Technical Report

SIMILARITY-LAW PROFILES IN THERMALLY STRATIFIED SHEAR FLOWS

by

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ABSTRACT

Mean wind velocity and temperature profiles were measured in a wind-tunnel boundary layer made thermally stable or unstable (corresponding to inversion and lapse conditions) by flow over a horizontal flat plate (29 m long and 2 m wide) which was either cooled or heated along the downstream 13 m. These profiles for heights from just above the viscous region up to half the boundary-layer thickness (about 55 cm) were fitted to the log-plus-linear and log-plus-linear-plus-square relationships. Field data consisting of mean temperature and wind speeds taken at heights from 0.25 m up to 16 m were also plotted. Similarity between the mean wind velocity and the mean temperature profiles for both laboratory and field data was found to exist. Retainment of the square term in the series approximation of the dimensionless wind shear for the case of $(z/L)^2 < 1$, where z is the height and L is the Monin-Obukhov length scale, did not substantially reduce the data scatter. For thermally unstable flows, the dimensionless wind shear is expressed by the form $(1+z/L)^n$ with $n = 3/2$.

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INTRODUCTION

A log-plus-linear law has been used extensively in the study of atmospheric surface-layer flows. This law, based on the similarity theory of Monin and Obukhov[1], holds only in the region where the Monin-Obukhov length scale L, the friction velocity u_*, and the friction temperature T_*, are essentially invariant with height and where the ratio of height under consideration to the length scale must be much smaller than unity. These assumptions are not always completely met by the field data. In order to stress the range in which the law is valid, the dimensionless wind shear will be expanded in a Taylor's series. In addition, it will be assumed to have a binomial form. (This assumption is based on the fact that in the neutral air flow the velocity profile is logarithmic in height while in the extremely unstable thermal flow it conforms to a power law.)

A log-plus-linear-plus-square profile was investigated. This is done under the assumption that when the height approaches the order of magnitude of the length scale, it may improve the data fitting by retaining the square term in the approximation. Experimental data show that retainment of the square term in the series approximation does not necessarily reduce the data scatter.

In plotting the dimensionless wind shear versus Richardson number, the significant effects of thermal stratification on the mean wind velocity and the mean temperature profiles are revealed. Moreover, data comparisons show that laboratory data agree with the field data. Finally, the universal function for free convection is reviewed.

The study of mean wind speed and mean temperature variation in the thermally stratified turbulent boundary layer presented in this paper is part of a long-range effort by personnel of the Fluid Dynamics and Diffusion Laboratory to reach a better understanding of such shear flows. In this effort the primary interests are an investigation of the turbulence structure and the similarity between laboratory and atmospheric-surface-layer flow characteristics. A knowledge of the mean flow behavior is essential to the effort and the companion effort to study turbulent diffusion in the atmospheric surface layer by laboratory simulation.

BASIC EQUATIONS

The principles of the similarity theory introduced by Monin and Obukhov[1], require that the profiles of a mean wind velocity and a mean temperature, expressed in a dimensionless form, be universal functions of $\zeta = z/L$, where z is the height of observation and L is the Monin-Obukhov length scale. Following the similarity theory, a dimensionless wind shear S was introduced by Lumley and Panofsky[2] as

$$S = \frac{kz}{u_*} \frac{\partial U}{\partial z} = S(\zeta) , \tag{1a}$$

where k is the von Karman constant, u_* is the friction velocity, and U is a mean wind velocity expressed as a function of height z. By analogy, a dimensionless lapse rate was defined in reference 2 as

$$R = \frac{\alpha z}{T_*} \frac{\partial T}{\partial z} = R(\zeta) , \tag{1b}$$

where T_* is the friction temperature, T is a mean wind temperature expressed as a function of height, and α is defined as the ratio of eddy conductivity K_h to eddy viscosity K_m in the following form:

$$\alpha = \frac{K_h}{K_m} = \frac{kT_*}{u_*} \frac{\partial U}{\partial z} \Big/ \frac{\partial T}{\partial z} .$$

When the above equation for α is substituted into Eq. (1b), the resulting equation will show that Eqs. (1a) and (1b) are indeed identical, in accord with the similarity sypothesis.

Equation (1a) can be expanded in a Taylor's series

$$S(\zeta) = S(\zeta_o) + \sum_{p=1}^{n} \frac{\zeta^p}{p!} S^p(\zeta_o) ,$$

where $S^p(\zeta)$ is the pth derivative of S with respect to ζ. Supposing that $S = (1+\zeta)^n$ and $n > 0$, where n is an integer, the above equation assumes the following form:

$$S(\zeta) = 1 + \sum_{p=1}^{n} \frac{n!}{(n-p)!\ p!} \zeta^p ,$$

where $\zeta^2 < 1$. Combining the above equation and Eq. (1a) and integrating over z , yields

$$U(z) - U_o(z_o) = \frac{u_*}{k} f_1(\zeta) ,$$

and

$$f_1(\zeta) = \ln \frac{\zeta}{\zeta_o} + \sum_{p=1}^{n} \frac{n!}{p(n-p)!\ p!} \zeta^p + \text{const.} \qquad (2)$$

Similarly, from Eq. (1b) for R having the same binomial form as S , one obtains the result

$$T(z) - T_o(z_o) = \frac{T_*}{\alpha} f_1(\zeta) .$$

Therefore, $f_1(\zeta)$ is a universal function of ζ. When $\zeta^2 << 1$, Eq. (2) is approximated by

$$f_1(\zeta) \approx \ell n \frac{\zeta}{\zeta_o} + n\zeta + \text{const.} ,$$

which is the familiar "log-plus-linear" relationship.

If there is no limitation on ζ, namely ζ can be any value, then the universal function has the following from when $S = (1+\zeta)^n$:

$$f_2(\zeta) = \ell n \frac{\zeta}{\zeta_o} + \sum_{p=1}^{n} \frac{1}{p}(1+\zeta)^p + \text{const.} \tag{3}$$

Eq. (3) reduces to Eq. (2) for $\zeta^2 < 1$. When $n < 0$ and the dimensionless wind shear is assumed to be $S = (1-\zeta)^n$, Eq. (3) should become

$$f_3(\zeta) = \ell n \frac{\zeta}{\zeta_o} - \ell n \frac{(1-\zeta)}{(1-\zeta_o)} + \sum_{p=1}^{m-1} \frac{1}{p}(1-\zeta)^{-p} + \text{const.},$$

where $m = -n$, or $m > 0$. Again, for $\zeta^2 << 1$, the above equation approximates the log-plus-linear law.

