DEVELOPMENT AND EVALUATION OF A STREAM TEMPERATURE COMPONENT WITHIN THE PRMS WATERSHED MODELING PROGRAM

by

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A thesis submitted to the Faculty and Board of Trustees of the Colorado School of Mines in partial fulfillment of a Masters of Science Hydrology.

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ABSTRACT

Stream temperature is becoming a very important factor in water quality and the health of many aquatic ecosystems. Computer modeling software can help predict the response of watershed and stream systems to changes in climate or other conditions. This thesis project concerns the development of a new module for the deterministic prediction of stream temperature within the United States Geological Survey's (USGS) Precipitation-Runoff Modeling System (PRMS) watershed surface hydrology model. This module is based on the solution found in the United States Fish and Wildlife Service Stream Network Temperature model (SNTemp), coupled with PRMS meteorologic and hydrologic inputs. The module is called within PRMS to predict average daily stream temperature values. The model was validated in the Potato Creek watershed and matched all parameters of a regression curve fit of natural data to within 6 percent with a determination coefficient ($R^2$) of .77. A sensitivity analysis run using the Fourier Amplitude Sensitivity Testing (FAST) technique suggested that the most sensitive factors are solar radiation, air temperature, and rainfall amount. It was concluded that these will be the strongest factors in terms of propagation of errors in the model.
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CHAPTER 1
INTRODUCTION

Stream temperature is an important factor in the overall water quality of natural streams. Some of the earliest temperature measurements were made on the River Nile by Coutelle from 1799 to 1801 [Webb et al., 2008]. Much more recently, stream temperature changes due to management decisions and natural changes have become more of a focus [Null et al., 2013]. These effects on stream temperatures are often due to problems such as climate change and landscape alterations such as clear-cutting of forested land that can raise temperatures in stream channels [Isaak et al., 2011; Lynch et al., 1984].

Changes in stream channel temperature can have several adverse environmental effects. Temperature rises can result in chemical changes including decreases in dissolved oxygen, and increases in toxic metal solubility [Norton and Bradford, 2009]. Also, the health and reproduction of salmonid fish can be harmed by higher water temperatures. For these reasons, stream temperature has been found to be a major limiting factor for salmonid fisheries and fish populations [Bartholow, 1991]. There are some cases where stream temperature is already limiting these populations and could result in loss of habitat if the trend in water temperature rises. One such case is that of pacific salmon species, responding to regional warming trends caused by climate change [Farrell, 2009; Isaak et al., 2011]. Consequently, accurate prediction of the effect on stream temperatures by future atmospheric and land use change will be necessary for conservation efforts in these and many other watersheds.

Several different models and methods have been used to predict stream temperatures in the past. Some deterministic models use a theoretically-based treatment of the energy budget of a stream segment, whereas others have just calibrated to a single representative variable such as air temperature at the surface. Many commonly-used mechanistic models such as the United States Fish and Wildlife Service (USFWS) Stream Network Temperature Model (SNTemp) and the Army Corps of Engineers CE-QUAL-W2 do not include a rigorous hydrologic component, and thus do not simulate the runoff events. Flows must be preprocessed for input to these models [Norton and Bradford, 2009]. In order to improve upon the processes in such models, this project involves developing and testing a working stream temperature module within the USGS Precipitation-Runoff Modeling System (PRMS).

The PRMS model is developed and supported by the USGS. The model is deterministic, and relies on a modular application of physical process models with most important parameters distributed by Hydrologic Response Units (HRUs)[Markstrom et al., 2008]. PRMS is widely used to predict effects of climate change or land cover change on watershed surface outflows, because it includes meteorological and land cover data in water budget calculations. Due to the distributed parameter design of the model, many important meteorological quantities such as solar radiation and air temperature are already spatially
designated by HRU. As a result, this data can be designated fairly easily by stream segment and used in stream temperature calculations. PRMS has also been coupled with MODFLOW 2005 to create a coupled model called GSFLOW [Markstrom et al., 2008]. MODFLOW is a three dimensional finite difference groundwater flow simulation program also developed and supported by the USGS. It is widely used for a variety of groundwater modeling assessments [Markstrom et al., 2008]. This provides capability in the future to perhaps couple the PRMS module with a model that includes a more rigorous theoretical treatment of subsurface flow processes, but that is beyond the scope of this project.

The USFWS SNTemp model was used as the guide for the calculations and concept of the stream temperature module developed for this project. The result was essentially a coupling of the SNTemp solution with the meteorological and hydrologic processes of PRMS. Several factors led to it being the best fit for type of model to run inside PRMS. The controls of stream temperature include energy transfer across the water-air and water-streambed interfaces as well as lateral inflows at differing temperatures [Mayer, 2012]. The flows are handled by the existing parts of PRMS, so an energy budget type model was needed for the stream temperature module. The daily time step and spatial resolution scale of PRMS was best suited to an equilibrium temperature solution of the type developed by Edinger et al., [1968] or the one in Theurer et al. [1984], from which much of the module structure and equations was taken. This type provided a deterministic model based on the energy budget operable on the spatial and temporal resolution of PRMS watershed models. The module is adaptable and able to be used on a variety of watersheds, not built from a calibration on any specific stream network. Finally, the model would not add a lot of computational expense to what is supposed to be a model that can run on a normal personal computer.

The goal for this project was to create a viable tool for evaluation of climate and landscape change risk factors in watersheds where stream temperature is of concern. This tool would be fully integrated and supported within PRMS, and available to anyone who had the desire to use it. The goal was also to provide prospective users with a guide so they would know how to use the program and apply the model to real-world problems, and to provide some preliminary testing and analysis to evaluate how the coupled model works. This testing and analysis aimed to find to what parameters and quantities the module is particularly sensitive, in order to rank importance to error propagation.
References Cited


CHAPTER 2
PRMS STREAM TEMPERATURE MODULE: PRMS_STRMTEMP
A paper to be submitted as a USGS Open File Report
By Michael Sanders and Steve L. Markstrom

2.1 Introduction

Stream temperature is an important factor that can have a variety of effects on water quality. Water temperature can affect solubility and mobility of toxic metals [Norton and Bradford, 2009] and is also of particular importance in maintaining healthy populations of cold water fish species [Bartholow, 1991]. Consequently, stream temperature modeling predictions can be very important to the management of natural watershed systems. The stream temperature module (PRMS_STRMTEMP) was created to provide users a way to predict stream temperatures in segments within the Precipitation Runoff Modeling System (PRMS).

The purpose of this module is to compute the average stream temperature in each segment of a PRMS surface hydrology model. This temperature is defined as the daily average temperature at the outflow of a particular segment. It can be turned on as an option in the control file and extra parameters will need to be added to get the parameter file. It will then be called and run on each daily time step to put out an average temperature for each stream segment defined in the model.

2.2 Model theory

The stream temperature solution in the module is based on the one found in the US Fish and Wildlife Service Stream Network Temperature Model (SNTemp) [Theurer et al., 1984]. This solution is coupled with the PRMS model, drawing any necessary input available from other PRMS modules and variables. The stream temperature solution is based on an equilibrium temperature concept developed by Edinger et al. [1968]. The equilibrium temperature is defined as the water temperature at which the net energy fluxes across the interfaces of the water with the air and the stream bed will be zero. Thus, for a given set of daily meteorological energy inputs, surface water will tend toward this equilibrium temperature. The equilibrium temperature has been shown to be directly related to average stream temperatures on the daily to monthly time-scale [Bogan et al., 2003], and the net heat flux can be defined in terms of thermal exchange coefficients and this equilibrium temperature [Edinger et al., 1968; Theurer et al. 1984]. The PRMS stream temperature module calculates daily average stream temperatures using this method, relying on calculation of two thermal exchange coefficients as well as the daily equilibrium temperature in each segment. The final stream temperature solution in the module depends on the inflow sources and temperatures as well as the net heat flux estimated by the equilibrium temperature method.
2.3 Energy flux models and equilibrium temperature

The equilibrium temperature calculation in the stream temperature module is based on full stream channel energy balance as shown in Figure 1 below. The energy balance used in the model is given by:

\[
H_n = H_a + H_c + H_d + H_e + H_s + H_v + H_w + H_f
\]

where \(H_n\) is the net heat flux into the stream, \(H_a\) is the flux due to atmospheric emitted longwave radiation, \(H_c\) is the conventional heat flux due to conduction and convection at the air-water interface, \(H_d\) is the heat flux due to conduction at the stream bed-water interface, \(H_e\) is the evaporation heat flux due to the latent heat of vaporization, \(H_s\) is the flux due to solar radiation, \(H_v\) is the riparian vegetation emitted longwave radiation, \(H_w\) is the longwave radiation emitted by the water, and \(H_f\) is the heat dissipated from potential energy by friction. All energy fluxes are given in \(\text{W m}^{-2}\).

Figure 1: Depiction of the possible energy fluxes at the interfaces of a stream channel. From (http://www.fort.usgs.gov/products/Publications/3910/chapter4.html) accessed February 2014.

The equilibrium condition is of course when net heat flux is set equal to zero. Each of these energy exchanges has been shown to be a significant contributor to the energy balance of a stream channel, though to different extents in different cases [Webb et al., 2008]. Thus, it is important to include reasonable models for each term in representing the correct equilibrium temperature.

Some models for energy balance terms were taken from Theurer et al. [1984], whereas others, such as evaporation and solar radiation, can be converted from potentials supplied by the other required modules in PRMS. The PRMS-calculated evaporation rate can be multiplied by the latent heat of vaporization to get the energy flux. Potential solar radiation, however, needs to be modified to account for shading from topographic and riparian vegetation sources. A Bowen Ratio approach is used to generate conventional heat flux from the evaporative heat flux. All of the longwave radiation sources depend on the temperature, emissivity of the source material, and the Stefan-Boltzmann constant.
Atmospheric longwave

The atmosphere scatters a lot of incoming solar radiation and emits longwave radiation according to its temperature and composition. The model used for atmospheric longwave emission is given by:

$$H_a = (1 - r_l)(1 - Sh)(1 + kC_l^2)\varepsilon_a \sigma (T_a + 273.16)^4$$  \hspace{1cm} (2)

where $H_a$ is the energy flux in W m$^{-2}$, $r_l$ is a decimal quantity of the longwave reflection, $Sh$ is the shade factor, $k$ and $C_l$ together constitute the decimal cloud fraction, $\varepsilon_a$ is the emissivity of the air, $\sigma$ is the Stefan-Boltzmann constant, and $T_a$ is the air temperature in degrees C. The emissivity of air depends on the humidity according to:

$$\varepsilon_a = 0.61 + 0.05 \sqrt{e_a}$$  \hspace{1cm} (3)

where $e_a$ is the vapor pressure at the current air temperature.

Vegetation-emitted longwave radiation

In riparian vegetation-shaded zones, shortwave radiation is blocked by the vegetation. However, the vegetation will emit some longwave radiation as a blackbody. The model for this is:

$$H_v = \varepsilon_v \sigma Sh_v (T_a + 273.16)^4$$  \hspace{1cm} (4)

where $\varepsilon_v$ is the emissivity of the vegetation, and $Sh_v$ is the shade due to riparian vegetation. The emissivity of riparian vegetation is a decimal fraction and is taken from a tabulated value; the value used is 0.9526 [Theurer et al., 1984].

Water radiation

The stream water itself also emits longwave radiation as a blackbody. Note that this is a negative flux, or energy leaving the stream. It should also be noted that this depends on our quantity of interest – the water temperature. The equation is another simple formulation of the Stefan-Boltzmann law:

$$H_w = \varepsilon_w \sigma (T_w + 273.16)^4$$  \hspace{1cm} (5)

where $\varepsilon_w$ is the emissivity of water, which is about 0.9526 [Theurer et al., 1984], and $T_w$ is the temperature of the water in degrees C.

Shortwave radiation

The potential solar radiation is already calculated in PRMS, but not all of the potential solar radiation reaches and is absorbed by the stream. The model is given by:

$$H_s = (1 - \alpha)(1 - Sh)H_{sw}$$  \hspace{1cm} (6)

where $\alpha$ is the albedo or fraction reflected by the stream, $Sh$ is the total shade factor, and $H_{sw}$ is the
potential solar radiation reaching the stream.

