MODELING AND IDENTIFICATION OF
MULTISTORY BUILDINGS WITH
SEISMIC RECORDINGS

by
Lotfi O. Gargab
A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Engineering).

Golden, Colorado
Date ______________________

Signed: ____________________
Lotfi O. Gargab

Signed: ____________________
Dr. Ray R. Zhang
Thesis Advisor

Golden, Colorado
Date ______________________

Signed: ____________________
Dr. Tissa Illangasekars
Professor and Acting Head
Department of Civil and Environmental Engineering
ABSTRACT

This study proposes a continuous-discrete model for one-dimensional wave propagation in a multi-story building with seismic excitation and shows its applications in forward predicting analysis and inverse system identification. In particular, the building is modeled as a series of continuous shear-beams for columns/walls in inter-stories and discrete lumped-masses for floors. Wave response at one location of the building is then derived from an impulsive motion at another location in the time and frequency domains, termed here as wave-based or generalized impulse and frequency response functions (GIRF and GFRF). The GIRF and GFRF are fundamental in relating seismic wave responses at two locations of a building structure subjected to seismic excitation that is not fully known due to the complicated soil-structure interaction. Additionally, they play a key role in characterizing seismic structural responses, as well as in identifying dynamic parameters and subsequently diagnosing local damage of the structure.

For illustration, this study examines the ten-story Millikan Library in Pasadena, California with recordings of the Yorba Linda earthquake of September 3, 2002. With the use of the proposed continuous-discrete model as well as its degenerated ones, seismic wave responses are interpreted from the perspective of wave propagation, and more importantly, validated with the recordings and pertinent discrete-model-based results. Finally, a wave-based approach for system identification with a limited number of seismic recordings is presented, which can be used to evaluate structural integrity and detect damage in post-earthquake structural condition assessment.
TABLE OF CONTENTS

ABSTRACT ............................................................... iii
LIST OF FIGURES ...................................................... vii
LIST OF TABLES ....................................................... xiv
ACKNOWLEDGMENTS ................................................... xv
DEDICATION ........................................................... xvi
CHAPTER 1 INTRODUCTION ............................................. 1
  1.1 Background ...................................................... 1
    1.1.1 Mathematical Modeling of High-Rise Buildings .......... 3
      1.1.1.1 Discrete Parameters Model ......................... 4
      1.1.1.2 Distributed Parameters Model .................... 5
    1.1.2 System Identification Using Seismic Excitation ........ 5
  1.2 Problem Statement ............................................ 8
  1.3 Literature Review ............................................ 11
  1.4 Thesis Objectives ............................................ 12
  1.5 Expected Contribution ....................................... 14
  1.6 Thesis Overview ............................................. 14
CHAPTER 2 MODELING AND IDENTIFICATION WITH UNIFORM
          SHEAR BEAM MEDIUM ......................................... 17
  2.1 Modeling of Wave Propagation in Uniform Medium .......... 17
  2.2 Response of Uniform Medium ................................ 25
2.3 Response Features of Uniform Medium ........................................... 32
2.4 Application of System Identification using Seismic Recordings ........... 37
2.5 Usefulness and Limitation of Uniform Shear Beam Model .................. 42

CHAPTER 3 MODELING AND IDENTIFICATION WITH A PIECEWISE CONTINUOUS MEDIUM ........................................... 45
3.1 Modeling Wave Propagation in a Piecewise Continuous Medium .......... 45
3.2 Response of Piecewise Continuous Medium ...................................... 51
3.3 Response Features of a Piecewise Continuous Medium ....................... 60
3.4 Application of System Identification using Seismic Recordings ............ 64
3.5 Usefulness and Limitations of Piecewise Continuous Model ............... 71

CHAPTER 4 MODELING AND IDENTIFICATION WITH A CONTINUOUS-DISCRETE MEDIUM ........................................... 74
4.1 Modeling Wave Motion in a Continuous-discrete Medium ................... 74
4.2 Response of a Continuous-discrete Model ........................................ 79
4.3 Response Features of a Continuous-discrete Model .......................... 85
4.4 Simple Versions of a Continuous-discrete Model ............................. 88
   4.4.1 Degeneration to a Piecewise Continuous Model ......................... 89
   4.4.2 Degeneration to Discretized Mass Model ................................. 92
4.5 Application of System Identification using Seismic Recordings .......... 93
4.6 Usefulness of a Continuous-discrete Model ..................................... 96

CHAPTER 5 PARAMETRIC SENSITIVITY ANALYSIS OF MODEL PROPERTIES ON FEATURES OF DYNAMIC RESPONSE ........ 99
5.1 Parametric Analysis of Uniform Shear Beam Model ......................... 99
5.2 Parametric Analysis of Piecewise Continuous Model ....................... 106
LIST OF FIGURES

Figure 1.1 Structural monitoring / updating process. .............................. 4

Figure 1.2 Robert A. Milikan Library building: (a) North-South elevation section; (b) typical floor plan. ............................................................... 8

Figure 1.3 The Millikan Library building with recorded response: (a) location of accelerometers; (b) North-South seismic response due to the Yorba Linda earthquake of 03 September, 2002. ................................ 9

Figure 1.4 The Millikan Library building wave-based response in the time domain at selected floors due to the Yorba Linda earthquake of September 3, 2002 with respect to the basement floor. ........... 11

Figure 2.1 Mechanical, physical, and geometrical characteristics of a uniform shear beam model. ................................................................. 18

Figure 2.2 Dynamic equilibrium of a 1D elastic uniform source-free segment. 18

Figure 2.3 Wave motion through a uniform medium: transmission coefficients in the up-going and down-going propagation direction. 21

Figure 2.4 Relationship between wave motion components: (a) up-going and down-going wave motion components at levels $L$ and $H$ and related transmission and reflection coefficients. (b); equivalent reflection coefficient. ................................................................. 25

Figure 2.5 Non-dimensional model response of uniform shear beam parameters: shear displacement components at $z_r$ and $z_R$ levels and equivalent transmission and reflection coefficients. ....................... 26

Figure 2.6 Complex integration plane: schematic diagram explains location of poles and path of integration. ...................................................... 29

Figure 2.7 Schematic diagram of uniform medium model demonstrates model characteristics, response and reference levels. ......................... 33

Figure 2.8 Model response in the time domain: GIRF at response levels $z_{R_1} = 20\ m$ and $z_{R_2} = 30\ m$ with respect to an impulsive motion at referenced level located at $z_r = 0$ using Equation 2.47. ............... 33
Figure 2.9 Model response in the frequency domain: GFRF at the response levels $z_{R_1} = 20 \, m$ and $z_{R_2} = 30 \, m$ with respect to an impulsive motion at referenced level $z_r = 0$ using Equation 2.33.

Figure 2.10 Model response in the time domain: GIRF at $z_R=20 \, m$ with respect to impulsive motion at two referenced levels: $z_{r_1} = 0$ and $z_{r_2} = 5 \, m$.

Figure 2.11 Model response in the frequency domain: GFRF at $z_R = 20 \, m$ with respect to impulsive motion at two referenced levels: $z_{r_1} = 0$ and $z_{r_2} = 5 \, m$.

Figure 2.12 Uniform medium model of the Millikan library building showing the reference, considered floors, and height levels.

Figure 2.13 Seismic recordings of the Yorba Linda earthquake at the selected floors: North-South acceleration response at the west side of the Millikan building at the basement, 3rd, and 8th floors.

Figure 2.14 Recording-based GFRF of the Yorba Linda earthquake at the basement, 3rd and 8th floors with respected to seismic motion at the basement and 3rd floors.

Figure 2.15 Recording-based GIRF of the Yorba Linda earthquake at the basement, 3rd and 8th floors with respected to the seismic motion at the basement floor.

Figure 2.16 Matching model- and recording-based responses in the frequency domain: GFRFs at the 3rd and 8th floors with respect to a motion at the basement and the 3rd floors using the identified parameters: $H = 48.2 \, m$, $(v, \eta)_B = (326 \, m/s, 0.028)$, and $(v, \eta)_3 = (289 \, m/s, 0.044)$, where the subscripts $B$ and 3 are used to indicate the referenced levels.

Figure 2.17 Model-based response in the time domain at the selected floors: GIRFs at the 3rd and 8th floors with respect to the basement motion using the identified parameters: $H = 48.2 \, m$, $v = 326 \, m/s$, and $\eta = 0.028$.

Figure 2.18 Limitations of the uniform medium model: GFRF at the 3rd floor with respect to a motion at the basement floor.

Figure 3.1 Mechanical, physical, and geometrical characteristics of a piecewise continuous model.
Figure 3.2 Equivalent transmission and reflection coefficients a composite segment: (a) transmission and reflection coefficients in sub-segments; (b) equivalent transmission and reflection coefficients. 50

Figure 3.3 Schematic diagram of a piecewise continuous model illustrates model response parameters in terms of transmission and reflection coefficients. 53

Figure 3.4 Response parameters of a 2-layer continuous medium for the case where the response level is located at the lower layer: $0 < z_R h_1$. 54

Figure 3.5 Response parameters of a 2-layer continuous medium for the case where the response level is located at the upper layer: $h_1 < z_R \leq h_1 + h_2$. 56

Figure 3.6 Schematic diagram illustrates poles of integration located in the upper half complex plan. 57

Figure 3.7 Schematic diagram demonstrates general configuration of: (a) a 2-layer continuous model, (b) a uniform medium model. 61

Figure 3.8 Model response in the time domain: early part of the GIRF at $z_{R_1}$ and $z_{R_2}$ with respect to a motion at the model’s base using the 2-layer and uniform medium models with the indicated parameters. 62

Figure 3.9 Model response in the frequency domain: GFRF at $z_{R_1}$ and $z_{R_2}$ with respect to a motion at the model’s base using the 2-layer and uniform medium models with the indicated parameters. 64

Figure 3.10 The 2-layer model of the Millikan Library building: preliminary suggested model parameters. 65

Figure 3.11 GIRFs at the 4th and 7th floors with respect to impulsive acceleration at the basement obtained based on Equation 3.11a and 3.15 with $r_{I_1} = 0.04$, $v_1 = 345.18 \ m/s$, $v_2 = 10.32 \ m/s$, $h_1 = 46.98 \ m$, $h_2 = 1.22 \ m$, and $\eta_1 = \eta_2 = 0.03$. 66

Figure 3.12 Comparison of GFRF amplitudes at the 4th and 7th floors with respect to impulsive basement acceleration obtained from seismic recordings and model Equation 3.28. 67

Figure 3.13 Comparison of GIRF at the 4th floor with respect to the basement acceleration motion obtained from seismic recordings and the model. 69
Figure 3.14 Comparison of GIRF at the 7th floor with respect to the basement acceleration motion obtained from seismic recordings and the model. .............................................................. 70

Figure 3.15 Comparison of GFRF amplitudes at the 4th and 7th floors with respect to impulsive basement acceleration obtained with uniform (one-layer), two-layer, and 11-layer models from seismic recordings and models. .............................................................. 71

Figure 3.16 Comparison of GIRFs at the 4th and 7th floors with respect to impulsive basement acceleration obtained with one- and two-layer models. .............................................................. 72

Figure 4.1 Continuous-discrete model of N-story high rise building: (a) geometrical configuration; (b) deformed shape. .................. 75

Figure 4.2 Free body diagram demonstrates shear force equilibrium at the jth floor. .............................................................. 76

Figure 4.3 Schematic diagram demonstrates 1-layer continuous-discrete model. 80

Figure 4.4 Complex integration plane: schematic diagram demonstrates the location of poles and path of integration. .......................... 83

Figure 4.5 Continuous-discrete and uniform medium models: schematic diagram demonstrates geometrical configuration, response levels, and primary identification characteristics. .............................. 86

Figure 4.6 Model response in the time domain: early part of GIRF at response levels $z_{R_1}$ and $z_{R_2}$ with respect to motion at the base level. 87

Figure 4.7 Model response in the frequency domain: GFRF at response levels $z_{R_1}$ and $z_{R_2}$ with respect to motion at the base level. ................. 89

Figure 4.8 Model response in the frequency domain of a continuous-discrete model with small $r_m$ compared to a uniform model with $r_m = 0$: $v = 300 \text{ m/s}, \eta = 0.030, \eta_f = 0.035, r_1 = 1$ ................................. 91

Figure 4.9 Model-based and recording-based GFRF at the 8th floor with respect to a unit impulsive motion at the basement level. .......... 94

Figure 4.10 Model-based and recording-based GFRF at the 8th floor with respect to a unit impulsive motion at the basement level. ................. 95
Figure 4.11 Model response in the time domain: GIRF at the 3rd and 8th floors with respect to impulsive motion initiated at the basement at time $t = 0$.

Figure 4.12 System identification using the proposed continuum models with seismic recordings at the basement and 8th floors matching two modal frequencies at (10.77 and 14.21) rad/sec.

Figure 5.1 Amplitude of transmission coefficient in the frequency domain for different values of shear wave velocity through a uniform segment of $h = 30$ m and $\eta = 0.035$.

Figure 5.2 Amplitude of transmission coefficient in the frequency domain for different values of hysteretic damping ratio through a uniform segment of $h = 30$ m and $v = 150$ m/sec.

Figure 5.3 Influence of shear wave velocity on model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the bottom level with $z_R/H = 1$ and $\eta = 0.035$.

Figure 5.4 Influence of shear wave velocity on model response in the time domain: GIRF at the top level with respect to impulsive motion at the bottom level with $z_R/H = 1$ and $\eta = 0.035$.

Figure 5.5 Effect of hysteretic damping ratio on model response in the frequency domain: GFRF at the top level with respect to impulse motion at the bottom level with $v = 250$ m/sec and $z_R/H = 1$.

Figure 5.6 Effect of hysteretic damping ratio on model response in the time domain: GIRF at the top level with respect to impulse motion at the bottom level with $v = 250$ m/sec and $z_R/H = 1$.

Figure 5.7 Effect of response level to height ratio on model response in the frequency domain: GFRF at the top level with respect to impulse motion at the bottom level for $v = 250$ m/sec and $\eta = 0.035$.

Figure 5.8 Effect of response level to height ratio on model response in the time domain: GIRF at the top level with respect to impulse motion at the bottom level for $v = 250$ m/sec and $\eta = 0.035$.

Figure 5.9 Transmission coefficient amplitude through an interface characterized with different $r_I$-ratio values with $\tau = 0.2$ sec and $\eta = 0.030$. 

xi
Figure 5.10 Reflection coefficient amplitude through an interface characterized with different $r_I$-ratio values with $\tau = 0.2 \ sec$ and $\eta = 0.030$. .... 108

Figure 5.11 Schematic diagram of the 2-layer model: (a) preliminary model characteristics; (b) model characteristics changing at the upper layer; (c) model characteristics changing at middle section; (d) model characteristics changing at the lower layer. .......... 109

Figure 5.12 Model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the bottom level for different pairs of $(v, \eta)$ assumed taking place at the upper layer. ......... 110

Figure 5.13 Model response in the time domain: GIRF at the top level with respect to motion recorded at the base for different pairs of $(v, \eta)$ assumed taking place at the upper layer. .................. 111

Figure 5.14 Schematic diagram demonstrates scattering of impulse wave inside the model explaining the first four peaks. ................. 112

Figure 5.15 Amplitude of upward reflected wave off the interface towards the top level for the simulated decrease in stiffness of the upper layer. ......................... 114

Figure 5.16 Model response in the time domain: enlarged time-zone included the first two inverted peaks for the simulated decrease in stiffness of the upper layer. .................. 114

Figure 5.17 Model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the base level for the assumed different $(v, \eta)$ at the middle section. ...................... 117

Figure 5.18 Model response in the time domain: GIRF at the top level with respect to impulsive motion at the base level for the assumed different $(v, \eta)$ at the middle section. ...................... 118

Figure 5.19 Schematic diagram demonstrates scattering of an impulse wave inside the model explaining the pattern for the first five peaks. 118

Figure 5.20 Amplitude of reflected wave by the lower interface towards the upper interface level for the assumed different $(v, \eta)$ at the middle section. ...................... 119

Figure 5.21 Amplitude of reflected wave by the upper interface towards the top level for the assumed different $(v, \eta)$ at the middle section. 120
Figure 5.22 Model response in the time domain: enlarged time-zone included the first five upright and inverted peaks. ................................. 120

Figure 5.23 Model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the base level for the assumed different \((v, \eta)\) at the lower layer. ........................................... 124

Figure 5.24 Model response in the time domain: GIRF at the top level with respect to impulsive motion at the base level for the assumed different \((v, \eta)\) at the lower layer. ........................................... 125

Figure 5.25 Schematic diagram shows scattering of impulse wave into transmission and reflection waves inside the model. .......................... 125

Figure 5.26 Amplitude of upward reflected wave by the interface towards the top level for the assumed different \((v, \eta)\) at the middle section. 126

Figure 5.27 Model response in the time domain: enlarged time zone that includes small inverted peaks of the Figure 5.24. ......................... 127

Figure 5.28 Amplitude of transmission coefficient at a floor-story boundary changes with frequency at selected \(r_m\) with \(\tau = 0.015\) sec, \(r_I = 1\), \(\eta = 0.030\), and \(\eta_f = 0.035\). ................................. 129

Figure 5.29 Amplitude of transmission coefficient at a floor-story boundary changes with frequency at selected \(\eta_f\) with \(r_m = 0.25\), \(\tau = 0.015\) sec, \(r_I = 1\), and \(\eta = 0.030\). ................................. 129

Figure 5.30 Schematic diagram demonstrates the continuous-discrete model of the 10-story building. ...................................................... 130

Figure 5.31 Model response in the frequency domain: GFRF at the top floor level with respect to the bottom for different pairs \((v, \eta)\) taking place at the top four floors. ........................................... 131

Figure 5.32 Model response in the time domain: enlarged time-zone included the first two inverted peaks. ................................. 132
LIST OF TABLES

Table 2.1 Identification of the Millikan library building due to different earthquakes ........................................... 43

Table 3.1 Influence of impedance ratio on the transmitted and reflected displacement amplitudes. .......................... 49

Table 3.2 Identification of the Millikan library building due to different earthquakes ........................................... 72

Table 4.1 Identification of the Millikan library building due to different earthquakes ........................................... 97

Table 5.1 Effect of model characteristics on dynamic response features. ............................................................... 105

Table 5.2 Wave travel time vs. relative changes in model dynamic features. ............................................................ 115

Table 5.3 Wave travel time vs. relative changes in model dynamic features. ............................................................ 122

Table 5.4 Wave travel time and its relative change vs. relative change in model dynamic features. ......................... 127

Table 5.5 Wave travel time and its relative change vs. relative change in model dynamic features. ......................... 132
ACKNOWLEDGMENTS

First and foremost, I wish to express my sincere gratitude to Professor Ray R. Zhang, my advisor for his continuous support. I have benefited from his guidance, great kindness and patience and I wish to say a heartfelt thank-you to him. I would like to thank all the committee members: Professors Derrick Hudson, Bernard Bialecki, Panos Kiousis D., Judith Wang for taking time to read and revise this document. A special thank-you to Professors Panos Kiousis D. and Judith Wang for they were always there for me whenever I needed advice to succeed and complete some of my courses. I also want to thank Sara Atkins (Graduate Program administrator) for her cooperation and help. Lastly, I would like to thank my family for their unwavering support and love.
CHAPTER 1
INTRODUCTION

1.1 Background

Civil structures are frequently subjected to unknown excitations under uncontrollable conditions; therefore, the necessity of ensuring public safety and examining the integrity of structural stiffness is important [1–3]. Detecting structural damage at its earliest possible stages—especially for the existing old structures that were built without considering seismic resistance provisions—could be utilized to plan maintenance schedules and consider different feasible solutions to retrofit/replace damaged parts [4–8]. Building structures located in/or near earthquake-prone areas are particularly the most susceptible structures to damage [9]; therefore, monitoring these structures in real-time using proper structural monitoring and effective damage detection approaches is very important.

In general, engineering structures are typically subjected to damage—either naturally or human-made—during their lifetimes, which introduces changes into their physical/mechanical/geometrical properties and can adversely affect their current and future performance [4, 10, 11]. In fact, damage has no absolute meaningful measurement and thus damage detection techniques depend upon comparing two different condition states of the structure; one of which represents an initial condition or undamaged status [4, 10, 12]. Generally, a damage detection technique is considered effective if it is able to satisfy the following stages, that is [4, 13–17]:

- Stage I: determination of damage occurrence
- Stage II: determination of spatial distribution of damage
- Stage III: determination of damage severity in a quantifiable manner
The process of monitoring engineering structures—subjected to operational loadings/extreme environmental conditions—and tracking changes in their dynamic properties to detect/locate damage is called Structural Health Monitoring (SHM) [2, 10, 11, 18]. The target goal of SHM is to enhance safety and reliability of engineering structures by detecting damage at its earliest possible stage [16, 17]. In particular, monitoring civil structures can be implemented continuously and/or periodically as in the case of monitoring bridges to detect damage caused by aging and traffic. Monitoring in this case can be used for detecting stiffness degradation and updating of the structural model. Monitoring can also be applied intermittently, which is triggered at a predefined threshold level of excitation, as in the case of monitoring high-rise buildings to detect damage caused by sudden and extreme events such as earthquakes and explosions. Monitoring in this case is mainly used to determine the safety and integrity of the structure.

Damage detection approaches applied to building structures can be included in one of two approaches: local or global [6, 17]. A local approach, represented by non-destructive tests (NDT) and visual inspection procedures, is frequently used to detect damage in a qualitative manner provided that the damaged vicinity is known in advance and accessible [4, 8, 10, 19]. This procedure, however, is expensive and requires a longer time, especially in cases where access to damaged areas involves a disassembly process [20]. In addition, the increased size and complexity of today’s civil structures reduces the efficiency of the local approach, which is becoming a less effective tool to detect/evaluate structural damage.

The necessity of assessing structural integrity and safety as well as estimating damage in a quantifiable manner has led to the development of a more global approach using dynamic features of the structure as damage indices [10, 17, 20, 21]. A global approach is mainly developed to evaluate large and complex structures through measuring changes in their dynamic features/parameters using vibration-based meth-
ods [8, 12]. In particular, a global approach (vibration-based) is conceptualized on the basis that dynamic parameters of a building structure (modal frequencies, modal shapes, and modal damping) are functions of structure properties (mass, stiffness, and damping). Thus, changes in dynamic parameters of a structure can be attributed to changes in structural properties [4, 15, 19]. Consequently, a global approach could be used to detect damage of structures by monitoring their dynamic parameters, or their derived quantities, which are extracted from system response and serve as damage indicators [7].

Many damage detection schemes applied to civil structures are dependent on vibration-based approach. Such schemes are basically dependent on one of these concepts: model-based damage identification, neural networks, genetic algorithms, and changes in modal frequencies and mode shapes and their derivatives [22]. While these methods essentially show potential capability of detecting structural damage, difficulties are found when applying these approaches in cases of large scale, sparsely instrumented civil structures [23]. It has been mentioned that structural damage can be detected by recognizing changes in the structure’s response, and therefore, with the aid of a model-based damage detection scheme, structural damage can be identified by comparing updated modal parameters to baseline status. This process can best be described by Figure 1.1.

Figure 1.1 shows that damage detection essentially depends on adopting an appropriate structural model and applying an efficient system identification approach. The following sections provide a review of commonly used structural models and system identification approaches applicable for high-rise buildings.

1.1.1 Mathematical Modeling of High-Rise Buildings

Unlike static analysis methods where the resulting response quantities: displacement, moment, stress, etc. are functions of the applied load only, the output quantities in a dynamic analysis are dependent on inertia force as well. In general, two differ-
ent approaches are used to represent structural properties in the dynamic analysis, namely, discrete parameter and distributed parameter models [24]. Below is presented a brief description and classification of each model demonstrating its basis and typical use.

1.1.1.1 Discrete Parameters Model

Structural building properties in this model are discretized at predefined points called the coordinates. The number of displacement components at each coordinate is commonly termed the degree of freedom. The governing equation of motion can be simplified to an ordinary differential equation, in which dynamic response is determined at the chosen coordinate system. The subsections hereafter are oriented to describe common types of this modeling system used for high-rise buildings.

I: Lumped-mass Procedure

This procedure is useful in cases where the considerable portion of the total mass is concentrated at a few locations of the structure; therefore, inertia forces will be assumed to develop only at those locations, and structural response will be solved for
those coordinates. In this case, the mass of a supporting member of the structure can be added to those concentrated/ discretized points and itself be considered massless members.

II: Finite element procedure

This procedure expresses the resulting response of a structure in terms of a finite number of discrete response coordinates. In particular, each structural member is divided into a number of segments of arbitrary length, in which the ends of these segments are denoted as the nodal points. It should be noted that applicable displacement compatibility and stress continuity are supposed to be satisfied at those nodal points. With advances in computer technology, this approach has become the most convenient by far for the most common types of structures and provides a dependable idealized approach for dynamic structural analysis.

1.1.1.2 Distributed Parameters Model

In general, structural systems in civil engineering are of distributed parameters with infinite degrees of freedom and infinite dimensions [25]. High-rise buildings can be modeled as a uniform shear beam [26–28] or as piecewise continuous shear beams [29]. A dynamic problem, in this case, can be defined by the number of variables needed to describe the distribution of properties of the structure. The one dimension (1D) is the minimum class that can be used, in which structure properties are assumed to be distributed along the long axis; however, extended dimensions can also be implemented. For instance, deformation of thin plates is a 2D problem, while 3D is used to describe wave propagation in solid media [24].

1.1.2 System Identification Using Seismic Excitation

Investigating conditions and properties of a structural system by analyzing its dynamic response to external excitation has received considerable attention in the last two decades [6, 30]. In addition, instrumentation and collection of real responses
of full scale civil structures has helped validate currently used system identification algorithms and develop new ones [18]. The general approach used to address this problem is called system identification.

In general, system identification can best be defined as the techniques used to build mathematical models of systems/structures based on available theoretical knowledge and observation [25, 31, 32]. Validated and updated models, with the aid of recorded observations, can then be used for simulation, design, damage detection, and predicting future performance [1, 7, 9, 31, 33, 34]. It could be noted that selecting model parameters in terms of physical quantities can help interpret changes in the model response by changes in building properties [25]. Moreover, identification of model parameters at two distinctive time windows can be used to assess damage occurrence and its severity [19, 35, 36].

