OPTIMAL ACTIVE AND REACTIVE POWER DISPATCH IN A DISTRIBUTION NETWORK WITH HIGH PV PENETRATION

by

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Electrical Engineering).

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ABSTRACT

Integration of rooftop photovoltaic (PV) panels at the distribution network is on the rise. This trend is in part due to the global concerns for climate change as well as the need to find an alternative source of energy. However, if not properly coordinated, deploying renewable energy resources such as solar energy in the power grid can introduce challenges of its own. For instance, high penetration levels of PV may lead to overvoltage conditions, which can in turn cause additional stress on electrical components. In order to cope with such challenges, the operation of the distribution system has to be optimized while considering PV resources. Since irradiance at the ground level and daily load variations are stochastic in nature, a probabilistic approach is needed in order to provide a complete picture. This is what has been proposed in the current document. In this proposal, PV panels are assumed to contribute to reactive power support of the power grid, in addition to active power injection. Also, a centralized control of the distribution system is adopted. The controllable elements considered are voltage regulating transformers (voltage regulators), switching capacitors, load curtailment (or demand response), and the active and reactive powers provided by PV panels. The objective is to control the above devices in order to achieve operational goals such as improving system losses and the voltage profile, to name a few. A metaheuristic approach is used first to solve the problem. However, in order to improve the accuracy and reduce the convergence time, an analytical alternative is then proposed to optimize the performance of distribution system while considering photovoltaic integration.
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CHAPTER 1

THESIS SUMMARY

Due to the environmental concerns and the need for energy independence, governments and policymakers are encouraging power utilities and costumers to rely more on renewable energy resources. At the distribution level in the United States and many other countries around the world, photovoltaic (PV) panels are the fastest growing renewable energy source. Despite the fact that PVs bring economic and environmental benefits to the electric grid, numerous challenges emerge once their deployment reaches a high penetration level. Historically, researchers have concluded that when PV deployment exceeds a critical level at the distribution network, the system will no longer be able to withstand random variations of the power injected by these devices into the grid.

As a result, many researchers have proposed mitigation techniques to combat the challenges of high PV penetration. All these mitigating techniques are designed to utilize the existing resources in order to accommodate more PVs in distribution system. This would involve maximization of PV penetration level while the overall financial and operational costs are minimized. This optimization problem is slightly different from the well-known Voltage Var Optimization (VVO) whose goal is to coordinate only the voltage controlling devices in the system. VVO aims to optimally utilize reactive power and manipulate voltage magnitude in order to correct the voltage at buses while losses are minimized. The reason of utilizing reactive power and excluding active power is historic. The distribution system was designed to deliver active power to consumers without interruption. Thus, curtailment of active power could not be considered as a mitigating technique but instead an interruption to the service. Nowadays, the
situation has changed since the active power injected by the DGs (or PVs) can be curtailed without interrupting the service to the customers. Therefore, utilization of active power has been added as a tool to the VVO problem, introducing a new avenue in optimization referred to as Voltage, Var and Watt Optimization (VVWO).

In this proposal, VVWO is first performed using Evolutionary Algorithms (EA). Since EA would consume a relatively long time to converge, the analysis has been limited to deterministic data in order to reduce the number of samples. The deterministic VVWO proposed here involves five objectives to optimize: line losses, load curtailment, PV active power curtailment, and tap changing operations of voltage regulators (VRs) and switching capacitors (SCs). The simulations have been conducted using a modified version of the IEEE 123 bus test distribution system and the results are promising. The main finding of VVWO using EA is that PV penetration in the distribution system can be limitless if all resources are controlled centrally and if the uncertainty of irradiance is discarded.

However, it is very hard to predict solar energy at the ground level. Although the amount of solar irradiance itself is deterministic, cloud coverage is what makes this resource stochastic at the ground level. Thus, the element of uncertainty cannot be discarded. To resolve the issue of uncertainty of irradiance, VVWO has been performed in a probabilistic fashion. In this probabilistic analysis, various scenarios have been implemented to perform VVWO under different penetration levels of PVs while the uncertain variables are treated as random variables. EA cannot be efficiently used for this purpose since it is very slow and it may take years of simulations to perform probabilistic VVWO analysis! Therefore, an analytical model of distribution system and the optimization problem has been developed to speed up the simulation time and also improve accuracy. To do this, it is necessary to make sure the model and equations
are convex so that global optimum solution can be guaranteed. This can significantly improve the simulation time.

This proposal is organized as follows:

- Chapter 1 presents a summary of the thesis.
- Chapter 2 summarizes the findings from the literature review related to the impacts of rooftop PVs on the distribution system.
- Chapter 3 presents a macro-view to VVWO.
- Chapter 4 presents the deterministic approach for modeling VVWO using EA.
- Chapter 5 presents the analytical model for the VVWO problem, where power system has been modeled in detail. Other related topics such as the convexity of the model and ensuring Pareto optimality are also discussed here.
- Chapter 6 contains the results of the probabilistic analysis of VVWO.
- Finally, concluding remarks, contributions of the work, and the findings are presented in Chapter 7.
PV IMPACT ON DISTRIBUTION SYSTEM

PVs impact the distribution system in a positive and negative ways. In this chapter, we are going to focus on the negative impact of PVs on voltage and related issues. It has been observed that PVs influence steady state and dynamic (transient) stabilities of power system [1]–[3]. The severity of PVs’ impact on distribution system is dependent on: penetration level, spatial distribution of PVs in the grid, and the control type of PVs. Below is just a sample of PV impact;

- voltage fluctuation
- reverse power flow
- line losses
- electrical equipment ratings
- poor power quality
- unbalancing
- malfunction of protection scheme
- increase in the number of operations of OLTC, VRs, etc.
- reliability and regulation issues

The transient impacts of PVs on distribution system may include the following;

- Islanding effect
- Transients change in power due to cloud cover
In order to keep this review focused, only PV impact on distribution system is included (no transmission). Also, the following items are excluded: impact of PVs on harmonics, reliability, protection, islanding, and security. All mitigation techniques are excluded except for the capabilities of the interfacing inverter. The publications that are included in this work are the ones that studying the impact of Rooftop PVs or the distributed small-sized PV – PV plants are excluded. Economic analysis and environmental issues are excluded too. Also, any publication studying the impact of PV but it involves batteries, or any mitigation means, is excluded.

There have been many review papers about the impact of renewable energy (RE) resources and/or distributed generators (DGs) on power system since 1987 [4]. It’s worth emphasizing that REs are not the same as DGs because the latter comprises the former [5] – figure (2.3) shows the taxonomy of DGs and how they relate to REs. However, the impact of RE on electric system involves numerous angles. The current review publications incorporate a wide range of perspectives related to RE impact on power system. In such a situation, authors may restrict themselves to generalizations because they are covering many perspectives in few pages. Thus, in order to offer a macroview to the state-of-art in this topic, a subjective comparison among review papers is provided in Table (2.1). Main observations from Table (2.1) are as follow:

- Majority of existing review papers do not discriminate between PV’s impact on distribution system and transmission system.
- From the rated power perspective, PV power could be small-scale as in Rooftop PV panels or could be high-scale as in PV plant (PV plant and PV farm are used interchangeably in this document). Majority of review papers do not discriminate between the impact of Rooftop PVs and the impact of PV plants.
• The space allocated to review PV impact on electric system is small and not sufficient because the current review papers adds more unrelated topics, such as mitigating techniques, standards and policies of PV, and the projected growth of PVs, see miscellaneous section in Table (2.1).

In [29], the authors gave sufficient space to review the impact of PVs on reliability of electric system. The same thing can be said for [30] but it reviews the impact of all RE technologies, not PVs only. The review in [31] is mainly dedicated for surveying the hosting capacity of the grid to DGs but they added some section to discuss their impact too. In [32], a review about how artificial intelligence (AI) is utilized and about mitigation technique to DG integration are provided but they dedicated few pages to speak about their impact. In [5], [16], [24], the authors have included optimization as an additional mitigation technique to DGs impact. The topics covered by the review paper in [1] are very wide while their references are less than 250 sources and the same argument can be said about the following reviews [5], [8], [18]–[21], [9], [10], [12]–[17]. Also, some of the review papers examine the potential impacts of PVs in a specific country as in [14], [34] or for islands such as in [6].

2.1 Quantitative Assessment In Literatures

The publications that address the impact of rooftop PV on distribution system have started in late 70’s. Then, their number has increased exponentially since the start of twenty first century, see figure (2.2). In this section, the metadata of these publications are going to be analyzed.

The definition of penetration in publications is defined in various ways. Thus, it is critical to know how the penetration is defined because 7% penetration of PV with respect to conventional
generation might exceeds 100% penetration that is defined with respect to the demanded load over a feeder. Table (2.2) summarizes the definition of penetrations among publications.

A general inference about the definition of penetration is that publications that are treating the main transformer of the feeder as an infinite bus tend to define penetration as the total rated value of PVs with respect to the total rated value of loads. Also, publications that study the impact of rooftop PVs on transmission system tend to define penetration as the total rated value of PVs to the total conventional generations. Also, it has been observed that the publications from Australia, UK, and Canada tend to define penetration as the number of houses that has PV with respect to the total number of houses on the feeder. On the contrary, publications from United states, South America, and Africa defines penetration as the total PV to the total load.

One of the factors that received too many attentions is the impact of PVs on voltage imbalance. The well-known definition to voltage imbalance which is the ratio of negative sequence to the positive sequence has been adopted by majority of publications. However, this definition does not take into account the imbalance in voltage magnitude. Thus, some researchers who are interested in voltage imbalance from magnitude perspectives, rather than from phase perspective, utilized other definitions. Table (2.3) summarizes voltage imbalance that has been adopted by different publications.

Researchers, who studied the impact of PV, developed different scenarios in their research. For instance, some of the publications made a survey by engineers who are working in utilities that have PVs installed at consumers side. Then, based on engineers’ responses, they formed a conclusion about PV impact. Nevertheless, majority of publications simulated either a synthesized
feeder or an actual feeder. Then, based on simulation result, a conclusion about PV impact is formed.

Obviously, the simulated feeder is different from paper to another. For instance, some paper assumed that there are loads and PVs installed at PCC. In some papers, other DGs has been included in their simulation beside PVs. Some literatures assumed the load involves electric vehicles. Also, it has been observed that PV participation in reactive power support tend to be ignored in older publications. On the contrary, recent publications tend to simulate PVs at different fixed levels of non-unity power factor.

Decentralized control of the interfacing-inverter allows PVs to actively participate in reactive power support. Publications that assumed PVs are equipped with a decentralized controller lean towards the realm of optimization which is beyond the scope of this work. Thus, few papers have been found that study the impact of rooftop PVs that are equipped with decentralized controllers but their work is not formulated as an optimization problem.

Neither distribution system is balanced nor insolation is deterministic. Yet, many publications assumed in their simulation that the system is balanced and insolation is deterministic. To cope with the stochasticity of irradiance, some papers performed probabilistic simulation to find PV impact. Above discussion are listed in detail in Table (2.4) where the publications are sorted in an ascending order according to year of publication.

Above discussion are summarized into the following few pages. Both tables and figures are listed in order. Some tables are lengthy but they explain many aspects that might take pages to explain. These tables are classified subjectively because researchers would disagree on classification matters.
Figure 2.1  Classification of DG technologies, redrawn from [16]

Figure 2.2  Evolution of publications that has been included in this paper
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<td>Capacitor Banks</td>
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<td>Control Scheme/ Network Management</td>
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<td>Dump Load and/or Demand Response</td>
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<td>SVC</td>
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<tr>
<td>Case study is Included &amp; Case study is Included</td>
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<tr>
<td>Liberalized Market / benefits of GDs</td>
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<td>Historical Insight/ future perspectives</td>
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<td>Software Computational Tools</td>
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<td>Cloud Pattern</td>
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</table>

Table 2.1: Topics Covered by Various Review Papers on RE/DG Impact
Table 2.1 (continued)

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
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Notes:
Impact factor – RE integration to electrical system could impact various electrical quantities or devices. The “impact factor” is about the electrical quantities or devices that have been considered in a review paper.

Scope of review – Any research about RE integration presumes certain environment. The “scope of review” lists the test environment that have been considered in a review paper.

Mitigating techniques – Many review papers considered reviewing the mitigating techniques to RE impact on electric system. The “Mitigating techniques” lists mitigating techniques that have been considered by a review paper.

Miscellaneous – Some review paper provide additional valuable reviews that has no direct relation with RE impact on electric system.

Installed capacity – some review papers present the current installed capacity of renewable energy worldwide, or nationally. Also, it might provide a forecast to future growth of RE.

Indices – Author uses different quantification to electrical quantities to study the impact of RE on the grid. Some review papers provide a comparison to these quantifications.

Control Scheme – one of the mitigation techniques to RE impact is to propose new control scheme or a coordination among electric devices. This could involve active network management too.

PV alone – RE can take various forms, such as winds or geothermal energies. However, some review papers consider the impact of PV technology alone on distribution system.

SVC – stands for Static Var. Compensator. It is considered one of the mitigating techniques to RE impacts on electric system.

System Monitoring – Some papers utilize communication infrastructure in order to prevent islanding such as SCADA (Supervisory Control and Data Acquisition), PLCC (Power Line Carrier Communication), etc. An interested reader to all techniques that prevent islanding is advised to refer to [25].

Power Ramp – Due to volatility in RE power, conventional generation may ramp its output fast enough to balance the difference between load demand and power of RE.

MPPT – It refers to the Maximum Power Point Tracking converter.

Concept of RE – some review papers dedicates few pages to explain how RE works and how they are connected to the grid.

Case study is included – some review papers include in their work a new solution to mitigate RE impact.

Liberalized Market – Some review papers added a section about the relation between RE and market liberalization.

Hosting Capacity – penetration of RE resources cannot exceed a certain level in electric grid. Otherwise, problem emerges due to high penetrations. Some review papers involve hosting capacity into their work.

PV power scale distinction? – PVs could be of a small-scale, such as Rooftop PV, or it could be of large-scale, such as PV power plant. Some of the review papers segregate the impact of PVs based on their scale size. However, some review papers lumped sum the effect of both scales of PVs.

UPFC – it stands for unified power flow controller. This device provides reactive power compensation to the grid but with fast response.

DVR – it stands for Dynamic Voltage restorer. It can regulate both voltage magnitude and phase angle.

Software Computational Tools – Some review papers included a short review on power system software and provided a comparison to their capabilities.

SSSC – it stands for Static Synchronous Series Compensator.
Table 2.1  (continued)

√= indicated that an item has been included.
blank= indicated that an item has not been included.

Table 2.2  Penetration Indices

<table>
<thead>
<tr>
<th>Description of penetration index</th>
<th>References use this index</th>
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</thead>
<tbody>
<tr>
<td>Penetration index is defined as the ratio of total PV power relative to either load power, peak load demand, feeder capacity, or main transformer capacity. This could involve either the instantaneous or rated powers.</td>
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<td>[3], [7], [43], [133], [142], [44], [143], [147], [45], [52], [35], [53], [62], [36], [63], [72], [37], [73], [82], [38], [83], [92], [59], [93], [102], [40], [103], [112], [41], [113], [122], [42], [123], [132]</td>
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<td>Penetration index is defined as the number of the houses that have PVs relative to the total number of the houses.</td>
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<td>[148], [149], [158], [163], [150], [157]</td>
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<tr>
<td>Penetration index is not defined. Alternatively, the total kW of PVs and loads are reported.</td>
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<td>[164], [165], [174], [264], [273], [175], [274], [278], [176], [183], [166], [184], [193], [167], [194], [203], [168], [204], [213], [169], [214], [223], [170], [224], [233], [171], [234], [243], [172], [244], [253], [173], [254], [263]</td>
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<td>Penetration index is defined as the ratio of total PV power relative to the sum of losses and load powers.</td>
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<td>[279]</td>
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<td>Penetration index is defined as the ratio of total RE power relative to the total conventional generation.</td>
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<tr>
<td>[280]–[283]</td>
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<tr>
<td>Penetration index is about how much area of the roof is covered by PV relative to the total area.</td>
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<td>[284]</td>
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<tr>
<td>Penetration index is defined as the ratio of the reverse power at the main transformer to the total power of the system.</td>
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<td>[285]</td>
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<tr>
<td>The penetration index is defined as the ratio of the instantaneous power by PV relative to the sum of both instantaneous PV and load powers, which is called Self Consumption Rate (SCR).</td>
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<td>[286]</td>
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Table 2.3  Voltage Imbalance Adopted by Publications.

<table>
<thead>
<tr>
<th>Description of voltage imbalance</th>
<th>References use this definition</th>
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</thead>
<tbody>
<tr>
<td>The percentage of negative sequence component of the voltage relative to the positive sequence component.</td>
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<tr>
<td>[71], [81], [82], [93], [226], [260], [269], [274], [277], [287]</td>
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<td>The percentage of the maximum deviation of a single-phase voltage magnitude relative to the average value of all phases.</td>
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<tr>
<td>[45], [84], [107], [118], [160], [209], [253], [255]</td>
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<tr>
<td>Uses both above definitions.</td>
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<tr>
<td>[153]</td>
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<tr>
<td>They just provided a plot of voltages, but voltage imbalance has not been quantified.</td>
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<tr>
<td>[86], [102], [110], [159], [196], [210], [223], [228], [237]</td>
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<tr>
<td>Voltage magnitude difference between two phases.</td>
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Table 2.4 Metadata of Publications.
Table 2.5 Impact Factors by Publications

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<th>Year</th>
<th>Factor Description</th>
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<td>2009</td>
<td>Connecting PVs to different locations in the grid that has different voltage levels</td>
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<tr>
<td>2013</td>
<td>Reactive power reverse</td>
</tr>
<tr>
<td>2015</td>
<td>Impact of PV on unbalance of voltages</td>
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<tr>
<td>2019</td>
<td>System behavior in summer vs winter (seasonal response)</td>
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Note: Factors marked with √ indicate a significant impact.
Table 2.5  (continued)

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### 2.2 Historical Glance

In 1839, Edmond Becquerel observed the photovoltaic effect which is the conversion of light to electricity. In 1954, Bell Laboratory manufactured a solar cell with efficiency of 6%. In 1987, the mass-production of PVs become economically viable. The first mention of grid-connected PV appeared in the IEEE Transactions in 1974 [303]. Also, the discussion regarding the possibilities of moving toward solar energy was a hot topic in 1976 [304].

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The technical aspects of PV energy and its sufficiency to supply residential load first appeared in 1975 [303]. The University of Delaware had a PV array to charge batteries, not connected to the grid. Then, the batteries supply power to a DC-load for residential purposes. The purpose of the study is to find the correlation between daily residential demand and solar energy [305]. Then, in 1978, another paper appeared about the economic aspects of connecting PVs to the grid [303], [306]. The impact of PV on distribution system first appeared in 1977 [56], [303]. It modeled PVs as a conventional generation and discussed some reliability issues. Then, the detailed issues of connecting PVs to the grid such as safety, protection, and quality issues in 1980 [303], [307]. The main concern during the 80’s is how much PV penetration can be inserted to the system such that the ramp rate of conventional generation can withstand the fluctuations of PV output power [35], [308]–[310].

Between 1977 and 1979 [139]–[141], researchers performed economic analysis to PVs integration with the grid. They found no barrier to resume installation of PVs and endorsed their integration from economic standpoint. However, they forecasted that there could be issues from utility side. Another study in 1979 [311] emphasized that PV integration could bring challenges and careful studies should be performed. In a study conducted in 1977 [56], PVs are modeled as a conventional generation and their reliability issues are discussed. Then, another study in 1980 analyzed the issues of connecting PVs to the grid such as safety, protection, and quality issues [307]. Another study in 1981 forecasted that PV integration could bring instability at transmission level and or it could exacerbate voltage unbalance [312]. Safety was (and still) an issue as in [313]. However, some studies had recommended PV integration to grid regardless of the anticipated problems as in [142], [143], [314]–[316]. Another interesting study that is conducted in 1990 [222] on an actual feeder that had no PV but it is forecasted that its PV capacity might reach to 3MW by
2018; in their simulation, they found no barrier to integrate PVs even if PV penetration reaches the forecasted limit in 2018.

It had been believed that (i) cloud pattern and (ii) penetration level of PVs and (iii) geographical dispersion of PVs over the grid determine how much PVs can be installed in power system. The two former factors determine whether a reverse power could occur in power system. Reverse power is problematic in radial system because it causes faulty operations of protective devices.

The cost of generation units to cope with fluctuations in PV generation is insignificant when PV penetration is low. This is because the system can cope with PV fluctuations in a similar manner to the fluctuations in load. However, under intense cloud movement, once PV penetration exceeds 13.27% the cost of coping with PV fluctuations outweighs the economic benefits of PVs [309].

To parametrize the fluctuations in PVs, the change in PV output power over a unit of time to the total installed capacity is included in [310] to investigate PV problems. For instance, if the installed capacity in the system is 1150MW while penetration level of PVs is 5%, the installed capacity of PVs would be 58MW. Then, 10% of PV fluctuations would mean 5.8MW change in PV output per minute. The study in [310] concluded that when fluctuations decreases, the permissible PV penetration increases from economical point of view. According to the result reported by [310]; when PV fluctuations decreases to 10%, the allowable penetration level increases up to 10%. It is important to realize this result was based on 80’s technology where PVs were not uncontrollable [308].
2.3 Design Issues

In conventional distribution system, the power is supposed to be flowing in one direction, toward the load. Voltage drop over lines is expected to increase when the distance between the load and the main transformer increases. However, voltage drop is limited to a certain limit, such that the voltage over buses shouldn’t exceed ±5% of the rated value. The connected loads are expected to follow a general pattern too. Consequently, all of the components in distribution system (conductors, transformers, coils, etc) has been selected and designed to satisfy a unidirectional power flow and certain ratings of voltages. In addition, voltage controlling devices has been designed and allocated in distribution system to maintain voltage within the permissible limits. These controlling devices are designed and allocated provided that the loads are following a certain behavior that is bounded by an upper and lower limits. Also, the controlling devices are selected to control the voltage of downstream side which should be less than the voltage of the upstream side due to voltage drop.

Once PVs are incorporated into distribution system, the expected system behavior has changed. The active power consumption of the loads are going to follow a different trend that is a combinations the load pattern and insolation. The insolation is subjected to many variables, such as wind and dust, which makes its forecasting is a cumbersome task. Thus, in most of the engineering studies it is convenient to treat insolation as random variable. The voltage drop over lines is dependent on the amount of power consumption at load end. On the other hand, PVs take care of the local load partially which might decrease the current magnitude passing through lines. This would lead to a lower voltage drop across lines, or a voltage rise at the point of common coupling (PCC). The situation gets more complex when the power injected by PVs exceeds the power consumed by the load which lead to reverse power flow in distribution system. This mostly
happens during the noon time when load is at its minimum and insolation at its maximum. The current voltage controlling devices are not designed to combat with a reverse power. For instance, VRs are changing their tap position based on the voltage on the downstream side. Once a reverse power flow happens, the voltage at the downstream side of VRs would be greater than the voltage at the upstream side. In such a case, VRs would try to change its own tap position to control the voltage at the upstream side (for some operating modes, VR controls the upstream side if a reverse in power flow is detected). However, this action would not lower the voltage at the downstream but increasing it, which is the opposite of what VR should do. A similar argument goes for the capacitor banks. They are designed to supply a local reactive power to increase (not to decrease) the voltage over nearby buses. This is because a voltage drop is expected if PVs are not present. However, PVs are actually causing an increase of the voltage over buses because the amount of voltage drop over lines has decreased (less current magnitude passes through the conductors). Therefore, there would be a need to decreases the voltage over buses by absorbing the reactive power which the complete opposite of what capacitor banks do. In such a case, voltage controlling devices are no longer perfectly fit for the current distribution system unless they have been recalibrated, or may be redesigned, to satisfy the new random power consumption [317].

2.4 rooftop PV impact

Publications have studied PV impact on distribution system from various angles as shown in table (2.5). Although table (2.5) provide a detailed classification to the studied-factors and the corresponding publications, most of the lessons that have been reported in publications are repeated. For instance, majority of publications came to the same conclusion about voltage-rise as a consequence of PV penetration. Regardless of the repeated conclusions, each paper adds a new perspective in terms of the impact factors or in terms of the simulation and testing environment.
Thus, the following subsection are summarizing the main lessons that have been inferred from these publications.

![Diagram of a typical distribution feeder](image)

Figure 2.3 A typical distribution feeder.

### 2.4.1 Voltage and Reverse Power Flow Issues

The following subsections explain impact of PVs on voltage and power flow.

#### 2.4.1.1 Conceptual Rationale

In conventional distribution system, voltage drop over lines makes the voltage over buses decrease monotonically from main transformer to the farthest bus at the end of the feeder. This monotonic behavior of voltage profile is taken into consideration provided that the voltage at the farthest bus do not fall under 0.95 p.u. and do not exceeds 1.05 p.u. at the main transformer. Thus, it is most common to have a voltage at main transformer that is above the nominal value in order to compensate for voltage drop over conductors. Figure (2.3) shows a typical distribution feeder where the loads are evenly allocated over the feeder. If PV is not operating, the voltage decreases when we move to the farthest bus, as shown in figure (2.4). Notice, the voltage at the main transformer is set at the maximum level in order to let the voltage of farthest bus stays at the minimum level.
When PVs are added to some buses, the injected power by PVs reduces the total load at PCC. Thus, the voltage drop across lines decreases since the current magnitude passing through lines has decreased. If the PV is installed at the end of the feeder, the voltage at PCC increases above its original value and this is applicable to all buses. Consequently, when the bus become closer to the main transformer, the chance of its voltage to exceed maximum limit increases (because their old values are already high). Generally, the voltage at the main transformer (which is already preset to be above the nominal value) will increase and could exceed the permissible limits. Figure (2.4) shows how the voltage over the feeder would behave if the PV is injecting active (no reactive) power that would reduce consumed load at PCC bus of the system in figure (2.3). Notice the voltage at PCC has increased which led to an overvoltage at the main transformer.

When PV penetration is high, a reverse power occurs across the lines. In such a case, the voltage profile over all buses increases. Depending on the configuration of the grid, some buses may experience an over voltage problem. A reverse active power creates an over voltage at PCC bus and main transformer as shown in figure (2.4).

In general, it can be concluded from above discussion that PVs may cause a higher voltage at PCC and neighboring buses [317].

To resolve the overvoltage problem, PVs has to participate into voltage regulation. The PVs can do two things to bring the voltage to the permissible range: to curtail its active power and to absorb reactive power from the grid.

Voltage regulation can be attained by two mainstream ways[317]: either by the design of the system itself (e.g., conductor selection, substation and distribution transformer tap settings and fixed capacitor banks) or by controlling devices such voltage regulators (VRs), on-load tap changer
(OLTC), etc. The former method assumes that load profiles are decreasing monotonically toward the end of the feeder. The latter method is based on the continual monitoring of voltage in order to acts accordingly. Once PVs are installed in the distribution system, they interfere with the aforementioned methods which would affect voltage level in the feeder.

![Diagram showing voltage profile](image.png)

Figure 2.4 Voltage profile of the distribution feeder shown in figure (2.3). The vertical axis shows the voltage over the buses. The horizontal axis shows bus name. The square dots show the voltage profile when PVs are injecting active power such that a reverse flow of current present. The plus “+” dots show voltage profile when PVs are injecting active power but not up to the level of having a reverse current over the lines. The cross dots show the normal voltage profile of distribution feeder when PVs are not operating.

Once PVs are installed, power flow in the feeder is no longer necessarily exhibiting monotonic pattern. Also, if penetration level is low enough such that there is no reverse power, power flow over the feeder tend to decrease which would decrease voltage drop. Consequently, voltage profile of the feeder tends to increase which will be exacerbated at low loading period of the day.
The interference of PV with control mechanism of VRs, switching capacitors (SCs), OLTCs etc. is detrimental to voltage control. As a remedy to such a problem, the recent IEEE 1547 standards allowed PVs to actively regulate the voltage at point-of-common-coupling (PCC) by absorbing (injecting) reactive power from (to) the grid in order to decrease (increase) voltage.

If PV power factor is fixed, voltage rise has a direct proportionality to penetration level: higher penetration causes higher voltage level. However, some publication (e.g. [170]) claimed that at very extreme penetration levels, voltage level becomes inversely proportional to penetration level. They supported their findings with analytical analysis and simulation.

2.4.1.2 Literatures

Distribution system has many devices that are responsible for maintain the voltage profile within the permissible limitations such as VRs and OLTC. These devices are mostly equipped with a decentralized control that has been calibrated based on the expected loading at different load buses. However, when PV is connected to a bus, it will change the expected loading at that bus [339]. Nowadays, the change of the expected loading cannot be controlled because that the active power injected by PVs is uncontrollable (not dispatchable) [166]. Once the PV changes the expected loading at a bus, two issues arise: decentralized control of the related devices could be impacted and voltage drop will decrease which might increase the voltage at PCC. Consequently, voltage profile is going to be impact severely. However, the severity of voltage fluctuations is dependent on weather conditions, geographical location of PV and system topology [1], [340].

