Combined Numerical and Experimental Analysis of the Mechanical and Thermal Stress Distribution in a Rubber-Aluminum Roll

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1. Abstract

Structural components made of rubber play an important role in engineering processes. In spite of the widespread and necessary use of rubber in engineering, a general working knowledge of rubber properties and technology is rather uncommon. A typical application in ropeway engineering is the rubber-aluminum roller subjected to very high thermal and stress gradients during its service life. An attempt is made to describe chemical constitution and compounding of natural rubber as well as the mechanical properties of the elastomer. Material model characterization is summarized and hyperelastic models are reviewed. Important factors influencing energy conversion into heat are introduced and an attempt is made to analyze the thermal pressure generated during oscillating loads. It is shown that the thermal load case is important in determining the stress distribution in the roller components and must be included during the design process.

2. Introduction

Structural design and analysis routines applied to ropeway engineering have developed to a mature science over the last thirty years. The use of advanced materials and state of the art quality control allowed developing rope transportation systems with a very high standard of safety. However, even with the advancement in computer technology and the development of faster numerical methods, testing of full-scale systems is still the method of choice to verify and validate new components.

The continuous increase in transportation speed and applied loads requires a more accurate analysis of the rubber-aluminum roller typically used in ropeway systems. In this paper, the thermal and mechanical properties of rubber as well as the interactions of the rubber-aluminum components are described in detail. First, the physical properties of the elastomer are reviewed, including vulcanization, compounding and determination of mechanical properties. Stress-strain tests are performed on rubber specimens to determine the modulus of the elastomer at different strains. In addition, it is recognized that deformations in rubber like materials are associated with significant thermal effects.

Typically, during dynamic loading of rubber, some energy is constantly converted to heat. Thus, the high thermal expansion of rubber coupled with the high bulk modulus generates a substantial thermal pressure if the elastomer is confined. In this study,

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numerical methods and experimental verifications are used to determine the stress distribution in the aluminum body generated by the thermal pressure in the elastomer and the cyclic loading conditions generated by the cabin weight.

3. Chemical Constitution and Compounding

Natural rubber is a hydrocarbon described by the chemical formula $\left(C_5H_8\right)_n$. The molecule C_5H_8 is known as an isoprene and forms the basic building block of natural rubber. They are arranged in cis-configuration forming long chains with high elasticity, see Figure 1. The chains are kinked and arranged in the material like agitated snakes, they are perfectly regular and have freely rotating links at given distance. Raw rubber has only few weak crosslinks between chains as in Figure 2. When subjected to an external force the chains disentangle and some crosslinks break. Thus, the material deforms by viscous flow and undergoes large displacements until breakage of all links or alternatively an equilibrium state is reached. It crystallizes readily at temperature of 0°C and below. At higher temperature the rubber becomes soft and sticky.

During vulcanizing [4], [7] additional crosslinks are established and a coherent network is formed to influence the mechanical behavior of the elastomer. Rubber molecules are linked along their length by short chains of sulfur atoms. Vulcanization or curing gives the material strength, elasticity and a mechanical behavior less sensitive to temperature. The more crosslinks are formed in the network the greater the stiffness of the rubber, at the same time it becomes less hysteretic. If then subjected to an external load the response is (visco-) elastic. When the rubber is deformed, a part of the energy is stored as elastic energy, recoverable upon unloading or breakage of crosslinks. Thermal effects dissipate the remainder of the energy. With an increase of cross-link density the network becomes tighter and the motion of the chains becomes more impeded. The network is then incapable to dissipate large amounts of energy. Hardness increases and the material undergoes low elongation and brittle fracture might occur. Thus, rubber has an optimal cross-link density range for practical use, high enough to prevent failure by viscous flow, but low enough to avoid brittle failure.

Particulate fillers, such as carbon black or silica are chemically linked to rubber to improve mechanical properties as well as tear and abrasion. The majority of rubbers contain filler whose particles are very small and posses a high specific surface area.
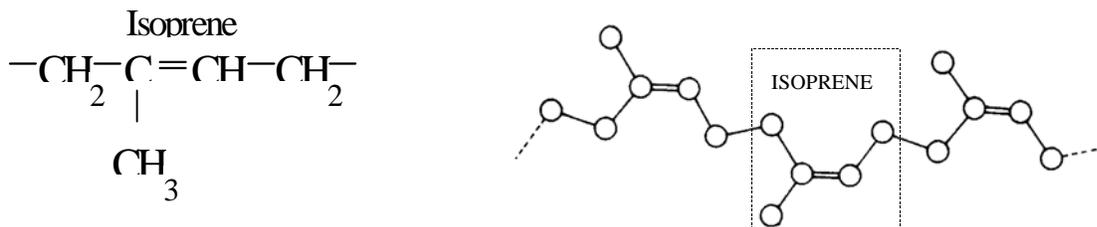

Figure 1: Structural formulae of one isoprene molecule (left) and structure of a chain molecule (right), (after Treloar [7], 1975).

Filled compounds subjected to large strains show a high degree of non linearity in the stress-strain response including strain softening known as Mullins effect, [ ]. The strain

softening effect is probably caused by the breakdown of weak chemical bindings between filler and rubber molecules. Unfilled or lightly filled compounds do not show a significant degree of strain softening and when deformed in shear remain essentially linear up to strains of about 200%.

Depending on temperature and/or on compounding rubber can assume a crystalline, a rubbery or a glassy state. By lowering the temperature to a point where any motion of the rubber molecules are eliminated the rubber becomes like glass and brittle. This temperature is called the glass transition temperature and is –71°C for unvulcanized and a few degrees higher for the vulcanized rubber. On the other side rubber can be safely used up to an ambient temperature of about 100°C. The process of low temperature crystallization as well as the change of properties associated with high temperature are completely reversible.

To determine durability and longevity of rubber, tests were performed on two bridge bearings in use for over 30 years [2]. The tests showed that the stiffness has increased by only 5 % for one and 15 % for the other bearing. Accelerated aging tests on high damping rubber specimens showed that the oxidative aging extends only 2 mm into the rubber block [3].

4. Environmental Factors

Elastomeric materials are very sensitive to environmental effects and can rapidly deteriorate when kept under severe conditions. In fact, oxygen, ozone, sunlight, heat, and humidity are factors that together or separately can provoke progressive and irreversible damage, generally known as aging. The destructive effect of these factors is greater if static deformations or dynamic excitations of the rubber article accompany them. Nevertheless, present technology has the capacity to notably reduce the effects of environmental conditions through suitable formulation of the compound. It is thus possible to obtain a high level of durability for rubber products, at least for those exposed to normal environmental conditions.

Figure 2: Sketch of a molecular entanglement, after Gent [4].
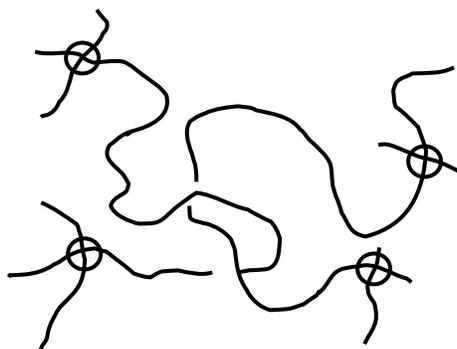
5. Mechanical Behavior of Natural Rubber

A list of representative mechanical properties of a high damping natural rubber provided by "The Tun Abdul Razak Research Center" (TARRC) are summarized in Table 1. The principal mechanical properties of rubber are bulk modulus, shear modulus at a given strain, tensile strength, elongation at break, damping and hardness. Typical for rubber like materials, the maximum extensibility falls within the range of 500-1000% and that the shear modulus is much smaller than the bulk modulus.

