AN ELECTRONIC ANALYZER
OF
AUDIO-MAGNETOTELLURIC SIGNALS

by

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ABSTRACT

An electronic instrument which measures magnetotelluric signals in the frequency range between 1.78 Hz and 715 Hz is described. Based on a design proposed by Berdichevskii (1969), the instrument computes the autocorrelation of a horizontal component of the magnetic field and the crosscorrelation of this magnetic component with an orthogonal, horizontal component of the electric field.

Field tests of the analyzer, near Livermore, Colorado, over Precambrian granite, and at the Lowry Bombing Range, Colorado, over a thick sedimentary section, are described. The apparent resistivities determined by the AMT measurements at frequencies above 152 Hz agree within an order of magnitude with DC resistivity measurements made previously at these sites. However, the apparent resistivities for frequencies below 152 Hz are rejected because of the observed incoherence of the signals.

The design objectives of the analyzer were met. However, if the full potential of the instrument is to be realized, artificially generated source fields should be used.
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INTRODUCTION

This Master of Science thesis describes an instrument built by the author: an electronic analyzer of audiomagnetotelluric signals in the frequency range from 1.78 Hz to 715 Hz.

The funds for the electronic parts and the author's assistantship came from Air Force Research Contract no. F 19628-69-C-0281 (Keller, Lebel, and Ausman, 1970). Valuable help in designing, building, and testing this instrument was provided by Drs. Keller and Merideth of the Colorado School of Mines' Geophysics Department. Evan L. Ausman, Jr. made the DC resistivity soundings which are compared to the AMT results in the section of the thesis describing the Livermore test area.

The objective of the program is to determine the earth's resistivity. The frequency range specified provides field penetrations of interest in mining exploration for the resistivities usually encountered in such applications.

Resistivities are obtained from perpendicular measurements of the horizontal electric and magnetic
fields. Measurements are reduced to final values for $\mathbf{P}$ in the field, and no further data processing is required.

Early, classical papers on magnetotellurics include Cagniard's (1953) on M.T. theory and applications, Wait's (1954) on the source-field requirements, and Price's (1962) on M.T. when the source-field is considered. In 1958, Ward and others described AFMAG, a method of electrical prospecting which uses the earth's natural magnetic field at audio frequencies. This thesis is a continuation of the initial work done at the Colorado School of Mines by N. Harthill (1967).
THEORY

To provide a basis for the design of the analyzer, the source of the AMT signals is described. Then, expressions for the resistivities of simple earth geometries are derived from the field components sensed by the analyzer.

Source

It is generally agreed that, for frequencies between 1.78 Hz and 715 Hz, almost all audio-magnetotelluric signals are generated by thunderstorms and/or by power distribution systems (Keller and Frischknecht, 1966, p. 200; Ward, 1967, p. 251; Strangway and Vozoff, 1970, p. 116). The thunderstorms form more or less well-defined centers of activity coinciding with the onset of the monsoon seasons in these areas of the globe: Central Africa, Central-South America, and the East Indies. The peak of secular activity from these sources occurs in June and July. Since the thunderstorms take place in late afternoon and early evening, a diurnal curve of activity for Colorado has a nearly uniform amplitude except for a trough between 8 p.m. and midnight.
The signals generated by the lightning strokes within the thunderstorms, although distance dependent, can be represented by typical curves as in figure 1.

![Variations of magnetic field](image)

**Figure 1.** Variations of earth's magnetic field vs frequency.

This graph, from Keller and Frischknecht (1966, p. 200), presents the variations of the magnetic field vs frequency in gammas. Note the 60-db minimum at 1 Hz.

At least two possibilities explain this minimum: that the lightning discharges do not contain much energy at these low frequencies and that resonance effects, such as the Schumann resonances between 8 and 40 Hz, and waveguide effects, such as the earth surface-E layer waveguide, are effective only at frequencies well above 1 Hz. Both
explanations probably are correct. Below 1 Hz, the rise in field strength represents energy arriving from outside the earth, rather than energy from sferics.

Although we know that lightning strokes can be represented by both vertical and horizontal current flow components and that the thunderstorms are not a single unit but a cell, we assume that audio-magnetotelluric signals originate from a single source. Plane-wave theory is used throughout the theoretical developments culminating in the description of $\rho_a$, the apparent resistivity, as a function of two orthogonal, horizontal components of the electric and magnetic fields. These assumptions are made: (1) the source is unique; (2) the source is distant (Harthill, 1967, computed a distance of four wavelengths in the earth as a minimum); and (3) the vertical component of the magnetic field at the receiver is zero. Farsstad (1970), working with a controlled-source modeling system, found that the fields at the receiver behaved as plane waves for source-receiver separations greater than three skin depths.

Although it is well known that plane-wave theory is inadequate in some situations, it provides a reasonable starting point for describing field measurements. More exact analysis of actual field behavior often becomes unmanageable mathematically.
Derivation of the Expression for the Apparent Resistivity of a Uniform Earth

Neglecting displacement currents, assuming sinusoidal time variations as well as an incident field which varies only with $z$, and given this coordinate system,

\[ \nabla \times \mathbf{E} = -i \omega \mu \mathbf{H} \]  \hspace{1cm} (1)

\[ \nabla^t \mathbf{E} = i \omega \mu_0 \sigma \mathbf{E} \]  \hspace{1cm} (2)

Figure 2. Coordinate system.

Maxwell's equations can be written:
\[
\frac{\partial E_x}{\partial z} = i \omega \mu \sigma E_x \tag{3}
\]

where

- \( \vec{E} \) = electric field intensity (volt/m)
- \( \vec{H} \) = magnetic field intensity (amp/m)
- \( \omega \) = angular frequency (rad/sec)
- \( \mu \) = permeability (henry/m)
- \( \sigma \) = conductivity (mho/m) = \( 1/\rho \)
- \( \rho \) = resistivity (ohm-m).