In fact, based on representation of model response, system identification can be classified into time-domain system identification and frequency domain system identification. However, system identification based on frequency domain is more frequent use due to its convenience [32, 34]. Moreover, system identification techniques could be classified into linear, nonlinear, hybrid, parametric, and non-parametric procedures. Parametric system identification, for instance, involves establishing a mathematical model for the structure, which is characterized with a set of physical parameters to be determined [37]. Non-parametric system identification, on the other hand, could be used to estimate characteristics of operational dynamic features [3, 7, 25]. It should be mentioned that the process of establishing the model and the related parameters as well as the system identification algorithm is a problem-specific technique. Additionally, a complete identification of the model and determination of its parameters, especially relies on the amount of furnished input-output measurements, which could be acquired by performing test procedures to structural systems [25, 38].
In general, modal properties of a structure extracted using dynamic tests are valuable for calibration of mathematical models, which could be utilized to improve damage detection procedures [38, 39]. There are two distinguished dynamic test approaches that are generally considered appropriate for civil structures: the Forced Vibration Test (FVT) and the Ambient Vibration Test (AVT) [1]. In particular, the AVT is considered more preferable for identifying civil structures than FVT because of their large size and the more energy needed to excite the structure at higher frequencies. This is besides that AVT does not need machineries/equipments to perform the test (inexpensive) and does not interrupt the functionality of the structure during test application as well as requires no knowledge of excitation force [39]. Additionally, the recorded response using AVT represents valuable information in that it is recorded under the effect of realistic conditions and is considered sufficient for subsequent system analysis, identification, and damage detection [1, 18, 35]. In particular, AVT recorded data is considered representative in cases where the ambient excitation has a broad band range in the response domain. This ensures that all natural frequencies of the structure are excited [33].

However, the presence of other factors than the considered ambient excitation and the difficulties of eliminating their influence on the resulting system response, make the recorded data not purely related to a specific excitation force [2, 3, 22, 40]. This problem is overcome by using recorded output as input, which is considered sufficient for subsequent system analysis [41]. This is especially applicable due to the fact that modal parameters (modal frequencies, modal shapes, and modal damping) are operating parameters, that is, functions of structure properties [25, 33, 35]. For instance, seismic time history response recorded at different locations along the structure height can be normalized to the recorded data at the base floor (floor in contact with the soil). Normalized data can then be used for analyzing the structure where the influence of soil-structure interaction on the resulting response is eliminated [10, 36]
1.2 Problem Statement

For performance-based structural design, vibration control, and damage diagnosis of high-rise structures, response analysis and system identification are important and typically carried out with a discrete Multi-Degree-of-Freedom (MDOF) model, as far as 1D horizontal motion is concerned. For example, the Millikan Library building, shown in Figure 1.2a can be modeled as 10-DOF system with each floor mass and inter-story stiffness (i.e., physical parameters) calculable based on design configuration and materials properties. This model can be calibrated in terms of identified vibratory features (i.e., modal frequencies, damping and shapes—a function of physical parameters) through Fourier spectral analysis of a 10-recording set of Yorba Linda earthquake of September 3, 2002 shown in Figure 1.3.

Figure 1.2: Robert A. Milikan Library building: (a) North-South elevation section; (b) typical floor plan.

Subsequently, seismic demand such as structural peak acceleration to a scenario earthquake is predictable, which is useful for seismic design/retrofit and vibration
control. Similarly, a change of some physical parameters or higher-order modal parameters is identifiable with a 10-recording set of a new earthquake, which is a detection and quantification of local, minor damage in post-earthquake structural conditions evaluation. Furthermore, implementing a damage mechanism, such as material hysteresis, plastic hinge, and crack into the linear 10-DOF model would make the modeling more rigorous in simulating nonlinear vibratory features, thus enhancing credibility in forward analysis and inverse identification, among many other broad-based applications.

Figure 1.3: The Millikan Library building with recorded response: (a) location of accelerometers; (b) North-South seismic response due to the Yorba Linda earthquake of 03 September, 2002.

While the aforementioned vibration-based (or discrete-modeling-based) approach is overwhelmingly used in structural engineering, it has theoretical drawbacks due to the implicit assumption that motion at different heights is synchronous. This assumption distorts the time-space characterization of seismic motion in building structures. For illustration, the seismic response obtained with the 10-DOF model will never show authentic floor-to-floor propagation features of high-frequency, dominant-
energy waves, observed prominently in the 10-12.5 sec time-window in Figure 1.3b. In other words, the vibration-based approach captures major motion features as a function of time and distorts the floor-to-floor motion relationship or wave features. Note that modal shapes essentially characterize floor-to-floor motion relationship of modified seismic responses with re-aligned time, thus not the true wave features.

Wave-propagation features are even clearly exposed in the floor-to-floor time shift of the first peak motion in 0-0.2 sec time window demonstrated in Figure 1.4, which depicts pure structural acceleration responses at selected floors to a band-limit impulsive acceleration at the basement floor, denoted here as floor 0, extracted from recordings in Figure 1.3b with the use of seismic interferometry (SI) [27, 42], by removing influences of seismic input and soil-structure interaction.

Distorting the aforementioned wave features would falsely predict, and likely underestimate, the maximum inter-story drift, a key index of seismic demand for structural design. This is due to the fact that the time-delay peak waves at two neighboring stories would have the drift calculated as the difference between one peak amplitude and one non-peak value, which is typically larger than the difference between two peak values without a time-delay effect. Similarly, this time-delay feature would also affect the efficacy of vibration control if actuators installed in different floors are operated with a central feedback-control device. More importantly, understanding and utilizing the wave features could create an alternative wave-based (or continuous-modeling-based) approach for system identification of high-rise buildings, which can be used to improve greatly the efficiency of post-earthquake structural condition assessment, in comparison with a traditional, vibration-based approach.

As well known, effectiveness of vibration-based system identification in general, and recognition of local physical parameters in particular, relies on a large number of recordings exemplified as 10-set recordings for the Millikan Library, which is neither common nor practical for most structures currently or in the near future. In contrast,
Figure 1.4: The Millikan Library building wave-based response in the time domain at selected floors due to the Yorba Linda earthquake of September 3, 2002 with respect to the basement floor.

The wave-based approach requires only a few recordings. For instance, consider again the Millikan Library building, for which three recordings are available at the basement, 4th, and 7th floors. The pure structural responses, in the time domain, or GIRFs (to be elaborated) at the 4th and 7th floors are obtainable, as shown in Figure 1.4.

Then, the 1st peak-to-peak wave traveling time from the 4th to 7th floors is measurable, which is directly related to the wave velocity of the building segment. Similarly, the corresponding peak-amplitude reduction is associated with the segment damping. Both identified velocity and damping can then be related to local physical parameters such as shear modulus and hysteresis damping if the building segment is modeled as a continuum medium.

1.3 Literature Review

Recent studies show the advantages of a wave-based approach over a vibration-based one in some seismic response analysis and damage diagnosis of building struc-
tures. In particular, recognizing the deficiency of discrete modeling in addressing seismic drift demand for buildings, Iwan [43] proposed to use a 1D uniform shear-beam model for buildings and obtained a seismic drift spectrum for design. Inspired by Iwan's study and also from research in other disciplines, Safak [44] introduced a 1D lumped-continuous modeling for a structure-soil system with impulsive seismic excitation in bedrock. With the model, he solved for wave responses with the time-domain analysis methodology, compared them with MDOF structural modeling with ground excitation, and revealed wave propagation features and influences of soil-structure interaction in seismic structural responses, among others. Independently, Todorovska et al. [45] modeled 2D anisotropic wave propagation for a real seven-story building. While developed over the past decade for exploration seismology, ultrasound and hazard studies, SI was first employed by Snieder and Şafak [27] to extract pure structural responses from seismic recordings. This SI methodology was not only used well for explaining wave phenomena in buildings, but also easily for system identification with a 1D uniform shear-beam model. Following Snieder and Şafak [27]'s work, Kohler et al. [46] studied seismic propagating waves in 3D steel, moment-frame building and verified with ETABS finite-element modeling. Recently, Todorovska [47, 48] was further applied SI for damage detection based on 1D wave traveling times. Moreover, Todorovska and Rahmani [29] using 1D continuous shear layers for seismic response analysis of the Millikan Library building and Zhang et al. [28, 49] with continuous-discrete building models among others.

1.4 Thesis Objectives

In this study, a number of objectives are set to achieve the main target of analyzing high-rise building structures using continuum models and identifying their dynamic features using available seismic recordings. Additionally, after accomplishing these two goals, the approach can be used for detecting local changes in the building properties. In order to achieve these goals, the following objectives are to be carried out
• The first objective is to construct the mathematical continuum model: this objective aims to introduce the fundamental and theoretical framework as well as the relevant assumptions to model the seismic motion of high-rise buildings. Three different models, starting from simple to complicated, are presented and shear deformation due to one dimensional seismic wave propagation in the vertical direction is considered in detail.

• The second objective is to derive the model response and explore dynamic features: this objective aims to apply results obtained from the first objective to determine the model response in the frequency and time domains. In addition, similarities and differences between the continuum and discrete model response formulations are described.

• The third objective is to carry out system identification: this objective aims to identify the proposed dynamic features defined in the second objective using seismic recordings. In particular, it is to show that normalizing seismic records data with respect to seismic response at the very bottom level of the building can eliminate effects of soil-structure interaction. Therefore, identified model parameters can be related directly to building properties.

• The fourth objective is to perform parametric analysis: this objective aims to examine effects of the proposed model characteristics of the general model and its degenerated forms on features of wave propagation as well as model response. Additionally, it aims to investigate model response and extracting sensitive dynamic features that could be related quantitatively and qualitatively to changes in the model characteristics. Consequently, this could help determine sensitive features to local changes in model characteristics, which can be used as
damage indices and thus enhance model-based damage identification techniques, especially in cases of limited seismic recordings.

1.5 Expected Contribution

Achievement of the above mentioned objectives helps in improving the modeling of seismic wave motion of high-rise buildings with the continuum medium and enhancing the system identification using the real-time response, i.e., seismic recordings. Consequently, this will effectively assist in advancing broad-based application areas such as in-depth understanding the response of high-rise buildings to seismic excitations and increasing the accuracy of system identification, which could be applied to post-earthquake damage detection among others.

1.6 Thesis Overview

The thesis is organized as follows: Chapter 1 introduces the research topic by describing discrete and distribute (continuum) modeling of high rise building structures and the advantages of using recorded vibration data of structure response for system identification. Application of system identification using limited seismic recordings is presented and the promising applicability towards post-earthquake damage detection using model-based approach is also mentioned. In addition, it introduces a literature review of the thesis problem along with thesis objectives and the expected contribution of the study.

Chapter 2 presents the theoretical framework and essential assumptions for modeling high-rise buildings as a uniform medium. The representation of model response in the frequency domain and time domain as well as dynamic features are determined using a wave-based approach, which is then compared to traditional formulation of a vibration-based approach. As an application part, the proposed model is used to identify the Millikan Library building with recordings of the Yorba Linda earthquake of September 3, 2002. The chapter concludes with a discussion of the usefulness and
limitations of the uniform medium model.

Chapter 3 presents the theory and assumptions that are made for modeling high-rise building structures as a piecewise continuous system. In this model, columns and/or walls are modeled as shear beams, whereas floor masses are ignored. Continuity and boundary conditions are used to model wave motion through model interfaces. The model response is determined using a wave-based approach and common features with a vibration-based approach are highlighted. To demonstrate the system identification process, the Millikan Library building is identified with a set of recordings of the Yorba Linda earthquake of September 3, 2002 using a 2-layer model. The chapter ends with an outline of the advantages and disadvantages of the piecewise continuous model.

Chapter 4 presents the fundamental theory and assumptions that are made for modeling high-rise buildings as continuous-discrete system where the inter-story space (columns, walls, shear walls) is modeled as a series of shear beams and the floors as lumped masses. Model response is determined using a wave-based approach where the features of 1D-wave propagation are revealed and explored. In particular, it is shown that the continuous-discrete model can be degenerated to a uniform medium and discretized mass models. The model is then used to identify the Millikan Library building with the aid of the Yorba Linda earthquake of September 3, 2002. The chapter ends with an outline of the significant improvements on modeling high-rise buildings using the continuous-discrete model.

Chapter 5 presents parametric analysis of the proposed continuum modeling discussed in Chapters 2, 3, and 4. In particular, the dominant properties of each model are analyzed and its influence on wave propagation features as well as on model response in the frequency and time domains are explored and detailed. Additionally, sensitive dynamic features to local changes in shear stiffness of the building are examined and proposed to be used as wave-based damage indices.
Chapter 6 presents a summary of the results discussed so far through the presented chapters of the thesis, which pertain to continuum modeling and identification of high-rise buildings using seismic recordings. It also outlines conclusions based on demonstrating advantages of the proposed modeling and its superiority in cases of sparsely instrumented high-rise buildings. Finally, the chapter ends with a recommendation to continue this research by using the results obtained so far from Chapters 2, 3, and 4 for improving model-based damage detection.
CHAPTER 2
MODELING AND IDENTIFICATION WITH UNIFORM SHEAR BEAM MEDIUM

This chapter presents theoretical framework and essential assumptions for modeling high-rise buildings as a uniform shear beam. In particular, model response representation in the frequency domain and time domain as well as wave-based dynamic features are determined and compared to a traditional formulation of vibration-based approach. Furthermore, the application of system identification with seismic recordings is considered where the proposed model of the Millikan library building is identified with recordings of the Yorba Linda earthquake of September 3, 2002. Finally, the chapter is concluded with discussing usefulness and limitations of the uniform shear beam model.

2.1 Modeling of Wave Propagation in Uniform Medium

This section introduces a 1D mathematical model to describe the propagation of seismic waves through high-rise buildings. With this objective, an N-story building is modeled as a uniform shear beam, which is characterized by shear modulus \( G \), mass density \( \rho \), hysteretic damping ratio \( \eta \), and height \( H \) as shown in Figure 2.1. It should be noted that the described model parameters are assumed to represent the average properties of the building structure. In addition, shear deformation of the model is assumed due to the propagation of shear waves in the vertical direction.

Dynamic equilibrium of an infinitesimal elastic uniform source-free segment, shown in Figure 2.2, in terms of shear force \( F_S \), and inertia force \( F_I \), can be expressed as

\[
(F_S + \frac{\partial F_S}{\partial z} dz) - F_S = F_I.
\]  

Equation 2.1 states that unbalanced shear forces at the ends of the considered segment are balanced by the inertial force generated by acceleration of the segment’s
Figure 2.1: Mechanical, physical, and geometrical characteristics of a uniform shear beam model.

Figure 2.2: Dynamic equilibrium of a 1D elastic uniform source-free segment.
mass $m$. Assuming that cross-sectional planes remain plane after deformation and shear stress is uniformly distributed over cross sections, Equation 2.1 can then be expressed in terms of physical quantities of $F_S$ and $F_I$, that is

$$\frac{\partial \tau_S}{\partial z} Adz = m \frac{\partial^2 u(z,t)}{\partial z^2}, \quad (2.2)$$

where $\tau_S$ is the shear stress and $u(z,t)$ is the horizontal shear displacement. Recalling that $m = \rho Adz$ and $\tau_S = G\gamma$ where $\gamma = \partial u(z,t)/\partial z$ is the shear strain, then one can substitute these quantities back into Equation 2.2 and simplify, yielding

$$\frac{\partial^2 u(z,t)}{\partial z^2} = \frac{1}{(G/\rho)} \frac{\partial^2 u(z,t)}{\partial t^2}. \quad (2.3)$$

The quantity $(G/\rho)$ has the dimensions of $(L/T)^2$ where $L$ and $T$ are, respectively, the distance and time dimensions. Thus, this quantity can be interpreted as the velocity squared at which shear waves propagate in the building. Therefore, shear deformation $u(z,t)$, due to wave propagation in the vertical direction is governed by

$$\frac{\partial^2 u(z,t)}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 u(z,t)}{\partial t^2}. \quad (2.4)$$

With the aid of Fourier representation of wave motion

$$u(z,t) = \int_{-\infty}^{\infty} U(z,\omega) e^{i\omega t} d\omega, \quad (2.5)$$

$$U(z,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u(z,t) e^{-i\omega t} dt, \quad (2.6)$$

one can solve for shear displacement $u(z,t)$, by substituting Equation 2.5 into Equation 2.4 to obtain

$$\int_{-\infty}^{\infty} \left[ \frac{\partial^2 U(z,\omega)}{\partial z^2} + \left( \frac{\omega}{v} \right)^2 U(z,\omega) \right] e^{i\omega t} d\omega = 0, \quad (2.7)$$

where $i$ is the imaginary unit and $\omega$ is the angular frequency. Note that for a given frequency content, Equation 2.7 can be reduced to an ordinary differential equation and solved for the independent variable, $z$. The general solution can be put in the
exponential form, $U(z, \omega) = e^{\lambda z}$, in which $\lambda$ is constant. Substituting $U(z, \omega)$ and its derivatives, back into Equation 2.7 yields $\lambda_{1,2} = \mp i \omega / v$. Consequently, The general solution of Equation 2.7 can be given as

$$ U_z \equiv U(z, \omega) = C_1 e^{-i \omega \frac{z}{v}} + C_2 e^{i \omega \frac{z}{v}} \equiv U^u_z + U^d_z. \tag{2.8} $$

Shear displacement in the time domain can then be obtained by substituting Equation 2.8 back into Equation 2.5 which results in

$$ u_z \equiv u(z, t) = \int_{-\infty}^{\infty} \left( C_1 e^{i \omega (t - \frac{z}{v})} + C_2 e^{i \omega (t + \frac{z}{v})} \right) d\omega $$

$$ = g_1 \left( t - \frac{z}{v} \right) + g_2 \left( t + \frac{z}{v} \right) \equiv u^u_z + u^d_z, \tag{2.9} $$

where $U_z$ and $u_z$ are used to represent the compact form of $U(z, \omega)$ and $u(z, t)$, respectively, the superscripts $u$ and $d$ correspond to up-going and down-going propagation directions, and the notation " $\equiv$ " is used to indicate identical quantities.

Equations 2.8 and 2.9 demonstrate the general solution of Equation 2.4 in the frequency domain and time domain, respectively. This solution can be interpreted as the sum of two constant-shape displacement waves traveling in opposite vertical directions with the same velocity, i.e., $C_1 e^{-i \omega z / v}$ or $g_1(t - z / v)$ propagating upward in the positive $z$-direction and $C_2 e^{i \omega z / v}$ or $g_2(t + z / v)$ propagating downward in the negative $z$-direction [50].

Incident waves, in general, are turned into transmitted and reflected waves when propagating across a boundary distinguished by two different media. The ratio of amplitudes of transmitted and reflected waves to amplitude of incident wave are, respectively, given by transmission and reflection coefficients, denoted here as $T$ and $R$. To investigate these coefficients in the case of the uniform shear beam model, one can consider the uniform segment shown in Figure 2.3. For simplicity, shear wave components at each level will be indicated by level notation, i.e., $U^u_l \equiv U^u_{z_l}$. With this in mind, shear waves traveling in the positive $z$-direction at levels $z_m$ and $z_l$ can be related as
\[ U_m^u = C_1 e^{-i\omega \frac{z_m}{v}} \left( C_1 e^{-i\omega \frac{z_l}{v}} \times \frac{1}{C_1} e^{i\omega \frac{z_l}{v}} \right) \quad (2.10a) \]
\[ = e^{-i\omega \frac{(z_m-z_l)}{v}} C_1 e^{-i\omega \frac{z_l}{v}} \quad (2.10b) \]
\[ = e^{-i\omega \frac{(z_m-z_l)}{v}} U_l^u \quad (2.10c) \]
\[ = T_{ml} U_l^u, \quad (2.10d) \]

**Figure 2.3:** Wave motion through a uniform medium: transmission coefficients in the up-going and down-going propagation direction.

Similarly, down-going shear waves at levels \( z_l \) and \( z_m \) can be related as

\[ U_l^d = C_2 e^{i\omega \frac{z_l}{v}} \left( \frac{1}{C_2} e^{-i\omega \frac{z_m}{v}} \times C_2 e^{i\omega \frac{z_m}{v}} \right) \quad (2.11a) \]
\[ = e^{-i\omega \frac{(z_m-z_l)}{v}} C_2 e^{i\omega \frac{z_m}{v}} \quad (2.11b) \]
\[ = e^{-i\omega \frac{(z_m-z_l)}{v}} U_m^d \quad (2.11c) \]
\[ = T_{lm} U_m^d, \quad (2.11d) \]
Equations 2.10d and 2.11d show that up-going waves as well as down-going waves are related, in the direction of motion, through the transmission coefficient, only. Therefore, one can conclude that in the uniform medium case, propagating waves continue traveling in the same direction of motion, in which this can be explained, physically, is that the energy of propagating waves are entirely transmitted into the direction of motion without reflection. Mathematically, this means that reflection coefficient, in either direction, is equal to zero. With the aid of Equations 2.10 and 2.11, one can express transmission coefficients explicitly as

\[ T_{ml} = \frac{U_m^u}{U_l^d} = e^{-i\omega \frac{(z_m - z_l)}{v}} = T_{lm}. \]  

Equation 2.12 shows the transmission coefficient representation in the frequency domain, which reveals that it is function of section properties that bounded by levels \( z_l \) and \( z_m \), i.e., \( G, \rho, \) and \( h \). Using a matrix analogy, one can express wave motion between the considered levels in compact form as given below

\[
\begin{pmatrix}
U_m^u \\
U_l^d
\end{pmatrix}
= \begin{bmatrix}
T_{ml} & 0 \\
0 & T_{lm}
\end{bmatrix}
\begin{pmatrix}
U_l^u \\
U_m^d
\end{pmatrix}.
\]  

To examine the transmission coefficient in the time domain, one can perform an inverse Fourier transform by applying Equation 2.5 to Equation 2.12, namely

\[ \tilde{T}_{ml} = \int_{-\infty}^{\infty} e^{-i\omega \frac{(z_m - z_l)}{v}} e^{-i\omega t} d\omega = \delta (t - \tau). \]

Equation 2.14 shows a delta function \( \delta(t) \), with a time delay \( \tau = (z_m - z_l)/v \), meaning the upward transmission coefficient acts as an impulsive wave propagating through a segment height \( (z_m - z_l) \), as shown in Figure 2.3, without amplitude decay. The attenuation of impulse wave amplitude due to damping, however, can be taken into consideration by replacing the real shear wave velocity with a complex one, i.e., \( v^* = v(1 + i\eta |\omega|) \), [28, 51], that is

\[ T_{ml} = e^{-i\omega \frac{(z_m - z_l)}{v}} = e^{-i\omega (z_m - z_l)(1 + i\eta |\omega|)} = e^{-\frac{i\omega (z_m - z_l)(1 + i\eta |\omega|)}{v(1 + \eta^2 |\omega|^2)}}. \]
For small hysteretic damping ratios, Equation 2.15 can be simplified to

\[ T_{ml} = e^{-i\omega \left( \frac{z_m - z_l}{v} \right)(1-i\eta|\omega|)} \]

Equation 2.16 is equivalent to multiply Equation 2.12 with a frequency-dependent attenuation factor \( B = e^{-\eta|\omega| \frac{(z_m - z_l)}{v}} \).

It has been explained that propagating waves in a uniform medium are completely transmitted upward in the direction of motion without reflection, meaning transmission and reflection coefficients are equal to one and zero, respectively. At the free-top and fixed-bottom levels, transmission and reflection coefficients can be determined using displacement compatibility and shear force equilibrium conditions applicable to those boundaries. In particular, at the free-top level, a zero-shear force condition is dominant where no shear strain can be held in the space. Referring to Figure 2.1, this condition can, mathematically, be expressed as

\[ F_s(H^+, t) = 0, \]

and, the shear force just underneath the free surface is found as

\[ F_s(H^-, t) = \tau_s A = G\gamma A, \]

where the superscripts (\( +, - \)), indicate the up and down sides of the level \( z_H \), respectively. For convenience, the elevation \( z \), hereafter will indicate the positive side of the considered level and thus, the superscript (\( + \)) will be dropped.

Equation 2.18 can be simplified further, using Fourier-transform and wave motion representation in the frequency domain given, respectively, by Equations 2.5 and 2.8. Recall also that the shear strains for small deformations can be expressed as \( \gamma = \partial u(z, t)/\partial z \). With this in mind, shear force equilibrium at the free top level can then be written as

\[ GA \int_{-\infty}^{\infty} i\omega \left( U^n_{H^+} - U^d_{H^-} \right) \ e^{i\omega t} \ d\omega = 0. \]

Equation 2.19 is satisfied non-trivially when \( U^d_{H^-} = U^n_{H^-} \), meaning upward propagating waves approaching the free end are completely reflected back to propagate
downward with the same motion direction, i.e., reflection coefficient at the free-top level \( R_{HH^-} = \frac{U_{dH^-}}{U_{hH^-}} = 1 \). Owing to the fact that no waves propagate in the vacuum, i.e., \( U_{dH} = 0 \), thus, displacement compatibility condition at the free-top level can be written as

\[
U_H^u = U_{H^-}^u + U_{H^-}^d \\
\Rightarrow \frac{U_H^u}{U_{H^-}^u} = 2. \tag{2.20}
\]

Comparing Equations 2.12 and 2.20 suggests the transmission coefficient at the free-end level \( T_{HH^-} \), is equal to 2, which means that propagating waves approaches the top level are transmitted to the free surface with twice the amplitude of the approaching wave.

Similarly, due to the fact that base of the building is stationary at any time, the zero-displacement condition is dominant at the fixed-bottom level. This can be expressed mathematically as

\[
U_{0^-}^u + U_{0^-}^d = 0 \Rightarrow U_{0^-}^u = -U_{0^-}^d. \tag{2.21}
\]

Equation 2.21 suggests down-going waves approaching the base are completely reflected upward with the opposite motion direction, i.e., reflection coefficient at the fixed-base level \( R_{00^-} = \frac{U_{0^-}^u}{U_{0^-}^d} = -1 \).

With Figure 2.4 as reference, one can relate the down-going component of shear displacement at level \( L \) to the up-going component at the same level using transmission and reflection coefficients of the section bounded by level \( L \) and the free-end level. In particular, Equation 2.12 can be used to relate up-going waves between levels \( L \) and \( H^- \), and the reflection coefficient at the free-end level \( H^- \), can be used to relate down-going and up-going waves, that is

\[
U_{L}^{d} = T_{LH^-} U_{H^-}^{d} \\
= T_{LH^-} R_{HH^-} U_{H^-}^{u} \\
= T_{LH^-} R_{HH^-} T_{H^-L} U_{L^-}^{u} \\
= R_{HL} U_{L^-}^{u}. \tag{2.22}
\]

Obviously, Equation 2.22 describes that the reflection coefficient, \( R_{HL} \), combines effects of transmission and reflection coefficients between levels \( L \) and \( H \).
Figure 2.4: Relationship between wave motion components: (a) up-going and down-going wave motion components at levels $L$ and $H$ and related transmission and reflection coefficients. (b): equivalent reflection coefficient.

