OPEN PIT MINE SCHEDULING BASED ON FUNDAMENTAL TREE ALGORITHM

BY

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A thesis submitted to the Faculty and Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Mining and Earth System Engineering).

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ABSTRACT

Long-term production scheduling design is a very important part of mining because it determines the economic outcome of a project. It is a very complex and difficult problem basically due to its large scale. Much effort has been devoted to solving the optimum pit scheduling problem, but there has been no success in developing a scheduling method to give optimum results in maximizing net present value of a mining project.

It is already known that the optimum result for the scheduling problem can be obtained using a mathematical programming method such as Mixed Integer Programming. However, it is not possible to formulate the scheduling problem as a mathematical programming model since the number of variables required for the mathematical model is too great to be solved by today's available computer technology. Therefore, a methodology is required to combine, or aggregate, the mining blocks and to decrease the number of variables in scheduling without losing the optimality. Therefore, fundamental tree concept is introduced in this thesis research to combine the blocks.

A fundamental tree is defined as any combination of blocks such that:

1. the blocks can be profitably mined,
2. the blocks obey the slope constraints and,
3. there is no proper subset of the chosen blocks that meets 1 and 2.

Linear Programming (LP) formulation is developed as a mathematical model to find a set of fundamental trees that exist for a deposit. Since the blocks are combined to form the fundamental trees, the number of variables required for the scheduling model is decreased significantly. This decrease in the number of variables makes it possible to mathematically formulate the scheduling problem.
Mixed Integer Programming (MIP) formulation is further developed to determine the yearly mine schedule from the fundamental trees. The material scheduled for each year is constrained to have the attributes of a real mine operation such as mine production capacity, processing mill capacity, grade of ore, etc.

The fundamental tree mathematical programming model is applied to a multi-mineral copper deposit in Peru, South America. The mine is scheduled to meet yearly production requirements using fundamental trees such that net present values (NPV) of the cash flows are maximized.

NPVs of annual after-tax cash flows are generated based on the schedule provided by the “Fundamental Tree Algorithm” developed in this research.

The deposit is also scheduled using three of the computer packages available to the mining industry. These are Mintec’s M821V, Earthwork’s NPV Scheduler, and Whittle’s Milava open pit mine schedulers.

The results obtained in this study indicate that CSM-LP Scheduler gives the highest NPV among alternatives.
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ACKNOWLEDGMENTS

I am very grateful to my thesis advisor, Dr. Kadri Dagdelen, for his guidance and financial and intellectual support at all times during this thesis study. Without his support and guidance, it would not have been possible to complete this dissertation.

I would like to thank my committee members, Dr. Hugh King, Dr. Mark Kuchta, and Dr. Levent Ozdemir, and committee chair, Dr. John Emerick, for their time and considerations on this research.

I would like to acknowledge Natural Resources Research Institute of Minnesota for providing financial support for this research study.

I would also like to express my special thanks for Dr. Thys B. Johnson for his efforts and time in this research. His suggestions and advice were very helpful in completion of this research.

I am very thankful to my fellow graduate students for their sincere friendship. I would like to especially thank Dario Bernabe for his help on the haul road design.

I would like to acknowledge The Ministry of The National Education of The Republic of Turkey for providing financial support for six years for my studies.

I dedicate this thesis to my wife, Masako, for her understanding and encouragement. Without my wife's encouragement and support, I could not have completed this dissertation.
CHAPTER 1
INTRODUCTION TO MINE PLANNING

1.1 INTRODUCTION

Mining can be defined as an excavation in the earth made for the purpose of removal and sale of an economically valuable mineral or material. In a mining project, the first step is the exploration by which the orebody is located and outlined, and its tonnage and grade determined by a sampling process (such as drilling).

After determining an outline of a massive deposit, a block model is developed to represent the deposit (see Figure 1.1). The size of a block for a typical open pit mine can be 50'x50'x20' depending on geology and mining method. The size of the blocks is generally considerably smaller than the drill hole spacing. The grade of each block in the model is estimated using one of the estimation techniques such as distance weighting or Geostatistics. The block model concept is discussed in more detail in section 1.3.

An open pit mine can be designed using circular analysis as shown in Figure 1.2. The mineable reserves can be determined by assuming physical capacities (ore and waste production capacity, milling and refinery capacity) and production costs (mining and processing cost). The mineable reserves are determined by defining final pit limits. This is normally done by using the moving cone method, or the Lerchs and Grossmann’s method. The whole deposit is divided into two subgroups by final pit limits as shown in a 2D example in Figure 1.3. Only the material inside the final pit limit can be mined economically under the current economical conditions.
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The next step is to divide the whole portion of material within the ultimate pit limits into smaller volumes to be used in scheduling the mine as shown in Figure 1.4. These smaller nested pits are called “pushbacks,” or “incremental cuts,” or “phases.” Each increment can be formed by generating the optimum pit limits for different market conditions such as price of the product, or stripping cost, or processing cost. These phases can be obtained using any of the traditional pushback design algorithms such as Whittle’s method (maximizes undiscounted dollar value of each phase), Seymour’s method (maximizes dollar per ton value of each phase), Dugdelen and Bongarcon’s double parameterization method (maximizes amount of metal contained within each phase), etc. Each pushback configuration generates different cash flows resulting in different Net Present Value (NPV). Two different possible configurations of phases are illustrated in Figure 1.5.

After designing pushbacks, the next step in mine planning is to determine a sequence of extraction for each year for the period of mine life. The scheduling is done applying some constraints such as maximum ore production capacity, maximum waste stripping capacity, minimum ore production requirement, minimum and maximum grade of the ore that can be processed at the mill, etc.

1.2 PIT SLOPE CONSTRAINTS IN MINING

The pushbacks that are designed during mine planning must obey the maximum allowable pit slope constraint. In order to achieve this, a 3D - cone model is used. The angles of a cone and horizontal and vertical planes are shown in Figure 1.6. Mining slope angle is generally defined from the azimuth angle, which is the angle measured from North in the
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Figure 1.7. An example 3D cone figure used in designing pushbacks. $R_1$ is the radius of the top of the cone, $R_2$ is the radius of the bottom of the cone.
clockwise direction. A sample slope angle representation is shown on a cone in Figure 1.7. At \( \gamma \) - degree of azimuth angle in the figure, the slope angle is 90-\( \theta \) degree. The angle between the sides of the cone and the horizontal plane is the same as the pit's maximum allowable slope angle. The slope angle may be different at different directions of azimuth as shown in the figure.

For example, in the given figure \( \alpha \) is the slope angle along East – West direction, and \( \beta \) is that along North – South direction, and \( \alpha \) may not be equal to \( \beta \). The sides of the cone at different azimuth directions are set at the same slope as the slope angle of an actual pit. The bottom of the cone is called “apex.” The apex of the cone is set on a block (or node) in the block model, and all the blocks within the cone are identified as the overlying blocks. That means all the blocks within the cone must be mined before the block sitting at the apex of the cone can be mined out. Since the cone has the same slope in all directions as the actual pit, mining the blocks using this cone ensures the slope constraint for the open pit mining operations.

In order to divide the mineral deposit into smaller volumes, the apex of the cone is set over a positive node. The location of the positive node chosen depends on the pushback design method, but mostly it is started from the positive node on the most upper level of the deposit. All the nodes inside the cone set over the positive nodes must be removed from the deposit to be able to remove (or mine) the positive node. By following certain rules (the rules are different for different methods), these nodes are connected to each other by arcs. The set of connected nodes are considered, or planned to be mined all together starting from the most top level nodes and proceeding downward.
1.3 BASIC DECISION PROBLEM IN MINE PLANNING

Sometimes when the apex of the cone is set over an ore block (positive node), some or many of the overlying nodes inside the cone will also fall inside another cone whose apex is set over a different ore block. An example for this situation is illustrated in Figure 1.8. As shown in this figure, node C falls inside both of the two cones whose apex are set over two different nodes, node A and node B. There are generally many blocks within the area overlapped by more than one cone. The problem is deciding which block should be mined together, or choosing the right connections between nodes to achieve an objective such as maximization of NPV, or maximization of metal content, or minimization of stripping ratio. For example, in the figure, should node C be connected with node A or node B?

In the given example in Figure 1.8. assume that node B has higher dollar value than node A. If an algorithm aims to maximize the undiscounted value of a project, it will choose to connect node C with node B. This connection will allow node B to be mined before node A. This means that all the nodes within the cone whose apex is set over node B have to be scheduled in an earlier period than block A and all the blocks that fall in the cone of block A.

![Diagram](image.png)

Figure 1.8. Node C is inside both of the two cones whose apex are set over two different nodes, A and B.)
1.4 PURPOSE OF THE STUDY

The purpose of this study is to develop a methodology for offering "optimum" open pit mine schedules for the mine life. The term optimum is defined herein to mean maximizing NPV of cash flows while meeting mine, mill, refinery capacity, grade, and pit slope constraints. It is believed that the NPV maximization objective with the required constraint can be achieved by programming the open pit scheduling problem as a mathematical formulation model. In order to be able to develop a mathematical formulation model of the scheduling problem that can be used in actual mine deposits, the blocks need to be combined in such a way that the optimality will not be lost.

Since the overall NPV defines the whole economic outcome of a mining project, NPV maximization is the main objective of most of the mining companies. Much effort has been devoted to solving the optimum pit scheduling problem, but there has been no success in developing a scheduling method to give optimum results in maximizing NPV of a mining project. So, NPV maximization in open pit mine scheduling is the ultimate goal in this thesis research.

In order to achieve the ultimate goal in this dissertation, a mathematical programming model for the open pit mine scheduling problem is considered to be necessary. It is already known in the industry that the open pit mine scheduling problem can be formulated as a Mixed Integer Programming (MIP) - a mathematical programming model - to obtain the optimum scheduling result. However, the number of variables required to formulate the scheduling problem as a mathematical model for a real mine deposit using the mine blocks is too great to solve the formulation within a reasonable time using the computers available today.

In order to overcome the problem of the number of variables in developing a mathematical formulation model for the scheduling problem, the blocks can be combined into
larger volumes. There are many different methods used in the industry to aggregate the blocks that generate larger volumes called “pushbacks” (discussed in section 2.5). However, these traditional methods combine the blocks into too big volumes to get an optimum result from the scheduler. Since these volumes of pushbacks are so big, the optimum extraction sequence of the blocks cannot be determined by any of the known schedulers.

In order to combine the blocks properly to obtain the optimum scheduling result, the fundamental tree concept will be developed in this dissertation. A fundamental tree is defined as a set of aggregated blocks with the following properties:

1. the blocks belonging a fundamental tree can be profitably mined,
2. these blocks obey the slope constraints, and,
3. there is no subset of the chosen blocks that meets 1 and 2.

In the definition of the fundamental trees, property 1 ensures that each aggregated set of blocks is economically worthy to mine. Property 2 is necessary for the safe extraction of the combined blocks. Property 3 is required to ensure that the blocks will not be combined into too large volumes, which makes it impossible for scheduler to generate the extraction sequence to give the optimum result in NPV maximization. The objective of the fundamental tree concept is to combine the blocks into volumes that are large enough to decrease the number of the variables required in mathematical formulation of the scheduling problem to a practical level, and small enough for the mathematical formulation model to generate an optimum solution in NPV maximization. A Linear Programming (LP) mathematical model will be developed to generate the fundamental trees. It will be shown that the LP model provides aggregations of blocks that have the same properties as the fundamental trees for any mine deposit.

A mathematical formulation, Mixed Integer Programming (MIP), will be developed to determine the yearly production schedule of the mine based on the fundamental trees. The MIP scheduling model will further aggregate the fundamental trees to generate the
yearly production schedule in such a way that the overall NPV of the project will be maximized satisfying all the operation constraints.

Fundamental trees will be found for a multi-mineral copper deposit in Peru, South America, using the LP model. Then, the deposit will be scheduled to annual production satisfying the mine operational constraints. The scheduling results based on the fundamental tree model will be compared with the three most widely used scheduling programs: Mintec’s M821V, Earthwork’s NPV Scheduler, and Whittle’s Milava mine scheduler.

1.5 SCOPE OF WORK

A general overview of the mine planning process will be presented in Chapter 2. Previous attempts to solve the production scheduling problem in order to maximize the overall NPV of a mining project will also be discussed in this chapter.

The LP model formulation of the fundamental tree algorithm will be discussed in Chapter 3. The formulation will be presented using two example 2D block models. It will be shown in Chapter 4 that the aggregation of blocks produced by fundamental tree algorithm are fundamental trees.

The MIP formulation of the open pit mine scheduling problem will be discussed in Chapter 5. The objective function and the formulation of the required production constraints will be presented. The MIP formulation will also be illustrated on a 2D example data set.

The implementation of the fundamental tree algorithm on a multi-mineral copper deposit in Peru, South America, will be discussed in Chapter 6. Once fundamental trees are found for the deposit, the MIP scheduling model formulation will be used to schedule the mine to yearly production. This production scheduling formulation will include all the
applicable production constraints for the mine. Scheduling outcome - such as annual ore and waste tonnages, cash flows, and NPVs - will be discussed.

The scheduling results will be compared with those coming from the schedules of Mintec's M821V, Earthwork's NPV Scheduler, and Whittle's Milava mine scheduler programs in Chapter 7.

The conclusions of the dissertation will be summarized, and recommendations for future work will be discussed in Chapter 8.
CHAPTER 2

OVERVIEW OF THE MINE PLANNING PROCESS

2.1 INTRODUCTION

Historical developments in mine planning can be classified into two major groups as "traditional mine planning" and "computerized mine planning." Computerized mine planning includes four major stages:

1. development of block model (geological and economical),
2. determination of the final pit limits,
3. generating mineable volumes within the final pit limits, and,
4. production scheduling for certain time intervals (yearly, monthly, weekly, or daily) throughout mine life.

2.2 TRADITIONAL MINE PLANNING

Before the development of computer technology in the 1960's, mine planning was done by hand calculation and trial and error methods using cross-sectional maps (Koskineniemi 1979, Soderburg 1968). The incremental pit limits were drawn on cross-sectional maps of the mineralized area according to grades observed and considering the pit slope angles, geological and economical conditions. The final pit limits were expanded or con-
tracted at each trial to find the incremental pit limits that satisfy a minimum profit requirement.

The profitability of an increment was directly related to the ratio of tons of waste to tons of ore. This ratio is called the breakeven stripping ratio or BESR (Soderburg 1968, Dagdelen 1985) and defined as:

$$\text{BESR} = \frac{(\text{Recoverable Value/ton ore}) - (\text{Production Cost/ton ore})}{\text{Stripping Cost/ton waste}}$$

Sometimes a minimum acceptable profit was also included as a cost in the above formula. This stripping ratio defined the limits where it was uneconomical to remove ore given considering the amount of waste that must be removed in order to reach the ore.

The ore and waste classification between the incremental pit limits were based on a constant breakeven cutoff grade. The tons of ore were calculated based on this constant breakeven cutoff grade by planimetering the areas on the cross-sections. The workable pits were obtained by putting these cross-sections together and smoothing between them (Pana 1965, Johnson 1968).

It has been discussed by Lane 1964, Johnson 1968, and Dagdelen 1985, that there are many limitations and faults in mine planning with the traditional methods. One of these limitations is that these methods are based on a trial and error process which can not ensure optimality. Another one is that application of constant cutoff grade and breakeven stripping ratio does not maximize discounted cash flows.

In the 1960s, computers began to be used in mine planning, replacing the traditional, hand calculated trial and error methods. Some of the limitations of the traditional method were eliminated by the computerized approach. The two-dimensional cross-sections were replaced by three-dimensional geologic and economic block models. The trial and error breakeven stripping ratio analysis of ultimate pit limits were replaced by three-
dimensional ultimate pit limit algorithms. Development of extraction sequences were carried out interactively using computers (Dagdelen 1985).

2.3 BLOCK MODEL DEVELOPMENT

Since the development of computer technology, most of the open pit mine plans are based on geologic and economic block models. The block model is generated by dividing the mineral deposit into rectangular blocks (Barnes 1982, Johnson 1968, Pana 1965, Dagdelen 1985). The size of block is generally determined considering the equipment size to be used for excavation. The block height is generally the same as the bench height. The rectangular block model is the most readily acceptable in the computer environment and the easiest to process.

Grades of the different minerals in the deposit, recoveries or volumes when a block is processed can be assigned to each block. An example block model is shown in Figure 1.1. A distance weighting techniques such as Polygon Method, Inverse Distance Power Method, and Triangulation Method, or Geostatistical methods such as Ordinary Kriging, Indicator Kriging, and Disjunctive Kriging can be used to interpolate the data available at certain points throughout the whole deposit.

The geological block model is converted into economical block model using the parameters given in Table 2.1 as a final step in the block model generation. The final block model is usually called the “economic block model.”
Table 2.1. List of unit cost items for block evaluation (Crawford and Dawey, 1979)

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drilling</td>
<td>$/ton ore and waste</td>
</tr>
<tr>
<td>Blasting</td>
<td>$/ton ore and waste</td>
</tr>
<tr>
<td>Loading</td>
<td>$/ton ore and waste</td>
</tr>
<tr>
<td>Hauling</td>
<td>$/truck hour</td>
</tr>
<tr>
<td>Haul roads</td>
<td>$/truck hour</td>
</tr>
<tr>
<td>Waste dumps</td>
<td>$/ton waste</td>
</tr>
<tr>
<td>Pit pumping</td>
<td>$/ton ore</td>
</tr>
<tr>
<td>Mine general</td>
<td>$/ton ore, or ton waste</td>
</tr>
<tr>
<td>Ore reloading</td>
<td>$/ton ore</td>
</tr>
<tr>
<td>Ore haulage</td>
<td>$/ton ore</td>
</tr>
<tr>
<td>Concentrating</td>
<td>$/ton ore</td>
</tr>
<tr>
<td>Concentrate delivery</td>
<td>$/ton concentrate</td>
</tr>
<tr>
<td>Smelting</td>
<td>$/ton concentrate</td>
</tr>
<tr>
<td>General plant</td>
<td>$/ton ore</td>
</tr>
<tr>
<td>Blister casting, loading and freight</td>
<td>$/ton blister</td>
</tr>
<tr>
<td>Refining</td>
<td>$/ton blister</td>
</tr>
<tr>
<td>Selling and delivery</td>
<td>$/lb refined copper</td>
</tr>
</tbody>
</table>

Metal Prices

- Copper: $/lb
- MoS₂: $/lb
- Gold: $/oz
- Silver: $/oz
2.4 ULTIMATE FINAL PIT LIMITS

The optimum ultimate pit limit is defined as the contour after extracting the volume of material that provides the total maximum profit obeying only the geometric constraints of safe wall slope (Johnson and Sharp 1971). A feasible solution of a problem can be defined as the solution that obeys all the constraints of a given problem. There may be many feasible solutions that do not violate the slope constraint for the ultimate pit limit problem. Each feasible solution draws boundary lines around some blocks, or volume, within the specified blocks and each of these volumes obey the geometric constraints (slope constraints). The term optimum refers to the feasible solution that has the maximum profit among all the possible feasible solutions.

