EFFECTS OF ANISOTROPIC MAGNETIC SUSCEPTIBILITY
IN DATA INTERPRETATION AND ITS POTENTIAL IN
APPLICATION

by

Zhuo Liu
A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science (Geophysics).

Golden, Colorado
Date ______________________

Signed: ______________________
Zhuo Liu

Golden, Colorado
Date ______________________

Signed: ______________________
Dr. Yaoguo Li
Thesis Advisor

Golden, Colorado
Date ______________________

Signed: ______________________
Dr. John H. Bradford
Professor and Head
Department of Geophysics
ABSTRACT

Anisotropic magnetic susceptibility (AMS) contains information about the internal structure of magnetic geologic units and may have the potential to provide information about the formation history and subsequent geologic processes to which these units were subjected. Its application in the geological study has been established as a reliable tool in formation analyses. However, in the geophysical study, especially in magnetic data interpretation, there has not been much work on AMS, although its potential has been known for decades. Based on the current research status, the goal of this thesis is to assess the influence of AMS on induced magnetization and to explore the possibility of extracting structural information from magnetic data containing the influence of AMS.

In this work, I review the basic mathematical expression of AMS and examine the rotation matrix through the derivation of Euler parameters. I show that the matrix can be directly formed by using the principal directions of the AMS within the user-defined coordinate system. Forward modeling of magnetic response over a synthetic AMS model shows that even at low susceptibility values, the induced magnetization direction can be rotated significantly away from the inducing field direction. A series of numerical experiments demonstrate that estimating source parameters directly from magnetic data in the presence AMS can recover the information of AMS and help to identify different magnetic units.

The comparison between different constructions of rotation matrix provides us with a direct means to rotate AMS tensor for modeling purpose, while the forward modeling result shows that the interpretation of AMS-influenced data is faced with similar challenges as found in the cases of self-demagnetization due to high magnetic susceptibility or strong remanent magnetization. Parameter estimation studies show that it is possible to recover structural information through AMS from magnetic data without the assistance of laboratory measurements. This research opens the door for future work on magnetic data interpretation.
based on AMS and its potential use in differentiating and characterizing geologic units of different compositions and origins.
TABLE OF CONTENTS

ABSTRACT ................................................................. iii
LIST OF FIGURES .......................................................... vii
LIST OF TABLES ............................................................ xi
LIST OF SYMBOLS ........................................................... xii
LIST OF ABBREVIATIONS .................................................. xiv
ACKNOWLEDGMENTS ....................................................... xv

CHAPTER 1 INTRODUCTION ................................................... 1

CHAPTER 2 OVERVIEW OF ANISOTROPY MAGNETIC SUSCEPTIBILITY TENSOR ............................................................... 5

2.1 Fundamental Mathematics for Anisotropic Magnetic Susceptibility ....................................................... 5

2.2 Construction of Rotation Matrix ...................................... 8

2.2.1 Rotation by Euler Angles ............................................ 9

2.2.2 Correspondence of the rotation matrix with principal directions ........................................................... 11

2.3 Validation of Rotation Matrix .......................................... 12

2.4 Summary ................................................................. 17

CHAPTER 3 UNDERSTANDING THE EFFECTS OF ANISOTROPIC MAGNETIC SUSCEPTIBILITY ON MAGNETIC ANOMALIES ....................................................... 18

3.1 General Calculation of Total-field Anomaly ........................ 18

3.2 Influence of AMS on magnetic data ................................... 19

3.3 Distinguishing from remanent and self-demagnetization effect ............................................................... 25

3.4 Summary ................................................................. 26
CHAPTER 4 PARAMETER ESTIMATION FOR ANISOTROPIC MAGNETIC SUSCEPTIBILITY ........................................... 28

4.1 Method of Parameter Estimation ............................................. 28

4.2 Necessity of Parameter Estimation Using AMS Models .................. 29

4.3 Parameter Estimation for additional 2D Models .......................... 35

4.4 Summary .......................................................................... 39

CHAPTER 5 CONCLUSIONS ......................................................... 41

5.1 Future Work ....................................................................... 42

REFERENCES CITED ................................................................. 44
LIST OF FIGURES

Figure 2.1 Geometric representation of anisotropy in 3D by an ellipsoid. The semi-axis lengths are proportional to the magnitude of corresponding principal susceptibilities, and the orientation of axes are parallel to principal directions. ................................................................. 6

Figure 2.2 Synthetic model designed to test the method for calculating the rotation matrix. The shaded cuboid region represents the magnetic prism. The observation grid is from -400 m to 400 m with a 25-m interval on the x-y plane. The source is a 300 m × 300 m × 50 m prism buried 100 m below the center of the observation grid. ................................. 12

Figure 2.3 Illustration of principal directions (blue) and inducing field direction (red). For principal direction, the longest arrow pointing north-east into the depth is $\hat{p}_1$ direction, the intermediate-length arrow pointing to the south sub-horizontally is $\hat{p}_2$ direction, and the shortest arrow pointing to the west sub-horizontally is the is the $\hat{p}_3$ direction. .............................................. 13

Figure 2.4 Difference between total-field anomalies with full susceptibility tensor rotated by matrices obtained from principal directions and the calculation of Euler parameters. Since we cannot present the difference of rotation matrices by a figure, this result is obtained by subtracting the total-field anomaly of the isotropic source from that of the anisotropic source. Note that the difference shown on the colorbar is on the order of $10^{-13}$. This difference is small enough that we believe the two rotations are equal as they yield the same result. .......................................... 15

Figure 2.5 Total-field anomaly generated by different sources and their differences. Inducing fields are the same for all cases, as the inclination is 60° and declination 25°. (a) Total-field anomaly generated by isotropic source; (b) Total-field anomaly generated by anisotropic source; (c) Difference between two anomalies. In three figures, the pink line indicates the inducing field declination, and the yellow line in the third figure marks the trend of anomaly produced by the anisotropic medium (max-min values). ................................................................. 16
Figure 3.1  Total-field anomaly produced by a dyke with AMS of different anisotropy degrees: a) The source is a 45° dipping dyke located 50 m below the ground surface. The red arrow shows the direction of the inducing field, and green arrows show the principal directions of AMS. b) The total-field magnetic anomalies generated by the dykes with different anisotropy degrees. ................................................................. 21

Figure 3.2  Angular differences between the inducing field and induced magnetization. The contour lines show the difference between inducing field direction and orientation of induced magnetization. The dashed line to the left marks the threshold in anisotropy degree below which there is no noticeable effect of AMS. ................................................................. 22

Figure 3.3  The angular difference between the inducing field and induced magnetization in 3D. Results are controlled by the inclination and declination of the inducing field and anisotropy degree of the source, and presented as dots only with an angular difference greater than 15°. The color of scattered dots shows the difference between inducing field direction and orientation of induced magnetization. The light gray plane at the bottom of the space marks the threshold in anisotropy degree, 1.7, below which there is no noticeable effect of AMS. ................................................................. 24

Figure 4.1  Synthetic model and simulated observed data. a) The cross-section of the 2D dyke model. The black parallelogram indicates the source location. The horizontal line represents the ground surface, and the red arrow is the direction of the inducing field. The green arrows show the principal directions, with maximum susceptibility parallel to the dip direction, and minimum susceptibility pointing normal to the dip. b) Simulated observed data with 1-nT Gaussian noise. The observation grid is in the x-direction from -100 m to 1000 m, with a 50-m station interval. . 30

Figure 4.2  Parameter estimation results from treating a data as if they were from the isotropic medium. The color contours show the data misfit $\phi_d$ in log$_{10}$ scale. The “x” marks the minimum value of data misfit, and two dashed lines mark the corresponding estimated effective susceptibility, 0.011, and estimated dip angle, 62°. The “+” marks the true parameters. The true effective susceptibility is 0.0079, and the true dip angle is 45°. From this map, it is evident that when inverting parameters for an isotropic source based on data produced by the anisotropic source, the recovered parameters cannot describe the true model. ................. 31
Figure 4.3  Comparison between true source and data with recovered source and data. a) Location of the true source (green) and recovered source (black) that described by the estimated parameters. Arrows represent corresponding principal directions; b) Observed data (green), predicted data (black), and the difference between them (red). Although the recovered data has a similar pattern and intensity compared to the true data, the recovered model is significantly different from the true model. Note that the differences between true and recovered data is well above the added noise level (1 nT).  

Figure 4.4  Extended tests over a 45° dyke model with different anisotropic degrees. These tests show the differences between the estimated and true parameters when treating the data produced by the anisotropic medium as being from the isotropic medium. a) The data generated by a dyke model with different anisotropy degree, as indicated by the vertical axis. The magnitude of data decreases with the increasing anisotropy degree because of corresponding decreasing effective susceptibility; b) The true effective susceptibility, estimated effective susceptibility, and their differences; c) The true and estimated dip angle and their differences. The vertical dashed light green line in the middle and the right panels indicate the 1.7 threshold on the anisotropy degree.  

Figure 4.5  Parameter estimation with assumption of AMS model. In both panels, the horizontal-axis represents the anisotropy degree, and the dashed red vertical line indicates the true anisotropy degree based on true principal susceptibilities. The upper panel shows the difference between true and recovered dip. The lower panel shows the minimum data misfit in $\log_{10}$ scale.  

