NUMERICAL MODELING AND RAY TRACING
OF SPATIAL FREQUENCY MODULATION
FOR IMAGING

by

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ABSTRACT

This thesis presents the first full geometric and physical optics characterization of aberrations and the performance of a simple system that implements SPatIal Frequency modulation for Imaging (SPIFI). SPIFI utilizes a chirped frequency mask that modulates the intensity of light along a single axis at a linear frequency sweep giving each lateral position a unique spatial frequency. By taking the Fourier transformation of the voltage verses time readout of a single element detector position data can be extracted for each point along the modulation axis. The geometric analysis is performed using Fraunhofer diffraction and conjugate imaging planes in a Fourier optics based approach. The ray tracing analysis is performed using the ZEMAX software for configurations involving achromatic lenses, flat field lenses, and a specialized lens array setup. Lastly, a unique SPIFI design setup is presented making use of a reflective scanned SPIFI mask to achieve random access imaging using three distinct spots for simultaneous imaging.
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CHAPTER 1
INTRODUCTION

In traditional wide field imaging techniques the illumination of biological samples result in scattering of light both in the bulk and off of surface features in the sample. The resulting 2D image captured using a CCD or CMOS camera will result in image degradation due to the scattering of light throughout the sample medium. The final image is therefore a combination of this scattered light and the in focus light limiting the utility of the image, especially in thick samples where there is an increased probability for light to be scattered.

In 1957 Marvin Minsky patented the first confocal scanning microscope [1]. The principle underlying a confocal microscope incorporates the restriction of light rays that do not originate from the sample focal plane. As can be seen in Figure 1.1 the illumination light first passes through a pinhole and then is focused with a lens onto the sample. The light that is then transmitted through the sample is focused once again by a lens through another pinhole and finally onto the detector. Light rays that do not originate from the focal volume in the sample will be blocked by the pinhole eliminating the chance for detection.

Confocal microscopy is a method thus devised to reduce image degradation by blocking out-of-focus light through the implementation of a pinhole allowing for a fixed depth of field and with the ability to optically section a sample at controlled depths. The use of confocal microscopy is however limited in its applications as this method predominately relies on single photon fluorescence which has a squared dependence on the incident light intensity. The implementation of confocal microscopy systems use short wavelength sources that contribute to photo damage and exhibit low penetration depths due to the ultraviolet wavelengths of light needed for single photon fluorescence. Additionally, as confocal microscopy is a point source illumination method scanning the incident focus or translation of the sample in the focal plane is required to build up a full 2D image [2].
Figure 1.1: Setup for confocal microscope. The red lines are shown to pass through both pinholes while the blue lines, representing scattered light, does not pass through the final pinhole since it does not originate from the focal plane.

The use of Multiphoton Microscopy in biological laboratories is highly prevalent for its benefits in restricting photo-toxicity to only the focal volume of the sample. Additionally, the use of near infrared wavelength excitation light (1280 nm) results in increased tissue penetration depths compared to visible light (775 nm) used in single photon excitation methods giving characteristic attenuation lengths of 425 \( \mu \text{m} \) at 1280 nm and 232 \( \mu \text{m} \) at 775 nm [3]. Additionally, the scattering of light in media can be roughly approximated by Rayleigh scattering which is inversely proportional to the the fourth power of the wavelength of light, thus for a material with an index of refraction similar at both wavelengths a doubling of the wavelength used for excitation will result in close to 16 times the reduction in off axis scattered light.

The drawbacks of using longer wavelength light for excitation broadens the focal volume in both the axial and transverse directions for the excitation beam according to the Abbe limit; however, using the nonlinear intensity dependence where the emitted signal is related to the power of the non-linearity we are able to achieve higher resolution than with linear
excitation of the fundamental excitation frequency.

The intensity of light can be shown to decay exponentially at increased axial depths in the sample according to the equation

\[ I(z) = I_0 e^{-\frac{z}{\alpha(\lambda)}} \] (1.1)

where \( I_0 \) is the intensity of light at the surface of the sample (where \( z = 0 \)), and \( \alpha(\lambda) \) is the attenuation coefficient for the material at the given wavelength of light, \( \lambda \). The maximum imaging depth is the axial distance at which the intensity of the signal light becomes equal to the background light.

It has been shown that the maximum imaging depth is linear to the attenuation coefficient of the sample medium and logarithmic to the average incident surface power, the duty cycle, and the collection efficiency of the detector [3].

One disadvantage to point source scanning microscopy is the reduction in sample illumination which is concentrated primarily at the focus. The reduction in field illumination results in the need for either scanning of the beam focus through the sample or translation of the sample through the focal plane to reconstruct the full objective field of view. The need for a large field scan results in slower acquisition rates of the 2D focal plane image than compared to wide field illumination, which is able to image the full objective field of view simultaneously.

SPatIal Frequency modulated Imaging (SPIFI) is a single-element detection imaging technique that is achieved through the use of an extended line source for illumination. The line source is modulated laterally so that each point along the source has its intensity modulated at a unique temporal frequency. With each spatial point along the illumination source corresponding to a particular temporal frequency through time integration of the signal on the single element detector a Fourier transform can be taken to turn the temporal signal into a spatial mapping corresponding to the unique spatial locations of each point along the excitation line source.
The first published work outlining the technique of SPatIal Frequency modulated Imaging was published by Futia et al [4], their work outlines the use of a rotating reticle with a chirped radial frequency mask which was based on the previous work of Sanders et al with the use of frequency-modulated reticles for imaging [5, 6]. In the article published by Futia et al, the use of a spatial-chirped mask to image an illumination line source was achieved with a resolution of 177 \( \mu \text{m} \) in the direction of the line source and 150 \( \mu \text{m} \) in the orthogonal direction with the capability to image 38 distinct lateral pixels at a rate of up to 130 Hz [4].

The next series of articles published about SPIFI were on detailing the computational tomography for image reconstruction [7], demonstrating the use of a pulsed laser source for SPIFI imaging using multiple beams for volumetric scanning and the use of multiphoton fluorescence with SPIFI [8], and deriving the diffraction theory of SPIFI [9].

The next article outlines the use of a line-diffuser and a line-scan camera to achieve 2D SPIFI images. This method uses a modified SPIFI setup that extends the modulated light sheet back into a uniform Gaussian profile with modulation maintained in the lateral direction. The sample is then fully illuminated by this modulated profile and light is then passed through a line-diffuser laterally perpendicular to the axis of modulation and collected on the line-scan camera. This SPIFI setup achieves 2D images without the need for scanning optics or translation of the sample [10].

Work published in [11] demonstrated that super-resolution images can be obtained using SPIFI and virtual energy states of atomic transitions to reach twice the diffraction limited spatial frequencies through the use of multiphoton photon excitation. Previous work with super-resolution techniques relied on real energy states for atomic transitions which limited the utility of these methods to specially designed illumination sources or specific fluorescent dyes. This work further demonstrated a method to increase the resolution by \( 2\eta \) below the diffraction limit, where \( \eta \) is the order of the non-linear intensity response of the sample media.
The work of Hwang et al demonstrated a method to increase the sample acquisition from a 1D line cursor to a 2D map using a combined method of SPIFI with a tunable frequency source to image a 2D focal plane simultaneously without the need for scanning or translation of the sample [12].

Another article outlining 2D SPIFI acquisition was published by Winters et al [13]. In this paper the use of two orthogonal SPIFI masks was used to add frequency modulation in both the x and y lateral directions thus mapping an array of frequencies to the 2D pixel locations in the samples focal plane.

The implementation of SPIFI imaging coupled with SSTF for micro-machining demonstrates further the use of SPIFI as a method for real time imaging paired with a high intensity beam [14].

One proposed method to optimize the acquisition rate of multiphoton microscopy is by extending the focal volume from a focal point to a focal line. By extending the area that the excitation beam can cover in a single illumination leads to a reduction in the total number of scan regions needed to build up a full 2D image thereby reducing the overall scan time compared to single point source illumination methods. This method would include the reduction in photon intensity through the sample thereby reducing the likelihood for the sample to be damaged by high intensity light; however, the probability of excitation is proportional to the illumination intensity, if the intensity is reduced too much the collected signal will also be reduced by a corresponding amount while the background signal will remain relatively unchanged. The limitation on the signal-to-noise ratio will greatly affect the extent to which the illumination source can be extended.

To combat both of these extremes the use of several point sources rather than a full line cursor would result in a reduction in the acquisition rate while still mitigating the reduction in photon intensity. An additional benefit to this method allows for the capability to control the focal depth of each of these point sources independently, thus allowing for scanning at multiple depths in the sample simultaneously to build up a 3D image of the sample.
For example, in a system that has an average laser power of 300 mW with a pulse duration of 150 fs and a repetition rate of 60 MHz. We can then calculate the average pulse power by dividing the total power by the repetition rate to get 5 nJ of power per pulse. If we then assume that our system has a power loss of 50% than we have an illumination power of 2.5 nJ of power per pulse. For efficient imaging we will assume that the needed illumination intensity is 100 GW/cm$^2$, thus we can calculate the energy per pulse that this corresponds to for a spot size of 1 $\mu$m. Using the intensity equation below

$$I = \frac{E}{AT}$$

(1.2)

where $E$ is the energy per pulse, $A$ is the area of the pulse ($A = \pi r^2$ for a radius of $r$), and $T$ is the pulse duration. Solving we get the energy to be 471.2 pJ per pulse. Dividing the energy per pulse of our laser by energy per pulse of our efficient imaging system we get 5.305, which means that our laser system can support up to 5 distinct spots for effective imaging. If the area of our spot would be extended as a line source for imaging we would have a line area of 16.66 $\mu$m$^2$ at a width of 1 $\mu$m the length would then only be 16 $\mu$m in length. Thus it can be seen that separating the "light sheet" into as much as five distinct points would allow for increased control in imaging over a simple line configuration.

