MECHANICAL PROPERTIES OF
THE BAKKEN FORMATION

by
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ABSTRACT

The Bakken Formation is located in the Williston Basin in North Dakota, Montana, and up into southern Saskatchewan, Canada. The Bakken Formation lies unconformably over the Upper Devonian Three Forks Formation and is conformably overlain by the Lodgepole Formation. Production in the Bakken depends on horizontal wells with multistage fracture stimulations. The effective minimum horizontal stress is a primary controller of fracture growth. Knowledge of the elastic properties and Biot’s poroelastic coefficients is required to accurately determine the effective minimum horizontal stress.

In this study I have measured dry rock velocities for four geologic facies from the middle Bakken interval and one from the Lodgepole Formation. Mineralogy data was also obtained for many samples including the rocks measured in the laboratory. This data along with available literature measurements in the Bakken Shales allows for estimation of dry rock elastic constants and Biot’s coefficients in-situ.

The dry rock stiffness tensor was determined by treating the dipole shear log as a dry rock measurement. Empirical equations were then derived from laboratory data and applied to the shear waves to predict the remaining components of the stiffness tensor. Fluid substitution was performed with Gassmann’s equation to acquire the saturated stiffness tensor.

Biot’s coefficients were calculated using the dry rock stiffness tensor and an estimate of the mineral bulk modulus by assuming a Voigt-Reuss-Hill effective medium for the pure grain moduli. Biot’s coefficients describe the ability of the pore pressure to counteract the outward stresses on the rock and will be between zero and one. The values for all formations were well below one and ranged from 0.15-0.75 over the unit of interest.

The saturated stiffness tensor and Biot’s coefficients were input into the effective minimum horizontal stress equation assuming uniaxial strain. The stress profile showed no major contrast over the area of interest. A slight decrease in horizontal stress was observed in the common reservoir facies of the middle Bakken, but the remaining units all had similar horizontal stress.

Mini-Frac tests were performed in the Upper Bakken Shale and the Scallion member of the Lodgepole Formation. The tests provide estimates of reservoir pressure, total minimum horizontal stress, tensile strength, and total maximum horizontal stress. The interpreted stress from the Mini-Frac tests matched well with the modeled results, and showed low stress contrast between the Upper Bakken Shale and Scallion member.
A total minimum horizontal stress profile was provided by Schlumberger for the same well along with transversely isotropic elastic properties. The stress profile was only able to predict the Mini-Frac test in the Upper Bakken Shale, and a calculation of Thomsen anisotropy parameters (Thomsen, 1986) showed the δ parameter in the Bakken Shales ranged from 0.5-1.2. These δ values are much higher than any existing anisotropy measurements in the Bakken Shales (Vernik & Nur, 1992). The likely cause for the high values was an attempt by Schlumberger to match the Mini-Frac tests by adjusting the anisotropy and disregarding the possibility of Biot’s coefficients less than unity. This lead to a massive stress contrast in the Bakken Shales that would ultimately be interpreted as a strong fracture barrier.

The in-situ pressure testing and modeled results show low stress contrast throughout the unit of interest. The analysis by Schlumberger predicted a contrasting stress profile, but the input parameters were unrealistic and the profile did not match in-situ pressure testing. This demonstrates the importance of estimating accurate Biot’s coefficients and realistic anisotropy parameters. A poor interpretation will impact completion strategies and potentially damage resource recovery.
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To myself.
CHAPTER 1
INTRODUCTION

This chapter provides a basic introduction to the thesis.

1.1 Geologic Background

The Bakken Formation is located in the Williston Basin in North Dakota, Montana, and up into southern Saskatchewan, Canada. The Bakken Formation lies unconformably over the Upper Devonian Three Forks Formation and is conformably overlain by the Lodgepole Formation. The core samples in this study came from Freda Lake Field in Saskatchewan, Canada. Deposition occurred during the Tamaroa sequence (Wheeler, 1963) during a series of onlap-offlap cycles. Time-equivalent shale units include the Exshaw/Banff in the Alberta basin, the Woodford shale in the Anadarko Basin, the Chattanooga in the Southern Appalachian Basin, the Antrim in the Michigan Basin, and the New Albany in the northern Appalachian Basin (Meissner, 1978). Figure 1.1 shows a cross section of the Bakken and bounding formations. The approximate location of the Freda Lake Field is shown in Figure 1.2 to be in the Northwest corner of the Williston Basin away from the Nesson Anticline located in the central part of the basin. There are no major structural features near the Freda Lake Field.

![Figure 1.1: Cross sectional view of the Bakken formation and the bounding Lodgepole and Three Forks formations. Modified from (Meissner, 1978).](image)

The shale maturity, or generation of fluid hydrocarbons, varies within the Bakken. Abnormally high pore pressures in the Bakken shales have been attributed to hydrocarbon generation associated with the thermal anomaly in the central part of the Williston Basin (Price et al., 1984). Rock-Eval pyrolysis is a method of determining the type and maturity of organic matter in a rock specimen through controlled heating (Katz, 1983). Rock-Eval analysis was performed on samples from the
Figure 1.2: A map view of the Williston Basin with the approximate location of the Freda Lake Field highlighted in red (Pitman et al., 2001).
study well and each of the shale units was within the immature window, suggesting no hydrocarbons have been generated. Since hydrocarbon generation is low, the pore pressure gradient is expected to be lower in the Freda Lake field than in other Bakken plays. The pore pressure will be determined from in situ pressure testing.

The middle Bakken unit has been broken into seven lithofacies (Simenson, 2010). Figure 1.3 has the facies descriptions used in this study (CSM, 2010) and a comparison with other popular facies descriptions of the middle Bakken (Canter & Sonnenfeld, 2009; Kohlruss & Nickel, 2009; Nordeng & LeFever, 2008). The study well had Facies A-F present, and laboratory measurements were taken on C-F.

**MIDDLE BAKKEN LITHOFACIES**

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<tr>
<td>L5 - Siltstone, gray-green, massive, reftled, dolomitc, Nertes ichnofacies.</td>
<td>A0 - Patterned pyritic dolostones.</td>
<td>F1 - Pyritic dolostones.</td>
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<tr>
<td>C - Siltstone, laminated, argilaceous, &amp; vfg sandstone, bioturbated, soft sediment deformation. Phycosphiton, Planolites &amp; Teichichnus.</td>
<td>L4 - Interbedded dark-gray shale and buff, silty sandstone, coarssens upward, moderately bioturbated (Cruziana ichnofacies)</td>
<td>A1 - Calcalc, whole fossil, dolo- to lime wackestones fossil-rich beds.</td>
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<tr>
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<td>L3 - Sandstone, upper &amp; lower vary to flaser silty sandstone. Skolithos ichnofacies. Middle coarse-grained, masssive to xbedded.</td>
<td>B1 - Highest energy, coarsest grain alternated cross-bedded bioclast, v.f.g. sandstone.</td>
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<tr>
<td>A - Siltstone, gray-green, argilaceous, abundant bioturbation, Nertes &amp; Phycosphiton.</td>
<td>L2 - Interbedded dark-gray shale and buff, silty sandstone, moderate to intense bioturbation (Cruziana ichnofacies), fossiliferous.</td>
<td>C - Rhythmic, varve-like, mm to cm laminated, well sorted, v.f.g. sandstone and siltstone with calcite cement. Hummocks and wave ripples.</td>
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<tr>
<td>B - Bioturbated, argilaceous, calc. poorly sorted, vfg sandstone/siltstone with hermiphostoicinctubia.</td>
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<td>E - Intraclastic-skeletal lime wackestone, 1-4 ft thick.</td>
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Figure 1.3: A comparison of the facies description applied in this study (CSM, 2010) with other popular facies descriptions for the middle Bakken. From Steve Sonnenberg, personal communications.

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Figure 1.4: The gamma ray signature for the study well. The facies are labeled to the right. The gray highlighting represents shale units, yellow indicates the middle Bakken, and light blue indicates the Lodgepole Formation.

1.2 Elastic Properties

Velocities and strains have been measured on four samples from the middle Bakken and a sample from the upper bounding Lodgepole limestone. The purpose is to establish the elastic properties of the Bakken petroleum system. Extensive measurements have been performed on the Bakken shales (Vernik & Nur, 1992), but the primary reservoir interval has been largely overlooked.
The middle Bakken is generally considered elastically isotropic. In an isotropic medium the velocities measured in any direction will be equal and there are only two independent elastic constants required to describe the elasticity of the material. In an anisotropic medium the velocities will be different depending on the direction of propagation, and for shear waves, the polarization i.e. the direction of particle motion. The most general case is a triclinic medium with 21 independent elastic constants (Musgrave, 1970). The model that will be considered in this study is for transversely isotropic (TI) media. In this model there is a single axis of rotational symmetry and the independent elastic constants is reduced to five. The wave velocity will depend on the angle between the direction of propagation and the symmetry axis (Tsvankin, 2001). This model is commonly applied to horizontally layered media and rocks with a single set of aligned fractures or microcracks. A vertically transversely isotropic (VTI) symmetry is shown in Figure 1.5.

The Bakken shales have been tested for orthorhombic symmetry (Vernik & Nur, 1992). An orthorhombic medium has three orthogonal planes of mirror symmetry. This model is common in seismic exploration for media with a transversely isotropic background and a single set of parallel vertical fractures. Referring to Figure 1.6, in an orthorhombic medium a vertically propagating shear wave with polarization in the x1 direction will have a lower velocity than a wave with the same propagation direction but polarization in the x2 direction. The study by Vernik & Nur (1992) measured orthogonally polarized vertical shear wave velocities within \( \pm 0.02 \text{km/s} \) for all but one sample. The sample was marked inhomogeneous and was not adequately described by a TI model, but the remaining samples suggest the Bakken shales will be in large part accurately characterized with a TI model. However, the laboratory measurements are performed on intact rock samples; the presence of in-situ fractures may reduce the symmetry to an orthorhombic or even lower symmetry model.
In the study well a dipole sonic log was recorded. This will measure the vertical fast and slow shear velocities. From Figure 1.6, the fast shear wave will be aligned with the $x_2$ axis. The separation of the two shear waves will occur when they intersect a vertical fracture. Figure 1.7 has the fast and slow shear waves recorded in the Lodgepole Formation through the lower Bakken shale. The main interval of interest is from 2035 – 2070m (refer to Figure 1.4). The 2000 – 2020m interval in the Lodgepole Formation has large shear wave separation, but in the interval of interest the shear waves are nearly equal. This is further evidence that a TI model will be appropriate for this well.

In the degenerate case of isotropy, some of the elastic constants in the TI model will be equal.

### 1.3 Fracture Mechanics

Production in the Bakken is dependent on horizontal drilling and hydrofracture stimulation. A primary application of this work is to aid in hydrofracture modeling. Laboratory measurements of dry rock elastic properties can aid fracture modeling in multiple ways:

- The anisotropic stiffness tensor can be calculated from velocity measurements that are typically unavailable in-situ

- The dry rock stiffness tensor and mineralogy data allows estimation of the effective stress coefficients, often referred to as Biot’s coefficients
Figure 1.7: The shear slowness (inverse of velocity) measured with the dipole sonic log. In the Lodgepole Formation (2000-2035m) the shear waves show large separation, but in the Bakken interval (2040-2070m) the shear waves are nearly equal.
The elastic properties, minimum horizontal stress, and strength parameters are the primary inputs for fracture models. In this section I will introduce Biot’s coefficients and discuss the fundamental assumptions in the minimum horizontal stress calculation.

Biot’s coefficients determine the effect pore pressure will have on the effective stress. This will be a function of the dry rock stiffness tensor and the pure mineral bulk modulus. If the dry rock stiffnesses are near the pure mineral bulk modulus, then Biot’s coefficient will approach zero and the pore pressure will not have any impact on the effective stress. The effective stress controls both the elastic properties (Nur & Byerlee, 1971) and the fracture properties (Brace & Martin, 1968; Bruno & Nakagawa, 1991; Jaeger et al., 2007), so accurate determination of Biot’s coefficients is of critical importance.

The minimum horizontal stress is also among the primary factors influencing hydraulic fracture growth. In a normally stressed environment the overburden due to the overlying rock mass will constitute the maximum stress. The horizontal stresses will be a product of the deformation due to the overburden, and tectonic stresses. In-situ open fractures will generally be vertical and aligned parallel to the maximum horizontal stress. In Figure 1.6 the maximum horizontal stress would be in the x2 direction. The fractures align themselves such that the displacement is parallel to the minimum stress plane (x1). The initiation of a tensile fracture will then require the pore pressure to overcome the minimum horizontal stress and the tensile strength of the rock.

In the presence of tectonic stresses the equation for minimum horizontal stress includes lateral strain. Experiments by Karig & Hou (1992) have shown even under uniaxial compression and total lateral strain fixed to zero, there will be inelastic processes that control the horizontal stresses rather than purely elastic deformation. Although Katahara (2009) argues if the rock mass can be considered as a thermodynamically closed system, meaning no exchange of heat or mass with the surrounding rock, then horizontal strain energy has to approach zero. The strain energy reduction mechanism suggested in the paper is a solution/reprecipitation cycle that ultimately brings the horizontal strain energy to zero.

In-situ studies that may determine lateral strain have been heavily clouded by faulty preconditions such as Biot’s coefficients equal to unity (Schmitt & Zoback, 1989). The lateral strain terms in practice have been relegated to mere calibration parameters (Higgins et al., 2008). Given that there is no direct method of measuring lateral strain in-situ, we must focus our energy on accurately determining the other parameters in the horizontal stress equation. When all other explanations are
exhausted, and in-situ observations do not coincide with modeled results, then we should introduce the lateral strain terms. However, they should not be included as mere calibration parameters; lithology and stress path should be considered (Jones & Addis, 1986).

In this study I will disregard lateral strain of any kind and calculate minimum horizontal stress with the uniaxial strain assumption. Biot’s coefficients and the saturated stiffness tensor are the input parameters. The minimum horizontal stress calculation is covered in Chapter 4.

The strength of the formation is the remaining unknown term that will determine fracture growth. The tensile strength may be explained as the extensional stress required to fracture the material (Jaeger et al., 2007), and is typically determined with a Brazilian test. Steady-state fracture growth will depend on the fracture toughness, or stress intensity factor of the formation. This parameter will depend on the specific fracture geometry and applied load. Due to core availability these tests have not been conducted on the Bakken samples. General values of tensile strength will be inferred from in-situ stress measurements, but additional studies should be conducted to determine more robust strength parameters. This study will focus on the accurate determination of minimum horizontal stress.

1.4 In-situ Stress Testing

Mini-Frac tests have been performed in the Scallion member and the Upper Bakken shale. Multiple tests were performed on other parts of the Bakken and Three Forks, but the containment mechanisms apparently failed before a fracture formed in the formation. The basic premise of a Mini-Frac test is to isolate a portion of the formation and raise the pressure in the well until the formation fractures. Then the well is shut in and the pressure readings are interpreted for various properties such as closure stress, reservoir pressure, and tensile strength.

The closure stress is the stress at which a crack is closed in the formation. Crack closure will occur when the pore pressure drops below the total minimum stress in the system (as opposed to effective minimum stress). Therefore, the closure stress is equal to the total minimum stress (Jaeger et al., 2007). In reality there will not be a single closure stress, but a distribution of closure stresses dependent on the aspect ratio of the crack (Mavko et al., 1998), but the primary crack closure will occur when the total minimum stress surpasses the pore pressure. The closure stress provides in-situ data to compare with the modeled horizontal stress profile.

The breakdown pressure (pressure at which the formation fractures) is a factor of the hoop stress surrounding the borehole and the tensile strength of the material. After the initial breakdown of the formation, the pressure may be raised until the previously formed fractures are open again. This is
referred to as the reopening pressure. The breakdown pressure minus the reopening pressure will provide an estimate of the tensile strength of the formation. The closure stress and tensile strength may then be applied to equations for breakdown pressure to estimate the maximum horizontal stress (Jaeger et al., 2007), although near-borehole effects will have a large impact on the recovered values (Sayers, 2010).