Figure 4.6  Simple dyke model test. This test is designed to examine if the parameter estimation can recover the parameters describing the anisotropy and recover the presence of the magnetic dyke. a) The true and recovered model. b) The observed data, predicted data and the difference between them. In this test, the parameters are recovered with tolerable error level. During the estimation, the estimation of recovered principal susceptibilities is more robust and fairly close to the true values, while the dip angle and principal direction are more sensitive to the noise.
Figure 4.7 Two-dyke model test. This test is designed to examine if the parameter estimation can recover different parameter sets describing different anisotropic medium, and hence identify different magnetic bodies and recover the presence of the magnetic dykes. a) The green arrows and parallelograms are the true model and principal directions, while the pink arrows and parallelogram are the recovered model and principal directions. b) The blue line is the true data, the orange line is the recovered data based on estimated parameters, and the green line is the difference between them.

Figure 4.8 Extended tests for assessing uncertainty. The true model is shown in green, the recovered models are shown in pink, and the brown line on the top indicates the ground surface. When comparing between the true and recovered models, the recovered principal directions hold their pattern with relatively high consistency, but the recovered dip angle varies around the true angle with a slightly larger range.
LIST OF TABLES

Table 4.1 True and Recovered Parameters of One-Body Test . . . . . . . . . . . . . . 36
Table 4.2 True and Recovered Parameters of Two-Body Test . . . . . . . . . . . . . . 37
Table 4.3 True Parameters and RMSE of Each Parameter Based on 50 Tests . . . . . 38
LIST OF SYMBOLS

Magnetic Permeability in the Vacuum .................................................. \( \mu_0 \)
Magnetization ......................................................................................... \( \vec{M} \)
Inducing Field ....................................................................................... \( \vec{H}_0 \)
Magnetic Flux of Inducing Field ............................................................ \( \vec{B}_0 \)
Principal susceptibility Tensor ................................................................. \( \kappa' \)
Full Susceptibility Tensor ..................................................................... \( \kappa \)
Scalar susceptibility value ...................................................................... \( \kappa \)
Principal Direction in local coordinate system ...................................... \( \hat{\hat{p}} \)
Principal Direction in user coordinate system ....................................... \( \hat{x}, \hat{y}, \hat{z} \)
Effective Susceptibility ......................................................................... \( \kappa_{\text{effect}} \)
Anisotropy Degree ................................................................................ \( P_2 \)
Euler Angles .......................................................................................... \( \phi, \theta, \psi \)
Rotation Matrix from Euler angles ....................................................... \( \mathbf{R} \)
Rotation Matrix from Euler Parameters ............................................... \( \mathbf{R}_{\text{ep}} \)
Rotation vector ..................................................................................... \( \mathbf{r} \)
Eigenvector ............................................................................................ \( \mathbf{v} \)
Diagonal Matrix of Eigenvalues ............................................................ \( \Lambda \)
Matrix of Eigenvectors .......................................................................... \( \mathbf{V} \)
Euler Rotation Axis ............................................................................... \( \vec{s} \)
Euler Rotation Angle ............................................................................ \( \phi \)
<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler parameters</td>
<td>$a, b, c, d$</td>
</tr>
<tr>
<td>Unit Quaternion</td>
<td>$Q$</td>
</tr>
<tr>
<td>Total-Field Anomaly</td>
<td>$\Delta T$</td>
</tr>
<tr>
<td>Number of sources</td>
<td>$M$</td>
</tr>
<tr>
<td>Green’s Tensor</td>
<td>$\Gamma$</td>
</tr>
<tr>
<td>Model Objective Function</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Data Misfit</td>
<td>$\phi_d$</td>
</tr>
<tr>
<td>Vector of Observed Data</td>
<td>$d^{obs}$</td>
</tr>
<tr>
<td>Vector of Predicted Data</td>
<td>$d^{pre}$</td>
</tr>
<tr>
<td>Estimated Standard Deviation of Noise</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>Number of observed data</td>
<td>$N$</td>
</tr>
<tr>
<td>Number of parameter realizations</td>
<td>$n$</td>
</tr>
<tr>
<td>Recovered parameter</td>
<td>$m^{est}_i$</td>
</tr>
<tr>
<td>True parameter</td>
<td>$m^{true}_i$</td>
</tr>
</tbody>
</table>
# LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMS</td>
<td>Anisotropic magnetic susceptibility</td>
</tr>
<tr>
<td>nT</td>
<td>Nanotesla</td>
</tr>
<tr>
<td>IGRF</td>
<td>International Geomagnetic Reference Field</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root-Mean-Square Error</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

I would like to express my appreciation to my adviser, Dr. Yaoguo Li, for his continuous encouragement and mentoring during this research, and his support in my growth as a graduate student. This research would not happen without his enthusiasm, patience, and guidance. I also would like to thank my thesis committee members, Dr. Richard Krahenbuhl, and Dr. Jeffrey Shragge, for their guidance and insight during this research.

I am grateful for the support of my parents, Dr. Jianxin Liu and Mrs. Fan Lin, and the Departments of Geophysics at Colorado School of Mines. I would like to send gratitude to my parents for creating a firm financial backing and for establishing a loving home for me in the pursuit of my Master degree. I appreciate the Department for offering me the TA support. I have learned a great deal from the TA experience.

I would also like to thank all my friends and colleagues in CGEM for their generous help and advice, especially Elizabeth Maag-Capriotti, Andy McAlile, and Qian Yin, along with Dr. Zhaowang Zong from Department of Chemical and Biological Engineering, and Dr. Hongzhu Cai from China University of Geosciences (Wuhan) for their assistance in the writing.

I would like to express special thanks to my fianceé, Miss Zhen Ren from George Mason University, for her support and companion that have very much provided me with the steadiness in this foreign land.
CHAPTER 1
INTRODUCTION

Anisotropic magnetic susceptibility (AMS) as a characteristic of rocks has long been used in geology due to its availability from laboratory measurements. Publications on AMS are primarily focused on five major areas: structural-tectonics, igneous processes, sedimentology, alteration processes, and rock mechanics.

In structural geology, Borradaile and Henry [1] use AMS as an indicator of the strain direction in ductile deformation and stress directions in neotectonic environments. Winkler et al. [2] also distinguish strain patterns based on AMS data, while Pares and van der Pluijm [3] illustrate that strain is one of several factors affecting the magnetic lineation. Glen et al. [4] provide support for continental drift by presenting the consistency of principal directions of AMS between the west coast of Namibia and the east coast of Brazil. Chadima et al. [5] use AMS as a complementary method to magnetostratigraphy studies for establishing a correlation between different rock groups. Mantani [6] uses the angle between the direction of magnetic foliation and the orientation of extensional flow apophysis as an indicator to determine the kinematic vorticity number. Dudzisz et al. [7] apply AMS to determine the orientation of the principal tectonic strain direction developed during the formation of the West Spitsbergen Fold-and-Thrust belt (WSFTB).

In igneous petrology, Ernst and Baragar [8] use AMS measurements to reconstruct the pattern of magma flow in a dyke swarm. In sedimentology, Wassmer et al. [9] apply AMS to the study of unconsolidated deposits to find that the vertical variation of magnetic foliation indicates the rapid change in under-water transport mechanisms from rolling to gradual or uniform suspension during each tsunami wave cycle. In distinguishing alteration processes, Lapointe et al. [10] first use the relationship between the bulk susceptibility and the intensity of alteration to qualitatively evaluate the alteration level of the rock. This method was
further developed by Sheibi et al. [11] who use AMS measurements to aid mapping of igneous rock type based on the relationship between principal axis and alteration type.

In the study of rock mechanics, Lebedev et al. [12] and Vishnu et al. [13] establish a relationship between elastic anisotropy and AMS. They discover that rocks are weaker in directions parallel to magnetic foliation than perpendicular to it, and assume that the anisotropy of acoustic and magnetic susceptibility originated from the same source.

The use of AMS in all geological studies listed above is based on laboratory measurements of the rock samples. The majority of structural analyses only compare direct AMS measurements with structural information geometrically. While in the mineral exploration, AMS has only been applied in rare cases. Although Hrouda [14] points out that AMS can work as a tool in applied geophysics due to its influence on the orientation of magnetization, no significant work has been reported on applying magnetic anisotropy in the geophysical analysis. Mukherji et al. [15] and O’Driscoll et al. [16] compared the degree of anisotropy in drill cores extracted from banded iron formations in eastern India and indicated the possibility of using measured anisotropy degree to infer the concentration of ferromagnetic minerals, but neither gave a quantitative expression of this relationship. Starostenko et al. [17] summarize the 3D forward calculation of the magnetic field of a folded layer model with AMS, but focus more on meshing the model domain in the cylindrical coordinate system.

Based on the current status, the use of AMS in geophysical data interpretation is lacking. This leads to my research interest and the question of whether it is possible to use this property to infer AMS of a geologic unit indirectly from the magnetic data, and what auxiliary information is needed to achieve this goal.

In the next chapter, I briefly discuss the basic mathematical expression of AMS and statistical parameters that quantitatively describe AMS. Before forward modeling the magnetic response produced by the sources with AMS, I examine the construction of the rotation matrix that rotates the AMS tensor measured in the laboratory to the user coordinates. Next, I design a synthetic model and vary the parameters to numerically validate the approach
for rotation matrix construction. The chapter concludes that the rotation matrices formed
directly from the principal directions are the same as those obtained from Euler angles.