In [149], [211], when the ratio of PVs total injected power to the total demanded load is high, a reverse power flow could occur at the main transformer. Therefore, the transmission grid
could be impacted because the normal behavior of the distribution grid has changed. This change will not impact voltage profile only but the rest of the system [1].

It has been mentioned above that the injected power by PVs increases bus voltages. This will degrade network stability because the stochastic analysis of distribution system in [96] shows that the chance of bus voltage to exceed its permissible limits increases if the penetration level of PVs increases.

Voltage profile can be improved by utilizing the reactive power resources efficiently. However, most of existing PVs are operating under unity power factor [1] in order to maximize the benefits of PVs. Operating PVs under unity power factor makes sense if penetration of PVs is at its minimum level [341]. Once penetration level has exceeded certain threshold [340], voltage problems would emerge. The solution to this problem is to inject (absorb) reactive power to increase (decrease) voltage over buses. Consequently, the power factor of the grid would decrease which implies an inefficient system [103].

Based on above discussion, there should be an optimum penetration level of PVs in distribution system. Therefore, numerous studies have been carried out to find the optimum percentage of PV penetration on a distribution feeder. For instance, a study from the UK [149] shows that when PV penetration is less than 33%, no voltage violation over buses is noticed. Once PV penetration level exceeds 33%, voltage profile may exceed the permissible limits. However, most of the studies that are similar to [149] performed their analysis on a specific network topology, PV rating, insolation, loads etc. which make their conclusions are case dependent. However, the consensus among these researches is that each feeder has a threshold of PV penetration that should be maintained, assuming no centralized control. Another study in Japan
[342] indicated that tolerable PV penetration level ranges between 5% and 20%. Notice that different conclusion has been reached by [342] and [149] about the maximum penetration level.

Therefore, some literature tried to come up with a general rules that would help engineers to make a quick judgement about maximum penetration level based on general description of the grid. For instance, the authors in [166] concluded that the rural feeder tends to experience voltage rise problems than the urban feeders to their long span. Their reasoning is that the long span of rural feeders increases their impedance.

In [218], [331], the authors performed three-phase analysis to investigate the impact of PV on voltage profile and losses of a distribution feeder. Their result agree with other literatures: PV reduces voltage drop and line losses but once PV exceeded a certain penetration level, over/under voltage over buses may occur.

When PV operate under unity power factor, the active power flow in lines is correlated with penetration level. Active power flow decreases when PV penetration increases until penetration reaches a threshold level before the power flow increases again but in a reverse direction. However, once PV starts participating in reactive power support, they tend to absorb reactive power from the grid in order to combat voltage rise. In such a situation, reactive power flow increases. However, penetration level is not the only factor that determine power flow in feeders but irradiance, temperature, conductor size, and load profile and others [300].

2.4.2 Voltage Sag/Dip Issue

Voltage sag could be thought of as a momentarily interruption. It is a reduction of RMS value of source voltage between 10% and 90% of nominal voltage for 10 ms up to 60s according to IEEE Std. 1159–1995 [1].
According to [343], the increase of DG penetration is associated with the frequency of voltage sag. The rationale of having voltage sag is that DGs causes reverse power flow in distribution system. This reverse of power results in malfunctioning of protection devices which may lead to a voltage sag.

If the cause of voltage sag is not related to the malfunctioning of protection devices but to an actual fault, a study [343] showed that DGs have a positive impact on voltage sag provided that the fault ceases no longer than 2 seconds.

### 2.4.3 Voltage Fluctuations and Flicker Issue

The irregular variations of voltage magnitude and frequency is called “voltage fluctuations” provided that its magnitude is within the permissible limits [1]. However, if the frequency of voltage fluctuation ranges from 0.05 to 42 Hz is called “voltage flicker” [1].

The stochastic nature of insolation makes PVs a potential source of voltage fluctuations in distribution system. In addition to stochasticity of PV power, the interfacing inverter of PVs contributes in generating unacceptable voltage flicker [344]. There is an association between DG penetration level and voltage fluctuation and flicker in distribution system is reported in [242]. Of course, an increase in penetration level would bring more voltage fluctuations and flicker.

In [1], [345], it has been found that the sudden change of cloud condition is an additional cause for flicker. However, the spatial distribution of PVs in distribution system is significantly correlated with cloud transient’s impact on flicker and voltage profile. For instance, the authors in [346] found that if PVs are concentrated in a specific geographical area in distribution system, the chance of observing flickers due to cloud transient increases.
Cloud transient’s increases usually during cloudy and foggy days. So, the power fluctuations of PVs due to clouds may cause an excessive operations of VRs and OLTCs. These excessive operations may be considered as another source of voltage flickers as in [1], [138], [280], [346].

2.4.4 Rapid Changes in Power

PV output power is dictated by the fluctuations in insolation, or the irradiance at ground level. Once PV penetration level exceeds a threshold value, power fluctuations become a problem because researches has shown the following: (i) fluctuations in power that is caused by PVs is going to surpass the normal load variations [54], [88], [167], [176] and (ii) voltage profile at PCC tends to follow insolation profile[76], [288], [300]. Thus, voltage fluctuations are manifestation of fluctuation in power. Conversely, no association between flicker and power fluctuation is found according to [291].

The research in [35], [78] showed that there is an inverse proportionality between the intensity of power fluctuation and spatial dispersion of PVs over the grid: less fluctuations tends to correlate with more dispersion of PVs. A similar correlation is observed in [330] between fluctuations and the geographical area of the grid.

The fluctuation in net power (absorbed load minus the power injected by PV) at PCC is what counts when it comes to voltage fluctuation. Thus, in [51], the simulation shows that load profile interrelates with power fluctuation even when PVs are included. Temperature adds additional layer of complications because it associates with power fluctuations according to [320].

The authors in [320] observe that power fluctuations becomes less intense when the sky transitions from being sunny to cloudy. On the other hand, when the sky transitions from being
cloudy to sunny, power fluctuation tends to be more intense. The authors have rationalized this observation by associating it with temperature as follow:

- When the sky transitions from being sunny to cloudy, not only insolation decreases but temperature too. A decrease in insolation decreases output power of PVs. However, PV power tends to increase if temperature decreases. Thus, a less intense fluctuation in PV power occurs.

In [60], [61], [300], authors claim that reconfiguration of the grid is the best approach to resolve voltage fluctuations relative to other approaches such as installing SVC or capacitor banks.

2.4.5 Voltage Imbalance Issue

Voltage unbalance simply implies unequal voltage magnitude among phases. PVs are mostly a single phase generators that are installed randomly in distribution systems. The randomness of PV installations could increase/decrease unbalance in power which will lead to an increase/decrease in voltage unbalance. In [188], the authors found that the PV penetration level is highly associated with voltage unbalance index in distribution system.

2.4.6 Rated Voltage Level of The Grid

Some of the publications[53], [57], [89], [180], [192], [289] investigated the impact of PVs if they are installed at PCCs that have different voltage levels. Authors in [53] found that when the rated voltage of the grid increases, its sensitivity to the power factor of PVs decreases. Thus, PV participation in voltage regulation in low voltage grid is more effective than in medium voltage grid. On the contrary, if PV does not participate in reactive power support, the hosting capacity to PV of medium voltage grid is higher than the low voltage grid. Also, in [192], they found that the cost of operation is significant by simply decreasing the rated voltage from 240V to 230 V.
The rated voltage of the grid has an impact on tap operations too. When PV does not participate in reactive power support, a general conclusion about rated voltage, and tap operations according to [62] are as follow:

- If the VRs are located at higher (lower) voltage level, they tend to fluctuate less (more). However, VRs would deliver a poor (an excellent) voltage regulation. The researchers explained this by saying that the current variations tend to decrease (increase) when voltage level increase (decrease) which led to less (more) fluctuations in VRs.

2.4.7 Line Losses

Generally, PVs reduce line losses but with exceptions. Many factors that determine whether PVs reduce or increase line losses. In other words, if PVs aren’t properly coordinated, they could have a negative impact on line losses.

In distribution system, line losses consist of two components, namely $R I^2$ and $X I^2$ losses. In conventional distribution system, $X I^2$ are ignored in literature due to two reasons: $X/R$ ratio in distribution system is low and utilities mostly don’t charge for VAR consumption. Once PVs are introduced to distribution system, should we include $X I^2$ losses?

Notice that both $R I^2$ and $X I^2$ are dependent on the magnitude of the current. If current magnitude has increases or decreased, $R I^2$ and $X I^2$ losses are going to increases or decreased proportionally. PVs have no impact on the values of line’s resistance or reactance but they impact the magnitude of the current. Thus, the impact of PVs on line losses can be analyzed through the analysis of PVs’ impact on current magnitude.
The current magnitude consists of two components: active component and reactive component [347], \( I^2 = (I_{\text{active}})^2 + (I_{\text{reactive}})^2 \). In order to reduce \( I \), both \( I_{\text{active}} \) and \( I_{\text{reactive}} \) has to be reduced. The active component \( I_{\text{active}} \) accounts for the active power that is consumed by loads. The active power consumption can be offset by injecting an active power from PVs. The optimum value of \( I_{\text{active}} \) occurs when it becomes zero which can be attained by making the active power injected by PVs exactly equal to the active power of loads. Once the active power of PVs exceeds the active power of loads, a reverse active power is going to flow in lines which would increase the value of \( I_{\text{active}} \). Consequently, the current magnitude increases (decreases) if the difference between the active power injected by PVs and consumed by loads increase (decrease), assuming the reactive power is fixed. A similar discussion could be applied to the reactive power. The reactive component \( I_{\text{reactive}} \) accounts for the algebraic sum of the reactive power absorbed and/or by loads and PVs. Thus, if we assume the active power is fixed, the current magnitude can be minimized by making PVs supply the reactive power of load locally.

In summary, our dream is to make PVs supply an apparent power that is exact equal to the apparent power consumed by the loads. Unfortunately, there is no free lunch. Contradictories of voltage rise, insolation, network configurations etc. limit our dream.

As mentioned in section (2.4.1), the injection of an active power of PVs cause a rise in bus voltage. In order to bring the voltage down to the permissible limit, the PV has to absorb a reactive power from the grid [317]. Once PVs start absorbing reactive power, the current magnitude increases and line losses too. However, the PV can neither inject or absorb reactive power unless it curtails some of its active power. A curtailment of PV active increases the consumption of active
power which will increase current magnitude and line losses too. Obviously, VVWO is necessary is such scenario.

Active and reactive power of PVs are dependent on PV penetration level and insolation which makes them additional stressing factors to line losses. Also, an excess active power of PV at arbitrary node “X” could be dispatched to supply demand at node “Y” which makes network configuration and load diversity additional stressing factors to line losses.

2.4.8 Tap Operating Devices

The tap operating devices are the devices that equipped with a control circuit to change its tap position in a response to the input signals. In electrical system, voltage regulators (VR) and switching capacitors (SC) are considered tap switching devices.

The impact of PVs on VRs and SCs depends on control mode, location, control settings, and instantaneous load [317]. Participation of PVs in voltage regulation implies that PVs equipped with a decentralized control that respond to the change in the voltage at PCC. VRs and SCs are, also, equipped with a decentralized control that could be sensitive to either current or voltage of the grid. If the control VRs and/or SCs are sensitive to bus voltages, their control is going to interfere with the control PVs which may lead to “voltage hunting” [317]. Thus, time delay and setpoints of control circuits has to be recalibrated in order to avoid “voltage hunting” between SCs, VRs, and PVs. Alternatively, the control of SCs should be sensitive to current (instead of voltage) to avoid “voltage hunting” with PVs.
2.4.8.1 Switching Capacitors

Distribution system is radial in nature and the current flowing at an upstream bus is proportionally related to the current that is consumed by the load at a downstream bus. SCs are installed usually at an upstream bus to control the voltage at a downstream bus. If SCs are operating using current control, their set points involve the ratio between the current passing through SCs and load bus. The purpose of the SCs current control is to offset the current at load bus. If PV is installed at load bus, the current ratio set point of VR is no longer reflecting the exact current at load bus. Therefore, SCs may fail to operate properly if they are current-controlled (decentral control) due to PVs presence in the grid [317]. Alternatively, the control of SCs should be based on reactive power and neither on current nor on voltage.

When SCs are controlled based on the reactive power, they sense the local reactive power to estimate the reactive power at the downstream bus. Similar to current, the reactive power passing at SCs is proportional to the reactive power at load bus. Therefore, the control of SCs is adjusted with a set point that represent the ratio of reactive power at SCs and load bus. If the control of SCs found that the reactive power at the downstream bus exceeds a threshold value, they change their tap position accordingly [317]. If PVs are not operating under unity power factor (which is the case), the control set point at SC do not adequately represent the reactive at load bus.

If the PV is operating under unity power factor, the injected active power by PV increases the voltage at load bus. Once the SC senses that the reactive power at load exceed a threshold value, it will inject more reactive power to the grid. This reactive power would increase the voltage at load bus too. Therefore, the voltage at load bus is going to be aggravated due to the active power
injected by the PV and the reactive power injected by SC which may make the voltage level exceed the permissible level [317].

2.4.8.2 Voltage Regulators

Generally, VRs in distribution grid are equipped with a decentralized control. Thus, the control mechanism of VR is going to be explained before indulging into DGs impact.

2.4.8.2.1 Control mechanism of VRs

Control mechanism of VRs are explained in the following subsections.

Figure 2.5 Basic Diagram of VR decentralized Control. Redrawn from [348].

2.4.8.2.1.1 Basic Principle

The essence of VR control is to estimate the voltage at the load bus and adjust its tap position accordingly. Figure (2.5) shows the basic construction of VR decentralized control. There is going to be a current transformer (CT) that is installed at the secondary side of VR and a potential transformer (PT). The CT and PT find the instantaneous current ($I_{sec}$) and voltage ($V_{sec}$) at the secondary side of VR respectively. Line impedance is already known to VR because the VR has to compensate for the voltage drop. Then, the VR uses simple KVL principle to estimate the voltage at PCC, $V_{PCC} = V_{sec}^{sec} - I_{sec}^{sec} Z_{line}$. This simple explanation are not an exact representation
of how VR control works but it serves the purpose of explaining the basic principle of VR decentralized control.

In practice, the VR is an autotransformer that change its tap position. Therefore, the exact model of VR would involve the parameters of the autotransformer. However, the VR regulator can be approximated (it is good approximation) to the following model [348], [349]:

\[
\begin{align*}
V_{VR}^{sec} &= k_{VR} V_{VR}^{pri} \\
I_{VR}^{pri} &= k_{VR} I_{VR}^{sec} \\
k_{VR} &= 1 + 0.00625 \text{Tap}
\end{align*}
\] (2.1)

In equation (2.1), the primary and secondary voltages and currents \(V_{VR}^{pri}\), \(V_{VR}^{sec}\), \(I_{VR}^{pri}\), and \(I_{VR}^{sec}\) of VR are given in p.u. if the VR is stepping up the voltage at the secondary side, the multiplicative factor \(k_{VR}\) would be greater than 1. The tap position \(Tap\) is an integer that is positive when the VR is stepping up position, a negative when VR is stepping down position and zero at nominal position [349].

CAVEAT: the system in (2.1) is just one variation of VR modeling.

2.4.8.2.1.2 Components of VR Circuit

In practice, the VR consists of an autotransformer with a tap changing mechanism. Mostly, VRs are equipped with 32 tap positions (16 positions to step up the voltage) and each position amounts to \(\pm0.625\%\) or 0.75 Volts on 120Volt scale. There are few parameters and terminology associated with the VR;
- Load Center (LC): it is either the output terminal voltage of VR or the voltage at a remote bus on the feeder.
- Line Drop Compensator (LDC): it is a circuit that compensates for the voltage drop between VR and LC. Also, it controls the tap position of VR. This circuit is shown in figure (2.5) while its detailed wiring is shown in figure (2.6).
- Desired Voltage Level (DVL): the control mechanism estimates the voltage level at LC. Once the voltage at LC is out of range, the control circuit decides to bring it to a certain level, which is called the Desired Voltage Level.
- Bandwidth (BnD): When VR changes its tap position to bring the voltage at LC to DVL level, the voltage can’t stay exactly at DVL but fluctuates up and down. These fluctuations has to stay within upper and lower bounds which are called Bandwidth.

![Figure 2.6](image)

**Figure 2.6** Line Drop Voltage Compensator (LDC) circuit from figure (2.5).

Let us choose LC in figure (2.5) to be at the end of the feeder, the bus where the load and PV are connected. The turns ratio of PT is denoted as \( \frac{N^P_{PT}}{N^P_{PT}} \) and the turns ratio of CT are denoted as \( \frac{N^C_{CT}}{N^C_{CT}} \). The PT reduces the line-to-Neutral voltage to a fixed value which is typically 120V if the VR is a single phase. Therefore, it would be more convenient to make the turns ratio of PT as \( \frac{N^P_{PT}}{1} \) while...
the primary turns could be calculated as follow; \( N_{PT}^p = (V_{VR}^{sec})_{\text{rated}} / 120 \). The value \( Z_{LDC(\text{volt})} \) in figure (2.6) are calibrated in volts to represents the equivalent impedance of the line, and they can be calculated as follow;

- Select the rated current of CT at the primary side as the base current. Since the rated values of CT are chosen as base, we could represent the base current in terms of CT turns. Therefore, the turns \( N_{CT}^p \) can considered as the base current at the line side \( (I_{\text{base(line)}} = N_{CT}^p) \) while turns \( N_{CT}^s \) is the base current at LDC side \( (I_{\text{base(LDC)}} = N_{CT}^s) \).

- Select the line-to-neutral voltage at the secondary side of VR to be the base voltage\( (V_{\text{base(line)}} = (V_{VR}^{sec})_{\text{rated}}) \). Thus, the base voltage at LDC side will be \( V_{\text{base(LDC)}} = \frac{(V_{VR}^{sec})_{\text{rated}}}{N_{CT}^p}. \)

- The base impedance at the line side will be; \( Z_{\text{base(line)}} = \frac{(V_{VR}^{sec})_{\text{rated}}}{N_{CT}^p} \)

- The base impedance at LDC side will be; \( Z_{\text{base(LDC)}} = \frac{(V_{VR}^{sec})_{\text{rated}}}{N_{CT}^p N_{CT}^s} \)

- The line impedance in p.u. will be; \( Z_{\text{line(p.u.)}} = Z_{\text{line}} \frac{N_{CT}^p}{N_{CT}^p N_{CT}^s} \)

- The p.u. impedance at LDC has to be equal to the p.u. impedance at line side;

\[
Z_{\text{LDC(p.u.)}} = Z_{\text{line(p.u.)}} = Z_{\text{line}} \frac{N_{CT}^p}{N_{CT}^p N_{CT}^s}
\]

- The p.u. at LDC side in volts is; \( Z_{\text{LDC(volts)}} = Z_{\text{LDC(p.u.)}} N_{CT}^s = Z_{\text{line}} \frac{N_{CT}^p}{N_{PT}^p} \)

From above discussion, by knowing \( Z_{\text{line}} \), the equivalent impedance at LDC side in volts can be calculated using the following equation;
\[
Z_{LDC(\text{volts})} = Z_{\text{line}} \frac{N_P^p}{N_P^P}
\]

\[
Z_{LDC(\Omega)} = Z_{\text{line}} \frac{N_P^p}{N_P^P N_C^C}
\]

(2.2)

The value of \(Z_{LDC(\text{volts})}\) is precalibrated in VR and kept fixed based on LC’s location.

### 2.4.8.2.1.3 How it works?

In this section, the operation of VR is going to be explained using an example. Thus, let us assume the base voltage in the following analysis at LDC circuit is chosen to be 120V.

In figure (2.6), the voltage over the relay \(V_{\text{relay}}\) represents the scaled voltage level at LC. For instance, if the voltage at LC is 400 V and this value is exactly what VR wants it to be, the value of \(V_{\text{relay}}\) would be equal to 120V. If the voltage at LC is less than DVL level, the voltage over the relay would be less than 120 V \((V_{\text{relay}} < 120)\).

The VR determines tap position based on \(V_{\text{relay}}\), as follow:

- Tap position will be calculated; 
  \[
  \text{Tap} = \frac{120 - V_{\text{relay}}}{0.75}.
  \]
  - In above equation, the DVL is chosen to be 120V.
  - One tap position is equivalent 0.625%. On 120V scale, 0.75 V is about 0.625%. Thus, above equation is divided by 0.75 V.

- Then, the multiplicative factor \(k_{VR}\) would be calculated as follow; 
  \[
  k_{VR} = 1 + 0.00625 \text{Tap}.
  \]

Then, the system in (2.1)) can be utilized to calculate the primary and secondary voltage and current of VR.
Now, the following simple example helps the reader to grasps VR operation. Let us assume that DVL is 120 V and BnD is 2 V. The parameters in figure (2.5) are; \( Z_{\text{line}} = 0.3 + j0.9 \) Ohm, the rated voltage at VR is 2400 V, the power at LC is consuming 700 kW and 300kVars. The rated current of the line would be \( \sqrt{\frac{700^2 + 300^2}{2400}} \approx 300 \) A. Thus, The primary CT ratio will be selected to be \( N_{CT}^P = 300 \). If the line current is going to be reduced to 5 A, then the secondary ratio of CT will be selected to be \( N_{CT}^S = 5, \frac{N_{CT}^P}{N_{CT}^S} = \frac{300}{5} = 60 \). Also, the \( N_{PT}^P \) is selected to be 20 because line voltage has to be reduced to 120V level. From (2.2)), \( Z_{LDC(Ohm)} = 0.9 + j2.7 \) Ohm.

If we use FB (forward backward sweep) method, the voltage at LC would be \( 2165 \angle -5.96 \) V and \( \|_{\text{line}} = 307.1 - j171.4 \) A. The voltage at LC is about 10% less than the rated value, 2400 V. Thus, we expect the VR to raise its tap position.

The VR would react as follow;

- \( V_{\text{reg}} = \frac{2400}{20} = 120 \) V
- \( \|_{\text{LDC}} = \|_{\text{line}}/\left(\frac{N_{CT}^P}{N_{CT}^S}\right) = 5.12 - j2.86 \) A
- \( V_{\text{drop}} = Z_{LDC(Ohm)}\|_{\text{LDC}} = 12.32 + j11.25 \) V.
- \( V_{\text{Relay}} = V_{\text{reg}} - V_{\text{drop}} = 108.3 \angle -6 \)
  - Notice that \( V_{\text{Relay}} \) is out of BnD because it should be between 119V and 121V.
- The magnitude of the voltage at the relay is less than 120 V. Thus, the tap position has to be changed.
- \( Tap = \frac{120 - 108.3}{0.75} = 15.6 \approx 15 \)
As shown above, the VR will change its tap position to a raise position. It will be at position number 15. Now, using (2.1), let us check the voltage at LC.

Figure 2.7 Voltage Regulator tap position with the corresponding $k_{VR}$ and power flow direction. In cases (a), (b), and (c) the downstream side of VR is regulated when the power is flowing in forward direction. In cases (d), (e), and (f) the upstream side of VR is regulated when the power is flowing in the reverse direction.
\[
k_{VR} = 1 + 0.00625 \quad \text{Tap} = 1 + 0.00625 (15) = 1.09375
\]

- The voltage at the secondary side of VR will be 2400 * 1.09375 = 2625 V.

- The current at the secondary side: \( \frac{307.1 - j171.4}{1.09375} = 280.8 - j156.71 \) A.

- Using KVL, the voltage at LC will be 2409\( \angle -4.9 \) V.

This is acceptable level because it is about 0.4% higher than the rated voltage. Now, let us check what would be the voltage at the relay after correction.

- \( I_{line} = 280.8 - j156.7 \) A
- \( I_{LD} = 4.68 - j2.61 \) A
- \( V_{drop} = 11.26 + j10.3 \) V.
- \( V_{reg} = \frac{2400 + 1.1}{20} = 131.25 \)
- \( V_{relay} = [V^r_{reg} - V^r_{drop}] + j[V^i_{reg} + V^i_{drop}] = 120.43 \angle -4.9 \)
  - \( V_{relay} \) is within the BnD 119 < \( V_{relay} < 121 \)

### 2.4.8.3 Impact of PVs on VRs

As shown in section (2.4.8.2.1), the VR tries to keep the voltage at remote bus within a specific limit. Thus, assuming PV operating under a unity power factor, a shorter (longer) distance between VR and the designated bus results in a better (worse) voltage regulation but the number of tap changing operation increases (decreases) [62]. Similar phenomena is observed when voltage level of the grid is changed: a higher (lower) voltage of grid where VR is installed results in a poor (excellent) voltage regulation of the designated bus but the number of tap changing operation decrease (increase) [62].
In some situations, a reverse power flow could happen to the feeder. One of the reasons to have a reverse power flow is the high penetration of PVs. VRs determine the direction of power flow based on the direction of active power in the line [317]. Once a reverse power flow occurs, the VR would react based on its operational mode. There are three main modes of operations of VRs, namely Normal Bidirectional Mode, Co-Generation Mode, and Reactive Mode [317], [350]. The following sections would explain how each mode works and how VR deals with the reverse power.

2.4.8.3.1 Normal Bidirectional Mode

VR is designed to control the downstream side in distribution system. Thus, the tap of VR can be modeled at nominal, stepping-up, or stepping-down positions as shown in figure (2.7,a) (2.7,b), or (2.7,c) respectively if the VR is controlling the downstream side. When VR is operating under “Normal Bidirectional Mode”, it can control the upstream side too once it senses a reverse in active power. Therefore, during reverse power flow, the tap of VR can be modeled at nominal, stepping-up, or stepping-down positions as shown in figure (2.7,d) (2.7,e), or (2.7,f) respectively [317].

When the power is moving in forward direction, the voltage at downstream bus changes as expected according to VR tap position. For instance, if the power is flowing from point “A” to “D” in the system shown in figure (2.8,a) and the VR is at nominal position as shown in figure (2.7,a), the voltage over “A”, “B”, “C”, and “D” buses could be plotted [the circular dots] as shown in figure (2.8,b). Notice that the voltage over bus “D” is below the minimum threshold. Thus, the VR should switch to a raise position, figure (2.7,b), in order to bring the voltage over bus “D” to a higher value. Once the VR steps up its secondary voltage, the voltage over bus “C” and “D” would
follow the square dots that are shown in figure (2.8,b). However, if PVs are connected to bus “D”, they would improve its voltage and bring it above threshold limit and the voltage profile could follow the crossed dots that are shown in figure (2.8,b). In figure (2.8,b), notice that the VR could stay at its nominal position while voltage over all buses remain within the permissible limits if PVs

Figure 2.8 Voltage correction of VR during forward and backward power flow in a feeder. (a) a distribution feeder where location “A” represent the upstream side, “B” primary side of VR, “C” secondary side of VR, and “D” downstream side of the feeder. (b) voltage over buses “A”, “B”, “C”, and “D” when the power flowing from bus “A” to bus “D”, forward direction. (c) voltage over buses “A”, “B”, “C”, and “D” when the power flowing from bus “D” to bus “A”, reverse direction. The circular dots represent the voltage over buses when VR is at its nominal position. The square dots represent the voltage over buses after the VR changes it tap position when PVs are disconnected. The crossed dots represent the voltage over buses after the VR changes it tap position when PVs are connected.
are connected. Also, notice that the PVs do not impact the voltage at bus “A” significantly because it is connected to larger grid (an infinite bus).

During contingencies, the conventional distribution system could be reconfigured such that the downstream side of VR becomes the supply bus [67], [68], or the infinite bus. In such contingencies, a reverse power occurs, and the VR that is operating in “Normal Bidirectional Mode” would control the upstream side. When the grid is reconfigured (the downstream side becomes the supply side) and the upstream bus voltage is above threshold limit for some reason as shown in figure (2.8,c) [circular dots], the VR will switch to a stepping-down position, see figure (2.7,f), in order to bring the voltage at the upstream within the permissible limits. Unfortunately, bus “A” is an infinite bus (connected to a larger grid) and its voltage is going to be mainly affected by the larger grid (in many situations, the voltage of bus “A” is assumed fixed). Thus, the VR would have a very insignificant effect to improve the voltage at bus “A”. Consequently, the net effect of VR is going to raise the voltage at bus “D” [317]. In such a situation, the voltage profile could follow the crossed dots that are shown in figure (2.8,c).

2.4.8.3.2 Co-Generation Mode

When the power is moving forward [from bus “A” to bus “D” in figure (2.8, a)], the Co-Generation Mode operates same as the Normal Bidirectional Mode. However, once the active power flows in the forward direction and the VR is operating under Generation Mode, the VR will remain controlling the voltage at the downstream bus [317], [350].

The first shortcoming of Co-Generation Mode occurs when the network is reconfigured such that the downstream bus becomes the supply bus. In this situation, the voltage at the downstream bus tends to exceed the threshold value. Then, the VR will switch to a step-down
position, as in figure (2.7, c), in order to lower the voltage at the downstream bus. Unfortunately, the voltage at the downstream bus is almost fixed (an infinite bus). Consequently, the net effect of VR is to increase the voltage at the upstream bus which could make its voltage exceeds the permissible limit. In other words, the exact opposite of the situation that has been discussed in figure (2.8, c) is going to be observed. If the upstream bus has no PV connected to it, most probably

Figure 2.9 Voltage correction of VR during Co-Generation Mode of VR where LC is selected to be bus “D”. (a) a distribution feeder where location “A” represent the upstream side, “B” primary side of VR, “C” secondary side of VR, and “D” downstream side of the feeder. (b) voltage over buses “A”, “B”, “C”, and “D” when the power flowing from bus “A” to bus “D”, forward direction, and the PV power do not exceed the load power at bus “D”. The circular dots represent the voltage over buses when VR is at its nominal position. The crossed dots represent the voltage over buses after the VR changes it tap position. (c) voltage over buses “A”, “B”, “C”, and “D” such that PV power at bus “D” exceeds the load power.
its voltage is going to be less than the downstream bus which would lessen the problem of Co-Generation Mode.

