MINERAL ASSET VALUATION UNDER PRICE UNCERTAINTY USING REAL OPTIONS

by

Marko Visnjic
A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science (Mining Engineering).

Golden, Colorado
Date ______________________

Signed: ____________________
Marko Visnjic

Signed: ____________________
Dr. Kadri Dagdelen
Thesis Advisor

Golden, Colorado
Date ______________________

Signed: ____________________
Dr. Priscilla Nelson
Professor and Head
Department of Mining Engineering
ABSTRACT

Production scheduling under commodity price uncertainty has suffered from an exponentially increasing problem size as simulations and real options flexibility are used to generate and evaluate production schedules. The use of unsupported commodity price behavior mechanisms and fundamental problems in risk adjustment have resulted in incorrect treatment of commodity price risk in production scheduling. Previous work on this subject has considered both the real options and production scheduling components yet has failed to honor both simultaneously and integrate them completely. The proposed methodology maintains a problem size similar to that of a deterministic solution yet fully adjusts the production schedule for market attitudes towards commodity price risk. Five economic scenarios consisting of the proposed methodology, price simulations and traditional discounted cash flow (DCF) are explored using a commercially available production scheduling package. It is concluded that the proposed methodology provides a supportable and risk adjusted basis for production scheduling. Using simulations shows that mine plans cannot be evaluated against price paths as has been done in previous work; yet results in an impossibly large problem size when done correctly. As the traditional DCF scenarios are heuristically selected they are unsupportable and consequently result in unsupportable production schedules and valuations.
TABLE OF CONTENTS

ABSTRACT ................................................................................................. iii
LIST OF FIGURES ....................................................................................... vii
LIST OF TABLES .......................................................................................... ix
ACKNOWLEDGMENTS ................................................................................ x

CHAPTER 1 INTRODUCTION ................................................................. 1
  1.1 Problem Statement ........................................................................... 2
  1.2 Scope of Work .................................................................................. 4
  1.3 Methodology .................................................................................... 8
  1.4 Objective ........................................................................................... 9

CHAPTER 2 LITERATURE REVIEW .................................................. 10
  2.1 Previous Work ................................................................................ 10
  2.2 Real Options ................................................................................... 19

CHAPTER 3 ECONOMIC PARAMETERS ............................................. 23
  3.1 Introduction ...................................................................................... 23
  3.2 Real Options Valuation .................................................................... 23
  3.3 Geometric Brownian Motion Commodity Price Movement .......... 24
  3.4 Discounting ...................................................................................... 26
    3.4.1 Discounting for Time Value of Money ...................................... 26
    3.4.2 Risk Adjustment ........................................................................ 27
    3.4.3 The Binomial Lattice Method .................................................... 32
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Binomial lattice depicting probabilistic realization of a state variable &quot;X&quot; and the number of downstates in the first index followed by the period in the second index.</td>
</tr>
<tr>
<td>2.1</td>
<td>Depiction of traditional DCF analysis.</td>
</tr>
<tr>
<td>2.2</td>
<td>Depiction of real options cash flow analysis.</td>
</tr>
<tr>
<td>3.1</td>
<td>Example binomial tree of state variable &quot;X&quot; with the first index indicating the number of down moves and the second index being the period. The true probabilities of up and down moves are given as $\theta_u$ and $\theta_d$ respectively.</td>
</tr>
<tr>
<td>3.2</td>
<td>Cash flow binomial tree corresponding to the behavior of the state variable in Figure 3.1.</td>
</tr>
<tr>
<td>3.3</td>
<td>Binomial lattice showing risk adjusted probabilities being solved by backward induction.</td>
</tr>
<tr>
<td>3.4</td>
<td>Gold Price from August 2nd, 2010 to November 3rd, 2017. (Slight offset in the time scale due to non equal number of trading days per year.)</td>
</tr>
<tr>
<td>3.5</td>
<td>Futures contract prices, actual and extrapolated.</td>
</tr>
<tr>
<td>3.6</td>
<td>Gold Price Simulation 7, deviation from the average: 1.77%.</td>
</tr>
<tr>
<td>3.7</td>
<td>Gold Price Simulation 5, deviation from the mean: 0.22%.</td>
</tr>
<tr>
<td>4.1</td>
<td>McLaughlin Deposit Ultimate Pit.</td>
</tr>
<tr>
<td>4.2</td>
<td>Block Destination Flowchart.</td>
</tr>
<tr>
<td>4.3</td>
<td>Geometric Phasing.</td>
</tr>
<tr>
<td>4.4</td>
<td>Grade cross section.</td>
</tr>
<tr>
<td>4.5</td>
<td>Geometric Phases with 0.05 oz/ton grade.</td>
</tr>
<tr>
<td>4.6</td>
<td>Geometric Phases with 0.1 oz/ton grade.</td>
</tr>
</tbody>
</table>
Figure 4.7  Period 3 Cross Section ..................................................... 57
Figure 4.8  Period 5 Cross Section ..................................................... 57
Figure 4.9  Period 3 and 5 Cross Section .......................................... 57
Figure 4.10 Surface map depicting ore tons processed by each production schedule at each period .................................................... 59
Figure 4.11 Surface map depicting waste tons mined by each production schedule at each period ..................................................... 59
Figure 4.12 Surface map depicting strip ratio processed by each production schedule at each period ................................................. 60
Figure 4.13 Average grade per period for each mine plan ...................... 60
Figure 4.14 Total material movement for the futures curve production schedule. Excluding stockpile reclaim to the mill, all material movement satisfies the yearly mining fleet capacity (shown as horizontal line). This is important for ensuring efficient use of capital. ................................. 61
Figure 4.15 Total material movement for the Price Path 1 production schedule. Excluding stockpile reclaim to the mill, all material movement satisfies the yearly mining fleet capacity (shown as horizontal line). This is important for ensuring efficient use of capital. ................................. 62
Figure 4.16 Total material movement for the futures curve production schedule. Due to the higher strip ratio of this schedule relative to that of the futures curve, a large mining fleet is needed to fill the mill in each period. ......................................................... 62
Figure 4.17 Price paths that generate mine plans which perform as expected when evaluated against that price path. .............................. 65
Figure 4.18 Price paths that generate mine plans which do not perform as expected when evaluated against that price path. .............................. 66
LIST OF TABLES