In this thesis a comprehensive ray trace analysis of a simple SPIFI imaging system is performed for the first time. Consideration is given to both geometric ray tracing analysis via ZEMAX and physical optical transformations via Fourier analysis in Mathematica. Additionally, a SPIFI system utilizing a spatial light modulator (SLM) is presented that allows for 3D random access scan imaging.
CHAPTER 2
THEORY OF SPATIAL FREQUENCY MODULATION FOR IMAGING

The theory of coherent SPatIal Frequency modulation for Imaging (SPIFI) is developed in [5]. SPIFI uses an extended line source to illuminate a sample with signal light collected on a single element detector. A schematic showing the reduced setup for SPIFI can be seen in Figure 2.1. Incident light is first focused into a "light sheet" by a cylindrical lens and transmitted through the mask located at the focal plane of the cylindrical lens. The light is then diffracted in the lateral direction and collimated by the focusing lens onto the objective lens. The beams are then focused onto the sample where they interfere and create a corresponding modulation in the objective plane. Emitted light can then be collected, either by epi- or transmission modes, by the collecting lens onto the single element detector. The mask used for this setup can be seen in Figure 2.2.

Figure 2.1: Setup for SPIFI using a rotating transmission mask. Each diffracted order \{-1, 0, 1\} is focused onto the objective plane.

The mask is modulated in radial coordinates such that there is variation in both the radial and azimuthal directions. The equation describing the binary mask is given as:

\[ m(r, \phi) = \frac{1}{2} + \frac{1}{2} \text{sign} \left[ \cos \left( 2\pi \Delta k r \phi \right) \right] \] (2.1)
where $\Delta k$ is the spatial chirp parameter of the mask for both the radial and azimuthal directions. The frequency modulation rate is the equivalent line spacing between adjacent transmission peaks in the mask. A value of 1 for the equation represents full transmission and a value of 0 is for zero transmission.

The mask is aligned with the optical axis co-linear with the radial axis such that with an offset in the x-direction, $x_c$, we can write the mask equation in terms of our optical coordinates. We will also introduce a time variation on the radial coordinates such that the mask will rotate with a constant radial frequency, $\nu_r$. The substitution equation becomes $\phi(t) = \nu_r t$ and the final modulation on the optical axis can be re-written as:

$$m(x, t) = \frac{1}{2} + \frac{1}{2} \text{sign} \left[ \cos \left( 2\pi \Delta k (x - x_c) \nu_r t \right) \right]$$ \hspace{1cm} (2.2)

At any fixed time, $t$, the mask is modulated in the x-direction with a periodic binary signal that will diffract an axial propagating beam in the lateral direction. Since the binary mask is modulated at a 50% duty cycle only odd diffraction orders and the fundamental will be observed [15]. The angle of diffraction for a given order, $m$, can be expressed through the
The diffraction equation is given by:

$$\sin(\theta_m) = \frac{m\lambda_p}{d}$$  \hspace{1cm} (2.3)$$

where $\lambda_p$ is the central peak wavelength of the incident light and $d$ is the spacing between adjacent peaks in the diffraction mask. The spacing parameter can be re-written in terms of the mask modulation as:

$$d(t) = \frac{1}{\Delta k \nu_r t}$$ \hspace{1cm} (2.4)$$

Substituting the new expression into the diffraction angle equation we get the time-dependent diffraction angle expressed as:

$$\sin(\theta_{1,m}(t)) = m\lambda_p \Delta \nu_r t$$ \hspace{1cm} (2.5)$$

When the diffracted beams cross in the sample plane they will have incident angles proportional to the diffracted angle as:

$$\sin(\theta_{2,m}(t)) = \frac{n_1}{n_2} M \sin(\theta_{1,m}(t))$$ \hspace{1cm} (2.6)$$

where $n_1$ and $n_2$ are the refractive indexes for the mask and object plane respectively, and $M$ is the system magnification given by the ratio of the geometric focal lengths of the focusing lens to the objective lens as $M = F_f/F_o$.

For a beam of light with carrier frequency, $w_0$, and field amplitude, $E_0$, we can write the field as it propagates as:

$$E(x, t) = E_0 u(x) e^{i(w_0 t - k_x x)}$$ \hspace{1cm} (2.7)$$

where $u(x)$ is the normalized beam profile in the lateral direction and where $k_x$ is the propagation wave vector for the beam. We can relate the propagation wave vector to a diffracted beam in the object plane as:

$$k_x(t) = \frac{2\pi}{\lambda_p} \sin(\theta_{2,m}(t)) = 2\pi \frac{n_1}{n_2} M m \Delta \nu_r t$$ \hspace{1cm} (2.8)$$
From this expression we can re-write the field equation as a function of the diffracted order $m$ as:

$$E_m(x, t) = E_0 u(x) e^{i\omega_0 t} e^{-i2\pi\frac{m}{2} M m \Delta k \nu_r x t}$$  \hspace{1cm} (2.9)

Introducing the spatial frequency component, $\kappa = (n_1/n_2)M \Delta k \nu_r$, we can simplify the above equation to the form:

$$E_m(x, t) = E_0 u(x) e^{i\omega_0 t} e^{-i2\pi m \kappa x t}$$  \hspace{1cm} (2.10)

The combined electric field for two diffracted orders $n$ and $m$ is given as:

$$E_{n,m}(x, t) = E_n(x, t) + E_m(x, t) = E_0 u(x) e^{i\omega_0 t} \left( e^{-i2\pi n \kappa x t} + e^{-i2\pi m \kappa x t} \right)$$  \hspace{1cm} (2.11)

where $E_n$ and $E_m$ are the field amplitudes for the diffraction orders $n$ and $m$ respectively.

The intensity from the interference of two diffracted orders is given as:

$$I_{n,m}(x, t) = \frac{\epsilon_0}{2} |E_{0,n}(x)|^2 (2 + e^{-i2\pi (n-m) \kappa x t} + e^{i2\pi (n-m) \kappa x t})$$  \hspace{1cm} (2.12)

using the Euler Trig relationship $e^{ix} + e^{-ix} = 2\cos(x)$ the above equation can be simplified to the form:

$$I_{n,m}(x, t) = \frac{\epsilon_0}{2} |E_0 u(x)|^2 (1 + \cos(2\pi (n-m) \kappa x t))$$  \hspace{1cm} (2.13)

The total intensity profile in the objective plane is a linear superposition of all combined diffracted orders that pass through the objective aperture. In the case of the fundamental and first order diffracted beams $m \in \{-1, 0, 1\}$ we have the total illumination intensity as:

$$I_{tot}(x, t) = I_{0,1}(x, t) + I_{0,-1}(x, t) + I_{-1,1}(x, t) = \frac{\epsilon_0}{2} |E_0 u(x)|^2 (3 + 2 \cos(2\pi k \kappa x t) + \cos(4\pi k \kappa x t))$$  \hspace{1cm} (2.14)

It is evident that this illumination intensity has three distinct spatial frequencies that are all harmonics of the fundamental frequency, $f_0(t) = 2\pi \kappa t$. The three harmonics are $\{0, f_0(t), 2f_0(t)\}$ for fluorescent excitation. The intensity of emitted light depends on the spatial distribution of probe molecules in the sample objective plane, given by the transmission function $T(x)$, and in general for nonlinear processes as a function of the illumination...
intensity, $I_{tot}$, which can be expanded in a Taylor series of integer order, $\eta$. The signal measured on the single pixel detector is given by the equation as:

$$s(t) = \gamma \langle [I_{tot}(x,t)]^\eta T(x) \rangle_x$$

(2.15)

where $\gamma$ is the detector efficiency, and $\langle \cdot \rangle_x$ is the spatial integral taken over all lateral space. For a simple first order process we have the measured signal on the three diffracted orders $m \in \{-1, 0, 1\}$ given as:

$$s(t) = \gamma \int I_{tot}(x,t)T(x)dx$$

(2.16)

From this signal we can separate the signal by its three harmonic frequencies, $s(t) = \gamma [s_0(t) + s_1(t) + s_2(t)]$, where the three signals are given as:

$$s_0(t) = \frac{3}{2} \epsilon_0 |E_0|^2 \int |u(x)|^2 T(x)dx$$

(2.17)

$$s_1(t) = \epsilon_0 |E_0|^2 \int |u(x)|^2 \cos (2\pi \kappa x t) T(x)dx$$

(2.18)

$$s_2(t) = \frac{1}{2} \epsilon_0 |E_0|^2 \int |u(x)|^2 \cos (4\pi \kappa x t) T(x)dx$$

(2.19)

The functions above can be cast into a general form using the Euler Formula $e^{ix} = \cos(x) + i \sin(x)$ as:

$$s_m(t) = s_m \text{Re} \left[ \int f(x)e^{i2\pi f_{x,m}x}dx \right]$$

(2.20)

where $s_m$ is the amplitude of the signal for the $m$-th harmonic, $f(x) = |u(x)|^2 T(x)$ and $f_{x,m} = \kappa x t$. The above formula can be seen as a Fourier transform in the lateral direction of the function $f(x)$ which is the combination of the transmission function of the sample $T(x)$ and the normalized beam profile $|u(x)|^2$. Thus through a Fourier transform on the signal collected by the single element detector we can recover the spatial distribution of fluorescent probe molecules represented by the transmission function $T(x)$. The above derivation can be extended to the reflective mask with modulation in the lateral and vertical directions as opposed to the radial and azimuthal directions. The modified setup to include the reflective mask can be seen in Figure 2.3. Here the incident light is first
reflected on a scan mirror array and through a F-Theta lens whose flat-field focal plane is coincident with the reflective SPIFI mask. The F-Theta lens is designed to have a flat focal plane where displaced light is linear to the incident angle rather than the to the tangent of the incident angle. The diffracted light is then back propagated through the scan mirror array and focused through the objective lens to the sample and finally collected by the focusing lens onto the single element detector. An image of the reflective mask used in this setup can be seen in Figure 2.4.