The Chapter 3 begins with a summary of the procedure for calculating the total-field anomaly from a rectangular prism. Then, I forward model the total-field anomaly produced by different anisotropic sources in both 2D and 3D and illustrate how the AMS will change the resultant total-field anomaly. All forward modelings are performed without the presence of remanence or self-demagnetization effect. With the forward results, I examine the difference between the directions of inducing field and the induced magnetization produced by the source with AMS, where the AMS has fixed principal directions but varies as a function of anisotropy degree. The modeling results reveal how the magnetic response is affected by the anisotropy, and the angular difference between the inducing field and induced magnetization provides guidance for subsequent interpretation.

In Chapter 4, I design a 2D inverse problem to assess the question whether the structure information can be inferred from magnetic data with AMS. I describe the inversion method applied to the synthetic data, discuss the necessity of inverting for anisotropy models when AMS is present, and estimate the parameters for both single and multiple sources in 2D to test the possibility of inferring AMS from magnetic data. The parameter estimation results show that the grid search is valid for recovering the information of AMS and capable of identifying different magnetic units in such cases. To assess the error during the estimation process, I perform repeated tests over one model and one set of data with different noise realization and quantify the Root-Mean-Square error (RMSE). The results indicates that, although the recovered parameters are sensitive to the noise, the RMSE of the recovered results with the assumption of the anisotropic model is still lower than that when inverting for isotropic source when data were produced by an anisotropic source.

I conclude my thesis in Chapter 5 with a summary of the calculation and the behavior of total-field anomaly in the presence of AMS. Then, I discuss the effectiveness of structural (dip) parameter estimation based on total-field anomaly data. Finally, I propose a few
possible research extensions and the potential scenarios that require further development of AMS in the geophysical study.
CHAPTER 2
OVERVIEW OF ANISOTROPY MAGNETIC SUSCEPTIBILITY TENSOR

The AMS is highly sensitive to geological structure in the presence of magnetic minerals, and a small perturbation can affect its properties. In order to use AMS in geophysical data interpretation, it is necessary to understand the fundamental mathematical expression of the AMS, its quantitative analysis, and how it can affect the induced magnetic field.

This chapter begins with the discussion of mathematical expression of AMS based on the laboratorial measurement results on oriented samples. A few parameters are introduced to quantitatively describing the characteristics of the AMS. I then briefly review the Euler’s rotation theory and validate the approach for obtaining the rotation matrix without the recovery of three Euler angles by a series of tests on a single prism model in 3D for both isotropic and anisotropic medium with randomly generated parameter values. The results form this chapter establish the equivalence between the rotation matrices constructed by different approaches and provide us a simpler way to model AMS responses.

2.1 Fundamental Mathematics for Anisotropic Magnetic Susceptibility

The induced magnetization of rock is mathematically expressed by the product of the magnetic susceptibility and inducing field,

\[ \vec{M} = \kappa \cdot \frac{\vec{B}_0}{\mu_0} = \kappa \cdot \vec{H}_0, \]  

(2.1)

where \( \vec{M} \) is the induced magnetization, \( \vec{B}_0 \) the magnetic flux density of the inducing field, \( \mu_0 \) the magnetic permeability in the vacuum, and \( \vec{H}_0 \) the inducing magnetic field. Mathematically, the magnetic susceptibility can be expressed as a 3 × 3 symmetric second-rank tensor:

\[ \kappa = \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} \\ \kappa_{21} & \kappa_{22} & \kappa_{23} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix}, \]  

(2.2)
where $\kappa$ is the symmetrical full magnetic susceptibility tensor. The non-diagonal elements are non-zero in general. In a Cartesian coordinate system that is aligned with the principal directions, the magnetic susceptibility tensor becomes diagonal:

$$\kappa' = \begin{pmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_2 & 0 \\ 0 & 0 & \kappa_3 \end{pmatrix},$$

(2.3)

where $\kappa'$ is the principal susceptibility tensor, and $\kappa_1$, $\kappa_2$, and $\kappa_3$ the principal susceptibilities ($\kappa_1 \geq \kappa_2 \geq \kappa_3$). The principal directions, $\hat{p}_1$, $\hat{p}_2$ and $\hat{p}_3$ are the orientations of principal susceptibilities (Winkler et al. [2]). In practice, three principal susceptibilities are defined in the tilted local coordinate system whose axis directions are parallel to three principal directions, while principal directions are defined in the user-defined coordinate system.

Geometrically, the anisotropy can be represented by an magnitude ellipsoid (Figure 2.1) whose semi-axes’ lengths are proportional to the magnitude of principal susceptibility, and axes’ orientations are parallel to principal directions (Heij et al. [18]).

![Figure 2.1: Geometric representation of anisotropy in 3D by an ellipsoid. The semi-axis lengths are proportional to the magnitude of corresponding principal susceptibilities, and the orientation of axes are parallel to principal directions.](image)

Figure 2.1: Geometric representation of anisotropy in 3D by an ellipsoid. The semi-axis lengths are proportional to the magnitude of corresponding principal susceptibilities, and the orientation of axes are parallel to principal directions.
When $\kappa_1 = \kappa_2 = \kappa_3$, the magnetic susceptibility is isotropic and can be geometrically represented as a sphere. When anisotropy is present, the AMS ellipsoid has a prolate shape when $\kappa_1 > \kappa_2 \approx \kappa_3$, an oblate shape when $\kappa_1 \approx \kappa_2 > \kappa_3$, and tri-axial when $\kappa_1 > \kappa_2 > \kappa_3$.

In practice, although the coordinate system may not be aligned with the principal directions, the three principal susceptibilities can be measured on oriented samples. For laboratory measurements, the collected samples should be free of weathering and other chemical processes that may have altered its composition and AMS. Before collection, the in-situ orientation of the rock sample should be marked on its surface and documented explicitly, and should be presented as its natural orientation during measurements (Dubey [19]). The directional susceptibility is measured under a low field in at least six directions (Tarling and Hrouda [20]). There are two common instruments for measuring AMS: A.C. bridge and balance-transformer equipment (Stephenson and De Sa [21], Christie and Symons [22]). Both of them measure the difference in inductance of the measuring coil before and after the insertion of the rock sample. Before the measurement, calibration must be done with samples having known magnetic susceptibilities.

When three principal susceptibilities are obtained, statistical analyses can be applied to quantitatively describe the shape of the AMS ellipsoid. There are many ways to define the parameters that describe the shape of the AMS ellipsoid. The effective, or bulk susceptibility, is used to approximate the overall susceptibility intensity of an anisotropic medium:

$$\kappa_{\text{effect}} = \frac{|\vec{M}|}{|\vec{H}_0|},$$

(2.4)

where $|\vec{M}|$ is intensity of induced magnetization, and $|\vec{H}_0|$ the magnitude of inducing field.

Given that the magnetization is linearly related with magnetic susceptibility and inducing field for most geological scenarios, the intensity of induced magnetization is directly proportional to the susceptibility. Thus, if I have the same effective susceptibilities, I should have the resultant anomalous magnetic fields with the same amplitude. In this research, I shall use this property in the comparison between the total-field anomalies generated by
isotropic and anisotropic media.

Other parameters are also useful in geophysical interpretation. To quantitatively describe the AMS ellipsoid by the difference between the longest and shortest axis, we use the degree of anisotropy (Nagata [23]):

\[ P_2 = \frac{\kappa_1}{\kappa_3}. \]  

(2.5)

Geometrically, the anisotropy degree describes the eccentricity of AMS ellipsoid, while mathematically, the anisotropy degree describes how strongly anisotropic the medium is.

### 2.2 Construction of Rotation Matrix

The preferred orientation of magnetic fabric is formed during formation and altered during subsequent deformation, and the principal axes are in general not parallel to the axes of the user coordinate system. To perform forward modeling, we need to rotate the AMS tensor to the user coordinate system.

The common approach to define a rotation matrix is using three Euler angles \((\phi, \theta, \psi)\) and multiplying three rotation matrices defined about each axis. However, three Euler angles are not measured in laboratorial tests, and the only directional information available is the principal directions. To fully utilize the laboratory test results and avoid the calculation to recover three Euler angles, I need to establish the relationship between vectors representing principal directions and the rotation matrix.

I designate a user-defined coordinate system in advance. In most cases the user-defined coordinate system is a right-hand coordinate system with \(\hat{x}\) pointing to the north and \(\hat{z}\) pointing vertically down. It can be expressed mathematically by three column unit vectors:

\[
\begin{bmatrix}
\hat{x} \\
\hat{y} \\
\hat{z}
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]  

(2.6)

Given an oriented rock sample, three principal directions can be measured under the user-defined coordinate system by three unit vectors: \(\hat{p}_1, \hat{p}_2,\) and \(\hat{p}_3\), which define the the local coordinate directions.
2.2.1 Rotation by Euler Angles