Typical stress-strain curves for rubber are highly nonlinear, hence it is not possible to assign a definite value to Young's modulus except in the region of small strain, see Figure 3 and Figure 4. The shape of the response curves is influenced among others by the type and amount of filers and by the amount of cross-linking. In general the presence of fillers increases the stiffness of rubbers but introduces additional non-linearity in the stress-strain response. The high initial stiffness decreases with increasing strain, remains approximately constant for intermediate strain values and increases again towards the end. The initial stiffness reduction is associated with breakage of rubber-filler bonds. The final increase is due to the limited extensibility of the polymer chains and possibly also to strain crystallization. The material is initially isotropic, but straining reorients molecule, and the rubber becomes anisotropic. With repeated loading, crosslinks may break and reduce the tensile strength. This effect known as strain softening or "Mullins effect" occurs mainly during the first few cycles, see also Figure 5b.

The shear modulus as well as the damping ratio of rubber depends also on the strain amplitude and loading history. In Figure 5, changes up to a strain magnitude of 150% are given. The shear modulus reduces from 5.5 MPa at 5% strain to 0.5 MPa at 100%. The dependence of the damping ratio from the strain magnitude is less pronounced; the value remains approximately between 15% and 20% for strains up to 150%.

Property	HDR
Bulk modulus (MPa)	2500
Tensile Strength (MPa)	13.5
Elongation at Break (%)	700
Shear modulus[a] (MPa)	0.55
Damping[a] (%)	14
G(-20)/G(20)	2.2
[a] at 100% strain and 0.5 Hz.	

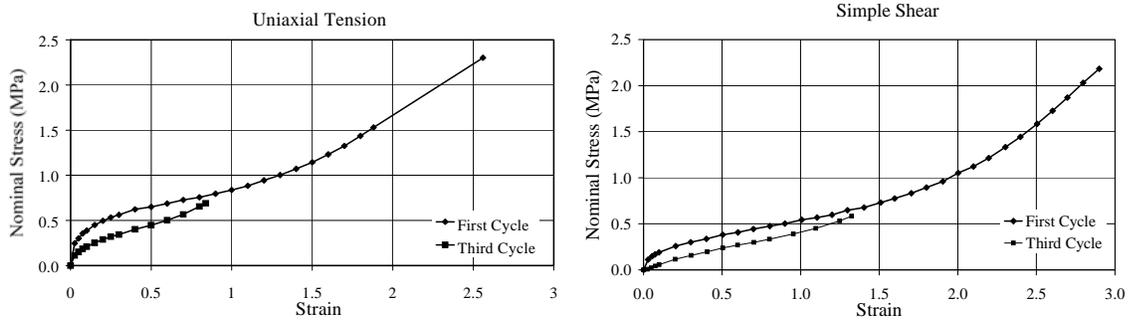**Table 1: Mechanical properties of high damping rubber on a testpiece: at T=23°C.**

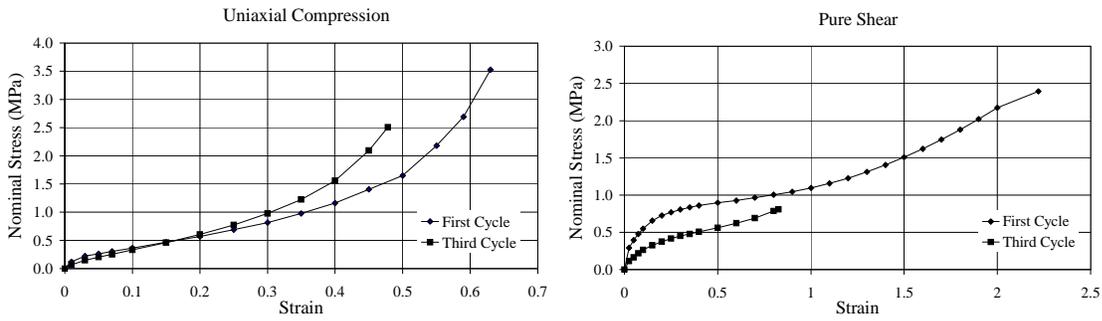**Figure 3: Uniaxial Tension and Simple Shear Test of High Damping Natural Rubber.**

Figure 4: Uniaxial Compression and Pure Shear (planar) Test of High Damping Natural Rubber.
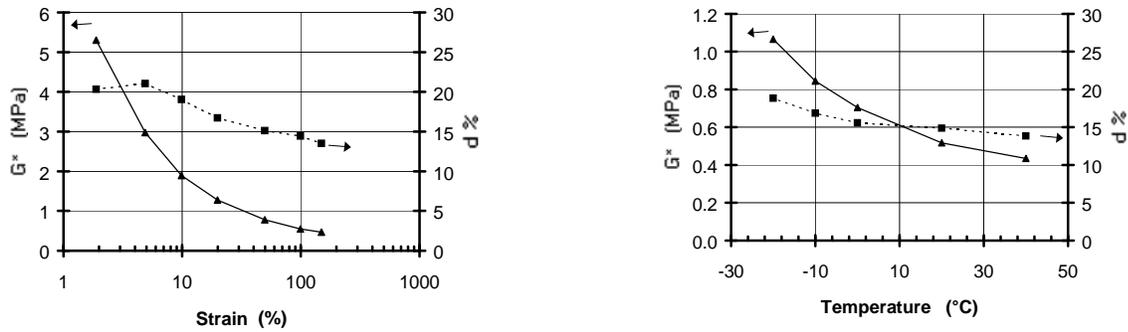

Figure 5: Effect of strain amplitude and temperature on shear modulus and damping (3rd cycle data, strain sweep at 0.5 Hz) of High Damping Natural Rubber.
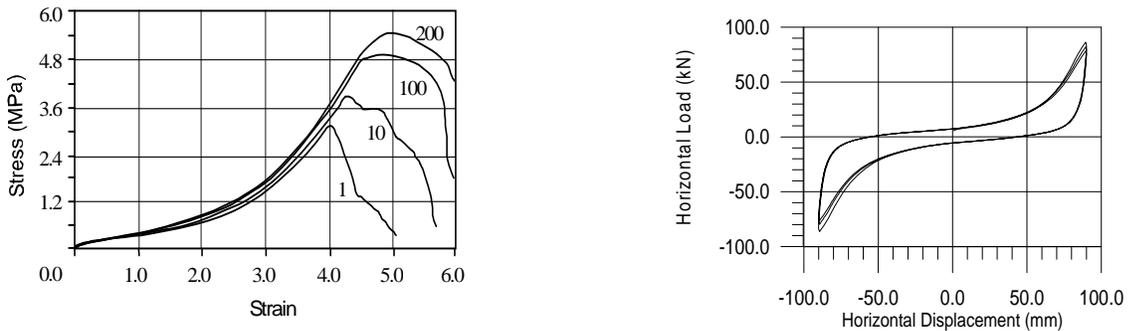

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To evaluate the effect of strain rate on rubber, elastomeric bearings are subjected in [6] to shear tests at rates equal to 1, 10, 100, 200 $\% s^{-1}$. In these tests, a vertical compression load is applied first and kept constant while the bearings are subjected to simple shear. In general, the shear strain in rubber layers is calculated simply as the quotient of the horizontal displacement and the total rubber height.