1.5 Thesis Overview

The following chapters of this thesis cover the laboratory measurements that have been performed, effective medium modeling of the results, and application of the results in predicting fracture barriers.

Chapter 2 describes the experimental set-up and results. Elastic properties of four facies from the middle Bakken and one sample from the Lodgepole Formation were measured. Literature data on the Bakken shales is also introduced and analyzed. Since no shale measurements were performed in this study, I include the literature data in later modeling schemes.

In Chapter 3 I model part of the data with Hudson’s crack model (Hudson, 1981). I draw conclusions about the cause of anisotropy observed in the samples based on this modeling. Estimates of crack aspect ratio and crack porosity are also determined. Backus averaging (Backus, 1962) is applied to the literature shale data to determine appropriate kerogen elastic properties.

Chapter 4 is a comprehensive discussion on potential fracture barriers. The beginning of the chapter introduces the minimum horizontal stress equation. The drawbacks are then discussed of a popular technique referred to as the ANNIE approximation (Schoenberg et al., 1996) that estimates the TI stiffness tensor. The laboratory results and well log data are then used to estimate the TI stiffness tensor with an alternative method. The model is then verified with in-situ stress testing. The chapter concludes with a calculation of the minimum horizontal stress. Conclusions are drawn about the likely fracture barriers in the Bakken petroleum system.

In Chapter 5 the conclusions and future work are discussed.
CHAPTER 2
EXPERIMENTAL SPECIFICATIONS AND DATA OVERVIEW

This chapter serves as an introduction to the experimental data taken in the laboratory and the basic principles that should be understood before moving on to more advanced analysis. The main topics are the experimental set-up, static moduli, and isotropic versus anisotropic sample characterization. Additional data reported for the Bakken shales is analyzed in Section 2.5 for application in later chapters.

2.1 Experimental Set-up

The general experimental set-up is shown in Figure 2.1. Oriented strain gages are attached to the sample with K20 epoxy. There are two main types of strain gages: foil gages that provide accurate, but less sensitive results, and semiconductor gages that have higher sensitivity, but also suffer from environmental effects (temperature, electrical heating, etc.). The semiconductor gages were used on most of the samples to retain space for the velocity crystals. A set of strain gages is also attached to an aluminum end piece to provide an accurate measure of changes in stress. The aluminum standard is an invaluable tool to remove systematic error in the apparatus. Since the aluminum is measured with the same system, errors due to the electronics are removed. The Young’s modulus \( E \) of aluminum is known to be 70\( GPa \). Defining the Young’s modulus \( E \) as:

\[
E_a = \frac{\sigma_a}{\varepsilon_a}
\]  

(2.1)

The compressional and shear crystals are attached to the sample with conductive epoxy to couple the system and ground the electrical circuit. The crystals can also be housed in the aluminum end pieces allowing a velocity to be acquired along the maximum stress direction. The housing and piezoelectric crystal combination is referred to as a transducer.

The epoxy jacket is a flexible polymer made by Resinlab. The jacket separates the confining oil from the sample. The system has the ability to also control the pore pressure of the sample, but that has not been utilized for this study.

The geometry limitations have a large impact on anisotropic samples. Since the triaxial system requires a cylinder (as opposed to a square), we will not be able to change the maximum stress direction on any given sample. This means that we cannot acquire all the necessary Young’s moduli
Figure 2.1: The general triaxial system set-up. The sample is encapsulated in a flexible epoxy and maximum and minimum stresses are applied. The brass piston pushes the sample up through hydraulic drive providing the maximum stress, and the system is filled with oil surrounding the sample to produce the minimum stress.
and Poisson’s ratios to describe the elastic properties with a single sample. However, we will be able to measure all the necessary velocities if we align the maximum stress with the bedding direction and attach velocity crystals to the sides of the sample. This will limit us to hydrostatic loading ($\sigma_{\text{min}} = \sigma_{\text{max}}$) since the maximum in-situ stress is perpendicular-to-bedding, and increasing the maximum stress parallel-to-bedding would perturb the true anisotropy.

The samples that have been measured with the bedding parallel to the maximum stress directions are from Facies E and D. Facies E had measurements at 0°, 30°, 60°, and 90°, but negligible anisotropy was recovered ($< 2\%$ P-wave anisotropy) so the velocities have been averaged. Facies D had measurements at 0°, 30°, 45°, 60°, and 90°, and showed strong velocity anisotropy (11 – 17% P-wave anisotropy). The full transversely isotropic (TI) stiffness matrix was calculated from the velocities, but only one Young’s modulus and Poisson’s ratio was recovered from the strain measurements.

The anisotropic measurements of Facies E and D were prompted by the results of Facies C. In this sample the maximum stress was perpendicular-to-bedding (in-situ condition). We measured velocities in the symmetry planes, but not in oblique-angle directions. This allows for most of the stiffness coefficients to be calculated, but not the crucial $C_{13}$ stiffness closely related to minimum horizontal stress. The P-wave anisotropy ranged from 5 – 12%, which is ample reason to consider the specimen weakly anisotropic.

The last two measurements were from Facies F, and the upper bounding Lodgepole. Both samples were assumed isotropic with only a single set of velocity measurements taken parallel to the maximum stress and perpendicular to the bedding. Facies F was assumed isotropic due to the lack of continuous bedding. Bioturbation and whole fossil inclusions will disrupt the bedding planes, causing effectively isotropic elastic properties. The Lodgepole Formation primarily consists of limestone mud. Visual inspection suggested no layering of any kind and bench top (no applied pressure) velocity measurements showed velocities near the pure mineral velocities, so bedding-parallel microcracking was deemed insignificant.

### 2.2 Isotropic Data

Elastic isotropy signifies that the elastic properties will not show any directional dependence. The generalized Hooke’s Law is shown in 2.2 in the reduced two-index notation and demonstrated in 2.3 for the special case of isotropy. Table 2.1 shows the transformation from the complete four index notation to the condensed two-index notation. The numbers 1 – 3 represent compressive or tensile stress/strain along the three principle axes (e.g. $x, y, z$), and 4 – 6 represent shear stress/strain (e.g.
The strain of a material is defined as the change in length over the total length \( \Delta L/L \). The stress represents a unit force per unit area \( F/A \). Stiffness coefficients are generally given in Gigapascal (GPa). For anelastic material the stiffness coefficients \( (C_{ij}) \) allow conversion from stress to strain or vice versa, and are connected to the velocity of a material as well as many common engineering terms used to describe the elasticity of a medium. If the material contains no symmetry planes then 21 stiffness coefficients \( (C_{ij}) \) are required to completely describe the elastic properties. The independent stiffness coefficients required to describe an isotropic medium are reduced to two. The transition from no internal symmetry (triclinic) to isotropy is shown in 2.3.

**Table 2.1: Conversion from four-index notation to two-index notation**

<table>
<thead>
<tr>
<th>( ij(kl) )</th>
<th>( I(J) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>33</td>
<td>3</td>
</tr>
<tr>
<td>23,32</td>
<td>4</td>
</tr>
<tr>
<td>13,31</td>
<td>5</td>
</tr>
<tr>
<td>12,21</td>
<td>6</td>
</tr>
</tbody>
</table>

\[
\sigma_i = C_{ij}\varepsilon_j \tag{2.2}
\]

\[
\begin{pmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\
C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\
C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}
\end{pmatrix} \tag{2.3}
\]

Here \( \sigma \) and \( \varepsilon \) are the directional stress and strain of the material. The isotropic non-zero stiffness coefficients are related to the velocities and density \( (\rho) \) by the following equations:

\[
C_{11} = \rho V_P^2 \tag{2.4}
\]

\[
C_{44} = \rho V_S^2 \tag{2.5}
\]

\[
C_{12} = C_{11} - 2C_{44} \tag{2.6}
\]

Equation 2.6 shows there are only two independent stiffness coefficients for isotropic materials. In this case, the engineering terms Young’s modulus \( (E) \), Poisson’s ratio \( (v) \), bulk modulus \( (K) \), and
shear modulus \((G)\) have the following relationships with velocity, density, stress, and strain given a stress in an arbitrary plane \(a\), and perpendicular planes \(b\) and \(c\):

\[
E = \rho V_S^2 \left(\frac{3V_P^2 - 4V_S^2}{V_P^2 - V_S^2}\right) = C_{44} \frac{3C_{11} - 4C_{44}}{C_{11} - C_{44}} = \frac{9KG}{3K + G} = \frac{\Delta \sigma_a}{\Delta \varepsilon_a} \tag{2.7}
\]

\[
v = \frac{1}{2} \left(\frac{V_P^2 - 2V_S^2}{V_P^2 - V_S^2}\right) = \frac{1}{2} \frac{C_{11} - 2C_{44}}{C_{11} - C_{44}} = \frac{3K - 2G}{2(3K + G)} = -\frac{\Delta \varepsilon_{b,c}}{\Delta \varepsilon_a} \tag{2.8}
\]

\[
K = \rho(V_P^2 - \frac{4}{3}V_S^2) = C_{11} - \frac{4}{3}C_{44} = \frac{E}{3(1 - 2v)} = \frac{\Delta \sigma_a}{\Delta \varepsilon_a + \Delta \varepsilon_b + \Delta \varepsilon_c} \tag{2.9}
\]

\[
G = \rho V_S^2 = C_{44} = \frac{E}{2(1 + v)} = \frac{\Delta \tau_{ab}}{\Delta \varepsilon_{ab}} \tag{2.10}
\]

The velocities \(V_P\) and \(V_S\) are the compressional and shear wave velocities. \(\tau\) represents a shear stress or the tangential stress. In practice the shear modulus is calculated from the other elastic properties; direct shear stress is rarely applied to rock samples. The last term in each series of equations gives the physical definition of each property. Figure 2.2 is a visual representation of the stress-strain experiment. The final terms hold true for any degree of anisotropy, causing the planes \(a\), \(b\), and \(c\) to become increasingly unique and the Young’s moduli and Poisson’s ratios to increase in number.

In anisotropic materials the other terms in equations 2.7-2.10 will not be as simple. The addition of one unique plane \((a \neq b = c)\) significantly complicates the relationships and the equations must be re-derived from equation 2.2 i.e. equations 2.7-2.10 are not valid for anisotropic materials. This fact has not been well understood in the industry, leading to misinterpretation of anisotropic elastic properties. The correct expressions are given in Section 2.2.

The Lodgepole Formation and Facies F are both assumed isotropic and the following data interpretation will contain errors if they are proven to be anisotropic in the future. In Facies E directional velocities were measured and showed negligible anisotropy supporting the hypothesis of isotropy. The measured P-wave anisotropy determined by subtracting the slow P-wave from the fast P-wave and dividing by the slow P-wave was 0.6 – 1.9% nearly within the 1% velocity error. The calculated Thomsen anisotropy parameter \(\varepsilon\) (discussed in Section 2.2) ranged from 0.002 – 0.016. This is not enough anisotropy to warrant any special treatment; therefore the velocities have been averaged and included in the isotropic sample analysis.
Figure 2.2: A graphical explanation of the stress-strain relationships. In an isotropic sample the Young’s modulus, Poisson’s ratio, and bulk modulus can be determined from a single sample by applying a uniaxial stress ($\sigma_a$) and measuring two strains ($\varepsilon_a$ and $\varepsilon_b$ or $\varepsilon_c$). The shear modulus was not directly measured in the laboratory, but calculated from the other elastic terms.

The methodology for recovering the static moduli was to raise the axial pressure by a small increment ($1 - 2MPa$) repeatedly for each confining pressure. For the samples that utilized semiconductor gages environmental effects strongly influenced the recovered values. Spurious readings also occurred from random noise and had to be completely disregarded. The foil gages did not suffer from environmental effects, but stiffness of the samples caused the signal to be similar in magnitude to the electrical noise of the system. Based on the repeatability of the measurements, the error in Young’s modulus is estimated at $\pm 6GPa$ and the error in Poisson’s ratio is $\pm 0.05$ for both types of gages.

Since the gages had such large inaccuracies, the interpretation will be based on the average response at each confining pressure. Figure 2.3 shows the static moduli for the Lodgepole Formation. The dynamic data in Figure 2.4 has high values for Young’s modulus and Poisson’s ratio with relatively minor variance over the entire measurement interval. This suggests most of the changes in Figure 2.3 are likely long-term damage or drifting of the semiconductor gages. The amount of error in the static moduli is not surprising considering the Young’s modulus of this rock is greater than the Young’s modulus of aluminum. As the strain level is decreased the environmental drift and
random noise have larger effects on the derived elastic properties.

Figure 2.3: Measured static moduli for the Lodgepole. The circles are Young’s moduli, stars are bulk moduli, squares are shear moduli, and blue triangles are Poisson’s ratios. The Poisson’s ratios are plotted with the secondary y-axis. The error in $E$ and $\nu$ is estimated at $\pm 6 \text{GPa}$ and $\pm 0.05$, respectively.

Figure 2.5 has the static moduli for Facies F in the middle Bakken. This facies is located directly below the upper Bakken Shale. This was the only sample measured with foil gages. The variance in Poisson’s ratio was the main cause for changing to semiconductor gages. There is a general upward trend with increasing confining pressure, but precise evaluation is not possible with this data. The dynamic moduli in Figure 2.6 show a larger deviation from the static moduli than the data sets from the Lodgepole. If the relative changes were the same in both, then the dynamic moduli could be used directly for the static values.

The static moduli for Facies E are shown in Figure 2.7. This sample had strong visible layering and a high gamma ray signal in the well log. I suspected the clay volume and visible layering
Figure 2.4: Dynamic moduli and velocity measurements for the Lodgepole Formation. (a,b) has the P and S velocities, (c,d) has the derived Young’s moduli and Poisson’s ratios, and (e,f) has the bulk moduli and shear moduli. The legend gives the confining pressures of each measurement (increments are 500psi).
Figure 2.5: Static moduli for Facies F in the middle Bakken. The circles are Young’s moduli, stars are bulk moduli, squares are shear moduli, and blue triangles are Poisson’s ratios. The Poisson’s ratios are plotted with the secondary y-axis.
Figure 2.6: Dynamic moduli and velocity measurements for Facies F in the middle Bakken. (a,b) has the P and S velocities, (c,d) has the Young’s moduli and Poisson’s ratios, and (e,f) has the bulk moduli and shear moduli. The legend gives the confining pressures of each measurement.
would cause significant anisotropy, but after velocity analysis the sample showed minimal anisotropy. Figures 2.8-2.10 have the dynamic properties versus confining pressure. The data shows a similar response to Facies F.

![Graph showing dynamic properties versus confining pressure](image)

Figure 2.7: Static moduli for Facies E in the middle Bakken. The circles are Young’s moduli, stars are bulk moduli, squares are shear moduli, and blue triangles are Poisson’s ratios. The Poisson’s ratios are plotted with the secondary y-axis.

### 2.3 Transversely Isotropic Elastic Properties

A transversely isotropic (TI) material is azimuthally symmetric about a single axis. Examples of TI materials are layered media or isotropic media with a single set of oriented fractures. A layered medium with fractures parallel to the bedding can also be approximated with a TI model. There are five stiffness coefficients required to completely describe the elastic properties. The stiffness matrix is given below:
Figure 2.8: Dynamic P and S wave velocity for Facies E in the middle Bakken.
Figure 2.9: Dynamic Young’s modulus and Poisson’s ratio calculated from velocities for Facies E in the middle Bakken.
Figure 2.10: Dynamic bulk and shear moduli calculated from the velocities for Facies E.
\[
\begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}
\] (2.11)

Figure 2.11 has the labeled axes for a layered medium.

