USE OF COHERENCE IN SEISMIC

VELOCITY DETERMINATION

By

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Seismic data can be used to determine an approximation of the $v_{RMS}$ versus $T_o$ function for an earth section where $v_{RMS}$ is defined as the root mean square velocity and $T_o$ is the two-way normal incidence time. In this paper the simple theory for making the hyperbolic travel time-distance approximation is developed with an explanation of the meaning of the associated $v_{RMS}$ and its use in determining the average velocity function.

A multifold seismic record corrected for constant $v_{RMS}$ within a prescribed range of values is scanned using the following coherence techniques to determine the $v_{RMS}$ versus $T_o$ function:

1. Summation
2. Polarity coincidence
3. Unnormalized crosscorrelation
4. Semblance
5. Energy normalized crosscorrelation
6. Semblance-weighted summation

The $v_{RMS}$ versus $T_o$ function as determined by each coherence technique is compared to the function determined from a nearby sonic log. It is concluded from this data that the coherences semblance, unnormalized crosscorrelation and energy normalized crosscorrelation are better in establishing the $v_{RMS}$ versus $T_o$ function than summation, polarity coincidence, semblance-weighted summation.
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INTRODUCTION

An uncorrected seismic record represents a wealth of information. A number of seismic records arranged in a common depth point format can yield important information about the velocity and structure of a sedimentary basin. These records must be corrected for non-uniform weathering and topography. Records so corrected cannot be used in a structural interpretation; they must be further corrected to remove the arrival time variation of events due to wave front curvature. This additional correction is called normal moveout and, in the case of a horizontally stratified earth model, represents the event time shift that must be made to produce the effect of plane waves at normal incidence. Normal incident time for a horizontally stratified model is the vertical raypath travel time. After correction for moveout, a structural interpretation can be made.
DERIVATION OF THE MOVEOUT EQUATION

Normal moveout can yield important velocity information about a more or less horizontally layered earth. This velocity information can be used in structural interpretation and with some success in lithologic studies. The assumptions for the physical model are as follows:

1. Seismic energy is considered as travelling along ray paths through a horizontally layered earth.
2. Each horizontal layer is considered homogeneous and isotropic and is characterized by a velocity and thickness.
3. No lateral variation in velocity exists within the layers.
4. Deviations from the predictions derived from this model are due to structure.

For the model in figure 1, each ray path will be linear in each layer and will have an angle of incidence \( \theta_k \) in each layer \( k \). The total time \( t \) for the ray to travel from the source \( S \) to reflection point \( R_p \) is the sum of the travel times through the separate layers. Since time \( t_k \) through any one layer of thickness \( h_k \) is

\[
t_k = h_k \sec \theta_k / V_k = h_k / V_k \cos \theta_k = h_k / V_k (1 - \sin^2 \theta_k) \frac{1}{2} = h_k / V_k (1 - \phi_k^2) \frac{1}{2}
\]

(1)

where \( \sin \theta_k = \phi_k \), \( k = 1, 2, 3, \ldots n \)

the total travel time \( t \) through \( n \) layers is
FIGURE 1
HORIZONTALLY LAYERED MODEL
FOR
DERIVATION OF MOVEOUT EQUATION
The ray in travelling from the source to reflection point \( R_p \) will have traveled an accompanying horizontal distance \( x \) where \( x = X/2 \). This distance \( x \) is the sum of the horizontal projections of the ray paths in each of the layers.

\[
x = \sum_{k=1}^{k=n} \frac{h_k}{\cos \theta_k} = \sum_{k=1}^{k=n} \frac{h_k \sin \theta_k}{\cos \theta_k} = \sum_{k=1}^{k=n} \frac{h_k \phi_k}{(1-\phi_k^2)^{1/2}}
\]  

The \( \phi_k \) for a given model are implicit functions of \( x \) defined by equation (3). Snell's law implies the constraint \( \phi_k/V_k = \phi_n/V_n \) and hence any \( \phi_n \), say \( \phi_j \) may be considered as the single independent variable of \( t = t(\phi_j) \) and \( x = x(\phi_j) \). This complicated definition of \( t(x) \) is circumvented by expanding \( t \) in a Taylor's series in \( x \) in the neighborhood of \( x = 0 \).

\[
t = t(0) + \frac{t'(0)x}{1!} + \frac{t''(0)x^2}{2!} + \frac{t'''(0)x^3}{3!} + \ldots + \frac{t^n(0)x^n}{n!}
\]  

The first six terms of the series will be calculated to yield an approximation and to demonstrate the influence of the higher power \( x \) terms; the immediate problem is to determine the various order derivatives of \( t \). From the definitions of \( t = t(\phi_j) \) and \( x = x(\phi_j) \),

\[
\frac{dt}{d\phi_j} = \frac{dt}{dx} \frac{dx}{d\phi_j}
\]
From the Snell's law constraint,
\[
\frac{d\phi_k}{d\phi_j} = \frac{V_k}{V_j}
\]
and so
\[
\frac{dt}{d\phi_j} = \sum_{k=1}^{k=n} \frac{\partial t}{\partial \phi_k} \frac{d\phi_k}{d\phi_j}
\]
becomes
\[
\frac{dt}{d\phi_j} = \sum_{k=1}^{k=n} \frac{h_k(-1/2)(-2\phi_k)}{V_k(1-\phi_k^2)^{3/2}} \cdot \frac{V_k}{V_j} = \sum_{k=1}^{k=n} \frac{h_k\phi_k(1-\phi_k^2)^{-3/2}}{V_k} \frac{V_k}{V_j}
\]
and
\[
\frac{dx}{d\phi_j} = \sum_{k=1}^{k=n} \frac{\partial x}{\partial \phi_k} \frac{d\phi_k}{d\phi_j} = \sum_{k=1}^{k=n} \frac{h_k}{V_j} \left[ \frac{(1-\phi_k^2)^{1/2}}{-\phi_k(1/2)(-2\phi_k)(1-\phi_k^2)^{-1/2}} \right] \frac{V_k}{V_j}
\]
\[
= \sum_{k=1}^{k=n} \frac{h_k(1-\phi_k^2)^{-1/2}}{V_j} \left[ (1-\phi_k^2) + \phi_k^2 \right] \frac{V_k}{V_j}
\]
\[
= \sum_{k=1}^{k=n} \frac{h_k(1-\phi_k^2)^{-3/2}}{V_j} \frac{V_k}{V_j}
\]
(7)