EXPERIMENTAL RESULTS AND DISCUSSION

The meteorological wind tunnel[3] was specifically designed and constructed in an effort to obtain flows simulating those in the atmospheric surface layer. The tunnel as shown in Fig. 1 has a test section of approximately 29 m long with a cross-section of nearly 2 m by 2 m. The wind-tunnel boundary layer can be made thermally stable or unstable (corresponding to inversion and lapse conditions) by flow of heated or cooled air over a horizontal flat plate which was either cooled or heated along the downstream 13 m. All measurements were made at the section approximately 26 m downstream from the test-section entrance. The thermal and momentum boundary-layer thickness were both approximately equal to 70 cm at this section. A range of Richardson number from 0.5 to -0.5 was achieved by cooling or heating the wind-tunnel floor and heating or cooling, respectively, the ambient air.

Mean-velocity profiles were measured by means of a calibrated pitot-static probe having an outside diameter of about 3mm. The probe was used with a capacitance-type pressure transducer (Trans-sonics type 120 Equibar Meter) and the dynamic pressure head was plotted automatically as a function of height z. Velocities were calculated with the air density corrected to the local temperature conditions.

Mean-temperature profiles were measured with a thermocouple mounted on an actuator. Data were taken point by point in the z-direction so that no time-lag effect of the thermocouple occurred in the measurements.

Measured mean wind velocity and temperature at height z_i in the region where $z_i << |L|$ can in general be expressed by

$$U_i(z_i) - U_o(z_o) = A_1 \ln z_i + B_1 z_i + C_1 , \tag{4a}$$

and

$$T_i(z_i) - T_o(z_o) = A_2 \ln z_i + B_2 z_i + C_2 . \tag{4b}$$

Constants in the above equations are defined as $A_1 = u_*/k$, $A_2 = T_*/\alpha$; B_1, C_1, B_2, and C_2 are constants. In order to have a similar mean wind velocity and temperature profile, it is necessary to have

$$\beta'/L' = \frac{B_1}{A_1} = \frac{B_2}{A_2} ,$$

where β' is an arbitrary constant and L' is defined as

$$L' = \alpha L = \frac{T_m}{g} \frac{u_*(\partial U/\partial z)}{k(\partial T/\partial z)} = \frac{T_m}{g} \frac{A_1^2}{A_2} .$$

Because B_1/A_1 is not always exactly equal to B_2/A_2, it is, therefore, assumed that

$$\beta' = \frac{L'}{2}\left(\frac{B_1}{A_1} + \frac{B_2}{A_2}\right) .$$

When the A's and B's are known, L' and β' can be obtained from the above equations, where T_m is the mean temperature, in absolute temperature scale, averaged over the profile and g is the gravitational acceleration. A digital computer (IBM 1620) was used to calculate all the results reported here.

There are two methods by which the A's and B's can be determined statistically. The first method is as follows: If a reference point is taken at height z_j (where $z_j \ll |L|$) and if the corresponding mean wind velocity and temperature are taken to be the reference velocity and temperature, respectively, then Eqs. (4a) and (4b) can be rewritten as

$$(U_i - U_j)/(z_i - z_j) = A_1(\ln z_i/z_j)/(z_i - z_j) + B_1 ,$$

and

$$(T_i - T_j)/(z_i - z_j) = A_2(\ln z_i/z_j)/(z_i - z_j) + B_2 .$$

The second method utilizes the linear regression with reference to the ensemble means. Equations (4a) and (4b) may be rewritten as

$$(U_i - U_m)/(z_i - z_m) = A_1(\ln z_i - \frac{1}{N}\sum_{i=1}^{N} \ln z_i)/(z_i - z_m) + B_1 ,$$

and

$$(T_i - T_m)/(z_i - z_m) = A_2(\ln z_i - \frac{1}{N}\sum_{i=1}^{N} \ln z_i)/(z_i - z_m) + B_2 ,$$

where the subscript m refers to the population mean. The regression line determined by the least square method will give the best fit of experimental data and correspondingly the best estimate of the A's and B's can be obtained. Consequently, L', β', u_*/k, and T_*/α are calculable and some of them are tabulated in Table I. The dimensionless forms of the above equations are then given by

$$\frac{kL'}{u_*} \frac{U_i - U_j}{z_i - z_j} - \beta' = L' \frac{\ln z_i/z_j}{z_i - z_j} , \tag{5a}$$

and

$$\frac{\alpha L'}{T_*} \frac{T_i - T_j}{z_i - z_j} - \beta' = L' \frac{\ln z_i/z_j}{z_i - z_j} , \tag{5b}$$

or

$$\frac{kL'}{u_*} \frac{U_i - U_m}{z_i - z_m} - \beta' = L' \frac{\ln z_i - \frac{1}{N}\sum_{i=1}^{N} \ln z_i}{z_i - z_m} , \tag{5c}$$

and

$$\frac{\alpha L'}{T_*} \frac{T_i - T_m}{z_i - z_m} - \beta' = L' \frac{\ell n\, z_i - \frac{1}{N} \sum_{i=1}^{N} \ell n\, z_i}{z_i - z_m} \qquad (5d)$$

For convenience, the following abbreviations are made:

$$RU = \frac{kL'}{u_*} \frac{U_i - U_j}{z_i - z_j} - \beta' ,$$

$$RT = \frac{\alpha L'}{T_*} \frac{T_i - T_j}{z_i - z_j} - \beta' ,$$

and

$$RZ = L' \frac{\ell n\, z_i / z_j}{z_i - z_j} .$$

Figure 2 shows both the Project-Prairie-Grass[4] and the wind-tunnel data. The wind-tunnel data taken at heights from 0.2 to 30 cm were obtained in the meteorological wind tunnel of the Fluid Dynamics and Diffusion Laboratory.

