CHARACTERIZATION AND MODELING OF THE STRESS AND PORE-FLUID DEPENDENT ACOUSTIC PROPERTIES OF FRACTURED POROUS ROCKS

by

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ABSTRACT

Characterizing the subsurface will continue to be a very active research area for hydrocarbon exploration and production, water utilization, underground contaminant transport and waste storage, mineral extraction and geothermal energy. Seismic surveys will continue to be the backbone of subsurface characterization. This thesis study has uncovered several important aspects of the acoustic properties of reservoir rocks through experimentation and modeling. The thesis presents the results of a study of the characterization and modeling of the stress and pore-fluid dependent acoustic properties of fractured porous rocks. A new laboratory High Pressure and High Temperature (HPHT) triaxial testing system was developed to characterize the seismic properties of sandstone under different levels of effective stress confinement and changes in pore-fluid composition. An intact and fractured of Berea sandstones core samples were used in the experimental studies. The laboratory test results were used to develop analytical models for stress-level and pore-fluid dependent seismic velocity of sandstones. The results of the research are expected to contribute to the development of improved methods to characterize sedimentary rock formations and their pore-fluid compositions.

Models for stress-dependent P and S-wave seismic velocities of sandstone were then developed based on the assumption that stress-dependencies come from the nonlinear elastic response of micro-fractures contained in the sample under normal and shear loading. The contact shear stiffness was assumed to increase linearly with the normal stress across a micro-fracture, while the contact normal stiffness was assumed to vary as a power law with the micro-fracture normal stress. Both nonlinear fracture normal and shear contact models were validated by experimental data available in the literature.

To test the dependency of seismic velocity of sandstone on changes in pore-fluid composition, another series of tests were conducted where P and S-wave velocities were monitored during injection of supercritical CO₂ in samples of Berea sandstone initially saturated with saline water and under constant confining stress. Changes in seismic wave velocity were measured at different levels of supercritical CO₂ saturation as the initial saline water as pore-fluid was displaced by supercritical CO₂. It was found that the P-wave velocity significantly
decreased while the S-wave velocity remained almost constant as the sample supercritical \( \text{CO}_2 \) saturation increased. The dependency of the seismic velocity on changes on pore fluid composition during injection of supercritical \( \text{CO}_2 \) in Berea sandstone was modeled using a re-derived Biot-Gassmann substitution theory. In using the Biot-Gassmann substitution theory, it was found necessary to account for the changes in the pore-fluid compressibility in terms of the volumetric proportion and distribution of saline water and supercritical \( \text{CO}_2 \) in the sample pore space. This was done by using the empirical model of Brie et al. to account for the compressibility of mixtures of two-phase immiscible fluids. The combined Biot-Gassman and Brie et al. models were found to represent adequately the changes in P-wave velocity of Berea sandstone during displacement of saline water by supercritical \( \text{CO}_2 \).

The third experimental and modeling study addressed shear-wave splitting due to the presence of fractures in a rock mass. Tests were conducted using the high temperature and high pressure (HPHT) triaxial device on samples of Berea sandstone, containing a single induced tensile fracture running along the height of the sample. The fracture was created via a modified Brazilian Split Test loading where the edges of cylindrical samples were loaded on diametrically opposite two points by sharp guillotines. The Joint Roughness Coefficient (\( JRC \)) values of the fractured core samples were determined by profilometry and tilt test. The effect of mismatching of the fracture surfaces on shear wave splitting was investigated by applying different amounts of shear displacements to three core samples. The degree of mismatching of the fracture surfaces in the core samples was evaluated using the Joint Matching Coefficient (\( JMC \)). Shear-wave splitting, as measured by the difference in magnitudes of shear-wave velocities parallel and perpendicular to the fracture, \( V_{s1} \) and \( V_{s2} \) respectively, increases with increasing mismatch of the fracture surfaces and decreases with increasing effective stress, and approaches zero in the effective stress range tested. A model for the stress and \( JMC \) dependent shear-wave splitting was developed based on the experimental observations.

Finally, the magnitude of shear-wave splitting was correlated with the permeability of the fractured porous sandstone for fluid flow parallel to the induced fracture.
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CHAPTER 1
INTRODUCTION

1.1 Background

Understanding of the geology, structure and fluid flow characteristic of fractured reservoir is important in petroleum engineering, ground water resource, and environmental engineering. One of the most extensively used techniques to obtain information on the geologic structure of rock formations is seismic velocity measurements. A large number of geoscience, geo-engineering and geo-resource disciplines utilize seismic surveys to obtain maps of the distribution of seismic velocities in a formation, interfaces between rock units and, ideally, of reflection coefficients at these interfaces. Common examples of seismic methods used in the petroleum industry are: 1) Large-scale measurements at low frequencies (10-100 Hz) for estimating interval velocity, reflection amplitude, and AVO (amplitude vs. offset) analysis for a layered medium; 2) Intermediate scale with frequency range of 10-20 KHz for cross borehole measurements, VSP (vertical seismic profile) and sonic log analysis; 3) Small scale measurements using ultrasonic frequency (0.1-1.0 MHz), including velocity and attenuation measurements and core samples in the laboratory.

The use of seismic velocity is the most common form of geophysical survey. Other methods of geophysical survey use other petrophysical properties in combination with magnetic, electric, gravitational, and thermal theories. Seismic surveys are based on theory of elastic wave propagation in materials and try to deduce elastic properties of materials by measuring their elastic response to disturbances in the form of elastic seismic waves. Seismic waves can be generated in two ways: actively or passively. Historically, most of seismic surveys have been of the active type. Active seismic waves are generated from a source (e.g., from a sledgehammer) which are then received and sensed by receivers or a receiver array deployed along preset distances and locations from the source. The received signals are recorded using seismograph. The time of arrival over a given distance determines the seismic velocity.

The type of seismic survey depends on the mode of seismic wave propagation, which are grouped primarily into direct, reflected, refracted and surface waves. When the seismic source is close to the receiving point, reflected waves usually reach the receiving point first. At
greater distances, the seismic wave travels faster by the refraction path because its velocity is greater along the top of the lower, denser layer than it is through the upper layer. In this case, the refracted wave arrives first. The results of a seismic survey are presented as seismic profiles in the form of a cross-sectional drawing of the subsurface structures.

The two main imaging methods to produce models for the underground seismic velocities are: 1) time migration, which takes seismic data in time coordinates, and produces images and time-migration velocities that are an averaged velocity of a particular type; and 2) depth migration, which takes seismic data in depth coordinates and produces seismic images in depth coordinates.

Seismic velocities depend on the elastic moduli and density, and in turn, the elastic constants and densities depend on many factors including wave frequency, heterogeneity and discontinuities, rock properties (such as permeability, porosity, compressibility, density, tortuosity, mineralogy, grain sorting and clay content), pore fluid properties (such as viscosity, density and compressibility) and stress level. These dependencies of seismic velocity on rock properties are what geoscientists and geoengineers use to characterize rocks in terms of such properties as porosity, fluid saturation, texture, shear strength, fracture distribution. Knowledge of the relationships between elastic moduli and rock properties is needed before seismic survey results can be interpreted quantitatively. Many of these relationships are empirical and seismic velocities, and are correlated to rock units in a formation based on laboratory seismic and rock mechanical measurements on core samples. The effects of the different important parameters on seismic velocity are discussed further below.

1.1.1 Elastic Wave Propagation in Porous Media

Propagation of seismic wave isotropic elastic materials are characterized by two elastic wave speeds: Compressional or P-wave, \( V_p \), and shear or S-wave, \( V_s \) (Ewing et al., 1957; Aki and Richards, 1980).

These seismic wave velocities are defined as:

\[
V_p = \sqrt{\frac{(\lambda + 2\mu)}{\rho}}
\]  \hspace{1cm} (1.1)
\[ V_s = \sqrt{\frac{\mu}{\rho}} \quad (1.2) \]

where the constant \( \lambda \) and \( \mu \) are Lamé elastic and \( \rho \) is the density of the material. The Lamé parameters are related to the elastic Young’s modulus \( E \), Poisson’s ratio \( \nu \), bulk modulus \( K \), and shear modulus \( G \) as:

\[ \lambda + 2\mu = K + \frac{4}{3} G = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad (1.3) \]

\[ \mu = G = \frac{E}{2(1+\nu)} \quad (1.4) \]

P-waves travel longitudinally along a body and S-wave travel transversely along the propagation direction (Figure 1.1)

Figure 1.1 Propagation of P and S-waves in an elastic body (from Global seismology and wave propagation EPS 122: Lecture 9)

An important parameter in the characterization of the acoustic properties of rocks is the ratio of P and S-wave velocities, \( V_p / V_s \). Using the above equations, it can be shown that \( V_p / V_s \) can be directly related to Poisson’s ratio as:

\[ \frac{V_p}{V_s} = \sqrt{2 \left( \frac{\nu}{1-2\nu} + 1 \right)} \quad (1.5) \]
As can be seen, $V_p$ is always larger than $V_s$ for the range of values of Poisson’s ratio $\nu$ of 0 to 0.5. Typical range of values of the seismic velocities of different rock types are shown below in (Figure 1.2).

The compressional wave is a longitudinal wave that has the faster first arrival time than the shear wave. It has the direction of particle motion coincident with the wave propagation. The shear wave velocity identified as the lower wave velocity has particle motion in the plane perpendicular to the direction of wave propagation. Shear wave velocity has two basic types; the SH-wave that has particle motion parallel to the boundary and the SV-wave, which has particle motion perpendicular to both the wave propagation direction and the particle motion of the SH-wave. However, when shear wave velocity passes through anisotropic fractured petroleum reservoirs, it will split into fast $q_{s1}$ and slower $q_{s2}$ polarized components (Martin and Davis, 1987), giving evidence about reservoir fracturing character and principal stress direction. The effects of the compressive stress on the elastic properties of rocks, and that change of properties related to the behavior of the micro-cracks under load has been recognized since early 1900s. Due to stress effect, the largest velocity change took place in the direction of the applied stress (Nur and Simmons, 1969). Nur (1971) have observed velocity anisotropy caused by the stress effects on crack closure in lab experiments.

![Figure 1.2 Measurment of velocity in different rock types (from Telford W.M., Applied Geophysics, 2nd edition).](image-url)
1.1.2 Stress-Dependent Seismic Wave Velocities

Seismic wave velocity is strongly dependent on the effective stress level, and this stress level dependency partly accounts for the increase of the seismic velocity with depth. In general, seismic velocity increases non-linearly with increasing effective confining stress and then levels off to an “asymptotic velocity” when the effective confining stress becomes high. The asymptotic velocity occurs at the effective stress causing closure of cracks and other defects. At low effective stresses, cracks and defects are open and slow down seismic wave propagation, and then become closed with an increase in effective stress thereby increasing the seismic velocity. Figure 1.3 shows examples of stress dependent P and S-wave seismic wave velocities for three rocks (Eberhart-Phillips et al., 1989). Since effective stress is the difference between the total stress (overburden and horizontal) and the pore pressure, anomalously low seismic velocities can be used to detect overpressured zones.

As the pore pressure increases above hydrostatic, the effective stress will decrease and this will be reflected by lower seismic velocity in an overpressured zone. Rock mass stress-strain behavior is nonlinear and dependent on effective stress and strain level. Elastic bulk and shear moduli are also strongly dependent on stress level. Seismic wave velocity also increases with decrease in rock mass porosity. While porosity is a function of effective stress, effective stress has an effect on elastic wave velocity that is independent of the stress-induced porosity change. The dependence of elastic wave velocity on effective stress has been confirmed by many experiments on rock core samples in the laboratory. Rock samples of known porosities are tested in testing equipment that allows for measurement of elastic wave velocity at different confining pressures.

Fundamentally, the stress-dependency of elastic wave velocity is a result of the stress-dependency and nonlinearity of the elastic moduli of rocks. In turn, the nonlinear elastic behavior can be attributed to the non-linear behavior, particularly closure, of the discontinuities in a rock sample. As the discontinuities close, seismic wave propagates faster in a rock mass, and the maximum seismic velocity is obtained when stresses can no longer induce further closure of discontinuities. Strictly, elastic wave velocity should depend not just on the effective mean or octahedral stress but on the shear or deviatoric stress as well. This is the case, when seismic propagation is direction dependent resulting in anisotropic elastic behavior.
Several analytical models have been proposed for the effective stress dependent seismic wave velocities of rocks. Stress-dependent models have been developed based on the elastic response of idealized materials and using more rigorous approaches based on nonlinear elasticity (e.g., Johnson and Rasolofosaon, 1996; Winkler and Liu, 1996; Sarkar et al., 2003). These models derive the elastic moduli and the corresponding seismic velocities assuming idealized geometries of the pore space or discontinuities in a rock mass, or the contact between the individual grains that constitute the rock mass. An example of the latter is the use of the classical spherical Mindlin contact model (Duffy and Mindlin 1957). Crack contact models have been
developed by Mavko et al. (1995), and Gangi and Carlson (1996). However, most of stress dependent seismic velocity models are empirical based on curve-fitting of analytical expressions through experimental elastic wave velocity data using two or more curve-fitting parameters. Often, it is difficult to assign physical meanings to these empirical curve-fitting parameters.

The most commonly used empirical curve-fitting that relates wave velocity with and effective stress was first proposed by Zimmerman et al., (1986). It has the form:

\[
V = A + Cp' - B \exp(-Dp')
\]  

(1.6)

where \(V\) is the P- or S-wave velocity, \(p'\) is the effective mean stress, and \(A, B, C,\) and \(D\) are curve-fitting parameters. Different sets of curve-fitting parameters are used for \(V_p\) and \(V_s\). The above equation indicates that seismic velocity increases rapidly as the effective confining stress \(p'\) increases. Eq. (1.6) was used by other researchers (e.g., Carcione and Tinivella, 2001; Eberhart-Phillips et al., 1989; Freund, 1992; Jones, 1995; Kaselow and Shapiro, 2003; Khaksar et al., 1999; Kirstetter and MacBeth, 2001; Prasad and Manghnani, 1997) and was shown to fit seismic velocities for different rock types over a wide range of effective stress levels.

Han et al. (1986) formulated the following more general empirical model to represent the combined dependence of seismic velocity on effective stress, porosity and clay content:

\[
V = B_0 + B_1\phi + B_2\sqrt{c} + B_3[p' - \exp(-Dp')]
\]  

(1.7)

where \(c\) is clay content, \(\phi\) is the porosity, and \(B_0, B_1, B_2, B_3,\) and \(D\) are curve-fitting parameters. Different sets of curve-fitting parameters are used for \(V_p\) and \(V_s\). Eq. (1.7) is based on the P- and S-wave velocity data, which were measured from 64 fully water saturated sandstones samples with porosity ranging from 2 to 30% over an effective pressure range of 2 to 49 MPa. The entire set of velocity measurements for all 64 rock samples was used in the derivation to provide a useful description of the influence of three parameters: clay content, porosity, and effective pressure on seismic velocity. To investigate whether the effect of effective stress on seismic velocity may affect the dependency on porosity \(\phi\) and clay content \(c\), regression analysis was applied to the coefficients in Eq. (1.7) and the parameters \(\phi\) and \(c\). The effects of these two parameters were shown to be independent and can be separated from the stress dependency. Based on the experimental data, it was also found that seismic velocity fit significantly better with \(\sqrt{c}\) than with \(c\).
Using the experimental data from Han et al. (1986), Eberhart-Phillips et al. (1989) developed the following simplified version of Eq. (1.7) for a given values of porosity and clay content:

\[ V = A + K p' - B e^{-D p'} \]  

(1.8)

High values of \( B \) and \( D \) correspond to the case when cracks close rapidly as \( p' \) increases. Han and Zoback (1989) used Eq. (1.7) and provide specific values for the curve fitting parameters. Other even simpler have been developed by Mavko et al. (1998), and Bowers (1995). The model of Mavko et al. (1998) is given as:

\[ V = A - B e^{-p'^C} \]  

(1.9)

while the model of Bowers (1995) is expressed as:

\[ V = V_o - A (p')^B \]  

(1.10)

As can be seen, all the models presented above are very empirical involving equations that are curve-fitted through experimental data. Although the equations are much simpler that those based on complicated theories of elastic behavior of idealized materials, the empirical equations involve parameters are difficult to assign physical meanings.

In addition to empirical approaches, there have been several attempts to model stressdependent seismic velocities using more rigorous approaches based on nonlinear elasticity (e.g. Johnson and Rasolofosaon, 1996; Winkler and Liu, 1996; Sarkar et al., 2003). These models derive the elastic moduli and the corresponding seismic velocities assuming idealized geometries of the pore space or discontinuities in a rock mass, or the contact between the individual grains that constitute the rock mass. An example of the latter is the use of the classical spherical Mindlin contact model developed by Duffy and Mindlin (1957) and the crack contact models developed by Gangi and Carlson (1996), Mavko et al (1995).

### 1.1.3 Effects of Pore-Fluid Composition on the Acoustic Properties of Porous Rocks

The ability to detect seismic velocity changes because of variations in pore fluid composition is important in monitoring of underground fluid flow (e.g., oil exploration, contaminant flow and transport, waste disposal, etc.) using seismic methods. Seismic surveys can be used for economical, large-scale field underground fluid monitoring. Time-lapsed seismic
survey mapping can provide spatial distribution of seismic wave velocity which can be correlated with fluid migration and volumes in underground formations with time. The acoustic velocity response of rock media filled with fluids is primarily a function of rock mass elasticity, porosity, and pore fluid bulk modulus. These components, related to acoustic velocity, are affected by temperature, rock mass effective stresses, fluid pressure, and pore fluid composition (saturation). Previous studies (Kim et al. 2010; Shi et al. 2007) have shown that the acoustic velocity response of sandstone cores changes with the relative volumes of water and gas in the pore space. The change in acoustic velocity is a result of the change in the bulk modulus of the pore fluid as gas displaces water. However, the evaluation of the bulk modulus of mixture of multiphase fluid is still a challenge. A major reason is that the bulk modulus of a fluid mixture should also depend on the degree of uniformity of the fluid mixture in the rock pore space (Brie et al. 1995; Endres and Knight, 1991).

For porous and fractured rocks containing fluids, wave propagation occurs though both the solid and fluid components. There are two limits in the propagation of seismic waves in fluid saturated porous media, and these are: 1) Relaxed mode in which fluid flows freely between pore space (large permeability) and fluid pressure is nearly uniform through the rock (low frequency); and 2) Unrelaxed mode at which fluid is prevented to flow due to high induced pressure (high frequency and/or small permeability), thus, the material in this case is stiffer and the velocity is relatively large. The low frequency limit in which fluid is relaxed and in the equilibrium state is covered within the theory of Gassmann(1951). Models of the high frequency limit in which the fluid is unrelaxed include those of Mavko and Jizba (1991), and Mavko and Nolen-Hoeksema (1993). The slow fluid flow and low frequency limit is the most common situation found in flow of fluids in hydrocarbon extraction, underground waste disposal and storage, carbon sequestration and underground water utilization.

In the low frequency limit, due to the higher bulk modulus of fluids compared to gases, rocks saturated with fluids generally exhibit higher compressional velocity that dry rocks. Examples of experimental data showing the effects of pore fluid composition on the seismic velocity of porous rocks are shown in Figure 4. In general, the bulk modulus, and correspondingly the P-wave velocity, of rocks saturated with water are higher than the bulk modulus of rocks saturated with oil and dry rock. The total density (rock plus fluid) also changes
as fluid composition changes, but the influence of density changes on seismic velocity is minimal. For rocks containing mixtures of fluids and gases, theoretical and experimental studies indicate that compressional wave velocity decreases with decreasing fluid saturation as the fraction of the gas in the pore space increases. On the other hand, shear wave velocity appears to relatively unaffected by pore fluid composition and is relatively constant when fluid saturation decreases. This is due to the very low shear modulus of fluids. Theoretical and experimental studies also show that the distribution of the fluids in the pores also affects seismic velocity. For instance, rocks with two-phase immiscible pore fluids with piston-like displacement show different compressional wave velocity in comparison to rocks with fully mixed two-phase fluids in the pore space.

The simplest approach to detect pore fluid changes in rocks is to use the ratio of compression wave velocity and shear wave velocity, $V_p/V_s$ (e.g., Hamada, 2004). The sensitivity of this ratio to pore fluid composition is because the compression wave velocity decreases while the shear wave velocity is nearly independent with increasing of gas phase in the pores. This ratio also increases rapidly as the pore fluid becomes less compressible. However, more rigorous models for seismic velocity of fluid saturated rocks have been developed. Gassmann (1951) developed the earliest theory on the effects of pore fluid composition on the seismic velocity of rocks. This is now called Gassmann’s Substitution Theory. The original derivation of Gassman’s equation is very involved and complicated. However, it has been shown that this theory can be derived as well from poro-elastic equations of Biot (1941) as was done for instance by Berryman (1999). Biot’s coupled poroelastic equations provide a rigorous mathematical treatment of fluid flow in deformable porous media, where fluid flow affects mechanical response and vice versa, and fluid flow field cannot be analyzed separately from the mechanical response except under simple boundary conditions. One of the key assumptions in Gassman’s Substitution Theory is that the shear wave velocity and the shear modulus of a rock are not affected by the pore fluid composition.

For a porous medium containing gas and liquid mixed in a heterogeneous manner, Gassmann’s fluid substitution model reflects the dependency of the fluid content on seismic wave velocity in terms of the bulk modulus:
\[ K_{\text{sat}} = K_{\text{dry}} + \frac{\alpha^2}{\phi + 1 - \phi - \frac{K_{\text{dry}}}{K_s^2}} \]  

(1.11)

where \( K_{\text{sat}} \) is the bulk modulus of the fluid saturated medium, \( K_{\text{dry}} \) is the dry bulk modulus, \( \phi \) is the porosity, \( K_f \) is the bulk modulus of the pore fluid, \( K_s \) is the bulk modulus of the solid grains of the porous medium, \( \alpha \) Biot’s poroelastic parameter defined as:

\[ \alpha = 1 - \frac{K_{\text{dry}}}{K_s} \]  

(1.12)

![Figure 1.4 Examples showing the effects of the type of pore fluid (air, oil and water) on the stress dependent P- and S-wave seismic velocities of two rocks (Mavko and Mukerji, 1998).](image)
The bulk modulus given in Eq. (1.11) is the one substituted in Eq. (1.1) to obtain the P-wave velocity of the fluid saturated medium. On the other, it is assumed that the shear modulii of the fluid saturated and dry material are the same, i.e., \( G_{\text{sat}} = G_{\text{dry}} \). The dependency of the P-wave velocity on pore fluid composition is evident since the fluid bulk modulus \( K_f \) depends on the types, compositions and distributions of fluids in the pore space. Murphy (1982) experimentally showed that for Massillon sandstone the variation of P-wave velocity due to water saturation change is satisfactorily described by using the Gassmann’s equation. Previous studies by Kim et al. (2010), and Shi et al. (2007) have also shown that the change of acoustic velocity response of sandstone cores with water/ CO\(_2\) saturation change can be predicted by using the Gassmann’s fluid substitution equation.