### 2.2 Response of Uniform Medium

Shear deformation at level $z$ due to propagation of shear waves in the vertical direction is represented in the frequency domain and the time domain by Equations 2.8 and 2.9, respectively. A non-dimensional model response, hereafter called the motion ratio and denoted as $D_{Rr}(\omega)$, can be obtained by relating shear displacement at response level, denoted as $U_{R}$, to shear displacement at reference level, denoted as $U_{r}$, as detailed in Figure 2.5. It is interesting to note that reference position can be located at any level within the model’s height. With the aid of Equation 2.22, shear displacements at response and reference levels can, respectively, be expressed as

\[
U_{R} = U_{R}^{u} + U_{R}^{d} = U_{R}^{u} + R_{HR} \frac{U_{R}^{u}}{1 + R_{HR}} \tag{2.23}
\]

\[
\Rightarrow U_{R}^{u} = \frac{U_{R}}{1 + R_{HR}}
\]

\[
U_{r} = U_{r}^{u} + U_{r}^{d} = U_{r}^{u} + R_{Hr} \frac{U_{r}^{u}}{1 + R_{Hr}} \tag{2.24}
\]

\[
\Rightarrow U_{r}^{u} = \frac{U_{r}}{1 + R_{Hr}},
\]
where $U^d_R = R_{HR} U^u_R$ and $U^d_r = R_{Hr} U^u_r$. Furthermore, up-going waves at levels $z_R$ and $z_r$ can be related through Equation 2.12, namely

$$U^u_R = T_{Rr} U^u_r.$$  

By inserting the expressions for $U^u_R$ and $U^u_r$ given by Equations 2.23 and 2.24, respectively, into Equation 2.25 and simplifying, one can get

$$\frac{U_R}{(1 + R_{HR})} = T_{HR} \frac{U_r}{(1 + R_{Hr})}.$$  

Then, motion ratio can be found as

$$D_{Rr}(\omega) = \frac{U_R}{U_r} = \frac{(1 + R_{HR}) T_{HR}}{(1 + R_{Hr})}.$$  

Equation 2.27 shows the general wave-based model response formulation in non-dimensional form in the frequency domain. In particular, this formula clearly demonstrates that the model response is completely dependent on transmission and reflection coefficients above the referenced level, $z_r$, which are functions of model properties.
Furthermore, this formulation eliminates the effects of soil-structure interaction, suggesting that changes in model response is entirely due to changes in the parameters of the segment of the model immediately above reference level.

On the other hand, model response in the time domain can be obtained by applying Equation 2.5 to Equation 2.27, namely

\[ d_{Rr}(t) = \int_{-\infty}^{\infty} D_{Rr}(\omega) e^{i\omega t} \, d\omega. \]  

To show the similarities of model response formulation between vibration-based and wave-based approaches, one can examine the case where model response is considered at reference level, i.e., \( z_R = z_{sr} \), in which Equations 2.27 and 2.28 lead to \( D_{Rr} = 1 \) and \( d_{Rr} = \delta(t) \). This suggests that \( D_{Rr} \) and \( d_{Rr} \) are, respectively, frequency and time shear displacement responses at \( z_R \) to displacement impulse at \( z_{sr} \). Subsequently, the wave response representation in general and displacement response at \( z_R \) to input displacement at \( z_{sr} \) in particular, is found as

\[ u(z_R, t) = \int_{-\infty}^{\infty} U_r e^{i\omega t} \, d\omega = \int_{-\infty}^{\infty} D_{Rr}(\omega) U_r e^{i\omega t} \, d\omega. \]  

\[ u(z_R, t) = \int_{-\infty}^{\infty} d_{Rr}(t-\tau) \, u(z_{sr}, \tau) \, d\tau. \]

Equations 2.29 and 2.30 have the same mathematical form as the traditional vibration response representation in the frequency domain with \( D_{Rr} \) as the frequency response function FRF, and in the time domain with \( d_{Rr} \) as the impulse response function IRF, Duhamel’s or convolution integral.

While the aforementioned derivation is for displacement \((u(t), U(\omega))\), it is straightforward to extend it to velocity input \((v(t) = \frac{du}{dt}, V(\omega) = (i\omega)U(\omega))\) and acceleration input \((a(t) = \frac{d^2u}{dt^2}, A(\omega) = (i\omega)^2U(\omega))\), with \( D_{Rr} \) and \( d_{Rr} \) remaining the same. For ground acceleration input at \( z_{sr} \) and displacement response at \( z_R \), which is the typical case for displacement response to earthquake, Equations 2.29 and 2.30 can be modified as
\[ u(z_R, t) = \int_{-\infty}^{\infty} H_{Rr} A_{z_r} e^{i\omega t} d\omega \]

\[ u(z_R, t) = \int_{-\infty}^{\infty} h_{Rr}(t - \tau) a(z_r, \tau) d\tau, \]

where \( H_{Rr} = \frac{-D_{Rr}}{\omega^2} \) and \( h_{Rr} \) have the conventional meanings for the frequency response function and the imimpulse response function, respectively. Because of the aforementioned difference from FRF and IRF, \( D_{Rr} \) and \( d_{Rr} \) are, respectively, referred to as the wave-based or generalized frequency response function GFRF and, the generalized imimpulse response function GIRF.

In particular, features of the model response in terms of model characteristics can be obtained by inserting expressions for \( T_{Rr}, R_{HR}, \) and \( R_{Hr} \) back into Equation 2.27 and simplifying, yielding

\[ D_{Rr}(\omega) = \frac{1 + e^{-i\omega(2H_r - 2z_r)} e^{-\eta|\omega|(2H_r - 2z_r)} e^{-i\omega z_r} e^{-\eta|\omega| z_r}}{1 + e^{-i\omega 2H_r} e^{-\eta|\omega| 2H_r}} e^{i\omega t}, \]

where \( H_r = (z_H - z_r) \) denotes the effective height of the model’s segment bounded by \( (z_H, z_r) \).

Equation 2.33 demonstrates model response in terms of model parameters, in the frequency domain, due to a unit imimpulse wave hitting the model at the referenced level. Not only does this formula eliminates effects of soil-structure interaction, but it also makes it possible to calculate the response of any segment of the model by adjusting the reference level at the bottom of that segment.

Shear response in the time domain can be found by substituting Equation 2.33 into Equation 2.28 to obtain

\[ d_{Rr}(t) = \int_{-\infty}^{\infty} \left( 1 + e^{-i\omega(2H_r - 2z_r)} e^{-\eta|\omega|(2H_r - 2z_r)} e^{-i\omega z_r} e^{-\eta|\omega| z_r} \right) e^{-i\omega^2 t} e^{-\eta|\omega|^2 t} e^{i\omega t} d\omega. \]
integration need to be found first, and then the value of the function is determined at those points. In particular, the integrand given by Equation 2.34, which is a function of real frequency \( \omega \), is replaced with a variable \( y \). In particular, the dominator of Equation 2.34 can be written in the form \((1 + e^{-y^*})\) where \( y^* \) is set equal to \((iy + \eta|y|)\frac{2H_r}{v}\), and then one can solve for \( y^* \) which yields

\[
1 + e^{-y^*} = 0
\]

\[
\Rightarrow y^* = i\pi(2j - 1) = (iy + \eta|y|)\frac{2H_r}{v}.
\]  

(2.35)

Solving for \( y \) taking into account the cases \( y > 0 \) and \( y < 0 \), yields

\[
y_j = \omega_j(\pm 1 + i\eta), \quad \text{for} \quad j=1,2,\ldots,\infty,
\]  

(2.36)

where

\[
\omega_j = \omega_0(2j - 1)
\]  

(2.37a)

\[
\omega_0 = \frac{\pi v}{2H_r}.
\]  

(2.37b)

Equations 2.36 and 2.37 show that the integral given by Equation 2.34 has infinite number of poles, \( y_j(j = 1,2,\ldots) \) in the upper half complex plane as shown in Figure 2.6.

Figure 2.6: Complex integration plane: schematic diagram explains location of poles and path of integration.

With the aid of Equation 2.36, the integral given by Equation 2.34 can be rewritten in the form given below
\[ d_{Rr}(t) = \int_{-\infty}^{\infty} \frac{f_1(y_j)}{f_2(y_j)} e^{iy_j t} \, dy, \quad (2.38) \]

where \( f_2(y_j) = 1 + e^{-iy_j \bar{\tau}} \) and \( \bar{\tau} = \frac{2H}{v} \) is the travel time for the wave to propagate the model’s effective height twice. One should note that \( f_2(y_j) \) equals zero when \( e^{-iy_j \bar{\tau}} = -1 \). Setting \( \bar{y} = y + \psi \) where \( \psi \) is a small positive number, and using a first order Taylor’s expansion in \( \psi \), one can simplify \( f_2(y_j) \) as

\[ f_2(y_j) = 1 + e^{-i\bar{\tau} \psi} \approx 1 + e^{-i\bar{\tau}(1 - i\psi \tau - 0(\psi^2))} \approx i\psi \bar{\tau}. \quad (2.39) \]

This proves that poles of integration are simple. Applying contour integral along the route described in Figure 2.6, using the residue theorem, one can rewrite the integral given by Equation 2.38 as shown below

\[ d_{Rr}(t) = 2\pi i \sum_{j=1}^{\infty} \text{Res}_{\bar{y} \to y} \frac{f_1(y_j)}{f_2(y_j)} e^{iy_j t} \]

\[ = 2\pi i \sum_{j=1}^{\infty} \frac{(\bar{y} - y) f_1(y_j)}{i\psi \bar{\tau}} e^{iy_j t} \]

\[ = 2\omega_0 \sum_{j=1}^{\infty} f_1(y_j) e^{iy_j t}. \quad (2.40) \]

To facilitate determining \( d_{Rr}(t) \), given by Equation 2.40, the term \( f_1(y_j) e^{iy_j t} \) will be evaluated first and the final result can then be substituted back into Equation 2.40. It should be noted that the function \( f_1(y_j)e^{iy_j t} \) is two-sided function, i.e., \( y_j = (\pm \omega_j + i\eta \omega_j) \), and with this in mind, the considered term can be rewritten as

\[ f_1(y_j) e^{iy_j t} = e^{i(\pm \omega_j + i\eta \omega_j)(t - \frac{z_R}{v})} + e^{i(\pm \omega_j + i\eta \omega_j)(t - \frac{(2H - z_R)}{v})} \]

\[ = e^{-\eta \omega_j(t - \frac{z_R}{v})} \left( e^{-i\omega_j(t - \frac{z_R}{v})} + e^{i\omega_j(t - \frac{z_R}{v})} \right) + e^{-\eta \omega_j(t - \frac{(2H - z_R)}{v})} \]

\[ \left( e^{-i\omega_j(t - \frac{(2H - z_R)}{v})} + e^{i\omega_j(t - \frac{(2H - z_R)}{v})} \right). \quad (2.41) \]

Recalling that the complex exponential function \( (e^{-i\alpha} + e^{i\alpha}) \) can be expressed in trigonometric form \( (2 \cos \alpha) \), one can then rewrite Equation 2.41 in simplified form as given below

\[ f_1(y_j) e^{iy_j t} = 2 e^{-\eta \omega_j t} \left[ \cos \omega_j \left( t - \frac{z_R}{v_c} \right) + \cos \omega_j \left( t - \frac{2H - z_R}{v_c} \right) \right]. \quad (2.42) \]
Using the trigonometric identity \( \cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) \), one can further simplify Equation 2.42 as

\[
f_1(y_j) e^{iy_j t} = 2 e^{-\eta \omega_j t} \left[ \cos(\omega_j t) \left( \cos(\omega_j(\frac{z_R}{v}) + \cos(\omega_j(\frac{2H - z_R}{v})) \right) + \sin(\omega_j t) \left( \sin(\omega_j(\frac{z_R}{v}) + \sin(\omega_j(\frac{2H - z_R}{v})) \right) \right]. \tag{2.43}
\]

With the use of the trigonometric identities \( \cos(\alpha) + \cos(\beta) = 2\cos(\frac{1}{2}(\alpha + \beta)) \cos(\frac{1}{2}(\alpha - \beta)) \) and \( \sin(\alpha) + \sin(\beta) = 2\sin(\frac{1}{2}(\alpha + \beta)) \cos(\frac{1}{2}(\alpha - \beta)) \), Equation 2.43 can be put in more simplified form as given below,

\[
f_1(y_j) e^{iy_j t} = 4 e^{-\eta \omega_j t} \left[ \cos(\omega_j t) \left( \cos(\frac{\omega_j H}{v}) \cos(\frac{\omega_j (H - z_R)}{v}) \right) + \sin(\omega_j t) \left( \sin(\frac{\omega_j H}{v}) \cos(\frac{\omega_j (H - z_R)}{v}) \right) \right]. \tag{2.44}
\]

With the aid of Equations 2.37, the terms \( \cos(\frac{\omega_j H}{v}) \) and \( \sin(\frac{\omega_j H}{v}) \) can respectively be evaluated as detailed below

\[
\cos \left( \frac{\omega_j H_r}{v} \right) = \cos \left[ \omega_0 (2j - 1) \frac{H_r}{v} \right] = \cos \left[ \frac{\pi}{2} (2j - 1) \right] = 0 \tag{2.45a}
\]

\[
\sin \left( \frac{\omega_j H_r}{v} \right) = \sin \left[ \omega_0 (2j - 1) \frac{H_r}{v} \right] = \sin \left[ \frac{\pi}{2} (2j - 1) \right] = (-1)^{j+1}. \tag{2.45b}
\]

Using the results obtained from Equations 2.45(a,b), Equation 2.44 can be reduced to the following form

\[
f_1(y_j) e^{iy_j t} = 4 e^{-\eta \omega_j t} (-1)^{j+1} \cos \left( \omega_j \frac{H - z_R}{v} \right) \sin(\omega_j t). \tag{2.46}
\]

Substituting Equation 2.46 back into Equation 2.40 and rearranging, the shear response in time domain, \( d_{Rr}(t) \), can then be expressed in the following form

\[
d_{Rr}(t) = 8\omega_0 \sum_{j=1}^{\infty} (-1)^{j+1} e^{-\eta \omega_j t} \cos \left( \omega_j \frac{H_r - z_R}{v} \right) \sin(\omega_j t). \tag{2.47}
\]

Equation 2.47 shows that \( d_{Rr}(t) \) consists of infinite number of motion modes, each of which has exponentially decaying damping factor, modal shape with cosine factor,
and sinusoidal motion with modal frequency $\omega_j$. Fundamental, or first mode, with $j = 1$ has period time given as

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/2H_r} = 4 \frac{H_r}{v} = 4\tau,$$

(2.48)

where $\tau$ is the flight time for the impulse wave to travel from referenced level to the free-top end. Equation 2.48 suggests that fundamental period of a uniform model equals the time elapsed for waves to propagate up and down the height $H_r$ twice. Equations 2.33—2.48 are first derived by[27].

2.3 Response Features of Uniform Medium

This section is intended to show features of wave propagation in uniform medium as well as reveal response characteristics. For a better illustration, response of uniform medium model shown in Figure 2.7 is detailed using a wave-based approach, i.e., GIRF and GFRF. The model is subjected to fixed-base and free-end boundary conditions and characterized with 40 $m$ height and 0.035 hysteretic damping ratio. Shear deformation is assumed due to 1D shear waves propagating with 300 $m/s$ in the vertical direction. Wave features are demonstrated through pointing out amplitude and wave arrival time at two levels located at 20 $m$ and 30 $m$ with respect to two references positioned at 0 and 5 $m$ respectively.

Model response in the time domain with respect to referenced level located at the model’s base in terms of GIRF is graphically shown in Figure 2.8. The response is due to a unit imimpulse displacement wave produced by virtual source connected to referenced level at time $t = 0$. This impulse wave propagates upward with a shear wave velocity of 300 $m/s$ to arrive at level $z_{R_1}$ which is indicated by the first peak at $d_{10}$ with time shift corresponding to the travel time between referenced and $z_{R_1}$ levels. Then, the impulse wave continues propagating upward to reach level $z_{R_2}$ which is visualized by the first peak at $d_{20}$ with further time delay and amplitude decay. The time difference between the two mentioned peaks gives the elapsed time for the
Figure 2.7: Schematic diagram of uniform medium model demonstrates model characteristics, response and reference levels.

impulse wave to travel between the response levels $z_{R_1}$ and $z_{R_2}$.

Figure 2.8: Model response in the time domain: GIRF at response levels $z_{R_1} = 20$ m and $z_{R_2} = 30$ m with respect to an impulsive motion at referenced level located at $z_r = 0$ using Equation 2.47.
Then, the impulse wave continues propagating upward to be completely reflected back as it reaches the free-end level and starts traveling downward with the same motion direction, as the up-going wave does. The reflected impulse wave propagates downward to pass through response levels \( z_{R_2} \) and \( z_{R_1} \) as demonstrated by the second peaks at \( d_{20} \) and \( d_{10} \) respectively with further time shift and amplitude decrease. At the time the impulse wave reaches the reference level, an opposite impulse wave with the same amplitude is generated at the reference level propagates upward to maintain the base in motionless condition at that time instant. Therefore, the reference level with impulse displacement input is equivalent to a fixed-base end, which causes wave motion to disappear at any time other than \( t = 0 \).

This phenomenon can also be explained with transmission and reflection coefficients. In particular, at the free-end level, \( z_H \), the transmission and reflection coefficients are respectively given as \( T_{HH^-} = 2 \) and \( R_{HH^-} = 1 \), meaning that the impulse wave approaches the free-end is transmitted to the free surface with double amplitude and reflected back with the incident wave’s amplitude, which gives maximum displacement at the free end level. At the referenced level, the transmission and reflection coefficients are given as \( T_{bb^-} = 0 \) and \( R_{bb^-} = -1 \), suggesting that all down-going waves are completely reflected to up-going waves. This proves that the base is fixed for \( t > 0 \), i.e., fixed-base boundary.

The opposite impulse wave, generated at the referenced level, then continues propagating upward to pass through the response levels \( z_{R_1} \) and \( z_{R_2} \), respectively, with further time shift and amplitude decay as graphically demonstrated by the first inverted peaks at \( d_{10} \) and \( d_{20} \). The impulse wave keeps on traveling upward until it reaches the free-end level where it reflects back to propagate downward passing through response levels \( z_{R_2} \) and \( z_{R_1} \) with even more time shift and amplitude decrease as indicated by the second inverted peaks at \( d_{20} \) and \( d_{10} \) respectively. As the impulse wave approaches the reference level, an opposite impulse wave with the same amplitude is generated
at that time instant maintaining the model’s reference motionless. This impulse wave then propagates upward to repeat the same cycle. It should be noted that this cycle will be continued until the impulse’s wave energy is being dissipated.

The aforementioned cycle of wave propagation continues as time goes on. For the earlier time \((0 - 1s)\), the GIRF consists primarily of superposition of up-going and down-going traveling shear waves. For the later time \((t > 1s)\), where traveling waves can be regarded as standing waves, the GIRF develops the model dynamic vibration features characterized by modal frequencies, primarily by the 1st mode or the mode resonance with more reduction in the amplitude as time goes on. From perspective of vibration, the first couple of peaks are the result of initial condition, for the im impulse response in a system is equivalent to the free vibration with non-zero initial condition. Subsequently, as time goes on, the vibration features at that response level are dominated by the character of the resonance of the whole building, while high-order motion modes are also involved.

Figure 2.9 illustrates model response in terms of GFRF at the response levels \(z_{R1} = 20 \text{ m}\) and \(z_{R2} = 30 \text{ m}\) with respect to referenced level located at the model’s base. The first peak corresponds to fundamental natural frequency of the model and subsequent peaks are the higher modal frequencies. It is obviously shown that peak amplification decreases as natural frequencies gets higher. In addition, Figure 2.9 shows that higher frequencies are just a multiple of the fundamental frequency in the order of 3, 5, 7, \ldots, as per Equation 2.37a.

Owing to the fact that referenced level, \(z_r\), can be located at any level within the model’s height makes it efficient and simple to determine dynamic features of that part of the model immediately above the referenced level. Figure 2.10 and Figure 2.11 show, respectively, the GIRF an GFRF of the model shown in Figure 2.9 at \(z_R = 20 \text{ m}\) with respect to two referenced levels located at \(z_{r1} = 0\) and \(z_{r2} = 5 \text{ m}\).
Figure 2.9: Model response in the frequency domain: GFRF at the response levels $z_{R_1} = 20 \text{ m}$ and $z_{R_2} = 30 \text{ m}$ with respect to an impulsive motion at referenced level $z_r = 0$ using Equation 2.33.

Figure 2.10: Model response in the time domain: GIRF at $z_{R}=20 \text{ m}$ with respect to impulsive motion at two referenced levels: $z_{r_1} = 0$ and $z_{r_2} = 5 \text{ m}$. 
Figure 2.11: Model response in the frequency domain: GFRF at \( z_R = 20 \, m \) with respect to impulsive motion at two referenced levels: \( z_{r1} = 0 \) and \( z_{r2} = 5 \, m \).

It is clear that increasing reference level, \( z_r \), decreases effective height of the model, i.e., \( H_r = H - z_r \), which leads to an increase in fundamental frequency and a decrease in fundamental period for the same shear wave velocity.

2.4 Application of System Identification using Seismic Recordings

This section is aimed to demonstrate application of identifying dynamic features of Millikan library building with seismic recordings of September 3, 2002 Yorba Linda earthquake using wave-based approach. For this purpose, the Millikan library building is modeled as uniform shear beam, as shown in Figure 2.12, and identified with a set of three seismic recordings located at the basement, 3\(^{rd}\), and 8\(^{th}\) floors, as shown in Figure 2.13.

Recording-based GFRF of Yorba Linda earthquake at the selected floors is graphically shown in Figure 2.14, which is calculated using Fourier spectral ratio, namely

\[
\tilde{D}_{jb} = \frac{\tilde{U}_j \tilde{U}_b^*}{|\tilde{U}_b|^2 + \varepsilon} \rightarrow \frac{\tilde{U}_j}{\tilde{U}_b},
\]

\( (2.49) \)
Figure 2.12: Uniform medium model of the Millikan library building showing the reference, considered floors, and height levels.

Figure 2.13: Seismic recordings of the Yorba Linda earthquake at the selected floors: North-South acceleration response at the west side of the Millikan building at the basement, 3rd, and 8th floors.
where $\varepsilon$ is a positive small number, implying the added white noise, which is used primarily to avoid unstable calculation of GFRF at notches in the spectrum. It should be noted that as $\varepsilon$ approaches zero, Equation 2.49 is reduced to a Fourier spectral ratio or the definition of GFRF as given by Equation 2.27.

![Amplitude GFRF](image)

Figure 2.14: Recording-based GFRF of the Yorba Linda earthquake at the basement, 3rd and 8th floors with respected to seismic motion at the basement and 3rd floors.

Figure 2.14 shows that motion ratio at the 8th floor is calculated with respect to two referenced levels located at the basement and 3rd floors, where dynamic features of the model’s segment above the basement and 3rd floors are respectively visualized by $|\tilde{D}_{8b}|$ and $|\tilde{D}_{83}|$. Changing the referenced level from the basement floor to the 3rd floor caused the effective height to decrease which results in an increase in the fundamental frequency of the model.

Recording-based GIRF at the selected floors is shown in Figure 2.15 which is obtained by using numerical Fourier transform of GFRF. While features of wave propagation in the building can be observed from the arrival time of traveling waves from floor to floor in the 10-11 s interval shown in Figure 2.13, it can clearly be seen through the GIRF. At the basement, the GIRF is impulse acceleration, $\delta(t)$.
function, with $\varepsilon$ is selected as 5% of total power spectrum of the basement motion. As a fictitious input or virtual source to the building, the impulsive acceleration at the basement, or the peak for visual convenience, at $t = 0$ is propagated upward at building shear wave velocity, and time delay of the peak at increasing height is well observed.

Figure 2.15: Recording-based GIRF of the Yorba Linda earthquake at the basement, 3rd and 8th floors with respect to the seismic motion at the basement floor.

After hitting the top free-end level, the peak is reflected completely without changing its motion direction due to the reflection feature at the free-end, i.e., $R_{HH^-} = 1$. Time delay of waves traveling upward from the 8th floor to the top and then downward to the 8th floor generates the second peak at the 8th floor. The 2nd peak is then propagated downward and disappeared at the basement due to the fact that the basement with impulse acceleration is equivalent to the fixed basement end, which makes the wave motion to disappear at time other than $t = 0$.

The reflected waves with opposite direction are propagated upward and then reflected by the top floor to reflect again by the fixed-basement level and recovering its original direction. At that point, a standing wave phenomenon is established and the total elapsed travel time is equal to the fundamental period. The first part can
be considered as superposition of different modes of motion however, as time goes on effects of higher modes are damped out and then the later part of response is dominated by the fundamental mode.

As an application of system identification, parameters of the uniform shear beam model of the Millikan building, Figure 2.12, can be determined using seismic recordings, which can then help identify some dynamic features of the building exemplified as fundamental frequency, period, and damping. First, with the aid of the identified fundamental frequency from recordings-based GFRF, Figure 2.14, shear wave propagation velocity can be determined using Equation 2.36b and the corresponding effective height, \( H_r \). Then, damping can be obtained by minimizing mean squared error between recordings- and model-based GFRF in the frequency range 5 to 22 rad/sec. Figure 2.16 shows the best-match of the recordings- and model-based GFRFs with respect to referenced motion at the basement and 3\(^{rd}\) floors using the identified model parameters.

Figure 2.16: Matching model- and recording-based responses in the frequency domain: GFRFs at the 3\(^{rd}\) and 8\(^{th}\) floors with respect to a motion at the basement and the 3\(^{rd}\) floors using the identified parameters: \( H = 48.2 \) m, \((v, \eta)_B = (326 \text{ m/s, 0.028}), \) and \((v, \eta)_3 = (289 \text{ m/s, 0.044}), \) where the subscripts \( B \) and 3 are used to indicate the referenced levels.
Figure 2.17: Model-based response in the time domain at the selected floors: GIRFs at the 3rd and 8th floors with respect to the basement motion using the identified parameters: $H = 48.2 \, m$, $v = 326 \, m/s$, and $\eta = 0.028$.

Figure 2.17 shows model-based GIRF at the 3rd and 8th floors with respect to the basement floor using the identified model properties, i.e., shear wave velocity and hysteretic damping ratio. One can clearly see wave features in the early part, $t = 0 \sim 1 \, sec$, exemplified by time shift and amplitude decay as waves propagate up and down the considered levels. At the latter part, vibration features dominant the response, i.e., $t > 1 \, sec$. Identification of the Millikan library building using different earthquakes records data is summarized Table 2.1 and compared to the already obtained results.

### 2.5 Usefulness and Limitation of Uniform Shear Beam Model

It has been demonstrated that uniform shear beam is able to model high-rise building structures and reveal their fundamental dynamic features. Additionally, owing to the fact that the model is established using mathematical-based formalism and physical-oriented parameters, this makes selected parameters have clear physical
Table 2.1: Identification of the Millikan library building due to different earthquakes

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>$\omega_1$ (rad/sec)</th>
<th>$\omega_2$ (rad/sec)</th>
<th>$v$ (m/sec)</th>
<th>$\eta$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lytle [52]</td>
<td>12.08</td>
<td>52.36</td>
<td>-</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>San Fernando [52]</td>
<td>10.13</td>
<td>48.33</td>
<td>-</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td>Yorba Linda [27]</td>
<td>10.77</td>
<td>-</td>
<td>322</td>
<td>0.024</td>
<td>uniform shear beam model: $H=48.20$ m</td>
</tr>
<tr>
<td>Yorba Linda [28]</td>
<td>10.62</td>
<td>-</td>
<td>326</td>
<td>0.028</td>
<td>uniform shear beam model: $H=48.2$ m, $z_R=(z_3, z_8)$, $z_r=z_b$</td>
</tr>
<tr>
<td>Yorba Linda [28]</td>
<td>13.16</td>
<td>-</td>
<td>290</td>
<td>0.040</td>
<td>uniform shear beam model: $H=48.2$ m, $z_R=(z_3, z_8)$, $z_r=z_b$</td>
</tr>
</tbody>
</table>

meaning and help understanding changes in model behavior. In particular, it is important to note that the model can be identified with a set of two seismic recordings only. This can help in in-depth understanding of real structure behavior under effect of seismic excitation especially in case of limited seismic recordings.

