ESTIMATING THE INFLUENCE OF STRESS ON ELASTIC WAVE VELOCITIES FROM MEASUREMENTS MADE AT DIFFERENT SCALES ON SANDSTONE FACIES

by

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Geophysics).

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ABSTRACT

Knowing how stress influences velocity is crucial for seismic reservoir monitoring because seismic reflection signatures are sensitive to the dynamic processes related to production or injection. In this thesis, I address this issue by conducting research to quantify the influence of stress on elastic wave velocities at different measurement scales. I compare the influence of stress on elastic wave velocities measured from borehole sonic data, time-lapse multicomponent surface seismic data, and laboratory core experiments. To make this comparison, I develop a methodology by combining a series of existing techniques to estimate the influence of stress on shear-wave velocity acquired from crossed-dipole borehole sonic data. By combining the crossed-dipole radial profile of shear wave velocity with the redistribution of stress caused by the presence of a borehole, I establish a relationship between velocity and stress from borehole measurements made within the reservoir. Previously, information about the influence of stress on elastic wave velocity has been primarily obtained through laboratory measurements.

To estimate the magnitude of far-field principal stresses at the reservoir level in the subsurface I first integrate the formation bulk density well log to approximate the stress exerted by the overburden. Then, I use radial profiles of shear-wave slowness obtained from borehole sonic data to estimate the far-field principal horizontal stresses. Estimates of the far-field principal stresses are needed to model the stress redistribution caused by the drillout of a borehole.

To study the influence that borehole stress redistribution has on shear-wave velocity, I combine the borehole stresses that influence a vertically propagating fast shear wave and the radial profile of the fast shear wave velocity. I use an excess compliance rock physics model to extrapolate the borehole shear wave velocity/stress relationship to predict the influence stress has on compressional wave velocity.
Time-lapse multicomponent surface seismic data were recorded to monitor a water alternating gas (CO$_2$) injection scheme for enhanced oil recovery. Interpretations of the time-lapse seismic data, in conjunction with a reservoir simulation model, indicate regions of increased reservoir pore pressure. To simulate the reservoir conditions during a water alternating gas injection scheme, I conducted laboratory core experiments in which I measured compressional and shear-wave velocities as a function of confining pressure, pore pressure, fluid type (which included CO$_2$ in the gas and supercritical phase), and lithology.

The shear wave velocity from the borehole sonic data and the time-lapse multicomponent surface seismic data show a very similar response to changes in stress. The laboratory experiments indicate the core velocity measurements are less sensitive to a change in stress than both the borehole and seismic data. Borehole sonic data provide an alternative and, in this case, a more accurate method than laboratory core measurements for estimating stress-induced changes in shear-wave velocity within a reservoir. An accurate rock-physics relationship between shear-wave velocity and effective stress is required for a quantitative interpretation of pore pressure changes from multicomponent time-lapse seismic data.
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<tr>
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Elastic Wave Velocity \( v_{ij} \)

Excess Borehole Pressure \( P_b \)

Fast Shear Wave Velocity \( V_{S, Fast} \)

Grain Discontinuities \( \alpha_{ij} \)

Grain Discontinuities \( \beta_{ijkl} \)

Gravitational Acceleration \( g \)

Lamé Parameter \( \lambda \)

Normal Compliance of Grain Boundaries \( Z_N \)

Normal Stress \( \sigma_n \)

Poisson’s Ratio \( \nu \)

Pore Pressure \( P_p \)

Shear Compliance of Grain Boundaries \( Z_T \)

Shear Modulus \( \mu \)

Shear Wave Impedance \( Z_{V_S} \)

Slow Shear Wave Velocity \( V_{S, Slow} \)

Stiffness Coefficient \( c_{ijkl} \)

Stoneley Wave Velocity \( V_{Stoneley} \)

Third-Order Stiffness Coefficient \( c_{ijklmn} \)

Total Maximum Horizontal Stress \( S_{H_{\text{max}}} \)

Total Minimum Horizontal Stress \( S_{h_{\text{min}}} \)

Total Vertical Stress \( S_V \)

Total Stress Tensor \( S_{ij} \)
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<td>API</td>
<td>American Petroleum Institute</td>
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<tr>
<td>BHA</td>
<td>Bottomhole Assembly</td>
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<td>CO₂</td>
<td>Carbon Dioxide</td>
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<td>P-wave</td>
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<td>Photoelectric</td>
</tr>
<tr>
<td>RCP</td>
<td>Reservoir Characterization Project</td>
</tr>
<tr>
<td>S-wave</td>
<td>Shear Wave</td>
</tr>
<tr>
<td>SRVP</td>
<td>Shear Wave Radial Variation Profile</td>
</tr>
<tr>
<td>STC</td>
<td>Slowness-Time Coherence</td>
</tr>
<tr>
<td>NaCl</td>
<td>Sodium Chloride</td>
</tr>
<tr>
<td>scf</td>
<td>Standard Cubic Feet</td>
</tr>
</tbody>
</table>
Stock Tank Barrel ................................................................. bbl

Quantitative Evaluation of Minerals by Scanning Electron Microscopy . . . . . QEMSCAN

Water Alternating Gas ......................................................... WAG
ACKNOWLEDGMENTS

First, I thank my wife, Jessica, my daughter, Julianna, and my son, Mato, for all of their love, inspiration, and encouragement. I want to thank my father, Donald, my mother, Lydia, and my brother, Damian, for their love, support, and prayers. I want to thank my father-in-law, Jeff Bachmann, and my mother-in-law, Jo Ann Bachmann, for their understanding and encouragement.

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Most importantly, I thank the Great Spirit.
Dedicated to Jessica Eileen, Julianna Avellina and Mato Joaquin
CHAPTER 1
INTRODUCTION

As of 2010 there are 128 carbon dioxide (CO₂) enhanced oil recovery (EOR) projects worldwide using carbon dioxide CO₂ injection as a tertiary recovery method, with 113 in the United States (Kootungal, 2010). Often the criterion for successful seismic monitoring of CO₂ projects is the ability to collect high resolution seismic data because since seismic methods have been shown to be very effective in detecting time-lapse changes (Davis et al., 2003; Landro, 2001; Pawar et al., 2006; Robinson and Davis, 2011; Tura and Lumley, 1999). The effectiveness of seismic monitoring relies upon the changes in acoustic and elastic impedance caused by injection and production. These changes are then observed in the amplitude and travelt ime of seismic waves.

Time-lapse seismic methods, utilizing compressional waves and shear waves, can ideally improve identification of flooded or pressurized compartments within the reservoir during CO₂ EOR operations. The use of compressional wave data is not a definitive method for detection of pressured regions, but the addition of shear wave data may provide a distinction between a gas-bearing zone and a zone of high pore pressure (Nur and Wang, 1998), this is a good motivation for using shear waves as a time-lapse tool. An example provided by Xue et al. (2005) explains if an observed time-lapse compressional wave anomaly is caused by a decrease in velocity without an associated shear wave anomaly, the zone has most likely been flushed with CO₂ without any changes in pore pressure. However, if the shear wave data show a similar anomaly because of a reduction in shear wave velocity, the zone may have undergone a pore pressure buildup in addition to being flushed with CO₂. Identifying the impact of changing stresses within the reservoir is as important as recognizing the effects of saturation.
Saturation effects are typically estimated from fluid substitution modeling (e.g. Gassmann, 1951). Since fluid substitution methods do not directly account for changing rock properties with respect to changes in stress, additional information must be used to account for the effects of an altered stress state. This additional information may be directly obtained from crossed-dipole sonic well logs and experimental core studies. The purpose of this research is to show how a crossed-dipole sonic well log and an experimental core study are used to estimate the influence of stress on elastic wave velocities.

The contribution of this study applies directly to a major ongoing research project at Colorado School of Mines (CSM). The Reservoir Characterization Project (RCP) is an industry-funded academic consortium at CSM with a primary focus on applied research and knowledge dissemination in integrated reservoir characterization. For the RCP’s Phase XII project at Postle Field the research goal is to monitor a CO₂ EOR project by way of incorporating all geoscience and engineering data to effectively manage and enhance the oil recovery process. The RCP has a history of using compressional, shear and converted-wave time-lapse seismic acquisition to monitor CO₂ floods for EOR. The RCP conducted the first ever time-lapse, multicomponent seismic survey in conjunction with a CO₂ huff-n-puff project at Vacuum Field, New Mexico. Multicomponent time-lapse seismic data was used to monitor the EOR project at Weyburn Field, Saskatchewan, Canada. The current research project at Postle Field is also a first ever time-lapse study of a water alternating gas (i.e. CO₂) injection scheme for EOR.

1.1 Postle Field Overview

Postle Field, a large mature oil field in Anadarko Basin, was discovered in 1958 in the Oklahoma Panhandle (Texas County) near Guymon. Primary production began in 1961 and waterflood was instigated in 1967. The field produced 92 million barrels of oil under primary and waterflood production from the Morrow A sandstone. Oil production peaked at about 23,000 barrels of oil per day in 1970 and averaged about 2,000 barrels of oil per day when CO₂ injection began on November 15th at a rate of 35 million standard cubic feet per day.
Postle Field is divided into four producing units one of which is the Hovey Morrow Unit (HMU) that is addressed in this study. Primary production of oil and gas from Postle Field began in 1961. Starting in 1974, waterflooding was implemented to improve declining primary production. WAG flooding using CO$_2$ as a tertiary method to enhance oil recovery started in 2007 and has pushed production to an all time high. Waterflooding led to early water breakthrough and very high water-cut in producing wells. Both the early water breakthrough and the high water-cut suggest poor sweep efficiency and poor waterflood performance which are attributed to high permeability channels and permeability baffles in the reservoir (Heris, 2011).

Analyzing production performance for of various production units in Postle Field is complicated as described by Wright (2007):

Postle Field carbon dioxide flood marks the *first time anyone in the industry* had attempted a major CO$_2$ enhanced oil recovery project at such a late stage of a waterflooded field. Prior to CO$_2$ injection Postle Field produced oil with a 98% water cut.

A general production timeline on Postle Field and the average reservoir properties of the Morrow A sandstone is summarized in Table 1.1 (modified from Heris, 2011).

WAG injection at Postle Field for EOR is the volume of water to CO$_2$ injected into the reservoir and is referred to as the WAG ratio. The WAG ratio is expressed in reservoir volumes at reservoir conditions. The average WAG ratio implemented at Postle Field is 0.35 to 1. The reservoir pressure is maintained above the minimum miscibility pressure of 14.48 MPa (2100 psi) by injecting water and CO$_2$ at 29.65 (4300 psi) and by keeping the bottom hole pressure of a producing well higher than 6.89 MPa (1000 psi) (Heris, 2011).
Table 1.1: Major production timelines and average reservoir properties of Morrow A sandstone at Postle Field (modified from Heris, 2011).

<table>
<thead>
<tr>
<th>Event</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Discovery</td>
<td>1958</td>
</tr>
<tr>
<td>Start of waterflood</td>
<td>1967</td>
</tr>
<tr>
<td>Start of CO$_2$ WAG flood</td>
<td>1995</td>
</tr>
<tr>
<td>Average reservoir depth</td>
<td>6100 feet</td>
</tr>
<tr>
<td>Average reservoir thickness</td>
<td>45 feet</td>
</tr>
<tr>
<td>Average reservoir porosity</td>
<td>17%</td>
</tr>
<tr>
<td>Average reservoir permeability</td>
<td>47 mD</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>64°C (147 °F)</td>
</tr>
<tr>
<td>Initial reservoir pressure</td>
<td>11.67 MPa (1695 psi)</td>
</tr>
<tr>
<td>Oil gravity</td>
<td>38.8 API</td>
</tr>
<tr>
<td>Bubble point pressure</td>
<td>11.65 MPa(1690 psi)</td>
</tr>
<tr>
<td>Minimum miscibility pressure</td>
<td>14.48 MPa(2100 psi)</td>
</tr>
</tbody>
</table>

1.2 Data

The data used in this thesis to characterize the Morrow A sandstone at Postle Field from the RCP study area of the HMU unit.

- One core from well HMU 35-3
- Well log and core porosity and permeability from well HMU 13-2
- Twelve recent well logs, the same well logs used to characterize the well log facies model
- Monopole and crossed-dipole borehole sonic well log from well HMU 24-4
- Compressional wave and shear wave time-lapse interpretations for the time period between the baseline seismic survey (March, 2008) and the first seismic monitor survey (December, 2008). Additional information on the time-lapse multicomponent seismic surveys and their interpretation can be found in Zerpa (2011) and Singh (2013).
- Reservoir simulation model results for reservoir pressure change for the time period between the baseline seismic survey (March, 2008) and the first seismic monitor survey
(December, 2008). Additional information on the geologic modeling and reservoir flow simulation can be found in Jobe (2010) and Heris (2011).

1.3 Postle Field Geologic Environment

The geologic environment of the producing Upper Morrow Sandstone Formation reservoir at Postle Field is a Pennsylvanian-age siliciclastic incised valley-fill system. Typical Morrow sediments consist of shales punctuated by valley-fill sand deposits (Sonnenberg, 1985). The Morrow Formation is encased between two high stand limestones (Benton, 1973). The upper boundary is a conformable surface at the base of the Atoka Thirteen Fingers Limestone. The lower boundary of the Morrow Formation is an unconformable surface above the Mississippian Chester Limestone. The deposition of Postle Field during Pennsylvanian time was a relative sea-level lowstand shoreline where the Morrow Formation is much sandier (Jobe, 2010). The primary producing units in the Morrow Formation are the Upper Morrow A, A1, and A2 sandstones. At Postle Field, the producing formation that is undergoing the CO\textsubscript{2} flood is the Morrow A sandstone. A generalized stratigraphic column for Postle Field is shown in Figure 1.1.

Jobe’s (2010) recent study on the Morrow A sandstone includes core descriptions and a sedimentary facies analysis based on four well cores from Postle Field. Sedimentary facies associations were characterized based on lithology, grain size, texture, and sedimentary and biogenic structures observed in the core. The four sedimentary facies associations are shown in Table 1.2 (modified from Jobe, 2010).

Table 1.2: Sedimentary facies association of the Morrow A sandstone at Postle Field (modified from Jobe, 2010).

<table>
<thead>
<tr>
<th>Facies Association</th>
<th>Depositional Environment</th>
<th>Depositional System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominant Discharge Basal Lag</td>
<td></td>
<td>Active channels</td>
</tr>
<tr>
<td>Dominant Discharge Channel Fill</td>
<td></td>
<td>Braided River System</td>
</tr>
<tr>
<td>Subordinate Discharge Channel Fill</td>
<td></td>
<td>Abandonment Areas</td>
</tr>
<tr>
<td>Intra Channel Belt Abandonment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1.1: Generalized stratigraphic column for Postle Field, the gas producing formations are indicated by the red markers and the oil producing formations are indicated by the green markers. The reservoir formation at Postle Field is the Upper Morrow A sandstone.
The sedimentary facies associations suggest the Morrow A sandstone at Postle Field as being deposited by a braided river contained by three structurally controlled cut and fill packages within a larger incised valley fill system (Jobe, 2010).

1.4 Research Objectives

Production at Postle Field comes from complex reservoir compartments involving several different flow units related to the sedimentary facies associations. Properly understanding the dynamic behavior of a stratigraphically complex stress-sensitive reservoir like the Morrow A sandstone at Postle Field requires an integrated methodology that combines core, well log, and high resolution multicomponent seismic data. “Monitoring changes in the stress field and identifying reservoir connectivity can lead to increasing recovery efficiency by enabling conformance control and implementation of proactive reservoir management practice” (Davis, 2007).

This research addresses the importance of measurement scale by comparing the stress dependence of elastic wave velocities made at the laboratory scale, the borehole scale, and the surface seismic scale. For example the laboratory scale measurements are usually recorded at ultrasonic frequencies (≈20 kHz-10 MHz), borehole well logs are usually recorded at sonic frequencies (≈1-20 kHz) and surface seismic data are acquired over the frequency range of ≈10-200 Hz.

The relevance of this research lies in the application to reservoirs undergoing changes in pore pressure. It will extend research in the area of stress dependence of reservoir rocks, i.e. geomechanics. This study provides a reservoir level analysis of how shear wave velocity, from borehole sonic measurements, is influenced by the radial redistribution of stresses caused by the drillout of a borehole.

The objective of this research is to quantify the influence of stress on elastic wave velocities at different measurement scales. To address this issue I compare the influence of stress on elastic wave velocities measured from laboratory core experiments, borehole sonic data, and time-lapse multicomponent surface seismic data.
1.5 Thesis Summary

Chapter 2 is devoted to the application of using standard well logs to identify three different quality reservoir zones along with an interbedded shale at Postle Field, Oklahoma. The three reservoir well-log facies are interpreted as high-quality reservoir, intermediate-quality reservoir, and low-quality reservoir. The well log facies model that the high-quality reservoir correlates well with the high permeability zones; the intermediate-quality reservoir correlates to the moderate permeability zones; and the low-quality reservoir correlates to the low-permeability and cemented zones.

The drillout of a borehole leads to a redistribution of stresses causing the far-field tectonic stresses, $\sigma_V$, $\sigma_{H_{max}}$, and $\sigma_{h_{min}}$, to be transformed into borehole stresses, $\sigma_V$, $\sigma_{rr}$, and $\sigma_{\theta\theta}$. Stress redistribution near a borehole can be described by the Kirsch equations (Fjaer et al., 2008; Kirsch, 1898) which assume an infinite circular hole in a homogeneous, isotropic, linear elastic medium with principal stresses at infinity. To account for many of these assumptions results in adding additional parameters to the general solution given by Kirsch. It is difficult to obtain some of the necessary parameters used in the Kirsch equations, for example the far-field maximum horizontal stress.

Chapter 4 describes a method I used for estimating the far-field principal horizontal stresses. This method is linked to an approach from Lei et al. (2012) and consists of combining the radial profiles of shear-wave slowness along the direction of the fast and slow-shear wave polarization with acoustoelastic theory, the stress distribution around a borehole, and the three far-field moduli, $c^\infty_{44}$, $c^\infty_{55}$, and $c^\infty_{66}$. Acoustoelasticity relates the dependence of elastic wave velocities to changes in applied stress from a reference state. The inversion algorithm assumes the differences in $c^\infty_{44}$, $c^\infty_{55}$, and $c^\infty_{66}$ are the result of unequal far-field principal stresses. The acoustoelastic model assumes a reference state that is in a hydrostatically stressed isotropic state. Limiting the symmetry of the medium in the reference state is the number of measurements that can be made by the borehole sonic tool. To avoid zones that have been subjected to plastic deformation, radial positions near the borehole are excluded.
To establish a stress/velocity relationship from the borehole sonic data, I combine the borehole stresses that influence the vertically traveling fast shear wave and the radial profile of the fast shear-wave velocity (Chapter 5). In order to describe the influence borehole stresses have on the fast shear-wave velocity I use an excess compliance model to demonstrate that the compliance between sand grain boundaries is a function of stress. The excess compliance fitting parameters $Z_0$, compliance of grain boundaries, and $\sigma_c$, characteristic stress, did not show any variation relative to the three reservoir well log facies described in Chapter 2. This model is also capable of predicting the influence stress has on a compressional wave. Even in the absence of a radial profile of compressional-wave velocity an excess compliance model has the capability to establish a relationship between compressional-wave velocity and borehole stress. The model assumes an isotropic distribution of orientations, of grain boundary contacts. Once again, here the number of measurements that can be made by the borehole sonic tool limits the orientation distribution of grain boundaries.

In Chapter 6, laboratory core measurements show how differential pressure and fluid saturation influence the compressional and shear-wave velocity response of five Morrow A sandstone samples. In the laboratory experiments, I subjected the core to isostatic confining pressure and pore pressure. The samples were saturated with brine and flushed with live oil, an oil-CO$_2$ mixture and pure CO$_2$. The compressional-wave velocities show sensitivity to both fluid saturation and pressure, while the shear-wave velocities show a strong dependence to changes in differential pressure. Core samples taken from the high permeability zone show the largest velocity change with confining pressure and pore pressure, whereas the core sample from the cemented zone shows the least amount of velocity change. The overall velocity decrease was greater for shear-wave velocity compared to compressional-wave velocity. Average axial strain also showed the high permeability sample to be the most stress sensitive and the cemented sample the least sensitive to confining pressure. In addition, I attempted modeling the laboratory measured velocities using Gassmann’s equation (Gassmann, 1951) and the effective medium theory of Mori and Tanaka (Benveniste, 1987; Mori and Tanaka,
1973). Neither of these models successfully predicted the measured compressional-wave or shear-wave velocities.