The second shortcoming of Co-Generation Mode situation is related to the expectations of LDC circuit [317]. When the active power injected by PVs do not exceed the active power absorbed by the load, the LDC expects a monotonic decrease of the voltage magnitude across the feeder. However, once the active power injected by PVs exceeds the power absorbed by the load, the monotonic voltage profile disappears. Consequently, the LDC circuit of VR may take the wrong action. For instance, let us assume we have a distribution feeder that is shown in figure (2.9,a) and bus “D” is selected to be LC (refer to section 2.4.8.2.1). If the active power injected by PV is less than the active power absorbed by the load, voltage would follow the circular dots that are shown in figure (2.9, b). The VR may notice that the voltage at bus “D” is low and it has to be corrected. Then, the VR would step-up the voltage to follow the crossed dots in figure (2.9, b). However, if the active power injected by PV at bus “D” exceeds the active power absorbed by the load, the voltage profile could look like the one shown in figure (2.9, c). Notice that the voltage at bus “C” need to be corrected but the VR cannot see it.

2.4.8.3.3 Reactive Bidirectional Mode

The Reactive Bidirectional Mode operates similar to the Normal Bidirectional Mode. The only difference between these two modes is that Reactive Bidirectional Mode determines its controlling side based on the direction of reactive power through the line.

In some cases, the operations of VR can be affected by the presence of capacitor banks because their reactive power could affect the flow of reactive flow which may impact the VR controlling direction. Also, some PVs operate with a fixed power factor, which might affect the
operations of VR. Both of the aforementioned cases, the problem can be solved by calibrating the set points of VR such that the reactive power injected by capacitor banks of PVs is included.

When the PV is equipped with a decentralized control to regulate the voltage, it is going to change its reactive power injection or absorption continuously. Also, the switching capacitor banks will change its reactive power in a step-wise manner. In the previous two cases, the operations of VR is going to be impacted badly because its set points are fixed based on the reactive power absorbed by the load. Once the variations of reactive power by PVs and switching capacitors are added into the system, the setpoints of VR has to change accordingly.

2.4.9 Effect on Upstream Side.

At high penetration levels, impact of PV could encroach further to upstream side of the main transformer of a feeder. Some publications [113], [162], [175], [237], [280]–[283], [285], [312] explored the impact of PVs that are installed at low voltage level on the high voltage side of the grid. Most of these publications reached to a consensus that reverse power flow start happening once penetration level exceeds 30% approximately according to penetration index that defined as the ratio of total PVs to the total conventional generation, see table (2.2) for more information about penetration indices.

High level of PV penetration make the electric system more prone to instability due to main two reasons: system’s inertia reduces and wide disconnection of PVs due shading [175], [280], [281], [283], [285]. Another interesting observation in [175] is that although the increase in PV penetration makes transmission system more susceptible to perturbation following a fault, it become more damped (fewer oscillations). However, authors of [113] found no probable impact on transmission system from PVs that are installed at distribution level.
Simulation in [175] shows that when PV penetration reaches certain levels, phase angle difference among buses will reach higher values more frequently which would make it harder to maintain synchronization among generation units. Also, the reactive losses, or the $Xl^2$ losses, at transmission would follow η-shape: losses increase until penetration level reaches a certain level before they decrease again. Since the power injected by PV increases during the day and decreases during night, the conventional generation has to compensate for the change in reactive power due to PV.

Also, at high penetration levels of PVs [290], synchronous generator (SG) has to operate in a low power factor because its active power has been replaced by PVs’ active power. Regrettably, SG degrades and heats if it operates at low power factor. Also, the limitation on the stability curve of SG is another reason for not to operate under low power factor: for instance, the amount of available reactive power is limited by the amount of active power that is being injected by the SG. When PVs’ active power replaces active power by SGs, the margin of available reactive by SGs decreases which might increase the risk of not being able to satisfy reactive power requirements of the system.

The simulation in [175], [280] shows that fluctuations in voltage magnitude becomes more frequent since PV output power is weather-dependent. The obvious impact of rooftop PVs on voltage rise at transmission side is recognized but authors in [162] emphasized that voltage rise at distribution level precedes the one in transmission.

2.4.10 Hosting Capacity

Few review papers have tried to summarize the maximum penetration level reported in publications, namely [7], [8], [10], [11]. However, the consensus among publications is that
maximum penetration of PVs in a feeder is case dependent. Also, it is hard to build a rule of thumb to determine the maximum penetration of PVs in a feeder due to two main factors. The first factor is about the feeder itself which encloses feeder load, spatial installation of PVs, distance of the feeder, size of conductors, control scheme, voltage level etc. Second factor is about the limiting electrical quantity to PV penetration. For instance, maximum PV penetration level that is limited by voltage magnitude is different of the level that is limited by harmonics. Therefore, the maximum penetration level that is reported by publications varies widely. In brief, absolute penetration limit is very hard to determine [11] because it is case dependent.

Most of the publications investigated PV impact on distribution system at different penetration levels. Some papers tested PV impact at penetration levels less than 100% while other publications performed investigation at high penetration level, as high as 300%. Table (2.4) lists maximum penetration level that has been used by publications to investigate PV impact. Please keep in mind that penetration level listed in Table (2.4) is not the maximum-recommended-penetration level but the maximum-simulated-penetration level. An interested reader into the maximum-recommended penetration level is advised to refer to the following references [7], [8], [10], [11].

2.4.11 Miscellaneous Effects

One of the poor control strategies is to automatically disconnect PVs once the voltage at PCC reaches a certain limit in order to avoid overvoltage violations. If we have a high voltage on one of the phases while other phases are relatively low and the PVs that are connected to the high voltage phase disconnect momentarily, a good chance that the voltage of the other two phases will
rise. Then, the PVs that are connected to the other two phases will disconnect too and this is called the cascading effect according to [154].

Distribution system is radially designed. However, the authors in [190], [191] investigated the possible impact of PVs once the grid is configured as a mesh. They found that PVs will not create an overvoltage problem if they are installed at the main transformer regardless of grid topology (either radial or mesh). Also, they found that the impact of PV on line losses is mainly driven by penetration level and less sensitive to grid topology.

Reverse power is one of the consequences to PV penetration. However, the paper in [74] has addressed reverse power from different angle. The authors investigated the impact of PV if there is a reverse flow in active power but not in the reactive power which is called a “counter power flow”. They found no evidence to the impact of counter power flow on the grid.

Conservation Voltage Reduction (CVR) enables utilities to save energy. However, at high penetration of PVs, it is very hard to VRs and SCs to operate according to CVR principle. Authors in [174] claimed that if a proper control scheme is implemented, PV power can be utilized efficiently to achieve CVR.

The research in [73], [329] investigated whether PV would exhibit different impact on the grid if loads are modeled differently in computer. In their simulation, they made three assumptions: all loads are modeled as a constant power loads (CPL), constant impedance loads (CIL), and mixed load modeling (MLM). Then, the CPL modeling has been chosen as the reference case. Then, they run the simulation for different penetration levels. In the extreme case, they found 8.3% difference in losses with respect to the reference case. So, their first conclusion is that PV exhibit different impact on system’s losses if loads are modeled differently. Also, they investigated the effect of
load modeling on the threshold penetration level where the losses start increasing. They found that threshold penetration level does not change regardless of how the loads are modeled, and this is their second conclusion. Their third conclusion is about voltage; PV exhibits almost same impact on voltage profile regardless of how the loads are modeled. Thus, in general, load modeling does not have a significant association with PV impact on the system.

The geometry of three-phase lines, the relative distance between phases, PV penetration, and voltage profile are found to be dependent on each other in [59]–[61]. Although line geometry has an impact of voltage profile, this does not dismiss the influence of line distance, load imbalance, etc. on voltage profile under heavy penetration of PV.

In [48], [49], [293], authors found that PV could help in load shaving. However, unlike commercial customers, the peak load of residential customers does not coincide with the peak of PV output.

### 2.5 PVs According to Standards

The standards and guidelines of PVs covers many issues such as safety, protection, grounding, etc. The PV standards are different from country to another and usually are customized to fulfill certain considerations that are case dependent. In this document, we are going to focus on the famous standards that discusses any guideline that might affect VVWO. The following subsections are covering the issues that might be related.

#### 2.5.1 Voltage

According to IEC 61727 and IEEE 1547, the voltage of PV system is not regulated. However, IEEE1547a (2014) requires the PV to actively participate in voltage regulation by
changing both of its active and reactive powers [351]. This means the PV should injects an active power to the grid as much as possible. Once bus voltage exceeds the permissible limit, the PV should absorb reactive power from the grid to lower the voltage. However, the PV can neither inject nor absorb reactive power if it is injecting the whole available active power. In this situation, the PV has to curtail some of its active power in order to absorb reactive power from the grid. Curtailing the active power by itself decreases the voltage over buses. Thus, a trade-off has to be done between the active and reactive power of the PV such that it injects the most of its available active power. Another factor makes these constraints more complex. Once PV starts absorbing reactive from the grid, the magnitude of the current passing through conductor increases which might increases the losses. However, the current magnitude is also dependent on the active power. Thus, from line loss perspective, it desired to absorb zero reactive power. These factors are all complicating the optimization problem.

In addition, most of standards are limiting, such as IEEE 1547, the voltage magnitude at PCC to remain within ±5% of the nominal value. However, some standards have additional constraints to include the durations of voltage magnitude, such as GB/T 19964 standards [351].

2.5.2 Power Factor

Most standards require a minimum power factor of PV output power to be 0.9. Other standards, such as IEC 61727, adds an additional condition in which the output power of PV has to exceed its rated value [351]. IEEE 929 standard sets no minimum power factor requirements. However, if the output power of PV falls under 10% of its rated value, the power factor has to exceed 0.85 [351].
2.5.3 Voltage Unbalance

Standards in Germany, Canada and China require certain limitations on voltage unbalance of DGs. In general, most of these standards require voltage unbalance to be less than 2% [351].

2.6 PV Capability to Participate in Voltage Regulation

Rooftop PV panels can participate in voltage regulation by readjusting its active and reactive power. Since PV panel itself generates DC power, the interfacing inverter is the one that convert its power to AC. Also, it is the interfacing inverter that controls the amount of the active and reactive power generated by the PV [352]. A typical single-phase grid connected solar-PV system is illustrated in figure (2.10).

![Typical single-phase grid-connected PV. Redrawn from [352]](image)

There are many mitigation techniques to the impact of PVs as shown in table (2.1). In this section, we focus on mitigation techniques that can be accomplished using the interfacing inverter of PV only. Generally, the interfacing inverter can either curtail the active power and/or regulate...
the flow of the reactive power. Literatures that studied the capability of interfacing inverter are listed in table (2.5).

The following subsection shows a generic lesson of the benefits and shortcomings to utilization of the interfacing inverter. However, the working principle and mathematical modeling of PVs are introduced beforehand in the following subsection.

2.6.1 PV Model

In distribution systems, PV panels are mostly of the rooftop type. This means that majority of PV panels in distribution system are single phase. In the most general case, PV can be connected to the grid via a set of a DC/DC converter and a DC/AC inverter, as shown in figure (2.10). Due to the non-linear relationship between the output voltage and current of the PV, the DC/DC converter tracks the maximum power output of the PV, which is known as MPPT (maximum power point tracking). The DC/AC inverter receives the DC- power from MPPT converter and converts it to an AC-power. The phase angle \( \theta_{PV} \) and the magnitude of the output voltage \( V_{PV} \) can be controlled by adjusting the reference signal \( V_{ref} \). The output power of the inverter is calculated as follows:

\[
P_{PV} = mV_{PV}V_{PCC}[G \cos(\theta_{PV} - \theta_{PCC}) + B \sin(\theta_{PV} - \theta_{PCC})]
\]

\[
Q_{PV} = mV_{PV}V_{PCC}[G \sin(\theta_{PV} - \theta_{PCC}) - B \cos(\theta_{PV} - \theta_{PCC})]
\]

As shown in equation (2.3) and (2.4), three factors determine how much power can be injected by PV into the grid, namely the equivalent admittance \((G + jB)\) of the line between the inverter and the grid, the phase difference between PV and grid voltages \((\theta_{PV} - \theta_{PCC})\), and output voltage magnitude of the inverter \(mV_{PV}\) and the grid \(V_{PCC}\). The modulation index \(m\) of the inverter modulates the magnitude of the output voltage of PV, and it ranges from 0 to 1 \((0 \leq m \leq 1)\).
It should be noted that active power $P_{PV}$ and reactive power $Q_{PV}$ that are injected by the PV can be controlled indirectly by adjusting the magnitude and phase angle of $V_{ref}$. Therefore, the PV is equipped with a phase-locked-loop circuit to monitor the voltage at PCC and adjust $V_{ref}$ accordingly to control the output power of the PV.

The power injected by the PV is limited by grid voltage, rating of the inverter, and the allowed level of harmonics [353]. The limitation due to the power grid can be analyzed using the following equation [353]:

$$P_{PCC} + jQ_{PCC} = (V_{PCC} \angle \theta_{PCC}) \left\{ \frac{[(mV_{PV} \angle \theta_{PV}) - (V_{PCC} \angle \theta_{PCC})]}{Z \angle \theta_Z} \right\}^*$$  \hspace{1cm} (2.5)

Equation (2.5) models the complex power flowing into the PCC from the PV side. Above equation can be rewritten to become as follow;
If we assume all variables are fixed in equation (2.6) except for the modulation index \( m \), the diagram would look like a disk in PQ-plane, as seen in figure (2.11) with its center located at \((-V_{PCC}^2 (G - jB))\) and with a radius of \( \left( \frac{mv_{PCC}^2 V_{PV}}{Z} \right) \). Notice that variations in the value of \( m \) would change the radius of the circle. Similarly, if we assume all variables in equation (4.4) are fixed except for the angle difference \( (\theta_{PCC} - \theta_{PV}) \), the operating point on the circular path can be traced. For instance, for \( m = 1 \) and \( \theta_{PCC} - \theta_{PV} = 0 \), the PV would be operating in the first quadrant in figure (2.11) and it would inject active and reactive power to the grid.

The inverter’s rating imposes additional limitations on the power injected by the PV. The boundary of the operating region of the inverter would look like a circle in the PQ-plane with its center located at the origin, see figure (2.12). The overlapping region between the grid and the inverter operable region denotes the operable region of the PV, which is denoted as the light shaded region in figure (2.12). In addition, to meet the harmonic standards of the grid, the operable region is confined to what depicted as the dark shaded region in figure (2.12) [353].

Therefore, the model of PV panel in this work is formulated as follows:

\[
P_{PV} = I_{rr} P_{PV}^r
\]

\[
Q_{PV} = \begin{cases} \sqrt{(P_{PV}^r)^2 - P_{PV}^2}, & \text{if } P_{PV} > 0.1 P_{PV}^r \\ 0, & \text{otherwise} \end{cases}
\]

The available active power \( P_{PV} \) at any point in time of the PV is found by multiplying the normalized value of the irradiance \( I_{rr} \) (any value between 0 and 1) by the rated power \( P_{PV}^r \) of the
PV. Also, the available output reactive power $Q_{PV}$ at any point in time of the PV is calculated as in equation (2.8).

2.6.2 PV Participation in Voltage Pegulations (Pros vs Cons)

The interfacing inverter of a PV can control the phase shift of its output AC voltage with respect to current – an interested reader can refer to [354] for a thorough explanation about PV reactive power control. This ability to control phase shift means that the interfacing inverter can adjust the reactive power. Since overvoltage is a common impact, a PV can absorb reactive power from the grid in order to lower voltage level. Consequently, reactive power flow in the grid increases. Thus, main transformer will operate at lower power factor which might decrease its

![Figure 2.12 Operable region of the grid, inverter, and the PV on PQ-Plane. The lightly shaded region denotes the operable region of the PV. The dark shaded region denotes the operable region of the PV due to harmonics limitations. Redrawn from [353].](image)
efficiency [8]. Also, an increase in reactive component of current, increases current magnitude flowing in lines which would increases losses or reduce hosting capacity of the feeder.

The main impact of power fluctuations is the rapid changes in voltage level. This problem exacerbates at high penetration levels. However, the simulation in [37], [38] showed that this problem can be resolved by letting PV participate in reactive power support. In the simulation of [37], [38], power fluctuations do not impact voltage level significantly if PVs are operating under a fixed leading (absorbing) power factor of 0.9 albeit penetration is as high as 300%.

The performance of PV to regulate voltage has been compared with other voltage controlling devices in [64], [240], [243], [293]. In [293], PVs outperformed VRs and SCs in voltage regulation. In [64], the simulation shows that when PV penetration level exceeds 30%, they can replace voltage regulation equipment. Also, the simulation in [243] reaches to a similar conclusion. Optimization analysis in [341], [355] has reached to the same conclusion too but without a definite penetration level.

One way to eliminate the impact of PVs is by curtailing its active power partially or completely. In [197], the simulation shows that PV active power curtailment can mitigate power imbalance (that is caused by PVs) and improve voltage profile but this solution comes on the expense of line losses. On the contrary, in [196], it has been found that PV contribution in reactive power support is a more effective way to correct imbalance in distribution system than active power curtailment. However, they emphasized that the effectiveness of this solution is dependent on other dimensions such penetration, irradiance, cloud cover, and allocations etc.

Hosting capacity of the grid to PVs can be increased if active power curtailment is utilized [201]. Also, in [206], the reactive power margin by PVs increases if active power is curtailed which
would be useful mitigation source. Thus, this reactive power can be utilized to minimize tap operations of VRs. Also, voltage quality issues can be resolved such as voltage imbalance [210], voltage sag [204], voltage fluctuation [210], etc. However, active power curtailment is not recommended because it causes a significant energy losses [211], [256] and unfair curtailment to PV owners [219].
VVWO controls and/or coordinates the reactive and active power of the resources in electric system in order to optimize its operations. VVO is doing the same purpose of VVWO but without involving the active power. However, when penetration of DGs in distribution system reaches the level where VVO is no longer able to sustain the severe variations of the injected power, it became a good idea to involve the active power of DGs into optimization processes [356]. Thus, VVWO is an evolved version from VVO. In order to make a clear distinction between VVWO and other concepts, it is an optimization problem provided that its control variables manipulate voltage, active and reactive power to attain the objectives regardless of what would be the objectives or control variables.

In literature, it is very common to use the term “Optimal Power Flow (OPF)[357]” rather than VVWO. At conceptual level, both OPF and VVWO are dealing with active and reactive powers in order to optimize a single or multi-objectives. However, they differ from the historical perspectives. OPF is actually an evolved version from economic dispatch [358]. Economic dispatch was mostly about how to split the output power of units among loads such that fuel cost is minimized. This had been happening at transmission level because there were no generators at distribution level. On the contrary, VVWO is an evolved version of VVO because distribution system is transitioning to becoming an active grid. So, fuel cost was the trigger to OPF while the adoption of DGs gave birth to VVWO. Thus, when it comes to an optimization problem that
involves the utilization of DGs as a control variables, it would be more appropriate to use the term VVWO rather than OPF, which the case with this work.

Taxonomy of VVWO can be build based on three perspectives: control perspective, objective perspective, and research methodology perspective. The control perspective of VVWO entails the type of control employed in order to achieve a set of objectives. Generally, the control strategies of VVWO can be classified into three broad categories [356], [359]: Centralized, Decentralized, and Distributed Control. The VVWO differs, also, based on the objectives to optimize. Generally, from objective point of view, the VVWO can be classified into the following categories: allocation & reconfiguration perspective and utilization of existing resources.
perspective. Since DGs are stochastic in nature, VVWO can be performed with and without considering uncertainty of renewable resources, such as insolation etc. Therefore, VVWO can be classified from the perspective of research based on, for instance, whether it is deterministic or stochastic. Above discussion are plotted in figure (3.1) for simplicity.

3.1 Control Perspective

As mentioned earlier, control strategies that are employed to achieve VVWO can be classified into three categories: Centralized, Distributed, and Decentralized control. These classifications are borrowed from [352] but some parts of the Distributed control are considered to be Decentralized control for VVWO in this document.

The centralized control means that all the data are gathered at a master control center (MCC). Then, a processor at MCC makes the control decisions of the whole distribution network using optimization algorithms. Then, through communication medium, the control decisions are going to be transmitted to multiple controllers at various locations, each of which solves a smaller subpart of the problem. These processes are going to be repeated on a sampled time-wise basis [356]. An example of VVWO using centralized control are in [360], [361].

Distributed control entails the idea of dividing the grid into zones. DGs that belong to a zone can communicate between themselves in order to perform VVWO [359]. In a simple language, each zone has its own centralized control that is independent of neighboring zones. However, there could be a medium of communication between different zones to share data [356]. An example of VVWO using distributed control are in [362].

The control strategy becomes decentralized when each DG, VR, SC, etc. is having its own control that build its own decision based on the collected data from neighboring buses [359].
neighboring buses could involve DGs, which may allow two DGs to communicate as in [363]. In some cases, the DGs collects the data available at PCC only as in [364], [365].

For the sake of VVWO, the centralized control is the most expensive (because it is dependent on communication infrastructure) but the most efficient strategy [341].

3.2 Objective Perspective

Mostly, VVWO is a multi-objective problem. Objectives differ from research to another but line losses are mainly a common objective among all literatures. VVWO can be classified into two main categories based on type of objectives: allocation & reconfiguration and utilization of existing resources.

3.2.1 Allocation & Reconfiguration

Some literature employs VVWO to find the best allocations of voltage control equipment in the grid, for instance, such that line losses, switching operations, power curtailment, etc. are all minimized. Finding the best allocation could be for PVs, VRs, etc. Sometimes literatures use VVWO to find the optimum configuration of the grid by making the status of certain breakers be Normally Open or Normally closed or by including additional conductors into the grid. These kind of VVWO can be called “Allocation & Reconfiguration” because their objective is about finding the optimum settings of control variables by changing or modifying the existing system.

3.2.2 Utilization of Existing Resources

Unlike Allocation & Reconfiguration, some VVWOs utilize the existing resources in order to minimize the overall cost of the system. For instance, the researchers look for the best tap position of VRs, switching status of capacitor banks etc. in order to minimize line losses, switching
operations, power curtailments etc. This type of VVWO is called “Utilization of Existing Resources” because it deals with the existing system as it is and utilizes the available resources wisely to achieve its objectives.

3.3 Research Methodology Perspective

In order for VVWO to be solved, researchers pursue different strategies in their research. In this section, we present a general classifications of research strategies that have been employed in literatures.

A general taxonomy of research methodology could be classified into two genres: Heuristic methods and Analytic. A heuristic method mostly uses smart trial and error strategies in order to search for a solution that is good enough to the user. The analytical methods start by building models of the system and find a solution that should satisfy that model.

3.3.1 Analytical Approaches

Most of the existing analytical approaches to solve VVWO are based on convex modeling. Researchers have modeled their work into different scenarios which are listed in the following subsections.

3.3.1.1 Linear Programming

When VVWO is done using linear programming, the problem is formulated such that both system and constraints are linear. Linear programming is fast and reliable method to attain global optimum solution [366].
3.3.1.2 NonLinear Programming

KCLs and KVLs in power system are linear equations. However, power mismatch equations and voltage magnitude constraints, for instance, are quadratic. Thus, quadratic programming is the most practical NonLinear programming for VVWO [352]. The interior-point method are the mostly used non-linear programming solver to perform VVWO [352]. Some researchers used another solvers such as Newton Lagrangian method to solve VVWO as in [367].

3.3.1.3 Mixed Integer NonLinear Programming

Some of the NonLinear problems involves discrete variables. In power system, tap position of VRs exemplifies discrete variables that has to be dealt with when VVWO is implemented. These discrete variables could be modeled as either binary or integer variables in optimization problem which would increase simulation time and makes the problem harder to solve. These problems are called Mixed Integer NonLinear Programming (MINLP). For instance, in [368] the authors used sequential search strategy to solve VVWO where the system is modeled as MINLP. Branch flow model-based-relaxed OPF is utilized to formulate a mixed-integer second order conic programming problem to perform VVWO in [369] and [370]. The authors in [371] offered a control method to perform VVWO by finding the optimum sizing and placement of energy storage devices in distribution networks. Also, they considered the stochasticity of DGs and load in the problem.

3.3.1.4 Non-Linear Dynamic Optimization

The difference between Non-Linear optimization and Non-Linear Dynamic optimization is that the latter obtains the initial values by using linear optimization. This method has been implemented for VVO, [352], but it has not been implemented for VVWO yet.
3.3.2 Heuristic Approaches

Most of the search algorithms utilize heuristic approaches. For instance, evolutionary search, simulated annealing, particle swarm, etc. are all considered heuristic approaches in this document.

3.3.2.1 Simulated Annealing

The process of heating a material and cooling it slowly afterwards for manufacturing reasons has inspired simulated annealing methods. It is a search-based algorithm that can escape local minima by making “probabilistic moves” in order to find a better solution [352]. The “probabilistic move” means that a new point is randomly generated after each iteration. This random point should flow a proportional probability distribution scale of temperature. Simulated annealing method accept the new point that either increase or decrease the objectives, and this is how the local minima could be escaped. In [372], the authors implement simulated annealing to achieve VVWO.

3.3.2.2 Tabu Search

This method is developed by F. Glover [373] and P. Hansen [374]. Tabu search method ensembles human memory operation in terms of flexibility. Thus, this method is able to escape local minima and search beyond it [352]. Tabu search algorithm was used in [375] to perform VVWO in distribution networks in order to minimize power losses subjected to various network constraints.
3.3.2.3 Evolutionary algorithm

Evolutionary algorithms are developed to simulate the biological evolution, for instance, mutation, reproduction, recombination, and selection [352]. Authors in [376] used genetic algorithm for VVWO by using linear approximation of load flow equations, and heuristic selection of participating controls. The VVWO is performed in [377] to minimize the system losses using the model-based system. In [378], the authors formulated a multi-objective VVWO problem and they solved it using evolutionary algorithms. In [379], the authors utilized evolutionary algorithm to solve VVWO to minimize load curtailment in the system.
CHAPTER 4

DETERMINISTIC VVWO USING EA

The author has done some work on VVWO which is going to be presented in this chapter. Optimization problem at its basic level is about getting an input data, processing them, and producing an output. Therefore, a researcher who is doing an optimization has to design his optimization problem according to the control mechanism that is employed. Most of the contemporary control systems in distribution systems is a decentralized control. Literatures have shown excessively that once PVs’ penetration exceeds 25-30%, problems emerge in distribution system. This penetration limit remains a general guideline because that maximum penetration of PVs is a case dependent problem.

In this document, a centralized control has been utilized to perform VVWO. Our simulation shows that centralized control, if employed at distribution level, would eliminate the maximum penetration level of PVs. Therefore, we recommend centralized control to be implemented in distribution system not only because it mitigates the emerging problems in active grid but also because it allows utilities to maximize their profits and to reduce dependency on conventional generation due to environmental concerns.

Insolation is impacted by too many variables such as wind direction, dust, time, location, etc. It is permissible, according to Central Limit Theorem, to consider a variable that is dependent on an infinite number of variables to be a random variable. Therefore, insolation can be considered a random variable (literatures does this). In this case, the uncertainty of insolation complicates the
optimization problem. Thus, it would be wise to make sure that the centralized control of VVWO works perfectly on distribution system before including the uncertainty element into the problem. For these reasons and others, we assumed that our system is deterministic which is an enough assumption to test the concept of VVWO specifically.

The evolutionary algorithms (EA) has been utilized in order to do VVWO. Although EA is a very slow optimization algorithm, it is very robust and returns a satisfying result. At this stage of research, EA serves our purpose because simulation time is not the main concern (it is to make sure that distribution system that has high penetration of PVs can be optimized using centralized control). However, once we move ahead to next stage where uncertainty of insolation has been included into VVWO, simulation time will be considered.

The following subsections show our analysis of distribution system when VVWO is utilized assuming the following circumstances: 1) high penetration of PVs, 2) our data are deterministic, and 3) the system has a centralized control. Two types of EA algorithms had been used. The first algorithm is the classical EA which is presented in section (4.1). The second algorithm is called Non-dominated Sorting Genetic Algorithm III (NSGA-III) which is explained in section (4.2). We utilized both algorithms to perform a VVWO and our results and findings are presented in section (4.3).

