NUMERICAL MODELING OF HEAT TRANSFER IN A FURNACE WALL AT HIGH TEMPERATURE WITH CONVECTION AND RADIATION BOUNDARY CONDITIONS

by

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science (Applied Mechanics).

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ABSTRACT

Numerical heat transfer models for several industrial furnaces have been performed by running the computer codes DOT and TOPAZ using the finite element method. A variety of boundary conditions and material properties are considered in the models. The temperature distributions within walls and the temperature histories from the initial to steady state condition are obtained by modeling.

A modification of DOT code is presented for calculating the convective heat transfer coefficient $h_c$, which is either constant or a temperature dependent function. Both forced and free convection are considered. In addition, relevant parameters for radiation boundary conditions are determined, such as the flame temperature and emissivity, gas temperature and emissivity.

Based on the temperature distribution in the wall and temperature history during heating process, the energy savings are estimated. The results obtained from the models have shown that 11%-15% energy savings can be realized when an enhanced fiber is used over existing or commercially available fiber as the furnace wall materials. Likewise, 26%-35% energy savings are predicted when refractory fiber is used instead of the insulation fire brick.
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ACKNOWLEDGMENTS

The author wishes to express her appreciation to the following:

Dr. David R. Munoz for his constant availability, careful guidance, and unending encouragement during this research.

Dr. Gerald L. DePoorter for his efforts in providing financial support during this research.

The committee members Drs. Graham G. W. Mustoe and Phillip C. Martin for their help on this work.

The United States Department of Energy for funding this research under Contract Number DE-FC07-89ID12880.

Mr. James J. Ballard for his financial support and help.
PRINCIPAL NOTATION

α -- Thermal diffusivity; ratio of air-fuel
β -- Volumetric thermal expansion coefficient
ε_e -- Environmental surface emissivity
ε_{fi} -- Effective emissivity between the wall inner surface and the flame
ε_{f1} -- Flame emissivity near the inner surface
ε_{fo} -- Effective emissivity between the wall outer surface and the environmental surface
ε_{wi} -- Wall inner surface emissivity
ε_{wo} -- Wall outer surface emissivity
ν -- Kinematic viscosity
ρ --Bulk density
σ_0 -- Stefan-Boltzmann constant
A -- Surface area
c -- Specific heat of the insulation material
d -- Blaine fiber diameter
F_{e-wo} -- View factor of environmental surface toward the wall outer surface
F_{g-si} -- View factor of gas toward the inner surface
g -- Gravitational acceleration
h_a -- Air natural convection heat transfer coefficient
\( h_c \) -- Convective heat transfer coefficient
\( h_g \) -- Gas forced convection heat transfer coefficient
\( h_{rg} \) -- Gas radiation heat transfer coefficient
\( h_{rs} \) -- Surface radiation heat transfer coefficient
\( K_g \) -- Thermal conductivity of the gas
\( K_x, K_y \) -- Conductivity of material in \( x \), \( y \) directions
\( n_x, n_y \) -- The \( x \), \( y \) direction cosines of the outward drawn normal to the surface

\( Pr \) -- Prandtl number
\( Q_c \) -- Convective heat transfer
\( s \) -- Shot content in the fiber
\( t \) -- Time
\( T \) -- Temperature variable
\( T_a \) -- Air temperature; adiabatic flame temperature
\( T_e \) -- External environment temperature
\( T_f \) -- Film temperature
\( T_{fl} \) -- Effective flame temperature
\( T_g \) -- Fluid temperature near the wall; gas temperature
\( T_h \) -- Thickness of element
\( T_i \) -- Initial temperature
\( T_{si} \) -- Inside surface temperature
\( T_{so} \) -- Outside surface temperature
\( x, y \) -- Cartesian coordinates
CHAPTER 1
INTRODUCTION

1.1 Rationale

The general objective of the present work is to estimate the potential industrial energy savings when using high performance refractory fiber insulations for the furnace wall instead of conventional refractory insulations. To meet this goal, it is necessary to know how much energy is available for energy savings in the furnace wall.

Table 4 of a document of the Department of Commerce (DOC 1988) indicates the 1986 fuel energy consumption by various industrial groups in U.S.A.

Since furnaces are widely used in the iron and steel (IS), chemical (CH), glass and refractory (GR) industries (Thring, 1952), As a result of the high temperature requirements and large volume of processed materials, there is significant energy consumed in these industries. Therefore, Figure 1.1 shows the energy consumption in the above three industries, which is based on the data of Table 4 of the government document.
The total fuel energy consumption is 3.5 Quad (1 Quad = $10^{15}$ BTU), which can be considered the total fuel input into U.S. industrial furnaces. However, not all input energy is consumed in the furnace wall. The heat balance of a furnace as shown in Figures 1.2 to 1.6 illustrates the percentage of the energy consumption in the wall to the total input energy into the furnace.
Fig. 1.2 Heat balance of soaking pit (Hollander 1983)

Fig. 1.3 Energy balance -- five-zone pusher slab reheating furnace (Vance 1985)

Fig. 1.4 Heat balance of preheat furnace (Molloy 1985)

Fig. 1.5 Heat balance for continuous walking beam tempering furnaces (Casana and Stabile 1983)
Fig. 1.6 Energy flow diagram of a typical industrial furnace (Viskanta and Menguc 1987)

From above heat balance diagrams, the percentage of energy consumption through the furnace wall to total fuel input to the furnace for different furnaces is

Soaking pit 10%
Pusher slab reheating furnace 13%--20%
Preheat furnace 13.9%
Continuous walking beam tempering furnace 15.54%
A typical industrial furnace 14%

Roughly speaking, the average heat loss through the furnace wall is about 10% of the fuel input to the furnace (Hollander 1983). Therefore, the annual energy flow through and stored within the furnace wall is approximately 0.35 Quad. This is the energy available for energy savings in
the furnace wall. The task of this study is to estimate the energy savings among 0.35 Quad when using high performance refractory fiber insulations for the furnace wall instead of conventional refractory insulations through numerical modeling. To meet this goal, it is necessary to analyze the heat flow characteristics in the furnace wall.

1.2 Heat Transfer Mechanisms

Heat transfer in a furnace is complex and, as yet, not fully understood. The heat transfer conditions are markedly affected by such factors as the furnace dimensions, wall construction and insulation materials, the arrangement of the burners, the type of fuel burned, and the operational conditions of the furnace process. The furnace wall is usually made of refractory insulation materials.

Considering the porous or fibrous nature of the insulation material, heat is transferred through the wall by following modes: air natural convection, air and other gas conduction, solid-solid conduction and radiation. Natural convection is considered negligible due to the small size of the void in the insulation. Therefore, the dominant heat transfer modes in the insulation material are radiation and solid conduction.

The inside surface of the furnace wall is exposed to gas flame radiation. Because furnaces are usually equipped
with forced or induced induction fans, the inside surface is also subjected to forced convection. For the outside of the wall, the modes of heat transfer are the air natural convection and the radiation heat exchange between the outside surface of the wall and the surroundings.

Furnace operating conditions play an important role in heat transfer analysis. The industry usually uses a cyclical operation to control the heating process (Thring, 1952). Some cyclical operations can reach the steady-state condition, and some never do. Therefore transient phenomena must also be considered when analyzing a furnace wall.

1.3 Furnace Design Configuration

There are a variety of furnaces to satisfy the different requirements of industries. The following three furnace geometries are commonly used in industry: cylindrical furnaces, rectangular tunnel furnaces and annular furnaces (also called rotary hearth furnace).

Cylindrical geometries include the blast furnace for iron making, electric furnaces and ladles for steelmaking, cauldrons and stills for chemical processes, forge furnaces, heat treating furnaces, and firing furnace for ceramic products (Thring, 1952). Jaluria (1984) has modeled a cylindrical furnace. The cylindrical furnace that he chose to model was a batch annealing furnace for steel sheets.
Rectangular tunnel furnaces usually serve as kilns for refractory industrial clay product production, process heaters for chemical industries, and continuous furnaces for reheating steel billets. These are usually divided into several zones for control purposes and result in different heating stages. Braun (1982) has described a walking beam billet heating furnace that is used to heat steel stock prior to a forging operation in the railway products industry.

The rotary-hearth furnace has been successfully used for high temperature heating of forgings and billets which are square or round. Such a furnace is a batch type in which the products are charged and discharged at about the same point, but is continuous in that the pieces are carried through the furnace on a rotating floor and gradually achieve the desired temperature (Thring, 1952). Fried (1983) has introduced a rotary hearth forge furnace which is used to heat stock for closed die forging.

1.4 Properties of the Refractory Insulation Materials

Minimizing the energy consumption is an important consideration in the design and the installation of a furnace. To meet this objective, it is required that the furnace wall material should possess the following properties:
1) A low thermal conductivity in order to reduce heat loss by conduction through the wall.

2) A low density to reduce the heat storage allowing the furnace to more quickly reach the operating temperatures.

3) A good resistance to thermal shock and high temperature to extend the service life of the furnace.

4) A Low thermal shrinkage to reduce deformation of the wall and enhanced heat loss.

The heat transfer through the wall is significantly affected by the properties of insulation materials. Therefore, it is very important to select the best insulation materials for the particular furnace application. For many years, high temperature fire clay bricks have been used as furnace wall material. However, over the past two decades, low thermal mass materials such as ceramic fibers and fine powder insulations have been very successful in satisfying the above requirements. The American Society of Testing and Materials (ASTM) Committee, C-16 on Thermal Insulation has provided standard measuring methods on properties of the various refractory insulation materials at different temperatures (McElroy 1989). Numerous experiments (Chu and Tseng 1989; Crowley and Yong 1988; McElroy, Yarbrough and Tong 1986; Tong, Swathi and Cunnington 1987;
Yarbrough, Graces and McElroy (1987) have shown that the properties such as thermal conductivity \( K \), density \( \rho \), specific heat \( c \) and emissivity \( \epsilon \), are all dependent on temperature, which result in weak nonlinearity of the thermal analysis problem.

Thermal conductivity is one of the most important yet difficult to measure properties of a refractory material. The approved testing procedures and equipment are complex, costly and time consuming (John-Manville 1982b), and it is a function of many factors such as product type (chemistry and density), operating temperature and fiber orientation. Crowley and Young (1988) proposed a simple approach that uses a hot and cold face temperature to calculate thermal conductivity.

In general, the thermal conductivity measured for different insulations is the effective thermal conductivity (also called apparent thermal conductivity), which is the sum of the following mechanisms of heat transfer: thermal conductivity of air, thermal conductivity of the solid components of the insulating structure, the convective heat transfer within the pore structure, the radiation heat transfer within the structure and the interactions of these mechanisms. Therefore, it is important to obtain a reasonable thermal conductivity in the heat transfer
calculation in a furnace wall. Manville Corporation (1982) has developed an equation for calculating the effective thermal conductivity $k_e$ of refractory fiber products after performing a detailed statistical analysis of numerous test results obtained using the ASTM C-201 and ASTM-177 test procedures. The formula is

$$ k_e = 0.224 - 0.0316d + 0.0577T - 1.02 \times 10^{-5}T^3 + 1.39 \times 10^{-4}T^3 \frac{d}{\rho (1 - \frac{s}{100})} $$

1.1

The definition and range of variables for which this equation is valid are:

- $d$ -- Blaine fiber diameter, 2.5-5.2 μm
- $T$ -- Mean temperature in hundreds of degrees Rankine, 500 °F to 2000 °F
- $\rho$ -- Bulk density, 3.1-8.5 lb/ft³
- $s$ -- Shot content in the fiber, 25-53%

Bomberg and Klarsfeld (1983) have developed a semi-empirical model for the calculation of heat transfer in dry mineral fiber insulations, which takes into account the parameters characterizing the fiber structure and the boundary conditions. One of the parameters is the anisotropy factor $K_v/K_h$, which represents the effect of fiber orientation. Here, $K_v$ and $K_h$ is the perpendicular and parallel conductivity, respectively, to the planes of fiber
stratification. Figure 1.7 illustrates the relationship between $K_v$ and $K_h$ at a specified density.

![Graph showing thermal conductivity as a function of density and orientation](image)

Fig. 1.7 Thermal conductivity as a function of density and orientation (Bomberg and Klarsfeld 1983)

The effective or apparent thermal conductivity actually represents the combination of heat transfer effects due to conduction, convection and radiation. However, for the purposes of this study, I will simply consider the heat transfer through the wall as pure conduction represented by effective thermal conductivity. The thermal conductivity
measured by Manville (1982) and other companies or calculated from equation 1.1 will be taken as the effective thermal conductivity in the heat loss calculations.

1.5 Objectives

The work to be performed in this thesis will include developing two-dimensional, transient heat transfer models of furnace walls, exposed to different boundary conditions.

The specific objectives of the thesis are as follows:

1. Predict the temperature distribution in the furnace wall made of either multi-layered insulations or Z-Blok modules (Manville trademark) using the finite element method (FEM).

2. Write computer subroutines for calculating the convective heat transfer coefficient $h_c$ for different boundary conditions with various configurations of furnaces. Both natural and forced convection will be considered.

3. Calculate the heat loss for different refractory insulation materials installed in two furnace configurations to estimate the energy savings through using high performance, as opposed to commercially available, refractory fiber as the furnace wall material. The heat losses include the heat transferred through and heat stored within the wall.
CHAPTER 2
THEORETICAL CALCULATION

This chapter presents a basic mathematical model and
more commonly used theoretical methods for numerical
modeling the heat transfer process through the wall of an
industrial furnace. It also provides a step by step
computational procedure to solve the physical governing
equation subjected to a variety of boundary conditions using
the finite element method.

2.1 Formulation of the Mathematical Model

The heat balance of the furnace wall is that the input
energy is equal to the output energy plus the stored energy,
that is

\[ E_{in} = E_{out} + \Delta E_{stored} \]

Upon consideration of the heat transfer mechanisms
described in Chapter 1, the following assumptions have been
made:

1. No internal heat generation within the furnace wall.
2. No phase change within the wall.
3. The temperature is independent of the length of the
   furnace within the same zone. This assumption was
   applied because the furnace is usually divided into
   several zones according to process control
requirements. Therefore, the heat transfer model of a furnace wall is carried out zone by zone.

4. Heat transfer through the wall is purely conductive using the effective thermal conductivity, because the effective thermal conductivity represents the actual heat transfer properties of the wall material.

The governing differential equation in the cartesian coordinate system for heat conduction in a two dimensional furnace wall described in Appendix A is

$$\frac{\partial}{\partial x}(K_x \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial T}{\partial y}) = \rho_c \frac{\partial T}{\partial t}$$ \hspace{1cm} 2.1

Following the procedures in Appendix A, the governing differential equation in the cylindrical coordinate system for a 2-D cylindrical furnace wall is derived as equation 2.2, by assuming that the thickness of a small element in the cylindrical wall is a unit radian segment.

$$\frac{1}{r} \frac{\partial}{\partial r}(K_r r \frac{\partial T}{\partial r}) + \frac{\partial}{\partial z}(K_z \frac{\partial T}{\partial z}) = \rho_c \frac{\partial T}{\partial t}$$ \hspace{1cm} 2.2

The cartesian coordinate system is chosen for this presentation. However, the cylindrical coordinate is used in the computer code and furnace modeling examples.

The initial condition is:

$$T(x,y,0) = T_i(x,y), \hspace{0.5cm} x, y \hspace{0.5cm} \text{inside body at } t=0 \hspace{1cm} 2.3$$

$T_i$ is constant at each coordinate point.
At the inner surface $\Gamma_{si}$ of the wall, the following two types of boundary conditions are considered.

1. The boundary condition for the prescribed surface temperature $T_{si}$ which may be either constant or a function of the time is

$$T(x,y,t) = T_{si}(x,y,t), \quad x,y, on \Gamma_{si}$$

2. The convection and radiation boundary condition is

$$K_x \frac{\partial T}{\partial x} n_x + K_y \frac{\partial T}{\partial y} n_y = (h_g + h_{rg})(T_g - T_{si})$$

$$x, y, on \Gamma_i$$

At the outer surface $\Gamma_o$ of the wall, the convection and radiation boundary conditions are

$$K_x \frac{\partial T}{\partial x} n_x + K_y \frac{\partial T}{\partial y} n_y = h_a(T_a - T_{so}) + h_{rs}(T_e - T_{so})$$

$$x, y, on \Gamma_o$$

Where $K_x$, $K_y$ are the conductivity of material in $x$, $y$ directions, which are dependent on the temperature.

$n_x$, $n_y$ are the $x$, $y$ direction cosines of the outward drawn normal to the surface.

$h_g$ is the gas forced convection heat transfer coefficient, which is a function of the gas film temperature.

$h_a$ is the air natural convection heat transfer
coefficient, which is a function of the air film temperature.

$\text{h}_{rg}$ is the gas radiation heat transfer coefficient, which is equal to

$$
\text{h}_{rg} = \sigma_0 \varepsilon_{fg-si} F_{g-si} (T^2_g + T^2_{si})(T_g + T_{si})
$$

2.7

$h_{rs}$ is the surface radiation heat transfer coefficient which is equal to

$$
\text{h}_{rs} = \sigma_0 \varepsilon_{s0} F_{e-so} (T^2_e + T^2_{so})(T_e + T_{so})
$$

2.8

$\sigma_0$ is the Stefan-Boltzmann constant.

$F_{g-si}$ is the view factor of gas toward the inner surface of a furnace wall.

$F_{e-so}$ is the view factor of environmental surface toward the outer surface of a furnace wall.

$\varepsilon_{fl}$ is the effective emissivity between the inner surface and the flame inside the furnace. It equals

$$
\varepsilon_{fl} = \frac{1}{\varepsilon_{fl}} + \frac{1}{\varepsilon_{wi}} - 1
$$

2.9

$\varepsilon_{fl}$ is the flame emissivity near the inner surface.

$\varepsilon_{wi}$ is the inner surface emissivity.

$\varepsilon_{fo}$ is the effective emissivity between the outer and the environmental surfaces, which is equal to
\[ \epsilon_{fo} = \frac{1}{\frac{1}{\epsilon_e} + \frac{1}{\epsilon_{wo}} - 1} \]  

\( \epsilon_e \) is the environmental surface emissivity.