A particular solution of equation 3 is:

\[
E_x = E_{x_0} e^{\frac{-i \omega \mu \sigma z}{\rho}} e^{i \omega t}. \tag{4}
\]

From equation 1:

\[
\frac{\partial E_x}{\partial z} = -i \omega \mu H_y. \tag{5}
\]
Using equations 4 and 5,

\[
H_y = -E_x \frac{\sigma}{\sqrt{i \omega \mu}} e^{\frac{\sqrt{i \omega \mu}}{L} \cdot e^{i \omega t}}
\]  \tag{6}

and

\[
\frac{E_x}{H_y} = -\frac{1}{\sqrt{\frac{\sigma}{i \omega \mu}}}
\]  \tag{7}

\[
\rho = \frac{1}{\sigma} = \frac{1}{i \omega \mu} \left( \frac{E_x}{H_y} \right)^2.
\]  \tag{8}

**Derivation of the Asymptotic Conditions for a Two-Layer Earth**

Definitions of the skin depth and of the wave impedance are given in this section. The asymptotic conditions for a two-layer earth are then derived.

**Skin depth:** The skin depth is defined as the depth at which an electromagnetic wave is attenuated by 1/e or by 63.2 percent of its value at the surface. If displacement currents are neglected, the skin depth, \( \delta \) (meters), equals \( \left( \frac{2 \rho}{\mu_0} \right)^{\frac{1}{2}} \) which is the inverse of the real part of the wave number. Skin depth charts can be found.
in many publications (Keller and Frischknecht, 1966, p. 214; Vozoff and Strangway, 1970, p. 110, etc.).

**Wave impedance:** The wave impedance, $Z$ (ohms), is defined as the ratio of electric to magnetic field strength, $E_x/H_y$. For plane-wave propagation, and a two-layer earth, the wave impedance observed at the surface is (Keller and Frischknecht, 1966, p. 217):

$$Z_0 = \frac{i \omega \mu}{\sigma} \coth \left( \gamma_1 h_1 + \coth^{-1} \gamma_1/\gamma_2 \right)$$  \hspace{1cm} (9)

where $\omega$ = angular frequency (rps)
$\mu$ = permeability (henry/m)
$h_1$ = thickness of layer #1 (m)
$\gamma_1$ = complex wave number of medium #1 (m$^{-1}$)
$\gamma_2$ = complex wave number of medium #2 (m$^{-1}$)

and

$$\rho_a = -\frac{i \omega \mu}{\sigma} \coth^2 \left( \gamma_1 h_1 + \coth^{-1} \gamma_1/\gamma_2 \right)$$  \hspace{1cm} (10)

$$\rho_a = -\rho_1 \coth^2 \left[ \sqrt{\left( -\frac{i \omega \mu}{\rho_1} \right) h_1 + \coth^{-1} \sqrt{\rho_2/\rho_1}} \right]$$  \hspace{1cm} (11)

where $\rho_a$ = apparent resistivity (ohm-m)
$\rho_1$ = resistivity of medium #1 (ohm-m)
$\rho_2$ = resistivity of medium #2 (ohm-m).
For wavelengths much greater than the layer thickness, and rearranging terms in equation 9,

\[ Z(\omega) = \frac{-i\omega\mu}{i/\gamma + \gamma_1/\gamma_2} \]  

(12)

**Case 1:** A surface layer with a much larger resistivity than that of the second layer.

Then \( \gamma_1 \gg \gamma_2 \),

\[ Z(\omega) \approx i\omega\mu h_1 \]  

(13)

and

\[ f_2 = \frac{i\omega\mu h_1^2}{h_2} \]  

(14)

**Case 2:** A surface layer with a much lower resistivity than that of the second layer.

Then \( \gamma_1 \ll \gamma_2 \), \( \gamma_2 h_1 \ll 1 \),

\[ Z(\omega) = \frac{-i\omega\mu}{i/\gamma + \gamma_1/\gamma_2} \approx -\frac{f_2}{h_1} \]  

(15)

and

\[ f_2 = \frac{-i\beta^2}{h_1^2\omega\mu} \]  

(16)
The asymptotic expressions derived above are useful to make quick "order of magnitude" checks of the field data as they are collected.

Master curves for two and three layers are available in the literature. For example, Cagniard (1953, p. 523) derives a two-layer curve. Keller and Frischknecht (1966, p. 223) show a three-layer master curve. A set of three-layer master curves is derived and presented by Yungul (1961).

Correlation

In magneto-telluric measurements, the crosscorrelation between the electric and magnetic components is used to determine the coherency of the signals. Mathematically, the crosscorrelation function is:

$$ C_{AB}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_A(t) f_B(t - \tau) \, dt $$  \hspace{1cm} (17)

where

- $f_A$ = time function A
- $f_B$ = time function B
- $\tau$ = variable delay eg. introduced by delay line
- $T$ = period (sec).

The Fourier transform pair is:

$$ \Phi_{AB}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_{AB}(\tau) e^{-i\omega\tau} \, d\tau $$  \hspace{1cm} (18)
$$C_{AB}(\tau) = \int_{-\infty}^{\infty} \Phi_{AB}(\omega) e^{j\omega \tau} d\omega$$  \hspace{1cm} (19)

where \( \Phi_{AB} \) = spectrum of crosscorrelation
\( \omega \) = angular frequency (radians/sec)

In practice, the crosscorrelation function is obtained by averaging only for a time longer than the longest periods in the signals \( f_A(t) \) and \( f_B(t) \). If \( f_A(t) \) and \( f_B(t) \) arise from two unrelated processes, then \( C_{AB}(\tau) \) tends to zero for long averaging times. The crosscorrelation function thus represents the degree of conformity of two signals as a function of their mutual delay.

On the other hand, the autocorrelation is expressed mathematically as:

$$C_{AA}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_A(t) f_A(t-\tau) dt$$  \hspace{1cm} (20)

where \( f_A \) = time function A

\( \tau \) = variable delay eg. introduced by delay line
\( T \) = period (sec).

The cosine Fourier transform pair is:

$$\Phi_{AA}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_{AA}(\tau) \cos \omega \tau \ d\tau$$  \hspace{1cm} (21)
\[ C_{AA}(\tau) = \int_{-\infty}^{\infty} \Phi_{AA}(\omega) \cos \omega \tau \, d\omega \]  

(22)

where \( \Phi_{AA} \) = energy density spectrum of \( f_a(t) \).