Table 3.1  Futures contract prices and Treasury STRIPS ......................... 37
Table 3.2  Gold convenience yield ...................................................... 37
Table 3.3  Extrapolated futures .............................................................. 38
Table 3.4  Summary of futures contracts .................................................. 38
Table 4.1  Legend for the color coded grades for the McLaughlin Deposit ........ 44
Table 4.2  UPL Design parameters ........................................................ 45
Table 4.3  Production schedule design parameters ...................................... 46
Table 4.4  Cutoff grade parameters .......................................................... 47
Table 4.5  Block Destinations ................................................................. 47
Table 4.6  Traditional Scheduling Scenarios .............................................. 50
Table 4.7  Risk adjusted prices and time value of money .............................. 54
Table 4.8  Example NPV calculation ........................................................ 55
Table 4.9  Valuation results ................................................................. 55
Table 4.10  Production schedule summary ................................................. 58
Table 4.11  NPV comparative color map ................................................... 63
Table 4.12  Expected performance NPV comparative color map ...................... 64
Table 4.13  Unexpected performance NPV comparative color map ................... 65
ACKNOWLEDGMENTS

I would like to express my gratitude to the many people that have supported me throughout this process, without them none of this would have been possible. First and foremost I would like to thank my advisor, Dr. Kadri Dagdelen. It has been a privilege to work alongside such a gifted and bright man that has helped me develop greatly throughout our time working together. Mostly, I am ever grateful for his unwavering support and believing in me from day one. I wish to thank Dr. Thys Johnson for his support and invaluable ideas that have greatly assisted me with the development of this work; however, it is his kindness and ever-willingness to help that I will treasure most. Dr. Hugh Miller, it has been an honor to learn from your seemingly infinite experience and knowledge; combined with your immeasurable kindness you have been, a continue to be, a role model for me, thank you.

I wish to thank Dr. Graham Davis for his enthusiasm and assistance in helping me establish this thesis. His infallible knowledge of mining valuation and ability to convey it was instrumental in the successful completion of this work.

I would like to extend my deepest thanks to Robert Slade from Maptek for taking a strong interest in my project, I have greatly enjoyed our many days and nights working together. Your seemingly tireless dedication, enthusiasm and helpfulness not only helped me complete my thesis but has taught me more than I could have asked for.

I have been fortunate to have been surrounded by the very bright minds of the CSM faculty. At the risk of unfairly leaving some out I would like to thank, Dr. Ugur Ozbay, Dr. John Grubb, Dr. Jurgen Brune, Dr. Alexandra Newman, Dr. Jamal Rostami, Dr. Ben Gilbert, Professor William Wilson and Dr. Ian Lange.

To the fellow students at CSM that I have had the pleasure of meeting, thank you for inspiring, teaching and supporting me. In particular I would like to mention my two friends Raul Alejandro Chavez Trillo and Daniel Barba for their enduring friendship and for passing
along their wealth of knowledge and experience to me. While only colleagues for a short time, I am thankful to Ady Van Dunem for his help in starting my journey at CSM and for his friendship during our time together. I would like to thank my friend and colleague Tyler Rockley, I will cherish his selflessness and the tireless days and nights we spent working together. I am very fortunate to have met my friend and colleague Canberk Aras while at CSM. I will treasure our long insightful discussions, always being able to rely on you, and most importantly, your true friendship.

Finally, I would like to thank my parents Boris and Lidija and my brother Luka for their constant support and investment in me. Without the solid foundation they have always provided me, none of this would have been possible.
CHAPTER 1
INTRODUCTION

The mining production scheduling problem seeks an answer to a problem that asks three questions given a set of parameters: what to extract, when to extract it and where to send it. This problem has been approached by many yet remains to be solved to optimality successfully. Traditionally, the open pit mining production scheduling problem has been approached with the traditional discounted cash flow valuation methodology. Traditional discounted cash flow valuation assumes a constant price and a traditional discounted cash flow analysis to determine the net present value (NPV) of the asset. Problems arise with the traditional methodology when input parameters of the production scheduling problem exhibit stochastic behavior.

A large driver of mineral asset value is the commodity price. While impossible to predict the future, econometrically parameterized stochastic models have been developed that simulate the probabilistic behavior of mineral commodities (Dixit & Pindyck (1994)). However, the application of those models to the mining production scheduling problem to optimize under uncertainty has been a rather new topic. Concurrently, from an investment analysis standpoint, much work has been done in attempting to provide investors in mining projects the ability and tools to produce accurate valuations for their projects (Samis et al. (2006) and Davis (1998)). The most significant of these tools is the real options methodology which not only seeks to encompass management flexibility in project investment and operation but redefines the way projects are risk adjusted.

The combination of both fields of study can be categorized under the field of stochastic mine planning which seeks to maximize value from mining operations under uncertainty. While stochastic mine planning generally encompasses mine planning under grade and price uncertainty, the focus of this study is the relationship between commodity price uncertainty and the mine plan. This problem has been approached from multiple aspects by Espinoza
et al. (2013), Dimitrakopoulos & Abdel Sabour (2007), Ramazan & Dimitrakopoulos (2013), Del Castillo & Dimitrakopoulos (2014), Chatterjee et al. (2016), Salama et al. (2015) among others. Within each of their respective works there lie a few fundamental assumptions that bring attention to the real options methodology as it pertains to mine planning. The first of these is the risk adjustment of the revenue streams due to commodity price uncertainty; the second is the combination of price simulations with mine plans generated under traditional discounted cash flow assumptions or by a secondary and independent stochastic parameter.

