MODELING, IMAGING AND WAVEFORM INVERSION IN ATTENUATIVE ANISOTROPIC MEDIA

by

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ABSTRACT

Seismic attenuation, which is often anisotropic, has a profound influence on wave propagation and seismic processing. In addition to application of attenuation coefficients in lithology and fluid discrimination, compensation for angle-dependent attenuation and velocity dispersion is critically important in amplitude-variation-with-offset (AVO) analysis and imaging. In this thesis, I develop a waveform-inversion (WI) methodology for attenuation estimation, as well as an attenuation-compensated time-reversal (TR) imaging algorithm, for transversely isotropic media with a vertical symmetry axis (VTI). The attenuation model is assumed to have the same symmetry as the velocity function and is described by the Thomsen-style attenuation parameters.

First, the thesis introduces a time-domain finite-difference modeling technique for viscoelastic VTI media. Within the framework of the generalized standard linear solid (GSLS) model, I extend to anisotropic media the so-called $\tau$-method”, which produces a nearly constant quality-factor matrix $Q_{ij}$ within a specified frequency range. Then I present a system of anisotropic viscoelastic wave equations which include the memory variables that facilitate numerical implementation. Numerical examples for a range of TI models with different structural complexity confirm the accuracy of the proposed modeling scheme and illustrate the influence of attenuation and attenuation anisotropy on multicomponent wavefields.

Then this wavefield propagator is employed in a waveform-inversion algorithm for attenuation analysis in heterogeneous VTI media. Model updating is performed with the conventional $\ell_2$-norm objective function, and the inversion gradients for the viscoelastic parameters are derived from the adjoint-state method. Four VTI attenuation parameters for P- and SV-waves are updated simultaneously with a quasi-Newton optimization algorithm. The influence of velocity errors is mitigated by using a local-similarity technique. Transmission tests for a model with Gaussian anomalies in the VTI attenuation parameters and a reflection experiment for a modified BP TI section validate the developed WI algorithm.
The performance of waveform inversion may be strongly influenced by the accuracy of the estimated source wavelet. To mitigate the cross-talk between the attenuation parameters and source signature, I extend source-independent waveform inversion (SIWI) to anisotropic attenuative media. The corresponding objective function evaluates the difference between two convolved data sets that include reference traces from the observed and simulated data. As illustrated by numerical examples, the proposed SIWI method can produce sufficiently accurate attenuation parameters (albeit with reduced resolution) even for a substantially distorted source wavelet.

Accounting for attenuation anisotropy is essential in implementing attenuation-compensated time-reversal (TR) imaging for locating microseismic sources in unconventional shale reservoirs. TR imaging is carried out with a viscoelastic VTI propagator based on fractional Laplacians, which is designed to decouple the influence of dissipation from that of dispersion. The obtained viscoelastic equations make it possible to compensate for anisotropic attenuation and preserve time symmetry during back-propagation. The proposed $Q$-compensated TR imaging algorithm is tested on synthetic microseismic data from 2D VTI media. Taking attenuation anisotropy into account produces superior source images and more accurate excitation times compared to those obtained by purely elastic back-propagation or by the TR algorithm with isotropic $Q$-compensation.
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<td>Description of the tests for the BP model. Each test includes four TR experiments defined in Table 5.1. Note that “accurate $Q$” or “smoothed $Q$” in column 4 applies only to the IVTR and AVTR algorithms.</td>
<td>84</td>
<td></td>
</tr>
</tbody>
</table>
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CHAPTER 1
INTRODUCTION

Seismic waves encounter energy dissipation and velocity dispersion during their propagation in the subsurface. While it consists of both intrinsic (absorption) and scattering attenuation, “seismic attenuation” often refers to the former phenomenon, mainly because the conversion of energy (into heat) is sensitive to fluid type, saturation, and mechanical properties of reservoir rocks (Johnston et al., 1979; Tiwari and McMechan, 2007). In addition to its role in reservoir characterization, attenuation also needs to be compensated for in amplitude-variation-with-offset (AVO) analysis and imaging.

Angular velocity variation (velocity anisotropy) has been extensively studied and integrated into the standard industry processing flows. Formations that exhibit velocity anisotropy are often characterized by even stronger attenuation anisotropy (Best et al., 2007; Chichinina et al., 2009; Zhu et al., 2006). The physical mechanisms responsible for attenuation anisotropy include interbedding of thin layers (Carcione, 1992), directionally dependent stress (Deng et al., 2009), and preferential fluid flow (Akbar et al., 1993). Hence, attenuation anisotropy can potentially provide valuable information about the physical properties of the subsurface.

A prerequisite for accurate attenuation analysis is efficient viscoelastic modeling. Shekar and Tsvankin (2014) develop a modeling algorithm for attenuative anisotropic media based on the Kirchhoff scattering integral and summation of Gaussian beams. Compared to such ray-based techniques and the reflectivity method (e.g., Mallick and Frazer, 1987), finite-difference (FD) modeling can simulate full wavefields without restrictive constraints on the frequency range or medium structure. FD methods also are easier to implement and more computationally efficient than pseudospectral (e.g., Carcione, 2010) or finite-element (e.g., Komatitsch and Tromp, 1999) techniques. Although attenuation can be incorporated into frequency-domain finite-differences in a straightforward way, such algorithms are hampered by heavy memory requirements, especially
for large-scale 3D simulations (Operto et al., 2007). As a consequence, many publications focus on time-domain viscoacoustic/viscoelastic modeling (Blanch et al., 1995; Bohlen, 2002; Carcione, 1993; Day and Minster, 1984; Emmerich and Korn, 1987; Zhu et al., 2013). A generalized standard linear solid (GSLS) model is often adopted in time-domain finite-differences to simulate a nearly constant $Q$-factor within a specified frequency range.

Conventional methods for attenuation estimation include the centroid frequency shift method (CFSM) and the spectral-ratio method (SRM). For anisotropic media, Behura and Tsvankin (2009a) and Shekar and Tsvankin (2011) propose SRM-based layer-stripping schemes for P- and S-wave attenuation analysis, respectively. However, these conventional approaches are generally limited to structurally simple subsurface models and become unstable in the presence of interference.

Full-waveform inversion (FWI) can potentially provide more robust attenuation estimation for realistic subsurface structures. Originally introduced by Tarantola (1984) for high-resolution velocity model building, FWI has been also extended to attenuative media (e.g., Charara et al., 2000; Denli et al., 2013). By conducting synthetic transmission experiments, Brossier (2011) demonstrate the advantage of an hierarchical approach, in which the velocity parameters are recovered prior to attenuation analysis (Kamei and Pratt, 2008). Using the actual velocity field, Bai and Yingst (2013) apply multiscale FWI to estimate P-wave attenuation for a viscoacoustic version of the isotropic Marmousi model. Yet, attenuation estimation remains challenging because amplitudes are influenced by a number of other factors including geometric spreading, reflection and transmission coefficients, source radiation pattern, etc.

Attenuation breaks the time symmetry of wave equation, which hampers implementation of reverse-time migration (RTM) and time-reversal (TR) imaging. To compensate for attenuation and preserve the time symmetry, dissipation and dispersion effects should be treated separately (Zhu, 2014), which cannot be accomplished with the widely-adopted GSLS model discussed above. A constant-$Q$ propagator with the fractional Laplacian that decouples the two effects is developed by Zhu and Harris (2014). That propagator was successfully applied to imaging of subsurface structures (Zhu et al., 2014) and microseismic sources (Zhu, 2015) in isotropic media.
The goal of the thesis is to develop a robust waveform-inversion (WI) methodology for attenuation estimation, as well as a $Q$-compensated time-reversal imaging algorithm for transversely isotropic media. In chapter 2, I present a time-domain FD propagator for attenuative anisotropic models. Application of the generalized standard linear solid (GSLS) model makes it possible to simulate nearly constant quality-factor elements $Q_{ij}$ within a certain frequency range. Rotated staggered grids (RSG) and memory variables are introduced for numerical implementation of the time-domain FD method. The accuracy of the proposed algorithm is confirmed by reconstructing the angle-dependent (anisotropic) attenuation coefficients from synthetic reflection data. Numerical examples for VTI media with different structural complexity further illustrate the performance of the modeling code and the influence of attenuation and attenuation anisotropy on multicomponent wavefields. This work was presented at SEG annual meeting and published as a peer-reviewed article in *Geophysics* (Bai and Tsvankin, 2016).

The anisotropic viscoelastic propagator is employed in Chapter 3 to develop a WI algorithm that carries out attenuation analysis for heterogeneous VTI media. Based on the Born approximation, I derive the gradients of the objective function with respect to the viscoelastic parameters using the adjoint-state method. Four parameters describing the P- and SV-wave attenuation in VTI media are updated simultaneously with the L-BFGS (limited-memory Broyden-Fletcher-Goldfarb-Shanno) method. To remove the time shifts between the simulated and observed data caused by velocity errors, a local-similarity technique is incorporated into the inversion workflow, which substantially improves the convergence of the objective function. The inversion algorithm is validated by transmission tests for VTI models with Gaussian anomalies and a reflection experiment for a modified BP TI section. I presented this work at the International Workshop on Seismic Anisotropy and published it in *Geophysics* (Bai et al., 2017).

In addition to the velocity model, estimation of the attenuation parameters is sensitive to the accuracy of the source signature. In Chapter 4 I adopt a time-domain source-independent waveform inversion (SIWI) algorithm designed to reduce the influence of source signature on attenuation estimation. Synthetic tests for transmission and reflection data show that even relatively small
distortions of the source wavelet may prevent conventional waveform inversion from recovering attenuation anomalies. In contrast, the source-independent algorithm significantly improves data fitting and accuracy of parameter estimation, albeit at the cost of somewhat reduced spatial resolution. This work was submitted to the peer-reviewed journal *Geophysical Prospecting* (Bai and Tsvankin, 2019).

Chapter 5 is devoted to developing a $Q$-compensated time-reversal (TR) imaging algorithm for VTI media. With a new viscoelastic VTI propagator (different from that in Chapter 2) that decouples the effects of dissipation and dispersion, I compensate for anisotropic attenuation and restore the time symmetry of the wave equation by reversing the signs of the dissipation-dominated terms during back-propagation. Synthetic microseismic experiments show that anisotropic $Q$-compensation results in more accurate excitation times and microseismic source locations compared to those obtained without attenuation compensation or with purely isotropic quality-factors $Q_{P0}$ and $Q_{S0}$. This work was presented at the 2018 SEG annual meeting and was submitted to *Geophysics* (Bai et al., 2019).
CHAPTER 2

TIME-DOMAIN FINITE-DIFFERENCE MODELING FOR ATTENUATIVE ANISOTROPIC MEDIA

A paper published* in Geophysics
Tong Bai¹ and Ilya Tsvankin¹

Accurate and efficient modeling of seismic wavefields that accounts for both attenuation and anisotropy is essential for further development of processing methods. Here, we present a 2D time-domain finite-difference algorithm for generating multicomponent data in viscoelastic transversely isotropic media with a vertical symmetry axis (VTI). Within the framework of the generalized standard linear solid (GSLS) model, the relaxation function is expressed through the \( \tau \)-parameters (which quantify the difference between the stress and strain relaxation times) defined for anisotropic media. This approach produces nearly constant values of all components of the quality-factor matrix within a specified frequency band. The developed algorithm is based on a set of anisotropic viscoelastic wave equations parameterized by memory variables. Synthetic examples for TI models with different structural complexity confirm the accuracy of the proposed scheme and illustrate the influence of attenuation and attenuation anisotropy on multicomponent wavefields.

2.1 Introduction

Viscoelastic properties of subsurface formations have a profound influence on wave propagation and seismic processing. The attenuation-induced amplitude loss and velocity dispersion can cause distortions in amplitude-variation-with-offset (AVO) analysis and imaging. However, attenuation can also provide valuable information about lithology and fluids needed for reservoir characterization.

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A prerequisite for accurate attenuation analysis and estimation is efficient viscoelastic modeling (e.g., Shekar and Tsvankin, 2014). The main advantage of finite-difference (FD) methods compared to asymptotic algorithms is their ability to simulate the complete wavefield without sacrificing accuracy. In frequency-domain modeling, attenuation can be incorporated directly through the imaginary part of the stiffness coefficients. However, implementation of finite-difference methods in the frequency domain is hampered by the need to factorize a large sparse linear system of equations (Operto et al., 2007). As a result, many publications are focused on simulating wave propagation in attenuative media with finite-difference time-domain methods (Blanch et al., 1995; Bohlen, 2002; Carcione, 1993; Day and Minster, 1984; Emmerich and Korn, 1987; Zhu et al., 2013). A nearly constant quality factor $Q$ over a specified frequency range can be simulated by mechanical models. Memory variables, introduced into the corresponding convolutional stress-strain relationship, facilitate numerical implementation (Robertsson et al., 1994).

The attenuation coefficient for subsurface formations is often directionally dependent, and the magnitude of attenuation anisotropy is typically much higher than that of velocity anisotropy (Behura and Tsvankin, 2009a; Hosten et al., 1987; Zhu and Tsvankin, 2006). Červený (2005) and Vavryčuk (2007) present a detailed discussion of attenuation anisotropy based on ray theory. Behura and Tsvankin (2009b) show that the attenuation coefficient along seismic rays is close to the corresponding phase attenuation coefficient (computed for zero “inhomogeneity angle”) even for strong anisotropy. Shekar and Tsvankin (2014) develop an efficient Kirchhoff modeling algorithm for attenuative anisotropic media using Gaussian beams.

Although time-domain viscoelastic wave equations for general anisotropic media have been known for a long time (e.g., Tarantola, 1988; Komatitsch and Tromp, 1999; Charara et al., 2000; Fichtner and Van Driel, 2014), they were considered “too general for seismic purposes” (Tarantola, 1988) and thus seldom implemented. In one of the few published attempts to include attenuation anisotropy in time-domain FD modeling, Mittet and Renlie (1996) simulate acoustic full-waveform multipole logging. However, they do not give a clear description of employed attenuation anisotropy.
Here, we present a 2D time-domain FD algorithm designed to simulate P- and SV-waves for models with VTI symmetry for both velocity and attenuation. First, we discuss the rheology of an anisotropic viscoelastic model and a formalism for generating nearly constant $Q_{ij}$-values in a specified frequency band. Next, we present the viscoelastic wave equation for VTI media and describe its implementation in FD modeling. Finally, we demonstrate the accuracy and efficiency of the developed FD algorithm with numerical examples.

2.2 Methodology

2.2.1 Rheology of anisotropic viscoelastic model

The stiffness matrix $C_{ij}$ (in Voigt notation) for viscoelastic media becomes complex, and attenuation can be described by the quality-factor matrix $Q_{ij}$ (Carcione, 2007; Zhu and Tsvankin, 2006):

$$ Q_{ij} = \frac{\text{Re}(C_{ij})}{\text{Im}(C_{ij})}. $$

(2.1)

Attenuation can be easily incorporated into frequency-domain modeling through the imaginary parts of the stiffnesses or through the matrix $Q_{ij}$ (Gosselin-Cliche and Giroux, 2014; Operto et al., 2009; Shekar and Tsvankin, 2014). In the time domain, however, attenuation is typically introduced through the so-called relaxation function $\Psi$ (Carcione, 2007; Moczo et al., 2007):

$$ \Psi_{ij}(t) = F^{-1} \left\{ \frac{C_{ij}(\omega)}{i\omega} \right\}, $$

(2.2)

where $F^{-1}$ denotes the inverse Fourier transform, and both $\Psi_{ij}$ and $C_{ij}$ are expressed in the two-index Voigt notation. The generalized stress-strain relationship in linear viscoelastic media can be written as:

$$ \sigma_{mn} = \Psi_{mnpq} * \epsilon_{pq} = \dot{\Psi}_{mnpq} * \epsilon_{pq}, $$

(2.3)

where $\sigma_{mn}$ and $\epsilon_{pq}$ are the stress and strain tensor respectively, and the asterisk and dot denote convolution and time derivative, respectively. Equation 2.3 shows that the stress tensor is determined by the entire history of the strain field, rather than by just its current value (which is the case for
purely elastic media).

The relaxation function, which determines the viscoelastic behavior of the material, can be simulated by the so-called generalized standard linear solid (GSLS) model. A single standard linear solid (SLS) consists of two parallel mechanical systems, with one made of a spring and a dashpot in series and the other containing a single spring (Blanch et al., 1995). Several SLS’s in parallel constitute the GSLS, with each individual SLS called a “relaxation mechanism.” Most existing publications implement a relaxation function for isotropic media (e.g., Carcione, 1993; Moczo et al., 2007). For general anisotropic media, the function \( \Psi_{ij} \) is given by (Charara et al., 2000; Komatitsch and Tromp, 1999):

\[
\Psi_{ij}(t) = C^R_{ij} \left[ 1 - \frac{1}{L} \sum_{l=1}^{L} \left( 1 - \frac{\tau_{\sigma l}}{\tau_{ij}} \right) e^{-t/\tau_{\sigma l}} \right] H(t),
\]  

where \( C^R_{ij} = \Psi_{ij}(t \to \infty) \) is called the “relaxed modulus” corresponding to the low-frequency limit \( (\omega = 0) \), \( \tau_{ij} \) and \( \tau_{\sigma l} \) are the strain and stress relaxation times (respectively) for the \( l \)th mechanism, \( H(t) \) is the Heaviside function, and \( L \) is the number of mechanisms. The relaxed modulus \( C^R_{ij} \) is related to the real part of the corresponding complex modulus \( C_{ij} \) defined at the reference frequency \( \omega_r \):

\[
C^R_{ij} = \text{Re} \left( C_{ij} \right) \left[ \frac{1}{L} \sum_{l=1}^{L} \frac{\omega_r^2 \tau_{\sigma l} \tau_{\epsilon l}}{1 + (\omega_r \tau_{\sigma l})^2} \right]^{-1}. \tag{2.5}
\]

Generally, the more relaxation mechanisms (or SLS’s) are included, the wider is the frequency range in which it is possible to simulate a nearly constant \( Q_{ij} \). For different components of the anisotropic relaxation tensor \( \Psi \), the stress relaxation times can be identical, while \( \tau_{\epsilon l} \) generally differ (Komatitsch and Tromp, 1999).

### 2.2.2 The \( \tau \)-method

Blanch et al. (1995) demonstrate that the magnitude of attenuation in isotropic media is directly determined by the dimensionless parameter \( \tau \):

\[
\tau = \frac{\tau_{\epsilon l}}{\tau_{\sigma l}} - 1. \tag{2.6}
\]
For anisotropic media, $\tau$ becomes a matrix that we define as:

$$\tau_{ij} = \tau^{il}_{ij} \tau_{ll}^{-1}. \quad (2.7)$$

The quality-factor elements $Q_{ij}$ decrease with increasing $\tau_{ij}$. In the elastic case, the stress and strain relaxation times are equal, and $\tau_{ij}$ vanishes. Since $\tau_{ij}$ should remain constant for all relaxation mechanisms, the number of independent parameters ($\tau_{ij}$ and $\tau^{\sigma l}$) for each element of the relaxation function $\Psi_{ij}$ reduces from $2L$ to $L+1$ ($L$ denotes the number of relaxation mechanisms). For P- and SV-waves in a 2D viscoelastic VTI model, the total number of independent parameters is equal to $L + 4$ ($L$ for $\tau^{\sigma l}$ and 4 for $\tau_{ij}$).