Thus, the autocorrelation function, which has a maximum for \( \tau = 0 \), represents the coherence time of the original signal \( f_a(t) \). Autocorrelation and crosscorrelation are powerful tools in detecting very low-level signals in the presence of uncorrelated noise (Lubow, 1966).

The audio-magnetotelluric analyzer which is the subject of this thesis embodies a two-point crosscorrelator: \( E(\tau) \) and \( H(\tau - \tau) \) are multiplied once for \( \tau = 0 \) and again for \( \tau = T/4 \) where \( T \) is the period of the center frequency. These two products are then processed in averaging integrators. Because the crosscorrelation function of two sinusoidal signals having the same frequency is also a sinusoidal with this frequency, two points of the crosscorrelation function completely specify the function. For example, in-phase changes of the two products show that the magnitudes of \( E \) and \( H \) are changing relative to each other; out-of-phase changes of the two products show that the phases of \( E \) and \( H \) are rolling relative to each other.
The analyzer also computes the zero-delay autocorrelation of the magnetic signal: $H(\tau) \cdot H(\tau - \tau)$ for $\tau = 0$, followed by processing in an averaging integrator. Because the autocorrelation of a sinusoidal signal is a cosine function with the same frequency as the signal, the zero-delay autocorrelation of the magnetic signal specifies completely its autocorrelation function.

The ratios of the two previously described crosscorrelated results with the autocorrelated magnetic signals are calculated by divider networks and displayed on meters. One can then imagine $E(\tau) \cdot H(\tau)/H^2(\tau)$ as being the real part of the wave impedance and $E(\tau) \cdot H(\tau - \tau_4)/H^2(\tau)$ as being the imaginary part of the wave impedance.

The instrumentation mentioned above is fully described in the next section.
INSTRUMENTATION

The block diagram in figure 3, from Berdichevskii et al (1969), describes the instrument completely. Two identical channels are used for the E and H signals: preamplifiers, tuned amplifiers, phase shifter, multipliers, AC integrators, and finally dividers; only the sensors differ.

Sensors

The horizontal electric field is sensed with porous-pot electrodes filled with copper sulphate in contact with the ground. The far pot is connected to 100 m of unshielded wire, and the near pot is used as the ground reference for the whole system.

Variations in the horizontal magnetic field are sensed with a multiturn coil of wire. The "University of Texas" metal-core coil has an effective area at low frequencies of 20,000 m² and a resonant frequency of 250 Hz which was damped using 75 K-ohms across the coil's input. Its physical characteristics are as follows: (1) weight = 130 lbs, (2) length = 59" L and (3) height and width = 8½" H x 8½" W. The electrical
Figure 3. Block diagram of analyzer.
The characteristics of the coil are summarized in figures 4 and 5, which show the coil's effective area, impedance and sensitivity versus frequency.

The effective area is found by measuring the output voltage of the coil in a known magnetic field. The relevant formulas are:

\[ B = \frac{\mu_0 I A_{\text{source}}}{2\pi r^2} \]  

(23)

\[ A_{\text{receiver effective}} = \frac{\text{emf}}{\omega B} \]  

(24)

where

- \( B \) = magnetic flux density of source (weber/m\(^2\))
- \( \mu_0 \) = permeability (henry/m) = \( 4\pi \times 10^{-7} \)
- \( I \) = source current (amperes)
- \( A_{\text{source}} \) = area of source coil (m\(^2\))
- \( r \) = distance between source and receiver coils (m)
- \( A_{\text{receiver effective}} \) = effective area of coil being calibrated (m\(^2\))
- \( \text{emf} \) = voltage at output of coil being calibrated (volts)
- \( \omega \) = angular frequency (radian/sec).
Figure 4. Effective area and impedance of "U. of Texas" coil vs frequency.
Figure 5. Sensitivity of "U. of Texas" coil vs frequency.
The impedance is determined by injecting a known current into the coil and measuring the resultant voltage drop across the coil.

The sensitivity of the coil is derived from these formulas:

\[ \text{emf} = \omega \mu_0 H A_{\text{receiver effective}} \quad (25) \]

\[ \Sigma = 2\pi f A_{\text{receiver effective}} \times 10^{-3} \quad (26) \]

where

- \text{emf} = \text{voltage at output of coil (microvolts)}
- \omega = \text{angular frequency (radian/sec)}
- \mu_0 = \text{permeability (henry/m)}
- H = \text{magnetic field intensity (ampere-turn/m)}
- \Sigma = \text{coil sensitivity (microvolts/gamma)}
- A_{\text{receiver effective}} = \text{effective area of coil being calibrated (m}^2\text{)}
- f = \text{frequency (Hz)}.

The output of a coil responds to the time derivative of the magnetic field. For a sinusoidal excitation, this fact is of little importance since the output is then
proportional to "w H". However, the phase shifts introduced by differentiation of the magnetic field at the different frequencies make it difficult to use phase information in AMT measurements.

Preamplifiers (Figure 6)

The preamplifiers are Texas Instrument Inc., model RA 12, low frequency parametric amplifiers. Typical specifications are (1) differential input and output, (2) open loop voltage gain of 100 db, (3) bandpass of DC to 1 kHz for 60 db gain, (4) input impedance of \(10^9\) ohms in parallel with 1100 pf, and (5) equivalent noise voltage of 25 nanovolts rms measured in the passband from 1 Hz to 10 Hz.

In the circuit of figure 6, the amplifiers operate in the non-inverting mode and use only one of the differential outputs. The source resistance is approximately 100 ohms in parallel with the sensor's resistance. The maximum Johnson noise generated by this source for a 10-Hz bandwidth of the tuned amplifiers is 20 nanovolts. This value is comparable to the preamplifier's equivalent noise voltage. Therefore, the external resistance will add little to the preamplifier's internally generated noise.