Figure 2.11: Transversely isotropic medium with the three principle axes labeled. In this sample the 3-axis is azimuthally symmetric.

The velocities are related to the stiffness coefficients by the following equations (King, 1964):

\[ C_{11} = \rho V_{P90}^2 \]

\[ C_{33} = \rho V_{P0}^2 \]

\[ C_{44} = \rho V_{S0}^2 \]

\[ C_{66} = \rho V_{S90}^2 \]

\[ C_{12} = \rho (V_{P90}^2 - 2V_{S90}^2) = C_{11} - 2C_{66} \]

\[ C_{13} = \rho \left\{ \sqrt{\left[ \frac{4V_{P45}^2 - V_{P90}^2 - V_{P0}^2 - 2V_{S0}^2}{2} \right]^2 - \left[ \frac{V_{P45}^2 - V_{P90}^2}{2} \right]^2} - V_{S0}^2 \right\} \]

The numbers in the subscripts represent the phase angle with respect to the perpendicular-to-bedding plane (the vertical direction is 0°). To avoid any ambiguity, \( V_{S90} \) refers to the pure shear wave with particle motion parallel-to-bedding. Figure 2.12 shows the velocity directions and the particle motion with arrows for the shear waves. The 45° quasi-shear wave is also labeled; this is
an alternative velocity to the $45^\circ$ quasi-compressional wave that may be used to determine elastic properties. The ‘quasi’ qualifier is appropriate for oblique-angle P-waves and the vertically energized shear wave (Sv) in anisotropic media since the fastest waves will not be polarized normal to the slowness or ray directions (Ruger, 1996). I will drop the 'quasi' qualifier henceforth for conciseness.

Due to the complexity of the equations, the engineering terms will be given only through the stiffness coefficients, stress, and strain. The Young’s moduli, Poisson’s ratios, and bulk modulus are given by King (1964):

\[
\nu_{12} = \frac{C_{12}C_{33} - C_{13}^2}{C_{11}C_{33} - C_{13}^2} = -\frac{\Delta \varepsilon_2}{\Delta \varepsilon_1}
\]

\[
\nu_{13} = \frac{C_{13}(C_{11} - C_{12})}{C_{11}C_{33} - C_{13}^2} = -\frac{\Delta \varepsilon_3}{\Delta \varepsilon_1}
\]

\[
\nu_{31} = \frac{C_{13}}{C_{11} + C_{12}} = -\frac{\Delta \varepsilon_1}{\Delta \varepsilon_3}
\]

\[
K = \frac{C_{33}(C_{11} + C_{12}) - 2C_{13}^2}{C_{11} + 2C_{33} + C_{12} - 4C_{13}} = \frac{\Delta \sigma}{\Delta \varepsilon_1 + \Delta \varepsilon_2 + \Delta \varepsilon_3} = \frac{\Delta P}{\Delta V}
\]
\[ E_1 = \frac{(C_{11} - C_{12})(C_{11}C_{33} - 2C_{13}^2 + C_{12}C_{33})}{C_{11}C_{33} - C_{13}^2} = \frac{\Delta \sigma_1}{\Delta \varepsilon_1} \]

\[ E_3 = C_{33} - \frac{2C_{13}^2}{C_{11} + C_{12}} = \frac{\Delta \sigma_3}{\Delta \varepsilon_3} \]  \hspace{1cm} (2.13)

Figure 2.13 shows the two necessary measurements to recover the elastic properties, and which properties are recovered from each measurement. The bulk modulus requires uniform compression, which is not shown here and can be described simply as the change in pressure divided by the change in volume. As noted by King (1964) there is no dependence on \( C_{44} \) for any of the above relationships. To invert for the complete stiffness tensor a third sample with a principle stress at 45° with respect to the bedding must be measured (Tahini & Abousleiman, 2010).

As a comparison, let us consider the incorrect substitution of \((V_{P90}, V_{S90})\) and \((V_{P0}, V_{S0})\) into equations 2.13, \( E_1 \) coincides with \( E_{\text{FALSE}90} \), \( E_3 \) with \( E_{\text{FALSE}0} \), \( \nu_{12} \) with \( \nu_{\text{FALSE}90} \), and \( \nu_{13} \) with \( \nu_{\text{FALSE}0} \).

\[ E_{\text{FALSE}90} = \rho V_{S90}^2 \left( \frac{3V_{P90}^2 - 4V_{S90}^2}{V_{P90}^2 - V_{S90}^2} \right) = C_{66} \left( \frac{3C_{11} - 4C_{66}}{C_{11} - C_{66}} \right) \]  \hspace{1cm} (2.14)
\[ E_{FALSE0} = \rho V_{S0}^2 \left( \frac{3V_{P0}^2 - 4V_{S0}^2}{V_{P0}^2 - V_{S0}^2} \right) = C_{44} \left( \frac{3C_{33} - 4C_{44}}{C_{33} - C_{44}} \right) \] (2.15)

\[ \nu_{FALSE90} = \frac{1}{2} \frac{V_{P90}^2 - 2V_{S90}^2}{V_{P90}^2 - V_{S90}^2} = \frac{1}{2} \frac{C_{11} - 2C_{66}}{C_{11} - C_{66}} \] (2.16)

\[ \nu_{FALSE0} = \frac{1}{2} \frac{V_{P0}^2 - 2V_{S0}^2}{V_{P0}^2 - V_{S0}^2} = \frac{1}{2} \frac{C_{33} - 2C_{44}}{C_{33} - C_{44}} \] (2.17)

The false engineering terms clearly have strong differences from their correct counterparts, and it is difficult to extract any relation between the two. The problem with these equations is that they tend to over-predict horizontal stress due to anisotropy. For example, fluid-filled fractures may decrease the horizontal stress, but the above formulation would predict an increase.

The \( C_{13} \) stiffness coefficient contributes to all the engineering terms; therefore an oblique velocity is necessary to calculate each one. The \( \nu_{12} \) Poisson’s ratio has equal \( C_{13} \) terms in the numerator and denominator, so we may be able to approximate this value fairly well without an accurate \( C_{13} \) value. All the other terms should be determined with a reasonable estimate of \( C_{13} \).

### 2.4 Phase Versus Group Velocity

A question that arises for anisotropic samples is whether the phase velocity \( (V) \) or group velocity \( (v_{gr}) \) is measured for oblique bedding angles. In the symmetry planes, exactly perpendicular or parallel to bedding, the group and phase velocities and their respective angles will be equal. But in between the extreme angles they will generally be different. The group velocity refers to the energy propagation, whereas the phase velocity assumes a propagating plane wave. At any given point along the wavefront there is a representative phase and group velocity; the difficulty lies in determining the proper distance for the velocity calculation and the correct angle. In the laboratory if the flat part of the velocity surface contacts the receiver, the portion parallel to the receiver, then we will claim that we measured the phase velocity. If the receiver records a curved part of the velocity surface not parallel to the receiver then we claim that we have measured the group velocity. This is simply because the phase velocity and angle will be unknown if we contact a curved part of the velocity surface, and the group velocity and angle will be unknown if we contact the flat part of the velocity surface.

Figure 2.14a from Vestrum (1994) shows an example wavefront propagating from a long transducer (bold lines). If the black portion contacts the receiver then the velocity can be interpreted as a phase velocity (green arrow labeled \( \vec{v} \)). If the blue portion contacts the receiver then the velocity
should be interpreted as a group velocity (black arrow labeled $\vec{g}$). From Figure 2.14b the associated group velocity will clearly be larger than the phase velocity for a given angle. I also know if the receiving transducer misses the plane wave, then the phase angle ($\theta$) will be less than the bedding angle, and the group angle ($\psi$) will be greater than the bedding angle. This comes from the fact that the wavefront in Figure (2.14a) veers toward the parallel-to-bedding direction ($90^\circ$). If the blue portion of the wavefront is measured then the phase angle will be less than the bedding angle as shown in Figure 2.15 and the group angle will be greater than the bedding angle. To summarize, the physical constraints are:

- $v_{gr} > V$ at a given angle
- $\psi > \text{bedding angle} > \theta$ if the primary plane wave is missed

There is no simple closed-form solution for $C_{13}$ from group velocities. Cheadle et al. (1991) provides a scheme for iteratively calculating the $C_{13}$ stiffness given a 45° group velocity. But if the receiving transducer was attempting to measure the 45° phase velocity and missed the plane wave, the group angle will be greater than 45° and will depend on the specific experimental geometry. Vestrum (1994) was able to show that the Cheadle experiment, which assumed all group velocities were measured, actually had a mixture of group and phase velocity depending on the measurement plane. This stresses the responsibility of the experimenter to scrutinize each sample and velocity separately. In typical experimental geometries we are always in the gray area between phase velocity (infinitely long source and receiver) and group velocity (point source and receiver). We should not assume all the velocity measurements were one or the other.

For arbitrary group angles the simplest way to calculate $C_{13}$ is to guess a value and iteratively update the guess by fitting a curve to the converted phase velocity surface. When the converted phase velocity error is minimized and the physical constraints are met, then the correct $C_{13}$ value has been found. To improve the estimation we should sample the distinctive portions of the phase velocity surface, which include a low or high angle, and a mid-range angle. But measuring the $S_V$ wave for multiple oblique angles will improve the $C_{13}$ determination the most since the velocity surface will dramatically change depending on the material properties. For instance, if the Thomsen anisotropy parameters $\varepsilon$ and $\delta$ are equal (elliptical anisotropy), then the $S_V$ velocity surface is completely flat.
Figure 2.14: (a) Example wavefronts propagating from a long transducer. \( \mathbf{v} \) and \( \mathbf{g} \) are the phase and group velocity vectors, and \( \delta \) is the difference between group and phase angle. The black portion of the propagating wavefront is the plane wave. (b) An illustration of the lateral displacement \( D \) of the plane wave. \( D \) can be calculated if the height (\( H \)) and material properties are known. \( D \) represents the minimum transducer length necessary to measure the phase velocity. Image from Vestrum (1994).

Figure 2.15: (a) A representative 45\(^\circ\) sample consistent with the wavefront in Figure 2.14. (b) Snapshots of plane waves passing through a sample in blue and the associated energy propagation in red. The angle normal to the plane wave represents the phase angle (\( \theta \)) and the energy propagation angle is the group angle (\( \psi \)). The lowest plane wave parallel with the transducer will have a phase angle = 45\(^\circ\) and originate at an unknown location inside the source transducer causing the group velocity and angle to be unknown.
The conversions from phase to group velocity and angle are given by (Berryman, 1979; Tsvankin, 1996):

\[ v_{gr} = V \sqrt{1 + \left( \frac{1}{V} \frac{dV}{d\theta} \right)^2} \]  \hspace{1cm} (2.18)

\[ \tan\psi = \tan\theta \left[ 1 + \frac{1}{\sin\theta \cos\theta} \left( \frac{dV}{d\theta} \right) \right] \]  \hspace{1cm} (2.19)

The main issue that arises in equations 2.18 and 2.19 is the derivative of the phase velocity with respect to the phase angle. To assist with the explanation of the phase velocity equations, I will now introduce the Thomsen anisotropy parameters: the P-wave anisotropy \( \varepsilon \), S-wave anisotropy \( \gamma \), and the additional parameter controlling the oblique-angle \( P \) and \( S_V \) velocity response \( \delta \) (Thomsen, 1986):

\[ \varepsilon = \frac{C_{11} - C_{33}}{2C_{33}} \]  \hspace{1cm} (2.20)

\[ \gamma = \frac{C_{66} - C_{44}}{2C_{44}} \]  \hspace{1cm} (2.21)

\[ \delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} \]  \hspace{1cm} (2.22)

In Thomsen notation, the exact phase velocity equation becomes (Tsvankin, 1996):

\[ \frac{V^2(\theta)}{V^2_{P0}} = 1 + \varepsilon \sin^2\theta - \frac{f}{2} \pm \frac{f}{2} \sqrt{1 + \frac{4\varepsilon^2\sin^2\theta}{f} (\delta \cos^2\theta - \varepsilon \cos 2\theta) + \frac{4\varepsilon^2 \sin^4\theta}{f^2}} \]  \hspace{1cm} (2.23)

, where

\[ f = 1 - \frac{V^2_{S0}}{V^2_{P0}} \]

The plus or minus sign in front of the radical gives the P-wave velocity or the \( S_V \) wave velocity, respectively. The exact \( \frac{dV}{d\theta} \) derived in Mathematica is given by:

\[ \frac{dV}{d\theta} = \left( \frac{1}{2} \frac{V^2_{P0}}{2} \left( 2\varepsilon \cos \theta \sin \theta \pm \frac{\varepsilon f (\cos \theta (2L + 4M) + N)}{(1 + L + M)^{\frac{1}{2}}} \right) \right) \left( \frac{V^2_{P0}}{2} \left( 1 - \frac{L}{2} + \varepsilon \sin^2 \theta \pm \frac{1}{2} f (1 + L + M) \right) \right)^{-\frac{1}{2}} \]  \hspace{1cm} (2.24)
where
\[
L = \frac{4(2\delta \cos^2 \theta - \varepsilon \cos 2\theta) \sin^2 \theta}{f}
\]
\[
M = \frac{4\varepsilon^2 \sin^4 \theta}{f^2}
\]
\[
N = \frac{4\sin^2 \theta(2\varepsilon \sin 2\theta - 4\delta \cos \theta \sin \theta)}{f}
\]

The ± are positive for the P-wave and negative for the Sv-wave. The complexity of the phase velocity derivative has been a deterrent in seismic applications. The introduction of the linearized phase velocity significantly simplifies the expression (Tsankin, 2001):
\[
\frac{dV_p(\theta)}{d\theta} = V_{p0}\sin 2\theta(\delta \cos 2\theta + 2\varepsilon \sin^2 \theta) \quad (2.25)
\]

Likewise, the angle conversion simplifies to:
\[
\psi = \theta + (\delta + 2(\varepsilon - \delta) \sin^2 \theta) \sin 2\theta \quad (2.26)
\]

The popular linearized equations are justified in seismic surveys because the data samples primarily low angles. However, these should not be applied to laboratory data since we are calculating the material properties with large phase angles (\(\theta > 25^\circ\)). Once accurate anisotropy parameters are determined, then the linearized equations may be applied to seismic applications.

2.5 Transversely Isotropic Sample Data

The \(\delta\) parameter and \(C_{13}\) have not been calculated for Facies C since an oblique-angle velocity was not measured. The symmetry plane anisotropy parameters are shown in Figure 2.16 and Figure 2.17. In Facies D bedding angles of 0°, 30°, 45°, 56°, and 90° were measured to ensure accurate \(\delta\) and \(C_{13}\) determination. The anisotropy parameters are reported in Figure 2.18.

Facies D shows higher \(\varepsilon\) and \(\gamma\) values than Facies C, but both have moderate to high levels of anisotropy. Facies E had the strongest visible layering and moderate clay volume, but those factors did not cause any significant anisotropy. In a later section I will show that this anisotropy is consistent with bedding-parallel microcracks.

The complete set of static moduli could not be recovered from Facies D or C since I did not have additional samples. One Young’s modulus and Poisson’s ratio were recovered from each sample and
Figure 2.16: Facies C anisotropy parameter $\varepsilon$ for varying axial and confining pressures. The legend shows symbols for each confining pressure.
Figure 2.17: Facies C anisotropy parameter $\gamma$ for varying axial and confining pressures. The legend shows symbols for each confining pressure.
Figure 2.18: Facies D anisotropy values for varying confining pressures.
those results are shown in Figures 2.19 and 2.20. Notice the recovered properties are not the same in each sample; that is because the maximum stress direction for Facies C was perpendicular-to-bedding and for Facies D it was parallel-to-bedding. For Facies D I should have been able to recover an additional Poisson’s ratio, but the gages were damaged at the beginning of the measurement.