The computerized ultimate final pit limit algorithms can be classified into two groups as 1. heuristic, and 2. true optimizers. These two types of ultimate final pit limit algorithms are given by Kim 1979 as follows: heuristic methods are defined as the methods that work in most cases but lack rigorous mathematical proof. The true optimizing algorithms are the methods that are mathematically proven to find the optimum ultimate pit limits.

A list of some of the widely used ultimate pit limit algorithms are given by Dagdelen 1985 as follows:

a) Moving Cone Heuristic Algorithms

1. Moving Cone of Kennecott (Pana 1965, Pana and Carlson 1966),
2. Heuristic Moving Cone (Lemieux 1979),
3. Heuristic Moving Cone (Mario and Slama 1973),
4. Heuristic Algorithm (Phillips 1973),
5. Heuristic Algorithm (Korobov 1974).
b) Dynamic Programming Heuristics
   1. Lerchs and Grossmann 2-D Dynamic Programming (Lerchs and Grossmann 1965),
   3. Barnes' Best-Valued Dynamic programming (Barnes 1980 and 1982),
   4. Braticevic 3-D Dynamic Programming (Braticevic, 1984)

c) True Optimizing Algorithms
   1. Lerchs and Grossmann's 3-D Graph Algorithm (Lerchs and Grossmann's 1965, Gilbert 1966, Lipkewich and Borgman 1969, Chen 1976),

The Moving Cone Methods have been the most widely used algorithms to design the ultimate final pit limits in the heuristic category. Their popularity is due to their being easy to understand and implement on computers. These methods can be classified in two general groups in terms of their implementation as positive and negative moving cone methods. In the positive moving cone method, cones are generated and tested starting from the top level. The apex of the cone is put over positive value - blocks and the value of all the blocks within the cone are added. When a positive cone in which the total value of all the blocks is positive, is found, it is included in the ultimate pit. Searching process for positive cones continue until no more positive blocks can be found in the block model.

In the negative moving cone algorithm, negative cones are searched starting from the bottom level of the block model. When negative cones are found, they are excluded from the block model. The process continues until no more negative cones can be found.
One of the major disadvantages of cone methods is that the vertex of the cone can be put only on one ore block at a time. It can not consider the profitable combinations of two or more positive blocks. Since searching for all possible profitable combinations of positive blocks is not possible using cone method, generally the moving cone method doesn't find the true optimum. It misses profitable combination of the cones, and it can overextend the pit limits.

The dynamic programming heuristic methods are mostly based on the Lerchs and Grossmann's 2-D algorithm that finds the ultimate pit limits on a 2-D cross-section. These methods try to find 3-dimensional pit limits by applying 2-D dynamic programming algorithms repeatedly on cross sections. Barnes (1982) showed that even though these methods are not true optimizing techniques, they give very good results in finding the ultimate pit limits.

Lerchs and Grossmann's (LG) algorithm (1965) is one of the true optimizing methods based on the concepts of the graph theory in finding the ultimate final pit limit. A directed graph is created with a set of vertices $X_i$ and a set of arcs $A_{ij}$, $G(X_i, A_{ij})$. Each vertex $X_i$ represents a block with the volume $V_i$, and an arc $A_{ij}$ is formed from the block $X_i$ to $X_j$, if the block at the node $i$ can not be mined before mining the block at the $j$-node. Any feasible pit is represented by a closure of the graph $G$, that is a set of vertices $Y$ such that if a vertex $X_i$ belongs to $Y$ and if the arc $A_{ij}$ exist, then the vertex $X_j$ must also belong to $Y$. Maximum total mass, or maximum closure, is found in the graph $G$. A set of initial trees are first constructed to find the maximum closure of a directed graph. Then, each tree is transformed into new trees by using predetermined rules until no transformation is possible.

Bond (1995) has also explained the LG method in details and illustrated its proof of the optimality. He has formulated the LG algorithm as a Mixed Integer Programming (MIP) model to increase the efficiency of the method.
Johnson (1968) developed a maximum flow network model method to solve the ultimate pit limit problem. His method produces optimum results in maximization of the undiscounted dollar value in finding the ultimate pit limits for any deposit. He formulated the multi time period scheduling problem as a mathematical programming model.

Zhao and Kim (1992) simplified and improved the Lerch's - Grossmann algorithm. Their algorithm produces a true optimal solution and maximizes the total undiscounted net profit for a given 3-D block mine model. They tried to overcome joint support and reallocation problems by using new rules in generating and transforming trees. The three dimensional block model is formulated into a directed graph consisting of many trees. The vertices in the graph are equated to the blocks in the mine model and pit slope constraints are represented by arcs generated between either ore to waste, or waste to ore vertices. A positive upward arc is generated if the value of ore block is greater than the absolute value of a waste block. i.e., ore block can support the waste block completely. If the value of a positive block is less or equal to the absolute value of a negative block, a downward arc is generated, i.e., ore can partially support the waste block. They stated that it might be three times faster than the original LG algorithm.

2.5 PUSHBACK DESIGN

After the ultimate pit limit is defined the next step is to divide the material inside ultimate pit limits into smaller volumes called "pushbacks," or "incremental pits," or "nested pits." These push backs define the limits that can be combined into a sequence of extraction. The undiscounted economic value to be generated from the mining is determined by the total available economic value determined from the economic block model within the ultimate pit limits. However, the annual cash flow and the resultant NPV depends very much on the extraction sequence to be followed during mining operation.
There have been many attempts to determine the optimum extraction sequence of mining to maximize the overall NPV of the project.

The moving cone method is also used to determine the extraction sequence of a mine. After finding the ultimate final pit limits, cones are used to find the best pit for different cutoff grades. The optimum pit for the highest possible cutoff grade forms the most inner push back, and the optimum pit for the lowest possible cutoff grade is the most outer phase. The other possible cutoff grades in between these values make the increments of pushbacks.

Another way of finding pushbacks is the repeated use of an ultimate pit limit algorithm on a modified economic block model. The pit size can be enlarged or decreased by increasing or decreasing either cutoff grades or the price of metal (Crawford 1976, Mathieson 1982). By successively changing cutoff grades or the price a number of nested incremental pits can be obtained. For example, Lerchs and Grossmann’s algorithm (1965) is applied to find the pushbacks. One way of finding the incremental pushbacks is that one can use different alternative prices to obtain the value of blocks. By using the lowest possible price, the smallest pit is obtained, and the largest pit is obtained by using the highest possible price. The pits that can be obtained with the values of the price in between maximum and the minimum form the pushbacks.

Dagdelen and Francois - Bongarcon (1982) have introduced double parameterization of recovered reserves in open pit mining following Francois - Bongarcon and Marechal’s study. A pit belongs to the parameterization if and only if, among all of the possible pits of size (V, T), it contains the greatest quantity of recoverable metal (i.e., only technically optimum pits are included in the parameterization). The pits maximize the function $Q - \lambda V - \theta T$ for all the values of the parameters $\lambda$ and $\theta$, where V is the total tons, T is the ore tonnage.
The results do not parameterize the reserves in terms of economic parameters (e.g. metal price, mining cost, cutoff grades, etc.). Rather, reserves are parameterized into a number of technically optimal pits among which economically optimum pit will be searched independently of how these pits are obtained. Finding all the technically optimal pits for different pairs of λ and θ, automatically result in finding all the pits with maximum quantity of metal for all the different pairs (V, T). The study showed that either λ-parameterization or θ-parameterization could be used to come up with complete double parameterization. However, θ-parameterization is more costly to run. Because when θ changes the values for the recovery function RF(θ) and the metal quantity q(θ) for each block must be recalculated.

Whittle (1988) reduced the number of economic variables which affect open pit design to one major and one minor factor in his four - D model. Blocks are called ore if their value is positive, waste if negative, and air if zero. In estimation of a block value, the amount of metal (metal), ore and waste are considered to be constant. Variables are price (p), mining cost (costm), and processing cost (costp). A block value is estimated as:

\[ \text{metal} \times \text{Recovery} \times p - \text{ore} \times \text{costp} - \text{rock} \times \text{cost} \]

Dividing the whole expression by costm, two ratios are obtained: costp/costm (cratio) is not expected to change significantly unless there is a significant change in one of the cost components, or new mining or processing methods are introduced. Whittle assests that the only significant unknown is price/costm (mcostm), which is the amount of product that should be sold to pay for the mining of a tonne of waste. By using LG method, nested pits are generated for different values of mcostm.

Seymour (1995) applied Vallet's method (1976) to divide the ultimate pit into smaller sizes. Among the same size (volume) pits, the pit with the maximum value is obtained. These maximum value pits (pushback) are then sequenced according to their value. Strength of a branch is defined as the ratio of the cumulative value to the cumula-
tive mass. The arc is always generated from a lower strength node to a higher strength node. A high threshold cut-off is used for a branch to be considered as strong. Therefore more nodes became weak and dropped from consideration, and the solution is obtained more quickly. After the result is obtained a lower cut-off is applied and remaining blocks are considered. The process is repeated until the lowest possible cut-off is applied. Several runs are made for different prices and cost to get pushbacks.

In 1993, for the first time, Wang and Sevim put an upper limit into the size of the incremental pushbacks to solve the gap problem existing in the previous methods. Their idea is based on the assumption that finding the maximum metal pit of m blocks is the same as eliminating N the least-metal containing blocks out of M+N blocks. These N blocks are searched by using a downward cone template as opposed to upward cone in moving cone methods. They stored the cones which contain N blocks or less in an array with an ascending order of their average grade. So, N is the maximum number of blocks that form a push back. At the end the maximum metal blocks build the optimum pit and N or less number of blocks became the incremental pushbacks.

Ramazan and Dagdelen (1998) developed the minimum stripping ratio method to generate the pushbacks without creating a gap between them. The block model is modified by assigning indicator 1 to ore blocks and 0 to waste blocks. Then, Vallet's method is applied to this modified block model to generate pushbacks. The first pushback obtained has the lowest stripping ratio and the last push back has the highest stripping ratio among all the pushbacks. Indicators eliminated the effect of the high economic value blocks that are causing the gap problem. The minimum stripping ratio method is optimum in minimizing the stripping ratio of each pushback produced.
2.6 PRODUCTION SCHEDULING

After designing pushbacks, the next major step in mine planning is to determine a sequence of extraction for each year over the mine life. Before determining the extraction sequence, the cutoff grade to separate ore and waste is determined. For scheduling purposes, generally a fixed break-even cutoff grade is used (Milner 1977, Mathieson 1982). The break-even cutoff grade can be defined as the minimum grade that the revenues from mining that minimum grade ore can barely justify the mining and processing cost (Dagdelan 1985).

In determining the annual mining schedule, some constraints such as maximum ore production capacity, maximum waste stripping capacity, minimum ore production requirement, minimum and maximum grade of the ore that can be processed at the mill, etc. must be considered.

When using the pushback method production schedule for the mine life is determined by considering the amount of ore and waste tonnage within the pushbacks, or incremental cuts. The scheduling process generally starts from the top of the first pushback and goes down to the bottom bench of the first pushback. Then, it starts from the top bench of the second push back and moves down to the bottom bench of that push back. The process continues until the final pit limits are reached.

Johnson (1968) developed a mathematical model for the long-term scheduling problem of open pit mines which includes time value of money and different processing types. He decomposed the complex formulation into one full master problem and some smaller sub problems using Dantzig-Wolfe decomposition principles. Each of the sub-problems could be solved as a single time period problem, which has the same characteristics as the ultimate pit limit problem. He indicated that once the sub-problem was solved, solving the master problem was relatively simple, and each of the sub-problems were solved applying his maximum network flow algorithm. His model considered dynamic cutoff
grades rather than static cutoff grade, because he considered the mine optimization as a
time dependant process.

Dagdelen (1985) has formulated the long-term production scheduling problem as a
mathematical model. He applied the lagrangian method and subgradient method to solve
the multi-time period scheduling problem considering different types of materials in his
formulation. Using the lagrangian method, he decomposed the complex multi-time period
problem into smaller single time period problems that can be solved using optimum final
pit design methods such as the maximum flow algorithm. He applied the sub-gradient
method to find the Lagrangian parameters in his mathematical formulation.

Tolwinski (1998) has combined the blocks on the same bench and called them atoms.
Then, using the LG method, he generated pushbacks combining these atoms. After find-
ing pushbacks, he scheduled the mine using dynamic programming. His scheduling
method is a part of NPV scheduler program. He summarized his scheduling method as
follows:

"The scheduling algorithm, or "Scheduler" defines optimization objec-
tives and constraints in terms of target variables, where the target vari-
able are functions of block attributes. To find an optimal solution for
given objectives, scheduler divides the mine into elementary units called
atoms and builds a tree of potential solutions. The nodes in this tree de-
scribe the state of the mine at a given point of time and the arcs corre-
sponds to atoms. The information contained in the first tree is used to
built a second tree, where the arcs relate the time rather than atoms. The
second tree includes all schedules that satisfy the given constraints (if
such a schedule does not exist, the constraints are relaxed). Optimal
schedules are then found by dynamic programming."

Akaika (1999) has formulated the production scheduling problem as a mixed integer
programming model. His 4D relaxation method transforms the production scheduling
mathematical model. It is possible to apply the graph theory or network theory to this transformed form. After that he further transformed the network by considering the relaxed production capacity constraints, in order to reduce the effect of the gap problem. He improved the efficiency of the sub-gradient method in lagrangian relaxation problem significantly to reach the optimal solution much faster. The advantage of dynamic cutoff grade concept is considered in his 4-D network relaxation method.

2.7 PROBLEMS WITH CURRENT MINE PLANNING APPLICATIONS

The long-term production scheduling design is a very complex and difficult problem due basically its large scale and the restrictions on the extraction sequence not to validate the maximum allowable pit slope angle. Much effort has been devoted to solving the optimum pit scheduling problem, but there has been no success in developing a scheduling method to give optimum results in maximizing net present value of a mining project.

Johnson's mathematical model (1968) generates optimum results for each period to be scheduled individually. However, since its basis is linear programming, one runs into the problem of dealing with missing fractional blocks some being in the interior of mining unit.

Dagdelen (1985) stated that lagrangian method might not always converge an optimal solution if the multipliers that can result in a feasible solution for the constraint of the problem may not exist. The meaning of this is that the dual solution is not convex and there is a gap between the dual and primal solutions of the problem.

Tolwinski's (1998) application of the LG method for finding pushbacks does not generate an optimum result in NPV maximization due to the gap problem as discussed in the proceeding paragraphs. Dynamic programming gives the best results among some se-
lected alternative solutions. It does not consider all the possible alternatives and it does not result in a truly optimum solution to maximize NPV of a project.

The simplest case of an incremental pit is an ore block together with some waste blocks that have to be removed to reach the ore. The incremental pits can consist of more than one ore block, which may not have positive overall value within their cone when considered individually, but have positive total value within their cone when they are considered together. That is, if the apex of the cone is put on a single ore block at a time as in cone methods, none of these ore blocks may have a positive total value within the cone, but the total overall value of these ore blocks together can be positive. Searching for all possible combinations is not applicable with today's available computer technology due to the time and economic considerations. The moving cone method doesn't find the true optimum and considered to be a heuristic method. Drawbacks of the cone mining algorithms are discussed in Barnes (1980).

A common problem with the available methods in scheduling is the "gap problem." When the grade distribution of the ore body is not uniform, it is possible to have a big size difference between two adjacent pushbacks. This size difference may cause problems in production scheduling as illustrated in Ramazan (1996). There have been some attempts to solve the gap problem.

Wang and Sevim's (1993) approach to solve the gap problem is considered to be a heuristic approach in terms of the maximization of the metal content of the incremental pits due to the strict size limitation of the pushbacks. This method does not always give the highest cash flow stream (Ramazan, 1996).
CHAPTER 3
MATHEMATICAL FORMULATION OF THE FUNDAMENTAL TREE ALGORITHM

3.1 INTRODUCTION

A pure linear programming model is developed to provide fundamental trees. The model is tested on a variety of 2D example block models and every time it produced fundamental trees. Application of the model on a 3D data set will be discussed in the later chapters. In this chapter, the model will be explained using a 2D data set given in Figure 3.1 with 45 degree slope constraints. The slope constraints for ore blocks are illustrated in Figure 3.2.

![Diagram](image)

Figure 3.1. Example 2-D block model, (a) shows the block economic values in $/ton, and (b) shows the node numbers. Slope constraints are assumed to be 45 degree in all directions.
Figure 3.2. The network that shows the slope constraints of ore blocks for the 2D example block model given in Figure 3.1.

3.2 VARIABLE DEFINITIONS

The block model is represented by a network containing arcs, and nodes. A sink node, t, and a source node, s, are added to the block model. The network model is initially obtained by setting arcs in the system in 4 different ways:

1. from the source node towards all the positive nodes,
2. from each positive node to its overlying negative nodes according to the pit slope constraints,
3. from all the negative nodes towards the sink node, and,
4. from the sink node to the source node.

The amount of money that has to be spent from an ore block, i, to justify the cost of mining an overlying waste block, j, is represented as a flow, $f_{ij}$, going through an arc, $a_{ij}$, set from block i pointing to the block j. $X_{ij}$ is used in the network to activate an arc, $a_{ij}$. If there is a flow going through an arc, $a_{ij}$, the arc is activated by setting $X_{ij}$ parameter to a
number greater than zero. If there is no flow going through the arc, the arc is not activated setting $X_{ij}$ parameter to 0.