**Evaporation heat flux**

A large amount of energy is required for the phase change of water from the liquid to the gas phase. This energy does not go to raising the temperature of the water; it is stored in the vapor by virtue of the relative energy in that phase. This means that energy that would otherwise raise the temperature leaves the stream channel through this process, making this another negative energy flux. The evaporation rate is already calculated by PRMS. The evaporation rate is assumed to be the potential evaporation rate since the stream is a free water surface. In order to convert this to an energy flux, we need to simply determine the amount of energy needed to supply the latent heat of vaporization to the mass evaporating. The latent heat of vaporization is temperature dependent and can be approximated in the normal range of stream temperatures by:

\[ \lambda = 2495 \cdot 10^3 - 2360 \cdot T_w \]  

(7)

where \( \lambda \) is the latent heat of vaporization in joules per kilogram [Dingman, 2008]. The evaporation rate \( (E) \) is converted to m s\(^{-1}\). In order to get the mass flux, we can multiply by the density of water which is 1000 kg m\(^{-3}\). This can be multiplied by the latent heat to get energy flux in W m\(^{-2}\), resulting in the following model.

\[ H_e = (2495 - 2.36 T_w) E \cdot 10^6 \]  

(8)

**Conventional heat flux**

At the air-water interface of the stream, heat is also exchanged through the conventional transfer processes of conduction and convection. This transfer is related to the evaporation heat flux through a dimensionless quantity called the Bowen Ratio \( (Bo) \) as follows.

\[ H_c = Bo \cdot H_e \]  

(9)

The Bowen Ratio has a model based on air pressure, humidity, and temperature expressed as:

\[ Bo = \frac{0.00061 \cdot P}{(e_s - e_a)} \cdot (T_w - T_a) \]  

(10)

where \( P \) is the pressure in hPa and \( e_s \) is the saturation vapor pressure at the current \( T_a \). For our purposes, let the term \( 0.00061 \cdot P / (e_s - e_a) \) be called \( B_c \), such that the formula becomes:

\[ Bo = B_c \cdot (T_w - T_a) \]  

(11)

Then, combining equations 8, 9, and 11 our model becomes:
\[
H_c = 10^6 E B_c \left( -2.36 T_w^2 + (2495 + 2.36 T_a) T_w - 2495 T_a \right)
\]  

(12)

It should be noted that this is defined as a negative flux as well. The way it is defined, a positive \(H_c\) means that the water is warmer than the air and energy leaves the stream. Once again, this model depends on our variable of interest, \(T_w\).

**Streambed conduction**

Conduction can also occur at the streambed if a thermal gradient exists. This also depends on the conductivity of the streambed material. The model used to calculate this is:

\[
H_d = \frac{K_g}{\Delta Z} (T_g - T_w)
\]

(13)

where \(K_g\) is the thermal conductivity of the streambed in W m\(^{-1}\) C\(^{-1}\), and \(\Delta Z\) is the equilibrium depth from the water-ground interface at which the temperature is \(T_g\). Note that this is defined as a positive heat flux entering the stream, and that this also depends upon water temperature.

**Stream friction**

Friction can occur in streams as either internal fluid shear or as work done on the boundaries. This converts kinetic energy from the water into heat and warms the stream. The empirical stream friction model used is:

\[
H_f = 9805 \frac{Q \cdot S_f}{W}
\]

(14)

where \(S_f\) is the dimensionless stream gradient, \(Q\) is the discharge in cms, and \(W\) is the average top width in m.

**The energy balance in terms of \(T_w\)**

Now that the models of all energy fluxes are defined, one can see that four of them depend on the water temperature. These include the water-emitted longwave, evaporation heat flux, conventional heat flux, and ground conduction. We can sum all of the terms to create a total energy balance and determine the dependence on water temperature. The resulting energy balance equation has one term that depends on the converted Kelvin temperature raised to the fourth power, there is also a water temperature squared term from the conventional heat flux, as well as a few linear terms. Thus, we can express the energy balance equation as a polynomial function of water temperature as follows:

\[
-H_n = A (T_w + 273.16)^4 - C T_w^2 + B T_w - D
\]

(15)

where:

\[
A = 5.40 \cdot 10^{-3} , \quad \quad 15a
\]
\[ B = 10^6 E \left( B_c \left( 2495 + 2.36 T_a \right) - 2.36 \right) + K_g / \Delta Z , \quad 15b \]
\[ C = 10^6 \cdot E \cdot B_c \cdot 2.36 , \quad 15c \]
\[ \text{and } D = H_a + H_f + H_s + H_v + 2495 \cdot E \left( B_c T_a - 1 \right) + T_g \cdot K_g / \Delta Z . \quad 15d \]

This frames the energy balance in terms of our one unknown of water temperature. Since the equilibrium temperature is defined as the water temperature at which the energy flux is zero, it can be found by using a Newton's Method iteration to find the root of Equation 15 (\( H_n = 0 \)).

### 2.4 Solar shade model

The total solar shade factor in PRMS_Strmtemp is used as was presented by Theurer et al., [1984]. Two major parts are added together for the total shade factor. These are the topographic shade and the riparian vegetation shading, and each have their own factor computed for amount of shortwave radiation intercepted on a given day. This results in the following expression:

\[ Sh = Sh_t + Sh_v \quad (16) \]

where \( Sh \) is the total solar shade factor, \( Sh_t \) is the topographic shade factor, and \( Sh_v \) is the riparian vegetation shade factor.

As in SNTemp, the PRMS stream temperature module gives the user an option to either calculate solar shade within the module, or to simply input values of the total shade factor as a fixed segment parameter for each half of the year representing summer and winter. This option is provided in case the topography and land cover are not well known, or the actual shade factor is known from pre-processing or radiation measurements. Also in some cases, the user may deem an average shade factor sufficient and choose the constant shade factor option. In watersheds at high latitudes, the solar azimuth angle changes dramatically over the course of the year, which affects the amount of shade over time. Thus, it is recommended that the internal shade model be used in this scenario to account for these changes.

The topographic shade occurs when topography blocks direct sunlight from the stream reach for a portion of the day near sunrise and/or sunset. This generally occurs at streams flowing through canyons or hilly terrain and effectively changes the sunrise and sunset time at the location of the stream channel. This shade factor is modeled as a function of the time of year, latitude, stream segment azimuth angle, and topographic altitude angle from the stream channel. Latitude and time of year determine the daily path and timing of the position of the sun in the sky, whereas the stream segment azimuth angle and topographic altitude angle determine the portions of this arc that will be blocked by the surrounding topography. The segment azimuth angle \( A_r \) is the angle in the plane of the ground that the stream makes from the North-South line facing south for streams in the northern hemisphere. This azimuth angle
ranges from \(-\pi/2\) to \(\pi/2\) (all angles in this module are in radians). This defines an East bank and West bank of a stream segment, where the East bank is always on the left hand side facing southward. The topographic altitude angle \(\alpha_t\) is given as the vertical angle from the ground along the line perpendicular to the stream bank to the top of the topographic feature along a reach. A depiction of these angles along with the solar altitude and azimuth angles can be found in Figure 2 below. This can be specified for each of the banks as defined by the reach azimuth angle.

![Diagram of topographic shade calculation](image)

**Figure 2**: Definitions of altitude and azimuth angles in the topographic shade calculation. \(A_r\) and \(\alpha_s\) are the current solar azimuth and altitude respectively. Modified from Theurer et al., 1984.

The topographic shade factor is computed by subtracting the portion of sunlight between the local sunrise and sunset from the total amount of sunlight from sunrise to sunset given a level plain at the specified latitude and time of year. This can be found by relating the integrals of solar altitude \(\alpha_s\) for both cases as in:

\[
Sh_t = 1 - \left( \frac{\int_{h_{sr}}^{h_s} \sin \alpha_s \, dh}{\int_{h_{ss}}^{h_s} \sin \alpha_s \, dh} \right)
\]

where \(h_{sr}\) is the local sunrise hour angle, \(h_{ss}\) is the local sunset hour angle, \(h_s\) is the level-plain sunrise angle, \(h_s\) is the level-plain sunset angle, and \(\alpha_s\) is the solar altitude as a function of date and time. This effectively subtracts the fraction of the sunlight reaching the stream segment from the total potential, leaving the fraction of sunlight blocked by topography. Hour angles are have zero defined as solar noon,
or directly at midday at the top of the sun's arc, and one hour corresponds to $\pi/12$ radians. Equation 17 can be directly evaluated to yield:

$$Sh_t = 1 - \frac{[(h_{ss} - h_{sr})(\sin \phi \sin \delta)] + [(\sin h_{ss} - \sin h_{sr})(\cos \phi \cos \delta)]}{2[(h_s \sin \phi \sin \delta) + (\sin h_s \cos \phi \cos \delta)]}$$

(18)

where $\phi$ is the segment latitude, and $\delta$ is the declination of the sun at the time of year modeled. This declination is given by:

$$\delta = 0.40928 \cos \left( \frac{2 \pi}{365}(172 - D_j) \right)$$

(19)

where $D_j$ is the Julian day.

In order to compute this topographic shade factor, we need the local sunrise and sunset hour angles as well as the level-plain hour angle. The level-plain hour angle can be evaluated by:

$$h_s = \arccos \left[ -\left( \sin \phi \sin \delta \right) / \left( \cos \phi \cos \delta \right) \right]$$

(20)

Because the zero hour angle is set for midday, this has the same magnitude for sunrise and sunset in the level-plain case, and only changes sign. The local sunrise and sunset hour angles are more complicated to find. The sunrise hour angle is defined by

$$h_{sr} = -\arccos \left[ \sin \alpha_{sr} - \left( \sin \phi \sin \delta \right) / \left( \cos \phi \cos \delta \right) \right]$$

(21)

where $\alpha_{sr}$ is the local sunrise solar altitude. The sunset hour angle can be found by substituting the sunset solar altitude into Equation 21 for the sunrise altitude, and by changing the sign so the solution is positive. The sunset or sunrise solar altitude can be found by solving a system of two equations with two unknowns numerically. This system is given by:

$$\alpha_{sr} = \arctan \left[ \left( \tan \alpha_t \right) \left| \sin \left( A_{sr} - A_t \right) \right| \right]$$

(22)

$$A_{sr} = -\arccos \left[ \left( \sin \phi \sin \alpha_{sr} \right) - \left( \sin \delta \right) / \left( \cos \phi \cos \alpha_{sr} \right) \right]$$

(23)

where $A_{sr}$ is the sunrise solar azimuth angle and $\alpha_t$ is the topographic altitude on the sunrise-side bank. This $\alpha_t$ is determined to be the east bank unless $A_t$ is negative and less than the level-plain sunrise solar azimuth. In this case, the sun rises already to the west (or more likely south) side of the stream segment. The numerical solution for $\alpha_{sr}$ is constrained such that needs to be positive and cannot go above the maximum solar altitude for a given day. This is achieved by ensuring that:

$$\sin \alpha_{sr} \leq (\sin \phi \sin \delta) + (\cos \phi \cos \delta)$$

(24)

which is the condition for the maximum solar altitude.

This same system is applied to find the sunset hour angle, with a sign change in Equation 23.
giving the sunset solar azimuth equation. In the sunset model, $\alpha$ is determined to be the west bank, except in the case that the stream channel has a higher positive reach azimuth than the level-plain solar sunset azimuth angle for the day. Once the local sunrise and sunset hour angles as well as the level-plain hour angle have been computed for a segment, the topographic shade factor can be computed from Equation 18.

Once we have the topographic shade, the amount of shade cast by riparian vegetation between local sunrise and sunset can be computed. The riparian vegetation shade is a function of the length of shadows cast on the water, the total width of the stream, and the density of shadow-casting riparian vegetation along the bank of the stream segment. Again, the shade factor is a measure of the shadows cast over some width and percentage of the stream bank as a fraction of the total potential radiation into the stream segment. The riparian shade fraction can be expressed as:

$$Sh_v = \frac{\int_{h_s}^{h_a} (V_d W_s \sin \alpha_s) \, dh}{\int_{h_s}^{h_i} (W \sin \alpha_s) \, dh}$$

(25)

where $V_d$ is the vegetation density along the bank, and $W_s$ is the length of the shadows cast by riparian vegetation at a given solar altitude $\alpha_s$. In the stream temperature module, Equation 25 is numerically integrated for each stream segment using Gaussian quadrature. $W_s$ is constrained to be greater than zero and less than or equal to the width of the stream segment $W$. This instantaneous shadow length can be found using:

$$W_s = [(V_h \cot \alpha) \sin (A_s - A_r)] + \left[\frac{V_c}{2} - V_o\right]$$

(26)

where $V_h$ is the height, $V_c$ is the crown width, and $V_o$ is the offset of the riparian vegetation from the bank, all in units of meters. The vegetative shade calculation also allows for differing parameters on each of the east and west bank. Just like the topographic shade calculation, the sunrise-side bank parameters are used in Equations 25 and 26 before solar noon, and the sunset-side bank parameters used after. Finally, the vegetative shade factor from Equation 25 is added to the topographic shade factor from Equation 18 to get the total portion of potential sunlight blocked as in Equation 16.