Substitution of equations (6) and (7) into (5) yields
\[
\frac{dt}{dx} = \sum_{k=1}^{k=n} \frac{\phi_k}{V_k} \frac{V_k}{V_j} = \frac{\phi_k}{V_k} = \frac{\phi_j}{V_j}
\]
(8)
The second derivative is calculated using the chain rule and equation (8)

\[ \frac{dx}{d\phi_j} \frac{d}{dx} \left( \frac{dt}{dx} \right) = \frac{d}{d\phi_j} \left( \frac{dt}{dx} \right) = \frac{1}{V_j} \]  

\[ \frac{d^2t}{dx^2} = \frac{1}{V_j} \left( d\phi_j \right)^{-1} \]  

which upon substitution of equation (7) yields

\[ \frac{d^2t}{dx^2} = \frac{V_j}{V_j} \frac{1}{k=n} \sum \frac{h_k (1-\phi_k^2)}{V_k}^{-3/2} \]  

\[ \frac{d^2t}{dx^2} |_{x=0} = \lim_{\phi_j \to 0} \frac{1}{k=n} \sum \frac{1}{V_j} \frac{1}{V_j} \frac{2^{1/2}}{h_k V_k} \]  

All higher order derivatives are also calculated using the chain rule. For example, from equation (11)

\[ \frac{dx}{d\phi_j} \frac{d}{dx} \left( \frac{d^2t}{dx^2} \right) = \frac{d}{d\phi_j} \left( \frac{d^2t}{dx^2} \right) = \frac{d}{d\phi_j} \left( \frac{1}{V_j} \left( \frac{dx}{d\phi_j} \right)^{-1} \right) \]

\[ = - \frac{1}{V_j} \left( \frac{d^2x}{d\phi_j^2} \right) \left( \frac{dx}{d\phi_j} \right)^{-2} \]  

or
\[ \frac{d^3 \tau}{dx^3} = -\frac{1}{V_j} \left( \frac{d^2 x}{d\phi_j^2} \right) \left( \frac{dx}{d\phi_j} \right)^{-3} \]  

(14)

The second factor on the right hand side of the equation (14) is determined by differentiating equation (7)

\[
\frac{d^2 x}{d\phi_j^2} = \sum_{k=1}^{k=n} \frac{\partial}{\partial \phi_k} \frac{dx}{d\phi_j} \frac{d\phi_k}{d\phi_j} 
= \sum_{k=1}^{k=n} \frac{\partial}{\partial \phi_k} \frac{dx}{d\phi_j} \frac{d\phi_k}{d\phi_j} 
= \frac{3}{2} \frac{V_j^2}{V_j} \sum_{k=1}^{k=n} h_k v_k^2 (1-\phi_k^2)^{-5/2}
\]

(15)

which, with equation (7), upon substitution into equation (14) yields

\[
\frac{d^3 \tau}{dx^3} \bigg|_{x=0} = \lim_{\phi_k+0} \frac{-3}{\phi_k} \left[ \sum_{k=1}^{k=n} h_k v_k^2 (1-\phi_k^2)^{-5/2} \right].
\]

\[
\left[ \sum_{k=1}^{k=n} h_k (1-\phi_k^2) \frac{-3/2}{V_j} V_k \right]^{-3} = 0.
\]

The fourth derivative is now evaluated.
\[
\frac{d^3 x}{d\phi_j} \frac{d}{dx} \left( \frac{d^3 x}{dx^3} \right) = \frac{d}{d\phi_j} \left[ -\frac{1}{V_j} \left( \frac{d^2 x}{d\phi_j^2} \right) \left( \frac{dx}{d\phi_j} \right)^{-3} \right]
\]

\[
\frac{d^4 t}{dx^4} = \left[ -\frac{1}{V_j} \frac{d^3 x}{d\phi_j^3} \left( \frac{dx}{d\phi_j} \right)^{-3} + \frac{3}{V_j} \frac{d^2 x}{d\phi_j^2} \left( \frac{dx}{d\phi_j} \right)^{-4} - \frac{3}{V_j} \frac{d^2 x}{d\phi_j^2} \left( \frac{dx}{d\phi_j} \right)^{-5} \right] \left( \frac{dx}{d\phi_j} \right)^{-1}
\]

\begin{equation}
\frac{d^3 x}{d\phi_j^3} = \frac{3}{V_j^2} \left[ \sum_{k=1}^{k=n} h_k V_j^2 (1-\phi_k)-5/2 \frac{V_k}{V_j} + \sum_{k=1}^{k=n} h_k V_j^2 \phi_k^2 (1-\phi_k)^{-7/2} \frac{V_k}{V_j} \right]
\end{equation}

\begin{equation}
= \frac{3}{V_j^3} \sum_{k=1}^{k=n} h_k V_j^3 (1+4\phi_k^2)(1-\phi_k)^{-7/2}
\end{equation}

which upon substitution into equation (16) yields

\begin{equation}
\frac{d^4 t}{dx^4} \mid_{x=0} = \lim_{\phi_k \to 0} \left[ \sum_{k=1}^{k=n} h_k V_j^3 (1-\phi_k)^{-5/2} \frac{V_k}{V_j} \right]^{-4} \cdot \left[ \sum_{k=1}^{k=n} h_k V_j^3 \phi_k^2 (1-\phi_k)^{-7/2} \frac{V_k}{V_j} \right]^{-5}
\end{equation}

\[
= \sum_{k=1}^{k=n} h_k V_j^3 \phi_k^2 (1-\phi_k)^{-5/2}
\]

\begin{equation}
= \frac{3}{V_j} \left[ \sum_{k=1}^{k=n} h_k V_j^3 \phi_k^2 (1-\phi_k)^{-5/2} \right]^{-5}
\end{equation}
The fifth derivative is similarly determined.