A node, $n_i$, is the representation of a block $i$, which has an economical value to be generated by mining that block. It is assumed that the economic values of the nodes are predetermined and are known. Each node has an economic value of either positive or negative (zero block value is considered as negative).

Each node is assigned a parameter $V_i$ whose value is set to economic value of the block represented by that node.

Source node ($s$) is a node that is not in the block model and it has no value. It is an imaginary node that the arcs are generated from this node to the positive nodes.

Sink node ($t$) is a node that is not in the block model and it has no value. It is an imaginary node that the arcs are generated from the negative nodes towards the sink node.

An arc ($a_{ij}$) is a directed connection made from node $i$ to node $j$ that certain amount of flow can go through from node $i$ to node $j$.

Flow, $f_{ij}$, is the amount of flow on the directed arc going from node $i$ to node $j$.

Flow, $f_{u_{ij}}$, is the upper bound capacity of an arc, i.e. the maximum amount of flow that can be sent from node $i$ to node $j$, determined by the value of the node a particular arc is pointing to.

The coefficient $M$ is used as a big number (big $M$), which is bigger than any flow in the network.

Mutual support refers to the support of a waste block by more than one ore block.

The coefficient, or rank $C_i$, is obtained using the cone value of ore block $i$ as discussed in section 3.3. This coefficient assignment is to force the LP to start arc and flow setting from the highest cone value block. This procedure cooperates with other constraints (discussed in the proceeding sections) to result in trees that have all the properties of fundamental trees.
The cone value of node $i$, $CV_i$, is defined as the total value of all the blocks inside a cone whose apex is set on positive node $i$.

3.3 STEPS OF FUNDAMENTAL TREE ALGORITHM

The implementation of the fundamental tree algorithm is shown step by step in Figure 3.3. The first step is to find the ultimate pit limits of the block model. It is necessary to find the ultimate pit limits using one of the optimum ultimate pit limits algorithm such as LG, and Maximum Network Flow algorithms.

The second step is the determination of the cone values, $CV_o$, for all the ore blocks within the ultimate pit limits. To do this, the apex of a cone is set over an ore block and the economic values of all the blocks inside the cone are added. This procedure is repeated for all the ore blocks.

The third step is to assign coefficients to ore blocks. This process must start from the top bench and move down to the bottom bench. In the most top bench where some ore blocks exist, coefficients of the ore blocks in this bench are set starting from 1. The ore block that has the highest cone value among all the ore blocks on the same bench is set to 1. The ore block that has the second highest cone value on that bench gets 2, and the coefficients are assigned for all the ore blocks on the same bench. Then, the ore blocks on the one lower bench are assigned to coefficients. The smallest coefficient at this bench will be one plus the biggest coefficient assigned on the upper bench. The coefficients are assigned in the same way as upper bench. The coefficient assignment procedure is repeated for all the ore blocks.

If some ore blocks on the same bench have the same cone values, coefficients are assigned randomly.
Figure 3.3. The general flowchart showing the steps of the fundamental tree algorithm.
After coefficients are assigned, the mathematical formulation of the fundamental tree algorithm can be generated as discussed in the proceeding sections in this chapter. After setting up the objective function and the model constraints as Linear Programming (LP) formulation, the model is solved using CPLEX solver software (CPLEX, 1998).

After obtaining the solution, the problem is reformulated as discussed in sections 3.5 and 3.6. The formulation is solved again. If the number of trees obtained in this solution is the same as the previous one, the solution is optimum and the trees are fundamental trees. If the number of the trees is not the same as the previous solution, the problem reformulated and solved again until the same number of trees are reached. If the number of the trees found with the current solution is the same as the number of trees found from the previous solution, the result is optimum, and the algorithm is stopped.

3.4 LINEAR PROGRAMMING FORMULATION OF FUNDAMENTAL TREE PROBLEM

3.4.1 LP Objective Function

\[
\text{Minimize } \sum_{i}^{n} \sum_{j}^{w} C_{i} \cdot X_{ij}
\]

Where i is the ore block index, j is the overlying waste block index, n is the number of ore blocks, w is the number of waste blocks overlying a given ore block i according to pit slope constraints. \(C_{i}\) is the coefficient discussed in the previous section.

The mathematical model is established as a minimization problem which minimizes the connections between blocks. The meaning of the objective function is that it is always
more desirable to send flows from the nodes that have lower coefficients. Therefore, LP prefers to make arcs and send flows from the highest cone value (lowest coefficient) ore block. Since for every unit of flow, it is more desirable to send the flow from the highest cone value block, LP tends to send all the required flows from the highest cone value ore block on the same bench until either all the overlying waste blocks are totally supported or there is no more flow left on that ore block. Then, LP will move to the next higher cone value ore to send the flows that are required to support the overlying waste blocks.

This ranking \((C_i)\) in the mathematical formulation enables higher cone value ore blocks to be mineable before the lower cone value blocks.

### 3.4.2 Problem Constraints

The fundamental tree mathematical model requires the constraints discussed in the proceeding sections to be able to generate fundamental trees that are mineable. Here mineability refers to that none of the trees violates the pit slope constraints and all the trees must have positive total economic values.

#### 3.4.2.1 Upper Bound Constraints On Flows From Source Node To Positive Nodes

\[ f_{si} \leq V_i, \text{ for all } i's. \]

where \(s\) is the source node, \(i\) is the block identification number for a positive value (ore) block. \(f_{si}\) is the flow sent from source node to node \(i\), \(V_i\) is the economic value of block \(i\).

These constraints ensure that an ore block does not support more waste removal than its own economic value. Moreover, the parameters in these constraints may be viewed as identifiers of the fundamental trees in the model at the end of the solution. If there is a \(f_{si}\)
parameter whose value is strictly less than the value of ore block i, (i.e. \( f_i < V_i \)), this ore block is the root of a fundamental tree. The other ore blocks do not identify a fundamental tree, which means they are connected with other ore blocks to be able to support the cost of the overlying waste blocks.

3.4.2.2 Constraints On Flows From Negative Nodes To The Sink Node

\[ f_{jt} = -V_j + \xi, \]

where \( \xi \) is a small decimal number, \( j \) is a waste block identification number, and \( t \) is the sink node. \( \xi \) is assigned to the smallest possible number that will not be ignored by the solver used to solve the mathematical formulation. For the 2D example, 0.001 may be used.

The above flows are the flows between waste nodes, \( j \), and sink node, \( t \). These constraints ensure that the waste blocks are fully supported. The decimal value is used to establish the mutual support of the waste blocks whenever it is necessary. This epsilon also ensures that each fundamental tree will have a positive value.

3.4.2.3 Flow Constraints Around Each Node

There are four types of flow conservation equations in the model:

1. the flow balance around waste nodes,
2. the flow balance around the ore blocks,
3. the flow balance around the source node, and,
4. the flow balance around the sink node.
1. The total flow coming to a waste block must be equal to the flows leaving that waste block:

\[ \sum_{j=1}^{O} f_{ji} - f_{ji} = 0, \text{ where } O_j \text{ is the number of ore blocks underlying waste block } j. \]

2. The total flow coming to an ore block from the source node is equal to the total flow leaving that ore block:

\[ f_{si} - \sum_{j=1}^{W} f_{ij} = 0, \text{ where } w_i \text{ is the number of waste blocks overlying positive node } i. \]

3. The total flow going out of the source node is equal to the flow coming to the source node:

\[ f_{is} - \sum_{i=1}^{O} f_{si} = 0, \text{ where } O \text{ is the total number of ore blocks in the model.} \]

4. The total flow coming in to the sink node is equal to the flow going out of sink node:

\[ f_{ir} - \sum_{j=1}^{W} f_{jr} = 0, \text{ where } W \text{ is the total number of waste blocks in the model.} \]

The constraints at item 3 and 4 are not really needed. They are written just to see the full structure of the problem. In reality, we are only interested in the variables between waste and ore blocks. These two constraints are redundant constraints for "fundamental tree" model.
3.4.2.4 Active Arc Indicator Constraints

\[ f_{ij} - Mx_{ij} \leq 0, \] where \( i \) is an ore block, \( j \) is a waste block, and \( M \) is a big number, which is bigger than the biggest possible flow in the network. On the other hand, this number should not be too big which may cause the \( x_{ij} \) parameters be zero. Therefore, this number should be small enough so that when there is any flow on the \( a_{ij} \) arc, the \( x_{ij} \) will be a number greater than zero.

The big \( M \) in front of the \( X \)-parameter in these constraints is to ensure that if there is a flow on the arcs, \( X \)-parameter will be set to a number greater than 0, and if there is no flow on those arcs, \( X \) would be set to 0.

3.4.2.5 Model Solution

The optimum solution has two fundamental trees which are rooted at node 8 and node 9 since \( f_{8s} < 7 \) and \( f_{6s} < 6 \). Notice that \( f_{6s} = 2 \), which is the economic value of node 6, therefore it is not identified as a fundamental tree. The result is shown graphically in Figure 3.4.

It should be noticed that each of the two fundamental trees generated by LP has three properties:

1. each fundamental tree has positive combined value,
2. none of the trees violates the mining slope constraints, and,
3. there is no subset of the chosen blocks that meets 1 and 2.
Figure 3.4. LP solution provided two fundamental trees. One fundamental tree is rooted at node 8 and the other one is rooted at node 9.
3.5 MULTIPLE ITERATIONS OF THE LP MODEL

Sometimes more than one iteration for the formulation and solution may be necessary to ensure the optimality of the results. Optimality refers to the trees which have the three properties: 1. all of the trees have positive values, 2. none of them violates the slope constraints, and 3. none of them has any sub fundamental tree. The optimality of the fundamental tree result may fail when the initial coefficients obtained by ranking of the ore blocks in decreasing order of cone value change after some initial set of fundamental trees are generated. That means after some overlying blocks are connected to other blocks forming a fundamental tree, the cone value of the blocks that are located underneath those blocks may change. If the cone value changes, the ranking, or the objective function coefficients also need to be changed. This requires the problem to be reformulated and resolved. The following iteration steps are applied to obtain the optimum solution in generating fundamental trees:

Step 1. Set up the initial formulation as explained in section 3.4 and solve the problem.

Step 2. Re-identify the overlying waste blocks. If a waste block j overlying an ore block i, is included in a tree different than the tree in which the block i is included, block j is not considered as an overlying block of i. That means the node precedence is determined only by the activated arcs in the previous solution. After re-identifying overlying blocks, formulate and solve the problem again.

Step 3. If the new solution produced the same number of trees as the previous solution, the result is optimum, stop. Otherwise, repeat steps 2 and 3.
3.6 AN EXAMPLE OF THE LP MODEL FOR MULTIPLE ITERATIONS

A 2D example block model given in Figure 3.5 is used to apply the iterative solution technique of LP to reach the optimum result. The initial node precedence (overlying node relationship) assuming 45 degree slope constraints is illustrated as a network in Figure 3.6.

3.6.1 Application of Step 1

The first step is the initial LP formulation of the problem. The problem can be formulated as discussed in section 3.4 using the network given in Figure 3.6.

```
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<th>-5</th>
<th>-5</th>
<th>-10</th>
<th>-3</th>
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<th>3</th>
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</tr>
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<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a)   (b)
```

Figure 3.5. A 2D example block model requiring the iterative LP solutions. (a) is the block economic values, and (b) is the node numbers.

The intermediate solution of the problem is shown graphically in Figure 3.7. The two trees identified by this solution are rooted at node 7 ($f_{s7} < V_7$), and at node 8 ($f_{s8} < V_8$).
Figure 3.6. The node precedence relationship and the network model representation of the 2D example
Figure 3.7. The intermediate LP solution from the initial formulation obtained using CPLEX software for the example given in Figure 3.6.
3.6.2 Application of Step 2

The cone values (CV) and the objective function coefficients (C) of ore block are calculated as discussed in section 3.3. This time Figure 3.7, the updated network configuration, needs to be used in determination of these parameters.

The second Linear Programming formulation of the problem can be set up in the same way as discussed in section 3.4 using the new coefficients and the new network shown in Figure 3.7.

The solution of the problem is illustrated graphically in Figure 3.8. The three trees identified by this solution are rooted at node 7 ($f_{77} < V_7$), node 8 ($f_{88} < V_8$), and node 9 ($f_{99} < V_9$).

Numbering of the trees in this example is only based on the slope constraints. This numbering of the trees is not necessarily the sequencing of the trees. The sequencing is determined by MIP scheduling model as discussed in chapter 5.

3.6.3 Application of Step 3

In step 2, the new solution produced more trees (3 trees) than the initial solution. So the problem should be reformulated using the node precedence shown in Figure 3.8. But, for the given example, the repetition is not needed anymore. Because the tree I is already repeated and could not be divided in the second formulation, and trees 2 and 3 have only one ore blocks, which will not be able to produce more trees. Therefore, the solution illustrated in Figure 3.8 is the optimum solution to the problem and the trees can be called as "Fundamental Trees."
Figure 3.8. The final LP solution from the second formulation obtained using CPLEX software for the example given in Figure 3.6.
It should be noticed that these trees also have all the three properties of a fundamental tree:

1. each tree has positive overall value,
2. each tree can be mined without violating slope constraints, and,
3. no sub tree exist within a fundamental tree with properties 1 and 2.

3.7 EXISTENCE OF ALTERNATIVE SOLUTIONS

It should be noticed that there might be more than one unique solution to the mathematical programming model. For example, for the given problem in Section 3.6, an alternative solution is given in Figure 3.9. For the real mining block model, it may be difficult to identify all the possible solutions, but it may be possible to identify more than one solution. For example, in the block model at a given bench, if there are more than one ore block that have the same values, the objective function coefficients are assigned randomly. After one set of random coefficients are assigned, a solution will be obtained for the fundamental tree problem. After obtaining a set of fundamental trees, if the coefficients are switched another set of fundamental trees can be obtained. This different set of fundamental trees represents a different configuration of combined blocks, but the different set of fundamental trees for the same block model will have more or less the same number of trees in it.

In terms of NPV maximization, these different results are not considered to make a significant change, because the high grade material will always be able to support the overlying waste by itself and will be feasible to be mined before low grade material due to the nature of the objective function. Around high grade areas, the configuration of the trees will not change. Therefore some changes in the configuration of low grade areas will not significantly affect the NPV of the project.
Figure 3.9. The alternative LP solution for the example given in Figure 3.7.
CHAPTER 4
OPTIMALITY OF THE FUNDAMENTAL TREE FORMULATION

4.1 PROPERTIES OF A FUNDAMENTAL TREE

A tree can be defined as any combination, or aggregation, of blocks. A tree will be called as “fundamental tree” if it has all the three following properties as discussed in chapter 3:

1. the blocks can be profitably mined,
2. the blocks obey the slope constraints, and,
3. there is no proper subset of the chosen blocks that meets 1 and 2.

In this chapter, it will be shown that the aggregated blocks found by the linear programming model (discussed in Chapter 3) have the properties of the fundamental trees.

4.1.1 The Positive Value Property Of The Trees Found By the LP Model

The first property is that each fundamental tree must have a positive value. If a tree does not have a positive value, it cannot be economically mined. That means the ore blocks do not have enough value to justify the cost of mining the overlying waste blocks.

*Proposition 4.1.1.1:*
A tree obtained by the fundamental tree algorithm always has a total cumulative value greater than zero.

**Support 4.1.1.1**

A tree generated by the fundamental tree algorithm does not require any support from an ore block outside the tree. If required, it would already be supported by that ore block, and that ore block would also be included in this tree. The formulation contains the following model constraints:

\[ f_{j} = -V_{j} + \xi \quad \text{(discussed in section 3.4.2.2), } j \text{ is the waste block identification number} \]

\[ \sum_{\omega} f_{\omega} = 0 \quad \text{(discussed in section 3.4.2.3), } i \text{ is for all the underlying ore blocks of } j. \]

These model constraints ensure that all the waste blocks are totally supported by underlying ore blocks. The fundamental tree model formulation also includes the constraints discussed in section 3.4.2.1. \((f_{\omega} \leq V_{i})\) which mean that none of the ore blocks can send more flow to its overlying waste nodes than what it has. For a given tree, the above constraints can be written as follows:

\[ \sum_{i} V_{i} \geq \sum_{i} f_{i} \]

\[ \sum_{j} (-V_{j} + \xi) = \sum_{i} f_{i} \]

Therefore, \( \sum_{i} V_{i} \geq \sum_{j} (-V_{j} + \xi) \)

Where \( \sum_{i} V_{i} \) is the sum of all the ore blocks in a tree, and \( \sum_{j} (-V_{j} + \xi) \) is the sum of all the waste blocks and the small epsilons assigned to those waste blocks.

Therefore, the minimum value of a tree is the sum of all the epsilons assigned to the waste blocks within the tree. Since \( \xi \) is always a positive number, the total economic value of a tree generated by the fundamental tree algorithm is always greater than zero.

**Proposition 4.1.1.2:**
There always exists a basic feasible solution to the LP fundamental tree problem formulation.

Support 4.1.1.2:

In the previous chapters it has been stated that the fundamental tree model must be implemented only to the portion of the deposit that is within the ultimate final pit limits. The ultimate final pit limits must be determined using an optimum algorithm such as Maximum Network Flow Method (Johnson 1968), or the LG method (Lerchs and Grossmann, 1965). The total economic value of a deposit within the optimum ultimate pit limits is always greater than zero. Therefore, one can consider the entire deposit as a tree. It is not necessarily an optimum solution, but a feasible solution to the fundamental tree problem.

4.1.2 The Validation Of The Slope Constraints Property Of The Trees Found By LP Model

For the safety of a mine, the maximum allowable slope angle must be obeyed in mining operations. If the mining slope is greater than the maximum allowable slope angle, the mine collapses inside. If the mining slope angle is too flat, a lot of extra waste material has to be removed to reach ore and that decreases the profit of the mine. So, the slope constraints are very critical in mining operations.

Proposition 4.1.2.1:

Slope constraints in mining a given ore block are always satisfied within the trees found by the fundamental tree algorithm.
Support 4.1.2.1:

It is shown in proposition 4.1.1.1 that all the overlying waste blocks within the ultimate pit to be formulated must be supported by underlying ore blocks due to the model constraints. Since proposition 4.1.1.2 provides a feasible solution to the fundamental tree problem, all the overlying waste blocks will be supported.

