DETECTING ELECTRICAL AND HYDRAULIC HETEROGENEITIES
USING SEISMIC FOCUSING AND SEISMOELECTRIC
CONVERSIONS

by
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ABSTRACT

When a seismic wave propagates through a porous material and encounters a discontinuity in hydraulic or electrical properties, part of its energy is converted into an electromagnetic signal. Recording and interpreting this signal can be used to infer properties of the subsurface such as electrical conductivity, porosity, permeability, and saturation. However, a major issue of this method is that the amplitude of the converted electromagnetic signal is usually quite low and difficult to measure directly in the field. Sava & Revil recently proposed a seismic focusing technique (which will be referred to as “beamforming”), where multiple seismic sources located around the area of interest (usually placed in boreholes) are used to concentrate seismic energy at a desired location and time. If the focus point is located at a heterogeneity, a seismoelectric conversion will take place with a much greater amplitude than if a unique source had been used, making the converted signal easier to detect. So far, this new technique has only been tested on a simple case study and further investigations are needed to assess its strengths, weaknesses, and future potential applications. In this thesis, I develop a finite-element numerical model for the beamforming technique (coupled with seismoelectric conversions) in the frequency domain. I then perform a set of important numerical tests to assess the properties of the converted electromagnetic signals generated by the beamforming technique. Finally, the numerical beamforming model is successfully applied to track the location of a water saturation front inside an oil reservoir undergoing a water flooding process.
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# LIST OF SYMBOLS

Enhanced oil recovery ..................................................... EOR
Apparent mass density of the solid phase ............................. $\rho_s^\omega$
Solid phase displacement vector ........................................ $\mathbf{u}_s$
Hydro-mechanical coupling coefficient ................................. $\theta_\omega$
Pore fluid pressure .......................................................... $p$
Confining pressure ......................................................... $P$
Mass density of pore fluid ................................................ $\rho_f$
Permeability related constant .......................................... $k_\omega$
Forcing term applied on fluid phase ................................... $\mathbf{F}_f$
Effective stress tensor ..................................................... $\mathbf{T}$
Forcing term applied on solid phase ................................... $\mathbf{F}_s$
Lamé’s first parameter .................................................... $\lambda$
Shear modulus of solid frame .......................................... $G$
Biot coefficient #1 ......................................................... $M$
Biot coefficient #2 ......................................................... $\alpha$
Representative Elementary volume ..................................... REV
Filtration displacement .................................................... $\mathbf{w}$
Darcy velocity ............................................................... $\mathbf{\dot{w}}$
Bulk modulus of solid frame ............................................. $K_{fr}$
Fluid dynamic viscosity ................................................... $\eta_f$
<table>
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<th>Term</th>
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<td>$\rho$</td>
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<tr>
<td>Connected porosity</td>
<td>$\phi$</td>
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<td>Mass density of solid phase</td>
<td>$\rho_s$</td>
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<td>Mass density of fluid phase</td>
<td>$\rho_f$</td>
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<td>$K_s$</td>
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<td>$k_0$</td>
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<td>$m$</td>
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<td>Diffuse layer</td>
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Water saturation ............................................................. $S_w$
Mass density of oil phase .................................................. $\rho_o$
Mass density of water phase .............................................. $\rho_w$
Bulk modulus of oil .......................................................... $K_o$
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Magnetic permeability of the free space .......................... $\mu_0$
CHAPTER 1
INTRODUCTION

The seismoelectric method is a geophysical exploration technique that has been used to detect heterogeneities in the shallow subsurface. It consists in sending a seismic wave into the ground and remotely recording an electromagnetic response when the wave encounters a discontinuity in mechanical, hydraulic, or electrical properties of the porous rocks. The electromagnetic signals bear information about important characteristics of the subsurface such as permeability, porosity, and fluid content with a much better resolution than seismic waves. For instance, seismoelectric conversions have been used in laboratory experiments to detect mesoscopic fractures in the rocks (Jougnot & Rubino, 2013), sharp contrasts in fluid content and saturation (Revil & Mahardika, 2012), and image shallow targets in the subsurface (Haines, 2004).

The two main phenomena associated with seismoelectric conversions, which are called coseismic signal and interface response, will be discussed further in Chapter 2. However, standard seismoelectric methods focus primarily on the interface response, where a seismic wave incident upon an interface between two media creates a charge imbalance across that interface, which acts as a diffusing and oscillating electric dipole. This seismoelectric conversion can be detected remotely with an electrode and used to image heterogeneities in the subsurface. However, such a signal is usually difficult to record due to its relatively low amplitude, which is why it has not been more widely applied in geophysical exploration or environmental engineering. Field experiments aimed at imaging shallow subsurface targets (Butler et al., 1996; Mikhailov et al., 1997) show that seismoelectric conversions have a low amplitude and are difficult to measure. Butler et al. (1996) measured an interface response on the order of $0.5\, \text{mV} - 3\, \text{mV}$, generated at an interface located 7 m below the surface, while using a sledgehammer as a source. Mikhailov et al. (1997) also measured a seismoelectric
conversion, but on the order of 50 µV, which is generated at a soil/glacial till interface, 1.5 m down in the subsurface.

Recently, Sava & Revil (2012) found a way to obtain a conversion with a much higher amplitude. Instead of using a single seismic source, they developed a beam focusing technique where multiple sources are used to concentrate seismic energy at a desired location and time. If the point of focus is located at a heterogeneity, a seismoelectric conversion will occur and the diffused electric signal will be much stronger compared to the one excited by a single source.

In my thesis, I use the theoretical work of Sava & Revil and create the first numerical model for the beamforming technique, working in the frequency domain. I start by conducting a set of numerical tests to benchmark and evaluate the method on a simple case study. Then, I use my numerical beamforming model for a new cross-hole imaging application where I locate a water-oil saturation front inside an oil reservoir.

Please note that throughout this thesis, I use the terms “recorded wavefield” or “recorded potential” loosely. They do not correspond to real “recorded” data (nor real measurements), but refer to the synthetic signals modeled by a finite-element solver.

1.1 History of seismoelectric methods

Research on seismoelectric phenomena can be broken down into three main phases. According to Haines (2004), the first phase corresponds to simple observations and measurements of seismoelectric phenomena (Thompson, 1936 and Ivanov, 1939). Frenkel (1944) developed a first theory for porous media to explain Ivanov’s observations and managed to predict that the electric field in a compressional wave is directly proportional to the particle acceleration (Pride 2005). Even with the work of Frenkel, there was still a lack of understanding of the theory driving seismoelectric until the mid 90’s.

The second phase corresponds to a much more advanced understanding of the physics driving those phenomena with the theoretical work done by Pride (Pride, 1994; Pride & Haartsen, 1996; Haartsen & Pride, 1997). Pride was the first to derive an analytic expression
using a volume averaging method for the coupling mechanisms that occur in porous and elastic media between the mechanical set of equation or Biot theory (Biot, 1956a, b), and Maxwell’s equations. According to Revil et al. (2013), Pride’s way of modeling seismoelectric effects was done using three main assumptions: (1) solving the mechanical equations for the solid and fluid displacement vectors, (2) solving the electromagnetic problem in the diffusive limit of the Maxwell equations, and (3) linking the two sets of equations using an electrokinetic theory based on the use of the zeta potential, a local electrostatic potential defined inside the electrical double layer, at the interface between the grains and the pore fluid.

The last phase came with a better understanding of the micro-scale phenomena in porous materials (electrical double layer, complex conductivity, etc.), which gave rise to new simplified models for the coupling effects (Revil & Linde, 2006). Instead of using the zeta-potential to model the electrokinetic effect (coupling term in the two sets of equations), the theory is based on the volumetric charge density of the pore space in the porous material. This formulation has been developed and tested on experimental and field data by Revil and co-workers (Bolève et al., 2007; Revil et al., 2007). Throughout our work, we will use Revil’s approach to determine the streaming potential responsible for seismoelectric coupling and conversions in the quasi-static limit approximation of Maxwell’s equations (Revil & Linde, 2006). Another strength of this model is the reduction in the number of unknowns to solve for in Biot’s theory. In three dimensions, the classical method is to solve for six unknowns (three components of the solid phase displacement, and three components of the fluid phase). However, Revil and co-workers use Biot’s set of equations linking the filtration displacement to the fluid pore pressure and to the solid phase displacement. Therefore, by solving for the pore fluid pressure instead of the filtration displacement, they reduce the number of unknowns to four (three in the case of two-dimensional problems). This model also uses additional relationships between the excess charge density and the low-frequency permeability, decreasing the total number of unknowns also in the petrophysical model (Jardani et al.,
1.2 Research objectives and outline

My main objectives in this work can be described by the following steps:

(1) Build a 2D seismoelectric model in the frequency domain on a finite-element software (Comsol Multiphysics), using the full poroelastic theory.

(2) Follow the work of Sava & Revil (2012) by using an acoustic approximation of (1) and create the first numerical beamforming model in the frequency domain.

(3) Perform new fundamental numerical tests to assess the properties of the diffused electromagnetic signals arising from the beamforming technique, and benchmark my numerical code.

(4) Use my beamforming code for a new application where I locate a water-oil encroachment front inside a reservoir undergoing EOR.

Please note that all the modeling discussed in this thesis is limited to a 2D space.

Chapter 2 will discuss the seismoelectric theory, which is the coupling of Biot’s theory with Maxwell’s equations. I start by setting up the theoretical background, and I explain how and why those equations give rise to seismoelectric phenomena (coseismic signal and interface response).

In chapter 3, I present the beamforming technique developed by Sava & Revil (2012), the acoustic approximation I use to model it, and the extension to partially water saturated media. Finally, I provide a step by step methodology on how to practically apply this technique in the field.

In chapter 4, I show a simple 2D experiment where two anomalous bodies are imbedded into a homogeneous background. I explain in detail all the numerical modeling steps (mesh properties, solver type, PML parameters, seismic source, and FFT parameters). The results and conclusions I draw from these new fundamental tests are important to assess this new beamforming technique.
In chapter 5, I use my numerical model on a new cross-hole imaging application where I keep track of a water saturation front during the water flooding of an oil reservoir. At a given time (or snapshot), I scan the domain point by point and record the electrical potential at electrodes located on boreholes surrounding the area. When I focus seismic energy at a point located in the vicinity of the water-oil interface, I pick up a strong signal indicating the presence of a heterogeneous feature.

Finally, in chapter 6, I conclude and discuss future work going forward.
In this chapter, I discuss the two fundamental sets of equations needed to build a seismolectric theory. First, I summarize the dynamic poroelasticity theory (Biot’s theory) in fully water saturated media. Then, I discuss the dynamic electrokinetic theory (the coupling between the mechanical equations and Maxwell equations). Finally, I give a physical interpretation of these equations and I develop two numerical experiments to illustrate the two main seismolectric effects.

2.1 Dynamic poroelasticity theory in fully water saturated materials

The Biot poroelastic theory (Biot, 1956a, b) assumes an isotropic and fully-water saturated poroelastic material where the pore fluid is a viscous Newtonian fluid. Moreover, all pores are assumed to be connected. The mechanical disturbance applied to the medium is assumed to be harmonic (time dependence in the form of $e^{-j\omega t}$, where $j^2 = -1$ is the pure imaginary number, $\omega$ is the angular frequency, and $t$ is the time). As discussed previously, I chose to solve for the two components of solid phase displacement and for the pore fluid pressure, which leads to three unknowns. Assuming no electroseismic effect (seismic waves generated by the presence of an electric field), the mechanical modeling can be solved using the three main poroelastic equations of motion in the frequency domain (Jardani et al., 2010),

$$-\omega^2 \rho^*_s \mathbf{u}_s + \theta \omega \nabla p - \omega^2 \rho_f k \omega \mathbf{F}_f = \nabla \cdot \mathbf{T} + \mathbf{F}_s,$$

(2.1)

$$\mathbf{T} = (\lambda \nabla \cdot \mathbf{u}_s) \mathbf{I}_d + G \left[ \nabla (\mathbf{u}_s) + (\nabla (\mathbf{u}_s))^T \right],$$

(2.2)

$$\frac{p}{M} + \nabla \cdot [k \omega (\nabla p - \omega^2 \rho_f \mathbf{u}_s + \mathbf{F}_f)] + \alpha \nabla \cdot \mathbf{u}_s = 0,$$

(2.3)
where the superscript “T” denotes “Transpose”. Equation 2.1 corresponds to a macroscopic momentum conservation equation for the solid phase, equation 2.2 refers to a constitutive equation for the effective stress tensor (Hooke’s law for porous and elastic media) and equation 2.3 is a momentum conservation equation for the pore fluid. \( u_s \) corresponds to the solid phase displacement vector (each component has the unit of m), \( u_f \) is the fluid phase displacement, and \( w = \phi(u_f - u_s) \) is the filtration displacement vector (the relative fluid-solid displacement vector, also in m), \( p \) (in Pa) is the fluid pressure in the pore water phase. In 2D, I will only be solving for the two components of \( u_s \) and for the pore pressure \( p \), a total of three unknowns.

In equation 2.1, \( \rho_s^o \) (in kg m\(^{-3}\)) denotes an apparent mass density of the solid phase, the coupling term \( \theta_\omega \) (unitless) denotes a hydro-mechanical coupling coefficient, \( \rho_f \) (in kg m\(^{-3}\)) is the mass density of the pore fluid, \( k_\omega \) (in m\(^3\) s\(^{-2}\) kg\(^{-1}\)) is a permeability-related constant, \( F_f \) is a vector that corresponds to a force term applied only on the fluid phase (each component of \( F_f \) has a unit of N m\(^{-3}\)) and \( F_s \) is a force term applied on the solid phase of the material (each component of this vector is in N m\(^{-3}\)). \( \hat{T} \) (each element of the tensor is in Pa) is the effective stress tensor and is related to the total stress tensor \( T \) by equation 2.8.

In equation 2.2, \( \lambda \) (in Pa) is Lamé’s first parameter defined by \( \lambda = K - (2/3)G \) where \( K = K_{fr} \) and \( G = G_{fr} \) (both expressed in Pa) denote the bulk modulus and the shear modulus of the solid frame (solid skeleton without any fluid inside) respectively. \( I_d \) corresponds to the identity matrix. In equation 2.3, \( M \) (in Pa) and \( \alpha \) (unitless) are two Biot coefficients.

Please note that equations 2.1, 2.2, 2.3 and 2.11 have already been averaged over a representative elementary volume (REV) greater than the grain size and smaller than the wavelength of the seismic perturbation applied. Therefore, \( u_s, u_f, p \) and \( w \) have also been averaged.

All parameters and equations discussed above have been derived by Jardani et al. (2010) and Revil et al. (2013):

\[
k_\omega = \frac{1}{\omega^2 p_f + j \omega b} \quad \text{(2.4)}
\]
\[ \rho^S_\omega = \rho - \omega^2 \rho_f^2 k_\omega, \tag{2.5} \]
\[ \theta_\omega = \alpha - \omega^2 \rho_f k_\omega, \tag{2.6} \]
\[ \tilde{\rho}_f = F \rho_f, \tag{2.7} \]
\[ T = \tilde{T} - \alpha p I_d, \tag{2.8} \]
\[ M = \frac{K_f K_s}{K_f (1 - \phi - K_{fr}/K_s) + \phi K_s}, \tag{2.9} \]
\[ \alpha = 1 - \frac{K_{fr}}{K_s}, \tag{2.10} \]

and \( b = \eta_f/k_0 \) where \( k_0 \) (in \( m^2 \)) is the permeability for the quasi-static flow (DC permeability), and \( \eta_f \) is the fluid dynamic viscosity (in Pa s). In the following, I will note \( \rho = (1 - \phi) \rho_s + \phi \rho_f \) the average mass density of the material (in kg m\(^{-3}\)), \( \phi \) denotes the connected porosity and \( \rho_s \) and \( \rho_f \) denote the mass density of the solid and pore solution, respectively. \( K_s \) is the bulk modulus of the solid phase (in Pa), \( K_f \) is the bulk modulus of the fluid phase, and the formation factor \( F \) is related to the porosity by the first Archie’s law \( F = \phi^{-m} \), where \( m \) is called the cementation exponent (Archie, 1942).

Therefore, using the three equations 2.1, 2.2 and 2.3, I solve for the three unknowns \( \mathbf{u}_s(u_{s1}, u_{s2}) \) and \( p \). Eventually, equation 2.11 (generalized Darcy’s law) below is used to relate the pore pressure \( p \) and the solid phase displacement \( \mathbf{u}_s \) to the filtration displacement \( \mathbf{w} \):

\[ \mathbf{w} = k_w (\nabla p - \omega^2 \rho_f \mathbf{u}_s + \mathbf{F}_f). \tag{2.11} \]

### 2.2 Dynamic electrokinetic theory in fully water-saturated materials

In the following, I discuss the connection between the mechanical set of equations and Maxwell’s equations. Pride (1994) and Revil et al. (2013) showed that a coupling term existed and gave two different analytical expressions for it. This coupling coefficient gives rise to two main effects:
(1) Seismoelectric: a seismic perturbation creates an electromagnetic disturbance.