At Postle Field, the Morrow A sandstone is undergoing a water alternating gas (WAG) injection scheme using CO$_2$ as a tertiary method to enhance oil recovery. Therefore, the change in stress influencing the Morrow A sandstone observed in the time-lapse surface seismic data is a result of the EOR WAG process increasing pore pressure. Chapter 7 compares the influence of stress on elastic wave velocity measured from the time-lapse multicomponent surface seismic data, the borehole sonic data, and the laboratory core experiments. This comparison revealed a good agreement between the change in shear-wave impedance obtained from the borehole sonic data and observed in the time-lapse shear wave surface seismic data. Even though the laboratory core measurements capture an element of the stress influence on shear-wave velocity, the stress influence on shear-wave velocity is much less than that from both the borehole sonic and the time-lapse shear wave surface seismic data.
CHAPTER 2
WELL LOG FACIES MODELING

Much of the work done in reservoir modeling and geomechanics uses a cutoff value from a specific well log (i.e. a binary approach) to determine if a formation is sand or shale, or grain supported or clay supported.

By expanding the number of well logs input into a log facies model, the interpretation of what formations are reservoir versus non-reservoir can be extended to differentiate what is the same and what is different within a given geologic formation. For example, what zones in the Morrow A sandstone have a similar well log response. To fully understand why some zones show a similar well log response, the well log data needs to be integrated with the core data, petrology, special core analysis and seismic data.

Section 1.3 describes the complex depositional environment at Postle Field and naturally there is variability within the reservoir. The variability of the Morrow A sandstone is evident after a quick comparison of Figure 2.2 and Figure 2.3. The two reservoir intervals show different log responses, most notably a shale break and a difference in reservoir thickness. The well represented in Figure 2.2 is from the northern portion of the RCP study area, whereas the well shown in Figure 2.3 is in the southern portion of the RCP study area.

The results of the well log facies model will help quantify the variability of the Morrow A sandstone, provide insight as to how deposition changed spatially, and through integration with core data, characterize the different flow units.

I generated a well log facies model for Postle Field using 12 wells logged with the same tool technology and by the same service company. These well logs were corrected for environmental effects related to pressure, temperature, and borehole diameter. The well logs used as inputs for the well log facies model were gamma ray, neutron porosity, bulk density, and photoelectric logs. The reservoir zone from each well log were evaluated together to capture
the variability of the reservoir. To identify the different log responses within the reservoir zone, principal component analysis was applied to capitalize on variability in log response, followed by cluster analysis to group data by similar log response. Figure 2.1 shows the box and whisker plot for each of the four clusters and the well log response associated with each cluster. For each box, the median of the data is indicated by the green center line and the first and third quartiles are represented by the edges of the box. The difference between the first and third quartiles is known as the inter-quartile range (IQR). The length of the vertical lines, or whiskers, represent the extreme data values that are within 1.5 times the IQR. Data points considered outliers have a value more than 1.5 times the IQR range away from the top, third quartile, or bottom, first quartile, of the box and are plotted individually as red diamonds (Easton and McColl, 2013; MATLAB, 2004). Based on the trends of the four box and whisker plots for each log response log facies 1 is interpreted as high quality reservoir, log facies 2 as intermediate quality reservoir, log facies 3 as low quality reservoir, and log facies 4 represents an interbedded shale.

Once the log facies are established, a supervised classification method is then applied to any well with additional data such as crossed-dipole sonic or core data. Figure 2.2 shows where the well log facies model has been applied to a well with core measured porosity and permeability. From Figure 2.2, an inference can be made that the dark blue log facies 1 relates to high permeability zones, the light blue log facies 2 correlates to intermediate permeability zones and the green log facies 3 corresponds to zones of low permeability.

Applying the inferences made from the integration of the log facies model and the core measured porosity and permeability shown in Figure 2.2, Figure 2.3 compares the log facies model with the sonic and crossed-dipole sonic logs. Inspection of Figure 2.3 indicates that the low quality log facies 3 relates to the zones with high compressional wave ($V_P$), shear wave ($V_{S,\text{Fast}}$ and $V_{S,\text{Slow}}$) and Stoneley wave ($V_{\text{Stoneley}}$) velocities. Log facies 1 relates to the high quality reservoir and correlates to lower compressional wave ($V_P$), shear wave ($V_{S,\text{Fast}}$ and $V_{S,\text{Slow}}$) and Stoneley wave ($V_{\text{Stoneley}}$) velocities. In addition, log facies 3 associated
with the low quality reservoir shows a tendency to have more shear wave velocity anisotropy (track 7 in Figure 2.3), which is the percent difference between the fast ($V_{S,\text{Fast}}$) and slow ($V_{S,\text{Slow}}$) shear wave velocity, shown in track 6 of Figure 2.3.
Figure 2.1: Box and whisker plot of the four clusters for the input logs used to generate the log facies model.
<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>Facies</th>
</tr>
</thead>
<tbody>
<tr>
<td>6150</td>
<td>Log Facies</td>
</tr>
<tr>
<td>6160</td>
<td>1: High Quality Reservoir</td>
</tr>
<tr>
<td>6170</td>
<td>2: Intermediate Quality Reservoir</td>
</tr>
<tr>
<td>6180</td>
<td>3: Low Quality Reservoir</td>
</tr>
<tr>
<td>6190</td>
<td>4: Shale</td>
</tr>
<tr>
<td>6200</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.2: Well log facies model applied to a well with core measured porosity and permeability. The 1st track shows the gamma ray log. Track 2 plots the neutron porosity and bulk density. Track 3 represents core measured permeability and porosity. Track 4 corresponds to the well log facies. The reservoir zone (Morrow A sandstone) is between 6149 ft and 6208 ft.
Figure 2.3: Well log facies model applied to a well with a crossed-dipole sonic log. The 1st track shows the gamma ray log. Track 2 plots the neutron porosity and bulk density. Track 3 shows the photoelectric log (PEF). Track 4 corresponds to the well log facies. Track 5 represents the compressional wave sonic log. Track 6 displays the fast shear wave, slow shear wave and Stoneley wave sonic logs. Track 7 illustrates the anisotropy between the fast shear wave and slow shear wave sonic logs. The reservoir zone (Morrow A sandstone) is 6155 ft and 6220 ft.
CHAPTER 3
BOREHOLE WAVES

Elastic wave measurements are essential when estimating the mechanical properties of subsurface geologic formations. Open hole wire-line sonic tools measure the compressional and shear wave slowness of the geologic formation surrounding a borehole, generally based on the interval transit time of nondispersive plane waves (Sinha and Zeroug, 1999). Slowness is the reciprocal of velocity and is written as the interval transit time divided by the distance the elastic wave has propagated (e.g. $\Delta \text{time}/\Delta \text{distance}$); typical units for slowness are $\mu s/ft$.

In wireline sonic well logging there are two types of energy sources to generate elastic waves in the borehole, monopole and dipole. A monopole source has an omnidirectional energy pulse exciting compressional and shear headwaves followed by a relatively higher amplitude Stoneley wave mode, in a fast formation. A dipole source has a directional energy pulse, transmitting energy in a preferred direction, exciting flexural borehole wave modes along with compressional and shear headwaves. Typically, the first observed arrivals come from compressional headwaves; shear headwaves are not excited in slow formations. A formation is fast if the shear wave velocity is greater than the borehole-fluid compressional wave velocity; conversely, a formation is slow when the shear wave velocity is less than the borehole-fluid compressional wave velocity (Sinha and Zeroug, 1999; Tang and Cheng, 2004).

A Stoneley wave is best excited by a monopole source and is the last arrival; a schematic of the Stoneley wave mode is shown in Figure 3.1(a). A low-frequency Stoneley wave, also referred to as a tube wave (Sinha et al., 2006b; White, 1983), is responsive to the azimuthally averaged shear modulus, $c_{66}$, in the horizontal plane perpendicular to a vertical borehole axis (Norris and Sinha, 1993). Energy attenuation and velocity dispersion of Stoneley waves have been used to detect permeable formations. As low-frequency Stoneley waves propagate
axially along a borehole, they move fluid through porous rock formations (Sinha et al., 2006b; Winkler et al., 1989).

In slow formations, shear headwaves are not excited using a monopole source. Since shear wave velocity is an important attribute for characterizing formation lithology, dipole sources are designed to excite borehole flexural wave modes and have been shown to estimate the formation shear wave velocity (Sinha, 1997; Sinha et al., 2000). Figure 3.1(b) shows a schematic of a flexural wave mode; this illustration highlights that high frequencies investigate the volume near the borehole and that low frequencies investigate deeper into the rock formation. Dispersive flexural waves can be excited in both fast and slow formations; Sinha and Zeroug (1999) state that in addition to the bandwidth of the energy source, the following parameters that influence dispersion-curve modeling of formation flexural wave slowness are:

- Compressional wave velocity of the formation
- Shear wave velocity of the formation
- Formation bulk density
- Compressional wave velocity of the borehole fluid (or drilling mud)
- Density of the borehole fluid
- Diameter of the borehole

Sinha and Zeroug (1999) describe how a sensitivity analysis of the model parameters show that in a slow formation, shear wave velocity has the greatest influence on flexural wave dispersion. Whereas in a fast formation, the model parameters that mainly influence flexural wave dispersion are the shear wave velocity of the formation, the compressional wave velocity of the borehole fluid, and the diameter of the borehole. Additionally, vertically propagating flexural wave modes are mainly sensitive to the shear moduli, $c_{44}$ and $c_{55}$, in the two vertical planes perpendicular to the borehole axis.
By combining the monopole and dipole sources, one compressional and three shear moduli can be estimated (Sinha et al., 2006b). A geologic formation with orthorhombic, or lower symmetry, can be described in the three orthogonal planes by the three shear moduli, $c_{44}$, $c_{55}$, and $c_{66}$ (Sinha et al., 2006b; Thurston and Brugger, 1964). The stiffness coefficient $c_{33}$ can be estimated from the compressional headwave excited by the monopole source.

### 3.1 Sonic Log Processing

Waveforms recorded from a monopole source are processed using a slowness-time coherence (STC) method to identify the compressional, shear and Stoneley wave slowness for every depth (Kimball and Marzetta, 1984). The local maxima identified through a STC plot is then mapped into compressional, shear, and Stoneley wave logs as a function of depth (Sinha and Zeroug, 1999). See the compressional wave ($V_P$) and Stoneley wave ($V_{S,Stoneley}$) velocity logs in Figure 2.3.

Crossed-dipole energy sources are oriented orthogonally and positioned along the horizontal axis of the sonic tool. The dipole energy sources are triggered individually, exciting flexural waves in one direction and recording the waveforms before commencing the process in the orthogonal direction (Haldorsen et al., 2006). The process of generating a crossed-dipole log first requires the waveforms to be recombined into two inline and two crossline components corresponding to the inline and crossline dipole sources at every depth. Next, the waveforms are mathematically rotated along a natural coordinate system to minimize the energy on the off-diagonal components in order to determine the principal axis of symmetry (Alford, 1986). After the dispersive flexural-mode waveforms have been rotated to their principal azimuths, the slowness of the individual frequency components are estimated using a modified STC method that applies a variation of Prony’s pole estimation to isolate both dispersive and nondispersive first arrivals (Ekstrom, 1995; Kimball, 1998; Lang et al., 1987). A variation of Prony’s technique provides the slowness dispersions shown by the red and blue circles in Figure 3.2(a), Figure 3.3(a), and Figure 3.4(a). The slowness dispersions are obtained from after applying a variation of Prony’s technique on the various wavetrain
(a) Borehole deformation associated with the propagation of the Stoneley wave mode. The vertical arrow shows the propagation direction and the horizontal arrows represent the deformation of the borehole as the wave propagates upward (figure from Sinha et al. (2006b)).

(b) Borehole deformation related to the flexural wave mode. The high-frequency flexural waves probe the near borehole region whereas the low-frequency flexural waves probe deeper into the formation. The double-headed arrow signifies the diameter of the borehole (figure from Sinha et al. (2006b)).

Figure 3.1: Illustration of Stoneley and flexural borehole wave modes.
arrivals in the recorded waveforms (recorded waveforms are not shown). The sonic tool is capable of measuring the compressional wave and shear wave sonic slowness with an accuracy of between 2%-5%; the Stoneley wave slowness has an uncertainty of about 10% (Lei et al., 2012; Schlumberger, 2005).

3.2 Flexural Wave Dispersion

The primary purpose of flexural wave logging is to estimate the shear wave velocity of a subsurface geologic formation. Cross-plotting flexural wave slowness versus frequency generates a slowness dispersion curve. Figure 3.2(a), Figure 3.3(a), and Figure 3.4(a) show the slowness dispersion curves for a high quality, intermediate quality and low quality log facies obtained from the well HMU 24-4 (Figure 2.3). The low frequency asymptote (LFA) is an estimate of the zero frequency limit of the flexural wave slowness (red and blue dashed lines in Figure 3.2(a), Figure 3.3(a), and Figure 3.4(a)), which represents the “true” unaltered far-field formation slowness (Wang, 2006).

A slowness dispersion plot of the flexural wave modes provides information about the formation from near the borehole wall to the unaltered far-field region. Formations that exhibit a flexural wave slowness equal in perpendicular directions (after rotation) are considered to be isotropic (Plona et al., 2000). However, a difference in flexural wave slowness between the two principal directions is an indication of formation anisotropy, therefore identifying the fast and slow shear wave velocities and azimuths at a given depth. Dispersion analysis of the flexural wave mode is capable of identifying anisotropy from the formation surrounding a borehole. Elastic anisotropy can be categorized in a number of ways, such as either intrinsic or stress induced. Anisotropy from intrinsic structural effects can be described as aligned fractures or laminations and are identified by having parallel dispersion curves. Stress-induced anisotropy can be described as unequal principal stresses in the far-field which directly influences the near-field stress concentrations around the borehole. A slowness dispersion plot displaying a crossover of the two dispersion curves is a characteristic of stress-induced anisotropy (Sinha et al., 2000; Winkler et al., 1998). An example of both
intrinsic and stress induced anisotropy is shown experimentally and theoretically by Winkler et al. (1998).

Analysis of the crossed-dipole sonic well log slowness dispersion plots recorded at Postle Field (well HMU 24-4) show zones that have dispersion curve crossover. Figure 3.2(a) shows a difference in slowness between the two flexural wave dispersion curves at both the high and low frequencies, the crossing of the dispersion curves is indicative stress-induced anisotropy. A rose diagram displays where the difference in far-field flexural wave slowness is greater than 4% (Figure 3.5). The direction with the maximum frequency is an estimate of the fast shear wave azimuth. The fast shear wave azimuth is approximately N105°E/S75°E and is assumed to correspond to the direction of maximum horizontal stress.

3.3 Shear Wave Radial Variation Profiling

Flexural wave logging estimates the shear wave velocity of the formation obtained from the LFA of the dispersion curves (Wang, 2006). Radial variations in shear wave velocity in the two principal directions provide an estimate of the radial extent of mechanical alteration of the effective stiffness of the rock. The near borehole region is subjected to the borehole stress concentrations and drilling-induced damage, which alters the shear wave velocity of the formation relative to the far-field. Typically, the mechanical damage of the formation near the borehole wall reduces the formation stiffness and the shear wave velocity. A decrease or increase in shear wave velocity from the borehole wall to the undisturbed far-field is an indication that a mechanism is altering the formation in the vicinity of a borehole. Borehole flexural wave dispersions can be inverted to estimate the shear wave slowness of the formation relative to the radial distance from the borehole center (Sinha et al., 2006a; Sinha, 2004; Sinha et al., 2007, 2008; Sinha and Zeroug, 1999). The alteration of mechanical properties of the formation due to the redistribution of borehole stresses is of particular interest for understanding the influence stress has on elastic wave velocities.

The technique for inverting flexural wave slowness to obtain the radial variation of shear wave slowness of the formation is described by Burridge and Sinha (1996); Sinha (1997);
Sinha et al. (2006b). The method is based on the Backus-Gilbert inversion technique (Backus and Gilbert, 1970), where there is a trade-off between the radial resolution and the error in the estimated shear wave slowness at the various radial distances. A perturbation model predicts the flexural wave dispersion given a change in the formation properties, or in this case shear modulus. The difference between the measured flexural wave slowness, or Stoneley wave slowness, and an equivalent radially homogeneous and isotropic reference state is the underlying concept of the inversion scheme (Burridge and Sinha, 1996; Sinha et al., 2006b). This algorithm was applied to the fast and slow shear wave azimuth slowness dispersion curves from the Morrow A sandstone interval obtained from well HMU 24-4. The solid red and blue curves in Figure 3.2(a), Figure 3.3(a), and Figure 3.4(a) represent the dispersion for an equivalent isotropic and radially homogeneous formation. The results of the inversion algorithm are the shear wave radial variation profiles shown in Figure 3.2(b), Figure 3.3(b), and Figure 3.4(b)). The radial distance is defined by the unitless parameter of $r/a$, where $r$ is the distance from the center of the borehole and $a$ equals the borehole radius.
Figure 3.2: Shear wave slowness dispersion plots and shear wave radial variation profiles from a high quality well log facies zone. (a) Crossed-dipole flexural wave slowness dispersions are represented by the red and blue circles. The red circles indicate the fast shear wave dispersion and the blue circles indicate the slow shear wave dispersion. The naming convention is based on the slowness at the low frequency asymptote. The dashed red and blue lines denote the low frequency asymptote indicating the far-field slowness which is understood to be unaffected by the presence of the borehole. The solid red and blue curves represent the dispersion for an equivalent isotropic and radially homogeneous formation. (b) Red and blue curves signify shear wave radial variation profiles (SRVP) for the fast and slow shear wave slowness obtained from inverting the crossed-dipole flexural wave slowness dispersions in Figure 3.2(a). The dashed red and blue lines denote the low frequency asymptote (LFA), the estimated far-field slowness.
Figure 3.3: Shear wave slowness dispersion plots and shear wave radial variation profiles from an intermediate quality well log facies zone. (a) Crossed-dipole flexural wave slowness dispersions are represented by the red and blue circles. The red circles indicate the fast shear wave dispersion and the blue circles indicate the slow shear wave dispersion. The naming convention is based on the slowness at the low frequency asymptote. The dashed red and blue lines denote the low frequency asymptote indicating the far-field slowness which is understood to be unaffected by the presence of the borehole. The solid red and blue curves represent the dispersion for an equivalent isotropic and radially homogeneous formation. (b) Red and blue curves signify shear wave radial variation profiles (SRVP) for the fast and slow shear wave slowness obtained from inverting the crossed-dipole flexural wave slowness dispersions in Figure 3.3(a). The dashed red and blue lines denote the low frequency asymptote (LFA), the estimated far-field slowness.
Figure 3.4: Shear wave slowness dispersion plots and shear wave radial variation profiles from a low quality well log facies zone. (a) Crossed-dipole flexural wave slowness dispersions are represented by the red and blue circles. The red circles indicate the fast shear wave dispersion and the blue circles indicate the slow shear wave dispersion. The naming convention is based on the slowness at the low frequency asymptote. The dashed red and blue lines denote the low frequency asymptote indicating the far-field slowness which is understood to be unaffected by the presence of the borehole. The solid red and blue curves represent the dispersion for an equivalent isotropic and radially homogeneous formation. (b) Red and blue curves signify shear wave radial variation profiles (SRVP) for the fast and slow shear wave slowness obtained from inverting the crossed-dipole flexural wave slowness dispersions in Figure 3.4(a). The dashed red and blue lines denote the low frequency asymptote (LFA), the estimated far-field slowness.
Figure 3.5: Rose diagram obtained after Alford rotation of the crossed-dipole acoustic well log shows the maximum horizontal stress direction of about N105°E/S75°E for the depth range of 1744 ft. to 6303 ft.
CHAPTER 4
ESTIMATING PRINCIPAL STRESSES

The subsurface stress state is represented by three principal stresses (Figure 4.1): vertical stress, maximum horizontal stress, and minimum horizontal stress, in addition to the pore pressure. Typically, the vertical stress is assumed to be a principal stress and is equal to the weight of a column of overburden of unit area. This assumption is reasonable when considering large depths in regions with no structural bending or no residual stresses from previous tectonic activity. The vertical stress is dominated by the force of gravity, which points in the direction towards the center of the Earth.

As sediment is deposited and subsequently buried, the stresses during the formation process consist of lithostatic stress and tectonic stresses. Even in regions that show no seismic activity from shallow earthquakes it is possible to identify the direction of principal stress; it is also realistic to assume that the subsurface is acted on by tectonic stresses (Jaeger and Cook, 1976).

In order to calculate the principal effective stresses acting on a rock formation close to a borehole, first it is necessary to estimate the stresses that exist in its absence or at a distance where the borehole has no influence on the lithostatic and tectonic stresses.