The primary objective of this study is to provide an analysis of open pit mine production scheduling under commodity price uncertainty taking a fundamental approach to real options valuation, a technique that uses market securities to adjust for risk. Throughout the process, implications regarding value parameterized nested pits and evaluating price simulations against mine plans by Monte Carlo simulation will be explored.

1.1 Problem Statement

Due to the complexity of the combined problem of modern finance applied to production scheduling previous approaches tend to focus on either the mine planning aspect or the real options aspect. This results in a discrepancy in that either the mine planning is not done realistically, or the real options component is done without taking into consideration the correct method of discounting for risk in the project valuation. Often the focus of investigations into applying real options to mine planning is on the management flexibility available with the method. This flexibility is applied to delaying or abandoning the project in the face of uncertainty and in some works, temporarily closing the mine until more favorable market conditions exist. While the flexibility of the operation adds value, its application often overshadows the more fundamental and important difference in real options valuation, risk adjustment. Because of this overshadowing, this study will omit it entirely and focus on the more fundamental aspects to real options as applied to mineral asset valuation.

There are certain aspects to real options that pose a unique challenge to mineral asset valuation, a large source of these challenges is due to the heterogeneity of deposits, the
respective method of extraction and timing and sequencing constraints. These aspects of mining and the objective of maximizing value from these deposits gives rise to the production scheduling problem. Narrowing the problem scope to an open pit mining method results in the Open Pit Mine Production Scheduling Problem (OPM-PSP) which has been a focus of study since first being formalized by Johnson (1968). Methods to solve the OPM-PSP have generally used Mixed Integer Programming (MIP) models which are computationally costly and aim to return a global optimum given a set of input parameters. This solution methodology comes to odds with the real options solution method of binomial lattice reverse induction shown in Guthrie (2009) in that the OMP-PSP would need to be solved for all paths leading to the set of terminal nodes at the final period of the project life. For a mining asset with a life of 10 years and a single source of uncertainty following a Mean Reverting or Geometric Brownian Motion stochastic process, the binomial lattice would contain 2 to the 10 or 1024 paths, equating to 1024 OMP-PSP problems needing to be solved before a proper real options valuation can be made. Considering the large complexities of a mining operation with multiple sources of uncertainty it is easy to extrapolate the fact that the problem size becomes very large, very quickly. Methods to solve the OMP-PSP considering uncertainty include the works of many who have taken several different approaches to solve the problem in a computationally feasible and implementable manor without successfully obtaining a true optimum. Due to computational limitations an alternative method that approximates the real options solution must be explored. In this study, commercially available software that solves the OMP-PSP is used to demonstrate the agile and dynamic method that a fundamental real options valuation can done on a mining asset.

When exploring the topic of commodity price uncertainty impact on mine planning, often econometrically parameterized simulations of the commodity price behavior are used to analyze a mine plan’s robustness against different realizations of commodity price. This approach either makes use of Monte Carlo simulations to rapidly evaluate many mine plans against individual simulated commodity price paths or is considered in a risk parameterized
MIP or stochastic integer program (SIP). These methods allow for a computationally manageable and tractable approach to incorporating uncertainty in the mine planning problem. However, within these methods lies a significant problem, with each new commodity price path against which a mine plan is evaluated there is a unique mine plan corresponding to it as the definition of ore would fluctuate on a period by period basis. Therefore it is not possible to simulate a mine plan’s robustness against multiple commodity paths as the mine planners would respond to the changes in price with a new and optimal mine plan for each realization of price.

The problem that this work aims to address is how to perform a real options valuation on a mineral asset in a practical manor given computational limitations while honoring the fundamental risk adjusting principles of real options valuation. By attempting the solution to this problem, the parallel aspect of performing Monte Carlo evaluations of price path simulations against mine plans will be addressed as well in support of the hypothesized methodology.

1.2 Scope of Work

The study will focus solely on deposits that are chosen to be mined by means of open pit and thus applicable to OPM-PSP solution methods. As previously stated, the OPM-PSP is a well documented and understood problem that has relatively standard sequencing relationships. Conversely, the underground production scheduling problem is reliant on the mining method chosen and several other infrastructural components that make the problem harder to solve with fewer commercially available solutions to do so. Due to this, a method that tractably illustrates the objectives of both focus areas must be selected. While real options valuation can be applied to all mining assets and is method agnostic; the production scheduling problem is relatively uniform for open pit operations when compared to the underground production scheduling problem. For this reason, the study will be applied solely to an open pit operation.
To avoid any complexities arising from production scheduling from a complex of open pits, the deposit selected will only be mined by a single pit feeding a single mill. There will be five destinations that material from the pit can be sent to: the waste dump, a high and low grade stockpile, the mill overflow stockpile and the mill directly. By using a simple set of destinations as opposed to a larger network of processing options, the salient features of this study will be clearly demonstrated.

A key aspect to real options valuation is how risk adjustment is performed. All risky components of a project are adjusted independently at their location within the cash flow structure. To simplify this study, the sole source of uncertainty will be the commodity price. Focusing on the commodity price alone will provide a clear example for the methodology of real options valuation that is undiluted by other sources of uncertainty.

While optionality and management flexibility are powerful tools that real options valuation implements, their use often results in a distraction from the underlying fundamentals of the methodology. For this reason, all schedules and valuations generated will be assumed to be now or never investment opportunities that once operating cannot be suspended and resumed or terminated prior to the ten-year life of mine. The only exception to early termination is if the scheduler runs out of ore due to a low price realization and a consequently high cutoff grade.