\( \epsilon_{wo} \) is the outer surface emissivity.

\( T_i \) is the initial temperature.

\( T_g \) is gas temperature.

\( T_{si} \) is the inside surface temperature.

\( T_e \) is the external environment temperature.

\( T_a \) is the air temperature.

\( T_{so} \) is the outside surface temperature.

\( T \) is the temperature variable.

\( t \) is the time.

\( x, y \) are the cartesian coordinates

\( \rho \) is the density of the insulation material

\( c \) is the specific heat of the insulation material

The equations 2.1, 2.2 and their boundary conditions form the mathematical model of the heat transfer within the furnace wall. It is clear that this mathematical model is a two-dimensional nonlinear transient problem.

The nonlinearity is caused by the temperature dependent thermal conductivity and heat transfer coefficient as well as the radiation boundary conditions.
2.2 Literature Survey on the Theoretical Calculation

This section presents commonly used theoretical methods for solving the above mathematical model.

2.2.1 Analytical Method

Analytical methods can exactly express the relationship between the temperature, space and/or time. If the exact solution to a problem is required, it is necessary to use an analytical method. Exact analytical results, however, are only available for a limited number of special and simplified problems.

Ozisik (1968) has presented the analytical solution for the transient conduction problem with linear homogeneous and nonhomogeneous boundary conditions by means of the integral transform technique. He has also introduced an approximate analytical solution for a transient, nonlinear boundary value problem with a thermal radiation boundary condition and temperature dependent material properties in graphical form.

Tong, Birkebak, and Enoch (1983) first obtained an analytical expression for the dimensionless variables of radiative heat transfer rate $Q^+$ (at $x=0$) and $Q^-$ (at $x=L$) in terms of an unknown temperature distribution.

Tong, McElroy and Yarbrough (1989b) have analyzed transient radiation and conduction heat transfer through
planar porous materials, and solved the analytical expressions mentioned above and energy equations numerically for the temperature distribution.

Zerkle (1965) has presented an approximate analytical solution for the 1-D transient temperature distribution in a slab subjected to thermal radiation. Results obtained with a thermal-electrical analog computer are presented in dimensionless, graphical form for a wide range of variables.

Tao (1989) investigated the heat conduction problem with temperature-dependent material properties, and provided an analytical solution for nonlinear heat transfer problems.

2.2.2 Numerical Calculation

Because of the limited number of cases for which analytical solutions exist, numerical models have been developed for calculating heat transfer in industrial furnaces. The basic approach of a numerical model is to first develop a mathematical model of the physical processes and solve the governing equations with appropriate boundary conditions using a numerical technique. The simplification inherent in the use of algebraic equations to approximate differential equations makes numerical models powerful and widely applicable.

There are several numerical methods available for heat transfer calculations. The most popular methods are the:
Finite Difference Method (FDM), Finite Element Method (FEM), Boundary Element Method (BEM).

2.2.2.1 Finite Difference Method.  FDM is a numerical method to approximate the differential equation by simpler algebraic equation valid at series of nodes within the region. It is widely used in heat transfer calculations, especially for problems with simple geometries. Patankar (1980) is one of a list of many who have introduced a method of using FDM for heat transfer and fluid flow problems.

Jaluria (1984) has proposed a detailed numerical study of the combined modes of heat transfer in a furnace. The numerical simulation of the heat transfer in a batch annealing furnace for steel sheets was performed using a finite difference method. The thermal processes are coupled through the boundary conditions and the property variations. Strong non-linearity is introduced due to the radiative heat transfer mechanism. The non-linear coupled system of partial differential equations are solved numerically by time marching, which allows study of the time-dependent temperature fields in the system.

Barr, Brimacombe and Walkinson (1989) have presented a heat-transfer model for the rotary kiln by FDM. The refractory wall was divided into the two regions: an active layer at the inside refractory surface within which
temperature cycling occurs due to the kiln rotation, and a steady-state layer consisting of the remaining refractory and the shell where the temperature remains unchanged.

The transient portion of the governing differential equations are described in Appendix B using the finite difference method.

2.2.2.2 Finite Element Method. FEM is a numerical method in which the physical body is divided into finite elements which are assembled to provide an approximation to the real system. In recent years, the finite element method has become popular for heat transfer calculations. It can be used in solving problems which have more complicated geometries than those historically solved by the FDM. Rao (1982) introduced the application of finite element method to the determination of the temperature distribution within a conducting body. The finite element equation for heat transfer is derived by using variational or Galerkin approaches.

The personnel at INTERA (1982) have provided a report on a nonlinear heat transfer code (DOT, which stands for Determination of Temperature) for the analysis of two-dimensional planar and axisymmetric representations of structures. This code is a general purpose heat conduction code for both linear and nonlinear steady-state
or transient analysis. The solution technique employed is based on the finite element method. The DOT code can simulate radiative and convective boundary conditions although it can not be used to model radiative transfer through semi-transparent media. In addition, the DOT code allows the flexibility of working with constant or time dependant temperature and heat flux boundary conditions.

Orlandimic and Galluzzi (1981) have modified the DOT code for the analysis of the temperature distribution in a blast furnace refractory wall with flat cooling plates. The main purpose of their investigation was to determine the temperature distribution within the lining of the wall with several arrangements of cooling plates, in order to optimize the number and location of the cooling plates. Several subroutines were added to the DOT code: such as solid elements for a three-dimensional mesh of the furnace and convective and radiative surface elements for the boundary conditions. They considered that the conductivity of the refractory material wall and the convective heat transfer coefficients vary with the temperature of the inner and outer walls and the temperature of the surrounding fluids.

2.2.2.3 Boundary Element Method. BEM is a numerical method in which the discretization scheme only involves subdivisions of the boundary surface of a body to find a
solution of the differential equation by boundary integral equation techniques. Boundary element method has also been used for heat transfer calculations in furnace walls, which can reduce the dimensionality of the basic process by one but involves much more arithmetic calculation than finite element method on the calculating each component of the matrix. Meric (1986) has calculated optimal thermal insulation thicknesses over a conducting body using the boundary element method. The problem is steady-state and two-dimensional with nonlinear boundary conditions.

Among these numerical methods, the finite element method is completely general with respect to geometry, material properties and arbitrary boundary conditions. Complex bodies of arbitrary shape composed of several different anisotropic materials can easily be represented. Combinations of temperature-dependent material properties and nonlinear boundary conditions can also be readily studied. The implicit finite element method based on the mathematical model proposed by Rao (1982) was chosen to model heat transfer in the furnace walls.
2.3 Finite Element Model

2.3.1 Derivation of Finite Element Equation

The goal of the finite element method is to seek an approximate solution to the governing equations by the discretization of the physical domain into finite elements. There are two approximate approaches used to derive the finite element equation; the variational approach dealing with problems formulated in variational form and the weighted residual approach in which the finite element equation can be derived directly from the governing differential equations of the problem (Rao, 1982). The Galerkin method is one method of the weighted residual approach, which is widely used in the engineering problems. The Galerkin approach was chosen to obtain the approximate solution in this thesis, and it can be described by the steps outlined in Appendix B.

The finite element model, as derived in Appendix B, is the approximate solution of the governing equation 2.1. The matrix form of the finite element equations can be expressed as

\[ [C] \{ \dot{T} \} + [K] \{ T \} = \{ F \} \quad 2.11 \]

with
\[ [C] = [C_{ij}] = \int \int_{\Omega} N_i \rho c_p N_j \cdot J d\xi d\eta \]

\[ [K] = [K_{ij}] = \int \int_{\Omega} \left[ \frac{\partial N_i}{\partial x} K_x \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} K_y \frac{\partial N_j}{\partial y} \right] J d\xi d\eta + \int_{\Gamma_r} N_i (h_c + h_r) N_j d\Gamma \]

\[ \{F\} = \{F_i\} = \int_{\Gamma_r} N_i N_j (h_c + h_r) T_\omega d\Gamma \]

Where \([C]\) is the heat capacity matrix which is dependent on the density \(\rho\) and the temperature dependent specific heat capacity \(c\) of the material. \([K]\) is the temperature dependent thermal conductivity matrix including the convection and radiation heat transfer coefficients at the boundary.

\(\{T(t)\}\) = Vector of nodal point temperatures at time \(t\).

\(\{\dot{T}(t)\}\) = Vector of the time rate of change of the nodal point temperature at time \(t\).

\(\{F(t)\}\) = Vector of the externally supplied heat flux at time \(t\).

2.3.2 Computer Program Selection

In recent years, a number of computer programs using the finite element method have been developed for heat transfer modeling. Each program has its own features.
satisfying specific requirements. For example, ANSYS-PC/ED/TH (Finkel, Raichur and Gorman, 1986) is a thermal engineering analysis computer program developed by Swanson Analysis Systems. The version available for this study is an educational version that runs on PC computer, which contains steady state and transient heat transfer analysis capabilities, and can solve conduction, convection and radiation problems. However, no temperature dependent material properties are allowed.

TOPAZ (Shapiro 1986) is a two-dimensional implicit finite element code for heat transfer analysis. It completely follows the finite element procedures introduced in Appendix B. It can be used to solve for the steady-state or transient temperature field in 2-D planar or axisymmetric geometries. TOPAZ can be used to solve problems in which material properties are temperature dependent and either isotropic or orthotropic. A variety of time and temperature dependent boundary conditions can be specified including temperature, heat flux, convection and radiation. The program can be run on the CRAY and VAX computers. It is desirable for this project that the heat transfer model of the furnace walls be run on personal computers. This would allow routine demonstration to industry. TOPAZ does not satisfy the requirement. The DOT
code can satisfy the demonstration requirements of the project.

DOT is a nonlinear heat transfer code for analysis of two-dimensional planar and axisymmetric representations of structures. Capabilities are provided for modeling anisotropic heterogeneous materials with temperature dependent thermal properties and time dependent temperature, heat flux, convection and radiation boundary conditions. DOT code differs from TOPAZ code in the way in which the nonlinear problems are solved. DOT does not use the iteration technique to arrive at a solution. In the DOT program, the nonlinear term created by the temperature dependent convection and radiation boundary conditions are moved to the load vector $F_i$ from the thermal conductivity matrix $K_{ij}$. The finite element equation used in DOT is

$$[C] \{ \dot{T} \} + [K] \{ T \} = \{ F \} \quad 2.12$$

with

$$[C_{ij}] = \int \int_{\Omega^*} N_i \rho C_p N_j . \Xi \, d\eta$$

$$[K_{ij}] = \int \int_{\Omega^*} \left[ \frac{\partial N_i}{\partial x} K_x \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} K_y \frac{\partial N_j}{\partial y} \right] \Xi \, d\eta$$

$$\{ F_i \} = \int_{\Gamma^*} N_i N_j (h_c + h_r) (T_a - T_g) \, d\Gamma$$
Where $T_s$ is the value of the absolute surface temperature, which is determined at the midpoint of a particular time step by assuming a linear variation of the nodal temperature over the previous time step.

This method results in the significant savings in computer time and memory. The disadvantage is that the results are not as accurate as using the iteration technique.

Therefore, the DOT code can be used as a demonstration code for industry and TOPAZ can be used as a verification code for checking the accuracy of results from the DOT code. Although the DOT and TOPAZ codes provide a number of functions for heat transfer modeling, there still remain limitations for furnace wall heat transfer modeling. To overcome the current limitations of the DOT code, modifications have been made with regard to the boundary conditions and heat loss calculations. Sections 2.4 and 2.5 will present the modifications on convection boundary condition and energy losses calculation, respectively.
2.4 Convective Transfer Coefficient Determination

Forced and natural convection occur simultaneously at the boundary of the furnace walls. The inside surface of the wall is exposed to forced convection resulting from flowing gases driven by fans and by the combustion air swirl. It is also exposed to the natural convection induced by gravitational buoyancy forces. The outside surface of the wall is subjected to the natural convection of air. The convective heat transfer \( Q_c \) between the fluid and either of side of a furnace wall can be calculated by

\[
Q_c = h_c A (T_g - T_s)
\]

Where \( h_c \) is convective heat transfer coefficient, \( T_g \) is fluid temperature near the wall, \( T_s \) is surface temperature of a furnace wall, \( A \) is surface area.

It is necessary to know the convective coefficient \( h_c \) to find the heat transfer from formula 2.13. INTERA (1982) suggested using the average value of the heat transfer coefficient for laminar natural convection from an isothermal vertical plate of length \( L \) using empirical formula:

\[
h_c = \frac{0.555K Pr^{\frac{1}{4}} Gr^{\frac{1}{4}}}{L}
\]

\( Gr \) is the Grashof number which is equal to
\[
Gr = \frac{\rho g \beta (T_f - T_g) L^3}{v^2}
\]

Where \( K \) is thermal conductivity of the gas, \( Pr \) is Prandtl number, \( \rho \) is density of the gas, \( g \) is gravitational acceleration, \( \beta \) is volumetric thermal expansion coefficient, \( T_f \) is film temperature and \( v \) is the kinematic viscosity.

INTERA (1982) also recommended simplified correlations of the free convection heat transfer coefficients \( h_c \) for air as reference data to input for users. However, the reference did not include a forced convection heat transfer coefficient \( h_c \).

Therefore, a subroutine FTEMP for calculating the convective coefficient \( h_c \) as a function of temperature was added into DOT program. The computer program is listed in Appendix E. The \( h_c \) value at each surface node can be either input directly or calculated from the input fluid properties. The method for calculating \( h_c \) is different for each convective mode. This subroutine can handle the following convection modes using the methods summarized by Incropera and DeWitt (1985).

2.4.1 Forced Convection

If the fluid motion involved in the process is induced by external means (pump, blower, wind, vehicle motion,
etc.), the process is generally called forced convection (Kays and Crawford 1980). Two kinds of forced convection are considered in this study; external forced convection over a flat plate and internal forced convection in a tube. See a detailed description of the specific forced convection coefficients used in Appendix C.

2.4.2 Free Convection.

If the fluid motion arises from external body force fields, such as gravity, acting on density gradients induced by the thermal transport process itself, the process is usually called free convection. According to the geometry of the industrial furnace, two types of free convection are considered in the subroutine FTEMP. These are the free convection over vertical and horizontal plates and are described in detail in Appendix C.

The subroutine FTEMP was added to the subroutine FUNC in the DOT code. A input data manual for subroutine FTEMP is listed in Appendix D. Figure 2.1 is the flow chart of subroutine FTEMP for calculating temperature dependent functions. In the flow chart, a flag of function type NFLAG is input. If NFLAG is equal to zero, the function value is directly input at each selected temperature point in a subroutine GENER.
Otherwise, a different type of convection heat transfer coefficient $h_c$ is calculated at each selected temperature point by inputing related parameters and gas properties according to the methods presented in Appendix C.

The selected temperature point (refers to TFN in the subroutine FTEMP) range should be greater than the average temperature range between the fluid temperature and the boundary surface temperature. The function values $F_N$ (here refers to $h_c$) are calculated corresponding to the selected
temperature points TFN. At each time step, the $h_c$ is interpolated upon the average temperature.

The abbreviations of the type of convection in Figure 2.1 are described as following: E. FORCED refers to the external forced convection, I. FORCED refers to the internal forced convection, V. FREE refers to the free convection over a vertical plate, H. FREE refers to the free convection over a horizontal plate.

2.5 Radiation Heat Transfer Coefficient Determination

As Blokh (1988) has stated, the main mode of the heat transfer in the furnace at the high temperature is radiation. Most computer codes treat the radiation problem as being exposed to a blackbody radiation source. The TOPAZ code has been used to calculate the enclosure radiation by FEM. The solution procedure involves solving the enclosure radiation problem first for the net radiative flux on each surface using an estimated initial surface temperature distribution. The resulting fluxes are then used as boundary conditions for the solution of the wall conduction problem. The conduction solution then gives new estimates for the surface temperatures. The process is repeated until temperature convergence is achieved. The code FACET (Shapiro 1986) was used to calculate the radiation view.
factors between the surfaces. DOT code can be used to solve for the radiation exchange between two gray, infinitely large parallel plates.

Radiation heat transfer occurs between the outer surface and the surroundings as well as inside the furnace wall. Because the area of the surroundings is assumed much larger than that of the furnace surface, the surroundings can be considered a black body. Therefore, the emissivity of the surroundings $\varepsilon_e$ is 1 and the view factor of surrounding surface toward the outer surface of a furnace wall $F_{e-so}$ is 1. The emissivity of the furnace outer surface $\varepsilon_{wo}$ can be selected from Table A.11 (Incropera and DeWitt 1985) according to the surface properties. Therefore, the radiation heat transfer coefficient on outer surface $h_{rs}$ can be calculated from equation 2.8.

On the inner surface of a furnace wall, the radiation heat transfer mainly occurs between the combustion product flame and the inner surface of the furnace wall (Beer and Howarth 1968). There are three models to analyze the radiation and convection heat transfer inside the furnace (Viskanta and Menguc 1987). They are the well-stirred furnace model in which the gas temperature is assumed uniform throughout the furnace volume, the long furnace model in which the gas temperature is assumed to vary along
gas flow direction and the zonal method of analysis which provides the detailed heat-flux distribution in the furnaces (Beer and Howarth 1968). Since the objective of thesis is to model the heat transfer within furnace wall, the well-stirred model is selected in the thesis. This section focuses on calculating the radiation heat transfer coefficient between the flame and inner surface $h_{rf}$. To calculate $h_{rf}$, it is necessary to determine parameters such as the gas temperature $T_g$, the gas emissivity $\varepsilon_g$, the adiabatic flame temperature $T_a$, the effective flame temperature $T_{f1}$, the flame emissivity $\varepsilon_{f1}$ and the view factor of the surface from flame $F_{s-f1}$. Because most industrial furnaces use gases or oil as a fuel, calculation of the radiation parameters listed above is based on a gas or oil fired furnace.