Figure 6 shows the dependence of stress and strain from strain rates. For higher rates no change in stiffness can be observed, however the strain at failure increases with the strain rate. The stress at breakage for a strain rate of 200 $\% s^{-1}$ is about 1.7 times higher than for a strain rate of 1 $\% s^{-1}$. A typical load-displacement response of an elastomeric bearing subjected to shear deformation at a rate of 12 %s^{-1} is shown in Figure 5b. It is evident that the loading and unloading path do not coincide due to energy dissipation in the material. This effect is known as the hysteresis characteristic of the material and the area enclosed by the loading and unloading path is the dissipated energy of the material in the cycle; it is an import measure for the damping of the rubber.

6. Dynamic Properties

In ropeway engineering rubber-aluminum pulleys are subjected to stresses or strains that change periodically with time. For example, a continuous stress state in the aluminum body generated by different thermal expansion coefficients in the aluminum and the rubber, in addition to a periodic load due to the cabin weight. To introduce some of the concepts used in dynamic visco-elasticity, we review the special case of sinusoidal varying stress and strain.

Let a linear viscoelastic material be subjected to sinusoidal loading, both stress and strain will vary sinusoidally, but the strain will lag behind the stress by a phase angle δ, see Figure 6. The applied deformation varies in time with an angular frequency ω, i.e.

$$\varepsilon = \varepsilon_o \sin(\omega t)$$

then, the corresponding stress is given by

$$\sigma = \sigma_o \sin(\omega t + \delta)$$

expanding, the above equation yields

$$\sigma = \sigma_o \sin(\omega t) \cos \delta + \sigma_o \cos(\omega t) \sin \delta$$

$$\sigma = \varepsilon_o E_1 \sin(\omega t) + \varepsilon_o E_2 \cos(\omega t)$$

where E_1, E_2 and δ is defined below

$$E_1 = \frac{\sigma_o}{\varepsilon_o} \cos \delta, \qquad E_2 = \frac{\sigma_o}{\varepsilon_o} \sin \delta, \qquad \tan \delta = \frac{E_2}{E_1}$$

The above equation shows that the stress is described by modulus E_1 in phase with the applied strain and modulus E_2 that is 90° out of phase. E_1 is frequently called the "storage modulus" and is associated with the energy stored in the specimen due to the applied strain. E_2 is the "loss modulus" and describes the energy dissipated per cycle. The dissipated energy per cycle and per unit volume is given as

$$U_d = \int \sigma \, d\varepsilon = \int_0^{2\pi/\omega} \sigma \frac{d\varepsilon}{dt} dt = \pi E_2 \varepsilon_o^2$$

where the expression for stress and strain defined before have been used. The maximum energy stored in the system can be expressed as

$$U = \frac{1}{2} E_1 \varepsilon_o^2$$

The angle δ is if particular interest in the analysis of viscoelastic materials. It represents the phase angle.

7. Temperature Effects

The thermoelastic behavior of rubber has been investigated in great detail over the years. A detailed review of the thermoelastic behavior of cross-linked polymers is outside the framework of this paper. However, a few interesting aspects will briefly be mentioned in this section.

Early studies by Gough in 1805 on unvulcanized rubber showed that 1) the rubber held in a stretched state, under a constant load contracts (reversibly) when heated; 2) that

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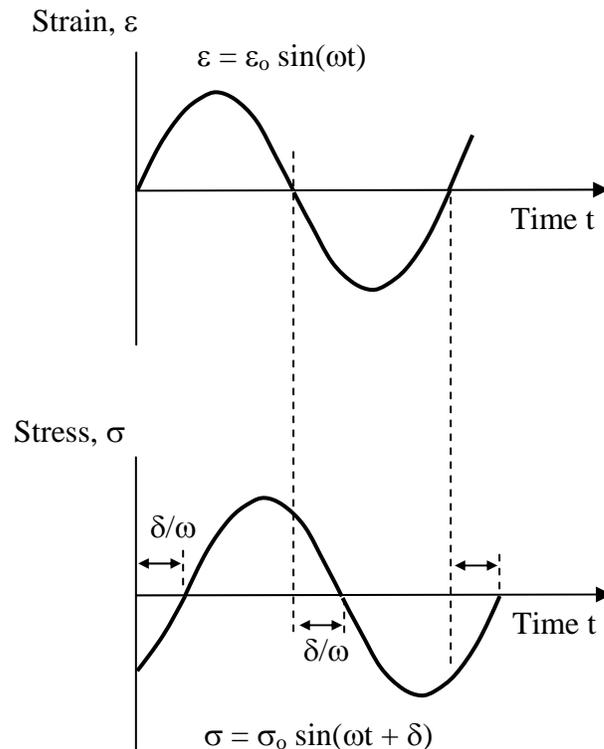
rubber generates heat when stretched. These findings where confirmed by

Figure 6: Sinusoidal Stress-Strain response for a perfect viscoelastic material.

Joule on vulcanized rubber about fifty years later, thus, the properties are known the Gough-Joule effects [7]. It is important to note that these observations are opposite in metals. In fact, 1a) an elastic metal spring under constant tensile force will expand by heating and 2a) an elastic metal spring cools on stretching.

The influence of temperature variation on rubber properties is important as well. It is considered to be the single most important environmental factor affecting the mechanical properties of rubber. In general four different regions can be observed, see Figure 7. In the so-called glassy region the material takes on its maximum value for the stiffness while having low values for the loss factor. The loss factor η or loss tangent (tan δ) determine the mechanical energy lossed in the material. It is also used to describe damping in viscoelastic materials. The second, the transition region, is characterized by having a modulus that decreases rapidly with increasing temperature, while the loss angle takes on its maximum value. The third is the rubber region where both the modulus and the loss factor usually take on somewhat low values and vary slowly with temperature. In the flow region, the material continuous to soften with increasing temperature, as it melts, whiles the loss factor takes on a very high value. For rubber-like material, such as cross-linked polymers, the forth region does not exits.

The influence of temperature on modulus and damping is given in Figure 5 for high damping natural rubber developed by the Malaysian Rubber Producers' Research Association. For filled natural rubber the stiffness falls with increasing temperature, the gradient being highest between -20°C and normal room temperature. Further, the non-linearity in the stress-strain response reduces as the temperature is raised because the stiffness is reduced more at low strains that at high strains. In Figure 5, the shear stiffness of 1.1 MPa at –20°C is 2.5 times the value at +40°C. Similarly the damping reduces from 19% to 14% for the same temperature interval.