Overall the static measurements were not accurate enough to warrant further modeling. The Poisson’s ratios were inconsistent in most of the samples. The Young’s modulus for Facies F nearly equaled the dynamic value, but showed a large difference in Facies E and the Lodgepole. Without additional static measurements I cannot determine the cause of these changes. In hindsight I should have continued using the foil gages and increased the stress increments. The semiconductor gages are frequently torn or partially damaged during the experiment causing the calculated properties to be unreliable.

Figure 2.19: Static moduli for Facies C. The black circles are $E_3$ and the blue triangles are $\nu_{31}$. Since this rock is anisotropic only a single Young’s modulus and Poisson’s ratio were recovered from the strain measurements. In this sample the uniaxial stress was applied perpendicular to bedding.
Figure 2.20: Static moduli for Facies D. The black circles are $E_1$ and the blue triangles are $\nu_{12}$. Since this rock is anisotropic only a single Young’s modulus and Poisson’s ratio were recovered from the strain measurements. A second Poisson’s ratio could have been recovered, but the strain gages were not functioning properly. In this sample the applied uniaxial stress was applied parallel to bedding.
In Figure 2.21 is a comparison between the $\delta_{gr}$ determined by assuming group velocities versus $\delta_{ph}$ assuming phase velocities. The transducer length was 6.5 mm and the distance between transducers was 3.64 cm. At the highest stress and lowest anisotropy level, $\delta$ is equal regardless of the technique applied. This suggests the plane wave is barely contacting the edge of the transducer, making the assumption of group or phase velocity equally acceptable. For any higher anisotropy, the $\delta_{gr}$ will yield an accurate $\delta$, whereas if lower anisotropy data were available $\delta_{ph}$ would provide the more accurate $\delta$. As the stress is decreased, the $\delta$ values deviate and at the lowest stress the $\delta_{ph}$ begins to dip downward. This dataset shows the danger in not properly evaluating the velocities. A high anisotropy sample could have a large $\delta$, but if $\delta_{ph}$ is calculated when $\delta_{gr}$ is appropriate, then $\delta$ may be grossly underestimated.

This may explain why we find in the literature that $\varepsilon$ and $\gamma$ show strong correlations, but $\delta$ exhibits almost no trend with the other anisotropy parameters. Figures 2.22 and 2.23 have the data from Vernik & Liu (1997), which spans over many shale formations including the Bakken Shales. The trend in Figure 2.22 is strong, but in Figure 2.23 there is no discernible correlation. It is not appropriate to consider all shale measurements at the same time. Mineralogy and depositional features may change the anisotropy characteristics. This study will incorporate only the data that was taken in the Bakken. The data will be analyzed to try and determine a general trend between the anisotropy parameters.

First, let us consider a sample with a height of 50 mm and a transducer width of 10 mm. The vertical velocities will be set at $V_{P0} = 3.5 \text{ km/s}$ and $V_{S0} = 2.0 \text{ km/s}$ and the anisotropy parameters will be set to $\varepsilon = 0.3$, $\gamma = 0.3$, and $\delta = 0.15$. The equation for displacement of a 45° P-wave ($D$ from Figure 2.14b) is given by (Dellinger & Vernik, 1994):

$$D = H \frac{(V_{P90}^2 - V_{P0}^2)}{2V_{P45}^2} \times \frac{V_{S90}^2 + V_{S0}^2 - 2V_{P45}^2}{V_{P90}^2 + V_{P0}^2 + V_{S90}^2 + V_{S0}^2 - 4V_{P45}^2} \quad (2.27)$$

Here $H$ is the height of the sample. The units of $D$ will equal the units of the input $H$. To calculate the $S_V$ displacement, simply replace $V_{P45}$ with $V_{S45}$. Given the sample geometry and material properties $D = 12.8 \text{ mm}$, which is greater than the transducer width of 10 mm. In this scenario the arrival must be interpreted as a group velocity contacting the edge of the transducer as in Figure 2.15b. The energy propagation distance would become the diagonal between transducer edges 50.99 mm, and the group angle ($\psi$) will equal $\cot \frac{D}{H}$ plus the bedding angle. If the above analysis is implemented, then a group velocity of 3.97 km/s would be determined for the group angle of 56°. $\delta$ could then be iteratively solved for with equations 2.18, 2.19, 2.22, 2.23, and 2.24.
Figure 2.21: A comparison of $\delta$ determined by assuming phase versus group velocity for Facies D. At high stress the methods coincide, suggesting at that point the first energy to reach the transducer is on the edge, and part of the plane wave. Assuming either group or phase velocity at this point will produce the same results.
Figure 2.22: $\varepsilon$ versus $\gamma$ for a wide range of organic rich shales. Data from (Vernik & Liu, 1997).
Figure 2.23: $\varepsilon$ versus $\delta$ for a wide range of organic rich shales. The data shows a poor correlation. Data from (Vernik & Liu, 1997).
Now let us consider the consequences of interpreting the arrival as the 45° phase velocity. The travel time will equal 12.84μs and the assumed distance is now equal to $H$. The new $\delta = 0.13$ making the error 0.02, which is within the experimental error of the velocities. However, if the same analysis is done with $\varepsilon = 0.4$ and 0.5 then $\delta = 0.07$ and 0.0, respectively.

Figures 2.24 and 2.25 have the velocity surfaces for the P-wave and Sv-wave, respectively with the material properties from the example above. The four curves shown are the phase velocity at the phase angle, group velocity at the phase angle, phase velocity at the group angle, and group velocity at the group angle. The variance in velocity surfaces may suggest the determination of phase or group velocity is a simple matter, but recall from the examples above that a group velocity interpretation requires a change in propagation distance and angle. Even if multiple velocity measurements are taken, the angle adjustments may cause a group velocity to appear as though it is a phase velocity. The group velocity at the group angle is a mirror image of the phase velocity at the phase angle.

Many past publications have velocity interpretation errors. In a paper by Johnston & Christensen (1994) the authors simply interchanged the velocities and assumed the bedding angle would be equal to the group or phase angle. Figure 2.15b shows that if a group velocity is measured the phase angle and group angle will not be equal to the bedding angle. The authors were able to come to the correct conclusion that they measured the phase velocity, but only with the aid of their Sv-wave data. Eaton (1992) took a similar approach by using the iteration scheme of Cheadle et al. (1991) on the dataset from Vernik & Nur (1992). The calculated $\delta$ values were as high as 0.98, and once again modeling the Sv-wave suggested the phase velocity had been measured. Dewhurst & Siggins (2006) made the same mistake with a single-plug method adapted from Wang (2002), but they did not measure an oblique angle Sv-wave velocity. The authors assumed a 45° group velocity was measured, and recovered $\delta$ values of nearly 0.5. In each of these cases if the group velocity was measured, the difference in properties will not be as large as the authors have suggested. However, a misinterpretation will lower the data quality and should be corrected. The study by Dewhurst & Siggins (2006) may have completely erroneous results due to the faulty interpretation.

I will further analyze the Vernik & Nur (1992) dataset since these measurements were taken in the Bakken Shales and will be used for modeling in Chapter 4.

To determine an accurate $\delta$ multiple oblique-angle P-waves and Sv-waves should be measured. The Sv wave is particularly useful since we generally cannot pick the exact onset of the wave, and the $\pm$ in equation 2.23 means the Sv and P-wave determined $\delta$’s will diverge if the wave onset is picked too late. In this case, an average of the two will provide an accurate $\delta$ value. However, the lateral displacement of the P-wave is much larger than the Sv-wave, so we are more likely to miss
Figure 2.24: The P-wave phase and group velocities at both the phase and group angles for a sample with $V_{P0}=3.5\text{km/s}$, $V_{S0}=2.0\text{km/s}$, $\varepsilon=0.3$, $\gamma=0.3$, and $\delta=0.15$.

Figure 2.25: The Sv-wave phase and group velocities at both the phase and group angles for a sample with $V_{P0}=3.5\text{km/s}$, $V_{S0}=2.0\text{km/s}$, $\varepsilon=0.3$, $\gamma=0.3$, and $\delta=0.15$. 
the P-wave phase velocity. In Vernik & Nur (1992) the P and Sv wave velocities have been reported, but in subsequent papers such as Vernik & Liu (1997) the $\delta$ determined from the P-wave is the value given. Figure 2.26 shows the difference between the two calculated $\delta$'s from the P ($\delta_P$) and Sv ($\delta_{SV}$) waves by forward modeling the velocity response with equations 2.23 and 2.24 and adjusting $\delta$ accordingly.

Figure 2.26: $\delta$ determined from a 45° P-wave and 45° SV-wave. If the difference were caused solely by picking the phase velocity too slow, then the Sv-wave should always give a higher $\delta$ than the P-wave. Original data from Vernik & Nur (1992).

For a portion of the data in Figure 2.26 the $\delta_{SV}$ is less than the $\delta_P$. An automatic picking scheme was applied to the waveforms that would pick the velocity when the deviation was 1% greater than the maximum value. This should always pick the onset of the wave slightly late; causing the Sv to over predict $\delta$ and P to under predict $\delta$. Assuming the TI model is valid, for the cases where $\delta_{SV}$ is less, the bedding angle of the sample may have been less than 45°. In Figure 2.25 the Sv velocity surface has a maximum value at approximately 40°. If this phase angle was measured by accident,
then the calculated $\delta_{SV}$ could be lower than the $\delta_P$ (high Sv leads to low $\delta$, and high P leads to high $\delta$). For these special cases the $\delta_{SV}$ is not reliable, the $\delta_P$ value should be considered even though the value is probably lower than the true $\delta$.

Based on the geometry reported in Dellinger & Vernik (1994) I calculated the displacement of the P-wave with equation 2.27 for $H = 40\,mm$. If the displacement is greater than the transducer width of 12$mm$, then the P-wave phase velocity was not measured. However, there is no guarantee that all the measurements strictly adhered to the geometric dimensions given in Dellinger & Vernik (1994). The Sv calculations are not shown since they are all well below the reported transducer width. Figure 2.27 has the displacement plotted against $\varepsilon$.

The data is highly questionable for $D > 12\,mm$, furthermore in a realistic setting the effective transducer width will be smaller than 12$mm$. The velocity crystal has to have enough space to generate a strong signal that can be picked. Dellinger & Vernik (1994) suggest any offset greater than half the transducer width may cause significant errors due to the low signal strength. The geometrical unknowns in the problem led me to conclude that if the separation in $\delta_{SV}$ and $\delta_P$ is larger than can be explained by a 1% velocity error, then it is probably due to a P-wave group velocity pick, and I should depend solely on the Sv-wave. If $\delta_{SV} < \delta_P$ then I should depend on the P-wave velocity. In the case where $\delta_{SV}$ and $\delta_P$ are close and $\delta_{SV} > \delta_P$ then I will take the average.
of the results. If I apply this methodology to the dataset then I get Figure 2.28. The dataset still shows considerable scatter, but there is a general increase in $\delta$ with increasing $\varepsilon$. By setting the linear trend intercept to zero in Figure 2.28 I obtain $\delta = 0.54\varepsilon$, which is within the bounds given by Vernik & Milovac (2011) of $0.4\varepsilon < \delta < 0.8\varepsilon$. These results will be used to model the anisotropy in the Bakken Shales.

![Figure 2.28: The best-determined values for $\delta$. A linear trend has been fit to the data. Source data from Vernik & Nur (1992).](image)

### 2.6 Conclusions

- The Lodgepole and middle Bakken Facies F were assumed isotropic. Facies E was experimentally determined to be isotropic, and Facies C and D were transversely isotropic

- The static Young’s moduli were close to the dynamic moduli for Facies F, E, and the Lodgepole, but errors in the Poisson’s ratios prevent any additional modeling
• An analysis of Bakken shale data from the literature lead to a modification of the key anisotropy parameter $\delta$. There is now a general trend between $\varepsilon$ and $\delta$. 
CHAPTER 3
EFFECTIVE MEDIUM THEORY

The following chapter is a description of the effective medium elastic theories that have been applied in this study. The models covered in this chapter are: Voigt-Reuss-Hill, Hudson’s crack model, Gassmann’s equation, and Backus averaging.

The Voigt-Reuss-Hill (VRH) medium will be applied multiple times for different applications. The model was chosen based on simplicity and commonality. A popular application of this model is to estimate the pure mineral properties in a mixed mineralogy system. Many of the proceeding models either require or are greatly simplified by considering a single grain modulus determined from the VRH medium and the pore space.

Hudson’s crack model has been chosen based on the assumption that anisotropy in the middle Bakken is from oriented microcracks. The stated limit for crack density is $< 0.1$ (Hudson, 1981), and the model requires relatively low aspect ratios, although no formal limit has been established. Cheng (1993) has provided a crack model capable of handling higher crack densities and any aspect ratio, but a comparison of Cheng and Hudson’s formulations with the aspect ratios modeled in this study suggested Hudson’s model was sufficient.

Gassmann’s equation (Gassmann, 1951) will be applied in Chapter 4 to saturate estimated dry rock properties. It may be more accurately described as a poroelastic model since the fluid is considered a viscous body rather than a purely elastic inclusion. The correct application of the model is for low frequency phenomena where the stress perturbation is slow enough to allow the pore pressure to equilibrate. Only the theory is discussed here.

Backus averaging (Backus, 1962) will be used to model the shale properties and determine appropriate kerogen elastic properties. Backus averaging models finely layered media and has been shown to adequately describe the elastic properties of shales. The inputs can be isotropic or transversely isotropic, and the output medium will be transversely isotropic.

3.1 Theory

Many effective medium theories are based on Eshelby’s work (Eshelby, 1957) regarding the effective elastic properties of a body containing inclusions. The idea is to represent mixtures of two or more materials by considering them as simple geometrical shapes and solving for an elastically equivalent medium. The primary assumption is that each inclusion is small in comparison to the
sampling interval. This is also a high frequency method, meaning pore fluid is not allowed to reach pore pressure equilibrium over the time period that a pressure wave is passing through. Eshelby (1957) and Nishizawa (1982) provide full derivations for the generalized effective medium theory. The proper applications for effective medium theory are for modeling laboratory ultrasonic data and most well log sonic data.

A simple expression for effective medium theory from Sayers & Kachanov (1995) is given by:

\[ \varepsilon_{ij} = (S^0_{ijkl} + \Delta S_{ijkl})\bar{\sigma}_{kl} \]  

(3.1)

Where \( \varepsilon_{ij} \) are the average strains, \( S^0_{ijkl} \) is the inclusion-free compliance matrix (inverse of the stiffness matrix), \( \Delta S_{ijkl} \) is the change in the macroscopic compliance matrix due to the inclusions, and \( \bar{\sigma}_{kl} \) are the average stresses. The matrix or pure grain properties will be determined with a Voigt-Reuss-Hill medium. The Voigt-Reuss-Hill medium is an average of the stiffest and softest possible material arrangement and is given by Hill (1965):

\[ M_V = \sum_{i=1}^{N} x_i M_i \]  

(3.2)

\[ \frac{1}{M_R} = \sum_{i=1}^{N} \frac{x_i}{M_i} \]  

(3.3)

\[ M_{VRH} = \frac{M_V + M_R}{2} \]  

(3.4)

, where \( M \) is the modulus of interest and subscripts \( V \), \( R \), and \( VRH \) stand for Voigt, Reuss, and Voigt-Reuss-Hill, respectively. This medium will not be applied until Chapter 4.

