650
	18.70	0.02266
	23.00	0.00142
Imbibition	18.10	0.00149
	14.75	0.00489
$C = 9.11 \times 10^{-3}$ cm/sec	13.35	0.01300
	11.55	0.04025
$p_b/\rho g = 8.2$ cm	9.80	0.16900
	8.45	0.37420
$\eta = 9.0$	7.75	0.52310
	5.37	0.54470
	2.65	0.53200
<u>Loveland Sand No. 3</u>		
Drainage cycle	11.20	1.00000
	12.80	1.00000
$C = 1.232 \times 10^{-2}$ cm/sec	16.65	0.58710
	17.40	0.48000
$p_b/\rho g = 17.2$ cm	18.45	0.32000
	18.90	0.24670
$\eta = 13.6$	19.90	0.14700
	22.20	0.04090
	24.40	0.00610
	27.60	0.00344
Imbibition cycle	18.15	0.00915
	15.70	0.01994
$C = 7.942 \times 10^{-3}$ cm/sec	14.95	0.03110
	14.15	0.05340
$p_b/\rho g = 10.50$ cm	13.45	0.08400
	12.05	0.19020
$\eta = 9.0$	10.65	0.35740
	8.35	0.62280
	6.50	0.63570
	4.90	0.64700
	2.45	0.643

TABLE 4

SUMMARY OF MAXIMUM RATES OF UPWARD FLOW,  
 COMPARED WITH THEORETICAL VALUES (EQUATION 9)

Soil	$qx10^2$ cm/sec	$Cx10^2$ cm/sec	$q_m$	Experimental d.	Eq. 9 d.
<u>Loveland Sand No. 1</u>					
Drainage cycle	0.80800	1.047	0.77200	0.647	0.639
	0.74060	"	0.70700	0.676	0.667
$p_b/\rho g = 18.00$ cm	0.61300	"	0.58600	0.718	0.723
	0.47300	"	0.45200	0.795	0.797
$\eta = 12.3$	0.40900	"	0.39030	0.843	0.837
	0.31000	"	0.30200	0.920	0.904
	0.25200	"	0.24130	0.977	0.959
	0.20300	"	0.19400	2.034	1.008
	0.13500	"	0.12800	1.112	1.093
	0.06200	"	0.05930	1.246	1.225
	0.00870	"	0.00830	1.456	1.495
	0.00200	"	0.00190	1.772	1.710
<u>Touchet Silt Loam</u>					
Drainage cycle	0.02663	0.0285	0.93500	0.625	0.667
	0.02062	"	0.72400	0.763	0.759
$p_b/\rho g = 72.2$ cm	0.01617	"	0.50800	0.874	0.888
	0.00857	0.0300	0.28400	1.120	1.098
$\eta = 6.2$	0.00291	"	0.09600	1.573	1.453
	0.00182	"	0.06100	1.650	1.596
<u>Loveland Sand No. 2</u>					
Drainage cycle	0.03610	0.943	0.03820	1.140	1.210
	0.02580	"	0.03660	1.223	1.219
$p_b/\rho g = 20.0$ cm	0.01590	"	0.01690	1.313	1.300
	0.00400	"	0.00428	1.407	1.436
$\eta = 15.2$	0.00160	"	0.00173	1.528	1.526
	0.00058	0.950	0.00061	1.625	1.637
Imbibition cycle	0.00080	0.943	0.00008	1.220	1.156
	0.00094	"	0.00010	1.148	1.123
$p_b/\rho g = 11.5$	0.00161	"	0.00017	1.045	1.065
	0.00370	"	0.00039	0.965	0.971
$\eta = 9.5$	0.00970	"	0.00104	0.865	0.869
	0.03340	"	0.00355	0.739	0.738
	0.06980	"	0.00743	0.650	0.648
<u>Crab Creek No. 2</u>					
Drainage cycle	0.40720	1.381	0.29500	0.919	0.912
	0.27130	"	0.19640	1.020	1.007
$p_b = 13.50$ cm	0.14640	"	0.10600	1.138	1.129
	0.09080	"	0.06520	1.216	1.211
	0.05900	1.393	0.04230	1.289	1.274
$\eta = 12.2$	0.03200	"	0.02297	1.377	1.360
	0.01400	1.381	0.01040	1.489	1.470
	0.99980	"	0.00709	1.542	1.513
	0.00365	"	0.00264	1.627	1.647
Imbibition cycle	0.00146	1.447	0.00103	1.443	1.270
	0.00318	"	0.00225	1.221	1.132
$p_b/\rho g = 8.2$ cm	0.00993	"	0.00684	1.040	0.985
	0.02340	"	0.00162	0.925	0.923
$\eta = 9.0$	0.04870	"	0.00337	0.827	0.836
	0.08340	"	0.00569	0.754	0.770