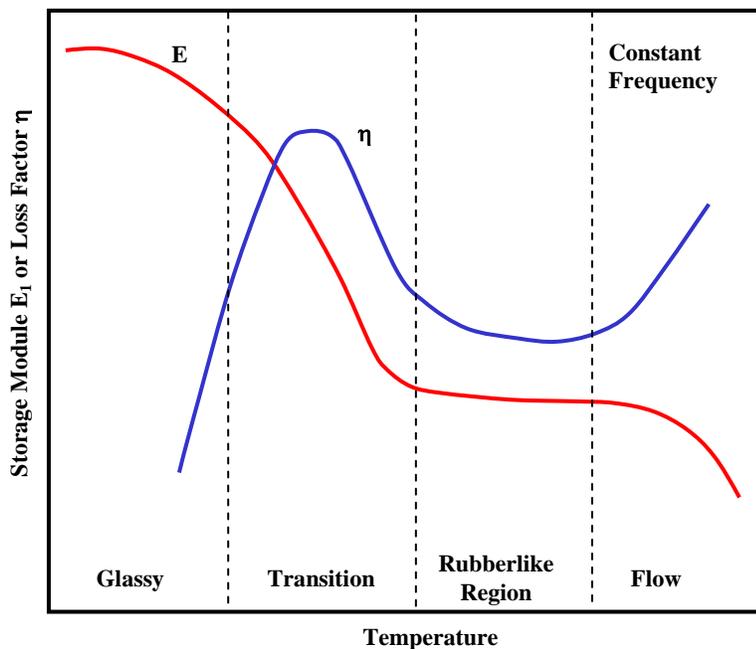Rubber subjected to cyclic loading dissipates energy in form of heat. The dissipation arises from internal friction due to the disentangling of the long chain molecules. It is well known that rubber is a very poor heat conductor with a thermal conduction coefficient of about $0.2 - 0.3\ W/m \cdot {}^{\circ}C$, a value about ten times lower then in metals. Therefore, during (dynamic) loading a heat build up can be induced. The heat generated depends on the strain amplitude, which is affected by the stiffness of the rubber and the load magnitude.

Finally, the coefficient of thermal expansion for rubber like materials is approximately 4.8 x 10^{-4} K^{-1}. For steel the thermal coefficient is 3.5 x 10^{-5} K^{-1}, an order of magnitude smaller than the value for rubber. Thus, even though rubber can be considered an almost incompressible material, the volume changes due to temperature variation are still existent and must be taken into consideration. In other words, the high thermal coefficient coupled with the high bulk modulus generates a situation where a very high thermal pressure exits if the elastomer is confined and subsequently heated. It is exactly this situation which arises in the aluminum-rubber pulley in ropeway engineering.

8. Constitutive Equations of Hyperelasticity

As discussed before, rubber is a highly nonlinear viscoelastic material, with properties strongly dependent on temperature and strain rate. Furthermore, the ratio of bulk modulus to shear modulus is very high and the material can thus be considered nearly incompressible in most cases. In this section constitutive equations for hyperelastic materials are described for quasi static loading conditions, e.g. strain rate independent. These material models are derived from a strain-energy-potential. The formulation defines the elastic energy stored in the material per unit reference volume, depending upon the strain-state at a given material point. The initial orientations of the long-chain rubber molecules are random and therefore the material is assumed isotropic. Due to loading the chains reorient along the direction of straining and induce a small anisotropy. In hyperelastic material models the assumption is made that the material is considered isotropic throughout the deformation history. As a consequence, the strain energy potential can be formulated as a function of strain invariants. For incompressible isotropic hyperelastic materials the strain energy density depends on the first and the second invariant only, the third invariant describing the volumetric change is always equal to unity. The deviatoric strain invariants and the volume change are defined as

$$I_1 = \lambda_1{}^2 + \lambda_2{}^2 + \lambda_3{}^2,$$
$$I_2 = \lambda_1{}^{-2} + \lambda_2{}^{-2} + \lambda_3{}^{-2},$$
$$I_3 = \lambda_1\,\lambda_2\,\lambda_3\,bzw.\,J = \sqrt{I_3}\ (=1\,if\ incompressible).$$

with the deviatoric stretches λ_i = J(-1/3) $\lambda_i{}^P$, J is the volume ratio and $\lambda_i{}^P$ are the principal values. To account for the compressibility of the material an additional term describing the volumetric part only is introduced. This term depends only on the third

invariant or the volume ratio, J. Therefore the strain energy potential, Ψ, is decoupled into a volumetric part $\hat{U}$, and an isochoric part $\hat{W}$ and has the general form

$$\Psi = \hat{W}(I_1, I_2) + \hat{U}(J).$$

The second Piola-Kirchoff stress tensor, **S**, is derived from the strain energy density function as

$$\mathbf{S} = 2\frac{\partial \Psi}{\partial \mathbf{C}},$$

where **C** is the right Cauchy-Green Tensor. A comprehensive description is given in many textbooks and dissertations such as Kaliske [10], Liu [8], Crisfield [11], ABAQUS Theory Manual [9].

The least squares error minimization method is used in ABAQUS to fit the strain energy function to the nominal stress–strain data from standard material characterization tests. Since the strain energy function is decoupled we can fit the isochoric and the volumetric terms separately. The isochoric part is fitted using uniaxial, planar and biaxial tension and compression data, neglecting compressibility of the material. The volumetric term in the energy formulation requires stress-strain data from hydrostatic compression tests. If such data is not available the first parameter D_1 can be calculated directly from an estimated constant bulk modulus, with the formulation given later. By using D_1 only ,the pressure-volume relation is linear.

If the material is not confined it may be quite satisfactory selecting a material model for a fully incompressible response. However for rope-way rollers highly confined situation does exist and the compressibility must be modeled correctly to obtain accurate results. The second part of the strain energy density function, describing the change in volume due to hydrostatic tension or compression is needed.

8.1 Polynomial Strain Energy Potential

There are two forms of strain energy potentials commonly used for modeling elastomers, the polynomial form and the Ogden form. The polynomial form is expressed in terms of deviatoric strain invariants and is

$$\textbf{Error! Bookmark not defined. } \Psi = \sum_{i+j=1}^{N} C_{ij}\left(I_1 - 3\right)^i \left(I_2 - 3\right)^j + \sum_{i=1}^{N} \frac{1}{D_i}\left(J_{el} - 1\right)^{2i}$$

where Ψ is the strain energy potential, J_{el} is the elastic volume ratio, I_1 and I_2 are the first and second invariants of the deviatoric strain, N, C_{ij} and D_i are material parameters, which may be functions of temperature. The C_{ij} describe the shear behavior of the material and the D_i introduces compressibility. For this model the initial shear modulus and bulk modulus are given by

$$\mu_o = 2\sum_{i+j=1}^{N} C_{ij} \qquad\qquad K_o = \frac{2}{D_1}$$

The elastic volume ratio J_{el} follows from the total volume ratio J and the thermal volume ratio J_{th} with the relation

$$J_{el} = \frac{J}{J_{th}}$$

and J_{th} is defined in terms of the linear thermal expansion ε_{th} as

$$J_{th} = (1+\varepsilon_{th})^3$$

where ε_{th} follows from the temperature and thermal expansion coefficient. Only isotropic thermal expansion can be used with this model.

8.2 Ogden Strain Energy Potential

Alternatively to the formulation in strain invariants, the Ogden strain energy density function is described in the principal stretches

$$\Psi = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_3^{\alpha_i}-3\right) + \sum_{i=1}^{N} \frac{1}{D_i}\left(J_{el}-1\right)^{2i}$$

where λ_i are the deviatoric principal stretches defined as

$$\lambda_i = J^{-\frac{1}{3}} \lambda_i^{P} \quad \rightarrow \quad \lambda_1 \lambda_2 \lambda_3 = 1$$

N, μ_i, α_i and D_i are material parameters, which may be temperature dependent. J_{el}, J and D_i have the same definitions as in the polynomial form. For this model the initial shear modulus and bulk modulus are given by

$$\mu_o = \sum_{i=1}^{N} \mu_i \qquad K_o = \frac{2}{D_1}$$

Because the powers α_i can be chosen by the user, the Ogden form usually provides a closer and more stable fit to the test data for a similar number of material constants in the strain energy function, especially at large strains. The value of N, and tables giving μ_i and α_i as functions of temperature can be defined explicitly in the Abaqus input file. Note that if $N = 2$, $\alpha_1 = 2$ and $\alpha_2 = -2$, the Mooney-Rivlin model is obtained. If $N = 1$ and $\alpha_1 = 2$, Ogden's model degenerates into a neo-Hookean material model.