\[
\frac{d^5 x}{d\phi_j} \frac{d}{dx} \left( \frac{d^4 t}{dx^4} \right) = 4 \frac{d^3 x}{d\phi_j^3} \left( \frac{dx}{dx} \right)^5 \left( \frac{d^2 x}{d\phi_j^2} \right) - 5 \frac{d^4 x}{d\phi_j^4} \left( \frac{dx}{dx} \right)^4 - 4
\]

\[
- \frac{15}{V_j} \left( \frac{d^2 x}{d\phi_j^2} \right)^2 \left( \frac{dx}{dx} \right)^5 - 6 \frac{d^2 x}{d\phi_j^2} - 6 \frac{d^2 x}{d\phi_j^2} \left( \frac{d^3 x}{d\phi_j^3} \right) \left( \frac{dx}{dx} \right)^6
\]

\[
\frac{d^5 t}{dx^5} = 4 \frac{d^3 x}{d\phi_j^3} \left( \frac{dx}{dx} \right)^6 - \frac{1}{V_j} \left( \frac{d^4 x}{d\phi_j^4} \right) \left( \frac{dx}{dx} \right)^5 - \frac{1}{V_j} \left( \frac{d^4 x}{d\phi_j^4} \right) \left( \frac{dx}{dx} \right)^5
\]

\[
- \frac{15}{V_j} \left( \frac{d^2 x}{d\phi_j^2} \right)^2 \left( \frac{dx}{dx} \right)^6 - 7 \frac{d^2 x}{d\phi_j^2} + 6 \frac{d^2 x}{d\phi_j^2} \left( \frac{d^3 x}{d\phi_j^3} \right) \left( \frac{dx}{dx} \right)^6
\]

(19)

Before substitution of terms, \(d^4 x/d\phi^4\) must be evaluated. Differentiate equation (17)

\[
\frac{d^4 x}{d\phi_j^4} = \frac{3}{V_j^3} \left( \sum_{k=1}^{k=n} h_k \right) \left( 1+\phi_k^2 \right)^7 \left( 1-\phi_k^2 \right)^{-9/2} + 8 \phi_k^2 \left( 1-\phi_k^2 \right)^{-7/2}
\]

\[
= \frac{3}{V_j^4} \left( \sum_{k=1}^{k=n} h_k \phi_k^4 \right) \left( 15\phi_k^2 + 20\phi_k^3 \right)
\]

(20)

Note that \(\phi_k\) is a multiplier common to each term of equation (20); thus for simplicity in evaluation \(d^5 t/dx^5\bigg|_{x=0}\), the above equation (20) equals zero in the limit as \(\phi_k \to 0\). By the same reasoning time

\[
\lim_{\phi_k \to 0} \frac{d^2 x}{d\phi_j^2} = 0
\]
Every term on the right hand side of equation (19) has a term which, in the limit as \( x \) and \( \phi_k \) both approach zero, equals zero.

\[
\frac{d^5 t}{d x^5} \bigg|_{x=0} = 0 \tag{21}
\]

The sixth derivative is determined by differentiating equation (19)

\[
\frac{d^6 t}{d x^6} = \frac{4}{V_j} \left[ \left( \frac{d^3 x}{d \phi_j^3} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-7} - 6 \frac{d^3 x}{d \phi_j^3} \frac{dx}{d \phi_j} \left( \frac{d^2 x}{d \phi_j^2} \right)^2 \right]
+ \frac{d^4 x}{d \phi_j^4} \left( \frac{dx}{d \phi_j} \right)^{-7} - \frac{1}{V_j} \left[ \frac{d^5 x}{d \phi_j^5} \frac{dx}{d \phi_j} - 5 \frac{d^4 x}{d \phi_j^4} \left( \frac{dx}{d \phi_j} \right)^{-7} \right]
- \frac{15}{V_j} \left[ \left( \frac{d^2 x}{d \phi_j^2} \right)^3 \left( \frac{dx}{d \phi_j} \right)^{-9} + 3 \frac{d^2 x}{d \phi_j^2} \frac{dx}{d \phi_j} \left( \frac{d^3 x}{d \phi_j^3} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-8} \right]
+ \frac{6}{V_j} \left[ \frac{d^2 x}{d \phi_j^2} \left( \frac{d^3 x}{d \phi_j^3} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-8} + \frac{d^2 x}{d \phi_j^2} \left( \frac{d^4 x}{d \phi_j^4} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-7} + \left( \frac{d^3 x}{d \phi_j^3} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-7} \right] \tag{22}
\]

Equation (22) is simplified by noting that both \( \frac{d^4 x}{d \phi_j^4} \) and \( \frac{d^2 x}{d \phi_j^2} \) equal zero as \( x \) approaches zero.

\[
\frac{d^6 t}{d x^6} \bigg|_{x=0} = \lim_{\phi_k \to 0} \left[ \frac{4}{V_j} \left( \frac{d^3 x}{d \phi_j^3} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-7} - \frac{1}{V_j} \frac{d^5 x}{d \phi_j^5} \left( \frac{dx}{d \phi_j} \right)^{-6} + \frac{6}{V_j} \left( \frac{d^3 x}{d \phi_j^3} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-7} \right]
= \lim_{\phi_k \to 0} \left[ \frac{10}{V_j} \left( \frac{d^3 x}{d \phi_j^3} \right)^2 \left( \frac{dx}{d \phi_j} \right)^{-7} - \frac{1}{V_j} \frac{d^5 x}{d \phi_j^5} \left( \frac{dx}{d \phi_j} \right)^{-6} \right] \tag{23}
\]
Before substitution of terms, \( \frac{d^5 x}{d\phi_j^5} \) must be evaluated. Differentiate equation (20).

\[
\frac{d^5 x}{d\phi_j^5} = 3 \sum_{j=1}^{\infty} \left[ \frac{\ln h_k v_k^5}{v_j^5} \left( 1 - \frac{\phi_k}{\phi_j} \right)^{\frac{-9}{2}} (15 + 60 \phi_k^2) + 9 \phi_k (15 \phi_k + 20 \phi_k^3) (1 - \frac{\phi_k}{\phi_j})^{\frac{-11}{2}} \right] 
\]

\[
\frac{d^5 x}{d\phi_j^5} = 3 \sum_{j=1}^{\infty} \left[ \frac{\ln h_k v_k^5}{v_j^5} (1 - \frac{\phi_k}{\phi_j})^{\frac{-11}{2}} (120 \phi_k^4 + 180 \phi_k^2 + 15) \right] 
\]

\[
\lim_{\phi_k \to 0} \frac{d^5 x}{d\phi_j^5} = \frac{45}{v_j^5} \sum_{k=1}^{\infty} \ln h_k v_k^5 
\]