TABLE 4--Continued

Soil	$qx10^2$ cm/sec	$Cx10^2$ cm/sec	$q_m$	Experimental d.	Eq. 9 d.
<u>Loveland Sand No. 3</u>					
Drainage cycle	0.26470	1.269	0.20860	0.938	0.975
	0.21860	"	0.17230	0.981	1.015
$p_b/\rho g = 17.2$	0.16950	"	0.13340	1.032	1.063
	0.12520	1.257	0.09960	1.090	1.099
$\eta = 13.6$	0.10400	"	0.08270	1.157	1.139
	0.03202	1.243	0.02580	1.278	1.299
	0.01856	"	0.01490	1.358	1.363
	0.00371	"	0.00300	1.549	1.542
Imbibition cycle	0.00018	"	0.00012	1.390	1.552
	0.00058	"	0.00037	1.303	1.368
$p_b/\rho g = 10.05$	0.00178	"	0.00115	1.221	1.205
	0.00475	"	0.00306	1.109	1.079
$\eta = 9.0$	0.00889	"	0.00574	1.040	1.006
	0.01267	"	0.00818	0.985	0.962

## EQUIPMENT AND PROCEDURES

### Design of Equipment

The individual components used in Duke's research and subsequent studies of a similar nature were those designed and built by Brooks and Corey. Figures 9, 10, and 11 show sketches of the several components. Most of the length of the soil columns was enclosed by acrylic plastic cylinders as shown in figure 9(a). Each of these sections had an annular groove machined into the wall at the bottom end of the section with small notches connecting this groove to both inside and outside the tubing. The purpose of these grooves was to provide a means for air to enter the column as drainage proceeded and to minimize the escape of liquid by capillary flow through the joints.

Inflow to the column was supplied through a section of acrylic tubing closed on the lower end and fitted with a capillary barrier as shown in figure 10. This section was machined on the inside to allow a thin sleeve of Selas filter material with a bubbling pressure of 10 psi, having the same inside diameter as the column sections, to be inserted. An annular space outside the barrier allowed free passage of liquid around its outer surface. This space was connected to an inflow siphon by means of a 1/4" drilled and tapped hole and plastic tubing.

Fluid was removed from the top of the soil column through a ceramic barrier of the same composition as that used for the inflow sleeve. The outflow barrier consisted of a disc of ceramic cemented to a plastic section milled from solid stock, having a center bore and provision for attachment of tubing at the upper end. This barrier was of such size as to slip freely into the column sections to allow good contact between barrier and soil, yet prevent excessive evaporation. This arrangement is shown in figure 11.

To provide for measurement of capillary pressures within the columns, sections of cylinders 2 cm in length were machined inside to form an annular groove to retain thin rings of ceramic, 1/2 cm wide, the arrangement being shown in figure 9(b). The annular groove was connected to a small displacement manometer. These tensiometer rings operated in the same manner as ordinary tensiometers except that, being flush with the inside wall of the column sections, they did not reduce the cross-sectional area of the soil column. All capillary barriers including those used in the tensiometers were cemented to the acrylic plastic using Armstrong A-1 industrial adhesive.

For the lower flow rates, the outflow was measured in a 4 mm O.D. glass tube calibrated in  $\text{cm}^3$  per cm of length. The higher flow rates were measured by allowing the liquid to drip from the end of the tube into a 5 ml burette. Figure 12 shows both the outflow measuring device and the constant head inflow reservoir.

For soil materials having a very high permeability and low bubbling pressure, Porvic plastic was substituted for Selas as barrier material to reduce the head loss through the barriers and to maintain better control of the capillary pressures.