The fluid bulk modulus of mixtures of different fluids and gasses is difficult to evaluate. The mixture bulk modulus should depend not only on the relative volumes of the fluids but also on the degree of uniformity of the fluid mixture (Brie et al., 1995; Endres and Knight, 1991). It is therefore important to properly quantify the bulk modulus of mixtures of fluids before the P-wave seismic velocity can be reliably predicted by the Gassmann’s substitution equation.

1.1.4 Shear Wave Splitting

Rock masses are generally heterogeneous and contain different types of discontinuities including bedding planes, joints and fractures. As a result, rock mass elastic behavior is expected to be anisotropic, and in turn, seismic wave propagation should be direction dependent as well. The most pronounced type of direction dependent seismic wave propagation is “shear wave splitting” or “seismic birefringence” when a shear wave encounters a discontinuity. Seismic shear wave propagates slower across than parallel to a discontinuity. As a result, a travelling shear wave is split into two perpendicular directions, one fast and slow, as it encounters a discontinuity (Figures. 1.5 and 1.6).

The incident shear wave splits into two polarized shear waves. The polarization of the fast S-wave is shown to correlate with the strike and the dip of the main fracture system traversed by the wave. The delay time between arrivals of the fast and the slow shear-waves is proportional to the fracture density. This phenomenon has been used to detect anisotropy in the underground formations using seismic surveys. In turn, measurements on degree of anisotropy are used to a
better understanding of the discontinuity density and orientations and alignments. More importantly, the information gathered on crack density also offers good indication of areas of increased permeability within the reservoir.

Figure 1.5 Dipole sonic log (a) fast shear wave azimuth; (b) traveltime of the fast and slow shear waves; and (c) shear wave birefringence. Three main zones of shear wave splitting are marked by Zone A, B, and C. Zone C corresponds to the first Frontier Sand. (From ARCO Schlumberger, 1989)
Figure 1.6 Illustration of shear wave splitting. A shear wave entering a vertically fractured medium is split into a fast ($S_1$) component parallel to the fracture, and slow ($S_2$) component perpendicular to the fracture (Martin and Davis, 1987).

The analysis for shear-wave splitting has two key steps. First, a pre-stack least-squares analysis of the transverse component amplitudes is used to determine the azimuth of the fast shear direction and the time delay between fast and slow shear waves (Bale et al., 2005). Second, the azimuth is rotated to the $S_1$-$S_2$ coordinate system, and is stacked to generate the $S_1$ and $S_2$ traces. These traces are then correlated to determine the total time delay between the $S_1$ and $S_2$ signals. There can be multiple layers each with a different orientations of the shear wave anisotropy. As a result, the shear waves starting from deep sources can become split in several layers before they propagate back to the receivers at the surface. These overburden effects must first be corrected by using a layer-stripping approach where a time-variant shift is applied to the $S_2$ shear for each overburden layer wave so it matches the $S_1$ time. This allows recombination of the split waves into new radial and transverse datasets corresponding to an equivalent isotropic layer with $S_1$ velocity. The process is repeated for each layer.

Despite the acknowledged value and importance of the phenomenon of shear wave splitting in seismic surveys of fractured rocks, knowledge of the relationship between rock microstructure and shear wave splitting has not been yet clarified sufficiently. In particular, there is currently limited understanding on the influence of effective stress level and shear displacement of fractures on the magnitude of shear wave splitting. Further studies are necessary for understanding the shear wave splitting response of rocks related to microstructural and stress changes in laboratories.
1.2 Research Objectives

The main aim of this research is to improve the characterization and modeling of the acoustic velocity of intact and fractured porous sandstones. The effects of stress confinement and pore fluid composition, particularly mixtures of saline water and supercritical CO₂, on acoustic velocity of micro-fractured sandstones are investigated experimentally. The research also aims to investigate the effects of stress and shear displacement on shear wave splitting in fractured rocks. Experiments will be carried out on intact (i.e., containing only micro-fractures) and fractured samples of sandstone using a newly developed High Pressure and High Temperature (HPHT) triaxial apparatus. Using the experimental results, improved models will be developed for the stress level and pore fluid dependent seismic velocity of sandstones, and the stress level and shear displacement dependent shear wave splitting in fractured sandstone. The specific objectives of the research are:

1. Develop a new advanced experimental High Pressure and High Temperature (HPHT) system for characterizing the stress level and pore-fluid content dependency of the acoustic wave velocity of intact and fractured sandstones. The system will have the capability to inject two immiscible phase fluids into core samples of sandstones and measure the seismic velocities while the sample is under triaxial condition, and stresses, pore pressures and temperatures corresponding to in situ conditions.

2. Perform experiments and develop a new model to characterize the stress dependent seismic wave velocities for porous rocks based the nonlinear elastic response of micro-fractures contained in the rock sample under normal and shear loading. The model will be validated against experimental and published results on stress dependent seismic velocity of rocks.

3. Perform experiments and develop a model on the variations in the seismic velocities of intact sandstones due to changes in the pore fluid composition, in particular during injection of supercritical CO₂ in core samples initially saturated with saline water. The pore fluid dependent acoustic velocity model will be based on the Biot-Gasmann poroelasticity theory.

4. Perform experiments and develop a model to characterize the effects of stress and shear displacement on the magnitude of shear wave splitting and permeability in fractured rocks, and the influence of shear wave splitting on the permeability of fractured porous rocks.
1.3 Scope of Work

It is envisioned that the results from the research can be used to improve the use of seismic surveys in characterizing underground formations and in the monitoring of fluid and migration in disparate energy and environmental applications including hydrocarbon exploration and production, geothermal energy utilization, and underground waste disposal and storage. However, the focus of the research, particularly of Chapter 3, is on the geological sequestration (GS) of carbon dioxide. With the increasing emission of greenhouse gas, the reduction of emission of anthropogenic carbon dioxide is a major concern. Carbon capture and sequestration in appropriate geologic formations (saline aquifers, coal seams, and gas reservoirs) is one of the promising methods to reduce the release of greenhouse gases in the atmosphere. To ensure success of CO\textsubscript{2} GS, field monitoring is essential on ascertaining movement, volumes and locations of injected CO\textsubscript{2} in sequestration reservoirs. One technique is to use time-lapsed seismic survey mapping to provide spatial distribution of seismic wave velocity as an indicator of CO\textsubscript{2} migration and volumes in a storage reservoir with time. Application of time-lapsed seismic survey mapping to monitoring of CO\textsubscript{2} GS projects require improved understanding of the acoustic properties of storage and how these properties are affected by different factors such as stress level, pore fluid composition, and fracture distribution and behavior.

1.4 Thesis Organization

This thesis consists of six chapters organized as follows:

Chapter 1 presents a literature review survey and introduction of the necessary scientific background on theory of propagation of elastic wave in porous media. Background materials on the effects of stress level and pore fluid composition acoustic velocity of rock as well as introduction to shear wave splitting are provided. This Chapter also lists the overall and specific objectives, and the scope of the research described in the thesis.

Chapter 2 presents the development of a testing system for the characterization of supercritical CO\textsubscript{2} injection in sandstone initially saturated with saline aquifers including the capability to monitor the acoustic properties of the tested material during injection of supercritical CO\textsubscript{2}. Test procedures and tests materials are also described. Preliminary experimental results to characterize two-phase immiscible fluid flow and seismic velocity
response of the tested sandstone samples are provided to demonstrate the capability of the testing system.

Chapter 3 presents the results of laboratory tests on the effects of stress level on the seismic velocities of micro-fracture sandstone core samples, as well as the characterization of the micro-fracture distribution in the tested materials using thin sections. Using the experimental data, a new analytical model to characterize stress dependent seismic wave velocities for randomly micro-fractured rocks is developed. The model ties the stress-dependent seismic velocities to the behavior and distribution of the micro-fractures in the tested samples.

Chapter 4 presents a brief introduction of CO$_2$ geological sequestration (GS) and related processes and the use of seismic velocity for field monitoring of CO$_2$ GS. Experiments are conducted where the seismic velocities were monitor during injection of supercritical CO$_2$ in core samples of sandstone initially saturated with saline water. The results of the experiments are modeled using the Biot-Gassmann poroelasticity. The focus of the mathematical modeling is on how the Biot-Gassmann theory can be modified to account for the distribution of two-phase fluids, particularly in terms of non-uniformity in the supercritical CO$_2$ displacement front, in addition to their relative volumes, in the pore space of sandstone.

Chapter 5 investigates experimentally the effect of the presence of a macroscopic fracture on ultrasonic wave velocity and shear wave splitting of core samples of sandstone. The experiment also characterizes the stress dependent permeability for fractured rocks. The methodology for creating artificial fractured cylindrical Berea core samples and the characterization of the morphology of the created fracture surface are presented. Models for stress level and shear displacement dependent shear wave splitting and permeability fracture sandstone are developed and validated against the experimental data.

Chapter 6 provides the main conclusions from the research as well recommendations for possible future work.

Appendix A: Hydro-mechanical for fractured porous rocks

Appendix B: Stress dependent absolute permeability for intact and fractured Berea sandstone.
1.5 References


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CHAPTER 2
DEVELOPMENT OF A NEW ADVANCED TESTING SYSTEM FOR
CHARACTERIZATION OF CO₂ INJECTION IN DEEP SALINE AQUIFERS

2.1 Introduction

Geological sequestration of carbon dioxide (CO₂) is acknowledged as one of the promising options to reduce the amount of CO₂ emission due to human activities. The estimated volume of potential sequestration reservoirs is abundant in the globe (Benson and Cole, 2008; IPCC-Intergovernmental Panel on Climate Change, 2005). Depleted oil and gas reservoirs and abandoned mines are important candidates for CO₂ storage. Still more promising are deep saline aquifers because of their huge estimated capacities and high connectivity of pore spaces (Bachu, 2003). CO₂ injected in geological formations is considered to be subsequently trapped due to different mechanisms. Trapping mechanisms include stratigraphic and structural trapping, residual CO₂ trapping, solubility trapping, and mineral trapping (IPCC72 Intergovernmental Panel on Climate Change, 2005). A large part of injected CO₂ is initially trapped due to the structural and stratigraphic trapping mechanisms.

The volume ratio of CO₂ trapped due to solubility and mineral trapping mechanisms is expected to increase with time. Along with this change, site integrity is expected to increase as the formation gets mineralized. To be economically feasible, CO₂ needs to displace in situ saline water efficiently. Injection of CO₂ is carried out in the form of a supercritical fluid as a result of high in situ fluid pressures and temperature. This is advantageous in terms of injectivity because supercritical CO₂ (referred to as scCO₂ in this study) is less viscous than liquid CO₂ and denser than CO₂ vapor. However, CO₂ injection pressures, which must be greater than in situ pore pressures, can induce adverse effects in the reservoir, such as seismic activity, deterioration of the capping and sealing properties of the cap rock layers, opening of fractures, and shear failure of faults (e.g. Sminchak and Gupta, 2001). Thus, one of the main technical issues in CO₂ geological sequestration is the prediction of the injectivity and prolonged stability of CO₂ in deep saline formations. Proper characterization of two phase relative permeabilities of CO₂ reservoir rocks for immiscible saline water and CO₂ flow at in situ conditions is indispensable to: 1) investigate the suitability of a storage site, 2) predict CO₂ migration in reservoirs, and 3) evaluate
site safety. Development of remote monitoring techniques for CO₂ migration is also an essential issue for safe and reliable operations of CO₂ sequestration. A typical direct monitoring technique uses observation wells and tracers to observe CO₂ migration in reservoirs. However, drilling observation wells is expensive, time consuming, provides only limited data which may not fully represent in situ conditions, and has the risk of deteriorating the sealing capacity of the cap rock (IPCC Intergovernmental Panel on Climate Change, 2005).

Indirect geophysical methods such as seismic and electric resistivity surveys can be used for economical, large-scale field underground fluid monitoring. Time-lapsed seismic survey mapping can provide spatial distribution of seismic wave velocity which can be correlated with CO₂ migration and volumes in a storage reservoir with time. The acoustic velocity response of rock media filled with water and CO₂ is primarily a function on rock mass elasticity, porosity, and pore fluid bulk modulus. These components, related to acoustic velocity, are affected by temperature, rock mass effective stresses, fluid pressure, and pore fluid composition (saturation). Previous studies (Kim et al., 2010; Shi et al., 2007) show that the change of acoustic velocity response of sandstone cores with water/CO₂ saturation change can be predicted by using the Gassmann’s equation (Gassmann, 1951). However, the evaluation of the bulk modulus of mixture of multiphase fluid is still problem. The bulk modulus of a fluid mixture should depend on the degree of uniformity of mixture (Brie et al., 1995; Endres and Knight, 1991). Acoustic velocity of porous rock filled with fluids can increase as the wave frequency becomes higher (typically in sonic to ultrasonic ranges) (e.g. Murphy et al., 1993).

This chapter presents the development and use of a laboratory testing system to clarify the hydraulic and seismic response of rocks to supercritical CO₂ injection under deep saline aquifer conditions. Thus, the goal is to find if changes in pore fluid content and distribution in a core sample particularly in terms of layering and non-uniformity in the displacement front can be detected from seismic velocity changes.

### 2.2 Testing System

The newly developed test system consists of a high-pressure high-temperature core holder having a maximum working confining pressure capacity of 70 MPa, two precision syringe pumps that control the back-pressure and injection pressure, a dome-loaded back pressure
regulator, a hydraulic pump for regulating cell pressure, and a differential pressure transducer. The core holder is capable of holding a cylindrical core sample under triaxial stress states. Two movable pistons controlled by hydraulic pressure are able to induce axial stress different from the confining stress.

One of the syringe pumps transmits a mixture of saline water and scCO$_2$ into the rock core sample at flow rates ranging from $1 \times 10^{-3}$ to cm$^3$/min and at pressures of up to 50 MPa. The values of the accuracy of the flow rate and pressure are 0.5% of setpoint and 0.5% of full scale, respectively. The resolution of fluid displacement is $16.6 \times 10^{-6}$ cm$^3$. Another syringe pump whose flow range and pressure range are from $1 \times 10^{-5}$ to 50 cm$^3$/min and from 0.07 to 70 MPa, respectively, acts as the back-pressure regulator with set-point pressure. The rated accuracy of flow rate and pressure are 0.3% of setpoint and 0.5% of full scale, respectively. Figure 2.1 shows a schematic diagram of the test system.

![Schematic diagram of the testing system](image)

Figure 2.1 Schematic diagram of the testing system

The temperature of the cell fluid is monitored by using a T-type sheathed thermocouple inserted into the core holder. The temperatures of fluids stored in the syringe pumps are regulated by using silicon rubber blanket heaters wrapped around the sidewalls of the syringes and the PID temperature controllers. The pressure and volume change of fluids in the syringe
pumps are monitored and acquired in a personal computer communicating with the pumps. The back-pressure regulator is equipped with a polyimide film diaphragm providing accurate regulation of outlet pressures. The differential pressure transducer is variable reluctance type having 860 kPa of capacity, working temperatures ranging from 255 to 344 K, and ±0.25% of full scale accuracy which includes the effects of non-linearity, hysteresis, and non-repeatability.

Saline water produced from the end face of rock core sample during scCO₂ injection is accumulated in a vessel placed on an accurate electronic balance. The mass data of saline water accumulated is acquired by using a computer via serial communication. The repeatability and linearity of the balance are 0.01 g and 0.03 g, respectively. The complete testing system is enclosed within a constant temperature air-circulating cabinet that is controlled with an on-off infrared heater and air-circulating fans. The end caps of the core holder are equipped with piezoelectric transducers for seismic compressional and shear wave velocity measurements. Figure 2.2 shows a schematic of the arrangement of piezoelectric transducers.

![Figure 2.2](image.png)

Figure 2.2 Arrangement and polarization of piezoelectric transducers for seismic wave velocity measurement. Arrows indicate the direction of oscillation of two different shear waves (horizontal “SH” and vertical “SV”), and “P” denotes compression wave.

Triads of shear wave transducers shake one of the end caps in two individual orthogonal directions. In the center of the cap is a compressional wave transducer. The natural frequencies of the transducers range from 0.25 to 1 MHz. Figure 2.3 shows a wiring diagram of the system used to acquire seismic data of rock core sample.
Figure 2.3 A wiring diagram of ultrasonic wave velocity measurement system.

The transducers are excited by using a square wave pulser/receiver capable of providing square pulses whose voltages are selectable between 100 and 400 V in increments of 100 V. The typical rise time of pulse is less than $1.0 \times 10^{-8}$ s. The maximum bandwidth and typical noise level of receiver are from 1 kHz to 35 MHz and 70 μV peak to peak, respectively. The seismic waveforms are converted into digital signals by using a digital oscilloscope having 100 MHz of bandwidth and $1 \times 10^9$ s of real time sample rate. The minimum time base and the vertical resolution of oscilloscope are $2 \times 10^{-9}$ s/div and 8 bits. The waveform data are acquired by using a personal computer communicating with the oscilloscope. Each waveform acquired is the average of 16 waveforms. The resultant minimum time interval of waveform acquisition in the personal computer is 6 s.

2.3 Testing Materials and Procedures

Cylindrical rock samples used for the experimental works are cored from a block of Berea sandstone. Subsequently, both ends of each core are ground to ensure that sample ends are parallel within required tolerances. The core samples are 38.1 mm in diameter, approximately 69 mm in length, and of 21% porosity. The absolute permeabilities of the core samples range from 31 to 43 mD. Carbon dioxide having 99.9999% of purity is used to prepare the scCO$_2$. Saline water used is a mixture of distilled water and sodium chloride at 3.4% of salinity. The procedure for the scCO$_2$ injection tests is described below. A core sample dried at $T = 383$ K is placed
inside the core holder then saturated with distilled water under vacuum by allowing the sample to imbibe distilled water. The axial and lateral pressures of the core sample are isotropically increased up to 12 MPa. The pore pressure ($P_{pore}$) is simultaneously raised to 10 MPa by keeping the effective stress less than 2 MPa. The core sample is consolidated at desired effective stress at $P_{pore} = 10$ MPa and $T = 313$ K to ensure that the CO$_2$ is kept in supercritical condition.

Distilled water filling the pore space is displaced with saline water saturated with scCO$_2$ at $P_{pore} = 10$ MPa and $T = 313$ K. The total volume of saline water injected is more than ten times of the pore volume of core sample. Supercritical CO$_2$ saturated with saline water is injected into core sample at a constant rate at the prescribed conditions. The injection rates of scCO$_2$ ($q_{inj}$) during the relative permeability characterization and ultrasonic wave velocity measurement are 2.0 and 0.20 cm$^3$/min, respectively. The pressure values used in the study are gage pressures.

2.4 Experimental Methods Overview

The relative permeability was measured in our lab by using unsteady state approaches. The experimental set ups consist of a core holder with a pressurized sleeve to exert confining pressure. The two porous disks capped the cylindrical cores at the both ends were used to provide a homogenous distribution flow at the inlet and outlet of the core. The core sample is 100% saturated of the wetting phase (drainage experiment process).

The outlet is usually maintained at a constant pressure, using a backpressure regulator. The fluid flow and pressure are controlled by pump B, and the temperature has to be maintained constant at 40 °C. The fluid saturations of scCO$_2$ and $S_w$ are measured during the displacement using the volumetric mass balance, where the volumes of fluids in vs. fluids out are compared.

2.5 Unsteady State Technique

In this experimental study unsteady state experiments technique was applied. In the unsteady state method, only one fluid phase is displaced from the core by injecting another fluid. Both, the wetting and non-wetting fluid will exist the core. Thus, the core sample is first saturated and then CO$_2$ is injected at supercritical conditions. A constant differential pressure is maintained between the outlet and the inlet. The core is placed horizontal position and the scCO$_2$ in injected laterally. The unsteady state experiments have a short duration.
The ratio of the gas versus water is driven from the cumulative scCO₂ injected over time and cumulative water produced over time. The pressure and saturation transients are usually analyzed by numerical modeling to determine the relative permeability. The capillarity pressure is neglected, and Buckley-Leverett displacement is assumed, thus, a numerical modeling derived the relative permeability of scCO₂/water. The unsteady-state techniques are perticulaly sensitive to heterogeneity.

### 2.6 Relative Permeability Measurements

Figure 2.4 shows the injection pressure during CO₂ injection at \( q_{\text{inj}} = 2.0 \text{ cm}^3/\text{min} \). The initial increase in injection pressure is due to increase in capillary pressure as the water saturation decreases. The injection pressure dropped after it reached 10.25 MPa at \( (N_{\text{CO₂, inj}}/N_{\text{pore}}) = 0.40 \). The subsequent injection pressure ranged from 10.19 to 10.22 MPa. The pore pressure is well regulated even when the volume fraction of supercritical CO₂ changes significantly.

![Injection pressure graph](image)

**Figure 2.4** Changes of injection pressure during CO₂ injection into Berea sandstone core sample at 2.0 cm³/min of flow rate.
Figure 2.5 shows the production behavior of saline water and the change in differential pressure at $q_{\text{inj}} = 2.0 \text{ cm}^3/\text{min}$. The differential pressure $\Delta P$ increased from the start of injection and then reached 65 kPa when the pore volume of CO$_2$ injected, ($N_{\text{CO}_2,\text{inj}}/N_{\text{pore}}$), is 0.53. Subsequently, it abruptly dropped to 17 kPa at ($N_{\text{CO}_2,\text{inj}}/N_{\text{pore}}$) = 1.21. The production rate of saline water significantly decreased at the same time. After this largest peak in differential pressure was reached, several small peaks were observed in the $\Delta P$-($N_{\text{CO}_2,\text{inj}}/N_{\text{pore}}$) relationship.

![Graph showing production behavior and differential pressure](image)

Figure 2.5 Production behavior of Berea sandstone core sample during immiscible displacement injecting supercritical carbon dioxide into saline-saturated Berea sandstone core sample.