Equations (5a) - (5d) can also be rewritten in another dimensionless form as follows:

$$k(U_i - U_j)/u_* - \beta'(z_i - z_j)/L' = \ell n\, z_i / z_j ,$$

$$\alpha (T_i - T_j)/T_* - \beta'(z_i - z_j)/L' = \ell n\, z_i / z_j ,$$

or

$$k(U_i - U_m)/u_* - \beta'(z_i - z_m)/L' = \ell n\, z_i - \frac{1}{N}\sum_{i=1}^{N} \ell n\, z_i \ ,$$

$$\alpha(T_i - T_m)/T_* - \beta'(z_i - z_m)/L' = \ell n\, z_i - \frac{1}{N}\sum_{i=1}^{N} \ell n\, z_i \ .$$

For a neutrally stratified flow, L' approaches infinity. Hence, the second term on the left-hand side of the above equations should be negligible in the near-neutral flows. For convenience, RUU represents the terms on the left-hand side of the first or the third equation; RTT, the terms on the left-hand side of the second or the fourth equation; and RZZ, the terms on the right-hand side of the last equation. Figure 3 shows the results of the first two equations while Fig. 4 gives the results of the last two equations. Data shown in these two figures are the same data as those presented in Fig. 2 with $N = 6$ for the field data and $N = 15$ for the laboratory data.

In the analysis of these data, it was found that the condition that ζ should be much less than unity in the range of heights considered, no longer held in some cases for both field and laboratory. It is conceivable that L' might diminish as a flow approaches the extremely stable or unstable thermal stratification. It can, therefore, be concluded that the log-plus-linear law should not be applied over the same height range for a diversity of thermal stabilities.

If the second-order term of ζ is retained in the series expansion, the measured mean wind velocity and temperature at height z can be expressed by

$$U - U_o = A_1 \ln z + B_1 z + C_1 z^2 + D_1 ,$$

and

$$T - T_o = A_2 \ln z + B_2 z + C_2 z^2 + D_2 ,$$

where the A's , B's , and C's are defined as before and D's are arbitrary constants. The A's , B's , and C's can be determined by means of the least-square method. The following dimensionless parameters are defined as

$$RUUU = k(U_i - U_j)/u_* - \beta_1'(z_i - z_j)/L' - \beta_2'(z_i^2 - z_j^2)/L'^2 ,$$

and

$$RTTT = \alpha(T_i - T_j)/T_* - \beta_1'(z_i - z_j)/L' - \beta_2'(z_i^2 - z_j^2)/L'^2 ,$$

where

$$\beta_1' = \frac{L'}{2}\left(\frac{B_1}{A_1} + \frac{B_2}{A_2}\right) ,$$

and

$$\beta_2' = \frac{L'^2}{2}\left(\frac{C_1}{A_1} + \frac{C_2}{A_2}\right) .$$

Figure 5 shows the log-plus-linear-plus-square profile. Only the wind-tunnel data were used for this analysis.

When the above equations are divided by $(z_i - z_j)/L'$, another form of dimensionless parameter can be obtained which will be defined as RUUUU for mean velocities and RTTTT for mean temperature. Figure 6 is a plot of these two parameters versus RZ which was defined before. Figures 2 to 6 show that little improvement on the data scatter was achieved by including the square term in the approximation of a universal function.

For a very unstable thermal stratification, the flow approaches a free convection and the universal function has the following form[5]:

$$f_4(\zeta) = C\zeta^{-1/3} + \text{const.} ,$$

where C is an arbitrary constant. The dimensionless wind shear is then given by

$$S(\zeta) = \frac{C}{3}\left|\zeta\right|^{-1/3} ,$$

which can be rewritten in terms of the gradient Richardson number as

$$S = \frac{C^{3/4}}{(27\alpha)^{1/4}} |Ri|^{-1/4} , \tag{6}$$

where

$$Ri = \frac{g}{T} \frac{(\partial T/\partial z)}{(\partial U/\partial z)^2} .$$

The dimensionless wind shear and Richardson number can be estimated from the measured mean wind velocity and temperature profiles in conjunction with Eq. (1a) and the last equation. Figure 7 shows the laboratory data as well as the field data for thermally unstable flows. Equation (6) is also shown. According to Gurvich[5] and Priestley[6], the transition from the forced convection to the free convection should occur at any Richardson number in the range from -0.03 to -0.05 and it should be a gradual rather than an abrupt transition. The present laboratory data agree very well with the field data. The deviation from Eq. (6) at the lower Richardson number or highly unstable region revealed in this figure is also present in the results of Gurvich.

Referring to the binomial form of the dimensionless wind shear presented in the last section, it may be assumed that n is a real number and $\xi^2 >> 1$. Then $f_4(\xi)$ is only a solution with $n = -1/3$ given by Gurvich[5]. For cases where ξ^2 approaches unity, it is assumed that the dimensionless wind shear is given by

$$S(\zeta) = (1 + \zeta)^n ,$$

where $\zeta = z/L = \alpha$ Ri S and $n > 0$. Therefore, the above equation can be rewritten as

$$Ri = -\frac{1 - S^{\frac{1}{n}}}{\alpha S} \tag{7}$$

Both the laboratory and the field data for thermally unstable flows conform to a line with $n = 1.5$, provided that α is considered to be unity. The above equation fits the data better than Eq. (6). Consequently, the universal function, for $\zeta < 0$ and n not an integer, should assume the form

$$f_5(\zeta) = \ln\left(\frac{(1+\zeta)^{1/2} - 1}{(1+\zeta)^{1/2} + 1}\right) + 2(1+\zeta)^{1/2} + \frac{2}{3}(1+\zeta)^{3/2} + \text{const.}$$

SUMMARY

Measured mean wind velocity and temperature profiles in the thermally stratified flow of a wind tunnel near the wall can be expressed in a log-plus-linear law which is also representative of the field data. Although both laboratory and field data show some scatter around the theoretical line, a similarity between the mean wind velocity and temperature profiles does seem to exist. The scatter can partially be attributed to the fact that in some cases the Monin-Obukhov length scale becomes comparable in magnitude to the height under consideration. This in turn breaks down the assumption that was made in order to obtain the law. Subsequently, log-plus-linear-plus-square profiles were tried and the results showed little imporvement on the scatter.

Free convection seems to prevail in some runs of both laboratory and field experiments. The transition from a forced convection to a free convection in the thermally stratified flow was shown to be gradual and occurred at a Richardson number in the same range as predicted. Both the laboratory and the field data for $\zeta < 0$ were best represented by a universal function of the form $f_5(\zeta)$ where $n = 3/2$.

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TABLE I. CHARACTERISTICS OF SOME WIND-TUNNEL AND PROJECT-PRAIRIE-GRASS DATA.