### Procedure

The columns were assembled by fastening together the acrylic sections mentioned previously. The inflow section was at the bottom of the column, with a tensiometer ring immediately above to provide a reference for the water table.

A 4 cm and a 2 cm cylinder section were placed above the lower tensiometer ring. A second tensiometer ring was placed above these two column sections. This portion of the column provided a convenient section for measuring conductivity as a function of capillary pressure. The latter measurements were made after the upward flow experiments were completed. The alternate placement of column sections and tensiometer rings until the desired length was attained completed the column arrangement for the upward flow tests.

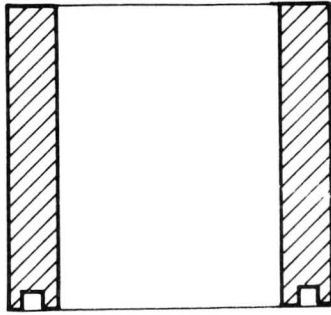
The sections were then taped together and packed using a mechanical packer of a type devised by Jackson et al. [11]. Experiments showed that this method resulted in a uniform bulk density throughout the column. Later experience has shown, however, that this packer (or any device which simultaneously vibrates and fills a soil column) causes large particles and aggregates to concentrate at the wall of the column thus producing non-homogeneity in horizontal planes. This circumstance adversely affected some of Duke's results.

After the columns were packed, a disc of fiberglass mat was placed over the top of the soil column to prevent erosion during the saturation procedure. A plastic plug containing holes to permit the escape of air held the fiberglass against the soil and retained the soil in the column. The column was then submerged in a container of hydrocarbon to which a vacuum was applied. The vacuum was maintained until air bubbles ceased to emerge from the soil and the capillary barriers. Afterwards the vacuum was removed allowing the liquid to fully saturate the soil and the capillary barriers.

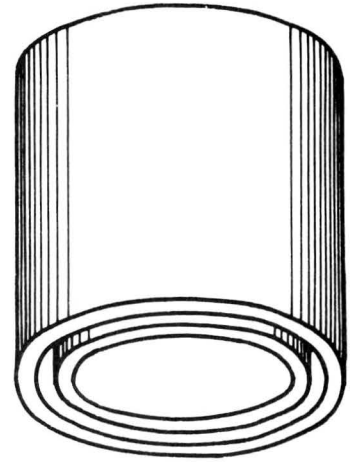
The saturated columns were removed from the liquid container and placed in a horizontal position to prevent desaturation. Short sections of oil-filled tubing were connected to all manometer taps and clamped to prevent air entering the capillary barriers. The column was then cut even with the top of the upper tensiometer section to give a smooth soil surface and fastened to a vertical channel iron near the manometer board.

The inflow reservoir was immediately attached to the inflow barrier to prevent the zone of zero

a.

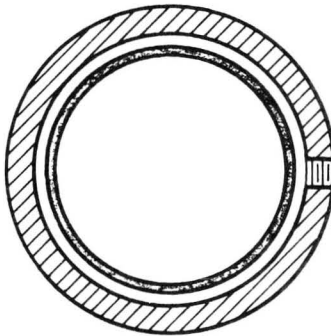


column section  
vertical cross-section

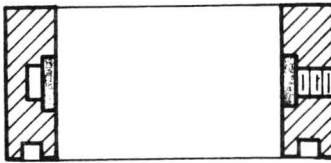


column section

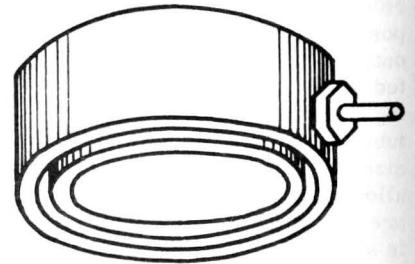
b.