Figure 4.1: Coordinate system of the three principal stress directions.
4.1 Vertical Stress

Following deposition, burial, and lithofication, the sediment is assumed to support the weight of the overlying formations. If the density varies with increasing depth, a good approximation for estimating the vertical stress is: (Fjaer et al., 2008; Jaeger and Cook, 1976).

\[ S_v(z) = \int_0^z \rho(z) g \, dz. \]  \hspace{1cm} (4.1)

In equation 4.1, \( S_v \) is the vertical stress, \( \rho(z) \) is the formation bulk density as a function of depth, \( z \), and \( g \) is the gravitational acceleration. Equation 4.1 can be used to approximate the vertical stress using the formation bulk density well log. However, the formation bulk density well log typically does not have continuous measurements from the surface to the depth of interest. In such cases the density can be approximated by

\[ \rho(z) = \rho_0 + az^b, \]  \hspace{1cm} (4.2)

in which \( \rho_0 \) is the estimated density at the surface (Sayars, 2010). A fit of equation 4.2 to the formation bulk density well log provides an estimate of the density for depths with no density data. Figure 4.2(a) shows the fit of equation 4.2 to the formation bulk density well log from well HMU 24-4, plotted as a function of vertical depth from the surface to just below the bottom of the well. This method, described in Sayars (2010), estimates the density in HMU 24-4 at depths with no data; the fit of equation 4.2 was achieved assuming a density of packed earth, \( \rho_0 \), at the surface to be 1.522 (gm/cc) (Aqua-Calc, 2012), the fitting parameters \( a \) and \( b \) are 0.34 and 0.13, respectively. Also shown in Figure 4.2(a) is the density of the drilling mud in the borehole, which is 1.1 (gm/cc) (equivalent to 9.2 lbs/gal).

The vertical stress calculated from equation 4.1, using the fit of equation 4.2 to the formation bulk density well log of HMU 24-4, is shown in Figure 4.2(b). Also shown in Figure 4.2 is the density of the drilling mud in the borehole (Figure 4.2(a)) and the internal
borehole stress due to drilling mud (Figure 4.2(b)). The force of the drilling mud exerted on the borehole wall is important when estimating the redistribution of principal stresses around a borehole, discussed in Section 4.3.

4.2 Horizontal Stresses

The primary focus for studying stress behavior observed around a borehole has been to approximate the orientation and magnitude of the far-field principal horizontal stresses based on examining borehole breakouts and assuming a failure model. More recently, advances in borehole sonic well logging technology have provided the capability to identify the azimuthal direction of the fast shear wave polarization and estimate the formation moduli $c_{33}$, $c_{44}$, $c_{55}$, and $c_{66}$. Knowledge of the formation moduli allows for estimates of the principal horizontal stress magnitudes to be made using borehole sonic data in the absence of borehole breakouts and tensile fractures.

Gough and Bell (1982) and Zoback et al. (1985) present a methodology to identify the mean azimuth of stress orientations from borehole breakouts and tensile fractures in a stress field with unequal horizontal stresses. In the absence of borehole breakouts and tensile fractures, an alternative non-destructive approach to estimate direction and magnitude of the principal horizontal stresses utilizes radial profiles of the shear-wave slowness. Such, radial profiles are capable of resolving the fast shear wave azimuth, which is assumed to coincide with the direction of present-day maximum horizontal stress. The three moduli $c_{44}$, $c_{55}$, and $c_{66}$, can be obtained by inverting the borehole flexural wave slowness dispersion for the radial variations of shear wave slowness in conjunction with the formation bulk density well log (see Chapter 3). Estimates of the moduli $c_{33}$, $c_{44}$, $c_{55}$, and $c_{66}$, allow for the principal horizontal stresses to be estimated based on the acoustoelastic effects of the formation (Lei et al., 2012; Sinha et al., 2010, 2008).

Acoustoelastic theory relates changes in effective shear moduli to incremental changes in biasing stresses and strains from a chosen reference state of the material (Norris et al., 1994; Sinha, 1982). Acoustoelastic theory provides a means to estimate the horizontal far-field
(a) Drilling mud density of 1.1 (gm/cc) (equivalent to 9.2 lbs/gal) is represented by the blue line. The bulk density well log from HMU 24-4 is represented by the black line; the red line represents the fit $\rho$ from equation 4.2 where $\rho_0 = 1.522$ (gm/cc), $a = 0.34$, $b = 0.13$ and $z$ is depth.

(b) Borehole pressure is estimated from the mud density and is represented by the blue line. The vertical stress, or overburden, is represented by the red line and is estimated using equations 4.1 and 4.2.

Figure 4.2: (a) Density of the drilling mud and the formation bulk density well log. (b) Estimated borehole pressure and vertical stress.
principal stress state and the two of the three third-order stiffness coefficients for isotropic materials. (Lei et al., 2012). Sinha et al. (2006b) and Sinha et al. (2008) describe how differences in effective shear moduli are related to differences in principal stress magnitudes through an acoustoelastic coefficient. An acoustoelastic coefficient relates differences in stiffness to differences in applied stress from a chosen reference state. Using the relationship between the far-field shear moduli, denoted by $c_{55}^\infty$ and $c_{66}^\infty$, the effective overburden stress ($\sigma_V$), and the effective minimum horizontal stress ($\sigma_{h_{min}}$), this technique can estimate an acoustoelastic coefficient and, therefore, the effective maximum horizontal stress ($\sigma_{H_{max}}$).

If no data have been collected to measure the magnitude of $\sigma_{h_{min}}$ (e.g., an in-situ stress test such as a mini frac), Sinha et al. (2010) and Lei et al. (2012) have constructed a series of two differencing equations at discrete radial positions that estimate both principal horizontal stresses, $\sigma_{H_{max}}$ and $\sigma_{h_{min}}$. In addition to estimating the principal horizontal stresses, two of the isotropic third-order elastic constants, $c_{144}$ and $c_{155}$, are calculated. This approach is based on the acoustoelastic theory presented by Sinha (1982) and Norris et al. (1994).

The method to estimate the horizontal stress magnitudes described by Lei et al. (2012), assumes that the three principal stresses are vertical stress, $\sigma_V$, maximum horizontal stress, $\sigma_{H_{max}}$, and minimum horizontal stress, $\sigma_{h_{min}}$. A Cartesian coordinate system defines the vertical axis as $x_3$ in the direction of the vertical stress, $x_1$ in the direction of maximum horizontal stress, and $x_2$ in the direction of minimum horizontal stress (Figure 4.1). This coordinate system allows the diagonal second-order stiffness coefficients along a principal axis to be written as follows:

\[
\begin{align*}
  c_{11} &= \rho v_{11}^2, \quad (4.3) \\
  c_{22} &= \rho v_{22}^2, \quad (4.4) \\
  c_{33} &= \rho v_{33}^2, \quad (4.5) \\
  c_{44} &= \rho v_{23}^2 = \rho v_{32}^2, \quad (4.6) \\
  c_{55} &= \rho v_{13}^2 = \rho v_{31}^2, \quad (4.7) \\
  c_{66} &= \rho v_{12}^2 = \rho v_{21}^2. \quad (4.8)
\end{align*}
\]
$v_{ij}$ is the velocity of an elastic wave propagating along the $x_i$-axis and polarized in the direction of the $x_j$-axis and $\rho$ represents the formation bulk density. It is convenient to describe the second-order stiffness tensor, $c_{ijkl}$, in the two-subscript $6 \times 6$ matrix notation, in which each pair of indices is replaced by a single index, such that $11 \rightarrow 1; 22 \rightarrow 2; 33 \rightarrow 3; 23 \text{ or } 32 \rightarrow 4; 31 \text{ or } 13 \rightarrow 5; 12 \text{ or } 21 \rightarrow 6$.

The acoustoelastic theory used by Lei et al. (2012) to estimate the horizontal stress magnitudes is based on the nonlinear acoustoelastic model described in Hughes and Kelly (1953), Thurston and Brugger (1964), and Norris et al. (1994). Lei et al. (2012) show that changes in moduli from a hydrostatically loaded reference state can be written in terms of changes in normal strain and effective normal stress. The hydrostatically loaded reference state of the formation is defined by the following equation (Lei et al., 2012):

$$\sigma_{\text{ref}} = \frac{\sigma_V + \sigma_{H_{\text{max}}} + \sigma_{h_{\text{min}}}}{3}. \quad (4.9)$$

Assuming the chosen reference state of the formation is a hydrostatically loaded isotropic material, the third-order elastic tensor is isotropic and is described by Thurston and Brugger (1964) as three independent coefficients chosen to be $c_{111}$, $c_{112}$, and $c_{123}$. Changes in the moduli $c_{44}$, $c_{55}$, and $c_{66}$ from a hydrostatically loaded reference state can be expressed in terms of $c_{144}$ and $c_{155}$ (Fuck and Tsvankin, 2009; Lei et al., 2012). The expressions for $c_{144}$ and $c_{155}$ given by Thurston and Brugger (1964) can be rewritten in the form of:

$$c_{144} = \frac{c_{112} - c_{123}}{2}, \quad (4.10)$$

$$c_{155} = \frac{c_{111} - c_{112}}{4}. \quad (4.11)$$

Changes in effective stress in a poroelastic rock formation is presented in the form of the effective stress tensor given by

$$\sigma_{ij} = S_{ij} - \eta P_p \delta_{ij}. \quad (4.12)$$
The effective stress tensor, $\sigma_{ij}$, is defined by the total stress tensor, $S_{ij}$, the effective stress coefficient, $\eta$, pore pressure, $P_p$ and the Kronecker delta, $\delta_{ij}$ ($\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ when $i \neq j$).

With respect to the previously defined coordinate system relative to a fixed set of axes, $x_1$, $x_2$, and $x_3$, the following equations show that $\sigma_{11}$ is equivalent to $\sigma_{H_{\text{max}}}$, $\sigma_{22}$ is equivalent to $\sigma_{h_{\text{min}}}$, and $\sigma_{33}$ is equivalent to $\sigma_V$

$$\sigma_{11} = S_{11} - \eta P_p \delta_{11} \equiv \sigma_{H_{\text{max}}} = S_{H_{\text{max}}} - \eta P_p, \quad (4.13)$$

$$\sigma_{22} = S_{22} - \eta P_p \delta_{22} \equiv \sigma_{h_{\text{min}}} = S_{h_{\text{min}}} - \eta P_p, \quad (4.14)$$

$$\sigma_{33} = S_{33} - \eta P_p \delta_{33} \equiv \sigma_V = S_V - \eta P_p. \quad (4.15)$$

In addition, the notation used in equations 4.13 - 4.15 indicates that the total stress $S_{11}$ is equivalent to $S_{H_{\text{max}}}$, $S_{22}$ is equivalent to $S_{h_{\text{min}}}$, and $S_{33}$ is equivalent to $S_V$. Since the effective stress coefficient, $\eta$, has been shown to deviate from unity, based on the experimental results of Xu et al. (2006) and Prasad and Manghnani (1997), I have used $\eta = 0.9$ for the Morrow A sandstone.

Borehole wireline sonic tools have the capability to measure the slowness of a rock formation around a borehole by exciting compressional, shear, and Stoneley wave modes from monopole and crossed-dipole sources (Pistre et al., 2005). The monopole and crossed-dipole waveforms provide estimates of the far-field compressional wave slowness, the Stoneley wave slowness, and the fast and slow shear wave slowness. Alford rotation (Alford, 1986) is applied to the dispersive crossed-dipole flexural waveforms to determine the fast and slow shear wave azimuth of the polarization vector. Additional processing produces corresponding fast and slow dipole flexural wave slowness dispersions (Lei et al., 2012). Sinha et al. (2006b) shows how radial variations of the Stoneley wave, fast shear wave, and slow shear wave slowness
dispersions can be inverted for their corresponding radial variation of velocity as a function of distance from the borehole wall to the far-field.

The vertical compressional wave velocity from borehole sonic measurements can be converted into the far-field modulus denoted by $c_{33}^\infty$, and the shear wave radial variations of velocity can be converted into the shear moduli, $c_{44}$ and $c_{55}$, by incorporating the formation bulk density. The fast and slow shear wave velocity radial profiles are primarily sensitive to the moduli $c_{44}$ and $c_{55}$ in the two perpendicular planes parallel to the borehole axis (Sinha et al., 2006b). For a vertical borehole parallel to the $x_3$-axis, the Stoneley shear wave velocity is sensitive to the azimuthally averaged far-field shear modulus in the horizontal plane and can be inverted for $c_{66}^\infty$, assuming the formation to be weakly anisotropic (Norris and Sinha, 1993).

The following equations in this section are taken directly from Lei et al. (2012) and are the key equations used for the estimation of the horizontal stress magnitudes. First, considering the stress change along the $x_1$-axis and in the direction of maximum horizontal stress, in which $\theta = 0^\circ$ (Figure 4.1), the following equation is the result of substituting the Kirsch equations (Kirsch, 1898) for estimating the principal stresses around a borehole into a nonlinear acoustoelastic model described in Lei et al. (2012) and is given by

$$c_{55}(r, \theta) \mid_{\theta=0} = m_1 \frac{a^2}{r^2} + \frac{3}{2} (c_{55}^\infty - c_{44}^\infty) \frac{a^4}{r^4} + c_{55}^\infty. \quad (4.16)$$

For the description of the stress distribution around a borehole as expressed by the Kirsch equations, see Section 4.3. Equation 4.16 is used for the regression of the radial variation in stiffness of $c_{55}$. Likewise, the expression representing the stress change along the $x_2$-axis in the direction of minimum horizontal stress in which $\theta = 90^\circ$ (Figure 4.1), is the result of substituting the Kirsch equations (Kirsch, 1898) for estimating the principal stresses around a borehole into a nonlinear acoustoelastic model described in Lei et al. (2012) and is given by
\[ c_{44}(r, \theta) \mid_{\theta = \frac{\pi}{2}} = m_2 \frac{a^2}{r^2} - \frac{3}{2} (c_{55}^\infty - c_{44}^\infty) \frac{a^4}{r^4} + c_{44}^\infty. \] (4.17)

Equation 4.17 is used for the regression of the radial variation in stiffness of \( c_{44} \). In equations 4.16 and 4.17, \( r \) represents the distance from the borehole axis, \( a \) is the radius of the borehole, \( \theta \) is defined as the polar angle relative to the major horizontal stress, and \( m_1 \) and \( m_2 \) are the model parameters. \( c_{55}^\infty \) and \( c_{44}^\infty \) are the far-field elastic moduli and assumed to be at a radial position undisturbed by the existence of the borehole where \( r \to \infty \). The model parameters \( m_1 \) and \( m_2 \) are given by

\[
m_1 = -\frac{1}{2} \sigma_{H_{\text{max}}} [4(C + D)\nu_{\text{ref}} + 4C + 5D] \\
+ \frac{1}{2} \sigma_{h_{\text{min}}} [4(C + D)\nu_{\text{ref}} + 4C + 3D] + DP_b, \tag{4.18}
\]

\[
m_2 = \frac{1}{2} \sigma_{H_{\text{max}}} [4(C + D)\nu_{\text{ref}} + 4C + 3D] \\
- \frac{1}{2} \sigma_{h_{\text{min}}} [4(C + D)\nu_{\text{ref}} + 4C + 5D] + DP_b. \tag{4.19}
\]

The model parameters \( m_1 \) and \( m_2 \) are functions of the effective maximum horizontal stress, \( \sigma_{H_{\text{max}}} \), the effective minimum horizontal stress, \( \sigma_{h_{\text{min}}} \), and the dimensionless acoustoelastic coefficients \( C \) and \( D \). \( P_b \) is the excess borehole pressure described as the difference between the drilling mud pressure and the pore pressure given by

\[ P_b = P_{\text{mud}} - P_p. \tag{4.20} \]

\( C \) and \( D \) are the acoustoelastic coefficients originally represented in terms of two of the isotropic third order elastic constants, \( c_{144} \) and \( c_{155} \), and the isotropic elastic moduli relative to a hydrostatically loaded reference state, expressed as \( M_{\text{ref}} \) and \( \mu_{\text{ref}} \)

\[ C = \frac{c_{155}(M_{\text{ref}} - 2\mu_{\text{ref}}) + c_{144}(\mu_{\text{ref}} - M_{\text{ref}}) + \mu_{\text{ref}}(2M_{\text{ref}} - 3\mu_{\text{ref}})}{\mu_{\text{ref}}(4\mu_{\text{ref}} - 3M_{\text{ref}})}. \tag{4.21} \]
The model parameters \( m_1 \) and \( m_2 \) can be approximated by simultaneously minimizing the difference between the modeled shear wave radial variation profiles using equations 4.16 and 4.17 and the inverted fast and slow shear wave radial variation profiles obtained from the crossed-dipole slowness dispersions of the flexural waves. After the model parameters \( m_1 \) and \( m_2 \) are approximated, explicit solutions for \( \sigma_{H_{\text{max}}} \), \( \sigma_{h_{\text{min}}} \), \( M_{\text{ref}} \), \( \mu_{\text{ref}} \), \( c_{144} \), and \( c_{155} \) can be written as follows:

\[
\sigma_{H_{\text{max}}} = \frac{c_{55}^\infty - c_{44}^\infty + 2DP_b - m_1 - m_2}{2D}, \tag{4.23}
\]

\[
\sigma_{h_{\text{min}}} = \frac{c_{44}^\infty - c_{55}^\infty + 2DP_b - m_1 - m_2}{2D}, \tag{4.24}
\]

\[
M_{\text{ref}} = 2\mu_{\text{ref}} \frac{1 - \nu_{\text{ref}}}{1 - 2\nu_{\text{ref}}}, \tag{4.25}
\]

\[
\mu_{\text{ref}} = \frac{c_{44}^\infty + c_{55}^\infty + c_{66}^\infty}{3}, \tag{4.26}
\]

\[
c_{144} = \frac{M_{\text{ref}} \left[ -2(D + 1)c_{44}^\infty + 2(D + 1)c_{55}^\infty + D(m_1 - m_2) \right]}{2(c_{44}^\infty - c_{55}^\infty) + \frac{\mu_{\text{ref}}(6c_{44}^\infty - 6c_{55}^\infty - Dm_1 + Dm_2)}{2(c_{44}^\infty - c_{55}^\infty)}}, \tag{4.27}
\]

\[
c_{155} = \frac{M_{\text{ref}} \left[ -2(D + 1)c_{44}^\infty + 2(D + 1)c_{55}^\infty + D(m_1 - m_2) \right]}{2(c_{44}^\infty - c_{55}^\infty) + \frac{D\mu_{\text{ref}}(4c_{44}^\infty - 4c_{55}^\infty - m_1 + m_2)}{2(c_{44}^\infty - c_{55}^\infty)}}. \tag{4.28}
\]

Where \( D \) is the acoustoelastic coefficient shown in equation 4.22 and can be rewritten in the form of
\[ D = \frac{-c_{44}^\infty - c_{55}^\infty + 2c_{66}^\infty + m_1 + m_2}{2P_b - 2\sigma_V}, \quad (4.29) \]

and \( \nu_{\text{ref}} \) is defined as the Poisson’s ratio in the hydrostatically loaded reference state:

\[
\nu_{\text{ref}} = \frac{-3c_{33}^\infty + 6D\mu_{\text{ref}} - 2DP_b + m_1 + m_2}{2\left[D\left(-3c_{33}^\infty + 3\mu_{\text{ref}} + 2(2D + 1)\sigma_V\right) + (2D + 1)(-2DP_b + m_1 + m_2)\right]}
+ \frac{2D\sigma_V(c_{44}^\infty - c_{55}^\infty + Dm_1 + Dm_2) + D(m_1 - m_2)(-2DP_b + m_1 + m_2)}{2(c_{44}^\infty - c_{55}^\infty)\left[D\left(-3c_{33}^\infty + 3\mu_{\text{ref}} + 2(2D + 1)\sigma_V\right) + (2D + 1)(-2DP_b + m_1 + m_2)\right]}.
\quad (4.30)

The regression method to estimate the model parameters, \( m_1 \) and \( m_2 \), simultaneously minimized the difference between the radial variation profiles of fast and slow shear wave slowness and equations 4.16 and 4.17. The fast and slow shear wave radial variation profiles shown as the red and blue curves in Figure 3.2(b), Figure 3.3(b), and Figure 3.4(b) are denoted as red and blue circles in Figure 4.3(b), Figure 4.4(b), and Figure 4.5(b) to highlight the regression curves. This minimization was accomplished using the MATLAB function \texttt{fminsearch} (MATLAB, 2004). Figure 4.3, Figure 4.4, and Figure 4.5 show examples of the regressions between the acoustoelastic model and the radial variation profiles of fast and slow shear wave slowness from a high quality, intermediate quality, and low quality well log facies (see Chapter 2 for the discussion on well log facies). The radial variation profiles of shear wave slowness exhibit a decrease in stiffness approaching the borehole wall (e.g. Figure 4.3(a)) suggesting that the region near the borehole wall has begun to show signs of yielding (Sinha et al., 2010). Any difference near the borehole wall between the measured shear wave radial profile and the acoustoelastic modeled radial profile is assumed to be caused by yielding of the rock formation. Only selected radial positions of the shear wave radial profile were used during the estimation of the model parameters \( m_1 \) and \( m_2 \), therefore neglecting the near borehole region.