To rotate a tensor from the local coordinate system with tilted axes to the user coordinate system, I construct a rotation matrix $R_{ep}$ based on Euler parameters and begin with calculating the cross product between the $\hat{x}$ and $\hat{p}_1$ axes (Mutze [24]). Take the $\hat{x}$-axis for instance:

$$\vec{s} = \hat{x} \times \hat{p}_1,$$  

where $\vec{s}$ is the Euler axis. Then I determine the rotation angle between $\hat{x}$ and $\hat{p}_1$ axes:

$$\varphi = \arctan \left( \frac{\|\vec{s}\|}{\hat{x} \cdot \hat{p}_1} \right),$$  

where $\varphi$ is the rotation angle about Euler axis from $\hat{p}_1$ to $\hat{x}$. Now I have all the components to compute four Euler parameters:

$$a = \cos \left( \frac{\varphi}{2} \right);$$  

$$(b, c, d) = \sin \left( \frac{\varphi}{2} \right) \vec{e} = \vec{q},$$

where $a$, $b$, $c$, and $d$ are four Euler parameters. The $\vec{e}$ is the unit vector along Euler axis direction,

$$\vec{e} = \frac{\vec{s}}{\|\vec{s}\|} = (e_1, e_2, e_3).$$

There is a relationship between four parameters:

$$a^2 + b^2 + c^2 + d^2 = 1.$$  

Moreover, they form a unit quaternion in scalar-vector representation (Mutze [25]):

$$Q = (a, \vec{q}) = a + bi + cj + dk,$$

where the $i$, $j$, and $k$ represent three Cartesian axes, and the scalar quantity, $a$, indicates the rotation angle about the rotation axis.
The rotation of a quaternion \( \mathbf{p} \) by another quaternion \( \mathbf{Q} \) can be expressed by evaluating the conjugate of \( \mathbf{p} \) by \( \mathbf{Q} \) (Spring [26]):

\[
\mathbf{p}' = \mathbf{QpQ}^{-1},
\]  

(2.14)

where \( \mathbf{p} \) is the original vector in the form of quaternion, or a point in 3D space if it is pure quaternion (imaginary part only), and \( \mathbf{Q} \) the rotation quaternion. For unit quaternion, the reciprocal of \( \mathbf{Q} \) is its conjugate:

\[
\mathbf{Q}^{-1} = \bar{\mathbf{Q}} = a - bi - cj - dk
\]  

(2.15)

Once the four Euler parameters are obtained, three Euler angles can be derived from the conversions. Blanco [27] provides one conversion to recover the yaw, pitch and roll angles:

\[
\begin{bmatrix}
\phi \\
\theta \\
\psi
\end{bmatrix} =
\begin{bmatrix}
\arctan \frac{2(ab+cd)}{1-2(b^2+c^2)} \\
\arcsin \left(2(ac-bd)\right) \\
\arctan \frac{2(ad+bc)}{1-2(c^2+d^2)}
\end{bmatrix}.
\]  

(2.16)

With three Euler angles having been calculated, the rotation matrix can be defined by three sub-rotations about three coordinate axes. However, we can also obtain the rotation matrix directly from these four Euler parameters calculated through Equation 2.7 to 2.10. The expression for the rotation matrix based on the Euler parameters is (Breckenridge [28]):

\[
\mathbf{R}_{\text{ep}} = \begin{bmatrix}
2(a^2 + b^2) - 1 & 2(bc - ad) & 2(bd + ac) \\
2(bc + ad) & 2(a^2 + c^2) - 1 & 2(cd - ab) \\
2(bd - ac) & 2(cd + ab) & 2(a^2 + d^2) - 1
\end{bmatrix}.
\]  

(2.17)

Theoretically, the components in \( \mathbf{R}_{\text{ep}} \) can be calculated using Euler parameters in only one direction. However, all three directions are recommended to be involved in practice for accuracy:

\[
\mathbf{R}_{\text{ep}} = \begin{bmatrix}
2(a_x^2 + b_x^2) - 1 & 2(b_xc_y - a_yd_y) & 2(b_xd_z + a_zc_z) \\
2(b_xc_y + a_xd_x) & 2(a_y^2 + c_y^2) - 1 & 2(c_yd_z - a_zb_z) \\
2(b_xd_x - a_xc_x) & 2(c_yd_y + a_yb_y) & 2(a_z^2 + d_z^2) - 1
\end{bmatrix}.
\]  

(2.18)

The approach in Equation 2.18 should be taken to avoid difficulties in scenarios such as when \( \hat{x} \parallel \hat{x}' \), \( \hat{y} \parallel \hat{y}' \), or \( \hat{z} \parallel \hat{z}' \) in which construction based on one direction may leave the other two axes rotated to incorrect directions.
2.2.2 Correspondence of the rotation matrix with principal directions

After the rotation matrix $R_{ep}$ is obtained, the principal susceptibility tensor $\kappa'$ can be rotated from the local coordinate system to the user-defined coordinate system by the tensor rotation equation (Lanczos [29]),

$$\kappa = R_{ep}\kappa' R_{ep}^T,$$

(2.19)

For Equation 2.19, an interesting relationship can be obtained. Let the eigenvalue decomposition of tensor $\kappa$ be:

$$\kappa = V\Lambda V^T,$$

(2.20)

where $V$ is the eigenvector matrix, and $\Lambda$ the diagonal eigenvalue matrix. From numerical experiments, I note that three principal susceptibilities are preserved in the $\Lambda$, and rotation vectors $r$ in matrix representation can be found in the eigenvectors, $V$. Although the order of the eigenvectors, $v_i$, may differ from that in rotation matrix $R_{ep}$, they are consistent with the order of eigenvalues. By reorganizing the eigenvalues in descending order in the form of a diagonal matrix, the following relation can be obtained:

$$\Lambda = \kappa'.$$

(2.21)

Since the eigenvalue decomposition is unique (Schütze et al. [30]), the corresponding rotation vectors can be found in eigenvectors as well:

$$\vec{v}_i = \pm \vec{r}_i.$$  

(2.22)

The negative sign is required when two vectors are parallel but in the opposite direction. However, it is important to note that the opposite in directions will not affect the integrity of tensor rotation as rotate $180^\circ$ twice will yield the rotated object back to its original state.

Given the correspondence above, combined with the relationship between eigenvectors and principal directions we discuss in Section 2.1, I can conclude that the vectors indicating principal directions are the vectors I need to construct a rotation matrix:

$$\begin{bmatrix} \hat{p}_1, \hat{p}_2, \hat{p}_3 \end{bmatrix} = \begin{bmatrix} \vec{r}_x, \vec{r}_y, \vec{r}_z \end{bmatrix} = R_{ep}.$$  

(2.23)
2.3 Validation of Rotation Matrix

To test the approached for constructing the rotation matrix, I designed a magnetic prism model and calculated its magnetic anomaly from varying AMS. The inducing field has a 60° inclination and 25° declination. The observation grid extends uniformly from -400 m to 400 m with a 25-m interval in both x- and y-direction on the x-y plane. The prism is located below the center of the observation grid and buried 100 m below the surface. The dimension of the prism is 300 m in both N-S and W-E direction, and 50 m thick in the vertical direction.

![Figure 2.2: Synthetic model designed to test the method for calculating the rotation matrix. The shaded cuboid region represents the magnetic prism. The observation grid is from -400 m to 400 m with a 25-m interval on the x-y plane. The source is a 300 m × 300 m × 50 m prism buried 100 m below the center of the observation grid.](image)

For the anisotropic prism, the principal directions are randomly generated. In this case, the principal directions are expressed by three column vectors and combined in the form of a matrix:

\[
\begin{bmatrix}
\hat{p}_1, \hat{p}_2, \hat{p}_3
\end{bmatrix} = \begin{bmatrix}
0.134430893 & -0.990268069 & 0.03602065 \\
0.25881905 & 0 & -0.96592583 \\
0.9565255 & 0.139173101 & 0.25630024
\end{bmatrix}, \quad (2.24)
\]
where \( \hat{p}_1 \) points to the north-east and downward, \( \hat{p}_2 \) to the south sub-horizontally, and \( \hat{p}_3 \) to the west sub-horizontally. Since the isotropic medium generates magnetization in all directions equally, there is no need to assign principal directions or perform rotation for an isotropic medium. Figure 2.3 visualizes the principal directions for anisotropic medium and the inducing field direction.

![Figure 2.3: Illustration of principal directions (blue) and inducing field direction (red).](image)

To examine the differences between total-field anomalies produced respectively by an isotropic and anisotropic prism \( kk \), I choose two sets of different susceptibilities. The isotropic magnetic susceptibility is given by:

\[
\kappa_i' = \begin{bmatrix}
0.083 & 0 & 0 \\
0 & 0.083 & 0 \\
0 & 0 & 0.083 \\
\end{bmatrix},
\]  

(2.25)
and anisotropic magnetic susceptibility by:

\[
\kappa'_a = \begin{bmatrix}
0.085 & 0 & 0 \\
0 & 0.065 & 0 \\
0 & 0 & 0.045
\end{bmatrix}.
\] (2.26)

To compare the two cases on a similar basis, I choose the isotropic susceptibility to be equal to the effective susceptibility of anisotropic susceptibility tensor. Given the principal directions in Equation 2.24, the rotation matrix can be calculated by Euler parameters through Equations 2.7 to 2.10:

\[
R = \begin{bmatrix}
0.134 & -0.99 & 0.036 \\
0.259 & 0 & -0.966 \\
0.957 & 0.139 & 0.256
\end{bmatrix}.
\] (2.27)

After applying the rotation matrix to the principal susceptibility tensor to rotate it from the tilted local coordinate system to the user coordinate system, I have the full susceptibility tensor defined in the user coordinate system:

\[
\kappa_a = R\kappa'_a R^T = \begin{bmatrix}
0.065 & 0.001 & 0.002 \\
0.001 & 0.048 & 0.01 \\
0.002 & 0.01 & 0.082
\end{bmatrix}.
\] (2.28)

If I apply eigenvalue decomposition to the full susceptibility tensor, the eigenvalues are presented as:

\[
\Lambda = \begin{bmatrix}
0.085 & 0 & 0 \\
0 & 0.065 & 0 \\
0 & 0 & 0.045
\end{bmatrix},
\] (2.29)

which is identical to the principal susceptibility values. The corresponding eigenvectors are:

\[
[\hat{v}_1 \ \hat{v}_2 \ \hat{v}_3] = \begin{bmatrix}
0.134 & 0.99 & 0.036 \\
0.259 & 0 & -0.966 \\
0.957 & -0.139 & 0.256
\end{bmatrix}.
\] (2.30)

When eigenvectors and corresponding eigenvalues are reorganized according to descending order of eigenvalues, the parallel relation between eigenvectors and rotation vectors - either in the same direction or opposite direction - in each direction can be obtained as described in Equation 2.22. In this test example, the angles between the three vector pairs...
are:
\[
\langle \hat{v}_1, r_x \rangle = 0^\circ; \\
\langle \hat{v}_2, r_y \rangle = 180^\circ; \\
\langle \hat{v}_3, r_z \rangle = 0^\circ.
\] (2.31)

By comparing Equation 2.24 and Equation 2.27, the equality can be observed between principal direction and rotation matrix. Thus, I have numerically verified the relation expressed in Equation 2.23.