Figure 2.6 shows the relative permeability curves of the sandstone core sample for saline water and scCO$_2$ calculated. The CO$_2$ saturation at the end point of displacement is 0.37. The relative permeability for CO$_2$ increased to 0.13 at the end point. It can be seen that dependency of relative permeability for CO$_2$ on CO$_2$ saturation is insignificant. The relative permeability for saline water is more sensitive to increase in CO$_2$ saturation. The relative permeability for saline water decreased to 0.16 when the CO$_2$ saturation is 0.15. The results indicate the very low mobility of sCO$_2$ injected in saline water.
From the test results, the calculation of the ratio of relative permeability for CO₂ to that for saline is obtained from where the relative permeability for carbon dioxide is obtained from

\[
\frac{k_{r,CO₂}}{k_{r,sln}} = \frac{1 - f_{sln,2} \mu_{CO₂}}{f_{sln,2} \mu_{sln}}
\]  

(2.1)

where relative the permeability for CO₂ is

\[
k_{r,CO₂} = \frac{\mu_{CO₂} L}{\Delta P A_k} q_{CO₂}
\]  

(2.2)

\(K\) is absolute permeability (Please see appendix B), the CO₂ saturation at outlet face of core sample, \(SCO₂,2\), can be calculated based on the Welge’s extension of Buckley-Leverett concept given as:

\[
S_{CO₂,ave} - S_{CO₂,2} = f_{sln,2} \frac{N_{CO₂,inj}}{N_{pore}}
\]  

(2.3)

\(S_{co₂, ave}\) is the average scCO₂ saturation in the core sample at any time, which is defined as the volume of supercritical CO₂ injected in the voids (reading volume of the pump B) divided by the volume of the voids (pore volume of the rock sample \(S_{CO₂,ave} = \frac{N_{CO₂,inj}}{N_{pore}}\)) where the fractional flow of saline is defined as:

\[
f_{sln,2} = \frac{q_{sln,2}}{q_{sln,2} + q_{CO₂,2}}
\]  

(2.4)

The best-fit curves of the Corey-type (e.g. Corey, 1954) model to the observed relative permeabilities for CO₂ and saline water are shown in the figure. The model is defined as:

\[
k_{r,sln} = k_{r,sln}^* \left( \frac{S_{sln} - S_{sln,r}}{1 - S_{sln,r} - S_{CO₂,2,r}} \right)^{m_{sln}}
\]  

(2.5)

\[
k_{r,CO₂} = k_{r,CO₂}^* \left( \frac{1 - S_{sln,r} - S_{CO₂,2,r}}{1 - S_{sln,r} - S_{CO₂,2,r}} \right)^{m_{CO₂}}
\]  

(2.6)

Where \(k_{r,sln}^*\) and \(k_{r,CO₂}^*\) are the end-point relative permeability for saline water and CO₂, \(S_{sln,i}\) is the irreducible saline water saturation, \(SCO₂, r\) is the residual CO₂ saturation, and \(m_{sln}\) and \(m_{co₂}\) are empirical parameters.
The values of the empirical parameters $m_{sln}$ and $m_{co2}$ determined by fitting Eqs. (2.5) and (2.6) to the experimental results are 2.5 and 1.1, respectively.

Figure 2.6 Relative permeability curves of a Berea sandstone core for supercritical carbon dioxide and saline.

2.7 Ultrasonic Wave Velocity Measurement During CO$_2$ Injection

Figure 2.7 shows the effective stress dependency of the P-wave (compression) and S-wave (shear) velocities, $V_p$ and $V_s$, of another dry core sample of Berea sandstone at $T = 313$ K. Both wave velocities increased with increasing effective stress linearly in semi-logarithmic plots. The slopes of regression lines for $V_p - \ln(\sigma_e)$ and $V_s - \ln(\sigma_e)$ plots are 0.35 and 0.25, respectively. P-wave velocity is more sensitive to effective stress than the S-wave velocity.

Figure 2.8 shows the injection pressure and saline water production behavior observed during CO$_2$ injection at a rate of 0.2 cm$^3$/min. The injection pressure is maintained at 10.00±0.05 MPa through displacement. The production behavior of saline water indicates that CO$_2$ saturation reaches 0.249 at the endpoint of displacement.
Figure 2.7 Effective stress dependencies of ultrasonic wave velocities of Berea sandstone core.

Figure 2.8 Injection pressure and saline production during CO$_2$ injection into Berea sandstone core at 0.2 cm$^3$/min flow rate, 40 MPa confining stress, 10 MPa pore pressure, and 313 K
Figure 2.9 shows some of waveforms of P-wave acquired during the CO2 injection. The values in the figures indicate the average CO2 saturation of the core sample when a P-wave velocity is measured. The arrival time of each wave has been determined at the first negative rise. The arrival time changed significantly from 36.0 to 36.6 ms and subsequently to 37.2 ms as the CO2 saturation increased from 0 to 0.151 and to 0.214.

![Waveforms of P-wave indicating delayed arrival time with increasing carbon dioxide saturation at 40 MPa of cell pressure, 10 MPa of pore pressure, and 313 K of temperature. The values shown in explanatory note are average carbon dioxide saturation of rock core sample.](image)

The CO2 saturation dependency of the bulk and shear moduli is summarized in Figure 2.11. The bulk and shear moduli, $K$ and $G$, for isotropic elastic solid can be calculated by

$$K = \rho \left( V_p^2 - \frac{4}{3} V_s^2 \right)$$  \hspace{1cm} (2.7)

$$G = \rho V_s^2$$  \hspace{1cm} (2.8)

where $V_p$ and $V_s$ are compression and shear wave velocities, and $\rho$ is the density of porous
medium. The bulk density of porous rock filled with saline and CO₂ is expressed as:

\[
\rho = \rho_{\text{dry}} + \phi [S_{\text{CO}_2} \rho_{\text{CO}_2} + (1 - S_{\text{CO}_2}) \rho_{\text{sal}}]
\]

(2.9)

where \(\rho_{\text{dry}}\), \(\rho_{\text{CO}_2}\), and \(\rho_{\text{sal}}\) are the bulk densities of the dry rock, CO₂, and saline, and \(\phi\) is the porosity of the rock sample.

Figure 2.10 S-waveforms propagating Berea sandstone core sample during immiscible displacement of saline with supercritical carbon dioxide at 40 MPa of cell pressure, 10 MPa of pore pressure, and 313 K of temperature. The values shown in explanatory note are average carbon dioxide saturation of rock core sample.

In Figure 2.11 and Eq. (2.9), \(S_{\text{CO}_2}\) is the scCO₂ saturation, which is defined as the volume of supercritical CO₂ in the voids (volume of CO₂ injected, which is directly reading from pump B) divided by the volume of the voids (pore volume of the rock sample). The bulk modulus decreases from 17.5 to 13.3 GPa during the displacement of saline. On the other hand, the shear modulus slightly increases from 12.5 to 12.7 GPa due to the increase in CO₂ saturation from 0 to 0.23.
Figure 2.11 Dependency of bulk and shear modulus of Berea sandstone core on carbon dioxide saturation during immiscible fluid substitution at 40 MPa of cell pressure, 10 MPa of pore pressure, and 313 K of temperature.

2.8 Discussion of Results

The testing system consists mainly of a high pressure triaxial core holder with rated confining capacity of 70 MPa, two high-pressure and precision syringe pumps transmitting saline water and supercritical CO₂ mixtures into a rock core sample, a differential pressure transducer, and a high-pressure hydraulic pump providing the cell pressure. The capabilities of the system are demonstrated in this study. Constant injection rate, immiscible displacement tests of saline water mixed with supercritical CO₂ were carried out on Berea sandstone core samples to characterize relative permeabilities for two-phase, saline water and supercritical CO₂ immiscible flow, and pore fluid content dependency of seismic wave velocity. The injected CO₂ will increase the compressibility and change the density-dependent pore pressure and fluid content. These changes will, in turn, affect propagation of the seismic waves response of rocks filled with
saline and CO₂. The latter is primarily a function on rock frame elasticity, porosity, and pore fluid bulk modulus. These components are affected by temperature, effective stress acting on rock frame, fluid pressure, and fluid composition (saturation).

2.8.1 Relative Permeability Characterization

As shown in Figure 2.6, small values have been observed for CO₂ saturation and relative permeability for CO₂ at the end point of displacement. Similar relative permeability curves and low CO₂ mobility were commonly observed in previous studies (Bennion and Bachu, 2005; Perrin et al., 2009). For instance, the end point CO₂ saturations of Ellerslie sandstone and Viking sandstone are 0.341 and 0.442, respectively (Bennion and Bachu, 2005). The corresponding relative permeabilities for CO₂ of the two sandstones are 0.116 and 0.332 at the end points, respectively. These values are comparable with the value shown in Figure 2.6 Therefore, the newly developed testing system is considered to be capable of reliably characterizing relative permeabilities of sandstone core sample for saline water and supercritical CO₂. One of the potential mechanisms of the high irreducible saline water saturation is the bypass of lower permeable areas in the rock core sample due to local heterogeneities (e.g., clay content and/or pore throat dimensions).

Another potential reason is the effect of buoyancy on displacement process. It is observed that the displacement efficiency of saline water increases with increasing CO₂ injection rate (Perrin et al., 2009). Buoyancy-induced flow resulting from injection of less dense CO₂ can prevent uniform displacement of saline water by CO₂. The buoyancy effect on heterogeneous CO₂ distribution should become more significant with decreasing CO₂ injection rate (Perrin et al., 2009). Therefore, when CO₂ is injected at the higher rates, it is expected that the CO₂ saturation at the endpoint becomes higher and thus the endpoint relative permeability for CO₂ increases.

Figure 2.12 illustrates the effect of scCO₂ injection rate on the displacement efficiency. The difference in testing conditions other than injection rate is the effective stress acting on rock frame; i.e., 30 and 5 MPa for \( q_{inj} = 0.2 \) and 2.0 cm\(^3\)/min, respectively. The dotted line in the figure is the ideal relationship between produced saline water and injected CO₂ that should be retained before breakthrough of CO₂ phase. The arrows in the figure are pointing to estimated
breakthrough points in the individual experimental runs. Interestingly, the CO\textsubscript{2} saturation of rock sample at breakthrough point is insensitive to the CO\textsubscript{2} injection rate. The values of CO\textsubscript{2} saturation at the breakthrough points are 0.179 and 0.176 at $q_{\text{inj}} = 0.2$ and 2.0 cm\textsuperscript{3}/min, respectively.

![Graph](image)

**Figure 2.12** Effect of injection flow rate of carbon dioxide on displacement efficiency of saline water in Berea sandstone cores. The cell pressures are 40 and 15 MPa at 0.2 and 2.0 cm\textsuperscript{3}/min of injection rates, respectively. Pore pressures and temperatures are 10 MPa and 313 K in both tests.

The effect of injection rate is seen clearly in the post-breakthrough saline water production behavior. The difference of produced saline water volume between these two runs increases as the volume of CO\textsubscript{2} injected increases. The production of saline water at $q_{\text{inj}} = 0.2$ cm\textsuperscript{3}/min becomes considerably small when $(N_{\text{CO}_2,\text{inj}}/N_{\text{pore}})$ is greater than 0.50. On the other hand, the saline water production at $q_{\text{inj}} = 2.0$ cm\textsuperscript{3}/min is still active at this range of $(N_{\text{CO}_2,\text{inj}}/N_{\text{pore}})$. The experimental result indicating that CO\textsubscript{2} saturation at breakthrough insensitive to the injection rate suggests that the pre-breakthrough CO\textsubscript{2} distribution in core sample is also insensitive to the injection rate. When breakthrough of CO\textsubscript{2} occurs, the local CO\textsubscript{2} saturation in the core sample
should be smaller as the distance from the inlet end face of core increases. The CO₂ phase injected at the lower rate cannot efficiently displace saline water occupying the pore spaces close to the outlet of core sample because flow paths of CO₂ should concentrate on the upper side of core sample, which was horizontally oriented.

### 2.8.2 Ultrasonic Wave Velocity Response During CO₂ Injection

The Gassmann’s equation (Gassmann, 1951) is used to evaluate the effect of pore fluid composition on the bulk modulus of a fluid saturated porous medium. This equation is given as:

$$K_{\text{sat}} = K_{\text{dry}} + \frac{\left(1 - \frac{K_{\text{dry}}}{K_m}\right)^2}{\frac{\phi}{K_{\text{fl}}} + \frac{1 - \phi}{K_m} - \frac{K_{\text{dry}}}{K_m}^2}$$

(2.10)

where $K_{\text{sat}}$ is the saturated and undrained bulk modulus of the rock sample, $K_{\text{dry}}$ is the rock frame or drained bulk modulus, $K_m$ is the mineral bulk modulus (bulk modulus of the rock grains), $K_{\text{fl}}$ is the pore fluid bulk modulus, and $\phi$ is the porosity.

The second term of the right-hand side of Eq. (2.10) represents the effect of pore fluid content on the bulk modulus $K_{\text{sat}}$. In the derivation of Eq. (2.10), it is assumed that the rock shear modulus $G$ is independent of fluid saturation, and this is verified in Figure 2.9, which shows a nearly constant $G$ as the CO₂ saturation changes. To calculate $K_{\text{sat}}$ in Eq. (2.10), it is necessary to determine the pore fluid bulk modulus $K_{\text{fl}}$. Two widely used equations to calculate the bulk modulus of mixtures of two-phase immiscible fluids are: (1) the serial Wood’s law (Wood, 1941), and (2) the parallel law Voigt-type model.

For a mixture of saline water and supercritical CO₂, Wood’s law gives the following equation (2.11) which, can be derived by assuming that the fluids are distributed in series

$$\frac{1}{K_{\text{fl}}} = \frac{S_{\text{sln}}}{K_{\text{sln}}} + \frac{1 - S_{\text{sln}}}{K_{\text{CO₂}}}$$

(2.11)

It was shown for Massillon sandstone that the variation of P-wave velocity due to water saturation change is satisfactorily described by Gassmann’s equation in combination with Wood’s law when the frequency of P-wave is in the seismic range (Murphy, 1982).
The CO₂ saturation dependency of P-wave velocity of the tested Berea sandstone core is shown in Figure 2.13. P-wave velocity decreases from 3.86 km/s to approximately about 3.65 km/s as CO₂ saturation increases from 0 to 0.24. The $V_p - \text{SCO}_2$ relationship calculated based on the Gassmann-Wood equation is shown in the figure. The value of $K_m$ used in the equation is assumed to be 38 GPa (Kim et al., 2010). The frame bulk modulus is 12.2 GPa that is obtained by using $V_p$ and $V_s$ at 30 MPa of effective stress shown in Figure 2.7. The bulk moduli of saline water and CO₂ for the pressure and temperature used in the tests are estimated to be 2.49 and 0.046 GPa, respectively, based on (Batzle and Wang, 1992) and (Kim et al., 2010). The Gassmann-Wood equation predicts drastic decrease of $V_p$ to 3.58 km/s as the CO₂ saturation increases from 0 to 0.24. As can be seen, the Gassmann-Wood equation does not adequately replicate the experimental results. It has been discussed that the deviation of $V_p - \text{fluid}$ saturation relationship given by the Gassmann-Wood equation from the experimental data is caused by unrelaxed pore pressure (Murphy et al., 1993).

Wave propagation through a porous medium whose pore space is filled with fluids can induce excess pore pressures whose intensity depends on local pore geometry, fluid mobility, and fluid compressibility (e.g. Brie et al., 1995). The excess pore pressure, which can lead higher local fluid bulk modulus, cannot relax at higher frequencies. The threshold of wave frequency at which the effect of unrelaxed porous medium on wave velocity starts to appear becomes lower as fluid mobility and/or rock permeability decrease (Batzle et al., 2006). However, in this experimental data, the effect of unrelaxed frame due to high frequency P-wave propagation seems to be small. This is because the predicted P-wave velocity at $\text{SCO}_2 = 0$, which is 3.83 km/s, is comparable to the observed P-wave velocity of 3.86 km/s.

A much-improved empirical equation to estimate the fluid bulk modulus of mixtures multiphase fluid is the following equation proposed by Brie et al. (Brie et al., 1995):

$$K_{fl} = S_{sln} K_{sln} + (1 - S_{sln}) K_{CO2}$$  \hfill (2.12)

where $n$ is an empirical exponential parameter.

As shown in Figure 2.11, the fluid bulk modulus decreases with increasing CO₂ saturation because CO₂ bulk modulus is lower than saline water bulk modulus. One of the special spatial distributions of fluid mixtures is the parallel distribution of two distinct fluid phases and it is characterized by using $n = 1$ in Eq. (2.12).
Figure 2.13 Applicability of Gassmann’s equation to P-wave velocity change during CO₂ injection at 40 MPa of confining stress, 10 MPa of pore pressure, 313 K, and 0.2 cm³/min of injection rate.

The effect of the lower bulk modulus of scCO₂ phase becomes predominant at lower CO₂ saturations as the value of $n$ increases. The value of $n$ determined by fitting the $K_{sat}-S_{CO₂}$ curve to the experimental one is 4.19. The value of the coefficient of regression $R^2$ of this fitting is 0.85. Despite the relatively high $R^2$-value, the observed $V_p-S_{CO₂}$ relationship is still not sufficiently replicated by the combined Gassmann-Brie et al. equation. The observed $V_p-S_{CO₂}$ relation is in good agreement with the Gassmann-Wood equation when $S_{CO₂} < 0.02$. However, the observed $V_p-S_{CO₂}$ relation moves closer to that given by the Gassmann-parallel law equation up to $S_{CO₂} = 0.15$. After that, the observed relationship again moves back closer to the curve given by the Gassmann-Wood equation especially when $S_{CO₂} > 0.20$. The transitions of the $V_p-S_{CO₂}$ relationship should correspond to the transitions of the fluid distributions in the core sample.
At the very early stage of displacement ($S_{CO2} < 0.02$), the observed $V_p - S_{CO2}$ relationship has changed along the curve given by the Gassmann-Wood equation. This change can be understood by considering the following displacement process. The injected CO$_2$ phase is likely to concentrate in the inlet end of core sample at this stage. The rest of pore space should not be invaded by CO$_2$. The fluids are therefore in the series distribution to which the Wood’s law is applicable. When $S_{CO2}$ is increased from 0.02 to 0.15, the $V_p - S_{CO2}$ relationship observed has moved close to that given by the Gassmann-parallel law model. This transition means that the fluid distribution is shifting to a layered distribution. This is reasonable because the injected CO$_2$ can concentrate in the upper part of the horizontally oriented core sample due to buoyancy effect and consequently the saline water and CO$_2$ phases flow in two distinct layers. After that, the observed $V_p - S_{CO2}$ relationship has gradually moved toward that predicted by the Gassmann-Wood equation. This transition is understandable by considering a process in which the layered fluid distribution is changing gradually to a homogeneous one as the fluid displacement proceeds. More specifically, the volume of the saline water occupying the outlet end of the core sample has now been gradually displaced by CO$_2$, and CO$_2$ occupies most of the pore space.

It can be seen that the observed $V_p$ remains higher than that predicted by the Gassmann-Wood equation even at the end point of displacement. This suggests that the fluid distribution still has some heterogeneity at the end point of displacement. On the average, the Gassmann-Brie et al. equation with an exponent of 4.19 provides a very good representation of the pore fluid dependent P-wave velocity of Berea sandstone.

2.9 Summary

Our recent experimental data show that, the spatial distribution of saline water and supercritical CO$_2$ in a core sample is shown to affect the relative permeability curves and ultrasonic wave velocity, and the compressional wave velocity changes during displacement of saline water with CO$_2$. The data indicated that compressional wave velocities were greatly decreased by CO$_2$ injected, while shear-wave velocities were, however, less affected by CO$_2$ injection. This can interpret the distribution behavior of the fluid during displacement that shifts from serial distribution toward a homogeneous phase then return to serial distribution as the displacement proceeds. The experimental results and theoretical analysis of seismic wave
velocity suggest that the decrease in compressional-wave velocity in saturated rocks during CO₂ injection may be seismically resolvable in-situ. Therefore, it is possible that seismic methods can be used in mapping and locating CO₂ regions, tracking CO₂ front movement, and monitoring CO₂ processes in reservoir rocks.

Relative permeabilities of Berea sandstone core sample for saline water and CO₂ mixtures were characterized in the drainage process carried out at constant confining stress, back pressure, temperature, and scCO₂ injection rate. The capability of the system for relative permeability measurement was validated by comparing the CO₂ saturation and relative permeability curves and values at the end point with those of the previous studies. Furthermore, the effect of CO₂ injection rate on the end point CO₂ saturation and relative permeabilities were discussed. The effective stress dependencies of ultrasonic P- and S-wave velocities of dry Berea sandstone core measured at the effective stresses not greater than 30 MPa at constant temperature were shown to follow linear relationships between seismic velocities and the logarithm of effective stress. The P-wave velocity is more sensitive to the change in effective stress. Ultrasonic wave velocity changes due to CO₂ saturation change were also measured using Berea sandstone core subjected to constant CO₂ injection rate. The P-wave velocity was observed to decrease with increasing CO₂ saturation whereas the S-wave velocity was almost constant.

The observed relationship between P-wave velocity and CO₂ saturation transitioned from the relation given by the Gassman-Wood model in the initial injection phase, to the Gassmann-parallel law model, then back to the Gassmann-Wood model towards the end of the displacement process. This should correspond to the transition of spatial distribution of saline water and CO₂ in core sample as the displacement of saline water proceeded. The results show the importance of pore fluid distribution in determining the effects of multiphase pore fluids on the seismic velocity of porous rocks. The Gassmann-Brie et al. equation with $n=4.19$ provides a very good representation of the effects of pore fluid composition on the P-wave velocity of Berea sandstone.
2.10 References


3.1 Introduction

Seismic velocity measurements are widely utilized by different geoscience, geoengineering, and georesource disciplines to obtain information on the geologic structure of rock formations. While directly related to the rock mass elastic moduli and density, seismic velocity is also affected by other parameters. One parameter that has strong effects on seismic velocity is the magnitude of the effective stress. In general, seismic velocity increases as the effective stress to which a rock mass is subjected increases. Seismic velocity also increases with decrease in rock mass porosity. While porosity is a function of effective stress, effective stress has an effect on seismic velocity that is independent of the stress-induced porosity change.

The dependence of seismic velocity on effective stress has been confirmed mainly by experiments on rock core samples in the laboratory. Rock samples of known porosities are tested in triaxial apparatus that allows for measurement of seismic velocity at different confining pressures. Fundamentally, the stress-dependency of seismic velocity is a result of the stress-dependency and nonlinearity of the elastic moduli of rocks. Mathematically, these stress dependencies can be expressed as:

\[ V_p = \sqrt{\frac{M}{\rho}}, \quad V_s = \sqrt{\frac{G}{\rho}} \]  \hspace{1cm} (3.1)

\[ M = M(p', K_{pf}), \quad G = G(p') \]  \hspace{1cm} (3.2)

where \( V_p \) is the P-wave or compressional wave velocity, \( V_s \) is the S-wave or the shear wave velocity, \( \rho \) is the density, \( G \) is the shear modulus, \( p' \) is the effective stress, \( K_{pf} \) is the bulk modulus of pore fluid, and \( M \) is the constrained modulus defined as:

\[ M = K + \frac{4}{3} G \]  \hspace{1cm} (3.3)

where \( K \) is the bulk modulus. In most formulations for stress-dependent seismic velocity, \( p' \) corresponds to the effective mean stress. The tensor summation is implied in Eq. (3.4).
\[ p' = \frac{\sigma'_{ii}}{3} = \left( \sigma_{ij} - \alpha \delta_{ij} p_{pf} \right) \]

where \( \sigma'_{ij} \) is the effective stress tensor, \( \sigma_{ij} \) is the total stress tensor, \( \delta_{ij} \) is the Kronecker delta, \( p_{pf} \) is the pore fluid pressure, and \( \alpha \) is Biot’s poroelasticity parameter.

The main objective of this study is to present more rigorous and physically based model for stress dependent seismic velocities of rocks. Although based on nonlinear elasticity, the model is much simpler than models based on idealized models of the rock microstructure. The stress dependency in seismic velocities is attributed to the presence of micro-cracks and fractures in rocks with reduced normal and shear stiffnesses in comparison to that of the host rock. Stress-induced deformation of the micro discontinuities contributes to the nonlinear stress-strain response of rocks and faster propagation of seismic waves across the rock. The resulting stress-dependent models are much simpler and fit experimental data well and using fitting parameters that difficult to assign physical meanings.