Table 4.1 shows the input and output parameters of the acoustoelastic model used to estimate the principal horizontal stresses for the three reservoir well log facies shown in
Figure 4.3, Figure 4.4, and Figure 4.5. Inspection of the the values for the stiffness coefficients $c_{44}$, $c_{55}$, and $c_{66}$ (Table 4.1) indicates the Morrow A sandstone to have orthorhombic or lower symmetry.

This non-destructive approach for estimating principal horizontal stresses assumes the differences observed between $c_{44}^\infty$, $c_{55}^\infty$, and $c_{66}^\infty$ are the result of unequal far-field stresses. As an unstressed medium, it is reasonable to assume that the Morrow A sandstone is homogenous and isotropic and any change in symmetry is caused by unequal principal stresses. However, the number of measurements that can be made by the borehole sonic tool limit our ability to estimate symmetry. Additional assumptions include no invasion of drilling fluid such that pore pressure remains constant, suggesting that the mud cake is an impermeable barrier between the drilling mud and the reservoir. Lastly, the pore pressure at the time of wireline logging is accurately represented by the simulation results obtained by Heris (2011).

As previously stated the wave moduli $c_{33}$, $c_{44}$, and $c_{55}$ have an accuracy of 2%-5%; the modulus $c_{66}$ (derived from the Stoneley wave velocity) has an uncertainty of about 10% (Lei et al., 2012; Schlumberger, 2005). Lei et al. (2012) determines that taking into account the uncertainties in $c_{33}$, $c_{44}$, $c_{55}$, $c_{66}$, overburden stress, pore pressure, the effective stress coefficient, borehole pressure, and the model parameters $m_1$ and $m_2$, the maximum and minimum horizontal stress magnitudes should have an uncertainty in the range of 4%-8%.

4.3 Stress Distribution Around a Borehole

The presence of a borehole in a rock formation alters the stress field in the surrounding region from that of the overburden and far-field tectonic stresses. The principal stresses are assumed to be vertical stress, $\sigma_V$, maximum horizontal stress, $\sigma_{H_{\text{max}}}$, and minimum horizontal stress, $\sigma_{H_{\text{min}}}$, and have an orientation defined by the coordinate system $x_1$, $x_2$, $x_3$ as shown in Figure 4.1. To align the coordinate system with the principal stress directions, a transformation from the in-situ subsurface coordinate system $(x_1, x_2, x_3)$ to the borehole coordinate system $(x'_1, x'_2, x'_3)$ can be represented using the direction cosines of the angle between the $x_2$-axis and the projection of the borehole axis on the $x_1$-$x_2$ plane, and of
Figure 4.3: (a) Regressions of the fast and slow shear wave radial variation profiles from a high quality log facies zone in order to estimate the two principal horizontal stresses, $\sigma_{H_{max}}$ and $\sigma_{h_{min}}$, using the acoustoelastic model described in Lei et al. (2012), based on equations 4.16 and 4.17. The red and blue circles represent the radial profiles for $c_{44}$ and $c_{55}$. The solid solid red and blue curves represent the regression results based on the acoustoelastic model, where $m_1$ and $m_2$ are the model parameters. The dashed red and blue lines show the far-field stiffness values obtained from the low frequency asymptotes (LFA) observed in the slowness dispersion plot. (b) is equivalent to (a) but shows the shear wave radial variation profiles and the regressions in slowness.
Figure 4.4: (a) Regressions of the fast and slow shear wave radial variation profiles from an intermediate quality log facies zone in order to estimate the two principal horizontal stresses, $\sigma_{H_{\text{max}}}$ and $\sigma_{h_{\text{min}}}$, using the acoustoelastic model described in Lei et al. (2012), based on equations 4.16 and 4.17. The red and blue circles represent the radial profiles for $c_{44}$ and $c_{55}$. The solid solid red and blue curves represent the regression results based on the acoustoelastic model, where $m_1$ and $m_2$ are the model parameters. The dashed red and blue lines show the far-field stiffness values obtained from the low frequency asymptotes (LFA) observed in the slowness dispersion plot. (b) is equivalent to (a) but shows the shear wave radial variation profiles and the regressions in slowness.
Figure 4.5: (a) Regressions of the fast and slow shear wave radial variation profiles from a low quality log facies zone in order to estimate the two principal horizontal stresses, $\sigma_{H_{max}}$ and $\sigma_{h_{min}}$, using the acoustoelastic model described in Lei et al. (2012), based on equations 4.16 and 4.17. The red and blue circles represent the radial profiles for $c_{44}$ and $c_{55}$. The solid solid red and blue curves represent the regression results based on the acoustoelastic model, where $m_1$ and $m_2$ are the model parameters. The dashed red and blue lines show the far-field stiffness values obtained from the low frequency asymptotes (LFA) observed in the slowness dispersion plot. (b) is equivalent to (a) but shows the shear wave radial variation profiles and the regressions in slowness.
Table 4.1: Input and output parameters used to estimate the principal horizontal stresses for the three reservoir well log facies shown in Figure 4.3, Figure 4.4, and Figure 4.5.

<table>
<thead>
<tr>
<th>Input Parameters¹</th>
<th>High Quality</th>
<th>Intermediate Quality</th>
<th>Low Quality</th>
<th>Output Parameters²</th>
<th>High Quality</th>
<th>Intermediate Quality</th>
<th>Low Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{33}^\infty$ (GPa)</td>
<td>34.8</td>
<td>38.7</td>
<td>58.3</td>
<td>$S_{H_{\text{max}}}$ (MPa)</td>
<td>39.5</td>
<td>40.3</td>
<td>39.7</td>
</tr>
<tr>
<td>$c_{44}^\infty$ (GPa)</td>
<td>11.9</td>
<td>11.5</td>
<td>19.1</td>
<td>$S_{h_{\text{min}}}$ (MPa)</td>
<td>36.0</td>
<td>35.0</td>
<td>37.1</td>
</tr>
<tr>
<td>$c_{55}^\infty$ (GPa)</td>
<td>13.0</td>
<td>12.9</td>
<td>21.5</td>
<td>$c_{144}$ (GPa)</td>
<td>-4030</td>
<td>-6560</td>
<td>-15,200</td>
</tr>
<tr>
<td>$c_{66}^\infty$ (GPa)</td>
<td>9.60</td>
<td>9.58</td>
<td>14.2</td>
<td>$c_{155}$ (GPa)</td>
<td>-15,000</td>
<td>-15,900</td>
<td>-50,800</td>
</tr>
<tr>
<td>$S_{V}$ (MPa)</td>
<td>45.8</td>
<td>45.9</td>
<td>45.7</td>
<td>$M_{\text{ref}}$ (GPa)</td>
<td>21.4</td>
<td>23.9</td>
<td>31.9</td>
</tr>
<tr>
<td>$P_{p}$ (MPa)</td>
<td>16.2</td>
<td>16.3</td>
<td>16.0</td>
<td>$\mu_{\text{ref}}$ (GPa)</td>
<td>12.0</td>
<td>11.8</td>
<td>18.8</td>
</tr>
<tr>
<td>$P_{\text{mud}}$ (MPa)</td>
<td>20.4</td>
<td>20.4</td>
<td>20.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$ (Unitless)</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_1$ (GPa)</td>
<td>-13.7</td>
<td>-14.5</td>
<td>-27.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_2$ (GPa)</td>
<td>-3.58</td>
<td>-0.394</td>
<td>-9.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

¹Positive values for total vertical stress ($S_{V}$), pore pressure ($P_{p}$), and borehole pressure ($P_{\text{mud}}$) indicate a compressive stress. For all other parameters the sign convention follows Lei et al. (2012).

²Positive values for total maximum horizontal stress ($S_{H_{\text{max}}}$) and total minimum horizontal stress ($S_{h_{\text{min}}}$) indicate a compressive stress. For all other parameters the sign convention follows Lei et al. (2012).
the angle between the borehole axis and the \( x_3 \)-axis (Fjaer et al., 2008).

Kirsch (1898) published the formulas that describe the region outside a circular hole with a principal stress at infinity assuming a homogeneous, isotropic, linearly elastic medium where plane strain is normal to the borehole axis. These formulas are often referenced as the Kirsch equations and have been simplified for a vertical borehole with unequal far-field principal stresses (see Fjaer et al., 2008; Hiramatsu and Oka, 1968). The Kirsch equations are written in terms of the cylindrical coordinates \( r \) and \( \theta \); the following equations are derived for a vertical borehole

\[
\sigma_{rr} = \left( \frac{\sigma_{H_{\text{max}}} + \sigma_{h_{\text{min}}}}{2} \right) \left( 1 - \frac{a^2}{r^2} \right) + \left( \frac{\sigma_{H_{\text{max}}} - \sigma_{h_{\text{min}}}}{2} \right) \left( 1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \right) \cos 2\theta + \frac{a^2}{r^2} P_{\text{mud}}, \tag{4.31}
\]

\[
\sigma_{\theta\theta} = \left( \frac{\sigma_{H_{\text{max}}} + \sigma_{h_{\text{min}}}}{2} \right) \left( 1 + \frac{a^2}{r^2} \right) - \left( \frac{\sigma_{H_{\text{max}}} - \sigma_{h_{\text{min}}}}{2} \right) \left( 1 + \frac{3a^4}{r^4} \right) \cos 2\theta - \frac{a^2}{r^2} P_{\text{mud}}, \tag{4.32}
\]

\[
\sigma_{zz} = \sigma_V - 2\nu (\sigma_{H_{\text{max}}} - \sigma_{h_{\text{min}}}) \frac{a^2}{r^2} \cos 2\theta, \tag{4.33}
\]

\[
\tau_{r\theta} = \left( \frac{\sigma_{h_{\text{min}}} - \sigma_{H_{\text{max}}}}{2} \right) \left( 1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta, \tag{4.34}
\]

\[
\tau_{rz} = 0, \tag{4.35}
\]

\[
\tau_{\theta z} = 0. \tag{4.36}
\]

The borehole stress in the vertical direction is referred to as axial stress, \( \sigma_{zz} \). In the horizontal plane perpendicular to the vertical borehole axis, the borehole stresses are radial.
stress and tangential stress (also referred to as the hoop stress) and are represented by $\sigma_{rr}$ and $\sigma_{\theta\theta}$, respectively. $\tau_{r\theta}$, $\tau_{rz}$, and $\tau_{\theta z}$ signify the shear stresses. The expressions represented in equations 4.31 - 4.36 are for a vertical borehole aligned along principal stress direction. Generally, the shear stresses are non-zero. However, for the case of a vertical borehole aligned along a principal stress direction the shear stresses vanish. The stress solutions given by equations 4.31 - 4.36 estimate the change in stress magnitude from the borehole wall to the far-field taking into account the internal borehole pressure, the borehole radius, the angle with respect to the major horizontal stress, the far-field principal stresses, and assume that the pore pressure remains constant. $r$ represents the distance from the center of the borehole axis, $\theta$ is defined as the polar angle relative to the $x_1$-axis (i.e. the azimuth of $\sigma_{H_{\text{max}}}$), $a$ is the radius of the borehole, $P_{\text{mud}}$ is the internal borehole pressure, and $\nu$ is Poisson’s ratio. The effect the borehole has on the redistribution of stress is expressed by the exponential terms $r^{-2}$ and $r^{-4}$; as $r \to \infty$ the borehole influence is shown to rapidly diminish. The boundary condition for the Kirsch equations states that the radial stress at $r = a$ is $\sigma_{rr} = P_{\text{mud}}$, assuming no invasion of drilling fluid such that pore pressure remains constant. Additional assumptions include constant temperature and porosity, which imply no volumetric strain. 

Figure 4.6 - Figure 4.8 illustrate the stress distribution around a borehole for examples from the three reservoir well log facies zones as estimated by the Kirsch equations for the parameters shown in Table 4.2. As the far-field principal stresses (at $r/a \geq 7$) transform in to the borehole stresses at the borehole wall ($r/a = 1$), the stress redistribution can cause the rock formation close to the borehole to yield or even fail (Lei et al., 2012; Sayers, 2010). When a borehole stress becomes greater than the far-field stress, the effect is often referred to as a stress concentration. Most often a stress concentration is observed in the tangential stress, $\sigma_{\theta\theta}$, in the region close to the borehole wall (Fjaer et al., 2008).

The change in stress distribution caused by a borehole has been shown to influence a vertically propagating shear wave with particle motion in the radial direction (Sinha and Kostek, 1996; Winkler et al., 1998). When Alford rotation (Alford, 1986) is applied to the
Table 4.2: The input parameters used to estimate the stress distribution around a borehole as shown Figure 4.6, Figure 4.7, and Figure 4.8.

<table>
<thead>
<tr>
<th>Well Log Facies</th>
<th>$S_V$ (MPa)</th>
<th>$S_{H_{max}}$ (MPa)</th>
<th>$S_{h_{min}}$ (MPa)</th>
<th>$P_p$ (MPa)</th>
<th>$P_{mud}$ (MPa)</th>
<th>$\nu_{dynamic}$ (Unitless)</th>
<th>$\eta$ (Unitless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Quality</td>
<td>45.8</td>
<td>39.5</td>
<td>36.0</td>
<td>16.2</td>
<td>20.4</td>
<td>0.220</td>
<td>0.9</td>
</tr>
<tr>
<td>Intermediate Quality</td>
<td>45.9</td>
<td>40.3</td>
<td>35.0</td>
<td>16.3</td>
<td>20.4</td>
<td>0.270</td>
<td>0.9</td>
</tr>
<tr>
<td>Low Quality</td>
<td>45.7</td>
<td>39.7</td>
<td>37.1</td>
<td>16.0</td>
<td>20.4</td>
<td>0.234</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The dynamic Poisson's ratio, $\nu_{dynamic}$, is estimated using the compressional wave velocity and the average of two far-field shear wave velocities.
crossed-dipole flexural wave forms and then the slowness dispersions are inverted for the fast and slow shear radial variation profiles, the fast and slow radial profiles are assumed to be in the direction of the far-field maximum and minimum horizontal stress (Sinha et al., 2006b). The velocity of the fast and slow shear wave radial variation profiles are influenced by the redistribution of stresses due to the borehole (Sinha and Kostek, 1996).

Sayers (2010) and Sayers et al. (2007) describe a perturbation theory in order to describe the principal stresses that influence the velocity of a vertical propagating compressional wave and a vertical propagating, radially polarized shear wave. The velocity for a vertically propagating compressional wave is influenced only by the change in radial and tangential stress:

\[ \Delta \sigma_{rr} + \Delta \sigma_{\theta\theta} \].  

(4.37)

The velocity for a vertical propagating and radially polarized shear wave is influenced by the change in vertical, radial, and tangential stress:

\[ 2\Delta \sigma_{rr} + 2\Delta \sigma_{zz} + \Delta \sigma_{\theta\theta} \].  

(4.38)

Consequently, a vertical propagating and radially polarized shear wave is influenced more by the radial and vertical stress than by the tangential stress (Sayers, 2010; Sayers et al., 2007).

Figure 4.6 - Figure 4.8 also display the change in stress magnitude influencing a vertically propagating compressional wave and a vertically propagating, radially polarized shear wave from the borehole wall to the far-field, based on equations 4.37 and 4.38. The change in stress magnitude that influences a vertically propagating and radially polarized shear wave aligned with the maximum horizontal stress, \( \sigma_{H_{max}} \) is greater than the change in stress magnitude that influences a vertically propagating compressional wave (Figure 4.6(c), Figure 4.7(c), and Figure 4.8(c)). The change in stress magnitude influencing both wave modes is greater in the direction of maximum horizontal stress. In the example from the intermediate quality well log facies zone shown in Figure 4.7, the change in stress magnitude increases when
approaching the borehole wall in the direction of minimum horizontal stress, $\sigma_{h_{\text{min}}}$, for both the vertically propagating compressional wave and the vertically propagating, radially polarized shear wave. This effect is the result of a stress concentration where the tangential stress exceeds the far-field stress.
Figure 4.6: Effective stress variation ($\sigma = S - \eta P_p$) from the far-field stresses ($\sigma_V$, $\sigma_{H_{\text{max}}}$ and $\sigma_{h_{\text{min}}}$) to the borehole stresses ($\sigma_{zz}$, $\sigma_{rr}$ and $\sigma_{\theta\theta}$) in the direction of maximum horizontal stress, $\sigma_{H_{\text{max}}}$ (a), and minimum horizontal stress, $\sigma_{h_{\text{min}}}$ (b), for the parameters in Table 4.2, from a high quality log facies zone. The horizontal axis is shown in the unitless parameter $r/a$ where $r$ describes the distance from the center of the borehole and $a$ describes the borehole radius. Effective stress influencing compressional wave and shear-wave modes from the far-field to the borehole wall in the direction of maximum horizontal stress, $\sigma_{H_{\text{max}}}$ (c) and minimum horizontal stress, $\sigma_{h_{\text{min}}}$ (d), based on equations 4.37 and 4.38 described in Sayers (2010) and Sayers et al. (2007).
(a) Effective stress redistribution in the direction of $\sigma_{H_{\max}} (\theta = 0^\circ)$.

(b) Effective stress redistribution in the direction of $\sigma_{h_{\min}} (\theta = 90^\circ)$.

(c) Effective stress influencing wave modes in the direction of $\sigma_{H_{\max}} (\theta = 0^\circ)$.

(d) Effective stress influencing wave modes in the direction of $\sigma_{h_{\min}} (\theta = 0^\circ)$.

Figure 4.7: Effective stress variation ($\sigma = S - \eta P_p$) from the far-field stresses ($\sigma_V$, $\sigma_{H_{\max}}$ and $\sigma_{h_{\min}}$) to the borehole stresses ($\sigma_{zz}$, $\sigma_{rr}$ and $\sigma_{\theta\theta}$) in the direction of maximum horizontal stress, $\sigma_{H_{\max}}$ (a), and minimum horizontal stress, $\sigma_{h_{\min}}$ (b), for the parameters in Table 4.2, from an intermediate quality log facies zone. The horizontal axis is shown in the unitless parameter $r/a$ where $r$ describes the distance from the center of the borehole and $a$ describes the borehole radius. Effective stress influencing compressional wave and shear-wave modes from the far-field to the borehole wall in the direction of maximum horizontal stress, $\sigma_{H_{\max}}$ (c) and minimum horizontal stress, $\sigma_{h_{\min}}$ (d), based on equations 4.37 and 4.38 described in Sayers (2010) and Sayers et al. (2007).
(a) Effective stress redistribution in the direction of \( \sigma_{H_{\text{max}}} (\theta = 0^\circ) \).

(b) Effective stress redistribution in the direction of \( \sigma_{h_{\text{min}}} (\theta = 90^\circ) \).

(c) Effective stress influencing wave modes in the direction of \( \sigma_{H_{\text{max}}} (\theta = 0^\circ) \).

(d) Effective stress influencing wave modes in the direction of \( \sigma_{h_{\text{min}}} (\theta = 0^\circ) \).

Figure 4.8: Effective stress variation \( (\sigma = S - \eta P_p) \) from the far-field stresses \( (\sigma_V, \sigma_{H_{\text{max}}} \text{ and } \sigma_{h_{\text{min}}}) \) to the borehole stresses \( (\sigma_{zz}, \sigma_{rr} \text{ and } \sigma_{\theta\theta}) \) in the direction of maximum horizontal stress, \( \sigma_{H_{\text{max}}} \) (a), and minimum horizontal stress, \( \sigma_{h_{\text{min}}} \) (b), for the parameters in Table 4.2, from a low quality log facies zone. The horizontal axis is shown in the unitless parameter \( r/a \) where \( r \) describes the distance from the center of the borehole and \( a \) describes the borehole radius. Effective stress influencing compressional wave and shear-wave modes from the far-field to the borehole wall in the direction of maximum horizontal stress, \( \sigma_{H_{\text{max}}} \) (c) and minimum horizontal stress, \( \sigma_{h_{\text{min}}} \) (d), based on equations 4.37 and 4.38 described in Sayers (2010) and Sayers et al. (2007).
CHAPTER 5
MODELING STRESS INDUCED EXCESS COMPLIANCE

Modeling the relationship between the fast shear wave radial profile and the borehole stress at reservoir conditions is accomplished using the excess compliance rock physics model described in Sayers and Kachanov (1991, 1995). The fast shear wave radial profile changes from the altered region near the borehole wall to the unaltered far-field region and is assumed to have the same azimuth as the maximum horizontal stress. The change in shear wave velocity, with respect to the center of the borehole, is in response to the radial redistribution of stresses caused by the drillout of a borehole. Elastic properties in sandstones, such as stiffness or compliance, are influenced by the change in effective stress due to any stress-sensitive discontinuities within the rock, for example, microcracks and grain boundaries (Sayers, 2002). This formulation, based on a perturbation theory, is considered to be very general in which few assumptions are needed to describe the deformation of any discontinuities. Excess compliance theory is one method of describing the influence effective stress has on elastic properties.