(2) Electroseismic: an oscillating electric field applied to a porous medium creates a mechanical disturbance (fluid flow) within the pores of the medium (this effect was neglected in the previous section).

Following the work of Revil et al. (2013), I discuss in this section the effect that this coupling terms will have on each set of equations (electromagnetic and mechanical).

2.2.1 Generalized Ohm’s law

Revil & Mahardika (2013) show that the total macroscopic electrical current density $J_{\text{total}}$ (each component of the vector has a unit of $A\,m^{-2}$) can be expressed by the sum of two dominant terms:

$$J_{\text{total}} = J_c + J_s,$$  

(2.12)

and $J_c$ is the conduction current density (in $A\,m^{-2}$) given by the local Ohm’s law:

$$J_c = \sigma E,$$  

(2.13)

where $\sigma$ (in S m) is the DC electrical conductivity. In the following, I will neglect surface conductivity and I model the electrical conductivity $\sigma$ using:

$$\sigma = \frac{1}{F} \sigma_w,$$  

(2.14)

where $\sigma_w$ is the electrical conductivity of water. Equation 2.14 is valid for a fully water saturated porous rock.

The last term of equation 2.12 is a source current density due to the advective drag of the excess charge of the pores space by the flow of the pore water (Pride, 1994; Revil & Mahardika, 2013). As the seismic wave passes through the pores, it compresses the grains and generates fluid flow within the pores, away from the seismic wave pressure peak. As the fluid flows, it drags part of the counterions present in the diffuse layer coating the grains (section 2.3). By doing so, the flow creates a local charge relative displacement giving rise to a dynamic streaming current density $J_s$ (in $A\,m^{-2}$) (Jardani et al., 2010),
\[ \mathbf{J}_s = \hat{Q}_v \dot{\mathbf{w}} = -j \omega \hat{Q}_v k_\omega (\nabla p - \omega^2 \rho_f \mathbf{u}_s + \mathbf{F}_f), \] (2.15)

where \( \hat{Q}_v \) (in \( \text{C m}^{-3} \)) denotes the volumetric charge density carried by the flow of pore water relative to the grains. This parameter is empirically related to the DC permeability by the following relation (Jardani et al., 2010; Revil & Jardani, 2010; Revil & Mahardika, 2013):

\[ \log(\hat{Q}_v) = -9.2349 - 0.8219 \log(k_0). \] (2.16)

Moreover, within the frequency range of seismic, I can neglect the frequency dependence and the imaginary part of \( \hat{Q}_v \) (Revil & Mahardika, 2013). Hence, equation 2.15 is where the coupling between mechanical and electromagnetic occurs. I can identify from this equation the first coupling coefficient to be:

\[ C_1 = -j \omega \hat{Q}_v k_\omega = C_1(\omega, F, \rho_f, k_0). \] (2.17)

It is clear from equations 2.16 and 2.17 that if I generate a pressure gradient or an acceleration of the solid phase (or both) by applying a pressure field on the material, I obtain a source current density and therefore, an electric field.

Moreover, as I consider the electromagnetic disturbances in the quasi-static limit of the Maxwell equations, all time derivatives vanish and the electric potential \( \psi \) (in V) can be obtained by solving Poisson’s equation:

\[ \nabla \cdot (\mathbf{J}_{\text{total}}) = 0, \] (2.18)

\[ \nabla \cdot (\sigma \nabla \psi) = \nabla \cdot \mathbf{J}_s, \] (2.19)

\[ \nabla \cdot (\sigma \nabla \psi) = \nabla \cdot \left[ -j \omega \hat{Q}_v k_\omega (\nabla p - \omega^2 \rho_f \mathbf{u}_s + \mathbf{F}_f) \right], \] (2.20)

where the electrical potential \( \psi \) and the electric field \( \mathbf{E} \) (in V/m) are related by the following equation:

\[ \mathbf{E} = -\nabla \psi. \] (2.21)
2.2.2 Generalized Darcy’s law

Equation 2.11 can also be written in the following form (Revil & Mahardika, 2013):

\[ \dot{\mathbf{w}} = -\frac{k_0}{\eta_f}(\nabla p + \rho_f \ddot{\mathbf{u}}_s + \rho_f F \dot{\mathbf{w}} + \mathbf{F}_r), \]  

(2.22)

where \( \dot{\mathbf{w}} \) is the Darcy velocity (in m/s). Moreover, as mentioned in section 2.1, \( \mathbf{F}_r \) is the body force applied to the pore water phase (or the electrical force acting on the excess of charges of the pore water), and can be expressed by Coulomb’s law,

\[ \mathbf{F}_r = \hat{Q}_v \mathbf{E}. \]  

(2.23)

After taking the Fourier transform of equation 2.22, I obtain:

\[ -j\omega \mathbf{w} = -\frac{k^*(\omega)}{\eta_f}(\nabla p - \omega^2 \rho_f \mathbf{u}_s) + \frac{k^*(\omega)\hat{Q}_v}{\eta_f} \mathbf{E}, \]  

(2.24)

where \( k^*(\omega) = \frac{k_0}{1 - j\omega \tau_k} \) is a complex value apparent permeability (in m²), and \( \tau_k = \frac{k_0 \rho_f F}{\eta_f} \) is a relaxation time (in s) (Revil & Mahardika, 2013).

Furthermore, from equation 2.24, I can identify the second coupling coefficient,

\[ C_2 = \frac{k^*(\omega)\hat{Q}_v}{\eta_f} = C_2(\omega, F, \rho_f, k_0). \]  

(2.25)

Equation 2.24 shows that an electric field can generate a relative displacement between the solid and the fluid phase, which is referred to as electroosmosis (flow of pore water in response to an electric field). As the positive counterions are flowing in under the presence of an electric field, they drag the water molecules along with them, creating fluid flow. However, it has been shown experimentally that the amplitude of the term \( C_2 \mathbf{E} \) is much smaller than the other terms in equation 2.24. In the following, I neglect this term in all computations (Revil et al., 1999).

Note that this implies that the mechanical set of equations (Biot’s theory) can be solved independently from Poisson’s equation. Practically, I first solve the system of equations 2.1, 2.2 and 2.3, which gives me \( \mathbf{u}_s \) and \( p \). Then, I can solve for the electric potential (equation...
2.3 Seismoelectric phenomena

This section is intended to give a more physical and intuitive explanation of the theory discussed above (sections 2.1.1 and 2.1.2). I start at the pore scale level by explaining the concept of electrical double layer (EDL). Then, to illustrate the two main seismoelectric effects, I show two simple numerical experiments performed on Comsol Multiphysics, a finite-element solver, using the full poroelastic equations described above. Please note that the parameters used for the numerical modeling on Comsol Multiphysics will be discussed in more detail in chapter 4.

2.3.1 Electrical Double Layer (EDL)

In the last fifteen years, Revil and co-workers have developed a more accurate and exhaustive understanding of the EDL at a pore scale level (Revil & Leroy, 2001; Leroy & Revil, 2004) than what had been done previously. This improvement was key to develop new simplified models for seismoelectric effects using a much more intuitive approach (Revil et al., 2007).

When a porous material is in contact with a fluid electrolyte (salt water for instance), the surface of its grains is the setting of electrochemical reactions, and it acquires a surface charge (Jardani et al., 2010). This excess charge attracts ions with opposite sign (counterions) present in the pore fluid. Those counterions form two adjacent layers around the surface of the grains. Coating the grains, we first encounter the Stern layer (SL) where counterions are sorbed and immobile. However, the counterions in the SL do not counterbalance exactly the charge at the grain surface. In order to ensure electro neutrality of a REV composed of \{grains + pore fluid\}, another layer called the diffuse layer (DL) is created further away from the grains, adjacent to the SL. Both layers form the EDL (Figure 2.1).

The key result here is that in the DL, the counterions are not sorbed, but mobile and free to move. Hence, if a seismic perturbation passes through and creates fluid flow, the
mobile counterions will be dragged along with the fluid, creating a local charge relative displacement.

### 2.3.2 Coseismic electric field

The passage of a seismic wave (fast P and S-waves) throughout a homogeneous poroelastic material generates fluid flow within the pores, which drags part of the counterions present in the DL. This relative charge displacement of the pore water with respect to the solid phase is responsible for a source current density $J_s$, called the streaming current density. Pride (1994) proved that in a homogenous medium, the conduction current counterbalances the streaming current, and we have:

$$J_{\text{total}} = J_c + J_s = 0.$$  \hfill (2.26)

From equations 2.12, 2.13, and 2.15, it is clear that in a homogeneous medium,

$$E = \frac{j \omega \hat{Q}_v k_w}{\sigma} (\nabla p - \omega^2 \rho_f u_s).$$  \hfill (2.27)
This electric field generated by the passage of the wave has an amplitude proportional to a linear combination of the pore pressure gradient and the acceleration of the solid phase. Therefore, it is only present and measurable within the support of the seismic wave and propagates at the same velocity. It is a local electric field.

In order to illustrate this phenomenon, I modeled a synthetic example where I placed a virtual seismic source 50 m below a reference level, at the top of a 500 m-thick and 500 m-wide fully water saturated sandstone layer (Figure 2.2). I place a virtual electrode and geophone at the bottom of the layer, and I propagate a seismic wave from the source (30 Hz-dominant frequency Ricker wavelet, time delayed by 50 ms). The mechanical, hydraulic, and electrical parameters used for this experiment are listed in Figure 2.3.

In Figure 2.4, Figure 2.5, and Figure 2.6, I display three sets of snapshots taken as the seismic wave propagates downwards into the sandstone layer. We note that no electric signal is recorded until the seismic wave reaches the bottom of the sandstone layer (Figure 2.4d, and Figure 2.5d). Once the wave reaches the receivers, the electrode picks up an electric signal corresponding to the coseismic field (Figure 2.6d). However, this phenomenon is of limited interest for geophysical exploration since it can not be measured remotely. The coseismic signal bears information on hydraulic and electrical properties, but only for the area located in the vicinity of the electrodes. Furthermore, this effect usually dominates other seismoelectric effects in terms of amplitude, and part of the difficulty in seismoelectric methods is filtering those coseismic signals. In chapter 4, I will give an example of a technique used to solve this issue.

2.3.3 Interface response

Another phenomenon occurs when a seismic wave passes through a interface with a sharp discontinuity in hydraulic or electrical properties. The seismic disturbances create fluid flow across the interface and generate a streaming current imbalance that takes place at the discontinuity. This process creates an oscillating electric dipole, whose moment is perpendicular to the interface. Therefore, an electric field (and an electric potential variation)
Figure 2.2: Numerical experiment setup. A virtual seismic source is placed at the top of a 500 m x 500 m homogeneous sandstone layer. At the bottom of the layer, I placed a geophone and an electrode (referred to as “receivers”) in order to measure the solid phase displacement, the fluid pore pressure, and the electric potential.
<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>SANDSTONE LAYER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk modulus of the solid phase $K_s$ (Pa)</td>
<td>36.5x10⁹</td>
</tr>
<tr>
<td>Bulk modulus of the fluid phase $K_f$ (Pa)</td>
<td>0.25x10⁹</td>
</tr>
<tr>
<td>Bulk modulus of the frame $K_{fr}$ (Pa)</td>
<td>2.22x10⁹</td>
</tr>
<tr>
<td>Shear modulus of the frame $G$ (Pa)</td>
<td>4.00x10⁹</td>
</tr>
<tr>
<td>Porosity $\varphi$</td>
<td>0.25</td>
</tr>
<tr>
<td>Material mass density $\rho$ (kg m⁻³)</td>
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</tr>
<tr>
<td>Excess charge density $Q_o$ (C m⁻³)</td>
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</tr>
<tr>
<td>Conductivity $\sigma$ (S m⁻¹)</td>
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</tr>
<tr>
<td>Hydraulic viscosity of water $\eta_f$ (Pa s)</td>
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</tr>
<tr>
<td>Log (DC permeability, $k_0$ in m²)</td>
<td>-12</td>
</tr>
</tbody>
</table>

Figure 2.3: List of mechanical, hydraulic and electrical properties of the fully water saturated sandstone layer (Araji et al., 2012).
Figure 2.4: Snapshots at $t = 88.5$ ms. (a) Vertical displacement distribution over the entire layer. (b) Electric potential distribution over the entire layer. (c) Vertical displacement at geophone. (d) Electric potential at electrode. Amplitudes have been scaled in a consistent manner.
Figure 2.5: Snapshots at $t = 219$ ms. (a) Vertical displacement distribution over the entire layer. (b) Electric potential distribution over the entire layer. (c) Vertical displacement at geophone. (d) Electric potential at electrode. Amplitudes have been scaled in a consistent manner.
Figure 2.6: Snapshots at $t = 268$ ms. (a) Vertical displacement distribution over the entire layer. (b) Electric potential distribution over the entire layer. (c) Vertical displacement at geophone. (d) Electric potential at electrode. Amplitudes have been scaled in a consistent manner.
is diffused away from the heterogeneity, and can be measured remotely with a set of electrodes (Araji et al., 2012). To better understand the nature of the electric field generated by the interface response, consider the Telegraphist equation (equation 2.28) expressed in the frequency domain (assuming a harmonic time dependence) by:

$$\nabla \times \nabla \times \mathbf{E} - \frac{j \omega}{D} \mathbf{E} - \frac{\omega^2}{c^2} \mathbf{E} - \mu_0 j \omega \mathbf{J}_s = 0,$$

(2.28)

where $D = \frac{1}{\mu_0 \sigma}$ denotes the electromagnetic diffusivity (in m$^2$/s), $c = \frac{1}{\sqrt{\mu_0 \epsilon}}$ is the electromagnetic velocity (speed of light), $\mu_0 = 4\pi \times 10^{-7}$ H m$^{-1}$ is the magnetic permeability of the free space, and $\epsilon$ is the electric permittivity of the medium (in F/m).

In equation 2.28, the second term in $-\frac{j \omega}{D} \mathbf{E}$ corresponds to the diffusion term, the third term is the propagation term in $-\frac{\omega^2}{c^2} \mathbf{E}$, and the last term in $-\mu_0 j \omega \mathbf{J}_s$ is the source term (streaming current) generated by the charge imbalance across the interface. For $f < f_{\text{critical}}$, where $f_{\text{critical}} = \frac{c^2}{2 \pi D}$, the diffusion term starts dominating the propagation term. Typically, with the frequency range I am using ($f < 1$ kHz), I can neglect the propagation term and assume the electric field generated by the interface response is purely diffusive. Another characteristic of the interface response is its quasi-instantaneous diffusion process. The diffusivity $D = \frac{1}{\mu_0 \sigma}$ is also linked to characteristic diffusion time $\tau$ and the characteristic diffusion length $L$, by the following expression:

$$D = \frac{L^2}{\tau}.\quad (2.29)$$

Hence, for typical values of $L = 200$ m, $\sigma = 0.01$ S m$^{-1}$, I obtain a characteristic diffusion time of $\tau \approx 0.50$ ms, which is smaller than the usual sampling rate used in seismic. Therefore, in the following, I will also neglect the diffusion term in equation 2.28, and assume that the interface response will have a quasi-instantaneous diffusion process for observation points located within a radius of 200 m. This confirms the validity the quasi-static limit approach of Maxwell’s equations (equations 2.18 and 2.21).

In order to illustrate this phenomenon, I use a similar setup to the one used in section 2.1.3, but now the upper half part of the sand layer is replaced by a shale layer (Figure 2.7).
Properties of both layers are listed in Figure 2.8. I propagate a seismic wave from a virtual point source located at the top of the shale layer, and display four sets of snapshots taken at different times during the wave propagation (Figure 2.9, Figure 2.10, Figure 2.12, and Figure 2.13).

Figure 2.7: Numerical experiment setup. A virtual seismic source is placed at the top of a 250 m-thick and 500 m-wide shale layer. Underneath the shale layer, a sandstone layer is modeled with the same properties as in the previous experiment. At the bottom of the layer, we placed a geophone and electrode (referred to as “receivers”) in order to measure the solid phase displacement, the pore pressure and the electric potential.