The expressions for the relaxation function (equation 2.4) and $\tau_{ij}$ (equation 2.7) allow us to find the complex modulus $C_{ij}$ from equation 2.2. Then the inverse of the quality factor is given by

$$Q^{-1}_{ij}(\omega) = \frac{\text{Im}(C_{ij})}{\text{Re}(C_{ij})} = \frac{\tau_{ij} \sum_{l=1}^{L} \omega^{\sigma_{il}}}{L + \tau_{ij} \sum_{l=1}^{L} \frac{\omega^{\sigma_{il}}}{1 + \omega^{2\sigma_{il}}}}. \quad (2.8)$$

Figure 2.1: Curve of $Q_{ij}$ (dashed line) simulated with three relaxation mechanisms in the frequency range from 2 to 200 Hz. The desired value of $Q_{ij}$ is 30 (solid line). The inverted parameters are: $\tau_{ij} = 0.2124$, $\tau^{\sigma 1} = 22.7$ ms, $\tau^{\sigma 2} = 1.3$ ms, and $\tau^{\sigma 3} = 2 \times 10^{-3}$ ms.
By applying least-squares inversion to equation 2.8, we can obtain the corresponding parameters \( \tau_{ij} \) and \( \tau_{\sigma l} \), which produce the desired nearly constant value of \( Q_{ij} \) in a specified frequency band (Bohlen, 2002). Figure 2.1 shows that the simulated \( Q_{ij} \)-curve using the inverted parameters \( \tau_{\sigma l} \) and \( \tau_{ij} \) is close to the desired \( Q_{ij} \)-value, when three relaxation mechanisms are used.

2.2.3 Viscoelastic VTI wave equation and FD implementation

Using equations 2.3, 2.4, and 2.7, the viscoelastic constitutive relation for 2D VTI media can be obtained as (Appendix A; Tromp et al., 2005; Fichtner and Van Driel, 2014):

\[
\dot{\sigma}_{mn} = \frac{1}{2} C_{mnpq}^U (v_{p,q} + v_{q,p}) + \sum_{l=1}^{L} \tau_{mn}^l, \quad (2.9)
\]

and

\[
\dot{r}_{mn}^l = -\frac{1}{\tau_{\sigma l}} \left[ \frac{1}{2L} \left( C_{mnpq}^U - C_{mnpq}^R \right) (v_{p,q} + v_{q,p}) + r_{mn}^l \right], \quad (2.10)
\]

where \( v_{p,q} \) is the derivative of the \( p \)th component of the particle velocity with respect to \( x_q \), \( C_{mnpq}^R \) are the relaxed moduli, \( C_{mnpq}^U \) are the unrelaxed moduli defined as \( C_{mnpq}^U = C_{mnpq}^R (1 + \tau_{mnpq}) \), and \( r_{mn}^l \) are the memory variables for the \( l \)th mechanism. The Einstein summation convention over \( p \) and \( q \) (\( p = 1, 3; q = 1, 3 \)) is assumed, and \( mn = 11, 13, 33; C_{mnpq}^U \) and \( C_{mnpq}^R \) can be expressed in the two-index notation using Voigt convention.

Equations 2.9 and 2.10 plus the momentum conservation law constitute the viscoelastic VTI wave equation, which allows us to carry out time-domain FD modeling for media with VTI symmetry for both velocity and attenuation. The stress-velocity formulation is adopted here because of its natural connection to staggered grids (Moczo et al., 2007), which generally provide high numerical accuracy. Our algorithm is based on a rotated staggered grid (RSG) (Saenger and Bohlen, 2004; Saenger et al., 2000, see Figure 2.2), which is preferable to the standard staggered grid (SSG) in anisotropic media. The particle velocity and density are defined at the center of each cell (staggered grid point), while other parameters including stress, memory variables, stress relaxation time, and \( \tau_{ij} \) are assigned to regular grid points. The two sets of parameters are related through FD operators in the auxiliary directions \( \tilde{x} \) and \( \tilde{z} \), as discussed by Saenger et al. (2000).
Figure 2.2: Scheme of a rotated staggered grid (RSG).

The time and spatial derivatives are approximated by the second-order and 12th-order centered differences, respectively, with the time derivative calculated using the leap-frog scheme (Kristek and Moczo, 2003). A sponge-layer absorbing boundary condition is applied to eliminate reflections from the model boundaries. To minimize numerical artifacts and avoid instabilities, we apply spatial and temporal (i.e., CFL) sampling criteria modified after Bohlen (2002):

\[ dh \leq \frac{\lambda_{\text{min}}}{n} = \frac{V_{S,\text{min}}}{nf_{\text{max}}} \]  

(2.11)

and

\[ dt \leq \frac{dh}{\sqrt{2} m V_{P,\text{max}}}, \]  

(2.12)

where \( \lambda_{\text{min}} \) denotes the minimum wavelength, \( V_{S,\text{min}} \) and \( V_{P,\text{max}} \) are the smallest S-wave velocity and largest P-wave velocity (taking into account anisotropy and dispersion), \( f_{\text{max}} \) is the maximum frequency in the source spectrum, and \( n \) and \( m \) are empirical parameters determined by the type and order of the FD scheme. In particular, \( m \) can be approximated by the sum of the absolute values of the FD coefficients. For our algorithm, these coefficients are set as \( n = 3 \) and \( m = 1.37 \).
2.2.4 Velocity dispersion

Physical dispersion refers to the velocity variation with frequency, which should be distinguished from numerical dispersion caused by discretization in FD computations. Attenuative media have to be dispersive to ensure causality (Futterman, 1962; Jacobson, 1987; Sun et al., 2009). In the GSLS model, the frequency-dependent P-wave vertical velocity for VTI media takes the form:

\[ V_{P0}(\omega) = \sqrt{\frac{\text{Re}(C_{33}(\omega))}{\rho}} = \sqrt{\frac{C_{33}^{R}}{\rho} \left[ 1 + \frac{\tau_{33}}{L} \sum_{l=1}^{L} \frac{\omega^{2} \tau_{l}^{2}}{1 + (\omega \tau_{l})^{2}} \right]}, \tag{2.13} \]

where \( C_{33}(\omega) \) is the complex modulus and \( C_{33}^{R} \) is the relaxed modulus defined in equation 2.5.

Figure 2.3 displays the dispersion curves of the GSLS model with three relaxation mechanisms and of the constant-\( Q \) model of Kjartansson (1979; see Carcione, 2007). The velocity in viscoelastic media is higher than the reference value for frequencies exceeding \( \omega_{r} \); for lower frequencies, the opposite is true.

![Figure 2.3: Dispersion curves of the GSLS model with three relaxation mechanisms and of Kjartansson’s constant-\( Q \) model, with \( Q = 30 \) in both cases. The reference velocity is 4 km/s at a frequency of 100 Hz.](image)
2.3 Synthetic examples

2.3.1 Validation test

To check the accuracy of the developed FD algorithm, we apply it to generate the wavefield in a two-layer viscoelastic VTI medium and then estimate the P-wave attenuation coefficient with the spectral-ratio method.

![Two-layer VTI model used for attenuation estimation.](image)

Figure 2.4: Two-layer VTI model used for attenuation estimation. The model size is 900 m × 300 m, with grid spacing Δx = Δz = 3 m. A horizontal reflector is located at a depth of 150 m. In the first layer, \( V_P^0 = 3.0 \) km/s, \( V_S^0 = 1.5 \) km/s, \( \rho = 2.0 \) g/m\(^3\), \( \epsilon = 0.2 \), and \( \delta = 0.1 \); in the second layer, \( V_P^0 = 2.0 \) km/s, \( V_S^0 = 1.0 \) km/s, \( \rho = 2.0 \) g/m\(^3\), \( \epsilon = 0.15 \), and \( \delta = 0.05 \). The attenuation parameters are the same for both layers and are listed in the first row of Table 2.1. An explosive source that excites a Ricker wavelet with a central frequency of 100 Hz is placed at the origin (white dot). The green line marks the receiver locations at the surface.

Using the vertical component of the reflection data generated for the model in Figure 2.4 and Table 2.1, we pick the PP events at different receiver locations and then obtain the frequency spectrum \( U^{(1)}(\omega) \) of that arrival. Similarly, we estimate the corresponding spectrum \( U^{(0)}(\omega) \) for the reference elastic medium. Then, according to the spectral-ratio method (Behura and Tsvankin, 2009a; Zhu et al., 2006), the logarithm of the frequency-domain amplitude ratio can be expressed as

\[
\ln \left| \frac{U^{(1)}(\omega)}{U^{(0)}(\omega)} \right| = G - 2\pi A_P ft, \tag{2.14}
\]

where \( G \), which is assumed to be frequency-independent, accounts for the source radiation pattern, geometric spreading, and reflection/transmission coefficients, \( t \) is the traveltine, and \( A_P \) is the
Figure 2.5: Logarithm of the amplitude ratio versus frequency for the PP reflection at an offset of 210 m (phase angle is about 28°) in the model from Figure 2.4.

P-wave group attenuation coefficient. As shown by Behura and Tsvankin (2009b), the coefficient $A_P$ is equal to the phase attenuation coefficient [which can be represented as $1/(2Q_P)$] computed for zero inhomogeneity angle (the angle between the real and imaginary parts of the wave vector).

Table 2.1: Actual and estimated attenuation parameters for the two-layer model from Figure 2.4. The quality factor $Q_{S0}$ was not estimated in this test because it requires special processing of mode-converted data (see Shekar and Tsvankin, 2011).

<table>
<thead>
<tr>
<th></th>
<th>$Q_{P0}$</th>
<th>$Q_{S0}$</th>
<th>$\epsilon_Q$</th>
<th>$\delta_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>30</td>
<td>30</td>
<td>0.4</td>
<td>1.2</td>
</tr>
<tr>
<td>Estimated</td>
<td>32.4</td>
<td>—</td>
<td>0.3</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 2.2: Parameters of a three-layer VTI model. The corresponding $\text{Re}(C_{ij})$ (related to the velocity parameters $V_{P0}$, $V_{S0}$, $\epsilon$, and $\delta$) are defined at a reference frequency of 100 Hz.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (km)</th>
<th>$V_{P0}$ (km/s)</th>
<th>$V_{S0}$ (km/s)</th>
<th>$\epsilon$</th>
<th>$\delta$</th>
<th>$\rho$ (g/m³)</th>
<th>$Q_{P0}$</th>
<th>$Q_{S0}$</th>
<th>$\epsilon_Q$</th>
<th>$\delta_Q$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>1.0</td>
<td>0.15</td>
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<td>-1.2</td>
</tr>
<tr>
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<td>3.0</td>
<td>1.5</td>
<td>0.2</td>
<td>0.15</td>
<td>2.2</td>
<td>30</td>
<td>30</td>
<td>-0.4</td>
<td>-1.2</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>4.0</td>
<td>2.0</td>
<td>0.3</td>
<td>0.2</td>
<td>2.5</td>
<td>30</td>
<td>30</td>
<td>-0.4</td>
<td>-1.2</td>
</tr>
</tbody>
</table>
Figure 2.6: P-wave quality factors obtained from the spectral-ratio method (stars) and the best-fit linearized approximation (equation 2.15, solid line).

Hence, the slope of the logarithmic spectral ratio yields the product $2\pi A_P t$. Figure 2.5 shows that the slope remains almost constant in a wide frequency range, as expected for a constant-$Q$ model. Some deviations from a straight line at high frequencies can be explained by the fact that we simulated a nearly constant-$Q_{ij}$ in the frequency band from 2 to 200 Hz. By applying the spectral-ratio method at different offsets, we invert for the attenuation parameters $A_{P0}$, $\epsilon_Q$, and $\delta_Q$ using the following linearized expression for the coefficient $A_P$ (Zhu and Tsvankin, 2006):

$$A_P(\theta) = A_{P0} \left(1 + \delta_Q \sin^2 \theta \cos^2 \theta + \epsilon_Q \sin^4 \theta\right),$$  \hspace{1cm} (2.15)

where $\theta$ is the phase angle with the symmetry axis, $A_{P0}$ is the P-wave vertical phase attenuation coefficient [close to $1/(2Q_{P0})$ for weak attenuation], $\epsilon_Q$ is the anisotropy parameter that quantifies the fractional difference between the horizontal and vertical attenuation coefficients, and $\delta_Q$ controls the curvature of $A_P(\theta)$ in the vertical direction. The attenuation-anisotropy parameters are defined as (Zhu and Tsvankin, 2006):

$$\epsilon_Q = \frac{Q_{33} - Q_{11}}{Q_{11}},$$  \hspace{1cm} (2.16)
\[
\delta_Q = \left. \frac{1}{2 A_P_0} \frac{d^2 A_P}{d\theta^2} \right|_{\theta=0^\circ}.
\] (2.17)

We process reflections in the offset range from 30 m to 840 m with an increment of 90 m and estimate the corresponding phase angles from the group angles using a linearized relationship (Tsvankin, 2012).

The inverted parameters, listed in the second row of Table 2.1, accurately reproduce the angle-dependent P-wave attenuation coefficient (Figure 2.6). The small discrepancies between the actual and inverted parameters in Table 2.2 are likely caused by the linearized approximations for the phase angle and the attenuation coefficient (equation 2.15), as well as by the deviations of the simulated \( Q_{ij} \) from the desired constant value. The inversion results can be further improved by using the exact attenuation coefficients obtained from the Christoffel equation (Carcione, 2007; Zhu and Tsvankin, 2006).

### 2.3.2 Examples for attenuative VTI models

Here, we present three modeling experiments to illustrate the performance of the algorithm and the influence of attenuation anisotropy. The snapshots of the amplitude of the particle velocity for a homogeneous VTI model are shown in Figure 2.7(a)-Figure 2.7(d). Compared to the wavefield for a nonattenuative medium in Figure 2.7(a), the P- and SV-arrivals in Figure 2.7(b)-Figure 2.7(d) exhibit clearly visible amplitude decay due to attenuation and attenuation anisotropy. The contribution of the coefficient \( \epsilon_Q \) in equation 2.15, in accordance with its definition, increases toward the isotropy (horizontal) plane (Figure 2.7(d) and Figure 2.7(f)). The parameter \( \delta_Q \) controls the angular variation of the P-wave attenuation coefficient near the vertical direction (Zhu and Tsvankin, 2006), so its influence is visible mostly at intermediate propagation angles (Figure 2.7(c) and Figure 2.7(e)). Note that the SV-wave attenuation anisotropy is largely controlled by the parameter \( \sigma_Q \) (which is a function of \( \epsilon_Q - \delta_Q \), see the expression in Zhu and Tsvankin, 2006), which is negative in Figure 2.7(c) and positive in Figure 2.7(d). When \( \sigma_Q \) is negative, the SV-wave attenuation coefficient decreases in the range of phase angles from 0° to 45° and then increases from
45° to 90° (Figure 2.7(c) and Figure 2.7(e)); for positive $\sigma_q$, the opposite is true (Figure 2.7(d) and Figure 2.7(f)).

Next, the three-layer model from Table 2.1 is used to simulate reflection data in the presence of attenuation anisotropy. We compare the modeled traces with ones from the reference medium with isotropic attenuation (Figure 2.8). As expected, the difference becomes more pronounced with larger offset. More interestingly, because of negative parameters $\epsilon_q$ and $\delta_q$, the energy is less attenuated at larger offsets despite a longer propagation path. This example illustrates the importance of taking attenuation anisotropy into account in AVO analysis (e.g., Samec and Blangy, 1992).

Finally, the anisotropic viscoelastic FD method is applied to a more complicated model with a salt body (Figure 2.9). This section is taken from the left part of the 2007 BP TTI model and is resampled with a coarser grid. We remove the tilt of the symmetry axis (i.e., turn the model into VTI) and make the section attenuative (Figure 2.10). The reflection energy is significantly damped due to attenuation (compare Figure 2.11(b) and Figure 2.11(c) with Figure 2.11(a) ). At large offsets (6-12 km), the diffraction from the left edge of the salt body (Figure 2.9(a) and Figure 2.9(b)) interferes with reflections from the thin layers in the overburden (Figure 2.9(c) and Figure 2.9(d)). This long-offset interference arrival is significantly influenced by attenuation anisotropy in the shallow (0-3 km) layers (Figure 2.11(d)). Although attenuation anisotropy is also pronounced at depth, the difference between the amplitudes of the deeper events for the isotropic and VTI models is much smaller because of a more limited range of propagation angles. The spectra of windowed traces (Figure 2.12) exhibit the amplitude decay and reduction in the dominant frequency caused by attenuation anisotropy.

### 2.4 Conclusions

We conducted time-domain FD modeling for 2D VTI attenuative media using the model of generalized standard linear solid (GSLS). The modified $\tau$-method was employed to obtain the stress relaxation times and $\tau_{ij}$-parameters and simulate nearly-constant $Q_{ij}$-behavior in a specified...
frequency range. The velocity dispersion produced by the GSLS model coincides with that for Kjartansson’s constant-$Q$ model. Efficient numerical implementation is based on rotated staggered grids (RSG) and introduction of memory variables.

To validate the algorithm, we reconstructed the attenuation parameters of a VTI layer by applying the spectral-ratio method to the simulated reflection data. The method was also tested on a more structurally complex attenuative TI model that contains a salt body. The presented algorithm can serve as the forward-modeling tool for anisotropic attenuation tomography.

2.5 Acknowledgments

I am grateful to the members of the A(nisotropy)-Team at CWP for fruitful discussions. This work was supported by the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP. The reproducible numeric examples in this paper are generated with the Madagascar open-source software package freely available from http://www.ahay.org. Some of the codes are modified after the sofi2D package (http://www.gpi.kit.edu/english/SOFI2D.php).
Figure 2.7: Snapshots of wavefields at 147 ms in a homogeneous medium. (a) Elastic VTI medium; (b) VTI medium with isotropic attenuation $Q_{P0} = Q_{S0} = 30$; (c) attenuative VTI medium with $Q_{P0} = Q_{S0} = 30$, $\epsilon_Q = 0$, $\delta_Q = 1.5$; (d) attenuative VTI medium with $Q_{P0} = Q_{S0} = 30$, $\epsilon_Q = 0.6$, $\delta_Q = 0$. (e) The difference between plots (b) and (c); and (f) the difference between plots (b) and (d). The model size is 1500 m $\times$ 1500 m, with grid spacing $\Delta x = \Delta z = 6$ m. Other parameters are: $V_{P0} = 4000$ m/s, $V_{S0} = 2000$ m/s, $\epsilon = 0.3$, $\delta = 0.2$, and $\rho = 2.0$ g/m$^3$. The phase velocities are defined at a reference frequency of 100 Hz. An explosive source that excites a Ricker wavelet with a central frequency of 100 Hz is placed at the center of the model.
Figure 2.8: Traces of the vertical component of the reflection data (red solid lines) for the three-layer VTI model from Table 2.2. (a) The near-offset trace at $x = 0$ km; (b) the intermediate-offset trace at $x = 0.2$ km; (c) the far-offset trace at $x = 0.4$ km. The black dashed lines are modeling results for the same VTI medium but with isotropic attenuation (i.e., $\epsilon_Q = \delta_Q = 0$). The model size is $400$ m $\times$ $600$ m, with grid spacing $\Delta x = \Delta z = 1$ m.
Figure 2.9: Velocity parameters of the salt section of the BP TI model: (a) $V_{P0}$, (b) $V_{S0}$ (both velocities are in km/s), (c) $\epsilon$, and (d) $\delta$. The modified model size is 11268 m $\times$ 13125 m, with grid spacing $\Delta x = \Delta z = 18.75$ m. An explosive source that excites a Ricker wavelet with a central frequency of 10 Hz is placed at the origin.
Figure 2.10: Attenuation parameters for the model from Figure 2.9: (a) $Q_{P0}$, (b) $Q_{S0}$, (c) $\epsilon_q$, and (d) $\delta_q$. 
Figure 2.11: Vertical component of the reflection data for the model from Figure 2.9 and Figure 2.10. The result of (a) elastic VTI modeling; (b) viscoelastic modeling with $\epsilon_\theta = \delta_\theta = 0$; and (c) viscoelastic VTI modeling. (d) The difference between plots (b) and (c).
Figure 2.12: Spectra of windowed traces (from 6.3 s \sim 8.1 s) at an offset of 10.1 km for the model from Figure 2.9 and Figure 2.10. The pink and blue curves correspond to the traces from Figure 2.11(b) and Figure 2.11(c), respectively.
CHAPTER 3
WAVEFORM INVERSION FOR ATTENUATION ESTIMATION IN ANISOTROPIC MEDIA

A paper published* in Geophysics
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Robust estimation of attenuation coefficients remains a challenging problem, especially for heterogeneous anisotropic media. Here, we apply waveform inversion (WI) to perform attenuation analysis in heterogeneous VTI (transversely isotropic with a vertical symmetry axis) media. A time-domain finite-difference algorithm based on the standard linear solid model simulates nearly constant quality-factor values in a specified frequency band. We employ the adjoint-state method to derive the gradients of the objective function based on the Born approximation. Four parameters describing the attenuation coefficients of P- and SV-waves are updated simultaneously with a quasi-Newton optimization algorithm. To remove the time shifts between the modeled and observed data caused by velocity errors, we apply a local similarity technique. Stable inversion still requires a sufficiently accurate velocity model to minimize the trade-off between the contributions of velocity and attenuation to amplitudes. The inversion algorithm is tested on homogeneous background models with a Gaussian anomaly in one of the attenuation parameters and on a realistic heterogeneous VTI model.