Source	U_2 cm/sec	T_2 ^{0}C	T_2-T_1 ^{0}C	z_2-z_1 cm	L' cm	β'	u_*/k cm/sec	T_*/α ^{0}C
WT	116	51.1	39.4	30.2	18.8	-0.015	23.6	9.42
WT	175	53.3	37.2	30.2	36.0	0.586	28.1	7.00
WT	284	59.5	35.6	30.2	54.9	2.60	32.7	6.35
WT	150	2.89	-26.7	30.3	-28.6	2.55	25.7	-6.61
WT	219	0.24	-25.7	30.3	-44.7	4.11	31.3	-6.23
WT	300	-1.94	-33.2	30.3	-64.0	4.30	40.4	-7.27
PPG	304	20.1	4.46	775	472	2.46	33.3	0.699
PPG	560	28.7	-4.39	1575	-1690	1.66	88.5	-1.43
PPG	445	21.0	6.73	1575	902	5.36	44.5	0.652
PPG	378	24.6	-4.03	1575	-579	0.793	57.5	-1.73
PPG	555	27.9	0.83	1575	7620	10.6	61.8	0.153
PPG	508	25.2	1.52	1575	2200	6.15	37.9	0.198
PPG	609	27.2	5.55	1575	1020	7.49	34.5	0.357
PPG	788	34.6	-1.23	1575	-5860	-3.83	72.5	-0.281
PPG	362	28.6	-4.38	1575	-399	0.549	46.6	-1.68
PPG	295	28.9	-3.68	1575	-437	0.620	38.0	-1.02

LIST OF SYMBOLS

Symbol	Definition
A	$A_1 = u_*/k$, cm/sec; $A_2 = T_*/\alpha$, ^{0}C
B	Arbitrary constants
c_p	Specific heat of air at constant pressure, calories/^{0}C/gm
C	Arbitrary constants
D	Arbitrary constants
f	Dimensionless function
g	Gravitational acceleration, cm/sec^2
H	Heat flux in the vertical direction, calories/cm^2/sec
k	von Karman constant
K_h	Eddy thermal conductivity, cm^2/sec
K_m	Eddy viscosity, cm^2/sec
L	Monin-Obukhov length scale, $L' = \alpha L$, cm
n	Integers or rational numbers
N	Total number of data collected in a profile
p	Integers
R	Dimensionless lapse rate
Ri	Richardson number
S	Dimensionless wind shear
T	Mean absolute temperature, ^{0}K

LIST OF SYMBOLS - continued

Symbol	Definition
T_*	$-H/(c_p \rho k u_*)$, friction temperature, ^{0}C
u_*	Friction velocity, cm/sec
U	Local mean velocity, cm/sec
z	Height, cm
α	K_h/K_m
β'	Arbitrary constant
ζ	z/L, dimensionless height
ρ	Density of air, gm/cm^3
$(\ )_i$	The variable at height z_i
$(\ )_o$	The variable at height z_o, an equivalent roughness height

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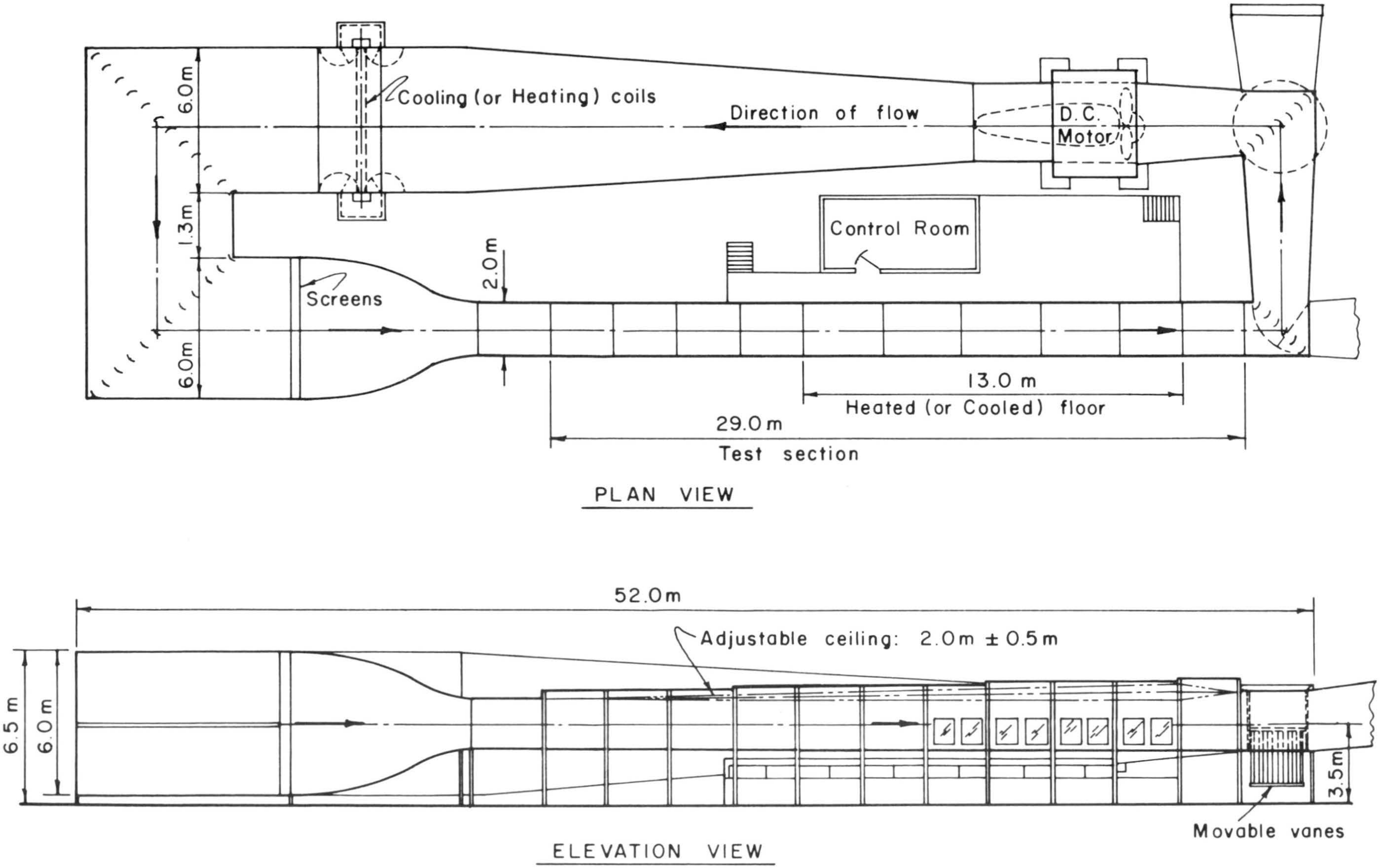