Alternative approaches that have successfully described the stress dependence of elastic wave velocities do so in terms of deformable interconnected porosity (i.e., compliant porosity) (Shapiro and Kaselow, 2005) or third-order elasticity (Sarkar et al., 2003). The stress dependence of compliant porosity can be described by equations taking on an exponential form (Eberhart-Phillips et al., 1989) which can be derived from first principles (Shapiro, 2003). Third-order elasticity provides a general way to model stress-induced changes in the stiffness of a rock formation (Prioul et al., 2004) by assuming linear relationships between the changes in stresses and strains from a chosen reference state (Malvern, 1969).
5.1 Theoretical Model

The laboratory core experiment described in Chapter 6 is a good example of how the elastic properties of the Morrow A sandstone exhibit stress dependence. Based on the results of the laboratory core experiment (see Chapter 6) the measured velocities are dependent on the stress load, and therefore related to the effective stress tensor.

The effective stress tensor, $\sigma_{ij}$, is defined by the total stress tensor, $S_{ij}$, the effective stress coefficient, $\eta$, pore pressure, $P_p$ and the Kronecker delta, $\delta_{ij}$, in which $\delta = 1$ when $i = j$ and $\delta = 0$ when $i \neq j$, given by

$$\sigma_{ij} = S_{ij} - \eta P_p \delta_{ij}.$$  \hspace{1cm} (5.1)

The stress sensitive discontinuities present within the Morrow A sandstone are responsible for the effect that varying effective stress has on elastic wave velocities. A good example of discontinuities within the Morrow A sandstone are displayed in the QEMSCAN images taken from the the three lithological zones, Chapter 6, Section 6.2, Figure 6.1. The stress-sensitive discontinuities displayed in the high permeability zone (Figure 6.1(b)) are likely to come from discontinuities between grain boundaries, in this case the discontinuities between quartz grain boundaries since most of the porosity occurs between quartz grains. Stress-sensitive discontinuities in the cemented zone and lower permeability zone are likely to occur as discontinuities between fine-grained kaolinite and chlorite clay (Figure 6.1(a) and Figure 6.1(c)). The cemented zone and lower permeability zone do not reveal as much porosity between quartz grains as the high permeability zone, the majority of the porosity for the cemented zone and lower permeability zone is interstitial between the fine-grained kaolinite and chlorite clay. This is not to say the Morrow A sandstone formation does not contain fractures or microcracks, only that no fractures or microcracks are observed in the core (Jobe, 2010) or the QEMSCAN images.

Following directly from the description given in Sayers (2007a); Sayers and Kachanov (1991, 1995); and Sayers et al. (2007), the elastic compliance tensor, $s_{ijkl}$, for a sandstone
can be written as

\[ s_{ijkl} = s_{ijkl}^0 + \Delta s_{ijkl}. \]  

(5.2)

\( s_{ijkl}^0 \) is the compliance of the host rock in the absence of any discontinuities or microcracks, such as at high confining stress where the discontinuities and microcracks are assumed to be closed. As stress is reduced, the discontinuities and microcracks in the rock begin to open, introducing an excess compliance term, \( \Delta s_{ijkl} \), leading to the exponential dependence of elastic wave velocity on stress (Sayers, 2002, 2006). The excess compliance term can be written as

\[ \Delta s_{ijkl} = \frac{1}{4} (\delta_{ik} \alpha_{jl} + \delta_{il} \alpha_{jk} + \delta_{jk} \alpha_{il} + \delta_{jl} \alpha_{ik}) + \beta_{ijkl}. \]  

(5.3)

\( \alpha_{ij} \) is a second-rank tensor and \( \beta_{ijkl} \) is a fourth-rank tensor, both have units of \( Pa^{-1} \). Sayers (2007a) explains that the tensors \( \alpha_{ij} \) and \( \beta_{ijkl} \) are related to the sum, orientation, normal compliance and shear compliance of the discontinuities of the grain boundaries. Sayers (2006, 2007a) explain that it is practical to describe the normal vector, \( \mathbf{n} \), to a grain boundary using spherical coordinates defined as the polar angle \( \theta \) and the azimuthal angle \( \phi \) relative to a fixed set of axes, such as \( x_1, x_2, x_3 \) (Figure 5.1). If the distribution and orientation of grain boundaries are assumed to be continuous then \( \alpha_{ij} \) and \( \beta_{ijkl} \) can be written as

\[ \alpha_{ij} = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} Z_T(\theta, \phi) n_i n_j \sin \theta d\theta d\phi, \]  

(5.4)

\[ \beta_{ijkl} = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} [Z_N(\theta, \phi) - Z_T(\theta, \phi)] n_i n_j n_k n_l \sin \theta d\theta d\phi. \]  

(5.5)

Previous work by Sayers (2006); Schoenberg (2002); and Schoenberg and Sayers (1995), shows that \( Z_N(\theta, \phi) \) and \( Z_T(\theta, \phi) \) correspond to the normal and shear compliances of all grain boundaries whose normal vectors lie in the polar angle range of \( \theta \) to \( \theta + d\theta \) and azimuthal angle range of \( \phi \) to \( \phi + d\phi \) relative to a reference axes (e.g. \( x_1, x_2, x_3 \)) where \( x_3 \) is aligned
The laboratory core experiment described in Chapter 6 shows a nonlinear increase in elastic wave velocity as the confining stress is increased. The rate at which the elastic wave velocity increases is lower at higher confining stresses. This nonlinear increase in elastic wave velocity as a function of confining stress is observed in many of the experimental studies discussed at the beginning of Chapter 6. Additionally, a similar nonlinear increase in the fast shear wave radial velocity profile is observed as the effective borehole stresses increase.

To account for the nonlinear increase in the elastic wave velocities as a function of confining stress, Schoenberg (2002) approximates such a dependence through an exponential decay function for the normal and shear compliances, $Z_N$ and $Z_T$. Sayers (2007a,b) expresses the exponential decrease of grain boundary compliance with respect to increasing stress through

$$Z_N = Z_T = Z_0 e^{-\sigma_n / \sigma_c}.$$  

(5.6)

$\sigma_n$ is the normal stress acting on the grain boundary and $\sigma_c$ is the characteristic stress that determines the rate of decrease of $Z_0$, the compliance of grain boundaries, with respect to
increasing stress. Mavko et al. (1995) introduced the general nonhydrostatic definition of
\( \sigma_n \), which is the effective normal stress acting on a grain boundary or microcrack. Following
Mavko et al. (1995), Sayers (2007a,b) defined the normal stress in the form of

\[ \sigma_n = n_i \sigma_{ij} n_j, \quad (5.7) \]

in which the components, \( n_i \), of the normal stress vector, \( \mathbf{n} \), can be written in terms of the
Euler angles \( \theta \) and \( \phi \) (in spherical coordinates, the Euler angles are equivalent to the polar
angle \( \theta \) and azimuthal angle \( \phi \)), shown as

\[ n_1 = \cos \phi \sin \theta, \quad n_2 = \sin \phi \sin \theta, \quad n_3 = \cos \theta, \quad (5.8) \]

\[ \sigma_n = n_i \sigma_{ij} n_j = \sigma_1 \cos^2 \phi \sin^2 \theta + \sigma_2 \sin^2 \phi \sin^2 \theta + \sigma_3 \cos^2 \theta. \quad (5.9) \]

\( \sigma_1, \sigma_2 \) and \( \sigma_3 \) represent the principal effective stresses relative to a fixed axes of \( x_1, x_2 \) and
\( x_3 \).

5.2 Inversion of the Fast Shear wave Radial Variation Velocity Profile

The elastic stiffness tensor, \( c_{ijkl} \), can be estimated by inverting the compliance tensor,
\( s_{ijkl} \), obtained from measured compressional and shear wave velocities, described in equations
5.2-5.9. Assuming that the normal and shear compliances of the grain boundaries are nearly
equal, equation 5.5 shows that the contribution from the fourth-rank tensor \( \beta_{ijkl} \) is minor
and that the elastic stiffness tensor will be mainly influenced by the second-rank tensor \( \alpha_{ij} \).

Sayers and Kachanov (1995) confirm this observation by modeling elastic stiffness using Brea
sandstone properties and show the contribution of the fourth-rank tensor, \( \beta_{ijkl} \), to be small.
In addition, inversion of measured velocity data made on Penrith sandstone for \( \alpha_{ij} \) and \( \beta_{ijkl} \)
(Sayers, 2002) shows it is reasonable to assume that the normal and shear compliances of
the grain contacts are nearly equal for a sandstone. Therefore, the following discussion will
omit the small contribution of the fourth-rank tensor \( \beta_{ijkl} \).
For the analysis of measured data, it is convenient to describe the elastic stiffness tensor, \( c_{ijkl} \), and elastic compliance tensor, \( s_{ijkl} \), in the conventional 6 x 6 matrix form in which a pair of indices are replaced by a single index such that \( 11 \rightarrow 1; 22 \rightarrow 2; 33 \rightarrow 3; 23 \) or \( 32 \) \( \rightarrow 4; 31 \) or \( 13 \) \( \rightarrow 5; 12 \) or \( 21 \) \( \rightarrow 6 \). In addition, Nye (1985) states that at the same time the factors of 2 and 4 are introduced as follows:

\[
\begin{align*}
s_{ijkl} &= s_{pq} \text{ when both } p \text{ and } q \text{ are } 1, 2 \text{ or } 3, \\
2s_{ijkl} &= s_{pq} \text{ when either } p \text{ and } q \text{ are } 4, 5 \text{ or } 6, \\
4s_{ijkl} &= s_{pq} \text{ when both } p \text{ and } q \text{ are } 4, 5 \text{ or } 6.
\end{align*}
\]

For the elastic stiffness tensor, \( c_{ijkl} \), there are no factors of 2 and 4 and it is simply written as follows (Nye, 1985):

\[
c_{ijkl} = c_{pq} \quad (i, j, k, l = 1, 2, 3; \ p, q = 1, ..., 6).
\]

For an isotropic medium the elastic stiffness tensor, \( c_{ijkl} \), can be written as

\[
c^{(isotropic)}_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \tag{5.10}
\]

or in the condensed two-index matrix form such that

\[
c^{(isotropic)} = \begin{pmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{pmatrix}, \tag{5.11}
\]

in which \( \lambda \) and \( \mu \) are the Lamé parameters (Tsvankin, 2005).

Since the direction of the fast shear wave radial velocity corresponds to the largest change in the effective stress (see Figure 4.6(c), Figure 4.7(c), and Figure 4.8(c)), only the fast shear wave radial velocity is inverted for \( \alpha_{ij} \). Thus, the Morrow A sandstone is assumed to be an isotropic medium and have an isotropic orientation distribution of grain contacts. For an
isotropic orientation distribution of grain contacts, the nonvanishing $\alpha_{ij}$ are obtained from equation 5.3 and are:

$$\alpha_{11} = \alpha_{22} = \alpha_{33} = \alpha. \quad (5.12)$$

The nonzero stiffness coefficients ($c_{ij}$), written in terms of the Lamé parameters are:

$$c_{11} = c_{22} = c_{33} = \lambda + 2\mu, \quad (5.13)$$

$$c_{44} = c_{55} = c_{66} = \mu, \quad (5.14)$$

$$c_{12} = c_{13} = c_{23} = \lambda. \quad (5.15)$$

The equations for compressional wave velocity, $V_P$, and shear wave velocity, $V_S$, in a homogeneous isotropic media are expressed as

$$V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad (5.16)$$

$$V_S = \sqrt{\frac{\mu}{\rho}}, \quad (5.17)$$

in which $\rho$ is the bulk density of the rock formation.

The blue curves in Figure 5.2(b), Figure 5.3(b) and Figure 5.4(b) show a fit of the excess compliance theory to the fast shear wave radial velocity profile (red circles) from a high quality, intermediate and low quality log facies zone obtained from the crossed-dipole sonic well log recorded in well HMU 24-4 shown in Figure 2.3. The fast shear wave radial velocity profiles (red circles) shown in Figure 5.2(b), Figure 5.3(b), and Figure 5.4(b) were converted to velocity (m/s) from the modulus $c_{55}$ (GPa), also denoted as red circles in Figure 4.3(a), Figure 4.4(a), and Figure 4.5(a), and the formation bulk density for the given depth shown in Figure 4.2(a). Equations 5.2-5.4 and the exponential form of equation 5.6 are used to fit the blue curves to the fast shear wave radial velocity profile. The excess compliance modeling of the fast shear wave radial velocity neglects the altered near borehole region and
the contribution of the fourth-rank tensor, $\beta_{ijkl}$. The blue curves shown in Figure 5.2(a), Figure 5.3(a) and Figure 5.4(a) represent the predicted compressional wave velocity response to increasing confining stress. This prediction is made using the inverted parameters, $Z_0$ and $\sigma_c$, obtained from the fast shear wave radial velocity profile. One possible explanation for the small deviation between the predicted compressional wave velocity at high confining stress and the compressional wave velocity from the monopole sonic well log is the omission of the fourth-rank tensor, $\beta_{ijkl}$ (Sayers, 2002).

Figure 5.5 shows the range of the inverted parameters, $Z_0$ and $\sigma_c$, with respect to the three log facies obtained from well HMU 24-4 (shown in Figure 2.3). The range of $Z_0$ and $\sigma_c$ appear to be independent of well log facies; this observation is in minor disagreement with the conclusions made from the laboratory core experiment discussed in Chapter 6. The high permeability samples used in the laboratory core experiment show a greater sensitivity to both confining pressure and pore pressure (see Table 6.5 and Table 6.6).

The coefficients, $s^0_{ijkl}$, shown in equation 5.2 were estimated from the formation bulk density well log, the compressional wave sonic well log, and the maximum shear wave velocity observed in the fast shear wave radial variation profile (from the crossed-dipole sonic well log). The double integral required to calculate $\alpha_{ij}$ was solved numerically using the MATLAB function `dblquad`; the minimization of parameters, $Z_0$ and $\sigma_c$, was accomplished using the MATLAB function `fminsearch`. The optimum values of $Z_0$ and $\sigma_c$ were obtained by minimizing the difference between the estimated shear wave velocity using the excess compliance model and the velocity from fast shear wave radial profile. The the estimated shear wave velocity is limited by the lower bound step size of the solver. If the step size of the solver is less than the lower bound, in this case $1e^{-6}$, the iterations end (MATLAB, 2004).
(a) Compressional wave velocity versus stress in the direction of \( \sigma_{H_{\text{max}}} \) (\( \theta = 0^\circ \)).

(b) Fast shear wave velocity versus stress in the direction of \( \sigma_{H_{\text{max}}} \) (\( \theta = 0^\circ \)).

Figure 5.2: (a) Predicted compressional wave velocity versus stress (blue curve) obtained from the excess compliance rock physics modeling of the shear wave radial variation profile (shown in (b)); the dashed blue line indicates the compressional wave velocity recorded by the monopole sonic well log. (b) Fast shear wave velocity radial variation profile (red circles) from a high quality log facies zone. The red curve represents the excess compliance rock physics modeling fit of equations 5.2-5.4 using the exponential form of equation 5.6, as described in Sayers (2007a).
Figure 5.3: (a) Predicted compressional wave velocity versus stress (blue curve) obtained from the excess compliance rock physics modeling of the shear wave radial variation profile (shown in (b)); the dashed blue line indicates the compressional wave velocity recorded by the monopole sonic well log. (b) Fast shear wave velocity radial variation profile (red circles) from an intermediate quality log facies zone. The red curve represents the excess compliance rock physics modeling fit of equations 5.2-5.4 using the exponential form of equation 5.6, as described in Sayers (2007a).
Figure 5.4: (a) Predicted compressional wave velocity versus stress (blue curve) obtained from the excess compliance rock physics modeling of the shear wave radial variation profile (shown in (b)); the dashed blue line indicates the compressional wave velocity recorded by the monopole sonic well log. (b) Fast shear wave velocity radial variation profile (red circles) from a low quality log facies zone. The red curve represents the excess compliance rock physics modeling fit of equations 5.2-5.4 using the exponential form of equation 5.6, as described in Sayers (2007a).
(a) Box and whisker plot of the compliance of grain boundaries, $Z_0$, for the three reservoir zones identified through the well log facies modeling.

(b) Box and whisker plot of the characteristic stress, $\sigma_c$, for the three reservoir zones identified through the well log facies modeling.

Figure 5.5: The box and whisker plots show the excess compliance rock physics model parameters $Z_0$ and $\sigma_c$, the compliance of grain boundaries and the characteristic stress, for the three reservoir zones identified through the well log facies modeling.
CHAPTER 6
LABORATORY CORE EXPERIMENT

Literature on studies addressing the effects of saturation, pressure and pore fluid have been documented as early as 1966 (King, 1966). King (1966) reported on the change in velocities of five sandstone samples saturated with air, an NaCl solution, and kerosene as a function of hydrostatic confining pressure and internal pore pressure. Furthermore, Domenico (1984) showed that shear wave velocity is more sensitive to increasing differential pressure than compressional wave velocity in three sandstone cores of low, intermediate and high porosity.

A series of papers have been published on the results of the compressional wave velocity response to the migration of CO$_2$ in water saturated sandstones using an experimental set-up that simulates cross-well seismic profiling. Xue et al. (2005) mapped the velocity changes caused by CO$_2$ injection in water saturated Tako and Shirahama sandstones at a constant pore pressure. In a similar experiment, Xue and Ohsumi (2004) mapped the velocity changes caused by the injection of gas, liquid and supercritical phase CO$_2$ into a water saturated Tako sandstone. They investigated the changes in velocity with increasing pore pressure and concluded that CO$_2$ injection causes larger velocity changes than the change associated with increasing pore pressure in their core samples. In addition to the compressional wave velocity, strain was measured as a function of hydrostatic pressure in a dry sample. Their observations showed how strain normal and parallel to the bedding plane are not equal and more deformation occurred normal to the bedding plane. To improve upon their cross-well seismic profiling experiment and the accuracy of the compressional wave velocity changes relative to the CO$_2$ injection, Xue and Lei (2006) applied a difference tomography method to the measured slowness. This technique correlated areas of higher porosity with greater velocity reduction. Velocity reduction when CO$_2$ was in liquid or supercritical phase was twice as much as when in the gas phase. From the same data set as Xue and Lei (2006), Shi
et al. (2007) focused on compressional wave velocity reductions due to super critical CO$_2$ displacement of pore water, where the velocity reductions deviated significantly from the Gassmann (Gassmann, 1951) predicted velocities for both patchy and uniform saturations. Lei and Xue (2009) expanded on the results of Xue and Lei (2006) and Shi et al. (2007) by showing how compressional wave velocity decreased, and the attenuation coefficient increased, due to gaseous, liquid, and supercritical CO$_2$ partially replacing pore water. Kim et al. (2010) also investigated the use of compressional wave velocity and resistivity as a way to monitor saturation of a sandstone during a CO$_2$ injection process. Kim et al. (2010) accomplished this by injecting supercritical CO$_2$ into a water saturated Berea sandstone sample at simulated reservoir conditions.

The effects of saturation, pore pressure and effective stress, studied in regard to sandstone core samples from the Northwest Shelf of Australia show that the $V_P/V_S$ ratio is an indicator for the saturation state between oil saturated and the dry state (Siggins and Dewhurst, 2003). Siggins (2006) also studied the response of CO$_2$ saturated sandstones at in-situ reservoir conditions for two synthetic sandstones and one reservoir sandstone/mudstone from the Otway Basin, C seam, Waarre Formation. His findings were that both compressional wave and shear wave velocities decreased when the samples were saturated with liquid CO$_2$, as compared to dry rock. Also, the synthetic sandstones showed better agreement with Gassman’s fluid substitution theory (Gassmann, 1951) than did the reservoir sandstone samples. Similarly Siggins et al. (2010) compared synthetic sandstones and Waarre C Formation reservoir sandstones when dry and saturated with gaseous and liquid phase CO$_2$. Larger changes were observed in both compressional wave and shear wave velocity when CO$_2$ was in the liquid phase.

The effects of saturation and pore pressure on the effective pressure law and the effective pressure coefficient have been studied by Christensen and Wang (1985) and Hofmann et al. (2005). The effective pressure coefficient is sensitive to the constituency of a rock (e.g. induced microfractures) and assuming the effective pressure coefficient to be unity could
lead to significant errors when predicting time-lapse changes (Xu et al., 2006).