3.1 Introduction

Viscoelasticity is a ubiquitous property of the subsurface, which produces energy dissipation and velocity dispersion during wave propagation. Attenuation coefficients can be utilized to obtain important information about reservoir rocks such as the fluid type, saturation, and mechanical properties of the rock matrix (Johnston et al., 1979; Tiwari and McMechan, 2007). Besides, correcting

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for attenuation helps improve the results of inversion and imaging in viscoelastic/viscoacoustic media (e.g., Causse et al., 1999, Zhu et al., 2014, Xue et al., 2016).

Formations that exhibit velocity anisotropy are often characterized by even stronger attenuation anisotropy (Best et al., 2007; Chichinina et al., 2009; Zhu et al., 2006). Laboratory experiments have shown that attenuation anisotropy may help estimate the orientation and properties of aligned fractures and the presence of organic laminae inside the rocks (Best et al., 2007; Chichinina et al., 2006; Clark et al., 2009; Ekanem et al., 2013).

Attenuation coefficients can be estimated from the reduction in the dominant frequency of the signal, which is the main idea behind the centroid frequency shift method (CFSM) and the peak frequency shift method (PFSM) (Quan and Harris, 1997). However, both methods are valid only for certain wavelet types characterized by a Gaussian amplitude spectrum (de Castro Nunes et al., 2011). The spectral-ratio method (SRM) (e.g., Sams and Goldberg, 1990) estimates the quality factor $Q$ in the frequency domain from the slope of the amplitude ratio of two arrivals. Although the performance of SRM depends on the chosen frequency band and on the clipping function applied at the edges of the bandwidth, it can separate attenuation from geometric spreading, reflection/transmission and other frequency-independent amplitude factors. Also, SRM is generally more stable than other conventional methods (e.g., CFSM or PFSM) for attenuation estimation (de Castro Nunes et al., 2011).

For anisotropic media, SRM-based layer-stripping schemes for P- and S-wave attenuation analysis are proposed by Behura and Tsvankin (2009a) and Shekar and Tsvankin (2011). However, all conventional methods are generally limited to structurally simple subsurface models and may give erroneous results in the presence of interference (Hackert and Parra, 2004; Reine et al., 2012).

Waveform inversion (WI) can potentially provide more robust attenuation estimation for realistic subsurface structures. Originally introduced by Tarantola (1984), WI has been widely used for building high-resolution velocity models, with the influence of attenuation often neglected for simplicity. However, applying purely acoustic WI to viscoacoustic data may distort the recovered velocity profile (e.g., Kurzmann et al., 2013). Significantly improved inversion results can
be achieved by including even smoothly varying $Q$-models in the simulation. In another example discussed by Causse et al. (1999), a viscoacoustic WI algorithm with preconditioning helps compensate for attenuation-induced phase and amplitude distortions, and consequently yields a more accurate velocity field.

Recent progress in multiparameter waveform inversion makes it possible to extend WI to anisotropic and/or attenuative models. Alkhalifah and Plessix (2014) present perturbation-based radiation patterns to analyze parameter trade-offs in WI for acoustic VTI media. Kamath and Tsvankin (2016) obtain radiation patterns for elastic VTI media and develop an elastic WI algorithm to resolve four Thomsen parameters simultaneously from P- and SV-wave data. To analyze the trade-offs in multiparameter viscoelastic WI for isotropic media, Brossier (2011) conducts synthetic transmission experiments and observes that while attenuation has little impact on velocity estimation, velocity and density can leave a strong imprint on the attenuation coefficients. Therefore, attenuation can be estimated using a hierarchical approach, in which the velocity parameters are recovered prior to attenuation analysis (Kamei and Pratt, 2008; Prieux et al., 2013). The inverted attenuation coefficients, however, usually have a lower resolution than their velocity counterparts (Barnes et al., 2014; Denli et al., 2013). Bai and Yingst (2013) apply multiscale WI to estimate the attenuation coefficients for a viscoacoustic version of the Marmousi model. Using the actual velocity field, they obtain an isotropic $Q$-image with acceptable resolution, which somewhat deteriorates at depth.

By incorporating the generalized standard linear solid model, Bai and Tsvankin (2016) devise a time-domain finite-difference algorithm to simulate nearly constant values of the components of the anisotropic quality-factor matrix $Q$ within a specific frequency band. Numerical examples for 2D models with VTI symmetry for both velocity and attenuation demonstrate that this method produces accurate wavefields even for complicated subsurface structures.

In this paper, we employ the modeling methodology of Bai and Tsvankin (2016) to perform WI for viscoelastic VTI media. First, we briefly discuss the time-domain viscoelastic modeling algorithm designed to produce frequency-independent $Q_{ij}$-values. Next, the gradients of the WI
objective function with respect to the TI attenuation parameters are derived by applying the adjoint-
state method. Assuming the velocity field to be known, we invert just for combinations of the
Thomsen-style attenuation parameters $A_{P0}$, $A_{S0}$, $\epsilon_{Q}$ and $\delta_{Q}$. We also describe application of a
local-similarity technique to reduce the sensitivity of inversion results to velocity errors. The
algorithm is tested on transmission data from models with Gaussian anomalies in the attenuation
parameters and reflection data simulated for a section of the modified BP TI model.

3.2 Methodology

3.2.1 Forward modeling for viscoelastic VTI media

Viscoelastic wave propagation can be simulated in a relatively straightforward way in the fre-
quency domain by making velocities or stiffness coefficients complex (e.g., Operto et al., 2007;
Shekar and Tsvankin, 2014). However, frequency-domain full-waveform modeling methods (e.g.,
finite-differences) are hampered by excessive memory requirements (Virieux et al., 2009). Aside
from their advantage in large-scale applications, time-domain simulations are generally more ro-
 bust for complex (e.g., elastic and anisotropic) models and facilitate selection of specific arrivals for
attenuation analysis (Kurzmann et al., 2013; Virieux et al., 2009). To generate constant $Q$-values
for the seismic frequency band in time-domain modeling, it is common to employ superposition of
several rheological bodies, each providing one relaxation mechanism (Blanch et al., 1995; Bohlen,
2002; Carcione, 1993; Emmerich and Korn, 1987). In most seismic applications, two to three re-
 laxation mechanisms are sufficient for a nearly constant-$Q$ simulation (Bai and Tsvankin, 2016;
Bohlen, 2002; Emmerich and Korn, 1987). Zhu et al. (2013) observe that even one mechanism
with properly chosen parameters can produce reasonably accurate results within the frequency
band typical for surface seismic surveys.

Here, primarily for purposes of computational efficiency, we employ a single relaxation mech-
anism. The expressions for the corresponding stress relaxation time and attenuation parameters $\tau_{ij}$
can be found in Appendix B. As discussed by Bai and Tsvankin (2016), anisotropic attenuation can
be described by the following relaxation function (shown here with one relaxation mechanism):

\[ \Psi_{ijkl}(t) = C_{ijkl}^R \left( 1 + \tau_{ijkl} e^{-t/\tau^\sigma} \right) H(t), \]  

(3.1)

where \( C_{ijkl}^R = \Psi_{ijkl}(t \to \infty) \) is called the “relaxed stiffness,” which corresponds to the low-frequency limit (\( \omega = 0 \)), \( \tau^\sigma \) denotes the stress relaxation time determined by the reference frequency (equation B.1), \( \tau_{ijkl} \approx 2/Q_{ijkl} \) (see equation B.2), which quantifies the magnitude of attenuation in anisotropic media, is commonly used in time-domain viscoacoustic/viscoelastic modeling, and \( H(t) \) is the Heaviside function. The relaxation function at zero time yields the “unrelaxed stiffness”:

\[ C_{ijkl}^U \equiv \Psi_{ijkl}(t = 0) = C_{ijkl}^R \left( 1 + \tau_{ijkl} \right). \]

(3.2)

The stiffness difference \( \Delta C_{ijkl} = C_{ijkl}^U - C_{ijkl}^R \) (equation B.3) is proportional to \( \tau_{ijkl} \) and, therefore, reflects the magnitude of attenuation.

The attenuation of P- and SV-waves in VTI media can be described by four Thomsen-style attenuation parameters, \( A_{P0}, A_{S0}, \epsilon_q \) and \( \delta_q \) (Zhu and Tsvankin, 2006). \( A_{P0} \) and \( A_{S0} \) denote the P- and S-wave attenuation coefficients in the vertical (symmetry-axis) direction (equations D.1 and D.2), the parameter \( \epsilon_q \) quantifies the fractional difference between the horizontal and vertical P-wave attenuation coefficients, and \( \delta_q \) controls the curvature of P-wave attenuation coefficients at the symmetry axis (equations D.3 and D.4). Combined with the unrelaxed stiffness coefficients \( C_{ijkl}^U \) (used as the reference elastic parameters), these attenuation parameters can be converted into the quality-factor elements \( Q_{ijkl} \) (or \( \Delta C_{ijkl} \), as in equation B.3).

The viscoelastic stress-strain relationship (an extension of Hooke’s law to attenuative models) can be expressed as

\[ \sigma_{ij} = C_{ijkl}^U \epsilon_{kl} + \Delta C_{ijkl} r_{kl}, \]

(3.3)

where \( r_{kl} \) are the memory variables, which satisfy the following partial differential equations (Bai and Tsvankin, 2016):

\[ \frac{\partial r_{kl}}{\partial t} = -\frac{1}{\tau^\sigma} (r_{kl} + \epsilon_{kl}). \]

(3.4)
3.2.2 Viscoelastic waveform inversion

WI utilizes the entire waveforms of certain arrivals (e.g., diving waves and/or reflections) to iteratively update the model parameters. The degree of data fitting is usually evaluated with an \( \ell_2 \)-norm objective function (e.g., Tarantola, 1988; Tromp et al., 2005):

\[
F(m) = \frac{1}{2} \sum_{r=1}^{N} \| u(x_r, t, m) - d(x_r, t) \|^2,
\]

(3.5)

where \( u(x_r, t, m) \) denotes the data computed for the trial model \( m \), \( d(x_r, t) \) is the observed data, \( r \) is the receiver index, and \( t \) is the time; summation over shots is implied. Instead of calculating the Fréchet derivatives, which can be prohibitively expensive, the gradient of the objective function is typically computed with the adjoint-state method (Fichtner, 2005; Tarantola, 1988; Tromp et al., 2005). Then just two simulations of wave propagation (forward and adjoint) are required to update the model at each iteration.

In viscoelastic media, the adjoint wavefield is “propagated backward in time, with numerically stable negative attenuation” (Tarantola, 1988). Tromp et al. (2005) and Fichtner and Van Driel (2014) present the adjoint equations for general anisotropic attenuative media, but implement them only for isotropic attenuation. Following Tarantola (1988) and applying the Born approximation, the gradients for the viscoelastic parameters \( \Delta C_{ijkl} \) (equation B.3) can be expressed as the cross-correlation of the memory variables from the forward simulation with the adjoint strain field (Appendix C):

\[
\delta \Delta C_{ijkl} = - \sum_{\text{sources}} \int_0^T \frac{\partial u_i^\dagger}{\partial x_j} r_{kl} \, dt,
\]

(3.6)

where \( u^\dagger \) denotes the adjoint displacement field.

In waveform inversion, it is convenient to choose parameters that have the same units and similar magnitude (e.g., Kamath and Tsvankin, 2016; Plessix and Cao, 2011). Also it is common to use the inverse quality factor \( Q^{-1} \) (i.e., the attenuation coefficient) in WI algorithms for attenuative media (Bai and Yingst, 2013; Liao and McMechan, 1995). Therefore, here we describe TI
attenuation by the P- and S-wave symmetry-direction attenuation coefficients $A_{P0}$ and $A_{S0}$ (equations D.1 and D.2) and two more parameters, $A_{Ph}$ and $A_{Pn}$, which quantify the magnitude of attenuation anisotropy. The P-wave horizontal attenuation coefficient $A_{Ph}$ is given by:

$$A_{Ph} = (1 + \epsilon_Q) A_{P0} \approx \frac{1}{2 Q_{11}}.$$  (3.7)

To account for the attenuation-anisotropy coefficient $\delta_Q$, we define the parameter $A_{Pn}$:

$$A_{Pn} = (1 + \delta_Q) A_{P0},$$  (3.8)

which governs the angular variation of the P-wave attenuation near the symmetry axis and has a form similar to the linearized normal-moveout (NMO) velocity for a horizontal VTI layer. As described in Appendix D, the gradients for the stiffness differences $\Delta C_{ijkl}$ can be converted into those for the attenuation coefficients $A_{P0}$, $A_{S0}$, $A_{Ph}$, and $A_{Pn}$ by applying the chain rule (equations D.6- D.8).

To reduce the ambiguity of the inverse problem, we assume the velocity parameters ($C_{ijkl}^V$) and density to be known. This prevents cycle-skipping in the inversion because the influence of attenuation-induced dispersion in the seismic frequency band is typically small (Kurzmann et al., 2013; Zhu and Tsvankin, 2006). Hence, the WI algorithm can operate with relatively high frequencies to increase the sensitivity of the wavefield to attenuation. As shown below, the influence of moderate velocity errors can be mitigated by applying the local similarity technique. Model updating is performed with the help of the limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithm, which scales the gradients by an approximate inverse Hessian matrix.

### 3.3 Synthetic examples

#### 3.3.1 Transmission tests

Here we conduct a series of transmission experiments for Gaussian anomalies in the Thomsen-style attenuation parameters embedded in a homogeneous VTI background. The wavefield is excited by displacement sources with the orientation specified in the figure captions. The velocity parameters $V_{P0}$, $V_{S0}$, $\epsilon$, and $\delta$ and the density are constant and kept at the actual values during
the inversion. The reference frequency, which determines the stress relaxation time (equation B.1) needed in the viscoelastic wave equation (equation B.8), is set equal to the central frequency of the wavelet (30 Hz in all transmission experiments). Starting from the homogeneous background model, we conduct simultaneous inversion for the attenuation parameters $A_{P0}$, $A_{S0}$, $A_{Ph}$, and $A_{Pn}$ using the gradients of the objective function obtained in equations D.5- D.8.

First, we introduce a Gaussian anomaly in $A_{P0}$ and place horizontal arrays of sources and receivers above and below the anomaly (Figure 3.1). Because the parameters $\epsilon_{Q}$ and $\delta_{Q}$ are constant, there are anomalies in $A_{Ph}$ and $A_{Pn}$ as well (equations 4.7-4.8). Although much of the $A_{P0}$-anomaly is well recovered (Figure 3.2(a)), its shape is somewhat distorted. The update in the shear-wave coefficient $A_{S0}$ is negligible (Figure 3.2(b)), as expected, because of the fact that P-wave attenuation in Thomsen-style notation is practically independent of $A_{S0}$ (Hao and Alkhalifah, 2016; Tsvankin and Grechka, 2011; Zhu and Tsvankin, 2006). The P-wave horizontal attenuation coefficient $A_{Ph}$ is only partially recovered, with the underestimated magnitude of the reconstructed anomaly (Figure 3.2(c)). The errors in $A_{Ph}$ are likely due to the limited ray coverage near the horizontal direction. Note that the difference between $\epsilon_{Q}$ and $\delta_{Q}$ is well constrained because it determines the magnitude of attenuation anisotropy for SV-waves (Zhu and Tsvankin, 2006), which significantly influences the multicomponent data for this model. According to equations 4.7 and 4.8, the ratio $A_{Pn}/A_{Ph}$ depends on $\epsilon_{Q} - \delta_{Q}$ and, therefore, the insufficient update in $A_{Ph}$ produces an underestimated value of $A_{Pn}$ (Figure 3.2(d)).

Next, we introduce a Gaussian anomaly in the shear-wave attenuation coefficient $A_{S0}$ (Figure 3.3). In this test, the inversion algorithm updates only $A_{S0}$, which indicates the absence of parameter trade-offs (Figure 3.4(b)). The peak of the reconstructed anomaly ($A_{S0} = 0.022$ or $Q_{S0} = 23$) is close to the actual value ($A_{S0} = 0.025$ or $Q_{S0} = 20$), although the shape is slightly distorted. Figure 3.5 demonstrates that the data residuals are substantially reduced after the inversion.

Similarly, in a crosswell survey for a model with a negative Gaussian anomaly in $\epsilon_{Q}$ (Figure 3.6), the algorithm accurately estimates the only perturbed parameter, $A_{Ph} = A_{P0}(1 + \epsilon_{Q})$. 32
Figure 3.1: Gaussian anomaly in the parameter $A_{P0}$ embedded in a homogeneous VTI medium. The plot shows the fractional difference between $A_{P0}$ and its background value [defined as $\left(\frac{A_{P0}^{\text{actual}} - A_{P0}^{\text{background}}}{A_{P0}^{\text{background}}}\right)$, 0.005 ($Q_{P0} = 100$); at the center of the anomaly, $A_{P0} = 0.025$ ($Q_{P0} = 20$). The other pertinent parameters are constant: $A_{S0} = 0.005$, $\epsilon_q = -0.2$, $\delta_q = -0.4$, $V_{P0} = 4000$ m/s, $V_{S0} = 2000$ m/s, $\epsilon = 0.15$, $\delta = 0.1$, and $\rho = 2.0$ g/m$^3$. The yellow dots denote the vertical displacement sources, which excite a Ricker wavelet with a central frequency of 30 Hz. The magenta line marks the receivers placed at each grid point.

Both the shape and peak magnitude of the anomaly ($\epsilon_q = -0.72$, about 90% of the actual minimum) are well resolved (Figure 3.7(c)).

3.3.2 Test for surface data

Next, we test the algorithm on surface data simulated for a modified section of the TI model generated by BP, which has been widely used in testing of anisotropic velocity-analysis techniques (Figure 3.8). The velocity parameters and density have a spatial distribution similar to that of the attenuation parameters and are not shown here; the tilt of the symmetry axis was set to zero to make the model VTI. At a depth of 0.3 km, we place 30 evenly spaced oblique displacement sources which excite a Ricker wavelet with a central frequency of 20 Hz; the receivers are located at each grid point (Figure 3.8(a)). Triangle filtering with a smoothing radius of 25 samples in both the vertical and horizontal directions is applied to obtain smooth initial models of the attenuation parameters suitable for the L-BFGS algorithm (Figure 3.9).

After 18 iterations, the objective function is significantly reduced (Figure 3.10), and the long-wavelength spatial distribution of the attenuation parameters (Figure 3.11) is recovered even in the deeper part of the section (Figure 3.11(a) and Figure 3.11(b)). The profiles in Figure 3.12 illustrate
Figure 3.2: Fractional differences between the inverted and initial parameters for the model from Figure 3.1: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$. The peak value of the recovered anomaly in $A_{P0}$ is 0.02 (about 80% of the actual maximum). The parameters $A_{Ph}$ and $A_{Pn}$ are underestimated (the updated maximum values are 0.011 and 0.006, whereas the actual peak magnitudes are 0.02 and 0.015, respectively).

The accuracy of the parameter updates obtained by WI. The estimated $A_{P0}$ and $A_{S0}$ generally follow the trends of the actual profiles, and $A_{Ph}$ is accurately recovered down to 5 km. In contrast, the reconstruction of $A_{Pn}$ is less accurate, likely due to the smaller wavefield sensitivity to the parameter $\delta_q$.