Equation (23) is then evaluated as

\[
\frac{d^6 t}{dx^6} |_{x=0} = \lim_{\phi_k \to 0} \left[ \frac{10}{v_j^6} \sum_{k=1}^{\infty} \ln h_k v_k^3 (1 + 4 \phi_k^2) (1 - \frac{\phi_k}{\phi_j})^{\frac{-7}{2}} \right]^2 
\]

\[
\left[ \frac{1}{\sum_{k=1}^{\infty} \ln h_k (1 - \frac{\phi_k}{\phi_j})^{\frac{-3}{2}} v_k^5} \right] \left[ \frac{45}{v_j^5} \sum_{k=1}^{\infty} \ln h_k v_k^5 \right] = 90 \left( \sum_{k=1}^{\infty} \ln h_k v_k^5 \right)^2 - 45 \left( \sum_{k=1}^{\infty} \ln h_k v_k^5 \right)^6 
\]

A six-term truncation of the Taylor's series may now be written as

\[
t = t(0) + \frac{x^2}{2} \sum_{k=1}^{\infty} \frac{\ln h_k v_k^3}{v_k} - \frac{x^4}{8} \left( \sum_{k=1}^{\infty} \frac{\ln h_k v_k^5}{v_k} \right)^4 + \frac{x^6}{16} \left( \sum_{k=1}^{\infty} \frac{\ln h_k v_k^7}{v_k} \right)^6 
\]
One may now introduce the source-receiver distances $X \triangleq 2x$ and the total ray travel time is $T_x \triangleq 2t$ to obtain

$$T_x = T_0 + \frac{X^2}{2} - \frac{1}{512} \left[ \frac{1}{k=n} \left( \sum_{k=1}^{k=n} h_k^3 \right)^2 \right] X^4$$

\[+ \frac{1}{64} \left( \sum_{k=1}^{k=n} h_k^3 \right)^2 - \frac{1}{k=n} \left( \sum_{k=1}^{k=n} h_k^5 \right)^2 \]

$$+ \frac{1}{2} \left( \sum_{k=1}^{k=n} h_k^3 \right)^2 - \frac{1}{k=n} \left( \sum_{k=1}^{k=n} h_k^5 \right)^2$$

$$+ \frac{1}{k=n} \left( \sum_{k=1}^{k=n} h_k^3 \right)^2 - \frac{1}{k=n} \left( \sum_{k=1}^{k=n} h_k^5 \right)^2$$

$$\times 6^6$$ (27)

The series is squared and truncated to two terms in most exploration work.

$$T_x^2 = T_0^2 + \frac{T_0 X^2}{2} \left( \sum_{k=1}^{k=n} h_k^3 \right)^2$$

(28)

The coefficient of the $X^2$ term is further modified.

$$\frac{T_0}{2} = \frac{1}{\sum_{k=1}^{k=n} h_k^3 V_k^2} \left( \sum_{k=1}^{k=n} h_k^3 V_k^2 \right)^2$$

(29)

The thickness $h_k$ of any one layer equals $V_k t_k$ where $t_k$ is the one-way normal incidence time for the layer. The term root-mean-square velocity is used in conjunction with the expression in equation (29).

Root-mean-square velocity or $v_{RMS}$ is defined by:
Equation (28) is thus

\[ T_x^2 = T_o^2 + \frac{x^2}{v_{RMS}^2} \]  

(31)

Normal moveout \( \Delta t \) is defined as the difference in the actual ray time \( T_x \) and the normal incidence time, \( T_o \).

\[ \Delta t = T_x - T_o = \left( T_o^2 + \frac{x^2}{v_{RMS}^2} \right)^{1/2} - T_o \]  

(32)

This equation is mathematically identical to the moveout equation for a one-layer, constant velocity model. It is of extreme importance in this study to note that the actual velocity that is measured by means of reflections in exploration seismic work is more nearly \( v_{RMS} \) and \( v_{RMS} \) should not be confused with the average velocity of the earth model (Schmitt, 1966).

Velocities determined from a normal moveout analysis must be corrected for dip. The assumption for dip correction is that the ray impinges on a dipping reflector. The effect of lateral gradient in \( v_{RMS} \) is neglected here for simplicity. In addition the ray is assumed to travel in a medium with a constant apparent velocity \( v_{NMO,\alpha} \) approximately equal to the root mean square velocity to a given reflector. An event for such a model will, in its unmigrated position, be located midway...
between the source and receiver. The actual reflection time $T_x$ for such a trace in terms of the dip $\alpha$, shot to receiver distance $X$, and vertical distance from receiver to reflector $h$ is determined from the following equation (see fig. 2).

$$v_{RMS}^2 T_x^2 = (2h - X \sin \alpha)^2 + X^2 \cos^2 \alpha$$

(33)

The normal incidence time $T_0$ at the $X/2$ location is

$$v_{RMS}^2 T_0^2 = [2h - 2(\frac{X}{2}) \sin \alpha]^2$$

(34)

Subtracting equation (34) from (33) yields

$$v_{RMS}^2 T_x^2 - v_{RMS}^2 T_0^2 = X^2 \cos^2 \alpha$$

or

$$T_x^2 = T_0^2 + \frac{X^2 \cos^2 \alpha}{v_{RMS}^2} = T_0^2 + \frac{X^2}{2} \frac{v_{RMS}^2}{\cos^2 \alpha}$$

(35)

In normal moveout analysis an apparent velocity $v_{NMO,\alpha}^2$ is actually determined from reflection times and receiver distances.

$$T_x^2 = T_0^2 + \frac{X^2}{v_{NMO,\alpha}^2}$$

(36)

This apparent velocity has to be multiplied by the cosine of the dip angle to yield the root-mean-square velocity as is shown in the comparison of equations (35) and 36).

$$v_{NMO,\alpha}^2 \cos \alpha = v_{RMS}^2$$

(37)
FIGURE 2
MODEL OF DIPPING REFLECTOR
DETERMINATION OF AVERAGE VELOCITY FROM $v_{RMS}$

Equation (27) is modified by using the following time averages of $v_k^j$.