tensiometer - horizontal cross-section

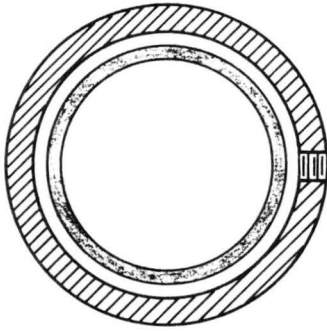


tensiometer - vertical cross-section

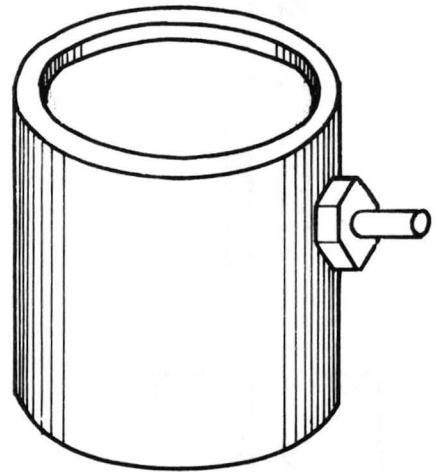


tensiometer

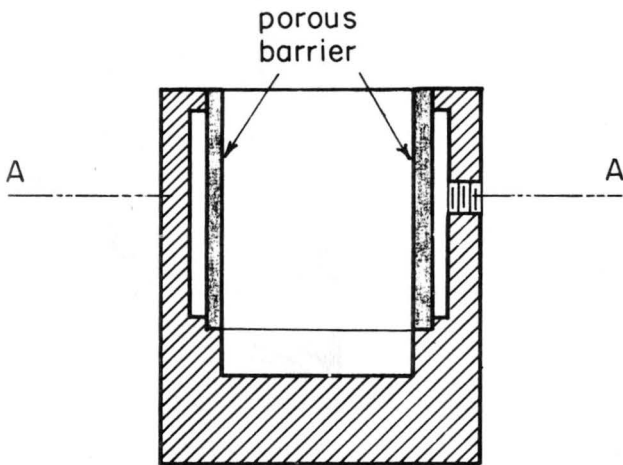
Figure 9. Tensiometer and column sections.



horizontal cross-section A - A



inflow pressure controller



vertical cross-section

Figure 10. Inflow pressure controller.

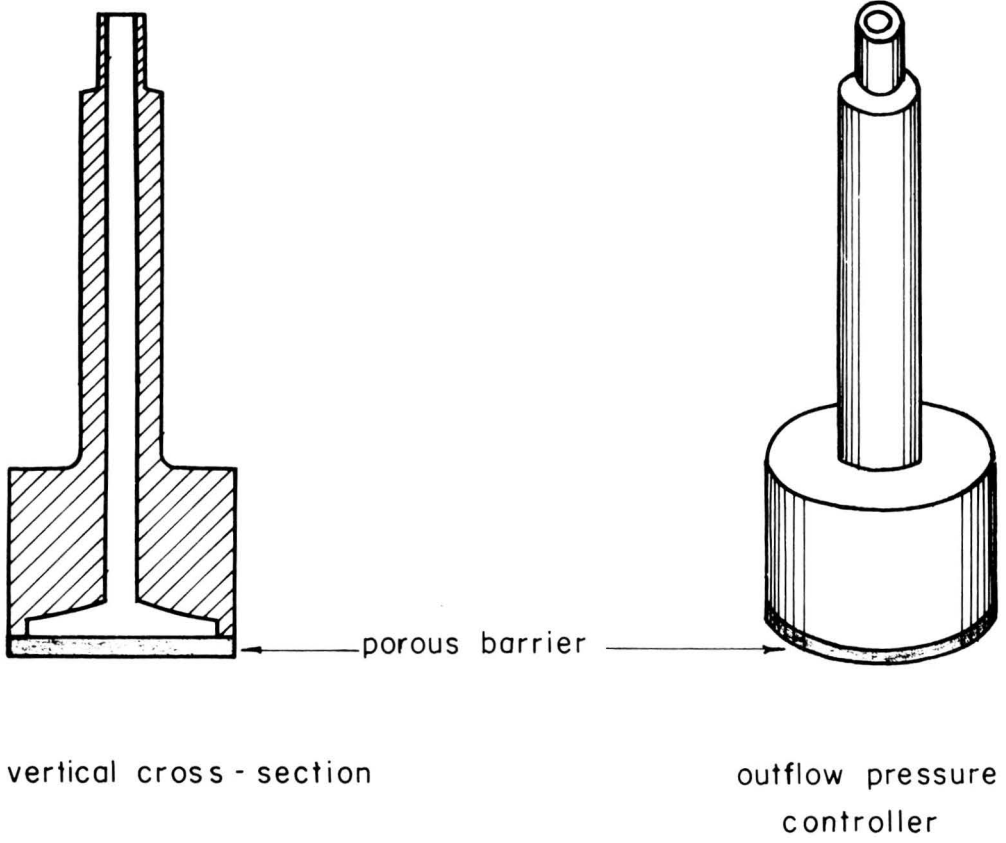


Figure 11. Outflow pressure controller.