### 3.2 Model for Fractured Rock Mass Deformation

The proposed model for stress dependent seismic velocity assumes that the stress dependencies come from the presences of micro-cracks or micro-fractures in the rocks mass based on the idealized geometry shown in Figure 3.1. For simplicity, the following assumptions are made in the derivation of the stress dependent seismic velocity models: (1) The principal stresses \( \sigma_1 \) and \( \sigma_3 \) are parallel to the reference axes \( x_V \) and \( x_H \), respectively. (2) The intact rock is isotropic. (3) The fracture lengths and spacings are much smaller than the volume of the rock mass being considered. (4) All fractures have very similar geometries and are uniformly distributed in the rock mass. (5) The rock mass has been under an isotropic stress condition in prior to the loading. (6) The deformation of rock mass in the positive direction of the \( x_H \) axis, i.e., the expansion in the lateral direction, is not allowed during vertical loading (oedometric loading condition). (7) The vertical deformation of rock mass is caused by a vertical stress increment sufficiently small compared to the initial isotropic stress. In addition to the above assumptions, it is postulated that the total deformation of the rock mass can be considered as the summation of the deformation of the rock matrix and the fracture deformation.
In terms of volumetric strains, this additively postulate can be written as:

\[ dv = dv_m + dv_f \]  

where \( dv \) is the volumetric strain increment of fractured rock mass, \( dv_m \) is the volumetric strain increment due to deformation of the rock matrix having no fractures, and \( dv_f \) is the volumetric strain increment due to deformation of the fractures.

Assuming oedometric compression condition, in which no lateral deformation is allowed, for a given stress increment \( d\sigma' \), the fractured rock mass volume change \( dv \), and the rock matrix volume \( dv_m \) change can be calculated as:

\[ dv = \frac{d\sigma'}{M}, \quad dv_m = \frac{d\sigma'}{M_m} \]  

where \( M \) is the constrained modulus of the fractured rock mass and \( M_m \) is the constrained modulus of the rock matrix or the rock mass in which all fractures are closed completely.

The volumetric strain due to fracture deformation requires a bit more careful consideration. As can be seen in Figure 3.1, during loading of the rock mass, the fractures are expected to undergo two types of deformations: normal deformation \( du_n \) (which can either be closure or dilation) and shear deformation \( du_s \).

For elastic loading of the fractures, shear induced dilation will not occur and only normal stress induced fracture closure or openings is expected. It is experimentally observed that shear...
deformations of fractures increase elastically without dilation when the shear displacement of fracture is less than 30% of displacement value observed at the peak of shear stress. Thus, the fracture deformations $du_n$ and $du_s$ can be directly related, respectively, to the normal and shear stress changes occurring across and along a fracture:

$$du_n = \frac{d\sigma'_n}{k_n}, \quad du_s = \frac{d\sigma_s}{k_s}$$  \hspace{1cm} (3.7)

where $d\sigma'_n$ is the effective normal stress increment across a fracture, $d\sigma_s$ is the shear stress increment across a fracture, $k_n$ is the fracture normal stiffness, and $k_s$ is the fracture shear stiffness.

In other words due to elastic response and lack of dilation, the normal and shear displacements are uncoupled from each other. Uncoupling the elastic normal and shear deformations of rock fracture as the components independently responding to normal and shear stress changes is previously proposed by Goodman’s (1974) hyperbolic model. This formulation is based on the widely-accepted concept of "deformable" fractures, which respond to the normal, and shear stress changes in an elastic manner even if no sliding of fracture occurs. Uncoupling the normal and shear deformations should be valid when the fracture shear deformations remain in the elastic range given as 30% of the shear displacement at peak. The volumetric strain of the rock mass attributed to the fracture closure/opening can be obtained as:

$$dv_f = \frac{1}{V} \sum_{j} N \cdot a_j \cdot du_{nj} \cos \theta_j$$  \hspace{1cm} (3.8)

where $N$ is the number of fractures distributed in rock mass volume $V$, and $a_j$, $du_{nj}$, and $\theta_j$ are the area of fracture, the normal deformation, and the orientation angle from of the $x_H$ axis of $j$-th fracture. Assuming that all fractures have very similar geometries, Eq. (3.8) can be rewritten approximately as:

$$dv_f \approx \frac{N}{V} \frac{\bar{a}}{\bar{\bar{a}}_n} \cdot d\bar{\bar{a}}_n \cos \bar{\theta} = F \cdot \bar{a} \cdot d\bar{a_n} \cos \bar{\theta} = \frac{1}{s} d\bar{a}_n$$  \hspace{1cm} (3.9)

where $\bar{a}$ and $d\bar{a}_n$ are the averages of all fracture areas and normal fracture deformations; and $F = N/V$ is the fracture frequency (i.e. the number of fractures per unit rock mass volume). $\bar{\theta}$ is the average orientation angle of fractures $\theta_j$ in which the angles exceeding $90^\circ$ are converted to be $180^\circ - \theta_j$. $s$ having the dimension of length is the average fracture spacing defined as:
\[ s = \frac{V}{N \bar{a} \cos \theta} \]  

Eq. (3.10) indicates that the value of \( s \) becomes shorter as the average fracture area increases and the average orientation angle decreases, as well as the number of fractures increases.

The definition of \( s \) is considered to be reasonable from the following insights of deformation mechanism: (1) The deformation attributed to closure of some fractures having larger average area can be substituted by the summation of deformations of the greater number of smaller fractures spaced at shorter intervals. (2) The deformation in \( x_V \)-axis of a rock mass having fractures inclined from the \( x_H \)-axis at the higher average orientation angle should be equivalent to the deformation of rock mass having fractures inclined at the smaller average orientation angle and spaced at the longer intervals. For a given fracture whose normal is oriented at an angle \( \theta \) from the \( x_V \)-axis, the normal stress across a fracture can be calculated as:

\[ \sigma'_{nj} = p' + \frac{q}{2} \cos 2\theta_j \]  

where \( q \) is the shear stress given by the principal stresses \( \sigma_1 \) and \( \sigma_3 \) as:

\[ q = \frac{1}{2}(\sigma_1 - \sigma_3) \]  

In the oedometric loading condition, the increment of effective horizontal stress is given by:

\[ d\sigma'_H = \frac{v}{1-v} d\sigma'_V = K_0 d\sigma'_V \]  

where \( v \) is Poisson’s ratio and \( K_0 \) is called coefficient of earth pressure at the rest.

The mean stress \( p \) can be calculated from Eq. (3.4) in 2D as follow:

\[ p = \frac{1}{2} \left( \delta_{ij} p_{pf} \right) = \frac{1}{2} \left( \sigma_{11} - \sigma_{22} \right) + \frac{1}{2} \left( \sigma_{22} - \sigma_{11} \right) = \sigma_v + \sigma_H \]  

where \( p_{pf} \) for dry rock is zero.

Substitute Eq. (3.13) in (3.14) leads to:

\[ p = \frac{d\sigma_v + K_0 d\sigma_v}{2} = \left( \frac{1 + K_0}{2} \right) d\sigma_v \]  

The shear stress can be then expressed as:

\[ q = \left( \frac{1 - K_0}{2} \right) d\sigma_v \]
The normal stress acting on the \( j \)-th fracture is:

\[
d\sigma_n' = \left[ \frac{1 + K_0}{2} - \frac{1 - K_0}{2} \cos 2\theta_j \right] d\sigma_v'
\]  
(3.17)

For the micro-fractures distributed uniformly as stated in the assumption (4), the average of normal stress increment acting on fractures can be represented by:

\[
d\sigma_n' = \frac{1 + K_0}{2} d\sigma_v'
\]  
(3.18)

because of \( \bar{\theta} = 45^\circ \). Combining Equations (3.7) and (3.9) yields the following equation for the volumetric strain from the fracture normal deformations:

\[
dv_v \approx F \cdot \bar{a} \frac{d\sigma_v'}{k_n \cos \bar{\theta}} = \frac{(1 + K_0)}{2} \frac{d\sigma_v'}{s \cdot k_n}
\]  
(3.19)

Substituting Equations (3.8) and (3.21) into Equation (3.7) yield to:

\[
\frac{d\sigma_v'}{M} = \frac{d\sigma_v'}{M_m} + \frac{(1 + K_0)}{2} \frac{d\sigma_v'}{s \cdot k_n}
\]  
(3.20)

Equation (3.17) assumes that the overall vertical stress change \( d\sigma_v' \) is the same in the rock matrix, fractures, and the fractured rock mass. From this equation, the fractured rock mass longitudinal modulus can be solved as:

\[
M = \frac{2(1 + K_0)}{M_m + 2(1 + K_0)^{-1} \cdot s \cdot k_n} \cdot M_m
\]  
(3.21)

Following the same procedure for elastic shear deformation caused by the shear stress increment \( dq \), the fractured rock mass shear modulus \( G \) can be derived as:

\[
G = \frac{s \cdot k_s}{G_m + s \cdot k_s} \cdot G_m
\]  
(3.22)

where \( G_m \) is the shear modulus of the rock matrix or the rock mass in which all fractures are closed completely.

### 3.3 Nonlinear Fracture Deformation

Extensive experimental data indicate that the load-deformation behavior of individual fractures is nonlinear. For instance, it is well known than during normal loading, fracture closure \( du_n \) is nonlinearly related to the normal stress \( \sigma_n \) acting across the fracture. One of the most
commonly used relations to model the normal stress induced closure of fracture is the hyperbolic law. An example is Goodman’s (1974) hyperbolic model function (Goodman, 1974) expressed as:

\[ u_n = \frac{\sigma'_n}{k_{ni}} \left( 1 - \frac{\sigma'_n}{\sigma'_n} \right) \]  \hspace{1cm} (3.23)

where \( k_{ni} \) is the normal stiffness corresponding to the initial effective normal stress \( \sigma'_n \). The hyperbolic law reflects the concave up plot of normal displacement vs. normal stress behavior of rock fractures. This concave up response is due to the fact as the fracture closes, the fracture locks and becomes stiffer and more difficult to close.

Differentiating Eq. (3.23) with respect to the normal stress \( \sigma'_n \) and taking the reciprocal yields the following nonlinear tangential normal stiffness:

\[ k_n = k_{ni} \left( \frac{\sigma'_n}{\sigma'_n} \right)^2 \]  \hspace{1cm} (3.24)

A comparison of Eq. (3.24) against extensive measurements of the normal compliance \( k_n^{-1} \) obtained by (Malama and Kulatilake, 2003) on macroscopic fractures of granodiorite are shown in Figure 3.2.

As can be seen, the hyperbolic law has one fundamental defect, that it gives almost zero normal fracture displacement, and correspondingly, quite small normal compliance \( k_n^{-1} \), even at \( \sigma'_n = 2 \) MPa. In other words, the hyperbolic law yields too large normal stiffness values at large Normal stresses. As a result, Eq. (3.26) does not match the data shown in Figure 3.2. To remedy the deficiency of the hyperbolic function model, a more general version of Eq. (3.24) is proposed using the power function (Appendix A):

\[ k_n = k_{ni} \left( \frac{\sigma'_n}{\sigma'_n} \right)^n \]  \hspace{1cm} (3.25)

where \( n \) is the empirical parameter. Comparison with experimental data in Figure 3.2 shows that Eq. (3.25) provides the better representation of the normal stress dependency of the fracture normal stiffness, the value of the power low model \( n \) obtained from the curve fitting is \( n=0.64875 \).
Figure 3.2 Comparison of the proposed power law model and Goodman’s Hyperbolic model with Malama and Kulatilake (2003) experimental data for a Granodiorite jointed rock tested under compressive loading.

In contrast to the fracture normal stress vs. normal displacement response, the relationship between fracture shear stress and shear displacement is generally linear at least before the fracture slips and plastic shear displacement occurs. The linear shear stress–displacement relationship is supported by many experimental studies. However, the shear stiffness of fractures is strongly affected by the normal stress. This is because as the normal stress increases, the fracture locks and it gets more difficult to shear the fracture. Based on an extensive study of experimental data, the following linear relationship was found to adequately represent the effects of normal stress on the shear stiffness of rock fractures:

$$k_s = k_{si} + k_{sn} \left( \frac{\sigma_n'}{\sigma_{ni}} - 1 \right)$$

(3.26)

where $k_{si}$ is the initial shear stiffness corresponding to zero normal stress and $k_{sn}$ is the slope of the $k_s$ vs. $\sigma_n'$ plot.
Parameter value of \( k_s \) is equal or greater than that of \( k_{sn} \). The fracture shear stiffness value at zero effective normal stress is given by \( k_u - k_{sn} \). For most fractures \( k_u - k_{sn} = 0 \) is assumable, but some fractures exhibit “lock in” behavior with some shear stiffness even at zero normal stress. Note that same parameter \( \sigma_{ni} \) is used to normalize \( \sigma_n \) in Eqs. (3.25) and (3.26), respectively to ensure that the units of normal and shear stiffnesses are independent of the unit of the stress used. Figure 3.3 compares Eq. (3.26) against experimental data for macroscopic fractures of oolitic limestone different in stress conditions (Pyrak-Nolte et al., 1996).

![Figure 3.3](image)

Figure 3.3 Normal stress dependency of shear stiffness with or without the presence of shear stress observed by (Pyrak-Nolte et al., 1996) experimental data and fitting result of proposed modul.

### 3.4 Stress Dependent Seismic Velocities Model

To propose the new model, the rock mass is assumed to have microscopic fractures randomly oriented. In addition, the characteristic length of micro-fractures is assumed sufficiently short compared to the rock mass dimensions. The macroscopic stress-dependent elastic wave velocity response is assumed to be attributed to the stress dependency of fracture
stiffness. The stress-dependent fracture normal stiffness is defined as a generalized power law function of effective normal stress, which is a modification of the Goodman’s model. On the other hand, the stress dependency of fracture shear stiffness is modeled as a linear function of normal stress based on experimental data.

3.4.1 Model of \( V_p \) and \( V_s \)

Substituting the fractured rock mass bulk and shear moduli \( K \) and \( G \) in Eq. (1) yields the following expressions for stress dependent P- and S-wave velocities.

\[
V_p = V_{pm} \sqrt{\frac{f_n(p'_{nor})}{1 + f_n(p'_{nor})}}, \quad f_n(p'_{nor}) = \frac{2}{1 + K_0} \frac{s \cdot k_n}{M_m} = \frac{2}{1 + K_0} \frac{s \cdot k_n p'_{nor}}{M_m}
\]

\[
V_s = V_{sm} \sqrt{\frac{f_s(p'_{nor})}{1 + f_s(p'_{nor})}}, \quad f_s(p'_{nor}) = \frac{s \cdot k_s}{G_m} = \frac{s[k_{si} + k_{sn}(p'_{nor} - 1)]}{G_m}
\]

where \( p'_{nor} \) is the effective mean stress normalized by the initial effective stress \( p'_i \), \( V_{pm} = \sqrt{M_m/\rho} \) and \( V_{sm} = \sqrt{G_m/\rho} \) are the seismic P and S-wave velocities of the intact and unfractured rock matrix.

The ratio of longitudinal modulus of fracture and that of rock matrix \( f_n \) can be termed fracture-matrix normal stiffness ratio. The ratio of shear component \( f_s \) is also termed fracture-matrix shear stiffness ratio. The values of \( f_n \) and \( f_s \) increase with \( p' \). Thus, as the effective stress increases, the values of \( V_p \) and \( V_s \) increase and approach the respective velocities of rock matrix \( V_{pm} \) and \( V_{sm} \). Figure 3.4 shows the relationships between the longitudinal modulus normalized by the rock matrix modulus \( M/M_m \) and the fracture-matrix longitudinal modulus ratio \( f_n \), and between the normalized shear modulus \( G/G_m \) and the fracture-matrix shear modulus ratio \( f_s \), which are given by:

\[
\frac{M}{M_m} = \frac{f_n}{1 + f_n}, \quad \frac{G}{G_m} = \frac{f_s}{1 + f_s}
\]

The \( M/M_m \) and \( G/G_m \) may be termed fractured-intact modulus ratio.

In the similar forms, fractured-intact velocity ratios can be given as:
\[
\frac{V_p}{V_{pm}} = \sqrt{\frac{M}{M_m}} = \sqrt{\frac{f_n}{1 + f_n}}, \quad \frac{V_s}{V_{sm}} = \sqrt{\frac{G}{G_m}} = \sqrt{\frac{f_s}{1 + f_s}}
\] (3.30)

It can be seen from Figure 3.4 that the values of the fractured-intact modulus ratio \( M/M_m \) and \( G/G_m \) reach 0.5 when the values of the fracture-matrix stiffness ratios \( f_n \) and \( f_s \) equal 1.0 respectively. The values of the fractured rock moduli \( M \) and \( G \) are seen to asymptotically approach the values of the respective intact moduli \( M_m \) and \( G_m \) as the values of fracture-matrix stiffness ratios increase. In the following discussion, the prime symbols of stress components denoting the effective stress will be abbreviated, because only dry condition will be discussed.

![Figure 3.4 Relationships between fractured-intact modulus ratios and fractured-intact stiffness ratios in longitudinal and shear modes.](image)

**3.4.2 Sensitivity of Stress Dependency of Elastic Moduli to Model Parameters**

The stress dependencies of \( M \) and \( V_p \) are characterized by \( k_{ni} \) and the exponential parameter \( n \) as shown in Eq. (3.29). The sensitivity of stress dependency of longitudinal modulus \( M \) to the parameter \( n \) is shown in Figure 3.5. The curves shown are computed by using Eq. (3.31) at
different values of $n$, which are 0.2, 0.4, 0.7, 1.0, and 2.0. The sets of other parameter values $s$, $k_{ni}$, and $M_m$ used are identical and have a relationship given by $2(1 + K_o)^{-1} s \cdot k_{ni}/M_m = 0.1$.

Figure 3.5 Sensitivity of stress dependency of fractured-intact longitudinal modulus ratio to material parameter $n$.

The parameter $2(1 + K_o)^{-1} s \cdot k_{ni}/M_m = 0.1$ defined as the ratio of the product of fracture spacing and initial fracture normal stiffness to matrix longitudinal modulus can be termed initial fracture-matrix normal stiffness ratio. Figure 3.5 indicates that the higher value of $n$ characterizes the stronger stress dependency of $M$ defined by the tangential slope of $M/M_m - p/p_i$ curve. However, this tendency is true only in the lower stress range. For instance, the tangential slope of the curve computed at $n = 2.0$ becomes smaller than that of the curve obtained at $n = 1.0$ when $p/p_i$ is higher than 10. When the Goodman’s model given by $n = 2.0$ is assumed for the normal contact stiffness, the value of normalized longitudinal modulus $M/M_m$ increases rapidly and reaches approximately 0.91 when the normalized effective stress $p/p_i$ is equal to 10. When
assuming a linear stress dependency for normal fracture stiffness characterized by $n = 1.0$, the value of the fractured longitudinal modulus reaches the half of the matrix modulus $M_m$ at $p'_{nor} = 10$. The sensitivity of stress dependency of the fractured-intact shear modulus ratio to the material parameter $k_{sn}$ is shown in Figure 3.6. The stress dependent shear fracture stiffness given in Eq. (3.28) can be written as:

$$\frac{k_s}{k_{si}} = 1 + \frac{k_{sn}}{k_{si}} (p'_{nor} - 1)$$

This non-dimensional relationship indicates that the rate of increase in fracture shear stiffness from the initial value due to stress change can be characterized by using the normalized parameter $k_{sn}/k_s$. The curves shown in the figure were calculated by using different values $1.0 \times 10^{-4}$, $1.0 \times 10^{-2}$, $5.0 \times 10^{-2}$, $2.0 \times 10^{-1}$, and $1.0$ for $k_{sn}/k_s$. The stress dependency of $G/G_m$ at lower stress level becomes stronger with increasing $k_{sn}/k_s$.

![Figure 3.6 Sensitivity of stress dependency of fractured intact shear modulus ratio to material parameter $k_{sn}$](image)

Figure 3.6 Sensitivity of stress dependency of fractured intact shear modulus ratio to material parameter $k_{sn}$. 
3.5 Model Validation and Experimental Studies

A model for stress-dependent elastic wave velocity response of fractured rock mass has been developed based on experimental evidence of stress-dependent fracture stiffness. Previously proposed models and previous experimental studies on stress-dependent fracture stiffness have been reviewed to propose the new model in a realistic yet simple form. The material parameters used in some empirical models widely used for stress-dependent elastic wave velocity response do not have clear linkages between physical properties of rock materials. Thus, the proposal model is validated by a published data and observed experimental data using curve fitting technique.

3.5.1 Validation of Stress Dependent Fracture Stiffness Model

In the proposed model, the stress dependency of fracture normal stiffness is formulated as Eq. (3.25). The equation is fitted to the experimental data of normal compliance depending on normal stress (Malama and Kulatilake, 2003) and the result is shown in Figure 3.2. It can be seen that the model curve is in a good agreement with the observed data. Compared to the Goodman’s hyperbolic model, Eq. (3.25) provides the better representation of the normal stress dependency of the fracture normal stiffness. The values of material parameters $k_{ni}$ and $n$ determined by the fitting are 2.56 GPa/m at $\sigma'_n = 0.236$ MPa and 0.801, respectively. Figure 3 compares the stress dependent shear stiffness model Eq. (3.26) against the experimental data for macroscopic fractures of oolitic limestone under different stress conditions (Pyrak-Nolte et al., 1996). The model has been fitted to the observed data. The stress dependency of $k_s$ observed at no shear stress condition, i.e., $\theta = 0^\circ$, is well described by the model. The values of material parameters determined for the data are shown in Table 3.1.

<table>
<thead>
<tr>
<th>$\theta$ (deg)</th>
<th>$\sigma'_n$ (Pa)</th>
<th>$k_{ni}$ (Pa/m)</th>
<th>$k_{sn}$ (Pa/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1.00 \times 10^6$</td>
<td>$1.21 \times 10^{13}$</td>
<td>$3.27 \times 10^{12}$</td>
</tr>
<tr>
<td>52.5</td>
<td>$1.00 \times 10^6$</td>
<td>$1.55 \times 10^{13}$</td>
<td>$4.59 \times 10^{12}$</td>
</tr>
</tbody>
</table>
The experimental data shown in Figure 3.3 indicate that the fracture shear stiffness becomes more sensitive to normal stress change when the shear stress exists on fracture joint surfaces. On the other hand, the proposed model Eq. (3.26) originally does not evaluate the effect of shear stress acting on fracture on shear stiffness. Nevertheless, it is interestingly observed that the proposed model sufficiently describes the stress dependency even when there exists the shear stresses on the fracture joint surface, i.e., $\theta = 52.5^\circ$. The value of material parameter $k_{sn}$ determined for $\theta = 52.5^\circ$ is $4.59 \times 10^{12}$ Pa/m, which is higher than that determined for $\theta = 0^\circ$ that is $3.27 \times 10^{12}$ Pa/m. These things suggest that the material parameter $k_{sn}$ in Eq. (3.26), which has been originally introduced to evaluate only the effect of normal stress on fracture shear stiffness, is capable of characterizing even the effect of shear stress generated on inclined fracture surface on fracture shear stiffness. One of the main reasons of this unexpected applicability is likely to be attributed to the fixed ratio of shear stress to normal stress arising from the loading on the fracture inclined at the fixed orientation angle.