In Figure 2.9b, we are not able to see the coseismic field generated by the passage of the wave in the shale layer. In fact, the coseismic field is definitely present, but for consis-
<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>SANDSTONE LAYER</th>
<th>SHALE LAYER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk modulus of the solid phase ( K_s ) (Pa)</td>
<td>36.5x10^9</td>
<td>6.9x10^9</td>
</tr>
<tr>
<td>Bulk modulus of the fluid phase ( K_f ) (Pa)</td>
<td>0.25x10^9</td>
<td>0.25x10^9</td>
</tr>
<tr>
<td>Bulk modulus of the frame ( K_H ) (Pa)</td>
<td>2.22x10^9</td>
<td>6.89x10^9</td>
</tr>
<tr>
<td>Shear modulus of the frame ( G ) (Pa)</td>
<td>4.00x10^9</td>
<td>3.57x10^9</td>
</tr>
<tr>
<td>Porosity ( \varphi )</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>Material mass density ( \rho ) (kg m(^{-3}))</td>
<td>2237.5</td>
<td>2485</td>
</tr>
<tr>
<td>Excess charge density ( Q_e ) (C m(^{-3}))</td>
<td>0.203</td>
<td>3.49</td>
</tr>
<tr>
<td>Conductivity ( \sigma ) (S m(^{-1}))</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>Hydraulic viscosity of pore fluid ( \eta_f ) (Pa s)</td>
<td>10(^{-3})</td>
<td>10(^{-3})</td>
</tr>
<tr>
<td>Log (DC permeability, ( k_o ) in m(^2))</td>
<td>-12</td>
<td>-16</td>
</tr>
<tr>
<td>Water saturation ( S_w )</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 2.8: List of mechanical, hydraulic, and electrical properties of the sandstone and shale layers chosen for numerical modeling (Araji et al., 2012).
Figure 2.9: Snapshot at $t = 88.5$ ms. (a) Vertical displacement distribution over the entire domain. (b) Electric potential distribution over the entire domain. (c) Vertical displacement at geophone. (d) Electric potential at electrode. Amplitudes have been scaled in a consistent manner.
tency purposes, I chose to keep the same normalizing factor and scale as in previous plots. Therefore, this shows that the coseismic field $\mathbf{E}_{\text{shale}}$ has an amplitude much smaller than in the sandstone layer. This result could have been anticipated since its amplitude can be expressed by:

$$||\mathbf{E}_{\text{coseismic}}|| \propto A L(\nabla p, \mathbf{u}_s, \omega, \rho_f),$$

(2.30)

where $A = \frac{k_0 \tilde{Q}_v}{\sigma}$ and $L(\nabla p, \mathbf{u}_s, \omega, \rho_f)$ is a linear combination of the remaining variables (equation 2.27). As a first order approximation, the greater $A$, the greater the amplitude of the coseismic field. In our example, $\frac{A_{\text{shale}}}{A_{\text{sand}}} \approx 6 \times 10^{-5}$. Therefore, the amplitude of the coseismic field in the sandstone layer should be approximately $6 \times 10^5$ times larger than the one in the shale layer, which explains why it is not visible on Figure 2.9b.

Snapshots displayed in Figure 2.10 were taken at the instant when the seismic wave reaches the shale/sandstone interface. At this moment, a seismoelectric conversion (interface response) is taking place. The passage of the wave across the interface creates a charge imbalance and diffuses an electric field that can be measured instantaneously and remotely (Figure 2.10d). Figure 2.10b and Figure 2.11 show the potential lines distribution, which is very similar to the ones of an electric dipole located at the interface, with a moment perpendicular to the interface. The two poles are located on each sides of the interface and each of them acts as a current point source with opposite signs, which confirms the similarity of this phenomenon to an electric dipole.

Since the electric signal diffused has a strong dipolar component, it is expected, for the 3D case, that its amplitude measured at an observation point in space should be inversely proportional to the square of the distance between the point and the interface where the conversion occurs. Also, as the medium is conductive, the amplitude should also be inversely proportional to its electrical conductivity. This is confirmed by Figure 2.11, where we can see that the electric potential distribution is not symmetrical in terms of amplitude with respect to the interface. In the shale layer, the conductivity is 10 times higher than in the sand layer and therefore, the electric potential is much more attenuated.
Figure 2.10: Snapshots at \( t = 133 \) ms. (a) Vertical displacement distribution over the entire domain. (b) Electric potential distribution over the entire domain. (c) Vertical displacement at geophone. (d) Electric potential at electrode. Amplitudes have been scaled in a consistent manner.
Figure 2.11: Snapshot at $t = 133$ ms. Interface response, potential lines distribution. This figure shows the same data as Figure 2.10b, but with a different color map and scale.
Figure 2.12: Snapshots at $t = 206$ ms. (a) Vertical displacement distribution over the entire domain. (b) Electric potential distribution over the entire domain. (c) Vertical displacement at geophone. (d) Electric potential at electrode. Amplitudes have been scaled in a consistent manner.
Figure 2.12 displays a set of snapshots taken after the seismic wave has crossed the interface. In Figure 2.12a, we can see the weak upgoing reflected wavefield in shale layer. Furthermore, the coseismic signal propagating along with the seismic wave is now visible, as its amplitude is much greater than in the shale layer (Figure 2.12b).

Figure 2.13: Snapshots at $t = 250\,\text{ms}$. (a) Vertical displacement distribution over the entire domain. (b) Electric potential distribution over the entire domain. (c) Vertical displacement at geophone. (d) Electric potential at electrode. Amplitudes have been scaled in a consistent manner.

The set of snapshots in Figure 2.13 was taken at the moment where the seismic wave reaches the bottom of the sand layer. The electrode can now pick up the coseismic signal
Finally, in Figure 2.14, I display the three time series associated with this experiment. In order to illustrate the instantaneous nature of the diffused electric potential, I displayed in Figure 2.14a the seismogram that I would have recorded had I placed a geophone at the shale/sandstone interface. It is clear that the arrival of the seismic wave at the shale/sandstone interface coincides with the interface response picked up at the electrode.

![Figure 2.14: (a) Time serie of the vertical displacement recorded by a geophone placed at the shale/sandstone interface. (b) Time serie of the vertical displacement recorded by the geophone placed at the bottom of the sandstone layer. (c) Time serie of the electric potential recorded by the electrode placed at the bottom of the sandstone layer.](image)

2.4 Summary

Throughout this chapter, I used the theoretical work done by Revil and co-workers to build my own full poroelastic and seismoelectric numerical model on a finite-element software. By testing it on a simple case study, I managed to observe the two main seismoelectric phenomena caused by the propagation of a seismic wave inside a porous material:

1. The local coseismic signal located inside the support of the seismic wave, whose amplitude is proportional to a linear combination of the solid phase acceleration and the
gradient of the pore fluid pressure.

(2) The interface response, which is an electric signal diffused when a seismic wave passes through a zone of sharp discontinuities in hydraulic or/and electrical properties. This phenomenon is of interest for geophysical exploration as it can be measured remotely and it bears information on the presence of a gradient of hydraulic and electrical properties within the subsurface (but not directly the values of those properties). Throughout my work, I focus on measuring this signal.

In this particular example, the amplitude of the interface response is comparable to the one from the coseismic signal picked up at the electrode. Unfortunately, field measurements (Mikhailov, 1997; Haines, 2004) show that the interface response usually has a relatively low amplitude compared to the coseismic signal, making it difficult to detect and interpret in the data. To solve this issue, Sava & Revil (2012) designed a method where seismic energy is focused at a location in order to increase the amplitude of the seismoelectric conversion. This is the method that I will use in the following and is explained in more detail in the next chapter.
CHAPTER 3
SEISMIC BEAMFORMING

The beamforming technique developed by Sava & Revil (2012) is designed to use seismic sources located in boreholes surrounding a zone of interest in the subsurface, and focus seismic energy at desired locations. The objective is to scan the rock volume point by point, and record the electrical potential using electrodes placed in the boreholes at each focusing time. When the seismic energy focuses at a heterogeneity, such as an interface between two media, I record a stronger seismoelectric conversion (interface response) than would have recorded had only one source been used. Therefore, this technique enables me not only to identify whether the point of focus is located at an interface, but also to make a map of the subsurface electrical and hydraulic structures. As I will discuss further in this chapter, this process can be very computationally intensive because it needs to be applied point by point in the area of interest. To reduce the computation time, I use an acoustic approximation, which I described in section 3.1, to model the seismoelectric problem (Sava & Revil, 2012). Then, I use the results from Revil et al. (2013) to extend the theory to partially water saturated porous rocks. Finally, the beamforming process is explained and discussed in detail in section 3.3.

3.1 Acoustic approximation for fully water saturated materials

Using the full poroelastic modeling (Biot, 1956a, b) is extremely expensive computationally. Therefore, Sava & Revil (2012) developed an acoustic approximation in order to optimize the beamforming process. However, this requires adapting and modifying the seismoelectric theory according to the new assumptions.

• **Step 1:** I solve for the confining pressure $P$ (in Pa) of the material using the following equation:
\[
\frac{\delta^2 P(x, t)}{\delta t^2} - K_u(x) \nabla \left[ \frac{1}{\rho(x)} \nabla P(x, t) \right] = s(x, t), \quad (3.1)
\]

where \(K_u\) (in Pa) denotes the undrained bulk modulus and can be expressed by (Gassmann, 1951):

\[
K_u = \frac{K_f (K_s - K_{fr}) + \phi K_{fr} (K_s - K_f)}{K_f (1 - \phi - K_{fr}/K_s) + \phi K_s}. \quad (3.2)
\]

Moreover, \(s(x, t)\) (in \(\text{Pa s}^{-2}\)) is the source term (seismic point source), and can be expressed by:

\[
s(x, t) = \delta(x - x_s) a(t), \quad (3.3)
\]

where \(\delta(x - x_s)\) is a 2D delta function, \(x_s = (x_s, z_s)\) is the source location, and \(a(t)\) is the source signal (in \(\text{Pa m}^2 \text{s}^{-2}\)). Revil et al. (2013) show that by neglecting the viscous coupling between the pore water and the solid phase, the velocity of the P-waves can be approximated by:

\[
V_p = \sqrt{\frac{K_u + \frac{4}{3}G}{\rho}}. \quad (3.4)
\]

However, by using the acoustic approximation in equation 3.1, the modeled P-wave velocity is \(V_p\), where \(V_p = \sqrt{\frac{K_u}{\rho}}\), which would not match field values (equation 3.4). In order to model more realistic P-wave velocity values, I modify equation 3.1 and replace the undrained bulk modulus \(K_u\), by \(K_u + \frac{4}{3}G\), where \(G\) (in Pa) is the shear modulus of the solid phase. Then,

\[
\frac{\delta^2 P(x, t)}{\delta t^2} - \left[ K_u(x) + \frac{4}{3}G(x) \right] \nabla \left( \frac{1}{\rho(x)} \nabla P(x, t) \right) = s(x, t). \quad (3.5)
\]

Indeed, adding the shear modulus in equation 3.5 does not violate the acoustic approximation, but only affects the kinematics of the acoustic wave equation in order to better fit the velocity values that we would observe in the field.

- **Step 2:** I express the confining pressure \(P\) as a function of the pore pressure \(p\) by using the Skempton coefficient \(B\):
\[ p = BP, \]  

where \( B \in [0; 1] \), and is given by:

\[ B = \frac{1 - K_{fr}/K_u}{1 - K_{fr}/K_s}. \]  

- **Step 3:** I update the model for the streaming potential.

The link between the confining pressure \( P \) and pore pressure \( p \) is key as the seismoelectric coupling is based on the gradient of the pore fluid pressure (equations 2.15 and 2.19). The passage of the seismic P-wave creates a confining pressure perturbation, which in turn generates a gradient in the pore fluid pressure. Hence, a flow of pore water relative to the solid phase is generated, which creates a streaming current density expressed by (Revil et al., 2013):

\[ J_s = \hat{Q}_v \hat{w} = -\frac{\hat{Q}_v k_0}{\eta_f} \nabla p, \]  

where \( \hat{w} \) (in m/s) is the time derivative of the filtration displacement, also called the Darcy velocity.

- **Step 4:** Poisson’s equation (analogous to equation 2.20) is simplified as I now neglect the term with the solid phase acceleration, and it can be rewritten as:

\[ \nabla \cdot (\sigma \nabla \psi) = \nabla \cdot J_s, \]

\[ \nabla \cdot (\sigma \nabla \psi) = \nabla \cdot \left[ -\frac{\hat{Q}_v k_0}{\eta_f} \nabla (BP) \right]. \]  

In the following, I will use the acoustic approach in order to reduce the calculation time. The full poroelastic theory can be applied similarly and I leave this more computationally heavy modeling for future testing.

### 3.2 Extension to partially water saturated materials

As I intend to apply the seismoelectric modeling to a case study (chapter 5) where I track the water saturation front of an oil reservoir undergoing a water flooding process, I need to
take into account the effect of water saturation in the seismoelectric equations. Revil et al. (2013) extended the seismoelectric theory to the case of partially water saturated media, which I will use for my modeling (chapter 4). As discussed in section 3.1, I will only show the equations extended to partially saturated rocks for the acoustic case.

Consider a partially water saturated material with two immiscible fluids, typically water for the wetting phase, and an insulating fluid phase, say oil. The combined oil phase and the water occupy the entire pore volume. Moreover, I can neglect the electrical double layer associated with the water-oil interface (Leroy et al., 2012) as its surface charge density contribution is much smaller than the one from the water-solid interface.

Changing the saturation of water changes not only the mechanical properties of the material, but also the hydraulic and electrical ones. Revil et al. (2013) show that those properties can be adjusted using the following equations:

\[
\rho_f(S_w) = (1 - S_w)\rho_o + S_w\rho_w, \tag{3.11}
\]

\[
\frac{1}{K_f(S_w)} = \frac{1 - S_w}{K_o} + \frac{S_w}{K_w}, \tag{3.12}
\]

\[
\eta_f(S_w) = \eta_o \left( \frac{\eta_w}{\eta_o} \right)^{S_w}, \tag{3.13}
\]

\[
\hat{Q}_v(S_w) = \frac{\hat{Q}_v(S_w = 1)}{S_w}, \tag{3.14}
\]

\[
\sigma(S_w) = \frac{1}{F} S_w^n \sigma, \tag{3.15}
\]

\[
k_0(S_w) = k_0(S_w = 1) \ast k_r(S_w), \tag{3.16}
\]

\[
k_r(S_w) \approx S_w^{n+2}, \tag{3.17}
\]

where \( S_w \in [0; 1] \) is the water saturation \( (S_w = 1 \) corresponds to a fully water saturated material), \( \rho_f, \rho_w \) and \( \rho_o \) are the mass densities (expressed in kg m\(^{-3}\)) of the fluid phase, water phase and oil phase respectively, \( K_o \) denotes the bulk modulus of the oil phase, and \( K_w \) the bulk modulus of the water phase (both expressed in Pa), and \( \eta_f, \eta_w \) and \( \eta_o \) are the dynamic
viscosities (expressed in Pa·s) of the fluid, water and oil phase respectively. In equations 3.15 and 3.17, \( n \) denotes the saturation exponent (Archie, 1942). I can now rewrite the full seismolectric set of equations using the acoustic approximation in a partially saturated porous medium:

\[
\frac{\delta^2 P(x, t)}{\delta t^2} - K_w(S_w) \nabla \left( \frac{1}{\rho(S_w)} \nabla P(x, t) \right) = s(x, t),
\]

\[
\nabla \cdot (\sigma(S_w) \nabla \psi(x, t)) = \nabla \cdot \left[ -\frac{\hat{Q}_w(S_w) k_0(S_w)}{\eta_f(S_w)} \nabla (B(x) P(x, t)) \right] .
\]

\( \text{3.3 Beamforming technique - Numerical approach} \)

In the following, I assume that the area of interest (reservoir, rock layer, etc.) is surrounded by boreholes in which I place \( N \) seismic sources, \( N \) geophones and \( N \) electrodes. I am interested in detecting the location of any anomalous features inside the survey area, such as an interface between two facies or a water saturation front. The algorithm works in two steps: (1) forward modeling and (2) time-reversal modeling.

\( \text{3.3.1 Forward modeling} \)

The goal of this step is to obtain the seismograms that I need to reinject into the medium to recreate a wavefield that will focus at a desired location and time. Using a finite-element software (Comsol Multiphysics), I choose the point of interest (point of focus that I want to scan) and insert a virtual seismic point source at that location. I typically inject a Ricker wavelet (chapter 4, section 4.3.4) and I record the seismograms at each virtual geophone located in the boreholes. This step does not need to include any seismic/electromagnetic coupling equations (assuming that I may neglect the electroseismic effect), and is fairly fast computationally.