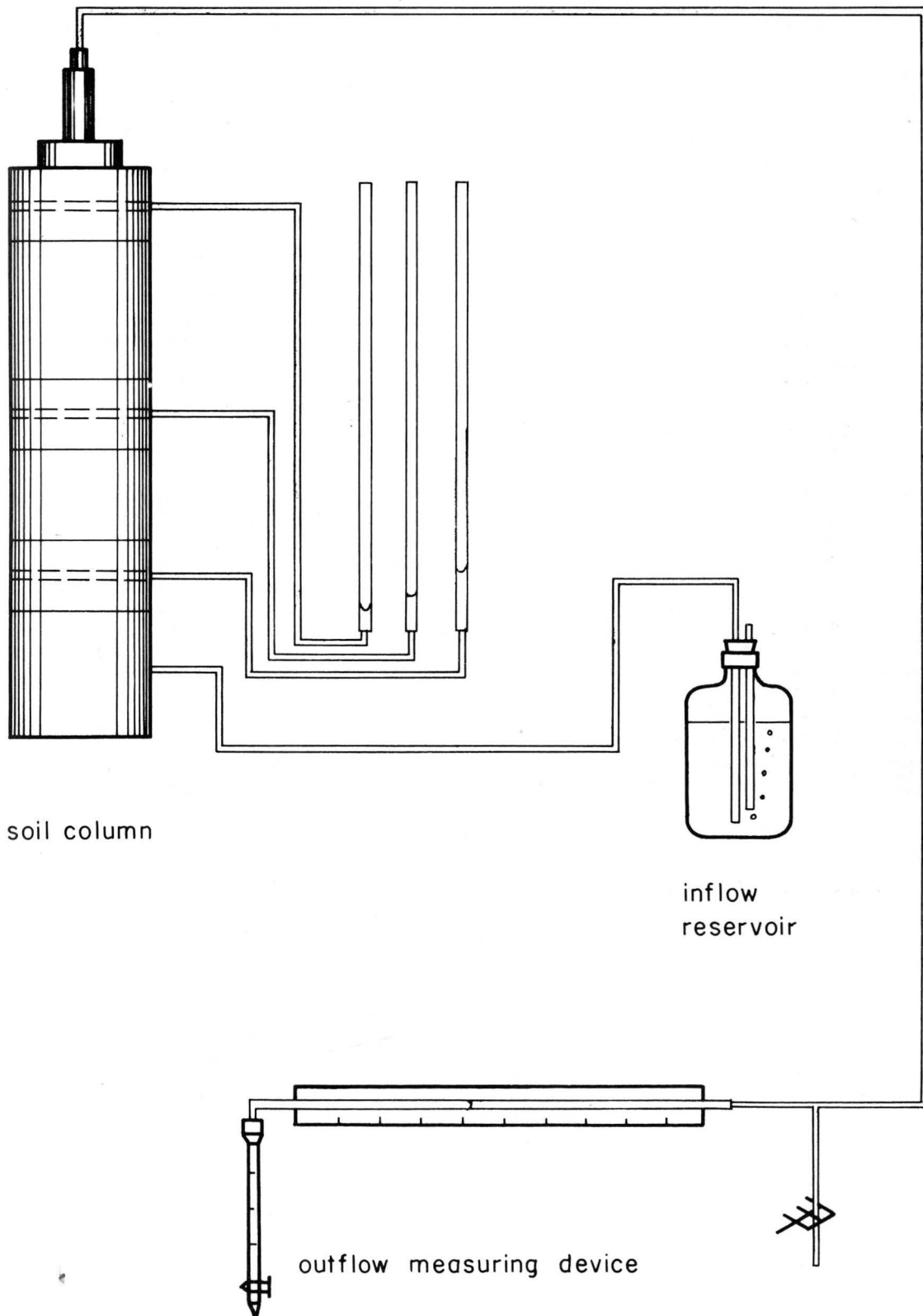


Figure 12. Schematic of apparatus for upward flow experiments.

pressure from dropping below the lower tensiometer ring. The upper barrier, having been previously saturated, was placed in position in contact with the soil at the top of the column and held firmly against the soil with a spring device. The set up was completely by clamping the outflow tube and connecting all manometers to their respective tensiometer rings. A schematic diagram of the apparatus is shown in figure 12.