3.5.2 Validation of Stress-Dependent Elastic Wave Velocity Model Against Published Data

Figure 3.7 compares experimental data of $V_p$ and $V_s$ of a dry core sample of Barre granite (Coyner, 1984) with the proposed model.
The material parameter values determined by fitting Eqs. (3.27) and (3.28) to the observed data are shown in Table 3.2. The bulk density of dry core is 2.65 g/cm$^3$. The S-wav velocity of intact rock matrix is determined to be $3.70 \times 10^3$ m/s and is comparable with typical high-pressure S-wave velocities of granite samples.

The value of intact P-wave velocity $V_{pm}$ determined to be $1.07 \times 10^4$ m/s is likely to be higher than typical values of higher pressure P-wave velocity of granite. However, the model is capable of describing the stress dependent wave velocities with 0.911 and 0.978 of the coefficients of determination $R^2$ for $V_p$ and $V_s$, respectively.

Table 3.2 Material Parameter Values for Dry Barre Granite

<table>
<thead>
<tr>
<th>$M_m$ (Pa)</th>
<th>$G_m$ (Pa)</th>
<th>$p'_i$ (Pa)</th>
<th>$2(1 + K_0)^{-1}s \cdot k_{ai}$ (Pa)</th>
<th>$s \cdot k_{si}$ (Pa)</th>
<th>$s \cdot k_{so}$ (Pa)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.0 \times 10^{11}$</td>
<td>$3.60 \times 10^{10}$</td>
<td>$6.30 \times 10^5$</td>
<td>$3.33 \times 10^{10}$</td>
<td>$4.5 \times 10^{10}$</td>
<td>$1.3 \times 10^9$</td>
<td>0.237</td>
</tr>
</tbody>
</table>

3.5.3 Experimental Validation of the Stress-Dependent Elastic Wave Velocity Mode – Laboratory Setup and Testing Procedures

The experimental setup used for the ultrasonic wave velocity measurement on a dry core of Berea sandstone has been illustrates in chapter 2 Figure 2.1. Figure 3.8 illustrate the schematic diagram of the High-pressure High-temperature triaxial core holder. The cylindrical core sample whose diameter is 38.1 mm and length is 70.1 mm is pressurized in a triaxial core holder having 70 MPa of pressure capacity. The lateral face of core is covered by Viton rubber sleeve. The endcaps of core holder are equipped with piezoelectric transducers having natural frequencies ranging from 0.25 to 1 MHz. The ultrasonic waves generated by exciting the transducers with square pulses at one end propagate the core and are received at another end transducers. The wave signals are monitored by using a 8-bit digital oscilloscope whose bandwidth and real time sample rate are 100 MHz and $1 \times 10^9$ s$^{-1}$.

Single waveform acquired in a personal computer communicating with the oscilloscope is average of 16 waveforms. The ultrasonic wave velocity measurement has been done at 313 K of the temperature and the effective stresses ranging from 2 to 55 MPa.
3.5.4 Visual Micro-Crack Observations and Stress Dependent Ultrasonic Wave Velocity Measurement

The core sample used was obtained by coring a block of Berea sandstone in the vertical direction of bedding plane. The dry density of the core sample used for ultrasonic wave velocity measurement is 2.215 g/cm$^3$. Figures 3.9a and 3.9c show thin section images obtained from different parts of the Berea sandstone. Traced drawings of micro-fractures observed in the thin sections Figure 3.9a and 3.9c are indicated in Figure 3.9b and 3.9d, respectively. The fracture patterns appear to be similar for the different thin sections. The orientations of the micro-fractures are likely to have no predominant direction, i.e., randomly oriented. In the thin section shown in Figure 3.9c, it is found that the average orientation angle of total 60 micro-fractures traced as shown in Figure 3.9d is 50° from the vertical axis of the figure. This value evidences the random orientation of micro-fractures. In addition, it is seen that typical scale of the micro-fractures and cracks should be several hundred microns or less, and sufficiently smaller than the
rock core dimensions. The average length of 60 micro-fractures seen in the thin section of Figure 3.9c is found to be approximately $2.1 \times 10^2 \mu m$. These visual observations confirm that the microstructure of the rock mass corresponds to what was assumed in the development of the stress-dependent elastic wave velocity model shown in Figure 1.

![Visual observation results of thin sections of Berea sandstone sample indicating presence of randomly-oriented micro-fractures.](image)

Figure 3.9 Visual observation results of thin sections of Berea sandstone sample indicating presence of randomly-oriented micro-fractures.

Figure 3.10 shows the stress dependencies of the P- and S-wave velocities of the dry Berea sandstone core. The P- and S-wave velocities increase from 2.409 to 4.144 km/s and 1.710 to 2.627 km/s as the effective stress increases from 2 to 55 MPa, respectively. The material parameter values determined by fitting Eqs. (3.29) and (3.30) to the observed data are shown in Table 3.3. The higher tangential slopes of $V_p-p$ and $V_s-p$ curves are observed at lower effective stress. The stress dependencies of $V_p$ and $V_s$ become stronger at lower stress. It is seen that the stress dependency of P-wave velocity is stronger than that of S-wave.
3.5.5 Validation of Stress-Dependent Elastic Wave Velocity Model

Table 3.3 shows the material parameter values determined for the dry Berea sandstone by fitting the proposed model to experimental data shown in Figure 3.10. The best fitting curves are shown in Figure 3.10 by using solid curves. The $R^2$-values for the curve-fitted $V_p$ and $V_s$ functions are 0.988 and 0.997, respectively. The intact rock matrix velocities $V_{pm}$ and $V_{sm}$ are estimated to be 4.75 and 2.81 km/s, respectively. The values of $V_p$ and $V_s$ of another dry core of Berea sandstone observed at 100 MPa by (Coyner, 1984) are 4.1 and 2.7 km/s.

**Table 3.3 Material Parameter Values for Dry Berea Sandstone Core**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_m$ (Pa)</td>
<td>$5.00 \times 10^{10}$</td>
</tr>
<tr>
<td>$G_m$ (Pa)</td>
<td>$1.75 \times 10^{10}$</td>
</tr>
<tr>
<td>$p'_i$ (Pa)</td>
<td>$1.00 \times 10^6$</td>
</tr>
<tr>
<td>$2(1+K_o)^{-1} s' k_{u_i}$ (Pa)</td>
<td>$1.22 \times 10^{10}$</td>
</tr>
<tr>
<td>$s' k_{si}$ (Pa)</td>
<td>$1.02 \times 10^{10}$</td>
</tr>
<tr>
<td>$s' k_{sn}$ (Pa)</td>
<td>$2.55 \times 10^9$</td>
</tr>
<tr>
<td>$n$</td>
<td>0.642</td>
</tr>
</tbody>
</table>
The value of estimated $V_{pm}$ seems to be overestimated. However, the $V_p$ in the Coyner’s observation continues to increase with approximately 3.9 m/(s·MPa) of the increasing rate of $V_p$ even when the effective stress increases from 90 to 100 MPa. Therefore, the comparable value of $V_p$ may be observed at the higher stresses. On the other hand, the estimated $V_{sm}$ is comparable with the $V_s$ by Coyner at $p = 100$ MPa. The value of exponential parameter $n$ determined to be 0.642 is higher than that of Barre granite. The higher value of $n$ is attributed to the higher stress dependency of $V_p$ at the lower stress range. The value of $s \cdot k_{sn}$ determined for the Berea sandstone $2.55 \times 10^9$ is approximately twice as much as that determined for Barre granite. The difference in $s \cdot k_{sn}$ reflects that the $V_s$ of the Berea sandstone is more sensitive to the stress change compared to that of Barre granite.

Figure 3.11 compares the experimental data and the computed curve of $V_p/V_s$ for the dry Berea sandstone core sample. The ratio $V_p/V_s$ is useful index as lithology discriminator. The $V_p/V_s$ is also used as an indicator of the existence of gas phase in pore space. The observed $V_p/V_s$ values slightly increase from 1.43 to 1.56 with increasing of effective stress from 2 to 55 MPa. It can be seen that the proposed model sufficiently describes the stress dependency of $V_p/V_s$ the $V_p/V_s$–$p$ relationship can be given from Eqs. (3.29) and (3.30) as:

$$
\frac{V_p}{V_s} = \frac{V_{pm}}{V_{sm}} \sqrt{1 + f_s^{-1}}
$$

(3.34)

The values of $f_n^{-1}$ and $f_s^{-1}$ in the right side of Eq. (3.34) approach zero as the effective stress increases, and therefore the value of $V_p/V_s$ approaches the velocity ratio of intact state $V_{pm}/V_{sm}$. For the dry Berea sandstone core used, the value of $V_{pm}/V_{sm}$ calculated by using the parameter values shown in Table 3.3 is 1.69. This value is within a range of $V_p/V_s$ value from 1.6 to 1.75 proposed by (Pickett GR. 1963) for sandstone. If the lithology of rock mass is known, the value of $V_{pm}/V_{sm}$ may be estimated. The estimated $V_{pm}/V_{sm}$ should be useful to determine the material parameters for the proposed model or to verify appropriateness of the material parameter values determined by fitting.
Interestingly, some material parameter values determined by fitting the model to \( V_p \) and \( V_s \) of the dry Berea sandstone are comparable with the parameter values determined based on directly-measured stiffnesses of macroscopic fractures, although the lithologies are different. The value of \( n \) determined for the micro-fractures the dry Berea sandstone that is 0.642 is comparable with that determined for the macroscopic fractures of the granodiorite shown in Figure 3.2 that is 0.801. The value of non-dimensional parameter \( k_{sn}/k_{si} \) characterizing stress dependency of shear stiffness of the micro-fractures of the Berea sandstone is 0.251. This \( k_{sn}/k_{si} \) value is comparable with that determined for the macroscopic fractures of the oolitic limestone shown in Figure 3.3 that is 0.269 when the fracture orientation angle \( \theta = 0^\circ \).

The spacing of micro-fractures \( s \) in the dry Berea sandstone may be able to be roughly evaluated to be 30 \( \mu \)m from Figures 3.9(c) and 3.9(d). This value is reasonable based on a calculation using Eq. (12). For the value of \( s \), 32 \( \mu \)m is obtained by using Eq. (12) with the following parameter values obtained from the image analysis on Figures 3.9(c) and 9(d), \( V = 2.61 \times 10^5 \ \mu \)m\(^3 \), \( \bar{a} = 2.1 \times 10^2 \ \mu \)m\(^2 \), \( \bar{\theta} = 50^\circ \), and \( N = 60 \). The depths of the section and micro-fractures are assumed to be identical and 1 \( \mu \)m. By using the estimated spacing value 30 \( \mu \)m, the

![Figure 3.11 Stress dependency of ratio of P-wave velocity to S-wave velocity](image-url)
values of $k_{si}$ and $k_{sn}$ for the micro-fractures in the dry Berea sandstone can be estimated to be $3.4 \times 10^{14}$ and $8.5 \times 10^{13}$ Pa/m, respectively. These values are approximately 20 times higher than those determined for the macroscopic fractures in the oolitic limestone shown in Table 3.1. The differences of $k_{si}$ and $k_{sn}$ between the micro- and macro-fractures seem to be reasonable because the initial degree of locking of micro-fractures should be much higher than that of macroscopic fractures and also the stress-induced locking of the micro-fractures should be mobilized with the smaller stress change.

The effect of initial normal stiffness of micro-fractures on the longitudinal stiffness is characterized in the form of $2(1+K_o)^{-1}s \cdot k_{n}$. The parameter value of $2(1+K_o)^{-1}s \cdot k_{n}$ was determined by fitting Eq. (3.29) to the observed data without knowing the value of $s$ and the Poisson’s ratio. Typical values of static Poisson’s ratio of Berea sandstone under drained condition seem to be in a range of 0.12 to 0.23 (Hart DJ, Wang HF 1995). Assuming $\nu = 0.20$ for the Poisson’s ratio of the Berea sandstone core tested under dry condition in this study, the value of $K_0$ can be estimated to be 0.25. Substituting $s = 30\mu m$ as well as $K_0 = 0.25$ into $2(1+K_o)^{-1}s \cdot k_{n} = 1.22 \times 10^{10}$ Pa gives $2.5 \times 10^{14}$ Pa/m as the value of $k_{n}$ at $p'_n = 1$MPa. For the granodiorite whose normal stiffness data is shown in Fig. 2, the value of $k_{n}$ is $1 \times 10^{10}$ Pa/m at $\sigma'_n = 1$MPa. The higher value of initial normal stiffness of micro-fractures of the Berea sandstone core compared to that of the macro-fracture of granodiorite sample can be justified from the very high degrees of interlocking and matching for the micro-fractures observed without external stress (Figure 3.9).

Compared to the other material parameters, the physical meanings of the intact moduli $M_m$ and $G_m$ are obvious and clear in macroscopic view, and thus these parameters are more determinable and appropriateness of determined parameter values can be easily checked. On the other hand, measuring directly the normal and shear stiffness of micro-fractures is difficult. Thus, further studies are necessary to relate the material parameters charactering stress dependency of micro-fracture stiffness $k_{ni}$, $n$, $k_{si}$, and $k_{sn}$ with material and physical properties of fractured rock mass. As the relationships between the material parameters and rock properties are becoming clear, the material parameter values can be determined based on smaller number of data points or may be indirectly determinable by using, for example, lithology data.
3.6 Summary

The proposed model for stress dependent seismic wave velocity has been found to be capable of describing the observed stress dependency of P- and S-wave velocities of Barre granite with sufficient accuracy. Ultrasonic wave velocity response of a dry core sample of Berea sandstone has been tested at effective stresses ranging from 2 to 55 MPa. Visual observation of thin sections obtained from the Berea sandstone confirms that the assumptions made for microstructure of rock mass model are appropriate. It is shown that the model can describe the stress-dependent ultrasonic wave velocity response of the dry Berea sandstone with a set of reasonable material parameter values.

Appropriateness of the material parameter values determined for the dry Berea sandstone by fitting the model to the observed relationship between wave velocity and effective stress has been discussed. It is shown that the parameter values characterizing the stress dependency of micro-fracture stiffnesses of dry Berea sandstone are comparable with those of the macroscopic fractures of other rock materials determined based on direct measurements of stress-dependent fracture stiffnesses. These comparable values suggests that the material parameter values determined by fitting the model to wave velocity data may be reasonable, although rigorous investigation is not available.
3.7 References


CHAPTER 4
EFFECTS OF CO$_2$ INJECTION ON THE SEISMIC VELOCITY OF SANDSTONE SATURATED WITH SALINE WATER

4.1 Introduction

There has been increasing evidence that anthropogenic release of carbon dioxide (CO$_2$) in the atmosphere has been one of the main causes of changes in global weather patterns. One major solution to reduce the further release of CO$_2$ in the atmosphere from human activities is geological sequestration (GS) whereby carbon dioxide (CO$_2$) is injected and permanently stored in underground geological formations. The estimated volume of potential sequestration reservoirs is abundant (IPCC, 2005; Benson, 2008). Depleted oil and gas reservoirs and abandoned mines are important candidates for CO$_2$ storage. Deep saline aquifers are also very promising because of their huge estimated capacities and high connectivity of pore spaces (Bachu, 2003). CO$_2$ injected in geological formations is trapped due to different mechanisms, including stratigraphic and structural trapping, residual CO$_2$ trapping, solubility trapping, and mineral trapping (IPCC, 2005). The advantageous in of supercritical CO$_2$ is used to displace CO$_2$ in situ saline water efficiently.

To examine the use of time-lapsed seismic survey mapping as a monitoring tool for CO$_2$ sequestration, this chapter presents mathematical and experimental studies of the effects of supercritical CO$_2$ injection on the seismic velocity of sandstone initially saturated with saline water. The mathematical model is based on poroelasticity theory, particularly the application of the Biot-Gassmann substitution theory in the modeling of the acoustic velocity of porous rocks containing two-phase immiscible pore fluids. The experimental study uses a high pressure and high temperature triaxial cell to clarify the seismic response of a sample of Berea sandstone to supercritical CO$_2$ injection under deep saline aquifer conditions. Measured ultrasonic wave velocity changes during CO$_2$ injection in the sandstone sample show the effects of pore fluid distribution in the seismic velocity of porous rocks. CO$_2$ injection was shown to decrease the P-wave velocity with increasing CO$_2$ saturation whereas the S-wave velocity was almost constant.

The focus of the mathematical modelling is on how the Biot-Gassmann theory can be modified to account for the distribution of two-phase fluids, particularly in terms of layering and
non-uniformity in the scCO$_2$ displacement front, in addition to their relative volumes, in the pore space of sandstone. The experimental study uses a laboratory testing system to clarify the acoustic response of rocks to supercritical CO$_2$ injection under deep saline aquifer conditions.

4.2 Biot-Gassmann Theory

Gassman’s substitution theory is the most widely used equation to describe the effects of the bulk modulus of the pore fluid on the seismic velocity of deformable porous media. The original derivation of Gassman’s equation is very involved and complicated. Here, it is shown that the equation can be derived very elegantly and succinctly using Biot’s poroelastic equations. The coupled poroelastic equations, which were first derived by Boit (1941), provide a rigorous mathematical treatment of fluid flow in deformable porous media, where fluid flow affects mechanical response and vice versa, and fluid flow field cannot be analysed separately from the mechanical response except under simple boundary conditions. (Notes: (a) subscript notation is used and repeated indices imply summation, (b) a dot over a symbol indicates differentiation with respect to time, and (c) all stresses are total). Biot’s theory couples the poroelastic stress-strain relation for fluid-saturated porous materials:

$$\dot{\sigma}_y = K \dot{\varepsilon}_y \delta_y + 2G \dot{\varepsilon}_y \alpha \delta_y \dot{p}$$

(4.1)

with the poroelastic fluid diffusion equation that quantifies fluid pressure changes in the same material (assuming single phase fluid flow and isotropic permeability):

$$\nabla \cdot \left( \nabla \frac{k}{\mu} p \right) = \alpha \dot{\varepsilon}_v + \left( \frac{\phi}{K_f} + \left( \frac{\alpha - \phi}{K_s} \right) \dot{p} + \dot{q} \right)$$

(4.2)

In the above equations $\nabla = \partial / \partial x_i$ , $\dot{\sigma}_y$ =stress tensor, $\dot{\varepsilon}_v$ =volumetric strain rate tensor, $\dot{\varepsilon}_y = \dot{\varepsilon}_\alpha = \dot{\varepsilon}_y \delta_y$ = deviatoric strain rate tensor, $\dot{\varepsilon}_v$ =strain rate tensor, $\dot{p}$ =pore pressure rate, $k$ =permeability, $\mu$ =fluid viscosity, $\dot{q}$ =fluid source or sink, $K$ and $G$ =elastic bulk and shear moduli of the porous medium, respectively, $\delta_y$ =Kronecker delta, $K_f$ and $K_s$ = bulk moduli of the pore fluid and the solid grains of the porous medium, respectively, $\phi$ =porosity, and $\alpha$ =Biot’s poroelastic constant defined as:

$$\alpha = 1 - \frac{K}{K_s}$$

(4.3)
In Eq. (4.2), single phase fluid flow and isotropic permeability were assumed. As will be shown below, permeability disappears in the derivation of the Biot-Gassmann equation. For this reason and by defining the pore pressure \( P \) as the average fluid pressure, the Biot-Gassman equation that will be derived below based on the Eqs (4.1) and (4.2) can also be used for two phase fluid flow. Note that Eq. (4.1) differs from conventional elasticity in that poroelastic deformation of the solid grains due to pore pressure changes is included in the total deformation of the porous medium. Eq. (4.2) also differs from conventional fluid diffusion equation in that it accounts for poroelastic deformation in the fluid flow in the porous medium.

Under undrained conditions, two conditions are achieved: (1) \( \nabla \left( kp / \mu \right) = 0 \) (i.e., no fluid gradients or Darcy fluid flow), and (2) \( \dot{q} = 0 \) (i.e., no fluid sink or source). Substituting these two conditions in Equation (4.2) and solving for \( \dot{p} \) gives:

\[
\dot{p} = - \frac{\alpha \dot{\epsilon}_v}{\left( \frac{\phi}{K_f} + \frac{\alpha - \phi}{K_s} \right)}
\]

(4.4)

Since the shear modulus \( G \) is independent of pore fluid composition and is the same for both drained and undrained conditions (Berryman 1999), Eq. (4.1) can be split into deviatoric and volumetric components:

\[
\dot{s}_{ij} = 2G \dot{\epsilon}_{ij} \quad \text{and} \quad \dot{\sigma}_m = K \dot{\epsilon}_v - \alpha \dot{p}
\]

(4.5)

where \( \dot{s}_{ij} = \dot{\sigma}_{ij} / 3 = \dot{\sigma}_y \delta_{ij} / 3 \) = mean stress rate, and \( \dot{s}_y = \dot{\sigma}_y - \delta_{ij} \dot{\sigma}_m \) = deviatoric stress rate tensor.

The validity of uncoupling the deviatoric and volumetric elastic response of fluid saturated porous media is investigated experimentally below. Substituting Eq. (4.4) in Eq. (4.5) yields:

\[
\dot{\sigma}_m = K \dot{\epsilon}_v + \frac{\alpha^2 \dot{\epsilon}_v}{\left( \frac{\phi}{K_f} + \frac{\alpha - \phi}{K_s} \right)}
\]

(4.6)

Dividing throughout by \( \dot{\epsilon}_v \) yields the undrained bulk modulus of a fluid-saturated porous rock \( K_u = \dot{\sigma}_m / \dot{\epsilon}_v \) as:

69
\[ K_u = K + \frac{\alpha^2}{\left( \frac{\phi}{K_f} + \frac{\alpha - \phi}{K_s} \right)} \]  

Eq. (4.7) was first derived by Gassmann (1951), however, as shown above this equation can also be derived from the more general and earlier Biot’s poroelasticity theory. Thus, it is only proper to call Eq. (4.7) as the Biot-Gassmann equation. Eq. (4.7) gives the undrained bulk modulus of fluid-saturated porous material as function of the drained or dry bulk modulus of the same material, the porosity, Biot’s poroelastic constant, and bulk moduli of the pore fluid and the solid grains of the porous medium. The different parameters in Eq. (4.7) are all constants for a rock specimen under the same effective stress and porosity except for the pore fluid bulk modulus which can change with fluid composition. It is important to note that the second term of the right-hand side of Eq. (4.7) represents the effect of pore fluid content on the undrained bulk modulus \( K_u \).