### 3.3.3 Mitigation of velocity errors

The above numerical experiments are carried out under the assumption that all velocity parameters are known. However, this assumption is not realistic in practice. To evaluate the sensitivity of the algorithm to velocity errors, we use the model in Figure 3.8 with distorted P- and S-wave vertical velocities $V_{P0}$ and $V_{S0}$ (set to 95% of the actual values throughout the section), while $\epsilon$ and
Figure 3.3: Gaussian anomaly in the parameter $A_{S0}$ embedded in a homogeneous VTI medium. The plot shows the fractional difference between $A_{S0}$ and its background value, 0.005 ($Q_{S0} \approx 100$); at the center of the anomaly, $A_{S0} = 0.025$ ($Q_{S0} \approx 20$). The other parameters are constant: $A_{P0} = 0.005$, $\epsilon_Q = -0.2$, $\delta_Q = -0.4$, $V_{P0} = 4000$ m/s, $V_{S0} = 2000$ m/s, $\epsilon = 0.15$, $\delta = 0.1$, and $\rho = 2.0$ g/m$^3$. The yellow dots denote the horizontal displacement sources and the magenta line marks the receivers placed at each grid point.

$\delta$ remain exact. As expected, velocity errors produce shifts between the observed and simulated events, leading to distorted adjoint sources (Figure 3.13) and deterioration in the inversion results (Figure 3.14).

To reduce the influence of velocity errors on the inversion, we estimate the local similarity map (Fomel, 2009; Fomel and Jin, 2009) from the recorded and modeled arrivals and pick the trend with the global maximum of the similarity values using ray tracing (Fomel, 2009). This helps calculate the time shifts and then align the corresponding events prior to WI (Figure 3.15). The original L2-norm objective function (equation 4.5) can be modified as

$$F(m) = \frac{1}{2} \sum_{r=1}^{N} \| u(x_r, t, m) - S(x_r, t)d(x_r, t) \|^2,$$

where $S(x_r, t)$ is a linear interpolation operator that shifts the recorded events based on the local similarity method. $S(x_r, t)$ is practically independent of the attenuation parameters ($m$) when the dispersion is relatively weak. Consequently, the gradient with respect to attenuation parameter $m$ becomes

$$\frac{\partial F(m)}{\partial m} = \sum_{r=1}^{N} \left[ \frac{\partial u(x_r, t, m)}{\partial m} \right]^T [u(x_r, t, m) - S(x_r, t)d(x_r, t)].$$

(3.10)
Figure 3.4: Fractional differences between the inverted and initial parameters for the model from Figure 3.3: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$. The peak value of the recovered anomaly in $A_{S0}$ is 0.022 (about 88% of the actual maximum).

The only difference between equation 3.10 and the gradient for the original data is in the form of the data residual. Expressions similar to equations 4.9 and 3.10 can be found in Luo and Hale (2014) who apply dynamic warping to mitigate the influence of velocity errors on least-squares migration.

After the correction, the algorithm generates more accurate adjoint sources and inversion gradients, which improves the parameter-estimation results (compare Figure 3.16 and Figure 3.14).

3.4 Discussion

The source-receiver configuration in the transmission experiments is designed to better constrain the perturbed attenuation parameters. The anomaly in the P-wave vertical attenuation coefficient $A_{P0}$ was recovered using near-vertical raypaths, whereas crosshole geometry was em-
Figure 3.5: Difference between the observed and simulated data for the model in Figure 3.3; the source is located at $x = 0.25$ km. The residuals for the initial model: (a) z-component; (b) x-component. The residuals for the inverted model after 40 iterations: (c) z-component; (d) x-component.

ployed for estimating the coefficient $A_{P_h}$, which is responsible for the P-wave attenuation near the horizontal isotropy plane. It would be beneficial to investigate the so-called “radiation patterns” (Alkhalifah and Plessix, 2014; Kamath and Tsvankin, 2016) of the attenuation parameters, which could help identify trade-offs for given acquisition geometries and devise a more robust multiparameter inversion strategy.

In addition to its influence on kinematics, an inaccurate velocity field also distorts recorded amplitudes through errors in geometric spreading and reflection/transmission coefficients. Compensation for such amplitude errors by our algorithm can cause distortions in the inverted attenuation parameters. The spectral-ratio and centroid-frequency-shift method separate attenuation from
Figure 3.6: Negative Gaussian anomaly in the parameter $\epsilon_Q$ embedded in a homogeneous VTI medium. The plot shows the fractional difference between the parameter $A_{Pb} = A_{P0}(1 + \epsilon_Q)$ and its background value, 0.025 (the background $\epsilon_Q = 0$); at the center of the anomaly, $A_{Pb} = 0.005$ ($\epsilon_Q = -0.8$). The other parameters are constant: $A_{P0} = 0.025$, $A_{S0} = 0.025$, $\delta_Q = 0.4$, $V_{P0} = 4000$ m/s, $V_{S0} = 2000$ m/s, $\epsilon = 0.15$, $\delta = 0.1$, and $\rho = 2.0$ g/m$^3$. The yellow dots denote the horizontal displacement sources and the magenta line marks the receivers placed at each grid point.

other factors by performing attenuation analysis in the frequency domain. A similar approach could be devised to improve the robustness of WI-based attenuation estimation (e.g., Dutta and Schuster, 2016), which is the topic of our ongoing work.

3.5 Conclusions

We presented a time-domain WI methodology for attenuation estimation in transversely isotropic media. The finite-difference modeling algorithm simulates a nearly constant $Q$-matrix in the seismic frequency band using one relaxation mechanism. By applying the adjoint-state method, we obtained the gradients of the objective function with respect to the viscoelastic parameters $\Delta C_{ijkl}$ through the cross-correlations of the memory variables (computed from the forward simulation) with the adjoint strain field. Then the gradients for the VTI attenuation parameters $A_{P0}$, $A_{S0}$, $A_{Ph}$, and $A_{Pn}$ are found using the chain rule.

The inversion algorithm was first tested on homogeneous VTI models with a Gaussian anomaly in one of the Thomsen-style attenuation parameters. A perturbation in $A_{P0}$ (with fixed $\epsilon_Q$ and $\delta_Q$)
leads to the corresponding anomalies in the parameters $A_{Ph}$ and $A_{Pn}$. With horizontal arrays of sources and receivers, the algorithm recovers most of the anomaly in $A_{P0}$, but $A_{Ph}$ and $A_{Pn}$ are only partially updated. The problem in estimating $A_{Ph}$ and $A_{Pn}$ is likely caused by insufficient illumination near the horizontal direction and trade-offs between the two parameters in the inversion of the multicomponent wavefield.

The algorithm was more successful in resolving the anomalies in $A_{S0}$ and $\epsilon_Q (A_{Ph})$. In the absence of measurable cross-talk with the other parameters, waveform inversion accurately estimated the shape of both anomalies, despite the slightly distorted peak magnitudes.

Finally, we tested the algorithm on surface data generated for a modified section of the BP TI model. Waveform inversion was able to reconstruct the long-wavelength distribution of the TI attenuation parameters, and the data misfit was reduced by 70%. We also confirmed the feasibility of mitigating the influence of velocity errors with the local similarity technique.

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Figure 3.7: Fractional differences between the inverted and initial parameters for the model from Figure 3.6: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$. The peak value of the recovered anomaly in $\epsilon_Q$ is $-0.72$ (about 90% of the actual minimum).
Figure 3.8: Attenuation parameters for a modified section of the BP TI model: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{P_{h}}$, and (d) $A_{P_{n}}$. The symmetry axis is vertical. The model size is $15000 \text{ m} \times 9250 \text{ m}$, with grid spacing $\Delta x = \Delta z = 25 \text{ m}$. The yellow dots on plot (a) denote the displacement sources, and the magenta line marks the receivers placed at each grid point.
Figure 3.9: Smoothed actual parameters used as the initial model: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$.

Figure 3.10: Change of the objective function with iterations for the reflection experiment in Figure 3.8
Figure 3.11: Inverted attenuation parameters: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$.

Figure 3.12: Profiles of the attenuation parameters at $x = 10$ km: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$. The blue, green, and red lines indicate the actual, initial, and inverted values, respectively.
Figure 3.13: Vertical displacement for the model in Figure 3.8; the source is located at $x = 0$ and receiver at $x = 7.5$ km. (a) The observed data, (b) the data simulated for the first iteration with an inaccurate velocity model ($V_{P0}$ and $V_{S0}$ are set to 95% of the actual values), (c) the difference between the traces on plots (a) and (b).
Figure 3.14: Attenuation parameters estimated with the distorted velocity model: (a) \(A_P^0\), (b) \(A_{S0}\), (c) \(A_{Ph}\), and (d) \(A_{Pn}\). The upper limit of all parameters (for the L-BFGS algorithm) is set to 0.04, which corresponds to the quality factor close to 12.5.
Figure 3.15: Same as Figure 3.13, but the observed arrivals on plot (a) are warped (shifted) using the local similarity technique. (b) The data simulated for the first iteration with an inaccurate velocity model ($V_{P0}$ and $V_{S0}$ are set to 95% of the actual values), (c) the difference between the traces on plots (a) and (b).
Figure 3.16: Attenuation parameters estimated with the distorted velocity model after applying the local similarity technique (Figure 3.15): (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$. 
CHAPTER 4
SOURCE-INDEPENDENT WAVEFORM INVERSION FOR ATTENUATION ESTIMATION IN VTI MEDIA

A paper submitted to Geophysical Prospecting
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In previous publications we presented a waveform-inversion (WI) algorithm for attenuation analysis in heterogeneous VTI (transversely isotropic with a vertical symmetry axis) media. However, WI requires an accurate estimate of the source wavelet, which is difficult to obtain from field data. To address this problem, here we adopt a source-independent waveform inversion (SIWI) algorithm that obviates the need for joint estimation of the source signal and attenuation coefficients. The objective function is obtained by convolving the observed wavefield with a reference trace from the modeled data and the modeled wavefield with a reference trace from the observed data. Because that function is defined as the $\ell_2$-norm of the difference between the two convolved data sets, the influence of source signature on attenuation estimation is mitigated. The inversion gradients for the viscoelastic VTI parameters have a form similar to that for conventional WI, with the exception of the adjoint sources computed by convolution and cross-correlation operations. The SIWI algorithm is validated using transmission tests for a homogeneous VTI model with a Gaussian anomaly in the shear-wave vertical attenuation coefficient. SIWI is also applied to the inversion of reflection data for a modified VTI model from Hess. It should be noted that due to the increased nonlinearity of the inverse problem, the SIWI algorithm requires a more accurate initial model to obtain inversion results comparable to those produced by conventional WI with the actual wavelet.

4.1 Introduction

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Seismic data are substantially influenced by intrinsic attenuation in the subsurface. The loss of high frequencies in attenuative media reduces the bandwidth of the recorded wavefields and, therefore, the resolution of seismic inversion and imaging. Reliable estimation of attenuation and compensation for its influence can improve the output of many seismic processing steps including amplitude-variation-with-offset (AVO) analysis and imaging (e.g., Zhu et al., 2014; Bai et al., 2018). In addition, attenuation coefficients can be employed in reservoir characterization because they provide information for fracture and fluid detection and estimation of permeability (Carcione et al., 2010; Donald et al., 2004; Müller et al., 2010).

Subsurface formations that exhibit velocity anisotropy are often characterized by directionally dependent attenuation coefficients (Best et al., 2007; Chichinina et al., 2009; Zhu et al., 2006). In particular, numerical and laboratory experiments have confirmed the link between attenuation anisotropy and parameters of aligned fractures (Chichinina et al., 2006; Ekanem et al., 2013; Guo and McMechan, 2017; Rao and Wang, 2009). To characterize the anisotropic attenuation coefficients of P- and SV-waves in thinly layered porous rocks, Krzikalla and Müller (2011) combine anisotropic Backus limits (under quasi-static and no-flow assumptions) with interlayer flow models.

The quality factor, which is responsible for attenuation, is often estimated by the frequency-shift (FSM) and spectral-ratio (SRM) methods based on the frequency dependence of attenuation coefficients (e.g., Quan and Harris, 1997; Sams and Goldberg, 1990). However, these methods may fail for realistic heterogeneous models and suffer from high sensitivity to noise and to event interference (de Castro Nunes et al., 2011).

A viable alternative to the conventional techniques is waveform inversion (WI), which is often used in high-resolution velocity analysis (e.g., Tarantola, 1984, Plessix et al., 2013). The influence of attenuation on both the amplitude and phase of seismic waves makes it an essential component of WI. Proper compensation for attenuation can significantly increase the accuracy of the estimated velocity parameters (Causse et al., 1999; Kurzmann et al., 2013; Xue et al., 2016). Some existing algorithms adopt an hierarchical strategy, in which velocity analysis is followed by attenuation
estimation (Kamei and Pratt, 2008; Prieux et al., 2013). Other implementations of WI for attenuative media rely on a priori knowledge of the velocity parameters (Bai and Yingst, 2013; Bai et al., 2017).

Errors in the source wavelet represent a serious challenge in implementing WI. Luo et al. (2014) demonstrate that an inaccurate wavelet phase may substantially distort the inverted velocity field. Using modeled elastic data for shallow subsurface, Groos et al. (2014) show that a properly designed source-wavelet correction can simulate most of the observed viscoelastic effect, which indicates a significant cross-talk between the source signature and attenuation parameters.

There are two main strategies to account for the influence of the source wavelet: joint inversion for the source signal and medium parameters (Sun et al., 2014; Wang et al., 2009) and so-called source-independent waveform inversion, or SIWI (Choi and Alkhalifah, 2011; Choi and Min, 2012). Due to the trade-offs between the source signature and attenuation, SIWI is better suited for attenuation analysis. Shigapov et al. (2013) compare three types of source-independent misfit functions in the frequency domain designed to remove the influence of the source wavelet from attenuation estimation. Their synthetic test for microseismic and crosswell perforation-shot data from a layered isotropic viscoelastic medium shows that the best inversion results are obtained with the convolution-based objective function. In the time domain, a source-independent objective function is introduced for acoustic media by Choi and Alkhalifah (2011) and for elastic isotropic models by Zhang et al. (2016).

In the framework of the generalized standard linear solid (GSLS) model, Bai and Tsvankin (2016) develop a time-domain finite-difference modeling algorithm for anisotropic attenuative media, which produces nearly frequency-independent elements \( Q_{ij} \) of the quality-factor matrix. Employing that simulator, Bai et al. (2017) propose a time-domain WI methodology for estimation of the VTI attenuation parameters. The gradients of the objective function are computed using the adjoint-state method. The four Thomsen-style parameters (Zhu and Tsvankin, 2006) describing the attenuation of P- and SV-waves are updated simultaneously. The influence of velocity errors is mitigated by employing the local-similarity technique.
Here, we incorporate the time-domain source-independent objective function proposed by Choi and Alkhalifah (2011) into the viscoelastic WI algorithm of Bai et al. (2017). First, we briefly review the methodology of time-domain modeling and waveform inversion in anisotropic attenuative media. Next, we introduce the source-independent WI objective function and the corresponding adjoint sources. Finally, synthetic tests confirm the ability of the developed methodology to estimate the VTI attenuation parameters without knowledge of the source signature.

4.2 Methodology

4.2.1 Forward modeling for viscoelastic VTI media

We simulate wave propagation in viscoelastic VTI media with a time-domain finite-difference code described in Bai and Tsvankin (2016). To increase computational efficiency, only one relaxation mechanism is employed, which is generally sufficient for nearly constant-$Q$ simulation within the frequency band typical for seismic surveys (Zhu et al., 2013).

The relaxation function for an arbitrarily anisotropic attenuative media can be found in Bai and Tsvankin (2016). For a single relaxation mechanism, that function has the form:

$$
\Psi_{ijkl}(t) = C_{ijkl}^R \left(1 + \tau_{ijkl} e^{-t/\tau^\sigma}\right) H(t),
$$

(4.1)

where $C_{ijkl}^R = \Psi_{ijkl}(t \to \infty)$ is called the “relaxed stiffness,” $\tau^\sigma$ denotes the stress relaxation time determined by the reference frequency, the parameters $\tau_{ijkl}$ control the difference between the stress and strain relaxation time (and, therefore, they determine the magnitude of attenuation in anisotropic media), and $H(t)$ is the Heaviside function. At zero time, the relaxation function generates the “unrelaxed stiffnesses” $C_{ijkl}^U$:

$$
C_{ijkl}^U \equiv \Psi_{ijkl}(t = 0) = C_{ijkl}^R (1 + \tau_{ijkl}).
$$

(4.2)

The stiffness difference $\Delta C_{ijkl} = C_{ijkl}^U - C_{ijkl}^R$, which depends on $\tau_{ijkl}$, quantifies the magnitude of attenuation.

The P- and SV-wave attenuation in VTI media is conveniently described by the Thomsen-style parameters $A_{p0}$, $A_{s0}$, $\epsilon_Q$ and $\delta_Q$ (Bai and Tsvankin, 2016; Zhu and Tsvankin, 2006). $A_{p0}$ and $A_{s0}$
are the vertical (symmetry-axis) P- and S-wave attenuation coefficients, the parameter $\epsilon_Q$ depends
on the fractional difference of the P-wave attenuation coefficients in the horizontal and vertical
directions, and $\delta_Q$ controls the curvature of the P-wave attenuation coefficient at the symmetry
axis. The explicit expressions for these parameters in terms of the real-valued stiffnesses and
elements of the quality-factor matrix can be found in Zhu and Tsvankin (2006). Combined with
the unrelaxed stiffness coefficients $C_{ijkl}^U$ (used as the reference elastic parameters), the Thomsen-
style attenuation parameters can be converted into the quality-factor elements $Q_{ijkl}$ or the stiffness
difference $\Delta C_{ijkl}$.

The time-domain viscoelastic stress-strain relationship can be written as:

\[ \sigma_{ij} = C_{ijkl}^U \epsilon_{kl} + \Delta C_{ijkl} r_{kl}, \] (4.3)

where $r_{kl}$ are the memory variables, which satisfy the following partial differential equations (Bai
and Tsvankin, 2016):

\[ \frac{\partial r_{kl}}{\partial t} = -\frac{1}{\tau} (r_{kl} + \epsilon_{kl}). \] (4.4)

### 4.2.2 Viscoelastic waveform inversion for VTI media

The $\ell_2$-norm objective function is often employed to measure the quality of data fitting during
the model-updating process (e.g., Tarantola, 1988; Tromp et al., 2005):

\[ F(m) = \frac{1}{2} \| u(x_r, t, m) - d(x_r, t) \|^2, \] (4.5)

where $u(x_r, t, m)$ and $d(x_r, t)$ are the simulated and observed data, respectively, $m$ is the vector of
model parameters, and $t$ is the time. Summation over shots and receivers is implied. By applying
the adjoint-state method (Fichtner, 2005; Tarantola, 1988; Tromp et al., 2005), the gradient of the
objective function at each iteration is obtained from only two wavefield simulations (one forward
and one adjoint). The gradients for the viscoelastic parameters $\Delta C_{ijkl}$ can be computed in the Born
approximation as the cross-correlation of the memory variables from the forward simulation with
the adjoint strain field (Tarantola, 1988; Bai et al., 2017):

\[
\frac{\partial F}{\partial \Delta C_{ijkl}} = - \int_0^T \frac{\partial u_i^\dagger}{\partial x_j} r_{kl} dt, \quad (4.6)
\]

where \( u_i^\dagger \) denotes the adjoint displacement field.

Following Bai et al. (2017), the inversion algorithm operates with the vertical P- and S-wave attenuation coefficients introduced above \( (A_{P_0} \text{ and } A_{S_0}) \), the P-wave horizontal attenuation coefficient \( A_{P_h} \),

\[
A_{P_h} = (1 + \epsilon_Q) A_{P_0} \approx \frac{1}{2 Q_{11}}, \quad (4.7)
\]

and the coefficient \( A_{P_n} \), which governs the angular variation of the P-wave attenuation near the symmetry axis:

\[
A_{P_n} = (1 + \delta_Q) A_{P_0}. \quad (4.8)
\]

The form of \( A_{P_n} \) is similar to the weak-anisotropy approximation for the normal-moveout (NMO) velocity in a horizontal VTI layer (Thomsen, 1986; Tsvankin, 2012).