However, with the completeness picture of the uniform medium model, there are yet some limitations making the use of this model imperfect. These limitations are due to assuming that the whole building structure is made of uniform medium and ignoring lumped masses at floor levels. These two assumptions presume no reflections occurring inside the building and hence waves entering the model from the base will propagate upward and completely reflect back by the free-end level. In reality however, presence of lumped masses at floor levels and variance in material properties along the building height, tend to turn propagating waves, in either direction, into transmitted and reflected waves.

Figure 2.18 shows graphical comparison between model- and recording-based GFRF at the 3rd floor with respect to the basement motion. It is clear that both curves ex-
Figure 2.18: Limitations of the uniform medium model: GFRF at the 3rd floor with respect to a motion at the basement floor.

It is essentially the same general trend and that the model-based curve catches the fundamental frequency of the building as the recording-based does. However, the model fails to catch subsequent higher frequencies, which may be interpreted as due to wave propagation through part of the building.

In conclusion, it has been demonstrated that uniform shear beam model is reasonable for modeling and identifying high-rise building structures with only two seismic recordings. In particular, identified parameters represent only the average building properties which can accurately point out fundamental dynamic features of the building structure. On the other hand, real building structures are generally made of different materials and include floor masses at floor levels. Hence, one can conclude that wave propagation in real buildings is quite different than uniform shear beam model. This deficiency of uniform shear beam model makes development of a new model is necessary for better analysis and identification of high-rise building structures.
CHAPTER 3
MODELING AND IDENTIFICATION WITH A PIECEWISE CONTINUOUS MEDIUM

This chapter presents theory and assumptions for modeling high-rise building structures as a piecewise continuous medium—non-homogeneous continuous layers. In this modeling system, columns and walls are modeled as shear beams while floor masses are ignored. Especially, continuity and equilibrium conditions are used to model wave motion at layer interfaces. In addition, model response, in time domain and frequency domain, is determined using wave-based approach and common features with a vibration-based approach are highlighted. In particular, application of system identification to the Millikan library building using seismic recordings is presented, in which the building is modeled as a continuous layers and identified with the Yorba Linda earthquake of September 3, 2002. Finally, the chapter ends with an outline of the usefulness and limitations of the proposed model.

3.1 Modeling Wave Propagation in a Piecewise Continuous Medium

This section presents the theoretical framework for modeling an N-story building as a series of N-shear beams separated with massless interfaces at floor levels as shown in Figure 3.1. Shear deformation of the model is assumed due to 1D shear wave propagation in the vertical direction. In particular, each shear beam layer is characterized with shear modulus $G$, mass density $\rho$, hysteretic damping ratio $\eta$, cross sectional area $A$, and story height $h$.

For the $j^{th}$ source-free shear beam bounded with $(z_j^-, z_{(j-1)}^+)$, wave propagation in a vertical direction is governed by Equation 2.4, which is repeated here for completeness as
Figure 3.1: Mechanical, physical, and geometrical characteristics of a piecewise continuous model.

\[ \frac{\partial^2 u(z_j, t)}{\partial z^2} = \frac{1}{v_j^2} \frac{\partial^2 u(z_j, t)}{\partial t^2}, \]  

(3.1)

where \( v_j = \sqrt{\frac{G_j}{\rho_j}} \) is the real shear velocity of waves propagating in the vertical direction through the \( j^{th} \) layer. With the aid of the Fourier transform of wave representation given by Equations 2.5 and 2.6, shear displacement representation in frequency domain and time domain can, respectively, be found by Equations 2.8 and 2.9.

At the \( j^{th} \) interface, shear displacement compatibility and shear force equilibrium can, respectively, be expressed as

\[ u(z_j^+, t) = u(z_j^-, t) \]  

(3.2)

\[ F_S(z_j^+, t) = F_S(z_j^-, t), \]  

(3.3)

where \( u(z, t) \) is the shear displacement and \( F_S(z, t) = G \gamma A \) is the shear force due to shear deformation with \( \gamma = \partial u(z, t)/\partial z \) is the shear strain. The superscript signs \((+, -)\) indicate, respectively, the up and down sides of elevation \( z_j \). For convenience,
the elevation height \( z \) will indicate the positive side of that level, and hereafter the superscript (+) will be dropped in later use.

With the aid of the Fourier transform, Equation 2.5, and shear displacement representation in frequency domain, Equation 2.8, one can rewrite Equations 3.2 and 3.3 in terms of out-going and in-going waves at level \( z_j \) as shown below

\[
U^u_{z_j} - U^d_{z_j} = U^u_{z_j} - U^d_{z_j} \tag{3.4}
\]

\[
(GA)_{j+1} \left( i \frac{\omega}{v_{j+1}} \right) U^u_{z_j} + (GA)_{j} \left( i \frac{\omega}{v_{j}} \right) U^d_{z_j} = (GA)_{j} \left( i \frac{\omega}{v_{j}} \right) U^u_{z_j} + (GA)_{j+1} \left( i \frac{\omega}{v_{j+1}} \right) U^d_{z_j} \tag{3.5}
\]

furthermore, one can note that the quantity \((GA\omega/v)\) can be simplified as

\[
GA\frac{\omega}{v} = (\rho v^2) A = \omega(\rho v)A, \tag{3.6}
\]

where the term \((\rho v)\) is called medium impedance. Equations 3.4 and 3.5 can especially be rewritten in matrix form and solved for out-going waves in terms of in-going waves which yields

\[
\begin{bmatrix}
U^u_{z_j} \\
U^d_{z_j}
\end{bmatrix} = \frac{1}{(1 + r_{I_j})} 
\begin{bmatrix}
2 \\
(1 + r_{I_j})
\end{bmatrix} 
\begin{bmatrix}
U^u_{z_j} \\
U^d_{z_j}
\end{bmatrix}, \tag{3.7}
\]

where \(r_{I_j}\) is the impedance ratio, expressed mathematically as

\[
r_{I_j} = \frac{(\rho v)_{j+1}}{(\rho v)_j} \frac{A_{j+1}}{A_j}. \tag{3.8}
\]

For simplicity, using \(U^u_j \equiv U^u_{z_j}\). With Equation 2.13 as a reference, Equation 3.7 can be written in terms of transmission and reflection coefficients in symbolic form, namely

\[
\begin{bmatrix}
U^u_j \\
U^d_j
\end{bmatrix} = 
\begin{bmatrix}
T_{jj} & R_{j-j} \\
R_{j-j} & T_{j-j}
\end{bmatrix} 
\begin{bmatrix}
U^u_{j-j} \\
U^d_{j-j}
\end{bmatrix} \tag{3.9}
\]
from which transmission and reflection coefficients can be expressed explicitly as

\[ T_{jj^-} = \frac{2}{1 + r_{Ij}} \]  
\[ R_{jj^-} = \frac{r_{Ij} - 1}{1 + r_{Ij}} = T_{jj^-} - 1 \]

\[ T_{j^-j} = \frac{2r_{Ij}}{1 + r_{Ij}} = r_{Ij} T_{jj^-} \]  
\[ R_{j^-j} = \frac{1 - r_{Ij}}{1 + r_{Ij}} = T_{j^-j} - 1, \]

Equations 3.10 and 3.11 demonstrate that transmission and reflection coefficients at model layer interfaces are completely dependent on the impedance ratio pertained to that interface and motion direction.

Physically, the impedance ratio can be thought of as an indicator that describes the relative rigidity between the upper and lower segments in the direction of motion. For instance, an impedance ratio less than one, suggests that an incident wave approaches a softer medium and that the transmission and reflection coefficients at such interface can, respectively, be found to be greater than and less than unity. Accordingly, transmitted and reflected waves generated at this interface will have the same motion direction as the incident wave does.

On the other hand, an impedance ratio greater than one, suggests that an incident wave approaches a stiffer medium, and thus, the transmission and reflection coefficients are, respectively, found to be less than unity and less than zero. Therefore, transmitted and reflected waves generated at this interface will, respectively, have the same and the opposite motion directions with respect to the incident wave direction, i.e., the transmitted wave continues traveling with the same motion direction while the reflected wave continues propagating in the opposite direction with a 180-degree phase difference.
It should be noted that when the impedance ratio at the \( j^{th} \) interface is equal to one, the transmission coefficients, in both directions of motion, are equal whereas the reflection coefficients are equal to zero, i.e., the uniform medium case. Displacement amplitudes of transmitted and reflected waves for different impedance ratios as well as their physical meanings are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Impedance ratio ( (r_I) )</th>
<th>Physical meaning</th>
<th>Displacement amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Free-end</td>
<td>Incident wave: ( A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transmitted wave: ( 2A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflected wave: ( A )</td>
</tr>
<tr>
<td>( 0 &lt; r_I &lt; 1 )</td>
<td>Soft medium</td>
<td>Incident wave: ( A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transmitted wave: ( A &lt; T &lt; 2A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflected wave: ( 0 &lt; R &lt; A )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>Uniform</td>
<td>Incident wave: ( A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transmitted wave: ( A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflected wave: ( 0 )</td>
</tr>
<tr>
<td>( 1 &lt; r_I &lt; \infty )</td>
<td>Rigid</td>
<td>Incident wave: ( A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transmitted wave: ( 0 &lt; T &lt; A )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reflected wave: ( -A &lt; R &lt; 0 )</td>
</tr>
</tbody>
</table>

In particular, the transmission and reflection coefficients at extreme boundaries can be found using the impedance ratio given by Equation 3.8. At the free-end level, the quantities \( \rho_{N+1}, v_{N+1}, \) and \( A_{N+1} \) are equal to zero, which leads to a zero-impedance ratio case at this interface, i.e., \( r_{IN} = 0 \), and consequently, Equations 3.10 are degenerated to \( T_{NN} = 2 \) and \( R_{NN} = 1 \). Meaning an incident wave approaches the free-end level will be transmitted upward with twice the amplitude of the incident wave. Similarly, at the fixed-bottom level, or referenced level, for the downward motion direction case, the \( r_{In} = \infty \); therefore, Equations 3.10 are degenerated to \( T_{00} = 0 \) and \( R_{00} = -1 \). Suggesting propagating waves approach the referenced level will completely be reflected upward with the same amplitude, however, in the opposite motion direction.

For a composite section bounded with \((z_l, z_n)\), or simply \((l, n)\) as shown in Figure 3.2, with intermediate level \( z_m \), i.e., \((z_l < z_m < z_n)\), repeating the use of Equation 3.9 for \((l, m)\) and \((m, n)\) can lead to representation of the transmission and reflection coefficients between \((l, n)\) in terms of those in two sub-sections \((l, m)\) and...
For the section bounded, namely by \((l, m)\), the out-going waves are related to in-going waves as

\[
U_{u}^{m} = T_{ml} \, U_{l}^{u} + R_{lm} \, U_{m}^{d}
\]  
\[(3.12)\]

\[
U_{d}^{l} = T_{lm} \, U_{m}^{d} + R_{ml} \, U_{l}^{u},
\]  
\[(3.13)\]

and for the section bounded by \((m, n)\), the out-going waves are related to in-going waves as

\[
U_{u}^{n} = T_{nm} \, U_{m}^{u} + R_{mn} \, U_{n}^{d}
\]  
\[(3.14)\]

\[
U_{d}^{m} = T_{mn} \, U_{n}^{d} + R_{nm} \, U_{m}^{u}
\]  
\[(3.15)\]

Equations 3.13 and 3.15 can be solved together for \(U_{u}^{m}\) and \(U_{d}^{d}\) and then the results are used to relate out-going and in-going waves between levels \((l, n)\) in terms of equivalent transmission and reflection coefficients, that is.
\[
\begin{bmatrix}
U_n^u \\
U_l^d
\end{bmatrix} =
\begin{bmatrix}
T_{nl} & R_{ln} \\
R_{nl} & T_{nl}
\end{bmatrix}
\begin{bmatrix}
U_1^u \\
U_n^d
\end{bmatrix}.
\tag{3.16}
\]

Subsequently, equivalent transmission and reflection coefficients in terms of subsection coefficients between levels \((l, n)\) can then be expressed explicitly as

\[
T_{nl} = \frac{T_{ml} T_{nm}}{1 - R_{nm} R_{lm}} \tag{3.17a}
\]
\[
R_{nl} = R_{ml} + \frac{T_{ml} R_{nm} T_{lm}}{1 - R_{nm} R_{lm}} \tag{3.17b}
\]

\[
T_{ln} = \frac{T_{lm} T_{mn}}{1 - R_{nm} R_{lm}} \tag{3.18a}
\]
\[
R_{ln} = R_{mn} + \frac{T_{nm} R_{lm} T_{mn}}{1 - R_{nm} R_{lm}} \tag{3.18b}
\]

The above composition rule can be applied repeatedly to find all transmission and reflection coefficients between any two levels and ultimately for the whole model.

### 3.2 Response of Piecewise Continuous Medium

Response representation of a piecewise continuous model in the frequency domain can be put in non-dimensional form, called the motion ratio and denoted as \(D_{Rr}(\omega)\). This response can especially be obtained, at any level within the model height, by relating shear displacement at the response level, denoted as \(U_R\), to shear displacement at the referenced level, denoted as \(U_r\). In particular, shear displacement at the referenced level can be expressed as

\[
U_r = U_r^u + U_r^d = U_r^u + R_{Nr} U_r^u = U_r^u (1 + R_{Nr}) \Rightarrow U_r^u = \frac{U_r}{(1 + R_{Nr})},
\tag{3.19}
\]

similarly, shear displacement at the response level can be expressed as
\[ U_R = U_R^u + U_R^d \]
\[ = U_R^u + R_{NR} U_R^u \]
\[ = U_R^u(1 + R_{NR}) \] (3.20)
\[ \Rightarrow U_R^u = \frac{U_R}{(1 + R_{NR})} , \]
where \( U_R^d = R_{Nr} U_R^u \) and \( U_R^d = R_{NR} U_R^u \). With the aid of Equation 3.16 with \( l \) and \( n \) are replaced with \( r \) and \( R \) correspondingly, up-going waves at the response level \( U_R^u \),
can be expressed as
\[ U_R^u = T_{Rr} U_r^u + R_{rR} U_R^d \]
\[ = T_{Rr} U_r^u + R_{rR} R_{NR} U_R^u \] (3.21)
\[ \Rightarrow U_R^u (1 - R_{NR} R_{rR}) = T_{Rr} U_r^u . \]
Inserting the expressions for \( U_r^u \) and \( U_R^u \) given by Equations 3.19 and 3.20, respectively, into Equation 3.21, one can obtain model response in terms of motion ratio as
\[ D_{Rr}(\omega) = \frac{U_R}{U_r} = \frac{(1 + R_{NR}) T_{Rr}}{(1 - R_{rR} R_{NR})(1 + R_{Nr})} . \] (3.22)
Equation 3.22 demonstrates that model response is completely dependent on transmission and reflection coefficients of the part of the model that is bounded by referenced and free-end levels as illustrated in Figure 3.3. Moreover, since transmission and reflection coefficients are functions of model parameters, changes in model response can then be reasoned to the changes in model properties immediately above the referenced level. Furthermore, the flexibility of locating the referenced level at any position within the model’s height helps eliminate the effects of soil-structure interaction and excitation source and hence, changes in the model response can be directly related to changes in model characteristics.

Response representation of a piecewise continuous model in the time domain can be obtained by applying the Fourier transform, given by Equation 2.5, to model response in the frequency domain, given by Equation 3.22, that is
\[ d_{Rr}(t) = \int_{-\infty}^{\infty} D_{Rr}(\omega) e^{i\omega t} d\omega \]
\[ = \int_{-\infty}^{\infty} \frac{(1 + R_{NR}) T_{Rr}}{(1 - R_{rR} R_{NR})(1 + R_{Nr})} e^{i\omega t} d\omega . \] (3.23)
The input/output model response concept has been explained in Section 2.2. Consequently, wave-based representation of shear displacement at $z_R$ to input displacement at $z_r$ is then given by Equations 2.29 and 2.30. However, for the case of ground acceleration input, shear displacement response can be obtained using Equations 2.31 and 2.32.

To show response features of a piecewise continuous medium in terms of model parameters, a simple case of a two-layer continuous model is presented. For convenience, two cases are demonstrated considering model response at two different positions with respect to a reference level located at the model’s base. First, consider the case where response level is located at the lower layer, i.e., $0 < z_R < h_1$, as shown in Figure 3.4. In this case, model response in the frequency domain can be obtained using Equation 3.22 with $N$ and $r$ are replaced with 2 and 0 respectively, that is

$$D_{R0}(\omega) = \frac{(1 + R_{2R}) T_{R0}}{(1 - R_{0R} R_{2R}) (1 + R_{20})}.$$  (3.24)
Figure 3.4: Response parameters of a 2-layer continuous medium for the case where the response level is located at the lower layer: \( 0 < z_R h_1 \).

In order to express the model response in terms of model parameters, however, one needs to substitute symbolic form of transmission and reflection coefficients appear in Equation 3.24 with their explicit mathematical expressions. The transmission coefficient \( T_{R0} \), within the lower layer is given by

\[
T_{R0} = e^{-i\omega \frac{z_R}{v_1}} = e^{-i\omega \tau_z}.
\]  

(3.25)

Recalling that the reflection coefficient for propagating waves at the lower layer is equal to zero, i.e., \( R_{0R} = 0 \). The Composition rule, on the other hand, is used to determine equivalent reflection coefficient concerning the two layers, i.e., \( R_{2R} \) and \( R_{20} \), namely

\[
R_{2R} = \left( \frac{2}{1 + r_I} - 1 \right) e^{-2i\omega \frac{(h_1 - z_R)}{v_1}} + \frac{4r_I e^{-2i\omega \frac{h_2}{v_2}}}{(1 + r_I)^2} \left[ \left( 1 - \frac{2}{(1 + r_I)} \right) e^{-2i\omega \frac{h_2}{v_2}} \right]
\]  

(3.26)
\[ R_{20} = \left( \frac{2}{1 + r_I} - 1 \right) e^{-2i\omega \frac{h_1}{v_1}} + \frac{4r_I e^{-2i\omega \left( \frac{h_2}{v_2} + \frac{(h_1 - z_R)}{v_1} \right)}}{(1 + r_I)^2 \left[ 1 + \left( 1 - \frac{2}{(1+r_I)} \right) e^{-2i\omega \frac{h_2}{v_2}} \right]} , \]

Substituting Equations 3.25—3.27 into Equation 3.24 represents the response in terms of model parameters, namely

\[ D_{R0}(\omega) = \frac{1 + \alpha \left( e^{-2i\omega(\tau_1 - \tau_2)} + e^{-2i\omega\tau_2} \right) + e^{-2i\omega(\tau - \tau_2)} e^{-i\omega \tau_z}}{1 + \alpha \left[ e^{-2i\omega\tau_1} + e^{-2i\omega\tau_2} \right] + e^{-2i\omega\tau}} , \]

where the notations mentioned above can be defined as

\[ \tau_z = z_R / v_1, \tau_1 = h_1 / v_1, \tau_2 = h_2 / v_2, \tau = \tau_1 + \tau_2 \]

\[ r_I = (\rho v)_{2} A_2 / (\rho v)_{1} A_1, \alpha = \frac{(1 - r_I)}{(1 + r_I)} . \]

Similarly, the second case where the response level \( z_R \), is located at the upper layer, \( h_1 < z_R \leq h_1 + h_2 \), is illustrated schematically in Figure 3.5. For this case, the transmission and reflection coefficients that appear in Equation 3.24 are defined as

\[ T_{R0} = \frac{2}{(1 + r_I)} e^{-i\omega \left( \frac{h_1}{v_1} + \frac{(h_1 - z_R)}{v_2} \right)} \]  

\[ R_{0R} = \left( \frac{2r_I}{(1 + r_I)} - 1 \right) e^{2i\omega \frac{(h_1 - z_R)}{v_2}} \]

\[ R_{2R} = e^{-2i\omega \frac{(H - z_R)}{v_2}} \]

\[ R_{20} = \left( \frac{2}{(1 + r_I)} - 1 \right) e^{-2i\omega \frac{h_1}{v_1}} + \frac{4r_I}{(1 + r_I)^2} \left( 1 - \left( \frac{2}{(1+r_I)} - 1 \right) e^{-2i\omega \frac{h_2}{v_2}} \right) . \]

Substituting Equations 3.31—3.34 into Equation 3.24 yields the response in terms of model parameters, namely

\[ D_{R0}(\omega) = \frac{2 \left[ 1 + e^{-2i\omega(\tau - \tau_z)} \right] e^{-i\omega \tau_z}}{1 + \alpha \left[ e^{-2i\omega\tau_1} + e^{-2i\omega\tau_2} \right] + e^{-2i\omega\tau}} , \]

55
Figure 3.5: Response parameters of a 2-layer continuous medium for the case where the response level is located at the upper layer: \( h_1 < z_R \leq h_1 + h_2 \).

where \( \tau_z = h_1/v_1 + (z_R - h_1)/v_2 \).

Model response in the time domain can be obtained by substituting Equations 3.28 and 3.35 into Equation 3.23. The integration can specifically be evaluated using a method of residue to obtain a closed form solution for some special cases and numerically for general cases. A special case is presented below to help demonstrate the characteristics of wave propagation in building structures and subsequently aid in system identification for general cases.

In particular, the GIRF is determined for the two considered cases under the conditions: \( r_1 = 1, \tau_1 = \tau_2, \) and \( \eta_1 = \eta_2 = \eta \). This yields \( \alpha \) is equal to zero and accordingly, the denominator of Equations 3.28 and 3.35 is rewritten as

\[
1 + e^{-2i\eta\tau} = 0, \tag{3.36}
\]

where the real variable \( \omega \) is replaced with the variable \( y \). Solve Equation 3.36 for the variable \( y \) and taking into account the cases \( y > 0 \) and \( y < 0 \), yields

\[
y_j = \omega_j(\pm 1 + i\eta) \quad j = 1, 2, \ldots, \infty. \tag{3.37}
\]
This demonstrates that the denominator of the GFRF for the considered cases has an infinite number of poles, i.e., \( y_j(j = 1, 2, \ldots) \), in the upper half complex plane as described by the schematic diagram shown in Figure 3.6.

The natural frequency of the model under the considered conditions can be found by solving the denominator of Equations 3.28 and 3.35 for \( \omega \), that is

\[
\omega_j = \frac{(2\pi j - i\log(-\alpha \pm \sqrt{-1 + \alpha^2}))}{\tau}.
\]

Under the assumed condition of the impedance ratio \( r_I = 1 \), which does not necessarily mean a uniform medium case, the \( \log \) term will be reduced to \( \log(\pm i) = \pm \frac{i\pi}{2} \), in which Equation 3.38 can be further simplified to

\[
\omega_j = \frac{(2\pi j \pm \frac{\pi}{2})}{\tau}.
\]

Using the following trigonometric identities

\[
\alpha = \pm \cos \beta \tag{3.40a}
\]

\[
\sin^2 \beta + \cos^2 \beta = 1 \Rightarrow \sin \beta = \sqrt{1 - \alpha^2}, \tag{3.40b}
\]

then, one can find the angle \( \beta \) as given below:

\[
\tan \beta = \frac{\sin \beta}{\cos \beta} = \frac{\sqrt{1 - \alpha^2}}{|\alpha|} \Rightarrow \beta = \arctan \frac{\sqrt{1 - \alpha^2}}{\alpha}, \tag{3.41}
\]
with this in mind, Equation 3.39 can be rewritten in terms of the complement of
angle \( \beta \), in which one can readily find modal frequencies in the order of sequential
pairs, namely

\[
\omega_{2j-1} = \frac{[(2j - 1)\pi - \beta]}{\tau}
\]

\[
\omega_{2j} = \frac{[(2j - 1)\pi + \beta]}{\tau} \quad \text{for } j = 1, 2, \ldots, \infty
\]

Closed form solution of GIRF for the case of \( r_{I_1} = 1 \), which could be, but not
necessarily, the uniform or one-layer model, is derived as detailed below. The GFRF
for this case is obtained by substituting \( \alpha = 0 \) into Equations 3.28 and 3.35, in which
these two Equations will be reduced to

\[
D_{R0}(y_j) = \frac{[1 + e^{-2iy_j(\tau - \tau_z)}}{[1 + e^{-2iy_j\tau}}.
\]

With the aid of Equation 3.23, the GIRF corresponding to the GFRF given by Equa-
tion 3.43 is specifically written as

\[
d_{R0}(t) = \int_{-\infty}^{\infty} D_{R0}(y_j) e^{iy_jt} dy
\]

\[
= \int_{-\infty}^{\infty} \frac{f_1(y_j)}{f_2(y_j)} e^{iy_jt} dy,
\]

where \( f_1(y_j) \) and \( f_2(y_j) \) are, respectively, denote the numerator and denominator of
the \( D_{R0}(y_j) \) given by Equation 3.43. Using the residue theorem, the integration given
by Equation 3.44 can then be written in a summation form as

\[
d_{R0}(t) = 2\pi i \sum_{\tilde{y} \to y} (\tilde{y}_j - y_j) \frac{f_1(y_j)}{f_2(y_j)} e^{iy_jt},
\]

where \( \tilde{y} = y + \psi \) and \( \psi \) is a small positive number. Note that the denominator of
Equation 3.44 is equal to zero when \( e^{-2iy_j\tau} = -1 \). Then using Taylor expansion in \( \psi \),
it can be shown that to a first order approximation, the denominator \( f_2(y_j) \) can be
simplified as

\[
f_2(y_j) = 1 + e^{-2iy_j\tau} \approx 1 + e^{-iy_j\tau(1 - 2i\psi\tau - 0(\psi^2))} \approx 2i\psi\tau.
\]
which proves that poles of integration are simple. Substituting this result into Equation 3.45 and simplifying, yields

\[ d_{R0}(t) = \frac{\pi}{\tau} \sum_{j=1}^{\infty} f_1(y_j) \, e^{iy_j t}. \]  

(3.47)

for simplicity, the term \( f_1(y_j) \, e^{iy_j t} \) will be evaluated first and the final result will then be incorporated into Equation 3.47. Recall from Equation 3.37 that the \( y_j \) is two-sided function. With this in mind, the considered term can be rewritten as

\[ f_1(y_j) \, e^{iy_j t} = e^{i(\pm \omega_j + in\omega_j)(t-\tau_z)} + e^{i(\pm \omega_j + in\omega_j)(t-(2\tau-\tau_z))} \]

\[ = e^{-n\omega_j(t-\tau_z)} \left[ e^{-i\omega_j(t-\tau_z)} + e^{i\omega_j(t-\tau_z)} \right] + e^{-n\omega_j(t-(2\tau-\tau_z))} \]

\[ = e^{-n\omega_j(t-(2\tau-\tau_z))} + e^{i\omega_j(t-(2\tau-\tau_z))} \], \n
(3.48)

using the equivalence of a complex exponential function and trigonometric function identity in the form \((e^{-i\alpha} + e^{i\alpha}) \equiv (2 \cos \alpha)\), then Equation 3.48 can be simplified as

\[ f_1(y_j) \, e^{iy_j t} = 2 \, e^{-n\omega_j t} \left[ \cos \omega_j(t-\tau_z) + \cos \omega_j(t-(2\tau-\tau_z)) \right], \] \n
(3.49)

furthermore, the trigonometric identity \( \cos(\alpha\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) \), can be used to simplify Equation 3.49, that is

\[ f_1(y_j) \, e^{iy_j t} = 4 \, e^{-n\omega_j t} \left[ \cos(\omega_j t)(\cos\omega_j(\tau_z) + \cos\omega_j(2\tau-\tau_z)) + \sin(\omega_j t)(\sin\omega_j(\tau_z) + \sin\omega_j(2\tau-\tau_z)) \right], \] \n
(3.50)

with the aid of the identities: \( \cos(\alpha) + \cos(\beta) = 2 \cos\frac{1}{2}(\alpha + \beta) \cos\frac{1}{2}(\alpha - \beta) \) and \( \sin(\alpha) + \sin(\beta) = 2 \sin\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta) \), Equation 3.50 can be further simplified as

\[ f_1(y_j) \, e^{iy_j t} = 4 \, e^{-n\omega_j t} \left[ \cos(\omega_j t)(\cos\omega_j(\tau - \tau_z)) + \sin(\omega_j t)(\sin\omega_j(\tau - \tau_z)) \right]. \] \n
(3.51)

using Equation 2.37, the terms \( \cos(\omega_j \tau) \) and \( \sin(\omega_j \tau) \) can be simplified as
\[
\cos(\omega_0(2j-1)\tau) = \cos[\frac{\pi}{2}(2j-1)] = 0 \quad (3.52a)
\]
\[
\sin(\omega_0(2j-1)\tau) = \sin[\frac{\pi}{2}(2j-1)] = (-1)^{j+1}. \quad (3.52b)
\]

Equations 3.52 is substituted into Equation 3.50, which can simplified as
\[
f_1(y_j) e^{i\omega_j t} = 4 e^{-\eta\omega_j t}(-1)^{j+1}\cos[j(\tau - \tau_z)] \sin(\omega_j t). \quad (3.53)
\]

Equation 3.53 is substituted into Equation 3.47 yields
\[
\frac{dR_0}{\tau} = 4\pi \sum_{j=1}^{\infty} (-1)^{j+1} e^{-\eta\omega_j(t-\tau_z)} \cos_j(\tau - \tau_z) \sin(\omega_j t) \quad (3.54)
\]

Equation 3.54 is consistent with Equation 2.47 for the case of a uniform medium model. It also shows that the GIRF consists of an infinite number of motion modes, each of which has an exponentially decaying damping factor, modal shape with cosine factor, and sinusoidal motion with modal frequency \(\omega_j\). This is essentially the traditional vibration perspective for seismic motion in buildings, in which the fundamental modal frequency \(\omega_1\), similar to higher-order modal frequencies, is interpreted as a vibration feature of the building’s periodic horizontal motion. Since the fundamental frequency corresponds to the fundamental period through \(\omega_1 = \frac{2\pi}{T_1}\), the fundamental period is also corresponds to wave travel time as \(T_1 = 4\tau\) which is viewed as four times the wave traveling time through the building height, i.e., wave interpretation.