\( \text{3.3.2 Time-reversal modeling} \)

I now try to use the seismograms obtained in the previous step to focus seismic energy at the point of interest. Once the seismograms \( P_{i}^{\text{rec}}(t) \) have been recorded at each geophone \( i \),
I back propagate the signal in time. This method I first “flip” the signals recorded at each geophone in time,

$$\forall t \in [0; T_{\text{rec}}], \forall i \in (1, N), P_{i, \text{shifted}}(t) = P_{i, \text{rec}}(T_{\text{rec}} - t), \quad (3.20)$$

where $T_{\text{rec}}$ is the total recording time or listening time (chapter 4, section 4.3.5). I then create virtual seismic point sources located at the position of the geophones on the boreholes and reinject the flipped seismic signals $P_{i, \text{shifted}}$ into the medium (Figure 3.1). Those

![Figure 3.1: Numerical process of the beamforming technique: stages 1&2 are part of the forward modeling process whereas stages 3&4 are part of the time-reversal modeling. This is only for one geophone/seismic source and needs to be applied for all geophones/seismic sources. $T$ corresponds to the total recording time for the survey. Stage 1 shows the pressure field at the source location. Stage 2 is the pressure field recorded at geophone $i$. In stage 3, the recorded pressure $P_i$ is flipped. Finally, in stage 4, the flipped signal is time shifted and reinjected (the geophone is now used as a virtual seismic point source).](image)

flipped outgoing pressure fields will then propagate and interfere constructively at the original location (i.e., where I initially put the virtual seismic source). This part of the modeling requires two sets of equations, the mechanical acoustic wave equation (equation 3.18), and the coupled electromagnetic equations (equation 3.19), that will give rise to the seismoelectric conversions (coseismic field and interface response).
3.3.3 Strengths

The main strength of this technique comes from the fact that I know exactly when and where the seismic wavefields will focus and interfere constructively. If the point of focus is located at a heterogeneity, an electric signal (interface response) is recorded with a much greater amplitude than the one recorded had there been only one wavefield crossing the interface. In fact, I explained that the electric potential diffused by a seismoelectric conversion due to a discontinuity in the medium properties is usually much smaller than the coseismic field. This technique forces the interface response conversion to increase in amplitude due to the focusing of the different pressure fields coming from the multiple seismic sources, which makes it easier to detect sharp discontinuities with electrodes. By applying this technique to a grid of points within the survey area (by scanning the area point by point), I can then use the electrical response to map the discontinuities in terms of electrical and hydraulic properties of the material. It is also important to note that by using this technique, we do not need to invert for the location of the heterogeneity. Finally, I can also adjust and increase the resolution of the mapping by scanning over a denser grid of points around certain areas, or by using a higher frequency seismic source.

3.3.4 Limitations

During my work, I identified three main limitations of this technique.

(1) This technique could be difficult to implement practically in the field since it requires seismic sources that could inject complicated wavefields. At this point, I am not aware if such sources exist, but a way to partially solve this issue would be to use the traveltimes of the first arrivals. On the modeled seismograms (forward modeling step), I can pick the first arrivals of the wavelet, and use these traveltimes to determine the time shift needed to be applied to each source in order to refocus the wavefields at the desired point. Indeed, the waveform of the focused wavefield will not be the same as the injected wavelet, which could lead to destructive interferences.
(2) The imperfect acquisition. In practice, the wavefield needs to be sampled spatially at sparse locations, and the area of interest is not necessarily surrounded with a full-aperture coverage. Those flaws in the acquisition process increase the dimension of the focus point (point-spread function, Bazargani & Sneider (2013)). In a cross-hole configuration, however, the angular coverage of the study area would not be too problematic. Moreover, in my work, I chose to set the density and spacing of seismic sources placed in the boreholes to be no larger than a wavelength of the seismic signal injected.

(3) For dissipative media, the time-reversal invariance of the wave-equation does not hold (Bazargani & Sneider, 2013).

Finally, I acknowledge the need to refine the seismic focusing technique and that this could have an large impact on the quality of our results. However, in the following, I will assume that seismic focusing can be done and leave this issue for future investigations.

3.4 Beamforming technique - A typical field scenario

In this section, I consider a real-life scenario and illustrate how such a process could be implemented in the field. For instance, I assume that I am trying to monitor an oil reservoir where two vertical boreholes have been drilled on each side of the survey area. In each borehole, I place $N$ seismic sources and $N$ electrodes. First, I need to have a good idea of the reservoir geometry and of its velocity model. This technique should be used as a tool to refine the knowledge of the reservoir hydraulic and electrical properties, and not to build a velocity model.

- **Stage 1:** The geophysicist models numerically the experiment geometry with accurate reservoir mechanical properties (velocity model). At this stage, there is no need to have any information about hydraulic or electrical properties of the reservoir. This stage is crucial because any error in the geometry, velocity model, or other mechanical properties of the reservoir would compromise the accuracy of the seismic focusing.
• **Stage 2:** This step is purely numerical. The geophysicist in charge would pick a specific location inside the survey area (say point $M(x, z)$) and model numerically the forward propagation with the virtual source at that location. The signals obtained at the virtual geophones located in the boreholes are then flipped and time-shifted (equation 3.20). At this point, the geophysicist has $2N$ seismograms that he needs to reinject physically using the seismic sources in the boreholes.

• **Stage 3:** As discussed previously, reinjecting complicated wavefields into the medium is quite difficult to perform practically with seismic sources, and this issue still needs to be addressed in future research. However, I will assume for now that using the traveltimes will be sufficient to refocus the wavefields.

• **Stage 4:** Assuming that the wavefields are properly focused at point $M(x, z)$, the electrodes located in the boreholes record the electric potential up to few time samples after the focusing (there could be a phase shift of the interface response relative to the peak of the pressure field at the focusing point). The electrograms are then downloaded and sent back to the geophysicist who will interpret them.

• **Stage 5:** This process is reiterated for each point needed to be scanned in the reservoir.

It is important to realize that stage includes a strong assumption that the velocity model for the numerical forward propagation is well known. Clearly, more work needs to be done to evaluate the impact of an error in the velocity model on the focusing process. More generally, as I mentioned above, this technique is designed to refine our reservoir model in terms of electrical and hydraulic properties and should not be used as a primary velocity-estimation tool.

### 3.5 Summary

Even if some aspects of the beamforming process still need to be addressed (e.g., wavefields refocusing), the technique described in this chapter has the potential to solve some of
the main issues encountered in seismoelectric acquisition.

(1) By combining a set of seismic sources, it increases the amplitude of the interface response relative to the coseismic signals, making it easier to detect and interpret that response in the electrograms.

(2) By construction, the geophysicist knows exactly at what time and location the wavefields interfere constructively. Therefore, there is no need to perform inversion to locate the source of the diffused signal. The operator can just scan the area of interest point by point, record the electric potential at the time of focusing, and finally, interpret the presence of a heterogeneity for each case. The result of this scanning is a structure map of the hydraulic and electrical properties of the subsurface, such as a water-oil encroachment front inside an oil reservoir (chapter 5).

(3) The beamforming modeling process is a computationally intensive method as it needs to be applied point by point over the area of interest. To reduce the computing time, I use a simplified acoustic model. I first solve the acoustic wave equation to obtain the confining pressure and relate it to the pore fluid pressure. Then, the electrokinetic conversion is performed by neglecting the term responsible for the solid phase displacement/acceleration. Revil et al. (2013) performed a comparison of this approximation to the full poroelastic modeling using the same synthetic data set as the one in chapter 5. Even though this gave promising results, more testing needs to be done.
CHAPTER 4
MODEL BENCHMARK

The goal of this chapter is to benchmark the beamforming technique by applying it to a simple problem. First, I give a description of the finite-element parameters I use in my numerical modeling. Then, I present the experiment geometry, parameters and objectives. Finally, I show my results and draw my first conclusions on the validity of this technique, its strengths and weaknesses. This benchmarking step is extremely important as I will be using it on a more realistic example in chapter 5, where I locate an oil-water encroachment front during the water flooding of a reservoir.

4.1 Presentation and objectives

I consider a 2D case where I modeled a porous material with constant mechanical, hydraulic and electrical properties (I will refer to it as the background material), in which I embedded two rectangular heterogeneities (I will refer to them as anomaly 1 and anomaly 2). To reduce the computation time, all three features have the same mechanical properties and only the hydraulic and electrical properties differ from one to another (there will be no seismic wave reflection at the boundaries between the background and anomalies 1&2). My objective is to scan certain parts of the survey, record the electric potential at the time of focusing, and conclude on the presence of a heterogeneity. The experiment will include two cases:

• **Case #1:** The background material is a fully water saturated shale layer and the two anomalies represent two fully water saturated sand bodies. In this case, when I do beamforming at a point located at an interface, the recorded interface response dominates the electrograms and is easy to interpret. Therefore, a simple beamforming technique is applied, and is sufficient to detect the location of the heterogeneities embedded in the background material.
• **Case #2:** The background and the anomalies are all sandstone formations but their hydraulic and electrical properties differ from one to another because of a change in water saturation. In that case, even with the beamforming approach, the amplitude of the interface response is still too low to dominate the electrograms. In that case, a differential approach (similar to time-lapse imaging) is used to obtain optimal results.

4.2 Experiment setup

I created a simple 2D domain representing a fully water saturated porous material that will be referred to as the background material (Figure 4.1). It is 100 m thick (depth ranges from $z = -100$ m to $z = 0$ m), and 60 m wide (offset ranges from $x = 0$ m to $x = 60$ m). Within the area of study, I imbedded two homogeneous rectangular bodies (referred to as anomalies 1&2) with different hydraulic and electrical properties (Figure 4.2 for case #1, and Figure 4.3 for case #2). The two heterogeneities (shown on Figure 4.1) are 15 m thick and 20 m wide (anomaly 1), 20 m thick and 20 m wide (anomaly 2).

The survey is surrounded by three boreholes, two vertical and one horizontal. The left vertical borehole is located at an offset of 5 m ($x = 5$ m) and reaches a depth of 100 m ($z = -100$ m). The right borehole is identical to the left one but is located at an offset of 55 m ($x = 55$ m). Finally, the horizontal borehole is located at the top of the survey area, at a depth of 5 m below the reference level ($z = -5$ m), and is 50 m wide. In the boreholes, I placed a total of 46 virtual seismic sources, geophones and electrodes, spaced every 5 m (19 on the left borehole, 19 on the right, and 8 on the horizontal well). The red triangles on Figure 4.1 correspond to both seismic sources and receivers (i.e., geophones and electrodes).

4.3 Finite-element parameters for numerical modeling

All numerical modeling was performed using Comsol Multiphysics 4.3b, a finite-element package. In this section, I explain in detail how the modeling was done, the parameters chosen, and I discuss the issues that I encountered throughout this process. By doing so, I hope to give the reader enough information to reproduce these experiments and apply
Figure 4.1: Experiment geometry. The white zone corresponds to the homogeneous background material in which anomalous bodies 1 & 2 are imbedded. The survey area is surrounded by three boreholes: two vertical boreholes located on each side, and one horizontal borehole at the top of the domain. The red triangles correspond to the location of the seismic sources/geophones/electrodes. The spacing between two consecutive seismic sources is 5 m (both on vertical and horizontal boreholes). The two vertical boreholes each have 19 seismic sources, while the horizontal one has eight sources. The three red dots correspond to the focusing points used during our experiment.
Figure 4.2: **Case \#1**: Mechanical, hydraulic and electrical properties of the background and the anomalies. The background is modeled as a shale layer and the anomalies represent sand bodies. All three materials are fully water saturated (Araji et al., 2012).
Figure 4.3: **Case #2**: Mechanical, hydraulic and electrical properties of the background and the anomalies. All three media are modeled as sandstone formations, but the water saturation level differs from one to another (Revil et al., 2013).

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>BACKGROUND</th>
<th>ANOMALY 1</th>
<th>ANOMALY 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undrained bulk modulus $K_u$ (Pa)</td>
<td>22x10⁹</td>
<td>22x10⁹</td>
<td>22x10⁹</td>
</tr>
<tr>
<td>P-wave velocity $V_p$ (m s⁻¹)</td>
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<td>3093</td>
<td>3093</td>
</tr>
<tr>
<td>Excess charge density $Q_e$ (C m⁻³)</td>
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<td>2.00</td>
<td>6.70</td>
</tr>
<tr>
<td>Log (DC permeability, $k_o$ in m²)</td>
<td>-12</td>
<td>-14</td>
<td>-16</td>
</tr>
<tr>
<td>Skempton coefficient $B$</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Average mass density $\rho$ (kg m⁻³)</td>
<td>2300</td>
<td>2300</td>
<td>2300</td>
</tr>
<tr>
<td>Hydraulic viscosity of water $\eta$ (Pa s)</td>
<td>$10^{-3}$</td>
<td>$10^{-3}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Conductivity $\sigma$ (S m⁻¹)</td>
<td>1.00</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>Water saturation $S_w$</td>
<td>1.00</td>
<td>0.10</td>
<td>0.03</td>
</tr>
</tbody>
</table>
this methodology to other types of equations. The steps described in this section apply not only to the simple case study using the acoustic approximation (chapter 4), but also to the previous example showed in chapter 2 (modeled using full poroelastic theory), and to the cross-hole monitoring technique we develop in chapter 5 (modeled using the acoustic approximation).

4.3.1 Choice of mesh

For the modeling in this chapter, I use a triangular mesh of non-constant element size (minimum element size of 0.0024 m and maximum element size of 1.2 m). My choice was driven by the need to have a minimum of about five elements of mesh per wavelength. Moreover, to make sure that the mesh was fine enough, I decreased the maximum mesh size, verified that it had no effect on the final results, and that the Comsol solver would always converge towards the same solution.

4.3.2 Choice of solver

I chose to solve the system of equations in the frequency domain due to the complexity of the seismoelectric equations. I also acknowledge that the acoustic approximation to solve the seismoelectric problem would have enabled me to use the time domain solver (Sava & Revil, 2012). However, in anticipation to future modeling of the beamforming technique using full poroelastic theory (as in chapter 2), the frequency domain was preferred. In Comsol, the equations for the frequency solver in stationary mode (no time derivative) need to be expressed in the following form,

$$\nabla \cdot \Gamma = f.$$  \hspace{1cm} (4.1)

In the frequency domain, the acoustic wave equation for isotropic and non homogenous media (equation 3.18) takes the following form:

$$- \omega^2 \tilde{P}(x, \omega) - K_u(S_w) \nabla \left[ \frac{1}{\rho(S_w)} \nabla \tilde{P}(x, \omega) \right] = \tilde{S}(x, \omega),$$  \hspace{1cm} (4.2)
where $\tilde{P}$ and $\tilde{S}$ corresponds to the Fourier transforms (from time to frequency) of the the confining pressure and the source, respectively. From now on, I will drop the $\sim$ and will only work with the functions in the frequency domain (unless specified otherwise).

In Comsol, equation 4.2 needs to be rewritten in the form:

$$\nabla \cdot \begin{bmatrix} \frac{1}{\rho} \frac{\delta P}{\delta x} \\ \frac{1}{\rho} \frac{\delta P}{\delta y} \end{bmatrix} = -\frac{\omega^2}{K_u} P - \frac{1}{K_u} S,$$

hence, by identification with equation 4.1,

$$\Gamma = \begin{bmatrix} \frac{1}{\rho} \frac{\delta P}{\delta x} \\ \frac{1}{\rho} \frac{\delta P}{\delta y} \end{bmatrix},$$

and,

$$f = -\frac{\omega^2}{K_u} P - \frac{1}{K_u} S.$$

### 4.3.3 Perfectly Matched Layer (PML)

In the following, I try to give an intuitive explanation of how the PML operates in the frequency domain. This process was described in more detail in the course notes by Johnson (2010) and I will try here to summarize its mechanisms.

Since I wanted to model the domain as an infinite medium, it was necessary to use a boundary condition that would not generate any reflection from the boundaries in the survey area or any significant artifacts at the edges. For that, I used a PML approach with parameters taken from Jardani et al., 2010. Using the PML in the frequency domain is fairly simple to understand and to implement. The idea is to create an artificially absorbing layer (PML) around the area of interest such that when the wave (seismic or electromagnetic) enters this layer, it is attenuated and decays exponentially (Johnson, 2010). The key step
in this method is to create a smooth transition in the medium properties between the edges of the domain and the beginning of the PML so that no reflection takes place. This can be done by a complex coordinate stretching (or complex-coordinate approach). Intuitively, I perform a change of variable from a real spatial coordinate system to a complex spatial coordinate system, which creates an exponentially decaying factor as the wave enters the PML. Inside the domain of interest, nothing is changed and the solutions of the governing equations remain identical.

I will explain the mechanism for the special 2D case of an isotropic and non attenuative (stiffness tensor only composed of real values) medium. In his work, Johnson (2010) makes two assumptions: (1) the space far away from the region of interest is homogeneous and (2) the space far from the region of interest is linear and time invariant. Therefore, I can state that any radiation solutions in infinite space (i.e., far from the source) can be written as a finite linear combination of plane waves. Assuming that I am considering the propagation direction along the x-axis, such a plane wave can be written as:

$$\Psi(x, \omega) = A(y) e^{j(k_x x - \omega t)}, \quad (4.6)$$

where \(x = (x, y)\), \(k = (k_x, k_y)\) is the wave vector and \(||k|| = \frac{\omega}{c}\). Let us assume that \(k_x > 0\) (the wave propagates in the positive x-direction). \(A(y)\) is the amplitude of the plane wave and is independent of \(x\). Note that the y-component term of the wave vector can be included in \(A(y)\).

Johnson explains that since the function \(\Psi_x : x \mapsto \Psi(x)\) defined from \(\mathbb{R} \mapsto \mathbb{C}\) is analytical, I can freely analytically continue it, which means evaluating the function \(\Psi_x : \mathbb{C} \mapsto \mathbb{C}\). However, the imaginary part of \(x, x \mapsto Im(x)\) needs to be a “well-behaved function”.