Commodity prices behave according to various forms of stochastic partial differential equations that describe the evolution of a parameterized entity through time. Commodities are known to move according to two forms of these stochastic differential equations, the Mean Reversion Process (MRP) and Geometric Brownian Motion (GBM) also commonly known as a random walk (Dixit & Pindyck (1994)). The first step to modeling the behavior of the commodity price in question is to test whether it is a random walk or mean reverting. A unique feature of random walks is that they do not have a stable mean (Tsay (2010)). From a mineral investment perspective this is important in that planning around an average or expected price is very difficult being that there is no stable mean on which to base forecast
results on. In this study, gold is the primary commodity present in the deposit and therefore is tested for mean reversion. It is concluded that it is in fact a random walk and thus parameterized as such for calibrating the model used in this study for production scheduling and valuation purposes.

Within the deposit selected to exemplify the concepts in this study, there are two salable commodities, gold and silver. Generating production schedules using simulated price paths for more than one stochastic variable requires pairing the variables together in an equiprobable manner to generate a representative set of price realizations. Alternatively stated, any simulation outcome of gold is equi-probable with any simulated outcome of silver. If gold and silver prices are correlated, this interaction would need to be included in the model to replicate more representative combinations of gold and silver price to be used in generating the production schedule and asset values. It is apparent that the problem size grows quite quickly as more stochastic variables are added into the valuation. To simplify the problem at hand, any silver in the deposit is ignored and treated as waste.

To reach a valuation a time frame defining the life of mine must be established before any further steps can be taken. This study simulated the gold price over the course of ten years with the first year being the start of mine production. For the problem to remain computationally tractable, ten representative simulations of the risk adjusted gold price were selected, from which to generate production schedules from. Fifteen simulations of ten price paths were generated and one simulation of ten was selected for use in this study. By specifying a ten-year time frame for the Ito process solution, the simulated price paths only extended from year one to ten. An implication of this is that there is no way to account for any revenue or cost generating mining activity past the ten-year life of the simulations. To extend the simulation life by extrapolating values forward would violate the random walk nature of the simulation and invalidate the results. Consequently, any output from the scheduling software past ten years is ignored and not factored into value or cost calculations.
Continuing in the vein of computational limitations, executing a true real options valuation for mineral assets under commodity price uncertainty with the current solution methodology is extremely computationally costly if not impossible as stated previously due to the binomial lattice. The lattice is expanded out for the life of the project with a node at each time step, generally a year in the mining case. The nodes represent the different states of the stochastic entity in question, referred to as the state variable.

Figure 1.1: Binomial lattice depicting probabilistic realization of a state variable "X" and the number of downstates in the first index followed by the period in the second index

Figure 1.1 shows an example lattice illustrating the probabilistic movement of the state variable, labeled X, through time. The movements up or down in the lattice are characterized by the true probabilities of up and down moves $\theta_u$ and $\theta_d$ respectively. These probabilities stem from the underlying stochastic process driving state variable behavior. The first subscript indicates whether a down move has occurred and the second subscript indicates the period. As Figure 1.1 illustrates, there are two periods and 4 possible paths to arrive at each of the nodes. The number of paths in a binomial lattice is an exponential function of periods, such that a binomial lattice with $n$ periods will have $2^n$ paths. For the ten-period example done in this study, there would be $2^{10}$ or 1024 possible paths to reach the terminal nodes. With respect to period by period mine planning, this would equate to 1024 separate production schedules that would need to be generated to use a backward induction approach.
to value the asset. The task of generating 1024 production schedules given current scheduling algorithms and computational power required is very large. It is apparent that given the computational limitations at the time of this study, an alternative approximation approach will need to be taken.

1.3 Methodology

The deposit selected for this study is the historical McLaughlin orebody located in northern California. This orebody was mined by the Homestake Mining Company from 1985-1996 as an open pit. Once the deposit’s primary commodity is identified the next step is to gather the commodity price data. As McLaughlin’s primary commodity was gold, the gold price from a specified past date is retrieved from the Federal Reserve Economic Data (FRED) and processed through statistical programming language R. "R” allows for time series data to be imported from external sources and manipulate it using pre-programmed packages. Using the Augmented Dickey Fuller test the gold price time series is determined to be a random walk before being parameterized as a GBM process. Using gold futures contract data from the Chicago Mercantile Exchange (CME) the market risk adjustment for the gold price is established. As gold futures contracts are only available for five years another five years needed to be extrapolated by the method laid out in Guthrie (2009) to be able to parameterize the simulation for all ten years of mine life. Using the futures contract curve, the Ito process describing the behavior of the gold price was parameterized and fifteen simulations of ten price paths were generated. The most representative simulation of the simulation’s yearly expectation values was chosen. With the risk adjusted price paths determined, the mine’s ultimate pit was determined using the Lerchs Grossman algorithm (Lerchs & Grossman (1965)).

Given the ultimate pit, an initial attempt was made to generate mine plans using the MiningMath SimSched Direct Block Scheduler (DBS) production scheduling software. The attempt failed in that SimSched is not a period by period scheduler requiring that all block values for all destinations be predefined before being processed by the solver. By predefining
the block values a dynamic cutoff grade strategy that correlates with a yearly varying price was not implementable. Due to the nature of the research topic, a production scheduling program that can handle a dynamic price environment was needed. To accomplish this Maptek’s production scheduling software Evolution was used. Evolution is a meta-heuristic genetic algorithm production scheduling software with two levels of production schedule optimization. Evolution Strategy is a bench-by-bench scheduler that dynamically optimizes cutoff grade policies while maximizing NPV. Evolution Origin is a block-by-block production scheduler that allows for a variable definition of ore for each block destination on a period by period basis. Origin’s genetic algorithm uses a primary and secondary objective system to rank generated schedules. Primary objectives consist of operational targets such as material movement of either total tons or ore tons, or equipment utilization targets. Secondary objectives are NPV, blending and min-max boundary limits. The genetic algorithm will return several schedules that satisfy the primary objectives and then are ranked based on the performance of the secondary objectives. The two objectives used in this study are material movement as the primary and NPV as the secondary. Origin is used to generate production schedules for all economic scenarios explored. With all production schedules generated, the hypothesis of performing a fundamental real options valuation on a mining asset under current computational limitations and its implications with evaluating commodity price paths against individual production schedules is tested.