2.5.1 Adiabatic Flame Temperature Determination

Assuming complete combustion of natural gas with 10% excess air, the combustion equation is

$$CH_4 + 2(1.1)O_2 + 2(1.1)3.76N_2 \rightarrow CO_2 + 2H_2O + 0.2O_2 + 8.272N_2$$  \hspace{1cm} 2.16

The first law of thermodynamics (enthalpy balance) of reacting systems for a steady-state, steady-flow process involving ideal gases (except for the $CH_4$) is:
\[ H_R = H_P \quad \text{or} \]
\[ \sum n_i [h_i^0 + \Delta h]_i = \sum n_e [h_e^0 + \Delta h]_e \quad 2.17 \]

Assuming both air and methane enter the burners at 25°C (or 77 °F) and 1 atm pressure, the enthalpy of the air is equal to zero. The enthalpy of the reactants is therefore equal to that of the fuel (Wylen and Sonntag 1986).

According to the enthalpy of formation from Table A.13E for CH₄, CO₂ and H₂O (Wylen and Sonntag 1986), the enthalpy of reactants is:

\[ H_R = \sum n_i [h_i^0 + \Delta h]_i = (h_i^0)_{CH_4} = -32211 \text{ Btu} \text{lb-mole} \quad 2.18 \]

and the enthalpy of products is

\[ H_P = \sum n_e [h_e^0 + \Delta h]_e = n_{CO_2} [h_{CO_2}^0 + \Delta h]_{CO_2} \\
+ n_{H_2O} [h_{H_2O}^0 + \Delta h]_{H_2O} + n_{O_2} [\Delta h]_{O_2} + n_{N_2} [\Delta h]_{N_2} \\
= [-169297 + \Delta h_{CO_2}] + 2 [-104036 + \Delta h_{H_2O}] \\
+ 0.2 \Delta h_{O_2} + 8.272 \Delta h_{N_2} \quad 2.19 \]

Where the absolute enthalpy \( \Delta h \) at specific temperature for different ideal gas can be obtained from Table A.11E (Wylen and Sonntag 1986). By trial-and-error solution, based on the enthalpy balance, the adiabatic flame temperature \( T_a \) was found to be 2190 K (3942 R).
2.5.2 Gas Temperature Determination

Blokh introduced a formula (Blokh 1988, equation [6-11]) which establishes the relationship between the furnace exit temperature of the gases \( T_g \) and the adiabatic flame temperature. The formula is

\[
\frac{T_a - T_g}{T_g} = \frac{M}{\Pi^{0.6}} \tag{2.20}
\]

Where \( M \) is an empirical coefficient. Its value is established depending on the type of fuel burned and on the height of burners with respect to the height of a furnace \( X = \frac{H_{\text{burn}}}{H_f} \). For a fuel oil or a natural gas fired furnace, \( M \) is equal to

\[
M = 0.54 - 0.2X \tag{2.21}
\]

\( \Pi \) is the furnace parameter which is related to the effect of the furnace loading on heat transfer \( \text{Bo} \) (also called Boltzmann number), the radiative properties of a furnace chamber \( \varepsilon_f \) and the radiative properties of a furnace wall surface, \( \Phi_w \). That is

\[
\Pi = \left( \frac{1}{\varepsilon_f} \right) \left( \frac{1}{\Phi_w} \right) \text{Bo} \tag{2.22}
\]

Since calculating \( \text{Bo} \) is very complicated and it is dependent on empirical data, Blokh (1988) suggested that, to a good approximation, \( \Pi \) may be assumed to be equal to 1 for
simplicity. Therefore, the gas temperature $T_g$ from equation 2.20 is

$$T_g = \frac{T_a}{M+1} \quad 2.23$$

### 2.5.3 Effective Flame Temperature Determination

In calculating radiation heat transfer from a flame to the surface of the furnace wall, the effective flame temperature $T_{f1}$ should be used instead of adiabatic flame temperature $T_a$. The adiabatic flame temperature is the maximum temperature that can be achieved for the given reactants when combustion takes place adiabatically. Any heat transfer from the reacting substances or incomplete combustion would tend to lower the temperature of the products (Wylen and Sonntag 1986). Therefore, the adiabatic temperature does not represent the actual flame temperature or its effect on the wall surface. Polyak and Shorin (1988) suggested an equation for the effective flame temperature calculation.

$$\left( \frac{T_{f1}}{T_a} \right)^4 = m \left( \frac{T_g}{T_a} \right)^{4n} \quad 2.24$$

The exponent $n$ (the indicator of the thermal conditions of a furnace) and the empirical coefficient $m$ are
established on the basis of experiments. Empirical results indicate that the value of m and n is very weakly dependent on the operational conditions of a furnace. For the majority of fuels, the value of m is near 1, the value of n is assumed to be 0.8 according to the available experimental data (Blokh, 1988). From equation 2.24, the effective temperature of the furnace medium is

$$T_{fl} = T_a \left( \frac{T_g}{T_a} \right)^{0.8}$$

2.5.4 Gas Emissivity Determination

The gas emissivity calculation is based on the method introduced by Incropera and DeWitt (1985). From the combustion equation 2.16 the combustion products consist of 1 mole of CO$_2$ ($n_{CO_2}$ = 1), 2 mole of H$_2$O ($n_{H2O}$ = 2), 0.2 mole of O$_2$ and 8.727 mole of N$_2$. The total mole number of combustion products, n, is 11.472. The O$_2$ and the N$_2$ are transparent gases which are considered nonparticipating in the radiation heat transfer. CO$_2$ and H$_2$O are heteropolar gases with somewhat unsymmetrical molecules which are responsible for the radiation participation of the gas body. Therefore, the total gas emissivity for a mixture of carbon dioxide and water vapor with other nonradiating gases is

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta \varepsilon$$
Where $\varepsilon_w$ is water vapor emissivity, $\varepsilon_c$ is carbon dioxide emissivity, $\Delta \varepsilon$ is correction factor associated with mixtures of $H_2O$ and $CO_2$ (Incropera and DeWitt 1985).

Since the energy is emitted by the individual molecules in the body of gas, it is reasonable to expect that the proportion of the radiant energy emitted will depend on the number of molecules in the path of the beam. At a given temperature, this number is the function of the partial pressure $P$ of the emitting gas and the mean beam length $L$. The mean beam length $L$ from Table 13.4 (Incropera and DeWitt 1985) is

$$L = \frac{3.6V}{A} \text{ (ft)} \tag{2.27}$$

According to the Dalton Model for the mixtures of gases (Wylen and Sonntag 1986), the partial pressure for each component gas inside furnace is directly proportional to the number of molecules of elements. Assuming the total pressure inside furnace is 1 atm, the partial pressure of water vapor is

$$P_{H_2O} = \frac{n_{H_2O}P}{n} = \frac{(2)(1)}{(11.472)} = 0.174 \text{ atm} \tag{2.28}$$

The emissivity of water vapor $\varepsilon_w$ can be obtained from Figure 13.15 (Incropera and DeWitt 1985) with the gas
temperature $T_g$ and $P_{\text{H}_2\text{O}}L$. The correction factor for obtaining $\varepsilon_u$ at the pressure other than 1 atm can be obtained from Figure 13.16 (Incropera and DeWitt 1985).

The partial pressure of carbon dioxide is

$$P_{\text{CO}_2} = \frac{n_{\text{CO}_2}P}{n} = \frac{(1)(1)}{(11.472)} = 0.087 \text{ atm} \quad 2.29$$

The emissivity of carbon dioxide, $\varepsilon_c$, can be obtained from Figure 13.17 (Incropera and DeWitt 1985) with the gas temperature $T_g$ and $P_{\text{CO}_2}L$.

The correction factor $\Delta\varepsilon$ can be obtained from Figure 13.19 (Incropera and DeWitt 1985) with the gas temperature $T_g$, $L(P_{\text{H}_2\text{O}}+P_{\text{CO}_2})$ and $P_{\text{H}_2\text{O}}/(P_{\text{H}_2\text{O}}+P_{\text{CO}_2})$.

Therefore, the total gas emissivity $\varepsilon_g$ can be calculated by equation 2.26.

2.5.5 Flame Emissivity Determination

The radiative properties of a gas or oil flame are governed by the radiative properties of the furnace gases ($\text{CO}_2$ and $\text{H}_2\text{O}$) and soot carbon particles (Blokh 1988). Therefore, the effective emissivity of a flame when burning fuel oil or a gas is calculated by the following formula (Blokh 1988).

$$\varepsilon_{fl} = m\varepsilon_{s,g} + (1-m)\varepsilon_g \quad 2.30$$
Where \( m \) is the relative occupation of the furnace volume by the luminous portion of the flame. Blokh assumed that for gas \( m=0.1 \), and for fuel oil \( m=0.55 \). The value of \( \varepsilon_g \) obtained above is valid for radiation from nonluminous flames. The optical thicknesses of the layer for the gaseous combustion products (\( CO_2 \) and \( H_2O \)) can be calculated from equation 2.31 (Blokh 1988).

\[
\varepsilon_g = 1 - e^{-t_g} \tag{2.31}
\]

The soot emissivity \( \varepsilon_{s,g} \) from luminous flames represents the radiative properties of soot carbon particles suspended in the flow of gases. These soot particles emit continuously over the electromagnetic radiation spectrum covering both the visible and infrared regions. Thermal radiation from soot particles in a gas or oil flame depends on the concentration of particles, their size and optical thicknesses of layer for the flows of soot particles. The emissivity is calculated by the equation 2.32 (Blokh 1988).

\[
\varepsilon_{s,g} = 1 - e^{-(t_s + t_g)} \tag{2.32}
\]

Where \( t_s \) is the optical thicknesses of the layer for the flows of soot particles. Blokh (1988) also presented a formula for calculating \( t_s \).
\[ t_s = 0.03(1.6 \times 10^{-3} - 0.5)(2 - \alpha) \frac{C^R}{H^R} L \] 2.33

Where \( \alpha \) is the fuel-air ratio, and \( \frac{C^R}{H^R} \) is the ratio of the relative content of carbon and hydrogen in the working mass of the fuel (Blokh 1988), which is equal to

\[ \frac{C^R}{H^R} = 0.12 \sum \frac{n_C}{n_H} C_{n_C} H_{n_H} \] 2.34

Where \( n_C \) and \( n_H \) are the numbers of carbon and hydrogen atoms in a compound. For methane (\( CH_4 \)), \( \frac{C^R}{H^R} \) is 0.03.

Substituting \( \varepsilon_{s,g} \), \( \varepsilon_g \) and \( m \) into equation 2.32 to calculate the total flame emissivity \( \varepsilon_{t1} \).

Blokh (1988) also presented a condition of the soot formation, that is, in a reaction formula described as

\[ C_{n_C} H_{n_H} + yO_2 \rightarrow (n_C - 2y)C + 2yCO + \frac{n_H}{2}H_2 \] 2.35

only when \( n_C > 2y \), can soot be formed. The natural gas combustion equation 2.16 illustrates that, \( n_C = 1, y = 2.2, n_C < 2y \). From the soot formation condition, it can be concluded that the soot does not form when burning natural gas and the emissivity of soot \( \varepsilon_{s,g} \) is equal to zero. Therefore thermal radiation for the natural gas flame is mainly determined by the radiation of the gaseous products of complete combustion (\( CO_2 \) and \( H_2O \)), that is \( \varepsilon_{t1} = \varepsilon_g \) for
burning natural gas.

2.5.6 Flame View Factor Determination

Flame view factor depends on many factors such as structure, shape and size of the flame, type of the fuel and location of the burners. Since we selected the well-stirred furnace model, the flame diffuses to gases, and the gases mix uniformly so that all energy emitted from the gas body is incident on the enclosing furnace inner surface (Beer and Howarth 1968; Schunmann 1952). Therefore, we can take the view factor for the gas to the furnace wall $F_g-si = 1$.

2.6 Calculation of Heat Losses

The goal of the thesis is to estimate the energy savings using high performance insulation fibers as a furnace wall material. The DOT code did not have the function for calculating energy losses. Knowing the temperature distribution within the furnace wall, the energy losses are calculated. This section presents the energy losses calculated in the subroutine TLOSS added to the DOT code. The total energy losses $E_{total}$ through the wall are equal to the sum of the energy losses through the wall $E_{loss}$ and the energy stored within the wall $E_{stor}$.

2.6.1 Energy Loss Calculation

According to the heat balance at the outer boundary of the
furnace wall, energy losses through the wall is equal to the energy losses from the outer surface by convection and radiation to the surroundings. The outer boundary can be divided into a number of one dimensional boundary elements. For a boundary element, the energy balance equation within the time $\Delta t$ is

$$E_{loss} = \Delta t (Q_c + Q_{rad}) = T_h \cdot \Delta t \int_s (h_c + h_{rs}) (T^{(e)} - T_e) \, ds \quad 2.36$$

Where $T_h$ is the thickness of element. For a plane, $T_h$ is a unit length. For the axisymmetric ring element with inner radius $r_i$ and outer radius $r_o$,

$$T_h = \pi (r_o + r_i)$$

Selecting the boundary element as a linear element $L$ with two points, the shape function at a location $x$ in the linear element is

$$N_1 = \frac{L - x}{L}; \quad N_2 = \frac{x}{L} \quad 2.37$$

From equation 2.18, the temperature at the middle point of the element is

$$T^{(e)} \bigg|_{x = \frac{L}{2}} = \sum N_i T_i \bigg|_{x = \frac{L}{2}} = N_1 T_1 + N_2 T_2 \bigg|_{x = \frac{L}{2}} = \frac{T_1 + T_2}{2} \quad 2.38$$

Where the nodal point temperature $T_1$ and $T_2$ are already calculated from DOT code. The temperature at middle point of the element is the element average temperature $T_{avg}$. 
Thus, taking the known temperature $T_{avg}$, and the same heat transfer convection coefficient $h_c$ and radiation coefficient $h_{rs}$ at the outer surface as that calculated above, the total energy losses from the furnace outer surface from the initial time step to the final time step $NT$ is calculated by

$$E_{loss} = T_H \cdot \sum_{m=0}^{NT} \Delta t \sum_{n=1}^{NE} [(h_c + h_{rs})(T_{avg} - T_e)]_n L_n^m$$

Where $NE$ is the number of boundary elements. This calculation is performed within subroutine HLOSS.

### 2.6.2 Stored Energy Calculation

The energy storage from initial time step to final time step $NT$ is:

$$E_{stor} = T_H \cdot \sum_{n=1}^{E} [C_{ij}(TF_j - TI_j)]_n$$

with

$$C_{ij} = \int \int_{Q_e} N_i \rho C_p N_j J d\xi d\eta$$

Where $E$ is the number of the finite elements within furnace wall and $T_H$ is the thickness of the finite element. $TF_j$ is the nodal point temperature at final time step, $TI_j$ is the nodal point temperature at the initial time step. Both $TF_j$ and $TI_j$ are already obtained from DOT code. $C_{ij}$ is obtained by performing the integral in equation 2.40 over each element. Therefore, substituting all known parameters
listed above, the energy stored within furnace wall can be calculated by equation 2.40. The storage term is determined within subroutine HSTOR.

Fig 2.2 illustrates the flow chart of the total energy loss calculation.

![Flow chart of energy loss calculation](image)

In the flow chart, TLOSS is a subroutine for calculating the total energy losses. HLOSS is a subroutine for calculating the energy losses through the wall, in which the time dependent fluid or environment temperature is considered in subroutine INTERP and the temperature dependent convection heat transfer coefficient $h_c$ is calculated in subroutine HPROP. HSTOR is the subroutine for summing the stored energy within each element obtained from
subroutine QSTOR. Three subroutines are called in the subroutine QSTOR, these are subroutine SHAP1, for calculating the shape function of the element, subroutine DERIV1, for calculating the derivative of the shape function and subroutine MATRP for calculating the density \( p \) and specific heat \( c \) of the wall material. The computer program for energy loss calculation is listed in Appendix E.

The effect of the gas convection on the energy losses through a 1-D furnace wall is studied under the steady-state condition. Due to the energy balance at inner surface of the wall, the energy losses are equal to the sum of the energy losses by convection and radiation energy exchange between the flame and the inner wall surface. Assuming that the gas emissivity is constant, the convection energy losses are compared with the radiation energy losses in Figure 2.3 for different convective heat transfer coefficients \( h_c \).

Figure 2.4 is the comparison of the convective and the radiative heat losses with a constant convective heat transfer coefficient \( h_c \) versus the ratio of the radiative heat transfer coefficients \( h_r \) to its maximum \( h_{r\text{max}} \). The maximum \( h_{r\text{max}} \) is determined by setting the gas temperature \( T_g \) equal to adiabatic flame temperature. In addition, the following value are assumed; effective emissivity \( \varepsilon_{f_1} = 0.5 \) and view factor \( F_{g-si} = 1 \) (see equation 2.7).
Figure 2.3 illustrates that the convective energy losses increase while the radiative energy losses decrease as the $h_c$ increases. When $h_c$ is less than 10 W/M^2-K, the radiative energy losses are over 16 times the convective energy losses. Therefore, the convection heat transfer can be ignored at the inner surface in this case. On the other hand, the convective energy losses is significant and can not be ignored when $h_c$ is greater than 10 W/M^2-K.
Figure 2.4 indicates that the radiative energy losses increase and the convective energy losses decrease as the ratio of $h_r$ to $h_{r\text{ max}}$ increases. When this ratio is greater than 1, the convective energy losses are much smaller than radiative energy losses and can be ignored. When the ratio is less than 1, the convective energy losses approach the radiative energy losses and cannot be ignored.
2.7 Computer Code Verification

This section presents the comparison between the analytical solution and the results obtained from the DOT and the TOPAZ codes for a transient problem. This was done to verify the accuracy of the selected computer codes.

It is assumed that the 1-D furnace wall is 3 inch thick. A finite element mesh and a sketch of the wall are shown in Figure 2.5. The wall is divided into 32 nodes and consists of 15 equally spaced 4-node planar elements.

![Fig. 2.5 Sketch and mesh of a 1-D furnace wall](image)

The wall material is a fiber blanket with a bulk density $\rho=1.0$ sec$^2$-lb/in$^4$, thermal conductivity $K=1.0$ BTU/sec-in-F and specific heat $c=1.0$ BTU-in/sec$^2$-lb-F. Therefore, the thermal diffusivity $\alpha=K/\rho=1.0$ in$^2$/sec. The wall initial temperature $T(x,0)$ is 100 °F, and is exposed
to a constant surface heat flux of unit intensity $Q(0,t)=1.0$ at the inner surface of the wall ($x=0$). The outer surface ($x=3.0$ inches) is assumed to be insulated.