Before the fundamental tree LP formulation is generated, the deposit is represented by a network model. This network model is actually an illustration of the node precedence relationship between the nodes based on the slope angle requirements. The mathematical model formulation is based on the network model configuration. The arcs in the network are set from each ore block to all its overlying waste blocks. And all its overlying waste blocks are forced to be fully supported. Therefore, each ore block, or combination of ore blocks is forced to fully support all its overlying waste blocks. In a situation where an ore block cannot fully support all its overlying blocks, another underlying ore block must exist that can help the ore block to support the overlying waste blocks. This existence of another ore block comes from the fact that the model is applied to the blocks only within the ultimate final pit.

If there is no support to a waste block overlying an ore block, that ore block cannot be in a tree. This is due to the fact that the ore block cannot be included within the optimum ultimate pit. If a tree is removed as a fundamental tree, ore blocks in another tree do not have to support the waste blocks in this tree, but there is a precedence relationship determined as the first tree must be mined before any other tree can be mined. Therefore, the slope constraints cannot be violated by any fundamental tree.
4.1.3 Any Tree Found By the LP Model Does Not Contain Any Other Tree That Meets
The First Two Properties Of The Fundamental Trees

The mathematical programming model developed during this research divides a de-
posit into a set of volumes that are removable by themselves. Each volume is called a
“Tree,” and each tree is called a “fundamental tree” if it has the unique properties initially
defined.

Proposition 4.1.3.1:
The union of fundamental trees found by the fundamental tree algorithm is the ulti-
mate pit.

Support 4.1.3.1:
The fundamental trees found by the fundamental tree algorithm contain all the ore
blocks within the ultimate pit limits. Since all the overlying waste blocks must be sup-
ported due to the constraints discussed in proposition 4.1.1.1, all the waste blocks and
connected ore blocks within the ultimate pits are included in a tree, and none of the
blocks outside the ultimate pit limits are considered in the LP model. Moreover, if an ore
block is not connected to any of the overlying waste blocks, it is considered a tree. There-
fore, the union of the fundamental trees found by the fundamental tree algorithm is the
ultimate pit.

Proposition 4.1.3.2:
One block cannot belong to more than one fundamental tree.

Support 4.1.3.2:
If a block in a tree is connected with another block that belongs to another tree dur-
ing the formulation solution, these two trees are said to be a single tree. If there is no con-
nection between the members of one tree and the members of another tree, the intersection of the trees is said to be empty. Therefore, a block cannot belong to more than two trees at the same time.

**Proposition 4.1.3.3:**

A tree found by the fundamental tree algorithm cannot contain a subset of trees that can be fundamental trees.

**Support 4.1.3.3:**

If a sub-tree exists that is also a fundamental tree it means there is a set of one or more ore blocks in the sub-tree that supports all its overlying waste. If this were so the LP solution would have removed the sub-tree as a fundamental tree. Therefore, no fundamental tree contains a subset that is also a fundamental tree. This condition is set by the objective function.

For example, a 2D small block model has the slope constraints as shown in Figure 4.1. The figure also shows the economic value of blocks, the cone value of ore blocks, and the objective function coefficients of ore blocks. The objective function of the LP model formulation for the ore block variables 4 and 5, and the waste block 2 for this example is as follows:

**Objective function:**

Minimize \( \ldots + X_{42} + 2X_{52} \)
Figure 4.1. A Small 2D example block model configuration showing the slope constraints, economic values of blocks, cone values and coefficient parameters of ore blocks.

Assume that the solution to the above formulation is as shown in Figure 4.2. In that figure, the arc parameters of the shown arcs have to be a number greater than zero. This means that some flow, say $f>0$, is sent from node 5 to node 2, and $X_{52}$ had to be assigned to a positive value $f/100$. The cost of this flow to the objective function will be $2*f/100$. However, if that arc is not connected, and the same amount of flow is sent from node 4, the cost of the same support to the objective function would be only $f/100$. Since $f$ is greater than 0, $f/100<2*f/100$. This violates the objective function’s optimality, which is a minimization function of the $X$-parameters.
Figure 4.2. Assumed solution of the example shown in Figure 4.1. A tree contains a fundamental tree as a subset violating the optimality of the objective function.

Figure 4.3. True solution of the example shown in Figure 4.1. None of the trees generated by the LP model has a subset fundamental tree.
Since the solution to the objective function is optimum, the above assumption cannot be correct. Therefore, a tree obtained by the fundamental tree algorithm cannot contain another fundamental tree as a proper subset. The true solution for this example is given in Figure 4.3.

All the above propositions show that the trees found by LP model are fundamental trees.

It is shown in Chapter 3 that the fundamental tree algorithm contains iterations. As shown in Figure 3.8 that the intermediate solution of the LP model has sub fundamental tree. The reason is that after the trees are generated, the cone values and the ore block ranking have changed. Therefore, the objective function is not representing the current situation correctly. In order to correct the objective function according to the current situation of the model, cone values are recalculated and objective function coefficients are re-assigned through the iterative steps of the algorithm. The trees generated at the end of the algorithm after the iterations do not contain any sub fundamental tree. As shown in Figure 3.9, none of the trees contain a sub fundamental tree.

4.2 SUMMARY FOR THE PROPERTIES OF MATHEMATICAL FORMULATION

The Linear Programming Formulation provides the trees that have the same properties as the fundamental trees for any given block model.

The first property of the tree is that all the trees must have positive overall values. First of all, it should be kept in mind that the LP formulation is set for the blocks that are within the ultimate final pit limits. It is also stated that the ultimate final pit limits are obtained using an optimum ultimate pit limit design algorithm such as LG, or Maximum Flow Network Model methods. Having optimum pit limits ensures that there is a feasible
solution to the problem. The feasible solution means that all the waste blocks can be supported by ore blocks. If the ultimate pit limit is not optimum for the given deposit, the LP formulation of fundamental trees may produce an infeasible solution.

In the LP formulation of the fundamental tree problem, the constraint 3.4.2.1, \( f_{\kappa} \leq V_{\nu} \), ensures that none of the ore blocks can support more waste block value than their own economical value. The constraint 3.4.2.2, \( f_{\mu} = -V_{\nu} + \xi \), ensures that there will not be any waste block overlying an ore block unsupported. Then, constraints at section 3.4.2.3, \( \sum_{i=1}^{n} f_{i} - f_{\mu} = 0 \), establishes the relationships between ore blocks and waste blocks. Therefore, these constraints assure that there cannot be any unsupported waste block in the model, and also none of the ore block can support more than their economical value (i.e. there cannot be any tree with negative overall value). The minimum possible total value of a tree is the small epsilon times the number of overlying waste blocks. Since epsilon is a number greater than zero, the total value within a tree is always greater than zero. If there is no overlying waste block in the tree, the value of the tree will directly be the total value of the ore blocks, which is a positive number. Moreover, it is assumed that the final pit limits are determined using an optimum algorithm. Therefore, there is a feasible solution to this formulation, which means trees with positive values are available within the ultimate pit limits.

For each ore block \( i \), a flow variable, \( f_{s,i} \), is used between the ore block and the source node, \( s \), in the mathematical formulation. If this variable is less than the economic value of block \( i \), this ore block is the root of a fundamental tree. That means after sending all the required flows to support the overlying waste blocks, there is still extra value on this block making the tree economical to mine. Therefore, each fundamental tree provided by the mathematical model must have a positive value. So, the first property of a fundamental tree is achieved.
The second property of the tree is achieved in the mathematical model using a series of equation setups. The coefficient parameter, $C_{ij}$ in front of all arcs going from the ore block $i$ to its overlying waste blocks in the objective function forces LP to send as much flow as possible from the ore block $i$ towards its overlying waste blocks. If the ore block has enough value to support all the overlying waste nodes, the tree containing this ore block and all its overlying waste blocks will directly obey the slope constraints. If it cannot support all the overlying waste blocks by itself, then the overlying waste nodes will have to be supported by another ore block because of the constraints \( \sum_{\mu} f_{\mu i} - f_{\mu i} = 0 \), where $i$ goes from 1 to $n$ representing all the underlying ore blocks of waste block $j$, and $f_{\mu i}$ is the value of waste block (including very little extra negative value) that needs to be supported. Addition of a very small extra value to the waste blocks in the constraints, $f_{\mu i} = w + 0.001$ ($w$ is the waste block value to be supported) enables the mutual support whenever required. All these settings in the constraints and objective function make the produced fundamental trees by the mathematical model obey the slope constraints. The third property of fundamental tree is also achieved.

The Weighting factors in the objective function encourages LP to send as much flow as possible from one ore block once it starts arc-settings and flow-sending process from that block. This weighting factor is obtained by summing up the economical values of all the blocks within the cone set on that ore block. The ore block that has the highest cone value gets the smallest coefficient (details are discussed in Chapter 3). The smaller coefficient means the lower cost to the objective function. Therefore, LP prefers to set the arcs and send the flows from these ore blocks. The process continues until either all the overlying waste blocks are supported, or no more value is left in the ore block. That prevents unnecessary mutual supports during the ore block and waste block connection process. Because the highest cone value ore has the ability to support highest number of overlying waste blocks among all the ore blocks within the same bench. That's why the model
starts setting arcs from the highest cone value block. Therefore, the model generates the 
fundamental trees that do not have sub fundamental trees for a mining block model, 
which is the third property of a fundamental tree. This cone weighting system not only 
achieves the third property of the fundamental trees, but it also selects higher cone value 
fundamental trees to be feasible to mine before the lower value ones, which also contrib-
utes in NPV maximization objective.

Therefore, the optimum solution of the mathematical model has all the properties of 
fundamental trees. Since it has all the properties, the trees generated by the fundamental 
tree algorithm are fundamental trees.
CHAPTER 5
MIXED INTEGER MODEL FORMULATION FOR
THE PRODUCTION SCHEDULING PROBLEM

5.1 INTRODUCTION

In this chapter, Mixed Integer Programming (MIP) formulation of the scheduling program will be provided. In scheduling model formulation, each fundamental tree is treated as a block in setting up the objective function and the model constraints.

5.2 MATHEMATICAL FORMULATION OF THE SCHEDULING PROBLEM

After determining fundamental trees, each tree is treated like a block in the mathematical model. Mixed Integer Programming (MIP) is used to develop the formulation. The objective function is maximization of the Net Present Value (NPV) of the project constrained to satisfy the operational limitations as discussed earlier.

5.2.1 Objective Function

Before writing the objective function, NPV of the trees need to be calculated as discussed below:

In period 0, the average value of ore blocks in a given tree, t, is calculated as:

\[ \text{ValOre}[0][t] = \frac{\text{ValOreTot}[t]}{\text{OreTot}[t]}, \]
Where ValOre[0][t] is the average economic value of ore blocks within the tree, t, at the 0th period, ValOreTot[t] is the total economic value of ore blocks within tree t, and OreTot[t] is the total ore tonnage in tree t.

In period 0, the average economic value of all the waste blocks in a given tree, t, is:

$$\text{ValWaste}[0][t] = \text{ValWasteTot}[t] / \text{Waste}[t],$$

Where ValWasteTot[t] is the total economic value of the waste blocks within the tree t, and waste[t] is the tonnage of waste material in tree t.

In period p, average ore and waste values of a given tree t considering \((d*100)\%\) discount rate:

$$\text{ValOre}[p][t] = \text{ValOre}[0][t] / [1 + d]^p,$$
$$\text{ValWaste}[p][t] = \text{ValWaste}[0][t] / [1 + d]^p,$$

If O[p][t] is the amount of ore scheduled for period p from tree t, then the objective function is:

$$\text{Maximize} : \sum_{p=1}^{\text{periods}} \sum_{t=1}^{\text{trees}} \{(\text{ValOre}[p][t]) \cdot O[p][t] + (\text{ValWaste}[p][t]) \cdot W[p][t]\}$$

Where W[p][t] is the tonnage of the waste to be mined at period p from tree t.

5.2.2 Objective Function Constraints

Two constraints are applied for the objective function to increase the efficiency of the solution of the model.

$$Z \geq RV;$$

$$Z - \sum_{p=1}^{\text{periods}} \sum_{t=1}^{\text{trees}} \{(\text{ValOre}[p][t]) \cdot O[p][t] + \text{ValWaste}[p][t] \cdot W[p][t]\} \leq 0$$
The above equation can be simply written as follows:

\[ RV \leq Z \leq (\text{Objective Function}), \text{ and} \]

Where, RV is a randomly chosen variable. Initially, RV is assigned a very big number, which MIP is expected to result in infeasible solution. Then, this number is decreased to half. When MIP starts running (being sure that there is a feasible solution), the MIP solver is stopped and the value of RV is increased. When it is believed that the optimum result is bigger than this number, but it is close to it, the problem is let to run for the solution. This application decreases search radius of the MIP solver, and in some cases, it may be very helpful to increase the solver efficiency. It should be always kept in mind that setting the equality constraints before any other constraint in mathematical formulation may significantly decrease the solution time of the problem.

5.2.3 Sequencing Constraints

\[
\sum_{i=1}^{trees} \sum_{nov=1}^{Over(t)} \left\{ \sum_{p=1}^{periods-1} (O[p][t] - OreTot[t]*X[p][t]) \leq 0 \right\}
\]

\[
\left\{ -\sum_{m=1}^{nov} O[m][nov] + OreTot[nov]*X[p][t] \leq 0 \right\}
\]

Where,

over[t] is the total number of overlying trees of tree t,

X[p][t] is 0, 1 integer, if any tonnage is extracted from tree t at period p, it is set to 1. Otherwise it is set to 0.
This formulation can be opened up as follows:
Assume tree, \( t = 1 \), has 2 overlying trees, 2, and 3. Total ore tonnages (in ton) for each tree is:

\[
\text{OreTot}[1] = 15, \quad \text{OreTot}[2] = 10, \quad \text{OreTot}[3] = 20
\]

\( \text{O}[1][1] - 15X[1][1] \leq 0; \)
\(-O[1][2] + 10X[1][1] \leq 0,
-0[1][3] + 20X[1][1] \leq 0,
\)

\( \text{period}=2; \)
\( O[2][1] - 15X[2][1] \leq 0 \)
\(-O[1][2] - O[2][2] + 10X[2][1] \leq 0 \)
\(-O[1][3] - O[2][3] + 20X[2][1] \leq 0. \)

Meaning of these equations is that if 1 ton, or more ore is taken from tree 1 in period 1, all the ore tonnages must be extracted in period 1, from trees 2 and 3. If 1 ton of ore is taken from tree 1 in period 2, all the ore tonnages in trees 2 and 3 must be taken in periods 1 and 2.

5.2.4 Blending Constraints

\[
\sum_{p=1}^{\text{periods}} \left( \sum_{t=1}^{\text{trees}} \left( \text{AvGr}[t] \ast O[p][t] - \text{GrMax} \ast O[p][t] \right) \right) \leq 0
\]

\[
\sum_{\mu=1}^{\text{periods}} \left( \sum_{t=1}^{\text{trees}} \left( -\text{AvGr}[t] \ast O[p][t] + \text{GrMin} \ast O[p][t] \right) \right) \leq 0
\]

Where \( \text{AvGr}[t] \) is the average grade of tree, \( t \) and \( \text{GrMax} \) is the maximum average grade
of material that can be sent to the processing plant in a given period, and \( \text{GrMin} \) is the minimum average grade of material that can be sent to the processing plant in a given period. The first equation enables the scheduler not to send a higher grade than the maximum grade that can be processed at the plant. The second equation is the lower limit on the average grade of the material that can be send to the mill.

5.2.5. Processing Plant Capacity Constraints:

\[
\sum_{p=1}^{\text{periods}} \{ \sum_{t=1}^{\text{trees}} G[p][t] \} \leq \text{ProcMax}
\]

\[
\sum_{p=1}^{\text{periods}} \{ \sum_{t=1}^{\text{trees}} G[p][t] \} \geq \text{ProcMin}
\]

Where \( \text{ProcMax} \) and \( \text{ProcMin} \) are the maximum and the minimum amount of material that can be handled at the processing plant in a given year.

Constructing the mill plant for processing the ore material in mines is one of the most expensive parts of the operation. Once the mill is built, it has to be fed with as much ore as possible each year to be able to recover its cost. As a result, the mill has to be applied two constraints: one is the upper limit, which is the maximum amount of that can be processed in a year. The other one is the lower limit, which is the minimum amount of ore that has to be provided to mill to keep it profitable.

5.2.6 Reserve Constraint:

\[
\sum_{t=1}^{\text{trees}} \{ \sum_{p=1}^{\text{periods}} G[p][t] \} \leq \text{OreTot}[t]
\]
For Each Tree, t:

\[ \sum_{p=1}^{\text{periods}} W[p][t] \leq waste[t] \]

Where, W[p][t] is the tonnage of waste scheduled from tree t in period p.

This constraint is necessary to make sure the MIP solver will not assign more tonnage from a tree during mine life than what is available in a tree.

5.2.7 Maximum Stripping Ratio Constraint:

\[ \sum_{p=1}^{\text{periods}-1} \left\{ \sum_{t=1}^{\text{trees}} (W[p][t] - SrMax \cdot O[p][t]) \right\} \leq 0 \]

Where Waste[t] is the total amount of waste in tree t, and X[p][t] is the 0 1 indicator, which is 1 if any ore is taken from tree t at period p, otherwise 0. Srmax is the maximum stripping ratio in a given period, O[p][t] is the tonnage of ore scheduled to be mined during period p from tree t.

5.2.8 Maximum Waste Stripping Capacity Constraints:

\[ \sum_{t=1}^{\text{trees}} W[p][t] \leq \text{Capacity} \]
Where waste[t] is the total tonnage of waste in tree t, PerBeg is the first period to be scheduled. X's are 0 – 1 integers. Capacity is the maximum waste stripping capacity per period.

5.2.9 Ore – Waste Relationship Constraints:

If some of the ore blocks are mined out from a tree, all the waste tonnage from that tree must be mined out until the end of that period:

\[ X[p][t] \cdot \text{Waste}[t] - \sum_{p=1}^{n} W[p][t] \leq 0 \]

For example, if at period 3, X parameter of a tree is 1, which means some or all the ore tonnages from that tree is mined out, during periods 1, 2 and 3, all the waste tonnage from that tree must also be mined out. This formulation enables MIP the flexibility to mine some of the waste tonnage from a tree at previous periods.