2.5 The average stream temperature solution

The average temperature solution in the module utilizes the assumptions of steady flow throughout the day in a roughly prismatic (constant average width/depth) stream channel. This allows use of a steady state equation. In the steady state condition, it is simple to account for all the energy entering and leaving the segment by either advection or heat flux. This leads to the equation for change in temperature along the reach:
\[
dT_w/dx = \left[ (q_l/Q)(T_l - T_w) \right] + \left[ (WH_n)/(Q \rho C_p) \right]
\] (27)

where \( q_l \) is the flux lateral flow per unit length along the segment, \( Q \) is the total discharge at the segment outlet, \( T_l \) is the temperature of the lateral flow, \( T_w \) is the temperature in the channel, \( W \) is the average top width of the channel, \( H_n \) is the net heat flux into the channel, \( \rho \) is the density of water, and \( C_p \) is the specific heat of water [Theurer et al., 1984]. The first bracketed term in this differential equation represents the temperature change due to lateral inflows at a temperature that differs from the upstream inflow from the stream channel. This lateral temperature is calculated within the stream temperature as a weighted average of subsurface and groundwater flow components and their temperature. The temperatures of the lateral flow components are assigned as running averages of air temperature for their respective residence times. Groundwater inflow temperature is always taken to be a running yearly average air temperature throughout a basin, as it usually has a residence time greater than one year. Subsurface flow has an option for the user to set the residence time over which the running average is kept. In a losing stream \( T_l \) is equal to \( T_w \) and the first term is zero. Similarly, \( q_l \) is equal to zero in the case of no lateral flow. The second term tracks the temperature change due to net heat exchange across the boundaries of the stream channel. The entire equation represents the temperature change in a column of water as it moves down the channel. As we know from Equation 2, \( H_n \) is nonlinear with a fourth-order dependence on water temperature. This makes it difficult to solve directly. Thus, we use a solution based on the equilibrium temperature concept.

The equilibrium temperature of each segment as well as the thermal exchange coefficients are calculated in the stream temperature module from meteorological conditions. The modified second-order Taylor expansion between \( T_e \) and the initial water temperature \( T_o \) gives a solvable approximation for the net heat flux of a stream segment. The full derivation of the expansion can be found in Appendix C. This relies on thermal exchange coefficients, which help relate the difference between the actual \((T_w)\) and equilibrium \((T_e)\) water temperatures to the net heat flux across all stream boundaries as given by:

\[
H_n = K_1(T_e - T_w) + K_2(T_e - T_w)^2
\] (28)

where \( K_1 \) and \( K_2 \) are the first and second order thermal exchange coefficients respectively.

The first-order thermal exchange coefficient is represented by \( K_1 \) and is the first derivative of the energy balance Equation 15 evaluated with water temperature at the equilibrium temperature \( T_e \) given by:

\[
K_1 = 4A(T_e + 273.16)^3 - 2C T_e + B
\] (29)

where \( A, B, \) and \( C \) are the same constants from Equation 15. The second-order coefficient \( K_2 \) calculates the actual heat flux at the initial water temperature \( T_o \) of the upstream inflow and corrects the first-order
Taylor series expansion about \( T_e \), giving an approximation of the heat flux function between the two temperatures. This is given by:

\[
K_2 = \frac{H_i - \left[K_1(T_e - T_o)\right]}{\left[(T_e - T_o)^2\right]}
\]  

(30)

where \( H_i \) is the initial net heat flux at temperature \( T_o \), found by evaluating Equation 2 at this initial temperature.

The relation to the equilibrium temperature given by Equation 28 can be substituted into Equation 27 to create an ordinary differential equation given by:

\[
dT_w/dx = \left(\frac{q_i Q}{Q} (T_l - T_w)\right) + \left[\left(W \left( K_1(T_e - T_w) + K_2(T_e - T_w)^2\right)\right)/\left(Q \rho C_p\right)\right]
\]

(31)

which is the second-order Taylor series expansion. The second-order solution to this equation used in the PRMS_Strmtemp module was developed by Theurer et. al. (1984). The solution to Equation 31 has three different cases for gaining, losing, and zero lateral flow stream channels. It is presented this way because the first term goes to zero for some cases, and also because \( Q \) varies in its expression in terms of \( Q_o \) and \( q_i \) between these cases. The general form is given by:

\[
T_w = T_e' - \left(\frac{T_e' - T_o}{1 + \left(K_2/K_1\right)(T_e' - T_o)(1 - R)}\right)
\]

(32)

Where \( R \) is a characteristic temperature -independent constant that changes based on on the case and whether there is incoming lateral flow for the reach. \( T_e' \) is directly dependent on \( T_e \) in all cases, but also varies from case to case. This solution is based on the first-order solution and the zero lateral flow second-order solution, both of which can be directly solved. For the cases with lateral flow, the first-order solutions are corrected according to the form given by the second-order zero lateral flow solution.

If we let

\[
a = (q_i T_l) + \left[\left(K_1 W\right)/\left(\rho C_p\right)\right]T_e
\]

(33)

and

\[
b = q_i + \left(K_1 W\right)/\left(\rho C_p\right)
\]

(34)

then, for case one of a gaining stream with positive \( q_i \), we have:

\[
T_e' = a/b
\]

(35)

and

\[
R = \left[1 + (q_i L/Q_o)\right]^{-b/q_i}
\]

(36)

where \( Q_o \) is the initial channel flow from upstream and \( L \) is the length of the segment.

For case two, losing stream with negative \( q_i \):

\[
T_e' = T_e
\]

(37)
and \[ R = \left[ 1 + \left( \frac{q_l L}{Q_o} \right) \right]^{\frac{(Q_o - b) L}{q_l}} \] (38)

For case three, with zero lateral flow:

\[ T_e' = T_e \] (39)

\[ R = \exp\left[ -\frac{b L}{Q_o} \right] \] (40)

This approach is designed to create a solution that accounts for the heat flux by perturbing from the equilibrium temperature where heat flux is zero. It also adjusts for situations in which lateral flow affects temperature of a stream channel. The thermal exchange coefficients allow for the equilibrium temperature relationship to be calibrated to the specific energy exchange conditions of each individual stream reach. Due to the steady-state flow and thermal exchange assumptions, this model solution is not recommended for prediction of diurnal fluctuations or operation on a shorter than daily time step. It also is not well suited to prediction of temperatures in stream reaches with highly heterogeneous hydraulic properties since it assumes prismatic reaches. It should be noted that a significant change in hydraulic properties, shading, or lateral flow in a stream channel should warrant separation as a discrete segment within a model that is using the stream temperature module, assuming all segments maintain an appreciable travel time for flow.

### 2.6 Module operation and parameters

The PRMS_Strmtemp module is optional, and PRMS will not run it by default. Running the module also requires input of several stream temperature specific parameters, since other PRMS modules contain little in the way of channel hydraulic properties and specific vegetation types. The setup and use of the module should be undertaken only with a good understanding of the model parameters and data required for an accurate simulation.

#### Control file specification and data input

PRMS users are able to turn on the PRMS_Strmtemp module by setting the Stream_temp_flag to 1 in the control file. The cloud-cover module is also necessary for a PRMS run with stream temperature and can be activated by setting the cloud_cov_flag to 1. It is highly recommended to use time series humidity data in the model, although it can be run with a constant humidity. Input of this data requires that the humidity_module be set to climate_hru, and the input data must already be distributed by hru in the format described by Markstrom et al., 2008. The other climate data can be input using either traditional data files and any air temperature and precipitation distribution scheme can be used with the stream temperature module. Control parameters for the stream temperature module are shown in Table 1 below.
Table 1: Stream temperature module control parameters. Data type 1 specifies an integer input, and data type 4 specifies a string

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description</th>
<th>Number of values</th>
<th>Data Type</th>
<th>Default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stream_temp_flag</td>
<td>Designates stream temperature module as off (0) or on (1)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Cloud_cov_flag</td>
<td>Designates cloud cover module as off (0) or on (1)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Humidity_cbh_flag</td>
<td>Designates humidity input by hru as off (0) or on (1)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Humidity_day</td>
<td>Specifies the path of the humidity climate by hru input file</td>
<td>1</td>
<td>4</td>
<td>none</td>
</tr>
</tbody>
</table>

PRMS_Strmtemp variables and input parameters

Parameters required for input to the stream temperature module are given in Table 2. Most parameters in the stream temperature module are designated by segment and thus have a number of values specified by the dimension nsegment. Width_values creates an array of dimension nsegment by a user specified dimension width_dim. Width_flow specifies a range of outflows and width_dim equal intervals in flow using three values, a minimum, a maximum, and the outflow interval given by the range of flow divided by width_dim. This allows the user to put in different stream top widths for each segment for different amounts of outflow and account for changes in width at different stages of flow.

Table 2: Stream temperature module parameter file input parameters

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description</th>
<th>Units</th>
<th>Type</th>
<th>Range</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>albedo</td>
<td>The fraction of shortwave solar radiation reflected by the stream</td>
<td>decimal</td>
<td>real</td>
<td>0 to 1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>seg_length</td>
<td>Length of each segment</td>
<td>m</td>
<td>real</td>
<td>400.0 to 20000.0</td>
<td>1000.0</td>
</tr>
<tr>
<td>Mann_n</td>
<td>The Manning roughness factor for each segment</td>
<td>s m^{-1/3}</td>
<td>real</td>
<td>0.01 to 0.15</td>
<td>0.04</td>
</tr>
<tr>
<td>seg_slope</td>
<td>The stream bed slope of each segment</td>
<td>decimal</td>
<td>real</td>
<td>0.0001 to 0.5</td>
<td>0.015</td>
</tr>
<tr>
<td>azrh</td>
<td>Stream reach azimuth for each segment</td>
<td>radians</td>
<td>real</td>
<td>-1.5708 to 1.5708</td>
<td>0.0</td>
</tr>
<tr>
<td>alte</td>
<td>East bank topographic altitude</td>
<td>radians</td>
<td>real</td>
<td>0.0 to 1.5708</td>
<td>0.0</td>
</tr>
<tr>
<td>Altw</td>
<td>West bank topographic altitude</td>
<td>radians</td>
<td>Real</td>
<td>0.0 to 1.5708</td>
<td>0.0</td>
</tr>
<tr>
<td>Parameter Name</td>
<td>Description</td>
<td>Units</td>
<td>Type</td>
<td>Range</td>
<td>Default Value</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------</td>
<td>-------</td>
<td>------</td>
<td>-------</td>
<td>---------------</td>
</tr>
<tr>
<td>vce</td>
<td>East side vegetation crown width</td>
<td>m</td>
<td>real</td>
<td>0.0 to 20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vde</td>
<td>East bank vegetation density</td>
<td>decimal</td>
<td>real</td>
<td>0.0 to 1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vde_win</td>
<td>Winter vegetation density</td>
<td>decimal</td>
<td>real</td>
<td>0.0 to 1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vhe</td>
<td>East bank vegetation height</td>
<td>m</td>
<td>real</td>
<td>0.0 to 30.0</td>
<td>0.0</td>
</tr>
<tr>
<td>voe</td>
<td>East bank vegetation offset</td>
<td>m</td>
<td>real</td>
<td>0.0 to 30.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vcw</td>
<td>West side vegetation crown width</td>
<td>m</td>
<td>real</td>
<td>0.0 to 20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vdw</td>
<td>West bank vegetation density</td>
<td>decimal</td>
<td>real</td>
<td>0.0 to 1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vdw_win</td>
<td>Winter west bank vegetation density</td>
<td>decimal</td>
<td>Real</td>
<td>0.0 to 1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vhw</td>
<td>West bank vegetation height</td>
<td>m</td>
<td>real</td>
<td>0.0 to 30.0</td>
<td>0.0</td>
</tr>
<tr>
<td>vow</td>
<td>West bank vegetation offset</td>
<td>m</td>
<td>real</td>
<td>0.0 to 30.0</td>
<td>0.0</td>
</tr>
<tr>
<td>gw_init</td>
<td>Initial ground water temperature</td>
<td>Degrees Celsius</td>
<td>real</td>
<td>0.0 to 45.0</td>
<td>1.0</td>
</tr>
<tr>
<td>ss_init</td>
<td>Initial subsurface flow temperature</td>
<td>Degrees Celsius</td>
<td>real</td>
<td>0.0 to 45.0</td>
<td>0.0</td>
</tr>
<tr>
<td>shadeflg</td>
<td>1 for seg_shade input, 0 for internal model calculated shade</td>
<td>None</td>
<td>integer</td>
<td>0 or 1</td>
<td>0</td>
</tr>
<tr>
<td>seg_shade</td>
<td>Constant total shade factor</td>
<td>decimal</td>
<td>real</td>
<td>0.0 to 1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>seg_shade_win</td>
<td>Constant total shade factor for winter vegetation</td>
<td>decimal</td>
<td>real</td>
<td>0.0 to 1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>width_flow</td>
<td>Specifies 3 values: the maximum, minimum, and flow interval for the width values array</td>
<td>m^3/s</td>
<td>real</td>
<td>0.0 to 200,000.0</td>
<td>50</td>
</tr>
<tr>
<td>width_values</td>
<td>Specifies a width for each segment at each interval in flow</td>
<td>m</td>
<td>real</td>
<td>0.0 to 200.0</td>
<td>10</td>
</tr>
<tr>
<td>ss_tau</td>
<td>Basin average subsurface flow residence time</td>
<td>days</td>
<td>integer</td>
<td>0 to 250</td>
<td>30</td>
</tr>
</tbody>
</table>
All declared module variables are listed in Table 3, and are designated by segment. Most are converted by averaging values of variables computed in other PRMS modules from the contributing (and therefore connecting) hru's. There are also two segment parameters that are similarly created by the stream temperature module from other hru parameters, listed in Table 3.