3.3 Response Features of a Piecewise Continuous Medium

This section intends to show dynamic features of wave propagation as well as vibration phenomenon of a piecewise continuous medium response, in the time domain and frequency domain, to a unit displacement pulse wave, generated at the referenced level at time \(t = 0\), using the wave-based approach. For the sake of illustration, a numerical example considers a 50 m height 2-layer continuous model, hereafter called the 2-layer model, with the boundary conditions of fixed-base/free-end, as shown in Figure 3.7a, is analyzed using the wave-based approach.
In particular, the lower layer is characterized with a 30 m height, 300 m/sec shear wave velocity, and 0.030 hysteretic damping ratio. And, the upper layer is characterized with a 20 m height, 200 m/sec, and 0.035 hysteretic damping ratio. In addition, the interface between the two layers is characterized with a 2/3 impedance ratio. In order to compare the dynamic behavior of the 2-layer continuous model to a uniform medium one, the response of a fixed-base and free-end uniform model with a 50 m height, as shown in Figure 3.7b, has been determined and incorporated as a reference. Model response has been examined at two levels located at 10 m and 40 m measured from the referenced level. It should be noted that the uniform medium model is characterized with the average 2-layer model properties, i.e., a 250 m/s shear wave velocity and 0.037 hysteretic damping ratio.

Figure 3.7: Schematic diagram demonstrates general configuration of: (a) a 2-layer continuous model, (b) a uniform medium model.

Response representation in time domain of the 2-layer and uniform medium models is graphically shown in Figure 3.8 where the GIRFs at response levels $z_{R_1}$ and $z_{R_2}$ are, respectively, indicated by $d_{10}$ and $d_{20}$. The response could be thought of as a due
to a unit displacement pulse wave, generated at the referenced level at model’s base at time \( t = -0 \). Then, the pulse wave starts propagating upward with the assumed shear wave velocity to reach \( z_{R_1} \), which is visualized by the first peak at \( d_{10} \). The difference appears at the first peak at \( d_{10} \) between the two models, in arrival time and amplitude, can be reasoned to the variance in wave propagation velocity.

![Figure 3.8: Model response in the time domain: early part of the GIRF at \( z_{R_1} \) and \( z_{R_2} \) with respect to a motion at the model’s base using the 2-layer and uniform medium models with the indicated parameters.](image)

In the 2-layer model case, the pulse wave reaches the interface will be split into a transmitted wave, which continues propagating beyond the interface in the upper layer and a reflected wave, which reflects back by the interface and continues propagating in the lower layer. In the uniform model case, however, the pulse wave continues propagating upward without splitting. In the case of impedance ratio, \( r_I = 2/3 \), one can recall Equation 3.10, in which transmission and reflection coefficients are equal to 6/5 and 1/5 respectively. This leads to the amplification of the transmitted pulse wave amplitude at the interface level with a 20% more than the counterpart of the uniform medium at the same level as obviously shown by the first peak at \( d_{20} \).

After that, the transmitted and reflected pulse waves continue traveling each in its own direction. The transmitted pulse wave is then reached at \( z_{R_2} \) which is demon-
strated by the first peak at $d_{20}$ and continues propagating upward to reach the free-end level. Due to reflection properties at the free-end level, the pulse wave is reflected back with the same motion direction and propagates downward towards $z_{R2}$, which is demonstrated by the second peak. Again, the time shift between the arrival times between the models is attributed to the difference in the wave propagation velocity.

At the same instant, the reflected pulse wave, reflected back by the interface, propagates downward with the same motion direction as the original pulse wave does, and reaches $z_{R1}$, which is demonstrated by the second small peak at $d_{10}$, where the time shift and amplitude decrease are observed. However, the uniform model response does not show this behavior. The pulse wave then continues propagating towards the model’s base, in which, and due to the referenced level properties, it reflects with an opposite motion direction and begins traveling upward towards $z_{R1}$, where the first inverted peak at $d_{10}$ indicates the arrival time for the opposite pulse wave.

The reflected pulse wave propagating downward through the $z_{R2}$ level arrives at the interface and splits into transmitted and reflected waves. Due to the fact that the motion direction is reversed, Equation 3.11 will be used instead to determine transmission and reflection coefficients, which are found as $4/5$ and $-1/5$, respectively. In particular, the transmitted wave continues propagating towards $z_{R1}$, which is schematically shown by the second higher amplitude. Meanwhile, the reflected wave starts propagating upward towards $z_{R2}$ with opposite motion direction. The reflected wave continues propagating towards the free-end level to be completely reflected back with the same motion direction and reaches at $z_{R2}$ again as schematically demonstrated by the first two inverted peaks at $d_{20}$. This cycle will continue and repeated over time until wave’s energy dissipates. It should be noted that the existence of the interface at some level between the extreme boundaries shortness the cycle of the fundamental period and hence, the 2-layer model has the smaller fundamental period than the uniform one.
The model response in the frequency domain is demonstrated in Figure 3.9. Obviously, the 2-layer model has a higher fundamental frequency than the uniform model does. Furthermore, one can notice that higher frequencies are not multiples of the fundamental frequency as in the case of the uniform model.

In addition, amplitude response at $d_{10}$ is higher in case of uniform model than the case of the 2-layer model, however at $d_{10}$, the 2-layer model shows a slight higher in the amplitude response than the case of the uniform model. This behavior could be attributed to the pulse wave amplitude as it evidently demonstrates by the GIRF shown in Figure 3.8.

3.4 Application of System Identification using Seismic Recordings

This section presents the application of using a piecewise continuous model to identify dynamic response features of the Millikan library building using wave-based approach with the Yorba Linda earthquake of September 3, 2002. In particular, the building is modeled as a 2-layer continuous model, with one soft-thin layer over a rigid-thick layer. The following model parameters—mechanical, physical, and geometrical
properties—were selected using a trial and error approach for the proposed model, i.e., $r_1 = 0.04$, $v_1 = 345.18\ m/s$, $v_2 = 10.32\ m/s$, $h_1 = 46.98\ m$, $h_2 = 1.22\ m$ and $\eta_1 = \eta_2 = 0.030$. The geometric configuration of the model using the above mentioned properties is shown in Figure 3.10.

![Figure 3.10: The 2-layer model of the Millikan Library building: preliminary suggested model parameters.](image)

To further clarify the wave propagation features, one can look at Figure 3.11, which demonstrates model-based GIRFs at the 4\textsuperscript{th} and 7\textsuperscript{th} floors, denoted as $d_{40}$ and $d_{70}$ respectively, due to an impulsive acceleration at the basement floor, from which wave propagation with damping-related amplitude attenuation is clearly observed. In particular, the first peak of $d_{40}$ is rooted from the impulse at the basement (level 0), which is propagated to the first peak of $d_{70}$ with reduced amplitude. That first peak at the 7\textsuperscript{th} floor is further propagated to the building top and then reflected to travel downward to the 7\textsuperscript{th} and subsequently 4\textsuperscript{th} floors, and generates the second peaks with further reduced amplitudes. The second peak at the 4\textsuperscript{th} floor continues the downward propagation to the basement.
Figure 3.11: GIRFs at the 4th and 7th floors with respect to impulsive acceleration at the basement obtained based on Equation 3.11a and 3.15 with $r_{t_1} = 0.04$, $v_1 = 345.18 \, m/s$, $v_2 = 10.32 \, m/s$, $h_1 = 46.98 \, m$, $h_2 = 1.22 \, m$, and $\eta_1 = \eta_2 = 0.03$

Since the motion disappears at the basement at $t \neq 0$ (due to an impulse feature at the basement which is proved to be a fixed boundary), a negative, same-amplitude peak, balancing the positive one at the basement, is generated and propagated upward. That negative peak is propagated to the 4th and then 7th floors with sequentially-reduced amplitude (shown as the third negative peaks in $d_{40}$ and $d_{70}$, and continues with the previous wave propagation pattern. As time goes on, wave response at the 7th floor (similar to the 4th floor) is then dominated by the vibration character of a resonance for the whole building, which has a fundamental period equal to $4\tau$, or four times the wave traveling time through the building height. It is verified theoretically and numerically that the peak-to-peak time elapse (or simply flight time) between the two locations is the wave traveling time. It is also noted that wave reflection and transmission at the connection of layers 1 and 2 do affect the wave amplitude reduction and flight time, which will be discussed later.

In short, the above shows wave and vibration features in GIRF and GFRF, confirming the aforementioned clarification in determining the GIRF/GFRF, i.e., the
wave-based or generalized version of the traditional impulse response function, IRF, and the frequency response function, FRF, with discrete multi-degree of freedom (MDOF) modeling. For illustration, this study shows the parametric identification of the Millikan Library with the use of the piecewise continuous model and a pair of seismic recordings after the Yorba Linda earthquake of September 3, 2002.

For system identification, recording-based GFRF/GIRF is required. One can first calculate the recording-based GFRF as given by Equation 2.49. For a pair of recordings available at the basement and floor 7, the GFRF with $\varepsilon = 5\%$ of the total power spectrum of basement motion can be found in Figure 3.12. In principle, all the frequencies corresponding to the spectral peaks in Figure 3.12 can be regarded as modal frequencies and used for system identification. For simplicity in practice and also for illustration with the use of a two-layer model, parametric identification is carried out here based on two modal frequencies identified from Figure 3.12 as $\omega_1 = 10.62 \text{ rad/sec}$ and $\omega_2 = 14.21 \text{ rad/sec}$.

With the use of Equations 3.42(a,b), the following two parameters are found

![Figure 3.12: Comparison of GFRF amplitudes at the 4th and 7th floors with respect to impulsive basement acceleration obtained from seismic recordings and model Equation 3.28.](image-url)
\[
\tau = \frac{2\pi}{(\omega_1 + \omega_2)} \\
\beta = \frac{\pi(\omega_2 - \omega_1)}{(\omega_1 + \omega_2)}
\]

(3.55a) (3.55b)

which yields \(\tau = 0.253\) sec and \(\beta = 0.454\) rad, and subsequently with the aid of Equations 3.28, 3.29, and 3.30, one can obtain the following parameters: \(r_{11} = 0.053\), \(v_1 = 361.85\) m/sec, \(v_2 = 19.18\) m/sec, \(h_1 = 45.77\) m, \(h_2 = 2.43\) m. These identified parameters are quite close to the aforementioned, pre-selected ones based on the structural configuration shown in Figure 1.2a. This partially confirms the appropriateness of the identification approach.

It should also be noted that the aforementioned identification is under special condition, i.e., \(\tau_1 = \tau_2\) and \(\eta_1 = \eta_2\). In general, parameters \(r_{11}, v_1, v_2, h_1\) and \(h_2\), together with \(\eta_1\) and \(\eta_2\), can be found by minimizing the mean squared error of the model-based GFRF from recording-based GFRF in a certain frequency range (say 5-22 rad/sec), among many other identification algorithms. While it is doable, this study instead presents the comparison of model-based GFRFs with the aforementioned, pre-selected parameters with recording-based ones, aiming to show the influences of some parameters in GFRF and GIRF.

Figure 3.12, Figure 3.13, and Figure 3.14 show respectively the comparison of recording-based GFRF and GIRF at the 4th and 7th floors with respect to band-limited (\(\varepsilon = 5\%\)) impulsive motion at the basement against model-based counterparts with respect to pure (\(\varepsilon = 0\%\)) impulsive motion at the basement. These three figures indicate that the two-layer model is able to capture the fundamental wave and vibration features shown in the recordings, exemplified as the first and second modal frequencies in Figure 3.12, and the proximity of the first couple of wave arrival times and well-matched resonant vibration features in Figure 3.13 and Figure 3.14.
The major difference in spectral amplitudes at the first modal frequency in Figure 3.12 and in wave amplitude and arrival time in the 0–0.5 sec time window in Figure 3.13 and Figure 3.14 can be minimized with an appropriate system-identification algorithm for identifying layer parameters and band limited $\varepsilon$.

While some fundamental characteristics of wave-based system identification are shown with the two-layer model, an increased number of layers in the model would be, in principle, more appropriate to realistically capture the physical multi-story structure of the building. To see the influence of a multi-layer model in system identification, one can alternatively examine the difference of GFRF and GIRF with two-layer and 11-layer models. Based on the structural configuration in Figure 1.2a, the building can be modeled as 11 layers, with the top 11th layer being the same as the 2nd layer in the 2-layer model, and with the first ten layers having the same flight time as the first layer in the 2-layer model.

Due to the story-to-story proximity in structure in Figure 1.1a, the flight time for the first 10 layers is assumed to be equally shared with each of the ten layers in the 11-layer model. The minor difference in story height in Figure 1.2a then leads
to a slightly-different velocity in each layer and $r_{lj} \approx 1$ for $j=1-10$. This yields to $T_{jj} \approx T_{j-j} \approx 1$, $R_{jj} \approx R_{j-j} \approx 0$ for $j=1-10$ and also $T_{lm} \approx T_{lm}T_{mn}$ and $R_{ln} = R_{mn}$, for $l,m = 0 – 10$ With the aid of Equations 3.10, 3.11, and 3.18. Subsequently this simplification leads to the $Ts$ and $Rs$ in the first ten layers in the 11-layer model similar to those with the first layer in 2-layer model. With those $Ts$ and $Rs$, GFRF with the 11-layer model obtained from Equation 3.21 with $N=11$ is essentially similar to Equations 3.27 and 3.33 with the two-layer model.

Figure 3.15 shows that both models capture the two modal frequencies. The difference of the 11-layer model from the 2-layer model resides in the increased amplitude at the down shifted first modal frequency, which is attributed to the wave scattering (wave transmission and reflection at the first nine layer-to-layer boundaries) in the major portion of the 11-layer building below the soft-thin top. With adjusted parameters in the first ten layers, the difference can be eliminated, suggesting that there is no qualitative change of wave and vibration features in GFRF with the number of layers per stories.
Figure 3.15: Comparison of GFRF amplitudes at the 4\textsuperscript{th} and 7\textsuperscript{th} floors with respect to impulsive basement acceleration obtained with uniform (one-layer), two-layer, and 11-layer models from seismic recordings and models.

On the other hand, wave and vibration features in GFRF and GIRF with a uniform one-layer model is qualitatively different from those in a 2- or 11-layer model, for the former cannot capture the motions with the second modal frequency, as shown in Figure 3.15. The difference can also be seen in Figure 3.16, although it is not as qualitatively clear as in Figure 3.15. System identification of the Millikan library building with the 2-layer model using two recordings of the Yorba Linda earthquake and relevant studies using other earthquake recordings are shown in Table 3.2.

3.5 Usefulness and Limitations of Piecewise Continuous Model

Using a wave-based approach in response analysis of high-rise buildings with the aid of derived GFRF and GIRF reveals, beside the well-observed vibration-based features, dynamic features of the propagation seismic waves through the building, in which traditional vibration-based approach can not show. Comparing piecewise continuous and uniform medium models, one can clearly see that, based on Figure 3.15, a uniform medium model can not capture building response behavior at higher modal frequencies, which in principle, this model can do it. With these advantages, however,
Figure 3.16: Comparison of GIRFs at the 4th and 7th floors with respect to impulsive basement acceleration obtained with one- and two-layer models.

Table 3.2: Identification of the Millikan library building due to different earthquakes

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>$\omega_1$ (rad/sec)</th>
<th>$\omega_2$ (rad/sec)</th>
<th>$v$ (m/sec)</th>
<th>$\eta$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lytle [52]</td>
<td>12.08</td>
<td>52.36</td>
<td>-</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>San Fernando [52]</td>
<td>10.13</td>
<td>48.33</td>
<td>-</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td>Yorba Linda [27]</td>
<td>10.77</td>
<td>-</td>
<td>322</td>
<td>0.024</td>
<td>uniform shear beam model: $H=48.20$ m</td>
</tr>
<tr>
<td>Yorba Linda [28]</td>
<td>10.62</td>
<td>-</td>
<td>326</td>
<td>0.028</td>
<td>uniform shear beam model: $H=48.2$ m, $z_R=(z_3, z_8)$, $z_r=z_b$</td>
</tr>
<tr>
<td>Yorba Linda (This study)</td>
<td>10.62</td>
<td>14.21</td>
<td>$v_1=345$</td>
<td>$\eta_1=0.03$</td>
<td>2-layer model: $v_2=10$, $\eta_2=0.03$, $h_1=47$ m; $h_2=1.2$ m; $z_R=z_7$; $z_r=z_b$</td>
</tr>
</tbody>
</table>

72
the piecewise continuous model is based on ignoring floor masses, in which presence of these structural elements can affect resulting in depth understanding high-rise buildings behavior.
CHAPTER 4
MODELING AND IDENTIFICATION WITH A CONTINUOUS-DISCRETE MEDIUM

This chapter is meant to provide theory and essential assumptions for modeling high-rise buildings as a series of shear beams for columns/walls and lumped masses for floors, hereafter called the continuous-discrete model. The model response is specifically determined using wave-based approach and response features of a one dimensional 1D vertical wave propagation are revealed. It has also been shown that the proposed model can be degenerated, under certain conditions, into some special modeling cases, namely piecewise, uniform, and discretized. As an application of system identification using seismic recordings, the Millikan Library building is modeled using a continuous-discrete model which is identified with the recordings of the Yorba Linda earthquake of September 3, 2002. The chapter is then concluded with an outline on significant improvements in using the continuous-discrete model to predict the seismic response of high-rise building structures.

4.1 Modeling Wave Motion in a Continuous-discrete Medium

In this chapter, an $N$-story building is modeled as a series of shear beams for columns and walls and lumped masses for floors as shown in Figure 4.1. It has been assumed that shear deformation of the model is due to a one-dimension shear wave propagation in the vertical direction. Each column/wall is characterized by shear modulus $G$, mass density $\rho$, hysteretic damping ratio $\eta$, cross sectional area $A$, and story height $h$. And each floor is characterized by lumped mass $m_f$ where overlapped parts with columns are excluded, and the hysteretic damping ratio $\eta_f = c_f/m_f$ where $c_f$ is the damping coefficient of lumped mass.
For a source-free uniform medium, the governing equation of shear displacement \( u(z_j, t) \), due to vertical motion of shear waves in the \( j^{th} \) column/wall bounded with \((z_j^-, z_j^+)\) is given by

\[
\frac{\partial^2 u(z, t)}{\partial z^2} = \frac{1}{v_j^2} \frac{\partial^2 u(z, t)}{\partial t^2},
\]

(4.1)

where the signs (+, −) indicate the level immediately above and below the \( j^{th} \) floor level respectively and \( v_j = \sqrt{\frac{G_j}{\rho_j}} \) is the shear wave velocity at the \( j^{th} \) column/wall.

Assume that slabs/girders are infinitely rigid in their own plane compared to columns/walls and then, one can only consider the horizontal displacement and ignore rotation at joints connecting slabs and columns. With this in mind, displacement compatibility at the \( j^{th} \) floor can mathematically be given as

\[
u(z_j^+, t) = u(z, t) = u(z_j^-, t).
\]

(4.2)

And shear force equilibrium at the \( j^{th} \) lumped mass due to model deformation is graphically demonstrated by the free body diagram shown in Figure 4.2 and can
mathematically be expressed as

\[
F_S(z^+, t) - F_S(z^-, t) - F_D(z_j, t) = F_I(z_j, t),
\]

(4.3)

where \( F_S \), \( F_D \), and \( F_I \) are respectively the shear, damping, and inertia forces. Using mathematical expressions for the above mentioned forces, one can rewrite Equation 4.3 as

\[
(GA)_{j+1} \frac{\partial u(z^+, t)}{\partial z} - (GA)_{j} \frac{\partial u(z^-, t)}{\partial z} - c_f \frac{\partial u(z_j, t)}{\partial t} = m_f \frac{\partial^2 u(z_j, t)}{\partial t^2}.
\]

(4.4)

With the aid of Fourier representation of wave motion, Equations 2.5 and 2.6, Equation 4.1 can be solved for shear displacement representation in the frequency domain as given by Equation 2.9. With this in mind, Equations 4.2 and 4.4 can be rewritten in terms of up-going and down-going waves, that is

\[
U_{z_j}^u + U_{z_j}^d = U_{z_j}^u + U_{z_j}^d = U_{z_j}^u + U_{z_j}^d
\]

(4.5)

\[
(GA)_{j+1} \frac{i \omega}{v_{j+1}} [-U_{z_j}^u + U_{z_j}^d] - (GA)_{j} \frac{i \omega}{v_{j}} [-U_{z_j}^u + U_{z_j}^d] - C_f \frac{i \omega}{U_{z_j}^u + U_{z_j}^d} = m_f (i \omega)^2 [U_{z_j}^u + U_{z_j}^d].
\]

(4.6)

For simplicity, wave motion at each level will be indicated using the level notation only, i.e., \( U_{j^+}^u \equiv U_{z_j}^u \). Equations 4.5 and 4.6 can then be rewritten in terms of in-going and out-going waves at level \( z_j \), the \( j^{th} \) floor, as

\[
U_{j^+}^u - U_{j^-}^d = U_{j^-}^u - U_{j^+}^d
\]

(4.7)
\begin{align*}
(GA)_{j+1} \frac{i \omega}{v_{j+1}} [-U_{j+}^u + U_{j+}^d] - (GA)_{j} \frac{i \omega}{v_{j}} [-U_{j-}^u + U_{j-}^d] &- C_{f_j} i \omega [U_{j}^u + U_{j}^d] \\
&= m_{f_j} (i \omega)^2 [U_{j}^u + U_{j}^d]. \quad (4.8)
\end{align*}

It has been proven that the quantity $GA \frac{\omega}{v}$ can be simplified to $\omega (\rho v A)$ as per Equation 3.6. Recalling also that the hysteretic damping coefficient, in terms of rate-independent damping ratio, is given by $C_{f_j} = \frac{\eta_j}{\omega}$. With this in mind, Equation 4.8 can be rewritten in a compact form as

\begin{equation}
\begin{align*}
&\quad r_{I_j} U_{j+}^u + U_{j-}^d + r_{D_j} U_{j-}^u - i r_{M_j} U_{j}^u = U_{j+}^u + r_{I_j} U_{j+}^d - r_{D_j} U_{j-}^d - i r_{M_j} U_{j-}^d, \quad (4.9)
\end{align*}
\end{equation}

where the coefficients $r_I$, $r_D$, and $r_M$ can be determined in terms of column impedance $\rho v$, column cross-sectional area $A$, floor-to-column mass ratio $r_m = \frac{m_j}{m}$, and wave travel time $\tau = h/v$ for a column height $h$, that is,

\begin{align*}
&\quad r_{I_j} = \frac{(\rho v)_{j+1}}{A_{j+1}} \frac{A_j}{A} \quad \quad (4.10a) \\
&\quad r_{M_j} = r_{m_j} \frac{h_j}{v_j} \quad \quad (4.10b) \\
&\quad r_{D_j} = \eta_{f_j} r_{M_j}. \quad \quad (4.10c)
\end{align*}

In particular, Equations 4.7 and 4.9 can be rewritten in matrix form, namely

\begin{equation}
\begin{bmatrix}
1 & -1 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
U_{j+}^u \\
U_{j-}^d
\end{bmatrix} =
\begin{bmatrix}
1 & -1 \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
U_{j+}^d \\
U_{j-}^u
\end{bmatrix},
\end{equation}

(4.11)

with the aid of matrix inverse, Equation 4.11 can be solved for out-going waves in terms of in-going waves at the $j^{th}$ floor, that is,

\begin{equation}
\begin{bmatrix}
U_{j+}^u \\
U_{j-}^d
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{1+r_{I_j} - r_{D_j} + ir_{M_j}} & \frac{2r_{I_j}}{1+r_{I_j} - r_{D_j} + ir_{M_j}} - 1 \\
\frac{2r_{I_j}}{1+r_{I_j} - r_{D_j} + ir_{M_j}} - 1 & \frac{1}{1+r_{I_j} - r_{D_j} + ir_{M_j}}
\end{bmatrix}
\begin{bmatrix}
U_{j+}^d \\
U_{j-}^u
\end{bmatrix}.
\end{equation}

(4.12)

Using the transmission and reflection coefficients concept, one can specifically relate out-going and in-going waves at the $j^{th}$ floor as

\begin{equation}
\begin{bmatrix}
U_{j+}^u \\
U_{j-}^d
\end{bmatrix} =
\begin{bmatrix}
T_{j+} & R_{j-} \\
R_{j+} & T_{j-}
\end{bmatrix}
\begin{bmatrix}
U_{j+}^d \\
U_{j-}^u
\end{bmatrix}.
\end{equation}

(4.13)
Equating coefficient matrices of Equations 4.12 and 4.13, one can then express transmission and reflection coefficients in the upward direction explicitly, namely

\[ T_{j^+j^-} = \frac{2}{1 + r_{I_j} - r_{D_j} + ir_{M_j}} \]  \hspace{1cm} (4.14a)

\[ R_{j^+j^-} = \frac{2}{1 + r_{I_j} - r_{D_j} + ir_{M_j}} - 1, \]  \hspace{1cm} (4.14b)

and transmission and reflection coefficients in the downward direction

\[ T_{j^-j^+} = \frac{2r_{I_j}}{1 + r_{I_j} - r_{D_j} + ir_{M_j}} \]  \hspace{1cm} (4.15a)

\[ R_{j^-j^+} = \frac{2r_{I_j}}{1 + r_{I_j} - r_{D_j} + ir_{M_j}} - 1. \]  \hspace{1cm} (4.15b)

By analogy with Equation 2.16, Equation 4.14a can also be expressed in terms of the up-going transmission coefficient for a uniform medium multiplied by frequency-dependent attenuation factor, that is

\[ T_{j^+j^-} = B_{f_j} e^{-\omega \frac{h_{e_j}}{r_{I_j}}}, \]  \hspace{1cm} (4.16)

where \( B_{f_j} \) and \( h_{e_j} \) are respectively the transmission coefficient amplitude and equivalent height of the \( j^{th} \) lumped floor mass and can mathematically be expressed as

\[ B_{f_j} = \frac{2}{\sqrt{(1 + r_{I_j} - r_{D_j})^2 + (r_{M_j})^2}} \]  \hspace{1cm} (4.17)

\[ h_{e_j} = \frac{v_j}{\omega} \tan^{-1} \left( \frac{r_{M_j}}{1 + r_{I_j} - r_{D_j}} \right). \]  \hspace{1cm} (4.18)

With Equations 2.13 and 2.16 as reference, Equations 4.12 and 4.16 indicate that the lumped floor mass can be treated as a column-type continuum medium with equivalent height and damping but with non-zero reflection coefficients.