Figure 1. Meteorological wind tunnel

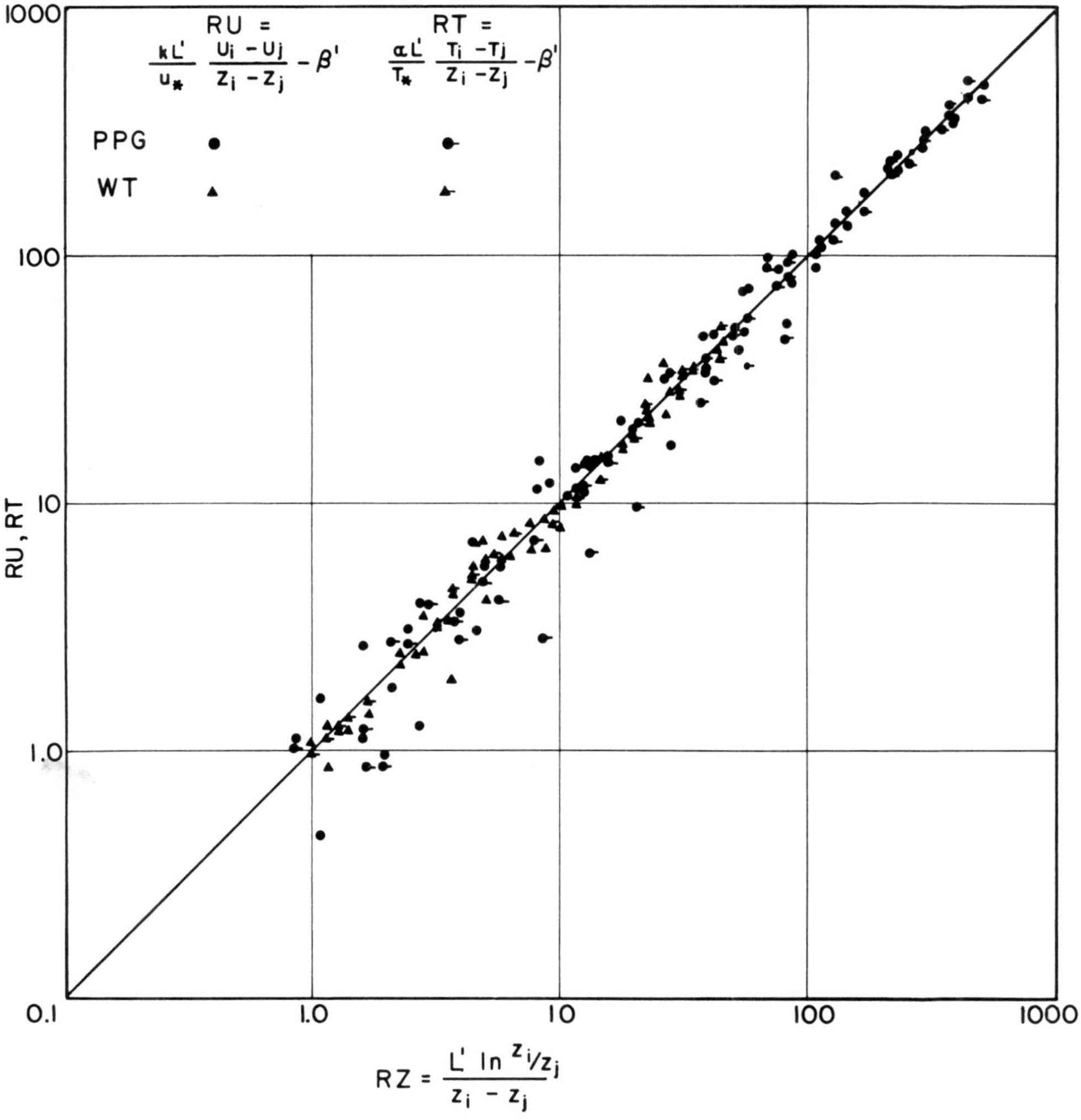
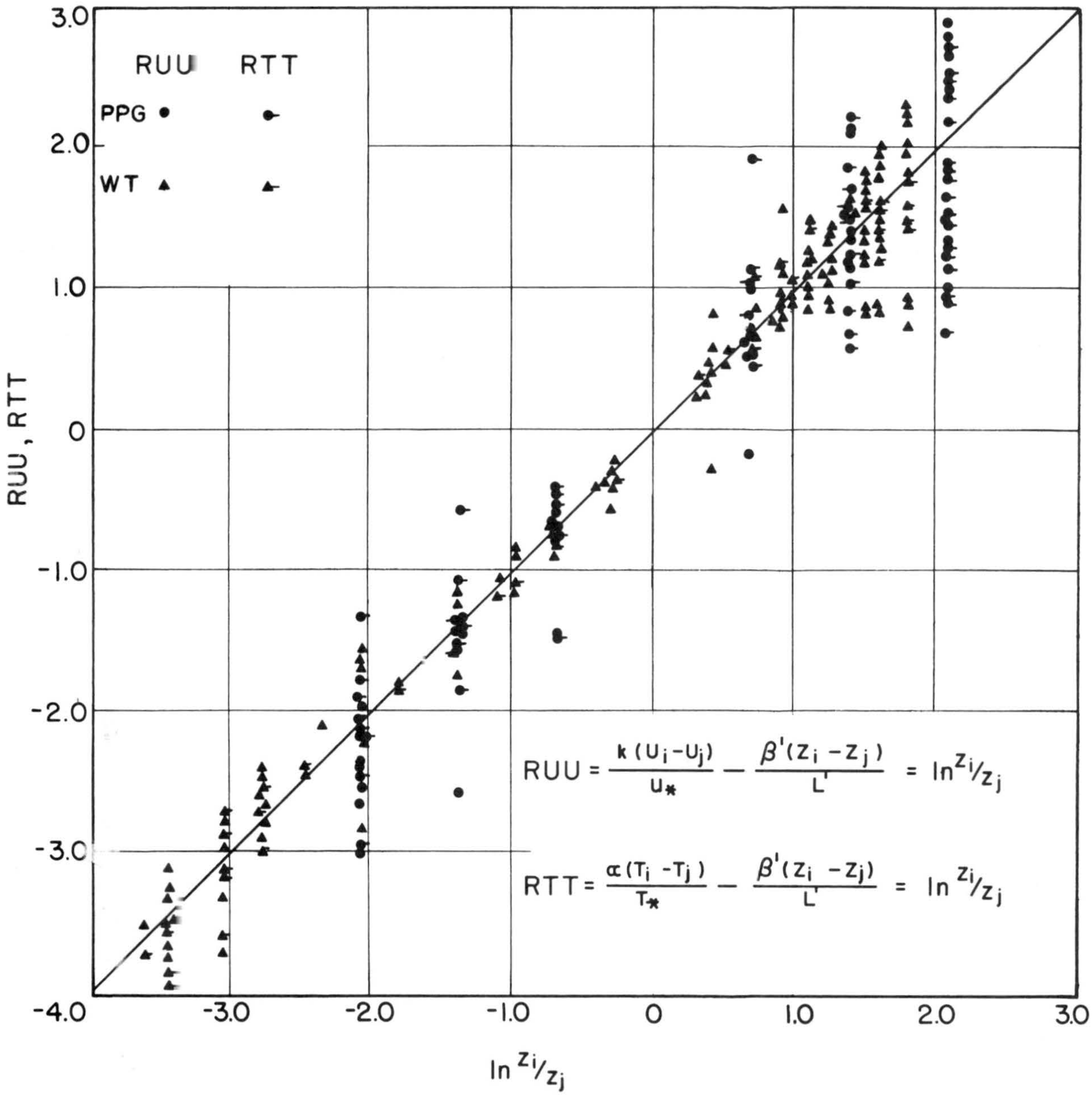