The laboratory data will be modeled with Hudson’s crack model. Hudson’s model represents cracks as penny-shaped inclusions aligned along a single plane. The model requires an isotropic background, and in the simplest case the cracks are perfectly aligned in one direction. Here I will give the equations for cracks aligned normal to the 3-axis (horizontal) in the condensed two-index notation (Hudson, 1981; Mavko et al., 1998):

\[ C^{eff}_{ij} = C^{iso}_{ij} + \Delta C^{crack}_{ij} \]  

(3.5)

The superscript \( eff \) represents the stiffness matrix of the medium containing cracks, \( iso \) is the isotropic background stiffness, and \( crack \) is the first and second order corrections from Hudson (1981).
for the added compliance of cracks. Below are the effective stiffnesses in the condensed two-index notation described in Chapter 2 (Hudson, 1981; Mavko et al., 1998):

\[ C_{11}^{\text{eff}} = C_{11} - \frac{C_{12}^2}{C_{44}} \varepsilon_{cr} U_3 + \frac{q}{15} \frac{C_{12}^2}{C_{11}} (\varepsilon_{cr} U_3)^2 \tag{3.6} \]

\[ C_{13}^{\text{eff}} = C_{12} - \frac{C_{12} C_{11}}{C_{44}} \varepsilon_{cr} U_3 + \frac{q}{15} C_{12} (\varepsilon_{cr} U_3)^2 \]

\[ C_{33}^{\text{eff}} = C_{11} - \frac{C_{12}^2}{C_{44}} \varepsilon_{cr} U_3 + \frac{q}{15} C_{11} (\varepsilon_{cr} U_3)^2 \]

\[ C_{44}^{\text{eff}} = C_{44} - C_{44} \varepsilon_{cr} U_1 + \frac{2}{15} \frac{C_{44}(3C_{11} + 2C_{44})}{C_{11}} (\varepsilon_{cr} U_1)^2 \]

\[ C_{66}^{\text{eff}} = C_{44} \]

\[ q = 15 \frac{C_{12}^2}{C_{44}} + 28 \frac{C_{12}}{C_{44}} + 28 \]

\[ U_1 = \frac{16 C_{11}}{3(C_{12} + 2C_{11})} \frac{1}{1 + M} \]

\[ U_3 = \frac{4 C_{11}}{3(C_{12} + C_{44})} \frac{1}{1 + k} \]

\[ M = \frac{4 C_{44}^{\text{inc}}}{\pi \alpha C_{44}} \frac{C_{11}}{(C_{12} + 2C_{11})} \]

\[ k = \frac{(C_{12}^{\text{inc}} + 2C_{44}^{\text{inc}})C_{11}}{\pi \alpha C_{44}(C_{12} + C_{44})} \]

The superscript \textit{inc} denotes the inclusion properties. Inclusion properties of dry cracks will equal zero causing \( k \) and \( M \) to equal zero. In this case the aspect ratio (\( \alpha \)) and porosity may be left unspecified, but a crack density must be stated.

The crack density (\( \varepsilon_{cr} \)) is given in terms of the crack porosity (\( \phi \)) and aspect ratio (\( \alpha \)) by:

\[ \varepsilon_{cr} = \frac{3 \phi}{4 \pi \alpha} \tag{3.7} \]
The closure stress ($\sigma_{cl}$) is given by (Mavko et al., 1998):

$$\sigma_{cl} = \frac{\pi}{2(1-\nu)} \alpha \mu$$

(3.8)

Where ($\nu$) and ($\mu$) are the Poisson’s ratio and shear modulus of the isotropic matrix. The closure stress is important when applying Hudson’s model to limit the distribution of aspect ratios. As stress is increased the amount of aspect ratios that remain open will decrease. The aspect ratio will not be required for Hudson’s model unless the rock is fluid saturated or to calculate the crack porosity.

In Chapter 4 Gassmann’s equation will be employed to fluid saturate estimated dry rock properties. Gassmann’s equation models the low frequency response of poroelastic media where there is enough time for the wave-induced pore pressure to come to equilibrium within the pore space. The equation can be efficiently described with the full, non-condensed notation as (Gassmann, 1951; Mavko & Bandopadhyay, 2009):

$$C_{ijkl}^{sat} = C_{ijkl}^{dry} + \frac{(K_m \delta_{ij} - C_{ijaa}^{dry} / 3)(K_m \delta_{kl} - C_{\beta \beta kl}^{dry} / 3)}{(K_m / K_{fl}) \phi (K_m - K_{fl}) + (K_m - C_{ppqq}^{dry} / 9)}$$

(3.9)

where

$$\delta_{ij} = \begin{cases} 
1, \text{ for } i = j \\
0, \text{ for } i \neq j 
\end{cases}$$

$K_{fl}$ is the fluid bulk modulus and $K_m$ is the mineral bulk modulus estimated with the VRH model and mineralogy data acquired by X-ray diffraction (XRD) and quantitative evaluation of minerals by scanning electron microscopy (QEMSCAN). A repeated index implies a sum over 1-3 i.e. $C_{ijaa} = C_{i11} + C_{i22} + C_{i33}$. As with the isotropic Gassmann’s equation, the shear stiffnesses are unaffected by fluid saturation.

Backus averaging will be applied to the data from Vernik & Liu (1997), which includes the subset of Bakken Shale data reported in Vernik & Nur (1992). The purpose is to acquire kerogen properties for later application. Backus averaging models a finely layered medium made of isotropic or transversely isotropic material. The constitutive equations for the Backus model are given in Backus notation by (Backus, 1962):

$$C_{11}^* = \langle C_{11} - C_{13}^2 C_{33}^{-1} \rangle + \langle C_{33}^{-1} \rangle^{-1} \langle C_{13} C_{33}^{-1} \rangle^2$$

(3.10)

$$C_{12}^* = \langle C_{12} - C_{13}^2 C_{33}^{-1} \rangle + \langle C_{33}^{-1} \rangle^{-1} \langle C_{13} C_{33}^{-1} \rangle^2$$
\[
C_{33}^* = \langle C_{33}^{-1} \rangle^{-1}
\]

\[
C_{13}^* = \langle C_{33}^{-1} \rangle^{-1} \langle C_{13} C_{33}^{-1} \rangle
\]

\[
C_{44}^* = \langle C_{44}^{-1} \rangle^{-1}
\]

\[
C_{66}^* = (C_{66})
\]

, where brackets indicate the volumetrically weighted average of each phase \( x_i \),

\[
\langle \ldots \rangle = \sum_{i=1}^{N} x_i \langle \ldots \rangle
\]

This section has introduced the effective mediums that will be used in this study.

### 3.2 Hudson’s Crack Model

In this section I will apply Hudson’s model to Facies D and estimate the crack porosity to determine if microcracks add significantly to the overall storage capacity of the middle Bakken. First I must determine if Hudson’s model is appropriate for the measured anisotropy then I will estimate the crack porosity. Since the laboratory measurements were performed on dry rock samples only the crack density is unique to the data set. Limits on the crack porosity are obtained by bounding the aspect ratios with the closure stress of the material. This analysis was adopted from Vernik (1993) where the author modeled microcracks in the Bakken shales and concluded there was significant storage capacity in the microcracks.

To apply Hudson’s model to the laboratory data, I considered the 90° plane parallel-to-bedding as the isotropic background matrix representing the overall stiffening of the rock frame. Then the P-wave anisotropy parameter \( \varepsilon \) was matched by adjusting the crack density \( \varepsilon_{cr} \). The modeling results are shown in Figure 3.1. The modeled crack density ranged from 0.04-0.05. The \( \delta \) parameter determined by assuming phase velocities is shown in blue. The discrepancy between the Hudson model predictions and the measured data may be due to the transducer length of 6.5\( mm \) that was assumed for all the group velocity measurements. The effective transducer length may be smaller.
than 6.5mm causing the energy propagation distance to decrease. This would lead to a slower P-wave and a lower $\delta$ value. However, Figure 3.2 has the three $\delta$ values with a 1% velocity error considered for $\delta_{gr}$ and the Hudson model is still within the error bars for all confining pressures. In this instance interpreting velocities as group or phase has little significance when the error bars of the measurement are considered.

![Figure 3.1: Anisotropy parameters for Facies D versus the Hudson model. The black points are the measured data with $\delta$ determined by assuming group velocity. The blue points were recovered by assuming phase velocities.](image)

The ability of Hudson’s model to adequately fit the data suggests the dominant cause for anisotropy in Facies D is consistent with horizontal microcracking. In Chapter 2 I stated that Facies E showed negligible anisotropy (1 – 2% P-wave variation). Facies E was the strongest prospect for having intrinsic anisotropy in the middle Bakken due to pervasive laminations and high clay content (10 – 15%). From this data I am concluding that the dominant cause for anisotropy in the middle Bakken interval is horizontal microcracking. Hudson’s crack model is the appropriate effective medium to represent this interval.

To determine the crack porosity I will consider the crack closure stress of Facies D. The matrix properties for this sample have been derived from XRD/QEMSCAN data and the VRH model given by equation 3.4. The determined properties are $E= 87.0 GPa$ and $\nu= 0.24$. Figure 3.3 is plotting
Figure 3.2: The measured $\delta$ values assuming phase (blue) and group (black) velocities and the Hudson model predictions. The error bars are shown for the $\delta_{gr}$ with a 1% error added to the 45° group velocity measurement.
the minimum aspect ratio that will remain open under the stress conditions based on 3.8. This allows me to limit the distribution of aspect ratios between 1 (spherical pore) and the minimum aspect ratio (thin crack). In Figure 3.4 I have taken the crack density to be 0.4 (approximate crack density at in-situ stress conditions) and calculated with equation 3.7 the crack porosity for the range of aspect ratios $10^{-4}$ to $10^{-1}$. The aspect ratios larger than 0.05 should not be considered cracks. The porosity assuming all cracks have an aspect ratio of 0.05 is 0.8%. Since many of the cracks will have a lower aspect ratio, the crack porosity will be lower than this amount and will not contribute significantly to the overall storage capacity. However, the microcracks may enhance reservoir quality by increasing the permeability. Figure 3.5 is showing a permeability measurement in Facies D from a sample taken from the Elm Coulee Field (Sarker, 2010). The differential pressure is equal to the confining pressure minus the pore pressure. As the differential pressure is increased the microcracks in the rock are closing and the permeability is substantially decreased.

### 3.3 Backus Averaging

The purpose for modeling shale properties with Backus averaging is to acquire approximate kerogen properties. These properties will be necessary for the estimation of the mineral bulk modulus in Chapter 4. The model here consists of a two-phase system of background shale and kerogen. In the data set from Vernik & Liu (1997) the Lockatong and Woodford shales have low porosities similar to the Bakken. The Woodford is also time-equivalent to the Bakken, so they have been included in this analysis. The background shale properties in GPa have been estimated by plotting each property versus the kerogen content and extrapolating the values to zero kerogen. The retrieved properties are:

$$
C_{11} = 86\text{GPa}, \ C_{12} = 36\text{GPa}, \ C_{33} = 76\text{GPa}, \ C_{44} = 25\text{GPa}, \ C_{13} = 27\text{GPa}, \ C_{66} = 32\text{GPa}
$$

and the best-fit kerogen values require an isotropic material with stiffnesses:

$$
C_{11} = C_{33} = 9.4\text{GPa}, \ C_{12} = C_{13} = 2.8\text{GPa}, \ C_{44} = C_{66} = 3.3\text{GPa}
$$

In terms of the bulk modulus ($K$) and shear modulus ($G$) the properties are $K = 6.0\text{GPa}$ and $G = 3.3\text{GPa}$.

The match to the $C_{33}$, $C_{44}$, and $C_{13}$ stiffness coefficients is shown in Figure 3.6. $C_{13}$ has been adjusted according to the anisotropy parameters determined in Section 2.5 (for the Bakken Shale only). Figure 3.7 has the corrected $C_{13}$ versus the original $C_{13}$. There is an overall increase in $C_{13}$, but the change is not sufficiently large to warrant different kerogen properties.

55
Figure 3.3: The minimum pore aspect ratio for Facies D versus the closure stress.
Figure 3.4: The porosity derived from equation 3.7 plotted against the assumed aspect ratios. The higher aspect ratios >0.05 should not be considered since they will be part of the rock frame.
Figure 3.5: Directional permeability measurements in Facies D at Elm Coulee Field. As differential pressure is increased the permeability substantially decreases. From (Sarker, 2010).

Figure 3.8 shows the $C_{11}$ and $C_{66}$ stiffness coefficients matched with the standard Backus averaging equations and a VRH medium. The Backus averaging equation for $C_{11}$ is nearly equivalent to a simple Voigt medium with modulus $M = C_{11}$ from equation 3.2 given in Backus notation by $C_{11}^* = \langle C_{11} \rangle$, and for $C_{66}$ Backus averaging is exactly a Voigt medium. Figure 3.8 shows that the Backus prediction does not adequately match the stiffnesses, and the VRH medium is more appropriate for estimating the parallel-to-bedding stiffness coefficients.

In this section Backus averaging has been applied to shale data with varying organic content to determine appropriate kerogen properties. The modeling results show for kerogen $K = 6.0 GPa$ and $G = 3.3 GPa$.

### 3.4 Conclusions

- The laboratory data suggests the anisotropy in the middle Bakken is consistent with bedding-parallel microcracks

- The derived range of crack porosity for Facies D suggests cracks do not add significantly to the storage capacity, but may still enhance reservoir quality by increasing permeability (Sarker, 2010)
Figure 3.6: The Backus averaging match to the $C_{33}$, $C_{44}$, and $C_{13}$ stiffness coefficients.
Figure 3.7: The $C_{13}$ stiffness coefficient before and after the corrections from Section 2.5. The change in stiffness does not warrant an adjustment to the kerogen properties.
Figure 3.8: The parallel-to-bedding stiffness coefficients modeled with the standard Backus averaging and a VRH medium. The VRH medium accurately predicts the data. Data from (Vernik & Liu, 1997).
• Backus averaging has been performed on a data subset from Vernik & Liu (1997) and the best-fit Kerogen properties are $K = 6.0 \text{GPa}$ and $G = 3.3 \text{GPa}$
CHAPTER 4
HYDRAULIC FRACTURE CONTAINMENT

The following chapter outlines a primary application of this work. Production in the Bakken is dependent on horizontal wells and hydrofracture stimulation. Identification of fracture barriers is important for modeling hydrofracture growth and planning production strategies. The following chapter will cover: the minimum horizontal stress equation, obtaining the formation stiffness tensor with well log and laboratory data, in-situ stress analysis, and fracture growth.

4.1 Minimum Horizontal Stress Equation

The minimum horizontal stress is a crucial parameter for any hydrofracture model. In this study I assume uniaxial strain i.e. the overburden stress is the only force acting on the system and there is no lateral strain. If the uniaxial strain assumption is valid then equation 4.1 will accurately predict the minimum horizontal stress. The possible fallacies in the uniaxial strain model have been addressed by Katahara (2009). Laboratory measurements have shown a build-up of lateral strain energy due to uniaxial compression upon initial loading and sediment compaction (Karig & Hou, 1992). This will cause a higher horizontal stress than is predicted by equation 4.1. As discussed in Chapter 1, Katahara argues there may be mechanisms that minimize the strain energy in the system. These mechanisms can over time counteract the non-elastic deformation that causes inaccuracies in equation 4.1.

To correct for the non-elastic deformation would require an experiment that takes sediments through the burial history of the formation while replicating the in-situ processes such as hydrocarbon generation, precipitation/dissolution, and temperature variation. Perhaps in the future there will be advancements in theory or laboratory procedures to allow us to complete this work, but as of now the practical limitations are too great. Before we concern ourselves with the errors of the uniaxial strain assumption we must accurately apply this model, and this study has recognized a host of misconceptions that have propagated throughout the oil industry. A primary misconception is related to the three measures of anisotropy \( \varepsilon, \gamma, \) and \( \delta \), which have varying effects on the horizontal stress. The least understood parameter \( \delta \) has the strongest and most coherent effect, yet the industry attempts to correlate the other anisotropy parameters with changes in horizontal stress.