Figure 2.3: Setup for SPIFI using a static reflective mask. Each diffracted order \{-1, 0, 1\} is focused onto the objective plane resulting in interference of the corresponding frequency modulation.

The equation describing this binary reflective mask is given as:

\[
m(x, y) = \frac{1}{2} + \frac{1}{2} \text{sign} \left[ \cos (2\pi \Delta k_x x \Delta k_y y) \right]
\]

where \(\Delta k_x\) and \(\Delta k_y\) are the frequency modulation rates of the mask in the lateral and vertical directions respectively. The frequency modulation rate is the equivalent line spacing between adjacent transmission peaks in the mask. A value of 1 for the equation represents full reflection and a value of 0 is for zero reflection.

A line cursor is again used in this setup and is scanned across the vertical axis with a constant frequency \(\nu_s\). The scanning is done by sweeping the line cursor across the mask from left to right then from right to left and repeating in this fashion thus it will have a
positive sweep from left to right and a negative sweep from right to left. This modifies the vertical position to be time dependent on the scan rate as \( y(t) = \Delta y f_\Delta(\nu_s t) \) where \( \Delta y \) is the maximum vertical displacement for the mask oscillation and where the function \( f_\Delta \) is the triangle wave function which is given by the equation \( f_\Delta(x) = 4\Delta y (|x \mod 1| - (1/2)) - 1 \). Here the vertical position oscillates between \( +\Delta y \) and \( -\Delta y \) in a triangle wave which is linear to the parameter \( f_\Delta(\nu_s t) \propto \nu_s t \) just as the time variation in the azimuth component is linear to the time scan rate of the mask rotation. For this derivation the triangle function will be replaced by the linear slope equation \( f_\Delta(\nu_s t) = 4\nu_s t \) which gives the same result when inserted into the periodic cosine function. The modulation on the line cursor then is expressed as:

\[
m(x, t) = \frac{1}{2} + \frac{1}{2} \text{sign} \left[ \cos \left( 8\pi \Delta k_x x \Delta k_y \Delta y \nu_s t \right) \right]
\]  

(2.22)

From the above equation we can see that this is similar to the form of the rotating mask with the rotation frequency \( \nu_r \) replaced with the scanning frequency \( \nu_s \) resulting in a
modulation that is also linear with time. Just like the transmission mask the reflective mask also has a 50% duty cycle resulting in only odd diffracted orders along with the fundamental. However, since the mask is being scanned the incident angle of light on the mask is not always normal to the mask surface this modifies the angle of diffraction equation for a given diffraction order, \( m \), as:

\[
\sin(\theta_m) = \frac{m \lambda_p}{d} - \sin(\theta_i) \tag{2.23}
\]

where the incident angle is given as \( \theta_i \). The spacing between adjacent peaks \( d \) are of the same form as the transmission mask. The spacing parameter can be re-written in terms of the mask modulation as:

\[
d(t) = \frac{1}{4 \Delta k_x \Delta k_y \Delta y \nu_s t} \tag{2.24}
\]

Similarly the incident angle can be rewritten in terms of the vertical position \( y \) where the normal component is coincident with \( y = 0 \). The equation for the vertical position on the mask is given as:

\[
y = F_{\Theta} \theta_i \tag{2.25}
\]

where \( F_{\Theta} \) is the geometric focal length of the theta-lens used to focus on the mask. For a mask that is limited in vertical extent to the maximum displacement \( \Delta y \) as long as \( \Delta y \ll F_{\Theta} \) than the paraxial approximation can be made \( x \approx \sin(x) \). Using this expression and the definition derived using the scan rate we get the equation for the incident angle as:

\[
\sin(\theta_i(t)) = \frac{4 \Delta y \nu_s t}{F_{\Theta}} \tag{2.26}
\]

Then the final equation for the diffracted angle is given as:

\[
\sin(\theta_{1,m}(t)) = 4 \Delta y \left( m \lambda_p \Delta k_x \Delta k_y - \frac{1}{F_{\Theta}} \right) \nu_s t \tag{2.27}
\]

The equation for the incident angle on the sample plane is similar to the transmission mask except the refractive index for the mask \( n_1 \) is set to 1 as the reflective mask is a mirror.
The incident angle in the sample plane is given as:

$$\sin(\theta_2, m(t)) = \frac{M}{n_2} \sin(\theta_1, m(t)) \quad (2.28)$$

where $n_2$ is the index of refraction in the sample focal plane and $M$ is the ratio of the geometric focal length of the focusing lens to the objective lens $M = F_f/F_o$.

Given the same expression for the propagating wave form as Equation 2.7. The spatial frequency in the lateral direction $k_x$ is then given as:

$$k_x(t) = \frac{2\pi}{\lambda_p} \sin(\theta_2, m(t)) = 8\pi \frac{M}{n_2} \Delta y \left( m\Delta k_x \Delta k_y - \frac{1}{F_\Theta \lambda_p} \right) \nu_s t \quad (2.29)$$

Here it can be seen that a similar spatial frequency component appears as $\kappa = 4\Delta y(M/n_2)\Delta k_x \Delta k_y \nu_s$. There also exists an offset to this spatial frequency component based on the off axis contribution to the ray vector $\kappa_\Theta = 4\Delta y(M/n_2)(1/F_\Theta \lambda_p)\nu_s$. We can then re-write the wave vector as:

$$k_x(t) = 2\pi (m\kappa - \kappa_\Theta) t \quad (2.30)$$

Here is the wave vector previously derived with an offset component $\kappa_\Theta$ which gives a shift to all the diffracted orders. The field equation for a diffracted wave of order $m$ is given as:

$$E_m(x, t) = E_0 u(x) e^{i\omega t} e^{-i2\pi(m\kappa - \kappa_\Theta)xt} \quad (2.31)$$

The combined electric field for two diffracted orders $n$ and $m$ is given as:

$$E_{n,m}(x, t) = E_n(x, t) + E_m(x, t) = E_0 u(x) e^{i\omega t} \left( e^{-i2\pi n\kappa xt} + e^{-i2\pi m\kappa xt} \right) e^{i2\pi\kappa_\Theta xt} \quad (2.32)$$

The intensity from the interference of these two waves is given as:

$$I_{n,m}(x, t) = \frac{\epsilon_0}{4} E_{n,m}(x, t) E_{n,m}^*(x, t) = \frac{\epsilon_0}{4} \left| E_0 u(x) \right|^2 \left( 2 + e^{-i2\pi(n-m)\kappa xt} + e^{i2\pi(n-m)\kappa xt} \right) \quad (2.33)$$

This is the same result we arrive at with rotation mask setup for the equation of illumination intensity. The use of a static reflective mask as opposed to the traditional rotating
transmission mask are first the reduction in size for these optical components. A static mask of similar diffraction resolution as a rotating mask is much more compact in size and removes the presence of wobble in the rotating mask which results from the motors rotation axis not being completely centered to the mask.
CHAPTER 3
DIFFRACTION THEORY OF SPIFI

The results gained from the plane wave derivation of SPIFI can be reproduced using the Fraunhofer diffraction equation. Fraunhofer diffraction models the transformation of plane wave light far from the diffracting object and at the focal plane of a focusing lens. The Fraunhofer diffraction equation is a special case of the Kirchhoff diffraction formula. Given an aperture in the x’y’-plane with the normalized aperture function \( A(x', y') \) the complex amplitude \( U(x, y, z) \) can be found for an incident plane wave normal to the x’y’-plane with a complex field amplitude \( E(x', y') \) and wavelength \( \lambda \). One form that the Fraunhofer diffraction equation can take is given as:

\[
U(x, y, z) \propto \int \int_A \ A(x', y') E(x', y') e^{-\frac{i 2 \pi}{\lambda z}(x'x + y'y)} \, dx' \, dy' \tag{3.1}
\]

where the function is integrated over the whole x’y’-plane. In the above equation we can make the substitution \( f_x = x/(\lambda z) \) and \( f_y = y/(\lambda z) \). This substitution transforms the equation into the Fourier transform of the aperture function multiplied by the complex field amplitude.

\[
U(x, y, z) \propto \mathcal{F}\{A(x', y')E(x', y')\}_{f_x,f_y} \tag{3.2}
\]

Using the convolution theorem which states that the Fourier Transform of two functions multiplied point-wise is equivalent to the convolution between the individual Fourier Transform of the two functions. Using the shorthand for the Fourier Transform as \( \mathcal{F}\{f(x', y')\}_{f_x,f_y} = \{\hat{f}\}[f_x,f_y] \) gives the rewritten equation as:

\[
U(x, y, z) \propto \{\hat{A} \otimes \hat{E}\}[f_x,f_y] \tag{3.3}
\]

To get the Fraunhofer equation for the focal plane of the lens set the axial length \( z \) equal to the focal length of the lens, \( F \), assuming the lens has a positive focal length. Applying this equation to the setup for the rotating transmission SPIFI mask we get the input beam
profile $E_0(x_0, y_0)$ will be transformed by the cylindrical lens with aperture $A_{cyl}(x_0, y_0)$ in only the vertical direction such that the profile at the focus will be given as:

$$E_1(f_{x,1}, f_{y,1}) \propto \{\hat{A}_{cyl}(f_{x,1}) \otimes \hat{E}_0(f_{x,1})\} [f_{y,1}]$$ (3.4)

Here the parameter for $x_0$ has been scaled into the Fourier plane to the parameter $f_{x,1}$ to keep the coordinates consistent between object planes and Fourier planes in the lens focus.