The Ogden model and the polynomial model use the same formulation for describing the compressibility in the strain energy density function. The changes in volume are very small and the pressure volume relation is approximately linear in hydrostatic compression and tension. A critical assessment of isotropic compression and tension for this and other models can be found in [8].

8.3 Particular Cases of the Strain Energy Potentials

If all of the D_i are zero, the material is fully incompressible. If D_1 is equal to zero, all of the D_i must be equal to zero.   For cases where the nominal strains are small or only moderately large (<100%), the first terms in the polynomial series usually provide sufficiently accurate model. The simplest form of the polynomial function is the form with $N = 1$, which is the compressible form of the classical Mooney-Rivlin law shown below

$$U = C_{10}(J_1 - 3) + C_{01}(J_2 - 3) + \frac{1}{D_1}(J_{el} - 1)^2$$

When $C_{01} = 0$, the strain energy function corresponds to the compressible form of the neo-Hookean law. The initial shear modulus and bulk modulus are given by

$$\mu_o = 2(C_{10} + C_{01}) \qquad K_o = \frac{2}{D_1}$$

The Ogden form is general in nature but particular cases are obtained for special choices for μ_i and α_i. The neo-Hookean form is obtained with N = 1, α_1 = 2 and can be made equivalent to the polynomial form through the relation $\mu_1 = 2C_{10}$. The Mooney-Rivlin form is obtained when N = 2, α_1 = 2 and α_2 = -2 and is equivalent to the polynomial form when $\mu_1 = 2C_{10}$ and $\mu_2 = 2C_{01}$

$$U = \frac{\mu_1}{2}\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right) + \frac{\mu_2}{2}\left(\lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} - 3\right) + \frac{1}{D_1}(J_{el} - 1)^2$$

The initial shear modulus and bulk modulus are given by the same expressions as for the Ogden strain energy potential.

9. Numerical Example

9.1 Problem Statement

Rubber-like materials are widely used in industry. A typical example for their use is the rubber component in the aluminum roller in ropeway systems. As mentioned before, the natural rubber has several characteristic properties. First, the stress-strain behavior is highly non-linear depending among others from the type and amount of fillers; secondly the material usually undergoes large strains, and finally, the material may be considered incompressible or nearly incompressible because of the large bulk modulus over the shear modulus. Even though the bulk modulus greatly exceeds the shear stiffness, the volume changes due to temperature variations are still existent and need to be accounted for. The high coefficient of thermal expansion creates a real possibility for substantial thermal pressure if the rubber components are confined and subsequently heated. These properties govern the selection of the appropriate constitutive models. In the present analysis a generalized Ogden model is used to describe the rubber in the thermoelastic domain and to evaluate the induced thermal stresses in the aluminum body.

Figure 8 shows the finite element model of a roller section to analyze the temperature-displacement interaction of the rubber and aluminum components. The cable is represented by a rigid surface in frictionless contact with the rubber. This constrain is necessary to simulate the resistance to thermal expansion of the elastomer by the cable and therefore obtain a better approximation for thermal pressure. Contact between a rigid surface and a deformable body is allowed to undergo finite sliding, however no heat generation is assumed at the contact. More complicated interaction contacts in coupled temperature-displacement simulations are possible, i.e. heat generation due to dissipation of energy created by the mechanical contacting surface. However,

simplifications are introduced whenever possible without compromising the stated problem. For the same reason, no contact interaction is included between the rubber and the aluminum body, i.e. no sliding and no separation of the rubber from the aluminum surface is allowed to occur.

Figure 8 shows that the rubber is highly confined and therefore modeling the compressibility is important to obtain accurate results. The solution cannot be obtained using a formulation based on the displacement based theory alone. In fact, for an incompressible material, a purely hydrostatic pressure may be added without changing the displacements. For nearly incompressible material, a very small change in displacement produces extremely large changes in pressure. A purely displacement-based solution is therefore too sensitive and round-off errors would produce a diverged and useless solution. This can be eliminated by considering the pressure variable as an independently interpolated quantity, coupled to the displacement solution through the constitutive theory and compatibility condition, with this coupling implemented by a Lagrange multiplier. This independent interpolation of the pressure variable is the basis of the "hybrid element" formulation. In other words, they are mixed formulation elements, using a mixture of displacement and stress variables with an augmented variational principle to approximate the equilibrium equations and compatibility conditions.

9.2 Rubber Characterization

The mechanical response of rubber is defined by choosing the parameters in the strain energy potential to fit the particular compound. Since hyperelastic materials are usually almost incompressible, the approach is to use data from experiments involving simple deformations. The parameters required to define the volume -preserving part of the strain energy function are defined first assuming that the material is fully incompressible. Then volumetric compression data is used to
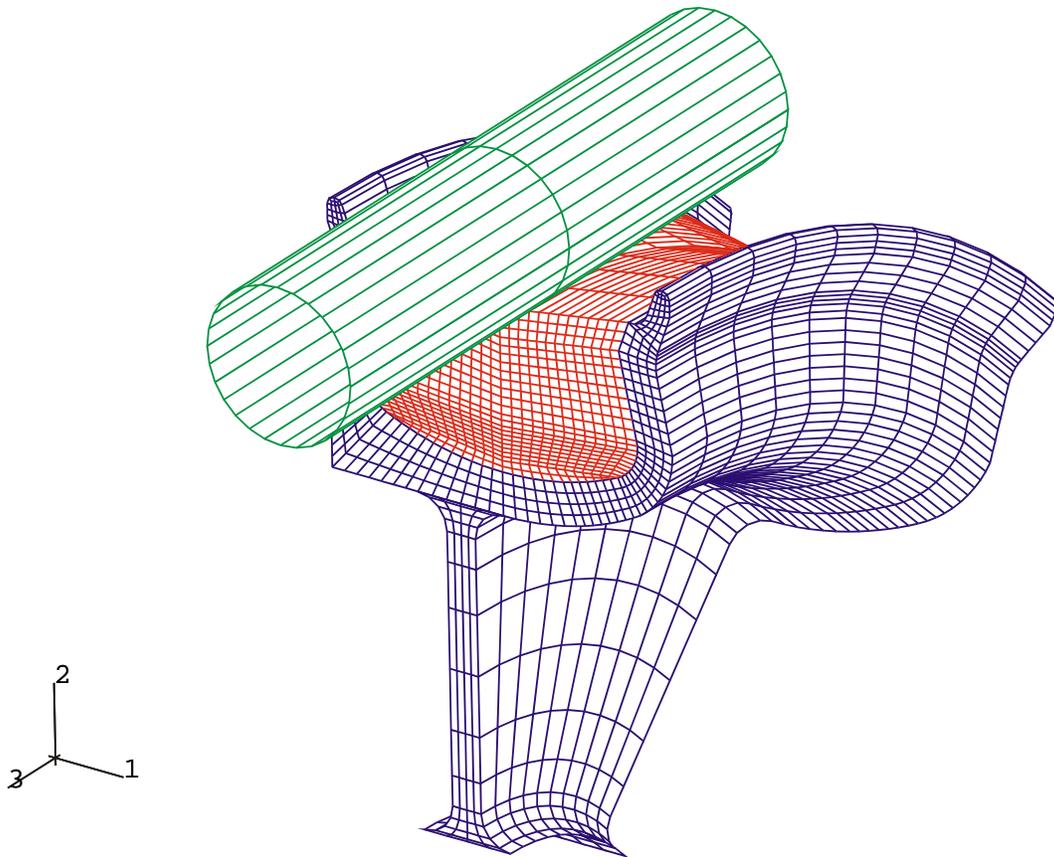
Figure 8: Finite Element Model of Rubber-Aluminum Roller and Rigid Rope