Temperatures were computed using the DOT and TOPAZ codes with $\Delta t=0.025$ seconds.

The analytical solution for the temperature at time $t$ and at any distance $x$ within the wall subjected a unit surface heat flux is given by Carslaw and Jaeger (1959) as

$$T(x, t) = T(x, 0) + \frac{2}{K} \left( \frac{\alpha t}{\pi} \right)^{\frac{1}{2}} e^{-\frac{x^2}{4\alpha t}} - \frac{X}{2} e\text{rfc}\left[\frac{X}{2\sqrt{\alpha t}}\right] \quad 2.41$$

The temperature distributions within the wall obtained from the DOT and TOPAZ codes are compared with the analytical solution in Figure 2.6 for the time $t=1$ second. The solid curve represents the analytical solution, the cross symbol points are the nodal temperatures obtained from the DOT program and the diamond symbol points are the nodal temperatures obtained from TOPAZ code.
As shown in Figure 2.6, the numerical results obtained from DOT and TOPAZ agree well with the analytical solution. Therefore, the selected computer codes DOT and TOPAZ are believed reliable for modeling the heat transfer in the wall of an industrial furnace. Chapter 3 will provide the results of several typical furnaces models using the DOT and TOPAZ codes.
CHAPTER 3
MODELING RESULTS FOR SEVERAL FURNACE WALLS

This chapter presents the modeling results of two furnace walls; the cylindrical furnace and the walking beam furnace. These furnace configurations represent most furnace geometries used in the industry as described in Chapter 1. This chapter also provides a comparison of energy losses for furnace walls insulated with different materials.

3.1 Cylindrical Furnace

Cylindrical furnaces can be divided into two categories. One category is the horizontal furnace such as vacuum heat treatment furnace (Walker 1974) and the rotary kiln (Barr 1989). Vertical furnaces, such as the coil steel sheet coil annealing furnace (Jaluria 1984), ammonia reformer (FPDT 1983) and intermittent ceramic firing kilns (Choudhury, Kayal, and Chakravarty 1990) form the other major category.

3.1.1 Structure of the furnace

The structure of the steel coiled sheet annealing furnace is shown in Figure 3.1 (Jaluria 1984).
Fig. 3.1 Sketch of steel sheet coil annealing furnace

All dimensions in Figure 3.1 are in meters. The total height and outer diameter of furnace are 4.8 meter and 2.4 meters, respectively. In this furnace, rolled cylindrical coils of steel sheet are stacked vertically, and separated by convertor plates. A stainless steel cover encloses the coils. The burners are located circumferentially around the
furnace wall, and are approximately 1.2 meters above the bottom of the furnace. The natural gas enters the burners and is directed tangentially into furnace to cause a swirl in the flow. The wall and the roof of the furnace are made of the multi-layer composite of refractory materials which are high performance insulation fibers (model 1) or insulating fire brick (model 2). The thickness of the wall is 0.32 meters, and the thickness of the roof is 0.4 meters. This thesis presents both models under the same conditions (including the mesh of the furnace, boundary conditions, initial condition and time steps) to compare the energy losses through the wall for two kind materials.

3.1.2 Finite Element Mesh

Since the furnace is axisymmetrical, the axis of the cylinder is designated an adiabatic surface and only the half furnace mesh must be generated. The mesh is divided into 300 nodal points, and is composed 236 uniform elements. Each element is composed of 4 nodes, which is 0.1 m high, 0.08 m wide with unit radian length thickness. Figure 3.2 shows the cylindrical furnace mesh.
3.1.3 Input Data Preparation

According to the requirement of DOT and TOPAZ codes, the following data are prepared for the cylindrical furnace model.
3.1.3.1 Time Data. This furnace is a batch furnace. Each cycle from heating to soaking is about 24 hours (86400 seconds). The time interval was \( \Delta t = 200 \) seconds. At time \( t = 0 \), the temperature at each nodal point is room temperature 27 °C.

3.1.3.2 Boundary Conditions. The inner surface of the wall is exposed to flame radiation and gas forced convection. The radiation parameters used in the model are selected as follows: the gas temperature \( T_g \) is 1470 K obtained from equation 2.59 with \( M = 0.49 \) and \( T_a = 2190 \) K. The effective flame temperature \( T_{fl} \) is 1592 K (Equation 2.61) and the flame emissivity \( \varepsilon_{fl} \) is found to be 0.1852 from equation 2.66 with \( m = 0.1 \), \( \varepsilon_g = 0.185 \) and \( \varepsilon_{sg} = 0.185 \). The surface emissivity \( \varepsilon_w \) depends on surface materials, and will be selected in next section.

Since the boundary layer thickness \( \delta = 0.035 \) meter (equation 2.34) is much smaller than the distance between surfaces (1.76 meter). The convection between the gas and inner surface is modeled as external forced convection. The forced convection heat transfer coefficient \( h_c \) at each temperature point is calculated by subroutine FTEMP by inputing gas velocity \( V = 10 \) m/s and gas properties. These are temperature dependent functions shown in Figure 3.3.
Fig. 3.3 Temperature dependent heat transfer coefficient $h_c$ at inner wall surface for a cylindrical furnace

The outer surface of the wall is exposed to air free convection and radiation exchange with the surroundings. The convection between the air and outer surface of the wall is similar to the free convection over a vertical plate while the convection between the air and the outer surface of the furnace roof is modeled as free convection over the horizontal upper surface of a heated plate. Both heat transfer coefficients $h_{cv}$ for vertical wall and $h_{ch}$ for horizontal roof at each temperature point are calculated in
subroutine FTEMP with air properties, which are the temperature dependent functions shown in Fig. 3.4. The air and surrounding temperatures are selected to be 27 °C.

![Fig. 3.4 Temperature dependent heat transfer coefficient at outer wall surface for a cylindrical furnace](image)

**Fig. 3.4** Temperature dependent heat transfer coefficient at outer wall surface for a cylindrical furnace

**3.1.3.3 Material Property Determination.** Bulk material properties include thermal conductivity \( k \), density \( \rho \), and specific heat \( c \), which are selected for two material models (fiber model and insulating fire brick model) in this section.
For the fiber model, both the wall and the roof are assumed to be composed of three layers of Cerachrome fiber blankets with different densities as shown in Figure 3.1. The thermal conductivity is temperature dependent as shown in Figure 3.5.

![Thermal Conductivity of Cerachrome](image)

**Fig. 3.5** Cerachrome thermal conductivity at different densities

The specific heat $c$ is also a temperature dependent function as indicated below (Manville 1984):

- $c = 1046.6 \text{ J/kg.K at } T = 550 \degree C$
- $c = 1130.4 \text{ J/kg.K at } T = 1100 \degree C$
The composition of Cerachrome fiber is 40% $\text{Al}_2\text{O}_3$, 53% $\text{SiO}_2$ and 7% $\text{Cr}_2\text{O}_3$ and the structure is anisotropic. From Figure 1.7, the anisotropy factor $K_{\text{paral}}/K_{\text{perp}}$ is selected to be 1.23, 1.37 and 1.5 for 4 pcf (64 kg/m$^3$) density, 6 pcf (96 kg/m$^3$) density and 8 pcf (128 kg/m$^3$) density, respectively (Bomberg and Klarsfeld 1983). A input file for the Cerachrome fiber model is chosen as a sample input file for the DOT code in Appendix F.

For the case in which the furnace is lined with insulating fire brick (IFB), the wall and the roof are equally divided into two layers of insulation bricks. At the hot surface, the density of the IFB is 384 kg/m$^3$ (24 pcf) while the density at cold surface is 288 kg/m$^3$ (18 pcf). The specific heat is selected to be 960 J/kg.k for both layers. The thermal conductivity is also a temperature dependent function as shown below (Manville 1982a).

\[
\begin{align*}
K &= 0.133 \text{ W/m.k} \quad \text{at } T = 500 ^\circ \text{F} (260 ^\circ \text{C}) \\
K &= 0.163 \text{ W/m.k} \quad \text{at } T = 1000 ^\circ \text{F} (538 ^\circ \text{C}) \\
K &= 0.196 \text{ W/m.k} \quad \text{at } T = 1500 ^\circ \text{F} (816 ^\circ \text{C}) \\
K &= 0.228 \text{ W/m.k} \quad \text{at } T = 2000 ^\circ \text{F} (1093 ^\circ \text{C})
\end{align*}
\]

3.1.4 Results

After entering the data tabulated above into the programs DOT and TOPAZ, the following results are obtained.
3.1.4.1 Temperature Distribution. The steady state wall temperature contours are plotted in Figures 3.6-3.8. The criterion of the steady-state is that the temperature at each point within the wall is independent of time (Incropera and DeWitt 1985), that is, the temperature at the present time step should be equal to that at the previous time step at each point in the wall. It is difficult for numerical calculation to obtain the same temperature at different time. Therefore, an approximate criterion for steady-state is set in DOT code as

$$0.9999 \leq \text{Minimum} \left| \frac{T_{i-1}}{T_i} \right| \leq 1$$

The minimum ratio of the temperature $T_{i-1}$ at the previous time step to the temperature $T_i$ at the present time step among all points within the wall is between 1 and 0.9999.

Figure 3.6 reveals the temperature contours at the midplane and outer surface from the DOT program for the fiber model and the brick model. Figures 3.7 and 3.8 illustrate the temperature contours obtained from the TOPAZ program for the fiber and brick models, respectively.

The nodal temperatures of the fiber wall are much lower than that of the brick wall. It is clear that the energy transferred from the fiber wall to the surroundings is less
than that from the brick wall.

Fig. 3.6 Temperature contours of the fiber and brick wall obtained from the DOT code
Fig. 3.7 Temperature contours of the fiber wall obtained from the TOPAZ code
3.1.4.2 Temperature History. Temperature history represents the thermal response of the wall to a change in boundary conditions and is used to determine the energy...
required to heat the furnace wall to steady state conditions. Figures 3.9 and 3.10 are comparisons of the temperature response for the fiber wall and the brick wall at the center and the outer surface of the wall.

![Graph showing temperature history curves for fiber and brick walls](image)

**Fig. 3.9** Temperature history curves for the fiber and the brick wall obtained from DOT code

It took about 12 hours of heating the fiber wall to reach steady state from the initial condition while heating the brick wall required approximately 24 hours, as shown in both Figures 3.9 and 3.10.
Fig. 3.10 Temperature history curves for the fiber and the brick wall obtained from TOPAZ program

Therefore, the fiber wall requires less energy to heat it to steady state conditions than the brick wall.

3.1.4.3 Energy Savings. The energy savings of using fiber instead of brick are calculated from the temperature profiles obtained from DOT and TOPAZ programs. The heat transfer rate, the total energy transferred through and the total energy stored within the wall are listed in Table 3.1. The total energy represents the sum of the transferred and
stored energy columns. The percent energy savings are equal to the ratio of the difference in energy losses between the brick wall and the fiber wall to the energy losses from the brick wall.

Due to the strong nonlinearity caused by radiation and the temperature dependent material properties, the results obtained from the DOT code are not as accurate as that obtained from the TOPAZ code, as mentioned in section 2.3.2.

**TABLE 3.1**
Energy Losses through the Fiber Wall and the Brick Wall

<table>
<thead>
<tr>
<th>Material</th>
<th>Code</th>
<th>Heat Rate (W)</th>
<th>Transferred Energy (J)</th>
<th>Stored Energy (J)</th>
<th>Total Energy (J)</th>
<th>Energy Saving Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>DOT</td>
<td>1730.8</td>
<td>9.39E08</td>
<td>0.95E08</td>
<td>10.34E08</td>
<td>27.4</td>
</tr>
<tr>
<td>Brick</td>
<td>DOT</td>
<td>2384.5</td>
<td>12.94E08</td>
<td>3.01E08</td>
<td>15.95E08</td>
<td>68.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>35.2</td>
</tr>
<tr>
<td>Fiber</td>
<td>TOPAZ</td>
<td>2718.4</td>
<td>14.76E08</td>
<td>0.99E08</td>
<td>15.74E08</td>
<td>18.1</td>
</tr>
<tr>
<td>Brick</td>
<td>TOPAZ</td>
<td>3320.6</td>
<td>18.03E08</td>
<td>3.25E08</td>
<td>21.27E08</td>
<td>69.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>26.0</td>
</tr>
</tbody>
</table>

As indicated in Table 3.1, using fiber as the furnace wall material can save 26% to 35% of the energy consumed when brick is used in a cycle for the cylindrical furnace.
3.2 Walking Beam Furnace Model

The walking beam furnace is one type of a rectangular tunnel furnace which is widely used in industry as mentioned in Chapter 1. The walking beam furnace is used as a reheat and a heat treatment furnace in the metal industries and it is usually operated continuously. This section presents a detailed heat transfer model for a walking beam furnace wall consisting of commercially available (existing) fiber or a future fiber with improved properties (enhanced fiber). Energy savings are predicted when an enhanced fiber is used instead of commercially available fibers.

3.2.1 Structure of the Furnace

A walking beam furnace, fired by natural gas for reheating steel billets (Braun 1982), was taken as an example of a continuous rectangular furnace for heat transfer modeling purposes. The furnace internal dimensions are 52' (15.85 m) long by 10' (3.048 m) wide by 5' (1.524 m) high. Billets are transported through the length of the furnace by the walking beam mechanism in about two hours. The two outer hearth sections remain stationary, while the center section moves in a rectangular pattern: up, forward, down and back. The billets are picked up by the upward movement of the beam, move forward about 0.3048 m (12 inches), and are then redeposited on the stationary
sections with the downward movement. The furnace is divided into three zones along the furnace length: a preheat zone, a heating zone and a soak zone. Table 3.2 lists the operating temperature, fiber type and Z-Blok size in each zone.

TABLE 3.2
Operating Conditions in Three Zones

<table>
<thead>
<tr>
<th>Zone</th>
<th>Operating Temperature</th>
<th>Fiber Type</th>
<th>Blok Size (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soak</td>
<td>2350 °F 1288 °C</td>
<td>SAFFIL 2600 °F</td>
<td>9</td>
</tr>
<tr>
<td>Heat</td>
<td>1850 °F 1010 °C</td>
<td>FIBERMAX 3000 °F</td>
<td>9</td>
</tr>
<tr>
<td>Preheat</td>
<td>1400 °F 760 °C</td>
<td>KAOWOOL 2600 °F</td>
<td>9</td>
</tr>
</tbody>
</table>

Z-Blok modules are formed from a continuous length of refractory fiber blanket which is accordion folded into a square or rectangular configuration. All of the integral stainless steel reinforcement and mounting components are located entirely within 1.25 inches of the cold face to protect them from corrosive gases and high face temperature. The modules are held in a state of compression by restraint banding which is cut and removed after installation. The modules are installed in the arched roof and sidewalls. The movement of the flue gases is similar to the internal flow along the axial direction of the furnace.
A sketch of the walking beam furnace is shown in Figure 3.11, in which the unit of dimension is the meter.

Fig. 3.11 Sketch of a walking beam furnace

Burners are roof mounted and fan flames out parallel to the roof. This type burner heats the roof first, then the roof radiates the heat down to billets. A fan is used to circulate furnace air to provide a well mixed environment.
3.2.2 Finite Element Mesh

Since the cross section of the furnace is symmetrical about the vertical midplane, only half of furnace mesh need be modeled. The mesh is divided into 215 nodal points, and is composed of 168 elements of different dimension. Each element is composed of 4 nodes with a unit thickness.

The gap between the stationary beam and the moving beam is considered as an air conduction body subjected to high temperature air convection. The finite element method requires that the body must be analyzed as a continuum, and the element must be interconnected at nodal points. Therefore, the continuum body in this model consists of the air conduction body, the moving and stationary beams.

The geometry of the Z-Blok is complicated, and the way in which the Z-Blok is installed in the wall makes the thermal conductivity different in all three directions. Therefore, in this thesis, the Z-Blok is modeled as a homogeneous body with an effective thermal conductivity. Figure 3.12 illustrates the walking beam furnace mesh. The Z-Blok modules are installed in the roof and sidewall of the furnace. On the other hand, the bottom is assumed to be made of insulating fire brick (IFB). Because the thermal gradients in the parties insulated by fiber are expected to be larger than those insulated by IFB, the mesh for upper
part of the furnace is fine whereas that for the lower part is rough.

**Fig. 3.12 Finite element mesh of a walking beam furnace**

### 3.2.3 Input Data Preparation

According to the requirements of the DOT and TOPAZ codes, the following data were prepared for the walking beam furnace model.
3.2.3.1 Time Data. The furnace was modeled for one 24 hour (86400 seconds) period. At time t=0, the temperature at each nodal point is room temperature 27 °C.

3.2.3.2 Boundary Conditions. This furnace is modeled as a continuous body. The inner surface of the wall is exposed to flame radiation and gas forced convection. The relevant radiation parameters are the following: the gas temperature $T_g$ is taken as the operating temperature listed in Table 3.2, the flame emissivity $\varepsilon_{fl}$ was found to be 0.22 for the soak zone, 0.26 for the heat zone, and 0.37 for the preheat zone. The inner fiber surface emissivity $\varepsilon_w$ is selected to be 0.5 (Incropera and DeWitt 1985).

Since the length of the furnace is 7.8 times the hydraulic diameter, the convection between the gas and inner surface is similar to the internal forced convection. The average forced convection heat transfer coefficients $h_c$ at selected temperature points (the maximum selected temperature should be greater than maximum average temperature of gas and inner surface of the wall) are calculated from subroutine FTEMP by inputing gas properties at selected temperature points and applying the following gas flow at each zone.

- Soak zone: 6.53 m³/sec. or 830,000 SCFH
- Heat zone: 4.523 m³/sec. or 575,000 SCFH
Preheat zone 1.699 m³/sec. or 216,000 SCFH

The convective coefficient $h_c$ is dependent on temperature at each zone as shown in Figure 3.13. At each step $h_c$ is interpolated upon the average temperature.