The amplifiers are operated with a low-frequency gain of 100 db decreasing by 20 db/decade above 10 Hz.
Figure 6. Preamplifiers.
Their output is AC coupled to a unity-gain buffer amplifier to avoid loading the preamps and to provide a low input impedance to the twin-T, 60 Hz, band-reject filter. This filter attenuates 60 Hz noise by 50 db. The output of the 60 Hz filter, \( E(\pi) \) or \( H(\pi) \), goes to a 100-Kohm voltage divider with eight 10-db steps of signal attenuation.

**Tuned Amplifiers (Figures 8 and 9)**

The preamplified, filtered, and attenuated signals are then fed to the tuned amplifiers. The central operational amplifier in the tuned part of the circuit is an "ideal-current-inversion negative immittance converter". To isolate the INIC unit, two non-inverting, high input-impedance operational amplifiers are used at the input and output of the stage. The input unit has unity gain, and the output unit has a gain of 7.8.

To realize the general transfer function of a bandpass network, \( \frac{E_{out}}{E_{in}} = \frac{s \omega_c}{s^2 + \omega_c^2 + \omega_o^2} \), the INIC is used in the following configuration:

![Figure 7. INIC bandpass active filter.](image)
Figure 8. INIC tuned amplifier (E).
Figure 9. INIC tuned amplifier (H).
where

\[ s = j\omega \]

\[ H = \text{positive real constant (gain)} \]

\[ \alpha = \text{positive real constant (pole position)} \]

\[ \omega_0 = 2\pi f_0 = \text{center frequency (rps)}. \]

In this case,

\[ \frac{E^*}{E_i} = \frac{-sKC_1G_1}{s^2C_1C_2 + s (C_1G_1 + C_2G_2 - KC_1G_1) + G_1G_2} \]

(27)

where

\[ K = \text{gain constant} = 1.98 \]

\[ G_1 = G_2 = (42.2K)^{-1} \]

\[ C_1 = C_2 = 2.15 \text{ MFD (at 1.78 Hz)} \]

\[ \alpha = 0.02 \text{ (pole position)}. \]

This network has a zero at the origin and two conjugate poles near the imaginary axis in the negative half of the complex plane. The constant, \( \alpha \), sets the distance of the poles from the imaginary axis and therefore the \( Q \) and gain of the active filter. Referring back to equation 27, we see that the gain constant, \( H \), appears only in the first degree term of the denominator. This means that the \( Q \) and the gain of the circuit can be varied without greatly changing the magnitude of the pole positions, i.e. the resonant frequency. Furthermore, we can see that \( C_1 \) and \( C_2 \), and \( G_1 \) and \( G_2 \) can have the same
values, and that changing \( C_1 \) and \( C_2 \) changes the resonant frequency without affecting the \( Q \) of the network. The need to change only two components to vary the pass-band center frequency is a major advantage of the INIC.

In the practical realization of the INIC described in figures 8 and 9, the gain is adjusted by varying the 2K potentiometer at the output of the INIC. A ratio of 1.98 between the 20 K + pot and the 10 K + pot resistors results in a gain of 100 and a \( Q \) of approximately 50. The center frequency is varied by changing \( C_1 \) and \( C_2 \), which have identical values. To ease the requirement for matching of the two tuned amplifiers, slight adjustment of the tuning resistors is provided in the E tuned amplifier (figure 8). Provision is also made for two external plug-in filters whose center frequency can be chosen at will.

The channel-to-channel match is made by injecting identical signals into both channels and varying the tuning resistors in the E channel. A match is accepted when the Lissajous figure of the two channels degenerates to a 45-deg line at the center frequency and remains a 45-deg line for test signals, on each side of the center frequency, which cause 6-db drops in magnitude of the output signals. The match can be made with a precision of approximately 5 percent.

The single frequency output of the E tuned amplifier,
$Z(\omega)$ is fed directly to the multipliers. A portion of the $H$ tuned amplifier's output, $I(\omega)$, goes to the phase shifter for further processing.

At this point in the circuitry, the maximum voltage gains at the different center frequencies are as shown in table 1.

Table 1: Maximum voltage gain at the output of the tuned amplifiers vs frequency.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Voltage gain (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.78</td>
<td>138.6</td>
</tr>
<tr>
<td>3.83</td>
<td>138.3</td>
</tr>
<tr>
<td>8.15</td>
<td>138.2</td>
</tr>
<tr>
<td>17.0</td>
<td>135.6</td>
</tr>
<tr>
<td>36.9</td>
<td>132.7</td>
</tr>
<tr>
<td>76.2</td>
<td>128.3</td>
</tr>
<tr>
<td>152</td>
<td>125.1</td>
</tr>
<tr>
<td>310</td>
<td>125.1</td>
</tr>
<tr>
<td>715</td>
<td>117.6</td>
</tr>
</tbody>
</table>

Phase Shifter (Figure 10)

This stage shifts the phase of the $H$ signal by $-90$ deg with no attenuation. The input divides equally between inverting and non-inverting operational amplifiers.
Figure 10. 90° phase shifter.
For the capacitors and resistors chosen, a total phase shift of \(-90\) deg results since each leg of the bridge arrangement contributes \(-45\) deg. The variable resistors can provide a total phase shift from \(-44\) deg to \(-180\) deg if required; however they are set at \(-90\) deg by detecting circular Lissajous figures on an oscilloscope.

The output of the phase shifter, \(H^*(\omega)\), goes to the \(E(\omega) \cdot H^*(\omega)\) multiplier.

**Multipliers (Figure 11)**

The three multipliers compute \(E(\omega) \cdot H(\omega)\), \(E(\omega) \cdot H^*(\omega)\), and \(H^2(\omega)\). They are Burr-Brown #4096/15, variable-transconductance multipliers which have as output the function \(\frac{XY}{10}\) to an accuracy of \(\pm 100\) mv. Operation is in the non-inverting mode.