The Fourier Transform of a focusing object can better be described by the impulse response of the object referred to as the Point Spread Function (PSF). Here we make the substitution in terminology where in general the PSF is equivalent to the corresponding Fourier Transform $\{\hat{A}_{cyl}\} [f_{y,1}] \rightarrow PSF_{cyl}(f_{x,1}, f_{y,1})$. Another substitution is the use of the Optical Transfer Function rather than the aperture $A_{cyl}(x_0, y_0) \rightarrow OTF_{cyl}(x_0, y_0)$.

$$E_1(f_{x,1}, f_{y,1}) \propto PSF_{cyl}(f_{x,1}, f_{y,1}) \otimes \{\hat{E}_0(f_{x,1})\} [f_{y,1}]$$ (3.5)

Next the incident field on the rotating mask is multiplied in the $(f_{x,1}, f_{y,1})$ - plane to give the light transmitted through the mask. The mask modulation is given by the function $M_{rot}(f_{x,1}, f_{y,1}, t)$ which is time dependent. Then this gives the equation of the transmitted light as:

$$E'_1(f_{x,1}, f_{y,1}, t) \propto M_{rot}(f_{x,1}, f_{y,1}, t) E_1(f_{x,1}, f_{y,1})$$ (3.6)

The light is then transformed once again to the collimated output of the focusing lens which is transformed in both the lateral and vertical directions and multiplied by the OTF for the focusing lens $OTF_{foc}(x_2, y_2)$ to give the output as:

$$E_2(x_2, y_2, t) \propto OTF_{foc}(x_2, y_2) \{\hat{E}'_1(t)\} [x_2, y_2] = OTF_{foc}(x_2, y_2) \{\hat{M}_{rot}(t) \otimes \hat{E}_1\} [x_2, y_2]$$ (3.7)

here we make use of the scaling of one objective plane to the next where $x_2 = x_0$ and $y_2 = (F_{foc}/F_{cyl}) y_0$ where $F_{foc}$ the focal length of the focusing lens and $F_{cyl}$ is the focal length of the cylindrical lens in the vertical direction. The effect is a scaling of the vertical axis but not the horizontal axis.
Next the field propagates to the objective lens picking up a magnification factor \( m = (F_{\text{obj}}/F_{\text{foc}}) \) which is the ratios to the two focal lengths such that \( x_3 = mx_2 \) and \( y_3 = my_2 \). This gives the field on the back aperture of the objective lens as:

\[
E_3(x_3, y_3, t) \propto E_2(x_3/m, y_3/m, t) \quad (3.8)
\]

The field is then Fourier Transformed by the objective lens to give the field amplitude in the sample plane as:

\[
E_4(f_{x,4}, f_{y,4}, t) \propto PSF_{\text{obj}}(f_{x,4}, f_{y,4}) \otimes \{\hat{E}_3(t)\} [f_{x,4}, f_{y,4}] \quad (3.9)
\]

where the magnification of the focusing and objective lens also gives the relations \( f_{x,4} = mf_{x,1} \) and \( f_{y,4} = mf_{y,1} \). This allows us to rewrite the equation in terms of the PSF of the focusing and objective lenses the mask modulation and the line focus.

\[
E_4(f_{x,4}, f_{y,4}, t) \propto PSF_{\text{obj}}(f_{x,4}, f_{y,4}) \otimes PSF_{\text{foc}}(f_{x,4}/m, f_{y,4}/m)
\]

\[
\otimes (M_{\text{rot}}(f_{x,4}/m, f_{y,4}/m, t)E_1(f_{x,4}/m, f_{y,4}/m)) \quad (3.10)
\]

here we can see that the field amplitude on the sample plane is proportional to the modulation mask function \( M_{\text{rot}} \) and the line focus field amplitude \( E_1 \). The illumination intensity of light \( I_{\text{ill}}(f_{x,4}, f_{y,4}, t) \) can then be expressed as:

\[
I_{\text{ill}}(f_{x,4}, f_{y,4}, t) = \frac{\epsilon_0}{4} E_4(f_{x,4}, f_{y,4}, t)E_4^*(f_{x,4}, f_{y,4}, t) \quad (3.11)
\]

If we assume that the PSF of both the focusing and objective lenses are diffraction limited such that each can be approximated with a delta-function than the intensity can be simplified to the form:

\[
I_{\text{ill}}(f_{x,4}, f_{y,4}, t) = \frac{\epsilon_0}{4} |M_{\text{rot}}(f_{x,4}/m, f_{y,4}/m, t)E_1(f_{x,4}/m, f_{y,4}/m)|^2 \quad (3.12)
\]

The emitted light from the sample \( S(x, y, t) \) will in general be functionally dependent on the incident field intensity and the spatial distribution of probe molecules \( C(x, y, t) \) given as:

\[
S(x, y, t) = f [I_{\text{ill}}(x, y, t)] C(x, y, t) \quad (3.13)
\]
In general the functional form of sample response can be expanded in a Taylor series previously discussed to give a single dependent on the non-linear process of order $\eta$ which re-writes the above equation as:

$$S_\eta(x, y, t) = [I_{id}(x, y, t)]^\eta C(x, y, t)$$  \hfill (3.14)

Integration of the emitted light over the spatial domain gives the time dependent signal measured by the single element detector as:

$$s_\eta(t) = \gamma \langle S_\eta(x, y, t) \rangle_{x,y} = \gamma \langle [I_{id}(x, y, t)]^\eta C(x, y, t) \rangle_{x,y}$$  \hfill (3.15)

where $\gamma$ is the detector efficiency and $\langle \cdot \rangle_{x,y}$ indicates a spatial integral over the whole sample plane. Taking a Fourier Transform in time on this spectrum gives the following result:

$$\mathcal{F}\{s_\eta(t)\}_\omega = \{\hat{s}_\eta\}[\omega] = \gamma \frac{\epsilon_0}{4} \langle |E_1(x, y)|^{2\eta} \mathcal{F}\{ |M_{rot}(x, y, t)|^{2\eta} C(x, y, t) \} \rangle_{x,y}$$  \hfill (3.16)

In the case of the rotating SPIFI mask which has the mask modulation equation defined in radial coordinates as:

$$m_{\text{rot}}(r, \phi, t) = \frac{1}{2} + \frac{1}{2} \cos(\Delta \nu_r t + \phi_0)$$  \hfill (3.17)

where $\phi(t)$ is linear in time with $\phi(t) = \nu_r t + \phi_0$ and whose output is modulated to the range $(-\pi, \pi)$ via the modulation function $\text{mod}^{\pi}_{-\pi}(\phi(t))$. The rotation rate $\nu_r$ is given in radians per second and $\phi_0$ is a phase offset for when $t = 0$. Next, changing variables to Cartesian coordinates we have the form given as:

$$m_{\text{rot}}(x, y, t) = \frac{1}{2} + \frac{1}{2} \cos \left( \Delta k \sqrt{x^2 + y^2} \text{mod}^{\pi}_{-\pi}(\tan^{-1}(x, y) + \nu_r t + \phi_0) \right)$$  \hfill (3.18)

where $\tan^{-1}(x, y)$ is a modified inverse tangent function that gives the angle between the coordinates $(x, y)$ and the origin $(0, 0)$ with the $+x$-axis as the zero radian output. Since the Fourier transform of this spectrum gives a fundamental frequency that is dependent on both $x$ and $y$ coordinates we then take into consideration the field amplitude of the line
cursor which restricts output to the x-axis first by being separable in both coordinates such that \( E_1x, y = E_{x,1}(x)E_{y,1}(y) \). Next, we assume that the extent of the vertical component is sufficiently limited that it is restricted to a single y-value of \( y = 0 \). This reduces the combined equation of the two as:

\[
E_{ill}(x, t) = E_{x,1}(x)E_{y,1}(0)m_{rot}(x, 0, t) = \frac{1}{2} + \frac{1}{2} \cos \left( \Delta k|x|\text{mod}_\pi^\nu_r t + \phi_0 \right) \tag{3.19}
\]

This function can be re-expressed without the use of the \text{mod} function in the argument as:

\[
E_{ill}(x, t) = \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \Delta k|x|\left( \nu_r t + \phi_0 \right) \right) \right] \otimes \text{rect} \left( \frac{\nu_r t + \phi_0}{2\pi} \right) \text{comb} \left( 2\pi(\nu_r t + \phi_0) \right) \tag{3.20}
\]