For a composite building section, for instance a lumped mass over a shear beam, bounded with \((z_l, z_n)\), or simply using level notation \((l, n)\), with intermediate level \(z_m\) in between such that \((l < m < n)\). Then, repeating use of Equation 2.16 for
the lower uniform medium and Equations 4.14 and 4.15 for the upper lumped-mass
will lead to the representation of transmission and reflection coefficients through the
section \((l, n)\) in terms of transmission and reflection coefficients through the sub-
sections \((l, m)\) and \((m, n)\). Mathematically, equivalent transmission and reflection
coefficients are expressed as given by Equations 3.16 and 3.17. It should be noted
that Equations 3.16 and 3.17 can be used repeatedly to find equivalent transmission
and reflection coefficients between any two levels inside the building.

4.2 Response of a Continuous-discrete Model

Shear displacement response in the frequency domain at the response level, \(z_R\),
to unit displacement pulse wave at the referenced level, \(z_r\), can be obtained in a
non-dimensional form, hereafter called the motion ratio and denoted as \(D_{Rr}\). General
formulation of the motion ratio has been derived in section 3.2 and is mathematically
expressed by Equation 3.22, which is repeated here for completeness,

\[
D_{Rr} = \frac{U_R}{U_r} = \frac{(1 + R_{NR})T_{Rr}}{(1 - R_{rR}R_{NR})(1 + R_{Nr})}. 
\tag{4.19}
\]

Equation 4.19 shows that the model response in the frequency domain is completely
dependent on transmission and reflection coefficients between referenced and free-
end levels. It should be noted that floor properties, floor mass and damping, are
incorporated in transmission and reflection coefficients in this case.

The model response in the time domain can specifically be obtained by applying
Fourier representation of wave motion given by Equation 2.5 to Equation 4.19, that
is

\[
d_{Rr} = \int_{-\infty}^{\infty} D_{Rr} e^{i\omega t} \, d\omega. \tag{4.20}
\]

Subsequently, wave response representation, in general, and displacement response
at \(z_R\) to input displacement at \(z_r\), in particular, are then given by Equations 2.29 and
2.30. For seismic excitation input, however, one can use the modified forms of these
Equations as given by Equations 2.31 and 2.32.
To show representation of the continuous-discrete model response, in the time domain and frequency domain, as well as dynamic features using a wave-based approach, a simple one layer model shown in Figure 4.3 is analyzed, and its dynamic features are revealed. In particular, the model is characterized with fixed-base/free-end boundary conditions and consists of lumped mass located at the top of continuous medium. The model response, in the frequency domain and time domain, is specifically determined and analyzed showing wave propagation features.

Figure 4.3: Schematic diagram demonstrates 1-layer continuous-discrete model.

To obtain the motion ratio response in terms of model parameters, one needs to replace symbolic forms of transmission and reflection coefficients, $T_s$ & $R_s$, with their corresponding expressions, namely

$$D_{R0} = \frac{U_R}{U_0} = \frac{(1 + R_{1R})T_{R0}}{(1 - R_{0R}R_{1R})(1 + R_{1r})}.$$  

(4.21)
\( T_{R0} = e^{-i\omega \tau_z} \) \hspace{1cm} (4.22a)

\( R_{0R} = 0 \) \hspace{1cm} (4.22b)

\( R_{1R} = (\alpha - 1) e^{-2i\omega (\tau - \tau_z)} \) \hspace{1cm} (4.22c)

\( R_{10} = (\alpha - 1) e^{-2i\omega (\tau - \tau_z)} \), \hspace{1cm} (4.22d)

where the coefficients mentioned above are expressed as

\[ \tau_z = \frac{z_R}{v} \] \hspace{1cm} (4.23a)

\[ \tau = \frac{H}{v} \] \hspace{1cm} (4.23b)

\[ \alpha = \frac{2}{1 - r_D + i r_M}, \] \hspace{1cm} (4.23c)

where \( \tau_z \) and \( \tau \) are, respectively, the flight time of propagation waves to reach response and free-end levels starting from reference level. Note that Equation 4.23c represents the special case of upward transmission coefficient through lumped mass—Equation 4.14a—for the free-end boundary condition, i.e., \( r_I = 0 \). Substituting Equations 4.22 into Equation 4.21, one can then obtain the model response in terms of model parameters, namely

\[ D_{R0} = \frac{e^{-i\omega \tau_z} \left[ 1 + (\alpha - 1) e^{-2i\omega (\tau - \tau_z)} \right]}{\left[ 1 + (\alpha - 1) e^{-2i\omega \tau} \right]}. \] \hspace{1cm} (4.24)

Equation 4.24 shows that the model response, in terms of motion ratio, is a function of transmission coefficient through the lumped mass, i.e., \( \alpha \). In addition, examining transmission coefficient amplitude reveals that it is inversely proportional to the factor \( r_M \)—Equation 4.10b—for a given floor hysteretic damping ratio, that is \( \alpha \propto \frac{1}{r_M} \). Accordingly, the model response given by Equation 4.24 will be examined at the extreme limits of the factor \( r_M \) taking into account the physical meaning.

In particular, the zero-value of the factor \( r_M \) necessitates that zero floor-to-column mass ratio, i.e., \( r_m = 0 \), meaning that propagating waves approaching the free end will be transmitted to the free surface with the twice the amplitude. Thus, Equation 4.24
can be simplified as

\[ D_{R0} = \frac{e^{-i\omega \tau_s} \left[ 1 + e^{-2i\omega(\tau-\tau_s)} \right]}{\left[ 1 + e^{-2i\omega \tau} \right]} . \]  

(4.25)

It should be noted that Equation 4.25 is identical to Equation 2.33, suggesting that the considered model in Figure 4.3 is reduced to a uniform medium model. Consequently, the model response in the time domain could be found using Equation 2.47. Dynamic features of the model represented by the fundamental frequency and the fundamental period are, respectively, given by Equations 2.37b and 2.48.

On the other hand, as the factor \( r_M \) approaches infinity, practically \( r_M >> 1 \) due to the increase in the multiplication of the quantities \( r_m, \omega, \) and \( \tau \), the transmission coefficient \( \alpha \) will approach zero. With this in mind, Equation 4.24 could then be rewritten as follows

\[ D_{R0} = \frac{e^{-i\omega \tau_s} \left[ 1 - e^{-2i\omega(\tau-\tau_s)} \right]}{\left[ 1 - e^{-2i\omega \tau} \right]} . \]  

(4.26)

The model response in the time domain, \( d_{R0} \), for the former case can be obtained by applying Equation 4.20 to Equation 4.26, namely

\[ d_{R0} = \int_{-\infty}^{\infty} \frac{e^{-i\omega \tau_s} \left[ 1 - e^{-2i\omega(\tau-\tau_s)} \right]}{\left[ 1 - e^{-2i\omega \tau} \right]} e^{i\omega t} d\omega . \]  

(4.27)

The closed form solution of Equation 4.27 could specifically be obtained by evaluating the given integral in the complex plan by contour integration using a residue technique. In particular, the given integrand, which is a function of the real variable \( \omega \), can be rewritten in terms of the variable \( y \), in which one can solve for poles of integration at which the integrand is not defined by setting the dominator of the integrand equal to zero, namely \( 1 - e^{-y^*} = 0 \) where \( y^* \) is set equal to \( i2\omega \tau \). One can then solve for the variable \( y \) taking into account the cases \( y > 0 \) and \( y < 0 \) yields

\[ y_j = \tilde{\omega}_j(\pm 1 + i\eta), \quad j=1,2,\ldots, \infty, \]  

(4.28)

where
\[ \tilde{\omega}_j = \tilde{\omega}_0 \, j \quad (4.29a) \]
\[ \tilde{\omega}_0 = \frac{\pi}{\tau} = \frac{\pi v}{H}. \quad (4.29b) \]

Equations 4.28 and 4.29 show that the integral given by Equation 4.27 has an infinite number of poles in the upper complex plane, as shown in Figure 4.4, where the fundamental frequency is twice the one for the uniform medium model, i.e., \( \tilde{\omega}_1 = 2\omega_1 \).  

![Figure 4.4: Complex integration plane: schematic diagram demonstrates the location of poles and path of integration.](image)

The integral given by Equation 4.27 in terms of \( y_j \) can be written in a compact form as

\[
d_{\mathcal{R}0}(t) = \int_{-\infty}^{\infty} \frac{f_1(y_j)}{f_2(y_j)} e^{iy_jt} \, dy, \quad (4.30)\]

where \( f_2(y_j) = 1 - e^{-2iy_j\tau} \). In particular, examining Equation 4.30 reveals that the denominator, \( f_2(y_j) \), is equal to zero when \( e^{-2iy_j\tau} = 1 \). Setting \( \tilde{y} = y + \psi \) where \( \psi \) is a small positive number and using a first order Taylor’s expansion in \( \psi \), one can simplify \( f_2(y_j) \) as

\[
f_2(y_j) = 1 - e^{-2i\tilde{y}_j\tau} \approx 1 - e^{-2i\tilde{y}_j\tau} (1 - 2i\tau - 0(\psi^2)) \approx 2i\tau, \quad (4.31)\]

which proves that poles of integration are simple. With this in mind, one can then perform the integration given by Equation 4.20 along the route described in Figure 4.4 using the residue theorem, namely

83
\[ d_{R0}(t) = 2\pi i \sum_{j=1}^{\infty} \frac{\text{Res}_{y \to y} f_1(y_j)}{f_2(y_j)} e^{iy_j t} \]

\[ = 2\pi i \sum_{j=1}^{\infty} \frac{\bar{y} - y_j}{2\psi\tau} e^{iy_j t} \]

\[ = \omega_0 \sum_{j=1}^{\infty} f_1(y_j) e^{iy_j t}. \quad (4.32) \]

In order to facilitate the evaluation of the summation given by Equation 4.32, the term \( f_1(y_j) e^{iy_j t} \) can first be simplified and then substituted back into the Equation 4.32. One should note that the considered term \( f_1(y_j) e^{iy_j t} \) is a two-sided function as per Equation 4.28. With this in mind, the considered term can be written as

\[ f_1(y_j) e^{iy_j t} = e^{i(\pm \omega_j + i\eta \omega_j)(t - \tau_z)} - e^{i(\pm \omega_j + i\eta \omega_j)(t - (2\tau - \tau_z))} \]

\[ = e^{-\eta \omega_j (t - \tau_z)} [e^{-i\omega_j(t - \tau_z)} + e^{i\omega_j(t - \tau_z)}] - e^{-\eta \omega_j (t - (2\tau - \tau_z))} \]

\[ = [e^{-i\omega_j(t - (2\tau - \tau_z))} + e^{i\omega_j(t - (2\tau - \tau_z))}]. \quad (4.33) \]

Bear in mind that complex exponential function \( e^{-i\alpha} + e^{i\alpha} \) can be expressed in trigonometric form \( 2 \cos \alpha \). Subsequently, one can use this trigonometric identity to further simplify Equation 4.33, namely

\[ f_1(y_j) e^{iy_j t} = 2 e^{-\eta \omega_j t} [\cos \omega_j (t - \tau_z) - \cos \omega_j (t - (2\tau - \tau_z))]. \quad (4.34) \]

With the use of the trigonometric identity \( \cos (\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) \), one can further simplify Equation 4.34 in the following form:

\[ f_1(y_j) e^{iy_j t} = 2 e^{-\eta \omega_j t} [\cos(\omega_j t) (\cos(\omega_j (\tau_z)) - \cos \omega_j (2\tau - \tau_z)) + \sin(\omega_j t) (\sin(\omega_j (\tau_z)) - \sin \omega_j (2\tau - \tau_z))]. \quad (4.35) \]

Using the identities: \( \cos(\alpha) - \cos(\beta) = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta) \) and \( \sin(\alpha) - \sin(\beta) = 2 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta) \), one can further simplify Equation 4.35 in the following form.
\[ f_1(y_j) e^{i y_j t} = 4 e^{-\eta \omega_j t} [\cos(\omega_j t) (-\sin\omega_j \tau \sin\omega_j (\tau_z - \tau)) + \sin\omega_j t (\cos\omega_j \tau \sin\omega_j (\tau_z - \tau))]. \]  

Equation 4.36 can be simplified further with the aid of Equations 4.29, namely

\begin{align}
\sin(\omega_j \tau) &= \sin(\tilde{\omega}_0 j \tau) = \sin(\pi j) = 0 \quad (4.37a) \\
\cos(\omega_j \tau) &= \cos(\tilde{\omega}_0 j \tau) = \cos(\pi j) = (-1)^j. \quad (4.37b)
\end{align}

Consequently, the results just obtained from Equations 4.37 can be incorporated into Equation 4.36 and be substituted into Equation 4.32, namely

\[ d_{R0}(t) = 4 \tilde{\omega}_0 \sum_{j=1}^{\infty} (-1)^j e^{-\eta \omega_j t} \sin[\omega_j (\tau_z - \tau)] \sin(\omega_j t). \]  

Equation 4.38 shows that \( d_{R0}(t) \) consists of an infinite number of motion modes, each of which has an exponentially decaying damping factor, modal shape with sinusoidal factor, and sinusoidal motion with modal frequency \( \omega_j \). The fundamental, or first, mode with \( j = 1 \) has the period time given as

\[ T_0 = \frac{2\pi}{\tilde{\omega}_0} = 2\tau. \]  

Equation 4.39 suggests that the fundamental period of the continuous-discrete model equals the time elapsed for waves to propagate up and down the model’s height once.

### 4.3 Response Features of a Continuous-discrete Model

This section shows the response features of a continuous-discrete model in the time domain and frequency domain. For this purpose, the response of a one dimension uniform medium with lumped mass located at the top is examined, and response features are revealed. In particular, the model is characterized with 50 m height, 300 m/s wave velocity, and 0.030 hysteretic damping ratio. The model response is determined at two locations with respect to assumed seismic motion at the referenced level, that is, the base or zero level. For comparison purposes, however, the response of
a uniform medium model, characterized with the same parameters as the continuous-discrete model, is incorporated as reference. Figure 4.5 describes the assumed models together with the characterized geometrical, physical, and mechanical properties.

![Figure 4.5: Continuous-discrete and uniform medium models: schematic diagram demonstrates geometrical configuration, response levels, and primary identification characteristics.](image)

Model response in the time domain in terms of GIRF is shown in Figure 4.6. The response can be thought of as being due to a unit displacement pulse wave generated at the model’s base at time $t = 0$. The pulse wave will be propagated upwards with a shear wave velocity of 300 $m/s$ to reach the response levels $z_{R1}$ and $z_{R2}$ which are demonstrated by the first peaks at $d_{10}$ and $d_{20}$ respectively. Time shift between the two peaks is equal to the elapsed time for the pulse wave to travel from $z_{R1}$ to $z_{R2}$. Moreover, pulse wave amplitude is obviously less than the one at $d_{10}$, in which it is decreased due to the presence of hysteretic damping. It should be noted that up to this point, models response in the time domain of both models is identical.
Figure 4.6: Model response in the time domain: early part of GIRF at response levels $z_{R_1}$ and $z_{R_2}$ with respect to motion at the base level.

The pulse wave will continue propagating upward to reach the free-end and then be completely reflected back to reach the response levels $z_{R_2}$ and to $z_{R_1}$. Obviously, GIRF at the response levels shows that wave arrival time in case of a uniform medium is shorter than the counterpart of the continuous-discrete model. This can be explained by the fact that the lumped mass, in the continuous-discrete model, acts as a continuum medium, with equivalent height as per Equation 4.18, which requires more time for the pulse wave to reach the response levels. In addition, one can note that more energy is dissipated in case of a continuous-discrete model.

The first negative peak at $d_{20}$ is basically generated by reflecting propagating waves approaching free end by the continuum/discrete mass interface. This negative pulse wave will continue propagating downward to reach the response level $z_{R_1}$ at $d_{10}$ with further time shift and amplitude decay. The positive upright peak, however, represents the arrival time of the reflected wave by the free-surface. Similarly, this pulse will continue traveling downward to reach response level $z_{R_1}$ with a further time delay and more amplitude decay, which is demonstrated by the second positive peak at $d_{10}$. 

87
The pulse wave will then continue traveling towards the model’s base and at the instant it reaches the base, an opposite-the same amplitude pulse wave will be generated at the base and starts traveling upward maintaining the base, or referenced level in general, stationary at that time. The negative pulse wave will then propagate upward towards the response levels $z_{R1}$ and $z_{R2}$, respectively, which are demonstrated visually by the first large inverted peaks. The pulse wave will continue traveling upward and will be split, in case of the continuous-discrete model, into a positive reflected wave and negative transmitted wave at the time the pulse wave reaches the lumped mass. The transmitted wave will then continue travelling towards the free surface and be completely reflected downward with the same wave motion direction. The reflected wave will reach the response level $z_{R2}$ with further time shift and amplitude decay. On the other hand, does not show a positive reflection wave and results in a shorter time to reach the mentioned response levels.

One should note that the transmitted wave takes more time than the reflected pulse wave to reach the response levels, in case of continuous-discrete model, due to the reason that the transmitted wave is delayed during traveling through the lumped mass. The later part of the GIRF is dominated by the fundamental mode where the effect of higher frequencies is damped out. The pulse wave will continue propagating up and down until its energy is dissipated. In fact, the continuous-discrete model has an extended wave period and hence a longer fundamental period.

Figure 4.7 shows the model response in the frequency domain in terms of GFRF. The figure clearly shows that the fundamental frequency of the continuous-discrete model is less than the one for the uniform medium model, in which one can relate this decrease in fundamental frequency to the extending in wave travel time.

### 4.4 Simple Versions of a Continuous-discrete Model

With practical design considerations of high-rise buildings associated with particular physical conditions, some simplifications can be made so that simplified versions
of the continuous-discrete model can be generated. This section discusses in detail the applicable conditions at which the continuous-discrete model can be simplified to either a piecewise continuous model or a discretized mass model.

4.4.1 Degeneration to a Piecewise Continuous Model

For most ordinary and regular high-rise buildings, one can consider that wave-based model characteristics, such as shear wave velocity \( v \), shear area \( A \), column impedance \( \rho v \), and column height \( h \), are not changed significantly from floor to floor. In such cases, the impedance ratio given by Equation 3.9 can be approximated as

\[
 r_I^j = \frac{(\rho v)_{j+1}}{(\rho v)_j} \frac{A_{j+1}}{A_j} \approx 1 \quad \text{for} \quad j = 1, 2, \ldots, N - 1. \tag{4.40}
\]

In general, an \( N \)-story building modeled with \( N \)-degrees of freedom will have \( N \) natural frequencies with \( N \omega_1 < \omega_N \). The practical design consideration of high-rise buildings subjected to earthquake-related excitation motion considers that the largest dominant frequency of interest, \( \tilde{\omega} \)

\[ \text{mas, istypically less than} \omega_0 \text{ with } n < N. \] With these approximations, one can apply the following simplification for the case of \( r_m << 1 \), namely
Recall that fundamental frequency $\omega_1$ for the uniform medium model can be found using Equations 2.37. Also model height $H$ could be approximated as $H \approx Nh_j$.

With this in mind, Equation 4.41 can be simplified further as

$$r_{M_j} < r_{mj} \frac{\pi v}{2Nh_j} \frac{h_j}{v_j}.$$  \hfill (4.42)

Based on the assumption that the building properties are uniformly distributed along its height, therefore, one can reasonably assume that $v \approx v_j$. Thus, Equation 4.42 can be rewritten as

$$r_{M_j} < r_{mj} \frac{\pi n}{2N} << 1.$$  \hfill (4.43)

Equation 4.43—based on the considered assumptions and simplifications mentioned above—states that the quantity $r_{M_j}$ can be approximated to zero. Accordingly, one can apply the above mentioned approximations to show the following conclusions.

With the aid of Equation 4.40, Equations 4.14a and 4.15a can be related as $T_{j+j-} \approx T_{j-j+}$; Equation 4.17 can be written as $B_{fj} \approx 1$, and Equations 4.14b and 4.15b can be related as $R_{j+j-} \approx R_{j-j+}$. This suggests that incident waves propagate through floors with a small floor-to-column mass ratio, $r_m << 1$, are transmitted upward and downward without loss of amplitude, i.e., no energy loss. Moreover, one can apply small-angle approximation to Equation 4.18, taking into account that $r_{D_j} = \eta_{fj}r_{M_j} \approx 0$, to obtain

$$h_{e_j} \approx \frac{v_j}{\omega} \frac{r_{M_j}}{1 + r_{fj}} \approx \frac{1}{2} r_{mj} h_j \quad \text{for } j = 1, 2, \ldots, N - 1.$$  \hfill (4.44)

Equation 4.44 suggests that floor mass acts as an extended column type with equivalent height, $h_{e_j}$, and zero damping.

To show the effect of the above mentioned approximations on equivalent transmission coefficient through the $j^{th}$ column-floor system, one can write the following
\[ T_{j}^{+} = T_{j}^{+} - T_{j}^{-} \approx e^{-i\omega \frac{h_{e}}{v_{j}}} e^{-i\omega \frac{h_{j}}{v_{j}}} \]

Alternatively, the equivalent transmission coefficient can be viewed as the transmission coefficient in column type with a decreased velocity and reduced damping factor. With Equation 2.37b as reference, fundamental frequency, \( \Omega_{1} \), of a high-rise building with \( r_{m} \ll 1 \) is less than the fundamental frequency with uniform model, \( \omega_{1} \). On the other hand, the corresponding response amplitude will be higher than the one for the uniform model due to damping reduction at interfaces. Figure 4.8 demonstrates the model response in the frequency domain showing the dynamic response features of the continuous-discrete model with \( r_{m} \ll 1 \) comparing to uniform model \( r_{m} = 0 \). Therefore, the equivalent transmission coefficient, in the upward and downward directions, for the \( j^{th} \) floor-column system can be viewed as those for column type with extended height \( h_{j} + h_{e,j} \) and reduced damping factor \( (e = c/(1 + h_{e}/h)) \).