The uniaxial strain approximation for minimum horizontal stress is given by:

\[
(\sigma_h - \alpha_h \sigma_p) = K_0(\sigma_v - \alpha_v \sigma_p)
\]  

(4.1)
where $\sigma_h$ is the total minimum horizontal stress, $\sigma_v$ is the total vertical stress, $\sigma_p$ is the pore stress (pressure), $\alpha_h$ is the horizontal Biot’s coefficient, $\alpha_v$ is the vertical Biot’s coefficient, and $K_0$ is the stress coupling coefficient. $K_0$ can be simply described as the horizontal to vertical stress ratio. For a vertically transversely isotropic (VTI) system where the elastic properties are symmetric about the vertical axis, the subscripts $h$ and $v$ may be replaced with 1 and 3, respectively. In the case of isotropy there is only a single Biot’s coefficient and $K_0 = \nu/(1 - \nu)$. If the medium is transversely isotropic $K_0$ is given by:

$$K_0 = \frac{C_{13}}{C_{33}} = \frac{E_1}{E_3} \left( \frac{\nu_{31}}{1 - \nu_{12}} \right)$$ (4.2)

The $K_0$ as a function of Young’s moduli and Poisson’s ratios is commonly reported as

$$\frac{E_h}{E_v} \left( \frac{\nu_v}{1 - \nu_h} \right)$$ (4.3)

Leading to the misconception that a simple substitution of $V_{P0}$ and $V_{S0}$ into the isotropic equations will yield the vertical Young’s modulus and Poisson’s ratio. Refer to Section 2.3 for a complete explanation of the Young’s moduli and Poisson’s ratios. Figures 4.1 and 4.2 have a comparison of the false versus real terms using the data from Facies D. $E_1$ and $\nu_{12}$ have minor errors, but $\nu_{31}$ and $E_3$ have large errors. Small errors in elastic constants may also lead to much larger errors in the Thomsen anisotropy parameters.

An approximate solution for $K_0$ is given by (Thomsen, 1986):

$$K_0 \approx 1 - 2\frac{V_{S0}^2}{V_{P0}^2} + \delta$$ (4.4)

To test the accuracy of this approximation I will consider the shale data set from Vernik & Liu (1997). Figure 4.3 shows the Thomsen approximation versus the true value for $K_0$. This dataset covers a wide range of shale formations with varying organic content and porosity. The approximation is accurate over the entire dataset.

Since the isotropic $K_0 = \nu/(1 - \nu) = 1 - 2\frac{V_{S0}^2}{V_{P0}^2}$ I can rewrite equation 4.4 as:

$$K_0^{anis} \approx K_0^{iso} + \delta$$ (4.5)

Equation 4.5 should be considered whenever a horizontal stress profile is created. The laboratory derived measurements of $\delta$ have significant errors, but we can determine reasonable limits on $\delta$ and rule out erroneous models.
Figure 4.1: The Young’s moduli for Facies D compared with the false Young’s moduli calculated with equations 2.14 and 2.15.
Figure 4.2: The Poisson’s ratios $\nu_{31}$ and $\nu_{12}$ for Facies D compared with the false Poisson’s ratios calculated with equations 2.16 and 2.17.
Figure 4.3: The stress coupling coefficient $K_0$ versus the Thomsen approximation for $K_0$. The dataset is taken from Vernik & Liu (1997).
I will explain the Biot’s coefficients in terms of the constitutive equations. Under the assumptions of homogeneity and linear elasticity, the constitutive relationship for stress ($\sigma_{ij}$) and strain ($\varepsilon_{kl}$) of a poroelastic medium can be expressed with Hooke’s law as (Biot, 1955; Thompson & Willis, 1991):

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} + \alpha_{ij} p$$  \hspace{1cm} (4.6)

Here $C_{ijkl}$ is the stiffness tensor, $\alpha_{ij}$ is Biot’s coefficient tensor, and $p$ is the pore pressure. I am providing the full non-condensed form here for conciseness in equation 4.8. I will revert back to the condensed notation once I am finished describing Biot’s coefficients. The stiffness coefficients can be determined from velocities that were discussed in Chapter 2. The effective stress may be separated from equation 4.6 and written as:

$$\sigma_{ij}^{\text{eff}} = \sigma_{ij} - \alpha_{ij} p$$  \hspace{1cm} (4.7)

From equation 4.7, the Biot’s coefficients will determine the effect pore pressure will have on the effective stress. Biot’s coefficients can be expressed through the dry rock stiffness tensor and the solid material bulk modulus $K_m$ (Biot, 1962; Gassmann, 1951):

$$\alpha_{ij} = \delta_{ij} - \frac{C_{ijkl}^{\text{dry}}}{3K_m}$$  \hspace{1cm} (4.8)

, where $\delta_{ij}$ is the Kronecker delta defined as

$$\delta_{ij} = \begin{cases} 
1, & \text{for } i = j \\
0, & \text{for } i \neq j 
\end{cases}$$

The repeated index in the stiffness tensor represents a summation over the indices 1 – 3 i.e. $C_{ijkl} = C_{ijkl}^{11} + C_{ijkl}^{22} + C_{ijkl}^{33}$. For the TI model that will be considered in this study the non-zero Biot’s coefficients are:

$$\alpha_{11} = 1 - \frac{C_{1111} + C_{1122} + C_{1133}}{3K_m}$$  \hspace{1cm} (4.9)

$$\alpha_{33} = 1 - \frac{2C_{1133} + C_{3333}}{3K_m}$$  \hspace{1cm} (4.10)

The remaining terms in equation 4.1 are the vertical stress and pore pressure. The vertical stress $\sigma_{33}$ is determined by integrating the density log, and the pore pressure will be interpreted from
Mini-Frac tests performed in-situ. This section has explained all the terms required to calculate minimum horizontal stress with the uniaxial strain approximation.

4.2 The ANNIE Approximation

The ANNIE approximation (Schoenberg et al., 1996) is a method for determining TI properties in a vertical wellbore for the purpose of building a horizontal stress profile. In a typical well logging scenario the axis of symmetry will be vertical and the bedding planes of the formation will be horizontal. The recoverable stiffness coefficients in this case from vertical velocities would be $C_{33}$ and $C_{44}$. In Chapter 2 I showed that there are five independent stiffness coefficients for a TI material. In addition to $C_{33}$ and $C_{44}$, if a Stoneley tube wave is measured, then $C_{66}$ can be estimated. That still leaves $C_{12}$, $C_{13}$, and $C_{11}$ undetermined, with $C_{13}$ and two of the three remaining stiffness coefficients required to completely describe the elastic properties.

To determine the remaining coefficients the ANNIE approximation assumes that $\delta$ is small and can be set to zero. More precisely, it states from equation 2.22 (Schoenberg et al., 1996):

$$C_{13} + 2C_{44} - C_{33} = 0 \quad (4.11)$$

From equation 4.11 there is clearly a fallacy in this assumption. If $\delta = 0$ then the isotropic stress profile will equal the anisotropic profile. The next assumption states for most laboratory data (Schoenberg et al., 1996):

$$C_{13} = C_{12} = C_{11} - 2C_{66} \quad (4.12)$$

In a vertical wellbore with a Stoneley wave measurement $C_{33}$, $C_{44}$, and $C_{66}$ are known ($C_{66}$ is estimated from the Stoneley wave). $C_{13}$ is solved from equation 4.11 then $C_{11}$ is determined from equation 4.12. In Figure 4.4 I have plotted these approximations versus the real data for Facies D. Figure 4.5 shows the approximations for the Vernik & Nur (1992) dataset. In both cases the ANNIE approximation works fairly well, although the critical $C_{13}$ stiffness is substantially under-predicted in Facies D. This would lead to a low $K_0$ value from equation 4.2. The assumption that $C_{12} = C_{13}$ is generally respectable for dry shale measurements, but a problem arises since $C_{12}$ will not be known in the wellbore. The critical assumption is actually from equation 4.11.

The two datasets shown in Figure 4.4 and Figure 4.5 are both dry measurements. In Figure 4.6 is an oil-saturated Mancos B shale sample from Sarker (2010). The ANNIE approximation does a poor job of matching the $C_{13}$ stiffness coefficient. One cause of this erroneous approximation may be that
Figure 4.4: Facies D data versus the ANNIE approximation.
Figure 4.5: The Vernik & Nur (1992) dataset versus the ANNIE approximation.
it is based on the publicly available anisotropic laboratory data, which are almost exclusively dry shale measurements. In oil-saturated rocks $C_{12}$ will rarely equal $C_{13}$. Saturation will preferentially change the soft direction perpendicular to bedding more than the stiff direction parallel to bedding. This will cause the $C_{13}$ stiffness to change more than $C_{12}$.

Figure 4.6: The stiffness coefficients versus the ANNIE approximation for a Mancos B shale. On the left is the $C_{13}$ coefficient and on the right is the $C_{11}$ coefficient. Data is from Sarker (2010).

The final analysis I will do on the ANNIE approximation is to determine the $\delta$ parameter from equations 4.4 and 4.5 that matches the $K_0$ value given by the approximation. Schlumberger has provided the $E_h$, $E_v$, $\nu_h$, and $\nu_v$ obtained with some form of the ANNIE approximation. The exact equations are unknown due to local formation variables that are applied e.g. equation 4.12 may become $A \times C_{12} + B = C_{13}$.

Given the Young’s moduli and Poisson’s ratios I can determine $K_0$ independently with equation 4.2. In Figure 4.7 I am showing the $K_0$ value calculated from the properties given, and in Figure 4.8 I have solved for $\delta$ with equation 4.5. Under high stress conditions (70 MPa) when microcracks are presumed closed and the intrinsic anisotropy is measured, the highest $\delta$ value recovered was 0.3. At low stress (5 MPa) the largest $\delta = 0.4$. Given the available laboratory data, the values in Figure 4.8 are unrealistically high. Herein lies the major fallacy with the ANNIE approximation: it assumes a $\delta = 0$, but when the approximation is applied to the horizontal stress calculation, the implied $\delta$ is often very large.

4.3 An Alternative Method

The flaws in the ANNIE approximation warrant a different approach for recovering the TI stiffness tensor. Based on equations 4.5, 4.11 and 4.12 I calculated the stiffness coefficients and anisotropy.
Figure 4.7: The stress coupling coefficient calculated from anisotropic elastic properties provided by Schlumberger.
Figure 4.8: The $\delta$ values calculated from elastic properties given by Schlumberger and equation 4.4. The maximum dry $\delta$ value measured in the lab was 0.4.
parameters. I also calculated $C_{11}$ and $C_{12}$ from the Young's moduli and Poisson's ratios and found the two calculations did not match. The mismatch suggests Schlumberger modified the ANNIE approximation. $C_{11}$ and $C_{12}$ in terms of the mechanical properties are given in equations 4.13 and 4.14 (Tahini & Abousleiman, 2010). $C_{66}$ can be calculated by $C_{66} = (C_{11} - C_{12})/2$. My attempt to recover the stiffness coefficients assumed there were no fitting parameters added to equations 4.11 and 4.12. Two Mini-Frac tests were performed: one in the Upper Bakken Shale and another 1.5m above in the Scallion member of the Lodgepole Formation. Schlumberger may have adjusted fitting parameters to match the Mini-Frac tests, thus altering the stiffness coefficients. I will only consider the $C_{66}$ stiffness coefficient determined from the Stoneley wave since this has not been modified by Schlumberger. The rest of the coefficients I will disregard. I can use equations 4.13 and 4.14 to recover the original $C_{66}$.

$$C_{11} = \frac{E_3(E_1 - E_3\nu_{12}^2)}{(1 + \nu_{31})(E_1 - E_1\nu_{31} - 2E_3\nu_{12}^2)}$$  \hspace{1cm} (4.13)$$

$$C_{12} = \frac{E_3(E_1\nu_{31} + E_3\nu_{12}^2)}{(1 + \nu_{31})(E_1 - E_1\nu_{31} - 2E_3\nu_{12}^2)}$$  \hspace{1cm} (4.14)$$

The available stiffness coefficients from the well log are $C_{33}$, $C_{44}$, and $C_{66}$. However, I will not consider the $C_{33}$ stiffness since I would have to model the high frequency fluid response. If I assume the shear stiffness is independent of saturation, I will have the dry $C_{44}$, $C_{66}$, and $\gamma$. The remaining parameters will be calculated with laboratory-derived empirical equations from this study and the Vernik & Nur (1992) dataset. Then I will saturate the dry rock properties with Gassmann’s equation. The advantages to this method are:

- Continuous Biot’s coefficients from dry rock properties

- Gassman’s equation does not require geometrical information about the pore space (Hudson’s model requires an aspect ratio for fluids with a non-zero bulk modulus)

- The relaxed fluid response will have greater relevance for mechanical applications than the high frequency models (albeit not a true static model)
The disadvantages are:

- Blocky mineralogy and grain density models that may cause spiky Biot’s coefficients and porosity particularly along formation boundaries

- Unknown errors in the empirical equations due to laboratory conditions i.e. core damage, preferential sampling (intact samples), limited sample mineralogy, etc.

Gassmann’s equation will model the undrained moduli for low frequency wave propagation. The static moduli necessary for mechanical applications are drained (the fluid may escape the pore space). In addition the static strain amplitude can be orders of magnitude higher. There will be unknown errors based on these differences, but the relaxed fluid response should offer a closer representation of the mechanical properties.

The rock properties required by Gassmann’s equation are: the dry TI stiffness tensor, the mineral bulk modulus, and the porosity. The available stiffness coefficients $C_{44}$ and $C_{66}$ are mildly altered by fluid saturation through the density effect and potentially a small amount of squirt flow in compliant pores (Mukerji & Mavko, 1994), but this will cause only minor error. They will be considered dry rock properties. The anisotropy parameter $\gamma$ can be calculated with these terms and will be independent of the density effect. The laboratory measurements on dry samples provide trends amongst the anisotropy parameters for both the shales and the middle Bakken.

The anisotropy in Facies D was higher than Facies C for most of the experiment, but at high stresses the $\gamma - \varepsilon$ trend coincides (Figure 4.9). A single trend will be applied to the middle Bakken and the upper and lower bounding Lodgepole and Three Forks Formations based on the $\gamma - \varepsilon$ trend from Facies C and the $\varepsilon - \delta$ trend from Facies D (Figure 4.10) assuming a group velocity was measured. These formations have been grouped together based on the assumption that none of them contain intrinsic anisotropy.

The correlations have been put into power functions to ensure when $\gamma = 0$ the other anisotropy parameters will equal zero. The inherent assumption here is that the $\gamma$ values are all positive. This assumption is true within the unit of interest. This does not mean the final anisotropy parameters must all be positive, only the dry anisotropy values. The relationships for the middle Bakken, Lodgepole, and Three Forks are as follows:

$$
\varepsilon = 2.04\gamma^{1.09}
$$

(4.15)
Figure 4.9: The $\gamma - \varepsilon$ trend from Facies C and D. The trends coincide at low stress for Facies C and high stress for Facies D. The trend from Facies C has been considered since the in-situ $\gamma$ is closer to these values.
Figure 4.10: The $\varepsilon - \delta$ trend from Facies D. This trend will be applied to the middle Bakken, Lodgepole, and Three Forks. The grouping of these formations is based on the assumption that they do not contain intrinsic anisotropy.
\[ \delta = 6.14 \varepsilon^{1.87} \]  

(4.16)

The corresponding relationships for the shales are given by:

\[ \varepsilon = 0.71 \gamma^{0.69} \]  

(4.17)

\[ \delta = 0.43 \varepsilon^{0.82} \]  

(4.18)

Equations 4.17 and Figure 4.12 are based on the Vernik & Nur (1992) dataset and the modifications from Chapter 2. The data trends are shown in Figures 4.11 and 4.12.