$$
\overline{v_k^j} \triangleq \frac{2}{T_o} \sum_{k=1}^{k=n} v_k^{j-1} h_k \quad \text{and} \quad \frac{T_o}{2} \overline{v_{RMS}^2} = \sum_{k=1}^{k=n} h_k v_k^2
$$

(38)

The modified equation is

$$
T_x = T_o + \frac{x^2}{2T_o v_{RMS}^2} - \frac{\overline{u^4_{RMS}}}{8 v_{RMS}^3 T_o} + \left[ \frac{\overline{u^2_{RMS}}}{8 v_{RMS}^3} - \frac{6}{v_{RMS}} \right] \frac{x^6}{16 v_{RMS}^5 T_o^5}
$$

(39)

Brown outlined a method of approximating $\overline{v_t} \triangleq \overline{v_t^1}$ or average velocity from $v_{RMS}$ using a correction to the fourth order $x$ term of equation (39) assuming that velocity is a function of depth only. If one assumes that all $n$ layers have the common velocity $V$, then equations (38) become

$$
\overline{v_t^j} \triangleq \frac{2}{T_o} v_t^{j-1} h \quad \text{and} \quad \frac{T_o}{2} \overline{v_{RMS}^2} = V h
$$

(40)

If $V \triangleq v_{RMS}$ then, since $T_o V/2 = h$, equation (40) becomes $\overline{v_t^n} \triangleq v_t^n$. Thus equation (39) for the one uniform layer with velocity $V \triangleq v_{RMS}$, and equation (39) for the $n$ layers have the same first term. If $\Delta \Delta T$ represents the excess of the $n$-layer $\Delta T$ over the one layer $\Delta T$, then
\[
\Delta \Delta T = - \frac{\chi^4}{8\nu_{\text{RMS}}^3 T^2} \left[ \frac{\bar{v}^4_t}{\nu_{\text{RMS}}} - 1 \right] \tag{41}
\]

Equation (41) is used in terms of the following definition.

\[
\mu_k \triangleq v_k - \bar{v}_t \tag{42}
\]

Note that \( \mu_k \) is the deviation of the instantaneous velocity \( v_k \) from the average velocity. Squaring equation (42) yields

\[
v_k^2 = \bar{v}_t^2 + 2\mu_k \bar{v}_t + \mu_k^2 \tag{43}
\]

which when time weighted and time averaged yields

\[
\frac{2}{T} \Sigma_{k=1}^{k=n} v_k^2 = \frac{2}{T} \Sigma_{k=1}^{k=n} \bar{v}_t^2 \bar{v}_t + \frac{2}{T} \Sigma_{k=1}^{k=n} \mu_k^2 \tag{44}
\]

If \( \sigma^2 \triangleq 2/T \Sigma_{k=1}^{k=n} \mu_k^2 \) then equation (44) becomes

\[
v_{\text{RMS}}^2 = \bar{v}_t^2 + \sigma^2 \tag{45}
\]

Equation (43) when squared, time weighted, summed for an \( n \) layer model and then time averaged yields

\[
\bar{v}_t^4 = \bar{v}_t^4 + 4\bar{v}_t^3 \mu_t + 6\bar{v}_t^2 \mu_t^2 + 4\bar{v}_t \mu_t^3 + \mu_t^4 \tag{46}
\]

where

\[
\mu_t \triangleq \frac{2}{T} \Sigma_{k=1}^{k=n} \mu_k^t \tag{47}
\]

By definition \( \mu_t = 0 \). If \( \mu_t^3 \) is assumed to be small because of cancellations within the sum, then
\[
\overline{\frac{v_t}{v_{RMS}}} - 1 = \frac{1}{4} \left[ \overline{\frac{v_t}{v_{RMS}}} - v_{RMS}^4 \right] = \frac{1}{4} \left[ \frac{v_t + 6v_t^2 + \mu_t + \sigma^2}{v_{RMS}} - (v_t + \sigma^2)^2 \right]
\]

(48)

\[
\frac{1}{4} \left[ 4v_t^2 - \mu_t + \sigma - 4 \right] = \frac{4\sigma^2}{v_{RMS}} \quad (49)
\]

It is assumed that the first term in the square brackets of equation (49) is much larger than the other two. The validity of the assumption is based upon several two-layer model studies which I made in which the first term was two orders of magnitude greater than the other two terms. The assumption validity can be calculated for any sonic log. Equation (49) when substituted into equation (41) yields

\[
\Delta \Delta T = -\frac{\sigma^2}{2v_{RMS}^2} \quad (50)
\]

If \( \Delta T \) is approximated by \( \frac{X^2}{2v_{RMS}^2} T_o \) then

\[
\frac{\Delta \Delta T}{\Delta T} \approx -2 \left( \frac{\sigma}{v_{RMS}} \right)^2 \quad (51)
\]

A computation of moveout for a model is approximated using the straight ray method

\[
\Delta T_s = \left( \frac{X^2}{2v_{RMS}^2} + T_o^2 \right)^{1/2} - T_o \quad (52)
\]
To more accurately determine $\Delta T$ for an actual section the straight ray computation may be corrected by the term $\Delta \Delta T$

$$\Delta T = \Delta T_s + \Delta \Delta T = \Delta T_s - 2 \frac{\Delta T_s}{T_o} \left( \frac{\sigma}{v_{\text{RMS}}} \right)^2$$  \hspace{1cm} (53)

Actual $\Delta T$ and $T_o$ values can be used to determine an apparent velocity $v_a$ defined as

$$\Delta T \triangleq \left( \frac{x^2}{v_a^2} + T_o^2 \right)^{1/2} - T_o$$ \hspace{1cm} (54)

or

$$v_a = \frac{\Delta T}{\Delta T (2T_o + \Delta T)}$$ \hspace{1cm} (54.1)