4.1 Classical Evolutionary Algorithm

In this section, the concept of classical evolutionary algorithm is going to be explained. Also, we will explain how multi-objective problem can be optimized using EA.
4.1.1 Optimization Problem Definition

Let us say we have the following optimization problem;

\[
\text{Minimize } \mathcal{Q} = \{\varphi_1(x), \varphi_2(x), \ldots, \varphi_M(x)\}
\]

Subject to

\[
\begin{align*}
    g_j(x) &= 0, \ j = 1,2,\ldots,J \\
    h_k(x) &\geq 0, \ k = 1,2,\ldots,K \\
    x^{(L)} &\leq x \leq x^{(U)}
\end{align*}
\]

Above system shows \(M\) number of objectives that are contained in the objective set \(\mathcal{Q}\). The goal is to find the solution \(x\) that minimizes all objectives \(\varphi\)’s. The solution \(x\) is a vector that contains all of the control variables. However, the control variables must be within their lower \(x^{(L)}\) and upper \(x^{(U)}\) bounds. Also, the optimization problem is subjected to \(J\) number of equality constraints \(g\) and \(K\) numbers of inequality constraints \(h\).

![Figure 4.1 The implementation of classical EA](image-url)
For the sake of simplicity, let us denote the $i^{th}$ objective, $k^{th}$ equality, and $j^{th}$ inequality as $\phi_i$, $h_k$, and $g_j$ respectively. Also, let us denote the objective set that is generated by $i^{th}$ solution $x_i$ as $\Omega_i$. Also, let $\phi_{k,i}$ denotes the $k^{th}$ objective generated by $i^{th}$ solution $x_i$.

4.1.2 EA and Objective Definition

In evolutionary algorithm (EA), the objective set has to be converted to a single objective as follow;

$$\phi = w_1\phi_1 + w_2\phi_2 + \cdots + w_M\phi_M$$  \hspace{1cm} (4.2)

The weights $\{w_1, w_2, \cdots, w_M\}$ are constants that are selected by the user to represent the cost of individual objectives. Selection of weights is very crucial in EA because an objective that is associated with a high weight is going to be optimized on the expense of other objectives. Another way to get around assigning weights is to normalize all objectives as follow;

$$\phi = \overline{\phi}_1 + \overline{\phi}_2 + \cdots + \overline{\phi}_M$$ \hspace{1cm} (4.3)

$\overline{\phi}_i$ is the normalized version of $\phi_i$ such that $0 \leq \overline{\phi}_i \leq 1$ which could (because there are many ways) be found as follow;

$$\overline{\phi}_i = \frac{\phi_i - \phi_i^{\text{min}}}{\phi_i^{\text{max}} - \phi_i^{\text{min}}}$$ \hspace{1cm} (4.4)

The maximum $\phi_i^{\text{max}}$ and minimum $\phi_i^{\text{min}}$ values of $\phi_i$ are predetermined by the user. Sometimes the values of $\phi_i^{\text{min}}$ and $\phi_i^{\text{max}}$ are obvious to the user. For instance, if $\phi_i$ represents the tap position of a voltage regulator that has $\pm 16$ taps, the values of $\phi_i^{\text{min}}$ and $\phi_i^{\text{max}}$ are going to be equal to “-16” and “16”, respectively. However, there are occasions when the user can’t
predetermine the values of $\phi_i^{min}$ and $\phi_i^{max}$. In the latter case, the user may find the value of $\phi_i^{min}$ and $\phi_i^{max}$ based on other methods. One of these methods are called Pareto Corner Search Evolutionary Algorithm (PCSEA), [380], which is going to be explained in the following section.

The classical EA receives two inputs: the initial population and the input data. Initial population consists of many individuals. Each individual contains, simply, a distinct setting of the control variables. The input data are the circumstances where the optimization problem in equation (4.1) has to be solved. For instance, the input data could be the parameters of irradiance or load. Then, these input data will be used to find the initial population. Then, the fitness of the initial population will be evaluated. The fitness for each individual can be found by finding $\mathcal{Q}$ and $\nu$ for each individual ($\nu$ is defined in NSGA-III section, it is the number of violations to the constraints). Then, a group of individuals are going to be selected from the initial population to create an offspring.

### Table 4.1 The initial Population Evaluation of PCSEA example

<table>
<thead>
<tr>
<th>solutions</th>
<th>Objectives $\phi_1(x)$</th>
<th>Objectives $\phi_2(x)$</th>
<th>Objectives $\phi_3(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0.0617</td>
<td>0.1561</td>
<td>0.1173</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.3924</td>
<td>0.5660</td>
<td>0.0336</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.5446</td>
<td>0.0183</td>
<td>0.4089</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.6359</td>
<td>0.2619</td>
<td>0.0731</td>
</tr>
<tr>
<td>$x_5$</td>
<td>0.0365</td>
<td>0.7365</td>
<td>0.6474</td>
</tr>
<tr>
<td>$x_6$</td>
<td>0.2322</td>
<td>0.4008</td>
<td>0.0357</td>
</tr>
<tr>
<td>$x_7$</td>
<td>0.2440</td>
<td>0.3225</td>
<td>0.1113</td>
</tr>
<tr>
<td>$x_8$</td>
<td>0.6014</td>
<td>0.0876</td>
<td>0.1886</td>
</tr>
<tr>
<td>$x_9$</td>
<td>0.9205</td>
<td>0.1960</td>
<td>0.1153</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>0.7453</td>
<td>0.0277</td>
<td>0.2315</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>0.0617</td>
<td>0.1561</td>
<td>0.1173</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>0.3924</td>
<td>0.5660</td>
<td>0.0336</td>
</tr>
</tbody>
</table>
The process of tournament selection operation (TSO), cross over and mutations are going to be utilized to create an offspring. These processes are explained in NSGA-III section. Above discussion is summarized in figure (4.1).

Once an offspring is created, it is going to be considered the current population. Then, the previous steps are going to be repeated to create another population and so on, until the convergence criteria is met [355].

<table>
<thead>
<tr>
<th>solutions</th>
<th>Objectives</th>
<th>Second norm of objectives, excluding one</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_1(x)$ $\rho_2(x)$ $\rho_3(x)$</td>
<td>$|{\rho_2, \rho_3}|$ $|{\rho_1, \rho_3}|$ $|{\rho_1, \rho_2}|$</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.06170 1.5610 0.1173</td>
<td>0.19526</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.39240 0.5660 0.0336</td>
<td>0.566996</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.54460 0.0183 0.4089</td>
<td>0.409309</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.63590 0.2619 0.0731</td>
<td>0.27191</td>
</tr>
<tr>
<td>$x_5$</td>
<td>0.03650 0.7365 0.6474</td>
<td>0.980591</td>
</tr>
<tr>
<td>$x_6$</td>
<td>0.23220 0.4008 0.0357</td>
<td>0.402387</td>
</tr>
<tr>
<td>$x_7$</td>
<td>0.24400 0.3225 0.1113</td>
<td>0.341166</td>
</tr>
<tr>
<td>$x_8$</td>
<td>0.60140 0.0876 0.1886</td>
<td>0.207951</td>
</tr>
<tr>
<td>$x_9$</td>
<td>0.92050 0.1960 0.1153</td>
<td>0.227399</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>0.74530 0.0277 0.2315</td>
<td>0.233151</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>0.06170 1.5610 0.1173</td>
<td>0.19526</td>
</tr>
<tr>
<td>$x_{12}$</td>
<td>0.39240 0.5660 0.0336</td>
<td>0.566996</td>
</tr>
</tbody>
</table>

To assess convergence, fitness value of the best solution of the recent population is compared with that of the best solution of few populations back. If there is no improvement in the fitness of the recent generation with respect to the selected $i^{th}$ population, convergence criterion is met and the solution is reported.
4.1.3 Pareto Corner Search Evolutionary Algorithm (PCSEA)

The purpose of PCSEA is to find the global solutions of individual objectives. The values of $\varphi_l^{\text{max}}$ and $\varphi_l^{\text{min}}$ represents the maximum and minimum global optima of $\varphi_l$, respectively. PCSEA is a search algorithm that can be explained using an example consisting of 3 objectives, [380], to find $\varphi_l^{\text{min}}$. The same process can be followed to find $\varphi_l^{\text{max}}$.

First, an initial population $\mathbb{P}_0$ with size $N = 12$ is generated. Then, the objectives of all solutions are going to be evaluated. Let us assume that the objectives of all populations are as shown in Table (4.1).

<table>
<thead>
<tr>
<th>Objectives $\varphi_1(x), \varphi_2(x), \varphi_3(x)$</th>
<th>Second norm of objectives, excluding one $|{\varphi_2, \varphi_3}|, |{\varphi_1, \varphi_3}|, |{\varphi_1, \varphi_2}|$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_5, x_3, x_2$</td>
<td>$x_1, x_1, x_1$</td>
</tr>
<tr>
<td>$x_{12}, x_{10}, x_6$</td>
<td>$x_8, x_6, x_1$</td>
</tr>
<tr>
<td>$x_1, x_8, x_4$</td>
<td>$x_9, x_7, x_7$</td>
</tr>
<tr>
<td>$x_{11}, x_{11}, x_7$</td>
<td>$x_{10}, x_{11}, x_6$</td>
</tr>
<tr>
<td>$x_6, x_1, x_9$</td>
<td>$x_4, x_2, x_3$</td>
</tr>
<tr>
<td>$x_7, x_9, x_1$</td>
<td>$x_{11}, x_8, x_{12}$</td>
</tr>
<tr>
<td>$x_2, x_4, x_8$</td>
<td>$x_7, x_4, x_8$</td>
</tr>
<tr>
<td>$x_3, x_7, x_{10}$</td>
<td>$x_6, x_5, x_4$</td>
</tr>
<tr>
<td>$x_8, x_6, x_{11}$</td>
<td>$x_3, x_3, x_2$</td>
</tr>
<tr>
<td>$x_4, x_2, x_3$</td>
<td>$x_2, x_{10}, x_5$</td>
</tr>
<tr>
<td>$x_{10}, x_{12}, x_5$</td>
<td>$x_{12}, x_{12}, x_{10}$</td>
</tr>
<tr>
<td>$x_9, x_5, x_{12}$</td>
<td>$x_5, x_9, x_9$</td>
</tr>
</tbody>
</table>

The next step is to take the second norm of all objectives except the first objective $\|\{\varphi_2, \varphi_3, \ldots, \varphi_M\}\|$. Then, take the second norm of all objectives except the second
objectives \[ \|\{\mathcal{O}_1, \mathcal{O}_3, \mathcal{O}_4, \cdots, \mathcal{O}_M\}\| \], and so on. Keep taking the second norm of all objectives while one of them is excluded until all objectives are exhausted \[ \|\{\mathcal{O}_1, \mathcal{O}_2, \cdots, \mathcal{O}_{M-1}\}\| \]. Then, add them to Table (4.1), which is redrawn in Table (4.2) for convenience. The purpose of taking the second norm of all objectives except one is well explained in [380].

The next step is to sort each column in Table (4.2) in an ascending order. The sorting process for this example is shown in Table (4.3). For instance, the minimum value of \( \mathcal{O}_1 \) is 0.0365, which is generated by solution \( x_5 \). Thus, the first row of \( \mathcal{O}_1 \) column in Table (4.3) should contain \( x_5 \).

**Pseudocode 4.1: Algorithm of PCSEA**

\[
\begin{align*}
1: & \text{Set } j = 1 \\
2: & \text{For each } x_i \in \mathbb{P}_j \\
3: & \quad \text{Evaluate } \mathcal{O}_{k,i} \text{ where } k = \{1,2,\cdots,M\} \\
4: & \text{For each } k \text{ where } k = \{1,2,\cdots,M\} \\
5: & \quad \text{Find } \|\{\mathcal{O}_1, \mathcal{O}_2, \cdots, \mathcal{O}_M\} - \{\mathcal{O}_k\}\| \\
6: & \quad \text{Sort each } x_i \in \mathbb{P}_j \\
7: & \text{Set } \mathbb{P}_{j+1} = \{\text{empty}\} \\
8: & \text{Select } \text{Opt} = \{x: \min(\mathcal{O}(x))\} \\
9: & \text{Select } \text{Opt} = \text{Opt} \cup \{x: \min(\|\ast\|)\} \\
10: & \text{Update } \mathbb{P}_{j+1} = \mathbb{P}_{j+1} \cup \{\text{Opt}\} \\
11: & \text{While } \text{size}\left(\mathbb{P}_{j+1}\right) < N \\
12: & \quad \text{Update } \mathbb{P}_{j+1} = \mathbb{P}_{j+1} \cup \{\text{mutate}(x \in \mathbb{P}_{j+1})\} \\
13: & \text{Update } j = j + 1 
\end{align*}
\]

Now, the first row of Table (4.3) contains the solutions that would be selected to the next generation \( \mathbb{P}_1 \). If one of the solutions is repeated in the first column, the solution of the underneath
row is going to be selected to join \( \mathbb{P}_1 \). The selected solutions in Table (4.3) are shaded. The set \( \mathbb{P}_1 \) is going to be;

\[
\mathbb{P}_1 = \{x_5, x_3, x_2, x_1, x_6, x_{11}\}
\] (4.5)

Notice that the size of \( \mathbb{P}_1 \) is equal to 6 which is less than \( N, (N = 12) \). Now, the process of the cross over and mutations, [381], are going to be implement on \( \mathbb{P}_1 \) to increase its size in order to make it equal to 12. Then, the same process is going to be repeated to generate \( \mathbb{P}_2 \) for predetermined number of generations. The last generation would contain the pareto minima of \( \varnothing_i^{min} \). The pseudocode (4.1) summaries PCSEA algorithm.

4.2 NSGA-III

The following subsections explain NSGA-III.

4.2.1 Optimization Problem Definition

NSGA-III is a short abbreviation of Non-Dominated Sorting Genetic Algorithm III. It is called III because NSGA’s authors have edited it through time and released three versions of it. Let us start by the system that is shown in equation (4.1). Above system shows \( M \) number of objectives that are contained in the objective set \( \mathcal{Q} \). The goal is to find the solution \( x \) that minimizes all objectives \( \varnothing \)’s. The solution \( x \) is a vector that contain the parameters of the control variables. However, the control variables must be within their lower \( x^{(L)} \) and upper \( x^{(U)} \) bounds. Also, the optimization problem is subjected to \( J \) number of equality constraints \( g \) and \( K \) numbers of inequality constraints \( h \). For convenience, the notations that has been already defined in section (4.1) are going to be reiterated here: let us denote the \( i^{th} \) objective, \( k^{th} \) inequality, and \( j^{th} \) equality
as $\varnothing_i$, $h_k$, and $g_j$ respectively. Also, let us denote the $i^{th}$ objective set that are generated by solution $x_i$ as $\mathcal{Q}_i$. Also, let $\varnothing_{k,i}$ denotes the $k^{th}$ objective set that is generated by $i^{th}$ solution $x_i$.

### 4.2.2 NSGA-III Description

In multi-objective optimization problems, the objectives contradict each other. This means that the optimization problem reaches to a level where a simultaneous optimization of all objectives cannot be reached. The classical way to deal with this problem is to convert all objectives to a single objective by either assigning weights to each objective or by moving all objectives to the constraint sets except one objective. The shortcomings of the aforementioned solutions are: the difficulty of selecting the proper weights and/or the increasing number of the constraint values because moving an objective to the constraint sets requires establishing an additional constraint value [382].

![Macro view of the NSGA-III algorithm](image)

To cope with the aforementioned problems, Pareto optimality is proposed extensively in literature [382]. The essence of Pareto optimality is to find a set of solutions that don’t dominate
one another such that no objective can be improved without damaging other objectives. One of the optimization methods that adopt Pareto optimality approach is the Non-dominated Sorting Genetic Algorithm-III (NSGA-III) [383], [384]. NSGA-III is an optimization method that belongs to the genre of evolutionary algorithms.

The following subsection presents an overall view on NSGA-III. The next subsection explains how to sort the solutions to different fronts. Third subsection explains how to find whether solution X dominates Y or not. Fourth subsection explains how to create an offspring. Fifth subsection explains how the elite solutions are selected of each population.

4.2.2.1 Macro View on NSGA-III

This subsection presents a macro view on how NSGA-III works while the following subsections will explain them in detail.

NSGA-III starts with a set $\mathbb{P}$ of initial solutions that contains $N$ number of solutions. Then, an offspring set $\mathbb{Q}$ with size $N$ is created (explained later). Then, both sets are combined into set $\mathbb{R}$ ($\mathbb{R} = \mathbb{P} \cup \mathbb{Q}$). The solutions that are contained in set $\mathbb{R}$ are going to be classified to different fronts. A front is a set that contain solutions that don’t dominated one another (concept of domination will be explained later). Also, if the rank of front X is lower than the rank of front Y, this means all of the solutions that belong to front X dominate all of the solutions that belong to front Y. For instance, the solutions that belong to third front $\mathcal{F}_3$ are dominated by the solutions that belong to the first $\mathcal{F}_1$ and second $\mathcal{F}_2$ fronts. The first front $\mathcal{F}_1$ would contain the solutions that don’t dominate one another but they dominate the remaining solutions in $\mathbb{R}$. The process of classifying solutions to different fronts is called sorting. Then, $N$ number of elite solutions are selected from $\mathbb{R}$ and added to set $\mathbb{S}$. The solutions in set $\mathbb{S}$ are simply the next generation. These
processes are going to be repeated for a certain number of times (or generations) before the algorithm is terminated. The solutions that belong to set $F_1$ of last generation are the Pareto optimal solutions. Above discussion are sketched in figure (4.2).

4.2.2.2 Sorting process

To explain sorting process according to NSGA-III let us assume that the $i^{th}$ solution that belongs to the $j^{th}$ front $F_j$ is denoted as $x_{i,j}$.

---

**Pseudocode 4.2: Sorting Algorithm of NSGA-III**

1: For each $x_i \in P$
2: Set $S_i = F_1 = \{\text{Empty}\}$, and $n_i = 0$
3: For each $x_j \in P$ such that $i \neq j$
4: \hspace{1em} if $x_i$ dominates $x_j$
5: \hspace{2em} $S_i = S_i \cup \{x_j\}$
6: \hspace{1em} if $x_j$ dominates $x_i$
7: \hspace{2em} $n_i = n_i + 1$
8: For each $x_i \in P$
9: \hspace{1em} if $n_i = 0$
10: \hspace{2em} $F_i = F_i \cup \{x_i\}$
11: Set $j = 1$
12: While $F_j \neq \{\text{Empty}\}$
13: Set $F_{j+1} = \{\text{Empty}\}$
14: For each $x_i \in F_j$
15: \hspace{1em} For each $x_q \in S_i,j$
16: \hspace{2em} $n_q = n_q - 1$
17: \hspace{1em} if $n_q = 0$
18: \hspace{2em} $F_{j+1} = F_{j+1} \cup \{x_q\}$
19: \hspace{1em} Set $j = j + 1$

---

Two entities $n$ and $S$ are developed for all solutions. The solution $x_i$ is dominated by $n_i$ number of solutions, and the solution set $S_i$ contains all solutions that are dominated by the solution $x_i$. The entities $n_i$ and $S_i$ of solution $x_i$ can be found by comparing $x_i$ with all solutions that belong to population $P$, except $x_i$ itself. The entities $n_i$ and $S_i$ of $i^{th}$ solution that belongs to front $F_j$ are denoted as $n_{i,j}$ and $S_{i,j}$.
The first step is to find the entities $n$ and $\mathcal{S}$ of all solutions in $\mathcal{P}$. Then, any solution with $n_t = 0$, it has to be assigned to the first front $\mathcal{F}_1$.

The second step is to go after each solution inside $\mathcal{F}_1$. For each $\mathbf{x}_{t,1}$ (all solutions that belong to $\mathcal{F}_1$), update solutions inside the set $S_{t,1}$ as follow; if $\mathbf{x}_q$ belongs to $S_{t,1}$, decrease $n_q$ by 1 ($n_{q,new} = n_{q,old} - 1$). If the new value of $n_q$ is zero, the solution $\mathbf{x}_q$ will be assigned to $\mathcal{F}_2$.

The third step is to repeat the second step for the remaining fronts. The algorithm of sorting process is presented in Pseudocode (4.2).

---

**Pseudocode 4.3: Domination test according to NSGA-III**

1: If each $\nu_j > \nu_i$
2: $\mathbf{x}_i$ dominates $\mathbf{x}_j$
3: If each $\nu_j = \nu_i$
4: For all objective $k \in M$
5: Set $c_i = c_j = 0$
6: If $\emptyset_{k,i} < \emptyset_{k,j}$
7: $c_i = c_i + 1$
8: If $\emptyset_{k,j} < \emptyset_{k,i}$
9: $c_j = c_j + 1$
10: Otherwise
11: no update to $c_j$ or $c_i$
12: If $c_j > c_i$
13: $\mathbf{x}_i$ dominates $\mathbf{x}_j$
14: If $c_i < c_j$
15: $\mathbf{x}_j$ dominates $\mathbf{x}_i$
16: If $c_i = c_j$
17: no solution dominates

---

**4.2.2.3 Domination**

In this section, the criteria to determine whether solution $\mathbf{x}_i$ dominates solution $\mathbf{x}_j$ or not is explained. Two factors determine whether a solution dominates other solution or not: number of constraint violations and improvements in the objectives. Let $\nu_i$ denotes the number of violations to the constraints of $i^{th}$ solution $\mathbf{x}_i$. The violations $\nu_i$ can be found by testing $\mathbf{x}_i$ against the
constraints \( g, h, x^{(l)} \) and \( x^{(u)} \) in equation (4.1). Also, let \( \mathcal{O}_i \) denotes the set of objectives that are generated by solution \( x_i \), as shown in equation (4.1). Let \( \wp_{k,i} \) denotes the value of \( k^{th} \) objective that is generated by solution \( x_i \), while \( k = \{1,2,\ldots,k,\ldots,M-1,M\} \).

The solution \( x_j \) dominates \( x_i \) whenever \( \wp_i > \wp_j \). If \( \wp_i = \wp_j \), then \( x_j \) and \( x_i \) are going to be compared based on \( \mathcal{O}_i \) and \( \mathcal{O}_j \). To test dominance using the objective sets \( \mathcal{O}_i \) and \( \mathcal{O}_j \), the individual objectives of both solutions are going to be compared. This can be performed by introducing two counters \( c_i \) and \( c_j \) for both \( x_i \) and \( x_j \) solutions, respectively. Also, let us assume that the optimization problem is about minimizing the objectives. To compare \( x_j \) and \( x_i \) based on \( \mathcal{O}_i \) and \( \mathcal{O}_j \), we start by the first objective as follow;

- If \( \wp_{1,i} < \wp_{1,j} \), the counter \( c_i \) is updated by increasing its value by 1 \( (c_{i,\text{new}} = c_{i,\text{old}} + 1) \).
- If \( \wp_{1,i} = \wp_{1,j} \), no updates happen to either counters.

Above process is going to be repeated for all objectives \( M \) (the size of \( \mathcal{O} \) is \( M \)). Once all objective has been exhausted, dominance will be determined based on the following criteria;

- If the \( c_i > c_j \), \( x_i \) dominates \( x_j \)
- Both solutions \( x_i \) and \( x_j \) don’t dominate each other if \( c_i = c_j \).

The latter case is an indication that both solutions belong to the same front. The domination test is presented Pseudocode (4.3).
4.2.2.4 Offspring creation

The \( i^{th} \) population \( \mathbb{P}_i \) is a set containing \( N \) number of solutions. According to NSGA-III, it is recommended to make \( N \) equal to the following:

\[
N = 4 \left( \frac{M + p - 1}{p} \right)
\]  
(4.6)

The constant \( M \) is the number of the objectives while \( p \) is an integer represents the selected number of divisions per an objective (choosing a value for \( p \) and the reason for having it will be explained later).

---

**Pseudocode 4.4: TSO, crossover, and mutation process**

1: Set \( Q = \{\text{empty}\} \)
2: For 1 to \( N/2 \):
3: \hspace{1em} Select randomly \( x_k \) and \( x_y \) from \( \mathbb{P}_i \)
4: \hspace{2em} If \( \sigma_y = \sigma_k \)
5: \hspace{3em} \( Q = Q \cup \{x_k \text{ or } x_y\} \)
6: \hspace{2em} If \( \sigma_y < \sigma_k \)
7: \hspace{3em} \( Q = Q \cup \{x_y\} \)
8: \hspace{2em} If \( \sigma_k < \sigma_y \)
9: \hspace{3em} \( Q = Q \cup \{x_k\} \)
10: For \((N/2) + 1\) to \( N \):
12: \hspace{1em} Select randomly \( x_k \) and \( x_y \) from \( Q \)
13: \hspace{2em} \( x_i = \text{crossover and mutation}\{x_k, x_y\} \)
14: \( Q = Q \cup \{x_i\} \)

To create the next population \( \mathbb{P}_{i+1} \) (or the offspring), NSGA-III selects two solutions \( x_k \) and \( x_y \) from the population \( \mathbb{P}_i \). If both solutions are infeasible (meaning both \( \sigma_k \) and \( \sigma_y \) aren’t equal to zero), the solution with smaller \( \sigma \) is added to the set \( Q \). If one solution is feasible while the other infeasible, the feasible solution is added to the set \( Q \). If both solutions are feasible, one of them is randomly added to \( Q \). This process is called tournament selection operation (TSO). Once the size of \( Q \) becomes equal to \( N/2 \), the TSO process has to be terminated.
Then, two solutions are selected randomly from $\mathbb{Q}$ set. Then, the crossover and mutation process, as described in [381], are going to be applied to the selected solutions to create a new solution (a child). Then, the new solution (or the child) is added to $\mathbb{Q}$. This process has to be repeated until the size of $\mathbb{Q}$ becomes equal to $N$. Crossover and mutation process are omitted in this document and the interested reader is advised to refer to [381]. Offspring creation is summarized Pseudocode (4.4).

4.2.2.5 Elite Selection

The elite solutions will be selected from the set $\mathbb{R}$ and dumped into set $\mathbb{S}$. During selection process, we give the priority to the solutions that belong to a lower rank front. For instance, the solutions that belong to $\mathcal{F}_1$ is favored over to those belonging to $\mathcal{F}_2$. However, only $N$ number of solutions has to be selected from the set $\mathbb{R}$. Consequently, we may end up with a set of solutions that belong to the same front $\mathcal{F}_u$ and we have to select few of them to be dumped into set $\mathbb{S}$. To resolve this issue, we can select the solutions from $\mathcal{F}_u$ based on their location in the objective space. Solutions that are located in less density areas are going to be favored to preserve diversity over the search space. Figure (4.3) helps the reader to visualize the effect of maintaining spatial diversity of solutions in search space.

To select among solutions based on their dispersion, we have to know where they are located in the objective space. NSGA-III adopted a method to know solutions’ locations in the objective space. The method is originally developed by Das in [385] and is called Normalized Boundary Intersection (NBI). NBI has been edited and improved by many researchers. The adopted NBI by NSGA-III is an edited version of the original one. The edited version of NBI is going to be denoted in this work as ENBI.
An outline of ENBI steps are going to be presented in the following paragraph. Then, a thorough explanation to each step will be provided afterwards, followed by an example.

To recapitulate, set $\mathbb{S}$ already contains the solutions that belong to fronts $\mathcal{F}_1$ up to $\mathcal{F}_{u-1}$ but we need to select some solutions from $\mathcal{F}_u$ using ENBI in order to make the size of $\mathbb{S}$ equals to $N$. ENBI starts by creating a hyperplane of the feasible solutions that belong to set $\mathbb{S}$. Then, normalize the hyperplane. Then, create a reference points on the hyperplane that are evenly distributed. Then, measure misalignment between each solution that belongs to $\mathcal{F}_1$ up to $\mathcal{F}_u$ and each reference point. Then, based on misalignment, assign each solution to its nearest reference point. The reference point that is associated with high number of solutions indicates that its vicinity

Figure 4.3 The solid circles represent the projected locations of a solution in the objective space. The example is for two objective problem. All solutions belong to the same front because no one dominate others. The figure on the left shows how the solutions that belong to the same front aren’t equally spaced. The figure on the right shows the feasible region and due to misplacement of the solutions there will be an explored area. This problem can be solved if we keep the solutions evenly distributed over the objective space as much as possible.
is highly crowded, and vice versa. Then, the solutions that belongs to \( \mathcal{F}_u \) and located in a less crowded area will be favored.

The hyperplane would take the following form;

\[
\alpha_1 \varphi_1 + \alpha_2 \varphi_2 + \cdots + \alpha_M \varphi_M = 1
\] (4.7)

To find the coefficients \( \alpha' \)'s in equation (4.7), we have to normalizing all objectives. This can be done, by finding the minimum value of each objective among the solutions available in set \( \mathcal{S} \), excluding the solutions that are infeasible (their \( \nu > 0 \)). If \( j^{th} \) solution \( (x_j) \) generates the minimum value of \( k^{th} \) objective \( (\varphi_{k,j}) \), they are going to be denoted as \( x_{k,min} \) and \( \varphi_{k,min} \) respectively. Therefore, a solution \( x_{i,min} \) would generate an objective set \( \Omega = \{\varphi_{1,i,min}, \varphi_{2,i,min}, \cdots, \varphi_{M,i,min}\} \), which can be denoted as \( \Omega(x_{i,min}) \). If we normalized the objectives set \( \Omega(x_{i,min}) \), it going to be denoted as \( \tilde{\Omega}(x_{i,min}) \).