The RCP has used experimental rock physics to understand how the changes in confining pressure, pore pressure and fluid saturation affect compressional wave and shear wave velocities. This knowledge has then been applied to the interpretation of multicomponent time-lapse surface seismic monitoring. Capello (1995) used rock physics as a feasibility study for seismic monitoring of a CO$_2$ huff-n-puff flood in the Permian San Andres Formation, a dolomitized carbonate deposit at Vacuum Field. Also using core samples from the San Andres Formation, Duranti (2001) investigated the coexistence of equant pores and fractures and found that a dual porosity rock physics model was more successful in predicting the changes in differential pressure and saturation observed in the experimental data.

Brown (2002) integrated experimental rock physics measurements from the carbonate Marly beds of the Mississippian Charles Formation to improve the interpretation of time-lapse seismic data for the monitoring of a CO$_2$ flood at Weyburn Field. Yamamoto (2004) included the rock physics results obtained by Brown (2002) to improve the flow modeling of CO$_2$ by minimizing the difference between calculated and observed acoustic impedance while simultaneously matching the production history.

Lastly, Rojas (2005) measured dry rock velocities for the purpose of analyzing the estimated responses from Gassmann fluid substitution. This study on tight gas sands linked $V_p/V_s$ sensitivity to pore pressure changes, lithology and fluid content to aid in identifying zones with a higher reservoir potential of the Late Cretaceous Williams Fork Formation at Rulison Field, CO.

Perhaps the earliest and most cited laboratory experiment related to monitoring a CO$_2$ EOR operation is by Wang and Nur (1989). They measured ultrasonic compressional wave and shear wave velocities on seven sandstones and one of unconsolidated sand, all of which were saturated with n-hexadecane and then flushed with CO$_2$. Their measurements were done at a constant confining pressure over a range of pore pressures and temperatures above and below the critical point of CO$_2$. Their findings show a significant decrease in com-
pressional wave velocity which was attributed to CO₂ saturation. The shear wave velocity exhibited a greater sensitivity to pore pressure and was shown to be much less sensitive to saturation.

Outside of research done on the Tako, Shirahama, Waarre C Formation, Williams Fork Formation sandstones and the carbonates of San Andres and Charles Formation, most of the experimental studies are not done on reservoir samples obtained from well core. This is evident in the statement made by Siggins et al. (2010), “Literature on laboratory measurements of the effects of CO₂ on the seismic properties of reservoir rocks is relatively sparse.”

The purpose of the rock physics laboratory experiment described in this chapter is to understand the effects different fluids and variations in pore pressure have on compressional wave and shear wave velocities. The results from this experiment and analysis reflect the use of reservoir core samples and actual reservoir fluids to simulate the EOR process taking place in the Morrow A sandstone at Postle Field. This work provides insight directly related to this specific type of tertiary recovery and geologic environment.

6.1 Sample Selection

I retrieved five core plug samples from three different lithological zones within the Morrow A sandstone interval. The three zones were a low porosity and low permeability cemented zone, a higher porosity and high permeability zone, and a higher porosity and lower permeability zone. Each sample was cut parallel to the bedding plane. The cylindrical dimensions of the samples were roughly 3.81 cm (1.5 inches) in length and 2.54 cm (1 inch) in diameter. The depth from where the core samples were retrieved, as well as density, porosity and permeability are listed in Table 6.1.

6.2 Mineralogy

I used QEMSCAN (Quantitative Evaluation of Minerals by Scanning electron microscopy) to quantitatively evaluate mineral content of each of the three lithological zones (Figure 6.1). The cemented zone (Figure 6.1(a)) consists of subrounded quartz grains cemented by ankerite
and kaolinite clay. This cemented zone shows a lower amount of grain to grain contact which suggests the possibility of early cementation. The lower amount of grain to grain contact is likely related to the presence of ankerite which may indicate the pore water was rich in iron. In addition, there are occurrences of angular overgrowths over subrounded quartz grains. Even though the QEMSCAN measured 14% porosity, most of the porosity for this cemented zone is interstitial between the fine-grained kaolinite clay.

The higher porosity and high permeability zone (Figure 6.1(b)) consists mostly of sub-rounded quartz grains with lesser amounts of kaolinite clay cement, where most of the porosity (25%) occurs between quartz grains. Unlike the other zones, only a trace amount of ankerite occurs as cement.

The higher porosity and lower permeability zone (Figure 6.1(c)) consists mostly of sub-rounded quartz grains with lesser amounts of ankerite and kaolinite clay cement. This zone is more similar to the cemented zone with respect to the presence of ankerite and interstitial porosity. Much like the cemented zone, most of the porosity (20%) is interstitial between the fine-grained kaolinite and chlorite.

Table 6.2 shows the mineral volume percentage related to each of the zones as described above. Additionally, Table 6.3 shows the density and moduli for each mineral, and Table 6.4 shows the average values for each of the different lithological zones.

### 6.3 Fluid Properties

The fluids used to saturate and flush through the core samples were reproduced to simulate the actual reservoir fluids at Postle Field. These fluids consist of brine with an NaCl concentration of 142,000 ppm, live oil with a gas oil ratio (GOR) of 103 L/L (578 scf/bbl), live oil with a 0.334 mole fraction of CO₂, and pure CO₂. The live oil has an API gravity of 38.6, and a gas gravity of 0.786. This information was obtained from a reservoir fluid study provided by the current field operator, Whiting Petroleum Corporation. Using the Fluid Acoustics for Geophysics (FLAG) calculator (Fluids/DHI Consortium, 2009), the live oil bubble point is estimated to be 13.33 MPa (1934 psi) at 63.9°C (147°F). The mixture
Figure 6.1: QEMSCAN images from the three lithological zones.
Table 6.1: Depth, density, porosity, and permeability of core samples from three different lithological zones.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Depth (ft)</th>
<th>Density (gm/cm$^3$)</th>
<th>Porosity (%)</th>
<th>Permeability (mD) Pre-experiment</th>
<th>Permeability (mD) Post-experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cemented</td>
<td>6152</td>
<td>2.394</td>
<td>11.54</td>
<td>10.6</td>
<td>6.67</td>
</tr>
<tr>
<td>High Permeability No. 1</td>
<td>6163</td>
<td>2.105</td>
<td>19.96</td>
<td>270</td>
<td>192</td>
</tr>
<tr>
<td>High Permeability No. 2</td>
<td>6164</td>
<td>2.067</td>
<td>22.64</td>
<td>123</td>
<td>105</td>
</tr>
<tr>
<td>Lower Permeability No. 1</td>
<td>6193</td>
<td>2.179</td>
<td>18.73</td>
<td>11.0</td>
<td>11.0</td>
</tr>
<tr>
<td>Lower Permeability No. 2</td>
<td>6194</td>
<td>2.104</td>
<td>20.84</td>
<td>68.8</td>
<td>26.8</td>
</tr>
</tbody>
</table>

Table 6.2: QEMSCAN mineral volume (%) of the three zones.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Quartz</th>
<th>Ankerite</th>
<th>Plagioclase</th>
<th>Kaolinite</th>
<th>Illite</th>
<th>Chlorite</th>
<th>Assesory Phases</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cemented</td>
<td>66</td>
<td>19</td>
<td>6</td>
<td>7</td>
<td>trace</td>
<td>trace</td>
<td>trace</td>
<td>1</td>
</tr>
<tr>
<td>High Permeability</td>
<td>84</td>
<td>trace</td>
<td>6</td>
<td>8</td>
<td>1</td>
<td>trace</td>
<td>trace</td>
<td>trace</td>
</tr>
<tr>
<td>Lower Permeability</td>
<td>75</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>1</td>
<td>3</td>
<td>trace</td>
<td>trace</td>
</tr>
</tbody>
</table>
Table 6.3: Mineral properties.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Grain Density (gm/cm$^3$)</th>
<th>Bulk Modulus (GPa)</th>
<th>Shear Modulus (GPa)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>2.65</td>
<td>37.0</td>
<td>44.0</td>
<td>(Carmichael, 1989)</td>
</tr>
<tr>
<td>Ankerite</td>
<td>3.05</td>
<td>91.0</td>
<td>45.0$^4$</td>
<td>(Barthelmy, 2010; Ross and Reeder, 1992)</td>
</tr>
<tr>
<td>Plagioclase</td>
<td>2.63</td>
<td>75.6</td>
<td>25.6</td>
<td>(Woeber et al., 1963)</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>2.59</td>
<td>11.0</td>
<td>6.0</td>
<td>(Vanorio et al., 2003)</td>
</tr>
<tr>
<td>Muscovite</td>
<td>2.79</td>
<td>61.5</td>
<td>41.1</td>
<td>(Aleksandrov and Ryzhova, 1961)</td>
</tr>
<tr>
<td>Chlorite</td>
<td>2.68</td>
<td>127.0</td>
<td>81.9</td>
<td>(Wang et al., 2001)</td>
</tr>
<tr>
<td>Mixed Clay$^5$</td>
<td>2.60</td>
<td>21.0</td>
<td>7.0</td>
<td>(Tosaya, 1982)</td>
</tr>
</tbody>
</table>

Table 6.4: Average mineral properties for each zone using the estimates obtained from Table 2.2 and Table 2.3.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Average Grain Density$^6$ (gm/cm$^3$)</th>
<th>Average Grain Bulk Modulus$^7$ (GPa)</th>
<th>Average Grain Shear Modulus$^8$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cemented</td>
<td>2.72</td>
<td>41.55</td>
<td>33.28</td>
</tr>
<tr>
<td>High Permeability</td>
<td>2.64</td>
<td>34.58</td>
<td>32.99</td>
</tr>
<tr>
<td>Lower Permeability</td>
<td>2.66</td>
<td>38.19</td>
<td>33.40</td>
</tr>
</tbody>
</table>

$^4$Shown is the shear modulus of dolomite (from Humbert and Plicque, 1972) since the author was unable to find published data on the shear modulus of ankerite. Ankerite is part of the dolomite mineral group and resembles the crystallographic and physical characteristics of dolomite.

$^5$Assessor Phases and Others from Table 6.2 and are treated as Mixed Clay.

$^6$Average grain density is calculated as the arithmetic average.

$^7$Average grain bulk modulus is calculated as the Voigt-Reuss-Hill average.

$^8$Average grain shear modulus is calculated as the Voigt-Reuss-Hill average.
consisting of live oil and a mole fraction of 0.334 CO₂ has an estimated bubble point of 17.14 MPa (2485 psi) at 63.9°C (147°F). The GOR and volume of gas for the live oil with a 0.334 mole fraction of CO₂ was calculated assuming a gas gravity of 1.53 and a molecular weight of 44.01 gm/mole for CO₂. This approach was used by Wang et al. (2009) based on experimental measurements of Postle Field oil and mixtures of approximately 10 mole% and 50 mole% CO₂. Figure 6.2 shows the estimated density (Figure 6.2(a)), bulk modulus (Figure 6.2(b)) and velocity (Figure 6.2(c)) of these fluids over the pore pressure range used in the experiment calculated with the FLAG calculator (Fluids/DHI Consortium, 2009). The phase of CO₂ for these pressures is supercritical fluid with the exception of the lowest pressure, 6.89 MPa (1000 psi), where CO₂ is in the gas phase.

6.4 Experimental Description

For the laboratory core experiments I used the direct pulse transmission technique to record compressional and shear wave velocities at ultrasonic frequencies (≈1 MHz). The experimental set-up included a pressure vessel to encase and apply an isostatic confining pressure to the core sample, heat tapes wrapped around the pressure vessel to control the temperature, independent pumps for regulating confining pressure and pore pressure, a pulse generator, and a digital oscilloscope for recording wave forms. A transducer was placed at each flat end of the core plug sample and was capable of recording a compressional wave and two shear waves in orthogonal directions. The two shear waves were oriented along the fast and slow velocity directions relative to the core sample at bench-top conditions. Each transducer casing had a fluid line for injecting the different fluids through the sample. In addition to recording the different wave modes, average axial strain was measured along the major axis of the sample. Uncertainties from the experimental set-up correspond to a velocity error of 0.3% to 0.4%. However, the actual error is higher due to handpicking the first arrivals.
(a) Fluid density as a function of pressure.

(b) Fluid bulk modulus as a function of pressure.

(c) Fluid compressional wave velocity as a function of pressure.

Figure 6.2: Properties of the fluids used to saturate and flush through the core samples, estimated using the FLAG calculator (Fluids/DHI Consortium, 2009).
6.5 Testing Sequence

I started the testing sequence with dry rock measurements in which I incrementally increased the confining pressure from 3.45 MPa (500 psi) to 41.37 MPa (6000 psi) at the estimated reservoir temperature of 63.9°C (147°F).

Following the dry rock measurements, I vacuum saturated the core sample with brine, then recorded measurements at different pore pressures. The fluids flushed through the samples following brine were live oil, live oil with a 0.334 mol fraction of CO$_2$, and pure CO$_2$. The pore pressure range for the brine saturated samples was 29.65 MPa (4300 psi) to 3.45 MPa (500 psi), to simulate the stress path from an injection well to a production well. For the other fluids, live oil, live oil with a 0.334 mol fraction of CO$_2$, and pure CO$_2$, the pore pressure range was 29.65 MPa (4300 psi) to 6.89 MPa (1000 psi). For these fluids, I did not record data below 6.89 MPa (1000 psi) since these pressures would be below the bubble point. For the fluid saturated and fluid flushed cases a constant confining pressure of 44.82 MPa (6500 psi) was maintained to simulate the stress of the overburden, and all the measurements were recorded at 63.9°C (147°F).

To verify that the pore fluids were being flushed, a few pore volumes were flowed through the sample. At least 10 minutes elapsed between a pressure step and the recording a measurement to allow for the pore pressure and pore fluids to equilibrate.

6.6 Ultrasonic Core Measured Velocities

First I measured compressional wave and shear wave velocities in air-saturated samples (dry rock) while loading and unloading an isostatic confining pressure. Following the dry rock testing, I measured compressional wave and shear wave velocities, as a function of pore pressure, on brine saturated core samples which were then flooded with live oil, live oil with a 0.334 mol fraction of CO$_2$ and pure CO$_2$, with a constant confining pressure of 44.82 MPa (6500 psi). The measured velocities are shown in Figure 6.3 – Figure 6.7 for the dry rock and fluid saturated core samples. The shear wave velocity shown in these figures is the average of
the two orthogonally measured shear waves. Differential pressure, $P_d$, is defined as confining pressure, $P_c$, minus pore pressure, $P_p$, shown by

$$P_d = P_c - P_p. \quad (6.1)$$

In general, the high permeability samples show a greater sensitivity to both confining pressure and pore pressure. Both compressional wave and shear wave velocities decrease with decreasing differential pressure (i.e. increasing pore pressure). The compressional wave velocities show a larger decrease in velocities due to pressure when flooded with CO₂, whereas pore fluid has little bearing on shear wave velocities. Table 6.5 and Table 6.6 show the percentage change in velocity for dry rock and fluid-saturated core samples over the differential pressure range of 15.17 MPa (2200 psi) to 37.92 MPa (5500 psi).

In terms of different lithological characteristics of the core samples with respect to velocity, the higher the permeability of the sample, the lower the compressional wave and shear wave velocities. This relationship cannot be extended to porosity since the porosity of the lower permeability samples was very similar to the high permeability samples.

The compressional wave velocities show a response to both pressure and fluid type. The compressional wave velocities systematically decrease, where the brine saturated core sample is the fastest and the CO₂ flushed core sample is usually the slowest. The shear wave velocities’ response is mainly dominated by pressure and generally the fluid effect is negligible.

As mentioned previously, shear waves were recorded in two orthogonal directions. Analysis of shear wave velocity anisotropy showed a general, though not always consistent, trend of decreasing with increasing differential pressure. This behavior may be caused by the closing of preferentially oriented compliant pores or the pore space becoming more uniform in the two shear wave directions.

Some specific observations with respect to each of the core samples are shown below:
6.6.1 Cemented Sample

Figure 6.3 shows the measured compressional wave (Figure 6.3(a)) and shear wave velocities (Figure 6.3(b)) of the cemented sample. The shear wave velocity for the brine saturated sample has a slightly higher velocity than when the sample is flushed with CO\textsubscript{2} (Figure 6.3(b)). More commonly the opposite behavior is observed.

6.6.2 High Permeability No. 1 Sample

Figure 6.4 shows the measured compressional wave (Figure 6.4(a)) and shear wave velocities (Figure 6.4(b)) of the high permeability number 1 sample.

The dry rock compressional wave velocities (Figure 6.4(a)) show an unusual pattern where the velocity, during loading, is higher than the velocity while unloading. The loading and unloading cycle may have caused permanent damage to the core, leading to this response. Additionally, the compressional wave velocities of the mixture, consisting of live oil with a mole fraction of 0.334 CO\textsubscript{2}, and pure CO\textsubscript{2} become nearly equal at differential pressures of 31.03 MPa (4500 psi) and 37.92 MPa (5500 psi) (equivalent to pore pressures of 13.79 MPa (2000 psi) and 6.89 MPa (1000 psi)). The estimated bubble point for the oil-CO\textsubscript{2} mixture is 17.14 MPa (2485 psi), suggesting the dominant phase of the oil-CO\textsubscript{2} mixture at these pressures is gas.

In regard to the shear wave velocities (Figure 6.4(b)), the dry rock velocities do not show the same pattern as the compressional wave velocities during the loading and unloading cycle. The dry rock shear wave velocities exhibit a response consistent with the other samples, where the velocity slightly increases during the unloading cycle. This sample shows the most shear wave velocity response to fluid; where the brine saturated velocity is slower than when flushed with the oil-CO\textsubscript{2} mixture and CO\textsubscript{2}.

6.6.3 High Permeability No. 2 Sample

Figure 6.5 shows the measured compressional wave (Figure 6.5(a)) and shear wave velocities (Figure 6.5(b)) of the high permeability number 2 sample. Much like the high
Figure 6.3: Compressional wave and shear wave velocities for the cemented sample versus differential pressure for dry rock, brine saturated, and when flushed with live oil and CO\textsubscript{2}. 

(a) Compressional wave velocities for the cemented sample versus differential pressure.

(b) Shear wave velocities for the cemented sample versus differential pressure.
(a) Compressional wave velocities for the high permeability number 1 sample versus differential pressure.

(b) Shear wave velocities for the high permeability number 1 sample versus differential pressure.

Figure 6.4: Compressional wave and shear wave velocities for the high permeability number 1 sample versus differential pressure for dry rock, brine saturated, and when flushed with the oil-CO$_2$ mixture and CO$_2$. 
permeability number 1 sample, the number 2 sample (Figure 6.5(a)) shows a very similar compressional wave velocity response to live oil with a mole fraction of 0.334 CO\(_2\) and pure CO\(_2\). These velocities nearly converge at differential pressures of 31.03 MPa (4500 psi) and 37.92 MPa (5500 psi). Also, the velocity of the sample when flushed with live oil begins to converge with the oil-CO\(_2\) mixture and CO\(_2\) at differential pressures of 31.03 MPa (4500 psi) and above. Again, this is most likely due to the effect of gas coming out of solution (live oil has a bubble point estimated to be 13.33 MPa (1934 psi) at 63.9\(^\circ\)C (147\(^\circ\)F)).

### 6.6.4 Low Permeability No. 1 Sample

Figure 6.6 shows the measured compressional wave (Figure 6.6(a)) and shear wave velocities (Figure 6.6(b)) of the low permeability number 1 sample. The most obvious difference between the low permeability number 1 sample and the other core samples is the much higher dry rock velocity for both compressional and shear waves when compared to the fluid saturated and fluid flushed velocities (Figure 6.6(a) and Figure 6.6(b)) . These higher dry rock velocities are the result of being recorded at room conditions, 20\(^\circ\)C (68\(^\circ\)F), while the fluid measurements were recorded at the estimated reservoir temperature of 63.9\(^\circ\)C (147\(^\circ\)F).

### 6.6.5 Low Permeability No. 2 Sample

Figure 6.7 shows the measured compressional wave (Figure 6.7(a)) and shear wave velocities (Figure 6.7(b)) of the low permeability number 1 sample. The low permeability number 2 sample shear wave velocities (Figure 6.7(b)) show a similar trend as the cemented sample where the brine saturated velocity is, at some pressures, marginally higher than when the sample was flushed with live oil and CO\(_2\). Again, typically the opposite effect is observed.

Unfortunately, the well from which the core samples used for the laboratory velocity measurements were retrieved did not have the necessary input well logs for the supervised classification of the log facies model, discussed in Chapter 2. To try and relate the core samples to the log facies model, I compared the permeability of the core samples pre-experiment (Table 6.1) to the permeability associated with each log facies taken from the well in
Figure 6.5: Compressional wave and shear wave velocities for the high permeability number 2 sample versus differential pressure for dry rock, brine saturated, and when flushed with live oil, the oil-CO$_2$ mixture and CO$_2$. 
(a) Compressional wave velocities for the low permeability number 1 sample versus differential pressure. Dry rock velocities were recorded at 20°C (68°F), fluid saturated velocities were recorded at 63.9°C (147°F).