The Fourier Transform of the first function will give three distinct peaks one at the DC value and one at both \( \pm \Delta k|x|\nu_r \). Thus the Transform is given as:

\[
\mathcal{F} \{ E_{x,1}(x)E_{y,1}(0)m_{rot}(x, 0, t) \}_\omega \propto s_0\delta(\omega) + s_{+1}\delta(\Delta k|x|\nu_r - \omega) + s_{-1}\delta(\Delta k|x|\nu_r + \omega) \tag{3.21}
\]

where the function \( \delta(\omega) \) is the Dirac-delta function which is singular when the argument given is equal to zero (\( \omega = 0 \)) and everywhere else zero. Here \( s_0 \) denotes the relative strength of the DC component and \( s_{+1} \) and \( s_{-1} \) for the first fundamental frequencies in the positive and negative values respectively. It can then be observed that for a given position \( x \) there is a unique frequency component that corresponds to that position giving us a one-to-one map from the spatial domain \( (x) \) to the spectral domain \( (\omega) \) such that we can recover spatial information through the mapping onto the spectral domain of the single element detector. The power on the Fourier Transform for distinct values defined by the Dirac-delta function will be convoluted in the spectral domain giving rise to \( 2\eta \) components in each of the positive and negative frequency values each with their relative amplitudes. These higher order frequencies allow for mapping to more discrete spatial elements thus increasing the effective resolution to reconstruct the spatial temporal distribution of probe molecules.
In practice the PSF for the objective lens and focusing lens with not be Dirac-delta point functions and instead will have some finite width that effectively limits the maximum frequency components that can be measured and consequently the limit to the spatial resolution. The limit is primarily defined by the objective lens numerical aperture through the relation:

$$\omega_{\text{max}} = \frac{NA}{\lambda}$$  \hspace{1cm} (3.22)

To illustrate the frequency modulation on the sample plane the following examples show how each optical element contributes to final illumination intensity on the sample plane. Each image is of 2048 by 2048 pixels which corresponded to discrete points in the respective image or focal plane. Each focal plane is the conjugate space corresponding to the Fraunhofer diffraction through the respective lens that transforms the light emitted from the image plane. The magnification factor for all lenses will be set to $M = 1$ such that there is no magnification present between objective planes in the same conjugate space.

The initial image seen in Figure 3.1 is of the Gaussian line cursor made by the focusing of light in the y-coordinate frame by the cylindrical lens.

Figure 3.1: A Gaussian line cursor for scanning across the rotating SPIFI mask. The Gaussian line has a standard deviation of 204.8 pixels in the x-direction and 2.048 pixels in the y-direction. The image was cropped from 896 to 1152 pixels in the x-direction and from 512 to 1536 in the y-direction.

Without the presence of the mask this line cursor will be transformed by the focusing lens to the conjugate space resulting in the rotated line cursor seen in Figure 3.2.
Figure 3.2: The transform of the line cursor resulting in a vertical line cursor which is the Fourier Transform in the Fraunhofer limit. Without the presence of the SPIFI mask. The image was cropped from 512 to 1536 in the x-direction and from 896 to 1152 pixels in the y-direction.
The mask that is used can be seen in Figure 3.3. The mask has an inner radius of 204.8 pixels and an outer radius of 1024 pixels. The spatial parameter is set to 0.1 and the mask has been shifted by 614.4 pixels so that the central radius of the mask is at the center of the 2048 by 2048 pixel image. The mask has been rated by one-eight to illustrate the modulation in the x-direction.

Figure 3.3: The rotating SPIFI mask which is offset to align the outer and inner diameters with the light sheet. The rotation angle of the mask is set at one-eight the full rotation of the mask.
The combination of the Gaussian line cursor and the rotating mask can be seen in Figure 3.4. Here the line cursor is modulated with the the mask and can be seen in the intensity plot of Figure 3.5.

![Figure 3.4: The rotating mask overlaid with the Gaussian line cursor. The transmission of the light that passes through the mask can be seen with a clear modulation along the lateral length of the line source. The image was cropped from 896 to 1152 in the x-direction and from 512 to 1536 pixels in the y-direction.](image)

The Fourier Transform of this modulated line cursor will give the light that exists in the conjugate plane of the focusing lens. This transform can be seen in Figure 3.6. Here it can be seen that all the diffracted orders are present except for the even orders which are not needed to reconstruct the binary mask modulation. The angular slant present is a result of the spatial phase angle present in the mask itself along the x-direction. A colorized image of these modes can be seen in Figure 3.8 where each mode is a distinct color. A line intensity plot is shown in Figure 3.7 where the peak intensity values though the central axis can be seen.

The light that can be focused onto the objective is limited by the aperture size of the objective lens. This aperture can be seen in Figure 3.9 with a radius given as 204.8 pixels. The Point Spread Function of this aperture is the Fourier Transform of the aperture and can be seen in Figure 3.10. A line intensity plot is shown in Figure 3.11 where the Airy disk profile can clearly be seen.

The diffracted light that passes through the objective lens aperture is given by multiplying the aperture function by the diffracted light to get the collected light seen in Figure 3.12. It
Figure 3.5: A line intensity plot through the center of Figure 3.4. The intensity follows the Gaussian intensity of the "light sheet" while being zero where the mask block transmitted light.
Figure 3.6: The diffracted rays of light which are copies of the un-diffracted light at each of the spatial frequencies of the modulated mask. The DC (un-diffracted light) and all the odd spatial frequencies are present to reconstruct the binary nature of the mask. The image was cropped from 512 to 1536 in the x-direction and from 512 to 1536 pixels in the y-direction.
Figure 3.7: A line intensity plot through the center of Figure 3.6. The intensity peaks at the un-diffracted mode and is repeated at odd intervals with decaying peak values.
Figure 3.8: The diffracted rays of light seen in Figure 3.6. Each diffracted mode is color coded to distinguish it from adjacent modes. The image was cropped from 512 to 1536 in the x-direction and from 512 to 1536 pixels in the y-direction.
Figure 3.9: The aperture of the objective lens as a simple circular aperture. The light transmitted is only the white pixels with grayscale value of 1. The image was cropped from 768 to 1280 in the x-direction and from 768 to 1280 pixels in the y-direction.
Figure 3.10: The Point Spread Function of the objective aperture. The PSF is the Fourier Transform of the aperture function and is an image of light focused at the focal plane. The image was cropped from 896 to 1152 in the x-direction and from 896 to 1152 pixels in the y-direction.
Figure 3.11: A line intensity plot through the center of Figure 3.10. The intensity plot shows the Airy disk pattern characteristic of circular aperture diffraction.
can be seen that not only do the DC and the two first order diffraction patterns pass, but part of the higher order diffracted light as well. A colorized image of these modes can be seen in Figure 3.13 where each mode is a distinct color.

Taking the Fourier Transform of this light will give us the light that will illuminate the sample at the focal plane of the objective lens. This light can be seen in Figure 3.14. A line intensity plot can be seen in Figure 3.15 which passes through the center of the x-axis of the beam, and in Figure 3.16 which passes through the y-direction offset to the right by 49 pixels.

As can be seen in Figure 3.14 the light that illuminates the sample will have rings present due to the limited spatial frequencies allowed by the objective lens. The osculations present on the sample mirror those on the mask with a reduced harmonic frequency which is a result of only allowing a finite number of fringes through the objective lens aperture.
Figure 3.12: The diffracted orders of light that are transmitted from the SPIFI mask and that are allowed to pass through the objective lens. The diffracted modes are angled as a result of the slight spatial slant present in the mask by the time varying phase component. The image was cropped from 768 to 1280 in the x-direction and from 768 to 1280 pixels in the y-direction.
Figure 3.13: The diffracted orders of light from Figure 3.12. The colors are used to distinguish each diffracted mode from adjacent modes. The image was cropped from 768 to 1280 in the x-direction and from 768 to 1280 pixels in the y-direction.
Figure 3.14: The light transmitted through the objective aperture and onto the sample focal plane. The rings present are a result of the limited spatial frequencies allowed through the objective lens resulting in Gibbs ringing. The image was cropped from 512 to 1536 in the x-direction and from 896 to 1152 pixels in the y-direction.

Figure 3.15: A line intensity plot through the center of Figure 3.14 in the x-direction. The intensity plot shows the Gaussian profile of the beam and the periodic modulation. The Gibbs phenomenon can be seen near the transitions of the on-off modulation and is a result of the finite spatial frequencies limited by the numerical aperture of the objective lens.
Figure 3.16: A line intensity plot of Figure 3.14 that passes through the peak value located at 49 pixels to the right of the central pixel location and in the y-direction. The intensity plot shows the sharp central peak located at the center of the "light sheet" with rippling on both sides.
CHAPTER 4
RAY TRACING OF A SIMPLE SPIFI SYSTEM

A full layout of the SPIFI system used for ray tracing can be seen in Figure 4.1. The specifications for the lenses used are given in Table 4.1. The illumination source is a fiber laser with a spectral peak at 1040 nm. The diffraction SPIFI mask used will be modeled as a diffraction grating with line spacing of 70 lines per millimeter. For the first time a comprehensive ray trace analysis of a SPIFI imaging system is performed.

Figure 4.1: Schematic diagram of simple SPIFI system with all lens specifications given in Table 4.1. All distances are approximate. Back propagated light (green) gets separated out by a beam splitter and focused onto a CCD camera.

First, lens placement was achieved through paraxial lens calculations to determine the relative inter-lens spacing between the cylindrical lens, focusing lens and objective achromatic lens. The input aperture was set to a 10 mm diameter with no additional input field angles. The image planes for the cylindrical lens and objective achromatic lens were located with a marginal ray solve (M-solve). The location of the focusing lens was found by taking a marginal ray solve from the lens, reversed and manually added and fixed to the lens spacing from the cylindrical lens’ focal plane.
Table 4.1: Optical lenses used and their corresponding specifications.