Now, assume that our area of interest ranges from \(x = 0\) to \(x = x_1\) and I want to add a PML to absorb plane waves propagating in the \(x > 0\) direction. Since I can extend \(\Psi_x\) to complex values of \(x\), I consider the following change of variable,

$$\tilde{x}(x, \sigma(x)) = x + j\sigma(x), \quad (4.7)$$
and,

\[
\forall x, \sigma(x) \in \mathbb{R}, \text{ and } \left\{ \begin{array}{ll}
\sigma(x) = 0 & \text{if } x < x_1 \\
\sigma(x) \geq 0 & \text{if } x \geq x_1
\end{array} \right. .
\tag{4.8}
\]

In that case, I have not created any discontinuity in the complex variable \(\tilde{x}\). Figure 4.4 shows the values of \(\tilde{x}\) in the complex plane (we chose \(x_1 = 500\) m as the inner boundary of the PML and \(x_2 = 780\) m as the outer boundary).

![Figure 4.4: Representation of the imaginary part of the new spatial variable in function of its real part.](image)

We can see that for \(x \leq x_1\), \(\tilde{x} = x\). For \(x > x_1\), the new spatial variable has an increasing imaginary part. In this 1D example, the PML ranges from \(x \in [x_1; x_2]\).

Now, I can rewrite the plane wave equation for the new variable \(\tilde{x}\), which gives me:

\[\Psi(\tilde{x}) = A(y) e^{j(k_x \tilde{x} - \omega t)} = A(y) e^{-j\omega t} e^{jk_x x} e^{-k_x \sigma(x)}. \tag{4.9}\]

Hence, it is clear that for \(x > x_1\), the plane wave becomes inhomogeneous due to the last term \(e^{-k_x \sigma(x)}\) (exponentially decaying for \(x \in [x_1; +\infty[\)). Moreover, the transition at \(x = x_1\) (interface between the area of interest and PML) is continuous and should not introduce any plane wave reflection. In Figure 4.5, I plot the real part of the plane wave \(\Psi(\tilde{x})\) as a function.
of $\tilde{x}$. It is clear that for $\text{Re}(\tilde{x}) > x_1$, the plane wave decays exponentially and reaches a quasi-null amplitude at $x = x_2$.

![Plane wave amplitude (real part)](image)

Figure 4.5: Plane wave amplitude (real part) as a function of the new spatial variable $\tilde{x}$. We can see the exponential decay as the wave enters the PML. With the adequate parameters for $\sigma$, the wave gets attenuated before reaching the end of the PML (located at $x_2 = 780$ m in our figure).

After performing the change of variables, I need to update the equations where the spatial coordinate $x$ appears.

Since, $\tilde{x} = x + j\sigma(x)$, I have $\frac{\delta \tilde{x}}{\delta x} = 1 + j \frac{\delta \sigma(x)}{\delta x}$. However, I am facing a problem as $x \mapsto \sigma(x)$ is not differentiable at $x = x_1$ (Figure 4.4) and therefore I am not able to express $\delta \tilde{x}$ as a function of $\delta x$. Even if the function $x \mapsto \sigma(x)$ did not introduce any discontinuities, it is still not “smooth” enough and I need to find a new function $\lambda \in C^1(\mathbb{R})$ with similar characteristics. If I choose $\tilde{x} = x + j \lambda(x)$ where $\lambda(x) = \int_0^x \sigma(u) \delta u$, I can now express $\delta \tilde{x}$ as a function of $\delta x$. $\lambda \in C^1(\mathbb{R})$ and has the same characteristics as $x \mapsto \sigma(x)$ in the sense that it is a positive and increasing function, null for $x < x_1$. It will therefore also create an exponentially decaying factor. Using the chain rule,

$$\frac{\delta \tilde{x}}{\delta x} = [1 + j \sigma(x)] = S_x,$$

(4.10)
and,
\[ \delta \tilde{x} = S_x \delta x. \]  \hspace{1cm} (4.11)

Please note that \( \forall x \in \mathbb{R}, S_x \neq 0 \) and \( \forall x \in D, S_x = 1 \) and \( \tilde{x} = x \), where \( D \) is the domain of interest.

I finally found an adequate change of variable (analytical continuation) that allows me to obtain the same plane wave (same solution) within the area of interest and an exponentially decaying plane wave outside domain. I performed this without creating any “jump” at the domain/PML boundary. Note that the exact same change of variable applies for waves propagating in the negative \( x \)-direction. Now, I apply this change for all spatial variables, \( x \) and \( y \) in both acoustic and Poisson’s equations as the spatial derivatives are now in function of \( \tilde{x} \) and \( \tilde{y} \) (for a 2D problem). Starting by equation 4.3, I have:

\[
\nabla \cdot \begin{pmatrix}
\frac{1}{\rho} \frac{\delta P}{\delta x} \\
\frac{1}{\rho} \frac{\delta P}{\delta y}
\end{pmatrix} = -\frac{\omega^2}{K_u} P - \frac{1}{K_u} S. 
\]  \hspace{1cm} (4.12)

Having done the change of variables, all modeled equations should still be verified but with the new set of coordinates: \( \tilde{x} \) and \( \tilde{y} \). Hence, \( P = P(\tilde{x}, \tilde{y}, \omega) \) and \( \psi = \psi(\tilde{x}, \tilde{y}, \omega) \). Equation 4.12 now becomes:

\[
\left( \frac{\delta}{\delta \tilde{x}} \right) \cdot \begin{pmatrix}
\frac{1}{\rho} \frac{\delta P}{\delta x} \\
\frac{1}{\rho} \frac{\delta P}{\delta y}
\end{pmatrix} = -\frac{\omega^2}{K_u} P - \frac{1}{K_u} S. 
\]  \hspace{1cm} (4.13)
However, such a change of variables is not realizable directly under the Comsol solver. Hence, I need to express all the partial derivatives using the real set of coordinates. By expanding the left hand side of equation 4.13, I obtain,

$$\frac{\delta}{\delta x} \left( \frac{1}{\rho} \frac{\delta P}{\delta \tilde{x}} \right) + \frac{\delta}{\delta y} \left( \frac{1}{\rho} \frac{\delta P}{\delta \tilde{y}} \right) = -\frac{\omega^2}{K_u} P - \frac{1}{K_u} S. \tag{4.14}$$

By substituting $\delta \tilde{x}$ by $S_x \delta x$ and $\delta \tilde{y}$ by $S_y \delta y$:

$$\frac{1}{S_x} \frac{\delta}{\delta x} \left( \frac{1}{S_x \rho} \frac{\delta P}{\delta x} \right) + \frac{1}{S_y} \frac{\delta}{\delta y} \left( \frac{1}{S_y \rho} \frac{\delta P}{\delta y} \right) = -\frac{\omega^2}{K_u} P - \frac{1}{K_u} S, \tag{4.15}$$

and by multiplying both sides of the equation by the product $S_x S_y$, I obtain,

$$S_y \frac{\delta}{\delta x} \left( \frac{1}{S_x \rho} \frac{\delta P}{\delta x} \right) + S_x \frac{\delta}{\delta y} \left( \frac{1}{S_y \rho} \frac{\delta P}{\delta y} \right) = -S_x S_y \left[ \frac{\omega^2}{K_u} P + \frac{1}{K_u} S \right]. \tag{4.16}$$

By construction, $S_x$ and $S_y$ are independent of $y$ and $x$, respectively. Therefore, I can set them inside the spatial derivatives $\frac{\delta}{\delta y}$ and $\frac{\delta}{\delta x}$, and I can now write equation 4.16 with the desired Comsol format (equation 4.1),

$$\nabla \cdot \begin{pmatrix} S_y & 1 \\ S_x \rho(x, S_w) & \delta P(x, \omega) \end{pmatrix} = -\frac{S_x S_y}{K_u(x, S_w)} \left[ \omega^2 P(x, \omega) + S(x, \omega) \right]. \tag{4.17}$$
In a similar fashion, equation 3.19 becomes,

\[
\nabla \cdot \left( \begin{array}{c}
\frac{S_y}{S_x} \sigma(x, S_w) \frac{\delta \psi(x, \omega)}{\delta x} + \frac{S_y}{S_x} \frac{\hat{Q}_x(x, S_w) k_0(x, S_w)}{\eta_f(x, S_w)} \frac{\delta (B(x, S_w) P(x, \omega))}{\delta x} \\
\frac{S_x}{S_y} \sigma(x, S_w) \frac{\delta \psi(x, \omega)}{\delta y} + \frac{S_x}{S_y} \frac{\hat{Q}_y(x, S_w) k_0(x, S_w)}{\eta_f(x, S_w)} \frac{\delta (B(x, S_w) P(x, \omega))}{\delta y}
\end{array} \right) = 0. \quad (4.18)
\]

Another difficulty is to find an adequate function \( \sigma \) (and therefore \( \lambda \) and \( S_i \)) so that the seismic or EM waves reach a null amplitude before the end of the PML. \( \sigma \) also needs to be taken such that it works equally for all frequencies. Typically, taking \( \sigma \) of the form \( \sigma = \frac{\omega}{k_x} \) can cancel out the frequency dependence as the exponentially decaying term is of the form \( e^{-k_x \sigma} \) where \( k_x = \omega/c \) in our case (assuming \( k_y = 0 \) here). Jardani et al. (2010) (see their appendix B) used and detailed an efficient set of parameters for \( \sigma, S_x \) and \( S_y \), which I used for my numerical modeling.

### 4.3.4 Choice of seismic source

In equation 4.17, I chose the seismic source to be a virtual point source,

\[
S(x, \omega) = \delta(x - x_s) A(\omega), \quad (4.19)
\]

where \( A(\omega) \) is the Fourier transform of a Ricker wavelet function. In the time domain, a Ricker wavelet is the second time derivative of a Gaussian wavelet, and can be expressed as:

\[
r(t) = \left[ 1 - 2\alpha(t - t_1)^2 \right] e^{-\alpha(t - t_1)^2}, \quad (4.20)
\]

where \( \alpha = (\pi f_d)^2 \), \( f_d \) (in Hz) is the dominant frequency of the wavelet, and \( t_1 \) (in s) is the time delay applied to the source.

The continuous Fourier transform of this function is,

\[
R(f) = \frac{(2\pi f)^2}{2\alpha} \sqrt{\frac{\pi}{\alpha}} e^{-\pi^2 f^2/\alpha} e^{-2\pi ft_1}. \quad (4.21)
\]
Throughout the experiment in this chapter, I picked a dominant frequency of $f_d = 500 \text{ Hz}$, which contains energy up to $f_{\text{max}} \approx 1450 \text{ Hz}$. Using a seismic velocity of around $V_p \approx 3100 \text{ m/s}$, I obtain a dominant wavelength of $\lambda_d \approx 6 \text{ m}$, and a minimum wavelength $\lambda_{\text{min}} \approx 2 \text{ m}$. That implies a dominant seismic resolution of $R_d = 1.55 \text{ m}$, and and maximum resolution of $R_{\text{max}} = 0.51 \text{ m}$. Moreover, I applied a time delay of $t_1 = 5 \text{ ms}$. Figure 4.6a shows the time domain function, and Figure 4.6b shows its amplitude spectrum.

![Figure 4.6](image)

**Figure 4.6:** (a) Time serie of a 500 Hz-Ricker wavelet with a 5 ms-delay. (b) Amplitude spectrum of a 500 Hz-Ricker wavelet.

### 4.3.5 Fast Fourier Transform (FFT) parameters

In this section, I discuss the different parameters that are key to solving the mechanical and electrical equations in the frequency domain using Fast Fourier Transforms (FFT) in Comsol. First, I give a brief overview of discrete-time signal processing theory (Oppenheim & Schafer, 2010). Then, I explain how I applied it to my work by using a simple example.

#### 4.3.5.1 Basic concepts of FFT

Consider a continuous and real and continuous signal $x(t)$ whose properties are shown on Figure 4.7. I sample it using an adequate sampling rate $T_s$ and I call $x[n]$ the new sequence,
where \( x[n] = x(nT_s) \). Here, the signal is causal and non-zero only for \( t \in [0, T_{\text{max}}] \). I call \( N \) the integer such that \( (N - 1)T_s = T_{\text{max}} \).

![Figure 4.7: Test signal \( x(t) \) composed as a sum of two sinusoidal functions with frequencies \( f_1 = 15 \text{ Hz} \) and \( f_2 = 30 \text{ Hz} \). (a) Signal time serie. (b) Signal amplitude spectrum.](image)

The Discrete Time Fourier Transform (DTFT) of the time sequence \( n \mapsto x[n] \) is given by the continuous function of \( \omega \),

\[
\forall \omega \in \mathbb{R}, \quad X(\omega) = \sum_{n=0}^{N-1} x[n] e^{-jn\omega},
\]

and \( X \) is \( 2\pi \) - periodic (discretizing in time creates periodicity in frequency). Hence, recovering the time series from the DTFT only requires recovering \( X(\omega) \) for \( \omega \in [-\pi; \pi] \). However, since \( n \mapsto x[n] \) is a real-valued sequence, \( X(-\omega) = X^*(\omega) \), where \( * \) denotes the complex conjugate. I can then limit my calculations for \( \omega \in I_\omega = [0; \pi] \). Since \( \omega = 2\pi f T_s \), the maximum value that can take \( f \) is for \( \omega_{\text{max}} = \pi = 2\pi f_{\text{max}} T_s \). So given a sampling rate \( T_s \), the maximum frequency \( f_{\text{max}} \) that can be recovered is for \( f_{\text{max}} = \frac{1}{2T_s} \) (Nyquist-Shannon sampling theorem).

Numerically, however, I am not able to solve for a continuous range of frequencies and I need to discretize interval \( I_\omega \). That implies that I have to consider the Discrete Fourier
Transform (DFT):

\[ \forall k \in (0, M - 1), \ X[k] = X(\omega_k = \delta \omega k) \text{ with } \delta \omega = \frac{2\pi}{M} \quad (4.23) \]

Moreover, \( \omega = 2\pi f T_s \), \( \delta \omega = 2\pi df T_s = \frac{2\pi}{M} \), and therefore, \( T_s df = \frac{1}{M} \). In Fast Fourier Transform (FFT) solvers, we usually set \( N \rightarrow N_{FFT} \), and \( M \rightarrow N_{FFT} \), where \( N_{FFT} = 2^p \) (we take \( N_{FFT} \) such that it is the next power of two bigger than \( N \) and all the sample values between \( N \) and \( N_{FFT} \) are “padded” with zeros). Taking \( N_{FFT} \) as a power of two makes the FFT algorithm run more efficiently. Moreover, the sequence \( k \mapsto X[k] \) only needs to be solved for \( k \in (0; \frac{N_{FFT}}{2}) \), and is related to the sequence \( x[n] \) by:

\[ \forall k \in (0; N_{FFT} - 1), \ X[k] = \sum_{n=0}^{n=N_{FFT}-1} x[n] e^{-j2\pi kn/N_{FFT}}. \quad (4.24) \]

When I use Comsol to solve an equation (for instance, a seismic wave equation) in the frequency domain, the software solves for each \( k \) in equation 4.24. Once all the frequencies have been solved for, I can reconstruct the signal using the inverse Fourier transform (IFFT) equation,

\[ \forall n \in (0, N_{FFT} - 1), \ \tilde{x}[n] \approx \frac{1}{N_{FFT}} \sum_{k=0}^{N_{FFT}-1} X[k] e^{j2\pi kn/N_{FFT}}. \quad (4.25) \]

Having sampled in frequency, the recovered sequence \( n \mapsto \tilde{x}[n] \) is \( N_{FFT} \) - periodic and is formed from the sequence \( n \mapsto x[n] \) by adding together an infinite number of shifted replicas of \( x[n] \). The shifts are all positive and negative integer multiples of \( N_{FFT} \) (which is also the period of the sequence \( k \mapsto X[k] \)). In the case where I chose \( N_{FFT} = 2^p > N \), the delayed replications of \( x[n] \) do not overlap and and one period of the periodic sequence \( \tilde{x}[n] \) is recognizable as \( x[n] \). Thus, I can recover the correct signal (time-sequence). In an analogous way as Nyquist-Shannon theorem, if I pick now \( N_{FFT} < N \), the replicas of \( x[n] \) will overlap and one period of \( \tilde{x}[n] \) is no longer identical to \( x[n] \) (Oppenheim & Schafer, 2010). This is analogous to frequency aliasing but in time.
The key conclusion here is that for a finite and causal time signal, the FFT/IFFT are controlled by three parameters:

1. $T_s$ the time sampling rate.
2. $N_{FFT}$ the total number of samples in the time sequence.
3. $df$ the frequency sampling, or frequency increment.

In practice, I will be solving (using the acoustic wave equation in the frequency domain, for instance) directly $X[k]$ and will try to get back to $\tilde{x}[n]$. Therefore, before I start solving for each frequency, I need to correctly set the three parameters $N_{FFT}$, $T_s$ and $df$.