5.2.10 Minimum Waste Stripping Requirement Constraints:

\[ \sum_{[p] \in tree} W[p][t] \geq \text{MinReq}, \text{ for all periods.} \]

Where MinReq is the minimum waste stripping requirement in period p.

In the application of this constraints, one should give special care. If there are many cases where a waste block is supported by more than one ore block within the fundamental trees, it is recommended that this constraint should not be implemented with this scheduling model. Because if there are some waste blocks under an ore block within a
tree, MIP formulation may schedule these waste blocks before the overlying ore blocks. This may happen when the cost of waste removal is very high.

5.3. MIP SCHEDULING ALGORITHM

The implementation of the MIP scheduling algorithm is illustrated in Figure 1. The first step is to determine a bench number, or a depth, up to which fundamental trees will be generated. This bench must be determined in a way that scheduling of the total material within the ultimate pit limits could be done by integer programming scheduling model. This bench number totally depends on the characteristics of the deposit such as number of ore and waste blocks up to that bench, the number of the trees that LP will generate and the tightness of the constraints needed in scheduling model, etc. This can be determined with a few trials on the deposit.

The second step is to find the fundamental trees by applying the fundamental tree algorithm as discussed in Chapter 3. After finding the fundamental trees, in the third step the MIP scheduling model is formulated.

MIP formulation is solved and the schedules of the blocks within the ultimate pit limits are obtained. If the last bench is not reached, the bench number is increased as in step 1. The problem is solved again by repeating all the steps starting from the second step. The algorithm stops when the blocks within the last bench are processed.

Special attention should be given to the bounds of the constraints in MIP formulation during the scheduling of the fundamental trees found for the incremented benches. The output of the last period which is scheduled during the previous scheduling must be taken into account for the current formulation. For example, the blocks within the first 10 bench is already scheduled for the first 3 periods. Assume that in the third period, 2 million tons of ore is scheduled for the mill plant, and the minimum and the maximum ca-
The capacities of the mill plant are 5 million and 6 million ton respectively. If it is decided to schedule the blocks up to bench 20 between periods 3 and 6, the minimum and maximum mill capacities in the MIP formulation must be constrained as 3 million and 4 million tons, respectively. This consideration should be given for all the related constraints in the MIP model.

Figure 5.1. Steps of the MIP scheduling algorithm
5.4 ILLUSTRATION OF THE MIP FORMULATION ON A 2D DATA

In this section, the mathematical formulation of the scheduling problem will be illustrated on a 2D example data set illustrated in Figure 5.2. The scheduling parameters are assumed to be as follows:

- Minimum waste stripping requirement: 1 block
- Maximum waste stripping capacity: 3 block
- Minimum mill production requirement (ore): 1 block
- Maximum mill production capacity (ore): 2 block
- Maximum stripping ratio: 4.0:1.0
- Minimum average grade at the mill plant: 2%
- Maximum average grade at the mill plant: 5%

One block is assumed to be a ton.

It is shown in Figure 5.2 that before mining the second and third trees, the first tree must be mined, and before the third tree, the second tree must be mined. Assume that the first tree has 5%, the second tree has 4%, and the third tree has 3% average grades. These grades are used to show the formulation rather than binding the constraints. According to the capacity assumption given above, the total periods for the deposit may be 3.

The average economic values of the trees for waste stripping and ore production is calculated at year 0 as follows:

- \( \text{ValOre}[0][0] = \text{ValOreTot}[0] / \text{OreTot}[0], \)
- \( \text{ValOre}[0][1] = 13/2 = $6.5 \text{ / ton} \)
- \( \text{ValOre}[0][2] = 11/1 = $11.0 \text{ / ton} \)
- \( \text{ValOre}[0][3] = 10/1 = $10.0 \text{ / ton} \)
Figure 5.2. Fundamental trees of a 2D example data set used to show the scheduling formulation. \( G_i \) is the grade of block \( i \).
\[
\text{ValWaste}[0][t] = \text{ValWasteTot}[t] / \text{Waste}[t]
\]

\[
\text{ValWaste}[0][1] = -11/3 = -3.67 \text{ / ton}
\]

\[
\text{ValWaste}[0][2] = -10/1 = -10.0 \text{ / ton}
\]

\[
\text{ValWaste}[0][3] = -3/1 = -3.0 \text{ / ton}
\]

After calculating the average economic values of the trees for year 0, NPV of these values needs to be calculated. In this example, it is assumed that discount rate is 12%.

\[
DF[p] = 1/(1.0+d)^p,
\]

Where DF is the discount factor (or multiplier), d is the discount rate, p is the period.

Therefore, DF for each period can be calculated as follows:

\[
DF[1] = 1/(1.12)^1 = 0.893
\]

\[
DF[2] = 1/(1.12)^2 = 0.797
\]

\[
DF[3] = 1/(1.12)^3 = 0.712
\]

\[
\text{ValOre}[1][1] = \text{ValOre}[0][1] \times DF[1] = 6.5 \times 0.893 = 5.80
\]

\[
\text{ValOre}[2][1] = \text{ValOre}[0][1] \times DF[2] = 6.5 \times 0.797 = 5.18
\]

\[
\text{ValOre}[3][1] = \text{ValOre}[0][1] \times DF[3] = 6.5 \times 0.712 = 4.63
\]

\[
\text{ValOre}[1][2] = \text{ValOre}[0][2] \times DF[1] = 11 \times 0.893 = 9.82
\]

\[
\text{ValOre}[2][2] = \text{ValOre}[0][2] \times DF[2] = 11 \times 0.797 = 8.77
\]

\[
\text{ValOre}[3][2] = \text{ValOre}[0][2] \times DF[3] = 11 \times 0.712 = 7.83
\]

\[
\text{ValOre}[1][3] = \text{ValOre}[0][3] \times DF[1] = 10 \times 0.893 = 8.93
\]

\[
\text{ValOre}[2][3] = \text{ValOre}[0][3] \times DF[2] = 10 \times 0.797 = 7.97
\]

\[
\text{ValOre}[3][3] = \text{ValOre}[0][3] \times DF[3] = 10 \times 0.712 = 7.12
\]

\[
\text{ValWaste}[1][1] = \text{ValWaste}[0][1] \times DF[1] = -3.67 \times 0.893 = -3.28
\]
ValWaste[1][2] = ValWaste[0][2]*DF[1] =-10*0.893 = -8.93
ValWaste[1][3] = ValWaste[0][3]*DF[1] =-3*0.893 = -2.68

The mathematical formulation can be written as follows:

Objective function:

Subject To
1. Objective function constraints:
\[ Z \geq 1.0 \]
2. **Sequencing constraints:**

\[
\begin{align*}
O[1][1] - 2.0*X[1][1] &\leq 0 & \text{The first period, first tree} \\
O[2][1] - 2.0*X[2][1] &\leq 0 & \text{The second period, first tree} \\
O[1][2] - 1.0*X[1][2] &\leq 0 & \text{The first period, second tree} \\
-O[1][1] + 2.0*X[1][2] &\leq 0 \\
O[2][2] - 1.0*X[2][2] &\leq 0 & \text{The second period, second tree} \\
-O[1][1] - O[2][1] + 2.0*X[2][2] &\leq 0
\end{align*}
\]

Since everything will be mined out at the end of mine life, last period does not need slope constraints.

\[
\begin{align*}
O[1][3] - 1.0*X[1][3] &\leq 0 & \text{The first period, third tree} \\
-O[1][1] + 2.0*X[1][3] &\leq 0 \\
-O[1][2] + 1.0*X[1][3] &\leq 0 \\
O[2][3] - 1.0*X[2][3] &\leq 0 & \text{The second period, third tree} \\
-O[1][1] - O[2][1] + 2.0*X[2][3] &\leq 0 \\
\end{align*}
\]

X-parameters are 0 – 1 integer variables, and number of integer variables is 9.

3. **Blending constraints:**

Upper bound constraints:

Lower bound constraints:

4. Processing plant capacity constraints:

Maximum capacity constraints:
O[1][1] + O[1][2] + O[1][3] ≤ 2.0

Minimum requirement constraints:
O[1][1] + O[1][2] + O[1][3] ≥ 1.0

5. Reserve constraints:
O[1][1] + O[2][1] + O[3][1] ≤ 2.0

6. Maximum stripping ratio constraints:

\[ W[1][1] + W[1][2] + W[1][3] - 4 \ O[1][1] - 4 \ O[1][2] - 4 \ O[1][3] \leq 0 \]

7. Maximum waste stripping capacity constraints:

\[ W[1][1] + W[1][2] + W[1][3] \leq 3.0 \]

8. Ore - waste relationship constraints:

For the first tree:

\[ 3X[1][1] - W[1][1] \leq 0 \] The first period,
\[ 3X[2][1] - W[1][1] - W[2][1] \leq 0 \] The second period,

For the second tree:

\[ X[1][2] - W[1][2] \leq 0 \] The first period,
\[ X[2][2] - W[1][2] - W[2][2] \leq 0 \] The second period,

For the third tree:

\[ X[1][3] - W[1][3] \leq 0 \] The first period,
\[ X[2][3] - W[1][3] - W[2][3] \leq 0 \] The second period,
9. Minimum waste stripping requirement constraints:

\[ W[1][1] + W[1][2] + W[1][3] \geq 1.0 \]
CHAPTER 6
A CASE STUDY ON A MULTI-MINERAL COPPER DEPOSIT

6.1 ORE CLASSIFICATION AND MATERIAL TYPE

The mathematical model is applied on a multi-mineral deposit that contains sulfide copper (can be processed at the processing plant), oxide copper (requires leaching), gold and silver in Peru, South America. Ore reserves are classified considering company process type suggested and grade and the following material types are taken into account:

1. Ore material types,
2. Oxide ore,
3. Mixed ore,
4. Mixed-sulfide ore
5. Waste material

The oxidic sources of copper are amenable to sulfuric acid leaching, and part of the ore is classified as oxide based on their soluble copper content. A mixture of sulfide and oxide ores has also been defined considering future use of this method in dual type processing (flotation and leaching, respectively). Sulfide ore is treated by flotation through the existing mill facility.

Ore reserves are also categorized in four different groups:

1. Proven
2. Probable
3. Possible
4. None

Each block in the block model is assigned a code from 1 to 13 as combination of material types and reserve categories as follows:
1. Proven oxide ore
2. Probable oxide ore
3. Possible oxide ore
4. Proven mixed ore
5. Probable mixed ore
6. Possible mixed ore
7. Proven mixed sulfide ore
8. Probable mixed sulfide ore
9. Possible mixed sulfide ore
10. Proven sulfide ore
11. Probable sulfide ore
12. Possible sulfide ore
13. Waste material

The following ore grade relationship is used to generate the above classification:

If soluble copper / total copper ≥ 90%, then the material type is oxide.
If oxide copper / total copper ≥ 10% then material type is sulfide.

To treat the mixed ore material type, the difference between total copper and oxide copper is considered as follows:
If (total copper – oxide copper) ≥ 0.59, and (total copper – sulfide copper) ≤ 0.50, then the material type is mixed sulfide.

If (total copper – oxide copper) < 0.59, and (total copper – sulfide copper) ≥ 0.50, then the material type is mixed oxide.

If (total copper – oxide copper) < 0.59, and (total copper – sulfide copper) < 0.50, then the material type is waste.

If (total copper – oxide copper) ≥ 0.59, and (total copper – sulfide copper) ≥ 0.50, then the material type is mixed.

The above classifications are taken from Bernabe 2001.

6.2 BLOCK MODEL

The limits of the mineralized area are given in Table 6.1. The overall dimensions of the block model are 3100m x 2500m x 450m along Easting, Northing and depth, respectively.

Table 6.1. The boundaries of the block model.

<table>
<thead>
<tr>
<th></th>
<th>Easting</th>
<th>Northing</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>33900</td>
<td>40500</td>
<td>3750</td>
</tr>
<tr>
<td>Maximum</td>
<td>37000</td>
<td>43000</td>
<td>4200</td>
</tr>
</tbody>
</table>
The block dimensions in the original model are defined as 5m x 5m x 10m along Easting, Northing, and vertical directions, respectively. Therefore, the deposit contains 620 x 500 x 45 blocks along Easting, Northing and vertical directions, which is a total of 13,950,000 blocks. The model is re-blocked as 20m x 20m x 10m block size, and the number of blocks in the re-blocked model is 155 x 125 x 45, which is 871,875.

The block economic values are calculated using the parameters given in Table 6.2 provided by mining company. Transportation cost is assumed to be variable and depends on the location of block and pit exit elevation: 3970 is the south pit elevation and 3985 is the North pit elevation.

The surface topography of the deposit is given in Figure 6.1. As can be seen from the figure, the mine is already in operation and some parts of the deposit have been mined out.
Table 6.2. Parameters that are used to generate the economic block model.

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>Minimum %Cu grade for calculating value</td>
</tr>
<tr>
<td>0.90</td>
<td>Base copper price ($/lb)</td>
</tr>
<tr>
<td>325.0</td>
<td>Gold price ($/ounce)</td>
</tr>
<tr>
<td>5.00</td>
<td>Silver price ($/ounce)</td>
</tr>
<tr>
<td>30.00</td>
<td>Copper concentrate grade (%Cu) area 1</td>
</tr>
<tr>
<td>28.50</td>
<td>Copper concentrate grade (%Cu) area 2</td>
</tr>
<tr>
<td>28.50</td>
<td>Copper concentrate grade (%Cu) area 3</td>
</tr>
<tr>
<td>28.50</td>
<td>Copper concentrate grade (%Cu) area 4</td>
</tr>
<tr>
<td>0.910</td>
<td>Copper recovery area 1</td>
</tr>
<tr>
<td>0.890</td>
<td>Copper recovery area 2</td>
</tr>
<tr>
<td>0.890</td>
<td>Copper recovery area 3</td>
</tr>
<tr>
<td>0.890</td>
<td>Copper recovery area 4</td>
</tr>
<tr>
<td>0.900</td>
<td>Copper recovery - mixed</td>
</tr>
<tr>
<td>0.360</td>
<td>Smelting refining freight area 1</td>
</tr>
<tr>
<td>0.360</td>
<td>Smelting refining freight area 2</td>
</tr>
<tr>
<td>0.360</td>
<td>Smelting refining freight area 3</td>
</tr>
<tr>
<td>0.360</td>
<td>Smelting refining freight area 4</td>
</tr>
<tr>
<td>0.620</td>
<td>Precious metal recovery</td>
</tr>
<tr>
<td>0.965</td>
<td>Cu Smelter recovery</td>
</tr>
<tr>
<td>0.841</td>
<td>Au Smelter recovery</td>
</tr>
<tr>
<td>0.828</td>
<td>Ag smelter recovery</td>
</tr>
<tr>
<td>2.700</td>
<td>G and A /tonne</td>
</tr>
<tr>
<td>1.200</td>
<td>Milling /tonne</td>
</tr>
<tr>
<td>0.992</td>
<td>Shipping “Recovery”</td>
</tr>
<tr>
<td>4100</td>
<td>Base Truck – area 1</td>
</tr>
<tr>
<td>4100</td>
<td>Base Truck – area 2</td>
</tr>
<tr>
<td>4040</td>
<td>Base Truck – area 3</td>
</tr>
<tr>
<td>4010</td>
<td>Base Truck – area 4</td>
</tr>
<tr>
<td>0.840</td>
<td>Ore base mining cost /tonne</td>
</tr>
<tr>
<td>0.010</td>
<td>Ore incremental mining cost / 10 meter bench below</td>
</tr>
<tr>
<td>0.005</td>
<td>Ore incremental mining cost / 10 meter bench above</td>
</tr>
<tr>
<td>0.500</td>
<td>Oxide copper cutoff</td>
</tr>
<tr>
<td>0.800</td>
<td>Oxide recovery SOLC</td>
</tr>
<tr>
<td>6.640</td>
<td>Oxide processing cost / tonne</td>
</tr>
<tr>
<td>0.128</td>
<td>Oxide SX and EW &amp; cathode freight</td>
</tr>
</tbody>
</table>
6.3 ULTIMATE PIT LIMITS

In order to be able to apply fundamental tree mathematical programming model, the final pit limits must be determined by an optimum method. In this case study, Whittle’s 3D software is used to determine the ultimate pit limits.

Initially, the ultimate pit limits are determined for the first 20 benches. Then, it is determined up to 27, 35 and 45 benches. The total tonnage of sulfide, oxide and waste material, and also the total number of ore (sulfide) and waste blocks are presented bench by bench in Table 6.3a through Table 6.3d for each of these pits. Each of these pits may be called as a pushback. As can be seen from Table 6.3a, the first pushback, which is found up to 20th bench, contains only about 3.4 million tons of sulfide. The first pushback has the highest oxide material tonnage, which is about 8.6 million tons, with the smallest overlying waste tonnage of about 28.9 million tons among all the four pits. From the first pushback to the last pushback, the waste tonnage steadily increases while the oxide tons decrease (see Tables 6.3a through 6.3b). Oxide is the material suitable for leaching process. The last pushback contains about 104.9 million tons of waste, and only 358,839 tons of oxide. The sulfide tonnages within the pushbacks increase until the last pushback. Pushback 3, which is determined up to bench 35, contains about 19.6 million tons sulfide material (suitable for mill processing plant).

As can be seen from Table 6.4, sulfide grades are increasing from top to down benches. However the grade of oxide material is very high on the upper benches and keeps decreasing towards the bottom of the pit.

The summary information of the four pits is given in Table 6.5.
Table 6.3a. Tonnages of sulfide, oxide, waste, and total material in tons and also average grade of sulfide and oxide in percent bench by bench within the first pushback up to 20th bench.