![Graph](image)

**Fig. 3.13 Convection heat transfer coefficient at the inner surface for the walking beam furnace**

The wall surface is exposed to free convection and radiation exchange with the surroundings. The calculation method used to determine both the heat transfer coefficient $h_{cv}$ for vertical wall and that ($h_{ch}$) for horizontal roof at
selected temperature points for the walking beam furnace is the same as that of the cylindrical furnace, which are temperature dependent functions shown in Figure 3.14.

![Graph showing convection heat transfer coefficient at outer surface for the walking beam furnace.](image)

**Fig. 3.14** Convection heat transfer coefficient at outer surface for the walking beam furnace

The air convection within the gap between the moving and stationary beam is similar to the convection between two parallel plates which are at the same temperature (Kays and Crawford 1980). The convective heat transfer coefficient is calculated based on the air properties at the average
temperature of gas inside furnace and air outside the furnace, which is found to be 10.64 W/m².K.

3.2.3.3 Material Property Determination. Material properties include thermal conductivity K, density ρ, specific heat c and shrinkage, which are selected for different fibers shown in Table 3.3.

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>Density (kg/m³)</th>
<th>Specific Heat (J/kg.K)</th>
<th>Thermal Conductivity (W/m.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KAOWOOL 2600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>185.8</td>
<td>1048.8</td>
<td>0.1262</td>
</tr>
<tr>
<td>Enhanced</td>
<td>184.2</td>
<td>1048.8</td>
<td>0.0936</td>
</tr>
<tr>
<td>Enhanced</td>
<td>197.1</td>
<td>1048.8</td>
<td>0.0927</td>
</tr>
<tr>
<td>Crystalline</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FIBERMAX</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>108.9</td>
<td>1246.0</td>
<td>0.1139</td>
</tr>
<tr>
<td>Enhanced</td>
<td>105.7</td>
<td>1246.0</td>
<td>0.102</td>
</tr>
<tr>
<td>Enhanced</td>
<td>113.2</td>
<td>1246.0</td>
<td>0.1004</td>
</tr>
<tr>
<td>Crystalline</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAFFIL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>123.4</td>
<td>1046.0</td>
<td>0.1121</td>
</tr>
<tr>
<td>Enhanced</td>
<td>112.1</td>
<td>1046.0</td>
<td>0.0989</td>
</tr>
<tr>
<td>Enhanced</td>
<td>120.1</td>
<td>1046.0</td>
<td>0.0963</td>
</tr>
<tr>
<td>Crystalline</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Three models are considered for each fiber in each zone as listed in Table 3.3: existing, enhanced and enhanced crystalline fiber models. The enhanced fiber properties represent those of "as received" fibers. The existing fiber
properties represent those of the "as received" fibers after heat treatment. The data for existing and enhanced fiber are obtained from measurements in the thermal conductivity laboratory at the Manville Corporation (Manville K-LAB Memo Report 1990). The enhanced crystalline fiber is assumed to have the properties of enhanced fiber with 10% shrinkage. This is assumed to occur only at the hot inner surface layer (about 0.051 m) because fiber will change from the amorphous phase into a crystalline phase at high temperature (Kingery 1963). After shrinkage, the thickness of the layer is 0.046 m. As the bulk fiber density increases, the thermal conductivity decreases. The density for the enhanced crystalline fiber is calculated based on the density of the enhanced fiber. The thermal conductivity for the enhanced crystalline fiber is then calculated from equation 1.1.

3.2.4 Results

After incorporating the above data into the program TOPAZ, the following results are obtained.

3.2.4.1 Temperature Distribution. The temperature contours are similar to that of the cylindrical furnace except that the temperatures are notably different. To compare the temperature distribution among three fibers within 3 zones, the nodal point temperatures within the furnace wall and roof are listed in the Table 3.4.
### TABLE 3.4

Nodal Point Temperatures within the Wall and the Roof

<table>
<thead>
<tr>
<th>Zone and Fiber</th>
<th>Temperature (K) at Distance from Inner Surface (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Soak Zone--SAFFIL</td>
<td></td>
</tr>
<tr>
<td>WALL</td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>1558</td>
</tr>
<tr>
<td>Enhanced</td>
<td>1558</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>1559</td>
</tr>
<tr>
<td>ROOF</td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>1558</td>
</tr>
<tr>
<td>Enhanced</td>
<td>1558</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>1559</td>
</tr>
<tr>
<td>Heat Zone--FIBER-max</td>
<td></td>
</tr>
<tr>
<td>WALL</td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>1279</td>
</tr>
<tr>
<td>Enhanced</td>
<td>1279</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>1279</td>
</tr>
<tr>
<td>ROOF</td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>1279</td>
</tr>
<tr>
<td>Enhanced</td>
<td>1279</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>1279</td>
</tr>
<tr>
<td>Preheat Zone--KAOWOOL 2600</td>
<td></td>
</tr>
<tr>
<td>WALL</td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>1028</td>
</tr>
<tr>
<td>Enhanced</td>
<td>1029</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>1029</td>
</tr>
<tr>
<td>ROOF</td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>1028</td>
</tr>
<tr>
<td>Enhanced</td>
<td>1029</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>1029</td>
</tr>
</tbody>
</table>
The temperature of the enhanced fiber wall is lower than that of the existing and crystalline fiber wall. The outer surface temperature of the crystalline fiber is near that of the enhanced fiber while the midplane temperature of the former is larger than that of the latter.

3.2.4.2 Temperature History. Temperature histories represent the response of the wall to thermal energy input and provide a means of calculating the energy stored within the furnace wall. The stored thermal energy depends on the bulk density and the specific heat of the wall materials. The following Figures will show how the stored energy relates to the fiber properties. Figures 3.15 and 3.16 illustrate the temperature histories at the midplane and at the outer surface of the furnace wall for different fibers in the soak zone, respectively.

It took about 3 hours more for the midplane of the wall to reach steady state conditions than it did for the outer surface of the wall in the soak zone. Since the midplane of the crystalline fiber model is subject to shrinkage and its thickness of the layer between midplane and inner wall surface is smaller than that of the existing and enhanced fiber models, the temperature of the enhanced crystalline SAFFIL fiber model at each time step is higher than that of the existing and enhanced SAFFIL fiber models.
Fig. 3.15 Temperature histories at the wall midplane in the soak zone

Fig. 3.16 Temperature histories of the outer wall surface in the soak zone
The temperature history curves for the midplane and the outer surface of three types of FIBERMAX fiber wall are plotted in Figures 3.17 and 3.18, respectively, for the heat zone of the furnace.

In the heat zone, it took about 12 hours for both midplane and outer surface of the furnace to reach steady state conditions. This is because the specific heat of FIBERMAX fiber is higher than that of the SAFFIL fiber in the soak zone (see Table 3.3), although the densities of both fibers are close to each other.

Fig. 3.17 Temperature histories at the wall midplane within the heat zone
Fig. 3.18 Temperature histories of the wall outer surface in the heat zone

The temperature history curves for the midplane and the outer surface of three types of FIBERMAX fiber wall are plotted in Figures 3.19 and 3.20, respectively, for the preheat zone of the furnace.

In the preheat zone, it took about 14 hours for both midplane and outer surface to reach the steady state conditions, because the density of the KAOWOOL 2600 fiber is higher than that of both SAFFIL and FIBERMAX fibers (see Table 3.3). Since the density of the enhanced crystalline (shrunk) KAOWOOL fiber is the largest among three KAOWOOL fibers, it required the greatest time to reach steady-state conditions (see Fig. 3.20). Also, it should be noted that
the temperature at the outer surface of the wall for existing fiber is much higher (Fig. 3.21) because its thermal conductivity is greater than the others.

In addition, the input file for the enhanced KAOWOOL 2600 fiber is chosen as a sample input file for the TOPAZ code in Appendix G.

Fig. 3.19 Temperature histories for midplane of the wall in the preheat zone
3.2.4.3 Energy Savings. In this section, the energy savings observed when the enhanced fiber is used instead of the commercially available fiber was estimated from energy losses obtained from the TOPAZ program listed in Table 3.5. The total energy in Table 3.5 represents the sum of the transferred and stored energy columns. In the section "Over Furnace" in Table 3.5, the value in each column is the sum of the data in each zone for each fiber model. For the existing fiber model, for example, the total energy losses
from whole furnace wall is 3965.7 megajoules.

The percent energy savings rates are equal to the ratio of the difference in energy losses between the existing and enhanced fibers to the energy losses from the existing (commercially available) fiber. The energy consumption rate (cost rate) from shrinkage was estimated as ratio of the difference in energy losses between the enhanced crystalline fiber and the enhanced fiber wall to the energy losses from the enhanced fiber.

Table 3.5 illustrates that using the enhanced fiber as the furnace wall material instead of the existing fiber can save 11.6% of the energy consumed by the wall of the walking beam furnace within a 24 hour cycle. However, if the furnace is operated continuously, only the energy transferred savings rate should be taken into account. Therefore, the energy savings rate for continuous operation is 15.5% of the energy consumed by the wall using a commercially available (existing) fiber.
<table>
<thead>
<tr>
<th>Zone and Fiber Type</th>
<th>Transferred Energy (MJ)</th>
<th>Stored Energy (MJ)</th>
<th>Total Energy (MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Soak Zone</strong> -- <strong>SAFFIL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>820.2</td>
<td>680.3</td>
<td>1500.5</td>
</tr>
<tr>
<td>Enhanced</td>
<td>731.0</td>
<td>613.4</td>
<td>1344.4</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>739.6</td>
<td>615.2</td>
<td>1354.8</td>
</tr>
<tr>
<td>Saving Rate %</td>
<td>10.9</td>
<td>9.8</td>
<td>10.4</td>
</tr>
<tr>
<td>Cost Rate %</td>
<td>2.9</td>
<td>0.3</td>
<td>1.5</td>
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<td><strong>Heat Zone</strong> -- <strong>FIBERMAX</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>651.0</td>
<td>555.8</td>
<td>1206.8</td>
</tr>
<tr>
<td>Enhanced</td>
<td>585.0</td>
<td>530.0</td>
<td>1115.0</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>595.7</td>
<td>532.7</td>
<td>1128.4</td>
</tr>
<tr>
<td>Saving Rate %</td>
<td>10.1</td>
<td>4.6</td>
<td>7.6</td>
</tr>
<tr>
<td>Cost Rate %</td>
<td>1.8</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>Preheat Zone</strong> -- <strong>KAOWOOL 2600</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>594.3</td>
<td>664.1</td>
<td>1258.4</td>
</tr>
<tr>
<td>Enhanced</td>
<td>429.0</td>
<td>616.1</td>
<td>1045.1</td>
</tr>
<tr>
<td>Enhanced Crystalline</td>
<td>446.7</td>
<td>620.8</td>
<td>1067.5</td>
</tr>
<tr>
<td>Saving Rate %</td>
<td>27.8</td>
<td>7.2</td>
<td>17.0</td>
</tr>
<tr>
<td>Cost Rate %</td>
<td>4.1</td>
<td>0.8</td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Overall Furnace</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Existing</td>
<td>2065.5</td>
<td>1900.2</td>
<td>3965.7</td>
</tr>
<tr>
<td>Enhanced</td>
<td>1745.0</td>
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<td>3504.5</td>
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<tr>
<td>Enhanced Crystalline</td>
<td>1782.0</td>
<td>1768.7</td>
<td>3550.7</td>
</tr>
<tr>
<td>Saving Rate %</td>
<td>15.5</td>
<td>7.4</td>
<td>11.6</td>
</tr>
<tr>
<td>Cost Rate %</td>
<td>2.1</td>
<td>0.5</td>
<td>1.3</td>
</tr>
</tbody>
</table>
Table 3.5 also indicates that the energy consumption rate (cost rate) from shrinkage is 1.3% when 10% shrinkage is assumed within the hot inner layer of the furnace wall of thickness 0.051 meter. As previously mentioned, when the shrinkage takes place, the increased density of the fiber increases the time required to reach the steady-state conditions, while the decreased volume reduces the stored energy and the decreased thermal conductivity (see Table 3.3), thereby resulting in decreasing the transferred energy. Because these opposing effects, therefore, the shrinkage does not significantly alter the total energy cost. However, this work did not involve a detailed shrinkage model. Future work should include more detailed studies on the relationship between the individual fiber and bulk fiber or that of temperature and shrinkage.
CHAPTER 4
CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

4.1 Conclusions

Typical industrial furnaces have been modeled by the finite element method. The results have shown that 11%-15% energy savings can be realized when an improved (enhanced) fiber is used over an existing or commercially available fibers as the furnace wall materials. Likewise, 26%-35% energy savings is estimated when refractory fiber is used instead of insulating fire brick. It was found that the annual energy flow through and stored within industrial furnaces represents approximate 0.35 Quad annually in the United States (Chapter 1). Therefore, the energy savings would be 0.0231-0.0315 Quad if the enhanced fiber is used in 60% of existing furnace walls instead of the commercially available fibers. The energy savings would be 0.0546-0.0735 Quad if 60% of the furnace walls were changed from insulating fire brick into the refractory fiber.

In addition to performing the energy savings calculations, a computer program for calculating the convection heat transfer coefficient was developed. This modification will aid industrial engineers involved in studying heat transfer problems associated with furnace
4.2 Suggestions for Further Work

As has been pointed out earlier, the work is based on several assumptions, such as the uniform gas temperature and gas velocity inside a furnace, the homogenous Z-Block module and shrinkage. To make the furnace model more realistic, the following further work is suggested:

4.2.1 Gas Temperature and Velocity Profile Model.
Inside a furnace, the gas temperature and velocity near the inner surface of a furnace wall are functions of time and position. It is necessary to use a more detailed model of the gas mixing to obtain a more reasonable boundary condition at the inner surface of the wall when the velocity is high.

4.2.2 Z-Blok Module Model. The Z-Blok module is a complicated geometry to model, because its material properties are nonhomogeneous in three directions. To effectively model the Z-Blok, a three dimensional heat transfer computer code will be required. In addition, the radiation propagation in fibrous materials is significant and should be included with the solid conduction and convective components.
4.2.3 Shrinkage Modeling. To see the effect of the fiber shrinkage on the heat transfer of the furnace, it is necessary to learn more about the shrinkage distribution within the furnace wall by analyzing the fiber thermal expansion and stress.
REFERENCES CITED


Hollander, Frans, and others. 1983. "Reheating processes and modifications to rolling mill operations for energy savings." Iron and Steel Engineer no. 6 (June): 55-62


For a small element of the furnace wall with a unit thickness shown in Figure A.1, the input energy within a time increment $dt$ is

$$E_{in} = (q_x + q_y)dt$$

and the output energy within a time increment $dt$ is

$$E_{out} = (q_{x, dx} + q_{y, dy})dt$$

The energy balance equation for the small element is

$$E_{in} = E_{out} + \Delta E_{stored}$$  \hspace{1cm} (A.1)

Fig. A.1 An element in a furnace wall

$\Delta E_{stored}$ is the heat storage per unit thickness during time $dt$, and is equal to

$$\Delta E_{stored} = \rho c \ dT \ dx \ dy$$  \hspace{1cm} (1)
Substituting $E_{in}$, $E_{out}$ and $\Delta E_{stored}$ into equation A.1, the heat balance equation for the small element is

$$(q_x + q_y) \, dt = (q_{x+dx} + q_{y+dy}) \, dt + \rho c dT \, dx \, dy(1) \quad A.2$$

From Fourier's law, the heat inflow rate is

$$q_x = -K_x \frac{\partial T}{\partial x} = -K_x \frac{\partial T}{\partial x} \, dy(1) \quad A.3$$

$$q_y = -K_y \frac{\partial T}{\partial y} = -K_y \frac{\partial T}{\partial y} \, dx(1) \quad A.4$$

and by the Taylor series approximation

$$q_{x+dx} \approx q_x + \frac{\partial q_x}{\partial x} \, dx = -K_x \frac{\partial T}{\partial x} - \frac{\partial}{\partial x} (K_x \frac{\partial T}{\partial x}) \, dx$$

$$= -K_x \frac{\partial T}{\partial x} \, dy(1) - \frac{\partial}{\partial x} (K_x \frac{\partial T}{\partial x}) \, dx \, dy(1) \quad A.5$$

$$q_{y+dy} \approx q_y + \frac{\partial q_y}{\partial y} \, dy = -K_y \frac{\partial T}{\partial y} - \frac{\partial}{\partial y} (K_y \frac{\partial T}{\partial y}) \, dy$$

$$= -K_y \frac{\partial T}{\partial y} \, dx(1) - \frac{\partial}{\partial y} (K_y \frac{\partial T}{\partial y}) \, dx \, dy(1) \quad A.6$$

Therefore, the governing differential equation in the cartesian coordinate system for heat conduction in a two dimensional furnace wall by Rao (1982) is

$$\frac{\partial}{\partial x} (K_x \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (K_y \frac{\partial T}{\partial y}) = \rho C_p \frac{\partial T}{\partial t} \quad A.7$$
APPENDIX B

Derivation of the Finite Element Equations

Procedures of the finite element formulation:

Step 1: Divide the furnace wall into $E$ elements. Each element has either four or eight nodes.