Figure 4.8: Model response in the frequency domain of a continuous-discrete model with small \( r_{m} \) compared to a uniform model with \( r_{m} = 0 \): \( v = 300 \text{ m/s}, \eta = 0.030, \eta_{f} = 0.035, r_{1} = 1 \)
4.4.2 Degeneration to Discretized Mass Model

The continuous-discrete model can be degenerated to lumped mass model, discretized mass, in cases where the quantity $\omega(h_j/v_j) << 1$. In such a case, the term $\omega(h_j/v_j)$ can be approximated as

$$\pm i\omega \frac{h_j}{v_j} \approx 1 - e^{\mp i\omega \frac{h_j}{v_j}}. \quad (4.46)$$

Multiplying the first and second terms of Equation 4.6 by $h/h$ and recalling that $k_{s,j} = G_jA_j/h_j$. With the aid of Equation 4.46, Equation 4.6 simplifies to

$$k_{s,j+1} \left[ - \left( (1 - e^{-i\omega (\frac{h}{v})_{j+1}})U^u_{j+1} + (e^{i\omega (\frac{h}{v})_{j+1}} - 1)U^d_{j+1} \right) \right] -$$

$$k_{s,j} \left[ \left( (1 - e^{i\omega (\frac{h}{v})_j})U^u_j + (1 - e^{-i\omega (\frac{h}{v})_j})U^d_j \right) - C_{f_j} (i\omega)U_j = m_{f_j} (i\omega)^2 U_j. \quad (4.47)$$

It should be noted that the quantities $e^{\mp i\omega (\frac{h}{v})_{j+1}}$ are, respectively, the upward and downward transmission coefficients in the $(j+1)$th column, and the quantities $e^{\pm i\omega (\frac{h}{v})_j}$ are, respectively, the upward and downward transmission coefficients in the $j$th column. With this in mind, Equation

$$k_{s,j+1} \left[ - \left( (U^u_{j+1} - U^u_{(j+1)-}) + (U^d_{j+1} - U^d_{j+1}) \right) \right] -$$

$$k_{s,j} \left[ \left( (U^u_j - U^u_{(j+1)-}) + (U^d_j - U^d_{(j+1)+}) \right) - C_{f_j} (i\omega)U_j = m_{f_j} (i\omega)^2 U_j. \quad (4.48)$$

Collecting up-going and down-going waves at each level, Equation 4.48 could be rewritten as

$$k_{s,j+1} \left[ \left( U^u_{j+1} - U^u_{(j+1)-} \right) - \left( U^u_j + U^d_j \right) \right] -$$

$$k_{s,j} \left[ \left( U^u_j + U^d_{j-} \right) - \left( U^u_{(j-1)+} + U^d_{j-1} \right) \right] - C_{f_j} (i\omega)U_j = m_{f_j} (i\omega)^2 U_j. \quad (4.49)$$

Recalling that shear displacement at any level is given by summing up-going and down-going waves at that level, Equation 4.49 could be rewritten as

$$k_{s,j+1} \left( U_{(j+1)-} - U_{j+} \right) - k_{s,j} \left( U_{j-} - U_{(j-1)+} \right) - C_{f_j} (i\omega)U_j = m_{f_j} (i\omega)^2 U_j. \quad (4.50)$$
Equation 4.50 shows that column-type continuum medium can be treated as a wave-independent discrete element or as a spring with a shear stiffness $k_s$, and accordingly, shear displacement response can only be considered at floor levels. With the aid of Fourier representation of wave motion, Equation 2.5, the quantities $(i\omega)U_j$ and $(i\omega)^2U_j$ can be represented in the time domain as $du(t)/dt$ and $d^2u(t)/dt^2$. Therefore, Equation 4.50 can then be expressed in an ordinary differential form as

$$m_f \frac{d^2u_j(t)}{dt^2} + c_f \frac{du_j(t)}{dt} + k_s \left( u_j(t) - u_{(j-1)}(t) \right) - k_{s_{(j+1)}} \left( u_{(j+1)}(t) - u_j(t) \right) = 0. \quad (4.51)$$

Equation 4.51 represents the equation of motion of a discrete mass model for the $j^{th}$ lumped mass. This clearly shows that the discrete mass model is a special case of the continuous-discrete model where it can be degenerated to the discrete-mass model under the considered condition.

4.5 Application of System Identification using Seismic Recordings

This section demonstrates application of system identification to the Millikan library building with the aid of seismic recordings of the Yorba Linda earthquake of September 3, 2002 using the continuous-discrete model. In particular, the building is modeled as a series of 11 shear beams for inter story space (columns, walls, shear walls) separated with 10 lumped masses corresponding to the floor levels where floor-to-floor height is detailed in Figure 1.2a. In general, the model of the Millikan library building could be identified with a pair of seismic recordings at locations. First, one can calculate recording-based GFRF, as given by Equation 2.49, at the selected locations. Matching the recording- and model-based GFRF at some frequency range using an appropriate system identification algorithm helps identify model parameters. Then, with the identified model parameters being identified GIRF can be constructed and used to interpret model response in terms of wave propagation features.

In particular, the building is identified with a pair of 2 seismic recordings located at the at the basement and 8th floors. With these two recordings and $\varepsilon = 5\%$, the model
parameters are identification using numerical minimizing the mean squared difference between the the recorded- and model-based algorithm at a frequency range of 5 to 20 rad/sec. It should be noted that the identified model parameters are based on two pairs of modal frequencies, i.e., (10.62, 11.67)rad/sec and (10.62, 14.26)rad/sec. Figure 4.9 and Figure 4.10 show the model-based and recording-based GFRF using the two modal frequencies sets with the frequency range of 5 to 20 rad/sec.

Figure 4.9: Model-based and recording-based GFRF at the 8th floor with respect to a unit impulsive motion at the basement level.

To clarify wave propagation features, one can look at Figure 4.11, which shows model-based GIRF at two locations, the 3rd and 8th floors, with respect to impulsive motion at the basement floor. The first peak at $d_{30}$ is rooted from a unit impulsive wave initiated at the base at time $t = 0$. This pulse wave will continue traveling upward to reach the 8th floor then the top-end and then be reflected back and hit the 8th floor again with time shift and amplitude decay, which is demonstrated graphically by the first and second peaks at $d_{80}$. The pulse wave will continue traveling downward to the 3rd floor and to the base, referenced level, which is shown by the second peak at the 3rd floor.
Figure 4.10: Model-based and recording-based GFRF at the $8^{th}$ floor with respect to a unit impulsive motion at the basement level.

Figure 4.11: Model response in the time domain: GIRF at the $3^{rd}$ and $8^{th}$ floors with respect to impulsive motion initiated at the basement at time $t = 0$. 
At the base, a negative same-amplitude pulse wave is generated to balance effects of the positive pulse wave, maintaining the base fixed at that time instant and satisfying the fixed-base boundary condition. The negative pulse wave will continue propagating upward to arrive at the 3rd and 8th levels respectively as indicated by the first inverted peaks at $d_{30}$ and $d_{80}$. The negative pulse wave will continue propagating upward to reflect back by the free end level to reach the referenced level back again passing through the 8th and 3rd levels as shown by the second inverted peaks at $d_{80}$ and $d_{30}$. At this time instant, the negative pulse wave approaching the basement will be balanced by a positive same-amplitude pulse wave generated at the base and will propagate upward to repeat the same cycle again. At latter time history response, the effect of higher frequencies are damping out and response will be dominated by the fundamental mode as shown by the GIRF at $t > 1$ sec where the building starts vibrating by fundamental mode. Different identified model parameters corresponding to the two sets of modal pairs used for the identification process are summarized in Table 4.1.

Table 4.1 shows that the continuous-discrete model is capable of capturing building response using any set of modal frequencies, in which the other model parameters could be identified accordingly. Moreover, Table 4.1 shows that floor mass playing a dominant role in identification of the mode.

4.6 Usefulness of a Continuous-discrete Model

Modeling high-rise buildings using the continuous-discrete model is effective, in which the influence of properties of elastic and rigid structural members are discussed and illustrated. It is clearly demonstrated that including floor mass could dramatically changes the other model parameters and hence affecting the identification of the building properties. Additionally, the continuous-discrete model considers effects of changing in structural properties along the building height. To show the superiority of the continuous-discrete model over the previously presented models, i.e. the
Table 4.1: Identification of the Millikan library building due to different earthquakes

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$v$</th>
<th>$\eta$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lytle [52]</td>
<td>12.08</td>
<td>52.36</td>
<td>-</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>San Fernando [52]</td>
<td>10.13</td>
<td>48.33</td>
<td>-</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td>Yorba Linda [27]</td>
<td>10.77</td>
<td>-</td>
<td>322</td>
<td>0.024</td>
<td>uniform shear beam model: $H=48.20 \text{ m}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$z_R=(z_3, z_8), z_r=z_b$</td>
</tr>
<tr>
<td>Yorba Linda [28]</td>
<td>10.62</td>
<td>-</td>
<td>326</td>
<td>0.028</td>
<td>uniform shear beam model: $H=48.2 \text{ m}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$z_R=(z_3, z_8), z_r=z_b$</td>
</tr>
<tr>
<td></td>
<td>10.62</td>
<td>14.21</td>
<td>$v_1=345$</td>
<td>$v_2=10$</td>
<td>2-layer model (This study): $h_1=47 \text{ m}$; $h_2=1.2 \text{ m}$; $z_R=z_7$; $z_r=z_b$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.62</td>
<td>11.67</td>
<td>$v_j=430$</td>
<td>$v_{11}=29$</td>
<td>11-layer model (This study): $h_j$ (see fig. 1-2); $r_{m_j}=0.44$ $(j=1-10)$; $r_{m_{11}}=0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.62</td>
<td>14.21</td>
<td>$v_j=345$</td>
<td>$v_{11}=27.8$</td>
<td>11-layer model (This study): $h_j$ (see fig. 1-2); $r_{m_j}=0.40$ $(j=1-10)$; $r_{m_{11}}=0$</td>
</tr>
</tbody>
</table>
uniform medium and piecewise continuous, one can look at GFRF of the Millikan Library building shown in Figure 4.12.

In particular, the Figure 4.12 demonstrates the model response in the frequency domain at 8th floor with respect to impulsive motion at the basement level. It is clearly shown that the continuous-discrete model can capture higher frequencies as well as the fundamental frequency. Comparing to the other two model, the continuous-discrete model particularly gives higher amplitude response at the fundamental frequency and less at higher frequencies. This can be attributed to the fact that the transmission and reflection coefficients at the floor levels, interfaces, in case of the continuous-discrete model, account for floor masses, which is represented by the coefficient $r_{M_j}$. Therefore, a more decay will be observed in response amplitude at higher frequencies.

![Figure 4.12: System identification using the proposed continuum models with seismic recordings at the basement and 8th floors matching two modal frequencies at (10.77 and 14.21) rad/sec.](image)

Figure 4.12: System identification using the proposed continuum models with seismic recordings at the basement and 8th floors matching two modal frequencies at (10.77 and 14.21) rad/sec.
CHAPTER 5
PARAMETRIC SENSITIVITY ANALYSIS OF MODEL PROPERTIES ON FEATURES OF DYNAMIC RESPONSE

This chapter is devoted to demonstrate influence of different properties of the continuous-discrete model, as well as its degenerated forms, on features of dynamic response. In particular, numerical simulation is carried out to illustrate the effectiveness of continuum modeling to detect, locate, and quantify local changes in the building properties. Subsequently, sensitive dynamic features to changes in model properties will be proposed as indices.

5.1 Parametric Analysis of Uniform Shear Beam Model

It has been mentioned in Section 2.1 that the uniform shear beam model can be characterized with shear stiffness $G$, mass density $\rho$, hysteretic damping ratio $\eta$, and height $H$. These characteristics are assumed to represent the average physical and mechanical properties of the building structure. This section is aimed to investigate the influence of these properties on the wave propagation features and model response characteristics as well.

Recall from Equations 2.27 and 2.28 that model response in the frequency and time domains is completely dependent on the transmission and reflection coefficients of that part of the model bounded by the reference and top levels. Investigating the effects of modal characteristics on these coefficients can, therefore, help understand wave propagation features through the uniform medium and hence, seismic motion response.

Figure 5.1 and Figure 5.2 show transmission coefficient amplitude for different values of shear wave velocity $v$, and hysteretic damping ratio $\eta$. In particular, Figure 5.1
shows that, at a given height, hysteretic damping ratio, and frequency, the transmission coefficient amplitude increases as shear wave velocity increases, i.e., decreasing wave travel time. However, Figure 5.2 demonstrates that the increase in hysteretic damping ratio decreases the transmission amplitude.

Figure 5.1: Amplitude of transmission coefficient in the frequency domain for different values of shear wave velocity through a uniform segment of $h = 30 \ m$ and $\eta = 0.035$.

Figure 5.2: Amplitude of transmission coefficient in the frequency domain for different values of hysteretic damping ratio through a uniform segment of $h = 30 \ m$ and $v = 150 \ m/sec$. 
To investigate the effect of model properties on the dynamic features, model response in the frequency domain, GFRF, and time domain, GIRF, are determined using Equations 2.33 and 2.47, respectively, for different assumed values of shear wave velocity $v$, hysteretic damping ratio, $\eta$, and response level to height ratio, $z_R/H$.

Figure 5.3: Influence of shear wave velocity on model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the bottom level with $z_R/H = 1$ and $\eta = 0.035$.

Figure 5.4: Influence of shear wave velocity on model response in the time domain: GIRF at the top level with respect to impulsive motion at the bottom level with $z_R/H = 1$ and $\eta = 0.035$. 
Figure 5.3 and Figure 5.4 demonstrate the effect of shear wave velocity \( v \), on dynamic features of a uniform shear beam model response in the frequency and time domains, respectively. In particular, Figure 5.3 illustrates that fundamental frequency, as well as the higher frequencies, of the model decrease as shear wave velocity decreases, while its effect on the amplitude response is almost negligible. Recall from Equation 2.37b that the modal frequencies are directly proportional to shear wave velocity, which is confirmed by the observation made by Figure 5.3.

Similarly, Figure 5.4 demonstrates arrival time and amplitude of an impulse wave at the top level, starting upward propagation from the bottom level at time \( t = 0 \). It is obvious that as shear wave velocity increases the arrival time decrease while the amplitude increases. Recall from Equation 2.48 that the impulse travel time is inversely proportional to shear wave velocity, and thus an increase in shear wave velocity decreases the wave travel time and the fundamental period as well. Furthermore, the conclusion drawn from Figure 5.1 can be used to explain that an increase in shear wave velocity increases the amplitude of transmission coefficient, which results in an increase in impulse wave amplitude.

Likewise, Figure 5.5 and Figure 5.6 show the effect of hysteretic damping ratio \( \eta \), on dynamic features of the model response in the frequency and time domains, respectively. In particular, the figures show that an increase in the hysteretic damping ratio has insignificant influence on the modal frequencies and the impulse arrival times, however, both amplitudes of the GFRF and impulse wave will be decreased.

Similarly, Figure 5.7 and Figure 5.8 show model response in the frequency and time domains for different values of response to height ratio \( (z_R/H) \) where the unit value corresponds to the case where response level is at the top level of the model. The figures show that the fundamental frequencies, as well as the higher ones, and fundamental period are not changed with changes in the \( z_R/H \). However, Figure 5.7 illustrates that the amplitude of GFRF increases as \( z_R/H \) increases.
Figure 5.5: Effect of hysteretic damping ratio on model response in the frequency domain: GFRF at the top level with respect to impulse motion at the bottom level with $v = 250 \text{ m/sec}$ and $z_R/H = 1$.

Figure 5.6: Effect of hysteretic damping ratio on model response in the time domain: GIRF at the top level with respect to impulse motion at the bottom level with $v = 250 \text{ m/sec}$ and $z_R/H = 1$. 
Figure 5.7: Effect of response level to height ratio on model response in the frequency domain: GFRF at the top level with respect to impulse motion at the bottom level for \(v = 250\, m/sec\) and \(\eta = 0.035\).

Likewise, Figure 5.8 demonstrates that GIRF shows two positive peaks for \(z_R/H < 1\), indicating arrival times of an impulse wave at response level as it propagates up and down the model height. However, at \(z_R/H = 1\), the figure shows only one positive peak, which indicating arrival time at the top level. A summary of effect of model characteristics on features of dynamic response is shown in Table 5.1.

Figure 5.8: Effect of response level to height ratio on model response in the time domain: GIRF at the top level with respect to impulse motion at the bottom level for \(v = 250\, m/sec\) and \(\eta = 0.035\).
Table 5.1: Effect of model characteristics on dynamic response features.

<table>
<thead>
<tr>
<th>Property</th>
<th>GFRF</th>
<th>GIRF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Modal Frequency</td>
<td>Amplitude</td>
</tr>
<tr>
<td>Modal Frequency $(\omega)$</td>
<td>$v \propto \omega$</td>
<td>-</td>
</tr>
<tr>
<td>Hysteretic damping $(\eta)$</td>
<td>$\eta \propto \frac{1}{A}$</td>
<td>-</td>
</tr>
<tr>
<td>Response to height ratio $(z/H)$</td>
<td>$\frac{z}{H} \propto A$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.1 clearly shows that the change in shear wave velocity results in significant changes in fundamental frequency and wave arrival time, while, the change in hysteretic damping ratio and response level to height ratio affects amplitudes of GFRF and an impulse wave. With this in mind, and based on Equations 2.37b and 2.48, one can conclude that changes in dynamic response features can be captured and related to changes in shear wave velocity in a measurable manner.

Consequently, with an appropriate system identification algorithm and available seismic records—obtained from at least two sensors installed at the bottom and top levels—it is possible to construct recording-based GFRF, which could be used to identify shear wave velocity and hysteretic damping ratio. With these characteristics being obtained, subsequently model-based GFRF and GIRF can be constructed, which reveal dynamic features of the model/building. It could be noted that constructing GFRF and GIRF with respect to seismic time history recorded at the base level of the building is especially effective in eliminating effects of soil-structure interaction, and thus the observed changes in response features could be related to changes in the model properties[27, 28, 53].

In fact, damage is a local phenomenon [5, 14, 54], which affects and alters properties of a damaged part—typically decreasing stiffness of that part [55]. This, however, leads to a decrease in shear wave velocity and increase in hysteretic damping [2, 53]. Consequently, it could be interpreted that prolongation in wave travel time can be
related to a decrease in shear wave velocity, i.e., damage occurrence [13]. For the case of 1D high-rise building model, the density is assumed to be uniformly distributed along the height [53]. Therefore, one can relate wave propagation velocity to shear stiffness of the building, or simply $v^2 \propto G$, or $G \propto \frac{1}{\tau^2}$. Moreover, with reference to Equation 2.37b, one can relate changes in fundamental frequency to changes in shear stiffness of the model, i.e., $G \propto \omega^2$. Therefore, observing and measuring changes in these dynamic features could be used to improve the detection and quantification of changes in the model properties, i.e., damage detection. In fact, this is an essential for evaluating structural integrity and safety after earthquake events.

Although the uniform shear beam medium has been shown to be simple and robustness for modeling seismic wave motion of high-rise buildings, it is not effective in detecting local changes in properties of the model/building. This is mainly due to the fact that the uniform shear beam model is characterized with the average properties of the building and therefore, changes in these characteristics cannot be related to a specific part of the model/building. Consequently, this can be used to infer damage presence but not to indicate its spatial distribution in the model/building.

5.2 Parametric Analysis of Piecewise Continuous Model

It has been explained in Section 3.1 that the piecewise continuous model consists of continuous shear layers, each of which is characterized with shear stiffness $G$, mass density $\rho$, hysteretic damping ratio $\eta$, cross-sectional area $A$, and height $h$. In addition, interfaces between successive layers are characterized with the ratio $r_I$, in which the influence of relative properties of successive layers is taken into account. This section is dedicated to study effect of model parameters on wave propagation features and model response as well in the frequency and time domains.

Equations 3.22 and 3.23 indicate that GFRF and GIRF formulation is completely dependent on the transmission and reflection coefficients of that part of the model bounded by the reference and the free-end levels. In order to comprehend effects
of model characteristics on model response, one should first examine the influence of these parameters on transmission and reflection coefficients. To show this effect, however, amplitudes of transmission and reflection coefficients are examined in the frequency domain for different $r_I$-ratio values as shown in Figure 5.9 and Figure 5.10.

Figure 5.9: Transmission coefficient amplitude through an interface characterized with different $r_I$-ratio values with $\tau = 0.2$ sec and $\eta = 0.030$.

In particular, Figure 5.9 shows amplitude of an upward transmitted wave through an interface characterized with $r_I$-ratio: 0, 1/2, 1, and 4. It is obvious that, at a given frequency, the amplitude of transmission coefficient reaches a maximum value of 2 at $r_I = 0$—free-end condition—and decreases as $r_I$ increases, in which these observations are consistent with Equation 3.10a. Likewise, Figure 5.10 demonstrates amplitude of downward reflected wave by the interface. In particular, the amplitude reaches its maximum value of one at zero $r_I$-ratio and tends to decrease as the $r_I$-ratio increases to reach zero value at $r_I = 1$, i.e., the uniform shear beam medium case where waves are transmitted upward without reflection according to Equation 2.13. With a further decrease in $r_I$-ratio, the reflection amplitude continues to decrease to reach its minimum value of -1 at $r_I = \infty$. These observations are consistent with Equation 3.10b. For the continuum medium, however, Figure 5.1 and Figure 5.2 can
Figure 5.10: Reflection coefficient amplitude through an interface characterized with different $r_I$-ratio values with $\tau = 0.2$ sec and $\eta = 0.030$. 

be used to demonstrate effects of shear wave velocity and hysteretic damping ratio on amplitudes of transmission and reflection coefficients. 

In fact, this could be physically interpreted as that for the case of smaller $r_I$-ratio (soft over rigid media), upward-propagating waves approaching the interface will be transmitted upward with higher amplitudes and reflected downward with the same motion direction and hence, a large model response is expected. On the other hand, for the case of larger $r_I$-ratio (rigid over soft media), upward-propagating waves approaching the interface will be transmitted upward with small amplitude and reflected downward with negative motion direction and hence, a small model response is expected. 

To investigate the influence of model characteristics on dynamic response features, a simple 2-layer model, shown schematically in Figure 5.11a, is analyzed using GFRF and GIRF at the top level with respect to an impulse motion at the bottom level. Each layer of the model is characterized with the average properties of the corresponding part of the building. In particular, sensitive response features to the changes in modal properties are depicted. It has been assumed that changes in the model properties at
the given layer is due to changes in the shear stiffness of that part of the model, which leads to a decrease in shear wave velocity and an increase in hysteretic damping. Three different scenarios will be considered hereafter, in which model-based-characteristic changes are assumed taking place at one layer at a time as schematically demonstrated in Figure 5.11(b,c,d).

5.2.1 Scenario I: Stiffness Decreasing at the Upper Layer

This scenario considers the case where a decrease in stiffness has been assumed taking place at the upper layer as schematically illustrated in Figure 5.11b. Physically, this will lead to changes in the upper layer properties, i.e., a decrease in shear wave velocity and an increase in hysteretic damping. Consequently, using observations made in Table 5.1, one can conclude that wave travel time through the upper layer will be prolonged and impulse wave amplitude will be decreased [53]. Furthermore, decreasing the shear stiffness at the upper layer leads to a decrease in $r_I$-ratio at the interface, which affects the features of wave propagation through the interface. Referring to Figure 5.9 and Figure 5.10, amplitudes of transmitted and reflected
waves will be increased as proven mathematically by Equations 3.8, 3.17, and 3.18. It could be noted that the downward reflected wave, $R_{11-}$, will propagate with the initial motion direction, however, the upward reflected wave, $R_{1-1}$, will propagate with an opposite motion direction. Capturing these features using GFRF and GIRF and relating them to a specific part of the model in a quantitative manner could be used for model-based damage identification.

Model response in the frequency domain for different pairs of $(v, \eta)$, representing a stiffness decrease at the upper layer, is illustrated in Figure 5.12. The figure shows that fundamental frequency did not change appreciably, however, higher frequencies appear to show changes for the assumed stiffness decrease scenarios. In particular, changes in the fundamental frequency can be related to the changes in the average properties of the model, i.e., shear wave velocity and damping amplitude. Though, the change in the higher modal frequencies—the second and third peaks shown in the figure—is due to change in the model/building properties, cannot be linked to the change in shear stiffness of a specific part of the model. Therefore, observation of these features can be used to indicate occurrence of stiffness decrease in the model/building.

![Figure 5.12: Model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the bottom level for different pairs of $(v, \eta)$ assumed taking place at the upper layer.](image)
Model response in the time domain, GIRF, at the top level (response level) with respect to impulse motion at the model bottom level is demonstrated in Figure 5.13. GIRF shows arrival times of transmitted and reflected waves at the top level, which are visualized as peaks on the figure. To understand arrival times depicted by GIRF, one can first investigate wave propagation routes inside the model. In particular, Figure 5.14 shows schematic diagram demonstrates scattering of an impulse wave inside the model initiated at time $t = 0$. It could be noted that black arrows indicate waves propagating with original motion direction, visualized with positive peaks in Figure 5.13, and gray arrows indicate waves propagating with opposite motion direction, visualized with negative peaks in Figure 5.13.

In particular, an impulsive wave generated at the base level at $t = 0$ starts propagating upward towards the interface with shear wave velocity of the lower layer. At the instant the impulse wave reaches the interface it will be split into a transmitted wave that continues propagating upward towards the top level with shear wave velocity of the upper layer and a reflected wave that starts traveling downward towards the base level with shear wave velocity of the lower layer.
Figure 5.14: Schematic diagram demonstrates scattering of impulse wave inside the model explaining the first four peaks.

At the instant the upward transmitted wave reaches the top-free level (visualized in Figure 5.13 with the first positive peak, denoted as $t_1$) the wave will be completely reflected back to propagate downward towards the interface with the same motion direction. By the time the reflected wave reaches the interface, it will be split into a transmitted wave that continues propagating downward with the shear wave velocity of the lower layer maintaining the same motion direction. And a reflected wave that starts propagating upward towards the top level with shear wave velocity of the upper layer with the opposite motion direction to reach the top level (visualized by the first small negative peak in Figure 5.13, denoted as $t_2$). In fact, referring to Figure 5.14, time shift between the first positive and negative peaks is equal to twice the time required for the wave to travel the upper layer, i.e., $\Delta t_1 = t_2 - t_1 = 2\tau_2 = 2h_2 v_2$. It is clearly shown that time shift, $\Delta t_1$, is completely dependent on properties of the upper layer, suggesting that changes in $\Delta t_1$ can be related directly to changes in the properties of the upper layer, i.e, $\Delta t_1 \propto \frac{1}{v_2}$. 

112
Similarly, the downward reflected wave by the interface will continue propagating towards the bottom level and will be completely reflected upward with an opposite motion direction and starting propagation upward towards the interface. At the interface, the impulse wave will be split into a transmitted wave, which continues propagating upward to reach the top level (visualized by the second negative peak and denoted as $t_2$). And a reflected wave, which continues propagating downward towards the bottom level, to be completely reflected upward with original motion direction and repeating the same cycle. The time shift between the first positive and the second negative peaks is equal to twice the time required for the wave to travel the lower layer, i.e., $\Delta t_2 = t_3 - t_1 = 2\tau_1 = \frac{2h_1}{v_1}$, meaning the time shift, $\Delta t_2$, is dependent on properties of the lower layer, i.e., $\Delta t_2 \propto \frac{1}{v_1}$. The third negative peak, denoted as $t_4$, is generated at the instant the downward transmitted wave through the interface reaches the bottom level to be completely changes its direction and starts traveling towards the top level. The time shift in this case reveals twice the time required by the wave to travel the total height of the model, i.e., $\Delta t_3 = t_4 - t_1 = 2(\tau_1 + \tau_2)$.

Moreover, decreasing stiffness of the upper layer causes $r_I$-ratio to decrease as well, which leads to an increase in amplitude of upward transmitted and reflected waves. Figure 5.15 demonstrates amplitude of upward reflected waves propagating towards the top level. To clearly the changes in wave arrival time and amplitude of reflected waves at the top level, the time zone including the first two inverted peaks is enlarged and shown in Figure 5.16.

Figure 5.16 illustrates that the increase in the amplitude and time shift, due to stiffness decrease at the upper layer, is clearly observable. While the amplitude increase is obvious at the first and second inverted peaks, the time shift due to prolongation of $\tau_2$ is more obvious at the first peak than the second one. Referring to Figure 5.14, this behavior could be explained by that the wave arrival time of the first negative peak, shown in Figure 5.16, can simply be expressed as $t_2 = \tau_1 + 3\tau_2$ and
Figure 5.15: Amplitude of upward reflected wave off the interface towards the top level for the simulated decrease in stiffness of the upper layer.