determine the D_l, if the compressibility is important, or to assume $D_l = 0$ if it not. It is usually best to obtain data from several experiments involving different kinds of deformation over the range of strains of interest in the actual application, and to use all of this data to determine the parameter of the strain energy formulation. The models are phenomenological, it has been observed that it is necessary to fit data from more than one deformation state in order to achieve good accuracy and stability of the numerical solution routine. As part of this study, actual rubber used in ropeway engineering was subjected at the TARRC laboratories to uniaxial tension, uniaxial compression and simple shear deformation, see Figure 9. The combination of results from all three tests will result in a good material model capable to describe rubber. In this study, the Ogden Strain Energy Potential is selected and the numerical results for the three characterization tests are compared to test values, see Figure 9. Finally in Table 2 coefficients for the Ogden model with N=3 are summarized.

9.3 Isothermal Loading

To verify the assumption and the selection of the model, static isothermal tests were performed at ambient temperature to determine the load-deflection response of a rigid rope pressed onto the pulley. The objective was to obtain experimental values of the confined rubber stiffness when static loads are applied. The corresponding stress distribution in the aluminum body is due to applied loading only, no energy
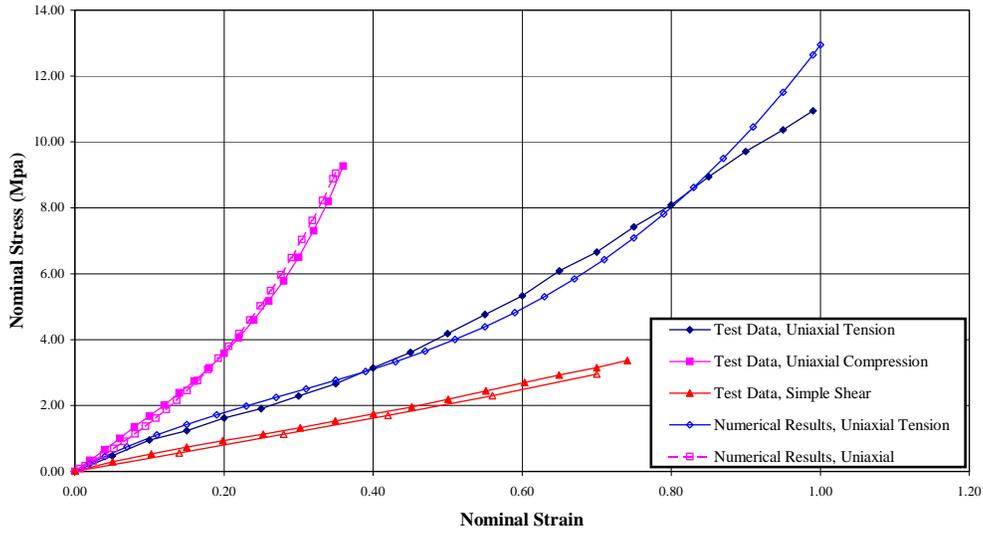
Figure 9: Uniaxial Tension, Compression and Simple Shear Data

Ogden	$\mu_1 = -884.083$ MPa	$\alpha_1 = 2.78745$	$D_1 = 0.0008$
Model	$\mu_2 = 421.868$ MPa	$\alpha_2 = 3.07091$	0
N=3	$\mu_3 = 466.241$ MPa	$\alpha_3 = 2.49572$	0

Table 2: Material Model Parameter for Ogden Strain Energy Function, N=3

dissipation in the rubber and therefore no thermal pressure is considered. Figure 10 shows the experimental load deflection response and the corresponding numerical results using the model in Figure 8.The maximum applied load in the numerical simulation is 7.5 kN corresponding to a 10 mm penetration into the rubber. The associated von Mises stress distribution is given in Figure 11. The highest stress state recorded is about 38 MPa.
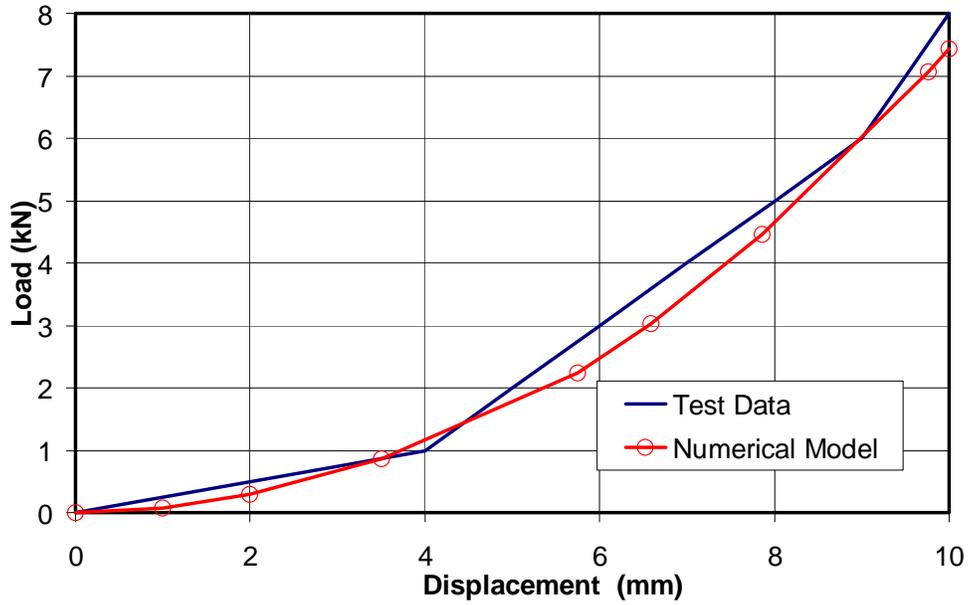
15