Upward flow was induced by releasing the clamp on the outflow tube and lowering the outflow end of the tube a short distance below the elevation of zero capillary pressure.

The level of zero capillary pressure for a particular flow was adjusted by varying the vertical position of the Mariotte siphon so that a zero pressure was maintained at the lower tensiometer ring. Increased flow rates required that the inflow reservoir be raised to compensate for increased head loss in the lower barrier and in the soil below the zone of zero pressure.

Repeated measurements of the flow rate showed that the time at which equilibrium was reached for a particular run could be judged by observing the piezometric head at the upper tensiometer. As the rate was increased, piezometric head at the upper tensiometers decreased. When the level in upper manometer tubes ceased to drop, it was assumed that steady flow had been established. This procedure was similar to the technique used by Gardner and Fireman [7] to determine when steady state had been reached. The flow rate was determined by measuring the discharge from the outflow siphon.

To determine the maximum flow rate (for a particular depth to the level of zero capillary pressure) the outflow tube was lowered in increments, allowing steady state to be established at each increment. The outflow tube was usually lowered several increments below the point at which the maximum flow rate was observed.

As the flow rate was increased the upper manometers initially indicated an increase in capillary pressure. After the maximum flow rate had been reached, however, further lowering the outflow tube often resulted in a lower capillary pressure and a decrease in the flow rate. This behavior seemed to confirm the theory of Schleusener and Corey [16] that a pressure reversal occurs near a surface when liquid is removed from the surface substantially faster than it can be supplied from the interior of the medium.

To investigate the effect of water-table depth upon the flow rate, a series of runs as described above was first made utilizing the entire length of the column as originally packed. After completion of these runs, the column was resaturated as before. Upon removing the saturated column from the saturation container, the column was shortened by cutting the tape at the top of the next lower tensiometer ring.

A thin metal strip was inserted between the sections at this point and the upper portion of the column was sliced off smoothly.

Having shortened the column, it was set up as previously described and a series of runs was made as before. This process was repeated until the column was reduced to about 9 cm length from the lower tensiometer to the soil surface.

#### Measurement of Conductivity as a Function of Capillary Pressure

The 9 cm-column remaining when the upward flow experiments were completed was used for the conductivity measurements. Other components of the apparatus were the same. After re-saturating the column, the lower barrier was connected to the outflow measuring device, and the Mariotte siphon was attached to the upper barrier. Flow was thus maintained in a downward direction.

The first conductivity measurements were made at capillary pressures sufficiently low that the sample remained completely saturated, but high enough to desaturate the annular space at the joints. The latter precaution is necessary to insure that liquid does not creep from the joints by capillarity. A uniform capillary pressure, as indicated by the manometers attached to the upper and lower tensiometer rings, was established by adjusting the elevation of the inflow and outflow devices. Flow was then in response to gravity only. The manometers served to indicate when steady state had been established for this case also. The flow rate, temperature, and capillary pressure were then recorded and the conductivity was computed. Subsequent data were obtained at increasingly higher capillary pressures by lowering both the inflow and outflow devices to obtain the desired capillary pressure and repeating the measuring procedure.

By the methods described above, Duke obtained curves of relative conductivity,  $C_e/C$ , as a function of capillary pressure (similar to those presented by Brooks and Corey [4] for soils that did not contain large separates or aggregates. Many of the soils studied by Duke, however, consisted of highly aggregated clay soils. Curves of relative conductivity as a function of capillary pressure for such soils invariably contained inflections such that the effective conductivity could not be accurately described by equation (4).