**Table 3:** Stream temperature module declared variables and internally calculated parameters

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
<th>Units</th>
<th>Data Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp_avg</td>
<td>Daily average stream-channel temperature by segment</td>
<td>Degrees Celsius</td>
<td></td>
</tr>
<tr>
<td>T_roff</td>
<td>Daily average air temperature by segment</td>
<td>Degrees Celsius</td>
<td>real</td>
</tr>
<tr>
<td>Seg_ccov</td>
<td>Daily cloud cover factor by segment</td>
<td>decimal</td>
<td>real</td>
</tr>
<tr>
<td>Seg_potet</td>
<td>Potential evaporation rate by segment</td>
<td>in d(^{-1})</td>
<td>real</td>
</tr>
<tr>
<td>Seg_melt</td>
<td>Daily snowmelt amount for each segment</td>
<td>in d(^{-1})</td>
<td>real</td>
</tr>
<tr>
<td>Seg_rain</td>
<td>Daily rainfall by segment</td>
<td>in d(^{-1})</td>
<td>real</td>
</tr>
<tr>
<td>Seg_upstream_inflow</td>
<td>Inflow to a segment from all connecting upstream segments</td>
<td>cfs</td>
<td>real</td>
</tr>
<tr>
<td>Seginc_sroff</td>
<td>Surface runoff entering each segment</td>
<td>cfs</td>
<td>real</td>
</tr>
<tr>
<td>seginc_sssflow</td>
<td>Subsurface flow entering each segment</td>
<td>cfs</td>
<td>real</td>
</tr>
<tr>
<td>seginc_gwflow</td>
<td>Ground water flow entering each segment</td>
<td>cfs</td>
<td>real</td>
</tr>
<tr>
<td>seginc_swrad</td>
<td>Potential shortwave solar radiation for each segment</td>
<td>Langley(s) d(^{-1})</td>
<td>real</td>
</tr>
<tr>
<td>seg_humid</td>
<td>Daily relative humidity by segment</td>
<td>decimal</td>
<td>real</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description</th>
<th>Units</th>
<th>Data Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seg_lat</td>
<td>Latitude of each stream segment</td>
<td>radians</td>
<td>real</td>
</tr>
<tr>
<td>Seg_elev</td>
<td>Elevation of each stream segment from sea level</td>
<td>m</td>
<td>real</td>
</tr>
</tbody>
</table>

These variables are computed and stored in the module, and are available for output as statistical or display variables. They can also be passed to other PRMS modules if needed.

**2.7 Guidelines and considerations for module use**

The PRMS stream temperature module provides a framework for deterministic prediction of daily average stream temperatures at a watershed scale. It is meant to provide a more accessible option for modeling stream temperature, with a full hydrologic model incorporated. The coupling of the SNTemp
solution with PRMS allows for joint calibration of the flows and temperatures. It also puts the model into a modular framework in which different modules used to compute and distribute the input variables found in Table 3 can be chosen and updated. There are many already calibrated PRMS models being used to assess climate change effects, such as the General Circulation Model simulations [Bjerklie et al. 2012, Battaglin et al., 2012]. Many are in areas where future stream temperature scenarios might be of interest. This module is meant to give researchers at the USGS and elsewhere an opportunity to adapt these models to predict stream temperatures.

The module is constrained to run on a daily time step since it is called within PRMS. Also, the equilibrium temperature concept of this module is best suited for average temperatures at daily or longer time-scales [Bogan et al., 2003]. Thus, this module is not suited to the prediction of diurnal fluctuations in stream temperature. Because of the assumptions made in the model, it is also not suited for very heterogeneous stream networks, where there is a lot of variability at small length scales along the stream channels.

Watershed and segment size is also a concern when building a PRMS model to predict stream temperatures. Herb and Stefan, 2011, analyzed the time and length scales at which stream temperature will reach equilibrium given constant conditions. Their study of the Vermillion River watershed found that times to equilibrium varied from about 7 to about 50 hours and lengths from 3 to 45 km, depending on flow and segment properties. While the solution of this module does not require the temperature to be at or very near equilibrium temperature, as a perturbation solution, it could suffer if the segments are much shorter than this length and time scale (for example 100 m segments). At the same time, very long segments could be detrimental to the solution by missing changes in hydraulic properties, shading, or meteorology. Very long segments would tend to make the model assumptions of a steady flow in a prismatic channel with even mixing less valid. The solution is less valid when the travel times of segments greatly exceed the time-step. Also, the stream temperature solution is contains non-linearities, and any errors in upstream segments may propagate unpredictably in watersheds with a very large number of segments. When used and calibrated carefully, the PRMS stream temperature module should prove a useful tool for management at a watershed scale.
2.8 References cited


3.1 Abstract

Stream temperature is becoming a very important factor in water quality and the health of many aquatic ecosystems. Computer modeling software can help predict the response of watershed and stream systems to changes in climate or other conditions. This paper details the development of a module for the deterministic prediction of stream temperature within the United States Geological Survey's (USGS) Precipitation-Runoff Modeling System (PRMS) watershed surface hydrology model. This module is based on the solution found in the United States Fish and Wildlife Service Stream Network Temperature model (SNTemp), coupled with PRMS meteorologic and hydrologic inputs. The module is called within PRMS to predict daily average stream temperatures in each stream channel of a watershed. The module was validated in the Potato Creek watershed and results matched all parameters of a regression curve of natural data to within 6 percent. A sensitivity analysis run using the Fourier Amplitude Sensitivity Testing (FAST) technique suggested that the most sensitive factors are those affecting solar radiation, air temperature, and rainfall amount. It was concluded that these will be the strongest factors in terms of propagation of errors in the model.

3.2 Introduction

Stream temperature can affect water quality and stream ecology in many ways, and is of growing interest in the era of climate change [Null et al., 2012; Isaak et al., 2012]. Changing thermal profiles in streams and watersheds are often the result of problems such as climate change and landscape alterations such as clear-cutting of forested land [Isaak et al., 2011; Lynch et al., 1984]. Fortunately, small and low-cost temperature sensors and loggers are widely available now to provide data to monitor changes [Webb et al., 2008].

Changes in stream channel temperature can have several adverse environmental effects. Temperature rises can result in chemical changes including decreases in dissolved oxygen, and increases in toxic metal solubility [Norton and Bradford, 2009]. Stream water temperatures can affect stratification upon entering a lake or reservoir, potentially altering the cycling of incoming nutrients [Sahoo et al., 2009]. Also, salmonid fish species are of special concern in many cases as their health and reproduction can be harmed by higher water temperatures [Farrell, 2009]. For these reasons, stream temperature has
been found to be a major limiting factor for salmonid fisheries and fish populations [Bartholow, 1991]. There are some cases in which stream temperature is already limiting these populations and could result in loss of habitat for if the water temperature rises. One such case is that of pacific salmon species, responding to regional warming trends due to climate change [Farrell, 2009; Isaak et al., 2012]. Consequently, accurate prediction of the effect on stream temperatures by future atmospheric and land use change will be necessary for conservation efforts in these and many other watersheds.

Several different models and methods have been used to predict stream temperatures in the past. Some models use a theoretically-based treatment of the energy budget of a stream segment, whereas others have just calibrated to a single representative variable such as air temperature at the surface [Caisse et al., 2001], though air temperature based models may have trouble with extrapolation along a linear model [Mohseni and Stefan, 1999]. Many commonly-used mechanistic models such as the USFWS Stream Network Temperature Model (SNTemp) and the Army Corps of Engineers CE-QUAL-W2 do not include a rigorous hydrologic component, and thus do not simulate the runoff events. Flows must be preprocessed for input to these models [Norton and Bradford, 2009]. In order to improve upon the processes in such models, a module for stream temperature predictions was developed within the USGS Precipitation-Runoff Modeling System (PRMS). The module was coded in Fortran 90 and is optionally called each time step by the model after flows have been computed by the other modules. The stream temperature portions of the model are largely borrowed from or inspired by those in the USGS SNTemp model [Theurer et al., 1984].

The PRMS surface hydrology model is continually developed and supported by the USGS. The model is deterministic, and relies on a modular application of physical process models with most important parameters distributed by Hydrologic Response Units (HRUs) [Markstrom et al., 2008]. These HRUs are user defined in area and shape as a contributing area to a single stream segment. The model uses a series of reservoirs of finite or infinite capacity to track the flow of water from precipitation until evaporation, storage, or outflow. In addition to calculating flows, other requisite quantities such as evaporation flux are also already calculated in the model. Due to the distributed parameter design of the model, many important meteorological quantities such as solar radiation and air temperature are already spatially designated by HRUs. This allows the meteorological data to be designated easily by stream segment. PRMS has also been coupled with MODFLOW 2005 to create a coupled model called GSFLOW [Markstrom et al., 2008]. MODFLOW 2005 is a three-dimensional finite-difference groundwater model, capable of simulating aquifer and subsurface flows. This provides capability in the future to perhaps couple the stream temperature module with a model that includes a more rigorous mechanistic treatment of subsurface flow processes, but that is beyond the scope of this project.
3.3 Model Theory and Equilibrium Temperature Concept

SNTemp is a stand-alone stream temperature model that was developed by the US Fish and Wildlife Service. It is a one-dimensional, steady state model that computes average stream temperatures at a daily to monthly timescale. It has been applied effectively in many studies including one by Norton and Bradford [2008]. They were able to calibrate the model to predict average temperatures in the Speed River basin in southern Ontario to a mean absolute error of between 0.2 and 1.8 degrees C. SNTemp operates on an equilibrium temperature concept. The theory behind this is that the average water temperature in a stream channel is related to the temperature of the water when an equilibrium with current meteorological conditions is reached. In other words, this is the temperature of the water in the stream channel when all of the energy fluxes in and out of the stream balance out to a net energy flux of zero. Theoretically, the temperature will asymptotically approach this equilibrium condition in a column of water as it moves downstream, assuming flow is constant and the meteorological and the energy flux conditions are unchanging [Theurer et al., 1984]. The temperature will rise (or fall) to a point at which the longwave radiation emitted by the stream water balances out the surplus (or deficit) incoming energy flux. The net heat flux can be defined in terms of thermal exchange coefficients and this equilibrium temperature can lead to an analytical solution for average stream temperature [Edinger et al., 1968; Theurer et al. 1984]. The solution in SNTemp and the PRMS stream temperature module is based on this expression of the net heat flux and also takes into account sources and temperatures of all inflows.

Many other deterministic temperature models have also been successfully based on this concept, including the Regional Equilibrium Temperature Model (RTEMP), which was developed by Watercourse Engineering, Inc. [Null et al., 2012]. A successful equilibrium temperature model was also implemented by Herb and Stefan, [2011]. Pike et al., [2013] used a stochastic dynamics approach, which allows for evaluation of uncertainty. This type of approach is expensive to run at a network scale and not appropriate for operation within PRMS. Some other models such as MNSTREM [Sinokrot and Stefan, 1993] and Heat Source [Boyd and Kasper, 2003] choose to employ a one-dimensional advection-dispersion type equation using a finite difference method, allowing for a much higher resolution of calculations in space and time as well as including mixing and dispersive properties. However, this sort of method is more computationally expensive, and also would be much more difficult to link directly into PRMS. This is due to the non-uniform spatial discretization of PRMS in HRUs as well as PRMS being more of a hydrologic than a hydraulic model. Also, much of the benefit of this approach would be lost at the daily time-scale for output from PRMS. For these reasons, the equilibrium approach used in SNTemp and outlined in Theurer et al., [1984] was used for the module created within PRMS.

Bogan et al. [2003] showed that average stream temperature was directly related to equilibrium
temperature for daily or longer time scales. As the time scale increases, so does the strength of the relationship between the two (Figure 3). Average temperatures were found to have a clear linear relationship with equilibrium temperatures, though in many cases there was a slope well below one. This is likely due to the strong effects of lateral inflows on the temperature in streams where these are more prevalent, reducing the relative effects of energy transfer at the air-water interface. Lateral flows such as groundwater come in at a relatively constant temperature, because they have a long residence time in the ground where they are shielded from meteorological fluctuations. In gaining reaches, groundwater inflows can significantly affect the temperature. The PRMS stream temperature module will run on a daily time step, since that is what PRMS already uses. This will also help account for daily fluctuations from storm event inflows that can cause large temperature swings on that timescale. The effects these event inflows are a likely source of some of the increased uncertainty in the average and equilibrium temperature relationship on a daily time step.

