Computational Analysis of a Nonlinear Driven Damped Harmonic Oscillator

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Overview

• Brief Overview of Project

• Definition of Key Aerodynamic Principles

• Description of Research

• Mathematical Models

• Results and Future Research Opportunities
Project Description

• Effect of Blade Design on Efficiency of Wind Turbines

• Sub-project to analyze model of harmonic oscillator, e.g. a mass on a spring
Definition of Key Aerodynamic Principles

• Aerodynamic Forces: Lift vs. Drag
  
• Angle of Attack $\alpha$
  
• Normal Stall vs. Dynamic Stall
  
• Rapid variations in $\alpha$
  
• Hysteresis
Original Wind Tunnel Model

- Ongoing project with Composite Materials Research Group (CMRG)
- Research regarding dynamic stall, effect of airfoil design
Experimental Problem

- How physical can we make this model?

- The windmill blades will experience torsional vibration in addition to the change in $\alpha$

- Need to find a way to model this vibration

- Solution: nonlinear spring coefficient
Mathematical Models

- One-dimensional harmonic oscillator in $\alpha = \Phi$

\[ \ddot{\Phi} + 2\xi \omega \dot{\Phi} + \omega^2 \Phi = F_0 \sin(\omega_f t) + Q(t) \]

\[ \omega = \sqrt{\frac{k}{I}} \]

\[ \dot{\Phi} = \frac{\partial \Phi}{\partial t} \]
Linear Solution
Nonlinear $k$ term

\[ k_\theta = r_1 k \frac{\sin(\theta)}{\theta} x \]

\[ x = \sqrt{(r_2 - r_1) + r_1 r_2 \sin^2(\theta)} \]
Nonlinear Solution
Hysteresis Curve
Hysteresis Split

Upper Hysteresis Curve: Average

\[ y = -1E-05x^3 + 0.0005x^4 - 0.0053x^3 + 0.0081x^2 - 0.1119x - 2.9604 \]

\[ \alpha \]

Lower Hysteresis Curve: Average

\[ y = 9E-06x^5 - 0.0004x^4 + 0.0042x^3 + 0.0206x^2 - 0.6883x - 4.3551 \]

\[ \alpha \]
Future Research Opportunities

• Comparison to hysteresis curves from unsteady model or quasi-steady model with Theodorsen’s function applied

• Improvement of future dynamic stall models, and wind foil analysis and design

• Potential for further analysis into the aerodynamics of the windfoil

• Calculation of variables of interest, such as power generation of the turbine