Now, we can find the normalized the objective sets that are generated by the solutions \( x_{1,min}, x_{2,min}, \ldots, x_{M,min} \) as follow;

\[
\tilde{\Omega}(x_{i,min}) = \{\varphi_{1,i,min} - \varphi_{1,min}, \varphi_{2,i,min} - \varphi_{2,min}, \cdots, \varphi_{M,i,min} - \varphi_{M,min}\} = \{0, \cdots, [0], \cdots, [0], \cdots, [0]\}
\] (4.8)

The symbols of above equation are;

- \( \tilde{\Omega}(x_{i,min}) \) represent the normalized objective set that is generated by solution \( x_{i,min} \).
- \( \varphi_{k,i,min} \) denotes the value of \( k^{th} \) objective that is generated by solution \( x_{i,min} \).
\( \varphi_k^{\text{min}} \) denotes the value of \( k^{\text{th}} \) objective that is generated by solution \( x_{k,\text{min}} \) provided that \( \varphi_k^{\text{min}} \) is the lowest value in set \( S \).

\( \overline{\varphi}_{k,i,\text{min}} \) denotes the normalized value of the objective \( \varphi_{k,i,\text{min}} \).

Now we are ready to find the coefficients \( \alpha' \)'s, using the following linear system;

\[
\begin{bmatrix}
\overline{\varphi}_{1,1,\text{min}} & \overline{\varphi}_{1,2,\text{min}} & \cdots & \overline{\varphi}_{1,M,\text{min}} \\
\overline{\varphi}_{2,1,\text{min}} & \overline{\varphi}_{2,2,\text{min}} & \cdots & \overline{\varphi}_{2,M,\text{min}} \\
\vdots & \vdots & \ddots & \vdots \\
\overline{\varphi}_{M,1,\text{min}} & \overline{\varphi}_{M,2,\text{min}} & \cdots & \overline{\varphi}_{M,M,\text{min}}
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_M
\end{bmatrix} =
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\] (4.9)

The shifted-pay-off-matrix \( \Phi \) is defined as the transpose of the system matrix in equation (4.9) as follow [386];

\[
\Phi =
\begin{bmatrix}
\overline{\varphi}_{1,1,\text{min}} & \overline{\varphi}_{1,2,\text{min}} & \cdots & \overline{\varphi}_{1,M,\text{min}} \\
\overline{\varphi}_{2,1,\text{min}} & \overline{\varphi}_{2,2,\text{min}} & \cdots & \overline{\varphi}_{2,M,\text{min}} \\
\vdots & \vdots & \ddots & \vdots \\
\overline{\varphi}_{M,1,\text{min}} & \overline{\varphi}_{M,2,\text{min}} & \cdots & \overline{\varphi}_{M,M,\text{min}}
\end{bmatrix}^T
\] (4.10)

Now, the objectives of all solutions that belong to set \( S \) should be normalized according to NSGA-III. However, the normalization step is already embedded into the previous steps and no need to perform any further normalization.

Then, create the reference points on the hyperplane. For each reference point we have to create a vector \( \beta \). Let the vector \( \beta_i \) denotes the vector \( \beta \) of \( i^{\text{th}} \) reference point. The \( j^{\text{th}} \) element of \( \beta_i \) is denoted as \( \beta_{i,j} \), where \( j = \{1,2,\cdots,M\} \). Each vector \( \beta \) must satisfy the following conditions

\( \sum_{j=1}^{M} \beta_{i,j} = 1 \)

\( 0 \leq \beta_{i,j} \leq 1 \)
Each reference point $\beta$ is going to have a location in the objective space. This can be done by multiplying the matrix $\Phi$ by $\beta$. For instance, we can find the location of $j^{th}$ reference point $\beta_j$ in the objectives space as follow;

$$
\begin{bmatrix}
\phi_{1,j,ref} \\
\phi_{2,j,ref} \\
\vdots \\
\phi_{M,j,ref}
\end{bmatrix} = [\Phi]
\begin{bmatrix}
\beta_{j,1} \\
\beta_{j,2} \\
\vdots \\
\beta_{j,M}
\end{bmatrix}
\tag{4.11}
$$

The $i^{th}$ coordinates in objective space of reference point $\beta_j$ is denoted in above equation as $\phi_{i,j,ref}$. Above equation can be represented in matrix form as follow;

$$
\phi_{j,ref} = \Phi\beta_j
\tag{4.12}
$$

Notice that the selected values of the vector $\beta_j$ determines its location $\phi_{j,ref}$ in objective space. However, we want the reference points to be distributed uniformly distributed over the objective space. Therefore, a careful selection of $\beta'$s is a must. Let us say we want to create $W$ number of reference points. This means we have to create $W$ number of $\beta$ vectors, which can be calculated as follow;

$$
W = \binom{M + p - 1}{p}
\tag{4.13}
$$

The parameters $p$ is an integer number and is selected by the user [it is exactly the same parameter that is mentioned in section (4.2.2.4)]. It represents the number of divisions per objective. Higher value of $p$ means more reference points are going to be created over the objective space.

Then, create matrix $Z$ of size ($W \times M$), where $Z_{(r),(c)}$ denotes the element of $Z$ that is located at $r^{th}$ row and $c^{th}$ column. To create the matrix $Z$, start by constructing the first row of $Z$. 89
Then, move to the next row and so on until you create \( W \) number of rows. The \( r^{th} \) row can be constructed as follow;

- Select randomly one element from set \( I_1 \) and assign it to \( Z_{(r),(1)} \). The set \( I_1 \) is formed as \( I_1 = \{0, \frac{1}{p}, \frac{2}{p}, \ldots, \frac{p}{p}\} \).

- Select randomly one element from set \( I_2 \) and assign it to \( Z_{(r),(2)} \). The set \( I_2 \) is constructed as follow \( I_2 = \{0, \frac{1}{p}, \frac{2}{p}, \ldots, \frac{k_2}{p}\} \) where \( k_2 = \lfloor (p - m_1) \rfloor \). The brackets \( [\ast] \) denotes the greatest integer less than or equal the values inside the brackets. The constant \( m_1 \) can be found as follow; \( m_1 = p \times Z_{(r),(1)} \).

- Generally, the value that is assigned to element \( Z_{(r),(j)} \) is going to be drawn randomly from the set \( I_j \). The set \( I_j \) is constructed as follow; \( I_j = \{0, \frac{1}{p}, \frac{2}{p}, \ldots, \frac{k_j}{p}\} \) where \( k_j = \lfloor (p - \sum_{i=1}^{j-1} m_i) \rfloor \) and \( m_{j-1} = p \times Z_{(r),(j-1)} \).

- Continue filling the remaining elements of \( r^{th} \) row up to \( Z_{(r),(M-1)} \) the same way as in the previous step.

- The last element \( Z_{(r),(M)} \) is filled differently using the follow formula; \( Z_{(r),(M)} = 1 - \sum_{j=1}^{M-1} Z_{(r),(j)} \)

Caveat: the rows of \( Z \) has to be formed such that they show different patterns. In other words, no duplicated rows. Therefore, the random selection of set \( I \) isn’t completely random but a careful selection.

Once matrix \( Z \) is created, \( \beta_j \) is going to be equal to the \( j^{th} \) column of \( Z^{transpose} \), as shown in below equation. Now we have to assign each solution to a reference point. This is can be done
by measuring how much the vector going from origin to the solution’s location in the objective space is aligned with the vector that is going to the reference point. This can be done using the following equations;

\[
\begin{bmatrix}
\beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,W_1} \\
\beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,W_2} \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{M,1} & \beta_{M,2} & \cdots & \beta_{M,W_M}
\end{bmatrix} = \mathbf{Z}_T \tag{4.14}
\]

\[
d_{j,\text{ref}(i)} = \left\| \varrho_j - \left( \mathbf{\varrho}_{l,\text{ref}} \right)^T \varrho_j \mathbf{\varrho}_{l,\text{ref}} \right\| \tag{4.15}
\]

\[
\varrho_{l,\text{ref}} = \frac{1}{\sqrt{\sum_{k=1}^{M} \mathbf{\varrho}_{k,\text{ref}}}} \begin{bmatrix}
\mathbf{\varrho}_{1,\text{ref}} \\
\mathbf{\varrho}_{2,\text{ref}} \\
\vdots \\
\mathbf{\varrho}_{M,\text{ref}}
\end{bmatrix} \tag{4.16}
\]

\[
\varrho_j = \begin{bmatrix}
\mathbf{\varrho}_{1,j} \\
\mathbf{\varrho}_{2,j} \\
\vdots \\
\mathbf{\varrho}_{M,j}
\end{bmatrix} \tag{4.17}
\]

The parameter \(d_{j,\text{ref}(i)}\) measures how much the vector going from origin to the projection of solution \(\mathbf{x}_j\) in the objective space (\(\varrho_j\) is the projection of \(\mathbf{x}_j\) in the objective space) is aligned with the vector going to reference point \(\varrho_{l,\text{ref}}\). The symbol \(\|\ast\|\) denotes the second norm. A higher value of \(d_{j,\text{ref}(i)}\) indicates more misalignment between \(\varrho_j\) and \(\varrho_{l,\text{ref}}\), and vice versa.

The solution \(\mathbf{x}_m\) is going to be assigned to \(\beta_i\) if \(d_{m,\text{ref}(i)}\) is the smallest in this set \(\{d_{m,\text{ref}(1)}, d_{m,\text{ref}(2)}, \ldots, d_{m,\text{ref}(W)}\}\). Let \(\rho_{i,\text{ref}}\) denotes the number of solutions that are assigned to \(\beta_i\). Then, let the set \(p\) equal to \(\{\rho_{1,\text{ref}}, \rho_{2,\text{ref}}, \ldots, \rho_{W,\text{ref}}\}\). Also, let \(p_{\text{min}}\) be a subset of \(p\) that
contains the minimum values of $\rho'$s. Also, let $p_{non}$ be a subset of $p$ that contains the non-zero values of $\rho'$s.

Let $p_{u-1, min}$ is $p_{min}$ that is created using the solutions that belong to fronts $F_1$ up to $F_{u-1}$. Also, let $p_{u, non}$ is $p_{non}$ that is created using the solutions that belong to front $F_u$. In a simple language, $p_{u-1, min}$ represent the reference points that are located in an area where there is no too many solutions around. Also, $p_{u, non}$ is telling us where is the spatial location of the solutions that belong to $F_u$ are located in the objective space.

Then, the remaining vacant positions in set $S$ can be filled as follow;

- Step. 1: If there is a common reference point $\beta_x$ between $p_{u-1, min}$ and $p_{u, non}$, it means $\beta_x$ is located in a less crowded location and one of the solutions that belong to $F_u$ is located in that region. In this case, select randomly one of the solutions that are assigned to $\beta_x$ provided that it belongs to $F_u$ and add it to $S$. Then, delete the selected solution from $F_u$. Then, go to step. 3.
- Step. 2: If there is no common reference point between $p_{u-1, min}$ and $p_{u, non}$, this means that all solutions that belong to $F_u$ are located in highly crowded place. In this case, select randomly one of the solutions that belong to $F_u$ and add it to $S$. Then, delete the selected solution from $F_u$. Then, go to step. 3.
- Step. 3: if the size of $S$ is equal to $N$, terminate the iterations. Otherwise update $p_{u-1, min}$ and $p_{u, non}$ and go back to Step. 1.

Note: above steps are meant to be general guidelines.
Figure 4.4  The location of objectives and reference points in the objective space of the given example. (a) The dots show the projection of solution \( x_1, x_2, \) and \( x_3 \) in the objective space. (b) The dots show the normalized location of solution \( x_1, x_2, \) and \( x_3 \) in the objective space. (c) The location of the reference points in the objective space.
### 4.2.2.6 Illustrative example

To explain how to generate the reference points and how to assign the solutions to them, an example with two objectives is presented in this section. Let us have three solutions \( \mathbf{x}_1, \mathbf{x}_2, \) and \( \mathbf{x}_3 \) and we want to construct a hyperplane for them and associate each one to a reference point. Let the first solution \( \mathbf{x}_1 \) generates the objective set \( \mathbf{Q}(\mathbf{x}_1) = \{ \varphi_{1,1}(\mathbf{x}_1), \varphi_{2,1}(\mathbf{x}_1) \} = \{ 1, 9 \} \), the second solution \( \mathbf{x}_2 \) generates the objective set \( \mathbf{Q}(\mathbf{x}_2) = \{ \varphi_{1,2}(\mathbf{x}_2), \varphi_{2,2}(\mathbf{x}_2) \} = \{ 2, 5 \} \), and the third solution \( \mathbf{x}_3 \) generates the objective set \( \mathbf{Q}(\mathbf{x}_3) = \{ \varphi_{1,3}(\mathbf{x}_3), \varphi_{2,3}(\mathbf{x}_3) \} = \{ 3, 4 \} \). The location of these solutions in objective space are shown in Figure (4.4,a). Notice that the minimum values of the first and second objectives are \( \varphi_1 = 1 \) and \( \varphi_2 = 4 \) and they are generated by solution \( \mathbf{x}_1 \) and \( \mathbf{x}_3 \), respectively. Therefore, \( \varphi_{1,min} = 1, \varphi_{2,min} = 4, \mathbf{x}_{1,min} \equiv \mathbf{x}_1, \) and \( \mathbf{x}_{2,min} \equiv \mathbf{x}_3 \). Now, we can find the normalized set \( \bar{\mathbf{Q}}(\mathbf{x}_{1,min}) \) and \( \bar{\mathbf{Q}}(\mathbf{x}_{2,min}) \) as follow;

\[
\bar{\mathbf{Q}}(\mathbf{x}_{1,min}) = \bar{\mathbf{Q}}(\mathbf{x}_1) = \{ \varphi_{1,1,\text{min}}, \varphi_{2,1,\text{min}} \} = \{ [1 - 1], [9 - 4] \} = \{ 0, 5 \}
\]

\[
\bar{\mathbf{Q}}(\mathbf{x}_{2,min}) = \bar{\mathbf{Q}}(\mathbf{x}_3) = \{ \varphi_{1,2,\text{min}}, \varphi_{2,2,\text{min}} \} = \{ [3 - 1], [4 - 4] \} = \{ 2, 0 \}
\]

The other solutions can be normalized as follow;

\[
\bar{\mathbf{Q}}(\mathbf{x}_2) = \{ \varphi_{1,2}(\mathbf{x}_2) - \varphi_{1,\text{min}}, \varphi_{2,2}(\mathbf{x}_2) - \varphi_{2,\text{min}} \} = \{ 2 - 1, 5 - 4 \} = \{ 1, 1 \}
\]

The normalized objectives are shown in figure (4.4,b). To find the coefficients \( \alpha's \) of above example, the problem is going to be formulated as follow;

\[
\begin{bmatrix}
\varphi_{1,1,\text{min}} & \varphi_{2,1,\text{min}} \\
\varphi_{1,2,\text{min}} & \varphi_{2,2,\text{min}}
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix}
= \begin{bmatrix}
1 \\
1
\end{bmatrix}
\]
If we substitute the numbers in above equation we would get the following result;

\[
\begin{bmatrix}
0 & 5 \\
2 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\Rightarrow
\alpha_1 = \frac{1}{2}
\quad
\alpha_2 = \frac{1}{5}
\]

Then, the shifted-pay-off-matrix would look like the equation below. This matrix is simply defined for the ease of explanation.

\[
\Phi = \begin{bmatrix}
\bar{\varphi}_{1,1,min} & \bar{\varphi}_{2,1,min} \\
\bar{\varphi}_{1,2,min} & \bar{\varphi}_{2,2,min}
\end{bmatrix}^T
= \begin{bmatrix}
0 & 5 \\
2 & 0
\end{bmatrix}^T
= \begin{bmatrix}
0 & 2 \\
5 & 0
\end{bmatrix}
\]

If \( p \) is chosen to be \( p = 3 \), then the number of reference points \( W \) will be

\[
W = \frac{(2 + 3 - 1)!}{(3)! (2 + 3 - 1 - 3)!} = 4
\]

Then, the set \( I_1 \) is going to be \( I_1 = \{0,0.333,0.667,1\} \). Now start filling the first row;

- Let us select \( Z_{(1),(1)} = 0 \).
- Then, \( Z_{(1),(2)} = 1 - 0 = 1 \)

The next row is going to be filled as follow;

- Let us select \( Z_{(2),(1)} = 0.333 \).
- Then, \( Z_{(2),(2)} = 1 - 0.333 = 0.667 \)

The next row is going to be filled as follow;

- Let us select \( Z_{(3),(1)} = 0.667 \).
- Then, \( Z_{(3),(2)} = 1 - 0.667 = 0.333 \)
The last row is going to be filled as shown in the following steps. Both $Z_{(4),(1)}$ and $Z_{(1),(2)}$ are going to be equal to 1 and 0 respectively. follow;

- Let us select $Z_{(4),(1)} = 1$.
- Then, $Z_{(1),(2)} = 1 - 1 = 0$

The resultant matrix $Z$ is going to be as shown below. Just an extra note: you could notice the symmetry of matrix elements.

$$Z = \begin{bmatrix} 0 & 1 \\ 0.333 & 0.667 \\ 0.667 & 0.333 \\ 1 & 0 \end{bmatrix}$$

Now the vectors $\beta'$s are going to be extracted from $Z^{\text{transpose}}$ as follow;

$$\beta_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} 0.667 \\ 0.333 \end{bmatrix}, \quad \beta_3 = \begin{bmatrix} 0.333 \\ 0.667 \end{bmatrix}, \quad \beta_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Now the coordinates, see figure (4.4,c), of the reference points are going to be;

$$\varphi_{1,\text{ref}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\varphi_{2,\text{ref}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0.667 \\ 0.333 \end{bmatrix} = \begin{bmatrix} 0.667 \\ 3.333 \end{bmatrix}$$

$$\varphi_{3,\text{ref}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0.333 \\ 0.667 \end{bmatrix} = \begin{bmatrix} 1.333 \\ 1.667 \end{bmatrix}$$

$$\varphi_{4,\text{ref}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \end{bmatrix}$$

Notice that above reference points are distributed uniformly over objective space. So, if we assign each solution to these references, we would get an idea about the spatial map of the solutions.
in the objective space. This technique is very helpful when we have an optimization problem that has high number of objectives.

4.3 Case Study and Results

In this section, both classical EA and NSGA-III are going to be used in order to optimize a distribution system that involves high penetration of PVs. The IEEE123 test system has been edited in order to perform the simulation.

Figure 4.5 The schematic diagram of the modified 123-bus test distribution system. The red triangular dots show the locations of the installed switching capacitor banks. The green square dots show the locations where PVs are installed. The dark blue parallelogram dots show the CLs.

The following section contains a full description of the edited IEEE123 in addition to how PVs are modeled. Then, the results of classical EA and NSGA-III are explained in section (4.3.2) and (4.3.3) respectively. Finally, a summary of the findings is presented in section (4.3.4).
4.3.1 System Description

The IEEE 123-bus test distribution feeder is used for simulating VVWO over one day using both classic EA and NSGA-III. The system has been edited by adding 42 PVs and two SCs (see fig. 4.5). The locations of the installed PVs are shown in Table (4.4). Two three-phase SCs are installed at buses 50 and 100. SCs are assumed to be able to inject reactive power between 10 kvar and 200 kvar with a step size of 10 kvar. All three phases of SCs are assumed to change their status together (ganged operation). Also, 5 loads are chosen to act as controllable loads (CL). The CLs should be able curtail about 10% of its consumption involuntarily. The CLs are selected to be the loads that are connected to phase A of buses 71, 42, and 114, to phase B of bus 43, and to phase C of bus 50. All voltage regulators and the OLTC are equipped with 32 tap positions, ±16.

4.3.2 Classical EA

The main objective is to reduce line losses and to minimize the operating cost of the system. Thus, the problem is formulated as in equations (4.18)), where the individual objective functions represent the system losses, the cost of using controllable loads, the cost of PV active power curtailment, the cost of fluctuations in the tap positions of VRs, and the cost of fluctuations in SC statuses, respectively. Since the various objective functions are of different sizes and units, they are normalized here to bring them within comparable ranges.

\[
\min \{ \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 \} \tag{4.18}
\]

The term \( \phi_1 \) denotes the active power losses of the system. It can be expressed as in (4.19)):

\[
\phi_1 = \frac{1}{L_{\text{base}}} \sum_{j=1}^{L} \left[ \sum_{i=1}^{L} L_{ij} \right] \tag{4.19}
\]
The $R I^2$ losses of the conductor connecting $i^{th}$ bus to $j^{th}$ node is denoted as $L_{ij}$. To normalize $\varphi_1$, we divide the total sum by $L_{base}$ which is the total line losses for the base case. The base case is defined here as the operating condition where the taps of all voltage regulators are at their nominal position, the load consumption is at its maximum, the switching capacitors are all disconnected, and the PVs are operating at their maximum capacity with a unity power factor.

Reduction in $i^{th}$ CL load can be modeled by multiplying its rated value by a multiplication factor $\ell_i$. However, there is a limit below which the CL load cannot be reduced. This has been modeled as:

$$
\varphi_2 = \frac{1}{T_{CL}} \left[ \sum_{all \, CLs} \left( \frac{1 - \ell_i}{1 - \ell_{i,\text{min}}} \right) \right]
$$

where $\ell_{i,\text{min}} < \ell_i \leq 1$. To normalize $\varphi_2$, we divide each CL by $(1 - \ell_{i,\text{min}})$, then we divide the summation by the total number of the CL loads $T_{CL}$.

The output power of PVs depends on solar irradiance levels during the day. Nevertheless, the interfacing inverter can curtail part or all of the available active power. The curtailment of $i^{th}$ PV active power can be modeled by multiplying its available power by a multiplication factor $\mathcal{T}_i$. Hence, the total curtailment $\varphi_3$ of all PVs is formulated as:

$$
\varphi_3 = \frac{1}{T_{PV}} \sum_{all \, PVs} \left( 1 - \mathcal{T}_i \right)
$$

Where $0 < \mathcal{T}_i \leq 1$. Parameter $T_{PV}$ represents the number of PV panels and is added here for normalization purposes.
It is preferable to minimize the daily variations of the tap position of the VRs and the switching operation of SCs in order to reduce wear and tear and minimize transients. To model the fluctuations of a tap/switch across the day, its position during time $t_n$ is compared with its position during time $t_{n-1}$. A difference in the position sets the state variable $S_i$ to 1. This has been modeled for VRs and SCs as:

$$\varphi_4 = \frac{1}{T_{VR}} \left\{ \sum_{all\ VRs} \left[ \sum_{24\ Hours} (S_{VR,i}) \right] \right\}$$  \hspace{1cm} (4.22)

And

$$\varphi_5 = \frac{1}{T_{SC}} \left\{ \sum_{all\ SCs} \left[ \sum_{24\ Hours} (S_{SC,i}) \right] \right\}$$  \hspace{1cm} (4.23)

Similar to above, the total number of the VRs, $T_{VR}$, and the total number of SCs, $T_{SC}$, are included in equation (4.22) and (4.23), respectively, for the purpose of normalization.

Above objectives are subjected to the following constraints;

- The maximum and minimum permissible voltage magnitude on all buses:

$$0.95 \leq V_{All\ Buses} \leq 1.05$$
- The line ampacity limits
- Maximum and minimum limits of PV active and reactive powers
- Maximum allowable number of switching operations for the OLTC, VRs, and SCs

Figure 4.6  Simulation output. (a) The shape of load and irradiance across the day for all loads and PVs, (b) The kW line losses for all six cases, (c) The ratio of the total injected kW of PVs to the total kW load demand of all six cases across the day, (d) The percentage of the total kW curtailment of all PVs across the day for all six cases, and (e) The percentage of the total reduction in kW of the CLs of all six cases.
• Maximum capacity of the electric devices such as the transformers, induction motors, generators, etc.

The interfacing inverters of the PVs allow for curtailing their active power and/or regulating the reactive power absorbed or injected by the device. Therefore, the output of PVs is considered a control variable. At the same time, tap positions of OLTC/VRs, the status of SCs, and load curtailment (demand response) are assumed to be controllable remotely.

Classical EA converts the multi-objectives to a single objective. In this work, the multi-objectives have been converted to single objective by assigning weights to each objective. Then, the fitness is going to be equal to the sum of all objectives. Equation (2.24) shows how the fitness of each individual has been formulated.

When above system has been implemented on IEEE123 system, the total sum of kW ratings of all PVs is equal to half the total sum of kW of the loads under the base case scenario. \( l_{\text{min}} \) is chosen to be 90%. The shape of load and solar irradiance across the day are shown in figure (4.6,a).

Six different cases are simulated in order to investigate the effect of high PV penetration. The case No. 5 considers PVs working under a unity power factor, whereas in other cases, PVs are allowed to contribute to reactive power support of the grid. The difference between cases 0-4 is how equation (4.24) is formulated.

\[
\text{Fitness} = w_1\phi_1 + w_2\phi_2 + w_3\phi_3 + w_4\phi_4 + w_5\phi_5
\]  

(4.24)

The cases that has been studied are designed as follow:

- Case No.0: all weights in equation (4.24) are set to be equal to “1”.
- Case No.1: all weights in equation (4.24) are set to be equal to “1” except \( w_3 = 10 \).
Case No.2: all weights in equation (4.24) are set to be equal to “1” except $w_4 = 10$.

Case No.3: all weights in equation (4.24) are set to be equal to “1” except $w_2 = 10$.

Case No.4: all weights in equation (4.24) are set to be equal to “1” except $w_1 = 10$.

Case No.5: same as case No.0 but the PVs are working under unity power factor.

Also, the value of $L_{\text{base}}$ in equation (4.18) is set to be 32,145 kW.

<table>
<thead>
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<th>Case No.</th>
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<tbody>
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<tr>
<td>5</td>
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<td>1</td>
</tr>
</tbody>
</table>

To parametrize PV penetration, the ratio of the total active power injected by all PVs to the total active power of all loads is recorded for each hour. This ratio is plotted in figure (4.6,c) for all six cases. If the ratio is higher than 1, the total injected active power by PVs exceeds the total active power absorbed by the loads. This is a surplus generation and can be seen to occur between 10 AM and 3 PM. Notice that all cases show almost equal ratio except case 4 (the green curve). More discussions appear below.

The total line losses decrease as PV penetration increases because the amount of the power flowing in the lines decreases. However, once the active power injected by the PV connected to a
certain node exceeds the load demand, the excess power flows back through the line, which in turn increases the line losses. So, the minimum line losses are attained when the amount of the active power injected by the PV at a node equals its load. This behavior is shown in figure (4.6,b) where it can be observed that:

a) line losses decrease during the sunny hours, and

b) when the ratio of the total PV active power to the total load exceeds “1” between 10AM and 3 PM (see figure (4.6,c)), line losses increase slightly (barely noticeable in the graph).

The fitness function in equation (4.24) for case No.4 penalizes line losses more than other objectives. Therefore, when the ratio exceeds 1, the active power injected by the PVs flows back through the lines, which increases line losses. In this situation, the EA favors to curtail the active power of PVs to minimize the line losses, which explains the behavior of case 4 in figure (4.6,c).

The curtailed active power of all PVs is shown in figure (4.6,d). It displays the percentage of the total curtailed active power of the 42 PVs connected to the network across the day relative to their total available active power. Again, the behavior of case 4 in figure (4.6,d) reinforces the conclusion above. Also, it can be seen that a high penetration of PVs in the distribution network is possible if properly allocated. Very little PV active power has been curtailed even when the amount of the PV active power exceeds the demand. It has been reported in [387] that problems emerge once the PV penetration exceeded approximately 20%. On the contrary, using the centralized approach we could allow up to 150% penetration of the PVs if the issue with weather uncertainty can be solved.
Figure 4.7 The output of IEEE123 system simulation: (a) shows normalized load and irradiance profile over one day, (b) ratio R, i.e. the ratio of total injected active power by the PVs to the total loads, (c) total line losses, (d) percent curtailment of PV active power, and (e) percent curtailment of CL active power. The classical EA results in a 0% active power curtailment of CLs. Thus, the result of classical EA is omitted in (e).
The percentage of load reduction in CLs (demand response) with respect to the rated values is shown in figure (4.6,e). Since a very small percentage has been reduced, we could conclude the CLs contribute insignificantly in optimizing the distribution network under heavy PV penetration. The fluctuations of the curves can be attributed to the local optimality of the solutions returned by EA. Also, we believe we could attain less fluctuations and less load reduction (approaches to zero) if the convergence criteria of EA is perfect (goes to infinity).

![Figure 4.8](image)

**Figure 4.8** Close-up of figures 5(b) and 5(c) during peak hours: (a) R ratio, and (b) line losses in watts.

The SCs have changed their statuses few times across the day, as shown in Table (4.5). Also, the reactive power supplied by the PVs shows insignificant association with SC statuses because case No.5 (unity PV power factor) demonstrates almost equal number of changes in SC statuses with respect to the remaining cases (we believe this happens because the PV reactive power has not been restricted by the amount of the available active power).
This issue has been considered in NSGA-III. However, the taps of the VRs have fluctuated more than the SCs. Also, we notice a significant increase in the fluctuations of the VRs once the PVs stop contributing to the reactive power compensation. This is noticeable by comparing the total number of fluctuations during case No.5 to the total number of fluctuations during the other cases.