Figure 2.4: Difference between total-field anomalies with full susceptibility tensor rotated by matrices obtained from principal directions and the calculation of Euler parameters. Since we cannot present the difference of rotation matrices by a figure, this result is obtained by subtracting the total-field anomaly of the isotropic source from that of the anisotropic source. Note that the difference shown on the colorbar is on the order of $10^{-13}$. This difference is small enough that we believe the two rotations are equal as they yield the same result.

Figure 2.4 shows the difference between the magnetic responses with full susceptibility tensor rotated by matrices obtained from principal directions and the calculation of Euler parameters. In this test, I keep all other conditions identical and only change the way of constructing rotation matrixes. The difference in this test has an order of $10^{-13}$. According to this low difference, we believe that the rotation matrices constructed by those two paths are the same.
Figure 2.5: Total-field anomaly generated by different sources and their differences. Inducing fields are the same for all cases, as the inclination is 60° and declination 25°. (a) Total-field anomaly generated by isotropic source; (b) Total-field anomaly generated by anisotropic source; (c) Difference between two anomalies. In three figures, the pink line indicates the inducing field declination, and the yellow line in the third figure marks the trend of anomaly produced by the anisotropic medium (max-min values).

With the rotation matrix established, I next examine the difference in total-field anomalies produced by a simple prism with isotropic susceptibility and AMS. Figure 2.5 shows the test results incorporating rotated susceptibility tensors into this model. Figure 2.5(a) shows the response from the isotropic medium, Figure 2.5(b) shows the response from the anisotropic medium. In this test, the magnetic anomaly is caused purely by the induced magnetization, and there is no remanence or self-demagnetization involved. The pattern of total-field anomalies in two cases are similar as both anomalies have the same range and...
their patterns, in general, parallel to the declination direction. However, when we compare them by taking the difference, the change stands out. Figure 2.5(c) shows the difference between these two cases by taking the difference. The difference shows that the anomaly produced by the anisotropic prism is changed and its pattern is moved away from where the anomaly of the isotropic source is located, and the maximum difference is 15 nT. Although there is no noise added in this test, we assume that the noise of modern magnetic data is 2 to 5 nT to simulate the practical scenario. In this test, the difference caused by AMS is well above the common noise level, which means that the AMS has a detectable signal.

2.4 Summary

In this chapter, I reviewed both the mathematical and geometrical expression of AMS, along with statistical parameters of laboratory measurements for the quantitative description of AMS. The relationship between measured vectors representing principal directions and the rotation matrix was established so that the principal directions can be used directly to construct a rotation matrix for tensor rotation from the local coordinate system to the user coordinate system. This equality will ensure us to directly use the laboratory measurements and avoid the need for calculating Euler angles. The rotation is needed to calculate the forward response generated by the anisotropic medium.

A series of forward calculations over a simple prism model were carried out. The results further showed that the rotation matrix constructed by the principal direction is valid in the calculation of the magnetic response of an anisotropic medium. The difference between the total-field anomaly produced by prisms with isotropic susceptibility and AMS were compared. With the assumption that the variation of noise in magnetic data is within 5 nT, we found that the AMS signal can be well above the noise level. Thus, we believe that the signal produced by the AMS is detectable. These conclusions paved the road for me to have a more detailed discussion on the influence of AMS on the magnetic data in Chapter 3.
CHAPTER 3
UNDERSTANDING THE EFFECTS OF ANISOTROPIC MAGNETIC SUSCEPTIBILITY ON MAGNETIC ANOMALIES

Forward modeling is crucial to understanding how AMS affects magnetic responses and also a key component in inversion. In this chapter, I begin with a discussion of calculation process for total-field anomaly generated by a magnetic source. Then, I forward model the total-field anomalies produced by the sources with a different degrees of anisotropy and examine the differences between them. For all 2D and 3D cases modeled, there is no remanence or self-demagnetization effect involved. Beyond the forward results, I compare the directions of the inducing field and the induced magnetization that are affected by AMS, where the AMS varies as a function of anisotropy degree. The modeling results reveal how the magnetic response changes in the presence of the anisotropy. While the angular difference between the inducing field and induced magnetization provides guidance for data processing.

3.1 General Calculation of Total-field Anomaly

When principal susceptibilities and principal directions are obtained from sample measurements, I first rotate the principal susceptibility tensor to the coordinate system under which the observation grid is defined prior to calculating the magnetic response. The rotation between tensors in different coordinate systems is completed by the tensor rotation I discussed in Equation 2.19:

$$\kappa = R\kappa'R^T.$$  \hfill (3.1)

According to the relationship discussed in Section 2.2, during the forward modeling, I construct the rotation matrix directly from the measured principal directions:

$$R = \left[ \hat{p}_1, \hat{p}_2, \hat{p}_3 \right].$$  \hfill (3.2)
After the full susceptibility tensor is obtained, I then define the orientation of the background field, $\hat{B}_0$, from given inclination and declination, and the elements of induced magnetization, $\vec{M}$, through Equation 2.1. Finally, the total-field anomaly of one source with a uniform magnetization is calculated by Green’s function:

$$\Delta T = \frac{\mu_0}{4\pi} \hat{B}_0^T \Gamma \vec{M}$$

(3.3)

where $\Delta T$ is the total-field anomaly in nanotesla (nT), $\hat{B}_0$ the inducing field direction, and $\Gamma$, the Green’s tensor containing the geometrical information between the boundary of the source and observation locations.

The Green’s tensor is defined by the equation:

$$\Gamma = \iiint_{\Delta V} \nabla \nabla^T \frac{1}{|\vec{r} - \vec{r}'|} \, dv,$$

(3.4)

where $\Delta V$ is the region occupied by the source, and $|\vec{r} - \vec{r}'|$ the distance between observation location and a point inside the source. In the Cartesian coordinate system, this distance can be expressed as:

$$|\vec{r} - \vec{r}'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}.$$  

(3.5)

In this expression, the $x'$, $y'$, and $z'$ are the source coordinates. Thus, the magnetic sources in our modeling are organized as a set of small squares in 2D or cubes in 3D.

For multiple-source problems, the total-field anomaly is given by the summation of anomalies from individual sources:

$$\Delta T = \sum_{i=1}^{M} \Delta T_i,$$

(3.6)

where $M$ is the number of source bodies, and $i$ is the index of the body being calculated.

### 3.2 Influence of AMS on magnetic data

For 2D forward modeling, I assume a background field intensity of 42,000 nT and an inclination of 90°. The magnetic dyke is placed 50 m below the ground surface with a 45° dip angle and depth extent of 500 m. The observation grid is 50 m above the ground surface
with observation stations from -1,000 m to 1,000 m at a 10-m interval in x-direction. To avoid self-demagnetization issue in this study, $\kappa_1$ is set to 0.01, and $\kappa_3$ varies from 0.01 to 0.001 according to a set of anisotropy degrees, $P_2$. The principal directions are fixed for all cases and linked with dip direction, with $\hat{p}_1$ being parallel to the dip direction, and $\hat{p}_3$ perpendicular to the dip direction. This scenario represents the case where one principal direction is parallel to the bedding of a layer.

Figure 3.1 shows the configurations of the model and the anomalies generated by the dykes with AMS having different anisotropic degrees. When anisotropy degree is 1, the medium is isotropic, and the direction of the resultant magnetization is the same as that of the inducing field. As anisotropy degree increases, the orientation of induced magnetization is rotated towards the dyke’s dip direction, which is the maximum principal direction. Along with the changing direction of the induced magnetization, the pattern of the magnetic anomaly also changes. With the increase of anisotropy degree, the magnitude of the responses is weakened as expected. The decrease in the magnitude of total-field anomaly is caused by a decrease in effective susceptibility due to the decreasing $\kappa_3$. What is important is the observation that the zero crossing points of the magnetic anomaly have also changed. These results show that even with a low susceptibility value, the induced magnetization direction can be deviated significantly from the inducing field direction solely by the presence of AMS.
Figure 3.1: Total-field anomaly produced by a dyke with AMS of different anisotropy degrees: a) The source is a 45° dipping dyke located 50 m below the ground surface. The red arrow shows the direction of the inducing field, and green arrows show the principal directions of AMS. b) The total-field magnetic anomalies generated by the dykes with different anisotropy degrees.

Based on the results shown in Figure 3.1, I examine different anisotropy degrees and inducing field directions to compare the differences between the directions of the inducing field and induced magnetization. At this stage, the direction of the induced magnetization is controlled by the inclination of the inducing field and the anisotropy degrees. The result is shown in Figure 3.2.