Figure 2. Similarity law profile in terms of the dimensionless mean wind velocity, RU , and temperature RT



Similarity law

Figure 3. Similarity law profile in terms of the dimensionless mean wind velocity, RUU, and temperature, RTT

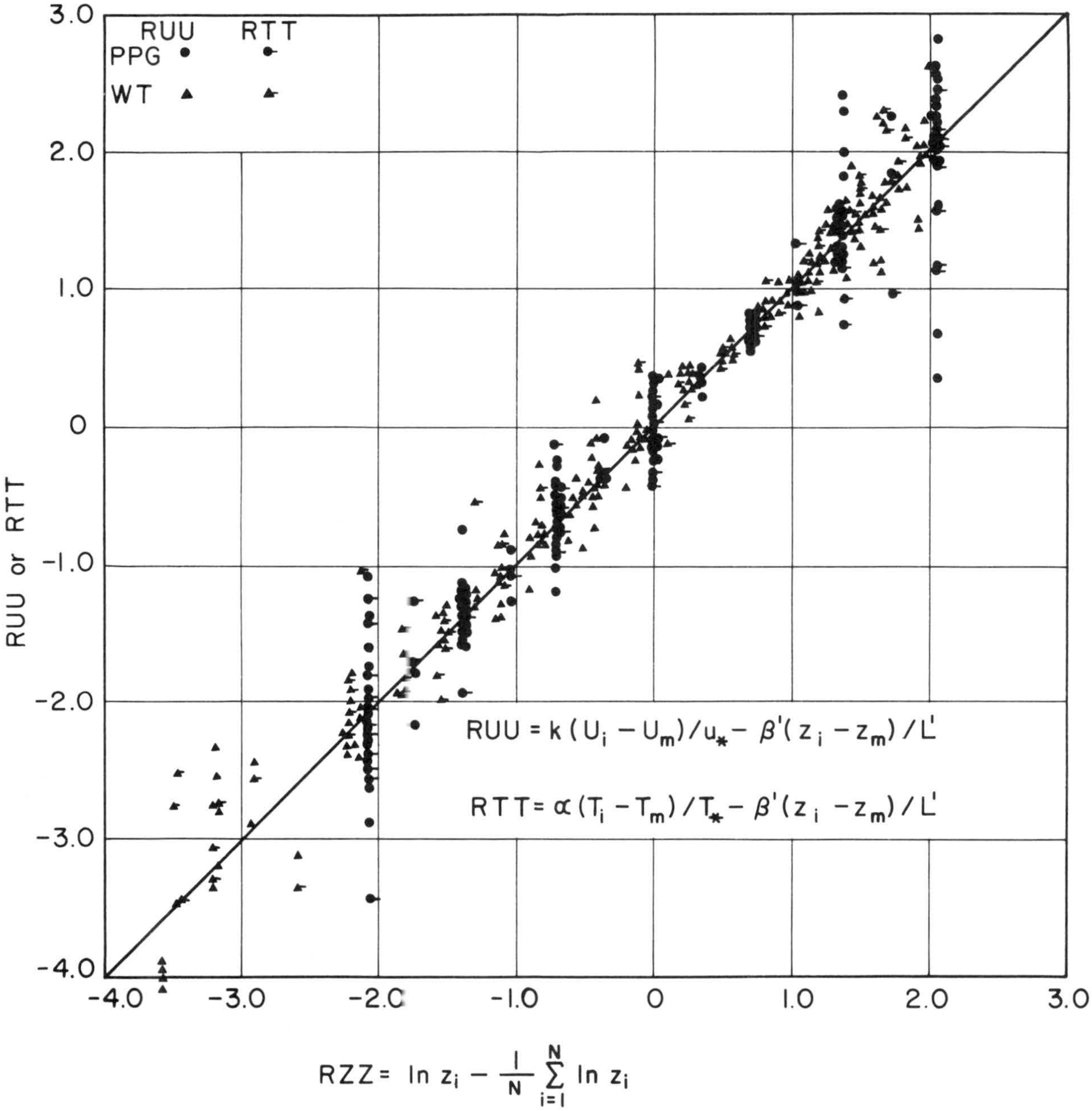


Figure 4. Similarity law profile by the method based on regression theory

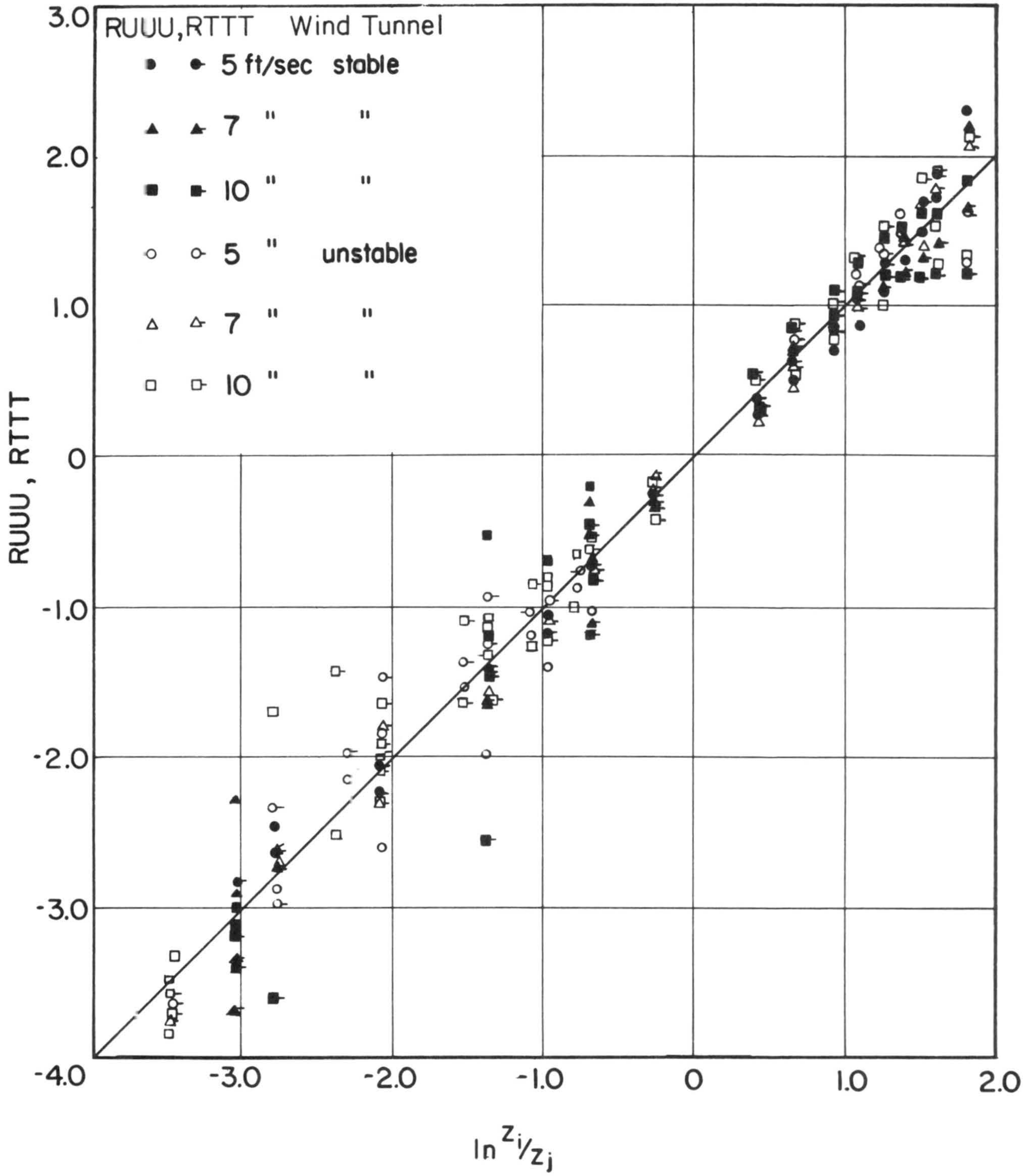


Figure 5. Log-plus-linear-plus-square profile in terms of the dimensionless mean wind velocity, RUUU, and temperature, RTTT