This apparent velocity is the velocity at which a straight ray would have to travel from the source in order to have the same travel time as that of a kinked ray having the same reflection point. If equations (52) and (54) are substituted into (53) then

$$\left( \frac{x^2}{v_a^2} + T_o^2 \right)^{1/2} - T_o = \left[ \left( \frac{x^2}{v_{\text{RMS}}^2} + T_o^2 \right)^{1/2} - T_o \right] \left[ 1 - \frac{2}{T_o} \Delta T_s \left( \frac{\sigma}{v_{\text{RMS}}} \right)^2 \right]$$ \hspace{1cm} (55)

If the spread length is small relative to the total travel path, then the binomial expansion yields

$$\frac{x^2}{2v_a^2 T_o} = \left( \frac{x^2}{2v_{\text{RMS}}^2 T_o} \right) \left[ 1 - \frac{2}{T_o} \Delta T_s \left( \frac{\sigma}{v_{\text{RMS}}} \right)^2 \right]$$ \hspace{1cm} (55.1)
Brown makes the approximation \( \frac{\Delta T}{S} \approx \frac{\Delta T}{v_{\text{RMS}}} \) and I have shown this to be adequate for a series of two-layer models. Thus

\[
v_{\text{RMS}}^2 = v_a^2 \left( 1 - \frac{2}{T_o} \frac{\Delta T}{v_a} \sigma^2 \right)
\]  

(55.2)

Brown has found in model studies that the 2 in the last expression is better replaced by 1.4. The equation is rewritten as

\[
v_a^2 = v_{\text{RMS}}^2 + \frac{1.4}{T_o} \Delta T \sigma^2
\]  

(56)

A plot of apparent velocity versus \( \frac{1.4 \Delta T}{T_o} \) yields \( \sigma^2 \) as slope. This slope, \( \sigma^2 \), is used to compute average velocities from root mean square velocities from the relationship in equation (45)

\[
\bar{v}_t^2 = v_{\text{RMS}}^2 - \sigma^2
\]

Root mean square velocity is in general greater than average velocity \( \bar{v}_t \).

The average velocities are used to compute depths associated with record times. An added benefit of the plot of apparent velocity versus \( \frac{1.4 \Delta T}{T_o} \) is that the intercept on the \( v_a \) axis is the \( v_{\text{RMS}} \) value (Brown, 1969).

The average velocity to a reflector can also be determined by modeling layers using Dix's equation (Dix, 1955). For a two-layer model in which the angles of ray incidence are small, the horizontally traversed distance is expressable as (see fig. 3).

\[
x_1 + x_2 + V_1 t \tan \theta_1 + V_2 t_2 \tan \theta_2
\]  

(57)
FIGURE 3
TWO-LAYERED MODEL FOR DERIVATION OF DIX'S EQUATION
Since for small angles $\tan \theta = \sin \theta$, equation (57) is approximated as

$$x_1 + x_2 = V_1 t_1 \sin \theta_1 + V_2 t_2 \sin \theta_2 = V_1 t_1 \sin \theta_1 + \frac{V_2^2 t_2}{V_1} \sin \theta_1 \quad (58)$$

The kinked ray path for near-normal incidence in the two-layer model is approximated by a straight ray through the layers, the angle of incidence of which is close to $\theta_2$. The associated velocity of the straight ray is designated as $v_{a_2}$ with an associated travel time $t_x$. Using Snell's law,

$$\frac{\sin \theta_1}{v_1} = \frac{x_1 + x_2}{v_{a_2} t_x} \quad (59)$$

which when substituted into equation (58) yields

$$x_1 + x_2 = \left( \frac{V_1^2 t_1 + V_2^2 t_2}{v_{a_2}^2 t_x} \right) \left( \frac{x_1 + x_2}{v_{a_2} t_x} \right) \quad (60)$$

Dividing both sides of equation (60) by $(x_1 + x_2)$ and taking the limit as $x_1 + x_2 \to 0$ yields

$$v_{a_2}^2 \approx \frac{V_1^2 t_1 + V_2^2 t_2}{t_1 + t_2} \quad (61)$$

For an $n$-layer model with near normal incidence, the above line of reasoning is extended.
Subtracting the two expressions in equations (62) yields \( v_n \) the interval velocity between the \( n \)th and \((n-1)\)th layer. Equation (62) demonstrates that \( \frac{v_{\text{rms}}}{n} = v_{\text{rms}} \). The interval velocity \( v_n \) is calculated from

\[
\begin{align*}
v_n^2 &= \frac{\sum_{k=1}^{n-1} v_{k}^2 t_k}{\sum_{k=1}^{n-1} t_k} = \frac{\sum_{k=1}^{n} v_{k}^2 t_k}{\sum_{k=1}^{n} t_k} \\
&\quad \text{(62)}
\end{align*}
\]

From a tabulation of \( v_{\text{rms}} \) and \( T_o \) values, a model consisting of a sequence of layers, whose interval velocities are calculated from equation (63) and whose thicknesses are calculated as \( \frac{v_n(T_{o_n} - T_{o_{n-1}})}{2} \), can be determined. The average velocity to any one of the reflecting horizons is determined by adding the thicknesses of the above layers and dividing this sum by the sum of their associated one-way times. The interval velocity from the reference plane to the first recorded reflection event is approximated by the \( v_{\text{rms}} \) to that event.
COHERENCE

Static-corrected multifold data that has been gathered in a common depth point (CDP) format will exhibit normal moveout for events; the greater the trace distance or source receiver distance for a particular trace within a CDP gather, the greater is the associated normal moveout. The event times for any one common depth point gather when arranged according to trace distance will appear hyperbolic.

The CDP traces must be corrected for moveout so as to make event times equal on all traces. The correction for a particular event with an associated $T_o$ or normal incident time is trace distance and velocity dependent; thus measurement of normal moveout and trace distance allows for computation of velocity. This velocity is referred to as stacking velocity, $v_s$, which in the limit as $x$ approaches zero equals the root mean square velocity, $v_{RMS}$, as is seen from equation (31)

$$\lim_{x \to 0} v_s = v_{RMS}$$

Several computation methods are used to estimate the best stacking velocity for multifold data; these methods are based on various ways of measuring coherence.