Step 2: Assume a suitable form of variation of $T$ in each finite element and express $T_e(x,y,t)$ in element $e$ as

$$T^{(e)}(x,y,t) = \sum_{j=1}^{n} N_j(x,y) T_j(t)$$  \hspace{1cm} \text{(B.1)}

Where $N_j(x,y)$ denotes the shape function corresponding to node $j$ of element $e$ while $T_j(t)$ denotes the temperature of node $j$. The shape function is a simple function to approximate temperature $T$ over an element. It depends on the element type, shape, size and the number of the nodes per element. Therefore, the temperature $T^{(e)}(x,y,t)$ at any point in the element obtained from equation 2.17 is the approximate solution to the exact solution at that point. The shape function is usually expressed in natural coordinates $(\xi,\eta)$. The cartesian coordinates $(x,y)$ can be transferred into natural coordinates by the Jacobian matrix (Rao, 1982). In general, the relationship between the two systems can be expressed as
\[ \iint_{\Omega} f(x,y) \, dx \, dy = \int_{-1}^{1} \int_{-1}^{1} f(x(\xi,\eta), y(\xi,\eta)) \, J \, d\xi \, d\eta \quad \text{B.2} \]

Where the Jacobian, \( J \), is defined by

\[ J = \begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{vmatrix} \]

Step 3: Develop the weak form of the given differential equation. The basic idea of the weighted residual approach is to make the residual \( R \) a minimum in some specified sense. A weighted residual form of the governing differential equation is obtained by multiplying the equation by a series of weighing functions \( W_1 \) and integrating over the element domain. In the Galerkin method, the integral of the weighted residual over the domain of the element is set equal to zero by taking the weighing functions \( w_i \) to be the same as the shape functions \( N_i \) (Rao, 1982). Substituting \( T^{(e)} \) into the differential equation \( 2.1 \), the integral of the weighted residual at any instant of time is

\[ R = \iint_{\Omega} \left[ \frac{\partial}{\partial x} \left( K_x \frac{\partial T^{(e)}}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial T^{(e)}}{\partial y} \right) \right. \]
\[ \left. - \rho C_p \frac{\partial T^{(e)}}{\partial t} \right] N_i \, dA = 0, \quad i=1,2,\ldots,n \quad \text{B.3} \]

Integrating the first and second term of equation \( \text{B.3} \) by parts, the weak form of differential equation is obtained
Step 4: Form the finite element equation

The second term of equation B.4 is the natural boundary condition term resulting from the integration by parts operation. Substituting the natural boundary conditions of the furnace wall and equation B.1 into this term and rearranging equation B.4, the finite element equation can be expressed as

\[- \iint_{\Omega^e} \left[ \frac{\partial N_i}{\partial x} K_x \frac{\partial T^e}{\partial x} + \frac{\partial N_i}{\partial y} K_y \frac{\partial T^e}{\partial y} \right] \cdot dA
\]
\[+ \int_{r^e} N_i \left[ K_x \frac{\partial T^e}{\partial x} n_x + K_y \frac{\partial T^e}{\partial y} n_y \right] \cdot ds
\]
\[- \iint_{\Omega^e} \rho C_p \frac{\partial T^e}{\partial t} N_i \ dA = 0, \quad i=1,2,\ldots,n \quad B.4\]

The integrals over the element domain \(\Omega^e\) and its boundary \(r^e\) are computed using numerical integration. Therefore, the numerical formula equation B.2 is used in equation B.5, upon rearrangement and simplification of equation B.5, the matrix form of the finite element equations can be expressed as
Where

\[ [C] = [C_{ij}] = \int\int_{\Omega} N_i \rho c_p N_j \, J d\xi d\eta \]

\[ [K] = [K_{ij}] = \int\int_{\Omega} \left( \frac{\partial N_i}{\partial x} K_x \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} K_y \frac{\partial N_j}{\partial y} \right) J d\xi d\eta \]

\[ + \int_{\Gamma} N_i (h_c + h_r) N_j d\Gamma \]

\[ \{F\} = \{F_i\} = \int_{\Gamma} N_i N_j (h_c + h_r) T_0 d\Gamma \]

Where \([C]\) is the heat capacity matrix which is dependent on the density \(\rho\) and the temperature dependent specific heat capacity \(C\) of the material. \([K]\) is the temperature dependent thermal conductivity matrix including the convection and radiation heat transfer coefficients at the boundary.

\{T(t)\} = Vector of nodal point temperatures at time \(t\).

\{\dot{T}(t)\} = Vector of the time rate of change of the nodal point temperature at time \(t\).

\{F(t)\} = Vector of the externally supplied heat flux at time \(t\).

Step 5: Assembly of the element equations to derive the
system equations.

Assembly of the element equations is based on the requirement of the continuity of temperature at the element joining nodes. This means that at the common node of the elements, the temperature is the same for all the elements joining at that node. When several elements are connected at a common node, the node conductivity matrix $K_{ij}$ and nodal load vector $F_i$ of each element are added to obtain the net conductivity matrix and the net load vector at that node. Having satisfied this requirement, the system equation is formed by direct summation of the element equations and the boundary conditions are incorporated.

$$\sum_{e=1}^{E} [C^e_{ij} \{ \hat{T}_j \}] + \sum_{e=1}^{E} [K^e_{ij}] \{ T_j \} = \sum_{e=1}^{E} [F^e_i]$$

$i=1,2,\ldots,n; \ j=1,2,\ldots,n$

Step 6: Solve the system equation by the numerical integration schemes.

System equation B.7 represents a set of first order nonlinear differential equations. At any instant "t" in time, the system equation can be expressed as following equation by replacing sum of $C$, $K$, $F$ with $\bar{C}$, $\bar{K}$, $\bar{F}$, respectively.

Similarly, the system equation at time $t+\Delta t$ is
\[
\begin{bmatrix}
C_t \\
K_t
\end{bmatrix}
\begin{bmatrix}
\dot{T}_t \\
\dot{T}_t
\end{bmatrix}
+
\begin{bmatrix}
C_t \\
K_t
\end{bmatrix}
\begin{bmatrix}
T_t \\
T_t
\end{bmatrix}
= \begin{bmatrix}
F_t \\
F_t
\end{bmatrix}
\]
\[
\begin{bmatrix}
C_{t+\Delta t} \\
K_{t+\Delta t}
\end{bmatrix}
\begin{bmatrix}
\dot{T}_{t+\Delta t} \\
\dot{T}_{t+\Delta t}
\end{bmatrix}
+
\begin{bmatrix}
C_{t+\Delta t} \\
K_{t+\Delta t}
\end{bmatrix}
\begin{bmatrix}
T_{t+\Delta t} \\
T_{t+\Delta t}
\end{bmatrix}
= \begin{bmatrix}
F_{t+\Delta t} \\
F_{t+\Delta t}
\end{bmatrix}
\]

Where:

- \([C_{t+\Delta t}]\) = system heat capacity matrix at time \(t+\Delta t\).
- \([K_{t+\Delta t}]\) = system thermal conductivity matrix at time \(t+\Delta t\).
- \(\{T_{t+\Delta t}\}\) = vector of nodal point temperatures at \(t+\Delta t\).
- \(\{\dot{T}_{t+\Delta t}\}\) = vector of the time rate of change of the nodal point temperatures at \(t+\Delta t\).
- \(\{F_{t+\Delta t}\}\) = vector of the externally supplied nodal heat flux at \(t=\Delta t\).

Assuming that the temperature variation over the time step \(\Delta t\) is linear, the time rate-of-change in temperature \(\dot{T}\) at \(t=\Delta t\) is

\[
\{\dot{T}_{t+\Delta t}\} = \frac{1}{\Delta t} \{T_{t+\Delta t} - T_t\}
\]

Because the \(K\) and \(C\) are temperature dependent functions, equation B.9 is still a nonlinear matrix equation. In order to solve it, this nonlinear matrix is linearized by making the following approximations:

Therefore, the equation B.9 becomes

To compensate for the error caused by above
approximations, iteration schemes must be used. Since the modified Newton-Raphson iteration involves few thermal conduction matrix \( K' \) ( \( K' = C/Δt + K \) ) reformations, this method was chosen in this work. According to Bathe (1982), the iteration scheme using the modified Newton-Raphson method is

\[
[K'_t]\{ΔT^{(i)}_{t+Δt}\} = \{F\}^{(i-1)}_{t+Δt} - [K_t]\{T_t\} \tag{B.12}
\]

Where \( K_t \) is the update thermal conduction matrix at the chosen time, which depends on the number of reformation times. The choice of time steps depends on the degree of nonlinearity in the system. For a strongly nonlinear problem containing radiation boundary conditions for example, it is necessary to update the matrix very often. This linear matrix is then solved for increments in the nodal point temperature \( ΔT^{(i)} \) at each iteration \( i \). The updated nodal point temperature is obtained by

\[
T^{(i)}_{t+Δt} = T^{(i,1)}_{t+Δt} + ΔT^{(i)}
\]

The iteration is continued until appropriate
convergence criteria are satisfied. Bathe (1982) presented a convergence criterion defined by

\[ \frac{\| \Delta T^{(i)} \|_2}{\| T^{(i)} \|_2 \epsilon \Delta t} \leq \epsilon_D \]  

B.13

Where \( \epsilon_D \) is a specified temperature convergence tolerance, and depends on the desired accuracy.

For each time step, the above procedures is repeated until the final time step is reached.
APPENDIX C

Convective Correlations Used in the Finite Element Model

C.1 External Forced Convection over Flat Plate

External flow refers to flow along a surface in which the boundary-layer thickness is small relative to the distance to any other surface. The boundary-layer thickness is defined as $\delta$ by (Incropera and DeWitt 1985)

$$\delta = \frac{5.0}{\sqrt{\frac{\mu}{v_x}}} = \frac{5X}{\sqrt{Re_x}}$$

Where $x$ is the length of flat plate and $Re_x$ is the Reynolds number at $x$. For many industrial furnaces, the air flow along outer surface of the furnace wall can be considered external since $\delta$ is usually much smaller compared with the distance between furnace outer surface and surrounding objects. Also, the gas flow along the inner surface of the wall is an external flow if the boundary-layer thickness is much smaller than the distance between the walls. Usually the gas is driven by fan or blower. Therefore, the external forced convection over flat plate should be considered. The $h_c$ value for external forced convection can be calculated by
\[ h_c = \frac{Nu_L K_f}{L} \]  

Where \( K_f \) is the fluid conductivity obtained from Table A.4 (Incropera and DeWitt, 1985), \( L \) is the length of a flat plate and \( Nu_L \) is Nusselt number. If the Reynolds number \( Re_L \) is equal to or less than \( 5 \times 10^5 \), the flow is laminar, and the Nusselt number is

\[ Nu_L = 0.664 Re_L^{\frac{1}{5}} Pr^{\frac{1}{3}} \]  

Otherwise, the flow is turbulent and the Nusselt number is

\[ Nu_L = (0.037 Re_L^{\frac{4}{5}} - 871) Pr^{\frac{1}{3}} \]  

Where \( Pr \) is the Prandtl number obtained from tables of thermal transport properties (Incropera and DeWitt 1985). The Reynolds number \( Re_L \) is ratio of the inertia and viscous forces, that is

\[ Re_L = \frac{VL}{v} \]  

Thus, the external forced convective coefficient \( h_c \) at each temperature point can be calculated in subroutine FTEMP by inputting the gas velocity near the wall \( V \) the length of the wall \( L \) and the gas properties including conductivity \( K_f \), Prandtl number \( Pr \) and kinematic viscosity \( v \).
C.2 Internal Forced Convection in a Tube

The internal forced convection in a tube refers to the heat transfer caused by the forced internal flow between the fluid and the tube surface. The feature of the internal flow is that the thermal and momentum boundary layers which develop along the surface of the body are influenced by the development of boundary layers on any adjacent surfaces (Kays and Crawford 1980). In some industrial furnaces such as a long tunnel furnace, the heat transfer between the flowing gas and the inner surface of the wall can also be considered as the internal forced convection. For noncircular tubes, the diameter is termed the hydraulic diameter and is defined as

\[ D_h = \frac{4A_c}{P} \]  

Where \( A_c \) and \( P \) are the flow cross-sectional area and the wetted perimeter, respectively.

The internal flow problem methodology not only involves determining whether the flow is laminar or turbulent, but also involves establishing the length of the entry region. For simplicity, the following assumptions have been made in the subroutine FTEMP.

a) Fully developed conditions exist over the entire length inside the furnace.
b) The heat flux is constant over the inner surface of the wall when the flow is laminar.

c) The average conditions are prevalent over the entire furnace, which means the $h_c$ value obtained from program is the average convective heat transfer coefficient.

Invoking the above assumptions, the internal forced convective heat transfer coefficient $h_c$ can be calculated by

$$ h_c = \frac{N_{ud} K_f}{D_h} \quad \text{C.7} $$

As mentioned above $N_{ud}$ is Nusselt number, which can be calculated by various correlations depending on whether the flow is laminar or turbulent. If the Reynolds number $Re_D$ is equal to or less than 2300, the flow is considered laminar. For the fully developed region ($x/D \geq 10$) of laminar flow, the Nusselt number is a constant and is equal to 4.36 (for constant heat flux boundary conditions).

For the thermal entry length ($x/D < 10$), the Nusselt number is obtained by the following equation presented by Kays (1955).

$$ Nu_D = 3.66 + \frac{0.0668 (D/L) Re_D Pr}{1 + 0.004 [(D/L) Re_D Pr]^{\frac{2}{3}}} \quad \text{C.8} $$

For Reynolds number larger than 2300 the flow is
turbulent and the Nusselt number is obtained from the Dittus-Boelter equation (1930).

\[ Nu_D = 0.023Re_D^{\frac{4}{5}}Pr^n \]  \hspace{1cm} C.9

Where \( n = 0.4 \) for \( T_s > T_g \) and \( n = 0.3 \) for \( T_s < T_g \). The Reynolds number \( Re_D \) is defined as

\[ Re_D = \frac{4PV_{vol}}{\pi D_h \mu} \]  \hspace{1cm} C.10

Thus, the internal forced convective coefficient \( h_c \) at each temperature point can be calculated in subroutine FTEMP by inputting the gas volumetric flow rate \( V_{vol} \), the hydraulic diameter of the furnace \( D_h \) and the gas properties including conductivity \( K_r \), Prandtl number \( Pr \), density \( \rho \) and dynamic viscosity \( \mu \).

C.3 Free Convection over the Vertical Plate.

The free convection is due to buoyancy forces within the fluid. Buoyancy forces that are induced by the combined presence of a fluid density gradient and a gravitation force. On a heated vertical plate immersed in an extensive, quiescent fluid, the fluid close to the plate is less dense than fluid that is far away from the plate. Buoyancy forces therefore induce a free convection boundary layer in which the heated fluid rises vertically. The convection
heat transfer coefficient $h_c$ can be also calculated by formula 2-35. The Nusselt number depends on the Rayleigh number $Ra_L$ range. If $Ra_L < 10^9$, the Nusselt number can be obtained for laminar flow by the following equation (Incropera and Dewitt 1985)

$$Nu_L = 0.68 + \frac{0.67 Ra_L^{\frac{1}{4}}}{[1 + (0.492 \frac{9}{16})^{\frac{3}{8}}]^{\frac{4}{9}}} \quad (C.11)$$

Otherwise, the Nusselt number for turbulent can be obtained by

$$Nu_L = (0.825 + \frac{0.387 Ra_L^{\frac{1}{6}}}{[1 + (0.492 \frac{9}{16})^{\frac{3}{27}}]} \quad (C.12)$$

Where the Rayleigh number $Ra_L$ is ratio of the buoyancy force and the viscous force, and is defined as

$$Ra_L = \frac{gL^3}{\alpha v} (T_s - T_f) \quad (C.13)$$

Where $\beta$ is the volumetric thermal expansion coefficient, which represents the density changes in response to a change in temperature at constant pressure. For a perfect gas, it is the reciprocal of the absolute film temperature $T_f$. 
Thus, the vertical plate free convective coefficient $h_c$ at each temperature point can be calculated in subroutine FTEMP by inputting the gravitational acceleration $g$, the length of the wall $L$, the gas temperature $T_g$, shift for absolute temperature and the gas properties including conductivity $K_f$, Prandtl number $Pr$, kinematic viscosity $v$ and thermal diffusivity $\alpha$.

C.4 Free Convection over the Horizontal Plate

Free convection over the horizontal plate occurs at the furnace roof. For the horizontal plate, the free convection heat transfer depends on whether the plate is warmer or cooler than the surrounding fluid and whether it is facing up or down. The length of the plate refers to the characteristic length, and is defined as

$$L = \frac{A_s}{P}$$  \hspace{1cm} C.15

Where $A_s$ and $P$ are the plate surface area and perimeter, respectively. The method of calculating $h_c$ and the Rayleigh number is the same as the method introduced in the free convection over a vertical plate. But the method for calculating Nusselt number is different.
For the upper surface of heated plate or lower surface of cooled plate

\[ \text{Nu}_L = 0.54 \text{Ra}_L^{\frac{1}{4}} \quad (10^4 \leq \text{Ra}_L \leq 10^7) \quad \text{C.16} \]

\[ \text{Nu}_L = 0.15 \text{Ra}_L^{\frac{1}{3}} \quad (10^7 \leq \text{Ra}_L \leq 10^{10}) \quad \text{C.17} \]

For the lower surface of heated plate or upper surface of cooled plate, the following equation is used.

\[ \text{Nu}_L = 0.27 \text{Ra}_L^{\frac{1}{4}} \quad (10^5 \leq \text{Ra}_L \leq 10^{10}) \quad \text{C.18} \]

It should be noted that all gas properties are evaluated at the film temperature \( T_f \). Therefore, the horizontal plate convection coefficient \( h_c \) at each temperature point can be calculated in subroutine FTEMP by inputting the same data as that in the free convection over a vertical plate, except for inputting a flag that indicates the plate is warmer or cooler than the surrounding fluid and whether it is facing up or down.
APPENDIX D
Imput Data Manual for Subroutine FTEMP

(Adding this document to the User's Manual of DOT program)

B. TIME AND TEMPERATURE FUNCTION DATA

Each set consists of a control card followed by as many cards as needed to define the function.