1.4 Objective

This study proposes a practical method for implementing the economic input parameters needed for solving the production scheduling problem. The methodology is compliant with real options valuation techniques and honors the production scheduling process; simultaneously reducing the problem size when considering mine planning under commodity price uncertainty.
CHAPTER 2  
LITERATURE REVIEW

2.1 Previous Work

An effort has been made to incorporate uncertainty in mine planning from various groups approaching the problem from different angles. The facets of uncertainty considered generally have been grade and geological uncertainty within the deposit as well as uncertainty in the commodity price. Often the proposed solution methodology considers formulating the PSP in a deterministic sense as well as with a stochastic component included. Once formulated the problem is attempted to be solved to optimality given the uncertain parameters. The objective of this is to create a single mine plan that minimizes risk while still seeking the highest value.

A prevalent theme throughout the works reviewed is the discrepancy between the production scheduling and application of uncertainty to the problem. While geologic and grade uncertainty is often well described and parameterized, commodity price behavior is often assumed to follow a particular stochastic mechanism without any supporting arguments to justify that assumption. Without a justified argument to characterize commodity price behavior, there is a risk that the selected mechanism will be inaccurate and thus yield an incorrect solution. Conversely, the uncertain behavior of the commodity price may be correct but the production scheduling problem may not be well defined and have oversimplifications that distort results. The works reviewed have several pertinent issues with the approaches taken, some of which this study aims to address. These topics include: the correct combination of simulation parameters, discount rate selection, market and technical risk discounting and pairing production schedules with simulation realizations.

Ramazan & Dimitrakopoulos (2013) approach the production scheduling problem under uncertainty by incorporating grade uncertainty as a stochastic mixed integer problem (SIP).
The objective of the SIP is to generate a production schedule that minimizes geological risk using the logic that by deferring the higher risk areas to later in the production life cycle, more drilling will reduce that uncertainty allowing mine planners to better respond. As an input into the SIP, the authors simulated 15 orebody realizations and accounted for uncertainty in the grades by using an orebody risk discounting concept that emulates the desire to mine areas with higher confidence first and delay the mining of less certain areas to future dates when it is assumed that more information will become available. In order to obtain the NPV, resultant cash flows were discounted at an arbitrary 10 percent per year. SIP generated mine plans were compared with a traditional mine planning approach that does not prefer to mine less risky areas earlier in the project life. The conclusion of the study is that using a risk averse mine planning strategy that prefers to mine areas of higher certainty and higher value first reduces the likelihood of production shortfalls in earlier years maximizing value.

The study done by Ramazan & Dimitrakopoulos (2013) examined generating mine plans that correspond to equiprobable orebody simulations and optimizing the production schedule to meet a mill feed target early in the project life cycle. Risk is discounted for in two ways, with the geological risk discounting (GRD) that penalized areas of less grade confidence by applying an orebody discount rate and by using a traditional risk adjusted discount rate of 10%. The GRD is a concept unique to mine planning based on orebody simulation and is outside of the scope of this study, however the cash flow discount rate is relevant to this study and will be addressed. By selecting 10% the authors are stating that there is additional risk in the model that is unaccounted for. Any discount rate that exceeds the pure time value of money discounting contains a penalization for risk. The origin of this risk is not stated in the study and it is assumed that it is selected as is consistent with common practice. This is illuminating because it shows that risk is not accounted for in the cash flows directly therefor later cash flows are reduced exponentially at a rate greater than the time value of money. If uncertainty in the commodity price is the desired target of the 10% rate, a more
appropriate method to address this would be to discount the commodity price using market
derived attitudes towards risk, such values can be retrieved from the commodity futures
markets. If technical risk is the target, that uncertainty would be penalized in the cash
flows directly by reducing their expected value. Implementing this methodology, discounting
occurs directly with the components that are risky assigning the correct discount magnitude
on a period-by-period, item-by-item, basis.

Haque et al. (2014) explore the use of a partial differential equation (PDE) to model
mining asset value using a hedging strategy and management optionality and flexibility.
The objective of the PDE is to minimize mining losses while maximizing profits. Haque
et al. (2014) state that their proposed method can better handle the financial fundamentals
of real option analysis and give insight as to when management should exercise their options
to delay, abandon, pause or accelerate the mine than a binomial lattice method can. A key
conclusion from their work is that mining project value may be impacted by commodity
price volatility and managers should initiate the project when commodity price volatility is
at or below average levels (Haque et al. (2014)).

Haque et al. (2016) explore a method of hedging against commodity fluctuations by
generating a portfolio composed of long sales and short positions in the futures market.
The authors assert that by using a risk free contract to lock in the commodity price for
a certain number of commodity units and selling the rest long will result in the risk free
rate of return. Using this hedging strategy and the analytical form of the commodity price
movement in question, the authors generate a partial differential equation whose numerical
solution provides a method to value the asset in question. By not using a traditional discount
rate and applying a time value of money discount rate equivalent to the risk free interest
rate, the authors correctly account for the time value of money without exponentially over
penalizing the project in later years for risk that should be accounted for in the cash flows
directly.
Haque et al. (2014) and Haque et al. (2016) bring attention to the application of risk discounting using a modern finance approach and the addition of management flexibility options. By using futures contracts to establish a risk free contract for gold, the authors are applying a commodity market risk discount methodology consistent with that used in modern finance. While management flexibility is a large component of real options analysis (Samis et al. (2006) Davis (1998) Guthrie (2009)) certain applications in mining such as the option to temporarily abandon are costly due to resuming mill operations as well as hiring labor along with social implications that are not yet well quantified financially. In the case study presented, the authors respond to low metal prices by suspending operations until more favorable conditions are foreseen. The viability of this has yet to be proven and therefore establishes a point of uncertainty in methodology. With the PDE developed neither study addressed the need for changing the mine plan under commodity price uncertainty. However, Haque et al. (2016) reference work done by Salama et al. (2015) addressing that it is inappropriate to continue operating under the same mine plan as commodity prices fluctuate. With respect to this acknowledgment, the authors focused more on using management flexibility to address the problem of the need for dynamic mine planning in their study as opposed to dynamically changing the mine plan.