### 4.3.5.2 Practical applications

In this section, I explain how I set the three parameters for the IFFT in my modeling. I keep the same notation as in section 4.3.5.1.

Consider the geometry of the problem described in Figure 4.1. For this example, assume I will only be interested in solving the pressure field $p(t, x, z)$ everywhere in the domain, in response to a seismic point source located at $S(x_s, z_s)$.

(1) To obtain $T_s$, I need to analyze and have an idea of the frequency content of the pressure field that will be recorded at any point in the domain. Since I am injecting a 500 Hz-Ricker wavelet (Figure 4.6), which does not contain energy higher than $f_{max} \approx 1450$ Hz, I can assume the maximum frequency content for the solution will also have no energy above $f_{max}$. Using Nyquist-Shannon sampling theorem, I can already set $T_s$.

(2) Determining $N_{FFT}$ needs more caution. As explained in section 4.3.6, choosing a wrong $N_{FFT}$ can lead to overlaps in time when we recover the time sequence. Hence, before solving for the pressure field in the frequency domain, I need to estimate how long (in time or in sample number) is the discrete sequence $\tilde{p}[n]$ that I expect to recover at any given observation point in the domain (or in an equivalent way, what is the maximum time after which the modeled seismogram/electrogram will have no more non-zero values). This requires prior knowledge and anticipation of the physical phenomena I am trying to model. Since I want to use the same parameters for all points in the domain, I must consider the
extreme case where I set the source at one corner of the domain $S(0, 0)$ (Figure 4.1), and I want to solve for the pressure at point $M(60, -100)$ at the opposite corner. Assuming there is no bouncing off the edges, this is the longest time that a wave from the source will take to reach a point in the domain. Therefore, I need to calculate and estimate $T_{max}$ or $N$ (from previous section). $N$ corresponds to the number of samples after which I should be confident that the sequence is null at a given location. In my case, the distance between the two points is 116 m, the background velocity is $V_p \approx 3100 \text{ m/s}$, and the source is time delayed by 5 ms with non-zero values until 7 ms (Figure 4.6). Therefore, the seismogram at point $M$ should be null after $t_{max} \approx 45$ ms. Once I obtained this number, I first calculate $N$ using $T_s$ and $t_{max}$, and then I obtain $N_{FFT}$ (the closest power of two bigger than $N$).

(3) Finally, the frequency step $df$ is easily obtained by $df = \frac{1}{N_{FFT}T_s}$

An issue that I encountered during my modeling is that the PML was not set up properly and the waves (both seismic and EM) would bounce back and forth off the edges of the domain. The modeled seismograms were “recording” signals for much longer than if the waves had been attenuated during their first passage inside the PML, without reflecting off the boundaries. This created major time overlaps effect in my work, which was later resolved.

To conclude, solving in the frequency domain on a finite-element software (typically for seismic but also for EM waves), requires prior work and analysis. The recovered time series can only make sense if we anticipate beforehand some of its properties from the physics that we are solving: duration, shape, maximum frequency content, etc. A major drawback is that we cannot truncate the computation of a propagating wave. The full wave propagation needs to be modeled until the waves decays totally (otherwise we would get time overlap), which implies more computing time than with time domain solvers. However, it allows us to solve more complex equations, such as the full poroelastic models we used in chapter 2.

Finally, I displayed in Figure 4.8 the parameters chosen for my benchmark problem.
<table>
<thead>
<tr>
<th>PARAMETER FOR TOY PROBLEM</th>
<th>VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum frequency contained in source $f_{\text{max}}$ (Hz)</td>
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</tr>
<tr>
<td>Sampling rate $T_s$ (ms)</td>
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</tr>
<tr>
<td>Nyquist frequency $f_{\text{Nyquist}}$ (Hz)</td>
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</tr>
<tr>
<td>Frequency step for FFT solver $df$ (Hz)</td>
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</tr>
<tr>
<td>$N_{\text{FFT}}$</td>
<td>256</td>
</tr>
<tr>
<td>Total recording time $T_{\text{max}}$ (ms)</td>
<td>85</td>
</tr>
</tbody>
</table>

Figure 4.8: Fast Fourier Transform (FFT) parameters chosen for the benchmark problem.
4.4 Numerical amplitude decay analysis

In chapter 2, section 2.3.3, I showed that the interface response results in the diffusion of an electric potential whose equipotential lines distribution resembles the one of an electric dipole (Figure 2.11). In 3D, an ideal electric dipole has a potential decaying in \( \frac{1}{R^2} \), where \( R \) denotes the distance between the observation point and the center of the dipole. However, since all the modeling is done in 2D, the decay rate will be different than \( \frac{1}{R^2} \). To get an idea of what the decay should be, consider a point charge \( q \), at location \( S \), in a homogeneous and isotropic free space (the proof is similar for conductive media). In 2D, the electric field \( \mathbf{E} \) (created by this point charge) at an observation point \( M \), located on a circle of radius \( SM = R \) (and center \( S \)), is of the form \( \mathbf{E}(\mathbf{R}) = E(\mathbf{R}) \mathbf{e}_r \), where \( \mathbf{e}_r \) is the radial unit vector, and \( \mathbf{R} = SM = SM \mathbf{e}_r \). The expression of \( \mathbf{E}(\mathbf{R}) \) in 2D is identical to the one for \( \mathbf{E}(\mathbf{R}) \) for a 3D case created by an infinite line charge, perpendicular to the 2D plane. \( \mathbf{E}(\mathbf{R}) \) will have no component along the third dimension (by symmetry of the problem), and can be expressed by:

\[
\mathbf{E}(\mathbf{R}) = \frac{q}{2\pi \epsilon_0 R} \mathbf{e}_r, \quad (4.26)
\]

and by integration, the electric potential is of the form:

\[
\psi = \frac{q}{2\pi \epsilon_0} \ln(R) + \text{const.} \quad (4.27)
\]

In the case of an ideal electric dipole, we have two identical charges of opposite polarity separated by a distance \( d \), whose electric potentials obey equation 4.27. If I observe the electric potential at a point \( M \) “far” away from the dipole (i.e., \( OM \gg d \), where \( O \) is the mid point between the two charges), the contributions of the first order terms in \( \ln(R) \) from each point charge cancel out. Therefore, after performing a Taylor expansion (for each the potential field generated by each point), the first non-zero term of the resulting potential is on the order of \( \frac{\delta \ln(R)}{\delta R} = \frac{1}{R} \). Therefore, the electric potential diffused from an ideal dipole in 2D has a decay rate proportional to \( \frac{1}{R} \), and not \( \frac{1}{R^2} \).
In order to verify if this decay rate also applies to the interface response, I create a synthetic example where I measure the interface response at various points in the survey. I model the problem using the acoustic approximation as a benchmark test for amplitudes. Figure 4.9 shows the experiment setup, which is similar to the setup in section 2.3.3 (a shale layer lies on top of a sandstone layer). I placed a seismic point source at the top of the shale layer and installed fifteen in-line virtual electrodes $E_i$ down in the sand layer with a spacing of 5 m. The first electrode $E_1$ is placed 10 m deeper than the interface.

![EXPERIMENT SETUP](image)

Figure 4.9: Experiment setup. 15 electrodes are placed in the sand layer with a spacing of 5 m. The first electrode $E_1$ is located 10 m below the shale/sandstone interface.
When the wave hits the interface, a seismoelectric conversion takes place and I record it with the set of electrodes. As I move away from the interface, the seismoelectric signal decays rapidly in amplitude. Figure 4.10a shows the seismic wavefield distribution, and Figure 4.10b displays the electric potential distribution over the domain when the seismoelectric conversion occurs.

Figure 4.10: (a) Pressure field distribution over the entire domain as it hits the interface. (b) Potential distribution at the same time. Both pressure and electric potential amplitudes have been normalized.

Figure 4.11a,b display the electrogram recorded at \( E_2 \) and \( E_5 \), respectively. The first event at around 7.5 ms – 8 ms (for both \( E_2 \) and \( E_5 \)) is the interface response (synchronized with the arrival of the pressure wave at the interface), while the second event (at around 12 ms for \( E_2 \) and 16 ms for \( E_5 \)) corresponds to the arrival of the seismic wave at the electrodes \( E_2 \) and \( E_5 \) (coseismic signal).

In Figure 4.12, I plotted the interface response recorded at various electrode locations, which shows the rapid decay in amplitude with increasing distance from the virtual dipole. In order to quantify the decay rate, I show on Figure 4.13 the modeled interface response amplitude as a function of distance from the interface. Then, I try to identify the exponent
Figure 4.11: (a), (b) Synthetic electrograms recorded at $E_2$ and $E_5$, respectively. (c) Synthetic pressure field obtained if a geophone had been placed at the shale/sandstone interface.

Figure 4.12: Synthetic electrograms at various electrodes. Time series are plotted around time of interface response.
Figure 4.13: Amplitude of the interface response as a function of distance from the interface.

\[ \alpha \] that maximizes the fit between a decay in \( \frac{1}{R^\alpha} \) and the synthetic data from the numerical experiment. I calculate the amplitude decay for a range of \( \alpha \) values, and for each \( \alpha \), I compute an RMS error (compared to the synthetic data). For each model, I use the potential \( \phi_1 \) as a reference value, where \( \phi_1(R = R_1) = 1 \), and where \( R_1 = 10 \text{ m} \) is the distance between \( E_1 \) and the interface. Moreover, for a given \( \alpha \), the predicted potential \( \phi_i \) at electrode \( E_i \) is given by \( \phi_i = \left( \frac{R_1}{R_i} \right)^\alpha \). Figure 4.14 shows that the optimal \( \alpha \) is \( \alpha_{\text{min}} \approx 1.1 \). Therefore, the decay rate for the interface response is close to \( \frac{1}{R^{1.1}} \). Finally, Figure 4.15 compares the decay of the interface response to two models with different \( \alpha \) values, \( \alpha = 1.1 \) and \( \alpha = 2 \). The decay rate for \( \alpha = 1.1 \) matches our anticipated value of \( \frac{1}{R} \) and confirms the dipolar behavior of the interface response in 2D.

4.5 Results and interpretations - case #1

In this experiment, I used the beamforming technique in order to focus seismic energy at various points of interests, and I recorded the electric potential with the 46 electrodes. However, I did not scan the entire survey area as my goal in this section was to simply get
Figure 4.14: RMS error plotted for each value of $\alpha$. We obtain a minimum value for $\alpha \approx 1.1$.

Figure 4.15: Interface response amplitude decay as a function of distance (blue). Amplitude decay as a function of distance for $\alpha = 1.1$ (green), and for $\alpha = 2$ (red).
a first assessment of the method. Therefore, I decided to focus my study around three main locations: A, B and C (Figure 4.1).

4.5.1 Interface between background and anomaly 1: Point A (30 m depth, 40 m offset)

The focus point A is located at an interface with a sharp discontinuity in conductivity and permeability, which are the main drivers of the interface response. Figure 4.16 shows a set of snapshots representing the wavefields propagating in the domain as they converge towards point A.

I expected to see a strong interface response (seismoelectric conversion) at this point, which was confirmed by my results. When the wavefields focus at point A (at about \( t = 27 \) ms), I can instantly and remotely pick up a spike in the electrical potential from the electrodes located on the various boreholes. Figure 4.17a displays the electric potential recorded at an electrode located on the right vertical borehole, at a depth of 35 m, and at an offset of 55 m, 16 m away from the focus point A. I can clearly see that the dominant spike in electric potential is perfectly synchronized with the focusing time of the seismic wavefields at point A (Figure 4.17b). This enables me to interpret the presence of a heterogeneity in the medium at point A.

Now, if the seismoelectric conversion dominates the electrogram for an electrode located 16 m away from the focus point, let us see how this distance influences the electric signal. On Figure 4.18a, the interface response does not dominate the signal as much as in Figure 4.17a, and we can see two large spikes before (around \( t \approx 20.5 \) ms) and after (around \( t \approx 30 \) ms – 34 ms) with similar amplitudes as the interface response. Those signals are coseismic signals arising from the direct arrivals of the wavefields coming from other seismic sources near the recording electrodes. In this case, the amplitude of the coseismic signals is small enough relative to the interface response, and both phenomena are detectable.

Figure 4.17a and Figure 4.18a show that the amplitudes of the interface response for electrodes located at \( R_1 = 16 \) m and \( R_2 = 55 \) m are \( \phi_1 = -2.7 \) (normalized value) and
Figure 4.16: Sequence of snapshots illustrating the seismic beamforming process.
Figure 4.17: Case 1. (a) Normalized electric potential recorded by electrode located at a depth of 35 m and an offset of 55 m, located on the right vertical borehole (16 m away from the focus point $A$). (b) Normalized pressure field at focus point $A$.

Figure 4.18: Case 1. (a) Normalized electric potential recorded by electrode located at a depth of 80 m and an offset of 55 m, in the right vertical borehole (52 m away from the focus point $A$). (b) Normalized pressure field at focus point $A$. 
\[ \phi_2 = -0.7 \text{ (normalized value), respectively. Here, we have } \frac{\phi_2}{\phi_1} \approx 0.26. \text{ This implies that the interface response creates a diffused electric potential which decays in } \frac{1}{R^\alpha} \text{ with } \alpha \approx 1.14, \text{ confirming the decay model obtained in section 4.4.} \]

However, no matter what value of \( \alpha \) is obtained, it is still important to have electrodes located close enough to the focus point. In fact, not having a dense enough distribution of electrodes in the borehole could lead to miss certain heterogeneities inside the survey area. The advantage of the technique is that since I know exactly where the focus point is, I know which electrodes will be more likely to detect the signal.

Another feature of the interface response is its polarity asymmetry due the dipolar nature of the equipotential lines. For instance, when a seismic P-wave propagating downwards into the earth passes through a horizontal interface between two media, the seismoelectric conversion will be similar to the one diffused by an oscillating electric dipole whose moment is perpendicular to the interface. Therefore, in my experiment, as the focus point \( A \) is located at an interface parallel to the vertical boreholes, the polarity of the electric potential recorded on the right borehole should be opposite to the one recorded on the left borehole. Indeed, this is also due to the geometry of the heterogeneity. Figure 4.19a shows the electric potential recorded at a depth of 35 m, and at an offset of 5 m (on the left borehole). As predicted, the polarity of the spike is reversed.

Finally, if I look at the recording of an electrode directly above the focus point on the top borehole (located at an offset of 40 m and a depth of 5 m), I am unable to identify a single clear spike corresponding to the interface response at the focusing time (Figure 4.20a).

This is due to the shape of the equipotential lines created by a diffusing dipole with a horizontal moment. Figure 4.21 is the electric potential distribution over the entire domain at the focusing time. From this image, we can see that an electrode located on the top borehole at an offset of 40 m would not be able to pick up a clear interface response signal. This example reinforces the importance to keep a dense distribution of electrodes around the survey area, and in all different angle directions in order to detect all shapes of
Figure 4.19: Case 1. (a) Normalized electric potential recorded by electrode located at a depth of 35 m and an offset of 5 m, on the left vertical borehole (35 m away from the focus point $A$). (b) Normalized pressure field at focus point $A$.

Figure 4.20: Case 1. (a) Normalized electric potential recorded by an electrode located at a depth of 5 m and an offset of 40 m on the top horizontal borehole (25 m away from the focus point $A$). (b) Normalized pressure field at focus point $A$. 
4.5.2 Interface between background and anomaly 2: Point B (60 m depth, 45 m offset)

This example is very similar to the one applied on point A. Here, I test the effect of the conductivity discontinuity on the amplitude recorded in the signal. Figure 4.22a shows the recorded potential at an electrode located on the right borehole at a depth of 50 m and an offset of 55 m, 14 m away from focus point B. I observe a clear spike corresponding to the interface response. Moreover, I can see that it has a higher amplitude than the one observed for point A with a similar focus point/electrode distance. This is not surprising since point B is located at an interface (background/anomaly 2) with a much bigger conductivity gradient than for background/anomaly 1.

4.5.3 No heterogeneity: Point C (90 m depth, 35 m offset)

In this example, I verify that focusing seismic energy at a location where there is no heterogeneity generates no interface response. Figure 4.23 shows the electric potential at an electrode located on the right borehole (90 m depth, 55 m offset, 20 m away from the focus point C). As expected, I see no spike at the focus time, which indicates a lack of heterogeneity at this location.

4.5.4 Moving the focus point away from the interface

In this section, I move the focus point slightly away from the interface where point A is located. I beamform at three new points located 2 m, 3 m and 4 m away and to the left of point A inside anomaly 1 (Figure 4.24).

My objective is to determine, for this simple case study, the maximum distance beyond which I am not able to detect the interface anymore using the recorded electric potential. Moving the focus point within a radius of 1.5-2 m should not affect the results drastically as I am using a 500 Hz-Ricker wavelet with a dominant wavelength of \( \lambda_d = 6.2 \) m (resolution of approximately 1.5 m). Results for 2 m, 3 m and 4 m are shown in Figure 4.25a-d.
Figure 4.21: Distribution of the electric potential at the moment of focusing at point A. The seismoelectric conversion is taking place and is diffusing with a strong dipolar behavior.
Figure 4.22: Case 1. (a) Normalized electric potential recorded by electrode located at a depth of 50 m and an offset of 55 m on the right vertical borehole (14 m away from the focus point $B$). (b) Normalized pressure field at focus point $B$.