The dry rock anisotropy parameters are shown in Figure 4.13. The middle Bakken interval 2044-2058m has markedly less anisotropy than the values measured in the laboratory (Figures 2.16-2.18). The lowest \( \gamma \) value measured in Facies C was approximately 0.04 and none of the middle Bakken unit has anisotropy this high (note: the anisotropy in the middle Bakken has thus been extrapolated from the laboratory data and may contain errors). This strongly suggests stress relief perpetuated microcracking in the core samples.
The anisotropy parameters alone do not provide enough information to calculate the stiffness tensor. \( C_{44} \) and \( C_{66} \) are still the only known stiffness coefficients and between \( \varepsilon \) and \( \delta \) there are three unknowns: \( C_{11}, C_{33}, \) and \( C_{13} \). The next step and possibly the most error prone, requires a correlation between \( C_{66} \) and \( C_{11} \). The changing mineralogy in the Bakken will cause this correlation to falter. Each Bakken facies should undergo multiple laboratory tests to capture the effects of changing mineralogy. The highly laminated Facies C and E may be strongly affected by an increase in clay content, whereas the clay may not have a large impact on the poorly sorted and bioturbated Facies B. With the limited laboratory data available I will create a block model for each facies. Since no measurements were taken in Facies A and B I will group them with Facies F. The depositional water depth and mineralogy is similar in these three facies. Since Facies C and D were the only samples with measured anisotropy, I will consider the values from the other samples to be the horizontal stiffnesses (\( C_{11} \) and \( C_{66} \)). The conversions are as follows:

\[
C_{11}^{A,B,F} = 5.4C_{66} - 47.3
\]

\[
C_{11}^{C} = 3.6C_{66} - 25.1
\]
Figure 4.13: The dry rock anisotropy parameters calculated with equations 4.15-4.18. The in-situ anisotropy is less than the laboratory measured anisotropy.
\[ C_{11}^D = 3.5C_{66} - 15.4 \]

\[ C_{11}^E = 3.0C_{66} - 9.3 \]

\[ C_{11}^{LP} = 2.8C_{66} \]

\[ C_{11}^{Shales} = 2.8C_{66} + 1.2 \quad (4.19) \]

The superscripts denote the applicable unit of rock. The Lodgepole has only a multiplier because the laboratory experiment hardly changed properties. The trend was essentially a vertical line, and any small change in \( C_{66} \) lead to an abnormal change in \( C_{11} \). Once \( C_{11} \) is determined the remaining coefficients can be calculated with the anisotropy parameter equations 2.12-2.14. The trends represent the changes in stiffness due to the effective stress. The unknown mineralogy variation will cause errors in these trends.

The TI Biot’s coefficients are attainable from equations 4.9 and 4.10 once the mineral bulk modulus is found. XRD and QEMSCAN data is shown in Table 4.1 along with the calculated mineral bulk moduli using the VRH effective medium. The pure mineral properties were taken from Mavko et al. (1998). Instead of the Illite properties given by Mavko (\( K = 25\text{GPa} \)), a bulk modulus of 10\( \text{GPa} \) was considered. The new value is based on hydrated clay measurements performed by Bathija (2009) that showed a significant decrease in bulk modulus with clay hydration. The mineral density is also included since that will be necessary to determine accurate porosities in the well log.

The mineral bulk moduli and densities are applied in a blocky fashion to the well log. The nearest value is taken for calculations; there is no interpolation scheme applied. A modeling process to predict mineralogy would allow continuous mineral bulk modulus estimation. This would provide more accurate Biot’s coefficients and fluid substitution results.

In the Bakken shales the mineral bulk modulus and porosity are heavily dependent on the kerogen content. The Backus averaging model in Chapter 3 provided the kerogen properties. For the background shale bulk modulus I have taken average XRD results for the Upper and Lower Bakken Shales provided through personal communication by Kene Mba and calculated a non-kerogen mineral modulus with the VRH medium. Table 4.2 has the normalized mineral volumes and the calculated bulk modulus. The effective modulus will be determined with the VRH medium from the two-phase system of kerogen and non-kerogen properties.
Table 4.1: Mineral bulk modulus, mineralogy, and density

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<th>Dol</th>
<th>Cal</th>
<th>Feld</th>
<th>Illite</th>
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<td>2.682</td>
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<tr>
<td>2044.5</td>
<td>F</td>
<td>36</td>
<td>45</td>
<td>1</td>
<td>8</td>
<td>7</td>
<td>97</td>
<td>57.0</td>
<td>2.707</td>
</tr>
<tr>
<td>2045.9</td>
<td>E</td>
<td>40</td>
<td>8</td>
<td>42</td>
<td>7</td>
<td>2</td>
<td>99</td>
<td>50.3</td>
<td>2.683</td>
</tr>
<tr>
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<td>E</td>
<td>47</td>
<td>25</td>
<td>1</td>
<td>13</td>
<td>10</td>
<td>96</td>
<td>46.9</td>
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<tr>
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<td>E</td>
<td>49</td>
<td>24</td>
<td>1</td>
<td>13</td>
<td>10</td>
<td>97</td>
<td>47.7</td>
<td>2.676</td>
</tr>
<tr>
<td>2048.46</td>
<td>E</td>
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<td>97</td>
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<td>2.684</td>
</tr>
<tr>
<td>2049.11</td>
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<td>37</td>
<td>9</td>
<td>46</td>
<td>6</td>
<td>1</td>
<td>99</td>
<td>55.8</td>
<td>2.689</td>
</tr>
<tr>
<td>2049.6</td>
<td>C</td>
<td>40</td>
<td>17</td>
<td>26</td>
<td>10</td>
<td>4</td>
<td>97</td>
<td>52.1</td>
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<td>58</td>
<td>13</td>
<td>1</td>
<td>15</td>
<td>8</td>
<td>96</td>
<td>42.3</td>
<td>2.656</td>
</tr>
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<td>2.680</td>
</tr>
<tr>
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<td>1</td>
<td>13</td>
<td>11</td>
<td>97</td>
<td>47.9</td>
<td>2.676</td>
</tr>
<tr>
<td>2057.24</td>
<td>B</td>
<td>42</td>
<td>24</td>
<td>8</td>
<td>13</td>
<td>10</td>
<td>97</td>
<td>48.9</td>
<td>2.677</td>
</tr>
<tr>
<td>2058.21</td>
<td>A</td>
<td>39</td>
<td>21</td>
<td>16</td>
<td>10</td>
<td>10</td>
<td>97</td>
<td>49.6</td>
<td>2.677</td>
</tr>
</tbody>
</table>

Table 4.2: Non-kerogen mineral volumes and bulk modulus (Bakken shales)

<table>
<thead>
<tr>
<th>Shale</th>
<th>Quartz</th>
<th>Calcite</th>
<th>Dolomite</th>
<th>Feldspar</th>
<th>Pyrite</th>
<th>Clay</th>
<th>$K_{VRH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper</td>
<td>0.35</td>
<td>0.02</td>
<td>0.09</td>
<td>0.04</td>
<td>0.03</td>
<td>0.46</td>
<td>25.9</td>
</tr>
<tr>
<td>Lower</td>
<td>0.52</td>
<td>0.01</td>
<td>0.05</td>
<td>0.05</td>
<td>0.02</td>
<td>0.35</td>
<td>26.9</td>
</tr>
</tbody>
</table>

The same approach was taken in the False Bakken interval, but with XRD results from the False Bakken microfacies (Stroud, 2011). The results are summarized in Table 4.3. The average bulk modulus of 49.95 GPa will be applied to the well log.

Table 4.3: Non-kerogen mineral volumes and bulk modulus (False Bakken)

<table>
<thead>
<tr>
<th>Quartz</th>
<th>Calcite</th>
<th>Dolomite</th>
<th>Feldspar</th>
<th>Pyrite</th>
<th>Clay</th>
<th>$K_{VRH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.41</td>
<td>0.06</td>
<td>0.14</td>
<td>0.04</td>
<td>0.19</td>
<td>0.15</td>
<td>48.3</td>
</tr>
<tr>
<td>0.19</td>
<td>0.61</td>
<td>0.09</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
<td>58.9</td>
</tr>
<tr>
<td>0.11</td>
<td>0.78</td>
<td>0.03</td>
<td>0.01</td>
<td>0.04</td>
<td>0.03</td>
<td>64.0</td>
</tr>
<tr>
<td>0.24</td>
<td>0.38</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.18</td>
<td>44.0</td>
</tr>
<tr>
<td>0.37</td>
<td>0.18</td>
<td>0.04</td>
<td>0.11</td>
<td>0.05</td>
<td>0.25</td>
<td>34.7</td>
</tr>
</tbody>
</table>

The kerogen content can be extracted from the density log through the simple density correlation shown in Figure 4.14. There will be errors introduced due to the porosity in the formation, but as shown in Figure 4.15, RockEval data taken on five samples suggests the density correlation is reasonably accurate. The RockEval recovers the weight percent kerogen. I have multiplied by two to approximate the volume of kerogen. This is generally a good approximation since the density of kerogen is about half the density of many of the most abundant minerals found in nature (Passey et al., 2010).
Figure 4.14: Density correlated with kerogen volume with a linear fit to the data. Data from Vernik & Nur (1992).
Figure 4.15: The kerogen content derived from the density log compared with the RockEval data shown with the blue dots. The correlation is only applied in the shale units. The over-prediction in the Upper Bakken Shale may be due to the high porosity of up to 6%.
The porosity was determined by again considering a two-phase kerogen and non-kerogen system. The kerogen has an average density of 1.3g/cm³ and the non-kerogen part is a mixture of clay, quartz, carbonate, and pyrite with an average density of 2.75g/cm³ (Vernik & Milovac, 2011). The matrix density can be determined by:

\[ \rho_m = \rho_{nk}(1 - K) + \rho_k K \]  

(4.20)

Here the subscripts \( m, nk, \) and \( k \) denote matrix, non-kerogen, and kerogen properties, and \( K \) represents kerogen volume. To avoid negative porosities, a non-kerogen density of 2.79g/cm³ was considered for this well. In the False Bakken a non-kerogen density of 2.85g/cm³ was determined with the mineralogy from Table 4.3. The higher carbonate and pyrite content caused the large density values.

The information described above allows the calculation of Biot’s coefficients along the wellbore. Figure 4.16 has the vertical and horizontal Biot’s coefficients calculated with the dry rock stiffness tensor and equations 4.9 and 4.10.

All the parameters are now available for Gassmann’s equation. Figure 4.17 has \( \delta \) before and after fluid substitution and the value calculated from Schlumberger’s mechanical properties. In Figure 4.18 I am showing \( K_0 \) for the dry and saturated data. The process of saturating the rock with Gassmann’s equation lowers the anisotropy parameters but increases the stress coupling coefficient. From equation 4.4, \( \delta \) will decrease, \( V_{S0} \) will remain the same and \( V_{P0} \) will increase. The increase of \( V_{P0} \) must dominate the fluid saturation signal.

This section has outlined a method of determining the in-situ TI stiffness tensor in the Bakken Formation. The dry rock stiffness tensor is estimated with the in-situ shear waves then the formation is fluid saturated with Gassmann’s equation. In the case of orthorhombic symmetry the two vertically propagating shear waves may be used to estimate the remaining stiffness coefficients.

### 4.4 In-situ Stress Analysis

One method for determining in-situ stress properties is to perform a Mini-Frac test. A Mini-Frac test schematic is shown in Figure 4.19 (Nolte et al., 1997). The formation is first isolated from the surrounding rock, then fluid is pumped into the isolated section increasing the pressure (bottomhole pressure) until the formation is fractured. The pressure at which this occurs is referred to as the breakdown pressure. The well is then sealed, allowing no fluid transfer with the surface. The fluid in the borehole will leak off into the formation and the bottomhole pressure will decline. The lowering
Figure 4.16: The horizontal (black) and vertical (blue) Biot's coefficients.
Figure 4.17: A comparison of the dry, saturated, and Schlumberger provided $\delta$’s. The Schlumberger interpretation is in strong disagreement with the model I have suggested. This may be due to the assumption of Biot’s coefficients equal to unity. The anisotropy in the shales will still affect the minimum horizontal stress, but the magnitude is much smaller than Schlumberger’s model.
Figure 4.18: $K_0$ dry (black) and saturated (blue). The saturation process decreases the anisotropy parameters, but increases the stress coupling coefficient. This may be another source of error and misunderstanding in the current methodology. Low frequency models should be employed to closer emulate static behavior.
of fluid pressure will allow the fractures in the formation to close, and when the fractures close at the borehole there will be a slight kink in the pressure decline curve. The bottomhole pressure at which this kink occurs is interpreted as the closure pressure of the formation at that depth. If pressure decline is allowed to continue until the bottomhole pressure stabilizes, this pressure is interpreted as the reservoir pressure. In practice the pressure decline curve is extrapolated to determine the reservoir pressure.

Figure 4.19: Schematic of a Mini-Frac test. The maximum bottomhole pressure is termed the breakdown pressure, the kink in the section labeled ‘Pressure Decline’ occurs at the closure pressure, and when the pressure stabilizes, this may be inferred as the reservoir pressure. Image from Nolte et al. (1997).

Mini-Frac tests have been performed at depths of 2040m and 2041.5m in the Scallion member of the Lodgepole Formation and the Upper Bakken Shale. The complete tests are shown in Figures 4.20 (Scallion) and 4.21 (Upper Bakken Shale). Analysis will be based on the blue curves, which are the bottomhole pressures.

The orange triangles after each cycle are the interpreted closure stress and reservoir pressure. The maximum pressure in each chart is the breakdown pressure (fracture initiation), and each subsequent cycle maxima is the reopening pressure. Since the tensile strength of the fractured formation will equal zero, the difference in breakdown pressure and reopening pressure will equal the tensile strength of the formation (Jaeger et al., 2007).

The breakdown pressure will also provide an estimate of the maximum horizontal stress. There are three rock models to consider when determining maximum horizontal stress from breakdown
Figure 4.20: The Mini-Frac test performed at a depth of 2040m in the Scallion member of the Lodgepole Formation. The blue curve is the bottomhole pressure in units of kilopascals.
Figure 4.21: The Mini-Frac test performed at a depth of 2041.5m in the Upper Bakken Shale. The blue curve is the bottomhole pressure in units of kilopascals.
pressure: non-porous and non-permeable, porous and non-permeable, or porous and permeable. The permeability is with respect to the fracturing fluid. A non-porous rock would suggest a Biot’s coefficient of zero (no pore pressure effect); since the Biot’s coefficients are all above zero the first model will not be discussed. The equations for the latter two models are given by Jaeger et al. (2007); Schmitt & Zoback (1989):

\[
\begin{align*}
\text{(non-permeable)} & \quad P_b = 3\sigma_h - \sigma_H + T_0 - \alpha P_p \\
\text{(permeable)} & \quad P_b = \frac{3\sigma_h - \sigma_H + T_0 - 2\eta P_p}{1 + \alpha - 2\eta}
\end{align*}
\]

where

\[\eta = \frac{\alpha (1 - 2\nu)}{2(1 - \nu)}\]

Here \(P_b, \sigma_h, \sigma_H, T_0,\) and \(P_p\) are the breakdown pressure, minimum horizontal stress, maximum horizontal stress, tensile strength, and pore pressure. The \(\alpha\) and \(\nu\) are the isotropic Biot’s coefficient and Poisson’s ratio. Based on Figure 4.13, the Scallion member is nearly isotropic. There should not be significant errors introduced by applying the isotropic formulations in equations 4.21 and 4.22. The Upper Bakken Shale is transversely isotropic so the above isotropic formulations may cause significant errors. In both cases the average Biot’s coefficients and Poisson’s ratios have been taken from the well log and laboratory derived elastic properties. The results are summarized in Table 4.4. The \(\sigma_H^{np}\) and \(\sigma_H^p\) denote the maximum horizontal stress determined assuming a non-permeable and permeable formation, respectively.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>(\sigma_h) (MPa)</th>
<th>(P_p) (MPa)</th>
<th>(T_0) (MPa)</th>
<th>(\alpha)</th>
<th>(\sigma_H^{np}) (MPa)</th>
<th>(\nu)</th>
<th>(\eta)</th>
<th>(\sigma_H^p) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2040</td>
<td>28.6</td>
<td>27.0</td>
<td>5.0</td>
<td>.53</td>
<td>42.5</td>
<td>.30</td>
<td>.15</td>
<td>42.7</td>
</tr>
<tr>
<td>2041.5</td>
<td>30.0</td>
<td>29.0</td>
<td>21.0</td>
<td>.45</td>
<td>39.4</td>
<td>.27</td>
<td>.20</td>
<td>48.4</td>
</tr>
</tbody>
</table>

The tensile strength in the Upper Bakken Shale is larger than reported values in the Woodford and other Devonian shales. Refer to Figure 4.22 and Table 4.5 for literature tensile strength values (Chong et al., 1984; Sierra et al., 2010). The vertical tensile strengths from literature range from 11-14MPa (1525-2030psi). The lack of any applied pressure and the in-situ temperature may have a large impact on the determined strength values.
Figure 4.22: Tensile strength for the Devonian age Tennessee Oil Shale versus organic volume. Image from (Chong et al., 1984).