1. CONTROL CARD - First card of set.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>NC</td>
<td>Function number (GE.1 and LE.NBCF, See note 1).</td>
</tr>
<tr>
<td>6-10</td>
<td>NPTS(NC)</td>
<td>Number of temperature points used to describe this function (GE.2 and LE.NPTM)</td>
</tr>
<tr>
<td>11-15</td>
<td>NFUNC</td>
<td>Flag of time or temperature function.</td>
</tr>
</tbody>
</table>

2. TIME DATA (8F10.0) - If NFUNC.EQ.0 skip these cards.

As many cards as needed to define NPTS(NC) pairs of points [TFN(NC,I), FN(NC,I)], four pairs per card. See Note 2.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>TFN(NC,1)</td>
<td>Time at point 1: t1</td>
</tr>
<tr>
<td>11-20</td>
<td>FN(NC,1)</td>
<td>Function value at point 1: f(t1)</td>
</tr>
<tr>
<td>21-30</td>
<td>TFN(NC,2)</td>
<td>Time at point 2: t2</td>
</tr>
<tr>
<td>31-40</td>
<td>FN(NC,2)</td>
<td>Function value at point 2: f(t2)</td>
</tr>
</tbody>
</table>
41-50  TFN(NC,3)  Time at point 3: t3
51-60  FN(NC,3)  Function value at point 3: f(t3)
61-70  TFN(NC,4)  Time at point 4: t4
71-80  FN(NC,4)  Function value at point 4: f(t4)

Next card(s) - if required
1-10  TFN(NC,5)  Time at point 5: t5
11-20  FN(NC,5)  Function value at point 5: f(t5)

3. TEMPERATURE DATA - If NFUNC.EQ.1 skip these cards.

3.1 FLAG CARD (I5)

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>NFLAG</td>
<td>Flag of temperature function.</td>
</tr>
</tbody>
</table>

Note: NFLAG.EQ.0 General temperature dependent functions.
NFLAG.eq.1 External forced convection of plates
NFLAG.eq.2 Internal forced convection of plates
NFLAG.eq.3 Free convection of vertical plates
NFLAG.eq.4 Free convection of horizontal plates

3.2 DIRECT TEMPERATURE FUNCTION CARD(S)

If NFLAG.EQ.0, input TFN(NC,K), FN(NC,K) directly, and skip card 3.3 and card 3.4. If NFLAG.NE.0, skip this (these) card(s). See Note 3. As many cards as needed to define NPTS(NC) pairs of points [TFN(NC,I), FN(NC,I)], four pairs per card.
<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>TFN(NC,1)</td>
<td>Temperature at point 1: T1</td>
</tr>
<tr>
<td>11-20</td>
<td>FN(NC,1)</td>
<td>Function value at point 1: f(T1)</td>
</tr>
<tr>
<td>21-30</td>
<td>TFN(NC,2)</td>
<td>Temperature at point 2: T2</td>
</tr>
<tr>
<td>31-40</td>
<td>FN(NC,2)</td>
<td>Function value at point 2: f(T2)</td>
</tr>
<tr>
<td>41-50</td>
<td>TFN(NC,3)</td>
<td>Temperature at point 3: T3</td>
</tr>
<tr>
<td>51-60</td>
<td>FN(NC,3)</td>
<td>Function value at point 3: f(T3)</td>
</tr>
<tr>
<td>61-70</td>
<td>TFN(NC,4)</td>
<td>Temperature at point 4: T4</td>
</tr>
<tr>
<td>71-80</td>
<td>FN(NC,4)</td>
<td>Function value at point 4: f(T4)</td>
</tr>
</tbody>
</table>

Next card(s) - if required

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>TFN(NC,5)</td>
<td>Temperature at point 5: T5</td>
</tr>
<tr>
<td>11-20</td>
<td>FN(NC,5)</td>
<td>Function value at point 5: f(T5)</td>
</tr>
</tbody>
</table>

### 3.3 PARAMETER CARD

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>NSURF</td>
<td>1--Cold surface up, hot surface down</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0--Cold surface down, hot surface up</td>
</tr>
<tr>
<td>11-20</td>
<td>VEL</td>
<td>Velocity of the gas. (NFLAG.EQ.1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Flow rate of the gas. (NFLAG.EQ.2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gravitative acceleration.</td>
</tr>
<tr>
<td>21-30</td>
<td>GEO</td>
<td>Plate length. Hydraulic diameter of tube (NFLAG.EQ.2).</td>
</tr>
<tr>
<td>31-40</td>
<td>TGAS</td>
<td>Temperature of the gas.</td>
</tr>
<tr>
<td>41-50</td>
<td>DLR</td>
<td>Ratio of diameter and length.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shift for absolute temperature</td>
</tr>
</tbody>
</table>
3.4 GAS PROPERTY CARD (5F10.0)- Need NPTS(NC) CARDS

Input gas properties at each point (I) to calculate convective heat transfer coefficient $h$, which is FN(NC,I).

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>TFN(NC,I) Temperature at point I: $T_i$</td>
</tr>
<tr>
<td>11-20</td>
<td>THMP(I,1) Thermal conductivity of the gas, $K_i$</td>
</tr>
<tr>
<td>21-30</td>
<td>THMP(I,2) Momentum rate of the gas, $v_i$</td>
</tr>
<tr>
<td>31-40</td>
<td>THMP(I,3) Prandtl number, $Pr_i$</td>
</tr>
<tr>
<td>41-50</td>
<td>THMP(I,4) Thermal diffusivity, $u_i$.</td>
</tr>
</tbody>
</table>

NOTES:

1. Time and temperature functions need not be input in order of increasing function number NC.

2. The function tables input in this section are used to prescribe time dependent boundary conditions, such as environmental or nodal point temperatures and nodal point heat flows. The first time point TFN(NC,1) must be less than or equal to the time at solution start TSTART and the final time point TFN(NC,NPTS(NC)) must be greater than or equal to the time at the end of solution TSTART+NDT*DT.

3. The function tables input in this section are used to prescribe temperature dependent boundary conditions, such as the variation of convection coefficient $h$, of
emissivity and the rate of internal heat generation with temperature. It is also able to calculate the different convection coefficients $h$ through the input gas properties. Time and temperature values at successive points must increase in magnitude. Values of the functions at times or temperatures other than $TFN(NC,I)$ are calculated within the program using linear interpolation.
APPENDIX E
Program Listing

C$Header: FUNC.f,v $
SUBROUTINE FUNC (TFN, FN, NPTS, NPTM1)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
C
C
C DEFINE ALL BOUNDARY CONDITION FUNCTIONS
C
C********************************************************************
C
DIMENSION TFN(NPTM1,1), FN(NPTM1,1), NPTS(1)
COMMON /JUNK/
NUMNP, NEG, MODEX, NPAR(10), NG, KBC, KST, NDT, DT, TSTART,
1 TAMB, NPRINT, NTSREF, TIME, KP, N1, N2, N3, N4, N5, N6, N7, N8, N9, N10, N11, N12
2, N13, N14, N15, NUMEST, MIDEST, MAXEST, HED(18), MTOT, NLINE, NNBC, NB CF,
3 NPTM, KINT, NEL, NODS, MTYPE, NND5
COMMON /WORK/ FORM(4), WORK(196)
C
WRITE (6,110)
NLINE=NLINE+3
C
DO 70 L=1,NBCF
READ (5,80) NC, NPTS(NC), NFUNC
WRITE (6,120) NC, NPTS(NC), NFUNC
NLINE=NLINE+1
IF (NPTS(NC).GE.2.AND. NPTS(NC).LE. NPTM) GO TO 10
WRITE (6,140)
STOP
C
C READ TIME FUNCTION VERSUS TIME TABLE
C
10 IF (NFUNC.EQ.1) CALL FTIME (NC, TFN, FN, NPTS, NPTM1)
C
C READ TEMPERATURE FUNCTION VERSUS TIME TABLE
C
IF (NFUNC.EQ.0) CALL FTEMP (NC, TFN, FN, NPTS, NPTM1)
C CONTINUE
C FORMAT STATEMENTS
C
RETURN
C
80 FORMAT (3I5)
90 FORMAT (8F10.0)
100 FORMAT (4I(1H*))/4I TIME AND TEMPERATURE DEPENDENT
FUNCTIONS/
141(1H*)//)
110 FORMAT (4X,8FUNCTION,4X,9HNUMBER OF,6X,10HTIME
POINT,T51,4HTIME,
1T71,8FUNCTION/5X,6HNUMBER,4X,11HTIME
POINTS,7X,6HNUMBER,T50,
25HVALUE,T72,5HVALUE/)
120 FORMAT (4X,I5,7X,I5)
130 FORMAT (31X,I6,2E22.3)
140 FORMAT (/49H **ERROR** (NPTS) MUST BE .GE. 2 AND
.LE. (NPTM))
150 FORMAT (/52H **ERROR** BC FUNCTION TIME POINTS ARE
OUT OF ORDER)
160 FORMAT (/40H **ERROR** TFN(N,1) MUST BE .LE. TSTART)
170 FORMAT (/43H **ERROR** TFN(N,NPTS) MUST BE .GE.
NDT*DT)
END

SUBROUTINE FTEMP (NC,TFN,FN,NPTS,NPTM1)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
PARAMETER(PIE=3.1415926,CONS=1000000.)
C
C DEFINE ALL TEMPERATURE DEPENDENT BOUNDARY CONDITION
FUNCTIONS
C NFLAG.EQ.0 --- GENERAL TEMPERATURE DEPENDENT
FUNCTIONS
C NFLAG.EQ.1 --- THE EXTERNAL FORCED CONVECTION OF
PLATE
C NFLAG.EQ.2 --- THE INTERNAL FORCED CONVECTION
INSIDE TUBE
C NFLAG.EQ.3 --- THE FREE CONVECTION OF THE VERTICAL
PLATE
C NFLAG.EQ.4 --- THE FREE CONVECTION OF THE
HORIZONTAL PLATE
C

DIMENSION TFN(NPTM1,1), FN(NPTM1,1), NPTS(1), THMP(20,10)
COMMON /JUNK/
NUMNP,NEG,MODEX,NPAR(10),NG,KBC,KST,NDT,DT,TSTART,
1
TAMB,NPRINT,NTSREF,TIME,KP,N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N1
1
2
,N13,N14,N15,NUMEST,MIDEST,MAXEST,HED(18),MTOT,NLINE,NNBC,NB
CF,
3 NPTM,KINT,NEL,NODS,MTYPE,NND5
COMMON /WORK/ FORM(4),WORK(196)
C

WRITE (6,100)
NLINE=NLINE+9
WRITE (6,110)
NLINE=NLINE+4
C

NT=NPTS(NC)
READ (5,*) NFLAG
WRITE(6,115) NC,NPTS(NC),NFLAG
NLINE=NLINE+3
IF (NFLAG.EQ.0) THEN
READ(5,90) (TFN(K,NC),FN(K,NC),K=1,NT)
GOTO 10
ENDIF
READ (5,71) NSURF,VEL,GEO,TGAS,DLR
IF (NFLAG.EQ.1) WRITE(6,121) VEL,GEO,TGAS
IF (NFLAG.EQ.2) WRITE(6,122) VEL,GEO,TGAS,DLR
IF (NFLAG.EQ.3) WRITE(6,123) VEL,GEO,TGAS
IF (NFLAG.EQ.4) THEN
IF (NSURF.EQ.0) WRITE(6,124) VEL,GEO,TGAS
    IF (NSURF.EQ.1) WRITE(6,125) VEL,GEO,TGAS
ENDIF
NLINE=NLINE+7
C
C    READ TEMPERATURE FUNCTION OF THERMAL PROPERTIES OF
C    MATERIALS
C    VERSUS TEMPERATURE TABLE
C    THMP(K,1) = THE THERMAL CONDUCTIVITY OF THE
C    GAS
C    THMP(K,2) = THE MOMENTUM RATE OF THE GAS
C    (VISCOUS FORCE)
WRITE(6,128)
NLINE=NLINE+3
DO 15 K=1, NT
READ (5,91) TFN(K,NC),(THMP(K,J),J=1,6)
WRITE (6,135) TFN(K,NC),(THMP(K,J),J=1,6)
NLINE=NLINE+1

EXTERNAL FORCED CONVECTION COEFFICIENTS

IF (NFLAG.EQ.1) THEN
   RELN=VEL*GEO*CONS/THMP(K,2)
   EX1=1./3.
   RNUL=(0.037*RELN**0.8-871.)*THMP(K,3)**EX1
   IF (RELN.LE.5.0E+05)
      RNUL=0.664*RELN**0.5*THMP(K,3)**EX1
   GOTO 15
ENDIF

INTERNAL FORCED CONVECTION COEFFICIENTS

IF (NFLAG.EQ.2) THEN
   RATE=VEL*THMP(K,5)
   REDN=4.*RATE*CONS/(PIE*GEO*THMP(K,4))
   EX2=2/3
   VALL=1+0.004*(REDN*THMP(K,3)/DLR)**EX2
   IF (DLR.LT.10) THEN
      RNUL=3.66+(0.0668/REDN*THMP(K,3))/VALL
   ELSEIF(REDN.GT.1.0E+04) THEN
      RNUL=0.023*REDN**0.8*THMP(K,3)**0.3
   ELSE
      RNUL=4.36
   ENDIF
   WRITE (*,*) K,vel,REDN,RNUL
   GOTO 15
ENDIF

FREE CONVECTION COEFFICIENTS FOR VERTICAL PLATES

IF (NFLAG.EQ.3) THEN
EX1=1./6.
EX2=9./16.
EX3=8./27.
EX4=4./9.
RN=(1+(0.492/THMP(K,3))**EX2)**EX3
RS=(1+(0.492/THMP(K,3))**EX2)**EX4
RNUL=(0.825+(0.387*RALN**EX1)/RN)**2.
IF (RALN.LE.1.0E+09)
RNUL=0.68+0.670*RALN**0.25/RS
GOTO 15
ENDIF

FREE CONVECTION COEFFICIENTS FOR HORIZONTAL PLATES

IF (NFLAG.EQ.4) THEN
EX1=1./3.
IF (NSURF.EQ.1) RNUL=0.27*RALN**0.25
IF (NSURF.EQ.0) THEN
RNUL=0.15*RALN**EX1
ENDIF
ENDIF
WRITE (*,*) NSURF,K,RaIN,RNUL

CHECK THAT TEMPERATURE POINTS ARE IN INCREASING ORDER

TOLD=-1.
DO 20 K=1,NT
IF (TFN(K,NC).GT.TOLD) GO TO 20
WRITE (6,150)
STOP
20 TOLD=TFN(K,NC)

IF (TFN(1,NC).LE.TAMB) GO TO 30
WRITE (6,160)
STOP
30 TEND=TAMB+NDT*DT
IF (TFN(NT,NC).GE.TEND) GO TO 40
WRITE (6,170)
STOP

WRITE(6,70)
NLINE=NLINE+4
40 DO 60 K=1,NT
IF (NLINE.LT.55) GO TO 50
CALL TITLE (HED)
WRITE (6,100)
WRITE (6,70)
NLINE=10
50 WRITE (6,75) K,TFN(K,NC),FN(K,NC)
60 NLINE=NLINE+1
C
C FORMAT STATEMENTS
C
RETURN
C
70 FORMAT ('TEMPERATURE TEMPERATURE
FUNCTION',/,
1'POINT NUMBER VALUE VALUE ')/
71 FORMAT (I10,4F10.0)
75 FORMAT (3X,I5,10X,F10.4,6X,F10.4)
80 FORMAT (5I5,3F10.0)
90 FORMAT (8F10.0)
91 FORMAT (7F10.0)
100 FORMAT (32(1H*)/32H TEMPERATURE DEPENDENT
FUNCTIONS/32(1H*)/1' NFLAG.EQ.0  --- GENERAL TEMPERATURE DEPENDENT
FUNCTIONS'/,2' NFLAG.EQ.1  --- THE EXTERNAL FORCED CONVECTION OF
PLATE'/,2' NFLAG.EQ.2  --- THE INTERNAL FORCED CONVECTION
INSIDE TUBE'/,2' NFLAG.EQ.3  --- THE FREE CONVECTION OF THE VERTICAL
PLATE'/,2' NFLAG.EQ.4  --- THE FREE CONVECTION OF THE
HORIZONTAL PLATE'/)
110 FORMAT ('FUNCTION NUMBER NUMBER OF POINTS FUNCTION
FLAG'/)
115 FORMAT (6X,I5,14X,I5,18X,I5)
130 FORMAT (3X,I5,13X,I5,11X,I5)
121 FORMAT('PROPERTIES FOR CALCULATING EXTERNAL FORCED
CONVECTION h',
1//,'THE VELOCITY OF THE GAS ....................... ',F10.3,/,
2'THE LENGTH OF THE BOUNDARY ................... ',F10.3,/,
3'THE TEMPERATURE OF THE DAS ................... ',F10.3,/)
122 FORMAT('PROPERTIES FOR CALCULATING INTERNAL FORCED
CONVECTION h',
1//,'THE FLOW RATE OF THE GAS .................... ',F10.3,/,
2'THE DIAMETER OF THE TUBE ..................... ',F10.3,/,
3'THE TEMPERATURE OF THE GAS ................... ',F10.3,/,
4'THE RATIO OF LENGTH AND DIAMETER ............ ',F10.3,/)
123 FORMAT('PROPERTIES FOR CALCULATING VERTICAL PLATE FREE
CONVECTION
1 h'//,'THE LOCAL ACCELERATION DUE TO GRAVITY .........
',F10.3,/
2'THE LENGTH OF THE BOUNDARY ..........................
',F10.3,/
3'THE TEMPERATURE OF THE DAS ..........................
',F10.3,/
4'SHIFT FOR ABSOLUTE TEMPERATURE ........................
',F10.3,/
124 FORMAT('PROPERTIES FOR CALCULATING HORIZONTAL PLATE
FREE CONVECTION
1 h'//,'WHEN THE COLD SURFACE IS DOWN OR THE HOT
SURFACE IS UP',/,
2'THE LOCAL ACCELERATION DUE TO GRAVITY .........
',F10.3,/
3'THE LENGTH OF THE BOUNDARY ..........................
',F10.3,/
5'THE TEMPERATURE OF THE DAS ..........................
',F10.3,/
5'SHIFT FOR ABSOLUTE TEMPERATURE ........................
',F10.3,/
125 FORMAT('PROPERTIES FOR CALCULATING HORIZONTAL PLATE
FREE CONVECTION
1 h'//,'WHEN THE COLD SURFACE IS UP OR THE HOT SURFACE
IS DOWN',/,
2'THE LOCAL ACCELERATION DUE TO GRAVITY .........
',F10.3,/
3'THE LENGTH OF THE BOUNDARY ..........................
',F10.3,/
5'THE TEMPERATURE OF THE DAS ..........................
',F10.3,/
5'SHIFT FOR ABSOLUTE TEMPERATURE ........................
',F10.3,/
128 FORMAT ('TEMPERATURE PROPERTIES OF THE
GAS',/
1,' VALUE K v Pr u P
2 a',/)
135 FORMAT (F10.2,6(F10.4))
140 FORMAT (//49H **ERROR** (NPTS) MUST BE .GE. 2 AND
.LE. (NPTM))
150 FORMAT (//52H **ERROR** BC FUNCTION TIME POINTS ARE
OUT OF ORDER)
160 FORMAT (//40H **ERROR** TFN(N,1) MUST BE .LE. TSTART)
170 FORMAT (//43H **ERROR** TFN(N,NPTS) MUST BE .GE.
NDT*DT)
END
C$Header: FTIME.f,v $
SUBROUTINE FTIME(NC, TFN, FN, NPTS, NPTM1)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)