Two in-phase sinusoidal signals, \(A \sin \omega t\) and \(B \sin \omega t\), are processed as: \(\frac{AB}{10} \sin^2 \omega t = \frac{AB}{20} (1 - \cos 2\omega t)\), providing a DC component equal to one twentieth of the product of the amplitudes and an AC component with a frequency double that of the original signals. Two, \(90^\circ\)-out-of-phase signals, \(A \sin \omega t\) and \(B \sin (\omega t + 90^\circ)\) are processed as: \(AB \sin \omega t \cos \omega t = \frac{AB}{20} \sin 2\omega t\). In this case no DC component is present; only the frequency is doubled.

The DC component out of the multipliers is therefore sensitive to the phase difference between the
Figure II. Multipliers and dividers.
two multiplied input signals.

**AC Integrators (Figure 12)**

The three AC integrators amplify the DC components of the multiplier's outputs and reject their AC portions. They are a form of low-pass filter, operating with a gain of 100 and in the inverting mode. Chopper-stabilized operational amplifiers, B.-B. #3195/15, are used because their low input bias currents add little noise to the DC signals of interest. The Burr-Brown design provides a test point in the amplifier to detect overload conditions. This is the circuit shown at the bottom of figure 12.

In the feedback network of the integrators, seven different capacitors determine integrating time constants between 0.1 and 100 seconds to filter most effectively the different frequencies. A tee-network (100 K, 1 K, 100 K) replaces a 10-megohm feedback resistor with an equivalent feedback effect, but with much less susceptibility to stray-leakage and stray-capacitance errors.

The network consisting of the ± 15-v supplies, the two 20-K resistors, and the 200-ohm potentiometer injects a small DC voltage to buck out all the noise components which have arrived at the input to an integrator. This zeroing procedure is done with no input signal applied to the preamplifiers.
Figure 12. AC integrators.
The outputs of the AC integrators, $E(*) \cdot H(t)$, $E(*) \cdot H^*(t)$, and $H^2(t)$ go to the dividers.

**Dividers (Figure 11)**

The two dividers compute $E(*) \cdot H(t)/H^2(t)$ and $E(*) \cdot H^*(t)/H^2(t)$. Units identical to the multipliers, Burr-Brown #4906/15, but wired slightly differently, compute the function $-\frac{10X}{Y}$. However, in this case, the accuracy is very dependent on the magnitude of the denominator since the error equals $\frac{10 \text{ volts x 100 mV}}{\text{Denominator (volts)}}$. Small denominator values cause serious problems.

The results are displayed on two 50-0-50 DC microammeters fitted with the appropriate scales.

**Accessory Circuits—Operating Tests**

This section includes the accessory circuits (power supply and monitor) and the test circuits (continuity, batteries, equivalent noise, etc.), concluding with a general description of the analyzer itself.

In the analyzer, four regulated supply voltages are required (figure 13). Two separate +24-v series-pass regulators supply power for the preamplifiers. The primary power source is mercury batteries with a nominal voltage of 33 v and a rated capacity of 1000 milliamper-hour. Since the preamplifiers require 10 ma of current, the batteries last for about 18 days at 5 hours of service per day. The ±15-v supply, which powers all the
Figure 13. Power supply.
operational amplifiers, multipliers and dividers, is a
Burr-Brown 520/25 DC to DC inverter with a 12-v input
from an automotive-type battery. The inverter has an
output adjustment range of ± 1 percent of 15 v and an
output voltage regulation against line of ± 0.05 percent.
However, in operational amplifier operations, the
tracking ability of the ± 15-v supplies seems more
important than their regulation (magnitude) because any
unbalance in the +15-v supply relative to the -15-v
supply results in an equivalent common-mode error fed
into the amplifiers. Tracking for the 520/25 supply is
said to be better than 0.01 percent. The total current
drawn from this supply by the nineteen Burr-Brown units
amounts to 150 ma. Because of the inefficiency and the
heavy regulation of the inverter, the 12-v primary
supply must deliver 1 ampere. At this rate, a 60-
ampere-hour battery has a useful life between charges of
about 10 days at 5 hours of service per day.

The signal-monitor circuit (figure 14) includes a
peak-reading AC voltmeter mounted on the same board as
the power supply, suitable voltage-dropping resistors
to measure DC voltages, and two 50-0-50 DC microammeters.
These circuits enable the user to monitor the outputs of
the preamplifiers, tuned amplifiers, phase shifter,
multipliers, integrators, and dividers.

Various test functions are built into the analyzer.
Figure 14. Signal monitor.
The continuity of the electric or magnetic sensors may be checked with an ohm-meter (figure 13). The regulated supply voltages can be verified with a voltmeter arrangement (figure 13). To simulate the Johnson noise generated in the sensors, equivalent resistors can be switched in at the input of the preamplifiers.

Internal calibration of the analyzer provides a go/no-go check of the main circuits. To this end, the tuned amplifiers are forced to go into self-oscillation and generate test voltages which are fed to the other circuits and checked with the monitor.

Most of the circuits just described are mounted on printed circuit boards which plug into the main instrument frame. The completed analyzer fits into a 14- by 20- by 15- in. metal case and weighs 45 lb.
Figure 15. Photograph of analyzer's front panel.

Figure 16. Photograph of analyzer's internal layout.
Specifications of a Transmitter for Controlled-Source AMT Measurements

A high-power and variable-frequency source can also generate the electric and magnetic fields ratioed by the analyzer. Figure 17 shows a suitable transmitter-receiver configuration.