For isotropic magnetic medium, when the difference between inducing field and induced magnetization is less than 15°, the recovered model from a generalized inversion assuming that induced magnetization direction is close to that of the inducing-field direction can still recover the value and distribution of susceptibility within tolerable error level (Shearer [31]). For this reason, I divide the contour map in Figure 3.2 with a 15° interval. From this result, I observed that if the anisotropy degree is less than 1.7, the generalized inversion based on the assumption that the magnetic medium is isotropic will still be valid in the recovery of
Figure 3.2: Angular differences between the inducing field and induced magnetization. The contour lines show the difference between inducing field direction and orientation of induced magnetization. The dashed line to the left marks the threshold in anisotropy degree below which there is no noticeable effect of AMS.
the target since the deviation in directions is less than 15°. In such cases, we may not be able to extract information of anisotropy, and such generalized inversion will not be able to tell whether the source is isotropic or anisotropic. However, this situation may be different with a parametric inversion.

When anisotropy degree is greater than 1.7, the differences between two directions will exceed the 15° threshold. In such situations, the inversion assuming that the direction of induced magnetization is parallel to the inducing field direction is no longer accurate enough for recovering both the value and distribution of magnetic susceptibility. One may need to formulate a new inversion or use tools developed for inversion with remanent magnetization. For data from low-geological-complexity areas, parameter estimation may be preferable when data is affected by AMS. For more complicated geology, modified generalized inversion may be used to recover different magnetic sources.

As expected, the AMS can rotate the direction of induced magnetization in 3D as well. Although in 3D cases, the complexity of susceptibility tensor is increased due to the introduction of $\kappa_2$, my evaluations indicate that the 1.7 threshold on the anisotropy degree is still valid. With the experience from 2D, I further examine the differences between the direction of inducing field and induced magnetization orientation in 3D. In this test, the principal directions are randomly generated, and the induced magnetization is affected by both inclination and declination of the inducing field and the anisotropy degree. The result is shown by the 3D plot in Figure 3.3.

In Figure 3.3, each point represents one inducing direction-anisotropy degree pair, and the color indicates the angular differences between the inducing field and induced magnetization. All the points with difference less than 15° are removed, and the 1.7 threshold on the anisotropy degree is marked by a light gray plane at the bottom. Below the gray plane, there is no point shown, which means in 3D, when anisotropy degree is less than 1.7, the deviation in the orientation of induced magnetization caused by the AMS is still tolerable for generalized inversion, and the recovered result could be valid for interpretation.
Figure 3.3: The angular difference between the inducing field and induced magnetization in 3D. Results are controlled by the inclination and declination of the inducing field and anisotropy degree of the source, and presented as dots only with an angular difference greater than 15°. The color of scattered dots shows the difference between inducing field direction and orientation of induced magnetization. The light gray plane at the bottom of the space marks the threshold in anisotropy degree, 1.7, below which there is no noticeable effect of AMS.

When principal directions are known, the anisotropy degree is the only controlling factor on the deviation of the direction of induced magnetization. This can be explained in a simple case. Assume we have a diagonal susceptibility tensor, which means the principal axes are parallel to the right-handed coordinate system. It can be expressed in the form of anisotropy degree:

\[
\kappa = \kappa' = \kappa_3 \begin{bmatrix} P_2 & 0 & 0 \\ 0 & \kappa_2/\kappa_3 & 0 \\ 0 & 0 & 1 \end{bmatrix},
\] (3.7)

and inducing field \( \vec{H}_0 \) which is presented by the vector:

\[
\vec{H}_0 = [ \ H_x, \ H_y, \ H_z \ ]^T.
\] (3.8)
The induced magnetization can be calculated by Equation 2.1 as:

\[ \vec{M} = \kappa_3 \begin{bmatrix} P_2 \cdot H_x \\
\frac{\kappa_2}{\kappa_3} \cdot H_y \\
H_z \end{bmatrix}. \] (3.9)

Equation 3.9 shows that when AMS is present, the difference between the directions of the inducing field and induced magnetization is solely affected by the anisotropy degree, \( P_2 \), and independent of the magnitudes of susceptibilities. In forward modeling, the components of the inducing field are known, and three principal susceptibilities are invariant when values are set. Thus, I can control the angular deviation by setting different anisotropy degrees. In inverse problems, components of inducing field, or the intensity of inducing field along with inclination and declination, can be calculated by International Geomagnetic Reference Field (IGRF) model (Thébault et al. [32]), and the range of susceptibility can be constrained by prior information. Hence, the approximation of true susceptibilities can be inferred from an inverted anisotropy degree.

### 3.3 Distinguishing from remanent and self-demagnetization effect

Like AMS, the remanent magnetization, and the self-demagnetization effect can also affect both the magnitude and the direction of the total magnetization. However, the origin and the manner by which total magnetization is changed in these two cases are different from that of the AMS. Although the influence from multiple factors can be coupled together, it is necessary to recognize the differences between them.

The self-demagnetization effect, remanent magnetization, and the AMS have different origins. The remanent magnetization, commonly found in ferromagnetic minerals, is normally caused by an external field aligning the magnetic fabric during the diagenetic process (primary remanence) or weathering process (secondary remanence) (Reynolds [33]). In contrast, the root of AMS is connected more to the direct physical contact caused by geological movement. At the macro level, the AMS originates from the alignment of the magnetic fabric caused by the external forces, including compression, shear, and extension. While at the
micro level, there are two different causes of AMS. The first kind is crystalline anisotropy, which is an intrinsic property and depends on the action of lattice force; whereas the second is the shape anisotropy that is controlled by the alignment of ferromagnetic grains (Hrouda [34], Dubey [19]). Self-demagnetization, or shape demagnetization, is caused by the shape of the high-susceptibility source body (Guo et al. [35]). Compared with AMS and remanence, self-demagnetization is ubiquitous in any magnetized body. However, unlike the distinction between the AMS and the remanence, the self-demagnetization effect has a more frequent connection to the other two. Shape is one factor, but its magnetization can be caused by both remanence and the AMS. The self-demagnetization effect can arise in highly magnetized isotropic medium (remanence) or happens with the help of the shape anisotropy when a high intrinsic susceptibility presents (Winkler et al. [2]).

Since the remanence is caused by the Earth’s field during the rock genesis, it usually reflects the direction of the Earth’s magnetic field at the time of formation. By comparing remanence from different stratum, we can recover the geomagnetic history. Self-demagnetization effect simply generates an induced field inclining to the direction of the longer axis of the rock body, and hence change the direction of total magnetization. So it can work as a strike indicator. The current usage of AMS is listed in Chapter 1, and there will be no more detailed discussion regarding this topic.

3.4 Summary

In this chapter, I applied tensor rotation calculation with the rotation matrix based on principal directions to the forward modeling of total-field anomaly produced by a source with anisotropic susceptibility. By comparing the data sets generated by a magnetic medium with different anisotropy degrees, I found that in the absence of remanent magnetization and self-demagnetization effect, the direction of induced magnetization can be rotated and the pattern of resultant total-field anomaly changed solely by the anisotropy even with a low susceptibility value. When anisotropy is present, the induced magnetization direction will be rotated toward the orientation of maximum principal direction. Such a shift in direction
is only related to the anisotropy degree, and is independent of the magnitude of magnetic susceptibility.

In extended tests, I found that the 1.7 in anisotropy degree is a threshold when choosing the processing path whether the data should be treated as from isotropic or anisotropic medium. Below this boundary, the result from generalized inversion treating the source as isotropic will be still valid for interpretation, while above this limit, treating the source as an isotropic medium will result in large errors.

Based on the results from this chapter, when dealing with magnetic medium with anisotropy degree higher than 1.7, parameter estimation may be preferable in recovering the information of anisotropy from magnetic data. I shall discuss this idea in the next chapter.
CHAPTER 4
PARAMETER ESTIMATION FOR ANISOTROPIC MAGNETIC SUSCEPTIBILITY

In the preceding chapter, I discussed the effect of AMS on the magnetic data, and observed that when the anisotropy degree is greater than 1.7, there is a possibility that the information of anisotropy could be extracted. Based on that observation, I run parametric inversions to recover the AMS parameters of the magnetic unit. I begin with a brief discussion of the method of parameter estimation involved in this research. Before conducting the numerical tests, I examine the necessity of parameter estimation for the data produced by anisotropy medium through the comparison between the results from inversions with different assumptions on the same data set. I then perform the parameter estimation for 2D models with a single body and multiple bodies to examine the hypothesis if it is possible to extract structural information that linked with the presence of the AMS from the geophysical data. At the end of this chapter, I present the idea of the recovering structural information from geophysical data through AMS without the help of direct laboratorial measurement that commonly used in structural analyses.

4.1 Method of Parameter Estimation

In this research, I invert synthetic total-field anomaly data for four parameters: dip angle of the dyke, maximum principal direction, and two principal susceptibilities. The synthetic total-field anomaly datasets are simulated over 2D dyke models with different dip angles and AMS. The goal of parameter estimation is to find parameters that best fit the observed data. As the number of observed data is far more than the number of parameters need to be solved, we are dealing with overdetermined problems. Thus, the inversion is formulated as a minimization problem. The data misfit function to be minimized is:

\[
\phi_d = \sum_{i=1}^{N} \left( \frac{d_{\text{obs}}^i - d_{\text{pre}}^i}{\varepsilon_i} \right)^2,
\]

(4.1)
where $d_{i}^{\text{obs}}$ is the observed data, $d_{i}^{\text{pre}}$ the predicted data obtained via the forward calculation, and $\varepsilon_{i}$ the estimated standard deviation of the noise in the data. The predicted data is calculated by the forward modeling shown in Equation 3.3. The best-fit parameters are normally found by searching through sets of possible parameters for the minimum data misfit. Theoretically, the best-fit parameters will produce the lowest misfit, and the reproduced data could replicate the true data.