It can be deduced from Table (4.5) that the fluctuations of the switching operations of SCs and the tap positions of VRs are mainly dependent on their geographical location in the network. If the switching devices are located in positions sensitive to the PV operation, they respond more to the reactive power injected by the PV. Also, reactive power support by the PVs seems to reduce the fluctuations in the switching devices during the day.

4.3.3 NSGA-III

The objective function in equation (4.1) has been inserted below for convenience, equation (4.25). Five objectives are defined for NSGA-III which are as follow;

\[
\mathcal{Q} = \{\varphi_1(\mathbf{x}), \varphi_2(\mathbf{x}), \ldots, \varphi_M(\mathbf{x})\}
\]

Minimize \( \mathcal{Q} \) subject to

\[
\begin{align*}
g_j(\mathbf{x}) & \geq 0, \ j = 1,2,\ldots, J \\
h_k(\mathbf{x}) & = 0, \ k = 1,2,\ldots, K \\
\mathbf{x}(L) & \leq \mathbf{x} \leq \mathbf{x}(U)
\end{align*}
\]

- Line losses in the system \( \varphi_1 \)
- The cost that is associated with CLs \( \varphi_2 \)
- Active power curtailment of PV panels \( \varphi_3 \)
- Cost of switching operations variations in the tap positions of OLTC and VRs \( \varphi_4 \)
- Cost of switching operations of SCs \( \varphi_5 \)
The goal is to find optimal settings of the control variables, i.e. vector $x$, that simultaneously minimize the objective set $\mathcal{O}$. Naturally, these control variables are bound within their individual lower limits $x^{(L)}$ and upper limits $x^{(U)}$. In addition, the optimization problem is subjected to $J$ number of inequality constraints $g$ and $K$ numbers of equality constraints $h$.

$\varphi_1$, representing power losses, is defined as the sum of line losses in the system (in watts). In $\varphi_2$, the involuntarily reduction in $i^{th}$ CL load is modeled by multiplying its rated value by a multiplication factor $\ell_i$ as shown below;

$$\varphi_2 = \sum_{\text{all CLs}} \left( \frac{1 - \ell_i}{1 - \ell_{i.min}} \right)$$

(4.26)

Where $\ell_{i.min} < \ell_i \leq 1$. The denominator is intended to normalize the objective function. The discrete value of $\ell$ could be any value contained in the set $\{0.9, 0.95, 1.0\}$. The value of $\ell_{i.min}$ adopted in this work is 0.9.

The output power of PVs depends on solar irradiance levels during the day. However, the interfacing inverter can curtail part or all of the available active power. The active power curtailment of $i^{th}$ PV panel is modeled by multiplying its available power by a multiplication factor $T_i$. Hence, the total curtailment $\varphi_3$ of all PVs is formulated as:

$$\varphi_3 = \sum_{\text{all PVs}} (1 - T_i)$$

(4.27)

In this work, the values of the $T$ factor are assumed to belong to the set $\{0, 0.1, 0.2, 0.3, \ldots, 0.9, 1\}$. 

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To model the variations of $i^{th}$ tap/switch across the day, its position during time $t_n$ is compared with its the position during the previous time $t_{n-1}$. A difference in the positions would set the state variable $S_i$ be equal to 1 if the tap has changed its position. Otherwise, $S_i$ would be equal to zero. This has been modeled for VRs and SCs as:

$$
\Phi_4 = \sum_{all\ VRs} \left[ \sum_{24\ Hours} (S_{VR,i}) \right]
$$

$$
\Phi_5 = \sum_{all\ SCs} \left[ \sum_{24\ Hours} (S_{SC,i}) \right]
$$

The optimization problem is solved subjected to the following operational constraints:

- The maximum and minimum permissible voltage magnitude on at all buses must lie within ANSI low and high: $0.95 \leq V_{All\ Buses} \leq 1.05$,
- The line ampacity limits cannot be violated,
- Maximum and minimum limits of PV active and reactive powers must be maintained,
- Maximum allowable number of switching operations for the OLTC, VRs, and SCs cannot be violated.,
positions of OLTC/VRs, the status of SCs, and load curtailment of CLs are assumed to be controllable remotely. Together, they form the control vector $\mathbf{x}$.

The penetration of PVs is considered to be 50% according to the base case scenario, which is defined as the operating condition when all loads are fully connected, all VRs and OLTC are at their nominal tap positions (0%), PVs are operating at their rated power with a unity power factor, SCs are disconnected, and there is no curtailment of CLs.

The simulation is performed in the Matlab environment. The Forward/Backward Sweep method is used to perform the distribution power flow [348]. Also, power system components are modeled according to [348].

It has been considered in this work that load and irradiance are fixed for each hour. Thus, a full-day simulation is modeled using 24 discrete samples for load and solar irradiance. The

<table>
<thead>
<tr>
<th>Table 4.6</th>
<th>SCs fluctuations of IEEE123 system over one day</th>
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<tr>
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<td>Cases</td>
</tr>
<tr>
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<tr>
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<td>Classical EA(unity)</td>
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The simulation is performed in the Matlab environment. The Forward/Backward Sweep method is used to perform the distribution power flow [348]. Also, power system components are modeled according to [348].

It has been considered in this work that load and irradiance are fixed for each hour. Thus, a full-day simulation is modeled using 24 discrete samples for load and solar irradiance. The
irradiance data over one day are adopted from the National Solar Radiation Data Base website, for the city of Golden, CO associated with the period 1991-2005. The data for extraterrestrial irradiation (W/m²) is chosen based on a horizontal surface, over one day during the month of July. The load data for residential houses is adopted from the publicly available data at Open Energy Information (OpenEI). Daily consumption data has been determined based on the average values of the hourly load over a one-year period. Both irradiance and load data are then normalized for the purpose of simulation. The normalized profiles are shown in figure (4.7,a).

The optimization problem in equation (4.25) has been solved using both NSGA-III and the classical EA in order to compare the performance of both methods. Two cases have been simulated. The first case is when PVs contribute to reactive power support. The second case is when PVs operate with a unity power factor. Thus, four cases are simulated (excluding base case) as follows;

• Case 1: Optimization using NSGA-III with PV reactive power support,
• Case 2: Optimization using NSGA-III with no PV reactive power support,
• Case 3: Optimization using classical EA with PV reactive power support,
• Case 4: Optimization using classical EA with no PV reactive power support.

An additional parameter has been defined to help assess the performances of the optimization algorithms for the four cases. This parameter, denoted as R and shown in figure (4.7,b), is the ratio of the injected active power by the PVs to the total demand. The results indicate that PV penetration could go up to almost 95% if the system is well coordinated centrally, without violating system constraints. However, a close-up of figure (4.7) during the peak hours, i.e. hours
10 AM to 5 PM, shows some interesting patterns in particular for high PV penetration levels, see figure (4.8,a). A significant difference can be seen in the performance of the classical EA when PVs do not contribute to reactive power support. On the contrary, NSGA-III can maximize the R ratio regardless of whether or not PVs contribute to reactive power support. This indicates that PV penetration in the system can be increased if the voltage controlling devices are present and are centrally controlled (regardless of whether or not PVs contribute to reactive power support).

Line losses are shown in figure (4.7,c) and figure (4.8,b), where two observations can be made. First, when the R ratio is high (or PV penetration is high), system losses decrease significantly. Second, as indicated in figure (4.8,b), NSGA-III outperforms classical EA in minimizing line losses during the peak hours. In the case of NSGA-III, the reactive power support of PVs contributes slightly in reducing line losses. On the contrary, reactive power support of PVs marks a significant difference in the performance of the classical EA in reducing line losses.

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<td>13</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>41</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7 VRs fluctuations of IEEE123 system over one day
Therefore, generally speaking, it can be concluded that reactive power support of PVs contributes to minimizing line losses.

PV active power curtailment, as shown in figure (4.7,d), can be calculated as follow;

\[
PV(\%) = 100 \left[ 1 - \frac{\sum_{all\ PVs} \left( (Irr_i)(P_{PV,i}) \right)}{\sum_{all\ PVs} \left( (Irr_i)(P_{PV,i}) \right)} \right]
\] (4.30)

PV rated power \( P_{PV,i} \) is multiplied by the normalized value of irradiance \( Irr_i \) \((0 \leq Irr_i \leq 1)\) in order to obtain the available active power of the \( i^{th} \) PV. Multiplying the available active power of \( i^{th} \) PV by parameter \( T_i \) determines the injected active power.

Notice that about 5% of PV active power is curtailed on average, as shown in figure (4.7,d). However, when using the classical EA, this number increases to around 20% when PVs do not contribute to reactive power support. In cases 1, 2, and 3, PV active power curtailment shows very low variations compared to the high variations in the R ratio. For instance, the curtailment between hours of 6 and 8 AM is around 3% while R ratio is also rather low, at around 0.2. However, even when R ratio increases to around 0.95 at 2 PM, curtailment still maintains a low value of 5%.

Therefore, the curtailment of PVs cannot be attributed to penetration level of PVs, especially when the curtailment is very low.

Case 4 in figure (4.7,d) can be interpreted by observing the performance of the classical EA in reducing losses, see figure (4.8,b). It has been observed that reactive power support by PVs helps reduce losses, and NSGA-III outperforms classical EA when PVs do not contribute to reactive power support. EA performs poorly when it comes to reactive power support and this may explain the 20% curtailment of PV active power.
The curtailment of CLs, presented in figure (4.7,e), is calculated as follows;

\[
CL(\%) = 100 \left[ 1 - \frac{\sum_{all\,CLs} (\ell_i)(P_{l,i})}{\sum_{all\,CLs} (P_{l,i})} \right]
\]  \hspace{1cm} (4.31)

The pre-curtailed load \( P_{l,i} \) of \( i^{th} \) CL is multiplied by the parameter \( \ell_i \) in order to obtain the load value after curtailment. The classical EA shows zero load curtailment regardless of whether or not PVs contribute into reactive power support or not (since it is zero, the plot of EA is omitted). On the contrary, NSGA-III shows a slight load curtailment (less than 5%, see figure (4.7,e)) for some hours of the day (zero for other hours). It can also be seen that NSGA-III tends to increase the curtailment of CLs when PVs do not contribute to reactive power support.

Table (4.6) shows the variations of SC statuses throughout the day. It can be seen that classical EA outperforms NSGA-III. However, both methods show that reactive power support by PVs reduces the variations. The same conclusion can be made about VRs, see Table (4.7). At first this may be interpreted as classical EA generally outperforming NSGA-III. However, it should be noted that in a Pareto optimal solution, not all objectives reach their absolute optimum (zero in this case). Instead, the solution converges to a point where no objective can be improved without worsening one or more others. It is obvious that several objectives attain better values with NSGA-III, while some do better at times with classical EA, but this behavior can be attributed to the Pareto optimality feature of NSGA-III.

\subsection{Summary and Conclusions}

From both simulations by classical EA and NSGA-III, someone can summarize the results as follow;
➢ Centralized control eliminates the PV penetration ceiling in distributions system.
   Of course, this conclusion is true if the system deterministic.

➢ Assuming everything is equal, minimum line losses are achievable when the power
   injected by PVs is exactly equal the power absorbed by the loads. If the power
   injected by PVs exceeds the absorbed power, the magnitude of the current flowing
   through conductors increases which might increase losses.

➢ Proper allocation of PVs in the grid reduces the cost of switching devices, such as
   VRs.

➢ High penetration of PVs reduces (or eliminate) the need to have CLs in the system.

➢ The contribution of PVs in reactive power support in the grid, reduces the number
   of switching operations of VRs and SCs.

➢ Also, reactive power support by PVs could reduce line losses. The reason behind
   this is that current magnitude

➢ is a combination of both active and reactive components of the current. Thus, there
   is an association between reactive power and current magnitude which is associated
   with line losses.

➢ For some objectives NSGA-III outperforms classical EA but for some objectives
   we observe the opposite. Two reasons can explain this observation:

   o Classical EA is dependent on the weight assigned to each objective. Thus,
     assigning a large weight of an objective in classical EA might provide a
     result better than the one obtained through NSGA-III but on the expense of
     the other objectives.
In a Pareto optimal solution, not all objectives reach their absolute optimum at the same time. Instead, the solution converges to a point where no objective can be improved without worsening one or more others. It is obvious that several objectives attain better values with NSGA-III, while some do better at times with classical EA, but this behavior can be attributed to the Pareto optimality feature of NSGA-III.
CHAPTER 5

VVWO ANALYTICAL FORMULATION

In the previous chapters, it has been shown that the problems that emerge of high PV penetration in distribution system are manageable if the system is deterministic and is controlled centrally. However, solar irradiance is, obviously, not deterministic. The stochastic nature of insolation makes it difficult to optimize the system. Therefore, VVWO cannot be implemented directly to distribution system. As one more step closer to solve this issue, a probabilistic analysis of the system is proposed in this work.

Usually, the probabilistic analysis that has been reported in literature involves a big collection of real data, could be synthetic data, that is used to perform the simulation. Then, the system response is analyzed due to the collected data. Some of these data represent the extreme cases with maximum and minimum solar irradiance and load demand. Such studies can tell the probability of how the system would behave under extreme situations. Thus, probabilistic studies return a probabilistic answer rather than a deterministic one. Also, a probabilistic distribution or a histogram will be developed for the variables of interest.

A challenge emerges once we move toward probabilistic VVWO analysis: simulation time. Optimizing a three-phase power system is computationally expensive. Therefore, heuristic computation, such as EA, is not a practical option in this case. One way to reduce computation is by switching to analytical optimization. Thus, in this chapter a deterministic VVWO is going to be presented but analytically.
5.1 Convex Optimization

In the previous chapter, VVWO is performed using EA. Despite the fact that EA is very easy to implement and it can deal with all kinds of systems, it is very slow. In order to perform probabilistic analysis using EA simulation might takes years to return a satisfying solution. For this reason, another mathematical approach has to be pursued to cope with the problem of time simulation.

\begin{equation}
P_j = V_j V_i \left[ G \cos(\theta_i - \theta_j) + B \sin(\theta_i - \theta_j) \right]
\end{equation}

\begin{equation}
Q_j = V_j V_i \left[ G \sin(\theta_i - \theta_j) - B \cos(\theta_i - \theta_j) \right]
\end{equation}

To speed up the simulation time, it is good to model the power system mathematically and find the optimum solution of the model. A feeder similar to the one that is shown in figure (5.1) can be modeled as follow:

The system in equation (5.1) and (5.2) is nonlinear. Optimizing a nonlinear system require an iterative solver which is subjected to convergence and time issues. In addition, it is difficult to find a global solution for nonlinear system.

Alternatively, power system can be modeled as a nonlinear system but quadratic. However, the current has to be added as variable. Thus, the number of variables has to increase in order to
make the power system quadratic. This can be achieved by finding KCL and KVL over all buses. For instance, the system in figure (5.1) can be modeled as follow;

$$\mathbf{V}_i = \mathbf{V}_j + \mathbf{I}_{line} \mathbf{Z}_{line}$$ \hspace{1cm} (5.3)

$$\mathbf{I}_{line} = \mathbf{I}_{load}$$ \hspace{1cm} (5.4)

Notice that above two equations are linear. Optimizing linear system returns a global solution quickly. However, the power equation at bus $j$ can be modeled as follow;

![Diagram](image.png)

Figure 5.2 A diagram to show how KCL is implemented at each node.
\[ s_j = v_j^l \] \hspace{1cm} (5.5)

Above equation is nonlinear but quadratic. Therefore, power system can be modeled as quadratic system. Unfortunately, two challenges have to be dealt with. First challenge is about nonconvexity of power system. Second challenge is about modeling switching devices such as VRs. The following sections explain how these problems can be dealt with.

5.1.1 Power System Formulation

In this section, whole power system is going to be formulateized in quadratic form as in equation (5.38).

5.1.1.1 Nomenclature

The nomenclatures are shown and explained in the following subsections.

5.1.1.1.1 Indices

\( J \) = number of inequality constraints

\( K \) = number of equality constraints

\( N_{CL} \) = number of controllable loads

\( N_{PV} \) = number of PVs

\( N_{SC} \) = number of switching capacitors

\( N_{VR} \) = number of VRs

\( T \) = time horizon of the problem

\( i \) = index used for nodes
\( j \) = index used for nodes

\( n \) = number of nodes (buses) in the network

\( q \) = index used for nodes

\( t \) = time index

\( w \) = index used for nodes

5.1.1.1.2 Parameters

\( B_{c,i} \) = susceptance of the static capacitor connected to bus \( i \).

\( B_i \) = susceptance of the shunt admittance connected to bus \( i \).

\( G_{c,i} \) = conductance of the static capacitor connected to bus \( i \).

\( G_i \) = conductance of the shunt admittance connected to bus \( i \).

\( I_{i,j}^{\text{max}} \) = maximum current magnitude that a line between bus \( i \) and \( j \) can carry.

\( P_{CL,i} \) = active power of the controllable load that is connected to bus \( i \).

\( P_{PV,i}^{\text{rated}} \) = rated active power of the PV connected to bus \( i \).

\( P_{load,i} \) = total active power of loads that are connected to bus \( i \).

\( Q_{CL,i} \) = reactive power of the controllable load that is connected to bus \( i \).

\( Q_{SC,i}^{\text{rated}} \) = rated reactive power of the switching capacitor connected to bus \( i \).

\( Q_{load,i} \) = total reactive power of loads that are connected to bus \( i \).
\( R_{i,j} = \) resistance of the line connecting \( i^{th} \) and \( j^{th} \) nodes.

\( \delta_{SC,i}^{\text{max}} = \) number of taps of the switching capacitor that is connected to bus \( i \).

\( T_{RV,i,j}^{\text{max}} = \) number of taps of the voltage regulator that is connected between buses \( i \) and \( j \).

\( X_{i,j} = \) reactance of the line connecting \( i^{th} \) and \( j^{th} \) nodes.

\( \xi = \) normalized irradiance level

\( u_{C,i} = \) binary parameter that is “1” if a static capacitor is connected to bus \( i \), and 0 otherwise

\( u_{CL,i} = \) binary parameter that is “1” if a controllable load is connected to bus \( i \), and 0 otherwise.

\( u_{PV,i} = \) binary parameter that is “1” if a PV panel is connected to bus \( i \), and 0 otherwise.

\( u_{SC,i} = \) binary parameter that is “1” if a switching capacitor is connected to bus \( i \), and 0 otherwise.

\( u_{VR,i,j} = \) binary parameter that is “1” if a voltage regulator is connected between bus \( i \) and \( j \), and 0 otherwise.

\( u_{i,j} = \) binary parameter that is “0” if there is no impedance connecting buses \( i \) and \( j \). Otherwise, if the two buses are connected, its value is equal to “1”.

\( u_{\text{load},i} = \) binary parameter that is “1” if a load is connected to bus \( i \), and 0 otherwise.

\( u_{y,i} = \) binary parameter that is “1” if a shunt impedance is connected to bus \( i \), and 0 otherwise.
5.1.1.3 Sets

$\mathbb{B} =$ set of all buses (nodes) in the network

$\mathbb{B}_C =$ a subset of $\mathbb{B}$ that contains all buses where the static capacitors are connected.

$\mathbb{B}_{CL} =$ a subset of $\mathbb{B}$ that contains all buses where controllable loads are connected.

$\mathbb{B}_{PV} =$ a subset of $\mathbb{B}$ that contains all buses where PVs are connected.

$\mathbb{B}_{SC} =$ a subset of $\mathbb{B}$ that contains all buses where the switching capacitors are connected.

$\mathbb{B}_{VR} =$ a subset of $\mathbb{B}$ that contains all buses where voltage regulators are connected.

$\mathbb{B}_{load} =$ a subset of $\mathbb{B}$ that contains all buses where loads are connected.

$\mathcal{Q} =$ set of all objective functions.

5.1.1.2 Variables

$\varnothing_1 =$ lines losses

$\varnothing_2 =$ cost of curtailing controllable loads

$\varnothing_3 =$ cost of curtailing active power of PVs

$\varnothing_4 =$ cost of switching operations of VRs and OLTCs

$\varnothing_5 =$ cost of switching operations of switching capacitors.

$\mathbf{x} =$ vector of control (or decision) variables.

$\mathbf{x}^{(l)} =$ lower bounds of $\mathbf{x}$
\(x^{(u)}\) = upper bounds of \(x\)

\(g(x)\) = system of equality constraints

\(h(x)\) = system of inequality constraints

\(V_{i,t}^{imag}\) = imaginary part of voltage over bus \(i\) at time \(t\)

\(I_{i,j,t}^{imag}\) = imaginary part of line current flowing from bus \(i\) to bus \(j\) at time \(t\)

\(I_{c,i,t}^{imag}\) = imaginary part of the current injected by the static capacitor connected to bus \(i\) at time \(t\)

\(I_{CL,i,t}^{imag}\) = imaginary part of the current absorbed by the controllable load connected to bus \(i\) at time \(t\)

\(I_{PV,i,t}^{imag}\) = imaginary part of the current injected by the PV connected to bus \(i\) at time \(t\)

\(I_{SC,i,t}^{imag}\) = imaginary part of the current injected by the switching capacitor connected to bus \(i\) at time \(t\)

\(I_{VR,pri,i,j,t}^{imag}\) = imaginary part of the current at the primary side of the VR that is connected between buses \(i\) and \(j\) at time \(t\). Bus \(i\) is the primary side.

\(I_{VR,sec,i,j,t}^{imag}\) = imaginary part of current at the secondary side of VR that is connected between buses \(i\) and \(j\) at time \(t\). Bus \(i\) is the primary side.

\(I_{load,i,t}^{imag}\) = imaginary part of the current consumed by the load connected to bus \(i\) at time \(t\)

\(V_{i,t}^{real}\) = real part of voltage over bus \(i\) at time \(t\)
\( I_{\text{real},i,j,t} \) = real part of line current flowing from bus \( i \) to bus \( j \) at time \( t \)

\( I_{C,i,t} \) = real part of the current injected by the static capacitor connected to bus \( i \) at time \( t \)

\( I_{\text{real},CL,i,t} \) = real part of the current absorbed by the controllable load connected to bus \( i \) at time \( t \)

\( I_{\text{real},PV,i,t} \) = real part of the current injected by the PV connected to bus \( i \) at time \( t \)

\( I_{\text{real},SC,i,t} \) = real part of the current injected by the switching capacitor connected to bus \( i \) at time \( t \)

\( I_{\text{real},VR,pr,i,j,t} \) = real part of the current at the primary side of the VR that is connected between buses \( i \) and \( j \) at time \( t \). Bus \( i \) is the primary side.

\( I_{\text{real},VR,sec,i,j,t} \) = real part of current at the secondary side of VR that is connected between buses \( i \) and \( j \) at time \( t \). Bus \( i \) is the primary side.

\( I_{\text{load},i,t} \) = real part of the current consumed by the load connected to bus \( i \) at time \( t \)

\( P_{PV,i,t} \) = injected active power of PV at bus \( i \) at time \( t \)

\( Q_{PV,i,t} \) = injected reactive power of PV at bus \( i \) at time \( t \). Positive values indicate injection

\( \delta_{SC,i,t} \) = integer associated with the switching capacitor that is connected to bus \( i \) at time \( t \)

\( \delta_{VR,i,j,t} \) = tap position of the VR connected between buses \( i \) and \( j \) at time \( t \)

\( \ell_{i,t} \) = An integer variable to model curtailment of controllable load at bus \( i \) at time \( t \).
5.1.1.3 Optimization Problem

Minimize  \( \mathcal{Q} = \{ \varphi_1(\mathbb{x}), \varphi_2(\mathbb{x}), \ldots, \varphi_5(\mathbb{x}) \} \)

Subject to  
\[
\begin{align*}
g_j(\mathbb{x}) &= 0, \quad j = 1, 2, \ldots, J \\
h_k(\mathbb{x}) &\geq 0, \quad k = 1, 2, \ldots, K \\
\mathbb{x}(L) &\leq \mathbb{x} \leq \mathbb{x}(U)
\end{align*}
\]  
(5.6)

5.1.1.4 Objectives

Line losses

\[
\forall t: \varphi_{1,t} = \sum_{i=1}^{n} \left[ \sum_{j=1}^{n} u_{i,j} (R_{i,j} + X_{i,j}) I_{i,j,t}^2 \right]
\]  
(5.7)

Controllable loads:

\[
\forall i \in \mathbb{CL}, \forall t: \varphi_{2,i,t} = P_{CL,i,t} (10 - \ell_{i,t})
\]  
(5.8)

PVs

\[
\forall i \in \mathbb{PV}, \forall t: \varphi_{3,i,t} = \gamma_{t} P_{rated}^{PV,i,t} - P_{PV,i,t}
\]  
(5.9)

VRs

\[
\forall (i, j) \in \mathbb{VR}, \forall t: \varphi_{4,i,j,t} = |S_{VR,i,j,t} - S_{VR,i,j,t-1}|
\]  
(5.10)

Switching capacitors

\[
\forall i \in \mathbb{SC}, \forall t: \varphi_{5,i,t} = |S_{SC,i,t} - S_{SC,i,t-1}|
\]  
(5.11)
5.1.1.5 Equalities

KCL over all buses are modeled based on figure (5.2). The following equations show how
the modeling of KCL over buses are decoupled into real and imaginary parts.

\[ \forall i, \forall j \in \mathbb{B} - \{i\}, \forall t: \]
\[ (I_{i,j,t}^{imag})u_{j,i} = (G_{i,j,t}^{imag} + B_{i,j,t}^{real})u_{y,j,i} + (I_{load,i,t}^{imag})u_{load,i} + (I_{SC,i,t}^{imag})u_{SC,i} \]
\[ + (I_{CL,i,t}^{imag})u_{CL,i} - (I_{PV,i,t}^{imag})u_{PV,i} - (G_{C,i,t}^{imag} + B_{C,i,t}^{real})u_{C,i} \]
\[ + \left[ \sum_{k=1}^{n} \left[ (i_{i,k,t}^{imag})u_{i,k} + (I_{VR,prl,i,k,t}^{imag})u_{VR,i,k} \right] \right] \]

\[ \forall i, \forall j \in \mathbb{B} - \{i\}, \forall t: \]
\[ (I_{i,j,t}^{real})u_{j,i} = (I_{load,i,t}^{real})u_{load,i} + (I_{SC,i,t}^{real})u_{SC,i} - (I_{PV,i,t}^{real})u_{PV,i} + (I_{CL,i,t}^{real})u_{CL,i} \]
\[ - (G_{C,i,t}^{real} - B_{C,i,t}^{imag})u_{C,i} + (G_{i,j,t}^{real} - B_{i,j,t}^{imag})u_{y,i} \]
\[ + \left[ \sum_{k=1}^{n} \left[ (i_{i,k,t}^{real})u_{i,k} + (I_{VR,prl,i,k,t}^{real})u_{VR,i,k} \right] \right] \]

KVL across all buses

\[ \forall t, \forall i, \forall j: \text{ such that } j \neq i \]
\[ V_{i,t}^{imag} - V_{j,t}^{imag} = R_{i,j}I_{i,j,t}^{imag} + X_{i,j}I_{i,j,t}^{real} \]

\[ \forall t, \forall i, \forall j: \text{ such that } j \neq i \]
\[ V_{i,t}^{real} - V_{j,t}^{real} = R_{i,j}I_{i,j,t}^{real} - X_{i,j}I_{i,j,t}^{imag} \]
Loads

\(\forall t, \forall i \in \mathbb{B}_{\text{load}}:\)

\[P_{\text{load},i,t} = V_{i,t}^\text{real} I_{\text{load},i,t}^\text{real} + V_{i,t}^\text{imag} I_{\text{load},i,t}^\text{imag}\] (5.16)

\(\forall t, \forall i \in \mathbb{B}_{\text{load}}:\)

\[Q_{\text{load},i,t} = V_{i,t}^\text{imag} I_{\text{load},i,t}^\text{real} - V_{i,t}^\text{real} I_{\text{load},i,t}^\text{imag}\] (5.17)

PVs

\(\forall t, \forall i \in \mathbb{B}_{\text{PV}}:\)

\[P_{\text{PV},i,t} = V_{i,t}^\text{real} I_{\text{PV},i,t}^\text{real} + V_{i,t}^\text{imag} I_{\text{PV},i,t}^\text{imag}\] (5.18)

\(\forall t, \forall i \in \mathbb{B}_{\text{PV}}:\)

\[Q_{\text{PV},i,t} = V_{i,t}^\text{imag} I_{\text{PV},i,t}^\text{real} - V_{i,t}^\text{real} I_{\text{PV},i,t}^\text{imag}\] (5.19)

Switching capacitors

\(\forall t, \forall i \in \mathbb{B}_{\text{SC}}:\)

\[0 = V_{i,t}^\text{real} I_{\text{SC},i,t}^\text{real} + V_{i,t}^\text{imag} I_{\text{SC},i,t}^\text{imag}\] (5.20)

\(\forall t, \forall i \in \mathbb{B}_{\text{SC}}:\)

\[Q_{\text{SC},i}^\text{rated} \left(\frac{S_{\text{SC},i}}{S_{\text{SC}}^\text{max}}\right) = V_{i,t}^\text{imag} I_{\text{SC},i,t}^\text{real} - V_{i,t}^\text{real} I_{\text{SC},i,t}^\text{imag}\] (5.21)
Controllable loads