(b) Shear wave velocities for the low permeability number 1 sample versus differential pressure. Dry rock velocities were recorded at 20°C (68°F), fluid saturated velocities were recorded at 63.9°C (147°F).

Figure 6.6: Compressional wave and shear wave velocities for the low permeability number 1 sample versus differential pressure for dry rock, brine saturated, and when flushed with live oil, the oil-CO\textsubscript{2} mixture and CO\textsubscript{2}.
Figure 6.7: Compressional wave and shear wave velocities for the low permeability number 2 sample versus differential pressure for dry rock, brine saturated, and when flushed with live oil and CO₂.
Table 6.5: Percentage change of compressional wave velocity with increasing differential pressure from 15.17 MPa (2200 psi) to 37.92 MPa (5500 psi)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Dry Rock Loading (%)</th>
<th>Dry Rock Unloading (%)</th>
<th>Brine Saturated (%)</th>
<th>Oil Flushed (%)</th>
<th>Oil-CO₂ Flushed (%)</th>
<th>CO₂ Flushed (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cemented</td>
<td>6.5</td>
<td>-3.6</td>
<td>0.94</td>
<td>2.4</td>
<td>-</td>
<td>3.2</td>
</tr>
<tr>
<td>High Permeability No. 1</td>
<td>5.4</td>
<td>-6.1</td>
<td>2.5</td>
<td>-0.24</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>High Permeability No. 2</td>
<td>8.0</td>
<td>-5.1</td>
<td>2.4</td>
<td>4.6</td>
<td>5.9</td>
<td></td>
</tr>
<tr>
<td>Lower Permeability No. 1</td>
<td>4.0</td>
<td>-3.3</td>
<td>2.8</td>
<td>3.0</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>Lower Permeability No. 2</td>
<td>5.4</td>
<td>-4.1</td>
<td>2.9</td>
<td>-</td>
<td>3.4</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.6: Percentage change of shear wave velocity with increasing differential pressure from 15.17 MPa (2200 psi) to 37.92 MPa (5500 psi)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Dry Rock Loading (%)</th>
<th>Dry Rock Unloading (%)</th>
<th>Brine Saturated (%)</th>
<th>Oil Flushed (%)</th>
<th>Oil-CO₂ Flushed (%)</th>
<th>CO₂ Flushed (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cemented</td>
<td>6.5</td>
<td>-4.3</td>
<td>2.3</td>
<td>2.3</td>
<td>-</td>
<td>2.9</td>
</tr>
<tr>
<td>High Permeability No. 1</td>
<td>10.2</td>
<td>-8.3</td>
<td>7.7</td>
<td>-</td>
<td>10.5</td>
<td>7.7</td>
</tr>
<tr>
<td>High Permeability No. 2</td>
<td>8.3</td>
<td>-6.4</td>
<td>8.7</td>
<td>9.7</td>
<td>10.3</td>
<td>9.1</td>
</tr>
<tr>
<td>Lower Permeability No. 1</td>
<td>6.4</td>
<td>-5.2</td>
<td>5.0</td>
<td>5.9</td>
<td>5.3</td>
<td>5.7</td>
</tr>
<tr>
<td>Lower Permeability No. 2</td>
<td>6.9</td>
<td>-5.5</td>
<td>3.4</td>
<td>3.9</td>
<td>-</td>
<td>4.4</td>
</tr>
</tbody>
</table>

83
Figure 2.2. Figure 6.8 shows this comparison between the core samples used in the laboratory core experiment (represented by the yellow stars) and the variability in permeability for each of the reservoir log facies obtained from well HMU 13-2, shown in Figure 2.2 (represented by the box and whisker plots). I assume each core sample used in the laboratory core experiment corresponds to a log facies. Hence, the cemented sample is associated with the low quality reservoir log facies, the high permeability number 1 and number 2 samples are related to the high quality log facies and the lower permeability number 1 and number 2 samples correspond to the intermediate quality log facies.

![Permeability of core samples](image)

**Figure 6.8:** Permeability of the core samples used in the laboratory velocity measurements (stars) superimposed on the box and whisker plot of the permeability related to each log facies (from the well in Figure 2.2).

### 6.7 Average Axial Strain

Average axial strain measured during the dry rock loading and unloading cycle for three samples, each from a different lithological zone, are shown in Figure 6.9. The isostatic confining pressure has a greater effect on the high permeability sample and the least affect on the cemented sample. We used these results to correct for sample length, porosity, and
density at each differential pressure when estimating the velocities shown in Figure 6.3, Figure 6.4 and Figure 6.6.

Figure 6.9: Average axial strain as a function of isostatic confining pressure for three dry rock core samples.
CHAPTER 7
COMPARISON OF SEISMIC, BOREHOLE SONIC WELL LOG, AND CORE DATA

Evaluating the influence of stress on velocity is important for seismic reservoir monitoring given that seismic signatures are sensitive to the dynamic processes related to production and injection. Information on how elastic wave velocity behaves due to a change in stress, either from pore pressure or confining pressure, has been primarily obtained through experimental laboratory core measurements. Holt et al. (2000) found the coring process to result in permanent damage to the core sample caused by the stress-release taking place during retrieval. Alteration introduced by the coring process can have an effect on the mechanical and elastic properties. This alteration may introduce a discrepancy between the mechanical and elastic properties measured from a core sample and the magnitude of change observed in time-lapse seismic monitoring of a reservoir (Furre et al., 2009).

In addition, the issue of scale needs to be considered when discussing the cause of discrepancies between the core measurements and the seismic data. For reservoir characterization, the frequency used to make the measurement dictates the detection limit of a rock volume. Laboratory scale measurements are usually recorded at ultrasonic frequencies (≈20 kHz-10 MHz) and surface seismic data are acquired over the frequency range of ≈10-200 Hz. Borehole sonic well logging provides an intermediary scale, which is recorded at sonic frequencies (≈1-20 kHz), and has the capability to estimate the influence borehole stress has on the shear wave velocity of a geologic formation. In order to test the hypothesis that the stress sensitivity observed from borehole sonic data may be more representative of the stress changes taking place within the reservoir, I compared the results of the laboratory core experiment (Chapter 6) to the results from the crossed-dipole borehole sonic well log data used for the excess compliance rock physics modeling (Chapter 5), and the multicomponent time-lapse seismic analysis of Postle Field done by Heris (2011), Zerpa (2011), and Singh (2013). The
comparison of the time-lapse seismic, borehole sonic well log, and the laboratory core data includes the following:

- The compressional and shear wave velocities versus differential pressure obtained from the laboratory core experiment on the Morrow A sandstone samples (Figure 6.3 to Figure 6.7).

- The velocity of the fast shear wave radial variation profile and the stress corresponding to the vertically propagating and radially polarized shear wave in the direction of maximum horizontal stress, $\sigma_{H_{\text{max}}}$, (Figure 5.2(b) to Figure 5.4(b)), for each depth as defined by the well log facies model for the Morrow A sandstone formation, from well HMU 24-4 (Figure 2.3). Also used is the excess compliance rock physics model of the predicted compressional wave and the stress corresponding to the vertically propagating compressional wave in the direction of maximum horizontal stress, $\sigma_{H_{\text{max}}}$ (Figure 5.2(a) to Figure 5.4(a)).

- The multicomponent time-lapse seismic recorded at Postle Field consists of compressional wave and shear wave time-lapse data which were interpreted by Zerpa (2011) and Singh (2013). The interpretations cover the time period between the baseline seismic survey, acquired in March, 2008, and the first seismic monitor survey, acquired in December, 2008. In addition, Zerpa (2011) inverted the multicomponent seismic data for compressional wave and fast shear wave impedance within the Morrow A sandstone interval. The time-lapse analysis corresponds to the time period between the baseline seismic survey and the first monitor seismic survey (Figure 7.1(b) and Figure 7.3(b)). During the time period between the baseline and the first monitor seismic survey Postle Field underwent CO$_2$ WAG injection for EOR.

- Jobe (2010) and Heris (2011) conducted geologic modeling and reservoir flow simulation for Postle Field. An outcome from Heris (2011) details the estimated change in reservoir
pore pressure between the time of the baseline seismic survey and the first monitor seismic survey (Figure 7.1(a) and Figure 7.3(a)).

7.1  Shear Wave Stress Sensitivity

The change in reservoir pore pressure at Postle Field is a result of CO₂ WAG injection for EOR; Figure 7.1(a) shows the change in reservoir pore pressure for the time period between March, 2008 to December, 2008, which corresponds to the acquisition of the baseline and first monitor seismic surveys, respectively. The change in fast shear wave impedance within the Morrow A sandstone interval for this time interval is shown in Figure 7.1(b). Zerpa (2011) calculated the change in fast shear wave impedance as the difference in impedance between the first monitor and the baseline seismic surveys divided by the impedance observed in the first monitor seismic survey, written as

\[
\Delta Z_{Vs}(\%) = \frac{Z^2_{Vs} - Z^1_{Vs}}{Z^2_{Vs}} \times 100. \tag{7.1}
\]

In which \(Z^1_{Vs}\) represents the impedance observed within the Morrow A sandstone interval from the baseline seismic survey and \(Z^2_{Vs}\) represents the impedance observed within the Morrow A sandstone interval from the first monitor survey.

In order to represent how a change in effective stress influences the fast shear wave impedance I simply divided the change in fast shear wave impedance by the change in effective stress. This ratio is a means to compare the data from the laboratory core experiment, the crossed-dipole borehole sonic well log, and the multicomponent time-lapse surface seismic. For example, with respect to the time-lapse seismic I divided the maximum change in fast shear wave impedance of -5% (Figure 7.1(b)) by the maximum change in pore pressure of 300 psi (2.07 MPa)(Figure 7.1(a)), which equals -0.0167 \(\left(\frac{\Delta Z_{Vs}(\%)}{\Delta P_{p}(\text{psi})}\right)\). Two common assumptions that are made when interpreting pure shear wave time-lapse seismic data are: 1) a change in impedance is primarily due to a change in pore pressure which affects the shear modulus and 2) the effect of any fluid replacement on the bulk density is negligible. These
assumptions are made by Zerpa (2011) and Heris (2011) in their interpretation of Figure 7.1.

The method I used to compare the laboratory core experiment (Figure 6.3 to Figure 6.7) to the shear wave time-lapse surface seismic is as follows:

1. The measured shear wave velocities were converted to shear wave impedance, assuming the samples were completely saturated and using the fluid properties shown in Figure 6.2.

2. To maintain consistency, the change in shear wave impedance is calculated using equation 7.1, in which $Z_{Vs}^1$ represents the shear wave impedance at the lowest differential pressure and $Z_{Vs}^2$ represents the shear wave impedance at the highest differential pressure. This operation was done for the dry, brine saturated and fluid flushed measurements.

3. The greatest change in shear wave impedance was divided by the corresponding change in differential pressure for each series of measurements to obtain the maximum variability for each core sample.

Figure 7.2(a) shows how the change in differential pressure influences the shear wave impedance for each sample in the form of a box and whisker plot. Also shown in Figure 7.2(a) is the effective stress influence on the fast shear wave impedance within the Morrow A sandstone interval from the time-lapse seismic, which is $-0.0167 \left( \frac{\Delta Z_{Vs}(\%)}{\Delta P_p(\text{psi})} \right)$.

A similar approach was used to calculate the influence effective stress has on the fast shear wave impedance obtained from the crossed-dipole borehole sonic well log from well HMU 24-4 (Figure 2.3). The method I used to compare the fast shear wave radial profiles to the shear wave time-lapse surface seismic is as follows:

1. The velocity was obtained from the fast shear wave radial profile (Figure 5.2(b) to Figure 5.4(b)) and then converted to the fast shear wave impedance, assuming the formation bulk density was radially constant.
2. The change in shear wave impedance was calculated using equation 7.1, in which $Z_{Vs}^1$ represents the fast shear wave impedance at the lowest differential stress used in the excess compliance rock physics modeling, neglecting the altered region near the borehole. $Z_{Vs}^2$ represents the fast shear wave impedance at the highest differential stress used in the excess compliance rock physics modeling.

3. The change in fast shear wave impedance was then divided by the maximum change in the differential stress that influences a vertically propagating and radially polarized shear wave in the direction of maximum horizontal stress, $\sigma_{H_{max}}$ (equation 4.38).

Figure 7.2(b) shows how the change in effective stress influences the fast shear wave impedance for each reservoir well log facies in the form of a box and whisker plot. Also shown is the effective stress influence on the fast shear wave impedance within the Morrow A sandstone interval from the time-lapse seismic of -0.0167 ($\frac{\Delta Z_{Vs}(\%)}{\Delta P_p(\text{psi})}$).

The most important observation that can be made from Figure 7.2 is that the crossed-dipole borehole sonic well log is in good agreement with the time-lapse surface seismic in terms of how changes in stress influence shear waves.

### 7.2 Compressional Wave Stress Sensitivity

As water and CO$_2$ are alternatively injected into Postle Field for EOR, the Morrow A sandstone is subjected to an increase in reservoir pore pressure and a complex fluid replacement scheme. Figure 7.3(a) shows the change in reservoir pore pressure for the time period between the acquisition of the baseline and first monitor seismic surveys.

The change in compressional wave impedance within the Morrow A sandstone interval for this time period is shown in Figure 7.3(b). Zerpa (2011) calculated the change in compressional wave impedance as the difference in impedance between the first monitor and the baseline seismic surveys divided by the impedance observed in the first monitor seismic survey, given by
Figure 7.1: (a) Predicted changes in reservoir pressure between the baseline and 1st monitor seismic surveys obtained from the reservoir simulation model (from Heris, 2011). (b) shows the change in fast shear wave impedance within the Morrow A sandstone interval between the baseline and first monitor seismic surveys obtained from seismic inversion (from Zerpa, 2011).
(a) Box and whisker plots represent the shear wave stress sensitivity obtained from the laboratory core experiment. The green line, at -0.0167, indicates the shear wave stress sensitivity observed in the time-lapse surface seismic.

(b) Box and whisker plots represent the fast shear wave stress sensitivity obtained from the crossed-dipole borehole sonic well log shear wave radial variation profile. The green line, at -0.0167, indicates the shear wave stress sensitivity observed in the time-lapse surface seismic.

Figure 7.2: Box and whisker plots in (a) represent the shear wave stress sensitivity obtained from the laboratory core experiment described in Chapter 6. The box and whisker plots in (b) represent the shear wave stress sensitivity obtained from the fast shear wave radial variation profile for the three reservoir well log facies zones. The green dashed line shown at -0.0167 in (a) and (b) represents the maximum time-lapse surface seismic percent change in fast shear wave impedance divided by the maximum change in pressure within the Morrow A sandstone interval for the time period between the baseline and 1st monitor seismic surveys as shown in Figure 7.1.
\[
\Delta Z_{VP} (\%) = \frac{Z_{VP}^2 - Z_{VP}^1}{Z_{VP}^2} \times 100. \tag{7.2}
\]

\(Z_{VP}^1\) represents the impedance observed within the Morrow A sandstone interval from the baseline seismic survey and \(Z_{VP}^2\) represents the impedance observed within the Morrow A sandstone interval from the first monitor survey.

In order to represent how a change in effective stress influences the compressional wave impedance I divided the change in compressional wave impedance by the change in effective stress. This ratio is a means to compare the data from the laboratory core experiment, the excess compliance rock physics model of the compressional wave based on the fast shear wave radial variation profiles from the crossed-dipole borehole sonic well log, and the multi-component time-lapse surface seismic. For example, with respect to the time-lapse seismic I divide the maximum change in compressional wave impedance of -7.5\% (Figure 7.3(b)) by the maximum change in pore pressure of 300 psi (2.07 MPa) (Figure 7.3(a)), which equals \(-0.025 \left( \frac{\Delta Z_{VP} (\%)}{\Delta P_p (\text{psi})} \right)\).

When interpreting the compressional wave time-lapse seismic response to a WAG injection scheme, such as at Postle Field, the challenge lies in translating the effects caused by an increase in pore pressure (from the injection process) and the fluid replacement of brine and CO\(_2\) in bypassed oil zones. The WAG injection process increases reservoir pore pressure and changes fluid saturations therefore altering the bulk modulus, shear modulus, and formation bulk density. However, Heris (2011) attributes a high water-cut in producing wells to poor sweep efficiency due to reservoir heterogeneity. This leads to the assumption that the compressional wave impedance anomalies observed in Figure 7.3(b) are primarily related to the WAG injection scheme increasing pore pressure.

I used the following method to compare the laboratory core experiment (Figure 6.3 to Figure 6.7) to the compressional wave time-lapse surface seismic.
1. I converted the measured compressional wave velocity into compressional wave impedance, assuming the samples were completely saturated and using the fluid properties shown in Figure 6.2.

2. The change in compressional wave impedance was calculated using equation 7.2, in which $Z_{AVP}^1$ represents the compressional wave impedance at the lowest differential pressure and $Z_{AVP}^2$ represents the compressional wave impedance at the highest differential pressure. This operation was done for the dry, brine saturated and fluid flushed measurements.

3. The greatest change in compressional wave impedance was divided by the corresponding change in differential pressure for each series of measurements to obtain the maximum variability for each core sample.

Figure 7.4(a) shows how the change in differential pressure influences the compressional wave impedance for each sample in the form of a box and whisker plot. Also shown in Figure 7.4(a) is the effective stress influence on the compressional wave impedance within the Morrow A sandstone interval from the time-lapse seismic, which is $-0.025 \left( \frac{\Delta Z_{VP}^p(\%)}{\Delta P_p(\text{psi})} \right)$.

A similar approach is used to calculate the influence effective stress has on the compressional wave impedance obtained from the excess compliance rock physics model of the fast shear wave radial profile acquired from the crossed-dipole borehole sonic well log from well HMU 24-4 (Figure 2.3). The method I used to compare the excess compliance rock physics model of the compressional wave to the compressional wave time-lapse surface seismic is as follows:

1. The compressional wave velocity is obtained from the excess compliance rock physics model (Figure 5.2(a) to Figure 5.4(a)) and it is then converted to compressional wave impedance assuming the formation bulk density was radially constant.

2. The change in the modeled compressional wave impedance was calculated using equation 7.2, in which $Z_{VP}^1$ represents the modeled compressional wave impedance at the
lowest differential stress used in the excess compliance rock physics modeling of the 
fast shear wave radial velocity profile, neglecting the altered region near the borehole. 
$Z_{VP}^2$ represents the modeled compressional wave impedance at the highest differential 
stress used in the excess compliance rock physics modeling of the fast shear wave radial 
velocity profile.

3. The change in the modeled compressional wave impedance was divided by the by 
the maximum change in the differential stress that influences a vertically propagat-
ing compressional wave in plane parallel to the maximum horizontal stress, $\sigma_{H_{\text{max}}}$, 
(Figure 5.2(a) to Figure 5.4(a)) described by equation 4.37.

Figure 7.4(b) shows how the change in effective stress influences the modeled compres-
sional wave impedance for each well log facies in the form of a box and whisker plot. Also 
shown in Figure 7.4(b) is the effective stress influence on the compressional wave impedance 
within the Morrow A sandstone interval from the time-lapse seismic of $-0.025 \left( \frac{\Delta Z_{VP}}{\Delta P_{p}} \right)$. 

Figure 7.4 shows that neither the core samples nor the modeled compressional wave 
represent how changes in stress influence the compressional waves acquired from the time-
lapse surface seismic. However, the difference between the modeled compressional wave and 
the time-lapse surface seismic may be the result of the complex CO$_2$ WAG injection scheme 
of brine and CO$_2$ replacing bypassed oil. Further investigation is necessary to confirm this 
hypothesis.
Figure 7.3: (a) Predicted changes in reservoir pressure between the baseline and first monitor seismic surveys obtained from the reservoir simulation model (from Heris, 2011). (b) shows the change in compressional wave impedance within the Morrow A sandstone interval between the baseline and first monitor seismic surveys obtained from seismic inversion (from Zerpa, 2011).
Figure 7.4: The box and whisker plots in (a) represent the compressional wave stress sensitivity obtained from the laboratory core experiment described in Chapter 6. The box and whisker plots in (b) represent the predicted compressional wave stress sensitivity based on the excess compliance rock physics modeling of the fast shear wave radial variation profile for the three reservoir well log facies zones. The green dashed line shown at -0.025 in (a) and (b) represents the maximum time-lapse surface seismic percent change in compressional wave impedance divided by the maximum change in pressure within the Morrow A sandstone interval for the time period between the baseline and first monitor seismic surveys as shown in Figure 7.3. Note: The vertical axis for (a) and (b) are not equivalent, they differ by an order of magnitude.
CHAPTER 8
CONCLUSIONS

Borehole sonic data have provided an additional way to study the influence stress has on elastic wave velocities in the Morrow A sandstone at Postle Field. I employ a methodology to estimate the influence of stress on shear-wave velocity acquired from borehole sonic data. By combining the techniques used for radial profiling of shear-wave slowness (Sinha et al., 2006b), estimating horizontal stress magnitudes (Lei et al., 2012), modeling the redistribution of stress caused by the presence of a borehole (Fjaer et al., 2008; Kirsch, 1898), and describing the stress dependence of the elastic wave velocities using an excess compliance model (Sayers, 2007b, 2010), I establish a shear-wave velocity/stress relationship from borehole sonic data.