4.2 Necessity of Parameter Estimation Using AMS Models

As I discussed in the preceding chapter, the information of anisotropy is not extractable unless the anisotropy degree is sufficiently large (e.g., greater than 1.7). In such scenarios, the inversions assuming that the magnetic medium is isotropic is no longer valid. Before I move on to the numerical tests on recovering anisotropy parameters, it is important to examine the necessity of inversion for anisotropic parameters in data interpretation by testing the error when treating the data from the anisotropic medium as if they were from the isotropic medium.

The left panel in Figure 4.1 shows the observed data produced by a synthetic dyke model. I use the same model shown in Figure 3.1, but with a 50-m observation interval. With this interval, the total number of observations is 41 in these tests. I choose a true mode with $\kappa_{1}$ to 0.01, $\kappa_{3}$ to 0.005, and dip angle of 45°. Thus, the effective susceptibility in this model is 0.0079, and the anisotropy degree is 2. To simulate observed data, I add Gaussian random noise with a zero mean and 1-nT standard deviation to the calculated data. The simulated observed data is shown in the right panel in Figure 4.1.
Figure 4.1: Synthetic model and simulated observed data. a) The cross-section of the 2D dyke model. The black parallelogram indicates the source location. The horizontal line represents the ground surface, and the red arrow is the direction of the inducing field. The green arrows show the principal directions, with maximum susceptibility parallel to the dip direction, and minimum susceptibility pointing normal to the dip. b) Simulated observed data with 1-nT Gaussian noise. The observation grid is in the x-direction from -100 m to 1000 m, with a 50-m station interval.

I perform two tests to examine the error produced by the assumption of the isotropic model when inverting data affected by AMS. In the first test, I treat the data as if they were produced by an isotropic source and only invert for scalar susceptibility value and the dip angle. For this two-parameter problem, I use the grid search to find the minimum data misfit and the corresponding dip and susceptibility.

The recovered parameters and its differences from the true parameters are shown in Figure 4.2. The red “x” marks the minimum data misfit, which is 3154.7. The dashed lines indicate that, based on the recovered parameters, the source has a dip of 62° and an effective susceptibility of 0.011 in SI unit. However, this result is significantly different from the true parameters marked by “+”, which has a true effective susceptibility as 0.0079 in SI unit and a dip angle of 45°. This result implies that if I incorrectly treat the data as being produced
by an isotropic source, I cannot recover the correct source dip and effective susceptibility.

Figure 4.2: Parameter estimation results from treating a data as if they were from the isotropic medium. The color contours show the data misfit $\phi_d$ in $\log_{10}$ scale. The “x” marks the minimum value of data misfit, and two dashed lines mark the corresponding estimated effective susceptibility, 0.011, and estimated dip angle, 62°. The “+” marks the true parameters. The true effective susceptibility is 0.0079, and the true dip angle is 45°. From this map, it is evident that when inverting parameters for an isotropic source based on data produced by the anisotropic source, the recovered parameters cannot describe the true model.

The recovered model and predicted data determined by estimated parameters from inversion for an isotropic source are shown in Figure 4.3, along with the true model and observed data for comparison. In the left panel, the black parallelogram shows the recovered model. The red curve in the right panel shows the difference between the observed and predicted data. Although the recovered model defined by estimated parameters reproduces the main feature in observed total-field anomaly with a similar pattern and overall magnitude, the difference is much greater than the 1-nT noise level and has a maximum value of 15 nT. The recovered model also poorly represents the true model. The locations of top vertices are provided, but they were not being estimated. The recovered dip is much greater than the
true value. This result shows that the inversion assuming an isotropic body cannot recover the true model from the data from the anisotropic source.

Figure 4.3: Comparison between true source and data with recovered source and data. a) Location of the true source (green) and recovered source (black) that described by the estimated parameters. Arrows represent corresponding principal directions; b) Observed data (green), predicted data (black), and the difference between them (red). Although the recovered data has a similar pattern and intensity compared to the true data, the recovered model is significantly different from the true model. Note that the differences between true and recovered data is well above the added noise level (1 nT).

Based on the result from the first test, I perform the second test to show the large errors in estimated parameters and how these errors change as a function of anisotropy degree when estimating isotropic parameters based on the data produced by different anisotropic mediums. The results are shown in Figure 4.4.

The left panel shows the synthetic data sets as a map of anisotropy degree and observation positions. Gaussian noise having a standard deviation of 1 nT has been added. These datasets are generated over a magnetic dyke with a dip angle of 45°. The middle and right panels respectively show the comparison between true and recovered parameters, and how the error changes as a function of anisotropy degree. In these tests, the anisotropy degree
changes linearly with a fixed interval of 0.1. The dashed green line in the middle and right panels mark the 1.7 threshold on anisotropy degree. For the recovered effective susceptibility, $\kappa_{\text{effect}}$, the difference between true and recovered values have already passed above 20% when anisotropy degree is 1.7. The difference in the recovered dip angle is greater than 18°. These test results show that inversions treating the anisotropic target as isotropic will yield large errors and hinder the interpretation.

Figure 4.4: Extended tests over a 45° dyke model with different anisotropic degrees. These tests show the differences between the estimated and true parameters when treating the data produced by the anisotropic medium as being from the isotropic medium. a) The data generated by a dyke model with different anisotropy degree, as indicated by the vertical axis. The magnitude of data decreases with the increasing anisotropy degree because of corresponding decreasing effective susceptibility; b) The true effective susceptibility, estimated effective susceptibility, and their differences; c) The true and estimated dip angle and their differences. The vertical dashed light green line in the middle and the right panels indicate the 1.7 threshold on the anisotropy degree.

With the results from preceding tests, I perform another inversion to show if the error will decrease with the assumption of AMS model. In this test, the data are the same as shown in Figure 4.1, and I invert for two principal susceptibilities and the dip angle. Same as in the first
test, the grid search method is used. The difference between true and recovered dip and the minimum data misfit are shown as a function of anisotropy degree in Figure 4.5. The dashed red vertical line indicates the true anisotropy degree. I observe that, with the assumption of AMS model, the minimum data misfit in this test is 48.1. The corresponding recovered dip and anisotropy degree are close to the true values. When comparing this minimum data misfit value with that from the first inversion, the result in the current inversion is much closer to the number of observation.

The results from this inversion show that the parameter estimation result can be reliable if I invert the data by accounting for the AMS model. The contrast between these two results demonstrate that when inverting the data produced by anisotropic sources, an AMS model is required.

Figure 4.5: Parameter estimation with assumption of AMS model. In both panels, the horizontal-axis represents the anisotropy degree, and the dashed red vertical line indicates the true anisotropy degree based on true principal susceptibilities. The upper panel shows the difference between true and recovered dip. The lower panel shows the minimum data misfit in $\log_{10}$ scale.
4.3 Parameter Estimation for additional 2D Models

Based on the results from Section 4.2, I now proceed to recover the parameters describing the anisotropy magnetic medium with grid search based on synthetic data generated by dyke models. The true model parameters are randomly chosen, and only the location of top vertices of the dyke and the extent in the vertical direction are set as known parameters. The data are calculated on the observation profile from -1000 m to 1000 m along x-direction with a 50-m interval, and zero-mean Gaussian noise with a 1-nT standard deviation is added. In this section, the principal directions are not linked to the dip angle, and tests will try to recover the dip angle, the inclination of maximum principal direction, and the value of two principal susceptibilities. During the inversions, the possible susceptibility values will be searched on a 0.01 interval, and both the dip angle and the inclination of maximum principal direction will be searched with a 0.5° interval.

The first test is on a simple dyke model to see if the parameter estimation can recover the anisotropic susceptibility and the presence of the magnetic dyke. The result is shown in Figure 4.6. In the left panel, the green arrows and parallelogram indicate the true model and principal directions respectively, while the pink arrows and parallelogram are the recovered model and principal directions. In the right panel, the blue graph represents the true data, the orange graph represents the recovered data based on estimated parameters, and the green line represents the difference. The true and recovered parameters are listed in Table 4.1.

Compare to the result shown in Figure 4.3, the major features of the observed data are better reproduced both in shape and magnitude. In general, the recovered parameters approximate the true parameters with an much lower level of errors. The recovered principal susceptibilities are more robust and closer to the true values, while the dip angle and principal direction are more sensitive to the data noise.
Figure 4.6: Simple dyke model test. This test is designed to examine if the parameter estimation can recover the parameters describing the anisotropy and recover the presence of the magnetic dyke. a) The true and recovered model. b) The observed data, predicted data and the difference between them. In this test, the parameters are recovered with tolerable error level. During the estimation, the estimation of recovered principal susceptibilities is more robust and fairly close to the true values, while the dip angle and principal direction are more sensitive to the noise.