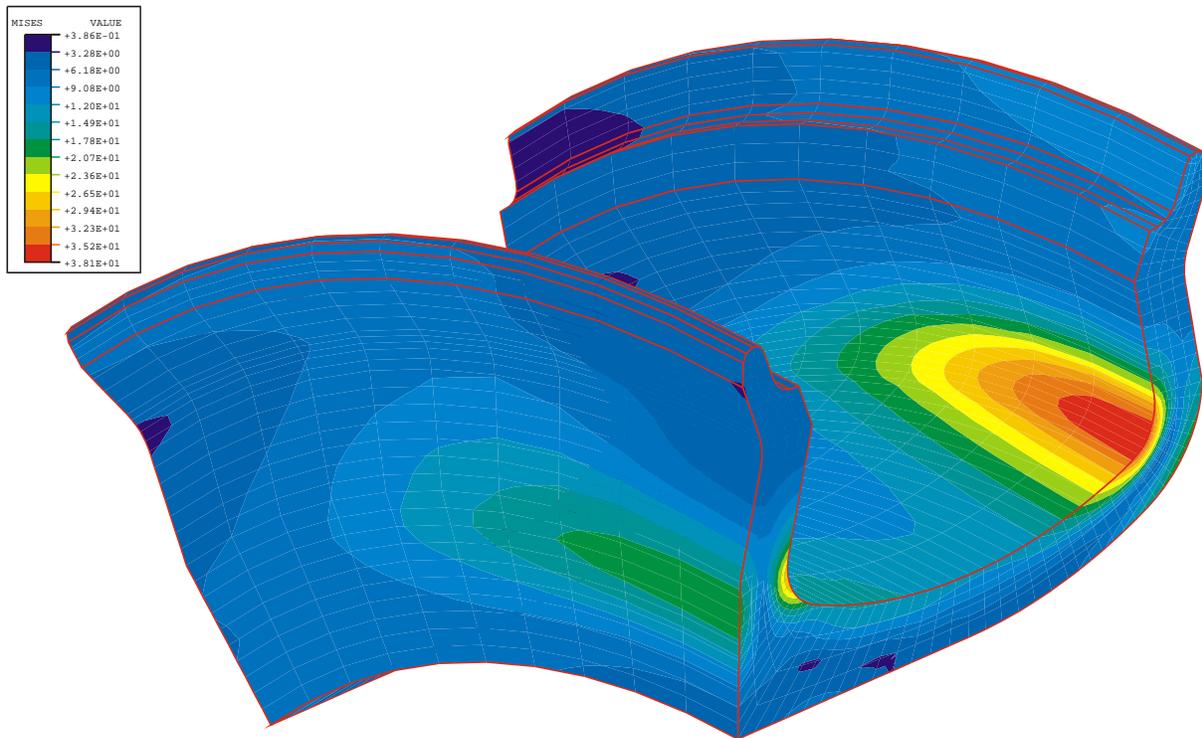**Figure 10: Static Load-Deflection Response**

Figure 11: Von Mises Stress Distribution for 7.5 kN Static Load Case

9.4 Coupled Temperature-Displacement Analysis

A steady state temperature gradient is established during operation in the rubber-aluminum roller. It is the objective to determine the stress distribution due to different

thermal expansion coefficients for the components of the roller. It is important to emphasis, that no additional static load is applied.

Experimental measurements on the temperature distribution provided reference values of 94 °C in the center of the rubber and 44 °C on the outside of the aluminum body. As noted before, the high coefficient of thermal expansion in the rubber will produce a substantial thermal pressure at the rubber-aluminum contact surface. A coupled temperature-displacement analysis is performed to model heat conduction in solids and the associated stress distribution.

Mechanical properties of the material are selected to model the physical response of the structure. The thermal expansion coefficient of aluminum is equal to 2.4E-5 /°C, the thermal conductivity of the same material is given as 200 W/m°C. For the rubber, the coefficients depend upon many factors, thus representative values are used. The thermal expansion for the elastomer is found in the literature as 5.12E-4 /°C, the thermal conductivity is assumed to be 0.2 W/m°C, see [4]. Comparing the expansivity of these materials, one begins to understand the built-in interfacial strains in the bonded rubber-metal structure.

The ultimate strength of the rubber is diminished at high temperature, therefore compounds should be selected which generate a minimum of heat. It is important to emphasis that the heat generation is given by the energy dissipation in the material and is thus proportional to the strain amplitude of oscillation. A stiffer compound will in general show less heat generation and a smaller temperature rise owing to the smaller amplitude of oscillation. Finally, the thermal boundary condition in the analysis is the experimentally determined temperature value of 94 °C in the center region and the 44 °C on the outside of the aluminum body. Figure 12 shows the temperature distribution in the structure after a steady state thermal equilibrium condition is found. The constraint provided by the rigid rope is clearly seen by the deformed rubber. In Figure 13 the von Mises stress distribution in the aluminum is given. The maximum value increases from the previous 38 MPa to 55 MPa. Thus, the results of this analysis suggest that the thermal load case is more severe that the static load.

In the last analysis the thermal pressure is combined with a statically applied load to the rigid rope. First, the thermal-displacement-coupled analysis establishes thermal equilibrium in the roller. Then, the static load of 9.3 kN is applied and the combined stress distribution determined. Figure 14 shows that the von-Mises stress increases to a maximum value of 98.7 MPa.