To attain plane-wave behavior of the fields at the receiver, the minimum separation, \( r \) (meters), between source and receiver is three skin depths, \( S \) (meters), (Harthill, 1967, and Farstad, 1970). For a uniform earth with a resistivity, \( \rho \), of 10 ohm-meters and for a transmitted frequency, \( f \), of 1.78 Hz,

\[
r_{\text{minimum}} = 3S = 3\sqrt{\frac{\rho}{\pi f \mu}} = 3\sqrt{\frac{10}{\pi \times 1.78 \times 4\pi \times 10}} = 3600 \text{ m} \quad (28)
\]
If a tangential electric field, $E_t$, of 2 $\mu$V/m is present at the receiver, an electrode separation, $a$, of 5 m will produce a more than adequate 10-$\mu$V signal at the input to the analyzer. The radial magnetic field corresponding to the 2-$\mu$V/m electric field can be computed as follows:

$$f = \frac{i}{2\pi f \mu_0} \left( \frac{E_t}{H_a} \right)^2$$

(29)

and

$$|H_a| = \frac{|E_t|}{\sqrt{\mu_\omega \mu}} = \frac{2 \times 10^{-6}}{\sqrt{10 \times 2 \times 10^5 \times 4 \times 10^3}} = 1.68 \times 10^{-4} \text{ amp-turn/m}$$

(30)

or

$$|H_a| = 1.68 \times 10^{-6} \times 4 \times 10^3 \times 10^5 = 0.211 \text{ gammas}$$

(31)

At 1.78 Hz and for a field of 0.211 gammas, the "University of Texas" coil has an output of 50 $\mu$V (figure 5). This voltage is an adequate input signal to the H channel of the analyzer.

To generate a tangential field of 2 $\mu$V/m at a distance of 3600 m from the transmitting dipole, in a medium with a resistivity of 10 ohm$\cdot$m, the required current by dipole-length moment, $I\cdotL$, is (Keller, 1968, p. 142)
I·L = \frac{n A^3 E_t}{\rho} = \frac{n \times 3.6 \times 10^5 \times 2 \times 10^{-6}}{10} = 29.3 \times 10^3 \text{ampere-m}; (32)

for a transmitting dipole of 1 km, 30 amperes of current are required.

To use the analyzer as the receiver of a controlled-source AMT system, the transmitter should have these specifications:

1. frequency range: 1 Hz to 1 kHz, continuously variable;
2. frequency stability: ± 1 percent of set frequency;
3. maximum output current: 50 amperes;
4. power required: 25 KVA;
5. wave-shape: sine or square.

A 4 to 1 reduction in transmitting power could be effected by requiring a field of only 0.5 \mu V/m at the receiver electrodes. One then runs the risk of including the natural electric fields in the measurements. In addition, for a frequency of 715 Hz and a resistivity of 10 ohm-meters, the magnetic field generated is just sufficient to provide an adequate output from the "University of Texas" coil. Another coil would have to be built or a different method of measuring the magnetic field would have to be used.
TEST RESULTS

Orthogonal measurements of E and H are made respectively with a 100-m electric sensor grounded at both ends through porous pots and with the "University of Texas" coil. The two leads from the electric sensor plug directly into the analyzer. The two leads from the coil are joined to the analyzer by an 8-ft extension cord to avoid seismic disturbances around the coil. A 25-ft extension cord joins the analyzer's power supply to a 12-v car battery so that movement of the car will not induce noise into the coil. After a five-minute warm up, the analyzer is operational. A sounding normally takes 3/4 of an hour.

The earth's apparent resistivity at the frequency used is calculated from the measured values of \( E(\omega) \cdot H(\omega)/H^2(\omega) \), \( E(\omega) \cdot H^*(\omega)/H^2(\omega) \), and the attenuator settings. Knowledge of the coil's area and of the separation between the electrodes is also required. Given these values,

\[
\rho = -\frac{\dot{E}}{\omega \mu_0} \left( \frac{E_0}{H_0} \right)^2 = -\frac{\dot{E}}{\omega \mu_0} \left( \frac{V_0/b}{-V/H/N} \right) \tag{33}
\]
\[ \rho_a = \frac{4\pi f \mu_0 (NA)^2}{b^2} \left( \frac{V_E}{V_H} \right)^2 \]  

and

\[ \rho_{dB} = 20 \log \frac{2\pi \mu_0}{b^2} + 20 \log f + 40 \log (NA) + \]

\[ 20 \log \left( \frac{[E(x) \cdot H(x)]^2}{\mu_0 H^2(x)} \right) \]

\[ 2 \times E_{att} - 2 \times H_{att} \]

where

- \( \rho_a \) = apparent resistivity (ohm-meters)
- \( \omega \) = angular frequency (rps)
- \( \mu_0 \) = permeability (henry/m) = \( 4\pi \times 10^{-7} \)
- \( E_x \) = x component of horizontal electric field (volts/m)
- \( H_y \) = y component of horizontal magnetic field (amp/m)
- \( b \) = electrode separation (m)
- \( NA \) = effective coil area \( m^2 \)
- \( f \) = frequency (Hz)
- \( E_{att} \) = setting of E channel attenuator (-db)
- \( H_{att} \) = setting of H channel attenuator (-db)
- \( V_E \) = voltage out of electric sensor (volts)
- \( V_H \) = voltage out of magnetic sensor (volts)
- \( \rho_{dB} \) = apparent resistivity (db relative to 1 ohm-meter)
\( \frac{\mathcal{E}(\mathbf{f}) \cdot \mathcal{H}(\mathbf{f})}{\mathcal{H}^2(\mathbf{f})} = 10^{-1} \) times output of one final divider
\( \frac{\mathcal{E}(\mathbf{f}) \cdot \mathcal{H}^*(\mathbf{f})}{\mathcal{H}^2(\mathbf{f})} = 10^{-1} \) times output of other final divider.

Measurements were made at two test sites: Livermore, Colorado and Lowry Bombing Range, Colorado. These sites were chosen because control data from DC resistivity surveys are available over them, because resistivities of the rocks at the two sites differ by 1000/1, and because the sites are easily accessible.

Tests near Livermore, Colorado

Livermore is 20 miles northwest of Fort Collins, Colorado. The locale around Livermore was studied by the Colorado School of Mines under the same Air Force Research contract which supplied the funds for the analyzer (Keller, Lebel, and Ausman, 1970). Measurements of wave-tilt and DC resistivity had been made in the area prior to the gathering of the audio-magnetotelluric data.

The survey area is partly underlain by Precambrian granite and partly by Pennsylvanian sandstones which are in fault contact with the granite. A\( ^{MT} \) measurements were made over both rock types.

The results of some of the audio-magnetotelluric soundings over granite near Livermore were as shown
in table 2 (see figure 18 for the location).

Table 2. Results of some of the AMT soundings near Livermore, Colorado.