### 4.3 Laboratory Testing Equipment

The main components of the laboratory testing system to study the effects of injection of scCO\(_2\) in the seismic velocity of porous rock are high pressure and high temperature (HPHT) triaxial cell. High pressure precision syringe pumps for back pressure and injection fluid pressure control, a hydraulic pump for cell pressure control, and a temperature control system allow for the injection of CO\(_2\) at supercritical conditions.

The end caps of the core holder are equipped with piezoelectric transducers to characterize ultrasonic wave velocity response of the rock core sample. Figure 4.1 illustrates the high-pressure and high-temperature (HPHT) triaxial cell. The triaxial core holder has a maximum working confining pressure capacity of 70 MPa, two precision syringe pumps that control the back pressure and injection pressure, a dome-loaded back pressure regulator, a hydraulic pump for regulating cell pressure, and a differential pressure transducer. The sample is enclosed in a Viton sleeve. Both ends of the core sample are in contact with high permeability porous metal filter discs in order to homogenize the fluid flow. Two movable pistons controlled by hydraulic pressure are able to induce axial stress different from the confining stress.

One of the syringe pumps transmits a mixture of saline water and scCO\(_2\) into the rock core sample at a accurate flow rates. Another syringe pump acts as the back-pressure regulator with
set-point pressure. The temperatures of fluids stored in the syringe pumps are regulated by using silicon rubber blanket heaters wrapped around the side walls of the syringes and the PID temp controllers. The pressure and volume change of fluids in the syringe pumps are monitored and acquired in a personal computer communicating with the pumps. The back-pressure regulator is equipped with a polyimide film diaphragm providing accurate regulation of outlet pressures.

Figure 4.1 High-pressure and High-temperature (HPHT) triaxial cell

Saline water produced from the end face of rock core sample during scCO$_2$ injection is accumulated in a vessel placed on an accurate electronic balance. The mass data of saline water accumulated is acquired by using a computer via serial communication. The complete testing system is enclosed within a constant temperature air-circulating cabinet that is controlled with an on-off infrared heater and air-circulating fans.

The end caps of the core holder are equipped with piezoelectric transducers for seismic compressional and shear wave velocity measurements. A schematic diagram of the arrangement of piezoelectric transducers is shown in chapter 2 Figure (2.2). Triads of shear wave transducers shake one of the end caps in two individual orthogonal directions. In the center of the cap is a compressional wave transducer.

The natural frequencies of the transducers range from 0.25 to 1 MHz. The transducers are excited by using a square wave pulser/receiver capable of providing square pulses whose
voltages are selectable between 100 and 400 V in increments of 100 V. The typical rise time of pulse is less than $1.0 \times 10^{-8}$ s. The maximum bandwidth and typical noise level of receiver are from 1 kHz to 35 MHz and 70 μV peak to peak, respectively. The seismic waveforms are converted into digital signals by using a digital oscilloscope having 100 MHz of bandwidth and $1 \times 10^{9}$ s$^{-1}$ of real time sample rate. The minimum time base and the vertical resolution of oscilloscope are $2 \times 10^{-9}$ s/div and 8 bits. The waveform data are acquired by using a personal computer communicating with the oscilloscope. Each waveform acquired is the average of 16 waveforms. The resultant minimum time interval of waveform acquisition in the personal computer is 6 s.

4.4 Test Material and Procedures

The study of the effects of scCO$_2$ injection in the seismic velocity of porous rock initially saturated with saline water was conducted using Berea sandstone, which is a well-known and widely used test material in the study of the rock mechanical and fluid flow behavior of sedimentary rocks. Cylindrical rock samples used for the experimental works are cored from a block of Berea sandstone. Both ends of the core samples are ground to ensure that sample ends are parallel within required tolerances. The petrophysical properties of the Berea sandstone used in the tests are summarized in Table 4.1. Carbon dioxide having 99.9999% of purity is used to prepare the scCO$_2$. Saline water used is a mixture of distilled water and sodium chloride at 3.4% of salinity.

<table>
<thead>
<tr>
<th>Dry bulk density (g/cm$^3$)</th>
<th>Porosity</th>
<th>Permeability (mD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.17</td>
<td>20 - 30</td>
</tr>
</tbody>
</table>

The procedure for the scCO$_2$ injection tests is described below. A core sample dried at $T = 383$ K is placed inside the core holder then saturated with distilled water under vacuum by allowing the sample to imbibe distilled water. The confining axial and lateral stresses are isotropically increased up to 12 MPa. The pore pressure $P_{pore}$ is simultaneously raised to 10 MPa by keeping the effective stress less than 2 MPa. Four core samples are consolidated at desired effective consolidation stress and with a pore pressure of $P_{pore} = 10$ MPa and a temperature of $T$.
= 313 K to ensure that the injected CO\textsubscript{2} is kept in supercritical condition. Distilled water filling the pore space is displaced with saline water saturated with scCO\textsubscript{2} at $P_{\text{pore}} = 10$ MPa and $T = 313$ K. The total volume of saline water injected is more than ten times of the pore volume of core sample. Supercritical CO\textsubscript{2} saturated with saline water is injected into core sample at a constant rate at the prescribed conditions. The injection rates $q_{\text{inj}}$ of scCO\textsubscript{2} ultrasonic wave velocity measurement 0.20 cm$^3$/min. The pressure values used are gage pressures.

4.5 Test Results and Discussions

The injection pressure is maintained at 10.00±0.05 MPa through displacement. The production behavior of saline water indicates that CO\textsubscript{2} reaches a maximum saturation of 0.25 at the end of displacement. Figure 4.2 shows the waveforms of P-wave acquired during the CO\textsubscript{2} injection. The values in the figures indicate the average CO\textsubscript{2} saturation of the core sample when a P-wave velocity is measured. The arrival time of each wave has been determined at the first negative rise in the waveform. The arrival time changed significantly from 36.1 to 36.4 ms and subsequently to 37.0 ms as the CO\textsubscript{2} saturation increased from 0.041 to 0.137 and to 0.195.

![Figure 4.2 Compression waveform change depending on CO\textsubscript{2} saturation observed during CO\textsubscript{2} injection into Berea sandstone core sample pre-saturated with saline water.](image-url)
Figure 4.3 shows the shear waveforms acquired during CO$_2$ injection. The arrival times are 64.2, 64.2, and 64.0 ms corresponding to the average CO$_2$ saturation values of 0.0, 0.004, and 0.184, respectively. The dependency of shear wave velocity on scCO$_2$ saturation seems to be very different to that of the P-wave, i.e., the S-wave velocity increases slightly with increasing the CO$_2$ saturation, and is almost insensitive to the scCO$_2$ saturation.

![Figure 4.3 Change of shear waveforms during CO$_2$ injection at 0.2 cm$^3$/min into Berea sandstone core sample pre-saturated with saline water.](image)

The CO$_2$ saturation dependency of the bulk and shear moduli of Berea sandstone is summarized in Figure 4.4 the bulk and shear moduli, $K$ and $G$ were calculated from:

$$K = \rho \left( V_p^2 - \frac{4}{3} V_s^2 \right)$$  \hspace{1cm} (4.8)

$$G = \rho V_s^2$$  \hspace{1cm} (4.9)

where $V_p$ and $V_s$ are the compressional and shear wave velocities, and $\rho$ is the density of porous medium.

The bulk density of porous rock filled with saline water and CO$_2$ is calculated from:
\[
\rho = \rho_{\text{dry}} + \phi[S_{\text{CO}_2} \rho_{\text{CO}_2} + (1 - S_{\text{CO}_2}) \rho_{\text{sln}}]
\]

where \(\rho_{\text{dry}}, \rho_{\text{CO}_2}\) and \(\rho_{\text{sln}}\) are the bulk densities of the dry rock, scCO\(_2\) and saline water, respectively, and \(\phi\) is the porosity of the rock sample.

\[\text{Figure 4.4 CO}_2\text{ saturation dependencies of bulk and shear moduli of Berea sandstone core samples.}\]

In Figure 4.4 and Eq. (4.7), \(S_{\text{CO}_2}\) is the scCO\(_2\) saturation, which is defined as the volume of supercritical CO\(_2\) in the voids divided by the volume of the voids. The measured irreducible saline water saturation for the tested samples is about 40\%, which means that the maximum volume of scCO\(_2\) that can be injected is about 60\% of the pore volume of the samples. For \(S_{\text{CO}_2} > 0.60\), scCO\(_2\) starts to bypass and can no longer displace the saline water. Thus, since pore fluid composition remains constant for \(S_{\text{CO}_2} > 0.60\), seismic velocity measurements were made only for
As can be seen in Figure 4.4, the average bulk modulus for the three samples decreased from about 21 GPa to about 19 GPa as \( S_{\text{CO}_2} \) is increased from 0 to 0.60. The differences in measured elastic moduli between the three samples are due to the slight variation in the sample porosities. It should be noted that there seems to be no discernible effect of the fluid flow direction in the measured seismic velocities and elastic moduli. This is possibly due to the very slow injection rates used in the tests with fluid flow and distributions that were apparently not affected by gravity. The rock undrained bulk modulus \( K_u \) decreased with increasing \( S_{\text{CO}_2} \) because the scCO\(_2\) bulk modulus is lower than the saline water bulk modulus, and as a result the pore fluid bulk modulus decreased as more scCO\(_2\) is injected into the samples. In contrast to the bulk modulus, Figure 4.4 shows that the shear modulus is nearly constant during the increase in scCO\(_2\) saturation from 0 to 0.60. This confirms the assumption used in Eq. (4.5) in the uncoupling of the elastic volumetric and deviatoric responses in a poroelastic medium.

To calculate the undrained bulk modulus \( K_u \) in Eq. (4.7), it is necessary to determine the pore fluid bulk modulus \( K_f \). Two widely used equations to calculate the bulk modulus of mixtures of two-phase immiscible fluids are: (1) the serial law, and (2) the Wood’s (1941) parallel law. For a mixture of saline water and scCO\(_2\), the serial law is given as:

\[
K_f = S_{\text{CO}_2} K_{\text{CO}_2} + (1 - S_{\text{CO}_2}) K_{\text{sh}}
\]

and Wood’s parallel law as:

\[
\frac{1}{K_f} = \frac{S_{\text{CO}_2}}{K_{\text{CO}_2}} + \frac{1 - S_{\text{CO}_2}}{K_{\text{sh}}}
\]

where \( K_{\text{CO}_2} \) is the bulk modulus of scCO\(_2\) and \( K_{\text{sh}} \) is the bulk modulus of saline water.

The serial law is applicable to the case where the two immiscible fluids are segregated perpendicular to the wave propagation direction (Figure 4.5a), and the parallel law is applicable to the case where the two immiscible fluids are segregated parallel to the wave propagation direction (Figure 4.5b). The predicted variations of the fluid bulk modulus \( K_f \) as function of CO\(_2\) saturation from the series and parallel models are illustrated in Figure 4.8 as function of \( S_{\text{CO}_2} \). As can be seen, the series law (Eq. 4.11) predicts linear decrease in \( K_f \) with increasing \( S_{\text{CO}_2} \). In comparison, the parallel law (Eq. 4.12) predicts a rapid decrease of the bulk modulus with
increasing $S_{CO_2}$. The series and parallel laws provide the upper and lower bound values of the seismic P-wave velocity and correspond to the so-called Voigt- and Reuss-bounds (Mavko et al. 1998). Experimental data indicate that during injection of supercritical CO$_2$ in porous rocks the two-phase fluids are distributed neither in parallel nor serial fashion in the test specimens as idealized in the models shown in Figure 4.5.

![Image of fluid distributions](image)

Figure 4.5 Illustration of (a) serial and (b) parallel distributions of two-phase fluids in respect to longitudinal direction.

X-ray CT (computed tomography) imaging of scCO$_2$ injection in Berea sandstone shows that the pore fluid distribution is more complicated than the ideal parallel or serial distributions and is dependent on the pore structure and the relative mobilities of the two fluids (Shi et al., 2011). To reflect this more complicated pore-fluid distribution, an improved empirical equation to estimate the fluid bulk modulus of mixtures two-phase fluids and account for their mixing is used. This equation is the one proposed by Brie et al. (1995):

$$K_f = S_{sl}^s K_{sl} + (1 - S_{sl}^s) K_{CO_2}$$  \hspace{1cm} (4.13)

where $S_{sl} = 1 - S_{CO_2} =$ saline water saturation and $n =$ empirical parameter. Note that $n=1$ corresponds to the serial fluid distribution.
The effect of the parameter $n$ on the shape of the $K_f$ vs. $S_{co2}$ curve is shown in Figure 4.6. As the value of $n$ increases, the fluid bulk modulus $K_f$ declines more rapidly with increasing $S_{co2}$. For $n>2$, the fluid bulk moduli $K_f$ values at high $S_{co2}$ even become lower than the lower bound values from the parallel law.

The dependency of $V_p$ on $S_{co2}$ of the tested Berea sandstone is shown in Figure 4.6. The $S_{co2}$-dependent $V_p$ is calculated and predicted from $K_s$ Eq. (4.7), with $K_f$ determined from either Eq. (4.11) or Eq. (4.12), and $G$ as:

$$V_p = \frac{1}{\rho} \left( \frac{K_s + \frac{4}{3}G}{\rho} \right)$$

(4.14)

Figure 4.6 Predicted variations of the bulk modulus $K_f$ of a mixture of saline water and supercritical CO$_2$ as function of CO$_2$ saturation from the series, parallel and Brie et al. (1995) models. Note: $K_{sln}$=bulk modulus of saline water.
As can be seen, $V_p$ decreases as $S_{CO_2}$ is increased to its maximum value of 0.60. The average $V_p$ for the three tested Berea samples decreased from about 4.15 km/s to about 4.02 km/s as $S_{CO_2}$ is increased from 0 to 0.60. For the experimental data, the values of $n$ are determined by fitting the experimental $V_p$ vs. $S_{CO_2}$ curves with the combined Biot-Gassmann and Brie et al. equations through the experimental results. The curve-fitted $n$-values, which range from 2.02 to 2.38 are summarized in Figure 4.9 together with the $R^2$-values as measures of the statistical fit of the Biot-Gassman-Brie et al. curves.

As can be seen, good agreements between experimental data and predicted curves are obtained, with $R^2$-values of 0.97 to 0.99 indicating the validity of the combined Biot-Gassmann and Brie et al. equations. The curve fitted Biot-Gassmann-Brie et al. equations predict an almost linear decrease in $V_p$ with increasing $S_{CO_2}$ for $S_{CO_2} < 0.4$ and a more gradual decrease in $V_p$ for $S_{CO_2} > 0.4$. The applicability of the patchy fluid bulk modulus model proposed by Brie et al. was
also previously investigated by Monsen and Johnstad (2005). It is reported that observed relationship between $V_p$ and CO$_2$ saturation can be produced by using the values of $n$ between 1.5 and 2. The higher value of $n$ determined in this study may suggest that the pore fluid distribution formed as a result of displacement is more homogeneous than those of Monsen and Johnstad (2005).

4.6 Summary

The experimental study uses a high pressure and high temperature triaxial cell to clarify the seismic response of a sample of Berea sandstone to supercritical CO$_2$ injection under deep saline aquifer conditions. Ultrasonic wave velocity changes due to CO$_2$ saturation change were measured using Berea sandstone core sample, which was initially saturated with saline water and was subjected to constant CO$_2$ injection rate. The results showed the effects of pore fluid distribution in determining the effects of multiphase pore fluids on the seismic velocity of porous rocks. Increasing CO$_2$ saturation affected the P-wave velocity which was observed to decrease whereas the S-wave velocity was almost constant during the CO$_2$ injection. The results confirm that the Biot-Gassmann theory can be used to model the changes in the acoustic P-wave velocity of sandstone containing different mixtures of supercritical CO$_2$ and saline water provided the distribution of the two fluids in the sandstone pore space is accounted for in the calculation of the pore fluid bulk modulus.

Two-phase fluids distributed parallel and in series in the voids relative to the wave propagation direction correspond to the lower and upper bound values, respectively, of the fluid saturation dependent P-wave velocity of sandstone. The observed relationship between P-wave velocity and CO$_2$ saturation transitioned from the relation given by the Biot-Gassman-Wood model in the initial injection phase, to the Biot-Gassmann-parallel law model, then back to the Biot-Gassmann-Wood model towards the end of the displacement process. This should correspond to the transition of spatial distribution of saline water and CO$_2$ in core sample as the displacement of saline water proceeded. The empirical relation of Brie et al. (Brie 1995) for the bulk modulus of mixtures of two-phase immiscible fluids, in combination with the Biot-Gassmann theory, was found to satisfactorily represent the pore-fluid dependent acoustic P-wave velocity of sandstone.
4.7 References


CHAPTER 5
STRESS DEPENDENT PERMEABILITY AND SHEAR WAVE SPLITTING IN FRACTURED ROCKS

5.1 Introduction

Evaluation of the effect of fractures on the thermo-hydro-mechanical response of rock bodies is of interest to civil, mining, and resource engineers. Hydraulic conductivity of fractured rock is important design parameter for dams, oil producing wells with optimized well arrangement, and CO₂ sequestration reservoirs having proper cap rock layer (IPCC 2005). Fractures distributed in rock masses cause anisotropy of the thermo-hydro-mechanical response. Response of hydraulic conductivity of rock mass to confining stress and/or pore pressure change changes due to the existence of cracks. The shear strength and deformability of rock joints are size-dependent parameters (Pratt HR), (Barton et al., 1985). Shear wave splitting in earth and rock materials has been observed in fields and laboratories (Crampin et al., 1990, Gao Y et al., 1995, Zhang et al., 2000, Miller et al., 2001, Cochran et al., 2004, Margheriti et al., and Stanchits et al., 2012). Shear wave splitting is caused by anisotropies related to rock microstructure and/or induced by anisotropic stresses. Shear waves entering in an anisotropic rock split into two orthogonal directions and propagate at different speeds.

The polarization of shear waves propagating in the vertical direction of the earth occurs in a way that the faster split wave polarizes in almost parallel direction to the maximum horizontal stress (Gao et al., 2008). Such polarization induced by anisotropic stress is caused by the distributions of stress-aligned fluid-saturated parallel vertical microcracks pervading almost in situ rocks (Crampin et al., 1994). Shear wave splitting is used to explore the degree of anisotropy by use of seismic survey. However, the relationship between rock microstructure and shear wave splitting has not been yet clarified sufficiently. Field measurements have often difficulties to directly measure the changes in pore pressure and earth pressure. Further studies are necessary for understanding the shear wave splitting response of rocks related to microstructural and stress changes in laboratories.

Hydromechanical response of rock joint is sensitive to the joint contact and matching Barton et al., 1973, Bandis et al., 1983, Revees et al., 1985). Zhao defines the joint matching coefficient
(JMC) to quantify the degree of matching (Zhao et al., 1997). When $JMC = 1$, the joint is completely matching and the highest shear resistance is obtained (Zhao et al., 1997). In such highly matched joint, a small degree of mismatching can cause significant reduction of shear resistance. As the JMC value decreases, the contact areas become smaller and shear resistance of joint decreases, when $JMC = 0$, the joint is completely mismatching.

The various geological process such as weathering causes reduction of JMC value of natural joint. The effect of joint matching on shear wave splitting has not been clarified yet. The stress dependency of shear wave splitting of joint should depend on the degree of joint matching. Permeability of rock joint is often described by using or modifying the parallel plate model:

$$k = \frac{d_{eh}^2}{12}$$

(5.1)

where $k$ is permeability and $d_{eh}$ is the hydraulic joint aperture equivalent to the smoothed wall assumed in the parallel plate model. Real joint has abrupt and rough surface. Thus, the equivalent hydraulic aperture of rock joint is largely different from the mechanical aperture. A common form to correct the mechanical aperture to the equivalent hydraulic aperture is:

$$d_{eh} = f(d_{em}, \text{Joint surface roughness})$$

(5.2)

where $d_{em}$ is the mechanical aperture of rock joint. Barton (1985) proposed the following model to convert the mechanical aperture of rock joint into hydraulic aperture by implementing the joint roughness coefficient (JRC):

$$d_{eh} = \frac{JRC^{2.5}}{(d_{em}/d_{eh})}$$

(5.3)

where the units of $d_{eh}$ and $d_{em}$ are microns.

The mechanical fracture opening/closure due to normal stress change seems to be estimated by knowing the contact normal stiffness. However, the modeling of normal stiffness of unmatched joint is not straightforward. Mismatched joints under normal loading indicate non-linear stress-deformation behavior. Bandis (1983) pointed out that the difference of joint normal stiffness between the interlocked (matched) and mismatched joints can be significant depending on the amount of joint opening and joint wall strength. Different approaches modeling rock joint permeability from that utilizing the joint normal stiffness should be attempted. This chapter
presents the effects of effective stress and the degree of joint matching on shear wave splitting and permeability have been experimentally studied by using high pressure triaxial core holder equipped with piezoelectric transducers at effective stresses up to 55 MPa.

5.2 Material and Testing Procedure

The core samples tested were obtained from a block of Berea sandstone. The core samples have 38.1 mm of diameter and approximately 70 mm of length. The porosity of these samples is 0.17. The core samples were fractured along the longitudinal axis of the samples by using two loading wedges generating indirect tensile stress in the sample as shown in Figure 5.1. A single macro-joint has been created in cylindrical core samples and different amounts of joint shear displacement, 0, 0.45, and 1.00 mm were given to the joints created to have rock joints different in the joint matching coefficient.

Figure 5.1 Loading wedges for creating artificial fracture along longitudinal axis of cylindrical sandstone core sample.
5.2.1 Core Sample and Fracture Profiling

Figure 5.2 shows the fracture surface of Berea sandstone core sample created by loading wedges. The Macro-fractured core sample preparation is illustrated in Figure 5.3. The fracture seems to be initiated from one of the contacts between rock sample and loading wedge and propagates to another contact. Figure 5.4 shows typical joint surface profiles traced by using a profilometer. There exists a tendency that the joint surface becomes rougher as the location is closer to the center of joint surface. The profiles of both edge of sample close to straight line. The core samples were successfully separated in two pieces. The joint roughness coefficient (abbreviated as $JRC$) of the created fracture joints was determined by carrying out tilt test (Barton et al., 1985) Figure 5.5. The created fractures are comparable because the values of $JRC$ are ranging in a narrow range of $6.5 \pm 0.1$. 

![Figure 5.2 Fracture surface of Berea sandstone core sample created by loading wedges](image)
Figure 5.3 Macro-fractured sample preparation

Figure 5.4 Aperture and joint matching coefficient of macro-fractured core samples
The mechanical and physical properties of the testing rock sample have been directly measured in the lab. The mechanical rock properties are one of the most important parameters that will be considered in the analysis structures in rock mass. Joint Compressive Strength (JCS) test has been performed in the lab using Schmidt Hammer and estimate to be about 84 MPa for the tested Berea sand stone block. Uncoffining Compressive Strength (UCS) test has been performed on a Berea sand stone core sample using the Loading Frame Stand Model 37-5570(CT-713A).