Replacing the attenuation-anisotropy parameters \( \epsilon_Q \) and \( \delta_Q \) by \( A_{P_h} \) and \( A_{P_n} \) is convenient for inversion purposes because \( A_{P_0}, A_{S_0}, A_{P_h}, \) and \( A_{P_n} \) have the same units and similar magnitudes. The gradients for the attenuation parameters can be obtained from those for the stiffness differences \( \Delta C_{ijkl} \) by applying the chain rule (Bai et al., 2017). The L-BFGS method (Nocedal, 1980) is used in parameter updating to scale the gradients by an approximate inverse Hessian matrix.

### 4.2.3 Source-independent viscoelastic WI

In the time domain, the displacement can be expressed as the convolution of the Green’s function and source wavelet (Choi and Alkhalifah, 2011). Then equation 4.5 can be represented as:

\[
F(m) = \frac{1}{2} \| G_{u} * s_{u} - G_{d} * s_{d} \|^2, \quad (4.9)
\]

where \( G \) denotes the Green’s function, \( s \) is the source wavelet, and the subscripts \( u \) and \( d \) refer to the simulated and observed wavefields, respectively. Because the source signature is difficult to estimate in practice (i.e., \( s_{u} \neq s_{d} \)), the conventional objective function (equation 4.5) introduces
distortions in the inverted model parameters represented by $G_u$ (see examples below).

To address this problem, Choi and Alkhalifah (2011) introduce a “source-independent” objective function, which has the form:

\[
F = \frac{1}{2} \| u * d^{ref} - d * u^{ref} \|^2, \tag{4.10}
\]

or

\[
F = \frac{1}{2} \| (G_u * s_u) * (s_d * G_d^{ref}) - (G_d * s_d) * (s_u * G_u^{ref}) \|^2, \tag{4.11}
\]

where the superscript “ref” denotes reference traces from the simulated and observed data. The new objective function is designed to remove the influence of source signature on parameter updating. The first-order data residual, which acts as the adjoint source, is derived by Choi and Alkhalifah (2011; see Appendix E):

\[
r = d^{ref} \otimes (u * d^{ref} - d * u^{ref}), \tag{4.12}
\]

where $\otimes$ denotes cross-correlation.

### 4.3 Synthetic examples

#### 4.3.1 Transmission tests

Here, we conduct a set of transmission experiments to evaluate the crosstalk between the source signature and model (i.e., attenuation) parameters and demonstrate that the source-independent algorithm can mitigate this crosstalk for VTI media.

First, a Gaussian anomaly in the shear-wave attenuation parameter $A_{S0}$ is inserted between displacement sources and receivers imbedded in a homogeneous VTI medium (Figure 4.1). The other three Thomsen-style attenuation parameters ($A_{P0}$, $\epsilon_Q$ and $\delta_Q$) are constant. The four VTI velocity parameters ($V_{P0}$, $V_{S0}$, $\epsilon$ and $\delta$) and density are also constant and kept at their actual values during the inversion. The source signal used to generate the observed data represents the first derivative of the Ricker wavelet (Figure 4.2(a)). The reference frequency, which determines the peak attenuation, is equal to the central frequency of the wavelet (30 Hz). The homogeneous VTI background is chosen as the initial model; the attenuation parameters $A_{P0}$, $A_{S0}$, $A_{Ph}$, and $A_{Pn}$ are
updated simultaneously.

Figure 4.1: Gaussian anomaly in the parameter $A_{S0}$ embedded in a homogeneous VTI medium. The plot shows the fractional difference between $A_{S0}$ and its background value, 0.005 ($Q_{S0} \approx 100$); at the center of the anomaly, $A_{S0} = 0.025$ ($Q_{S0} \approx 20$). The other medium parameters are constant: $A_{P0} = 0.005$, $\epsilon_\varphi = -0.2$, $\delta_\varphi = -0.4$, $V_{P0} = 4000$ m/s, $V_{S0} = 2000$ m/s, $\epsilon = 0.15$, $\delta = 0.1$, and $\rho = 2.0 \text{ g/cm}^3$. The blue dots denote the horizontal-displacement sources and the magenta line marks the receivers placed at each grid point.

Figure 4.2: (a) Source wavelet used to generate the “observed” data. (b) The trial wavelet used to obtain the inversion results in Figures Figure 4.5 and Figure 4.6.

Figure 4.3 shows the inversion result obtained with the actual wavelet. Similar to the transmission experiment in Bai et al. (2017), the conventional WI algorithm is able to reconstruct most of the anomaly in $A_{S0}$. The peak of the estimated anomaly is $A_{S0} = 0.021$ (or $Q_{S0} = 23.8$), whereas the actual value is 0.025 (or $Q_{S0} = 20$). The other three parameters are practically unchanged, which indicates the absence of crosstalk between $A_{S0}$ and other attenuation parameters in this (favorable) acquisition geometry. The objective function using the actual wavelet rapidly decreases
to less than 1% of the original value (Figure 4.4).

Figure 4.3: Fractional differences between the inverted and initial parameters for the model from Figure 4.1: (a) $A_P$, (b) $A_S$, (c) $A_{Ps}$, and (d) $A_{Pn}$. Waveform inversion is conducted with the actual wavelet (Figure 4.2(a)) using the conventional objective function in equation 4.5. The peak value of the recovered anomaly in $A_S$ is 0.021 (about 84% of the actual maximum).

However, in practice the source signature is seldom known and has to be estimated from the data. To test the sensitivity of the viscoelastic waveform inversion to the source signature, we replace the actual signal in Figure 4.2(a) with a Ricker wavelet that has a central frequency of 30 Hz (Figure 4.2(b)). The resulting attenuation parameters are strongly distorted; the anomaly in $A_S$ is completely smeared and there is a significant leakage from $A_S$ into the other parameters (Figure 4.5). In some parts of the model the inverted parameters reach their lower and upper limits set in the algorithm. These results confirm the strong crosstalk between the source signature and attenuation parameters. The distorted source wavelet prevents the conventional WI algorithm from converging towards the actual model (Figure 4.4).
Next, we apply the proposed SIWI algorithm with the reference trace from the observed and simulated data located at $x = 0.1$ km. After 10 iterations, the algorithm reconstructs most of the $A_{S0}$-anomaly, and there is almost no crosstalk with the other attenuation parameters (Figure 4.6). However, the coefficient $A_{S0}$ is estimated with less accuracy (Figure 4.6(b)) compared to the WI result obtained with the actual wavelet (Figure 4.3(b)). The peak of the $A_{S0}$-anomaly is substantially underestimated and the anomaly’s shape is distorted. This deterioration in the inversion results is due to the increased nonlinearity of the inverse problem caused by the cross-correlation and convolution operations in the SIWI algorithm (Choi and Alkhalifah, 2011).

Figure 4.7 illustrates the generation of the first-order data residual (or the adjoint source) in the first iteration of model updating. Using the simulated (Figure 4.7(a)) and observed (Figure 4.7(b)) data and the reference traces at $x = 0.1$ km (green line), we compute the convolved data sets $u \ast d^{\text{ref}}$ (Figure 4.7(c)) and $d \ast u^{\text{ref}}$ (Figure 4.7(d)). After the convolution we observe three events: ① P$\ast$P, ② P$\ast$S+S$\ast$P, and ③ S$\ast$S. Figure 4.7(e) displays the difference between the two convolved data sets (i.e. $u \ast d^{\text{ref}} - d \ast u^{\text{ref}}$). Our choice of the reference trace (green line) results in blank records at $x = 0.1$ km and at $x = 0.4$ km (because of symmetry).

Because the anomaly is introduced only in the parameter $A_{S0}$, which does not influence P-waves, there should be no data residual for events involving only P-modes. This explains the
elimination of the event corresponding to the P-P convolution after the subtraction (event 1 in Figure 4.7(e)). The oscillations at times close to 0.3 s (window size) in the simulated and observed data produce artifacts in Figure 4.7(c) and Figure 4.7(d) that are also visible in difference between the convolved data sets (see the red arrows in Figure 4.7(e)). Finally, Figure 4.7(f) shows the first-order data residuals \([i.e. \ d^{ref} \otimes (u \ast d^{ref} - d \ast u^{ref})]\), in which the cross-correlation correctes for the time shifts caused by the convolution operations. Reducing the time window to the original data size (0.3 s) allowed us to suppress most artifacts.

Figure 4.8 shows the initial and final data residuals for the conventional WI applied with the actual wavelet (Figure 4.8(a) and Figure 4.8(b)) and for the proposed SIWI method with the trial wavelet (Figure 4.8(c) and Figure 4.8(d)). The data residuals (or the adjoint sources) for SIWI have a more complex structure (compare Figure 4.8(c) with Figure 4.8(a)), but both inversion methods
Figure 4.6: Inversion results obtained by the proposed source-independent algorithm with a trial wavelet (Figure 4.2(b)). The inverted parameters (a) $A_{P_0}$, (b) $A_{S_0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$.

provide a comparable improvement in data-fitting (this is also reflected in the objective functions in Figure 4.4).

### 4.3.2 Test for surface data

Next, we test the performance of the SIWI algorithm on reflection data generated for the modified Hess VTI model, which includes anisotropic attenuation (Figure 4.9). The velocity parameters and density (not shown here) have a structure similar to that of the attenuation parameters (Han et al., 2001; Tsvankin, 2012). The model is strongly heterogeneous with such complex features as folding, a fault, and a salt body. The wavefield is excited by 15 oblique displacement sources evenly spaced at a depth of 75 m, which excite a wavelet with a central frequency of 40 Hz (the first derivative of the Ricker wavelet, Figure 4.10(a)). The initial attenuation parameters (Figure 4.11) are obtained by applying triangle filtering to the actual parameter field with a smoothing radius of
20 samples in the vertical and horizontal directions.

First, we mute the direct arrivals and perform conventional WI with the actual wavelet (Figure 4.10(a)). After 18 iterations, the algorithm produces sufficiently accurate estimates of the attenuation parameters, especially $A_{P0}$, $A_{S0}$, and $A_{Ph}$ (Figure 4.12). The boundaries of the salt body, the folded layers (especially the one at depths between 2 and 4 km), and the fault surface (at a horizontal distance of around 13 km and depth of 2 km) are well delineated. The vertical parameter profiles (Figure 4.13) illustrate the convergence toward the actual values down to the depth of 4 km; the objective function is shown in Figure 4.18.

Next, conventional WI is applied with a trial wavelet (Figure 4.10(b)). The wavelet distortion prevents the algorithm from updating the attenuation parameters for this model. To implement the proposed SIWI methodology, we choose the truncated near-offset (25 m) seismograms containing the direct P-arrival (Figure 4.14) as the reference traces. SIWI helps recover the long-wavelength model features in the shallow part of the section (Figure 4.15). Yet, the folded layers at depths between 2 and 4 km are mispositioned and the fault is somewhat smeared. These problems, along with the behavior of the objective function (the red curve in Figure 4.18), indicate that the SIWI model-updating process get trapped in local minima caused by the increased nonlinearity of the inverse problem.

To further validate our results, we repeat the SIWI experiment using somewhat better initial parameter fields (Figure 4.16). The improved initial model helps obtain more accurate attenuation parameters (Figure 4.17), which also provide better data fitting (see the corresponding objective function in Figure 4.18).

### 4.4 Discussion

Because attenuation estimation is performed here with the actual velocity model, the Green’s functions in equation 4.11 are mostly influenced by the attenuation parameters. However, the reference frequency, at which the peak attenuation is generated in time-domain simulations with one relaxation mechanism, also contributes to the Green’s function. Here, we facilitate the inversion by
choosing the same reference frequency for simulating both the “observed” and modeled data. As a result, the objective function in equation 4.11 is minimized for the actual attenuation parameters (i.e., for $G_u = G_d$). However, the inversion may become problematic if the reference frequency for the trial simulation differs from that for the observed data (i.e., if $f_{ref}^{u} \neq f_{ref}^{d}$). When the WI algorithm tries to find the desired value $A_1$ at $f_{ref}^{d}$ (Figure 4.19), it will instead obtain a different value $A_2$ from the attenuation curve for the assumed reference frequency $f_{ref}^{u}$. For instance, we repeated the transmission experiment in Figure 4.1 with $f_{ref}^{u} = 100$ Hz (whereas $f_{ref}^{d} = 30$), and obtained strongly distorted inversion results (not shown here).

The influence of the reference frequency can be mitigated by simulating a less variable or even constant quality-factor elements $Q_{ij}$ over the frequency band of the data. However, this requires the inclusion of several relaxation mechanisms and, consequently, several coefficients corresponding to the characteristic (reference) frequencies. To avoid estimating these extra parameters, it may be possible to employ a set of fixed coefficients for typical $Q$-values in the subsurface (e.g., use the same set of coefficients to simulate $Q = 20$ and $Q = 500$; see Fichtner and Van Driel, 2014).

### 4.5 Conclusions

We extended the source-independent waveform inversion (SIWI) methodology to attenuation estimation in heterogeneous VTI media. The method operates with two additional data sets which represent the convolution of (1) the observed data with the reference trace from the simulated data and (2) of the simulated data with the reference trace from the observed data. The SIWI objective function is defined as the $\ell_2$-norm of the difference between these data sets. The approximate adjoint source is obtained as the zero-lag cross-correlation of the reference trace from the observed data with the difference between the convolved data sets. The gradients for the attenuation parameters are computed with the adjoint-state method in the same way as in the conventional WI algorithm.

First, we examined the influence of the source signature on attenuation estimation using transmission data from a homogeneous background VTI model with a Gaussian anomaly in the shear-
wave attenuation parameter $A_{S0}$. Whereas conventional WI with the actual wavelet produces satisfactory inversion results, a slight distortion of the wavelet (mainly in its phase) results in strongly distorted attenuation parameters. In contrast, the proposed SIWI algorithm generates satisfactory inversion results (albeit with a somewhat lower resolution) despite the wavelet distortion.

The algorithm was also tested on synthetic reflection data from a modified Hess VTI model, which contains a fault, a salt body and folding layers. Muting the direct arrivals and conducting WI with the actual wavelet yields a sufficiently accurate long- and intermediate-wavelength attenuation model with well-delineated structural boundaries. However, when a distorted trial wavelet is used, the conventional algorithm completely fails to update the initial model. The proposed SIWI with the trial wavelet was conducted by employing the windowed direct P-arrivals as the reference traces. Our algorithm successfully reconstructed long-wavelength features of the attenuation model, although some short-wavelength components could not be resolved (e.g., the folded layers are mispositioned) due to the increased nonlinearity of the inverse problem. A better initial model made it possible for SIWI to avoid local minima of the objective function and obtain more accurate attenuation parameters.

4.6 Acknowledgements

We thank Hess for generating the synthetic model. This work was supported by the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP. The reproducible numeric examples in this paper are generated with the Madagascar open-source software package freely available from http://www.ahay.org.
Figure 4.7: (a) Simulated (at first iteration) and (b) observed vertical displacement for the experiment in Figure 4.6; the source is located at $x = 0.25$ km. The convolved data sets (c) $u \ast d^{\text{ref}}$, and (d) $d \ast u^{\text{ref}}$. (e) The difference between (c) and (d) $(u \ast d^{\text{ref}} - d \ast u^{\text{ref}})$. (f) The first-order residual $d^{\text{ref}} \otimes (u \ast d^{\text{ref}} - d \ast u^{\text{ref}})$. The green lines on plots (a) and (b) denote the reference traces and the red arrows in (e) and (f) point to induced artifacts. The marked events are: 1. $P \ast P$; 2. $P \ast S$ (also includes $S \ast P$); 3. $S \ast S$; 4. $P \otimes (P \ast P) + S \otimes (P \ast S)$; 5. $S \otimes (S \ast S) + P \otimes (P \ast S)$; 6. $P \otimes (S \ast S)$. 
Figure 4.8: Data residuals before [(a) and (c)] and after [(b) and (d)] the inversion. (a) and (b) are generated by the conventional WI with the actual wavelet, while (c) and (d) by the SIWI algorithm with the trial wavelet.
Figure 4.9: Attenuation parameters for the Hess VTI model: (a) $A_P^0$, (b) $A_S^0$, (c) $A_{Ph}$, and (d) $A_{Pn}$. The model size is $15000 \times 9375$ m, with grid spacing $\Delta x = \Delta z = 25$ m. The yellow dots in (a) denote displacement sources and the magenta line marks the receivers.

Figure 4.10: (a) Actual and (b) trial source wavelet used for the model in Figure 4.9. The central frequency is approximately 40 Hz.
Figure 4.11: Initial attenuation parameters for the model in Figure 4.9: (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$. The initial model is obtained by smoothing the actual parameters with triangle filtering that has a smoothing radius of 20 samples in both the vertical and horizontal directions.
Figure 4.12: Inverted attenuation parameters (a) $A_P$, (b) $A_S$, (c) $A_P$, and (d) $A_Pn$ obtained by the conventional WI with the actual wavelet (Figure 4.10(a)).

Figure 4.13: Profiles of the attenuation parameters at $x = 11.25$ km: (a) $A_P$, (b) $A_S$, (c) $A_P$, and (d) $A_Pn$. The red lines are the parameters estimated by the conventional WI with the actual wavelet (Figure 4.12). The blue and green lines mark the actual and initial parameters, respectively.
Figure 4.14: Reference traces (at $x = 7.025$ km) corresponding to the source at $x = 7$ km, which were extracted from the (a) observed and (b) simulated data.

Figure 4.15: Inverted attenuation parameters (a) $A_{P0}$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$ obtained by the proposed SIWI algorithm with the trial wavelet (Figure 4.10(b)).
Figure 4.16: Initial attenuation parameters (a) $A_P$, (b) $A_{S0}$, (c) $A_{Ph}$, and (d) $A_{Pn}$ obtained by smoothing with a triangle filtering that has a smoothing radius of 15 samples in the vertical direction and 20 samples in the horizontal direction.
Figure 4.17: Inverted attenuation parameters (a) $A_P$, (b) $A_S$, (c) $A_{Ph}$, and (d) $A_{Pn}$ obtained by the proposed SIWI algorithm with the trial wavelet using the initial model in Figure 4.16.
Figure 4.18: Normalized objective function for the reflection experiments (see the model in Figure 4.9). The conventional WI with the actual wavelet (Figure 4.12, blue curve); SIWI with the trial wavelet (Figure 4.15, red curve), which stopped after six iterations; SIWI with the trial wavelet using a better initial model (Figure 4.17, green curve).

Figure 4.19: Schematic diagram showing the influence of the reference frequency on WI-based attenuation estimation. The two curves have different peak magnitudes ($A_1$ and $A_2$) at their reference frequencies ($f_{\text{ref}}^d$ and $f_{\text{ref}}^u$) but generate the same magnitude of the attenuation coefficient at frequency $f_{\text{ref}}^d$. 
CHAPTER 5
ATTENUATION COMPENSATION FOR TIME-REVERSAL IMAGING IN VTI MEDIA

A paper submitted to Geophysics
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Time reversal is a key component in reverse-time migration (RTM) and source localization using passive seismic (e.g., microseismic) data. The successful implementation of time reversal depends on the time symmetry (reversibility) of the wave equation in acoustic and elastic media. This symmetry in time, however, is no longer valid in attenuative media, and attenuation is often anisotropic. Here, we employ a viscoelastic anisotropic wave equation that decouples the influence of energy dissipation and velocity dispersion. That equation helps compensate for anisotropic attenuation and restore the time symmetry by changing the signs of the dissipation-dominated terms in time-reversed propagation, while keeping the dispersion-related terms unchanged. We test the $Q$-compensated time-reversal imaging algorithm on synthetic microseismic data from a 2D transversely isotropic medium with a vertical symmetry axis (VTI). After back-propagating multicomponent data acquired in a vertical borehole, we image microseismic sources using wavefield focusing. The source excitation times are estimated by picking the maximum amplitude of the squared shear strain component $\epsilon_{13}$ at the source locations. Accounting for attenuation anisotropy produces superior source images and more accurate excitation times compared to those obtained without attenuation compensation or with a purely isotropic $Q$-factor. The algorithm is also applied to a modified BP TI model to investigate the influence of such factors as survey geometry, errors in velocity and attenuation, noise, and limited aperture.