<table>
<thead>
<tr>
<th>frequency (Hz)</th>
<th>Sounding no. 2 (ohm-m)</th>
<th>Sounding no. 3 (ohm-m)</th>
<th>Sounding no. 5 (ohm-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.78</td>
<td>4</td>
<td>9</td>
<td>---</td>
</tr>
<tr>
<td>3.83</td>
<td>0.1</td>
<td>0.6</td>
<td>---</td>
</tr>
<tr>
<td>8.15</td>
<td>0.1</td>
<td>0.9</td>
<td>--- Windy</td>
</tr>
<tr>
<td>17.0</td>
<td>3</td>
<td>0.9</td>
<td>---</td>
</tr>
<tr>
<td>36.9</td>
<td>120</td>
<td>14</td>
<td>---</td>
</tr>
<tr>
<td>76.2</td>
<td>300</td>
<td>450</td>
<td>1200</td>
</tr>
<tr>
<td>152</td>
<td>5620</td>
<td>2500</td>
<td>2300</td>
</tr>
<tr>
<td>310</td>
<td>22,000</td>
<td>17,800</td>
<td>1800</td>
</tr>
<tr>
<td>715</td>
<td>1</td>
<td>100</td>
<td>1</td>
</tr>
</tbody>
</table>

Figures 19 and 20, and table 3 present the DC resistivity data and interpretations. The surface resistivities varied between 10 and 100 ohm-m, in general, which might correspond to the resistivities measured with the 715 Hz setting of the analyzer. Near AMT sounding no. 2, DC sounding no. 28 detected a resistivity of 3500 ohm-m for the granite; at AMT sounding no. 3, DC sounding no. 22 measured a resistivity greater than
Figure 18. Location map of tests near Livermore, Colorado (from Keller, Lebel, and Ausman, 1970).
Figure 19. Summary presentation of resistivity sounding curves obtained in areas of sedimentary rock (from Keller, Lebel, and Ausman, 1970).
Figure 20. Summary presentation of resistivity sounding curves obtained in granite areas (from Keller, Lebel, and Ausman, 1970).
<table>
<thead>
<tr>
<th>Sounding number</th>
<th>Type of terrain</th>
<th>Soil number</th>
<th>Soil $\rho_1$</th>
<th>Soil $h_1$</th>
<th>Bedrock $\rho_2$</th>
<th>$\sigma_1h_1^2$</th>
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</thead>
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<tr>
<td>1</td>
<td>Sedimentary</td>
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<td></td>
<td></td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Sedimentary</td>
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<td></td>
<td></td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Granite</td>
<td>23</td>
<td>35</td>
<td>&gt;100</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Sedimentary</td>
<td></td>
<td></td>
<td></td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Sedimentary</td>
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<td></td>
<td></td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Sedimentary</td>
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<td></td>
<td></td>
<td>56</td>
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</tr>
<tr>
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<td></td>
<td></td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Sedimentary</td>
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<td></td>
<td></td>
<td>63</td>
<td></td>
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<td>Sedimentary</td>
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<td>410</td>
<td>0.020</td>
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</tr>
<tr>
<td>13</td>
<td>Sedimentary</td>
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<td></td>
<td></td>
<td>62</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Sedimentary</td>
<td></td>
<td></td>
<td></td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Sedimentary</td>
<td></td>
<td></td>
<td></td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Granite</td>
<td>13.5</td>
<td>17.</td>
<td>&gt;300</td>
<td>21.5</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Granite</td>
<td>58.9</td>
<td>1.7</td>
<td>500</td>
<td>0.050</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Sedimentary</td>
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<td></td>
<td></td>
<td>69</td>
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</tr>
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<td>19</td>
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<td></td>
<td>73</td>
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<td></td>
<td>81</td>
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</tr>
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<td></td>
<td></td>
<td>75</td>
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</tr>
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<td>22</td>
<td>Granite</td>
<td>33.2</td>
<td>3.0</td>
<td>&gt;3000</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>23</td>
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<td>550.</td>
<td>40.</td>
<td>&gt;2000</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td>24</td>
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<td>4.6</td>
<td>1500</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>25</td>
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<td>5.8</td>
<td>&gt;2000</td>
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<td></td>
</tr>
<tr>
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<td>100.</td>
<td>1.0</td>
<td>3000</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Granite</td>
<td>27.8</td>
<td>1.5</td>
<td>&gt;1000</td>
<td>0.084</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Granite</td>
<td>170.</td>
<td>2.3</td>
<td>3500</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Granite</td>
<td>120.</td>
<td>1.5</td>
<td>3000</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Granite</td>
<td>21.7</td>
<td>3.0</td>
<td>&gt;2000</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Granite</td>
<td>28.9</td>
<td>2.5</td>
<td>&gt;3000</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Granite</td>
<td>27.5</td>
<td>4.2</td>
<td>&gt;3000</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Granite</td>
<td>32.7</td>
<td>1.8</td>
<td>&gt;3000</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>

(Resistivities are expressed in ohmmeters, depths in meters)
3000 ohm·m; near AMT sounding no. 5, where the soil cover is greatest, the resistivity of the granite was greater than 3000 ohm·m according to DC sounding no. 33. These three values are of the same order of magnitude as the apparent resistivities measured with the AMT system at the higher frequencies and presented in table 2. In a uniform earth, resistivities of 10,000 ohm·m and frequencies of 152 Hz to 310 Hz produce skin depths of 3 to 4 km.

The apparent resistivities for the frequencies below 76.2 Hz are completely unreliable for the reasons outlined in the section on field errors.
Tests at Lowry Bombing Range, Colorado

The Lowry Bombing Range is 30 miles east of Golden in the extensively studied Denver sedimentary basin.

The geological section of interest at Lowry consists of the Pierre formation of Upper Cretaceous age. The first 1500 feet of the Pierre are interbedded siltstones and sandstones; these are underlain by about 3000 feet of shale, properly called the Pierre shale. The Niobrara-Benton formations of Upper Cretaceous age underlie the Pierre. They provide a good resistivity contrast with it since they are mainly limestones and sandstones (Harthill, 1967).