Figure 5.16: Model response in the time domain: enlarged time-zone included the first two inverted peaks for the simulated decrease in stiffness of the upper layer.
the wave arrival time given by the second negative peak is expressed by \( t_3 = 3\tau_1 + \tau_2 \), meaning that the travel time, \( \tau_2 \) is amplified with the factor 3 at the first peak. Additionally, it could be noted that model behavior did not change during the assumed response, i.e., for the considered simulated stiffness decrease, Figure 5.13 shows the same trend, meaning the model continues behaving as a 2-layer model. Furthermore, noticing that amplitude of reflected wave is increased at the first negative peak, shown in Figure 5.16, indicating that \( r_1 \)-ratio is decreased, meaning that the upper layer gets softer, i.e., a stiffness loss. Table 5.2 summarizes relative changes in the wave travel times through the considered model vs. relative changes in dynamic features.

Table 5.2: Wave travel time vs. relative changes in model dynamic features.

<table>
<thead>
<tr>
<th>Simulation case</th>
<th>Time shift ( \Delta t ) (sec)</th>
<th>Travel time ( \tau ) (sec)</th>
<th>Stiffness Decrease ( \Delta G ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (v_0, \eta_0) )</td>
<td>0.16</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>( (v = 97%v_0, \eta = 102%\eta_0) )</td>
<td>0.165</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>( (v = 95%v_0, \eta = 105%\eta_0) )</td>
<td>0.165</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>( (v = 92%v_0, \eta = 107%\eta_0) )</td>
<td>0.170</td>
<td>0.21</td>
<td>0.105</td>
</tr>
</tbody>
</table>

Table 5.2 shows that travel time through the lower layer, \( \tau_1 \), did not change for the assumed changes in \( (v, \eta) \) pairs, suggesting that no change in the stiffness of the lower part of the building. On the other hand, travel time through the upper layer, \( \tau_2 \), does show, which suggesting that a change in stiffness has occurred at the upper layer. Owing to the fact that \( \tau_2 \) is prolonged, it could be concluded that the stiffness of the upper layer is decreased. Moreover, relative changes in \( \tau_2 \) can be used to estimate the extent of stiffness change, i.e., percentage of stiffness decrease, \( \Delta G_2 \). Therefore, a decrease in stiffness of the upper layer can be detected and quantified by observing and measuring changes in \( \Delta t_1 \).
5.2.2 Scenario II: Stiffness Decreasing at the Middle Section

This scenario considers the case where stiffness decrease is assumed taking place at the top of the lower layer as graphically illustrated in Figure 5.11c. Physically, this will lead to a decrease in shear wave velocity and an increase in hysteretic damping of that part of the model. In fact, changes in properties of the middle section will act as if a new material is placed between the two layers and bounded by two interfaces marked in Figure 5.11c with numbers 1 and 2. The model in this case can then be described as a 3-layer model where the intermediate interfaces can be characterized with the upper-to-lower layer properties. It should be noted that \( r_{I_1} \)-ratio of the lower interface is initially equals to unity, however, as stiffness decreases at the middle section, \( r_{I_1} \)-ratio will be decreased accordingly. \( r_{I_2} \)-ratio of the upper interface, on the other hand, is initially less than unity and it will be increased as the stiffness of the middle-section is decreased.

Model response in the frequency domain is illustrated in Figure 5.17, which clearly demonstrates that the model response is less affected by changes in stiffness of the middle section. In fact, changes in the fundamental frequency can be related to changes in the average model/building properties, i.e., shear stiffness and shear wave velocity. Though changes in higher frequencies are due to changes in model properties, it cannot be used to pinpoint exact locations where the changes occurred. Therefore, GFRF can only be used to detect and estimate changes in the average properties of the model/building.

Model response in the time domain, GIRF, at the top level is demonstrated in Figure 5.18 where arrival times of transmitted and reflected waves at the top level are depicted visually by peaks. To understand arrival times of waves approaching the top level, one can trace the propagation of an impulse wave started the upward traveling from the bottom level at \( t = 0 \). Scattering of the impulse wave inside the model into transmitted and reflected waves are schematically demonstrated in Figure 5.19 where
Figure 5.17: Model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the base level for the assumed different \((v, \eta)\) at the middle section.

Black arrows indicate propagation of waves with original motion direction, visualized in Figure 5.18 with positive peaks, and gray arrows indicate propagation of waves with opposite motion direction, visualized in Figure 5.18 with negative peaks. For a better visualization, the zone includes the small negative peaks in Figure 5.18 is enlarged and graphically shown in Figure 5.22.

In particular, an impulse wave generated at the bottom level at \(t = 0\) starting propagation upward towards the lower interface, \(r_{l_1}\), with shear wave velocity of the lower layer. It could be noted that at the beginning where the shear stiffness of the top part of the lower layer is not changed, the impulse wave will be totally transmitted upward, \(r_{l_1} = 1 \Rightarrow T = 1, R = 0\), however, as the stiffness starts decreasing at that part of the model, the \(r_{l_1}\) starts decreasing and thus the impulse wave approaching the lower interface will be turned into transmitted and reflected waves. One can notice that a decrease in shear stiffness of the middle section leads to an increase in the amplitude of reflected waves for the upward wave propagation as illustrated in Figure 5.20.
Figure 5.18: Model response in the time domain: GIRF at the top level with respect to impulsive motion at the base level for the assumed different \((v, \eta)\) at the middle section.

Figure 5.19: Schematic diagram demonstrates scattering of an impulse wave inside the model explaining the pattern for the first five peaks.
At the instant the impulse wave reaches the lower interface, it will be split, assuming a stiffness decrease taking place at the middle section, into a transmitted wave, which continues propagating upward towards the upper interface with shear wave velocity of the middle section and a reflected wave that continues propagating downward towards the bottom level. It could be noted that $r_{I2}$ is initially assumed less than unity, soft over rigid case, however, as the stiffness decreases at the middle section, $r_{I2}$ will be increased, meaning that the amplitude of transmitted and reflected waves by the upper interface will be decreased as illustrated in Figure 5.21.

The upward transmitted wave through the second interface continues propagating to reach the top level with shear wave velocity of the upper layer (visually demonstrated by the first positive peak, denoted as $t_1$). This time represents the travel time between the bottom and top levels, $\tau$. Referring to Figure 5.19, suggesting that $\tau$ is the sum of travel times through each indicated layer. As the downward reflected wave by the lower interface reaches the bottom level, it will be reflected into an upward with opposite motion direction wave and continues propagating to reach the top level. The arrival time at the top level is depicted by the first negative peak, denoted as $t_2$, in Figure 5.22 where the time shift and the amplitude increase of the peak is

Figure 5.20: Amplitude of reflected wave by the lower interface towards the upper interface level for the assumed different $(v, \eta)$ at the middle section.
clearly indicated. In fact, the time shift between the first positive and negative peaks particularly indicates twice the time required by the wave to travel the unchanged part of the lower layer, i.e., $\Delta t_1 = t_2 - t_1 = 2\tau_1 = 2\frac{h_1}{v_1}$. Consequently, one can relate changes in $\Delta t_1$ to changes in shear stiffness of the lower layer, i.e., $\Delta t_1 \propto \frac{1}{v_1}$.

As the impulse wave reaching the top level, it will entirely be reflected downward and starts propagating towards the upper interface. At the instant it reaches the
upper interface, the impulse wave will be split into a transmitted wave that continues propagating downward towards the lower interface and a reflected wave that starts propagating upward with an opposite motion direction. Arrival time of the upward reflected wave to the top level is depicted in Figure 5.18 by the second negative peak—the first negative peak in Figure 5.22, denoted as $t_3$. It is interestingly to note that this peak shows no time shift but an amplitude decrease for the assumed stiffness decrease. Time shift between the first positive and the second negative peaks is equal to twice the time required by the wave to travel the upper layer, $\Delta t_2 = t_3 - t_1 = 2\tau_3 = 2\frac{h_3}{v_3}$. Noticing, therefore, no time shift in the amplitude of the second negative peak, suggesting no decrease in shear stiffness taking place at the upper layer.

The downward reflected wave by the upper interface will continue traveling downward and at the instant it reaches the base level, it will be reflected with opposite motion direction and starts propagating upward towards the top level. Arrival time of the impulse wave at the top level is visualized by the third negative peak as shown in Figure 5.18, denoted as $t_4$. Referring to Figure 5.19, it could be noted that time shift between the first positive peak and the third negative peak is equal to twice the time required by the wave to travel the lower layer, i.e., $\Delta t_3 = t_4 - t_1 = 2(\tau_1 + \tau_2) = 2\left(\frac{h_1}{v_1} + \frac{h_2}{v_2}\right)$. Knowing $\tau_1$, however, make it easy to get $\tau_2$, which can be related to the properties of the middle section. It could be noted that amplitude of the third negative peak decreases as stiffness decreases, which can be attributed to the influence of the upper interface on the reflected waves to the top level.

The upward reflected wave by the lower interface starts propagating with opposite motion direction towards the top level, which is visually illustrated by the fourth negative peak, denoted as $t_5$. Time shift between the first positive and the fourth negative peaks is equal to twice the time required for the wave to travel the height.
\((h_2 + h_3)\), i.e., \(\Delta t_4 = 2(\tau_2 + \tau_3) = 2\left(\frac{h_2}{v_2} + \frac{h_3}{v_3}\right)\), and since \(\tau_3\) is known, one can easily find \(\tau_2\). It could be observed that the amplitude of fourth negative peak is initially equal to zero, which represents unchanged stiffness case, i.e., \(r_{I_1} = 1\). However, as stiffness of the middle section starts changing, \(r_{I_1}\)-ratio decreases, which increases the amplitude of transmitted and reflected waves propagating upward. Table 5.3 summaries arrival times as well as the estimation of extent of stiffness changes through analysis of Figure 5.18.

Table 5.3: Wave travel time vs. relative changes in model dynamic features.

<table>
<thead>
<tr>
<th>Simulation case</th>
<th>Time shit</th>
<th>Travel time</th>
<th>Shear stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>((v_0, \eta_0))</td>
<td>(\Delta t_1)</td>
<td>(\Delta t_2)</td>
<td>(\Delta t_3)</td>
</tr>
<tr>
<td></td>
<td>(sec)</td>
<td>(sec)</td>
<td>(sec)</td>
</tr>
<tr>
<td>((v = 97%v_0, \eta = 102%\eta_0))</td>
<td>0.133</td>
<td>0.160</td>
<td>0.200</td>
</tr>
<tr>
<td>((v = 95%v_0, \eta = 105%\eta_0))</td>
<td>0.137</td>
<td>0.161</td>
<td>0.203</td>
</tr>
<tr>
<td>((v = 92%v_0, \eta = 107%\eta_0))</td>
<td>0.137</td>
<td>0.158</td>
<td>0.204</td>
</tr>
</tbody>
</table>

In Table 5.3, the first three columns indicate the time shift between arrival times at the top level, the first positive peak, and subsequent inverted peaks. Similarly, the next 3 columns indicate wave travel times though each layer obtained by analyzing arrival times using Figure 5.19 as a reference. \(\tau_1\) and \(\tau_3\) do not show changes in the magnitude with the considered scenarios, though \(\tau_2\) shows that it prolongs as stiffness decreases. \(\Delta G_1\) and \(\Delta G_3\) show zero changes whereas \(\Delta G_2\) indicates a little change due a decrease in the stiffness.

Physically, as stiffness decreases at the middle section, wave travel time through the upper and lower layers will remain the same, whereas travel time through the middle section will be prolonged and consequently, arrival time to the free level to be prolonged as well. Initially, the model will behave as 2-layer system, (for zero
stiffness change case), however, as a stiffness decrease starts taking place at the middle layer, the model will behave as a 3-layer model and thus the response will be changed accordingly.

5.2.3 Scenario III: Stiffness Decreasing at the Lower Layer

This scenario considers the case where a stiffness decrease is considered taking place at the lower layer as schematically shown in Figure 5.11d. This scenario tends to reduce shear wave velocity and increase hysteretic damping of the lower layer. Refer to Table 5.1, suggesting that the travel time and the amplitude of wave propagation through the lower layer will be prolonged and reduced, respectively. Furthermore, the model is assumed initially consisted of soft-over-rigid media, in which the $r_I$-ratio at the interface is less than unity. However, as stiffness of the lower layer decreases, $r_I$-ratio will be increased. This leads to a reduction in the amplitude of transmitted and reflected waves traveling in the upward direction. It should be mentioned that as the stiffness of the lower layer continues in decreasing, $r_I$-ratio will approach the unity, at which the model will behave as a uniform model. Though the graph shows this behavior, these observations can only be used to describe changes on model dynamic features and characteristics in more qualitative manner.

Model response in the frequency domain is demonstrated in Figure 5.23 where the decrease in modal frequencies due to stiffness reduction is more obvious than the previous two cases. This is maybe explained by that the lower part has a dominant effect on response behavior. While changes in fundamental frequency could be used to estimate changes in the average properties of the model, higher frequencies cannot be used to point out the part where the stiffness is decreased. Therefore, GFRF in this case can be used to detect changes in the average building properties.

Model response in the time domain, GIRF, at the top level with respect to an impulsive motion at the bottom level is demonstrated in Figure 5.24. In particular, Figure 5.25 shows schematic diagram demonstrates scattering of an impulse wave
Figure 5.23: Model response in the frequency domain: GFRF at the top level with respect to impulsive motion at the base level for the assumed different $(v, \eta)$ at the lower layer.

through the model, which starts propagating upward from the bottom level at time $t = 0$. Black arrows indicate waves propagate with original motion direction, visualized in Figure 5.24 with positive peaks, and gray arrows indicate waves propagate with opposite motion direction, visualized in Figure 5.24 with negative peaks.

In particular, at time $t = 0$, an impulse wave starts propagating upward with shear wave velocity of the lower layer. As it reaches the interface, however, the impulse wave will be split into a transmitted wave that continues propagating in the upward direction with shear wave velocity of the upper layer and the same motion direction. And a reflected wave that starts traveling in the downward direction towards the bottom level with the same motion direction. Figure 5.26 illustrates effects of decreasing shear stiffness of the lower part on amplitude of reflected coefficient. It is clearly shows a decrease in amplitude of reflected waves traveling in the upward direction.

The upward transmitted wave will continue propagating in the upper layer and at the instant it reaches the top level it will be reflected downward towards the interface with the same motion direction. Arrival time of the impulse wave at the
Figure 5.24: Model response in the time domain: GIRF at the top level with respect to impulsive motion at the base level for the assumed different $(v, \eta)$ at the lower layer.

Figure 5.25: Schematic diagram shows scattering of impulse wave into transmission and reflection waves inside the model.
top level is depicted in Figure 5.24 by the first positive peak, denoted as \( t_1 \). By the time the downward reflected wave reaches the interface, it will be turned into a transmitted wave that continues propagating in the lower layer with the same motion direction and a reflected wave that starts propagating upward towards the top level with opposite motion direction. Arrival time of the upward reflected wave at the top level is visualized in the Figure 5.24 by the first negative peak, denoted as \( t_2 \). For better visualization, however, the zone includes small negative peaks is enlarged as illustrated in Figure 5.27. Time shift between the first positive and the first negative peaks is equal to twice the time required for the wave to travel the upper layer, i.e., \( \Delta t_1 = t_2 - t_1 = 2\tau_2 = \frac{2h_2}{v_2} \). This shows that \( \Delta t_1 \) is directly related to shear wave velocity of the upper layer, or shear stiffness, i.e., \( \Delta t_1 \propto \frac{1}{v_2} \), which can be used to detect and quantify changes in stiffness of the upper layer.

The downward reflected wave by the interface will continue propagating towards the bottom level and at the instant it reaches the base level it will be completely reflected upward to start traveling in the upward direction with opposite motion direction. At the interface, the impulse wave will be turned into a transmitted wave, which continues propagating upward towards the top level. Arrival time at the top
level is illustrated in Figure 5.24 by the second negative peak, denoted as \( t_3 \). Time shift between the first positive and the second negative peaks is equal to twice the time required by the wave to travel the lower layer, i.e., \( \Delta t_2 = t_3 - t_1 = 2\tau_1 = 2 \frac{h_1}{v_1} \). Therefore, one can relate changes in \( \Delta t_2 \) to changes in shear wave velocity, or shear stiffness, of the lower layer, i.e., \( \Delta t_2 \propto \frac{1}{v_1} \). Table 5.4 summarizes arrival times of an impulse wave at the top level obtained from analyzing Figure 5.19.

Table 5.4: Wave travel time and its relative change vs. relative change in model dynamic features.

<table>
<thead>
<tr>
<th>Simulation case</th>
<th>Time shift</th>
<th>Travel time</th>
<th>Changes in shear stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta t_1 ) (sec)</td>
<td>( \Delta t_2 ) (sec)</td>
<td>( \tau_1 ) (sec)</td>
</tr>
<tr>
<td>( (v_0, \eta_0) )</td>
<td>0.16</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>( (v = 97% v_0, \eta = 102% \eta_0) )</td>
<td>0.155</td>
<td>0.205</td>
<td>0.1025</td>
</tr>
<tr>
<td>( (v = 95% v_0, \eta = 105% \eta_0) )</td>
<td>0.16</td>
<td>0.21</td>
<td>0.105</td>
</tr>
<tr>
<td>( (v = 92% v_0, \eta = 107% \eta_0) )</td>
<td>0.16</td>
<td>0.215</td>
<td>0.1075</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.27: Model response in the time domain: enlarged time zone that includes small inverted peaks of the Figure 5.24.
In Table 5.4, the first two columns indicate time shift between arrival times of transmitted and reflected waves at the top free level. The third and forth columns represent wave travel times through the lower and upper layers, respectively. In particular, wave travel time through the upper layer, \( \tau_2 \), did not show changes for the assumed different \((v, \eta)\) pairs, meaning that estimation of \( \tau_2 \) is entirely dependent on properties of the upper layer, in which no changing in stiffness has been assumed taking place in this simulated scenario. Conversely, wave travel time through the lower layer, \( \tau_1 \), does show a prolongation in wave travel time as stiffness decreases. Percentage decrease in shear stiffness can be estimated from relative changes in wave travel time through the lower layer.

5.3 Parametric Analysis of A continuous-discrete Model

It has been mentioned in Section 4.1 that a high-rise building can be modeled as a series of shear beams for the inter-story space (columns, walls, and shear walls) and lumped masses for floors at floor levels. Effect of continuum medium and massless interfaces on wave propagation as well as on response features have been discussed on Sections 5.1 and 5.2, respectively. In this section, however, influence of the lumped mass properties on wave propagation features and the subsequent effects on model response will be detailed in this section. In particular, influence of the floor mass, represented by floor-to-column mass ratio, \( r_m \) and hysteretic damping ratio, \( \eta_f \), on the amplitude of transmission coefficient wave propagating in the upward direction is demonstrated on Figure 5.28 and Figure 5.29 respectively.

In particular, Figure 5.28 demonstrates amplitude of transmission coefficient for different \( r_m \) values. The case where \( r_m = 0 \), especially represents the one layer uniform medium where the amplitude decay is due to hysteretic damping ratio of the continuum medium \( \eta \). However, as the lumped-mass increases, \( r_m \) will be increased and thus a more rapid decay in the transmission coefficient amplitude will be observed. Figure 5.29 illustrates the magnitude of transmission coefficient amplitude under the
effect of different $\eta_f$ values, which clearly shows that the amplitude increases as $\eta_f$ increases.

Figure 5.28: Amplitude of transmission coefficient at a floor-story boundary changes with frequency at selected $r_m$ with $\tau = 0.015$ sec, $r_I = 1$, $\eta = 0.030$, and $\eta_f = 0.035$.

Figure 5.29: Amplitude of transmission coefficient at a floor-story boundary changes with frequency at selected $\eta_f$ with $r_m = 0.25$, $\tau = 0.015$ sec, $r_I = 1$, and $\eta = 0.030$.

To examine response features under the effect of lumped mass properties, a simple continuous-discrete model of a 10-story building, shown in Figure 5.30, is analyzed in the frequency domain, GFRF, and time domain, GIRF, where the effect of a decrease
of shear stiffness of inter-story space is demonstrated and discussed.

Figure 5.30: Schematic diagram demonstrates the continuous-discrete model of the 10-story building.

It has been assumed that the model is identified with the following properties: 300 m/sec shear wave velocity, 0.15 floor-to-column mass ratio, 0.25 and 0.35 hysteretic damping ratios for column- and floor-type media, respectively. Additionally, it has been assumed that a decrease in stiffness in the building is considered taking place at the top four floors. Physically, this will reduce shear wave velocity and increase hysteretic damping at the upper part of the building. Numerical simulation of model response, in the frequency and time domains, for different pairs of \((v, \eta)\)—each of which represents a case of stiffness decrease—is presented.

Model response in the frequency domain, GFRF, is shown in Figure 5.31. In particular, fundamental frequency does not change due to stiffness decrease at the upper part of the model whereas higher frequencies indicate 4% and 3% decrease for the second and third modal frequencies, respectively. With these observations that may indicate a stiffness decrease occurrence—small changes—however, these features cannot be used to locate and estimate changes in stiffness.
Model response in the time domain is illustrated in Figure 5.32 where arrival times of transmitted and reflected waves at the top level are visualized by positive and negative peaks.

In particular, Figure 5.14 can be used to describe general pattern of scattering an impulse wave starting propagation upward from the bottom level at \( t = 0 \). In fact, presence of lumped masses at different floor levels will generate internal reflections as waves propagate in upward and downward directions. As stiffness decreasing starts taking place at the upper part, the upper four floors, \( r_{16} \) will be be decreased. This will increase amplitude of transmitted and reflected waves propagating in the upward direction. Table 5.5 summarizes analysis of Figure 5.31 where arrival times of transmitted and reflected waves at top level are captured and related to travel time through the lower and upper parts of the model.

In Table 5.5, the first two columns indicate time shift in arrival time between transmitted and reflected waves at the top level. As explained before, using time shift with the aid of Figure 5.14, one can determine wave travel time through the lower and upper layers, i.e., \( \tau_1 \) and \( \tau_2 \), respectively. While \( \tau_1 \) does not show changes
Figure 5.32: Model response in the time domain: enlarged time-zone included the first two inverted peaks.

Table 5.5: Wave travel time and its relative change vs. relative change in model dynamic features.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Time shift</th>
<th>Travel time</th>
<th>Changes in stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta t_1$</td>
<td>$\Delta t_2$</td>
<td>$\tau_1$</td>
</tr>
<tr>
<td>$(v_0, \eta_0)$</td>
<td>0.155</td>
<td>0.195</td>
<td>0.098</td>
</tr>
<tr>
<td>$(v = 97%v_0, \eta = 102%\eta_0)$</td>
<td>0.16</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>$(v = 95%v_0, \eta = 105%\eta_0)$</td>
<td>0.16</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>$(v = 92%v_0, \eta = 107%\eta_0)$</td>
<td>0.165</td>
<td>0.21</td>
<td>0.105</td>
</tr>
</tbody>
</table>
for the assumed changes in, \((v, \eta)\) properties of the upper layer stiffness decrease, \(\tau_2\) does show that and therefore one can conclude that the lower part attained no change in its properties. Moreover, relative changes in wave travel time of the upper layer can be used to estimate relative changes in local stiffness of that part separately from the global features effects.
CHAPTER 6
SUMMARY AND CONCLUSION

This chapter is aimed to summarize the results discussed so far through the presented chapters of this thesis, which pertain to continuum modeling and identification of multistory buildings using seismic recordings. In addition, conclusions based on demonstrating advantages of the proposed models and its superiority in detecting and quantifying local changes in model/building characteristics are made. Finally, the chapter is concluded with suggestions for future work, which can be used to enhance this area of research.

6.1 Summary of Results

In this work, three mathematical continuum models have been proposed for simulating seismic response of multistory buildings. For each model case, a set of parameters are introduced and identified using only one pair of available seismic records. In particular, model parameters are selected such that the changes in their values can have immediate physical significant meaning. This can ultimately help describing and explaining changes in model response.

Model response in the frequency and time domains, GFRF and GIRF, respectively, has been derived and proved to represent model response to a unit displacement impulse, not the traditional force input, hit the model’s at reference level. This response can be used to show influence of higher modal frequencies on model response at the early part of GIRF. Furthermore, it has been demonstrated that reference level can be located at any level within the building’s height, which can be used to obtain dynamic features of that part of the model immediately above the reference level. It has also shown that selecting reference level at the model’s base eliminating effects of soil-structure interaction, in which building response can be thought as a function

134
of building properties only. Subsequently, detecting changes in model response can be attributed/related to changes in the building properties. This technique, however, can be used to improve model-based damage detection procedures.

Closed form solution is obtained for the uniform shear beam and some special cases of the piecewise continuous and the continuous-discrete models and numerically for general cases. Additionally, it has also been shown that model response is completely dependent on transmission and reflection coefficients, which are used to help understanding seismic wave motion through real building structures. It has been illustrated and theoretically proven that GIRF can reveal features of wave propagation, arrival time and amplitude decay, between considered levels along model’s height.

Furthermore, sensitive analysis of different model properties and their unique effects on wave propagation features as well as on model response is investigated and detailed. In particular, wave propagation features are local in nature and hence, this unique property can greatly be used to detect local changes in building properties. This could be further used for practical application such as damage detection. In fact, these properties can particularly be treated as wave-based seismic damage indices, which could serve as alternative and/or supplementary to vibration-based ones.

6.2 Conclusions

In this study, continuum modeling has been proposed to model and analyze seismic response of multistory buildings, in which system identification can be carried out using available seismic records. In particular, model response is examined using generalized response functions in the time and frequency domains, GIRF and GFRF, respectively, which are fundamental to construct model response to unit displacement pulse wave at referenced level. It has been shown that generalized response functions not only reveal vibration features of the building, but it also shows unique perspective of wave propagation features relating to building response, in which traditional vibration approach can not expose. These features, however, have been proven to
have great impact in structural health monitoring (SHM) field for damage detection. Moreover, the unique capability to relate changes in wave propagation and dynamic response features to local changes in the mode/building properties can be used to improve and enhance abroad application-based on model-based damage detection. It should be mentioned that this study considers response of building structures due to shear deformation. Therefore, this approach could be considered applicable for structures where shear deformation is dominant.

6.3 Future Work

It has been mentioned that damage is local phenomenon, which alters and affects properties of a damaged section of the structure. In particular, detecting spatial distribution a damage needs more vibration sensors to be installed, which is not available now nor in the near future. On the other hand, updating model structure with a limited number of sensors provides non-unique solution, i.e., uncertainty of damage location. However, with the promising advantages of the wave-based approach for system identification and its ability to detect, locate, and quantify local changes in building properties, it is recommended to further investigate the applicability of using this approach for post-earthquake damage detection of high-rise buildings using seismic recordings.
REFERENCES CITED