Figure 4.23: Case 1. (a) Normalized electric potential recorded by electrode located at a depth of 90 m and an offset of 55 m on the right vertical borehole (20 m away from the focus point $C$). (b) Normalized pressure field at focus point $C$. 
Figure 4.24: Point of focus is moved away from point \( A \), inside anomaly 1.
Figure 4.25: Case 1. (a) Recorded potential for a focus point located at point A. (b) Recorded potential for a focus point located 2 m to the left of point A. (c) Recorded potential for a focus point located 3 m to the left of point A. (d) Recorded potential for a focus point 4 m to the left of point A. Electrograms (a)-(d) were recorded by the same electrode located in the right borehole at a depth of 30 m and an offset of 55 m.

I can see that at 3 m away from the interface (Figure 4.25c), the trough in electric potential corresponding to the interface response, splits into two lower amplitude troughs and becomes less dominant compared to the rest of the signal. Therefore, beyond 3 m, I start to lose accuracy and I am unable to clearly detect the presence of an interface. From this result, I can conclude (for this case study) that in order to carefully map the entire survey, one should probably use a focus point spacing of 2-3 m. Denser than this would not provide a better resolution due to the seismic wavelength, and sparser could lead to miss important features.

4.6 Results and interpretations - case #2

As discussed previously, this case uses the same experiment setup as case 1 but different electrical and hydraulic properties are assigned to the three media (Figure 4.3). They have been modified to model three sandstone formations with different water saturation levels. In this example, I will only focus on the three main points (A, B, and C), and see that
seismoelectric conversions at the interfaces are much smaller than coseismic signals. Therefore, a simple beamforming technique is not sufficient to observe the spikes in the synthetic electrograms. A differential method is applied where I simulate the same problem with no anomalies (simple homogeneous background), and I record the electric potential only due to the coseismic fields reaching the electrodes. Since the coseismic fields arriving at the boreholes should be the same whether the anomalies are present or not, I should be able to cancel them out by subtracting the response with just the homogeneous background to the response with the anomalies. This method is very similar to time-lapse imaging where I take a reference state (or baseline survey), and I monitor changes (or heterogeneities) occurring inside it by subtracting the two data sets.

4.6.1 Focus point A

I apply the same technique as in case 1 and I plot the electric potential response obtained at an electrode located on the right borehole, 16 m away from point A (Figure 4.26a,c). Unfortunately, I am not able to detect the spike on the top graph corresponding to the conversion at the interface (at \( t = 27 \text{ ms} \)) anymore. This is due to the fact that the amplitude of the coseismic fields is much greater than the interface response, and dominates the recorded potential, acting as noise in the data. As I back propagate the pressure fields during the beamforming process, each electrode receives direct seismic arrivals from its neighbors. Here, the number of sources is not sufficient to create an interface response that dominates the recorded signal as in case 1. However, since I know that the noise is due to the coseismic fields, I can recreate the same beamforming technique but in a reference medium without anomalies 1&2. I record the electric potential with the same electrode and I subtract the two time series. Figure 4.26b displays my results and I can now clearly see the seismoelectric conversion dominating the signal. This differential technique enables me to filter out the noise coming from the coseismic fields and to detect a clear spike in the data. Note that in this case, the interface response (even using 48 seismic sources) is 50 times smaller in amplitude than the coseismic signals.
Figure 4.26: Case 2. (a) Electric potential measured at an electrode on the right borehole (16 m away from point $A$). The interface response is not detectable anymore. (b) Electric potential recorded when the medium contains the two anomalies minus the potential recorded if we only had the background. The interface response is now visible. (c) Pressure field at point $A$.

4.6.2 Focus point B

I apply the differential technique on point $B$ but in this case, I am still unable to detect the spike (Figure 4.27a,b). The parameters chosen for this medium are such that the interface response is undetectable by the electrodes located on the boreholes. In fact, it seems counter intuitive at first: the conductivity and permeability discontinuities between background/anomaly 2 are much sharper than the one between background/anomaly 1. Therefore, one would expect the interface response to be even greater with anomaly 2. However, it is very difficult to anticipate the magnitude of the interface response since we do not have an easily interpretable analytic solution of the diffused potential. In this case, I have sharply decreased the permeability (compared to anomaly 1), but I also decreased the conductivity. It seems like those two parameters are acting in the opposite direction, canceling out each other’s effects for the interface response. For instance, if I repeat this experiment but with a reduced conductivity in the background medium, say $\sigma_{\text{background}} = 10^{-2}$ S/m, I am now able
Figure 4.27: Case 2. (a) Recorded potential at an electrode located on the right borehole (at a depth of 35 m, 16 m away from point A). (b) Difference of two time series: the electric potential in (a) minus the electric potential recorded at the same location but for a homogeneous background with no heterogeneity. (c) Pressure field at point B. Even after applying the differential method, I am still not able to detect the seismoelectric conversion at the interface.
to pick up a clear interface response with the differential method (Figure 4.28b).

Figure 4.28: Case 2 with a lower background conductivity of $10^{-2}$ S/m. (a) Recorded potential at an electrode located on the right borehole (at a depth of 60 m, 14 m away from point B). (b) Difference of two time series: the electric potential in (a) graph minus the electric potential recorded at the same location but for a homogeneous background, with no heterogeneity. (c) Pressure field at point B.

By decreasing the conductivity in the background medium (Figure 4.28), I reduced the conductivity gradient (at the background/anomaly 2 interface) that was acting in the opposite direction of the permeability discontinuity. This change increased the value of the potential at the receiver electrode (the electric potential due to a diffusing dipole in a conductive medium is inversely proportional to its conductivity).

One solution here would be to increase the number of seismic sources, but there are still a few issues that I need to consider. First, I should be concerned about the feasibility of increasing the number of seismic sources in the field: there is a limited number of sources that can be placed in a borehole. Moreover, adding more sources might sharply increase the cost if we need to realize this experiment in the field. Finally, I need to estimate how “small” the interface response is relative to the coseismic energy in order to determine how many extra sources need to added. For instance, if the amplitude of the coseismic field is $10^6$
times bigger than the interface response, I would need to add an enormous amount of seismic sources to start detecting a signal at the focusing time. In that case, even by applying the differential method, the small variations in the coseismic fields from the \{background\} case to the \{background + anomalies\} case dominate and mask the interface response. Hence, it is important to have an idea of the properties of the heterogeneities imbedded in the background before I launch the beamforming technique. This means that I need to have an intuition of what I am looking for beforehand. Hence, a preliminary modeling (or “feasibility analysis”) is needed to evaluate the potential of the method on the case study.

### 4.6.3 Focus point C

The same technique is applied to point C. In this case, I see no spike in the differential signal, which is what I expected due to the lack of heterogeneity (Figure 4.29a,b).

![Figure 4.29: Case 2. (a) Recorded potential at an electrode located on the right borehole (at a depth of 90 m, 20 m away from point C). (b) Difference between two time series: the electric potential in (a) minus the electric potential recorded at the same location, but for a homogeneous background with no heterogeneity. (c) Pressure field at point C.](image)
4.7 Summary

In this chapter, I conducted fundamental numerical tests that enabled me to better understand the strengths and weaknesses of the beamforming technique. First, I explained how to choose the right parameters for the finite-element modeling. Then, I confirmed that the distribution of the equipotential lines due to an interface response is similar to the one from an ideal electrical dipole with a decay rate of $\frac{1}{R}$ in 2D. Moreover, by applying the method to case 1, I was able to successfully detect interfaces between the background and the anomalies. In case 2, I had to use a differential technique where I assumed that the background properties were already known (similar to time-lapse imaging). However, I saw that one limitation of the method was the possibility of having an interface response much smaller than the coseismic signals. The key result here is that I need to estimate beforehand how much “bigger” (in terms of amplitude) are the coseismic signals relative to the interface response. In some cases, the number of sources needed to generate a detectable interface response might be too high and not realizable in the field. Therefore, the technique requires some prior knowledge of the type of heterogeneities we are trying to locate, and a feasibility analysis is highly recommended.
In this chapter, I use the beamforming technique on a new type of application where I monitor the position of a water-oil encroachment front during the water flooding of an oil bearing sand reservoir. I start by presenting in more detail the objective of my study and the experiment setup (two-phase flow modeling, petrophysical properties of the reservoir, and numerical model parameters), and finally, I present my results and conclusions.

5.1 Presentation and objective

In this experiment, I simulate an oil bearing sand reservoir undergoing a water flooding process ($EOR$). As the water comes down through the injecting well, it pushes the oil towards the producing well, and a water-oil encroachment front is created. A key challenge for the oil and gas industry has been to monitor with accuracy the position and evolution of this encroachment front over time in order to optimize costs and production. My objective here is to show that the beamforming technique can be very useful in detecting such kind of features.

During the water flooding process, the water penetrates into the reservoir and slowly replaces the oil in the pores, thereby creating a zone with high water saturation. By doing so, it changes the mechanical, hydraulic, and electrical properties of the material such that at the water-oil encroachment front, a sharp discontinuity in permeability and conductivity is created. I use the two-phase flow model discussed by Revil & Mahardika (2013) to simulate the advancement of the water-saturation front over time. At a given snapshot $i$ at time $t_i$, I beamform at a set of points located along a horizon of constant depth within the reservoir. By doing so, I will show that I am able to detect an interface response when the point of focus is located at, or near a heterogeneity. Finally, I use those electric responses to map the reservoir and identify the location of the water saturation front.
5.2 Reservoir modeling

I begin by describing the reservoir geometry and the experiment setup. Then, I show how the water-saturation distribution affects the reservoir mechanical, hydraulic and electrical properties. Finally, I discuss the numerical model parameters used to perform the beamforming technique.

5.2.1 Reservoir geometry

The area of study is 235 m wide (offset ranges from \( x = 10 \) m to \( x = 245 \) m), and 125 m thick (depth ranges from \( z = 2330 \) m to \( z = 2455 \) m). It is surrounded by two identical vertical wells: a water-injecting well located at an offset of \( x = 20 \) m, and reaching a depth of \( z = 2455 \) m, and an oil-producing well located at an offset of \( x = 235 \) m. Finally, a horizontal well has been drilled at the top of the reservoir (depth of \( z = 2335 \) m, ranging from \( x = 20 \) m to \( x = 235 \) m). In a similar fashion as in chapter 4, I placed 156 seismic sources/electrodes on the three boreholes (58 on each vertical borehole with a spacing of 2 m, and 40 on the horizontal, using the same spacing) (Figure 5.1).

5.2.2 Petrophysical model

The reservoir modeling was done using the approaches described in Revil & Cathles (1999) and Karaoulis et al. (2012) to generate a 2D heterogeneous sand reservoir in terms of porosity (Figure 5.2), and DC permeability at full water saturation (Figure 5.3). I assume that the initial saturation in place was \( S_o = 75\% \) (or \( S_w = 25\% \), as I only model two possible phases, oil and water). The water flooding of the reservoir is simulated in 2.5D by injecting water at a constant rate during 76 days in the left well (displayed in blue on Figure 5.1), and pumping oil from the right borehole (displayed in red on Figure 5.1) at a constant pressure (Revil & Mahardika, 2013). The two-phase flow computations are done according to Karaoulis et al. (2012) and Revil & Mahardika (2013), where the water is assumed to be the wetting phase, and the oil, the non-wetting phase. Figure 5.4 shows the mechanical, hydraulic and electrical properties used for the modeling of the reservoir.
Figure 5.1: Experiment geometry. The blue well is the water-injecting well and pushes the oil towards the producing well, displayed in red. At the top of the reservoir, a horizontal well (in green) has been drilled. In each well, we install a set of seismic sources and electrodes (marked at black triangles). For clarity purposes, we do not show all seismic sources/electrodes in the borehole.

Figure 5.2: Porosity distribution of reservoir model.
After performing the water flooding simulation, I selected four snapshots of the oil and water saturation distributions that I display on Figure 5.5.

The remaining reservoir properties, $\rho_f$, $\rho_s$, $K_f$, $K_u$, $B$, $\eta_f$, and $\dot{Q}_v$ are water saturation dependent and are calculated using equations in chapter 3, section 3.2. More specifically, for the relative permeability $k_r$, I use equation 3.17 with a saturation exponent $n = 2$, which gives $k_r \approx S_w^4$ (Revil et al., 2013). For the formation factor $F$, I use a cementation exponent $m = 2$, which gives $F = \phi^{-2}$. The saturation dependent electrical conductivity $\sigma(S_w)$ is calculated using equation 3.15, with $n = 2$.

Taking snapshot #3 at $t_3$, I compare the conductivity distribution at full water saturation (Figure 5.6a) to the one for a partially water saturated reservoir (Figure 5.6b). The water flooding introduces a sharp discontinuity in terms of electrical conductivity at the interface between the water saturated zone and the oil zone. As I saw in previous chapters, a sharp discontinuity in electrical conductivity is one of the key drivers of the interface response.
<table>
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<td>Bulk modulus of the water phase $K_w$ (Pa)</td>
<td>2.25x10^9</td>
</tr>
<tr>
<td>Bulk modulus of the oil phase $K_o$ (Pa)</td>
<td>1.50x10^9</td>
</tr>
<tr>
<td>Bulk modulus of the frame $K_f$ (Pa)</td>
<td>18.2x10^9</td>
</tr>
<tr>
<td>Shear modulus of the solid phase $G$ (Pa)</td>
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</tr>
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<td>Solid phase mass density $\rho_s$ (kg m^-3)</td>
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</tr>
<tr>
<td>Water phase mass density $\rho_w$ (kg m^-3)</td>
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</tr>
<tr>
<td>Oil phase mass density $\rho_o$ (kg m^-3)</td>
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</tr>
<tr>
<td>Dynamic viscosity of water $\eta_w$ (Pa s)</td>
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</tr>
<tr>
<td>Dynamic viscosity of oil $\eta_o$ (Pa s)</td>
<td>5x10^-3</td>
</tr>
<tr>
<td>Water electrical conductivity $\sigma_w$ (S m^-1)</td>
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Figure 5.4: List of mechanical, hydraulic and electrical properties used for reservoir modeling (Revil et al., 2013).
Figure 5.5: Evolution of the water/oil saturation distribution over four snapshots at $t_2$, $t_3$, $t_4$ and $t_5$.

Figure 5.6: (a) Electrical conductivity distribution calculated for a fully water saturated reservoir (Revil et al., 2013). (b) Effect of water saturation at snapshot #3 on the electrical conductivity distribution in the reservoir.
Another major parameter that contributes to the generation of an interface response is permeability. Figure 5.7 shows the effect of water saturation on the total reservoir permeability and stresses the sharp discontinuity at the water-oil interface.

![Figure 5.7: (a) Relative permeability distribution of the reservoir due to saturation at snapshot #3. (b) Total DC permeability calculated by taking into account the partial water saturation. By comparing it to the permeability at full saturation (Figure 5.3), we can now observe a sharp discontinuity at the water-oil interface.](image)

As mentioned previously, in order to perform the beamforming technique, I need to have a good knowledge of the velocity model in the reservoir. But when the water penetrates the reservoir, it changes the mechanical properties of the material and modifies its velocity model. However, in this case, the water saturation has a limited impact on the material velocity (Figure 5.8c). Since the beamforming technique is a tool used to refine the knowledge of the target/reservoir, it is fair to assume that a reservoir undergoing a water flooding process has already been studied and assessed by the operating company. Therefore, in this case, the assumption on the knowledge of the velocity model prior to (and during) the two-phase flow simulation is fair.

### 5.3 Numerical parameters

The numerical and finite-element parameters for this case study were carefully chosen using the methodology described in chapter 4, and are presented below.

- Mesh parameters are shown in Figure 5.9.
Figure 5.8: (a) P-wave velocity distribution assuming full water saturation. (b) P-wave velocity at saturation #3. (c) Difference between (a) and (b). The impact of water saturation on the P-wave velocity is not substantial.
Figure 5.9: Mesh parameters used for numerical modeling on Comsol.

- Seismic source parameters are displayed on Figure 5.10.

- FFT parameters have been chosen according to section 4.3.6 and are displayed on Figure 5.11.