The application of stochastic market price and its joint effect with geological uncertainty is studied by Del Castillo & Dimitrakopoulos (2014) as it pertains to taking a real options approach to mine planning. The authors lay out a three-step strategy to handle the joint effect of both sources of uncertainty. The first step is to establish the base case which established the financing requirements, costing structure and flexibility opportunities. The second step seeks to further capitalize on potential project flexibility opportunities and incorporate uncertainty through simulated orebodies and price paths. The third step develops a flexible mine planning evaluation model that reevaluates the mines operational state and reassigns blocks based on current economic and operational parameters. The authors implement a geometric Brownian motion process with Poisson exponential jump diffusion to model the
random walk nature and extreme volatility observed with gold prices (Del Castillo & Dimitrakopoulos (2014)). This methodology is consistent with the geometric Brownian motion behavior of the gold price. The authors used 20,000 simulations to describe the expected behavior of gold price over the life of mine in consideration. With this information and the inclusion of conditional simulation to describe the uncertainty in the underlying geology, the authors used a Monte Carlo simulation with optionality to determine expansion, contraction or early closure options of the operation. A probability curve is generated based on expected cash flows for a year in the mines life being either positive, negative or the mine is abandoned. The probability curve is generated by checking each of the 20,000 price path simulations against each one of the twenty-ore body conditional simulations. The conclusion drawn is that real options-based methods allow for the inclusion of optionality in the mine plan based on the stated sources of uncertainty.

The study done by Del Castillo & Dimitrakopoulos (2014) uses a Monte Carlo simulation approach to model the behavior of commodity price and its joint affect with grade uncertainty on mining asset values. Generating the probability curve and evaluating each of the 20,000 price path simulations against the twenty mine plans shows that the authors are only adjusting the mine plans to account for grade uncertainty and not respecting that the mine plan would change as the price path against which it is being evaluated changes. This creates a dynamic environment which Del Castillo & Dimitrakopoulos (2014) bypass and is a basis for the work done in this study. When generating the base case scenario Del Castillo & Dimitrakopoulos (2014) generate the mine plan based on an arbitrarily chosen discount rate of 8%. This discount rate is not explained in the study and leads to the inference that there are sources of risk unaccounted for in the analysis leading to the use of a risk adjusted discount rate. Examining the simulation method used for the gold price Del Castillo & Dimitrakopoulos (2014) parameterize the simulation based on historical data. While this method is acceptable for the volatility parameter, using the historical perceived mean of the process for the drift parameter does not account for the risk of gold in that time frame. The
appropriate way to handle the generation of the drift parameter is to extrapolate it from the behavior of the futures contract curve as is stated in Samis et al. (2006) and Guthrie (2009). By not parameterizing the model this way Del Castillo & Dimitrakopoulos (2014) did not appropriately discount the risky component of the project at the revenue level and resorted to a risk adjusted discount rate to discount the project value in a static discounted cash flow method.

To study the impacts real options analysis has on mine plans, Dimitrakopoulos & Abdel Sabour (2007) compared twelve different mine plans generated solely from grade uncertainty using both discounted cash flows and real options valuation. Using conditional simulation, twelve equally likely realizations of an ore body are generated and from those a corresponding mine plan is generated using value parameterized nested pits. The nested pits all use a deterministic gold price as is required by the Lerchs-Grossman algorithm (Lerchs & Grossman (1965)). Once all mine plans are created, NPV’s for each are generated using two variations of discounted cash flows and one of real options valuation to determine which plan yields the highest present value considering uncertainty. Through each plan, the stochastic price simulation paths are run throughout the life of mine to give a range of probable NPV realizations. Dimitrakopoulos & Abdel Sabour (2007) acknowledge that multiple realizations of the simulated variables can be generated after adjusting for market price risk associated with each variable. A case study is done using a small three-year life gold mine in Australia to compare the effectiveness of the three economic analysis methods. Actual gold price values were used in the analysis and the final assertion is that real options provided the highest value from the operation.

Dimitrakopoulos & Abdel Sabour (2007) evaluate the impact of management flexibility on a mining asset under grade and commodity price uncertainty. To test their methodology the authors perform a case study on an Australian gold mine where 12 fixed mine plans are generated (Dimitrakopoulos & Abdel Sabour (2007)). With this methodology, the mine designs are evaluated against the different grade and price simulations to determine which
one is the optimal one to pursue. The authors bypass the fact that under each combination of commodity price and orebody realization, a new production schedule will be generated, potentially altering the value of the asset. To account for this correctly, assuming $O$ orebody simulations and $P$ price simulations, a new production schedule would need to be generated for each combination of orebody realizations and price simulations. Taking the example given in Dimitrakopoulos & Abdel Sabour (2007) a study using 0,1T-1,T simulations and 12 orebody realizations would need to generate a mine plan for each combination of price path and orebody and evaluate them solely using their respective price and orebody realization. As the number of price simulations increases this becomes increasingly impossible with the limits of modern mine planning software and computing power.