Table 4.5: Woodford Shale tensile strength

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Load Direction (rel. to bedding)</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.81</td>
<td>⊥</td>
<td>12.8</td>
</tr>
<tr>
<td>33.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36.85</td>
<td>⊥</td>
<td>12.7</td>
</tr>
<tr>
<td>36.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>41.36</td>
<td>⊥</td>
<td>11.4</td>
</tr>
<tr>
<td>41.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>44.28</td>
<td>⊥</td>
<td>11.2</td>
</tr>
<tr>
<td>44.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.59</td>
<td>⊥</td>
<td>11.7</td>
</tr>
<tr>
<td>50.59</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The tensile strength in the Scallion member is lower than expected, considering Limestones are one of the strongest rock types found in nature. Mogi (2007) measured UCS values for the Solnhofen Limestone in the range 288-300MPa. Tensile strengths are generally 10% of the UCS strength (Costamagna et al., 2007) suggesting the value should be approximately 30MPa. Natural fractures or the previously fractured Upper Bakken Shale 1.5 meters below the Scallion may have caused the low tensile strength.

Estimates of maximum horizontal stress recovered from breakdown pressures are highly affected by near-borehole issues (Sayers, 2010). An alternative way to calculate the maximum horizontal stress is from the velocities where (Sinha, 2002):

\[ \Delta \sigma_H = \Delta \sigma_h + \frac{\Delta C_{55} - \Delta C_{44}}{\Delta C_{55} - \Delta C_{66}} (\Delta \sigma_v - \Delta \sigma_h) \] (4.23)

Figure 4.23 has the two vertical shear waves in which \( C_{44} \) and \( C_{55} \) may be calculated. Since the velocities are virtually equal, maximum horizontal stress will nearly equal the minimum horizontal stress based on equation 4.23. This formulation assumes that a variance in horizontal stresses would lead to \( C_{44} \neq C_{55} \); an assumption that should be experimentally verified.

The pore pressure results show moderate overpressuring. In the Scallion the pore pressure gradient is 0.58 psi/ft. and it increases to 0.65 psi/ft. in the Upper Bakken Shale. The pore pressure may decrease significantly away from the shales. I will consider a constant value in the shales of 0.65 psi/ft. and 0.55 psi/ft. in the middle Bakken and Lodgepole Formation.

Another important aspect of the in-situ stresses is the maximum horizontal stress azimuth. In a normally stressed environment, open vertical fractures will align with the maximum stress direction. The dipole sonic log will measure shear wave splitting due to vertical fractures, and the fast shear wave will be aligned parallel to the dominant fracture set. Since open fractures will be aligned with the maximum horizontal stress, the azimuth of the fast shear wave provides the maximum horizontal stress direction. Figure 4.23 has the shear waves measured in the Bakken unit and the Lodgepole Formation. There are no significant vertical fractures in the interval of interest (2035-2070m). The fast shear wave azimuth is shown in Figure 4.24. The azimuth in the Bakken interval is sporadic due to the lack of vertical fractures. The areas in the Lodgepole Formation with shear wave separation suggest an azimuth of 30° – 60°. These results are consistent with the inferred fracture direction from Olsen et al. (2009) of 54°.

From the in-situ stress testing I have estimated reservoir pressure and total minimum horizontal stress for the Scallion member and the Upper Bakken Shale. An approximate tensile strength has
Figure 4.23: The fast and slow shear waves. In the main interval of interest from 2035-2070m there is nearly no separation. Above the region of interest in the Lodgepole Formation there is significant separation, suggesting in-situ vertical fractures are open.
Figure 4.24: The fast shear wave polarization azimuth. The azimuth is rotating throughout the Bakken section due to the lack of open vertical fractures. In the Lodgepole Formation the areas with shear wave splitting (shown in Figure 4.23) suggest an azimuth of $30^\circ - 60^\circ$. 
been recovered in the Upper Bakken Shale that is above reported Devonian Shale tensile strengths. I have also estimated the maximum horizontal stress magnitude and azimuth, and the maximum stress azimuth is consistent with reported values.

4.5 Fracture Growth

In this section I will implement the analyses in the previous sections to locate potential fracture barriers. I will only consider the effects from the minimum horizontal stress. The effective and total minimum horizontal stresses can be calculated from equation 4.1 and the analyses from Section 4.3. The results along with the closure stresses determined from the Mini-Frac tests are shown in Figure 4.25. The closure stress will be equal to the total minimum horizontal stress (Jaeger et al., 2007). As a comparison, the reported anisotropic stress profile provided by Schlumberger is also shown. The modeled results here match well with the Mini-Frac results.

Figure 4.26 has the isotropic and anisotropic stress profiles from Schlumberger and the Mini-Frac test results. It appears that the anisotropic stiffness tensor was altered to match the Mini-Frac test. The best fit to Schlumberger’s model was acquired by calculating the effective horizontal stress rather than the total horizontal stress and setting Biot’s coefficients equal to unity. The remaining model misfit could be fully explained by a difference in pore pressure.

This method was only able to match the Mini-Frac test in the Upper Bakken Shale. No realistic anisotropy would explain the test result in the Scallion member of the Lodgepole Formation. The inherent assumption in Schlumberger’s model that Biot’s coefficients are equal to unity has caused a misinterpretation of the stress profile and an over prediction of the anisotropy. Any given Mini-Frac test may be matched by simply altering the anisotropy, but the continuous stress profile will be perturbed. The rock anisotropy will only provide a second order correction due to the low observed δ values; the Biot’s coefficients have a much larger impact on the horizontal stress profile.

Fracture growth will abide by the effective stress law so the effective stress should be considered for fracture propagation (Bruno & Nakagawa, 1991). The effective σh contrast is minor throughout the Bakken interval. A slight dip occurs near the reservoir Facies C and D, but there is not a significant stress contrast between the middle Bakken and the Bakken Shales. The Lodgepole Formation is also shown to be similar to the Bakken Shales. The fracture growth may ultimately depend on the fracture toughness and permeability due to the lack of a significant horizontal stress contrast.
Figure 4.25: The calculated effective and total minimum horizontal stress are shown in black and blue. The total minimum horizontal stress determined by Schlumberger is shown in light blue, and the Mini-Frac test results are the red dots.
Figure 4.26: The isotropic and anisotropic horizontal stress profiles provided by Schlumberger. The alteration of the stiffness coefficients does not allow the prediction of both Mini-Frac tests. As shown in Section 4.3 the change in the isotropic and anisotropic stress profiles suggests a δ value that is unreasonably large.
4.6 Conclusions

- The anisotropic stress coupling coefficient $K_0$ can be approximated by the isotropic $K_0$ plus $\delta$ (Thomsen, 1986)

- The ANNIE approximation may do a fair job predicting dry rock properties, but will poorly predict liquid saturated rocks

- The ANNIE approximation predicts $\delta$ values that are unrealistically high for the middle Bakken and shales

- A methodology for calculating Biot’s coefficients has been outlined that requires laboratory dry rock measurements and an estimation of $C_{66}$ from the Stoneley wave

- An alternative approach has been suggested to calculate minimum horizontal stress in the Bakken that utilizes Gassmann’s equation to closer emanate the static domain

- The in-situ stress analysis provides an estimate of tensile strength in the Upper Bakken Shale that is well above reported Devonian age shale tensile strengths

- The closure stress has been determined from two Mini-Frac tests and shows almost no stress contrast between the Scallion member and the Upper Bakken Shale

- The effective minimum horizontal stress suggests no major stress contrast throughout the Bakken interval

- The apparent adjustment of the stiffness tensor provided by Schlumberger is only able to match a single Mini-Frac test and will not provide an accurate stress profile
In this chapter I have provided concluding remarks and a few ideas for future work.

5.1 Conclusions

The elastic properties were measured for four facies in the middle Bakken and the Scallion member of the upper bounding Lodgepole Formation. Anisotropic velocity measurements were performed on Facies C, D, and E. The results showed negligible anisotropy in Facies E and moderate anisotropy in Facies C and D.

The Bakken Shale dataset from Vernik & Nur (1992) was analyzed and led to a modification of Thomsen’s $\delta$ parameter (Thomsen, 1986). The computed $\delta$ value using the $S_v$-wave showed different results than the $P$-wave, and the propensity for the $P$-wave to have large lateral displacement suggested the group velocity might have been measured for a number of the samples. The modifications to $\delta$ caused there to be a stronger correlation with the $\varepsilon$ anisotropy parameter.

The full stiffness tensor was recovered for Facies D. The estimated crack density determined from matching Hudson’s model to the data was 0.04 at reservoir conditions. By limiting the aspect ratio of the cracks with the closure stress, I determined that the crack porosity was $< 1\%$ and would not add significantly to the overall storage capacity. However, a permeability measurement by Sarker (2010) of Facies D suggests the permeability may be strongly affected by microcracks.

Backus averaging was applied to a subset of data from Vernik & Liu (1997) to determine appropriate kerogen properties. The recovered properties were $K = 6.0 GPa$ and $G = 3.3 GPa$. To match the parallel-to-bedding stiffness coefficients required an alteration to the Backus equations. The new calculation is a Voigt-Reuss-Hill medium with the appropriate stiffnesses. The modification to $\delta$ based on the $S_v$-wave was included in this analysis, but did not significantly impact the results.

An analysis of the ANNIE approximation, a method for determining the TI stiffness tensor with well log data, showed the model will have difficulties estimating saturated rock properties. Fluid saturation will preferentially affect the ‘soft’ plane perpendicular to bedding more than the ‘stiff’ plane parallel to bedding. Many of the available anisotropic laboratory data is from dry rocks and does not show the effects of fluid saturation.

Instead of applying the ANNIE approximation in the well log, I have used the laboratory data to build empirical equations predicting the dry rock properties from the dipole sonic log. Fluid
saturation will have a minor impact on shear wave propagation due to the change in density, but the effect is low enough to neglect and consider the shear wave data as purely dry rock measurements. The remaining elastic constants are determined solely from the shear waves. Over the unit of interest the dipole sonic log showed $C_{44} \approx C_{55}$ suggesting a VTI or isotropic model is sufficient. However, the same approach can easily be adapted to an orthorhombic medium in which an additional set of Thomsen parameters is required (Tsvankin, 2001).

To saturate the dry rocks I elected to apply Gassmann’s equation. Gassmann’s equation emulates the low frequency velocity response where the pressure wave frequency is low enough to allow the pore pressure to come to equilibrium over the measurement period. This was chosen, rather than attempting a high frequency fluid substitution, to predict the static response. Mechanical applications require the static moduli, and while this will not model the static strain amplitude or the drained properties, the undrained low frequency fluid response is a closer representation of the static properties.

The dry TI stiffness tensor and QEMScan and XRD mineralogy data allowed for an estimation of continuous Biot’s coefficients. Integration of the density log provided the total vertical stress, and by assuming uniaxial strain, the effective and total minimum horizontal stress was calculated. The stress profile showed no major stress barriers. Near the reservoir Facies C and D there was a dip in the horizontal stress, but the remaining units had similar horizontal stress.

Anisotropic Young’s moduli and Poisson’s ratios were provided by Schlumberger and showed a large horizontal stress contrast in the shales. A calculation of $\delta$ based on the provided elastic properties showed values greater than 1. The available laboratory data suggests this value is unrealistically large. Furthermore, two Mini-Frac tests were performed in the Lodgepole Formation and Upper Bakken Shale. The horizontal stress profile provided by Schlumberger matched the closure stress in the Upper Bakken Shale but not in the Lodgepole Formation. The presumed reason for the mismatch is that anisotropy was considered, but not Biot’s coefficients less than unity. Schlumberger apparently modified the ANNIE approximation until one of the Mini-Frac tests matched the total minimum horizontal stress.

The shear wave anisotropy $\gamma$ can be directly calculated from the dipole sonic log and the Stoneley wave estimation of $C_{66}$. The middle Bakken and Lodgepole Formation had $\gamma < 0.02$, but the $\delta$ calculated from elastic properties provided by Schlumberger was $\sim 0.2$. In the shales $0.03 < \gamma < 0.12$ and based on Schlumberger’s properties $0.5 < \delta < 1.2$. A more realistic explanation is that Biot’s coefficients in the shales are $\sim 0.45$ and $\delta < 0.05$, and for the middle Bakken and Lodgepole Formation Biot’s coefficients range from 0.4 – 0.6 and $\delta \sim 0$. 

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In hydrofracture modeling studies the stress variance will be more important than the absolute stress value. The properties provided by Schlumberger predict a large horizontal stress contrast in the shales, whereas the alternative method described in Chapter 4 shows no major stress contrast. The lack of any strong stress barrier may cause secondary predictors such as strength properties and permeability to control the fracture growth.

5.2 Future Work

The methodology for calculating Biot’s coefficients in this study is highly dependent on the pure mineral properties. (Sarker, 2010) manually changed the pore pressure and recovered the effective stress coefficients (change in properties with a change in pore pressure). In future studies dry rock measurements should be combined with fluid saturated experiments at different pressure conditions. The mineral modulus could then be recovered and compared with the calculated values from pure mineral properties. The saturating fluid could be compressed air to avoid fluid related changes. Additionally, heated experiments by Baharia (n.d.) show a change in organic matter properties with temperature. The high organic matter in the Bakken Shales may cause a significant change in elastic properties with temperature, leading to different stress characteristics inside and outside the thermal anomaly (Price et al., 1984). The shales in this study are from Saskatchewan, Canada outside the thermal anomaly. Comparisons of Mini-Frac tests inside and outside the thermal anomaly may show a predictable change in shale properties given the formation temperature.

The Bakken Shales in the Freda Lake Field were tested for thermal maturity and were within the immature window. Conventional wisdom suggests no hydrocarbons were produced in these shales. However, based on the Mini-Frac tests the Upper Bakken Shale was overpressured. A holistic approach should be undertaken to determine the cause of overpressuring. The field may have patchy maturity and specific migration paths for pore pressure. Predicting the maturity and migration paths may be the key to efficiently producing the field.

Production may also depend heavily on trapping mechanisms. The overlying Lodgepole Formation showed large shear wave splitting suggesting there are vertical fractures. In Chapter 4 I showed the horizontal stress profile provided by Schlumberger. This model would suggest the Bakken Shales are highly competent stress barriers and would likely provide the upper seal. I have indicated that they are not major stress barriers, and the characteristics of the Lodgepole Formation are equally important. The well in this study was water saturated, and the cause may have been vertical fractures in the Lodgepole Formation allowing hydrocarbons to escape into overlying formations. Fracture identification with seismic data may provide strong trends with oil saturated zones given
the migration paths are understood.
REFERENCES CITED