Normally, when  $C_e/C$  is plotted as a function of  $p_c$  on log-log paper, the result is a straight line for  $p_c > p_b$ ,  $p_b$  being the intercept of the straight line where  $C_e/C = 1$ , and  $\eta$  is the negative of the slope of the line. Because the curves for aggregated soils contained an inflection (as shown in figure 13(a), it was impossible to compare the measured flow rates with the theoretical values computed from equation (5) for these soils. Neither  $\eta$  nor  $p_b$  was uniquely defined by the measured data.

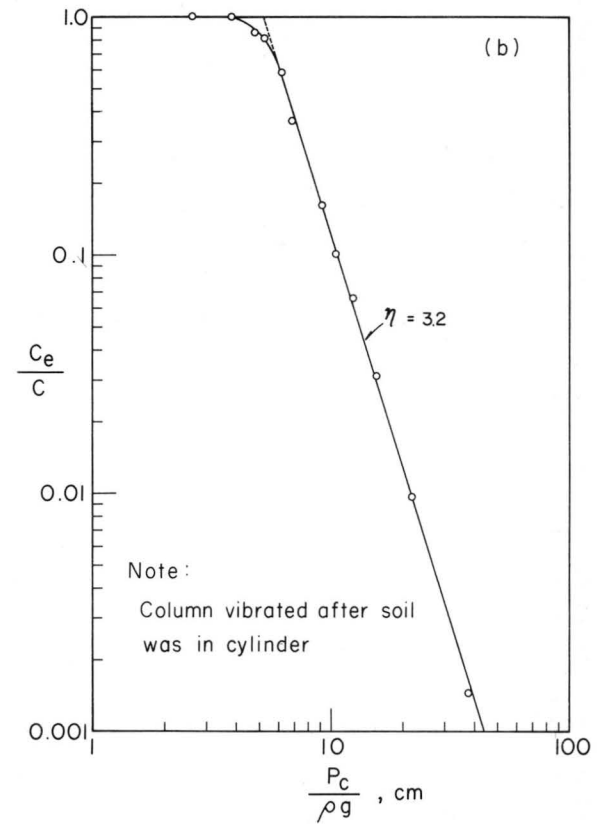
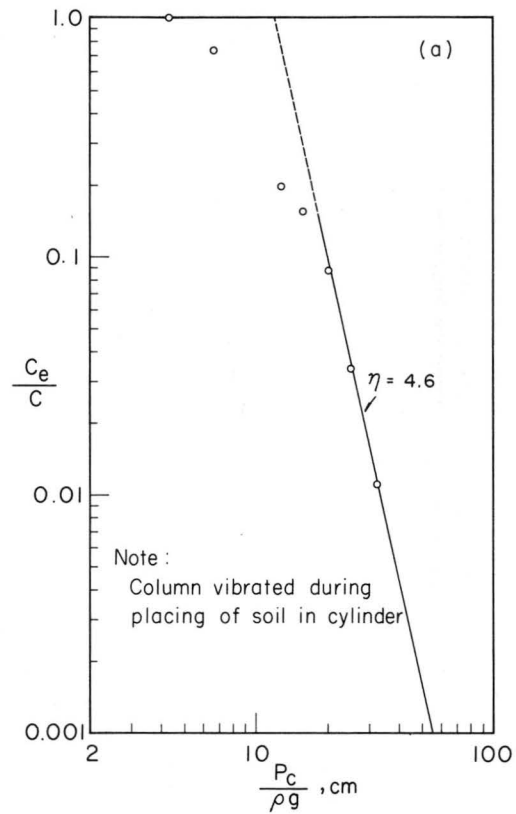


Figure 13. Relative conductivity - capillary pressure curves for granulated Pullman Clay as affected by method of packing.

The inflections shown in figure 13(a) were, at first, attributed to discontinuities in pore-sizes between primary and secondary pores. This theory has been disapproved, however, and it has been found that the inflections were caused by segregation of aggregates and large separates at the wall of the column due to the method of packing. Brooks [3],

using the same soil for which the curve shown in figure 13(a) was determined, obtained the curve in figure 13(b). The only difference in procedure was the method of packing. Brooks first filled the column with soil before starting vibration, whereas Duke used Jackson's procedure of vibrating during the process of filling.