Coherence is a measure of signal similarity between data channels. The coherence measurements to deduce stacking velocities in general are performed on signals with a common depth point. Coherence is measured on data within hyperbolic time gates. The hyperbolic time gates are functions of $v_{RMS}$, $t_o$ and gate length. A suite of $(T_o)s$ and $(v_{RMS})s$ are used to calculate the beginning times of the hyperbolic gates.
The most commonly used coherence measurement is truncated cross-correlation. This cross-correlation for two functions \( f_1(t) \) and \( f_2(t) \) is defined mathematically as

\[
\phi_{12}(\tau) = \frac{1}{T} \int_{-T/2}^{+T/2} f_1(t)f_2(t+\tau)dt
\]  

(64)

\( \tau \): time shift value for which the cross-correlation is computed.

Seismic velocity analyses involve the cross-correlation of digitized seismic signals. The cross-correlation between such signals involves processes of multiplication and summation; the above expression for digitized data is a summation. For a pair of digitized traces, the cross-correlation of signal within a hyperbolic time gate of width \((N-1)\) where \(N\) is the number of data samples within the time gate is approximated by

\[
\phi_{ip}(n,m) = \frac{1}{N-1} \sum_{j=-N/2}^{j=m+N/2} f_{i,j} f_{p,j+n}
\]  

(65)

\( f_i \): the \( i \)th channel in a data group

\( f_p \): the \( p \)th channel in a data group

\( j \): the \( j \)th sample within a time gate

\( n \): the shift value of the signal with respect to the central hyperbola

Cross-correlation is usually normalized; the two most common methods of normalization are geometric mean energy normalization and arithmetic means normalization. The geometric mean energy normalization normalizes the cross-correlation to values in the range \( \pm 1 \). Maximum correlation
occurs when the signals are identical in which case \( \phi_{12} \) normalized equals one. The energy normalized crosscorrelation is thus defined.

\[
\phi_{ip}^{\text{normalized}} = \frac{\sum_{j=m-N/2}^{j=m+N/2} f_{i,j} f_{p,j+n}}{\left( \sum_{j=m-N/2}^{j=m+N/2} f_{i,j}^2 \sum_{j=m-N/2}^{j=m+N/2} f_{p,j+n}^2 \right)^{1/2}}
\]  

(66)

For a set of \( M \) data channels, a reliable coherence measurement is the sum of all 0-lag crosscorrelations between all possible combinations of data channels. The sum is

\[
\phi_M = \sum_{p=1}^{p=M} \sum_{i=1}^{i=M} e^{ip} \phi_{ip}(o,m)
\]

\( e^{ip} = 1 \) for \( p > i \)

\( e^{ip} = 0 \) for \( i \geq p \)

(67)

The computation is simplified by considering the energy of the summation of \( M \) data channels, the energy of a finite discrete data sequence \( f_{i,j} \) being defined as

\[
\sum_{j=m-N/2}^{j=m+N/2} (f_{i,j})^2
\]

(68)

The summation energy for \( M \) channels is
The sum of all possible 0-lag crosscorrelations is

\[
\phi_0 = \sum_{p=1}^{M} \sum_{i=1}^{\frac{M}{2}} e^{i\theta_{ip}}(0,m) = \frac{1}{2(N-1)} \sum_{j=\frac{m-N}{2}}^{\frac{m+N}{2}} \left[ \sum_{i=1}^{M} f_{i,j} \right]^2 - \sum_{i=1}^{M} (f_{i,j})^2
\]

(70)

The sum of all 0-lag crosscorrelations is one-half the difference between the channel summation energy and the sum of the individual channel energies. The crosscorrelation sum is normalized as previously cited with normalization for M channels being performed in a manner such that if the M channels have identical signals then the crosscorrelation sum equals one. There are M combinations of correlation pairs taken two at a time; the number of combinations of M channels taken two at a time is \( M! = M(M-1) \). This sum must be multiplied by the inverse of this term. The denominator or normalization factor for each term of the sum is
The denominator is thus expressed as the mean energy of the signal system. The energy normalized crosscorrelation sum for \( M \) data channels is finally expressible as

\[
\phi_{M, \text{NORM}} = \frac{1}{(N-1)M(M-1)^2} \left( \sum_{j=m-N/2}^{j=m+N/2} \left( \sum_{i=1}^{i=M} f_{i,j} \right)^2 - \sum_{i=1}^{i=M} \sum_{i=1}^{j=m-N/2} f_{i,j}^2 \right) - \sum_{j=m-N/2}^{j=m+N/2} \left( \sum_{i=1}^{i=M} f_{i,j} \right)^2 - \sum_{j=m-N/2}^{j=m+N/2} \left( \sum_{i=1}^{i=M} f_{i,j}^2 \right)
\]

The maximum value is 1 when \( M \) channels consist of identical signals whereas the minimum possible value is \( -\frac{1}{M-1} \) for an \( M \) channel signal system such that

\[
\phi_{M, \text{NORM}} = \frac{1}{(N-1)M(M-1)^2} \left( \sum_{j=m-N/2}^{j=m+N/2} \left( \sum_{i=1}^{i=M} f_{i,j} \right)^2 - \sum_{i=1}^{i=M} \sum_{i=1}^{j=m-N/2} f_{i,j}^2 \right)
\]

one such signal system, for example, is that in which half of the signals are the negative of the other half of the signals. An outgrowth of this coherence measurement is semblance \( S_C \) defined by Taner, Cook and Neidell (1971).
The relationship between the energy normalized crosscorrelation and semblance is thus

\[ S_c = \frac{\sum_{j=m-N/2}^{j=m+N/2} \sum_{i=M}^{i=1} f_{i,j}^2}{\left( \sum_{j=m-N/2}^{j=m+N/2} \sum_{i=M}^{i=1} f_{i,j} \right)^2} \]  

\[ (73) \]

A somewhat less sophisticated coherence measurement is summation. This measurement is the maximum amplitude of the sum of M data channels; this maximum amplitude will be highest for similar signals added in phase. The more channels with similar signals that are added together, the more effective the coherence method.