<table>
<thead>
<tr>
<th>Lens Type</th>
<th>Model</th>
<th>Rectangular aperture</th>
<th>Effective focal length</th>
<th>Material no.1</th>
<th>Material no.2</th>
<th>R_1</th>
<th>R_2</th>
<th>R_3</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical Lens - Model LJ1567L1-B</td>
<td></td>
<td>32 by 30 mm</td>
<td>100 mm</td>
<td>N-BK7</td>
<td></td>
<td>51.68</td>
<td></td>
<td></td>
<td>5.22 mm</td>
</tr>
<tr>
<td>Focusing Lens no.1 - Model LA1050-B</td>
<td></td>
<td>Circles 25.4 mm</td>
<td>75 mm</td>
<td>N-BK7</td>
<td></td>
<td>51.5</td>
<td></td>
<td>-93.11 mm</td>
<td>9.69 mm</td>
</tr>
<tr>
<td>Objective Achromatic Lens - Model AC508-075-B</td>
<td></td>
<td>Circles 25.4 mm</td>
<td>75 mm</td>
<td>N-LAK22, R_1 = 51.8 mm, R_2 = -93.11 mm, thickness = 12 mm</td>
<td>N-SF6HT, R_2 = -93.11 mm, R_3 = -291.07 mm, thickness = 5 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Focusing Lens no.2 - Model AC254-060-B</td>
<td></td>
<td>Circles 12.7 mm</td>
<td>60 mm</td>
<td>N-LAK22, R_1 = 39.48 mm, R_2 = -33 mm, thickness = 6 mm</td>
<td>N-SF6HT, R_2 = -33 mm, R_3 = -165.2 mm, thickness = 1.7 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The SPIFI mask was inserted as a diffraction grating with line spacing of 70 lines per mm in the x-direction and placed at the focal plane of the cylindrical lens. The paraxial calculations can be seen in Figure 4.2 for the yz-plane where there is no diffraction off the SPIFI mask.

The paraxial analysis for the xz-plane of the optical system can be seen in Figure 4.3. The location of each component on the M-solves was fixed and the cylindrical lens was adjusted for the new rotation. The modulation for the SPIFI mask was also incorporated in the paraxial calculations using the modified Paraxial Ray Trace equation to account for the
diffractive orders. The zero and first order diffraction modes were used for calculation.

Figure 4.3: Excel table output of a paraxial ray trace using the lenses given in Table 4.1. Ray tracing is done in the xz-plane with M-solves denoted by brown lettering. Diffracted orders of the SPIFI mask are calculated using the grating equation for the zero and first order modes.

\[ n'u' - nu = \frac{m\lambda}{d} \]  \hspace{1cm} (4.1)

The above equation assumes no optical power contributed from the surface of the diffraction grating with index \( n' \) before the interface and \( n \) after the surface, with the paraxial rays \( u' \) before and \( u \) after. The diffracted order is given as \( m \), the peak wavelength as \( \lambda \) and the line spacing \( 1/d \) as the lines per mm of the diffraction grating.

All components were then placed in ZEMAX at the calculated paraxial locations with the SPIFI mask again modeled as a diffraction grating modulated in the x-direction at the specified 70 lines per mm. A layout of the ZEMAX setup can be seen in Figure 4.4. which shows the yz plane of the system and in Figure 4.5 which shows the xz-plane of the system.

A dummy surface was then inserted after the objective achromatic lens with a variable thickness and optimized for RMS spot size in the y-direction at the centroid for only the first diffracted orders \( (m = \pm 1) \). The relative shift in the paraxial focal plane to the optimized focal plane can be seen in Figure 4.6. The paraxial plane was calculated to be at a back focal distance of 65.985 mm and the optimized focal plane at distance 1.169 mm before the paraxial plane. The spot diagram for the optimized focal plane can be seen in Figure 4.7.
Figure 4.4: The ZEMAX layout for the yz-plane with all lenses placed at the paraxial locations determined in Figure 4.2. Overlap of all three diffracted orders ($m = 0, \pm 1$) can be seen after the mask plane.

Figure 4.5: The ZEMAX layout for the yz-plane with all lenses placed at the paraxial locations determined in Figure 4.3. All three diffracted orders can be seen with $m = -1$ (red), $m = 0$ (blue), and $m = 1$ (green).
Figure 4.6: The paraxial focal plane located on the right side and the calculated minimized focal plane for the first order diffracted modes ($m = \pm 1$). Seven marginal rays are drawn for each diffracted mode.
Figure 4.7: Spot diagram for the minimized focal plane with RMS spot size minimized for only the first diffracted order modes \((m = \pm 1)\). The three diffracted order modes are labeled with blue for the zeroth order mode \((m = 0)\), green for the positive first order mode \((m = 1)\), and red for the negative first order mode \((m = -1)\).
As can be seen from the spot diagram the focal spot size is almost at the diffraction limit in the y-direction for the first order modes. For the zero order mode however the spot size is far from diffraction limited. Adjusting the optimization for only the zero order mode we find the new focal plane at 0.348 mm before the paraxial focal plane. The spot diagram for this optimized focal plane can be seen in Figure 4.8.

Figure 4.8: Spot diagram for the minimized focal plane with RMS spot size minimized for only the zero diffracted order modes ($m = 0$). The three diffracted order modes are labeled with blue for the zeroth order mode ($m = 0$), green for the positive first order mode ($m = 1$), and red for the negative first order mode ($m = -1$).

As can be seen from the spot diagram the spot size is almost at the diffraction limit in the y-direction for the zero order mode and not at all near diffraction for the first order modes. It is evident that there is some spherical aberration present leading to different focal planes for the zero and first order diffracted modes. By optimizing for all three modes simultaneously
we find the focal plane is located at 0.897 mm before the paraxial plane. The spot diagram for the optimization of all three modes can be seen in Figure 4.9.

Figure 4.9: Spot diagram for the minimized focal plane with RMS spot size minimized for all three diffracted order modes \((m = \pm 1)\). The three diffracted order modes are labeled with blue for the zeroth order mode \((m = 0)\), green for the positive first order mode \((m = 1)\), and red for the negative first order mode \((m = -1)\).

Here we see that although all three modes have the same extent in the y-direction they are all far from diffraction limited.

We then introduce higher order diffraction into the paraxial analysis to see what affect these higher orders will have on the focal plane location and spot size. It can be seen in Figure 4.10 that for the given lenses the highest diffracted order allowed to pass through the focusing lens is the third order diffracted light.
Figure 4.10: System overview in the xz-plane for all diffraction orders up to the fourth order ($m = \pm 4$). Only axial rays are drawn for each diffracted order.

It is worth noting that for each higher order diffracted beam the focal plane shifts closer to the achromatic lens. This shift can be seen in Figure 4.11. Spherical aberration is responsible for the shift in focal plane location where each annular region of the lens that has a diffracted beam pass through it will focus at a different axial location.

Figure 4.11: Zoomed in at the focus of the system shown in Figure 4.10. The leftmost ray crossing is for the third order modes ($m = \pm 3$), the middle crossing is for the second order ($m = \pm 2$), and the right crossing is for the first order ($m = \pm 1$). The paraxial image plane is the right most vertical line with the minimized focal plane to the left.

Inserting a waveplate or other optical element in front of the achromatic lens the higher order modes will be blocked from reaching the focal plane eliminated the effects of the higher ordered light on the sample. With a diameter of 1-inch all but the zero and first order modes will be completely blocked form entering the achromatic lens and focusing onto the focal region as can be seen in Figure 4.12.
Figure 4.12: System overview in the xz-plane for all diffraction orders up to the fourth order \((m = \pm 4)\). The addition of a 1-inch aperture located 10 mm before the objective achromat lens can be seen to block out all the higher order modes above the first order. Only axial rays are drawn for each diffracted order.

Next, the alignment of the optical components that allow for effective collection of light from the sample plane back through the optical elements to a detector. This double pass analysis will assume that light will be emitted by the sample at the focal region which is minimized for only the first order modes. The final focusing achromatic lens will have its position shifted to be 155.86 mm after the cylindrical lens, the combined focal lengths of the cylindrical lens and the focusing achromatic lens.

An unfolded ZEMAX system layout can be seen in Figure 4.13, for the yz-plane and in Figure 4.14 for the xz-plane where the optical elements have been reversed to account for the back propagation of light through the system.

Figure 4.13: The ZEMAX layout for the yz-plane with lens elements duplicated and reflected to calculated back propagation of light reflected at the minimized focal plane. The rays are traced for the zero and first diffracted order modes.
Figure 4.14: The ZEMAX layout for the xz-plane with lens elements duplicated and reflected to calculate back propagation of light reflected at the minimized focal plane. The rays are traced for the zero and first diffracted order modes.

The focusing of this system can be seen enlarged in Figure 4.15 for the yz-plane and in Figure 4.16 for the xz-plane. From these two ray diagrams it is evident that the light is afocal and at different focal locations for the two diffracted modes present. The difference in the two focal lengths can be attributed to the lens separation between the first focusing lens and the objective achromatic lens which allows for divergent rays to shift the focal length in the xz-plane. Increasing the lens separation distance will shift the focus towards the lens for the xz-plane while leaving the focal length in the yz-plane unaffected. This will bring the two focal lengths to coincide with one another giving a minimum spot size.

Figure 4.15: Enlarged view of the focal plane region from Figure 4.13. The right most plane is located at a back focal distance of 65 mm and the left plane is locate at 5.736 mm to the left of the right focal plane.