![Figure 1](image)

**Figure 1:** The relationship between calculated equilibrium and average measured stream temperatures on record at a USGS gaging station on Yellow Creek in Alabama [Bogan et al., 2003]

### 3.4 Module Calculations

The equilibrium temperature calculation in the stream temperature module is based on full stream channel energy balance as shown in Figure 1 below. The energy balance used in the model is given by:
\[ H_n = H_a + H_c + H_d + H_e + H_s + H_v + H_f \]  

where \( H_n \) is the net heat flux into the stream, \( H_a \) is the flux due to atmospheric emitted longwave radiation, \( H_c \) is the conventional heat flux due to conduction and convection at the air-water interface, \( H_d \) is the heat flux due to conduction at the stream bed-water interface, \( H_e \) is the evaporation heat flux due to the latent heat of vaporization, \( H_s \) is the flux due to solar radiation, \( H_v \) is the riparian vegetation emitted longwave radiation, \( H_w \) is the longwave radiation emitted by the water, and \( H_f \) is the heat dissipated from potential energy by friction. All energy fluxes are in W m\(^{-2}\).

The equilibrium condition is of course \( H_n \) from Equation 1 is set equal to zero. Each of these energy exchanges has been shown to potentially be a significant contributor to the energy balance of a stream channel, though to different extents in different cases [Webb et al., 2008]. Thus, it is important to include reasonable models for each term in representing the correct equilibrium temperature.

![Figure 2: Depiction of the possible energy fluxes at the interfaces of a stream channel.](http://www.fort.usgs.gov/products/Publications/3910/chapter4.html) Accessed Feb. 2014

Some models for energy balance terms were used from Theurer et al. [1984], whereas others, such as evaporation and solar radiation, can be converted from potentials supplied by the other required modules in PRMS. The PRMS-calculated evaporation rate can be multiplied by the latent heat of vaporization to get the energy flux. Potential solar radiation, however, needs to be modified to account for shading from topographic and riparian vegetation sources. A full discussion of the shade model used in the PRMS stream temperature module can be found in Appendix A. A Bowen Ratio approach is used to generate conventional heat flux from the evaporative heat flux. All of the longwave radiation sources
depend on the temperature, emissivity of the source material, and the Stefan-Boltzmann constant. Because several of the heat flux components depend on the water temperature, Equation 1 can be expressed as a polynomial function of water temperature \( T_w \) as follows:

\[
-H_n = A(T_w + 273.16)^4 - C T_w^2 + B T_w - D
\]

(2)

where \( A, B, C, \) and \( D \) are constants based on average conditions for each time step. A derivation of this equation and its coefficients along with the models for each energy flux can be found in Appendix B. This frames the energy balance in terms of our one unknown of water temperature. Since the equilibrium temperature is defined as the water temperature at which the energy flux is zero, it can be solved for by using a Newton's Method iteration to find the root of Equation 2 \((H_n = 0)\).

The average temperature solution in the module utilizes the assumptions of steady flow throughout the day in a roughly prismatic (constant average width/depth) stream channel. This allows use of a steady state equation. In the steady state condition, it is simple to account for all the energy entering and leaving the segment by either advection or heat flux. This leads to the equation for change in temperature along the reach:

\[
\frac{dT_w}{dx} = \left[ \left( \frac{q_l}{Q} \right) \left( T_l - T_w \right) \right] + \left[ \left( \frac{W H_n}{Q \rho C_p} \right) \right]
\]

(3)

where \( q_l \) is the flux lateral flow per unit length along the segment, \( Q \) is the total discharge at the segment outlet, \( T_l \) is the temperature of the lateral flow, \( T_w \) is the temperature in the channel, \( W \) is the average top width of the channel, \( H_n \) is the net heat flux into the channel, \( \rho \) is the density of water, and \( C_p \) is the specific heat of water [Theurer et al., 1984]. The first bracketed term in this differential equation represents the temperature change due to lateral inflows at a temperature that differs from the upstream inflow from the stream channel. In a losing stream \( T_l \) is equal to \( T_w \) and the first term is zero. Similarly, \( q_l \) is equal to zero in the case of no lateral flow. The second term tracks the temperature change due to net heat exchange across the boundaries of the stream channel. The entire equation represents the temperature change in a column of water as it moves down the channel. As we know from Equation 2, \( H_n \) is nonlinear with a fourth-order dependence on water temperature. This makes it difficult to solve directly. Thus, we use a solution based on the equilibrium temperature concept.

The equilibrium temperature of each segment as well as the thermal exchange coefficients are calculated in the stream temperature module from meteorological conditions. The modified second-order Taylor expansion between \( T_e \) and the initial water temperature \( T_o \) gives a solvable approximation for the net heat flux of a stream segment. The full derivation of the expansion can be found in Appendix C. This relies on thermal exchange coefficients which help relate the difference between the actual \( (T_w) \) and equilibrium \( (T_e) \) water temperatures to the net heat flux across all stream boundaries as given by:
\[ H_n = K_1 (T_e - T_w) + K_2 (T_e - T_w)^2 \] (4)

where \( K_1 \) and \( K_2 \) are the first and second order thermal exchange coefficients respectively.

The first-order thermal exchange coefficient is represented by \( K_1 \) and is the first derivative of the energy balance Equation 2 evaluated with water temperature at the equilibrium temperature \( T_e \), given by:

\[ K_1 = 4A(T_e + 273.16)^3 - 2CT_e + B \] (5)

where \( A, B, \) and \( C \) are the same constants from Equation 2.

The second-order coefficient \( K_2 \) calculates the actual heat flux at \( T_o \) of the upstream inflow and corrects the first-order Taylor series expansion about \( T_e \) to estimate the heat flux function between the two temperatures. This is given by:

\[ K_2 = (H_i - [K_1(T_e - T_o)])/[((T_e - T_o)^2] \] (6)

where \( H_i \) is the net heat flux at initial temperature \( T_o \), found by evaluating Equation 2 at this temperature.

The relation to the equilibrium temperature given by Equation 4 can be substituted into Equation 3 to create an ordinary differential equation given by:

\[ dT_w/dx = [(q_l/Q)(T_i - T_w)] + [(W(K_1(T_e - T_w) + K_2(T_e - T_w)^2)]/[Q \rho C_p)] \] (7)

which is the the second-order Taylor series expansion. The second-order solution to this equation used in the PRMS_Strmtemp module was developed by Theurer et. al. [1984]. The solution to Equation 7 has three different cases for gaining, losing, and zero lateral flow stream channels. It is presented this way because the first term goes to zero for some cases, and also because \( Q \) varies in its expression in terms of \( Q_o \) and \( q_l \) between these cases. The general form is given by:

\[ T_w = T'_e - \frac{(T'_e - T_o) R}{1 + (K_2/K_1)((T'_e - T_o)(1 - R))} \] (8)

Where \( R \) is a characteristic temperature -independent constant that changes based on on the case and whether there is incoming lateral flow for the reach. \( T'_e \) is directly dependent on \( T_e \) in all cases, but also varies from case to case. This solution is based on the first-order solution and the zero lateral flow second-order solution, both of which can be directly solved. For the cases with lateral flow, the first-order solutions are corrected according to the form given by the second-order zero lateral flow solution.

If we let

\[ a = (q_lT_i) + [(K_1W)/\rho C_p]T_e \] (9)

and \( b = q_l+[K_1W]/\rho C_p \) (10) then, for case one of a gaining stream with positive \( q_o \), we have:
\[ T_e' = a/b \] (11)
and \[ R = \left[ 1 + \left( q_l L / Q_o \right) \right]^{-b/q_l} \] (12)
where \( Q_o \) is the initial channel flow from upstream and \( L \) is the length of the segment.

For case two, losing stream with negative \( q_l \):
\[ T_e' = T_e \] (13)
and \[ R = \left[ 1 + \left( q_l L / Q_o \right) \right]^{-b/q_l} \] (14)

For case three, with zero lateral flow:
\[ T_e' = T_e \] (15)
\[ R = \exp \left[ -\left( b L / Q_o \right) \right] \] (16)

This approach is designed to create a solution that accounts for the heat flux by perturbing from the equilibrium temperature, where net heat flux is zero. It also adjusts for situations in which lateral flow affects temperature of a stream channel. The thermal exchange coefficients allow for the equilibrium temperature relationship to be calibrated to the specific energy exchange conditions of each individual stream reach. Due to the steady-state flow and thermal exchange assumptions, this model solution is not recommended for prediction of diurnal fluctuations or operation on a shorter than daily time step. It also is not well suited to prediction of temperatures in stream reaches with highly heterogeneous hydraulic properties since it assumes prismatic reaches.

### 3.5 Coupling of SNTemp Solution to PRMS

The coupling of SNTemp to PRMS involves more than using the flows generated by PRMS in the original stand-alone model. The processes involved in the coupling of the two models are overviewed in Figure 3.

As shown in Figure 3, some of the major energy fluxes are determined by information calculated by other modules or received as input by PRMS. In the SNTemp stand-alone version of the model, the evaporation heat flux and conventional heat flux were computed by an empirical model based on the water and air temperatures. The solar radiation was also calculated and distributed by an internal model in this stand-alone program. PRMS has its own values for potential evaporation as well as daily solar radiation. In fact PRMS provides options in the form of other modules for calculating these important quantities. This is why the PRMS potential evaporation rate is used to obtain the value of \( H_r \) in the stream temperature module, as shown in Appendix B.
The evaporation heat flux as well as the air temperature is used to find the conventional heat flux $H_c$, since a Bowen Ratio approach is used for this model. The air temperature and solar radiation can be input and distributed in a variety of ways in PRMS. This improves one the system of using adiabatic corrections with elevation based on only one station, like in the SNTemp stand-alone. This gives the user much more freedom and control of the meteorology in the watershed being modeled. Thus, the calculations used for energy budget terms $H_e$, $H_c$, and $H_s$ (models presented in Appendix B) are fundamentally altered from the calculations used by the stand-alone temperature model. Since these are some of the most important terms in the energy balance [Sinokrot and Stefan, 1993], the equilibrium temperature and final solution could be significantly affected by differences in the models used.

The coupling also involves assigning temperatures to the inflows. The lateral inflow amounts are already designated by stream segment in another PRMS module. The temperatures of these inflows have to be estimated, however. Surface runoff is assumed to be at temperature equilibrium with the air and is assigned the average air temperature on the date of the rain event and runoff. The subsurface and groundwater temperatures are estimated using running averages of the air temperature over a certain residence time. Groundwater residence times are often greater than a year, and so the average temperature over the previous year is assigned to this lateral flow source. Subsurface interflow can vary
greatly in residence time. Because of this, the stream temperature module allows the user to specify a number of days over which to keep a running average of air temperature to be assigned to all subsurface inflows. This is one of the challenges of running the model in PRMS, since the spatial discretization scale and lack of explicit water table resolution do not allow for much tracking of a heat budget in aquifers and the soil zone. However, this system allows all lateral flow sources to be assigned different temperatures that are close and correlate to the actual temperatures of the inflows. The final, flow weighted average of lateral flow temperatures from these sources should provide a better system for estimating this lateral flow temperature than the single lateral flow input option in the stand-alone SNTemp model.

3.6 Model Validation

Following development of the module, it was tested on an idealized case in order to verify the temperature calculations. The test case was a small watershed containing two segments each contributed to by four HRUs of equal area. A specific equilibrium temperature of roughly 20 degrees celsius due to the constant meteorological conditions applied was targeted by a pre-defined calculation of the heat flux equation. The other terms including equilibrium temperature were pre-defined and the shortwave radiation input required for this equilibrium condition was solved for. Then, the outflow data was pulled from the model run, and the daily stream temperature calculations were re-done in an excel spreadsheet for each of the segments.

The results obtained from each calculation of the solution were very similar with both quickly settling at an equilibrium temperature near 20 Celsius and a constant flow rate near the target. Segment one showed a Root Mean Square Error (RMSE) of .004 degrees celsius between the two calculations of average temperature which is .02% of the steady temperature value, and segment two showed a .008 degree RMSE for a difference of .04%. Even though my model cannot be truly verified [Oreskes et al., 1994], this was a good double check that the model is reproducible and has no obvious discrepancies from the intended calculations.