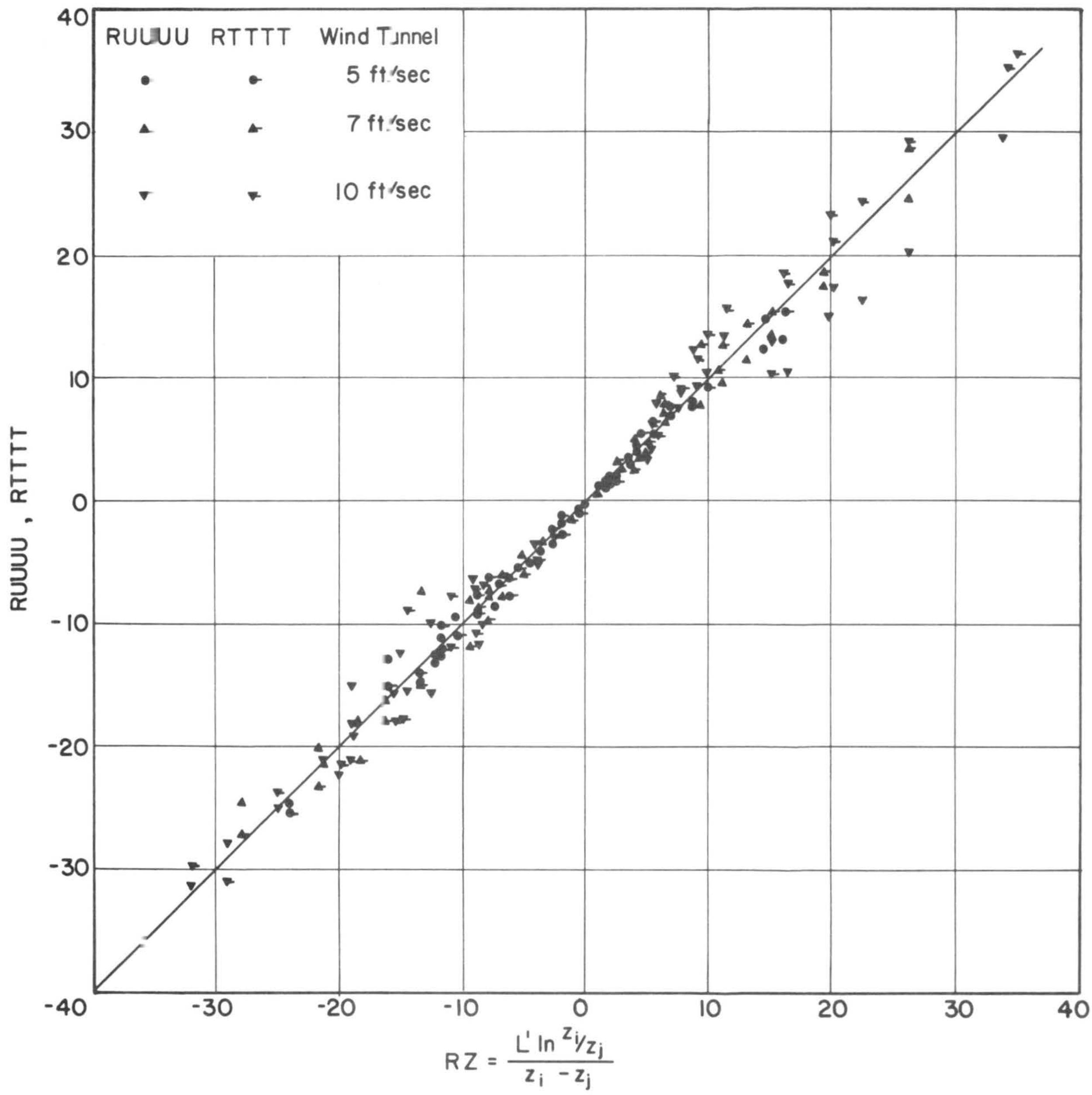


Figure 6. Log-plus-linear-plus-square profile in terms of the dimensionless mean wind velocity, RUUUU , and temperature, RTTTT

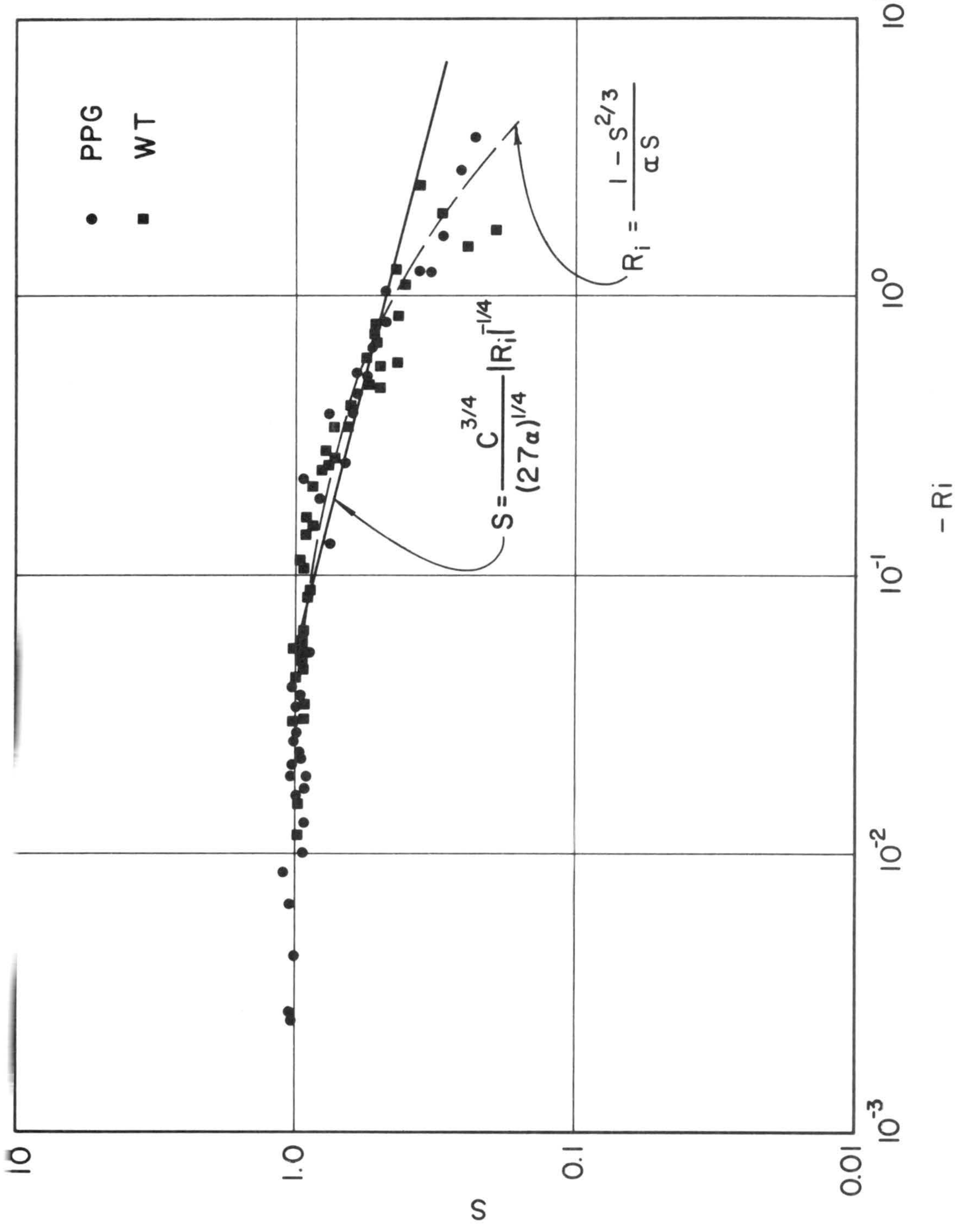


Figure 7. Dimensionless wind shear in thermally unstable flows as a function of Richardson number

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13. ABSTRACT

Mean wind velocity and temperature profiles were measured in a wind-tunnel boundary layer made thermally stable or unstable (corresponding to inversion and lapse conditions) by flow over a horizontal flat plate (29 m long and 2 m wide) which was either cooled or heated along the downstream 13 m. These profiles for heights from just above the viscous region up to half the boundary-layer thickness (about 55 cm) were fitted to the log-plus-linear and log-plus-linear-plus-square relationships. Field data consisting of mean temperature and wind speeds taken at heights from 0.25 m up to 16 m were also plotted. Similarity between the mean wind velocity and the mean temperature profiles for both laboratory and field data was found to exist. Retainment of the square term in the series approximation of the dimensionless wind shear for the case of $(z/L)^2 < 1$, where z is the height and L is the Monin-Obukhov length scale, did n ot substantially reduce the data scatter. For thermally unstable flows, the dimensionless wind shear is expressed by the form $(1+z/L)^n$ with $n=3/2$.

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