<table>
<thead>
<tr>
<th>Bench</th>
<th>Sulfide</th>
<th>S. Grade</th>
<th>Oxide</th>
<th>Ox. Grade</th>
<th>Waste</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>4170</td>
<td>0</td>
<td>0.000</td>
<td>0</td>
<td>0.000</td>
<td>6,742</td>
<td>6,742</td>
</tr>
<tr>
<td>4160</td>
<td>0</td>
<td>0.000</td>
<td>0</td>
<td>0.000</td>
<td>31,989</td>
<td>31,989</td>
</tr>
<tr>
<td>4150</td>
<td>0</td>
<td>0.000</td>
<td>0</td>
<td>0.000</td>
<td>125,990</td>
<td>125,990</td>
</tr>
<tr>
<td>4140</td>
<td>0</td>
<td>0.000</td>
<td>0</td>
<td>0.000</td>
<td>169,677</td>
<td>169,677</td>
</tr>
<tr>
<td>4130</td>
<td>0</td>
<td>0.000</td>
<td>0</td>
<td>0.000</td>
<td>193,127</td>
<td>193,127</td>
</tr>
<tr>
<td>4120</td>
<td>0</td>
<td>0.000</td>
<td>67,889</td>
<td>1.109</td>
<td>353,791</td>
<td>421,680</td>
</tr>
<tr>
<td>4110</td>
<td>34,941</td>
<td>1.099</td>
<td>286,999</td>
<td>1.281</td>
<td>784,338</td>
<td>1,106,278</td>
</tr>
<tr>
<td>4100</td>
<td>94,070</td>
<td>1.254</td>
<td>421,105</td>
<td>1.473</td>
<td>1,734,551</td>
<td>2,249,726</td>
</tr>
<tr>
<td>4090</td>
<td>142,422</td>
<td>1.295</td>
<td>781,545</td>
<td>1.645</td>
<td>2,842,621</td>
<td>3,766,588</td>
</tr>
<tr>
<td>4080</td>
<td>134,976</td>
<td>1.327</td>
<td>829,449</td>
<td>1.555</td>
<td>3,028,158</td>
<td>3,992,583</td>
</tr>
<tr>
<td>4070</td>
<td>166,832</td>
<td>1.270</td>
<td>852,361</td>
<td>1.592</td>
<td>3,048,382</td>
<td>4,067,575</td>
</tr>
<tr>
<td>4060</td>
<td>131,399</td>
<td>1.537</td>
<td>906,719</td>
<td>1.450</td>
<td>3,501,809</td>
<td>4,539,927</td>
</tr>
<tr>
<td>4050</td>
<td>230,395</td>
<td>1.367</td>
<td>844,423</td>
<td>1.487</td>
<td>3,450,156</td>
<td>4,524,974</td>
</tr>
<tr>
<td>4040</td>
<td>239,419</td>
<td>1.423</td>
<td>975,449</td>
<td>1.275</td>
<td>3,105,516</td>
<td>4,320,384</td>
</tr>
<tr>
<td>4030</td>
<td>324,193</td>
<td>1.424</td>
<td>980,130</td>
<td>1.138</td>
<td>2,685,859</td>
<td>3,990,182</td>
</tr>
<tr>
<td>4020</td>
<td>546,241</td>
<td>1.380</td>
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Table 6.3b. Tonnages of sulfide, oxide, waste, and total material in tons and also average grade of sulfide and oxide in percent bench by bench within the second pushback up to 27th bench.

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<th>Oxide</th>
<th>Ox. Grade</th>
<th>Waste</th>
<th>Total</th>
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<td>0</td>
<td>0.000</td>
<td>333</td>
<td>333</td>
</tr>
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</tr>
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<td>53,352</td>
</tr>
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<td>134,009</td>
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<td>280,829</td>
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Table 6.3c. Tonnages of sulfide, oxide, waste, and total material in tons and also average grade of sulfide and oxide in percent bench by bench within the third pushback up to 35th bench.

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<th>Waste</th>
<th>Total</th>
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<td>19,490</td>
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<td>57,695</td>
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Table 6.3d. Tonnages of sulfide, oxide, waste, and total material in tons and average grade of sulfide and oxide bench by bench within the fourth pushback up to 45th bench.

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<th>O. Gr (%)</th>
<th>Waste</th>
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<td>1,915,795</td>
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<tr>
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<td>0</td>
<td>0.000</td>
<td>640,487</td>
<td>1,348,497</td>
</tr>
<tr>
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<td>807,065</td>
<td>2.292</td>
<td>0</td>
<td>0.000</td>
<td>192,781</td>
<td>999,846</td>
</tr>
<tr>
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<td>2.111</td>
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<td>0.000</td>
<td>88,648</td>
<td>565,859</td>
</tr>
<tr>
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<td>0.000</td>
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<td>179,858</td>
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<td>0.000</td>
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<td>104,881,640</td>
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</table>
Table 6.4. Tonnages of sulfide, oxide, waste, and total material in tons and also average grade of sulfide and oxide in percent bench by bench within the final pit limits.

<table>
<thead>
<tr>
<th>Bench</th>
<th>Sulfide</th>
<th>S. Grade</th>
<th>Oxide</th>
<th>OX. Grade</th>
<th>Waste</th>
<th>Total</th>
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<td>62,862</td>
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<tr>
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<td>121,086</td>
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<td>0.000</td>
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<td>238,510</td>
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</tr>
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<td>423,868</td>
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<td>3,052,222</td>
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<tr>
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<td>1.290</td>
<td>781,545</td>
<td>1.645</td>
<td>4,062,861</td>
<td>4,988,545</td>
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<td>1.555</td>
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<td>1.590</td>
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<td>0.000</td>
<td>2,116,164</td>
<td>3,411,295</td>
</tr>
<tr>
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<td>0.000</td>
<td>1,435,731</td>
<td>2,577,340</td>
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<tr>
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<td>855,126</td>
<td>2.129</td>
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<td>0.000</td>
<td>1,060,569</td>
<td>1,915,795</td>
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<td>2.105</td>
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<td>0.000</td>
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<td>0.000</td>
<td>192,781</td>
<td>999,846</td>
</tr>
<tr>
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<td>477,211</td>
<td>2.111</td>
<td>0</td>
<td>0.000</td>
<td>88,648</td>
<td>565,859</td>
</tr>
<tr>
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<td>1.614</td>
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<td>0.000</td>
<td>26,235</td>
<td>179,858</td>
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<td>0.000</td>
<td>14,991</td>
<td>80,721</td>
</tr>
<tr>
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<td>16,956,765</td>
<td>1.209</td>
<td>299,974,906</td>
<td>361,532,021</td>
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Table 6.5. Tonnages and blocks within the final pit limits for the benches shown. The air blocks (the blocks that have 0 economic value) are treated as waste blocks.

<table>
<thead>
<tr>
<th>Bench #</th>
<th>Sulfide Ore</th>
<th>Oxide Ore</th>
<th>Waste</th>
<th>Total</th>
<th>Ore</th>
<th>Waste</th>
<th>Total</th>
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</thead>
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<td>5,778,852</td>
<td>66,697,526</td>
<td>82,330,933</td>
<td>2,349</td>
<td>6,712</td>
<td>9,061</td>
</tr>
<tr>
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<td>2,258,145</td>
<td>99,492,080</td>
<td>121,387,905</td>
<td>2,844</td>
<td>9,739</td>
<td>12,583</td>
</tr>
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<td>104,881,640</td>
<td>116,916,490</td>
<td>1,457</td>
<td>10,674</td>
<td>12,131</td>
</tr>
<tr>
<td>Total</td>
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<td>16,956,765</td>
<td>299,974,906</td>
<td>361,532,021</td>
<td>8,750</td>
<td>29,707</td>
<td>38,457</td>
</tr>
</tbody>
</table>

The outline of the ultimate pit limits for up to bench 20, 27, 35 and 45 are shown in Figures 6.2, 6.3, 6.4, and 6.5, respectively. As can be seen from the first push back in Figure 6.2., which is up to 20th bench, the mining area is spread throughout the whole mineralized area. South and Southeastern areas of the deposit contain mostly sulfide material while Northwestern region is mainly oxide. As can be seen from Figure 6.3, the second pit is much larger than the first pit. Table 6.5. also shows that the second pit has more than 9,000 blocks, while the first pit contains only about 4,700 blocks. As the pits move deeper, the size of the pits increases. It can be seen from Figure 6.4. that the third pit is much bigger than the second pit. As shown in Table 6.5, the third pit has about 12,600 blocks, which is approximately 30% more than the second pit. The size of the last pit illustrated in Figure 6.5. is very close to the third pit. Both the third and fourth pits have over 12,000 blocks.
6.4 APPLICATION OF FUNDAMENTAL TREE MODEL

All the model formulations are generated using C++ programs, and the solution analysis is also done with C++ programs. First, the blocks within the ultimate pit limits up to 20 bench are formulated and solved. All the mathematical formulations are solved using CPLEX software solver. Then, the blocks within the ultimate pit limits are treated as air blocks and surface topography is updated. Then the blocks that are in the second ultimate pit limits (27 benches) are processed. In similar way, all the deposit is processed and fundamental trees are found. The fundamental tree, and the mathematical programming information are given in Table 6.6. It is critical to notice that only positive economic value blocks are considered as ore in the fundamental tree model. During scheduling, the mill cutoff grade is used to distinguish ore and waste blocks.

Table 6.6. The fundamental tree model information for each incremental pit. Air blocks are considered as waste in fundamental tree model. #FT is the number of fundamental trees.

<table>
<thead>
<tr>
<th></th>
<th>Bench 20</th>
<th>Bench 27</th>
<th>Bench 35</th>
<th>Bench 45</th>
<th>Sum</th>
</tr>
</thead>
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<tr>
<td>Number of Ore Blocks</td>
<td>2,100</td>
<td>2,349</td>
<td>2,844</td>
<td>1,457</td>
<td>8,750</td>
</tr>
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<td>Number of Waste Blocks</td>
<td>2,579</td>
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<td>10,669</td>
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<td>3</td>
<td>5</td>
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<tr>
<td>Total Blocks</td>
<td>4,682</td>
<td>9,061</td>
<td>12,583</td>
<td>12,131</td>
<td>38,457</td>
</tr>
</tbody>
</table>

The Parameters For The First Iteration

<p>| | | | | | |</p>
<table>
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<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
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<td>637,213</td>
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<td>Objective nonzeros</td>
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<td>312,315</td>
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<tr>
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<td>9,061</td>
<td>12,583</td>
<td>12,131</td>
<td></td>
</tr>
</tbody>
</table>

Iteration1: # of Trees   | 1,883  | 1,644    | 1,824    | 321      |       |
Iteration2: # of Trees   | 1,883  | 1,661    | 1,640    | 328      |       |
Iteration3: # of Trees   |        | 1,661    | 1,640    | 328      | 5,512 |
The ratio of #FT/#ore blocks | 0.90  | 0.71    | 0.58     | 0.22     |       |
As can be seen from the above table, the iterations required for this deposit is mostly 3. The iterations of the fundamental tree algorithm are not really time consuming. Since all the variables in the LP fundamental tree formulation are linear, the solution time is usually less than a minute. In the block model the increase in the number of trees varies between 0% and 2.2%.

As discussed in Chapters 3 and 4, if there were no flow constraints, or slope constraints, the optimum result of the number of fundamental trees for a deposit would be the same as the number of ore blocks within the final pit limits. It should be noticed that the number of trees found is very close to the number of ore blocks. The difference between the number of ore blocks and the number of trees are because of that some ore blocks can not support the cost of all the waste block to be mined by themselves. Therefore, these blocks require mutual support with other ore blocks which decreases the number of fundamental trees identified for the model. When there is not a lot of waste block to be supported as in the first pit up to 20th bench, only 10% of the ore blocks are in mutual support, i.e. the number of the trees are 90% of the number of the ore blocks (see Table 6.6).

The mutual support of ore blocks occurs less when the stripping ratio is less within the pit that the model is applied. For example, within the first pit up to bench 20, there are 2100 ore blocks and 2582 waste blocks, stripping ratio is about 1.23. The number of trees generated is 1883, which is about 90%. Within the last pit up to 45 benches, the stripping ratio is about 7.32, the percentage of the number of fundamental trees to the number of ore blocks is only 22.5%. Figure 6.2. shows the stripping ratio versus the ratio of the number of trees to the ore blocks as percentage. That means there is a negative correlation between the stripping ratio and the number of fundamental trees produced. However, this relationship also strongly depends on the economic value of the ore blocks and mining cost of the waste material. If the mining cost is small and the economic values of the ore blocks are high, the model can still produce high percentage of fundamental trees.
Figure 6.2. The relationship between the ratio of waste to ore blocks, and the ratio of the number of the fundamental trees to the number of the ore blocks in percentage.
6.5 APPLICATION OF MIP SCHEDULING MODEL TO THE CASE STUDY

The mine is scheduled using MIP MODEL given in Chapter 5 for over seven years using the parameters given in Table 6.7 as the model constraints. The scheduling model formulations are discussed in Chapter 5. The mathematical model information is provided in Table 6.8.

Since there isn’t any integer variable used in the formulation of the fundamental trees, much bigger block models can be processed with the model and the answer can be obtained quickly. However, the block model cannot be formulated and solved for the scheduling problem at once. The scheduling problem requires integer variables. The number of integer variables in scheduling model can be written as follows:

Number of Integers in Scheduling Model = Number of Trees*(total periods-1).

According to the above relationship, as the deposit gets bigger, the number of periods that the material can be scheduled for will increase, and also the number of fundamental trees will increase. As a result, the number of integer variables in scheduling model will be too many to get any answer in a reasonable period of time. In order to keep the number of integers in a level possible to solve, the model is processed in parts which are defined by bench depth.

It should be noticed that the partial processing of the deposit does not violate the optimality of the model. Since there is node precedence within the block model due to the slope constraints, the blocks at higher benches have to be mined before the ones at lower benches can be mined whether the deposit is scheduled at once or in part.

As shown in Table 6.7, the mill in the scheduling model is constrained to produce at least 6.0 million tons of ore material (sulfide copper), and maximum 6.5 million tons. Oxide material production is constrained to have a capacity of 12 million tons. This is not a
Table 6.7. The parameters used to constraint the scheduling process.

<table>
<thead>
<tr>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.84</td>
<td>//cutoff grade of tcu (mill) and scu (leach), mine cost $/ton</td>
</tr>
<tr>
<td>6000000</td>
<td>6500000</td>
<td>12000000</td>
<td>//Processing plant capacity in 1000tons, max, min and max leach pad capacity</td>
</tr>
<tr>
<td>55000000</td>
<td>10000000</td>
<td></td>
<td>//waste stripping capacity, max and min in 1000tons</td>
</tr>
<tr>
<td>0.5</td>
<td>5.0</td>
<td></td>
<td>//minimum and maximum grades</td>
</tr>
<tr>
<td>10.5</td>
<td>0</td>
<td></td>
<td>//Maximum and minimum stripping ratios (minimum is not used as binding)</td>
</tr>
<tr>
<td>0.9</td>
<td></td>
<td></td>
<td>//price of copper, $/lbs</td>
</tr>
<tr>
<td>0.57</td>
<td>0.128</td>
<td></td>
<td>//selling cost for mill and leach produced copper, $/lbs</td>
</tr>
<tr>
<td>3.2</td>
<td>6.64</td>
<td></td>
<td>//milling cost, leaching cost, $/ton</td>
</tr>
<tr>
<td>0.89</td>
<td>0.80</td>
<td></td>
<td>//mill and leach recoveries</td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td></td>
<td>//stockpile cost, $/ton</td>
</tr>
<tr>
<td>22.406</td>
<td></td>
<td></td>
<td>//amount of copper in lbs in 1 ton of material with 1% grade</td>
</tr>
<tr>
<td>0.12</td>
<td></td>
<td></td>
<td>//percent discount rate</td>
</tr>
<tr>
<td>0.5</td>
<td>3.5</td>
<td></td>
<td>//min and max grades at the stockpile</td>
</tr>
</tbody>
</table>

Table 6.8. The scheduling model information.

<table>
<thead>
<tr>
<th></th>
<th>Bench 20</th>
<th>Bench 27</th>
<th>Bench 35</th>
<th>Bench 45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>5,719</td>
<td>10,158</td>
<td>41,256</td>
<td>3,171</td>
</tr>
<tr>
<td>Variables - Linear</td>
<td>5,711</td>
<td>10,459</td>
<td>13,557</td>
<td>1,335</td>
</tr>
<tr>
<td>Variables - Integer</td>
<td>-</td>
<td>3,322</td>
<td>4,920</td>
<td>328</td>
</tr>
<tr>
<td>Variables - Total</td>
<td>5,711</td>
<td>13,781</td>
<td>18,477</td>
<td>1,663</td>
</tr>
<tr>
<td>Objective Nonzeros</td>
<td>2,735</td>
<td>8,433</td>
<td>10,108</td>
<td>996</td>
</tr>
<tr>
<td>Best Integer Solution</td>
<td>109,295,238</td>
<td>124,895,327</td>
<td>99,787,429</td>
<td>21,650,915</td>
</tr>
<tr>
<td>LP Solution</td>
<td>109,295,238</td>
<td>125,720,000</td>
<td>105,564,339</td>
<td>21,651,748</td>
</tr>
<tr>
<td>Percent Optimality (%)</td>
<td>100.00</td>
<td>99.3</td>
<td>94.5</td>
<td>100.00</td>
</tr>
<tr>
<td>Run Time - hours</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Run Time - minutes</td>
<td>0</td>
<td>4</td>
<td>36</td>
<td>0</td>
</tr>
<tr>
<td>Run Time - seconds</td>
<td>1</td>
<td>40</td>
<td>24</td>
<td>4</td>
</tr>
</tbody>
</table>
binding constraint in this application. Since the stockpile of oxide material is assumed not to have an extra cost, it is mined when it exist on the way to reach the high grade sulfide. The partial ore, and waste mining in a given period is allowed from a fundamental tree, but partial block mining is not allowed.

The tonnage and average grades of the sulfide and oxide material, and the waste tonnage scheduled for mine production are illustrated in Table 6.9. The tonnage of the sulfide obtained from the scheduling model varies between 6 million and 6.5 million tons. The last year's sulfide production is about 5.8 million tons, which is the only year that production is below 6 million tons. The scheduled tonnages are the amounts that can be processed at the mill plant and does not require stockpile. The mill plant can handle the material with grades between 0.5% and 5% grade copper, which is not a binding constraint for this deposit. As shown in Table 6.9, the grade of the sulfide is not so much erratic to be binding for the processing plant.