3.6.1 Potato Creek Basin

The Potato Creek Basin is a small watershed located in the Flint River Basin in Western Georgia, USA (Figure 3). There was a USGS gage with id 02346500 located on Potato Creek near the outflow of the basin from the 1950's through Summer of 1976 [LaFontaine et al., 2013]. The drainage area of the basin is 482 square kilometers. This area was chosen because previous work had been done by Markstrom et al. [2014] on stream temperature in the area.
Figure 4: Shows the geographic location and extent (in green) of the Potato Creek Basin and its stream gage within the greater Apalachicola-Chattahoochee-Flint river basin. [Markstrom et al., 2014]

3.6.2 Validation Methods and Results

Dyar and Alhadeff [1997] analyzed stream temperature measurements taken at gage 02346500 in Potato Creek. The measurements used were a compilation of historical tests that had been taken at uneven intervals in the years between the 1950s and 1970s, and were deemed enough to model the yearly mean stream temperature pattern. They performed a regression analysis involving this and other streams in Georgia in which they fit the historical data with a sine curve to illustrate the seasonal periodicity of the stream temperature. The harmonic regression curve used to fit the model was of the form:

\[ T = M + A \sin (b t + c) \]  

(17)

where \( T \) is the average water temperature on day \( t \), \( t \) is the day of the current water year where \( t=1 \) is October 1, \( M \) is the yearly mean temperature, \( A \) is the amplitude of the sine curve, \( b \) is a constant converting day of year to arc length equal to \( 2\pi / 365 \), and \( c \) is the phase coefficient of the curve. A loosely-coupled sequential run of PRMS and SNTemp using a framework called P2S was calibrated to this match this regression by Markstrom et al. [2014]. Since recent and daily or distributed temperature data was unavailable in this watershed, Markstrom used the regression curve from historical data in order to calibrate his model runs. This model was chosen as a sort of quick validation test case to evaluate how the PRMS stream temperature module matched this seasonal pattern and the daily predictions from the P2S run.
For this validation case, a calibrated version of the Potato Creek PRMS watershed model was obtained from researchers working on the greater Appalachicola-Chattahoochee-Flint basin model and its sub-basins [LaFontaine et al., 2013]. This supplied the flow calibration for the validation simulation. Stream temperature module input parameters were adapted from those used by Markstrom et al. [2014] for the P2S simulations. The only available real temperature data was sparse and had uneven temporal sampling with 103 measurements occurring between July 1956 and June 1974 [Dyar and Alhadeff, 1997; Markstrom et al., 2014]. For this reason, the harmonic regression function given in Equation 17 was used to characterize the stream temperature in the P2S run and for this study. The P2S parameters adapted for this validation case were calibrated to minimize the difference in this seasonal pattern. The run of PRMS with the same parameter values used in the P2S run yielded a different set of daily average temperature data than that obtained by Markstrom et al. [2014]. The seasonal pattern closely matched, however. The results were compared to those of Dyar and Aldeheff, [1997] and Markstrom et al, [2014], and are illustrated in Table 1 and Figure 4.

Table 1: Compares regression parameters from fits to real data, PRMS stream temperature, and P2S simulations for the stream gage 02346500 in Potato Creek

<table>
<thead>
<tr>
<th>Harmonic properties computed from measured data (as reported by Dyar and Alhadeff, 1997, table 3)</th>
<th>Harmonic properties computed from PRMS with stream temperature simulation (percent bias of simulated value)</th>
<th>Harmonic properties computed from P2S simulation (percent bias of simulated value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ ($^\circ$C)</td>
<td>$A$ ($^\circ$C)</td>
<td>$C$ (radians)</td>
</tr>
<tr>
<td>16.7</td>
<td>9.8</td>
<td>2.8</td>
</tr>
</tbody>
</table>

(1%) (6%) (6%) (1%) (2%) (1%)

Figure 5: Plotted points and regression curves from the PRMS simulation ($r^2 = .77$) and the P2S simulation data from Markstrom et al [2014], compared with the fit from Dyar and Aldeheff, [1997].
The harmonic regression fit provided by the PRMS with stream temperature pretty closely matches the one fit to historical data [Dyar and Alhadeff, 1997]. The optimum curve fit of this form had an coefficient of determination ($r^2$) value of .7706. Shown in Figure 5, the data from each modeling effort follows the curve closely except in Winter, where there is a lot more variability in daily average temperatures. This is likely due to periods of low flow that occurred in Winter during the two year period shown in Figure 5.

It should be noted that the runs of PRMS used in this study and Markstrom, et al. produced different outflows at the gaged segment. This could be due to different versions of the model and the PRMS program used, with the original calibration used by Markstrom et al [2014] unable to be reproduced in this study. Markstrom et al. used an older released version of the PRMS model, whereas this study used the version yet to be released with the stream temperature module and other modifications. This affected flow and could have affected evaporation or other important quantities as well. The model run with the PRMS stream temperature module produced lower overall outflows almost across the board than the data used for the P2S simulation. This could account for the larger discrepancy in phase coefficient shown by the curve fit from this study, when using the same calibration parameters. Lower flows may have led to less lag between equilibrium temperature changes and average stream temperature changes shifting the phase of the curve. Also, this accounts for the daily differences between the P2S and PRMS- modeled average stream temperatures for this segment.

3.7 Sensitivity Analysis

Sensitivity analysis is a useful tool for deterministic models to estimate the relative effects of different input parameters on the model output. This estimation of importance can tell users which parameters can propagate the most error into the model output, and thus which are most important to use accurately. For the sensitivity analysis of PRMS with stream temperature, I used a global sensitivity analysis method called the Fourier Amplitude Sensitivity Test (FAST) method.

3.7.1 FAST Background

FAST is a global sensitivity analysis method that was developed by Cukier et al. in 1973. This sensitivity method computes what is known as the main effect of a parameter which is given by:

$$\frac{\text{var}_x[E(y|x)]}{\text{var}(y)}$$

(18)

where $x$ is a model parameter and $y$ is the model output value. Thus, the main effect can be described as the portion of the variance in output values that is accounted for by the variance of the parameter $x$.

FAST uses a unique method to globally sample a parameter space and isolate the main effect of each parameter by simultaneously varying all parameters. It isolates the effect of each parameter by sampling each at its own frequency out of a carefully varied set [McRae et al., 1982]. This allows the effect of each...
parameter on the variance of the model to be isolated by using the Fourier coefficient at its frequency.

3.7.2 FAST Methods and Results

The sensitivity analysis for the PRMS with stream temperature model was performed using the 'fast' package in R. Module parameters and other important parameters were chosen for which to evaluate the sensitivity of the model output. A set of 13 parameters were chosen, along with normal ranges they might cover within a watershed. A description of the parameters evaluated is given in Table 2. Then, the fast.parameters function was used to determine a set of frequencies according to the pattern reported in Mcrae et al., 1982 and to generate a series of parameterizations for 915 separate model runs. It should be noted that the set of parameter ranges are transformed to a unit hypercube in parameter space before the frequencies are assigned so that the resulting sensitivities are scaled. The models were all run over a two year time period and put out average stream temperature results for 103 stream segments within the Potato Creek model. The results were all compared to those of a single reference model and a file was made with the RMSE for each model output in relation to the reference model. Then, the sensitivity function in the R package was used to calculate the main effect of each parameter on the variance in the output errors (RMSE) in the set of model runs. The average main effect of each parameter over all output segments, and the range over which each parameter was varied is presented in Table 3.

Table 2: List of sensitivity analysis parameters with descriptions.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>alte</td>
<td>The altitude angle of the topography on the east bank. This affects the total shade factor (Appendix A).</td>
<td>radians</td>
</tr>
<tr>
<td>tmax_cbh_adj</td>
<td>An adjustment of the maximum temperature each day up (positive) or down (negative).</td>
<td>Degrees C</td>
</tr>
<tr>
<td>rain_cbh_adj</td>
<td>An adjustment of the daily rainfall amount during rain events</td>
<td>inches</td>
</tr>
<tr>
<td>azrh</td>
<td>The azimuth angle of the stream from the North-South direction (defined facing South). Affects shade (Appendix A).</td>
<td>radians</td>
</tr>
<tr>
<td>dday_slope</td>
<td>The slope of the relationship between daily air temperature and solar radiation.</td>
<td>decimal</td>
</tr>
<tr>
<td>seg_length</td>
<td>Gives the length of each segment defined by the model</td>
<td>meters</td>
</tr>
<tr>
<td>vde</td>
<td>Represents the fraction of east bank shaded by riparian vegetation (App. A)</td>
<td>decimal</td>
</tr>
<tr>
<td>width_values</td>
<td>Gives the width of each segment defined by the model</td>
<td>meters</td>
</tr>
<tr>
<td>albedo</td>
<td>Fraction of solar radiation reflected by the stream</td>
<td>decimal</td>
</tr>
<tr>
<td>jh_coef_hru</td>
<td>The Jensen-Haise coefficient used in calculating the potential evaporation</td>
<td>-</td>
</tr>
<tr>
<td>vhe</td>
<td>Average riparian vegetation height on east bank (Appendix A)</td>
<td>meters</td>
</tr>
<tr>
<td>vce</td>
<td>Average riparian vegetation crown width on east bank (Appendix A)</td>
<td>meters</td>
</tr>
<tr>
<td>voe</td>
<td>Average riparian vegetation offset from east bank</td>
<td>meters</td>
</tr>
</tbody>
</table>
Table 3: The ranges of sensitivity analysis parameters and the main effect on error of results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>alte (radians)</th>
<th>tmax_cbh_adj (degrees C)</th>
<th>rain_cbh_adj (inches)</th>
<th>azrh (radians)</th>
<th>dday_slope (decimal)</th>
<th>seg_length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>0.0 to 0.78</td>
<td>-10.0 to 10.0</td>
<td>.1 to 2.0</td>
<td>-1.57 to 1.57</td>
<td>0.3 to 0.8</td>
<td>500 to 8000</td>
</tr>
<tr>
<td>Main Effect</td>
<td>.433</td>
<td>0.09</td>
<td>0.0694</td>
<td>0.0334</td>
<td>0.0213</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>vde (decimal)</th>
<th>width_values (m)</th>
<th>albedo (decimal)</th>
<th>jh_coef_hru</th>
<th>vhe (m)</th>
<th>vce (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>0.1 to 0.9</td>
<td>1.5 to 8</td>
<td>.07 to .4</td>
<td>10.0 to 25.0</td>
<td>3.0 to 20.0</td>
<td>4.0 to 19.0</td>
</tr>
<tr>
<td>Main Effect</td>
<td>0.0053</td>
<td>0.0091</td>
<td>0.0077</td>
<td>0.00165</td>
<td>0.000118</td>
<td>0.00034</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>voe (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>1.0 to 5.0</td>
</tr>
<tr>
<td>Main Effect</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

The total of the main effects in this sensitivity run was equal to .689, indicating that there is variance in the uncertainties left unexplained by these main effects. This means that the analysis cannot be taken entirely quantitatively, but it can still be used to place the parameters in order of importance in a more qualitative fashion. The east side topographic altitude, given by alte was by far the largest contributor to differences between model runs according to the analysis. Other important parameters include the adjustments to daily high temperatures and precipitation amounts, tmax_cbh_adj, and rain_cbh_adj respectively. The stream reach azimuth angle (azrh), slope of the potential radiation-air temperature relationship (dday_slope), and segment length (seg_length) also had measurable effects. The riparian vegetation shade parameters and Jensen-Haise evaporation coefficient appeared to have created very little of the differences in the model runs. There are a few possible reasons for the failure of the FAST-generated main effects to describe the entirety of the model differences. One possibility is that the parameter space was not sampled rigorously enough. Another thing that might account for this discrepancy is that some of the parameters may be interacting with each other and not contributing to the differences in results independently enough to see all of the variance explained by main effects. Also, the stream temperature PRMS module is a complicated non-linear model, and perhaps errors are accumulating somehow in unpredictable ways and causing the variance to be difficult to model as a sum of the FAST frequencies.
3.8 Discussion and Conclusions

The results from the FAST sensitivity analysis give a fairly clear order in the sensitivity of errors to each stream temperature module parameter. The emergence of the topographic altitude angle as the largest contributor highlights the importance of shortwave radiation to the temperature solution. Inflow, flow rate, and air temperature are also some of the strongest contributors to the error between model runs. The overall average RMSE of all the segments and runs was 3.9 degrees, with a maximum RMSE of 22.9 degrees. This gives an idea of the scale of error that can propagate from varying certain parameters that affect solar radiation and air temperature. The sensitivity results tell us and potential users that the error propagation to the stream temperature output will be largest for errors in parameters with high main effects. The results suggest that one should be most careful when setting parameters pertaining to the amount of solar radiation reaching a stream segment, as well as strive for as accurate a distribution of precipitation and temperature data as possible for calibration of the model.

This result also highlights the applicability of the module in treatment of energy flux processes. This makes it a good choice for modeling temperature in stream networks where energy exchange dominates the thermal regime. The limiting assumptions and resolution of the flow output make it not as appropriate for streams in which mixing of inflows of different temperatures dominates. Assuming fully mixed steady flow and not having physical process-based groundwater and subsurface flow calculation limit the model in this area.