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5.1 Introduction

A fundamental property of wave propagation through elastic or acoustic media is reciprocity, which enables the application of time reversal (TR). By reversing the recorded data in time and then injecting them back into the medium, TR is supposed to focus the energy at the excitation location and time given a sufficiently wide acquisition aperture and knowledge of the medium parameters (e.g., velocities). TR is more suitable for data with a low signal-to-noise ratio than other event localization methods since it does not require picking of arrival times (Li and van der Baan, 2016). In addition to its applications in passive seismic surveys to locate and describe seismic sources (e.g., McMechan, 1982; Gajewski and Tessmer, 2005; Larmat et al., 2006; Steiner et al., 2008; Artman et al., 2010), time reversal is a crucial step in reverse-time migration (RTM). Time-reversed data are injected at the receiver locations and interact with the source (forward) wavefield through the imaging condition to produce reflecting interfaces (e.g., Baysal et al., 1983; McMechan, 1983).

The time symmetry (reversibility) of acoustic and elastic (nonattenuative) wave equations is explained by the presence of only even-order time derivatives. However, seismic waves propagating in the subsurface always experience energy dissipation and velocity dispersion. Attenuation-related terms break the time symmetry in the commonly used wave equations based on either the Generalized Standard Linear Solid (GSLS) model (e.g., Bohlen, 2002; Bai and Tsvankin, 2016) or Kjartansson’s constant-\(Q\) model (e.g., Carcione, 2008). These equations include first-order time derivatives or fractional time derivatives produced by the convolutional stress-strain relationship. As discussed by Zhu (2014), TR modeling of recorded viscoacoustic data through attenuative media generates a distorted source image because of additional attenuation during TR modeling. Therefore, to preserve the time symmetry and reconstruct well-focused source images, it is necessary to compensate for the influence of attenuation during TR modeling or back-propagation (e.g., Fink and Prada, 2001; Labyed and Huang, 2012; Ammari et al., 2013; Zhu, 2014).

Nearly-constant-\(Q\) (NCQ) models (e.g., Emmerich and Korn, 1987; Carcione, 1993; Bohlen, 2002; Bai and Tsvankin, 2016) are often adopted in simulating wave propagation in viscoacoustic
and viscoelastic media. A convolutional kernel, usually called the relaxation function, relates the stress and strain fields and ensures nearly invariant $Q$-values within a specified frequency band, given a sufficient number of relaxation mechanisms (memory variables). Based on Kjartansson’s constant-$Q$ model, Carcione et al. (2002) and Carcione (2008) proposed an alternative approach for $Q$-simulation that involves fractional time derivatives. Despite its advantages (accurate constant-$Q$ function and simple parameterization), numerical implementation of that method is hampered by the need to store at least certain parts of previously computed wavefields (Carcione, 2008), which entails excessive memory requirements.

The above propagators, however, are not suitable for $Q$-compensation because the dissipation and dispersion operators are coupled, and amplitude compensation is inevitably accompanied by a distortion of the velocity dispersion (Zhu, 2014; Guo et al., 2016). Using Kjartansson’s constant-$Q$ model, Zhu and Harris (2014) derive a decoupled constant-$Q$ acoustic wave equation with two separate fractional Laplacian operators accounting for amplitude dissipation and velocity dispersion. Zhu and Carcione (2014) generalize that approach for viscoelastic (but still isotropic) media. The viscoelastic propagator is implemented by Zhu (2015) and Zhu and Sun (2017) to restore the time symmetry in time-reversal imaging and RTM for isotropic attenuative models.

Laboratory experiments (Best et al., 2007; Zhu et al., 2006) confirm the existence of substantial attenuation anisotropy in the subsurface. In particular, the magnitude of the attenuation anisotropy parameters $\epsilon_Q$ and $\delta_Q$ for shale samples can exceed unity (Zhubayev et al., 2015). Therefore, it is imperative to account for attenuation anisotropy in seismic processing (e.g., microseismic imaging) for unconventional shale reservoirs. Estimation of attenuation anisotropy can provide new physical attributes for reservoir characterization and lithology discrimination (e.g., Behura et al., 2012; Guo and McMechan, 2017). Within the framework of the GSLS model, Bai and Tsvankin (2016) develop a time-domain finite-difference modeling algorithm for viscoelastic media with VTI symmetry for both velocity and attenuation. Employing that forward propagator, Bai et al. (2017) present a waveform-inversion methodology for VTI media, which can estimate the attenuation parameters required for anisotropic $Q$-compensation. Zhu (2017) extends the constant-$Q$ modeling
approach based on the fractional time derivatives (Carcione, 2008) to anisotropic viscoelastic media. However, these wave propagators include coupled effects of dissipation and dispersion, which poses a challenge for $Q$-compensated TR imaging. Zhu and Bai (2018) develop a formulation with fractional Laplacians for viscoelastic VTI media that decouples the attenuation terms into those responsible for amplitude loss and dispersion.

By employing the decoupled constant-$Q$ propagator of Zhu and Bai (2018), here we develop an algorithm for $Q$-compensated TR imaging in viscoelastic VTI media. In particular, we demonstrate that accounting for attenuation anisotropy could significantly improve TR imaging in shales. First, we briefly review the properties of the decoupled (in terms of dissipation and dispersion) viscoelastic anisotropic wave equation. Next, we show how that equation can be modified to restore the time symmetry for back-propagation in the presence of both attenuation and velocity anisotropy. A synthetic test demonstrates the decoupling of dispersion and dissipation phenomena in modeling of anisotropic wavefields based on the developed formulation. Then we implement anisotropic $Q$-compensation during back-propagation with the goal of spatial and temporal source localization using synthetic microseismic data. Finally, a modified BP TI model is employed to evaluate the influence of survey geometry, errors in the velocity and attenuation, etc., on microseismic source images obtained by the proposed $Q$-compensated TR algorithm.

5.2 Methodology

5.2.1 Anisotropic viscoelastic modeling based on fractional Laplacians

By employing the fractional Laplacian to approximate the fractional time derivatives, the stress ($\sigma_{ij}$)-strain ($\epsilon_{ij}$) relationship for attenuative VTI media can be written as (Zhu and Bai, 2018):

\begin{align}
\sigma_{11} &= \eta_{11} v_{11}^{2 \gamma_{11}} (\epsilon_{11} + \epsilon_{33}) + \left( \eta_{13} v_{55}^{2 \gamma_{13}} - \eta_{11} v_{55}^{2 \gamma_{11}} \right) \epsilon_{33} \\
&\quad + \tau_{11} v_{11}^{2 \gamma_{11}-1} \frac{\partial (\epsilon_{11} + \epsilon_{33})}{\partial t} + \left( \tau_{13} v_{55}^{2 \gamma_{13}-1} - \tau_{11} v_{55}^{2 \gamma_{11}-1} \right) \frac{\partial \epsilon_{33}}{\partial t}, \\
\sigma_{33} &= \eta_{33} v_{33}^{2 \gamma_{33}} (\epsilon_{11} + \epsilon_{33}) + \left( \eta_{13} v_{55}^{2 \gamma_{13}} - \eta_{33} v_{55}^{2 \gamma_{33}} \right) \epsilon_{11} \\
&\quad + \tau_{33} v_{33}^{2 \gamma_{33}-1} \frac{\partial (\epsilon_{11} + \epsilon_{33})}{\partial t} + \left( \tau_{13} v_{55}^{2 \gamma_{13}-1} - \tau_{33} v_{55}^{2 \gamma_{33}-1} \right) \frac{\partial \epsilon_{11}}{\partial t},
\end{align}

(5.1) (5.2)
and

\[ \sigma_{13} = \eta_{55} v_{55}^2 \varepsilon_{13}^{2} + \tau_{55} v_{55}^2 \varepsilon_{13}^{-1} \frac{\partial (2\varepsilon_{13})}{\partial t}, \]  

(5.3)

where

\[ \eta = C^{0}_{ij} \cos^{2}\left(\frac{\pi \gamma_{ij}}{2}\right) (\omega_{0})^{-2\gamma_{ij}} \cos(\pi \gamma_{ij}) \left(-\nabla^{2}\right)^{\gamma_{ij}}, \]  

(5.4)

\[ \tau = C^{0}_{ij} \cos^{2}\left(\frac{\pi \gamma_{ij}}{2}\right) (\omega_{0})^{-2\gamma_{ij}} \sin(\pi \gamma_{ij}) \left(-\nabla^{2}\right)^{\gamma_{ij}} \frac{1}{2}, \]  

(5.5)

and

\[ \gamma_{ij} = \frac{1}{\pi \tan^{-1}\left(\frac{1}{Q_{ij}}\right)}. \]  

(5.6)

Here \( \omega_{0} \) is the reference frequency, which should be larger than the dominant frequency of the source signal, \( C^{0}_{ij} \) are the stiffness coefficients defined at the frequency \( \omega_{0} \), \( Q_{ij} \) is the VTI quality-factor matrix (Zhu and Tsvankin, 2006), \( v_{11} = \sqrt{C^{0}_{11}/\rho} \), \( v_{33} = \sqrt{C^{0}_{33}/\rho} \), and \( v_{55} = \sqrt{C^{0}_{55}/\rho} \) \( (v_{11}, \) \( v_{33}, \) \( v_{55} \) are the velocities of the horizontally traveling P-wave and vertically traveling P- and S-waves, respectively). To describe the attenuation of P- and SV-waves in VTI media, it is convenient to use the Thomsen-style attenuation parameters \( A_{P0}, A_{S0}, \varepsilon_{Q}, \) and \( \delta_{Q} \) instead of the elements \( Q_{ij} \) (Bai and Tsvankin, 2016; Zhu and Tsvankin, 2006).

The terms multiplied with \( \eta_{ij} \) and \( \tau_{ij} \) in equations 5.1-5.3 account for the dispersion and dissipation, respectively. Note that with \( \gamma_{ij} = 0 \) in equations 5.4-B.2, we obtain \( \eta_{ij} = C^{0}_{ij} \) and \( \tau_{ij} = 0 \) \( (ij = 11, 13, 33, 55) \), and equations 5.1-5.3 describe purely elastic (nonattenuative) VTI medium. On the other hand, setting \( \gamma_{ij} = 0 \) in equation 5.4 eliminates velocity dispersion, while setting \( \gamma_{ij} = 0 \) in equation B.2 removes dissipation. A more detailed description of the decoupled viscoelastic VTI wave equation can be found in Zhu and Bai (2018).

### 5.2.2 Viscoelastic time-reversal imaging

To implement time reversal, we replace the time \( t \) in equations 5.1-5.3 with \( T - \hat{t} \), where \( T \) is the total recorded time and \( \hat{t} \) is the time variable for reverse propagation. The new system described by \( \hat{t} \) does not coincide with the original equations 5.1-5.3 because of the presence of the first-order
time derivative in the terms controlling the amplitude dissipation. To preserve time symmetry, we need to boost the amplitude during back-propagation, whereas the dispersion relationship should remain the same (Zhu, 2014). Hence, we change the sign in front of the dissipation-related operators (the terms containing $\tau_{ij}$) in equations 5.1-5.3:

$$\sigma_{11} = \eta_{11} v_{11}^{2\gamma_{11}} (\epsilon_{11} + \epsilon_{33}) + \left( \eta_{13} v_{55}^{2\gamma_{13}} - \eta_{11} v_{55}^{2\gamma_{11}} \right) \epsilon_{33}$$

$$- \tau_{11} v_{11}^{2\gamma_{11}} \frac{\partial (\epsilon_{11} + \epsilon_{33})}{\partial t} - \left( \tau_{13} v_{55}^{2\gamma_{13}} - \tau_{11} v_{55}^{2\gamma_{11}} \right) \frac{\partial \epsilon_{33}}{\partial t},$$

$$\sigma_{33} = \eta_{33} v_{33}^{2\gamma_{33}} (\epsilon_{11} + \epsilon_{33}) + \left( \eta_{13} v_{55}^{2\gamma_{13}} - \eta_{33} v_{55}^{2\gamma_{33}} \right) \epsilon_{11}$$

$$- \tau_{33} v_{33}^{2\gamma_{33}} \frac{\partial (\epsilon_{11} + \epsilon_{33})}{\partial t} - \left( \tau_{13} v_{55}^{2\gamma_{13}} - \tau_{33} v_{55}^{2\gamma_{33}} \right) \frac{\partial \epsilon_{11}}{\partial t},$$

and

$$\sigma_{13} = \eta_{55} v_{55}^{2\gamma_{55}} 2\epsilon_{13} - \tau_{55} v_{55}^{2\gamma_{55}} \frac{\partial (2\epsilon_{13})}{\partial t}.$$

(5.7)

(5.8)

(5.9)

If the time $t$ in equations 5.7-5.9 is replaced with $T - \hat{t}$, the expressions for $\sigma_{11}$, $\sigma_{33}$, and $\sigma_{13}$ become identical to equations 5.1-5.3. This means that the time-symmetry of the viscoelastic system can be restored by employing the modified equations 5.7-5.9 and amplifying the amplitudes during back-propagation.

To avoid instability that may be caused by enhancing high-frequency noise in the data, we apply a low-pass Tukey taper to the dissipation-related terms during time-reversal modeling. The taper parameters (cutoff frequency and taper ratios) are data-dependent. The maximum amplitude of the squared shear strain $\epsilon_{13}^2$ during reverse propagation is chosen as the imaging condition to focus energy at source locations.

5.3 Numerical examples

5.3.1 Decoupling of dissipation and dispersion

To study the decoupled dissipation and dispersion effects, we excite the wavefield by an explosive source embedded in a homogeneous VTI medium. Because the focus here is on the influence of attenuation (rather than velocity) anisotropy, we set the Thomsen velocity parameters $\varepsilon$ and $\delta$ at the reference frequency to zero, while the magnitude of the attenuation-anisotropy parameters...
is relatively large: $\varepsilon_Q = -0.6$ and $\delta_Q = -1.5$ (such negative values have been observed in laboratory experiments). The top two panels in Figure 5.1 show the displacement generated in the reference elastic medium (a) and in the fully attenuative model (i.e., that with both dispersion and dissipation, b). The bottom panels display the wavefield obtained by including only dissipation (c) or dispersion (d). As expected, the dissipation operator significantly reduces the amplitudes in Figure 5.1 (b) and Figure 5.1 (c). In contrast, the dispersion operator only delays the wavefronts in Figure 5.1 (b) and Figure 5.1 (d), especially near the vertical direction.

The variation of amplitude with angle shown in Figure 5.1 (b) and Figure 5.1 (c) is in good agreement with the linearized P-wave quality factor $Q_P = 1/(2 A_P)$ derived by Zhu and Tsvankin (2006):

$$Q_P(\theta) = Q_{P0} \left(1 - \delta_Q \sin^2 \theta \cos^2 \theta - \varepsilon_Q \sin^4 \theta\right).$$  \hspace{1cm} (5.10)

For negative $\varepsilon_Q$ and $\delta_Q$ used in Figure 5.1, the factor $Q_P$ increases away from the vertical up to angles close to $65^\circ$ and then decreases toward $90^\circ$ (Figure 5.2).

The dispersion-related wavefront delay in Figure 5.1 (b) and Figure 5.1 (d) is also anisotropic: the P-wavefront is visibly faster in the horizontal and oblique directions than in the vertical direction (Figure 5.3). Similar observations are made by Galvin and Gurevich (2015), who study dispersion due to wave-induced fluid flow in fractured media. In the intermediate frequency range where the dispersion is significant, the P-wave velocity differs in the directions parallel and perpendicular to aligned fractures, whereas at high frequencies the two velocities coincide (see Figure 2 in Galvin and Gurevich, 2015). Likewise, in our model there is no velocity anisotropy at the high reference frequency, for which $\varepsilon = \delta = 0$. Note that although we used an explosive source and there is no velocity anisotropy at high frequencies, attenuation anisotropy causes angle-dependent velocity dispersion, which produces a relatively weak SV-wave arrival (Figure 5.3).

Next, we conduct time-reversal imaging of synthetic microseismic data to demonstrate the need to apply anisotropic $Q$-compensation in attenuative VTI media. Time reversal is applied in four different ways (Table 5.1): (1) elastic TR of elastic data (ETR); (2) elastic TR (i.e., no $Q$-compensation) of viscoelastic data (NVTR); (3) viscoelastic TR with isotropic $Q$-compensation
using the actual quality factors $Q_{P0}$ and $Q_{S0}$, while $\epsilon_\varphi = \delta_\varphi = 0$ (IVTR), and (4) viscoelastic TR that compensates for the actual anisotropic attenuation (AVTR). The letter “V” stands for the viscoelastic VTI model used to generate the data.

### 5.3.2 Time-reversal imaging for a layered VTI model

First, we consider a layered VTI medium in Figure 5.4. All three layers have significant attenuation anisotropy, and layers 2 and 3 have moderate values of the velocity-anisotropy coefficients $\varepsilon$ and $\delta$ typical for shales (Table 5.2). The higher P-wave attenuation compared to that for S-waves in
Figure 5.2: Linearized P-wave quality factor as a function of the phase angle (equation 5.10) for the model from Figure 5.1 ($Q_{P0} = 20$, $\varepsilon_Q = -0.6$, and $\delta_Q = -1.5$).

the second and third layers may correspond to gas-saturated shales (Qi et al., 2017). The wavefields are excited by three dislocation sources with different magnitudes of the moment tensor. Figure 5.5 displays the horizontal displacement generated in the reference elastic medium (Figure 5.5(a)) and the actual viscoelastic VTI model (Figure 5.5(b)). Both energy dissipation and dispersion-caused time delay are clearly visible in Figure 5.5(b), especially for later arrivals.

Table 5.1: Types of time reversal used in the numerical examples. The viscoelastic data are computed for the models in Figure 5.4 and Figure 5.9, which include both velocity and attenuation anisotropy.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Data</th>
<th>Model for TR</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ETR</td>
<td>elastic</td>
<td>elastic</td>
</tr>
<tr>
<td>(2) NVTR</td>
<td>viscoelastic</td>
<td>elastic</td>
</tr>
<tr>
<td>(3) IVTR</td>
<td>viscoelastic</td>
<td>viscoelastic (isotropic $Q$)</td>
</tr>
<tr>
<td>(4) AVTR</td>
<td>viscoelastic</td>
<td>viscoelastic (anisotropic $Q$)</td>
</tr>
</tbody>
</table>

Next, we reverse the data in time and inject them back into the medium to localize the sources. Because we use dislocation-type sources described by the moment tensor with only one nonzero
Figure 5.3: Zoom of Figure 5.1 (d), which shows the wavefield in a dispersion-only VTI medium. The green dashed line marks the isotropic P-wavefront; the red arrow points to the SV-wave excited due to the angle-dependent velocity dispersion.

Table 5.2: Parameters of the VTI model from Figure 5.4. The velocity parameters $V_{P0}$, $V_{S0}$, $\varepsilon$, and $\delta$ correspond to the real parts of the stiffnesses $C_{ij}$ defined at an angular reference frequency of 10000 rad/s.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_{P0}$ (km/s)</th>
<th>$V_{S0}$ (km/s)</th>
<th>$\varepsilon$</th>
<th>$\delta$</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$Q_{P0}$</th>
<th>$Q_{S0}$</th>
<th>$\varepsilon_Q$</th>
<th>$\delta_Q$</th>
</tr>
</thead>
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<tr>
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<td>0.1</td>
<td>0.05</td>
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<td>30</td>
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<td>-0.2</td>
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<td>0.2</td>
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<td>-1.2</td>
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<td>1.5</td>
<td>0.25</td>
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<td>2.4</td>
<td>30</td>
<td>60</td>
<td>-0.4</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

component ($M_{13}$), the maximum amplitude of the squared shear strain $\varepsilon_{13}^2$ is chosen as the imaging condition (Kremers et al., 2011). Figure 5.6(a) shows the reference image with accurate source locations obtained by elastic TR of the elastic data (ETR, see Figure 5.5(a)). Next, we apply the same elastic TR algorithm to the viscoelastic data in Figure 5.5(b) (NVTR). As expected, the
Figure 5.4: Geometry of a synthetic microseismic survey in a three-layer VTI medium. The model size is 240 m × 320 m, with the grid spacing Δx = Δz = 0.4 m; the interval parameters are listed in Table 5.2. Three dislocation sources (marked by dots) with nonzero moment-tensor components \( M_{13} = 600 \) GPa, 800 GPa, and 1200 GPa (from left to right) are initiated at the origin times equal to 24 ms, 12 ms, and 3 ms, respectively; the central frequency of the source signal is 250 Hz. The green line at \( x = 20 \) m marks the receiver array.