C
C
C DEFINE ALL TIME DEPENDENT BOUNDARY CONDITION FUNCTIONS
C

C DIMENSION TFN(NPTM1,1), FN(NPTM1,1), NPTS(1)
COMMON /JUNK/ NUMNP, NEG, MODEX, NPAR(10), NG, KBC, KST, NDT, DT, TSTART, 1
TAMB, NPRINT, NTSREF, TIME, KP, N1, N2, N3, N4, N5, N6, N7, N8, N9, N10, N11, N12 2
N13, N14, N15, NUMEST, MIDEST, MAXEST, HED(18), MTOT, NLINE, NNBC, NB CF,
3 NPTM, KINT, NEL, NODS, MTYPE, NND5
COMMON /WORK/ FORM(4), WORK(196)

C
WRITE (6, 110)
NLINE=NLINE+3

C
C READ TIME FUNCTION VERSUS TIME TABLE
C
WRITE(6, 120) NC, NPTS( NC)
NLINE=NLINE+1
NT=NPTS( NC)
READ (5, 90) (TFN(K, NC), FN(K, NC), K=1, NT)

C
C CHECK THAT TIME POINTS ARE IN INCREASING ORDER
C
TOLD=-1.
DO 20 K=1, NT
IF (TFN(K, NC).GT.TOLD) GO TO 20
WRITE (6, 150) STOP
20 TOLD=TFN(K, NC)
C
IF (TFN(1, NC).LE.TSTART) GO TO 30
WRITE (6, 160) STOP
30 TEND=TSTART+NDT*DT
IF (TFN(NT, NC).GE.TEND) GO TO 40
WRITE (6, 170)
STOP

C
40  DO 60  K=1,NT
    IF (NLINE.LT.55) GO TO 50
    CALL TITLE (HED)
    WRITE (6,100)
    WRITE (6,110)
    NLINE=10
50  WRITE (6,130) K,TFN(K,NC),FN(K,NC)
60  NLINE=NLINE+1
C
C  FORMAT STATEMENTS
C
C
RETURN
C
90  FORMAT (8F10.0)
100  FORMAT (25(1H*)/25H TIME DEPENDENT
    FUNCTIONS/25(1H*)//)
110  FORMAT (4X,8FUNCTION,4X,9HNUMBER OF,6X,10HTIME
    POINT,T51,4HTIME,
    1T71,8FUNCTION/5X,6HNUMBER,4X,11HTIME
    POINTS,7X,6HNUMBER,T50,
    25HVALUE,T72,5HVALUE/)
120  FORMAT (4X,I5,7X,I5)
130  FORMAT (31X,I6,2E22.3)
150  FORMAT (//52H **ERROR** BC FUNCTION TIME POINTS ARE
    OUT OF ORDER)
160  FORMAT (//40H **ERROR** TFN(N,1) MUST BE .LE. TSTART)
170  FORMAT (//43H **ERROR** TFN(N,NPTS) MUST BE .GE.
    NDT*DT)
END

C$Header: TLOSS.f,v $
SUBROUTINE TLOSS( A,EMIN,HLOS,T0,STORG,KSTEP,NUMAT2)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

C
C
C CALCULATE THE HEAT LOSSES AND HEAT STORAGE
C
C
C
COMMON /JUNK/
NUMNP,NEG,MODEX,NPAR(10),NG,KBC,KST,NDT,DT,TSTART,
1 TAMB,NPRINT,NTSREF,TIME,KP,N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N1
1 ,N12
FIND THE MAXIMUM TEMPERATURE BETWEEN CURRENT TIME STEP AND PREVIOUS TIME STEP

if (kst.eq.-1) then
  emin=0.
goto 5
endif

E0=1.
DO 100 I=1,NUMNP
  II=N8+I-1
  JJ=N14+I-1
  EPS=A(JJ)/A(II)
  EMIN=AMIN1(EPS,E0)
  E0=EMIN
100 CONTINUE

LOOP OVER ALL ELEMENT GROUPS

DO 40 NG=1,NEG

READ (1) MIDEST,NPAR,NST,(A(I),1=1,MIDEST)

NGR=NPAR(1)
GO TO (10,20,30), NGR

ELEMENT GROUP 1 -- HEAT STORAGE FOR EACH ELEMENT

IF (KST.EQ.-1) GOTO 40
IF (((EMIN.GT.0.9999).OR.(KSTEP.EQ.NDT)) THEN
  MXNODS=NPAR(5)
  NUMAT=NPAR(7)
  MAXTP=NPAR(8)
NDM=2*MNXNODS
ND5DIM=MNXNODS-4
IF (ND5DIM.EQ.0) ND5DIM=1

CALL HSTOR
(A(M1),A(M2),A(M3),A(M4),A(M5),A(M6),A(M7),A(M8),A(N8),
1T0,MNXNODS,NDM,ND5DIM,MAXTP,NUMAT,STORG)
ENDIF
GO TO 40

C -----------------
C ELEMENT GROUP 2 -- HEAT LOSS OVER THE BOUNDARY SURFACE
C -----------------

20 NUMAT=NPAR(5)
CALL HLOSS
(A(M1),A(M2),A(M3),A(M4),A(M5),A(M6),A(M7),A(M8),A(N8),
1A(N1),A(N2),A(N3),A(N8),NUMAT,NPTM,HLOS)
30 GO TO 40

C -----------------
C ELEMENT GROUP 3
C -----------------

C30 CALL COND3 (A(M1),A(M2),A(M5),A(M6),A)

40 CONTINUE
C
RETURN
END

C$Header:  HLOSS.f,v $
SUBROUTINE HLOSS
(LM,XX,CL,MATP,SB,TSHIFT,NFH,NFTE,PROP,TFN,FN,NPT
1S,T,NUMAT,NPTM,HLOS)
PARAMETER(PIE=3.14159)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

C

******************************************************************************
C MODIFY FLUX VECTOR (Q) FOR CONVECTION AND RADIATION
C BOUNDARIES
******************************************************************************
C
DIMENSION LM(2,1), XX(2,1), CL(1), MATP(1), NFH(1),
NFTE(1), T(1),
PROP(5, NUMAT), TFN(NPTM, 1), FN(NPTM, 1), NPTS(1), Q(1), TT(1), hlos (1)
COMMON /JUNK/
NUMNP, NEG, MODEX, NPAR(10), NG, KBC, KST, NDT, DT, TSTART,
1 TAMB, NPRINT, NTSREF, TIME, KP, N1, N2, N3, N4, N5, N6, N7, N8, N9, N10, N11, N12,
2 N13, N14, N15, NUMEST, MIDEST, MAXEST, HED(18), MTOT, NLINE, NNBC, NB CF,
3 NDUM, KINT, NEL, NODS, MTYPE, NND5
C
NEL2=NPAR(2)
ITYP=NPAR(4)
do 100 L=1,numat
HLOS(L)=0.0
20 DO 80 N=1, NEL2
   if (matp(n).ne.L). goto 80
   II=LM(1,N)
   JJ=LM(2,N)
   XL=CL(N)
   MTYP=MATP(N)
   NFS=NFH(MTYP)
   NFT=Nfte(MTYP)
   HAM=PROP(1,MTYP)
   TEX=PROP(2,MTYP)
   V=PROP(3,MTYP)
C
C COMPUTE EXTERNAL TEMPERATURE
C
IF (NFT.NE.0) GO TO 30
VAL=1.0
GO TO 40
30 NPT=NPTS(NFT)
CALL INTERP (TFN(1,NFT), FN(1,NFT), NPT, TIME, VAL)
40 TE=VAL*TEX
   TS=0.5*(T(II)+T(JJ))
C
===================================
C CONVECTION BOUNDARY CONDITIONS
===================================
C
IF (NFS.GT.0) GO TO 60
C
LINEAR CONVECTION, H = CONSTANT
C
HL=HAM*XL
QC=HL*(TE-TS)
GO TO 70

COMPUTE TEMPERATURE DEPENDENT CONVECTION COEFFICIENT

NTP=NPTS(NFS)
CALL HPROP (TAVE,H,TFN(1,NFS),FN(1,NFS),NTP)
QC=HAM*H*XL*(TE-TS)

RADIATION BOUNDARY CONDITIONS

IF (V.EQ.0.) GO TO 80
TS=TS+TSHIFT
TS=TS*TS
TS=TS*TS
TE=TE+TSHIFT
TE=TE*TE
TE=TE*TE
EF=PROP(4,MTYP)
ES=PROP(5,MTYP)
QR=SB*V*XL*(1./(1./EF+1./ES-1.))*(TE-TS)
IF (ITYP.EQ.0) THEN
AXISYMMETRIC SOLID BOUNDARY ELEMENTS

XI=XX(1,N)
XJ=XX(2,N)
RAVA=(XI+XJ)/2.
QC=QC*RAVA
QR=QR*RAVA
ENDIF
HLOS(L)=HLOS(L)+QC+QR

CONTINUE
CONTINUE
RETURN
END

SUBROUTINE HSTOR
( LM,XY,IELT,NOD5,MATP,NTC,TPROP,DENS,T,TT,MAXNODS, 1NDM,ND5DIM,MATP,NUMAT,HSTORAGE)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C
******************************************************************************
C   CALCULATE THE HEAT FLUX OVER THE EVERY ELEMENT
C   THE EFFECTIVE HEAT CAPACITY VECTOR (C*)
C
******************************************************************************
C
DIMENSION LM(MXNODS,1), XY(NDM,1), IELT(1),
NOD5(N5DIM,1), MATP(1,1), NTC(1), TPROP(MAXTP,5,NUMAT), DENS(1), T(1), TT(1)
COMMON /JUNK/
NUMNP,NEG,MODEX,NPAR(10),NG,KBC,KST,NDT,DT,TSTART,
TAMB,NPRINT,NTSREF,TIME,KP,N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12
,N13,N14,N15,NUMEST,MIDEST,MAEST,HED(18),MTOT,NLINE,NNBC,NB
CF,
3 NPTM,KINT,NEL,NODS,MTYPE,NND5
COMMON /WORK/ DUM(10),TTNOD(8),TNOD(8),WORK(174)
DIMENSION A(1)
C
NEL1=NPAR(2)
IHFLG=NPAR(9)
C
HSTORAGE=0.0
DO 80 N=1,NEL1
NEL=N
NODS=IELT(N)
MTYPE=MATP(N)
NND5=NODS-4
NDOF=NODS*NODS
C
C   DETERMINE ELEMENT NODAL TEMPERATURE VECTOR
C
50 DO 60 I=1,NODS
   ND=LM(I,N)
   TTNOD(I)=TT(ND)
   60 TNOD(I)=T(ND)
C
C   CALCULATE HEAT FLUX AND HEAT STORAGE USING GAUSS QUADRATURE
C   OVER THE ELEMENT
C
CALL QSTOR
(QS,XY(1,N),NOD5(1,N),TNOD,TTNOD,NTC(MTYPE),
1DENS(MTYPE),TPROP(1,1,MTYPE),MAXTP,NODS)
C       CALCULATE THE HEAT STORAGE FOR ALL ELEMENTS
C
HSTORAGE = HSTORAGE + QS

80 CONTINUE
HSTORAGE = HSTORAGE * TIME
C
RETURN
END

C$Header: QSTOR.f,v $
SUBROUTINE QSTOR
(QS, XY, NOD5, TNOD, TTNOD, NTP, DE, TPROP, MAXTP, 1NODS)
PARAMETER (PIE = 3.14159)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
C
C
C       CALCULATE THE HEAT LOSSES AND HEAT STORAGE BY KNOWING
C       THE
C       TEMPERATURE FOR TWO DIMENSIONAL PLANAR AND
C       AXISYMMETRIC GEOMETRY
C
C
************ IMPORTANT CODE INFORMATION ************
C
C       DIMENSION XY(1), NOD5(1), TNOD(1), TTNOD(1),
C 1       TPROP(MAXTP,5)
C       COMMON /JUNK/
NUMNP, NEG, MODEX, NPAR(10), NG, KBC, KST, NDT, DT, TSTART,
1       TAM, NPRINT, NTSREF, TIME, KP, N1, N2, N3, N4, N5, N6, N7, N8, N9, N10, N11,
1       N12,
1       N13, N14, N15, NUMEST, MIDEST, MAXEST, HED(18), MTOT, NLINE, NNBC, NB
1       CF,
1       NPTM, KINT, NEL, NDUM, MTYPE, NND5
C       COMMON /WORK/
DUM(26), H(8), P(2,8), B(2,8), XJ(2,2), EK(3), WORK(127)
C       DIMENSION XG(4,4), WGT(4,4)
DATA
XG/0.,0.,0.,0.,-.5773502691896,.5773502691896,0.,0.,-.7745966692415,0.
1       ,0.,-.3399810435849,.3399810435849,.8611363115941,-.3399810435849/
1
1       ,0.,-.5773502691896,.5773502691896,0.,0.,-.7745966692415,0.
1       ,0.,-.3399810435849,.3399810435849,.8611363115941,-.3399810435849/
DATA
WGT/2.000,0.,0.,0.,1.0000000000000,1.0000000000000,0.,0.,.55
5
15555555556, .8888888888889, .5555555555556, 0., .3478548451375,
.652145
21548625, .6521451548625, .3478548451375/
C
ITYP2D=NPAR(4)
NINT=NPAR(6)
C
LOOP OVER ALL INTEGRATION POINTS
C
GS=0.0
VOL=0.0
C
DO 80 LX=1,NINT
R=XG(LX,NINT)
DO 80 LY=1,NINT
S=XG(LY,NINT)
WT=WGT(LX,NINT)*WGT(LY,NINT)
C
FIND INTERPOLATION FUNCTIONS (H) AND THEIR DERIVATIVES (P).
FIND JACOBIAN (XJ) AND ITS DETERMINANT (DETJ).
CALL SHAPE1 (R,S,XY,H,P,NOD5,XJ,DETJ)
C
EVALUATE JACOBIAN INVERSE (XJI) AND GLOBAL DERIVATIVE OPERATOR (B)
AT EACH INTEGRATION POINT (R,S) WITHIN THE ELEMENT
CALL DERIV1 (XY,H,P,B,XJ,DETJ,RAD,ITYP2D)
C
FAC=WT*RAD*DETJ
IF (ITYP2D.EQ.0) FAC=FAC
VOL=VOL+FAC
C
COMPUTE HEAT STORAGE FOR EACH ELEMENT
SUM=0.0
DO 30 I=1,NODS
SUM=SUM+H(I)*FAC*(TNOD(I)-TTNOD(I))
C
COMPUTE TEMPERATURE AT INTEGRATION POINT (R,S)
IF (NTP.EQ.1) GO TO 60
TIP=0.
DO 50 I=1,NODS
TIP=TIP+H(I)*TNOD(I)

C BASED ON TEMPERATURE AT INTEGRATION POINT (TIP),
C COMPUTE THE
C TEMPERATURE DEPENDENT CONDUCTIVITIES AND SPECIFIC HEAT
C
60 CALL MATPR
   (TIP,EK,SP,TPROP(1,1),TPROP(1,2),NTP,MAXTP,KST)
QS=QS+SUM*SP
80 CONTINUE

C AO=1./TIME
QS=DE*AO*QS

RETURN
END
APPENDIX F

Sample Input File for DOT Program

420--05C-02

START 2-D CYLINDRICAL FURNACE WALL MADE OF FIBERS

1  0.0  4.8  5
76  1.2  4.8
2  0.0  4.7  5
77  1.2  4.7
3  0.0  4.6  5
78  1.2  4.6
4  0.0  4.5  5
79  1.2  4.5
5  0.0  4.4  5
80  1.2  4.4
81  0.88  4.3  5
300  0.88  0.0
82  0.96  4.3  5
296  0.96  0.0
301  1.04  4.3  5
83  1.04  0.0
297  1.12  4.3  5
84  1.12  0.0
298  1.2  4.3  5
85  1.2  0.0
300  1.2  0.0

0  300  250.  0.  27.0
15  0
15  0
1

0  10.  5.28  1181.  2.5
0.  0.0263  15.89  0.707
27.  0.0263  15.89  0.707
127.  0.0338  26.41  0.690
227.  0.0407  38.79  0.684
327.  0.0469  52.69  0.685
427.  0.0524  68.10  0.695
527.  0.0573  84.93  0.709
627.  0.062  102.9  0.72
727.  0.0667  121.9  0.726
827.  0.0715  141.8  0.728
927.  0.0763  162.9  0.728
1027.  0.082  185.1  0.719
1127.  0.091  213.0  0.703
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STOP
APPENDIX G

Sample Input File for TOPAZ Program

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|---|---|---|---|---|---|---|
| 273.0 | 6.3197 |
| 300.0 | 5.2838 |
| 400.0 | 4.0139 |
| 500.0 | 3.2855 |
| 600.0 | 2.813  |
| 700.0 | 2.4866 |
| 800.0 | 2.2432 |
| 900.0 | 2.0499 |
| 1000.0 | 1.8982 |
| 1100.0 | 1.8693 |
| 1200.0 | 1.6625 |
| 1300.0 | 1.569  |
| 1400.0 | 1.5045 |
| 1500.0 | 1.4579 |
| 1600.0 | 1.4045 |
| 2000.0 | 1.4   |

out wall  h  2  6

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 273.0 | 4.2988 |
| 300.0 | 2.0498 |
| 350.0 | 4.5425 |
| 400.0 | 5.3914 |
| 450.0 | 5.8211 |
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