The sensitivity results are consistent with previous study on the primary controls of stream temperature in the top three parameters that emerged. It makes some sense that the stream temperature is most sensitive to radiation and air temperature, since many [Isaak et al., 2012; Caissie et al. 2001] have successfully modeled stream temperatures based on air temperature. Most studies of stream energy transfers have found the water-air interface to be the most sensitive, as was the case for Evans et al., 1998, who found an average of 82% of the energy transfer occurring at this interface of the stream. This supports the idea that radiation and air temperature could be some of the more important factors in the stream temperature solution of my model. Streamflow has also been shown to an important factor in determining stream temperatures, along with the energy exchange at the interface with the air [Mayer, 2012]. The rainfall amount affects this streamflow, so it also makes sense that this is one of the most sensitive quantities in the module.

It was interesting and somewhat unexpected, however, that a radiation parameter (alte) had the biggest main effect by an order of magnitude. Due to the well established and close relationship of stream temperature with air temperature [Caissie et al., 2001], it was expected that the temperature adjustment factor (tmax_adj) would be the most dominant term. This may be the case in other
watersheds. In the Potato Creek Model used for this analysis, solar radiation was the clear winner. This is perhaps because many of the stream channels in the watershed are wide and shallow with low gradients. This leads to a larger exposure to solar radiation per amount of volume flux of water, especially in dry years like 1998 and 1999 with the Winter low flow periods seen in the validation case. This may be contributing to the extra radiation sensitivity of this model and data. Future work could be done to compare the sensitivity to other watersheds of different types and in different locations. Other sensitivity methods with different sampling techniques or modifications to FAST could be used in order to compute interaction effects as well as main effects. Other techniques may be able to find the portions of the model variance unaccounted for by this analysis.

As shown by the quick validation, the PRMS stream temperature module can closely match the seasonal pattern in real stream temperature data. Further research needs to be done to assess other calibrations of PRMS with stream temperature and its predictive capacity in real watersheds. In the future, a validation case in which PRMS with stream temperature is calibrated by minimizing objective function that includes error from daily average stream temperature data as well as that from stream gage flow observations could be performed. This would provide further evaluation of the effectiveness of the module at predicting daily stream temperatures within PRMS. This sort of calibration could also provide an extra constraint to help hone the PRMS calibration in terms of flow partitioning between surface, subsurface, and groundwater flows. At the moment it represents an accessible tool that could easily be implemented and tested on a variety of watersheds.
3.9 References Cited


CHAPTER 4: GENERAL CONCLUSIONS

The new stream temperature module in PRMS was developed and tested with some success in the preliminary tests including the validation and sensitivity testing. This module provides a deterministic approach to modeling average daily temperature. This type of assessment could be useful for modeling future climate change scenarios, or testing the effects of other possible changes in weather or landscape.

There are several advantages and limitations to modeling stream temperatures within PRMS using the stream temperature module. Advantages include the free availability and prevalence of PRMS and calibrated watershed models. This could allow for an easy transition into modeling average stream temperatures in several watersheds in the future. This update also puts the SNTemp solution into a modular framework of meteorology and hydrology modeling that can adapt and update the options for and quality of input to the stream temperature module as other modules are continually updated or added. Other advantages include the fact that PRMS with stream temperature is not computationally expensive, yet provides plenty of options for calibration. It provides a full surface mass and heat transport model that can be calibrated simultaneously for both. It would best be applied for network or watershed scale assessments of meteorological, landscape, or hydraulic changes over long periods of time (several years). This makes it suited to assessing management or climate change scenarios over coming decades.

This modeling approach, like any, is ultimately limited in the types of applications for which it can be used. Model operation is limited to the daily time scale and only average stream temperatures are calculated. The partitioning of flow in PRMS is also not as rigorous as other models that include more explicit theoretical-based treatments of subsurface and groundwater flow. This makes it more difficult to accurately calculate the contributions of groundwater inflow to stream temperatures. The stream temperature model assumptions also limit the applications to watersheds and segments with enough homogeneity of channel hydraulic properties that can be represented accurately enough given these assumptions. It is not the best choice for networks and segments with a lot of mixing of lateral inflows, since it assumes fully mixed flow and does not have a theory based treatment of subsurface flow. Also, as with any deterministic modeling approach, the uncertainties are difficult to quantify.

In the future, PRMS with stream temperature can be used for watershed modeling assessments the accuracy of its calibrations can be tested. A good direction to go in the future would be to adapt the module to work as part of the GSFLOW model. This would give a more explicit theoretically-based treatment to groundwater flow, and maybe heat flow could even be traced along with water flow in the subsurface. This type of coupling could greatly improve the calibration of the lateral flow effects. Another possible direction for future study would be to attempt to use the stream temperature solution as a constraint in order to improve flow partitioning and calibrations in PRMS. This could help narrow a
field of non-unique solutions that could produce similar hydrograph results.
Appendix A: Solar shade model

The total solar shade factor in PRMS_Strmtemp is used as was presented by Theurer et al., 1984. Two major parts are added together for the total shade factor. These are the topographic shade and the riparian vegetation shading, and each have their own factor computed for amount of shortwave radiation intercepted on a given day. This results in the following expression:

\[ Sh = Sh_t + Sh_v \]  \hspace{1cm} (1)

where \( Sh \) is the total solar shade factor, \( Sh_t \) is the topographic shade factor, and \( Sh_v \) is the riparian vegetation shade factor.

As in SNTemp, the PRMS stream temperature module gives the user an option to either calculate solar shade within the module, or to simply input values of the total shade factor as a fixed segment parameter for each half of the year representing summer and winter. This option is provided in case the topography and land cover are not well known, or the actual shade factor is known from pre-processing or radiation measurements. Also in some cases, the user may deem an average shade factor sufficient and choose the constant shade factor option. In watersheds at high latitudes, the solar azimuth angle changes dramatically over the course of the year, which affects the amount of shade over time. Thus, it is recommended that the internal shade model be used in this scenario to account for these changes.

The topographic shade occurs when topography blocks direct sunlight from the stream reach for a portion of the day near sunrise and/or sunset. This generally occurs at streams flowing through canyons or hilly terrain and effectively changes the sunrise and sunset time at the location of the stream channel. This shade factor is modeled as a function of the time of year, latitude, stream segment azimuth angle, and topographic altitude angle from the stream channel. Latitude and time of year determine the daily path and timing of the position of the sun in the sky, whereas the stream segment azimuth angle and topographic altitude angle determine the portions of this arc that will be blocked by the surrounding topography. The segment azimuth angle \( A_s \) is the angle in the plane of the ground that the stream makes from the North-South line facing south for streams in the northern hemisphere. This azimuth angle ranges from \(-\pi/2\) to \(\pi/2\) (all angles in this module are in radians). This defines an East bank and West bank of a stream segment, where the East bank is always on the left hand side facing southward. The topographic altitude angle \( \alpha_t \) is given as the vertical angle from the ground along the line perpendicular to the stream bank to the top of the topographic feature along a reach. A depiction of these angles along with the solar altitude and azimuth angles can be found in Figure 1. This can be specified for each of the banks as defined by the reach azimuth angle.
Figure A-1: Depicts the definitions of altitude and azimuth angles in the topographic shade calculation. $A_s$ and $\alpha_s$ are the current solar azimuth and altitude respectively. Modified from Theurer et al., 1984.

The topographic shade factor is computed by subtracting the portion of sunlight between the local sunrise and sunset from the total amount of sunlight from sunrise to sunset given a level plain at the specified latitude and time of year. This can be found by relating the integrals of solar altitude $\alpha_s$ for both cases as in:

$$Sh_t = 1 - \left( \frac{\int_{h_s}^{h_u} \sin \alpha_s \, dh}{\int_{-h_u}^{h_u} \sin \alpha_s \, dh} \right) \tag{2}$$

where $h_u$ is the local sunrise hour angle, $h_s$ is the local sunset hour angle, $-h_u$ is the level-plain sunrise angle, $h_s$ is the level-plain sunset angle, and $\alpha_s$ is the solar altitude as a function of date and time. This effectively subtracts the fraction of the sunlight reaching the stream segment from the total potential, leaving the fraction of sunlight blocked by topography. Hour angles are have zero defined as solar noon, or directly at midday at the top of the sun's arc, and one hour corresponds to $\pi/12$ radians. Equation 2 can be directly evaluated to yield:

$$Sh_t = 1 - \left( \frac{(h_u - h_s)(\sin \phi \sin \delta) + [(\sin h_u - \sin h_s)(\cos \phi \cos \delta)]}{2[(h_s \sin \phi \sin \delta) + (\sin h_s \cos \phi \cos \delta)]} \right) \tag{3}$$

where $\phi$ is the segment latitude, and $\delta$ is the declination of the sun at the time of year modeled. This
declination is given by:

$$\delta = 0.40928 \cos \left( \frac{2 \pi}{365} (172 - D_j) \right)$$

(4)

where $D_j$ is the Julian day.

In order to compute this topographic shade factor, we need the local sunrise and sunset hour angles as well as the level-plain hour angle. The level-plain hour angle can be evaluated by:

$$h_s = \arccos \left[ -\frac{\sin \phi \sin \delta}{\cos \phi \cos \delta} \right]$$

(5)

Because the zero hour angle is set for midday, this has the same magnitude for sunrise and sunset in the level-plain case, and only changes sign. The local sunrise and sunset hour angles are more complicated to find. The sunrise hour angle is defined by

$$h_{sr} = -\arccos \left[ \frac{\sin \alpha_{sr} - \frac{\sin \phi \sin \delta}{\cos \phi \cos \delta}}{\cos \phi \cos \alpha_{sr}} \right]$$

(6)

where $\alpha_{sr}$ is the local sunrise solar altitude. The sunset hour angle can be found by substituting the sunset solar altitude into Equation 6 for the sunrise altitude, and by changing the sign so the solution is positive. The sunset or sunrise solar altitude can be found by solving a system of two equations with two unknowns numerically. This system is given by:

$$\alpha_{sr} = \arctan \left[ \left( \tan \alpha_t \right) \sin \left( A_{sr} - A_r \right) \right]$$

(7)

$$A_{sr} = -\arccos \left( \left[ \sin \phi \sin \alpha_{sr} \right] - \left[ \sin \delta \right] / \left[ \cos \phi \cos \alpha_{sr} \right] \right)$$

(8)

where $A_{sr}$ is the sunrise solar azimuth angle and $\alpha_t$ is the topographic altitude on the sunrise-side bank. This $\alpha_t$ is determined to be the east bank unless $A_r$ is negative and less than the level-plain sunrise solar azimuth. In this case, the sun rises already to the west (or more likely south) side of the stream segment. The numerical solution for $\alpha_{sr}$ is constrained such that needs to be positive and cannot go above the maximum solar altitude for a given day. This is achieved by ensuring that:

$$\sin \alpha_{sr} \leq (\sin \phi \sin \delta) + (\cos \phi \cos \delta)$$

(9)

which is the condition for the maximum solar altitude.

This same system is applied to find the sunset hour angle, with a sign change in Equation 23 giving the sunset solar azimuth equation. In the sunset model, $\alpha_t$ is determined to be the west bank, except in the case that the stream channel has a higher positive reach azimuth than the level-plain solar sunset azimuth angle for the day. Once the local sunrise and sunset hour angles as well as the level-plain hour angle have been computed for a segment, the topographic shade factor can be computed from Equation 3.
Once we have the topographic shade, the amount of shade cast by riparian vegetation between local sunrise and sunset can be computed. The riparian vegetation shade is a function of the length of shadows cast on the water, the total width of the stream, and the density of shadow-casting riparian vegetation along the bank of the stream segment. Again, the shade factor is a measure of the shadows cast over some width and percentage of the stream bank as a fraction of the total potential radiation into the stream segment. The riparian shade fraction can be expressed as:

\[
Sh_v = \frac{\int_{h_u}^{h_s} (V_d W_s \sin \alpha_s) \, dh}{\int_{-h_s}^{h_s} (W \sin \alpha_s) \, dh}
\]  

(10)

where \(V_d\) is the vegetation density along the bank, and \(W_s\) is the length of the shadows cast by riparian vegetation at a given solar altitude \(\alpha_s\). In the stream temperature module, Equation 10 is numerically integrated for each stream segment using gaussian quadrature. \(W_s\) is constrained to be greater than zero and less than or equal to the width of the stream segment \(W\). This instantaneous shadow length can be found using:

\[
W_s = [(V_h \cot \alpha_s) \sin (A_s - A_r)] + \left[\frac{V_c}{2} - V_o\right]
\]  

(11)

where \(V_h\) is the height, \(V_c\) is the crown width, and \(V_o\) is the offset of the riparian vegetation from the bank, all in units of meters. The vegetative shade calculation also allows for differing parameters on each of the east and west bank. Just like the topographic shade calculation, the sunrise-side bank parameters are used in Equations 10 and 11 before solar noon, and the sunset-side bank parameters used after. Finally, the vegetative shade factor from Equation 10 is added to the topographic shade factor from Equation 3 to get the total portion of potential sunlight blocked as in Equation 1.