Table 4.1: True and Recovered Parameters of One-Body Test

<table>
<thead>
<tr>
<th></th>
<th>Dip (°)</th>
<th>I_p1 (°)</th>
<th>κ'_1 (SI)</th>
<th>κ'_3 (SI)</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Parameters:</td>
<td>86</td>
<td>45</td>
<td>0.02</td>
<td>0.005</td>
<td>4</td>
</tr>
<tr>
<td>Recovered parameters:</td>
<td>91</td>
<td>34</td>
<td>0.02</td>
<td>0.006</td>
<td>3.33</td>
</tr>
</tbody>
</table>

The second test is on a model with two adjacent dykes with different AMS. This test examine whether these two dykes can be identified by different parameters sets recovered by the parameter estimation. The two-dyke model emulates the top of two adjacent dipping layers in a sedimentary basin. The two layers are in contact, have the same thickness, and share the same dip angle. The result is shown in Figure 4.7. We use the same symbols and
line colors as in Figure 4.6. The true and recovered parameters are listed in Table 4.2.

In this test, the recovered parameters for each single body represent the true parameters within an acceptable error level. While for identification of layers, the two layers are reasonably differentiated according to recovered principal directions and principal susceptibilities.

![Figure 4.7: Two-dyke model test. This test is designed to examine if the parameter estimation can recover different parameter sets describing different anisotropic medium, and hence identify different magnetic bodies and recover the presence of the magnetic dykes. a) The green arrows and parallelogram are the true model and principal directions, while the pink arrows and parallelogram are the recovered model and principal directions. b) The blue line is the true data, the orange line is the recovered data based on estimated parameters, and the green line is the difference between them.](image)

<table>
<thead>
<tr>
<th>Table 4.2: True and Recovered Parameters of Two-Body Test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Position</strong></td>
</tr>
<tr>
<td>True Parameters:</td>
</tr>
<tr>
<td>Left body</td>
</tr>
<tr>
<td>Right body</td>
</tr>
<tr>
<td>Recovered Parameters:</td>
</tr>
<tr>
<td>Left body</td>
</tr>
<tr>
<td>Right body</td>
</tr>
</tbody>
</table>
Having obtained the results from the above two tests, I perform another set of tests to assess the uncertainty of parameter estimation in recovering AMS information. Again, I use the model shown in the first test (Figure 4.6) and estimate the parameters fifty times with different noise realizations in the data to assess the influence of data noise on the recovered parameters. The Gaussian noise added to the calculated data has zero mean and 1-nT standard deviation, and fifty different realizations of noise with the same statistics level were added to the estimation process. In this test, four parameters are recovered: dip angle, the inclination of maximum principal direction, and two principal susceptibilities. The uncertainty of each parameter is quantified by calculating the RMSE:

\[
\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (m_{i}^{\text{est}} - m_{i}^{\text{true}})^2}{n}},
\]

where the \( n \) is the total number of realizations of one recovered parameter, \( m_{i}^{\text{est}} \) is recovered parameter, and \( m_{i}^{\text{true}} \) is the true parameter. The RMS errors are listed in Table 4.3.

**Table 4.3: True Parameters and RMSE of Each Parameter Based on 50 Tests**

<table>
<thead>
<tr>
<th></th>
<th>Dip (°)</th>
<th>( I_{\delta 1} (°) )</th>
<th>( \kappa'_{1} (SI) )</th>
<th>( \kappa'_{3} (SI) )</th>
<th>( \kappa_{\text{effective}} (SI) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>True parameter</td>
<td>86</td>
<td>45</td>
<td>0.02</td>
<td>0.005</td>
<td>0.0145</td>
</tr>
<tr>
<td>RMSE</td>
<td>9.362</td>
<td>6.176</td>
<td>0.00385</td>
<td>0.00475</td>
<td>0.00221</td>
</tr>
</tbody>
</table>

Figure 4.8 displays the true model and 50 recovered models. The green arrows and parallelogram are the true model, and the pink arrows and parallelograms are recovered models. When comparing the recovered models and true mode, the recovered principal directions hold their pattern with relatively high consistency, while the recovered dip angles vary around the true angle with a larger range. The RMSEs of two principal susceptibilities are at a similar level.

In this extensive test, the true anisotropy degree is 4. The variation of recovered dip angle is about 9°, and the deviation of recovered effective susceptibility value is less than
16%. When comparing with the results from the second test in Section 4.2, the advantage of assumptions for anisotropy susceptibility is clear. For the same anisotropy degree, when assuming the source is isotropic (Figure 4.4), the difference in the recovered dip is above 45°, while the difference in recovered effective susceptibility is 35%. The error differences in these two tests further show that the correct treatment on data produced by the anisotropic medium is important in reducing the error in interpretation.

Figure 4.8: Extended tests for assessing uncertainty. The true model is shown in green, the recovered models are shown in pink, and the brown line on the top indicates the ground surface. When comparing between the true and recovered models, the recovered principal directions hold their pattern with relatively high consistency, but the recovered dip angle varies around the true angle with a slightly larger range.

### 4.4 Summary

In this Chapter, I discussed the method of parameter estimation and illustrated the necessity of the assumption for anisotropy when dealing with the data produced by AMS. After that, I carried out inversion tests to examine the feasibility and effectiveness of parameter estimation in recovering information describing the occurrence of AMS. In the first test, I
found that the parameter estimation achieved through least-squares solution can quantify the anisotropy model within an acceptable error level while data are contaminated by noise at a level seen in practice. In the second test, the results showed that parameter estimation is capable of recovering the parameter for multiple magnetic bodies and hence identify different units based on recovered AMS parameters.

I performed a set of extended tests to examine the error caused by the realization of estimated noise during the grid search in parameter estimation. Fifty estimations were carried out on a single-body model with the different realizations of added noise, and the errors in recovered parameters were quantified by the RMSE. With the RMSE and comparison between true model and recovered models, I found that the recovered dip angle and principal direction are more sensitive to the noise, while recovered principal susceptibilities vary around the true value in a smaller range of deviation.

Given the simplicity of the objective function, such sensitivity to the noise might be caused by an over-fitting problem that resulted from the lack of regularization in the inversion. Nevertheless, when handling the data affected by AMS, the estimation assuming that the data is produced by anisotropic medium still has a higher accuracy than that assuming the medium is isotropic.
CHAPTER 5
CONCLUSIONS

The direct measurement of AMS has been used in structural geology studies because of the relationship between the geometrical presence of AMS and the external forces applied to the rock during the formation process. However, its application in geophysical data interpretation has not been done. In this research, I rekindled this nearly forgotten but important topic. I assessed the influence of AMS on magnetic anomaly and data interpretation and developed a preliminary interpretation path for total-field anomaly data affected by AMS.

I began with the discussion of the general path in the forward modeling of magnetic response with the influence of AMS. Through mathematical derivation and numerical tests, I showed that the rotation matrix formed directly by principal directions is the same as that calculated from traditional Euler angles. If the laboratory measurements are available when performing the forward modeling, we can use principal directions to construct a rotation matrices directly and avoid the complicated process to determine three Euler angles.

Based on the forward modeling results, I found that even at low susceptibility values, the anisotropy of magnetic susceptibility can still rotate the direction of resultant induced magnetization towards the orientation of maximum principal direction. I further examined the deviation between the inducing field direction and that of the induced magnetization. Results reveal that when anisotropy degree is greater than 1.7, the deviation would be too large. Previous work in remanent magnetization has shown that, in such scenarios, the inversions assuming the induced magnetization parallel to inducing field will fail in recovering susceptibility distribution. Thus, the effect of AMS must be considered in data interpretation when the anisotropy degree greater 1.7.

To examine the possibility of inferring AMS parameters from magnetic data, I adopt parametric inversion for simplicity and examine the influence of AMS in inversion. In para-
metric inversion tests that incorporate known position of the top of the dyke, the results indicate that we can recover the AMS parameters from the total-field anomaly data. For a model with two magnetic dipping layers where each layer has a different AMS, the parameter estimation can be used to differentiate between the two magnetic layers. Tests show that the accuracy of recovery is affected by data noise. The recovered dip angles differ from the true parameters as in any inversions, but the deviation does not exceed a few degrees.

This research describes the procedure for modeling the magnetic response from source with AMS and identifies a threshold on anisotropy degree that indicates under what circumstances we should take the influence of AMS into account in data interpretation. The inversion results show the possibility of recovering AMS parameters directly from magnetic data for characterizing geology without the help from laboratory measurement.

5.1 Future Work

Like the studies in the remanent magnetization inversion in the past decade, the research on the AMS may provide a pathway forward to advance the interpretation techniques for magnetic data. Future work on the inversion of magnetic data may benefit from this research in increasing the reliability of inverted susceptibility model.

It is also possible to use AMS to recover structural information solely from magnetic data. As I discussed in Chapter 1, based on published work, the relationship between the presence of AMS (e.g., principal directions, anisotropy degree) and subsurface structure has been used as a supplementary tool in structural geology studies, and I showed in Chapter 4 that AMS parameters can be inferred from magnetic data. Based on this work, the following research can focus on the development of generalized inversion algorithm to recover the distribution of AMS in tilted magnetized mediums from magnetic data.

However, due to the current understanding of the AMS, this possible application may be limited. One drawback of using AMS for geophysical data interpretation is that its effect can be coupled with the influence of remanent magnetization. The effect of remanent magnetization may be too strong and overshadows the influence of AMS on total-field anomaly.
Thus, in areas with strong remanence, the AMS may not be identified from magnetic data. However, in scenarios such as stratigraphy research over sedimentary areas, the remanence may be weak so that the effect of AMS could be clear enough for data interpretation.
REFERENCES CITED