Knowing how stress influences velocity is crucial for seismic reservoir monitoring because seismic reflection signatures are sensitive to the dynamic processes related to production or injection. Previously, information about the influence of stress on elastic wave velocity has been primarily obtained through laboratory measurements. The methodology developed in this thesis is necessary to compare the influence of stress on elastic wave velocities measured from time-lapse multicomponent surface seismic data, borehole sonic data, and a laboratory core experiment. An important conclusion from this multi-scale comparison is that the stress influence on shear waves estimated from the surface seismic data and the borehole sonic is in close agreement. In contrast, the stress influence on shear waves measured from the laboratory core experiments does not agree with the results from either the surface seismic or borehole sonic data.

The results of this thesis are specific to the Morrow A sandstone reservoir in Postle Field at well location HMU 24-4. However, the data, techniques and workflow proposed could be recorded and applied to other reservoirs with stress-sensitive characteristics. Furthermore, the proposed techniques and workflow are not limited to hydrocarbon bearing formations.
but could be adapted for reservoirs with potential for anthropogenic greenhouse gas sequestration.

The main conclusions from this research are as follows:

- The crossed-dipole borehole flexural waves estimate the maximum horizontal stress direction to be N105°E/S75°E.

- Stress induced anisotropy is observed by the crossover of the crossed-dipole flexural wave slowness dispersions, which is an indication that the Morrow A sandstone has orthorhombic or lower symmetry.

- The parameters $Z_0$, compliance of grain boundaries, and $\sigma_c$, characteristic stress, used to describe the stress dependence of the fast shear wave velocity radial profile (obtained from the crossed-dipole borehole sonic data) did not show any variation relative to the three reservoir well log facies.

- The laboratory core experiments show that samples taken from the high permeability zone exhibit the largest velocity change with confining pressure and pore pressure. Average axial strain also showed the high permeability sample to be the most stress sensitive to confining pressure.

- The laboratory core experiments show the sample from taken from the cemented zone to exhibit the least amount of velocity change with confining pressure and pore pressure.

- The laboratory core experiments show that the stress dependence was greater for shear-wave velocity compared to compressional-wave velocity.

- A comparison between the three measurement scales, time-lapse shear wave surface seismic data, crossed-dipole borehole sonic data, and laboratory core experiments revealed a good agreement between the change in shear-wave impedance obtained from the crossed-dipole borehole sonic data and observed in the time-lapse shear wave surface seismic data.
The stress influence on shear-wave velocity observed from the laboratory core experiments, is much less than both the crossed-dipole borehole sonic data and the time-lapse shear wave surface seismic data.

Lastly, my general observation about this multi-scale comparison is that the main mystery to be solved is not which type of measurement is more representative or accurate than the others, but rather what is the mechanism responsible for the discrepancies?
Results of my research are influenced by the experimental set up of the laboratory core experiment, the current state of knowledge on the formation stiffening observed in the borehole sonic data, and the processing workflow of the time-lapse multicomponent surface seismic data. To address the influence that these components may have, I recommend further research in the following areas.

9.1 Laboratory Core Experiment

The discrepancy between the laboratory core experiment and time-lapse surface seismic (Figure 7.2(a)) may originate from permanent damage to the core sample occurred during retrieval and the considerable difference in the scale of these two measurements. However, the laboratory core experiment described in Chapter 6 represents perhaps the most basic type of measurement system. According to Scott Jr. (2007) laboratory measurements taken under a hydrostatic stress load are not representative of the reservoir stress path. The stress path influences the stress sensitivity of a rock; therefore, an appropriate stress path representing the reservoir is necessary when predicting a time-lapse seismic response. A more advanced experimental method, such as the low-amplitude stress-strain technique described by Batzle et al. (2006), would provide laboratory measurements in the same frequency range as the surface seismic and the borehole sonic data. By combining the low-amplitude stress-strain technique with ultrasonic transducers, this experimental set up has the capability to measure compressional and shear properties over a frequency range of around 5 Hz to 800 kHz (Batzle et al., 2006). In addition, the laboratory core experiment oversimplified the confining stress by only applying an isostatic confining pressure to the sample, to simulate the estimated overburden stress. To apply a realistic stress regime on the core sample, a conventional triaxial system (Bardet, 1997) or the more recently developed true triaxial loading device
(Mandeville and Penumadu, 2004; Yin and Kumruzzaman, 2008) would be able to more accurately represent the three orthogonal subsurface principal stresses.

9.2 Surface Seismic Data Processing

Additional processing of the compressional and shear wave surface seismic data could have implications on where fluid replacement and changes in pore pressure are observed. Singh’s (2013) static characterization of the Morrow A sandstone using pure shear wave surface seismic data shows remarkable improvement in outlining the sand fairway at Postle Field. Extending this workflow of multicomponent data processing, inversion and interpretation to the baseline and second monitor surface seismic surveys could enhance the time-lapse anomalies. An updated time-lapse interpretation of Postle Field could better constrain the permeability modifications Heris (2011) applied to the reservoir model for improving the history match of total liquid, oil, and water production. Replacing the core derived stress dependence of elastic moduli in the reservoir model that Heris (2011) used, with the excess compliance rock physics model of the fast shear wave radial variation profile would also improve the estimated impedance change between the baseline and monitor surveys based on the flow simulation model.

9.3 Formation Stiffening Around the Borehole

When inverting the crossed-dipole slowness dispersions for the fast and slow shear wave azimuth radial profiles (as discussed in Chapter 3 (see Figure 3.2)), I encountered zones within the Morrow A sandstone interval where the rock formation appeared to be the most stiff at radial distances (i.e. $r/a$) between 2.5 to 4.5 from the center of the borehole. Figure 9.1 shows the stiffness obtained from the fast and slow shear wave radial variation profiles, $c_{55}$ and $c_{44}$, as red and blue markers. Close examination of Figure 9.1 shows the maximum stiffness for $c_{55}$ and $c_{44}$ occurs at an $r/a \approx 3.5$. Overlaying the stiffness coefficients, $c_{55}$ and $c_{44}$ (Figure 9.1) is the stress influencing a vertically propagating and radially polarized shear wave, represented by the red and blue curves, and calculated using equation 4.38 (Chapter 102).
The red curve represents the stress influencing a shear wave polarized along the fast shear wave azimuth and the blue curve represents the stress influencing a shear wave polarized along the slow shear wave azimuth, as a function of distance from the center of the borehole. Both the red and blue curves indicate that the maximum stress influencing a vertically propagating and radially polarized shear wave occurs at the far field, where $r/a \geq 7$.

A literature review did not reveal the mechanism responsible for the formation stiffening immediately after the altered near borehole region. The mechanism(s) controlling the formation stiffening may very well be dependent upon the azimuth relative to the principal horizontal stresses.

### 9.3.1 Formation Stiffening In the Direction of Minimum Horizontal Stress

In this section I describe areas for further investigation relative to how the borehole stress redistribution may alter the surrounding rock formation. The formation stiffness profile for $c_{44}$ in Figure 9.1, from the borehole wall to the far-field along the slow shear wave azimuth, is represented by the blue markers. The blue curve represents the stress profile influencing the vertically propagating shear wave which is assumed to be radially polarized in the direction of minimum horizontal stress, $\sigma_{h_{\text{min}}}$, where $\theta = 90^\circ$. Moreover, the blue curve is an estimate of the stress influencing the stiffness profile shown by the blue markers.

Two possible mechanisms for introducing formation stiffening at intermediate radial distances away from the center of the borehole can be described through an elastoplastic theory that accounts for the effects of rock yield and the formation of compaction bands in the direction of minimum horizontal stress. The elastoplastic theory described by Sayers et al. (2008) shows that in the direction of minimum horizontal stress, the maximum value for each of three principal borehole stresses is in the vicinity directly behind the radius of the yielded zone (see Figure 3 in Sayers et al., 2008). The maximum value for each of three principal borehole stresses corresponds to the maximum shear wave velocity in the direction of minimum horizontal stress (see Figure 4 in Sayers et al., 2008). Elastoplastic theory allows the borehole stresses to yield the rock formation near the borehole. Yielding alters the
stresses and the velocity of a vertically propagating radially polarized shear wave relative to the radial distance from the center of the borehole, when compared to the elastic solution.

Compaction bands develop in the direction of minimum horizontal stress, which in a normal faulting regime, is assumed to be the slow shear azimuth for a vertically propagating radially polarized shear wave. Compaction bands occur in sandstones consisting of large grains ($\geq 0.3$ mm) and high porosity. They form at the radial distances with the highest stress concentration (i.e. tangential borehole stress, $\sigma_{\theta\theta}$) perpendicular to the borehole wall (Haimson, 2003). For examples of compaction bands forming around a borehole see Haimson and Lee (2004) and Stanchits et al. (2009). Within quartz-rich sandstones, compaction bands substantially reduce porosity; the inelastic deformation mechanism is mainly attributed to grain crowding due to grain sliding and grain rotation (Haimson, 2003; Schutjens et al., 2004).

When large stress loads are applied to high porosity rocks they cause pore collapse and grain deformation; the process is described as compactive yielding. For rocks that display signs of compactive yielding during stress loading, a cap exists to define the onset of inelastic deformation (Olsson, 1999; Schutjens et al., 2004). The cap in stress space separates the near elastic domain from the inelastic domain (see Figure 1 in Schutjens et al., 2004).

Olsson (1999) illustrates the concept of how compaction bands form by analyzing a conceptual stress-strain curve of a conventional axis symmetric compression test (see Figure 2 in Olsson, 1999). The main feature in the stress-strain curve is a shelf showing zero modulus. The start of a shelf indicates pore collapse and grain deformation and marks the boundary between the near elastic and inelastic domains (see Figure 4 in Olsson, 1999). Following the shelf, a stress-hardening phase can occur. The microstructure of the rock in this second stress-hardening phase is completely different from the rock in the stress-hardening phase in the near elastic domain. Therefore, the compaction behavior during the second stress-hardening is also different from the first cycle of stress-hardening (Schutjens et al., 2004).
Further investigation is necessary to describe the observation of formation stiffening in the direction of maximum horizontal stress. In this section I postulate that the stress history of the reservoir and the machinery used to drill a borehole may induce compaction. The formation stiffness profile for \( c_{55} \) (Figure 9.1), from the borehole wall to the far-field along the fast shear wave azimuth, is represented by the red markers. The red curve represents the stress profile influencing the vertically propagating fast shear wave which is assumed to be radially polarized in the direction of maximum horizontal stress, \( \sigma_{H_{\text{max}}} \), where \( \theta = 0^\circ \). Moreover, the red curve is an estimate of the stress influencing the stiffness profile shown by the red markers.

Figure 9.1 shows the formation stiffness of \( c_{55} \) at a \( r/a \approx 3.5 \) to be significantly greater than the stiffness of the far-field \( (r/a \geq 7) \). Understanding the mechanism(s) responsible for formation stiffening along the fast shear wave azimuth is inchoate and not straightforward. The changes in effective stress over the history of a reservoir due to decreasing pore pressure during depletion and increasing pore pressure during water flooding may cause the reservoir to be more susceptible to compaction. The history match of the average reservoir pressure of Postle Field (see Figure 5.7 in Heris, 2011) shows variations of approximately 13.79 MPa (2000 psi) over the life cycle of the reservoir. The effective stress path or loading history of a sandstone reservoir may initiate near elastic and inelastic deformation. Reservoir depletion increases the effective stress and requires the rock frame to support a greater load. The deformation mechanisms leading to reservoir compaction depend on the microstructure of the rock, grain and fluid composition, porosity, tectonic stresses, pore pressure, and temperature (Schutjens et al., 2004).

In addition, the forces introduced to the surrounding formation during drilling are possible mechanisms of formation stiffening. The segment of the drillstring that affects the direction of the bit, and therefore the trajectory of the borehole, is the bottomhole assembly (BHA). A simple BHA consists of only a drill bit, collars and drillpipe; a more complicated
BHA can consist of multiple components (for further description see Figure 8.120 in Bourgoyne Jr. et al., 1991). Additionally, Bourgoyne Jr. et al. (1991) provides an analysis of the BHA assuming the drillstring is static and two-dimensional. The forces acting on the bit are the axial load (i.e. weight on bit) and the side forces caused by tangency of the BHA. The principle stress trajectories of the drill bit show a zone of triaxial compression orthogonal to the line load of the drillstring and BHA (Maurer, 1966; Reichmuth, 1963). To accurately estimate the stresses exerted on the rock formation during the drilling process the three dimensional forces and displacements of the drillstring and BHA while rotating must be considered. This type of analysis requires a dynamic drilling simulation of a borehole using appropriate formation properties and tectonic stresses.

Despite proposing directions for further study to describe the limitations of this research, uncertainty remains. The formation stiffening phenomena observed in the region around 2.5 to 4.5 borehole radii (Figure 9.1), may be best expressed through the statement made by Morgan (2012).

I was drawn to science out of curiosity. Any questions I had, the answers were already there. But even science is never certain. Even the soundest of theories is subject to new data... and there are some things, even science can’t explain.
Figure 9.1: The stress influencing a vertically propagating and radially polarized shear wave is represented by the red and blue curves, with their corresponding units on the left vertical axis. The curves are based on equation 4.38 described in Sayers (2010) and Sayers et al. (2007). The red curve indicates the stress influencing a vertically propagating shear wave polarized in the direction of maximum horizontal stress, $\sigma_{H_{\max}}$, where $\theta = 0^\circ$. Whereas the blue curve indicates the stress influencing a vertically propagating shear wave polarized in the direction of minimum horizontal stress, $\sigma_{h_{\min}}$, where $\theta = 90^\circ$. The red and blue circles represent the stiffness obtained from the fast and slow shear wave radial variation profiles with their corresponding units on the right vertical axis. The red circles, $c_{55}$, indicate the stiffness assumed to be in the direction of maximum horizontal stress, $\sigma_{H_{\max}}$, where $\theta = 0^\circ$. The blue circles, $c_{44}$, indicate the stiffness assumed to be in the direction of minimum horizontal stress, $\sigma_{h_{\min}}$, where $\theta = 90^\circ$. 

\[ \begin{align*} 
2\Delta\sigma_{rr} &+ 2\Delta\sigma_{zz} + \Delta\sigma_{\theta\theta} (\theta = 0^\circ) \\
2\Delta\sigma_{rr} &+ 2\Delta\sigma_{zz} + \Delta\sigma_{\theta\theta} (\theta = 90^\circ) 
\end{align*} \]
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APPENDIX - AN OVERVIEW OF SOME ROCK PHYSICS MODELS

A review of the more commonly used rock-physics models and a hybrid approach to modeling sands, sandstones, and shales has been published by Avseth et al. (2010). Grechka and Kachanov (2006) compare two crack models, Schoenberg’s linear slip theory (Schoenberg, 1980) and Hudson’s crack model (Hudson, 1980), for dry fractures. Schoenberg’s linear slip theory assumes non-interaction between dry fractures, while Hudson’s crack model assumes the crack density to be the dilute approximation. Grechka and Kachanov (2006) observed that the non-interaction approximation of Schoenberg’s linear slip theory is sufficiently accurate to model intersecting or flat cracks.

Some of the rock-physics models which have been successful in modeling the fluid substitution response at high frequencies under laboratory conditions are the crack models of O’Connell and Budiansky (1974) and Kuster and Toksoz (1974). A successful extension of the Kuster and Toksoz (1974) crack model was done by Toksoz et al. (1976) and Cheng and Toksoz (1979), in which an estimation of the pore aspect ratio spectrum was done as a function of pressure and saturation conditions. Again the assumption is that these models are for the dilute concentrations of inclusions; in other words, the pore space is perfectly disconnected. Also Sun and Goldberg (1997) successfully modeled laboratory data using a derivation which extends Biot’s theory for a two-phase fractured porous medium.

Even though Gassmann’s relation for fluid substitution (Gassmann, 1951) is the low frequency limit of Biot’s theory (Biot, 1956), Siggins (2006) and Siggins et al. (2010) have had success using Gassman’s relation to model laboratory data at ultrasonic frequencies. However, I achieved only limited success using Gassmann’s relation (Gassmann, 1951) and the effective medium theory of Mori and Tanaka (Benveniste, 1987; Mori and Tanaka, 1973) to predict velocity changes related to fluid saturation and differential pressure in the rock physics laboratory experiment. Generally Gassmann’s relation is not expected to model
fluid effects at ultrasonic frequencies. This result only means Gassmann’s relation is not appropriate for this laboratory data but may be suitable at the field scale (Mavko, 2005).

A.1 Gassmann’s Fluid Substitution Theory

The relation Gassmann (1951) derived, assuming linear elasticity, is often used to estimate the changes in the effective bulk modulus with different pore fluids. Gassmann’s relation and equivalent variations, such as A.1, can be found in Mavko et al. (2003), where $K_{sat}$, $K_{dry}$, $K_0$ represent the effective bulk moduli of the fluid saturated rock, dry rock and mineral material. $K_{fl}$ is the effective bulk modulus of the pore fluid and $\phi$ is the porosity of the rock. Equation A.2 states that the effective shear modulus of the saturated rock, $\mu_{sat}$, is equal to the the effective shear modulus of the dry rock, $\mu_{dry}$.

\[
K_{sat} = K_{dry} + \frac{\phi}{K_{fl}} + \frac{1 - \phi}{K_0} - \frac{K_{dry}}{K_0^2} \left(1 - \frac{K_{dry}}{K_0}\right)^2 \tag{A.1}
\]

\[
\mu_{sat} = \mu_{dry} \tag{A.2}
\]

For laboratory experiments the dry rock bulk modulus, $K_{dry}$, can be obtained using the equations for P- and S-wave velocity in a homogeneous isotropic media given by (A.3) and (A.4), in which $K$ and $\mu$ are the effective bulk and shear moduli and $\rho$ is the average density of the rock.

\[
V_p = \sqrt{\frac{K + (4/3)\mu}{\rho}} \tag{A.3}
\]

\[
V_s = \sqrt{\frac{\mu}{\rho}} \tag{A.4}
\]
A.2 Mori and Tanaka Effective-Field Theory

The effective-field theory of Mori and Tanaka (1973) assumes an average strain within the matrix, which is not the same as the applied strain, this assumption accounts for the interaction of inclusions for a non-dilute approximation of effective properties. Benveniste (1987) derived simple expressions for the effective bulk and shear moduli of a two-phase composite with randomly oriented ellipsoidal inclusions, given by equations (A.5) and (A.6).

\[
\frac{(K^* - K_m)}{K_i - K_m} = \frac{\phi P^ni}{(1 - \phi) + \phi P^mi} \quad (A.5)
\]

\[
\frac{(\mu^* - \mu_m)}{\mu_i - \mu_m} = \frac{\phi Q^ni}{(1 - \phi) + \phi Q^mi} \quad (A.6)
\]

\(K^*\) and \(\mu^*\) are the effective bulk and shear moduli, \(K_i, \mu_i, K_m, \mu_m\) are the bulk and shear moduli of the inclusions and matrix. \(\phi\) is the porosity of the matrix; \(P^ni\) and \(Q^ni\) represent the averaging process for the ellipsoidal inclusions of material \(i\) in a background medium \(m\). Explicit expressions for \(P^ni\) and \(Q^ni\) for any aspect ratio ellipsoid can be found in Berryman (1980).

The effective-field theory of Mori and Tanaka (1973), as explained in Benveniste (1987) and Benveniste and Dvorak (1990), considers the composite (e.g. the rock) to be binary with perfect bonding between constituents. The particulate phase consists of ellipsoidal particles, or voids, that have the same shape but can be of different sizes. These inclusions are uniformly and randomly distributed. Each phase, rock matrix and inclusions, is assumed to be homogeneous.

To estimate the effective moduli, the bulk modulus, shear modulus, and porosity are the inputs for the matrix parameters. For fluid filled inclusions, the properties can be estimated using the FLAG calculator (Fluids/DHI Consortium, 2009). For a dry rock estimation, \(K_i\) can be assumed to be zero and \(\mu_i\) can be assumed to be zero for all inclusions for both dry or fluid filled.
The pore aspect ratio, typically assumed to be oblate spheroids, for a rock can be determined by minimizing the sum of the difference squared between the predicted effective moduli and the measured dynamic moduli. Usually minimization is done simultaneously for bulk and shear moduli at each differential pressure.