Computer output data of the UCS test results following the portion of ASTM D7012-10 is estimated to be about 92 MPa. The Elastic modulus plot of the computer output data is shown in Figure 5.6. Three fractured core samples are prepared and they have different shear displacements of fracture joint artificially given. The given fracture shear displacement \( (d_{fs}) \) are 0, 0.45, and 1.00 mm. The core sample having 0 mm of fracture shear displacement is compressed by applying isotropic stresses up to 60% of the unconfined compression strength in the triaxial core holder and subsequently unloaded to erase the initial disturbance of fracture core sample induced by fracturing process. This cycle was repeated 3 times. The core samples having 0.45 and 1.00 mm of fracture shear displacement were compressed in the triaxial cell in the same
method to the 0 mm-displacement sample. To maintain the fracture displacement during loading/unloading cycles, some aluminum alloy sheets are placed between the end face of core sample and loading piston of the core holder. After loading/unloading cycles, the aluminum alloy sheets are removed and then the end faces are ground to obtain flat end faces. The effect of loading on fracture profile was investigated by using performing tilt test on the fracture joint of shear displacement after the elastic wave velocity and permeability measurements. The $JRC$ values decreases from 6.5 to 6.2. This means that the fracture joint does not stay intact after loading but the degree of change in roughness is not significant.

Figure 5.6 Computer output for best calibration Stress vs Strain relationship

Figure 5.7 indicates the aperture of fracture and joint matching coefficient (abbreviated as $JMC$) (Zhao et al., 1997). The aperture was obtained as the difference between the diameter prior to fracture creation and the diameter of fractured sample. The aperture increases with the shear displacement of fracture joint. The fracture aperture estimated for 0 mm-shear fracture displacement sample shown in the figure is obtained from the following empirical relation (Barton et al. 1985):
\[ d_e = \frac{JRC}{5} \left( 0.2 \frac{UCS}{JCS} - 0.1 \right) \quad (5.4) \]

where \( JRC \) is the joint roughness coefficient, \( UCS \) is the unconfined compression strength, and \( JCS \) is the joint compressive strength.

The measured value is comparable with the estimated value as seen in the figure. The \( JMC \) indicates the degree of matching of fracture joint. In this study, the \( JMC \) values were determined as the length of fracture profile matching to the whole length of fracture profile. The fracture joint profiles were obtained by using a profilometer. The \( JMC \) value decreases with fracture joint shear displacement.

![Graph showing aperture vs. fracture shear displacement](image)

Figure 5.7 Indicates the aperture of fracture and joint matching coefficient

### 5.2.2 Testing Set-up

Figure 5.8 illustrates End caps equipped with Piezoelectric transducer used for the experiments measurements. The cylindrical core plugs are pressurized in a triaxial core holder. The core is encased in a Viton rubber sleeve. The end-caps of the core holder are equipped with piezoelectric transducers having natural frequencies ranging from 0.25 to 1 MHz. The ultrasonic waves generated by exciting the transducers with square pulses at one end propagate the core and
are received at another set of transducers at the other end. The wave signals are monitored by using an 8-bit digital oscilloscope whose bandwidth and real time sample rate are 100 MHz and $1 \times 10^9$ s$^{-1}$. Single waveform acquired in a personal computer communicating with the oscilloscope to determine the wave travel time is average of 16 waveforms. The ultrasonic wave velocity measurement has been done by using core samples at 40 °C of the temperature and the effective stresses up to 55 MPa.

Figure 5.8 End caps equipped with Piezoelectric transducer

The permeability of core samples was measured by using saline water having 3.4% of salinity as pore fluid at 40 °C and 10 MPa of pore pressure (Appendix B). The pore fluid is transmitted from a precision syringe pump capable of sending fluids into the rock core sample at flow rates ranging from $1 \times 10^{-3}$ to 107 cm$^3$/min and at pressures of up to 50 MPa. The backpressure is controlled by using another syringe pump. The differential pressure between the inlet and outlet of core sample is monitored by using a differential pressure transducer. It is variable reluctance type transducer having ±0.25% of full scale accuracy, which includes the effects of non-linearity, hysteresis, and non-repeatability. As shown in Figure 5.8 Triads of shear wave transducers shake one of the end caps in two individual orthogonal directions. In the center of the cap is a compressional wave transducer.

To measure shear wave splitting, the fracture joint created are aligned parallel to the one of oscillation directions of shear wave. The faster shear wave velocity is obtained by the triad of
transducers oscillating parallel to the fracture joint. The slower velocity is measured by using another triad of shear wave transducers.

5.3 Shear Wave Splitting Depending on Effective Stress and Joint Matching Coefficient

The shear wave splitting is measured at effective stresses varying from 0.5 to 55 MPa. The shear wave splitting parameter observed at 8 MPa of effective stress increases with decreasing JMC value. The splitting parameter decreases with effective stress and finally becomes zero. The effective stress at which the splitting disappears increases with JMC value. The shear wave splitting observed was modeled as the monotonically decreasing function of effective stress by implementing JMC, fracture aperture, and JCS of fracture joints. Good agreements were observed between the observed and model curves by fitting only one material parameter at these wide-ranging JMC and effective stress.

5.3.1 Laboratory Measurement of Shear Wave Splitting

Figure 5.9 shows typical waveforms of the faster (S1) and slower (S2) shear waves for the macro-fractured core sample having JMC = 0.1 observed at 16 MPa of effective stress. The arrival times of S1 and S2 waves are determined to be 56.50 and 57.30 μs, respectively.
Figures 5.10, 5.11 and 5.12 show the shear wave splitting of fractured core samples having 0.9, 0.3, and 0.1 of \( JMC \) values respectively at 6 to 55 MPa of effective stresses. It can be seen that the shear wave splitting is becomes less pronounced as the effective stress increases. The shear wave splitting for the fractured core sample having \( JMC = 0.9 \) as shown in Figure 5.10 is weak and disappears when the effective stress is greater than 10 MPa. The faster and slower shear wave velocities (\( V_{s1} \) and \( V_{s2} \)) at 6 MPa of effective stress are 2.086 and 2.080 km/s, respectively. The shear wave splitting becomes pronounced as the \( JMC \) decrease. Figures 5.11 and 5.12 show that the threshold of effective stress values at which the splitting disappears for the sample having \( JMC = 0.3 \) and 0.1 are 20 and 30 MPa, respectively. Figure 5.13 summarizes the effects of \( JMC \) and effective stress on the degree of shear wave splitting evaluated as the slowness of shear wave propagation due to fracture presence normalized by the faster shear wave velocity. The dependency of shear wave splitting on \( JMC \) can be clearly observed in the figure. As discussed above, the shear wave splitting of core samples having \( JMC = 0.9, 0.3, \) and 0.1 disappears at 10, 20, and 30 MPa of effective stresses, respectively.

\[
\text{Figures 5.10 Stress dependency of shear wave velocities of macro-fractured core sample having } JMC = 0.9
\]
Figures 5.11 Stress dependency of shear wave velocities of macro-fractured core sample has 
$JMC = 0.3$

Figures 5.12 Stress dependency of shear wave velocities of macro-fractured core sample has 
$JMC = 0.1$
Figures 5.13 Stress dependency of shear wave splitting parameter of macro-fractured core samples.

### 5.3.2 Modeling Shear Wave Splitting

The experimental results shown above indicate that the shear wave splitting becomes pronounced as the JMC increases. In addition, the splitting fades with increasing effective stress. For the observed dependency of shear wave splitting, the following model has been proposed (Almrabat, Daisuke, and Gutierrez):

\[
\frac{V_{s1} - V_{s2}}{V_{s1}} = \alpha_s d_{e0} \frac{JCS}{P_{\text{eff}}} (1 - JMC)
\]

(5.5)

where \( \alpha_s \) is a fitting parameter in mm\(^{-1} \), \( d_{e0} \) is the initial fracture joint aperture in mm, \( P_{\text{eff}} = (P_{\text{cell}} - P_{\text{pore}}) \) is the effective stress where \( P_{\text{cell}} \) and \( P_{\text{pore}} \) are the cell and pore pressures, and \( JCS \) is the joint wall compressive strength. The term \( \alpha_s d_{e0} \) characterizes the effect of joint aperture.

Equation 5.5 calculates the higher shear wave splitting in cases where the aperture increases and/or the JMC value decreases. The splitting parameter value decreases as the effective stress
increases. Figure 5.15 compares the observed splitting parameter values and model curves fitted to the data.

![Graph showing the comparison of observed stress dependency of shear wave splitting parameter and proposed model curve.](image)

Figure 5.14 Comparison of observed stress dependency of shear wave splitting parameter and proposed model curve.

Table 5.1 shows the material parameter values used to calculate the model curves. The fitting parameter is only $\alpha_e$ and the other are measurable properties of fracture joint. Nevertheless, the model curves are in good agreement with the observed stress dependency curves. The effect of effective stress causing a decay of shear wave splitting is assumed to be stronger if the joint compressive strength is weaker. The value of $\alpha_e$ for the tested core samples is determined to be $1.96 \times 10^{-2}$ mm$^{-1}$ from the least-square method. The strong stress dependency of splitting parameter observed at the lower stress ranges for the fractured core samples having $JMC = 0.3$ and 0.1 is sufficiently described by the proposed model with 0.664 and 0.797 of $R^2$ values as shown in Figure 5.14.

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Application of an empirical model of stress-dependent elastic wave velocity proposed for micro-fractured porous rock to macro-fractured porous rock will be discussed by using the observed data. The proposed model of shear wave velocity is expressed as:

\[
V_s = V_{sm} \sqrt{\frac{f_s(p'_{nor})}{1 + f_s(p'_{nor})}} \tag{5.6}
\]

\(f_s(p'_{nor})\) is a non-dimensional function of effective stress given by:

\[
f_s(p'_{nor}) = \frac{s \cdot k_s}{G_m} = \frac{s [k_u + k_m (p'_{nor} - 1)]}{G_m} \tag{5.7}
\]

where \(s\) is fracture spacing, \(k_u\) is the material parameter related to the initial fracture shear stiffness, \(k_m\) is the material parameter representing the degree of stress dependency of fracture shear stiffness, \(p'_{nor}\) is the effective mean stress normalized by the initial effective stress \(p'_i\), and \(V_{sm} = \sqrt{G_m / \rho}\) are the elastic S-wave velocity of the intact and unfractured rock matrix.

Eq. (5.7) provides variation of S-wave velocity increasing with effective stress and asymptotically approaching that of the unfractured rock matrix, \(V_{sm}\), at very high effective stresses. Figure 5.15 shows the best fitting curves of Eq. (5.7) to the shear wave velocities observed for the fractured core sample having 0.1 of JMC. The agreements between observed and fitting curves are good.

The fitting parameters \(s \cdot k_u\) and \(s \cdot k_m\) for the \(V_{s1}\) were determined to be 8.99 and 2.94 GPa, respectively, by using the least-square method. For \(V_{s2}\), the values of these parameters are 3.73 and 2.76 GPa, respectively. It is seen that the difference of \(s \cdot k_u\) between \(V_{s1}\) and \(V_{s2}\) is significant while that of \(s \cdot k_m\) is negligibly small. Table 5.1 illustrates the material parameter values of empirical S-wave velocity model determined for micro-fractured dry Berea sandstone cores at different JMC values.

<table>
<thead>
<tr>
<th>(d_{fs}) (mm)</th>
<th>JMC</th>
<th>(d_{ef}) (mm)</th>
<th>(JCS) (MPa)</th>
<th>(\alpha_e) (mm(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.9</td>
<td>0.10</td>
<td>92</td>
<td>0.0196</td>
</tr>
<tr>
<td>0.45</td>
<td>0.3</td>
<td>0.17</td>
<td>92</td>
<td>0.0196</td>
</tr>
<tr>
<td>1.00</td>
<td>0.1</td>
<td>0.23</td>
<td>92</td>
<td>0.0196</td>
</tr>
</tbody>
</table>
To quantify the effect of JMC on the shear fracture stiffness parameters (Figure 5.16), the ratio of fracture stiffness parameter $k_{u,2}/k_{u,1}$ and $k_{u,2}/k_{u,1}$ are shown in Figure 5.17. $k_{u,2}/k_{u,1}$ is given as the ratio of initial shear stiffness parameter of $V_{s2}$ to that of $V_{s1}$. $k_{u,2}/k_{u,1}$ is the ratio of stress-dependency parameter of normal stiffness of $V_{s2}$ to that of $V_{s1}$. The insensitivity of $s \cdot k_u$ to the existence of macroscopic fracture is true at different JMC values for the tested core samples.

On the other hand, the sensitivity of $k_{u,2}/k_{u,1}$ to the existence of macro-fracture becomes greater as the JMC value decreases. The JMC-dependence of $k_{u,2}/k_{u,1}$ is likely to be given as a linear function of JMC shown in the figure. This may suggest that the stress-dependency of shear wave velocity slowness due to macro-fracture can be estimated from known JMC.
Figure 5.16 Dependency of fracture shear stiffness parameters on joint matching coefficient

Figure 5.17 Dependency of ratio of fracture shear stiffness parameters on joint matching coefficient
5.4 Effects of Fracture Unmatching and Stress on Permeability

Figure 5.18 shows the permeability of the intact and fracture rock cores decreasing with effective stress. The permeability of intact core is hardly dependent on the effective stress. The permeability of intact core at 1 MPa is 22.6 mD. It decreases to 19.3 mD at 55 MPa. The permeability of fractured core sample having $JMC = 0.9$ decreases at the higher rate compared to that of intact core. The effect of fracture on the permeability disappears when the effective stress is higher than 10 MPa. The initial permeability of fractured rock cores increases with decreasing $JMC$ values. The relationship between the degree of stress dependency of permeability and $JMC$ is not clear.

![Figure 5.18 Effective stress dependency of permeability of intact and macro-fractured rock core samples.](image)

The slope of $\log(k) - P_{\text{eff}}$ relationship representing the stress dependency of permeability seems not to depend on the $JMC$. The main mechanism of permeability reduction of fractured rock due to stress increase should be the closure of fracture joint as modeled in the parallel plate model. This suggests that the normal stiffness of fracture joint is not a simple function of $JMC$. 

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Figure 5.19 plots the data of stress-dependent fracture permeability in relation to the shear wave splitting parameter. The parameter of y-axis is given as the square root of permeability value of fracture core samples normalized by that of the intact core sample. They exist the following linear relationship for the observed relationship:

\[
\sqrt{\frac{k_F}{k_I}} = 1.358 + 42.16 \frac{V_{s1} - V_{s2}}{V_{s1}}
\]

(5.8)

where \( k_F \) and \( k_I \) are the permeability of fractured and intact rock cores. Applicability of Eq. (5.8) means that the shear wave splitting parameter is closely related to the fracture joint aperture.

Figure 5.19 Relationship between square root of ratio of permeabilities of macro fractured to intact core samples and shear wave-splitting parameter.
5.5 Summary

The stress dependencies of shear wave splitting and permeability of macro-fractured porous rock core samples were tested by using triaxial core holder. The cylindrical core samples of Berea sandstone were fractured by inducing tensile stress along their longitudinal axis. The fracture joints were characterized by using profilometer and tilt test. The fracture joints created are highly reproducible. JRC values for the created fracture joints are determined to be 6.5 ± 0.1. To investigate the effects of joint matching on shear wave splitting and permeability, 0.45 and 1.00 mm of different joint shear displacements were applied to core samples. The JMC values were determined to be 0.9, 0.3, and 0.1 for fractured core samples having 0, 0.45, and 1.00 mm of joint shear displacement, respectively.

Applicability of the previously proposed model for micro-fractured porous rock to the macro-fractured rock can be demonstrated if the model is capable of describing the observed stress dependency of shear wave splitting. The model can appropriately describe the stress dependency of shear wave velocity slowed by the existence of macro fracture without modification. Interestingly, the difference between the faster and slower curves can be characterized by adjusting only the initial shear stiffness parameters for the fractured core samples used regardless of JMC value.

The permeability of fractured rock cores is more strongly dependent on the effective stress. The stress dependency of permeability is not a simple function of effective stress. It is found that the square root of permeability linearly increases with shear wave splitting parameter under the effective stresses from 0.5 to 55 MPa. This implies that shear wave splitting parameter is a parameter that proportionally changes with fracture aperture.
5.6 References


12) Pratt, HR., Black, AD., Brace, WF., Friction and deformation of jointed quartz diorite. Paper presented at the 3rd Congress of ISRM, Denver, CO.


CHAPTER 6
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

6.1 Summary

The research described in this thesis developed improved characterization and modeling of the acoustic velocity of intact and fractured porous sandstones. The effects of stress confinement and pore-fluid composition, particularly mixtures of saline water and supercritical CO\textsubscript{2}, on acoustic velocity of micro-fractured sandstones were investigated experimentally. The research also investigated the effects of stress and shear displacement on shear wave splitting in fractured rocks. Experiments were carried out on intact (i.e., containing only micro-fractures) and fractured samples of sandstone using a newly developed High Pressure and High Temperature (HPHT) triaxial apparatus. Improved models were developed for the effects of stress level and pore fluid composition on the seismic velocity of sandstones, and the stress level and shear displacement on shear wave splitting in fractured sandstone based on the experimental results. The specific contributions from the research are:

1. A new advanced experimental High Pressure and High Temperature (HPHT) system for characterizing the stress level and pore-fluid content dependency of the acoustic wave velocity of intact and fractured sandstones was developed. The system is capable of injecting two immiscible phase fluids into core samples of sandstones and measuring the seismic velocities while the sample is under triaxial condition, and stresses, pore pressures and temperatures corresponding to in-situ stress, pore pressure and temperature conditions.

2. The stress dependent seismic wave velocities of porous rocks were characterized experimentally and modeled analytically based on the nonlinear elastic response of micro-fractures contained in the rock sample under normal and shear loading.

3. The changes in the seismic velocities of intact sandstones due to changes in the pore fluid composition, in particular during injection of supercritical CO\textsubscript{2} in core samples initially saturated with saline water, were tested experimentally. The pore fluid dependent acoustic velocity of the tested sandstone was modeled using the Biot-Gasmann poroelasticity theory in combination with the empirical model of Brie et al. (1995) for the compressibility of mixtures of two-phase immiscible fluids.
4. The effects of stress level and shear displacement on the magnitude of shear wave splitting in fractured rocks, and the influence of shear wave splitting on the permeability of fractured porous rocks, were investigated experimentally. Analytical models for the stress and shear displacement dependent shear wave splitting in fractured rocks were developed.

6.2 Conclusions

The major conclusions from the research described in this thesis are summarized as follows:

6.2.1 Stress-dependent seismic velocity of sandstone

The following major conclusions were obtained regarding the influence of the effective stress level on the seismic velocities of the tested sandstone samples.

The stress-dependent elastic wave velocity response of fractured rock mass was found to be attributable to the closure of micro-fractures in the rock mass due to effective stress changes. As the micro-fractures close, seismic waves propagate faster, and seismic velocity approaches an asymptotic value corresponding to the state when the micro-fractures can no longer close.

The micro-fractures were found to obey a generalized power-law model for non-linear stress-dependent normal contact stiffness, while the fracture shear stiffness was assumed to linearly increase with the effective normal stress acting on fracture. Based on the postulate of uniformly distributed random fractures in the rock mass, and the superposition of the rock matrix and micro-fracture incremental displacements for a given stress increment, a model for stress-dependent P- and S-wave velocities was proposed. The proposed velocity model was validated through curve-fitting of the model to stress-dependent elastic wave velocities observed by using dry Beria sandstone. The advantage of the proposed model in comparison to existing models that it may involve parameters that may have real physical meanings.

6.2.2 Pore-fluid dependent seismic velocity of sandstone

The following major conclusions were obtained regarding the influence of the pore fluid composition on the seismic velocities of the tested sandstone samples:

Ultrasonic wave velocity changes due to CO₂ saturation change were measured using Berea sandstone core samples which were initially saturated with saline water and were subjected to constant CO₂ injection rate. The results showed the effects of pore fluid distribution in
determining the effects of multiphase pore fluids on the seismic velocity of porous rocks. Increasing CO$_2$ saturation affected the P-wave velocity, which was observed to decrease, whereas the S-wave velocity on the other hand, was relatively unaffected by the changes in pore fluid composition, it was almost constant during the CO$_2$ injection. This latter result confirmed the validity in the requirement needed to uncouple volumetric from shear response from the poroelasticity equations, and to derive Gassmann’s substitution theory.

Gassmann’s (1951) fluid substitution equation was shown to be directly derivable from Biot’s (1941) poroelasticity theory. The derivation from Biot’s poroelasticity equation was less involved compared to the original derivation of Gassmann. The results confirmed that the Biot-Gassmann theory can be used to model the changes in the acoustic P-wave velocity of sandstone containing different mixtures of supercritical CO$_2$ and saline water, provided the distribution of the two fluids in the sandstone pore space is accounted for in the calculation of the pore fluid bulk modulus. The observed relationship between P-wave velocity and CO$_2$ saturation transitioned from the relation given by the Biot-Gassman-Wood model in the initial injection phase, to the Biot-Gassmann-parallel law model, then back to the Biot-Gassmann-Wood model towards the end of the displacement process. This should correspond to the transition of spatial distribution of saline water and CO$_2$ in core sample as the displacement of saline water proceeded. The Brie et al.’s (1995) empirical equation was found to represent adequately the compressibility of different mixtures of saline water and supercritical CO$_2$. The combined Biot-Gassmann-Brie et al. equation was shown to model adequately the changes in P-wave velocity of sandstone initially saturated with water and injected with supercritical CO$_2$.

The empirical relation of Brie et al. (Brie 1995) for the bulk modulus of mixtures of two-phase immiscible fluids, in combination with the Biot-Gassmann theory, was found to satisfactorily represent the pore-fluid dependent acoustic P-wave velocity of sandstone

6.2.3 Stress and shear displacement dependent shear wave splitting and permeability of fractured sandstone

The following major conclusions were obtained regarding the influence of the effective stress level and fracture shear displacement on shear wave splitting and permeability of fractured sandstone cylindrical core samples of Berea sandstone were fractured by inducing tensile stress

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along their longitudinal axis. The artificially created tensile-fractured samples were characterized by using profilometry and the tilt test, and were found to be highly reproducible. S-wave velocities parallel and perpendicular to the induced fracture, in addition to the P-wave velocity, were measured under different effective stress levels and magnitudes of fracture shear displacements. Thus, significant differences in S-wave velocity parallel and perpendicular to the fracture were detected, indicating clear signs of shear wave splitting.

The magnitude of the shear wave splitting was found to decrease rapidly with increasing effective stress and approaches zero at some threshold effective stress. While, the magnitude of the shear wave splitting was found to increase with increasing shear displacement along the fracture, and correspondingly, with increasing value of the Joint Matching Parameter (JMC). The Joint Matching Parameter was proposed by Zhao (1997) to provide of the mismatch between the two sides of a rough fracture, and as a result a high value of JMC indicates a high degree of fracture mismatch.

The change in the contact shear stiffness of fractures with different levels of mismatch was found to correlate well with the JMC. Based on the experimental results the shear wave splitting observed was modeled as the monotonically decreasing function of effective stress by implementing JMC, fracture aperture, and JCS of fracture joints. Good agreements were observed between the observed and model curves by fitting only one material parameter at these wide-ranging JMC and effective stress.