∀\(t, \forall i \in \mathbb{B}_{CL}\):

\[0.9 P_{CL,i,t} + 0.1 P_{CL,i,t} \left(\frac{\ell_{i,t}}{10}\right) = V_{i,t}^{\text{real}} I_{CL,i,t}^{\text{real}} + V_{i,t}^{\text{imag}} I_{CL,i,t}^{\text{imag}}\] (5.22)

∀\(t, \forall i \in \mathbb{B}_{CL}\):

\[0.9 Q_{CL,i,t} + 0.1 Q_{CL,i,t} \left(\frac{\ell_{i,t}}{10}\right) = V_{i,t}^{\text{imag}} I_{CL,i,t}^{\text{real}} - V_{i,t}^{\text{real}} I_{CL,i,t}^{\text{imag}}\] (5.23)

Voltage regulators

∀\(t, \forall (i, j) \in \mathbb{B}_{VR}\): bus \(i\) is the primary side.

\[V_{j,t}^{\text{real}} = \left(1 + 0.00625 S_{VR,i,j,t}\right) V_{i,t}^{\text{real}}\] (5.24)

∀\(t, \forall (i, j) \in \mathbb{B}_{VR}\): bus \(i\) is the primary side.

\[V_{j,t}^{\text{imag}} = \left(1 + 0.00625 S_{VR,i,j,t}\right) V_{i,t}^{\text{imag}}\] (5.25)

∀\(t, \forall (i, j) \in \mathbb{B}_{VR}\): bus \(i\) is the primary side.

\[I_{VR,\text{pri},i,j,t}^{\text{real}} = \left(1 + 0.00625 S_{VR,i,j,t}\right) I_{VR,\text{sec},i,j,t}^{\text{real}}\] (5.26)

∀\(t, \forall (i, j) \in \mathbb{B}_{VR}\): bus \(i\) is the primary side.

\[I_{VR,\text{pri},i,j,t}^{\text{imag}} = \left(1 + 0.00625 S_{VR,i,j,t}\right) I_{VR,\text{sec},i,j,t}^{\text{imag}}\] (5.27)
KCL at the secondary side of VR.

$$\forall (i, j) \in \mathbb{V}_R, \forall t: \text{bus } i \text{ is the primary side}$$  \hspace{1cm} (5.28)

$$i_{VR,sec,i,j,t}^{imag} = (i_{j,w,t}^{imag})u_{j,w} + (i_{load,j,t}^{imag})u_{load,j} + (i_{SC,j,t}^{imag})u_{SC,j} - (i_{PV,j,t}^{imag})u_{PV,j}$$

$$+ (i_{CL,j,t}^{imag})u_{CL,j} - (G_{C,j}V_{j,t}^{imag} + B_{C,j}V_{j,t}^{real})u_{C,j}$$

$$+ (G_{j}V_{j,t}^{imag} + B_{j}V_{j,t}^{real})u_{y,j}$$

$$\forall (i, j) \in \mathbb{V}_R, \forall t: \text{bus } i \text{ is the primary side}$$  \hspace{1cm} (5.29)

$$I_{VR,sec,i,j,t}^{real} = (I_{j,w,t}^{real})u_{j,w} + (I_{load,j,t}^{real})u_{load,j} + (I_{SC,i,j,t}^{real})u_{SC,j} - (I_{PV,j,t}^{real})u_{PV,j}$$

$$+ (I_{CL,j,t}^{real})u_{CL,j} - (G_{C,j}V_{j,t}^{real} - B_{C,j}V_{j,t}^{imag})u_{C,j}$$

$$+ (G_{j}V_{j,t}^{real} - B_{j}V_{j,t}^{imag})u_{y,j}$$

5.1.1.6 Inequalities

Voltage constraints over all buses;

$$\forall t, \forall i \in \mathbb{B}:$$  \hspace{1cm} (5.30)

$$0.95^2 \leq \left[ (V_{i,t}^{real})^2 + (V_{i,t}^{imag})^2 \right] \leq 1.05^2$$

Line ampacity limits;

$$\forall t, \forall i, \forall j \in \mathbb{B} - \{i\}: \text{such that } i \neq j$$  \hspace{1cm} (5.31)

$$0 \leq \left[ (I_{i,j,t}^{real})^2 + (I_{i,j,t}^{imag})^2 \right] \leq (I_{i,j}^{max})^2$$
PV constraints

\[ \forall t, \forall i \in B_{PV} : \]
\[ 0 \leq (P_{PV,i,t}) \leq (t_{PV,i} P_{PV,i}^{\text{rated}}) \]  \hfill (5.32)

\[ \forall t, \forall i \in B_{PV} : \]
\[ (Q_{PV,i,t})^2 \leq [(t_{PV,i} P_{PV,i}^{\text{rated}})^2 - (P_{PV,i,t})^2] \]  \hfill (5.33)

\[ \forall t, \forall i \in B_{PV} : \]
\[ 0 \leq t_{i,t} \leq 1 \]  \hfill (5.34)

Switching capacitor constraints;

\[ \forall t, \forall i \in B_{SC} : \]
\[ 0 \leq S_{SC,i,t} \leq S_{SC,i}^{\text{max}} \]  \hfill (5.35)

Voltage regulator constraints;

\[ \forall t, \forall i \in B_{VR} : \]
\[ -0.5 T_{RV,i,j}^{\text{max}} \leq S_{VR,i,j,t} \leq 0.5 T_{RV,i,j}^{\text{max}} \]  \hfill (5.36)

Controllable load constraints;

\[ \forall t, \forall i \in B_{CL} : \]
\[ 0 \leq \ell_{i,t} \leq 10 \]  \hfill (5.37)
5.1.2 Convexity in Quadratic System

In general, convex system has two mainstream topics. First topic is about modeling a system to be a convex one. Second topic is about the algorithms that solves the system [388]. In the section, we focus on modeling the power system in a convex way. Also, this section is not meant to be a thorough review of convex quadratic system modeling but it gives the reader an insight of the mainstream approaches in this filed.

A quadratic system can be formulated as follow [389];

\[
\min X^T A_{obj}X + D_{obj}X
\]

\[
X^T A_{const}X + D_{const}^T X + C_{const} \leq 0
\]

(5.38)

Such that

\[
X = [x_1 \ldots x_n]^T
\]

The vector \(X\) represent the variables of the quadratic system. The square matrices \(A_{obj}\)and \(A_{const}\) relate the quadratic terms in the objective function and the constraints respectively. The vector \(D_{const}\) and \(C_{const}\) represent the coefficients of the linear and constant terms in the constraints respectively.

In order to make sure the system in (5.38) is convex we have either to keep all quadratic terms convex or to keep its hessian matrix be a positive-semi-definite [389]. In quadratic power system, the convex terms could appear of the following form \(x_i^2\) but not like \(x_i x_j\). To verify that the hessian matrix of the system in (5.38) is positive-semi-definite, the eigenvalues of both matrices \(A_{obj}\) and \(A_{const}\) have to be not negative.
Convex systems can be solved efficiently because a solver does not need to check for numerous local solutions in order to find a global solution [388]. This sort of efficiency can be quantified by a concept called non-deterministic polynomial-time hardness which is known for short as NP-hard [388]. In other words, a solver takes longer time to find a solution for NP-hard problem than a convex one (because a solver has to search for in multiple points for a global solution for non-convex systems), which is the case with power system. Unfortunately, there is no guarantee for power system to obey the condition of convexity.

When the system in equation (5.5) is written in cartesian form as shown in equation (5.39), we end up with bilinear terms $V_j^{real}I_{load}^{real}$, $V_j^{imag}I_{load}^{imag}$, $V_j^{imag}I_{load}^{real}$, and $V_j^{real}I_{load}^{imag}$ (the superscript $real$ and $imag$ denotes real and imaginary parts respectively). These bilinear terms are not convex, such problems are called biconvex [390].

$$P_j = V_j^{real}I_{load}^{real} + V_j^{imag}I_{load}^{imag}$$

$$Q_j = V_j^{imag}I_{load}^{real} - V_j^{real}I_{load}^{imag}$$

(5.39)

Some of the approaches develop an “envelope” for non-convex terms and equations. The envelope results into a set of equations that define the feasible-solution region of the non-convex term provided that theses equations are convex [391].

A convex envelope for bilinear terms over a rectangle region is proposed in by McCormick in [392]. Then, in [390] the authors show that McCormick relaxation defines a convex/concave envelope. Their approach works by finding the maximum and minimum boundaries of bilinear terms. For instance, if we have a bilinear term $x_1x_2$ that has maximum boundaries $x_{1,max}$ and...
and minimum boundaries $x_{1,\text{min}}$ and $x_{2,\text{min}}$ respectively, their envelope can be constructed over a rectangular region as follow;

\[(x_{1,\text{max}} - x_1)(x_{2,\text{max}} - x_2) \geq 0\]
\[(x_1 - x_{1,\text{min}})(x_2 - x_{2,\text{min}}) \geq 0\]  \hspace{1cm} (5.40)
\[(x_{1,\text{max}} - x_1)(x_2 - x_{2,\text{min}}) \geq 0\]
\[(x_1 - x_{1,\text{min}})(x_{2,\text{max}} - x_2) \geq 0\]

Above system can be written as follow;

\[x_1 x_2 \geq x_{2,\text{max}} x_1 + x_{1,\text{max}} x_2 - x_{1,\text{max}} x_{2,\text{max}}\]
\[x_1 x_2 \geq x_{2,\text{min}} x_1 + x_{1,\text{min}} x_2 - x_{1,\text{min}} x_{2,\text{min}}\]  \hspace{1cm} (5.41)
\[x_1 x_2 \leq x_{2,\text{min}} x_1 + x_{1,\text{max}} x_2 - x_{1,\text{max}} x_{2,\text{min}}\]
\[x_1 x_2 \leq x_{2,\text{max}} x_1 + x_{1,\text{min}} x_2 - x_{1,\text{min}} x_{2,\text{max}}\]

Notice that the system in (5.41) replaces the terms $x_1 x_2$ with a rectangular envelope. This envelope consists of four inequalities that are linear. Therefore, the bilinear term $x_1 x_2$ can be replaced with a new term $x_3$ in the optimization system such that it is subjected to the envelope in (5.41).

\[\mathcal{Y} \equiv \{x_{1,\text{min}} \leq x_1 \leq x_{1,\text{max}}, \quad x_{2,\text{min}} \leq x_2 \leq x_{2,\text{max}}\}\]
\[SE \equiv \mathcal{Y} \cap \{(x_2 - x_{2,\text{min}})(x_{1,\text{max}} - x_{1,\text{min}}) \leq (x_{2,\text{max}} - x_{2,\text{min}})(x_1 - x_{1,\text{min}})\}\]  \hspace{1cm} (5.42)
\[NW \equiv \mathcal{Y} \cap \{(x_2 - x_{2,\text{min}})(x_{1,\text{max}} - x_{1,\text{min}}) \geq (x_{2,\text{max}} - x_{2,\text{min}})(x_1 - x_{1,\text{min}})\}\]
\[ SW \triangleq \mathbb{R} \cap \{(x_2 - x_{2,max})(x_{1,max} - x_{1,min}) \leq (x_{2,min} - x_{2,max})(x_{1} - x_{1,min})\} \]

\[ NE \triangleq \mathbb{R} \cap \{(x_2 - x_{2,max})(x_{1,max} - x_{1,min}) \geq (x_{2,min} - x_{2,max})(x_{1} - x_{1,min})\} \]

\[ N \triangleq NE \cap NW \]

\[ S \triangleq SE \cap SW \]

\[ E \triangleq NE \cap SE \]

\[ W \triangleq NW \cap SW \]

In [393], the authors divided the feasible-solution region (or the envelope) into the cartesian product of triangles and rectangles. The convex (concave is omitted here for brevity) envelope of the bilinear terms \(x_1x_2\) can be formulated as follow;

Figure 5.3 The triangular regions to define the envelope of bilinear term \(x_1x_2\). (a) the rectangular envelope of \(x_1x_2\). (b) triangular regions divisions of the rectangular region: North (N), East (E), South(S), and West (W). (c) triangular regions divisions of the rectangular region: North-West (NW), and South-East (SE). (d) triangular regions divisions of the rectangular region: North-East (NE), and South-West (SW).

- The rectangular envelope of \(x_1x_2\) is divided into eight triangular regions: North (N), South (S), East (E), West (W), South-East (SE), North-West (NW), South-
West (SW), and North-West (NW). These regions are shown in figure (5.3). The mathematical boundary definitions of these triangular regions are defined in equation (5.42).

- At the triangular region SE, S, or E the convex envelope \( \text{conv}(x_1, x_2) \) of \( x_1x_2 \) is defined in equation (5.43).
- At the triangular region NW, N, or W the convex envelope \( \text{conv}(x_1, x_2) \) of \( x_1x_2 \) is defined in equation (5.44).

\[
\text{conv}(x_1, x_2) = \begin{cases} 
  x_{1,\text{max}}x_{2,\text{min}}, & \text{if } x_1 = x_{1,\text{max}} \text{ and } x_2 = x_{2,\text{min}} \\
  \mathcal{N}/\mathcal{D}, & \text{otherwise}
\end{cases}
\]

Such that

\[
\mathcal{N} = (x_{2,\text{min}}^2 - x_{2,\text{min}}x_{2,\text{max}})x_1^2 + (x_{2,\text{max}}^2 - x_{1,\text{min}}x_{1,\text{max}})x_2^2
\]

\[+ (x_{2,\text{min}}x_{1,\text{max}} - x_{1,\text{min}}x_{2,\text{max}})x_1x_2
\]

\[+ (x_{1,\text{min}}x_{2,\text{min}}x_{2,\text{max}} + x_{2,\text{min}}x_{1,\text{max}}x_{2,\text{max}} - 2x_{2,\text{min}}^2x_{1,\text{max}})x_1
\]

\[+ (x_{1,\text{min}}x_{2,\text{min}}x_{1,\text{max}} + x_{1,\text{min}}x_{1,\text{max}}x_{2,\text{max}} - 2x_{1,\text{max}}^2x_{2,\text{min}})x_2
\]

\[+ (x_{2,\text{min}}^2x_{1,\text{max}} - x_{1,\text{min}}x_{2,\text{min}}x_{1,\text{max}}x_{2,\text{max}})
\]

\[
\mathcal{D} = (x_{2,\text{min}} - x_{2,\text{max}})x_1 + (x_{1,\text{max}} - x_{1,\text{min}})x_2
\]

\[+ (x_{1,\text{min}}x_{2,\text{min}} + x_{1,\text{max}}x_{2,\text{max}} - 2x_{1,\text{max}}x_{2,\text{min}})
\]

The symbol \( \text{conv}(x_1, x_2) \) denotes the convex envelope of bilinear term \( x_1x_2 \).

\[
\text{conv}(x_1, x_2) = \begin{cases} 
  x_{1,\text{min}}x_{2,\text{max}}, & \text{if } x_1 = x_{1,\text{min}} \text{ and } x_2 = x_{2,\text{max}} \\
  \mathcal{N}/\mathcal{D}, & \text{otherwise}
\end{cases}
\]
\[ N = (x_{2,\text{max}}^2 - x_{2,\text{min}} x_{2,\text{max}}) x_1^2 + (x_{1,\text{min}}^2 - x_{1,\text{min}} x_{1,\text{max}}) x_2^2 \\
+ (x_{2,\text{max}} x_{1,\text{min}} - x_{2,\text{min}} x_{1,\text{max}}) x_1 x_2 \\
+ (x_{2,\text{max}} x_{2,\text{min}} x_{1,\text{max}} + x_{2,\text{max}} x_{2,\text{min}} x_{1,\text{min}} - 2x_{2,\text{max}} x_{1,\text{min}}^2) x_1 \\
+ (x_{1,\text{min}} x_{2,\text{min}} x_{1,\text{max}} + x_{1,\text{min}} x_{2,\text{max}} x_{1,\text{max}} - 2x_{1,\text{min}}^2 x_{2,\text{max}}) x_2 \\
+ (x_{2,\text{max}}^2 x_{1,\text{min}} - x_{1,\text{min}} x_{2,\text{max}} x_{1,\text{max}}) \\
\]

\[ D = (x_{2,\text{max}} - x_{2,\text{min}}) x_1 + (x_{1,\text{min}} - x_{1,\text{max}}) x_2 \\
+ (x_{1,\text{min}} x_{2,\text{min}} + x_{1,\text{max}} x_{2,\text{max}} - 2x_{1,\text{min}} x_{2,\text{max}}) \\
\]

The interested reader into the proof of equation (5.43) and (5.44) is advised to read reference [393].

Another way to deal with the nonconvexity of quadratic system is called Semi Definite Relaxation (SDR). A crucial observation in equation (5.38) is that the diagonal elements in \( X^T A_{\text{obj}} X \) dominates the off-diagonal elements. Thus, we could generate the following equality:

\[ X^T A_{\text{obj}} X = \text{Trace}(X^T A_{\text{obj}} X) = \text{Trace}(A_{\text{obj}} XX^T) = \text{Trace}(A_{\text{obj}} X) \]

\[ X^T A_{\text{const}} X = \text{Trace}(X^T A_{\text{const}} X) = \text{Trace}(A_{\text{const}} XX^T) = \text{Trace}(A_{\text{const}} X) \]

Notice that a new variable \( X \) has been introduced in above equation. Also, the system \( A_{\text{obj}} X \) or \( A_{\text{const}} X \) are both looks fictitiously linear, which makes them convex.

Notice that the matrix \( X \) is real (in power system) and symmetric. According to [394], \( X \) is positive semidefinite if \( a^T X a \geq 0 \) for any real vector \( a \). Since \( a^T X a = a^T X X^T a = (a^T X)(a^T X)^T = (\sum a_i x_i)^2 \geq 0 \), thus the matrix \( X \) is a positive semidefinite. Therefore, the system in (5.38) can be converted to the following convex problem [395];
\[
\min \text{Trace}(A_{\text{obj}}X) + D_{\text{obj}}^TX
\]

\[
\text{Trace}(A_{\text{const}}X) + D_{\text{cont}}^TX + C_{\text{const}} \leq 0
\]

(5.46)

\[
X = [x_1 \cdots x_n]^T, \quad X = XX^T
\]

\[
X \succeq 0
\]

The symbol \(\succeq\) denotes PSD. According to [396], any matrix that is with size of \(i, j\) (\(i\) rows and \(j\) columns) would have a rank of \(r\) if this matrix is equal to the product of two matrices of size \(i, r\) and \(r, j\) respectively. Thus, from the aforementioned argument, the matrix \(X\) has a rank of 1. Therefore, above equation can be re-written as follow [397];

\[
\min \text{Trace}(A_{\text{obj}}X) + D_{\text{obj}}^TX
\]

\[
\text{Trace}(A_{\text{const}}X) + D_{\text{cont}}^TX + C_{\text{const}} \leq 0
\]

Such that

\[
X = [x_1 \cdots x_n]^T, \quad X = XX^T
\]

\[
X \succeq 0
\]

\[
\text{rank}(X) = 1
\]

Equation (5.47) is an exact transformation of in equation (5.38). The system in (5.47) is convex except for the condition \(\text{rank}(X) = 1\), [397]. Therefore, one way to relax the system in (5.47) is by dropping the condition \(\text{rank}(X) = 1\) according to [397]. Unfortunately, most literatures do not explain why the constraint \(\text{rank}(X) = 1\) has to be dropped, such as [397] and [398]. The reason for dropping the rank constraints is answered by [399]. According to [399], if
rank constraint in equation (5.47) is not dropped, it would have an infinite number of solutions that are of rank 2 (interested reader can refer to the proof in [399]).

### 5.1.3 Mixed Integer Problem

Almost any mixed-integer-nonlinear programming (MINLP) can be factorized and reduced to a mixed-integer-quadratic programming (MIQP). The system in (5.38) is a typical form of MIQP if it contains both continuous and discrete variables.

Although there has been a tremendous work dedicated to solve MIQP systems, still the existing approaches are rare and often restricted to special classes of objective functions or variable domain. The state-of-the-art of optimization techniques in MIQP perform weakly compared to MILP systems. Even if MIQP has a complete linear constraints, still it is hard to solve. The main two reasons that make MIQP hard to solve are the nonconvexity of quadratic terms and discontinuity caused by discrete variables. Thus, most of the existing algorithms and software tools are either restricted to convex-MIQP, or they non-convex-MIQP but without guarantee of global optimality. Most of existing methods that solve non-convex-MIQP are based on the idea of convex estimators that is combined with branching and range reduction techniques[400].

In power system, we have discrete variables, such as the taps of voltage regulators. This section shows a brief description of discrete optimization, which is known as Mixed Integer Programming (MIP). This section provides a general overview on MIP which is mainly based on Burer and Letchford work in [401].

A discrete variable could be a binary, which is the simplest form of MIP. One of the techniques to deal with MIP is to linearize it using a similar techniques to McCormick relaxation while maintain the discrete variables. This approach results in mixed-integer-linear-programming
(MILP) which can be solved easily by many solvers. However, authors came up with different linearization approaches which augmented the research in this field. Another way to deal with binary-MIP is to convexify it. One of the convexification approaches is by adding or subtracting appropriate multiple of terms of this form \((x^2 - x)\), which would be equal to zero if \(x\) is binary.

In many applications, the discrete variables are not binary. This type of non-binary MIP is dealt with by three mainstream ways: Reformation Linearization Technique (RLT), mixed integer semi definite programming (MISDP), and Polyhedral Theory (PT). An interested reader about RLT is advised to refer to source [402]. MISDP is simply a mix between RLT and SDP. PT is about studying the convex hull of feasible solutions in order to reach to either a linear system or convex quadratic relaxations. To reiterate, all aforementioned approaches are either restricted to special cases or does not guarantee global optimality.

In [400], proposed a convex approach to solve unconstrained MISDP. They dealt with discrete variables by embedding the unconstrained-SDP with branch-and-bound search algorithm. Also, their SDP relaxation is not relaxing the non-convexity of in the objective function but also the non-convexity in variable domain too. In [403], the authors used SDP to convexify MIQCP that has linear constraints. Then, they used Branch-and-Bound techniques in order to deal with discrete, or integer, variables.

Power system problem is a non-convex MIQP that has the following traits:

- Constraints are quadratic.
- Constraints that are not necessarily convex.
- The matrices \(A_{\text{const}}\) and \(A_{\text{obj}}\) (see equation (5.38)) are not necessarily symmetric.
• The variables are mix of both continuous and discrete.

Our short literature survey shows that no solution approach that can convexify such a system that has all of above traits. All existing approaches are, simply, either derivatives of McCormick relaxations or relaxations for special cases: power system has to be oversimplified in order to satisfy these special cases. Otherwise, we can retain exact modeling of power system on the expense of global optimality.
5.2 Goal Programming

The normal boundary intersection (NBI) belongs to goal programming family and it has been introduced by Das and Dennis in [404]. The basic idea behind NBI can be explained using figure (5.4). This figure shows the objective space of two-objective optimization problem. If the first objective $\mathbf{\varnothing}_1$ is minimized (a single objective), we would obtain $\mathbf{\varnothing}_1^{\text{min}}$ that is generated by the control variable $\mathbf{x}_{1,\text{min}}$. Similarly, the control variable $\mathbf{x}_{2,\text{min}}$ would generate the global minimum $\mathbf{\varnothing}_2^{\text{min}}$ of the objective $\mathbf{\varnothing}_2$. The global solutions $\mathbf{\varnothing}_1^{\text{min}}$ and $\mathbf{\varnothing}_2^{\text{min}}$ are called the corner solutions that can be found using any appropriate optimization method, such as PCSEA method which has been explained in section (4.1.3).

Now we can draw a line (this line is going to be a hyperplane in case of multi-objective problem) that connect the following two points:

- The location where $\mathbf{\varnothing}_1^{\text{min}}$ touches the objective space.
- The location where $\mathbf{\varnothing}_2^{\text{min}}$ touches the objective space, see figure (5.4).

In this document, this line is called NBI-line or NBI-hyperplane in case of multi-objective problem. The basic idea behind NBI is to construct a perpendicular vector on the NBI-line. This vector should point toward the origin in case of a minimization. This vector can be placed at any point over the line by selecting the proper values of $\mathbf{\xi}_1$ and $\mathbf{\xi}_2$, because it is going to lead us to one of the pareto optimal solutions. Now, the length of this vector has to be maximized in order to minimize all of the objectives. This way a multi-objective problem can be converted to a single objective problem.
The generation of \( \Sigma \)'s helps us to find a pareto set of solutions that are uniformly distributed over the objective space. Since we are looking in this document for only one pareto optimal solution, it suffices to select the solution that falls at the middle of the NBI-plane. Therefore, we could select \( \sum \Sigma_i = 1 \) and \( \Sigma_1 = \Sigma_2 = \cdots = \Sigma_N \) in order to position the vector at the middle of NBI-line.

Let us assume the coordinates of the point \( \mathbf{f} \) on the NBI-line is \((\varphi_1, \varphi_2)\). Then, the following equation will help us to determine the coordinates \((\varphi_1, \varphi_2)\) on figure (5.5), [386]:

\[
\begin{bmatrix}
\varphi_1 \\
\varphi_2
\end{bmatrix} = \begin{bmatrix}
\varphi_1^{min} \\
\varphi_2^{min}
\end{bmatrix} + \begin{bmatrix}
\varphi_1^{min} - \varphi_1^{min} & \varphi_1(x_{2.min}) - \varphi_1^{min} \\
\varphi_2^{min} - \varphi_2^{min} & \varphi_2(x_{2.min}) - \varphi_2^{min}
\end{bmatrix} \begin{bmatrix}
\Sigma_1 \\
\Sigma_2
\end{bmatrix} + \mathbf{U} \quad (5.48)
\]

Let the vector \( \mathbf{U} = \mathbf{L} \mathbf{n} \) where \( \mathbf{L} \) is a scalar and \( \mathbf{n} \) is a normal vector on NBI-plane pointing toward the origin. In above equation, if \( \mathbf{U} = 0 \), \((\varphi_1, \varphi_2)\) would fall exactly on the NBI-plane. However, if \( \mathbf{L} > 0 \), the location of \((\varphi_1, \varphi_2)\) would get closer to the origin. Thus, if we maximize \( \mathbf{L} \), we will be able to minimize our objectives [386]. Therefore, we could generalize equation (5.48) and optimization problem as follow;

\[
\max_{\mathbf{x}, \mathbf{L}} \mathbf{L} \quad (5.49)
\]

Such that

\[
\begin{bmatrix}
\varphi_1(x_1) \\
\varphi_2(x_2) \\
\vdots \\
\varphi_N(x_N)
\end{bmatrix} - \begin{bmatrix}
\varphi_1^{min} \\
\varphi_2^{min} \\
\vdots \\
\varphi_N^{min}
\end{bmatrix} - \mathbf{L} \mathbf{n}
\]

\[
= \begin{bmatrix}
\varphi_1^{min} - \varphi_1^{min} & \varphi_1(x_{2.min}) - \varphi_1^{min} & \cdots & \varphi_1(x_{N.min}) - \varphi_1^{min} \\
\varphi_2^{min} - \varphi_2^{min} & \varphi_2(x_{2.min}) - \varphi_2^{min} & \cdots & \varphi_2(x_{N.min}) - \varphi_2^{min} \\
\vdots & \vdots & \ddots & \vdots \\
\varphi_N^{min} - \varphi_N^{min} & \varphi_N(x_{2.min}) - \varphi_N^{min} & \cdots & \varphi_N(x_{N.min}) - \varphi_N^{min}
\end{bmatrix} \begin{bmatrix}
\Sigma_1 \\
\Sigma_2 \\
\vdots \\
\Sigma_N
\end{bmatrix} \quad (5.50)
\]
How to find the normal vector \( \mathbf{n} \) that is normal to the hyperplane \( \sum_{k=1}^{k=N} \alpha_k \phi_k - d_k = 0 \) such that it passes through the point \( \{d_1, d_2, ..., d_N\} \)?