Source images are blurry and mispositioned because of the uncompensated influence of attenuation (Figure 5.6(b)). Then, we compensate for attenuation during back-propagation, but under the isotropic \( Q \)-assumption (IVTR). We expect an overcompensation for P-waves and undercompensation for SV-waves based on the angle dependence of the quality factors of both modes in the second layer (Figure 5.7). The source images are somewhat smeared, have a lower magnitude, and are shifted (especially the right two) from their actual positions (Figure 5.6(c)). Finally, taking the attenuation anisotropy into account (AVTR) allows us to obtain well-focused and accurately positioned images of all three sources (Figure 5.6(d)), with the quality comparable to that of the reference ETR result (Figure 5.6(a)).

Using the obtained source locations, we can also estimate the corresponding excitation (origin) times. Here, following Kremers et al. (2011), the excitation time is found by picking the maximum
Figure 5.5: Horizontal displacement for the model in Figure 5.4. The data are computed for (a) the reference elastic medium, and (b) the viscoelastic medium.

value of the integral $E_S = \int_S \epsilon_1^2 dS$, where $S$ is the area surrounding the source location with a radius of 4 m (the approximate P- and SV-wavelengths are 10 m and 5 m, respectively). The time evolution of the field $E_S$ using different TR algorithms is displayed in Figure 5.8. In the reference ETR experiment, all three calculated peaks of the evolution curves are close to the actual excitation times (Figure 5.8(a)). Ignoring the influence of attenuation in TR (NVTR) leads to significant distortions (Figure 5.8(b)), and the magnitude of $E_S$ is much lower. By choosing a relatively small window to evaluate $E_S$, we assume a priori knowledge of the approximate source location. The small window size mitigates the influence of the amplified unfocusing energy outside the source area produced by inaccurate $Q$-compensation. This explains the acceptable result obtained by IVTR (Figure 5.8(c)), with the exception of the “red” source (Figure 5.4), which is most influenced by inaccurate $Q$-compensation. Finally, AVTR produces time-evolution curves (Figure 5.8(d)) that are similar to the reference ones (Figure 5.8(a)) and accurate estimates of the excitation times. Increased uncertainty in the source location (i.e., a larger window used to compute $E_S$) leads to significant deterioration in the excitation times estimated by IVTR, while the AVTR results remain accurate (not shown).
5.3.3 Time-reversal imaging for a modified BP section

Next, we test the $Q$-compensated TR imaging algorithm on microseismic data simulated for a modified section of the BP TI model. The attenuation parameters are generated by scaling the corresponding velocity parameters: $Q_{P0} = 1.25 Q_{S0} = 8 V_{P0}$ (in km/s), $\varepsilon = -2 \sqrt{\varepsilon}$, and $\delta_{Q} = -10 \delta$ (Figure 5.9). The magenta dots in Figure 5.9(a) are dislocation sources (with the only nonzero moment tensor component $M_{13}$ that ranges between 2800 and 5000 GPa), which excite a Ricker wavelet with the central frequency from 100 to 110 Hz. As in the previous example, we back-propagate the modeled data to focus the wavefield at the source locations. We conduct six different tests (Table 5.3) to assess the influence of such factors as the survey geometry (direction and aperture), noise, and errors in velocity and attenuation, on the performance of the TR imaging algorithm. As before, each test includes the four TR applications listed in Table 5.1. A Tucky taper with a cutoff frequency of 170 Hz and taper ratio of 0.2 is applied to stabilize back-propagation.

Table 5.3: Description of the tests for the BP model. Each test includes four TR experiments defined in Table 5.1. Note that “accurate $Q$” or “smoothed $Q$” in column 4 applies only to the IVTR and AVTR algorithms.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Monitoring array</th>
<th>Velocity</th>
<th>$Q$</th>
<th>Noise</th>
</tr>
</thead>
<tbody>
<tr>
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<td>vertical well</td>
<td>accurate</td>
<td>accurate</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>surface</td>
<td>accurate</td>
<td>accurate</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>vertical well</td>
<td>smoothed</td>
<td>accurate</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>vertical well</td>
<td>accurate</td>
<td>smoothed</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>vertical well</td>
<td>accurate</td>
<td>accurate</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>vertical well (limited aperture)</td>
<td>accurate</td>
<td>accurate</td>
<td>No</td>
</tr>
</tbody>
</table>

Test 1. Reference test

First, TR is applied to noise-free data recorded in a vertical well by a receiver array with wide aperture (see yellow dots in Figure 5.10(a)). The back-propagation is carried out with the actual velocity model. The reference source images computed by ETR (Figure 5.10(a)) coincide with the actual source locations. NVTR (i.e., no $Q$-compensation with viscoelastic data) severely under-estimates the wavefield energy and produces smeared source locations (Figure 5.10(c)). Isotropic
Q-compensation (IVTR) leads to overstated magnitudes and unfocused source images, especially for the two rightmost sources (Figure 5.10(e)). For example, the image of the third source from left is shifted by about 80 m (more than three times the average P-wavelength) from the actual location. Taking the actual attenuation anisotropy into account (AVTR) generates well-focused and accurately positioned source images (Figure 5.10(g)), which are almost indistinguishable from the reference ones (Figure 5.10(a)).

**Test 2. Surface array**

To evaluate the performance of the proposed TR algorithm for surface data, we place the receivers immediately below the surface. As expected, AVTR (Figure 5.11(h)) reconstructs source locations with high accuracy comparable to that of ETR (Figure 5.11(b)). NVTR (Figure 5.11(d)) distorts the source locations more significantly than in the previous test (Figure 5.10(d)) due to the high attenuation in the shallow layers (Figure 5.9(a) and Figure 5.9(b)). Isotropic Q-compensation (IVTR) produces better source images than in test 1 (compare Figure 5.11(f) with Figure 5.10(f)) because wave propagation for this configuration is predominantly near-vertical and the attenuation is relatively well-described by the parameters $Q_{P0}$ and $Q_{S0}$. Still, correcting for anisotropic attenuation (Figure 5.11(h)) enhances the source focusing and provides more accurate source locations.

**Test 3. Smoothed velocity model**

In the previous two tests, TR was performed with the actual velocity model. Next, we study the influence of velocity errors on TR images by back-propagating the wavefield through smoothed fields of the parameters $V_{P0}$, $V_{S0}$, $\epsilon$, and $\delta$ (Figure 5.12(b)); the maximum velocity error is about 8%. As expected, the distorted velocity leads to smearing of the source images (even with ETR) but the maximum focused amplitudes still approximately correspond to source locations (Figure 5.13(a) and Figure 5.13(b)). As before, AVTR’s source images are comparable to those by ETR, except for stronger unfocused artifacts produced by AVTR near the third source (we enumerate sources from left to right). IVTR generates much stronger artifacts (especially for the third and fourth sources), which could lead to serious errors in event location.
Test 4. Smoothed attenuation models

Here, we evaluate distortions in TR images due to inaccurate (smoothed) attenuation models, with the maximum error for all four attenuation coefficients close to 8% (Figure 5.14(b)). Similar to the results of the previous tests, AVTR produces high-quality source images (Figure 5.15(d)), while the IVTR images are smeared and mispositioned (especially for the third and fourth sources, see Figure 5.15(b)). This test shows that smoothed $Q$-models might be sufficient for time-reversal imaging, but attenuation anisotropy has to be accounted for.

Test 5. Noise

In the next test, we add band-limited random noise (with the same frequency band as the signal) to the simulated elastic and viscoelastic data (Figure 5.16). While ETR and NVTR produce almost the same source images (Figure 5.17(a) and Figure 5.17(c) or Figure 5.17(b) and Figure 5.17(d)) as the ones in the reference test 1, noise gets substantially amplified after source focusing in IVTR and AVTR, which masks the energy in the source area (Figure 5.17(e) and Figure 5.17(g)). However, if approximate source locations are known, one can still identify energy focusing in the source area using both AVTR and IVTR (Figure 5.17(h) and Figure 5.17(f)), although IVTR images are somewhat smeared and mispositioned. Hence, it is essential to suppress noise amplification during $Q$-compensation, which is essential for avoiding interpretation ambiguities in source localization.

Test 6. Limited aperture

Finally, we examine the dependence of the TR images on the aperture of the receiver array. The length of the receiver array in Figure 5.10(a) is reduced from 2.5 km to about 1 km which significantly limits the acquisition aperture (see yellow dots in Figure 5.18(a)); also, the spacing between adjacent receivers is increased from 40 m to 72 m. As a result, the source focusing using ETR slightly deteriorates, especially for the third and fourth sources from left (compare Figure 5.18(a) with Figure 5.10(a)). Similar degradation due to the limited aperture can be observed in the images computed by the other TR algorithms. Still, appropriate compensation for anisotropic attenuation (AVTR; Figure 5.18(g)) generates source images that are clearly superior to the output of NVTR (Figure 5.18(c)) and IVTR (Figure 5.18(e)).
5.4 Discussion and conclusions

We implemented time-reversal imaging with a viscoelastic VTI wave equation, in which the energy dissipation and velocity dispersion are separated. By reversing the signs of the dissipation-related terms while keeping those accounting for dispersion unchanged during back-propagation, we compensate for the $Q$-effect and preserve the time-invariance properties of the wave equation for attenuative anisotropic media. The presence of attenuation anisotropy causes angle-dependent amplitude and traveltime variation even without velocity anisotropy at the reference frequency. Numerical tests on synthetic viscoelastic microseismic data from a layered VTI model and modified BP TI section validate the $Q$-compensated time-reversal imaging algorithm. It should be emphasized that isotropic $Q$-compensation produces significant distortions in the spatial and temporal source localization. In contrast, accounting for attenuation anisotropy leads to superior source images and accurate excitation times, which are comparable to the reference ones obtained for purely elastic media. Therefore, our $Q$-compensated method can potentially provide more accurate microseismic source parameters (locations and excitation times) in unconventional shale plays.

We also used the modified BP model to evaluate the influence of survey geometry, errors in velocity and attenuation, noise, and receiver array aperture on the performance of the proposed method. Our main conclusions are as follows: (1) Attenuation anisotropy plays a more significant role in TR for borehole data than for surface surveys. (2) Distortions in the anisotropic velocity field can seriously reduce the quality of source focusing, which emphasizes the need to obtain a sufficiently accurate velocity model prior to TR. (3) The spatial resolution of the attenuation model generally does not have a large impact on the TR imaging results. (4) Random noise with the same frequency band as the data gets amplified during $Q$-compensation, which tends to mask the focused energy around the source locations and can cause stability problems. (5) Aperture limitations may lead to deterioration in the source focusing. It should be emphasized that in all these tests TR with anisotropic $Q$-compensation (AVTR) produces results comparable to the reference ones generated by elastic TR of elastic data (ETR), and consistently outperforms the NVTR (no $Q$-compensation) and IVTR (isotropic $Q$-compensation) implementations.
In addition to the proposed attenuation-compensated time-reversal algorithm for imaging microseismic sources, the developed back-propagation operator can be also combined with the source wavefield to perform so-called $Q$-compensated RTM (e.g., Zhu and Sun, 2017) in VTI media. Such $Q$-RTM with an appropriate imaging condition should produce superior migrated sections in the presence of anisotropic attenuation.

5.5 Acknowledgements

We thank Hemang Shah of BP for generating the TTI model. This work was supported by the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP. The reproducible numeric examples in this paper are generated with the Madagascar open-source software package freely available from http://www.ahay.org.
Figure 5.6: (a) Source images of the elastic data from Figure 5.5(a) obtained by elastic time-reversal (TR). (b-d) Source images of the viscoelastic data from Figure 5.5(b) obtained using (b) elastic TR (i.e., no $Q$-compensation), (c) viscoelastic TR with a purely isotropic $Q$-factor, and (d) viscoelastic TR with the actual anisotropic attenuation. A Tucky taper with a cutoff frequency of 800 Hz and taper ratio of 0.2 is applied to stabilize back-propagation. The red circles denote the actual source locations.
Figure 5.7: Linearized quality factors of the P- (blue curve) and SV-waves (red) as a function of the phase angle for the second layer from the model in Figure 5.4.
Figure 5.8: Time evolution of the field $E_S$ calculated by conducting (a) ETR, (b) NVTR, (c) IVTR, and (d) AVTR; $E_S$ is computed in a circle with a radius of 5 m. The exact excitation times for the three sources in Figure 5.4 are marked by the black lines, while the estimated times are marked by the red, green, and blue lines (same color as the corresponding sources in Figure 5.4).
Figure 5.9: Attenuation parameters for a modified section of the BP TI model: (a) $Q_P$, (b) $Q_S$, (c) $\varepsilon_Q$, and (d) $\delta_Q$. The model size is $8437.5 \times 11250\,\text{m} \times 2.4\,\text{km} \times 2.4\,\text{km}$, with grid spacing $\Delta x = \Delta z = 4\,\text{m}$. The black dots denote dislocation sources with the excitation time ranging from 72 to 714 ms and the central frequency from 100 to 110 Hz.
Figure 5.10: Source images for the model from Figure 5.9 obtained by (a) ETR, (c) NVTR, (e) IVTR, and (g) AVTR using a vertical receiver array. (b), (d), (f), (h) The corresponding zoomed-in source areas. The red circles mark the actual source locations. The wavefield is recorded by a vertical array of 60 evenly spaced receivers [yellow dots on plot (a)].
Figure 5.11: TR results for a horizontal receiver array [yellow dots on plot (a)]. Source images by (a) ETR, (c) NVTR, (e) IVTR, and (g) AVTR. (b), (d), (f), (h) The corresponding zoomed-in source areas.
Figure 5.12: (a) Actual and (b) smoothed fields of the P-wave vertical velocity $V_{P0}$. The smoothing was performed by a triangle filtering (with a smoothing radius of 20 samples in the vertical and horizontal directions). The same filtering is also applied to the other VTI velocity parameters $V_{S0}$, $\epsilon$, and $\delta$ (not shown here).
Figure 5.13: TR results for the smoothed velocity parameters (see Figure 5.12(b)). Source images obtained by (a) ETR, (c) NVTR, (e) IVTR, and (g) AVTR. (b), (d), (f), (h) The corresponding zoomed-in source areas. The wavefield is recorded by a vertical array of 60 evenly spaced receivers [yellow dots on plot (a)].
Figure 5.14: (a) Actual and (b) smoothed fields of the P-wave vertical quality factor $Q_{P0}$. The smoothing was performed by a triangle filtering (with a smoothing radius of 20 samples in the vertical and horizontal directions). The same filtering is also applied to the other VTI attenuation parameters $Q_{S0}$, $\epsilon_{Q}$, and $\delta_{Q}$ (not shown here).
Figure 5.15: Source images obtained by (a) IVTR, and (c) AVTR using the smoothed attenuation model (see Figure 5.14(b)). (b) and (d) The corresponding zoomed-in source areas.
Figure 5.16: Horizontal displacement for the modified BP section. The data are computed for (a) the reference elastic medium, and (b) the viscoelastic medium. (c) The elastic data from plot (a) after the addition of band-limited random noise. (d) The viscoelastic data from plot (b) with the addition of the same noise; the approximate signal-to-noise ratio is close to one.
Figure 5.17: TR results using the noise-contaminated elastic (Figure 5.16(c)) and viscoelastic (Figure 5.16(d)) data. Source images obtained by (a) ETR, (c) NVTR, (e) IVTR, and (g) AVTR. (b), (d), (f), (h) The corresponding zoomed-in source areas. The wavefield is recorded by a vertical array of 60 evenly spaced receivers [yellow dots on plot (a)].
Figure 5.18: TR results for a shorter receiver array with a larger distance between receivers [yellow dots on plot (a)]. Source images by (a) ETR, (c) NVTR, (e) IVTR, and (g) AVTR. (b), (d), (f), (h) The corresponding zoomed-in source areas.
CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

In this thesis, I developed new algorithms for forward modeling, full-waveform inversion, and time-reversal imaging in attenuative anisotropic media. Below I summarize the thesis results and provide recommendations for future work.

6.1 Conclusions

Within the framework of the generalized standard linear solid (GSLS) model, I presented a FD algorithm for simulating anisotropic attenuation in the time domain. An extension of the “$\tau$-method” helps generate nearly constant (i.e., frequency-independent) $Q_{ij}$ elements of the quality-factor matrix. The resulting velocity-dispersion curve coincides with that for the well-known Kjartansson’s constant-$Q$ model. The accuracy of the modeling algorithm is confirmed by reconstructing the attenuation parameters of a VTI layer from reflection data using the spectral-ratio method. The influence of attenuation and attenuation anisotropy on multicomponent wavefields is studied for VTI media with different structural complexity.

This wavefield simulator is used to develop a time-domain waveform-inversion (WI) algorithm for attenuation estimation in VTI media. The gradients of the objective function with respect to the viscoelastic parameters are obtained with the adjoint-state method. The inversion algorithm is first tested on transmission data for models with Gaussian anomalies in the attenuation parameters. Whereas reconstruction of the shape and peak amplitude of the anomalies is generally satisfactory, cross-talk between some attenuation parameters for specific survey geometries may lead to deterioration in the inversion results. The WI algorithm is also validated on reflection data from a modified version of the BP TI model. The inversion accurately estimates the long-wavelength spatial features of the attenuation parameters and reduces the data misfit by about 70%. The influence of velocity errors is successfully mitigated with the local similarity technique designed to correct for time shifts between the observed and modeled data.
WI-based attenuation estimation requires accurate knowledge of the source signature. To reduce the influence of errors in the source wavelet, I incorporated a source-independent objective function into the WI algorithm. Numerical examples for transmission and reflection data demonstrate the advantages of this source-independent technique. Even a small distortion of the source wavelet causes serious errors in conventional WI and prevents the algorithm from properly updating the model. The proposed source-independent method generates much more accurate inversion results, although the spatial resolution is somewhat lower compared to that of conventional WI with the actual wavelet.

I also implemented $Q$-compensated time-reversal (TR) imaging using a decoupled viscoelastic VTI wave equation based on the fractional Laplacians. Attenuation compensation is accomplished by reversing the signs of the dissipation terms during back-propagation while keeping those of the dispersion terms unchanged. Attenuation anisotropy causes angle-dependent amplitude and travel-time variations even without velocity anisotropy at the (high-limit) reference frequency. The proposed TR imaging algorithm is tested on synthetic viscoelastic microseismic data from a layered VTI model and from a modified BP TI section. Time reversal with anisotropic $Q$-compensation produces accurate source locations and excitation times (comparable to the output of elastic TR applied to elastic data) and consistently outperforms TR without $Q$-compensation or with purely isotropic $Q$-compensation. I also investigated the influence of survey geometry, errors in velocity and attenuation, noise, and limited aperture on the performance of the $Q$-compensated time-reversal imaging.

6.2 Recommendations for future work

Although it is possible to correct for traveltime shifts using the local-similarity technique, velocity errors also cause amplitude distortions through the geometric-spreading factor and reflection/transmission coefficients. Such amplitude errors can propagate into the attenuation parameters obtained by WI. The spectral-ratio method (and the central frequency-shift method) separates attenuation from other amplitude factors in the frequency domain. Integrating the spectral-ratio
method into the WI framework could potentially make $Q$-estimation more robust and less sensitive to velocity errors.