The permeability of fractured rock cores was found to be strongly dependent on the effective stress, but it was found difficult to correlate directly effective stress with fracture permeability. However, it was found that the square root of permeability increases linearly with the degree of shear wave splitting for different levels of effective stresses. This implies that shear wave splitting, which can be measured in the field, can be used to detect reservoir fracture permeability changes.

6.3  Recommendations for Future Work

Several important issues remain beyond the scope of the work presented in this thesis. The following recommendations are given for future work that can be performed to extend the outcomes of the studies presented in this thesis:
1) A more accurate modeling of the effects of pore fluid composition on the seismic velocity of porous rocks requires knowledge of the distribution of the fluids in the pore space of the rock. This can be characterized, for instance, by using advanced imaging techniques such as Computer Tomography (CT).

2) The results from the laboratory experiments have to be compared with the observed data from field seismic survey.

3) The developed modeling techniques have to be implemented in field seismic surveys in order to improve interpretation of field seismic data.

4) Conduct coupled hydro-thermo-mechanical laboratory testing to measure changes in intact and fractured rock single phase and two phase permeability as function of effective stress and temperature changes.

5) Correlated changes in hydraulic properties with seismic velocities to allow the use of seismic surveys to directly detect fluid flow property changes in underground res.

6) Conduct more experiments involving a wider range of parameters involving pore pressures, temperatures, pore fluid composition, fluid flow rates and rock types.
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APPENDIX A
HYDRO-MECHANICAL MODEL FOR FRACTURED POROUS ROCKS

For the hydro-mechanical modeling of fractured porous rocks, discontinuities will be treated statistically, and homogenization techniques will be used to treat discontinuous rock masses as equivalent continua. There are currently several techniques for the equivalent continuum representation of the hydro-chemo-mechanical behavior of fractured rock. One such technique, which will be adopted in the proposal, uses Oda’s crack tensor (Oda 1985; Oda et al. 1986) in the formulation of an equivalent anisotropic compliance matrix and (stress-dependent) permeability tensor for fractured rock masses.

Figure A-1 shows an illustration of the main elements of Oda’s crack tensor for two-dimensional stress and fluid flow situations. Realistic fracture scales and patterns of discontinuities will be entered into these models using the geostatistical package FracMan (Dershowitz et al. 1991).

Figure A-1 Representation of fractures in a fractured rock mass REV. \( \mathbf{n} \) = normal to fracture plane, \( r \) = size of fracture, and \( t \) = fracture aperture. These data may be obtained from borehole imaging or core analysis.
In the equivalent continuum technique, it is assumed that the size of a sampled volume $V$ exceeds the Representative Elementary Volume REV of the fractured rock mass. A REV of a fractured rock mass is defined as the smallest volume of the rock mass, which is large enough relative to the characteristic scale of the fractures in the volume, but small relative to the characteristic scale of interest.

The overall behavior of the REV is considered to be the material behavior at a point in the continuum theory, hence the name “equivalent continuum” (Cai & Horii 1992). The volume $V$ is homogeneously cut by $m^{(r)}$ fractures whose centers are randomly distributed. The volume associated with the fractures is denoted $V^{(r)}$, and $V^{(m)}$ is the volume associated with the intact rock. The fracture pattern is characterized by a probability density function (PDF) $E(n,r,t) = E(-n,r,t)$. Following (Oda 1984), if $(n,r,t)$ designates fractures whose unit normal vector $n$ is oriented inside a small solid angle around $n$, and the diameters $r$ and apertures $t$ and range from $r$ to $r+dr$, and from $t$ to $t+dt$, respectively, then $2E(n,r,t)d\Omega drdt$ gives the probability of finding $(n,r,t)$ fractures in an REV of a fractured rock mass.

To obtain an equivalent continuum stress-strain or compliance relation for the fractured rock mass, the principle of superposition is used. For a given stress increment $\{d\sigma\}$, the total strain increment $\{d\varepsilon\}$ is obtained from the summation of the matrix and the fracture strain increments $\{d\varepsilon^m\}$ and $\{d\varepsilon^f\}$:

$$\{d\varepsilon\} = \{d\varepsilon^m\} + \{d\varepsilon^f\}$$

(A-1)

The superposition principle implies that the stress increments for the equivalent continuum mass $\{d\sigma\}$, and stress increments for the matrix and the fractures $\{d\sigma^m\}$ and $\{d\sigma^f\}$, respectively, are all equal:

$$\{d\sigma\} = \{d\sigma^m\} = \{d\sigma^f\}$$

(A-2)

The matrix stress and strain increments are related by the compliance matrix $C^{m}_{ijkl}$, while the fracture stress and strain increments are related by the compliance matrix $C^{f}_{ijkl}$:

$$d\varepsilon_{ij} = C^{m}_{ijkl} d\sigma'_{kl}, d\varepsilon^f_{ij} = C^{f}_{ijkl} d\sigma'_{kl}$$

(A-3)
The fracture strain increment \( \{d\varepsilon_f^{ij}\} \) is defined as (Stietel et al. 1996):

\[
d\varepsilon_f^{ij} = \frac{1}{V} \int_\Omega \left( du_{i,j} + du_{j,i} \right) dV
\]

(A-4)

where \( du_{ij} \) are the increments of displacement jumps across the fracture surface. The increment of displacement jump can be split in two components \( du^n \) and \( du^s \), which are parallel to the normal and maximum shear stresses on a \((n,r,t)\) fracture, respectively. These displacement jumps can be related to the local fracture normal and shear stress or tractions \( d\sigma_n \) and \( d\sigma_s \) across the fracture by the fracture compliance relation:

\[
\begin{bmatrix}
    du^n \\
    du^s
\end{bmatrix} =
\begin{bmatrix}
    C_{nn} & C_{ns} \\
    C_{sn} & C_{ss}
\end{bmatrix}
\begin{bmatrix}
    d\sigma_n \\
    d\sigma_s
\end{bmatrix}
\]

(A-5)

Assuming that the stress is uniform in the domain, the stress vector on the fracture plane is \( d\vec{\sigma} \cdot n \) and the normal stress vector is \((n \cdot d\vec{\sigma} \cdot n)n\), so the total shear stress can be derived from vector subtraction as:

\[
d\sigma_n = d\vec{\sigma}_j n_j, \quad d\sigma_s = d\vec{\sigma}_n n_i - d\vec{\sigma}_m n_i n_j n_l
\]

(A-6)

Similar to the stress-strain response of the fractured rock mass, the equivalent continuum permeability tensor is found by superposition of the matrix permeability and the permeability for the fracture system.

\[
k_{ij} = k_{ij}^m + k_{ij}^f
\]

(A-7)

Comparison between Darcy’s law and the parallel plate model for fracture flow yields an equivalent permeability tensor \( k_{ij}^f \) of the fracture network:

\[
k_{ij}^f = \lambda (P_{ki} \delta_{ij} - P_{ij})
\]

(A-8)

where \( P_{ij} \) is a second-rank tensor, defined by:

\[
P_{ij} = \frac{\pi\rho}{4} \int_0^{\Omega} \int_0^{t} r^2 n_i n_j E(n,r,t) d\Omega dr dt
\]

(A-9)

**Hydro-mechanical Properties from Seismic Velocities**

Assimilation of seismic data from time-lapse seismic surveys requires a link between seismic velocities and hydro-mechanical properties. For seismic wave propagation analysis only the
elastic response of the fractured rock mass is required. For an elastic rock matrix the compliance matrix $C_{ijkl}^m$ is given by:

$$C_{ijkl}^m = \frac{1 + \nu}{E} \delta_{ik} \delta_{jl} - \frac{\nu}{E} \delta_{ij} \delta_{kl}$$  \hfill (A-10)$$

where $E$ is the Young’s modulus and $\nu$ is the Poisson’s ratio of the rock matrix. Assuming a simple fracture mechanical model, based on an uncoupled normal–tangential behavior, the elastic increments of jump fracture displacement vectors can be expressed as:

$$du_n = \frac{d\sigma_n'}{k_n^f}, \quad du_s = \frac{d\sigma_s'}{k_s^f}$$  \hfill (A-11)$$

where $k_n^f$ and $k_s^f$ are the normal and shear stiffness of the fracture.

Based on a statistical distribution of fractures, the fracture compliance matrix becomes:

$$C_{ijkl}^f = \frac{\pi D}{4} \int_0^{r_m} \int_0^{r_m} \int_0^{\Omega} \left[ \left( \frac{1}{K_n^f} - \frac{1}{K_s^f} \right) n_i' n_j' n_k' n_l' + \frac{1}{4K_s^f} \left( \delta_{ik} n_j' n_l' + \delta_{jk} n_i' n_l' + \delta_{il} n_j' n_k' + \delta_{lj} n_i' n_k' \right) \right] E(n,r,t) d\Omega dr dt$$  \hfill (A-12)$$

Extensive experimental results indicate that the normal and shear stiffnesses of fractures are stress-level dependent and the following empirical relations were found to be adequate in representing the stress-level dependencies (Gutierrez & Almrabat 2009):

$$k_n^f = k_{ni} \left( \frac{\sigma_n'}{\sigma_{ni}} \right)^n$$  \hfill (A-13)$$

$$k_s^f = k_{si} + k_{sn} \left( \frac{\sigma_n'}{\sigma_{ni}} - 1 \right)$$  \hfill (A-14)$$

where $k_{ni}$ is the normal stiffness corresponding to the initial effective normal stress $\sigma_{ni}'$, $n$ is the empirical parameter, $\sigma_n'$ is the effective normal stress across the fracture, and $k_{so}$ is the fracture shear stiffness at high effective stress (completely closed fracture). $k_{si}$ is the initial shear stiffness corresponding to zero normal stress and $k_{sn}$ is the slope of the $k_s$ vs. $\sigma_n'$ plot. For most fractures $k_{si} = 0$ is assumable, but some fractures exhibit “lock in” behavior with some shear stiffness even at zero normal stress. Note that same parameter $\sigma_{ni}$ is used to normalize $\sigma_n$ in Eqs.
(A-13) and (A-14), respectively to ensure that the units of normal and shear stiffnesses are independent of the unit of the stress used.

The validity of Eqs. (A-13) and (A-14) in describing the elastic compliance of fractured rock masses can be shown by obtaining the P and S wave seismic velocities from the above equations. In case of an isotropic stress condition, the stress-dependent seismic velocities $V_p$ and $V_s$ can be obtained from the above equations as:

$$V_s = V_{sm} \left( \frac{k_{si} + k_{si} \left( \frac{\sigma_n'}{\sigma_{ni}} - 1 \right)}{M_n + s \left( k_{si} + k_{si} \left( \frac{\sigma_n'}{\sigma_{ni}} - 1 \right) \right)} \right)^{\frac{1}{2}}$$  \hspace{1cm} (A-15)

$$V_p = \sqrt{\frac{M}{\rho}} = V_{pm} \left( \frac{s \cdot k_{ni} \left( \frac{\sigma_n'}{\sigma_{ni}} \right)^n}{M_m + s \cdot k_{ni} \left( \frac{\sigma_n'}{\sigma_{ni}} \right)^n} \right)^{\frac{1}{2}}$$  \hspace{1cm} (A-16)

where $V_{pi}$ and $M_i$ are the P-wave velocity and bulk modulus of the intact rock, $V_{si}$ and $G_i$ are the S-wave velocity and shear modulus of the intact rock, $\rho'$ is the effective mean confining stress, and $s$ is average fracture spacing. Eqs. (A-15) are compared with the laboratory measurements of stress-dependent shear wave splitting (birefringence) in foliated quartz phyllite obtained by Stanchits et al. (2008).

The comparisons shown in Figure A-2 illustrate the validity of Eq. (A-16) in linking the stress-dependent compliance behavior of fractured rock mass with seismic velocities. Since fracture mechanical response controls the changes in fracture asperity $t$ and permeability ($t^2 = t^2/12$), the stress-dependent permeability of the fractured rock mass can also be predicted from Eqs. (A-7) and (A-8). The effect of pore fluid saturation on seismic velocity will be accounted for by using Gassman’s (1951) fluid substitution equation.
Figure A-2 Comparisons of stress-dependent shear wave splitting (birefringence) in foliated quartz phyllite from model and experiments
References


APPENDIX B
STRESS DEPENDENT ABSOLUTE PERMEABILITY FOR FRACTURED SANDSTONE

B.1 Introduction

Permeability modulus has been defined by researchers following the linear poroelastic theory to investigate the stress-dependent fluid flow through porous media. However, permeability in materials varies exponentially with the applied pressure. In less permeable rocks, deformation due to applied load does not fit well with experimental data (Luo Ruilan, Feng Jinde 2007). However, permeability of the fractured rocks is more stress-sensitive, and, therefore, makes its exploitation a hard task. Many experimental studies discuss the effect of stress-sensitive mechanisms in low permeability. Several parameters control the permeability measurement in intact rocks, such as, tortuosity, distribution of pore throat size along with the presence of clay minerals, and the natural fracture system. In different fractured models, the main uncertainties in determining the fluid flow along fractures are the permeability of the individual fractures and the changes in fracture permeability including the possibility of fracture closure. Thus, the permeability of the individual fracture, \( k \), with a conduction aperture, \( e \), (with unit of length) is usually calculated from the parallel-plate law (Snow, D.T. 1969):

\[
k = \frac{e^2}{12}
\]  

The main objectives of the study described in this appendix are to provide direct experimental measurement of fracture permeability for Berea sandstone, and to investigate how it varies with mechanical loading.

B.2 Model for Fracture Permeability (Parallel Plate Model)

Fluid flow characteristics in fractures are strongly controlled by the roughness of the apertures and its relation to fluid flow in fractured rock. The reduction in aperture size, due to applied confining pressures, needs to be quantified. It is important to define the equation describing the change of the fracture parameters under different confining stresses. The fracture
permeability, $k_f$, is obtained by combining the viscous force and Darcy equation for flow through fractures:

$$k_f = 8.45 \times 10^9 w^2$$   \hspace{1cm} (B-2)

where $w$ is a fracture width in centimeters.

In order to determine the two unknown parameters ($k_f$, $w$), first, it is important to determine the contribution flow rate from the matrix ($q_m$) and fracture ($q_f$) from each zone by applying Darcy’s equation (Dicman et al., 2004). Therefore, the average flow rate of fracture and matrix from core flooding experiments can be given by:

$$q_{av} = \frac{k_{av} A \Delta p}{\mu l}$$   \hspace{1cm} (B-3)

The flow rate of the matrix can be determined from core flooding experiments for intact core as:

$$q_m = \frac{k_m (A - w l) \Delta p}{\mu l}$$   \hspace{1cm} (B-4)

where the flow rate in fracture $q_f$ can be expressed as:

$$q_f = \frac{k_f (w l) \Delta p}{\mu l}$$   \hspace{1cm} (B-5)

Thus, flow rate of fracture can be determined as:

$$q_f = q_{av} - q_m$$   \hspace{1cm} (B-6)

Substituting Eqs. (B.3), (B.4) and (B.5) in Eq. (B.6) gives:

$$k_f = \frac{k_{av} A - k_m (A - w l)}{w l}$$   \hspace{1cm} (B-7)

Inserting Eq. (B.2) in (B.7) and solving for fracture permeability gives the following

$$8.45 \times 10^9 w^3 l - k_{av} A + k_m (A - w l) = 0$$   \hspace{1cm} (B-8)

where $A$ is the matrix area (cm$^2$), and $l$ is the lateral extension of the fracture (cm), $\Delta p$ is pressure drop across the core (atm), $\mu$ is viscosity (cp) and L is core length (cm).

The flow through a smooth conduit can be expressed by involving the fracture width ($w$) and the pressure gradient $\Delta p$ as follow:
\[ q_f = 9.86 \times 10^9 \frac{w^3 l \Delta p}{12 \mu l} \]  \hspace{1cm} (B-9)

### B.3 Stress Dependent Permeability for Intact and Fractured Rocks

Many experimental studies discuss the effect of the stress-sensitive mechanism in low permeability rocks. The relationship between permeability and confining stress for low permeability is nonlinear elasticity because of increasing stress due to high clay and cement content, poor grain sorting, and narrow pore throat. As a result, these parameters control the permeability measurement in rocks. During the loading process for low permeability rocks, the deformation is not ideally linear elastic. Initially slightly plastic deformation is observed followed by elastic deformation; then, finally, plastic deformation again. Luo Ruilan and Feng Jind (2009) described this process by defining a new rock deformation coefficient \( S_p \) as follows:

\[ S_p = CK^{\frac{n}{p}} \]  \hspace{1cm} (B-10)

The relationship of \( S_p \) and initial permeability \( K_0^* \) at initial effective stress \( \sigma_{effo} \) follows a power function. Consequently, the relationship of permeability and effective stress can be expressed as a dual-power function as follows:

\[ K = K_0^* \left( \frac{\sigma_{eff}}{\sigma_{effo}} \right)^{S_p} \]  \hspace{1cm} (B-11)

Shi and Wang (2010) suggested that the relationship between effective pressure and rock permeability follows a power function based on the experimental data of Morrow et al (1984). The power law for describing the stress dependency of permeability is expressed as:

\[ K = K_0 \left( \frac{P_{eff}}{P_{effo}} \right)^{P} \]  \hspace{1cm} (B-12)

where \( P \) is a material constant that has different value ranges for different rock types. \( P \) is also dependent on clay and cement content, and the range of loading and unloading stress, while \( K_0 \) is the initial permeability at the initial effective pressure \( P_{effo} \).

### B.4 Experimental Approach

Permeability measurements on intact and fractured Berea sandstone core samples are done in the lab using a newly developed experimental system. The intact Berea sandstone core sample is placed into the core holder and dried at \( T = 313 \) K. The tested core sample is saturated with
distilled water under a vacuum. The axial and lateral pressures of the core samples are isotropically increased via Viton sleeve up to 12 MPa. Both ends of the core samples are contacted with porous metal filters aimed at homogenizing the fluid flow. In an attempt to study the movement of brine through intact and fractured core samples, the experimental process involved the injection of water at a rate ranging from 1 to 5 cm$^3$/min for the intact rock and 2 to 12 cm$^3$/min for the fractured rock.

**B.4.1 Test Method and Permeability Measurements**

Figure B-1 shows the injection flow profile in an intact Berea core sample, at a confining stress of 2 MPa, while, Figure B-2 shows the injection flow profile in a fractured Berea core sample, at a confining stress of 2 MPa. The total volume of saline water injected is more than ten times the pore volume of the core sample. The pore pressure ($P_{pore}$) is simultaneously raised to 10 MPa by keeping the effective stress less than 2 MPa. Accordingly, the distilled water filling the pore space is displaced by saline water.

![Figure B-1 Injection flow profile in intact Berea sandstone core sample](image-url)
The applied normal loads range from 10.5 to 65 MPa for the intact rock sample and from 10.5 to 55 MPa for the fractured rock sample. For each normal load, pressure drops across the core sample in the core holder are recorded under different injection rates of saline water. The Cartesian plot of $\frac{\Delta p}{l}$ versus $\frac{Q\mu}{A}$ is a straight line. The permeability is estimated from the slope of the straight line:

$$k_{abs} = \frac{Q\mu}{A} \frac{1}{\Delta p/l}$$

Absolute permeability for the intact Berea core sample has been measured at different normal loads to characterize stress-dependent permeability.

In order to control and adjust the movement of the brine flow through the fracture opening, in the core sample, the fluid flow is controlled by a lining up of two porous plate grooves with the core fracture opening. The sample is then loaded to a predetermined normal stress level, and then unloaded to zero stress. This procedure is used in order to erase the initial disturbance of the fracture core sample induced by the fracturing process and to mechanically seat the sample to the
initial zero stress aperture (Bandis et al., 1983). Thus, the sample is loaded under three cycles of normal loading and unloading. Each cycle involves increasing the effective stress approximately 60% of the UCS of the intact core sample, which is equal to 55 MPa, and the normal displacement of the fracture due to the maximum loading cycle is allowed to stabilize for each increment. The fracture aperture at the end of the first three cycles of normal loading and unloading is considered to be the initial aperture at the time of creation of the fracture. At the fourth cycle of normal loading and unloading, permeability measurements are performed at different normal stresses following the same procedure of as that of the intact rock.

Figure B-3 shows the absolute permeability for intact Berea sandstone at an effective stress of 1 MPa. This core has 22.6 mD absolute permeability under these conditions. The permeability for the intact sandstone rock \( k_m \) is determined by measuring the contribution flow rate balance from the intact rock by applying Darcy’s law. Figure B.4 shows the average absolute permeability for fractured Berea sandstone \( k_{ave} \) at an effective stress of 2 MPa. This core has 41.01mD absolute permeability under these conditions. The average permeability for the fractured Berea sandstone is determined by measuring the contribution flow rate balance from the fractured rock by applying Darcy’s law.

![Graph showing permeability](image)

**Figure B-3** Absolute permeability of intact Berea sandstone at \( \sigma_{conf} \) 11 MPa
Figure B-4 Absolute permeability of fracture Berea Sandstone at $\sigma_{\text{conf}}$, 11 MPa

### B.4.2 Permeability of Intact and Fractured Rock

Figure B-5 shows the experimental results of the deformation characteristics of the permeability ratio of initial to final absolute permeabilities and effective stress for an intact and fracture core samples. Thus, the experimental data of absolute permeability for intact and fractured Berea sandstone are measured under confining stress ranging from 11 to 65 MPa. As seen, the relationship between the absolute permeability ratio and the effective stress fits well with the empirical power law function Eq. (B.14) based on the Morrow et al (1984) model.

$$
\frac{K}{K_0} = \left( \frac{\sigma_{\text{eff}}}{\sigma_{\text{eff}0}} \right)^{-Sp}
$$

Thus, the permeability reduction is greatest at the lowest stress. The stress-sensitivity of the fractured Berea core sample is stronger than that of the intact Berea sandstone. Therefore, during the loading process, the fracture aperture, the micro-fracture structure, and the soft rock
materials, such as clay and cement, are initially compressed and deformed. The deformation of low permeability rocks is influenced by pore structure and rock compressibility.

![Graph](image)

Figure B-5 Relationship between the ratio of initial to final absolute permeabilities and effective stress

### B.5 Mechanism and Discussion

Low permeability rock deformation is a function of rock composition, grain size, clay content, and cement fill. Low permeability sandstone rocks are not necessarily dominated by muddy or clay matrix. Experimental data shows in Figure C.5 the relationship between permeability and effective stress. For the lower routine permeability $K_{o}$, permeability has a great reduction with increasing effective stress. The permeability reduction is greatest at the lowest stress. The stress-sensitivity of fractured Berea sandstone core sample is stronger than that of intact sandstone core sample. The deformation process of low permeability rocks is not ideally elastic deformation. Alternatively, it is starts as mildly plastic deformation and turns to be an
elastic deformation. The stress sensitivity of low permeability rocks is strong. Experimental results, gives that increasing the confining stress across the core sample reduced the permeability. The experimental results for stress dependency absolute permeability for intact and fractured Berea sandstone are validated by the theoretical power law model and give a good agreement curves fitting.
B.6 References