The lens separation distance between the first focusing lens and the objective achromat was set to a variable parameter along with the final focusing distance. Optimization was
Figure 4.16: Enlarged view of the focal plane region from Figure 4.14. The right most plane is located at a back focal distance of 65 mm and the left plane is locate at 5.736 mm to the left of the right focal plane.

then done with a merit function that minimized the RMS spot radius at the centroid for only the first order diffracted modes. The shift in position gives a separation distance between the first focusing lens and the objective achromat as 168.197 mm and a focal plane shift of 9.168 before the 65 mm plane resulting in a back focal length of 55.832 from the back of the focusing achromatic lens. The new focal length positions can be seen in Figure 4.17 for the yz-plane and in Figure 4.18 for the xz-plane. The spot diagram at the minimum focus can be seen in Figure 4.19 and a zoomed in version for just the first order diffracted modes in Figure 4.20

Figure 4.17: The enlarged focus from Figure 4.15 The new minimized focus is located to the left near the ray crossing and at a distance of 9.168 mm to the left of the focal plane to the right.
Figure 4.18: The enlarged focus from Figure 4.16. The new minimized focus is located to the left near the ray crossing and at a distance of 9.168 mm to the left of the focal plane to the right.

Figure 4.19: Spot diagram of the optimized focal plane minimized for RMS spot size at the centroid. The zero mode is seen to be far larger than the first order modes due to aberrations accumulated through the lens.
Figure 4.20: Enlarged view of Figure 4.19. The airy radius can be seen clearly showing that our spot focus is not diffraction limited.
Ray fan plots for this setup can be seen in Figure 4.21, Figure 4.22, Figure 4.23. All three modes are plotted showing that there is defocus present between the zeroth and first ordered modes. Additionally, due to the 5th order nature of the ray fan plot the dominant aberration is that of spherical aberration.

Figure 4.21: The ray fan plot at the paraxial focal plane for the lens setup for the zeroth order diffracted mode.

From Figure 4.17 and Figure 4.18 it can be seen that the afocal length for the first order modes are now overlapped and as can be seen in Figure 4.19 they are approximately circular in spot size at the focal plane for the first order modes. Looking at Figure 4.20 however we see that this spot size is not diffraction limited.

The next analysis will cover the effective change in the focal spot size at the sample region as the peak wavelength is varied to imitate a spectral bandwidth of the illumination
Figure 4.22: The ray fan plot at the paraxial focal plane for the lens setup for the positive first order diffracted mode.
Figure 4.23: The ray fan plot at the paraxial focal plane for the lens setup for the negative first order diffracted mode.
source. The two wavelengths that will be used are 1020 nm and 1060 nm both at 20 nm from the peak spectrum of 1040 nm previously used.

With the addition of the two secondary wavelengths the focal spot size at the sample plane minimized for the first order modes can be seen in Figure 4.24. As can be seen in Figure 4.24 the spot size varies for each of the three wavelengths while the shape remains the same for all the three diffracted modes. Optimizing separately for each wavelength we then recalculate the minimum RMS spot size at the centroid in the y-direction giving the values listed in Table 4.2 for the new focal length and RMS spot size in the y-direction.

From the data in Table 4.2 it can be seen that the focal system is robust to a bandwidth of light at the tested wavelengths where the effect on the focal length are change by approx-
Table 4.2: Focal plane for three wavelengths of light and their respective spot size in the y-direction.

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Focal Plane Location Relative to the Paraxial Plane</th>
<th>Spot Size in y-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1020 nm</td>
<td>-1.082 mm</td>
<td>16.52984 µm</td>
</tr>
<tr>
<td>1040 nm</td>
<td>-1.169 mm</td>
<td>16.53067 µm</td>
</tr>
<tr>
<td>1060 nm</td>
<td>-1.254 mm</td>
<td>16.53151 µm</td>
</tr>
</tbody>
</table>

imate 0.09 mm for a 20 nm shift in wavelength and the RMS spot size in the y-direction is changed by only 0.001 µm for the same given wavelength shift.

To mitigate the effects present with the defocus on the diffractive order modes changing the objective achromat for a more suitable lens that has reduced spherical aberration. The following analysis will look at replacing the objective achromat with the LSM05-BB objective lens from Thor Labs, the parameters of this lens can be found in Table 4.3. The next lens to be analyzed will be the objective lens system seen in Stirman et al [16]. with lens parameters found in Addendum A. The focusing characteristics of the Stirman lens setup is analyzed in Addendum B.

Table 4.3: Optical lenses used and their corresponding specifications.

<table>
<thead>
<tr>
<th>Objective Lens no.2 - Model LSM05-BB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective focal length = 110 mm,</td>
</tr>
<tr>
<td>Circular aperture radius = 25.8 mm,</td>
</tr>
<tr>
<td>Working Distance = 94 mm</td>
</tr>
</tbody>
</table>

The LSM05-BB lens was replaced in the focal setup to characterize the focus as can be seen in Figure 4.25 for the yz-plane and Figure 4.26 for the xz-plane. The focal plane was located via an M-solve which gives a back focal length of 94.110 mm at 1040 nm wavelength. The separation between the objective lens was set for the combined focal separation at 189.267 mm (95.157 mm + 94.110 mm). The minimized focal plane was once again found by minimizing the RMS spot size for the centroid in the y-direction for only the first order diffractive modes. The shift in focus was found to be 2.075 mm before the paraxial plane.
located at back focal length of 94.110 mm. The spot diagram for the minimum focus can be seen in Figure 4.27.

![Figure 4.25: The ZEMAX setup in the yz-plane with the objective achromat lens replace with the LSM05-BB Rays not shown for the LSM as the optical component is treated as a black box optic.]

![Figure 4.26: The ZEMAX setup in the xz-plane with the objective achromat lens replace with the LSM05-BB Rays not shown for the LSM as the optical component is treated as a black box optic.]

The spot size is comparable to that found in the first setup with the objective achromatic lens. The new RMS spot size in the y-direction for the LSM05-BB is 33.342 µm. The spot size for the achromatic lens was found to be 27.645 µm. From this information it can be seen that the correction in aberrations leaves the spot size no better off than with using the achromatic lens.

Next, the Stirman lens setup was added to ZEMAX replacing the LSM05-BB lens and moved to a separation distance of 106.371 mm. The separation for the first custom achromat was set to the determined zero field separation determined in Addendum B of 35.825 mm. The separation distance after both custom achromats in the Stirman lens setup were set to...
Figure 4.27: Spot diagram for the minimized RMS spot size in the y-direction for only the first order modes. The airy disk is shown where the first order modes are near the diffraction limit.
variable and optimized for the minimum RMS spot size in the y-direction at the centroid. The separation after the first custom achromat was found to be optimized at 35.210 mm and the back focal length at 11.160 mm. The separation distance between the focusing lens and the Stirman objective was thus modified to 106.317 mm to account for the new back focal length. The setup for the optimized lens setup can be seen in Figure 4.28, for the yz-plane and in Figure 4.29 for the xz-plane. The spot diagram for the focal plane can be seen in Figure 4.30.

![Figure 4.28](image)

Figure 4.28: The ZEMAX setup in the yz-plane with the objective achromatic lens replaced with the Stirman lens array from Addendum A.

![Figure 4.29](image)

Figure 4.29: The ZEMAX setup in the yz-plane with the objective achromatic lens replaced with the Stirman lens array from Addendum A.

As can be seen in Figure 4.30 the spot sized for the first order diffracted light is below the diffraction limit in the y-direction. The spot size in the y-direction was calculated to be at 3.8392 µm. This spot size is significantly improved over the objective achromatic lens and the LSM05-BB lens previously analyzed.

Once again we analyze the improved performance using the Stirman lens system in a double pass collection setup with the same focusing achromat previously used for the double
Figure 4.30: Spot diagram for the minimized RMS spot size in the y-direction for the first order modes. The airy disk shown encompassing the rays from the first order mode suggest diffraction limited performance for the first order modes.
pass analysis. The position of the focusing achromat is once again set to 155.86 mm after the cylindrical lens to ensure equal focusing in the yz-plane and the xz-plane. The separation distance between the Stirman lens system and the focusing lens was set to variable along with the dummy surface for the final focal plane. A 1-inch aperture is place 10 mm before the Stirman lens system to simulate the aperture of a waveplate. The lens setup was then optimized for the minimum RMS spot size on the first order modes only.

The unfolded setup including the Stirman lens setup can be seen in Figure 4.31 in the yz-plane and in Figure 4.32 for the xz-plane. The optimized separation distance from the focusing lens to the waveplate is 79.415 mm. The distance form the focusing achromat to the back paraxial plane is 56.533 mm with the minimized spot located at 0.689 mm before the paraxial plane. The spot diagram for this optimized spot size can be seen in Figure 4.33.

Figure 4.31: The double pass system layout for the Stirman lens in the yz-plane. The focusing achromatic lens is seen at the right focusing light to the focal plane.

Figure 4.32: The double pass system layout for the Stirman lens in the xz-plane. The focusing achromatic lens is seen at the right focusing light to the focal plane.