Chatterjee et al. (2016) use a variation of the minimum cut maximum flow algorithm to generate an ultimate pit and production phases for a given ore body under the effects of commodity price uncertainty. The approach taken uses the geostatistical method of Sequential Gaussian Simulation (SGS) to simulate commodity price and then processes it through the authors proposed minimum cut algorithm to return a single production phase sequence. With the introduction of capacity constraints into the problem, an optimal solution is not possible with this methodology (Chatterjee et al. (2016)). The implementation of Lagrangian relaxation of those constraints allows for the possibility of a solution, however a convergent or feasible solution is not guaranteed. Additionally, LP relaxation does not guarantee a feasible solution either (Chatterjee et al. (2016)). The study done by Chatterjee et al. (2016) uses sequential gaussian simulation to simulate commodity behavior which is unprecedented practice (Chatterjee et al. (2016)). This method may not lend itself to discounting the risky component of the mine plan, being the commodity, appropriately using certainty equivalents. The authors assumed a static discount rate of 10% in their case study indicating that further sources of risk were unaccounted for in the cash flows. The objective of the study is to determine a single phase design that may be operationally implementable considering uncertainty in the commodity price. While generating a phase design, the method does not
solve the production scheduling problem as is of interest in this study.

Salama et al. (2015) explored the impact of altering the mine plan in an underground copper mine in response to different values of the copper price. The authors varied the copper price between $5250/t Cu to $9750/t Cu in $500/t Cu increments. At each increment, the number of load haul dump (LHD) units needed to maintain mine production is recalculated and used as a parameter in determining the optimal mine plan under a commodity price realization. The authors used a mixed integer program (MIP) to determine the optimal extraction sequence in the sublevel stoping operation. The results of the study were that given a commodity price environment, a mine plan that is generated with that price as a parameter yields between a 3.52—7.25% gain in NPV for that project compared to a static mine plan.

While the study done by Salama et al. (2015) does demonstrate the value added in changing the mine plan in response to commodity price fluctuations, the authors made several assumptions in their model. The most apparent assumption is maintaining a constant price throughout the life of mine for each of the 10 price realizations. This method fails to capture the stochastic nature of metal prices. By incorporating this behavior the authors would be able to explore the impact that adjusting a life of mine schedule to optimally fit several different price simulations would have on the NPV of the project relative to a base case mine plan generated on the mean price. A second parameter used by the authors is the project discount rate. The selected rate is 25% as the project seemed apparently risky (Salama et al. (2015)). The authors do not justify the use of that discount rate or where it originates from. If the source of the uncertainty is technical risk, the authors should have incorporated that uncertainty into the cash flows directly for each year by reducing the expected cash flows in each year according to that years technical risk. If the source of uncertainty is market risk, the authors should have determined a certainty equivalent for the uncertain market variable which in this case is copper. This method avoids exponential penalization of the project in later years when the risk profile may not necessarily be exponentially increasing with project
periods. To account for time value of money, a market risk free rate should be considered as opposed to any internal metrics.

Espinoza et al. (2013) explore solving the Open Pit Mine Production Scheduling Problem (OPM-PSP) in a robust form to improve net present value under commodity price uncertainty. To approach this problem, the authors first define the traditional method for solving the OPM-PSP under deterministic circumstances, then the mathematical behavior of commodity price uncertainty is defined. The authors assert that all commodity prices can be considered to follow an Arithmetic Ornstein-Uhlenbeck process (Espinoza et al. (2013)) as established by Dixit & Pindyck (1994). The process describes that a commodity can fluctuate in the short term but in the long term will revert to the series mean. Espinoza et al. (2013) simulated the gold price from a starting value of $900/oz for twenty years with the estimated parameters $\eta = 0.1$ indicating the rate at which the gold price will revert to the mean and $\sigma = \frac{\mu_m}{100}$ being the volatility of the process as parameterized by the mean of the series pertaining to the commodity $m$ in question. Using a discretized version of the Ornstein-Uhlenbeck process Espinoza et al. (2013) simulate 100 realizations of the gold price. Within the simulation are ellipses of uncertainty, $\epsilon$, that vary between 0 and 1 from no uncertainty to high uncertainty respectively. The method of determining the ellipses is described in the paper and is out of the scope of this study. Using the ellipses, the authors are able to redefine the OPM-PSP to accommodate price paths within a specified level of uncertainty $\epsilon$. By setting the uncertainty tolerance of the problem the robust formulation of the OMP-PSP, the R-OMP-PSP, can be solved. The objective of the R-OMP-PSP is to maximize the NPV based on the worst possible price path contained within the uncertainty ellipse, thus making the schedule robust against any other price simulations contained within that ellipse (Espinoza et al. (2013)). The study concluded that after running the R-OPM-PSP with uncertainty ellipses set at 0.01,0.5,0.9 for low, medium and high levels of uncertainty respectively, the gains through robustness are marginal. A common trend is that the total and ore tonnage of the deposit decreased as the level of uncertainty increased.
Correspondingly the average grade increased with uncertainty.

Espinoza et al. (2013) approached the OMP-PSP with the attempt to generate schedules that are robust against uncertainty. The method begins with the characterization and parameterization of commodity price. An assertion made is that the commodity behaves according to mean reversion; this can have significant implications on the results of the study due to the behavior of the input price paths and may have been a result of why the gains were marginal when schedules are highly sensitive to price volatility Espinoza et al. (2013).

In order for a process to be classified as a mean reverting process, the historical trend upon which the forecasting equation is based must be proven to have a statistically significant mean. The authors did not perform a statistical test to prove that the process is stationary about a mean therefor the hypothesis that the two commodities used in the study, gold and copper, are mean reverting is unsupported. Espinoza et al. (2013) state that the discount factor has been omitted in their analysis but would be incorporated as $\mu_t^m = \frac{1}{1+\tau_t}\mathbb{E}(p_t^m)$. This states that given a discount rate $\tau$, the drift parameter of the process would be the discounted expectation value of the price of a given commodity $m$ at time $t$. The concept of this drift reduction is similar to the method seen in Guthrie (2009), where the drift parameter is risk adjusted. The difference between the method used by Espinoza et al. (2013) and Guthrie (2009) is that instead of using a certainty equivalent to discount for risk a risk adjusted discount rate is used.