The destination schedule of the material is presented in Table 6.10. As shown in the table, the oxide material is leached yearly with the average grade of the material mined rather than processing high grade material before low grade. Because generally, in mining the amount of copper that can be produced by leaching process in a given year by SX & EW is limited. Therefore, even if higher grade material is sent to the leach pad, the amount of copper produced will not be much higher than the copper amount produced with lower grade material. Considering the production limitation at the SX & EW, the oxide leached from the stockpile is considered to have an average grade of the material at the stockpile.

Table 6.11 shows the grades of the auxiliary minerals, mixed copper, gold and silver, within the material sent to the processing plant. Although these minerals are not the main interest to the company they have some contributions to the cash flow.
Table 6.9. The unsmoothed mine production schedule by year. Grades are in percentage.

<table>
<thead>
<tr>
<th>Years</th>
<th>Sulfide</th>
<th>Oxide</th>
<th>Waste</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tons</td>
<td>Grade</td>
<td>Tons</td>
<td>Grade</td>
</tr>
<tr>
<td>1</td>
<td>6,575,599</td>
<td>1.641</td>
<td>11,601,112</td>
<td>1.324</td>
</tr>
<tr>
<td>2</td>
<td>6,320,610</td>
<td>1.428</td>
<td>2,550,651</td>
<td>0.956</td>
</tr>
<tr>
<td>3</td>
<td>6,453,884</td>
<td>1.693</td>
<td>2,032,616</td>
<td>0.996</td>
</tr>
<tr>
<td>4</td>
<td>6,547,023</td>
<td>1.568</td>
<td>225,561</td>
<td>0.942</td>
</tr>
<tr>
<td>5</td>
<td>6,467,138</td>
<td>1.407</td>
<td>152,938</td>
<td>0.723</td>
</tr>
<tr>
<td>6</td>
<td>6,408,920</td>
<td>1.705</td>
<td>196,960</td>
<td>0.882</td>
</tr>
<tr>
<td>7</td>
<td>5,827,176</td>
<td>1.543</td>
<td>196,927</td>
<td>0.892</td>
</tr>
<tr>
<td>Total</td>
<td>44,600,350</td>
<td>1.570</td>
<td>16,956,765</td>
<td>1.209</td>
</tr>
</tbody>
</table>

Table 6.10. Unsmoothed destination schedule by year. Grades are given in percentage, tonnages are in thousands of tons.

<table>
<thead>
<tr>
<th>Years</th>
<th>Sulfide</th>
<th>Oxide</th>
<th>Waste</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mill</td>
<td>Leach Pad</td>
<td>Stockpile</td>
<td>Waste Dump</td>
</tr>
<tr>
<td></td>
<td>Tons</td>
<td>Grade</td>
<td>Tons</td>
<td>Grade</td>
</tr>
<tr>
<td>1</td>
<td>6,576</td>
<td>1.641</td>
<td>3,500</td>
<td>1.324</td>
</tr>
<tr>
<td>2</td>
<td>6,321</td>
<td>1.428</td>
<td>3,500</td>
<td>1.056</td>
</tr>
<tr>
<td>3</td>
<td>6,454</td>
<td>1.693</td>
<td>3,500</td>
<td>1.134</td>
</tr>
<tr>
<td>4</td>
<td>6,547</td>
<td>1.568</td>
<td>3,500</td>
<td>1.299</td>
</tr>
<tr>
<td>5</td>
<td>6,467</td>
<td>1.407</td>
<td>2,563</td>
<td>1.288</td>
</tr>
<tr>
<td>6</td>
<td>6,409</td>
<td>1.705</td>
<td>197</td>
<td>0.882</td>
</tr>
<tr>
<td>7</td>
<td>5,827</td>
<td>1.543</td>
<td>197</td>
<td>0.892</td>
</tr>
<tr>
<td>Total</td>
<td>44,600</td>
<td>1.570</td>
<td>16,957</td>
<td>1.209</td>
</tr>
</tbody>
</table>
Table 6.11. The grades of the auxiliary products of the material sent to the mill plant (unsmoothed).

<table>
<thead>
<tr>
<th>Period</th>
<th>Mixed Cu (%)</th>
<th>Au (gr/ton)</th>
<th>Ag (gr/ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.122</td>
<td>0.251</td>
<td>6.260</td>
</tr>
<tr>
<td>2</td>
<td>0.093</td>
<td>0.235</td>
<td>5.755</td>
</tr>
<tr>
<td>3</td>
<td>0.116</td>
<td>0.269</td>
<td>6.493</td>
</tr>
<tr>
<td>4</td>
<td>0.067</td>
<td>0.272</td>
<td>6.180</td>
</tr>
<tr>
<td>5</td>
<td>0.064</td>
<td>0.235</td>
<td>5.277</td>
</tr>
<tr>
<td>6</td>
<td>0.080</td>
<td>0.257</td>
<td>6.677</td>
</tr>
<tr>
<td>7</td>
<td>0.088</td>
<td>0.230</td>
<td>5.611</td>
</tr>
<tr>
<td>Average</td>
<td>0.090</td>
<td>0.250</td>
<td>6.043</td>
</tr>
</tbody>
</table>

After obtaining the scheduling results from the model, the haul roads are designed, the pits are smoothed to provide equipment access. In practice, it is not possible to mine the schedule with the exactly same tonnages obtained from MIP. Because the model did not include the haul roads and accessibility constraints. Therefore, the haul roads are designed after obtaining the true optimum schedule results. As shown in Figure 6.3a, the total material initially scheduled to be mined annually is erratic. It is high in the first year, about 63 million tons, and it gets down to about 32 million tons during the fourth and fifth years. The total tons of material to be mined after year five increases up to about 61 million tons at the last year. This tonnage variation is too much to be mined. In a mining operation, generally the number of trucks and shovels do not change every year due to their high costs.
Figure 6.3. Mine production by year a) unsmoothed, b)smoothed

Table 6.12. The smoothed mine production schedule tons (1000) and grades (%) by year.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Sulfide</th>
<th>Sulf. Grades</th>
<th>Oxide</th>
<th>Ox. Grades</th>
<th>Waste</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6,258</td>
<td>1.451</td>
<td>10,033</td>
<td>1.316</td>
<td>58,654</td>
<td>74,945</td>
</tr>
<tr>
<td>2</td>
<td>6,121</td>
<td>1.461</td>
<td>1,896</td>
<td>1.227</td>
<td>67,018</td>
<td>75,035</td>
</tr>
<tr>
<td>3</td>
<td>6,212</td>
<td>1.492</td>
<td>2,682</td>
<td>1.044</td>
<td>66,395</td>
<td>75,289</td>
</tr>
<tr>
<td>4</td>
<td>6,036</td>
<td>1.543</td>
<td>350</td>
<td>1.131</td>
<td>68,573</td>
<td>74,959</td>
</tr>
<tr>
<td>5</td>
<td>6,134</td>
<td>1.387</td>
<td>1,132</td>
<td>0.913</td>
<td>67,414</td>
<td>74,680</td>
</tr>
<tr>
<td>6</td>
<td>6,325</td>
<td>1.58</td>
<td>487</td>
<td>0.827</td>
<td>67,090</td>
<td>73,902</td>
</tr>
<tr>
<td>7</td>
<td>6,277</td>
<td>1.951</td>
<td>316</td>
<td>0.889</td>
<td>52,744</td>
<td>59,337</td>
</tr>
<tr>
<td>8</td>
<td>1,197</td>
<td>2.117</td>
<td>51</td>
<td>0.57</td>
<td>9,368</td>
<td>10,616</td>
</tr>
<tr>
<td>Total</td>
<td>44,560</td>
<td>1.568</td>
<td>16,947</td>
<td>1.209</td>
<td>457,256</td>
<td>518,763</td>
</tr>
</tbody>
</table>
The total mine production capacity is about 75 million tons. The mine production of the smoothed schedule is illustrated graphically in Figure 6.3b. As can be seen from the figure, the overall mine production is more or less constant until the end of mine life. The annual production are also given in Table 6.12. As shown in the table, the total waste tonnage increased from about 300 million tons to about 457.2 million tons. This increase in the amount of waste stripping is because of the haul roads and pit smoothing. Pit. As can be seen from the table the actual mine life is extended to year 8. This is because of that the extra waste material that must be mined due to the haul road design. Mining of the ore with the initially scheduled amount requires very high stripping due to the haul road design, which is above the overall mine stripping capacity. Therefore, the total ore tonnages that can be mined per year decreased and the mine life is extended. For example, in the first year MIP model scheduled about 6.57 million tons of sulfide and 11.60 million tons of oxide (see Table 6.9). But when haul roads are designed, only 6.26 million tons of sulfide and 10.03 million tons of oxide (see Table 6.12) could be mined.

The annual after tax cash flows and NPV’s before pit smoothing and after smoothing are illustrated in Table 6.15. The increase in the waste stripping due to the pit smoothing increased the operating cost of the mine. Therefore, the total of undiscounted cash flows, and also NPV’s coming from the smoothed pits are lower than the one coming from the raw schedule. The annual cash flows coming from the raw scheduling model, and from the schedule after haul road design are also illustrated graphically in Figure 6.4. The figure shows that the annual cash flow of the project is higher during the first four years of the mine, around $90 million. In years 4 and 5, it is down to around $64 million. It increased at year 7 up to $96 million, since there is high grade ore at lower benches. The total NPV is found to be $398.82 million from the project.
Table 6.13. Smoothed destination schedule by year. Grades are given in percentage, tonnages are in thousands of tons.

<table>
<thead>
<tr>
<th>Years</th>
<th>Sulfide</th>
<th>Oxide</th>
<th>Waste</th>
<th>Sum (mill, leach, waste)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mill</td>
<td>Leach Pad</td>
<td>Stockpile</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tons</td>
<td>Grade</td>
<td>Tons</td>
<td>Grade</td>
</tr>
<tr>
<td>1</td>
<td>6,258</td>
<td>1.451</td>
<td>3,500</td>
<td>1.316</td>
</tr>
<tr>
<td>2</td>
<td>6,121</td>
<td>1.461</td>
<td>3,500</td>
<td>1.268</td>
</tr>
<tr>
<td>3</td>
<td>6,212</td>
<td>1.492</td>
<td>3,500</td>
<td>1.108</td>
</tr>
<tr>
<td>4</td>
<td>6,036</td>
<td>1.543</td>
<td>3,500</td>
<td>1.298</td>
</tr>
<tr>
<td>5</td>
<td>6,134</td>
<td>1.387</td>
<td>2,093</td>
<td>1.098</td>
</tr>
<tr>
<td>6</td>
<td>6,325</td>
<td>1.58</td>
<td>487</td>
<td>0.827</td>
</tr>
<tr>
<td>7</td>
<td>6,277</td>
<td>1.951</td>
<td>316</td>
<td>0.889</td>
</tr>
<tr>
<td>8</td>
<td>1,197</td>
<td>2.117</td>
<td>51</td>
<td>0.570</td>
</tr>
<tr>
<td>Total</td>
<td>44,560</td>
<td>1.568</td>
<td>16,947</td>
<td>1.209</td>
</tr>
</tbody>
</table>

Table 6.14. The grades of the auxiliary products of the material sent to the mill plant (smoothed).

<table>
<thead>
<tr>
<th>Period</th>
<th>Mixed cu (%)</th>
<th>Au (gr/ton)</th>
<th>Ag (gr/ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.147</td>
<td>0.208</td>
<td>5.47</td>
</tr>
<tr>
<td>2</td>
<td>0.085</td>
<td>0.244</td>
<td>6.064</td>
</tr>
<tr>
<td>3</td>
<td>0.099</td>
<td>0.24</td>
<td>6.019</td>
</tr>
<tr>
<td>4</td>
<td>0.076</td>
<td>0.268</td>
<td>6.611</td>
</tr>
<tr>
<td>5</td>
<td>0.086</td>
<td>0.213</td>
<td>7.012</td>
</tr>
<tr>
<td>6</td>
<td>0.102</td>
<td>0.186</td>
<td>6.626</td>
</tr>
<tr>
<td>7</td>
<td>0.069</td>
<td>0.187</td>
<td>4.179</td>
</tr>
<tr>
<td>8</td>
<td>0.099</td>
<td>0.278</td>
<td>8.345</td>
</tr>
<tr>
<td>Average</td>
<td>0.074</td>
<td>0.193</td>
<td>5.286</td>
</tr>
</tbody>
</table>
Table 6.15. After tax cash flows and NPV’s (in millions of dollars) at 12% discount rate.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Unsmoothed Cash Flows</th>
<th>Unsmoothed NPV</th>
<th>Smoothed Cash Flows</th>
<th>Smoothed NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>117.88</td>
<td>105.25</td>
<td>90.65</td>
<td>80.94</td>
</tr>
<tr>
<td>2</td>
<td>95.47</td>
<td>76.11</td>
<td>90.68</td>
<td>72.29</td>
</tr>
<tr>
<td>3</td>
<td>118.26</td>
<td>84.18</td>
<td>88.54</td>
<td>63.02</td>
</tr>
<tr>
<td>4</td>
<td>129.41</td>
<td>82.24</td>
<td>96.54</td>
<td>61.36</td>
</tr>
<tr>
<td>5</td>
<td>103.30</td>
<td>58.62</td>
<td>64.79</td>
<td>36.76</td>
</tr>
<tr>
<td>6</td>
<td>81.61</td>
<td>41.34</td>
<td>63.62</td>
<td>32.23</td>
</tr>
<tr>
<td>7</td>
<td>58.69</td>
<td>26.55</td>
<td>96.13</td>
<td>43.49</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>21.62</td>
<td>8.73</td>
</tr>
<tr>
<td>Total</td>
<td>704.61</td>
<td>474.28</td>
<td>612.58</td>
<td>398.82</td>
</tr>
</tbody>
</table>

Figure 6.4. The annual cash flow comparison between the initial scheduling model and smoothed pits.
Table 6.16 shows the benches (m.) that some material is scheduled from each pit to be mined for a given period before smoothing. As can be seen from the table, the first pit is scheduled for the first year only. Because the amount of sulfide within its ultimate pit is only about half (3.5 million ton as given in Table 6.5) of what is required for the mill (6 million ton).

Table 6.16. The interval of benches (m.) that some material is scheduled in a given period from each pit (unsmoothed).

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Some of bench maps and cross sectional maps before and after designing haul roads and smoothing the pits are illustrated in Figures 6.9a through 6.17c. The figures named with “a” show the scheduling periods of the blocks before haul road design and smoothing. The figures named with “b” show the year that a block is mined. Ore blocks are illustrated in rectangular boxes. It is shown in these figures that after designing haul roads and smoothing the pits, many more waste blocks are added to be mined. In these bench maps and cross – sectional maps, the progress of the mining can be seen at those locations. The figures named with “c” illustrates the statistical information and distribution of the ore block grades for the benches and cross – sections.
Mine contour maps at the end of each period (year) are given in Figures 6.18 through 6.24. As shown in Figure 6.13, the haul roads are designed from East side of the pit towards the West. Sulfdic copper is mined at Eastern part of the pit where the previous mining operation has been already active. The oxidic copper is mined at the North-Western region of the deposit.

Figure 6.5a shows the plan view of the unsmoothed scheduling results at bench 4150m. On this bench, there isn’t any ore block, and there is only 238,510 tons of waste as shown in Table 6.4. Some of the blocks are scheduled as one block only and staying by themselves alone. These blocks cannot be mined with big mining shovels in the same way as the scheduler results. It is shown in Figure 6.5b that all the blocks at the same area are mined in the same period. For example, on the eastern part of the deposit between North 41800 and North 42100 area, all the blocks are assigned to 6th period during haul road design and pit smoothing process. There are still a few blocks by themselves on the Southern part of the deposit around East 35900 grid line. Some of these blocks are because of design errors, which happen if the design lines do not exactly match over each others. Some of the blocks are by themselves, because minimum mining width for the mine is 30m, but a block is 20m. The 30m width is not enough to reach the second block, so the designed map shows only the first block.

Figure 10a shows that at bench 4100, the unsmoothed scheduling results are not really bad, most of the blocks are scheduled together. For example, a group of blocks are scheduled on the Eastern part of the deposit mostly between North 41600 and 42100 for the first and 7th periods, another group around the gridlines East 35700 and 36000 and North 42100 for the 1st period, another large group of blocks are on the Western part around North 41800 for the 1st period. These blocks are suitable for mining machines access. There are only a few blocks by themselves, that are not practical to be mined. Figure 10b shows the same bench after haul roads are designed and pits are smoothed. Scheduling period of some of the blocks are different than that of MIP scheduling results. This is
because of the haul roads, which are set to have a minimum 100m width in this case study. Once a haul road is set in an area all the blocks must be mined within the road at the same period. Figure 6.6c shows the grade distribution of the blocks on that bench. As shown in the map the high grade blocks are on the Western part of the deposit, and Figure 6.6b shows that they are scheduled to be mined at the first period.

As can be seen in Figures named as “a” and “b”, haul road design and pit smoothing are grouping the blocks to be mined in a given period. The design is also adding a lot of waste material to the MIP schedule.

It is shown in Figure 6.17 that the first year the high grade leach material is scheduled from the west and North-Western part of the deposit. This map is obtained after the pit smoothing. The Figure 6.6b shows the mining outline at bench 4100m elevation. As shown in that figure in square that Western part has also some sulfide material. The grade distribution at bench 4100m level is illustrated in Figure 6.6c. The North-South cross sectional map between North 41000 and 41900m is generated at Easting 35000 in Figure 6.13a (unsmoothed) and 6.13b (smoothed). In the unsmoothed cross-section, the first period’s mining goes down to 3990m, while the smoothed map reaches down to 3980m. Figures 6.14a, b, and c shows the cross-sectional maps at Easting 36360m. As seen from these maps, the material at these locations is not scheduled at the first year. This Eastern area contains mostly sulfide ore.