It is well known that low frequencies play an important role in waveform inversion for velocity parameters. The influence of frequency on attenuation estimation is more complicated. On one hand, higher-frequency events are more strongly attenuated in the subsurface, which increases the sensitivity to the attenuation parameters. On the other hand, inversion for higher frequencies is more likely to be impeded by local minima of the objective function (as is the case for velocity estimation) and by the lower signal-to-noise ratio of the recorded events. The influence of frequency on WI-based $Q$-estimation should be more thoroughly analyzed in the future. Multiscale inversion algorithms that progress from low to high frequencies, which are widely used in velocity inversion, could make attenuation parameter estimation more robust.

In joint inversion for velocity and attenuation, a sufficiently accurate smooth background $Q$-model can substantially improve the inverted velocity parameters (Kurzmann et al., 2013). Hence, the relatively low-resolution attenuation model obtained by the spectral-ratio method can be employed in elastic WI for the VTI velocity parameters (Kamath and Tsvankin, 2016). Then the attenuation parameters can be refined using the proposed viscoelastic WI algorithm with the local-similarity technique.

For purposes of quantitative reservoir characterization, macroscale seismic attributes could be linked to microscale rock-physics properties through effective poroelastic parameters (e.g., Dupuy et al., 2016). While the constant-$Q$ assumption is widely adopted in surface seismic surveys, it becomes inadequate for a wider frequency range covering seismic, crosswell, sonic, and core measurements. A better understanding of the frequency-dependent $Q$-behavior could potentially help in filling the data gap between different frequency bands. For example, ultrasonic $Q$-measurements can provide constraint for seismic attenuation analysis. Also, mesoscale $Q$-values, which may be directly linked to crucial reservoir parameters like permeability, can be inferred from either microscale (rock-physics) or macroscale (seismic) measurements.
In addition to the proposed attenuation-compensated time-reversal algorithm for imaging microseismic sources, the back-propagation operator developed in Chapter 5 can be also combined with the source wavefield to perform so-called $Q$-compensated reverse-time migration (RTM; e.g., Zhu and Sun, 2017) in VTI media. Such $Q$-RTM with an appropriate imaging condition (e.g., that based on the energy norm; see Rocha et al., 2017) should improve the quality of migrated sections in the presence of anisotropic attenuation.
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Here, we adopt the particle velocity-stress scheme to derive the viscoelastic wave equation for P- and SV-waves in 2D VTI media. A similar formalism is presented by Tromp et al. (2005) who employ the displacement-stress scheme and Fichtner and Van Driel (2014) who use a different definition of the memory variables.

Using the definition of $\tau_{ij}$ (equation 2.7), the relaxation function (equation 2.4) can be rewritten in the four-index notation as

$$\Psi_{mnpq}(t) = C_{R}^{R_{mnpq}} \left( 1 + \frac{\tau_{mnpq}}{L} \sum_{l=1}^{L} e^{-t/\tau_{l}} \right) H(t),$$  \hspace{1cm} (A.1)

where there is no summation over the indices $m$, $n$, $p$ and $q$.

Substituting equation A.1 into the generalized stress-strain relationship (equation 2.3) and then taking the time derivative on both sides yields:

$$\dot{\sigma}_{mn} = \Psi_{mnpq}(t) * \dot{\epsilon}_{pq}$$

$$= C_{U}^{mnpq} \dot{\epsilon}_{pq} - \frac{1}{L} \left( C_{U}^{mnpq} - C_{R}^{mnpq} \right) \left( \sum_{l=1}^{L} e^{-t/\tau_{l}} \right) H(t) * \dot{\epsilon}_{pq};$$  \hspace{1cm} (A.2)

here $C_{U}^{mnpq}$ is the unrelaxed modulus defined as $C_{U}^{mnpq} = C_{mnpq}^{R}(1 + \tau_{mnpq})$.

Replacing the convolution terms with the memory variables $r_{mn}^{l}$, we transform equation A.2 into:

$$\dot{\sigma}_{mn} = \frac{1}{2} C_{U}^{mnpq} (v_{p,q} + v_{q,p}) + \sum_{l=1}^{L} r_{mn}^{l};$$  \hspace{1cm} (A.3)

where

$$r_{mn}^{l} = -\frac{1}{L} \frac{\tau_{l}}{\tau_{l}} \left( C_{U}^{mnpq} - C_{R}^{mnpq} \right) e^{-t/\tau_{l}} H(t) * \dot{\epsilon}_{pq}.$$  \hspace{1cm} (A.4)
Differentiating equation A.4 with respect to time, we find:

\[ \dot{r}_{lm}^{l} = \frac{1}{\tau^{\sigma l}} \left[ \frac{1}{L} \left( C_{\text{mpq}}^{U} - C_{\text{mpq}}^{R} \right) e^{-t/\tau^{\sigma l}} H(t) \right] \dot{p}_{q} + \frac{1}{\tau^{\sigma l}} \left( C_{\text{mpq}}^{U} - C_{\text{mpq}}^{R} \right) e^{-t/\tau^{\sigma l}} \delta(t) \dot{p}_{q}, \]  
\( A.5 \)

where \( \delta(t) \) is the 1D \( \delta \)-function.

Combining equations A.4 and A.5 yields the following differential equations:

\[ \dot{r}_{lm}^{l} = -\frac{1}{\tau^{\sigma l}} \left[ \frac{1}{L} \left( C_{\text{mpq}}^{U} - C_{\text{mpq}}^{R} \right) \dot{p}_{q} + r_{mn}^{l} \right] \]
\[ = -\frac{1}{\tau^{\sigma l}} \left[ \frac{1}{2L} \left( C_{\text{mpq}}^{U} - C_{\text{mpq}}^{R} \right) (v_{p,q} + v_{q,p}) + r_{mn}^{l} \right], \]  
\( A.6 \)

where \( v_{p,q} \) is the derivative of the \( p \)th component of the particle velocity with respect to \( x_{q} \); the Einstein summation convention over \( p \) and \( q \) and Voigt convention is assumed.

Equations A.3 and A.6 describe viscoelastic wave propagation in VTI media. The relevant stress elements for P- and SV-waves in 2D VTI media are (the relaxed and unrelaxed moduli are expressed in the two-index Voigt notation):

\[ \dot{\sigma}_{11} = C_{11}^{U} v_{1,1} + C_{13}^{U} v_{3,3} + \sum_{l=1}^{L} r_{11,l}, \]  
\( A.7 \)

\[ \dot{\sigma}_{33} = C_{13}^{U} v_{1,1} + C_{33}^{U} v_{3,3} + \sum_{l=1}^{L} r_{33,l}, \]  
\( A.8 \)

\[ \dot{\sigma}_{13} = C_{55}^{U} (v_{1,3} + v_{3,1}) + \sum_{l=1}^{L} r_{13,l}, \]  
\( A.9 \)

with

\[ \dot{r}_{11}^{l} = -\frac{1}{\tau^{\sigma l}} \left[ \frac{1}{L} \left( C_{11}^{U} - C_{11}^{R} \right) v_{1,1} + \frac{1}{L} \left( C_{13}^{U} - C_{13}^{R} \right) v_{3,3} + r_{11}^{l} \right], \]  
\( A.10 \)

\[ \dot{r}_{33}^{l} = -\frac{1}{\tau^{\sigma l}} \left[ \frac{1}{L} \left( C_{33}^{U} - C_{33}^{R} \right) v_{3,3} + \frac{1}{L} \left( C_{13}^{U} - C_{13}^{R} \right) v_{1,1} + r_{33}^{l} \right], \]  
\( A.11 \)

\[ \dot{r}_{13}^{l} = -\frac{1}{\tau^{\sigma l}} \left[ \frac{1}{L} \left( C_{55}^{U} - C_{55}^{R} \right) (v_{3,1} + v_{1,3}) + r_{13}^{l} \right]. \]  
\( A.12 \)
APPENDIX B
ANISOTROPIC VISCOELASTIC WAVE EQUATION IN THE TIME DOMAIN

Using the standard linear solid model, which includes only one relaxation mechanism, the stress relaxation time $\tau_\sigma$ and the attenuation parameters $\tau_{ij}$ (Bai and Tsvankin, 2016) can be approximated as

$$\tau_\sigma \approx \frac{1}{2 \pi f_0},$$

(B.1)

and

$$\tau_{ij} = \frac{2}{\sqrt{Q_{ij}^2 + 1} - 1} \approx \frac{2}{Q_{ij}},$$

(B.2)

where $f_0$ denotes the reference frequency, which is often defined as the central frequency of the source wavelet.

The $\tau_{ij}$-parameters, which quantify the magnitude of attenuation in anisotropic media, can be converted into another set of viscoelastic parameters denoted by $\Delta C_{ij}$ (Bai and Tsvankin, 2016):

$$\Delta C_{ij} = C_{ij}^U - C_{ij}^R = \frac{C_{ij}^U \tau_{ij}}{1 + \tau_{ij}} \approx \frac{2 C_{ij}^U}{2 + Q_{ij}},$$

(B.3)

where $C_{ij}^U$ and $C_{ij}^R$ denote the unrelaxed and relaxed stiffness coefficients, respectively. The elements $C_{ij}^U$, which define the velocity field, are assumed here to be known.

The stress ($\sigma_{ij}$) - strain ($\epsilon_{kl}$) relationship in anisotropic viscoelastic media can be written as:

$$T_{ij} = \sigma_{ij} - \dot{\Psi}_{ijkl} * \epsilon_{kl},$$

(B.4)

where $T_{ij}$ is the stress tensor and " * " denotes the time-convolution operator.

Parameterizing the relaxation function (equation 4.1 in the main text) in terms of $C_{ij}^U$ and $\Delta C_{ij}$ yields:

$$T_{ij} = \sigma_{ij} - C_{ijkl}^U \epsilon_{kl} + \Delta C_{ijkl} \frac{e^{-t/\tau^\sigma}}{\tau^\sigma} H(t) * \epsilon_{kl}.$$

(B.5)
Introducing the memory variables,

\[ r_{kl} = -\frac{1}{\tau_e} e^{-t/\tau_e} H(t) * \epsilon_{kl}, \]  

we rewrite equation B.5 as

\[ T_{ij} = \sigma_{ij} - C^{ij}_{ijkl} \epsilon_{kl} = \Delta C_{ijkl} r_{kl}, \]  

with

\[ \dot{r}_{kl} = -\frac{1}{\tau_e} (r_{kl} + \epsilon_{kl}). \]

Equations B.7- B.8 and the momentum conservation law constitute the viscoelastic wave equations for VTI media.
APPENDIX C
GRADIENTS FOR VISCOELASTIC PARAMETERS $\Delta C_{jklm}$

In this section, we follow the approach of Charara et al. (2000) to derive the gradients of the objective function with respect to $\Delta C_{jklm}$. Application of the Born approximation to the momentum conservation law and equation B.7 yields:

$$\delta f_i = \rho \delta \ddot{u}_i - \frac{\partial}{\partial x_j} \delta \sigma_{ij}, \quad \text{(C.1)}$$

and

$$\delta T_{ij} = \delta \sigma_{ij} - C_{ijkl}^U \delta \epsilon_{kl} - \Delta C_{ijkl} \delta r_{kl}, \quad \text{(C.2)}$$

where $\delta u_i$, $\delta \sigma_{ij}$, $\delta \epsilon_{kl}$, and $\delta r_{kl}$ are the perturbed wavefield variables, and $\delta f$ and $\delta T$ are the virtual force and stress, which are determined by the perturbed model parameters ($\delta \rho$, $\delta C_{ijkl}^U$, and $\delta \Delta C_{ijkl}$):

$$\delta f_i = -\ddot{u}_i \delta \rho, \quad \text{(C.3)}$$

$$\delta T_{ij} = \epsilon_{kl} \delta C_{ijkl}^U + r_{kl} \delta \Delta C_{ijkl}. \quad \text{(C.4)}$$

The solution of the perturbed viscoelastic wave equation can be written as

$$\delta u_i = \int_V dV \int_0^T G_{ij} (\delta f_j) \, dt - \int_V dV \int_0^T \frac{\partial G_{ij}}{\partial x_k} (\delta T_{jk}) \, dt$$

$$- \int_V dV \int_0^T G_{ij} (\delta \rho) \ddot{u}_i \, dt - \int_V dV \int_0^T \frac{\partial G_{ij}}{\partial x_k} \left[ (\delta C_{ijklm}^U) \epsilon_{lm} + (\delta \Delta C_{ijklm}) r_{lm} \right] \, dt, \quad \text{(C.5)}$$

where $G_{ij}$ denotes the elastic Green’s function.

Comparing equation C.5 with

$$\delta u = \int_V \frac{\partial u}{\partial \mathbf{m}} \delta \mathbf{m} \, dV, \quad \text{(C.6)}$$
where \( \partial u/\partial m \) denotes the Fréchet kernel (Tarantola, 1988), we obtain
\[
\frac{\partial u_i}{\partial \Delta C_{jklm}} = - \int_0^T \frac{\partial G_{ij}}{\partial x_k} r_{lm} \, dt. \tag{C.7}
\]

The adjoint of the operator in equation C.6 can be written as
\[
\delta m = \sum_{\text{sources}} \int_0^T \left[ \frac{\partial u_i}{\partial m} \right]^* \delta u_i \, dt, \tag{C.8}
\]
with the kernels \( \partial u_i/\partial m \) being identical for a linear operator and its transpose (Tarantola, 1988).

Then the derivatives of the objective function \( F \) with respect to the viscoelastic parameters take the form:
\[
\frac{\partial F}{\partial \Delta C_{jklm}} \equiv \delta \Delta C_{jklm} = \sum_{\text{sources}} \int_0^T \left[ \frac{\partial u_i}{\partial \Delta C_{jklm}} \right]^* \delta u_i \, dt' \nonumber \tag{C.9}
\]
\[
= \sum_{\text{sources}} \int_0^T \left[ \frac{\partial u_i}{\partial \Delta C_{jklm}} \right] \delta u_i \, dt'
\]
\[
= - \sum_{\text{sources}} \int_0^T \int_0^T \frac{\partial G_{ij}}{\partial x_k} r_{lm} \delta u_i \, dt \, dt'.
\]

By defining the adjoint wavefield as
\[
u_j^* = \int_0^T G_{ij} \delta u_i \, dt, \tag{C.10}
\]
we rewrite equation C.9 as
\[
\frac{\partial F}{\partial \Delta C_{jklm}} \equiv \delta \Delta C_{jklm} = - \sum_{\text{sources}} \int_0^T \frac{\partial u_j^*}{\partial x_k} r_{lm} \, dt. \tag{C.11}
\]

Therefore, the gradients for the viscoelastic parameters \( \Delta C_{jklm} \) are the zero-lag cross-correlations of the forward memory variables \( r_{lm} \) with the adjoint strain fields \( \partial u_j^*/\partial x_k \) (Charara et al., 2000).
The P- and S-wave attenuation coefficients in the symmetry direction can be expressed through the quality-factor matrix \( Q_{ij} \) as follows (Zhu and Tsvankin, 2006):

\[
A_{P0} \equiv Q_{33} \left( \sqrt{1 + \frac{1}{Q_{33}^2}} - 1 \right) \approx \frac{1}{2Q_{33}},
\]  
(D.1)

and

\[
A_{S0} \equiv Q_{55} \left( \sqrt{1 + \frac{1}{Q_{55}^2}} - 1 \right) \approx \frac{1}{2Q_{55}}.
\]  
(D.2)

The influence of attenuation anisotropy on P- and SV-waves can be described by the Thomsen-style coefficients \( \epsilon_Q \) and \( \delta_Q \) (Zhu and Tsvankin, 2006):

\[
\epsilon_Q \equiv \frac{Q_{33} - Q_{11}}{Q_{11}},
\]  
(D.3)

\[
\delta_Q \equiv \frac{1}{2A_{P0}} \frac{d^2 A_P}{d\theta^2} \bigg|_{\theta = 0^\circ}.
\]  
(D.4)

For inversion purposes, it is convenient to operate with parameters that have the same units and similar magnitudes. Here, we employ the horizontal P-wave attenuation coefficient \( A_{P_h} \) and a parameter denoted by \( A_{P_h} \) (equations 4.7 and 4.8), which absorb the influence of \( \epsilon_Q \) and \( \delta_Q \).

The gradients for the TI attenuation parameters are then obtained from those for \( \Delta C_{ijkl} \) (equation C.11) using the chain rule:

\[
\frac{\partial F}{\partial A_{P_h}} = \frac{\partial F}{\partial \Delta C_{11}} \frac{4C_{11}^U}{(1 + 4A_{P_h})^2},
\]  
(D.5)

\[
\frac{\partial F}{\partial A_{P0}} = \frac{\partial F}{\partial \Delta C_{33}} \frac{4C_{33}^U}{(1 + 4A_{P0})^2} + \frac{\partial F}{\partial \Delta C_{13}} \frac{4C_{13}^U}{(1 + \tau_{13})^2} \frac{a + b - 1}{b},
\]  
(D.6)

\[
\frac{\partial F}{\partial A_{S0}} = \frac{\partial F}{\partial \Delta C_{55}} \frac{4C_{55}^U}{(1 + 4A_{S0})^2} + \frac{\partial F}{\partial \Delta C_{13}} \frac{4C_{13}^U}{(1 + \tau_{13})^2} \frac{-a}{b},
\]  
(D.7)
\[ \frac{\partial F}{\partial A_{P_n}} = \frac{\partial F}{\partial \Delta C_{13}} \frac{4C_{13}^{U}}{(1 + \tau_{13})^2 b} \]  

(D.8)

where

\[ a \equiv \frac{C_{55}^{U}}{C_{33}^{U}} \left( \frac{C_{13}^{U} + C_{33}^{U}}{C_{33}^{U} - C_{55}^{U}} \right)^2 \]  

(D.9)

\[ b \equiv \frac{2C_{13}^{U} C_{13}^{U} + C_{55}^{U}}{C_{33}^{U} C_{33}^{U} - C_{55}^{U}} \]  

(D.10)

and

\[ \tau_{13} = \frac{4}{b} [A_{P_n} + (a + b - 1)A_{P_0} - a A_{S_0}] . \]  

(D.11)
APPENDIX E
DERIVATION OF THE FIRST-ORDER ADJOINT SOURCE

The gradient of the “source-independent” objective function (equations 4.10 and 4.11) with respect to the model parameters can be written as:

$$\frac{\partial F(m)}{\partial m} = \left[ \frac{\partial u}{\partial m} \right] \cdot \left[ u \right] \cdot \gamma \left[ d \right] \cdot \left[ d \right] \cdot \left[ u \right] \cdot \gamma \left[ d \right].$$  \hspace{1cm} (E.1)

The derivative $\frac{\partial u^{ref}}{\partial m}$ involves only the reference trace, which is often windowed to include just the direct P-arrival (especially for the reflection experiments). Because usually the direct arrivals are weakly sensitive to the model parameters (i.e., $\frac{\partial u^{ref}}{\partial m}$ is small), this term can be ignored (see Choi and Alkhalifah, 2011). Then equation E.1 becomes:

$$\frac{\partial F(m)}{\partial m} = \left[ \frac{\partial u}{\partial m} \right] \cdot \left[ u \right] \cdot \gamma \left[ d \right] \cdot \left[ d \right] \cdot \left[ u \right] \cdot \gamma \left[ d \right].$$  \hspace{1cm} (E.2)

where summation over shots and receivers are implied for the scalar expression, and $x = u \cdot d^{ref} - d \cdot u^{ref}$.

By introducing $\xi = t - \tau$ (so $\tau = t - \xi$), equation E.2 can be rewritten as:

$$\frac{\partial F(m)}{\partial m} = \int_{-\infty}^{\infty} x(t) \int_{-\infty}^{\infty} \frac{\partial u}{\partial m}(t - \tau) d^{ref}(\tau) d\tau dt,$$

It is straightforward to show that $r = d^{ref} \otimes x$ (see equation 4.12) can be treated as the adjoint source.
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Best luck,
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On Jan 14, 2019, at 5:45 PM, Tong Bai <tbai@mymail.mines.edu> wrote:

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