The speed of the coherence computations may be improved by decreasing word length. The data can be reformatted with amplitude values being assigned +1, -1, or 0 values. Coherence can then be performed on these data.
DATA ANALYSIS

A twenty-four trace, six-fold record spanning four common depth points was analyzed for $v_{\text{RMS}}$ versus $T_o$ using various coherence techniques. The record, chosen from a relatively flat dip area, was corrected for statics. The CDP gather traces were corrected for moveout using constant stacking velocities; stacking velocities were incremented in the range 5800-8500 ft/sec in 100 ft/sec increments. These corrected traces were scanned over 42 ms or 21-sample windows with 40 ms overlap of windows; the following coherencies were then plotted at the window midpoint times (figures 4):

1. Summation of all channels (fig. 4a)
2. Polarity coincidence (summation technique)(fig. 4b)
3. Unnormalized crosscorrelation between all possible channel pairs (using equation 70)(fig. 4c)
4. Semblance (fig. 4d)
5. Energy normalized crosscorrelation between all possible channel pairs (using semblance as input and equation 74)(fig. 4e)
6. Semblance-weighted summation of all channels (product of summation and semblance)(fig. 4f)

The plots were instrumental in establishing $T_o$ and $v_{\text{RMS}}$ ranges associated with high coherencies but ultimately the printout of coherence values was used in establishing $v_{\text{RMS}} - T_o$ pairs. A dashed curve was drawn through the average of these points for each coherency method (figure 5); on each graph a solid curve representing the $v_{\text{RMS}} - T_o$ function as determined from a nearby sonic log was drawn for comparison. It appears that each coherence method effectively determines a good
stacking velocity but that unnormalized crosscorrelation, energy normalized crosscorrelation and semblance are more effective than the other methods in the time range 1.7 - 2.9 sec.

The \( v_{\text{RMS}} \) versus \( T_{\text{o}} \) function as established by semblance was used to determine interval velocities between highly coherent events using Dix's equation (figure 6); this plot is compared to the interval velocities as determined from the sonic log. Note that the greater \( v_{\text{INT}} \) error in figure 6 is over time bands for which there is greater uncertainty in \( v_{\text{RMS}} \) in figure 4.

The \( v_{\text{INT}} \) versus \( T_{\text{o}} \) as established using Dix's equation was used to model the section. The average velocity function for this model was plotted as the seismic determined \( \overline{v}_t \) in figure 7. Brown's method for computing \( \overline{v}_t \) was determined for four events. Regression analysis was used to compute the coefficients of the first three terms of the time series in equation (27) for event times for each event. Values of \( T_x \) were then computed for \( X \) values from this series for these events. Apparent velocities were computed for these \( T_x \) versus \( X \) pairs using equation (54). A linear plot of \( v_a \) versus \( 1.4 \Delta T/T_o \) using a least squares fit established a slope \( \sigma^2 \). The \( \sigma^2 \) were subtracted from the \( v_{\text{RMS}}^2 \) values as determined from the intercept on the \( v_a \) axis to compute \( \overline{v}_t^2 \). The \( \overline{v}_t \) values were plotted in figure 7. The \( \overline{v}_t \) function as determined from the sonic was plotted for comparison.

The seven-layer model of the section as determined using sonic data (figure 6) was used in generation of \( X^2 \) versus \( T^2 \) values for the reflections from each interface. The \( X^2 \) versus \( T^2 \) plots demonstrated linearity for \( X \) values in the range 0 - 9000 ft and thus grossly established the validity of the time-distance hyperbolic approximation for the data.
It is concluded from the velocity analysis that the coherences semblance, unnormalized crosscorrelation and energy normalized cross-correlation are better than summation, polarity coincidence, and semblance-weighted summation in establishing the $v_{RMS}$ versus $T_o$ function. Semblance, unnormalized crosscorrelation and energy normalized crosscorrelation give better resolution and the $v_{RMS}$ versus $T_o$ values established from these methods more closely fit the actual $v_{RMS}$ versus $T_o$ function.
Figure 4a

Graph of summation coherence versus $T_0$ as a function of the parameter $v_{RMS}$
FIGURE 4a
Figure 4b

Graph of polarity coincidence (summation) coherence versus $T_o$ as a function of the parameter $v_{RMS}$
Figure 4c

Graph of unnormalized crosscorrelation coherence versus $T_o$ as a function of the parameter $v_{RMS}$.
Figure 4d

Graph of semblance coherence versus $T_o$ as a function of the parameter $v_{RMS}$
Figure 4e

Graph of energy normalized crosscorrelation coherence versus $T_o$ as a function of the parameter $v_{RMS}$
Figure 4f

Graph of semblance-weighted summation coherence versus $T_0$ as a function of the parameter $v_{RMS}$.
FIGURE 4g

RECORD ANALYZED FOR VELOCITY
(record is corrected for moveout and statics)
\[ \sqrt{\text{RMS}} (\text{ft/ms}) \]

**FIGURE 5a**

\[ \sqrt{\text{RMS}} - T_0 \text{ function as determined using summation coherence} \]

- **SONIC DETERMINED**
- **SEISMIC DETERMINED**
$\sqrt{\text{RMS}}$ (ft/ms)

- **SONIC DETERMINED**
- **SEISMIC DETERMINED**

**FIGURE 5b**

$\sqrt{\text{RMS}} - T_0$ FUNCTION AS DETERMINED USING POLARITY COINCIDENCE
\( \sqrt{\text{RMS}} \) vs. \( T_0 \) function as determined using unnormalized cross correlation coherence

- **SONIC DETERMINED**
- **SEISMIC DETERMINED**
\[ \sqrt{\text{RMS}} \text{ (ft/ms)} \]

**FIGURE 5d**

\[ \sqrt{\text{RMS}} - T_0 \text{ FUNCTION AS DETERMINED USING SEMBLANCE AND ENERGY NORMALIZED CROSS CORRELATION} \]
$\sqrt{V_{RMS}} (ft/ms)$

- **SONIC DETERMINED**
- **SEISMIC DETERMINED**

**Figure 5b**

$\sqrt{V_{RMS}} - T_0$ function as determined using semblance-weighted summation.
$\sqrt{\text{INT}} (\text{ft/ms})$

FIGURE 6

$\sqrt{\text{INT}} \text{ vs } T_0 \text{ as determined by Dix's equation}$
$V_{avg}$ (ft/ms)

- SONIC DETERMINED $V_{avg}$
- SEISMIC DETERMINED $V_{avg}$ (USING DIX'S EQUATION & MODELING)
- BROWN'S METHOD

FIGURE 7

$V_{avg}$ vs. $T_0$
BIBLIOGRAPHY