The second year’s scheduling results are shown on the contour map provided in Figure 6.18. As illustrated in the figure, mining operation is extended both at the Western, and Northwestern region of the deposit where a lot of leach material is available, and also Southern and Eastern part of the deposit where most of ore is sulfide. It can be seen in Figure 6.8a and b (smoothed and unsmoothed on bench 4000 most of the blocks are scheduled on the Northern part of the deposit for the second year. Figure 8c indicates that some sulfide ore also exist on the Western and North-Western part of the deposit at 4000m bench. In Figure 6.9a and 6.9b, at 3950m elevation, a few blocks can be seen in
the second year as indicated in Table 6.10 that the second period scheduling is actually going down to 3930m deep. However, as shown in the Figures 6.9a and 6.9b at 3950m elevation, there are not a lot of blocks scheduled for the second period. Figure 6.9c shows that the sulfide ore is distributed all over the deposit. The histogram in this figure shows a lognormal distribution for both oxide and sulfide material, which is generally a common distribution for gold, silver and copper deposits. Figure 6.13a (North-South cross-sectional view at Easting 35000) shows that some blocks are scheduled by MIP model for the second period at around North 41500 at elevation 3980m and higher. However, most of these blocks are smoothed and mined at the first year as shown in Figure 6.3b. Figure 6.3c indicates that some these blocks are sulfide ore, but they are not really high grade. It is shown in Figures 6.14a and 6.14b (North-South cross-section at Easting 36360m), there is only a few block scheduled by MIP model, but pit smoothing process has added some waste blocks to this period. The variability of the histogram in Figure 6.18c indicates that there may be more than one population. This means the copper grades may be coming from different geologic structures.

Figures 6.15a and 6.15b, East-West cross-sectional maps at Northing 41340, shows that the blocks are scheduled for the second and first period down to 3930m, which is also shown in Table 6.10. Figure 15c shows that most of the high grade sulfide is distributed towards the bottom of the pit. Sulfide grade at this section also shows a lognormal distribution. The variations in the histogram of the oxide gives the indication that the oxide grades may be coming from different geological structure, even though sulfide distribution does not give any indication of a different population.

Figure 6.19 shows that the pit is extended down to bench 3870m deep at the third year. Figure 6.13a and 6.13b shows that some blocks are scheduled for the third period at about 3950m deep at Easting 35000. Figures 6.15a and 6.15b shows location of some blocks scheduled for the third period at East-West cross-sectional view at Northing
41340m. North-South cross-sectional map at Easting 36360, Figures 6.14a and 6.14b show that some blocks are scheduled at about 3850m deep for the third period.

Figure 6.20 illustrates the fourth year’s mining contour map. It can be seen from North-South cross sectional maps the location of blocks scheduled for the fourth year in Figures 6.17a and 6.17b, and also a lot of blocks scheduled for the fourth period appear in Figures 6.15a, 6.15b, 6.16a, and 6.16b East West Cross sections at Northing 41340 and 42500m.

The pit contour maps at the end of years 5, 6, 7, and 8 are provided in Figures 6.21, 6.22, 6.23, and 6.24. As can be seen from Figures 6.11a and 6.11b at elevation 3850, the mining operation concentrates on Southeastern region and the Northwestern region of the deposit. Mining operation goes down to 3760m. deep.
Figure 6.5a. Plan view of the scheduling periods of blocks at bench 5 before smoothing the pits (elevation = 4150m)
Figure 6.6a. Plan view of the scheduling periods of blocks at bench 10 before smoothing the pits (elevation = 4100m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.6b. Plan view of the scheduling periods of blocks at bench 10 after smoothing the pits (elevation = 4100m), squares around the blocks identify the ore blocks at 0.5% cutoff grade.
Figure 6.6c. Grade distribution of blocks at bench 10 (elevation = 4100m).
Figure 6.7a. Plan view of the scheduling periods of blocks at bench 15 before smoothing the pits (elevation= 4050m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.7c. Grade distribution of blocks at bench 15 (elevation= 4050m).
Figure 6.7a. Plan view of the scheduling periods of blocks at bench 20 before smoothing the pits (elevation= 4000m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.8b. Plan view of the scheduling periods of blocks at bench 20 after smoothing the pits (elevation = 4000m), squares around the blocks identify the ore blocks at 0.5% cutoff grade.
Figure 6.9a. Plan view of the scheduling periods of blocks at bench 25 before smoothing the pits (elevation = 3950m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.9b. Plan view of the scheduling periods of blocks at bench 25 after smoothing the pits (elevation = 3950m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.9c. Grade distribution of blocks at bench 25 (elevation=3950m).
Figure 6.10a. Plan view of the scheduling periods of blocks at bench 30 before smoothing the pits (elevation = 3900m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.10b. Plan view of the scheduling periods of blocks at bench 30 after smoothing the pits (elevation= 3900m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.10c. Grade distribution of blocks at bench 30 (elevation=3900m).
Figure 6.11a. Plan view of the scheduling periods of blocks at bench 35 before smoothing the pits (elevation = 3850m), squares around the blocks identify the ore blocks at 0.5% cutoff grade.
Figure 6.11b. Plan view of the scheduling periods of blocks at bench 35 after smoothing the pits (elevation= 3850m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.11c. Grade distribution of blocks at bench 35 (elevation=3850m).
Figure 6.12a. Plan view of the scheduling periods of blocks at bench 40 before smoothing the pits (elevation = 3800m), squares around the blocks identify the ore blocks at 0.5% cutoff grade.
Figure 6.12b. Plan view of the scheduling periods of blocks at bench 40 after smoothing the pits (elevation= 3800m), squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.12c. Grade distribution of blocks at bench 40 (elevation=3800m).
Figure 6.13a. North – South cross-sectional view of the scheduling periods of blocks at Easting 35000m, before smoothing the pits, squares around the blocks identify the ore blocks at 0.5% cutoff grade.
Figure 6.13b. North – South cross-sectional view of the scheduling periods of blocks at Easting 35000m, after smoothing the pits squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.13c. North – South cross-sectional view of the grade distribution of blocks at Easting 35000m,
Figure 6.14a. North – South cross-sectional view of the scheduling periods of blocks at Easting 36360m before smoothing the pits, squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.14b. North – South cross-sectional view of the scheduling periods of blocks at Easting 36360m after smoothing the pits, squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.14c. North – South cross-sectional view of the grade distribution of blocks at Easting 36360m,
Figure 6.15a. East–West cross-sectional view of the scheduling periods of blocks at Northing 41,340m before smoothing the pits, squares around the blocks identify the ore blocks at 0.5% cutoff grade.
Figure 6.15b. East – West cross-sectional view of the scheduling periods of blocks at Northing 41340m after smoothing the pits, squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.15c. East – West cross-sectional view of the grade distribution of blocks at Northing 41340m.
Figure 6.16a. East – West cross-sectional view of the scheduling periods of blocks at Northing 42500m before smoothing the pits, squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.16b. East - West cross - sectional view of the scheduling periods of blocks at Northing 42500m after smoothing the pits, squares around the blocks identify the ore blocks at 0.5 % cutoff grade.
Figure 6.16c. East – West cross-sectional view of the grade distribution of blocks at Northing 42500m.
Figure 6.17 Mine progress contours at the end of year 1.
Figure 6.23. Mine progress contours at the end of year 7
CHAPTER 7
COMPARATIVE ANALYSIS

7.1 INTRODUCTION

The mine is also scheduled using MINTEC’s M821V, EARTWORK’s NPV Scheduler, and Whittle’s Milava mine scheduling programs. Scheduling details for these three methods can be found in Bernabe (2001). The same production and blending constraints are applied for all these cases as in the Fundamental Tree model, which will be called as CSM LP scheduler. The comparison is based on the smoothed pits, because that is the actual design for mining.

The tonnages and grades of sulfide, oxide, and waste and the total tonnages scheduled by each of the four methods are given in Table 7.1. The total copper grade with CSM scheduler is around 1.45% in the first two years and about 1.5% in the third year. All the other three methods are mining 1.5% grade copper in the first year, while during the second and third years, the grade of the Whittle’s schedule dropped to 1.43%. The same grade dropped appears in the NPV schedule also for the second and third years. The total copper grade with Mintec’s scheduler is dropped to 1.35% for the second year while it raised back to the 1.5% for the third year. Since the average grade of the copper increases by depth, the copper grades are increasing for all the scheduler for years 5, 6 and 7 up to about 2%.

The sulfide and the oxide tonnages are about the same for all the method, around 44.6 million tons of sulfide and around 16.8, 16.9 million tons of oxide. The waste tonnages have some differences between the methods. CSM scheduler design resulted of
about 457 million tons of waste, while NPV schedule has 455 million, Mintec’s schedule has 460 million and Whittle’s schedule has 462 million tons of waste. The difference in the waste tonnages is mostly because of the haul road design and pit smoothing, which is done by hand in the computer.

CSM scheduler is mining about 10 million tons of oxide in the first year with an average grade of about 1.32% Cu as shown in Table 7.1. The table shows that Mintec’s scheduler is mining only about 362000 tons of oxide with an average grade of about 1.23% Cu, and NPV scheduler is mining 4.8 million tons of oxide at 1.48% Cu, and Whittle’s scheduler is mining about 1.7 million tons of oxide at about 1.37% Cu. During the second and third years, the other three method’s oxide production is so low that leach operation could not be established until year 3. Leaching pad is assumed to process 3.5 million tons of oxide per year.

Sulfide ore production is kept constant for Whittle’s and Mintec’s schedulers at 6.205 million tons until the last year, while NPV scheduler 6.373 million tons of sulfide for the first year and 6.205 million tons after the first year until the last year. CSM scheduler’s sulfide production is varying between 6 million and 6.3 million tons.
Table 7.1. The scheduling results of four methods by years. Tonnages are in 1000 tons and grades are in percent.

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7.2 CASH FLOW AND NPV COMPARISONS

The calculation of the cash flow tables can be found in Bernabe (2001). The cash flows coming from each method are presented in Table 7.2. As can be seen from the table, CSM scheduler has higher cash flows than any other methods in the first two years, $90.65, and $90.68. During the third year, CSM scheduler’s cash flow, about $88.54 million, is higher than NPV scheduler’s, $84 million, and about the same as the Whittle’s scheduler $88.8 million. These high cash flows at early years generate high overall NPV for the project. Mintec’s scheduler’s cash flow at the first year is only about $47.5 million, the second year it goes down to $40.6 million and the third year’s cash flow is about $99 million, which is higher than the other three methods for this year. The cash flows coming from the schedules of earthwork is about $56 million in the first year, $46.7 million in the second year. Cash flows coming from Whittle’s scheduler is about $51 million in the first year, and $52.6 million for the second year. CSM scheduler generated a total undiscounted economic value of about $612.58 million from the deposit, and the Mintec’s method generated $620.2 million, earthwork’s schedule resulted $610.8 and Whittle’s method resulted about $614.6 million undiscounted total economic value. So, the total undiscounted cash flows coming from different methods are very similar.

The NPV’s at 12% discount rate are given at Table 7.3 for all the methods. CSM scheduler has a total NPV of about $398.82 million, while the highest NPV of the other three methods generated by Whittle’s method is about $373.8 million, which is $24 less than the CSM scheduler. Mintec’s method has generated about $371.8 million, and NPV scheduler has resulted in only $366.4 million. There are a few reasons for the high NPV result of the CSM scheduler comparing to the other three methods.
Table 7.2. Annual cash flows coming from four different approaches are given in million of dollars.

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<td>63.62</td>
</tr>
<tr>
<td>7</td>
<td>123.51</td>
<td>114.93</td>
<td>98.00</td>
<td>96.13</td>
</tr>
<tr>
<td>8</td>
<td>26.18</td>
<td>45.78</td>
<td>38.41</td>
<td>21.62</td>
</tr>
<tr>
<td>Total</td>
<td>620.20</td>
<td>610.84</td>
<td>614.58</td>
<td>612.58</td>
</tr>
</tbody>
</table>

Table 7.3. NPV's at 12% discount rate coming from four different approaches are given in millions of dollars.

<table>
<thead>
<tr>
<th>Periods</th>
<th>MINTEC</th>
<th>EARTHWORK</th>
<th>WHITTLE</th>
<th>CSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.44</td>
<td>50.21</td>
<td>45.62</td>
<td>80.94</td>
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<tr>
<td>2</td>
<td>32.36</td>
<td>37.24</td>
<td>41.93</td>
<td>72.29</td>
</tr>
<tr>
<td>3</td>
<td>70.63</td>
<td>59.79</td>
<td>63.18</td>
<td>63.02</td>
</tr>
<tr>
<td>4</td>
<td>51.32</td>
<td>49.69</td>
<td>64.57</td>
<td>61.36</td>
</tr>
<tr>
<td>5</td>
<td>57.02</td>
<td>49.33</td>
<td>50.56</td>
<td>36.76</td>
</tr>
<tr>
<td>6</td>
<td>51.63</td>
<td>49.68</td>
<td>48.13</td>
<td>32.23</td>
</tr>
<tr>
<td>7</td>
<td>55.87</td>
<td>51.99</td>
<td>44.33</td>
<td>43.49</td>
</tr>
<tr>
<td>8</td>
<td>10.58</td>
<td>18.49</td>
<td>15.51</td>
<td>8.73</td>
</tr>
<tr>
<td>Total</td>
<td>371.85</td>
<td>366.42</td>
<td>373.84</td>
<td>398.82</td>
</tr>
</tbody>
</table>
One of the superiority of the CSM scheduler comes from that the other methods are using pushbacks in scheduling process. Pushbacks are much bigger in volume than the fundamental trees which makes it very difficult to identify the best yearly scheduling sequence within them. Because within these large pushbacks, the high values are averaged out with low values. On the other hand, the small volumes of the fundamental trees allow the scheduler to identify the high economic value areas of the deposit selectively. Another disadvantage of having big volumes of push backs is that the total number of pushbacks that can be generated from the deposit will not be many. Usually, the number of pushbacks used in scheduling is less than 20. Since these push backs are nested (overlying one another), most of the time, a lot of material must be mined out from the first pushback, before the second push back can be considered for scheduling. This forces the scheduler to mine certain areas of the deposit even if it may not be the best thing to do.

One of the other important superiority of the CSM method is that CSM method uses integer programming to schedule the fundamental tree, which produces true optimum result in NPV maximization since integer programming can choose the truly best trees among very many trees. The other methods schedule the pushbacks basically by trial and error methods. They find more than one solution that satisfy the scheduling problem constraints. Among the solutions identified, they choose the best solution to schedule the deposit. Since the CSM method is using integer programming with the objective function that maximizes the NPV of the individual small trees, it considers all the possible options available in the deposit. Having many fundamental trees provides enough alternatives to generate truly optimum NPV.
8.1 CONCLUSIONS

There are three major contributions in this dissertation:

1. *The original contribution is that a mathematical programming model is developed with only linear variables to produce fundamental trees for any mining data set.* This model does not include any integer variables and as such it can solve large problems without running into time constraints. It is believed that the most important characterization of the model is that the pit slope constraints are not violated even though the formulation is a purely linear model.

2. *It is shown that fundamental tree algorithm generates fundamental trees within the pushbacks.* The number of integer variables required for the formulation is decreased significantly. Hence, the first time multi time period scheduling problem can be modeled by MIP to maximize NPV of a given mine project.

3. *A mathematical programming model (Mixed Integer Programming) is developed to solve the scheduling problem.* There are two main advantage of this model. The first one is that the number of integers used depends on the number of the trees found for a particular pit and the number of years the material can be scheduled. Since the number of integers does not depend on the number of waste blocks in the model, the mathematical model can produce the optimum solution within an acceptable time interval. The second advantage is that if current mining
and economical conditions change, it is very easy to change the constraints in the model and modify the mine plan according to the new conditions.

In this dissertation, a practical application of the model is illustrated using a multi-mineral deposit in Peru, South America. There is no problem in application of the model in real life; haul roads are designed, pit slope constraints are obeyed and all the production constraints are applied. The proposed scheduling method in this dissertation results in mineable schedule.

The main objective of this dissertation is considered as NPV maximization of a given mining project. It is shown that the proposed method improved the NPV that can be generated from a mining operation. The proposed scheduling method leads the mining operation towards the most profitable parts of the deposit at early years of mining. It is noticed that the other existing scheduling methods ignored high grade oxide copper that can be reached at early years of mining operations, and went after less profitable sulfide copper areas of the mine. In this specific case study, the proposed method appeared to be superior to other scheduling methods in providing higher NPV of the cash flows generated by the scheduler. The superiority of the CSM model mostly is because of that CSM model considers the NPV of the fundamental trees during scheduling while obeying scheduling constraints. The other methods do not have a real optimizing schedule, but they try to satisfy the scheduling constraints as the main objective. They may find more than one scenarios of mining to satisfy the production constraints. When they do find more alternatives, they choose the best option among these alternatives. On the other hand, CSM model considers directly NPV maximization as an objective and satisfies the mining constraints.
8.2 FUTURE WORK

In this dissertation, a fundamental tree model is developed and a scheduling formulation is provided in integer programming. It should be noticed that the fundamental tree model could provide trees for many more blocks than the scheduling model can handle. The MIP scheduling formulation based on fundamental trees still requires integer variables to sequence the trees. The number of integer variables limits the performance of the scheduler due to excessive run time. Therefore, more research should be done to develop schedules based on fundamental trees without using integer variables for the sequencing constraints of the trees.

The fundamental trees may be used to parameterize the resources for many different objectives such as minimum strip ratio. The fundamental tree algorithm can be easily modified in ranking ore blocks to make the minimum strip parts of the deposit to be feasible for extraction before other parts of the deposit. The resultant number of fundamental trees may not be the maximum number of possible trees, once the model is modified, but in terms of minimizing the stripping ratio objective, the modifications would lead to the optimality. The main point to be noticed is that both CSM’s fundamental tree model and the scheduling model can be modified for different objectives.

It is recommended that the proposed algorithm be tested to see if it continues to produce good results on large data sets and to confirm that the propositions always hold using smaller data sets. The programs should be further tested for both coding and logic errors to be sure results are valid in all cases.

Stronger mathematical supports should be established and more work establishing and validating the propositions of chapter 4 should be done.

A sensitivity analysis on big M and ε parameters in the fundamental tree LP formulation may result in very useful and interesting conclusions for mining industry. These pa-
rameters may have very important effects in the formation of the trees, and also in the resultant NPV of a mining project. It is not known whether these two parameters may be related or not. That is, \( \xi \) may need to be the reciprocal of big \( M \). Therefore, their relationship may also be analyzed.
REFERENCES