References
Appendix B: Energy Flux Models

Atmospheric longwave

The atmosphere scatters a lot of incoming solar radiation and emits longwave radiation according to its temperature and composition. The model used for atmospheric longwave emission is given by:

\[ H_a = (1 - r_l)(1 - S_h)(1 + kC_l^2)\varepsilon_a \sigma (T_a + 273.16)^4 \]  

(1)

where \( H_a \) is the energy flux in W m\(^{-2} \), \( r_l \) is a decimal quantity of the longwave reflection, \( S_h \) is the shade factor, \( k \) and \( C_l \) together constitute the decimal cloud fraction, \( \varepsilon_a \) is the emissivity of the air, \( \sigma \) is the Stefan-Boltzmann constant, and \( T_a \) is the air temperature in degrees C. The emissivity of air depends on the humidity according to:

\[ \varepsilon_a = 0.61 + 0.05 \sqrt{e_a} \]  

(2)

where \( e_a \) is the vapor pressure at the current air temperature.

Vegetation-emitted longwave radiation

In riparian vegetation-shaded zones, shortwave radiation is blocked by the vegetation. However, the vegetation will emit some longwave radiation as a blackbody. The model for this is:

\[ H_v = \varepsilon_v \sigma S_h (T_a + 273.16)^4 \]  

(3)

where \( \varepsilon_v \) is the emissivity of the vegetation, and \( S_h \) is the shade due to riparian vegetation. The emissivity of riparian vegetation is a decimal fraction and is taken from a tabulated value; the value used is 0.9526 [Theurer et al., 1984].

Water radiation

The stream water itself also emits longwave radiation as a blackbody. Note that this is a negative flux, or energy leaving the stream. It should also be noted that this depends on our quantity of interest – the water temperature. The equation is another simple formulation of the Stefan-Boltzmann law:

\[ H_w = \varepsilon_w \sigma (T_w + 273.16)^4 \]  

(4)

where \( \varepsilon_w \) is the emissivity of water, which is about 0.9526 [Theurer et al., 1984], and \( T_w \) is the temperature of the water in degrees C.

Shortwave radiation

The potential solar radiation is already calculated in PRMS, but not all of the potential solar radiation reaches and is absorbed by the stream. The model is given by:

\[ H_s = (1 - \alpha)(1 - S_h) H_{sw} \]  

(5)
where $\alpha$ is the albedo or fraction reflected by the stream, $Sh$ is the total shade factor, and $H_{sw}$ is the potential solar radiation reaching the stream.

**Evaporation heat flux**

A large amount of energy is required for the phase change of water from the liquid to the gas phase. This energy does not go to raising the temperature of the water, it is just stored in the vapor by virtue of the relative energy in that phase. This means that energy that would otherwise raise the temperature leaves the stream channel through this process, making this another negative energy flux. The evaporation rate is already calculated by PRMS. The evaporation rate is assumed to be the potential evaporation rate since the stream is a free water surface. In order to convert this to an energy flux, we need to simply determine the amount of energy needed to supply the latent heat of vaporization to the mass evaporating. The latent heat of vaporization is temperature dependent and can be approximated in the normal range of stream temperatures by:

$$\lambda = 2495 \cdot 10^3 - 2360 T_w$$  \hspace{1cm} (6)

where $\lambda$ is the latent heat of vaporization in joules per kilogram [Dingman, 2008]. The evaporation rate ($E$) is converted to m s$^{-1}$. In order to get the mass flux, we can multiply by the density of water which is 1000 kg m$^{-3}$. This can be multiplied by the latent heat to get energy flux in W m$^{-2}$, resulting in the following model.

$$H_e = (2495 - 2.36 T_w) E \cdot 10^6$$  \hspace{1cm} (7)

**Conventional heat flux**

At the air-water interface of the stream, heat is also exchanged through the conventional transfer processes of conduction and convection. This transfer is related to the evaporation heat flux through a dimensionless quantity called the Bowen Ratio ($Bo$) as follows.

$$H_c = Bo H_e$$  \hspace{1cm} (8)

The Bowen Ratio has a model based on air pressure, humidity, and temperature expressed as:

$$Bo = \frac{0.00061 \cdot P}{(e_s - e_a) (T_w - T_a)}$$  \hspace{1cm} (9)

where $P$ is the pressure in hPa and $e_s$ is the saturation vapor pressure at the current $T_a$. For our purposes, let the term $0.00061 \cdot P / (e_s - e_a)$ be called $B_c$, such that the formula becomes:

$$Bo = B_c \cdot (T_w - T_a)$$  \hspace{1cm} (10)

Then, combining equations 7, 8, and 10 our model becomes:
It should be noted that this is defined as a negative flux as well. The way it is defined, a positive $H_c$ means that the water is warmer than the air and energy leaves the stream. Once again, this model depends on our variable of interest, $T_w$.

**Streambed conduction**

Conduction can also occur at the streambed if a thermal gradient exists. This also depends on the conductivity of the streambed material. The model used to calculate this is:

$$H_d = K_g / \Delta Z (T_g - T_w)$$

where $K_g$ is the thermal conductivity of the streambed in W m$^{-1}$ C$^{-1}$, and $\Delta Z$ is the equilibrium depth from the water-ground interface at which the temperature is $T_g$. Note that this is defined as a positive heat flux entering the stream, and that this also depends upon water temperature.

**Stream friction**

Friction can occur in streams as either internal fluid shear or as work done on the boundaries. This converts kinetic energy from the water into heat and warms the stream. The empirical stream friction model used is:

$$H_f = 9805 \frac{Q \cdot S_f}{W}$$

where $S_f$ is the dimensionless stream gradient, $Q$ is the discharge in cms, and $W$ is the average top width in m.

**The energy balance in terms of $T_w$**

Now that the models of all energy fluxes are defined, one can see that four of them depend on the water temperature. These include the water-emitted longwave, evaporation heat flux, conventional heat flux, and ground conduction. We can sum all of the terms to create a total energy balance and determine the dependence on water temperature. The resulting energy balance equation has one term that depends on the converted Kelvin temperature raised to the fourth power, there is also a water temperature squared term from the conventional heat flux, as well as a few linear terms. Thus, we can express the energy balance equation as a polynomial function of water temperature as follows:

$$-H_n = A (T_w + 273.16)^4 - C T_w^2 + B T_w - D$$

where: $A = 5.40 \cdot 10^{-8}$

$$14a$$
\[ B = 10^6 E (B_c (2495 + 2.36 T_a) - 2.36) + K_g / \Delta Z , \]  
\[ C = 10^6 E \cdot B_c \cdot 2.36 , \]  
and \[ D = H_a + H_f + H_s + H_v + 2495 \cdot E (B_c T_a - 1) + T_g \cdot K_g / \Delta Z . \]

This frames the energy balance in terms of our one unknown of water temperature. Since the equilibrium temperature is defined as the water temperature at which the energy flux is zero, it can be found by using a Newton's Method iteration to find the root of Equation 14 \((H_n = 0)\).

**References**


Appendix C: Net heat flux at $T_w$ as a Taylor Series Expansion

Taylor Series Background and Form of Expansion

The net heat flux in this module is found by a modified Taylor polynomial expansion of the zero heat flux at the equilibrium temperature. A Taylor series expansion is an expression of a function $f(x)$ as a power series given by:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

(1)

where $f^{(n)}(x_0)$ is the nth derivative of the function evaluated at the point of expansion $x_0$ [Hazewinkel, 2001]. A full Taylor series or polynomial expansion can be used to represent any continuously differentiable function at any given $x$. However, the technique is often used out to first or second order derivative terms as a simplifying approximation for $x$ near $x_0$ as in this case.

From the paper and Appendix B, we recall that the full heat flux equation is a fourth-order polynomial in water temperature $T_w$, given by:

$$-H_n = A(T_w + 273.16)^4 - C T_w^2 + B T_w - D$$

(2)

with $A$, $C$, $B$, and $D$ constant. In order to enable a workable analytical solution to the differential equation for temperature, the heat flux at a given water temperature is found by using a second-order Taylor polynomial expansion about the equilibrium temperature. We find the value of $H_n$ at $T_w$ using this type of expansion around the equilibrium temperature $T_e$. Thus, from Equation 1, our $f(x)$ is $H_n(T_w)$, our $x_0$ is $T_e$, and our $x$ is $T_w$ yielding:

$$-H_n(T_w) = -H_n(T_e) + \frac{dH_n(T_e)}{dT}(T_w - T_e) + \frac{d^2 H_n(T_e)}{2 dT^2}(T_w - T_e)^2$$

(3)

for the form of a second-order expansion of our net heat flux. Choosing to expand about $T_e$, the zero order term $H_0(T_e) = 0$. Taking this into account and converting to positive heat flux, Equation 3 becomes:

$$H_n(T_w) = \frac{dH_n(T_e)}{dT}(T_e - T_w) + \frac{d^2 H_n(T_e)}{2 dT^2}(T_e - T_w)^2$$

(4)

which is the general form of the approximation used in this module.

Modification of Expansion to Span $T_o - T_e$

In this module, the water enters a stream channel at an initial inflow temperature $T_o$ and progresses toward $T_e$ by gaining or losing heat at a rate $H_n$ dependent on the current temperature. Because of this, we know the temperature of the water is between $T_o$ and $T_e$ at any point and can limit the
evaluation of our net heat flux to this range. The normal type of expansion given by Equation 4 would give a good approximation for the net heat flux assuming \( T_w \) is close to \( T_e \). However, the further \( T_w \) is away from \( T_e \), the further the solution to the expansion would potentially be from the actual solution of the full equation. The approximation in this module modifies the second-order term in order to fix the correct value of net heat flux at \( T_o \) as well as at \( T_e \). This correction factor type approximation assumes a constant second derivative between the two temperatures.

The module uses thermal exchange coefficient to represent the derivative terms of the Taylor series expansion. Using this, Equation 4 can be expressed:

\[
H_n = K_1 (T_e - T_w) + K_2 (T_e - T_w)^2
\]

where \( K_1 \) and \( K_2 \) are the first and second-order thermal exchange coefficients respectively. The first-order coefficient is given by

\[
K_1 = \frac{dH_n(T_e)}{dT} = 4A(T_e + 273.16)^3 - 2C T_e + B
\]

which yields the exact same first-order term in Equation 5 as in Equation 4. This first order term projects the first derivative out from \( T_e \) to \( T_w \) as shown by the dotted line in Figure C-1.

![Figure C-1](image)

**Figure C-1**: A graphical representation contrasting the first and second-order Taylor expansions about \( T_e \) with the correction factor approach used in this solution.

For the second-order term, instead of projecting out the local second-derivative from \( T_e \) as in Equation 4 and the dashed line in Figure C-1, a finite difference calculation of the average deviation from
the first-order approximation across the entire $T_o - T_e$ range is made. Thus the second-order exchange coefficient is given by:

$$K_2 = \frac{(H_i - [K_1(T_e - T_o)])/[(T_e - T_o)^2]}$$

(7)

where $H_i$ is the net heat flux from Equation 2 evaluated at $T_o$. This constitutes a correction factor to the first-order solution based on the difference found evaluated at $T_o$ and is shown by the solid line in Figure C-1. If we evaluate Equation 5 at $T_o$ using this definition of $K_2$, we now get $H_i$, which is the actual heat flux at $T_o$. If we evaluate elsewhere in the range, it gives a fraction of this difference that changes constantly in $T^2$. This basically models a smooth curve between zero at $T_e$ and $H_i$ at $T_o$, starting with the evaluated first derivative at $T_e$ and using a constant second derivative throughout. This is the most accurate way to cover this entire range with a second-order approximation, assuming a smooth, continuous heat flux function being approximated. The correction factor approach is ensured to yield the correct value at and close to both $T_e$ and $T_o$, whereas the typical second-order Taylor polynomial expansion may deviate more from the correct value the further out $T_o$ is from $T_e$. This is evidenced by the slight curve of the Taylor second-order solution under-predicting the value of $H_i$ as shown in Figure C-1. The heat flux equation should be monotonic in water temperature, so there should not be any local minima or maxima cut off by this modification of the Taylor series approximation. However, some sharper, more corner-like features may be smoothed by this type of approximation across the entire range.

References

Appendix D: Supplementary File: Module Source Code

Included as a supplement to this thesis is the Fortran 90 file that serves as the source code for the PRMS stream temperature module. When compiled in PRMS, this serves three main functions. The declare (stream_temp_decl) function declares the variables and parameters to be stored in the module. The initialization function (stream_temp_init) assigns initial values for parameters used within the module that are unique to this module. These are both called once at the beginning of a model run. The run function (stream_temp_run) serves as the main code that computes stream temperatures for each segment on each time-step. This code is provided so that the reader can check the operations against the calculations presented in this thesis and can see the structure of the module operation. The reader can also learn the PRMS variables contained in and used by the stream temperature module.