As can be seen in Figure 4.33 the focus is near diffraction limited in performance. The shape of the the focus is also improved by being more localized to the central point and with
Figure 4.33: Spot diagram at the minimized focal plane for the RMS radius at the centroid for the first order modes. The zero order mode is elongated in the lateral direction as a result of the afocal nature of the system.
minor lateral shift for the first order spots.
CHAPTER 5
RANDOM ACCESS SPIFI SETUP

Through the development of SPIFI, imaging has developed from point source scanning to "light sheet" scanning to increase the overall illumination area that can be scanned at a given scan cycle. By extending the point source to a "light sheet" there is an accompanying reduction in illumination intensity for any given laser light source that ultimately limits the extent to which the "light sheet" can be broadened. Additionally, previous SPIFI methods have relied on the sequential scanning of a sample through all areas of interest or by 2D methods that capture a wide field illumination of the sample. To further reduce total scan time the implementation of a random access scanning method would allow for scanning only the areas of interest. Presented here is a SPIFI setup that allows for random access scanning in 3D.

The illumination source is a mode-locked fiber laser operating at 1035 nm peak wavelength with a 10 nm bandwidth and a pulse rate of 71 MHz. The objective lens used is a Zeiss A-Plan (441050-9903-000 Objective A-Plan 40x/0.65). The theta-lens used was a ThorLabs F-Theta Lens (FTH100-1064) with a working distance of 97.8 mm and a maximum scan angle of ±28° resulting in a 70 mm x 70 mm scan field. The 2D scan mirror system is a Galvanometric silver coated mirror from ThorLabs (GVS012) with a voltage sweep of 0.8 Volts per degree.

A simplified diagram for the experimental setup can be seen in Figure 5.1 with the vertical scan setup in Figure 5.2. Light emitted from the fiber laser setup is passed through a half-wave plate adjusted to allow a small percent of the signal to be reflected off the polarizing beam splitter (PBS) onto a photodiode to measure the pulse characterization of the laser. Remaining light is then passed through a mirror array system for alignment and passed through another half-wave plate designed to prepare the polarization for the spatial light
modulator. Light is then passed through a telescope lens system consisting of a 35 mm focal length 1" diameter focusing lens and then through the edge of a 100 mm focal length 2" diameter lens. The expanded beam is enlarged by a magnification factor of $M \approx 2.86$ and is then focused onto the face of the spatial light modulator. The beam is then modified by the spatial light modulator reflecting back through the 100 mm focal length lens. The light then passes through another half-wave plate to allow full transmission through the second polarizing beam splitter. Light passes through the quarter wave plate and then through the 2D galvanometric mirror setup where light is scanned through the F-theta lens. Light is then focused through the theta lens onto the mask where the modulated light is then reflected back through the theta lens and realigned with the optical axis. The beam then passes through the quarter wave plate again to ensure total reflection off the polarizing beam splitter. Light is then directed into the vertical scan setup. The modulated beam that enters the vertical scan setup is then passed through the objective lens and focused onto the sample. Radiated light from the sample is then collected by the objective lens and reflected off a dichroic. The dichroic is designed to transmit light fully at the wavelength of the incident light and to reflect totally the light emitted by the sample for the particular multiphoton process being observed. The reflected light is then collected by a photo-multiplier tube and the time-voltage signal is relayed to the data acquisition module.

The need for precision and control of the objective focus for microscopy and laser machining is highly sought after. Current methods exist by which a conjugate mirror is shifted axially to create defocus in the propagated light that will result in a shift in the focal depth of the objective lens. This method often requires the reproduction of the objective lens and a larger design setup. Here we present a method that does not required the replication of an objective lens in the system and can be incorporated with less restriction on positioning and field of view of the objective lens.

A Spatial Light Modulator (SLM) is a reflective optical component that modulates incident light by creating spatial variations in the diffracted light. Modulation can either be on
Figure 5.1: Setup for reflective SPIFI measurements. The vertical scan setup can be seen in Figure 5.2

Figure 5.2: Vertical scan setup for SPIFI measurements. Modulated light enters vertically to the objective lens and is focused onto the sample. Fluorescent light is then collected by the objective lens and reflected off a dichroic onto the photo-multiplier tube.
the intensity of light, on the phase, or by some combination of the two. The use of SLMs is applied extensively to generate holographic displays that take advantage of the interference of diffracted light off the SLM and specialized patterns called computer generated holograms (CGH). The use of CGH to shape beams of light is called beam shaping. A common beam shaping method in use is in projectors to display images.

The conjugate domains present in the focal plane of a focusing lens and the far field allow for optical transformation into the Fourier Transform on the far field intensity distribution. Taking advantage of this property a SLM can be used for pulse shaping of ultra-short light pulses.

Creating uniform spatial variations on the SLM an incident beam will be diffracted in the same manner as a diffraction grating. This allows for an incident beam of light to have its power redistributed to any number of harmonic orders. By carefully choosing the modulation pattern a diffracted beam can be imparted with a curved wavefront, which when focused by a lens will defocus the beam. Combining the diffraction of a single beam into multiple beams that can each be remotely focused by imparting a defocus we can created a remote focusing setup that can be controlled digitally and without the need for moving parts, see Figure 5.3.

The modulation mask for an SLM that would give the desired three diffracted beams can be seen in Figure 5.4. The modulation is achieved by taking discrete values from the modulation function:

\[ m(x) = \frac{1}{2} + \frac{1}{2} \sin(2\pi kx) \]  

where the modulation is in the x-direction with the spacing parameter \( k \). All pixel values are between 0 and 1 corresponding to normalized phase values using the look-up table (LUT) calibrated for the specific mask.

A CCD camera image can be seen in Figure 5.5 demonstrating the three beams created from the diffraction pattern coded on the SLM. Using both a phase and amplitude mask the desired defocus can be encoded onto each of the three beams.
Figure 5.3: Diffraction of beam and wavefront shaping using an SLM. Each diffracted order \{-1, 0, 1\} is imparted with a curved wavefront that will focus to their respective focal planes \{f_{-1}, f_0, f_1\}.

This particular SPIFI setup allows for control in the number of diffracted beams and in their relative position and defocus imparted by the SLM. Through these variables any number of beams can be generated to scan at controlled positions and each at a tunable depth giving a high degree of freedom in random access scanning microscopy.
Figure 5.4: Modulation mask on the SLM that to give diffraction of three beams seen in Figure 5.5 The mask is an discrete sine wave on the mask with pixel values ranging from 0 to 1.

Figure 5.5: Three spots measured on the CCD camera using a diffraction mask on the SLM and having a mirror replace the SPIFI mask. The reduplication of the three spots vertically is a result of back reflection in the setup.
CHAPTER 6
CONCLUSION

In summation this thesis has outlined the numerical calculations for both simple ray tracing and diffraction based SPIFI approaches. The Fraunhofer diffraction analysis exactly replicates the optical response seen in the lab for rotating mask SPIFI systems. However, the ray tracing analysis done in ZEMAX has shown that simple optical components such as achromat and flat field lenses exhibit a high degree of aberrations that result in axial defocus of the diffracted modes from one another based on the order of diffraction. These aberrations can be corrected for with specially designed optics, such as the Stirman lens array system analyzed in this paper, to yield reduce axial spread of the diffracted modes.

Lastly, a novel SPIFI system was introduced that allows for random access imaging with three distinct spots which would increase the acquisition rate by up to three time that of a single spot scanning method. This setup would also allow for selective imaging of only the sample region of interest, thus eliminating the need for full range scanning as is used in most fixed scanning array setups.
REFERENCES CITED


The listed specifications in Figure A.1 are corrected for the new part designations for the OptoSigma lenses. Additionally, added precision and a correction to surface 71 radius are made the correct values are given in Table A.1. The materials are also updated for parts incorrectly labeled as N-BK7 to their correct material of S-BSL7.

Table A.1: Corrections to the specification listen in Figure A.1. Changes are in bold.

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<th>Thickness (mm)</th>
<th>Material</th>
<th>Semi-Diameter</th>
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</thead>
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</tr>
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<td>SLSQ-30-50N</td>
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</tr>
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<td></td>
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</tr>
<tr>
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Supplementary Figure 6
Stirman et al.

Objective
Surface #

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Supplementary Figure 6. Full prescription data for the objective lens.
The objective lens was constructed from COTS components and two custom cemented achromats.
The objective has ~8.5 mm of working distance. The axial separation on surface 76 and decenter of
custom lens 1 (red outline) are used as compensators in the objective assembly. Focus compensa-
tion occurs at surface 83.

Figure A.1: Lens prescription data for Stirman et al lens.
APPENDIX B
CHARACTERIZATION OF STIRMAN LENS ARRAY

The characterization of the Stirman lens array was done at a wavelength of 1040 nm with three field angles at zero and five degrees. The entrance pupil diameter was set to 10 mm. Adjustment of the compensators was made to optimize for the best RMS spot radius at the centroid by making the thickness after the two custom achromat lenses variable. The optimized thickness values are 14.008 mm after the first custom achromat lens and 11.841 mm after the second custom achromat lens where the focal plane is located. The diagram for the ray trace with optimized lens separations can be seen in Figure B.1 and the corresponding spot diagram in Figure B.2 for all three field angles.

![Figure B.1: ZEMAX layout for Stirman lens array with infinite conjugate objective. Three field angles are drawn with five rays each.](image)

The five degree field angles were then removed and optimization was done on only the zero field angle. The optimized thickness after the first custom lens is 35.825 mm and the back focal length is 11.214 mm. The spot diagram for the zero field angle optimization can
Figure B.2: Spot diagram at the minimized focal plane for the Stirman lens array. Three field angles are drawn.
be seen in Figure B.3

Figure B.3: Spot diagram at the minimized focal plane for the Stirman lens array for zero field angle.

As can be seen in Figure B.3 the focus for a zero field angle ray will be diffraction limited at 1040 nm wavelength.