To answer above question, I will start by a three-dimensional problem. Then, I will generalize it to \( N \) dimension. First, I will select the values of \( \mathbf{E}'s \) such that the normal vector \( \mathbf{n} \) passes through the center of the hyperplane. Therefore, the normal vector would pass through this point

\[
\begin{bmatrix}
\phi_1^{min} + \mathbf{E}_2(\mathbf{x}_2.min) - \phi_1^{min} + \mathbf{E}_3(\mathbf{x}_3.min) - \phi_2^{min} \\
\phi_2^{min} + \mathbf{E}_3(\mathbf{x}_3.min) - \phi_2^{min} \\
\phi_3^{min} + \mathbf{E}_3(\mathbf{x}_3.min) - \phi_3^{min}
\end{bmatrix}
\]

on the hyperplane. Select \( \mathbf{E}_1 = \mathbf{E}_2 = \mathbf{E}_3 = \frac{1}{3} \) to make \( \mathbf{n} \) passes through the middle of the hyperplane. In this case, the equation of our hyperplane would look like: \( \alpha_1(\phi_1 - d_1) + \alpha_2(\phi_2 - d_2) + \alpha_3(\phi_3 - d_3) = 0 \) where

\[
\begin{align*}
\alpha_1 &= \phi_1^{min} + \mathbf{E}_2(\mathbf{x}_2.min) - \phi_1^{min} + \mathbf{E}_3(\mathbf{x}_3.min) - \phi_1^{min} \\
\alpha_2 &= \mathbf{E}_1(\phi_1(\mathbf{x}_2.min) - \phi_2^{min}) + \phi_2^{min} + \mathbf{E}_3(\phi_1(\mathbf{x}_3.min) - \phi_2^{min}), \\
\alpha_3 &= \mathbf{E}_1(\phi_1(\mathbf{x}_3.min) - \phi_3^{min}) + \mathbf{E}_2(\phi_1(\mathbf{x}_3.min) - \phi_3^{min}) + \phi_3^{min}.
\end{align*}
\]

The next step is to find the coefficients \( \alpha' \)s. The following linear system can be constructed to find \( \alpha' \)s.

\[
\begin{bmatrix}
\phi_1^{min} - d_1 & \phi_2(\mathbf{x}_1.min) - d_2 & \phi_3(\mathbf{x}_1.min) - d_3 \\
\phi_1(\mathbf{x}_2.min) - d_1 & \phi_2^{min} - d_2 & \phi_3(\mathbf{x}_2.min) - d_3 \\
\phi_1(\mathbf{x}_3.min) - d_1 & \phi_2(\mathbf{x}_3.min) - d_2 & \phi_3^{min} - d_3
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\] (5.51)

Once the coefficients \( \alpha' \)s are calculated using above linear system, the coordinates of the normal vector \( \mathbf{n} \) is going to be \( \mathbf{n} = (\alpha_1, \alpha_2, \alpha_3) \). To generalize above system, the coordinates of the normal vector \( \mathbf{n} = (\alpha_1, \alpha_2, ..., \alpha_N) \) can be found using the following system;

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\[
\begin{bmatrix}
\varphi_{m_1}^{1} - d_1 & \varphi_2 - d_1 & \ldots & \varphi_{N} - d_N \\
\varphi_1 - d_1 & \varphi_{m_2}^{2} - d_2 & \ldots & \varphi_{N} - d_N \\
\vdots & \vdots & \ddots & \vdots \\
\varphi_1 - d_N & \varphi_{m_N}^{N} - d_N & \ldots & \varphi_{N} - d_N \\
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_N \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
\]

(5.52)

Such that \(d_k = \varphi_{m_k}^{k} + \sum_{i=1}^{(i=k)} \varepsilon_i [\varphi_k - \varphi_{m_k}^{k}]\) and \(\varepsilon_1 = \varepsilon_2 = \ldots = \varepsilon_N = \frac{1}{N} \).

If we insert the coefficients of \( \hat{n} \) in the constraint of (5.50), we would get the following;

\[
\begin{bmatrix}
\varphi_1 \\
\varphi_2 \\
\vdots \\
\varphi_N \\
\end{bmatrix}
- \begin{bmatrix}
\varphi_{m_1}^{1} \\
\varphi_{m_2}^{2} \\
\vdots \\
\varphi_{m_N}^{N} \\
\end{bmatrix}
- \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_N \\
\end{bmatrix}
\]

(5.53)

If all objectives \(N\) are nonnegative, make sure that all \(\alpha's \geq 0\) in order to keep the vector \(\hat{n}\) pointing toward the origin.
CHAPTER 6

VVWO PROBABILISTIC ANALYSIS (CASE STUDY)

In the previous chapters, it was concluded that VVWO is necessary to optimize the performance of the distribution system under high PV penetration. In addition, the analytical formulation for the VVWO problem was discussed. However, solar irradiance is clearly not deterministic, neither are the loads on the system. Therefore, the deterministic VVWO formulation does not provide a full picture. As one more step closer to solve this issue, a probabilistic analysis of the system is necessary. Therefore, in this chapter, a case study of VVWO have been analyzed probabilistically.

The edited IEEE 123-bus test distribution feeder that has been explained in section (4.3.1) is used for simulating the probabilistic VVWO. This system is coded in GAMS and has been solved using BONMIN solver [405]. The calculations are made in p.u. with base power and voltage of 5M V.A. and 4.16k V respectively.

The formulation of electric system that is shown in section (5.1.1) is adopted here but with two excepts, namely the objective formulation and the cost function of the switching devices. The multiobjectives are converted to a single objective by taking their weighted sum as in equation (6.1). Also, The objectives of VRs and SCs should be formulated such that $\varphi_4$ and $\varphi_5$ are equal to zero if their previous tap statuses ($\delta_{VR,i,j,t-1}$ and $\delta_{SC,i,t-1}$) are equal to their new status ($\delta_{VR,i,j,t}$ and $\delta_{SC,i,t}$). Otherwise, $\varphi_4$ and $\varphi_5$ should be equal to “1”. Such a formulation requires Boolean operator which might affect the complexity of the solution. Alternatively, in order to
avoid Boolean operators and to maintain the quadratic formulation, the objectives \( \varphi_4 \) and \( \varphi_5 \) have been approximated by the equations (6.2)-(6.9). The variables \( Z, J, \) and \( X \) in equations (6.2)-(6.9) are dummy variables.

\[
\begin{align*}
\text{Minimize} & \quad Q = \varphi_1(X) + \varphi_2(X) + \cdots + \varphi_5(X) \\
\text{Subject to} & \quad g_j(X) = 0, \ j = 1, 2, \ldots, J \\
& \quad h_k(X) \geq 0, \ k = 1, 2, \ldots, K \\
& \quad X^{(L)} \leq X \leq X^{(U)}
\end{align*}
\]

(6.1)

\[
\begin{align*}
\forall (i,j) \in \mathcal{VR}, \forall t: & \quad \varphi_{4,i,j,t} = 2Y_{VR,i,j,t} - (Y_{VR,i,j,t})^2 \\
\forall (i,j) \in \mathcal{VR}, \forall t: & \quad (Z_{VR,i,j,t} + 1)Y_{VR,i,j,t} = Z_{VR,i,j,t} \\
\forall (i,j) \in \mathcal{VR}, \forall t: & \quad Z_{VR,i,j,t} = (X_{VR,i,j,t})^2 \\
\forall (i,j) \in \mathcal{VR}, \forall t: & \quad X_{VR,i,j,t} = S_{VR,i,j,t} - S_{VR,i,j,t-1} \\
\forall i \in \mathcal{SC}, \forall t: & \quad \varphi_{5,i,t} = 2Y_{SC,i,t} - (Y_{SC,i,t})^2 \\
\forall i \in \mathcal{SC}, \forall t: & \quad (Z_{SC,i,t} + 1)Y_{SC,i,t} = Z_{SC,i,t} \\
\forall i \in \mathcal{SC}, \forall t: & \quad Z_{SC,i,t} = (X_{SC,i,t})^2 \\
\forall i \in \mathcal{SC}, \forall t: & \quad X_{SC,i,t} = S_{SC,i,t} - S_{SC,i,t-1}
\end{align*}
\]

(6.2)-(6.9)

Notice that the formulations in equations (6.2)-(6.9) are quadratic. In order to get a better insight on how the above approximations work, a graph of \( \varphi \) versus \( X \) is shown in figure (6.1). Notice that all points are close to 1. However, if \( X \) is equal to 1, the value of \( \varphi \) will be 0.75 which can still be considered an acceptable approximation if \( X \) is an integer.
It worth mentioning that above approximation can be improved dramatically. This improvement can be done by inserting a multiplication factor in equation (6.4) and (6.8). For instance, both equations can be written as follow:

\[ \forall (i, j) \in int_{VR}, \forall t: \quad Z_{VR,i,j,t} = 100(X_{VR,i,j,t})^2 \] (6.10)

\[ \forall i \in int_{SC}, \forall t: \quad Z_{SC,i,t} = 100(X_{SC,i,t})^2 \] (6.11)

In above equations, a multiplications factor is added. It will improve figure (6.1) dramatically. Unfortunately, this method has not been used in this work since it came to the mind of the author (Kamel Alboaouh) late.
6.1 Simulation Approach

Various scenarios have been simulated to assess VVWO thoroughly. Mainly two cases have been simulated:

- Case.1: The rated value of all loads are assumed to be equal to half of the original rated values of IEEE123 system.
- Case.2: The rated value of all loads are assumed to be equal to the original rated values of IEEE123 system.

Figure 6.2 The main four cases that has been simulated in order to account for loading and insolation.

Figure 6.3 This is a duplication of figure (6.4). The only difference is that this figure unifies the vertical axis
Above cases imitate the system under heavy and lightly loading situations. In order to simulate the clear and cloudy sky situations, above two cases have been simulated twice: one with clear sky situation and the other with cloudy sky. This would lead us to four cases, as shown in figure (6.2): case.1.a, case.2.a, case.1.b, and case.2.b. Then, each case of the four cases has been simulated under different penetration levels, namely 0%, 20%, 40%, 60%, 80%, and 100%. The penetration level is defined as the total sum of PV rated values to the total rated value of IEEE123 system rated loads. Therefore, this would lead us to 20 cases with different penetration levels, plus 2 cases without PV (The total is 22 cases).

6.2 Probabilistic Sampling

Each of the aforementioned 22 cases has been simulated using 500 random samples. Thus, the total samples are 11000 samples.
For both heavily and lightly loaded cases, each sample has different random load data set. Each random sample of a load have been generated by multiplying its rated value by a derating factor that has been drawn from a normal distribution with mean equal to 1 and variance of 0.1, \( \text{norm} \sim (1, 0.1) \).

In the situation of clear sky, all PVs experience an irradiance level that does not change from sample to another and high enough to allow all PVs to release its full capacity. Although all PVs experience the same level of irradiance during the cloudy situation, this level changes randomly from sample to another. The irradiance level has been drawn from uniform distribution with an interval \([a, b]\) where "b" represents the irradiance level that allow PVs to release its full capacity while "a" is defined s "a = 0.1b".

Figure 6.5 Box plot of the voltage of some selected remote buses. (a) Case2.a. (b) Case.1.a. (c) Case.2.b. (d) Case.1.b.
6.3 Results and Discussions

The following subsections explain the results. The following section is about losses in result of probabilistic VVWO.

6.3.1 Losses

When there is no irradiance (0% penetration), the mean of total line losses is about 65kVA and 18.5kVA for the heavily and lightly loaded cases respectively. If the penetration increases to 40%, the mean of line losses decrease to 27kVA and 5.5kVA for both heavily and lightly loaded cases respectively, see figure (6.4,6.3,a,b). For the heavily loaded cases, the mean of line losses keeps decreasing until penetration level reaches 80%. At 100% penetration, the mean of losses is similar to the mean at 80% penetration but with less frequency. Unlike the heavily loaded case, the lightly loaded case exhibits an increase in the mean of losses once penetration level exceeds

![Figure 6.6 Frequency plot of the voltage of some selected remote buses. (a) Case2.a. (b) Case.1.a. (c) Case.2.b. (d) Case.1.b.](image)

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40%. Therefore, from line losses point of view, it is recommended to maintain penetration level around 100%, assuming a clear sky situation.

During cloudy situations, the random samples of irradiance chops the power injected by PVs to the grid which results in a more variations in losses. For this reason, the histograms of line losses widen during cloudy situations, see figure (6.4, 6.3,c,d). Also, the mean of line losses has shifted to the right side (losses have increased) with respect to the clear sky situation. The impact of irradiance can be seen as a decrease in penetration, which would increase line losses as long as the total injected power by PVs is less than or equal to the absorbed power by loads. Therefore, the optimum penetration level that minimizes line losses can exceeds 100% depending on the intensity of the clouds. For an additional output to line losses, please, refer to figures (6.24, 6.25,6.26, 6.36,6.37, and 6.38).
Voltage of remote buses tends to be impacted by PV power more than any other bus. Thus, few remote buses have been selected to monitor their voltage at different loading and penetration levels. The frequency plot of the voltage level of bus No. 71, 85, 29, 33, 51, 66, 96, and 92 are shown in figure (6.6) and (6.5). Since the plot includes different buses, multi-peaks are observed for each curve. For heavily-loaded-clear-sky case, figure (6.6, 6.5, a), an increase in penetration level causes the peaks of the frequency plots to move closer to each other’s, shift to the right, and become taller. This means voltage profile tends to increase once penetration increases. The same pattern is noticed in the lightly loaded case, figure (6.6, 6.5, b), but beyond the 80% penetration level, voltage profile tends to decrease (instead of increasing). Thus, it can be deduced that voltage profile of the grid tends increase up to certain penetration level before it decreases again. Above discussion is applicable to the cloudy situations. However, in cloudy situations, figure (6.6, 6.5, c,d) the number of frequencies tends to be lower than the clear sky situations, figure (6.6, 6.5, a,b).
This decrease in frequency is happening because that the cloudy situations contain more heterogenous samples than in clear sky situations.

An additional output for voltage magnitude which include buses that are located in middle of the grid or closer to the main transformer are shown in figure (6.14,6.15,6.16,6.28,6.29, and 6.30).

6.3.3 PV Performance

One of the objectives is to minimize the curtailed active power by PVs, which is shown in figures (6.8, 6.7). The result of the heavily-loaded-clear-sky case, figure (6.8, 6.7,a), is inline with the expectations: if penetration increases, curtailment increases. The same thing is noticed with the lightly loaded case, figure (6.8, 6.7,b), but it shows higher magnitude of active power curtailment especially for the 80% and 100% penetration levels.

Figure 6.9 Frequency plot of the reactive power injected(+) or absorbed(-) by PVs. (a) Case2.a. (b) Case.1.a. (c) Case.2.b. (d) Case.1.b.
The cloudy cases, figure (6.8, 6.7,c,d), shows the same behavior of clear sky cases in terms of its association with penetration level. Unlike clear sky case, cloudy cases are not showing multi-peaks because their samples are more heterogenous.

The plot of reactive power by PVs tends to shift to the left and becomes taller if penetration decreases for clear sky cases, figure (6.9, 6.10, a,b), and the same thing is observed for the cloudy cases, figure (6.9, 6.10,c,d). This means that PVs tend to inject reactive power to the grid if penetration increases in order to attain VVWO objectives. However, at low loading and penetrations levels, figure (6.9, 6.10,b), PVs tend to absorb reactive power from the grid. Thus, in light of this simulation, the optimal behavior of the reactive power injected by PVs is dependent on loading and penetration levels; at low loading levels, PVs tend to absorb reactive power from the grid; higher penetration level tends to make PVs injects reactive power to the grid.

For reactive power, we notice that PVs tend to absorb (inject) reactive power from the grid at low (high) penetration. This behavior has been interpreted as follow:

Figure 6.10  A duplication of figure (6.9) where all the vertical axes have the same scale.
• At low penetration level, an increase in PV active power injection, reduces power flows in the lines. This in turn reduces losses, which is positive, but it also increases some voltage magnitudes. Once the voltage reaches its maximum value it might limit PV active power injection. One solution would be to lower voltage magnitude by allowing PVs to absorb reactive power from the grid in order to allow more active power injection by the PVs.

• At high penetration levels, an increase in PV active power injection could cause a reverse power flow in lines. This can increase line losses, which is not desired. One solution would be to increase reactive power injection by the PVs so that the reactive component of line corrects reduce, resulting in lower losses.

An additional metric has been added in order to measure how much PV active power has been injected to the grid with respect to the load. This metric is called R-ratio which is the ratio of the total sum of the active power injected by PVs to the total active power absorbed by loads. For
instance, when R-ratio equal to 1, the total injected power by PVs would be equal to the total absorbed power by loads. The R-ratio of the heavily-loaded-clear-sky case is shown in figure (6.11,6.12,a). It can be observed that the frequency plot moves to the left and gets shorter if penetration increases. This means that an increase in penetration of PVs will increase the chance of PV curtailment in order to attain VVWO objectives.

In heavily loaded case, the ideal situation is to have R-ratio equal to 1 at 100% penetration, and it seems that VVWO were able to bring R-ratio to unity as shown in figure (6.11, 6.12,a). What if the total available power by PVs exceeds the total load? The lightly-loaded-clear-sky case, figure (6.11, 6.12,b), can help us answering this question. For the 20%, 40%, 60%, 80%, and 100% penetrations of the lightly-loaded-clear-sky case, the ideal value of R-ratio should be around 0.4, 0.8, 1.2, 1.6 and 2 respectively. It seems from figure (6.11, 6.12,b) that the simulated value for the 20% and 40% penetration are very close to the ideal value but not for the 60%, 80%, and 100% penetrations. The reason for having a value of R-ratio less than the ideal value is that the available
power of PVs exceeds the demand which may cause a reverse power flow and increases losses eventually. Thus, from figure (6.11, 6.12,b), as long as the available power by PVs is less than or equal the rated load, the optimal decision is to inject almost all available active power of PVs. However, once the available power by PVs exceeds the demand, it is recommended to curtail them partially. For cloudy situations, figure (6.11, 6.12,c,d), the frequency plot get more flatter when penetration increases.

For more outputs about PV performance, please refer to Figure (6.17,6.18,6.19,6.31,6.32, and 6.33).

<p>| Table 6.1 Tap Changing Statistics |</p>
<table>
<thead>
<tr>
<th>VRs</th>
<th>SCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case.2.b</td>
<td>7</td>
</tr>
<tr>
<td>Case.1.b</td>
<td>4</td>
</tr>
<tr>
<td>Case.2.a</td>
<td>4</td>
</tr>
<tr>
<td>Case.1.a</td>
<td>1</td>
</tr>
</tbody>
</table>

| Total Number of Tap changing that has been recorded for all samples and all penetration levels |

### 6.3.4 Switching Operations

VVWO succeeded to keep VRs and SCs within the same position for majority of samples. Only few times a change in tap position has been recorded, as shown in Table (6.1). This means that the power injected by PVs, especially the reactive power, can contribute significantly in minimizing the switching operations of VRs and SCs.

For more outputs about the switching operations of VRs and SCs, please, refer to figure (6.20, 6.21, 6.34, and 6.35)
Figure 6.13  Lightly Loaded case Output: (a) CLs curtailment of Clear Sky, (b) CLs curtailment of Cloudy Sky,
Figure 6.14 Lightly Loaded case Output of voltage magnitude for some selected remote buses: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky,
Figure 6.15  Lightly Loaded case Output of voltage magnitude for some selected buses that are located in middle of the grid: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky,
Figure 6.16  Lightly Loaded case Output of voltage magnitude for some selected buses that are located at upstream side of the grid: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky.
Figure 6.17 Lightly Loaded case Output of curtailment of the active power by PVs: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky.
Figure 6.18 Lightly Loaded case Output of the injected reactive power PVs. (+) injecting /(-) absorbing: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky,
Figure 6.19  Lightly Loaded case Output of the ratio of the total active power injected by PVs to the total active power absorbed by loads: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky.
Figure 6.20 Lightly Loaded case Output of SCs statuses: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky.
Figure 6.21 Lightly Loaded case Output of VRs statuses: (a) Volt. of Clear Sky, (b) Volt. of Cloudy Sky.
Figure 6.22 Lightly Loaded case Output of the injected apparent power by PVs: (a) Clear Sky, (b) Cloudy Sky.
Figure 6.23  Lightly Loaded case Output of the “total” injected apparent power by PVs:
(a) Clear Sky, (b) Cloudy Sky.
Figure 6.24 Lightly Loaded case Output of the “total” line losses in kW: (a) Clear Sky, (b) Cloudy Sky.
Figure 6.25 Lightly Loaded case Output of the “total” line losses in kVAR: (a) Clear Sky, (b) Cloudy Sky.
Figure 6.26  Lightly Loaded case Output of the “total” line losses in kVA: (a) Clear Sky, (b) Cloudy Sky.
Figure 6.27  Heavily Loaded case Output: (a) CLs curtailment of Clear Sky, (b) CLs curtailment of Cloudy Sky,
Figure 6.28 Heavily Loaded case. Output of Voltage magnitude to some selected remote buses: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.29    Heavily Loaded case. Output of Voltage magnitude to some selected buses that are located in middle of the grid: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.30   Heavily Loaded case. Output of Voltage magnitude to some selected buses that are located at the upstream side of the grid: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.31  Heavily Loaded case. The curtailed active power of PVs: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.32 Heavily Loaded case. The injected reactive power of PVs. (+) injecting / (-) absorbing: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.33 Heavily Loaded case. The ratio of the total injected active power of PVs to the total active power consumed by PVs: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.34  Heavily Loaded case. The statuses of SCs: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.35 Heavily Loaded case. The statuses of VRs: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.36  Heavily Loaded case. Total kW line losses: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.37 Heavily Loaded case. Total kVAR line losses: (a) Clear Sky, (b) Cloudy Sky,
Figure 6.38  Heavily Loaded case. Total kVA line losses: (a) Clear Sky, (b) Cloudy Sky,
CHAPTER 7

CONCLUSIONS

In this work, the optimization of the distribution system has been considered under heavy penetration levels of rooftop PVs. The optimization and analysis has been conducted in different fashions: heuristic, deterministic, analytical, and probabilistic. All optimization scenarios have been implemented assuming a centralized control and an ideal communication infrastructure. Also, a literature review on the impact of PVs on distribution system has been presented.

The analytical formulation presented is comprehensive, i.e., discrete variables are included. The VVWO performance under various penetration levels has been tested using three methods: classic EA, NSGA-III, and analytic analysis. Then, these methods have been compared using the same electric distribution system. The comparisons provided almost the same general lessons which further underline the findings of this work.

➢ Findings

The literature review has shown that fit-and-forget approach is not recommended under high penetration levels of PVs even when they are equipped with a decentralized control. Alternatively, the simulation results of the centralized VVWO optimization has shown that a distribution system can accommodate high penetration levels of PVs without violating voltage limits.

If the only objective of VVWO is to maximize PV penetration, penetration levels would be limitless under the following assumptions:
The infinite bus is readily available to absorb the extra capacity of PVs or to supply load demand if PVs are not available.

- The system is deterministic.
- Cables and transformers are ideal.

The VVWO simulation has shown that once both objectives of maximizing PV penetration and minimizing line losses are combined together, the optimal PV penetration changes. In such a situation, the optimal PV penetration occurs when the injected PVs power equalizes the local load.
demand because the flow of power in lines would be zero which would minimize losses. However, this is not necessarily always true. In order to explain this idea, a simple example shown in figure (7.1, a) will be utilized. Let us assume the following three scenarios happened to the system in figure (7.1, a):

- Scenario .1: the power injected by $PV_{bus2}$ is equal to the power absorbed by $load_{bus2}$. Similarly, the power injected by $PV_{bus4}$ is equal to the power absorbed by $load_{bus4}$.

- Scenario .2: the power injected by $PV_{bus2}$ is equal to the power absorbed by $load_{bus2} + load_{bus3}$. Also, the power injected by $PV_{bus4}$ is equal to the power absorbed by $load_{bus4}$.

- Scenario .3: the power injected by $PV_{bus2}$ is equal to the power absorbed by $load_{bus2}$. Conversely, the power injected by $PV_{bus4}$ is equal to the power absorbed by $load_{bus4} + load_{bus}$.

In scenario No.1, we would be having losses on $line_{bus1,2}$ and $line_{bus2,3}$, see power flow in figure (7.1, b). In scenario No.3, we will have losses on $line_{bus2,4}$ and $line_{bus2,3}$, see power flow figure (7.1, d). However, in scenario No.2 we will be having losses on $line_{bus2,3}$ only, see power flow figure (7.1, c). Thus, it can be seen that the optimal penetration in this case is not when PVs’ power equals the local loads as one might intuitively think.

At the optimal penetration level of PVs, it has been found that voltage regulation equipment can maintain system’s voltage within the proper limitation if coordinated properly. However, this would happen at the expense of increased tap operations. It has been found that the fast response of PVs, provided they are allowed to participate in reactive power support, would minimize the
number of tap operations of VRs and switching operations of SCs. At high penetration levels, the reactive power support by PVs could replace or at least reduce the operation instances of VRs and SCs (of course, PV location plays a fundamental role here). However, when PVs injects/absorb reactive power in the grid, reactive component of current flowing in lines would increase which might increase line losses. Our simulation results show that PVs’ participation in reducing the active current component flowing through the lines will outweigh the increase in reactive current component. Thus, in general, PVs tend to reduce line losses.

➢ Contributions

Studies in the literature have warned utilities over the last four decades about the negative technical impact of high penetration levels of PVs if they are installed without modifying or upgrading the current electric system. Hence, the next obvious question is: what can be done once PV penetration reaches that problematic level? Naturally, budget restrictions prevent utilities from upgrading their entire network which could further complicate matters.

Our analysis and simulation have shown that a proper utilization of existing resources has the potential to eliminate the negative technical impact of PVs. On top of that, it has been observed that PVs at high penetration levels can act as a standby reactive power resource which can be either absorbed or injected with a fast response. This property flips the table and makes PVs beneficial to the grid if utilized properly. Therefore, it has been found that the optimum penetration level of PVs is higher than what has been thought in the literature. This makes solar energy a promising solution to preserve our environment and reduce dependence on fossil fuels.

However, it should be noted that the key statement here is “proper utilization of existing resources”. A proper utilization cannot be attained without a strong communication infrastructure.
and automated control of the system. Utilities would therefore need to install an effective communication & control infrastructure in order to materialize the benefits of solar energy. The existing resources, such as VRs, SCs, etc. can be maintained but their control scheme would need to change.

Studying the optimum performance of power systems at high penetration levels of PVs is a difficult task, especially due to the multi-objective nature of the problem. Thus, the simulations that have been carried out are intended to provide **an insight into the potential of PVs in the power system which might be useful to utilities and policy makers**. For instance, it has been found out that voltages at buses are not necessarily the limiting factor for PV penetration, instead system losses can be what limits PV deployment.

In this work, an attempt was made to convexify the analytic formulation of VVWO. Therefore, McCormick relaxation has been used for this purpose. McCormick relaxation does not approximate the model but the solution region for solvers. Thus, when solvers try to find a solution for a system that has been relaxed using McCormick, they would be confined to a smaller solution region and may converge faster. Unfortunately, BONMIN solver which was used in this work could not find an initial solution (because solvers are an iterative process that start with an initial feasible solution) to our relaxed system. The reason is likely to be either an intrinsic incompatibility between the solver and McCormick relaxation (non-electric realm) or the relaxation results in a non-overlapping solution regions for some of the constraints.

**Future Work**

We found that penetration level of PVs is limitless in terms of voltage violations. However, once we want to find the optimum penetration level to attain certain objectives, penetration level
decreases. The future work might find the optimum penetration level by including more practical factors.

In our simulation, we have assumed that the distribution system has an infinite bus. This bus is ideal and readily available once needed to supply power or absorb the reverse power without any limitations. In practice, this is not the case. On the upstream side of the infinite bus, there are utilities who are concerned about their reserve power, economic issues, stability, capacity of their power, contractual commitment, federal-policy challenges, etc. Of course, each of these concerns could be an interesting research area. However, we are concerned only about the most influential factor to PV penetration at the distribution level, which is the ability to supply the loads. This can be measured by monitoring the system’s frequency (we are referring to the fundamental frequency, 60Hz, and not the harmonics involved).

System’s frequency is dependent mainly on the balance between generation and demand. Also, the response speed and capacity of generation units to satisfy load is a critical factor in order to prevent the system from voltage or frequency collapse (this happens when generation units can no longer respond to the change in demand). Thus, once the penetration level of PVs at the distribution level reaches high levels, one might wonder whether existing generation units can withstand the fast variations of irradiance. What would be the optimum penetration level in this case? In other words, the VVWO becomes a dynamic-VVWO.

Dynamic-VVWO is not just about the stability of the system (the system becomes unstable when generation units lose their synchronization). To be specific, the objectives of Dynamic-VVWO would have the same objectives of VVWO plus fuel cost of generation units: The
constraints of Dynamic-VVWO would be the same as VVWO plus frequency limitation and synchronization.

The control system for dynamic-VVWO can be centralized at the transmission level side for the purpose of idealization. Then, this centralized control system communicates with the centralized control systems of various distribution systems. Thus, the control system would look like a zone-control. For instance, if we have a transmission network that serves 4 distribution systems, we would have 5 centralized control systems that share data via an ideal communication medium.

The analytical formulation of VVWO is algebraic in steady state, i.e. no differential variables. On the other hand, dynamic-VVWO would be modeled as a system of algebraic and differential equations in order to account for the response time of generation units and various decentralized control variables such as the response of interfacing-inverters, VRs, SCs, etc. (a delay time response is needed when these devices has their own decentralized control). Simulating a differential system is time consuming, which will be a challenge for any research project.

Once the system response has been investigated, more dimensions can be added. For instance, the protection scheme is another critical limiting factor. However, the inertia of the system precedes protection scheme limitations. Thus, it is recommended to study dynamic-VVWO beforehand. Then, once protection scheme is studied, the latency of communication system could incorporate into the study.

Another perspective that can be considered as a future work is the possible use of PVs at high penetration in the case of islanded operation of a network. Since the power supplied by PVs is volatile, the amount of available energy would be dependent on cloud coverage and movement
in the sky. Therefore, the amount of energy available from PVs would be stochastic. In this case, a stochastic VVWO must be performed in order to investigate whether or not the network can continue operating using solely the PV resource.
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