Six soundings were made at Lowry. Three are presented in Table 4 (see figure 21 for location).

Table 4. Results of some of the AMT soundings at the Lowry Bombing Range, Colorado.

<table>
<thead>
<tr>
<th>frequency (Hz)</th>
<th>Sounding no. 4 (ohm-m)</th>
<th>Sounding no. 5 (ohm-m)</th>
<th>Sounding no. 6 (ohm-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.78</td>
<td>2</td>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td>3.83</td>
<td>7</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>8.15</td>
<td>4.5</td>
<td>8</td>
<td>0.01</td>
</tr>
<tr>
<td>17.0</td>
<td>0.1</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>36.9</td>
<td>0.9</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>76.2</td>
<td>1.5</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>152</td>
<td>0.9</td>
<td>100</td>
<td>18</td>
</tr>
<tr>
<td>310</td>
<td>1</td>
<td>6</td>
<td>0.7</td>
</tr>
<tr>
<td>715</td>
<td>1.5</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 21. Location map of tests at the Lowry Bombing Range, Colorado.
Well-log data and dipole-dipole measurements made for the electrical prospecting courses give these resistivities at Lowry: (1) surface soil, 10 ohm-meters; (2) Pierre sandstones and siltstones, 5 ohm-meters; (3) Pierre shale, 3 ohm-meters; (4) Niobrara-Benton sandstones and limestones, 15 ohm-meters.

By comparison, referring to table 4, the apparent resistivities for the higher frequencies of the AMT system lie mainly between 1 and 10 ohm-meters, which is of the same order of magnitude. The skin depths for frequencies between 152 and 310 Hz and for a resistivity of 5 ohm-meters are 65 to 90 meters. These do not even penetrate the Pierre shale.

Once again, the data for the frequencies below 76.2 Hz have been rejected as unreliable for the reasons given in the section on field errors.

Field Errors

Field measurements of audio-magnetotelluric signals are known to be much more difficult than one would expect on the basis of theory.

The user quickly finds that wind noise is very bothersome at frequencies below 60 Hz specially if a vehicle is close by. The cure is to move away from the vehicle and use a heavy coil. Burying the coil seems inefficient for a measurement which is supposed to last
for 3/4 of an hour.

The user then notices "E bursts". These are signals which appear on the telluric line but which have no corresponding magnetic manifestation. Although "E bursts" have not been adequately studied, two explanations are commonly advanced to account for them: (1) the source of the signals is so local that the requirements for plane-wave behavior are not met; (2) local inhomogeneities in the earth's resistivities create certain polarizations in the incident field whereby cancellation of the magnetic fields and reinforcement of the electric fields could occur over small areas of the earth's surface. High amplitude bursts cause some ringing in the filters and probably evoke a non-linear response in the amplifiers. Both these phenomena degrade the quality of the correlation. A limiter circuit which could eliminate from the sample all signals markedly above the average amplitude might cure these problems.

Finally, the serious user notices the lack of coherence of the magnetic and electric signals at frequencies below 100 Hz. This absence of coherence manifests itself by more or less random swings of the meters at the output (dividers) of the analyzer. At frequencies between 1.78 Hz and 17.0 Hz, part of the problem might be insufficient voltage gain or inadequate sensitivity of the field sensor. It is possible that, at
these low frequencies, the only signals large enough to be seen by the analyzer are those poorly correlated signals produced by local sources. Lower-noise amplifiers, narrower-bandwidth filters, and magnetic sensors with greater S/N ratios will be needed to test this hypothesis.
This section includes a review of the possibilities inherent in the audio-magnetotelluric method of prospecting; some comments on the efficiency of the instrument; a proposal for improvements to the analyzer; and suggestions for more fertile fields of research into and use of the audio-magnetotelluric method.

The audio-magnetotelluric method seems particularly well suited to mining exploration. The frequency band covered, 1 Hz to 1000 Hz, produces more than adequate skin depths for the resistivity range from 1 ohm-meter to 1000 ohm-meters. Yet, because of the seeming incoherence of the magnetic and electric fields at frequencies below 100 Hz, only the upper frequencies can be used to measure the earth's resistivities. For a resistivity of 1 ohm-meter and a frequency of 100 Hz, the skin depth is 50 m; if the resistivity is 200 ohm-meters and the frequency is 100 Hz, the skin depth is 700 m. These depths are adequate only in high resistivity environments.

The impracticality of using the lower frequency
range is also unfortunate from the analyzer's point of view. Interwire capacitive coupling degrades its performance at the high frequencies: the gain is lower, and the $Q$ is less stable. One also notices that, because of the input wiring capacitance, system calibration depends on the length and type of the cables connected to the sensors.

Possible improvements to the instrumentation are (1) 5-db steps in the attenuators rather than 10-db steps, (2) a design change of the filter from the INIC realization to more stable, but more complicated and expensive, single feedback twin-$T$ filters, (3) additional circuitry to compute in analog form the apparent resistivity, (4) inverse threshold circuitry to eliminate signal bursts from the total sample.

Instrument problems, notwithstanding, the work done for this thesis project pointed out the following: (1) further instrumentation in the audio-magnetotelluric field should concentrate on the frequency band between 100 Hz and 10 kHz, if natural sources are to be used. (2) Alternatively, this analyzer could be used at frequencies down to 1.78 Hz, but with a controlled-source generating the measured fields. A design calculation shows that 25 KVA are required to produce an adequate signal for a plane-wave measurement at 1.78 Hz over a uniform earth with a resistivity of 10 ohm-meters.
(3) "E bursts" need to be better explained.

The stated objective, to measure apparent resistivities with the audio-magnetotelluric method, was met. However, the measurement accuracies are not as good as in controlled-source methods, and the measurements themselves could not be made reliably in the frequency band from 1.78 Hz to 100 Hz.


Madden, T., and Thompson, W., 1965, Low-frequency electromagnetic oscillations of the earth-ionosphere cavity: Rev. of Geophys., v. 3, no. 2, p. 211.