5.4 Results and interpretations

I start by doing a feasibility analysis of the beamforming technique for this synthetic example. I can then assess the preferred approach: direct or differential (i.e., time-lapse) (section 4.6). Then, in a similar fashion as in chapter 4, I apply the beamforming technique at a few points of interests and interpret the results. It is also assumed that during the beamforming process, the saturation front does not move substantially (i.e., at a given snapshot, I can perform my measurements while having a static saturation front). Finally, I apply the method at a set of 32 points located at a constant depth horizon, for snapshot #3. This enables me to map the water saturation front using the seismoelectric signals recorded at the electrodes.
<table>
<thead>
<tr>
<th><strong>SEISMIC SOURCE PROPERTY</strong></th>
<th><strong>VALUES</strong></th>
</tr>
</thead>
<tbody>
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<td>Seismic source type</td>
<td>Ricker wavelet</td>
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<tr>
<td>Dominant frequency $f_d$ (Hz)</td>
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<td>Maximum frequency $f_{max}$ (Hz)</td>
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<tr>
<td>Source delay $t_s$ (ms)</td>
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</tr>
<tr>
<td>Dominant wavelength $\lambda_d$ (m)</td>
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<tr>
<td>Dominant seismic resolution $R_d$ (m)</td>
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<tr>
<td>Minimum wavelength $\lambda_{min}$ (m)</td>
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<tr>
<td>Maximum seismic resolution $R_{max}$ (m)</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 5.10: Seismic source parameters.
In this analysis, I discuss two possible issues that I may encounter if I were to apply this method in the field. First, I verify that the interface response will be large enough in amplitude relative to the coseismic signals so that I will be able to detect it (using either the direct or differential approach). Then, I discuss the noise and the precision at which it is possible to measure electric signals in this type of setup.

### 5.4.1.1 Amplitude of the interface response

In the field, the first question is whether it is possible to get a measurable interface response by beamforming (considering the amplitude of the coseismic signals). To assess the feasibility of the beamforming technique in this scenario, I perform a simple test. I use a similar approach as Revil & Mahardika (2013) where I inject a seismic pulse (whose properties are described in Figure 5.10) from a source $S$ located in the injecting well, and I record the electric potential with an electrode $E_1$, located in the oil-producing borehole.
Both $S$ and $E_1$ are located at a depth of 2390 m. For educational purposes, I also place a virtual geophone $G$ at the oil-water interface, and at a depth of 2390 m. The setup is detailed on Figure 5.12. Figure 5.13a displays the electrogram recorded at $E_1$ and Figure 5.13b shows the seismogram at the geophone $G$. As shown on the electrogram, the interface response due to a single source is clearly detectable, which indicates that the beamforming technique seems adequate for this case study. Since the interface response generated by a single seismic source is visible and clearly detectable, a differential approach will not be necessary, and the direct approach will be sufficient to locate the saturation front. This analysis shows that in the field, the feasibility study is a key step as it is a cheap and easy way for the operator to assess the usefulness of the beamforming technique for the reservoir.
5.4.1.2 Measurements accuracy and noise

For seismoelectric experiments trying to detect shallow targets, typical sources of noise come from power lines and telluric signals, and electric field measurements for can be as accurate as $1 \mu$V/m. In some cases, such experiments resulted in the detection of an interface response of a few mV/m (Butler et al., 1996; Mikhailov et al., 1997). For deeper targets, electrical noise is largely reduced and measurements can be as accurate as $1 \text{nV/m}$. Therefore, it is reasonable to assume that for the experiment conducted in this chapter, it is possible to physically detect the interface response and the coseismic signals with an electrode located in the boreholes.

5.4.1.3 Acoustic model vs. poroelastic model

In Revil et al. (2013), we conducted a simple experiment on this data set, which is similar to the one performed in 5.4.1.1. At snapshot #4, we sent a seismic pulse from a source located in the injecting well (at a depth of 2390 m), and we placed a geophone and electrode on the producing borehole at $E_1$ (at a depth of 2390 m). Figure 5.14a,c show the
electrograms obtained using the full poroelastic modeling and the acoustic approximation, respectively. We essentially obtain the same signals, which confirms the validity of the acoustic approach for this case.

5.4.2 Test points

Now that the technique has been benchmarked, the beamforming process is straightforward. I consider the reservoir at snapshot #3 and I apply the method at three test points A, B, and C, located (1) in the water-saturated zone, (2) at the water-oil interface, and (3) in the oil-saturated zone (Figure 5.15). This allows me to test the method and obtain an idea of the amplitude and shape of the interface response (the responses obtained at E\textsubscript{1} for the three focus points are summarized on Figure 5.16).

Please note that for the points A and C, even if we do not observe any interface response at electrode E\textsubscript{1}, it is always important to check with electrodes located in different directions (left and horizontal boreholes, for instance) if still no signal is detected at the focusing time. In fact, we saw in chapter 4 that depending on the shape of the heterogeneity, some directions were not able to pick up the interface response due to the distribution of the electric equipotential lines.

5.4.2.1 Point A (65 m, 2390 m) - water saturated zone

Point A is located in the water saturated portion of the domain, about 35 m away from the interface. As expected, the recorded signal does not show any sharp spike at the focusing time (Figure 5.16a,d).

5.4.2.2 Point B (101 m, 2390 m) - water-oil interface

Point B is located on the saturation front and therefore, I expect a strong interface response. Figure 5.17a-e show five snapshots during the beamforming process at point B, and Figure 5.17f displays the electric potential distribution at the focusing time.
Figure 5.14: Comparison of the acoustic approximation to the full poroelastic numerical model. (a) Horizontal displacement at $E_1$. (b) Normalized electric potential recorded at $E_1$ using the poroelastic model. (c) Normalized confining pressure at $E_1$. (d) Normalized electric potential at $E_1$, using the acoustic approximation. SE stands for seismoelastic conversion (i.e., interface response), and CS stands for coseismic signal (Revil et al., 2013).
Figure 5.15: Location of the three focus points A, B, and C at snapshot #3 with various backgrounds. The blue line in (a) corresponds to the contour line with maximum gradient in terms of water saturation. (b) Background is water saturation. (c) Background is electrical conductivity. (d) Background is the saturation-adjusted permeability.
Figure 5.16: (a)-(c) Electric potential recorded at electrode $E_1$ during the beamforming process at point $A$, $B$, and $C$, respectively. (d)-(f) Seismograms at each focus point. Only point $B$ generates an interface response.
Figure 5.17: Beamforming sequence at point $B$ (marked at “FOCUS POINT” on the figures) located at the interface. (a)-(e) Propagating wavefields interfere constructively at point $B$. (f) Electric potential distribution at the focusing time.
The electrogram obtained at the focusing time is shown in detail on Figure 5.16b, and on Figure 5.18a. I can clearly interpret point $B$ to be located at the water-oil interface.

![Graph](image)

**Figure 5.18:** Focusing on point $B$. (a) Recorded electric potential at electrode $E_1$. The spike at around $t = 63$ ms corresponds to the interface response. (b) Seismogram at point $B$.

Figure 5.17f displays the electric potential distribution at the focusing time and clearly shows the dipolar behavior of the interface response. However, the center of the dipole seems shifted towards the water saturation zone, and the contour line where the electric potential is null does not coincide with the zone of steepest water saturation gradient (red line on Figure 5.17f). We saw that for the simple case studied in chapter 4, the contour line of null potential was located exactly at the interface between the two media. However, in this new case, there is a smoother gradient across the interface in terms of saturation, conductivity, and permeability (Figure 5.19). Hence, it is more difficult to anticipate where the center of the dipole will be located.

Figure 5.20 displays the 1D spatial distribution of the pressure field and electric potential at the time of focusing, along the constant depth horizon $z = 2930$ m. We can now better visualize the asymmetric profile (in terms of amplitude) of the interface response obtained
Figure 5.19: 1D spatial distribution of (a) water saturation, (b) permeability, and (c) conductivity along the $z = 2390\,\text{m}$ horizon. The properties’ gradients are smoother than in previous examples studied in chapters 2&4.
in Figure 5.17f. This can be explained by the much higher conductivity in the water saturated zone, which forces the electric potential to decay faster (the electric current does not penetrate the water saturated zone as much as the oil saturated zone).

Moreover, in Figure 5.18, we can observe a slight shift of about two time samples between the pressure peak at point $B$, and the trough in the electrogram corresponding to the interface response. In fact, we can show numerically that as we move away from point $B$, the electric signal generated by the interface response is slightly phase shifted. In Figure 5.21, I display the experiment setup conducted in order to assess this effect. I place five virtual electrodes inside the reservoir, and each electrode $B_i$ is placed at the same depth as point $B$, with a spacing of 20 m ($B_1$ is 40 m away from point $B$).

Figure 5.22 shows that as we move away from the focus point $B$, the interface response recorded is phase shifted, and the pressure peaks does not coincide perfectly with the troughs in the electrograms. However, this lag never exceeds two time samples in our case and does not substantially influence the interpretation of the data. Note that this phenomenon was also observed in Figure 5.13, where the interface response seemed out of phase compared to the pressure field at the interface.

5.4.2.3 Point $C$ (140 m, 2390 m) - oil saturated zone

Point $C$ is not located at a heterogeneity, and similarly to point $A$, I do not obtain a clear spike in the electrogram picked up by $E_1$ (Figure 5.16c,f and Figure 5.23).

5.4.3 1D analysis

I generate 32 synthetic electrograms by doing beamforming at a set of points located at a constant depth of 2390 m, with offsets ranging from $x = 65$ m to $x = 140$ m. The electric responses are recorded using a unique electrode $E_1$. The goal of this analysis is to interpret the synthetic data and try to recover the location of the water-oil encroachment front. In order to do so, I define an “interface function” $f_{\text{int}} = f_{\text{int}}(x)$ (where $x$ is the offset of the considered focus point) by the following:
Figure 5.20: 1D spatial distribution of (a) conductivity, (b) normalized pressure field, and (c) normalized electric potential along the horizon of constant depth ($z = 2390 \text{ m}$), at the focusing time. We can clearly see the asymmetric profile of the interface response that we obtained on Figure 5.17f.
Figure 5.21: Setup for the phase shift numerical analysis. Beamforming is done at point $B$, and the interface response is recorded at each $B_i$ and at $E_1$. Spacing between two consecutive $B_i$ is 5 m, and $B_1$ is 40 m away from point $B$.

Figure 5.22: (a)-(e) Recorded potentials at each $B_i$. (f) Recorded potential at electrode $E_1$. (g),(f) Pressure field at point $B$. 

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Figure 5.23: (a) Electric potential obtained at electrode $E_1$ for point $C$. As the focus point is not located near a heterogeneity, no spike appears in the signal. (b) Seismogram at point $C$.

(1) If I observe a clear and unique spike in the data at the time of focusing (or up to two samples before or after it), I consider the signal to be the result of an interface response, and I set the value of $f_{\text{int}}$ as the absolute value of the maximum amplitude of the spike.

(2) Otherwise, I assume the focus point to be located in a homogenous zone and I set the value of $f_{\text{int}}$ to zero.

In Figure 5.24, I show the synthetic electrograms for points located inside the water saturated zone. By following my interpretation method, none of those signals correspond to interface responses. Therefore, I set $f_{\text{int}}$ to zero for the following range of offsets: $\forall x \in [65\,\text{m}; 95\,\text{m}], f_{\text{int}}(x) = 0$. In Figure 5.25, I show the responses for points located around the water-oil interface, for $x \in [97\,\text{m}; 107\,\text{m}]$. For each focus point, I can observe a clear spike at the time of focusing, which indicates that the point is located near the water-oil encroachment front. I update the interface function as previously explained (case (1)).

Furthermore, as I move away from the interface into the more homogeneous oil saturated zone ($x \in [107\,\text{m}; 140\,\text{m}]$), I am not able to observe any clear spike in the data anymore.
Figure 5.24: Synthetic electrograms (recorded at electrode $E_1$) and seismograms (recorded at focus points) for target points located at a constant depth of 2390 m with offsets ranging for $x \in [67 \text{ m}; 91 \text{ m}]$. In all these figures, none of the focus points are interpreted as giving rise to an interface response.
Figure 5.25: Synthetic electrograms (recorded at electrode $E_1$) and seismograms (at focus points) for target points at a constant depth of 2390 m, and located close to the water-oil interface. In these figures, all focus points are interpreted as giving rise to an interface response.
Finally, on Figure 5.27c, I plot the 1D function $f_{\text{int}}$ with respect to offset, and I am able to locate the position of the water-oil interface, which confirms the usefulness of this technique for this case. In this example, I only performed beamforming for a set of points located at a constant depth horizon, at snapshot #3. Indeed, a more exhaustive mapping of the reservoir can be done by applying the same analysis to horizons at various depths for each snapshot.

5.4.4 Summary

In this chapter, I first modeled the water flooding process of an oil-bearing sand reservoir, and I performed a feasibility analysis to verify the potential of the method for this case study. Second, I tested it on three different benchmark points and obtained three synthetic and interpretable electrograms. Third, I performed a 1D analysis where I did beamforming for a set of 32 points with offsets ranging from $x = 65$ m to $x = 140$ m, and at a constant depth of 2390 m. I then generated 32 synthetic electrograms, which were easily interpretable, and I managed to recover the location of the water saturation front with accuracy. I used the beamforming method for a new type of problems with direct potential applications to the oil and gas industry. More specifically, I showed that it could be applied to time-lapse monitoring of a water-oil encroachment front inside a reservoir undergoing EOR.
Figure 5.26: Synthetic electrograms (at electrode $E_1$) and seismograms (at focus points) for target points located in the oil saturated zone, and at a constant depth of 2390 m, with $x \in [111 \text{ m}; 140 \text{ m}]$. In all these figures, none of the focus points are interpreted as giving rise to an interface response.
Figure 5.27: 1D interpretation of the water-oil interface location using synthetic data. (a) Spatial distribution of the water saturation (snapshot #3). (b) Water saturation adjusted electrical conductivity. (c) Interface function $f_{int}$ as a function of offset.
Here, I summarize the research described in the thesis, discuss my main contributions, and present conclusions on my results. Finally, I provide suggestions for future extensions of this work.

6.1 Results and conclusions

When I started working on this thesis, Sava & Revil (2012) had only recently published their theoretical work on the beamforming technique and the associated seismoelectric conversions, and had applied it to a simple case study. Much work still needed to be done to better understand the physical characteristics of the diffused electric signals (amplitude and decay rate, shape of the electric potential distribution at focusing time, etc.), the general strengths and weaknesses of the method, and its potential applications to the oil and gas industry. The main contributions of this thesis are as follows:

First, I used the theoretical work done by Sava & Revil (2012) to develop a finite-element numerical model for the beamforming technique in the frequency domain. This model was also extended to the full poroelastic theory. However, to reduce the computing time, I performed all my tests using the acoustic approximation. The validity of this approximation was later confirmed by the case study in chapter 5.

Second, after building the numerical model, I created a simple synthetic example to conduct fundamental tests that led to a better understanding of the beamforming technique. I showed the dipolar behavior of the interface response at the focusing time, and verified that its decay rate was identical to the one of an electric dipole, where the two poles are located on different sides of the heterogeneity responsible for the seismoelectric conversion. By doing so, I also confirmed that the acoustic approximation adequately describes the characteristics of the interface response. Moreover, I showed that in certain cases the interface response
generated by seismic focusing might still not be large enough relative to the coseismic signals. To solve this problem, I used a differential approach, similar to time-lapse imaging, in order to suppress the effect of the coseismic signals, and successfully detected the interface response in the synthetic electrograms.

Third, I used the numerical model to show potential applications of the beamforming method in the oil and gas industry, and more specifically for cross-hole imaging. I modeled an oil reservoir undergoing EOR, and performed beamforming at a set of focus points located at a constant depth with a range of offsets. I showed that the points that generate strong seismoelectric conversions are located at the water-oil interface, and successfully recovered the position of the saturation front.

6.2 Future work

Since the beamforming technique is fairly new, the next step would be to test it directly in the field with real data. However, I showed in this thesis that more work needs to be done to address some practical difficulties associated with this the implementation of the method.

(1) On the seismic side, the main issue that should be investigated further is how to find an efficient way to physically refocus the wavefields once the forward modeling has been performed. A lot of work has been done to focus wavefields at a certain subsurface point for imaging applications (Bazargani & Sneider, 2013; Broggini et al., 2012). However, these techniques are usually numerical (e.g., designed to estimate the locations of the sources of microseismic events), and therefore the wavefields do not need to be reinjected physically into the medium. So far, I do not know any technique that would enable seismic focusing of physical wavefields. As mentioned earlier, one way to refocus the wavefields would be to pick the traveltimes of the first arrivals and use them to time-shift the wavefields before retransmitting them into the medium (using standard seismic sources). Although that would definitely produce energy at the original location, there would still be some uncertainty about the shape of the waveform at the focus point. I believe this difficulty may be the main drawback of this method, and it needs to be addressed before conducting field experiments.
(2) Nevertheless, applying this method to a simple sand box experiment would be an interesting step to better understand the method’s potential, and using the traveltimes to refocus the wavefields would help assess the validity of this technique.

(3) Finally, another possibility would be to perform a joint iterative inversion to recover simultaneously the electrical conductivity distribution, the permeability distribution, and the seismic velocity model. However, as mentioned earlier, this technique is specifically devised to retrieve hydraulic and electrical properties and discontinuities in the area of interest. Therefore, accurate knowledge of the velocity field prior to applying this technique is needed, and the joint inversion could only be used to refine the velocity.
REFERENCES CITED