10. Conclusions

Rollers used in ropeway systems are typically made up of rubber and aluminum components. In spite of the widespread and necessary use of rubber in engineering, a general working knowledge of rubber properties and technology is rather uncommon. In this paper important concepts and associated mechanical properties
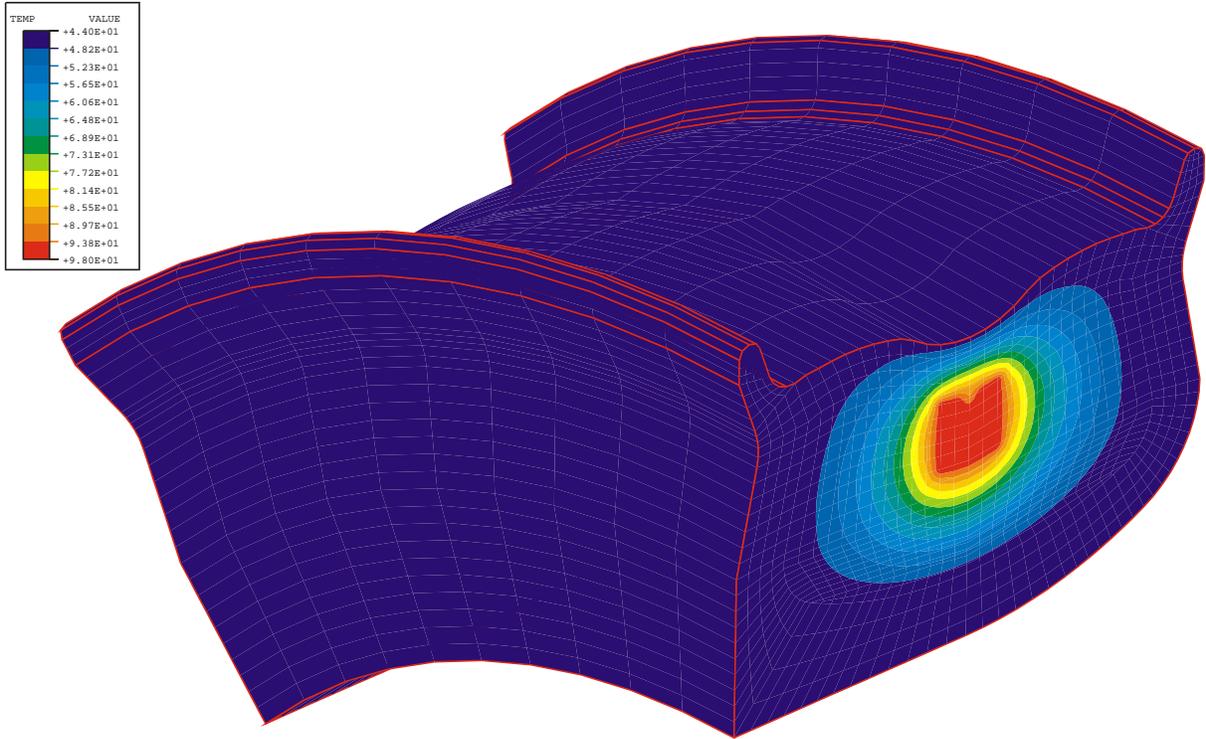
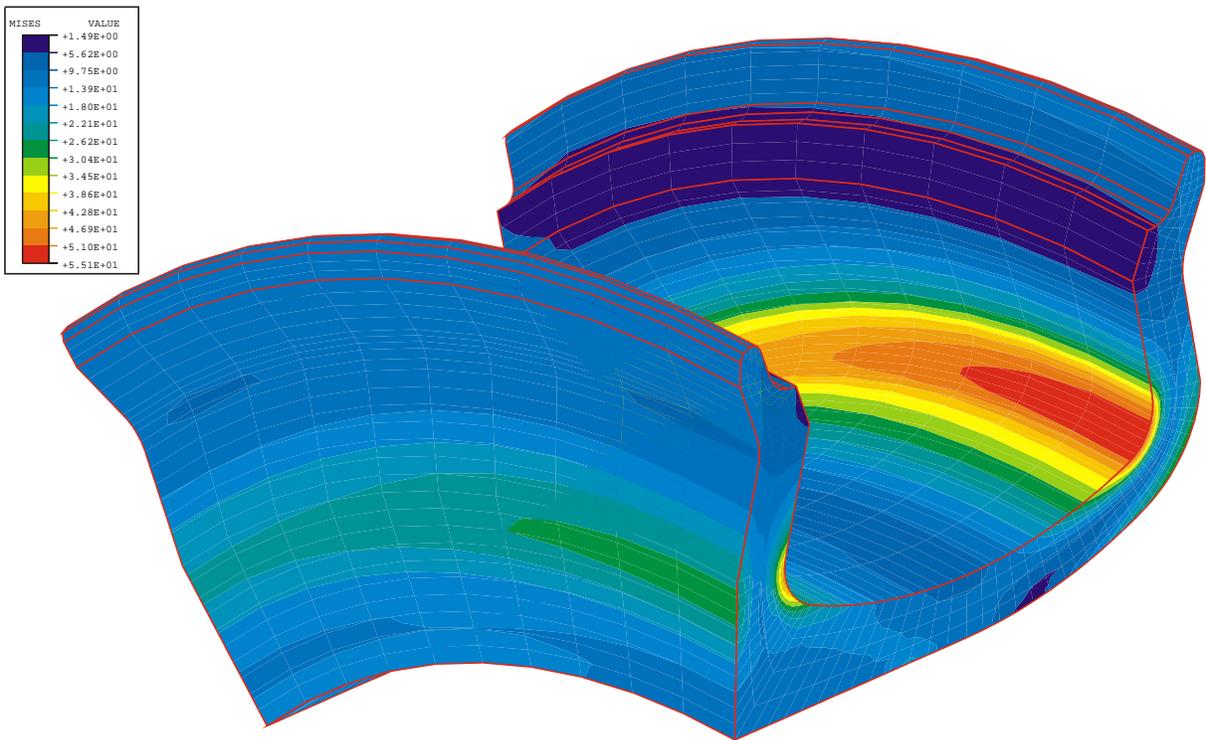**Figure 12: Steady State Temperature Distribution in the Roller**

Figure 13: Von-Mises Stress Distribution due to Thermal Pressure only

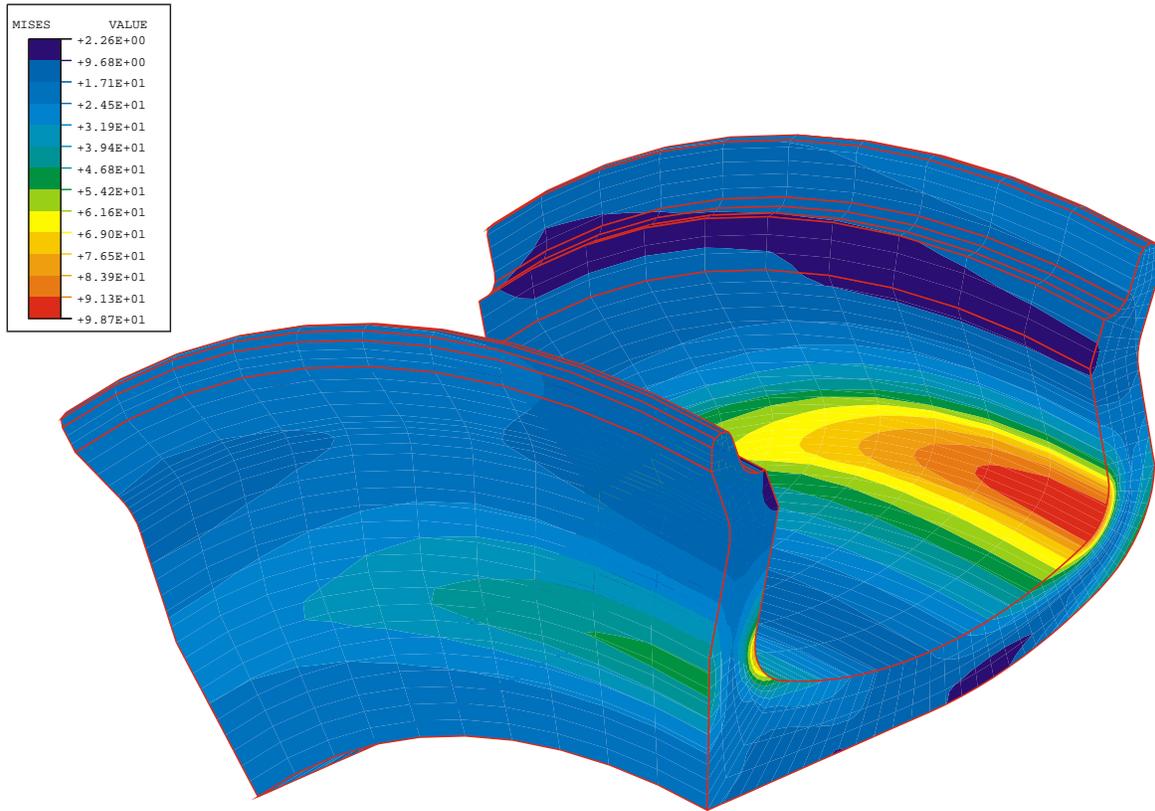| MISES	VALUE
	+2.26E+00
	+9.68E+00
	+1.71E+01
	+2.45E+01
	+3.19E+01
	+3.94E+01
	+4.68E+01
	+5.42E+01
	+6.16E+01
	+6.90E+01
	+7.65E+01
	+8.39E+01
	+9.13E+01
	+9.87E+01

Figure 14: Stress Distribution from Thermal Pressure and 9.3 kN Static Load

of rubber have been reviewed. It is shown that during dynamic loading of rubber, energy is constantly dissipated and converted into heat. It is important to emphasis that the dissipation is proportional to the strain amplitude of oscillation. Thus, a stiffer compound will in general show less heat generation and a smaller temperature rise owing to the smaller amplitude of oscillation. The ultimate strength of the rubber is diminished at high temperature, therefore compounds should be selected which generate a minimum of heat.

Comparing the expansivity of rubber and aluminum, one begins to understand the built-in interfacial strains in the bonded rubber-metal structure. Numerical analysis of rubber-aluminum roller show that the thermal load case generates a more severe stress distribution than a representative static load. The thermal pressure distribution is therefore an important load case in determining the stress distribution in the roller components and must be included during the design process.

11. References

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