2.2 Real Options

Work has been done in applying modern finance valuation techniques with respect to the specific considerations that investors look for in mineral assets. Samis et al. (2006) address the benefits and appropriate uses of real options valuation as well as point out incorrect applications and potential red herrings when considering management flexibility. With the increasing popularity of real options valuation as an alternative to traditional static discounted cash flow analysis, much of the focus is on how much value management flexibility adds to the valuation in question (Samis et al. (2006)). Although flexibility poses
a significant potential for value creation, the underlying fundamentals between traditional discounted cash flow analysis and real options analysis is the treatment of risk discounting (Samis et al. (2006)). Traditional DCF applies a risk adjusted discount rate (RADR) across the aggregate cash flows to account for both time value of money and risk; while real options valuation discounts risk at the cash flow level allowing for differentiation between risky components and assets with different pricing and cost structures (Samis et al. (2006)).

Figure 2.1: Samis et al. (2006) depiction of traditional DCF analysis

Figure 2.2: Samis et al. (2006) depiction of real options cash flow analysis

Figure 2.1 and Figure 2.1 illustrate the primary differences between traditional discounted cash flow analysis and real options analysis. In Figure 2.1 the expected cash flows are not risk adjusted and the aggregate cash flows are then uniformly discounted with a single constant risk adjusted discount rate. Figure 2.2 shows that the price per unit is risk adjusted to
calculate revenue and is used to calculate expected revenue. The only source of uncertainty in this example is the sale price and therefore it is the only component of the cash flows that incurs and risk adjusting. After subtracting operating costs and capital, which are assumed to be certain in this example, a final risk adjusted cash flow ($E_{RA}[\text{Net cash flow}]$) is discounted for time value of money. In the example used by Samis et al. (2006) in Figure 2.2, the $E[\tilde{S}]$ is the expected value of the commodity price for a mining project and is the sole source of uncertainty in the asset valuation. Samis et al. (2006) demonstrate that this method is consistent with the no arbitrage principle which states that assets with identical payoffs and identical levels of risk have the same value. This principle is demonstrated by Salahor (1998) in the form of the principle of value consistency which states that given sufficiently low transaction costs and barriers in financial markets assets with the same payoffs have the same price. The significance of this principle with respect to cash flow analysis is that contained within it is the principle of value additivity, stating that an assets value is the sum of all of the values of its individual cash flow components. The principle of value additivity is demonstrated in the valuation exercise shown in Figure 2.2. One of the significant aspects of value consistency deduced by Samis et al. (2006) is that the risk adjusted mineral price in the formulation $E_{RA}[\tilde{S}]$ is equivalent to the forward price of that mineral. From that the risk adjustment shown in Figure 2.2 can be determined to be $\frac{E_{RA}[\tilde{S}]}{E[S]}$ (Samis et al. (2006)). This concept is fundamental for the approach taken to value the mining asset in this study.

An assumption in real options analysis is that the price of the projects output behaves according to an Ito process (Davis (1998)).

$$dS = \alpha_S Sdt + \sigma_S Sdz$$  \hspace{1cm} (2.1)

Where $S$ is the unit price of the good, $\alpha_S$ is the drift at time $t$ of the good, $\sigma_S$ is the volatility at time $t$ of the good and $dz$ is a standard Weiner process (Davis (1998)). Extrapolating this concept, the value of the project producing good $S$ can be expressed as $V(S, t)$ (Davis (1998)). It follows that the value of the operation is a function of all goods sold variables which may be expanded out according to Ito’s Lemma (Davis (1998)). With regards to estimating the
drift and volatility parameters for each revenue adding component modeled stochastically, Davis (1998) proposes the methods needed to calculate the parameters to be used under a variety of operating conditions including operations with and without operational flexibility. The resulting partial differential equation indicates the value of a project according to its revenue generating components. The application of an accurate estimating technique is critical to obtain as accurate as possible representations of the behavior of each cost or revenue stream component. Guthrie (2009) illustrates how this is done for generating risk adjusted simulations of geometric Brownian motion behaving assets such as gold. With a risk adjusted revenue stream generated using the strategies and principles an accurate real options valuation can be carried out on the asset in question.
CHAPTER 3
ECONOMIC PARAMETERS

3.1 Introduction

Understanding the underlying mechanisms describing the behavior of the uncertain entity that introduces risk into asset valuation is fundamental to performing a real options valuation. In this chapter, the stochastic behavior of the commodity price will be described and from the key points of that mechanism, the approach to solving a real options valuation can be understood. It will become apparent that performing a valuation on an open pit mining asset using a traditional backwards induction binomial lattice method while also honoring the OMP-PSP becomes nearly computationally impossible. The first portion of the solution strategy to approximate the binomial lattice solution will be described, simulating the risk adjusted gold price. The risk adjusted price will be the driving input parameter in the final stage of the valuation procedure, generating the mine plans.

3.2 Real Options Valuation

The economic basis for this study is that of real options valuation. This method is often considered as an alternative to traditional discounted cash flow analysis (Samis et al. (2006)). One of the most familiar reasons is that real options valuation allows for the incorporation of management flexibility in determining an asset value (Samis et al. (2006)). While flexibility certainly adds value to assets, the fundamental difference between traditional discounted cash flow and real options valuation techniques is the method of risk adjustment (Samis et al. (2006), Guthrie (2009)). Traditional discounted cash flow techniques accommodate for risk by using a single risk adjusted discount rate (RADR) applied over the net aggregate cash flows. Contained within the RADR are discount rates for technical and market risk, as well as for time value of money. Applying this method exposes a fundamental flaw: how are all cash flows able to be correctly adjusted for risk when there are multiple variable cost
structures and revenue streams coming from a mineral asset? The answer is that it cannot. Finding the appropriate discount rate to correctly value a project is a nearly impossible task (Brealey et al. (2006)). To overcome this hurdle, real options takes a different approach, if there are separate cost and revenue structures that are components of a single cash flow structure, then they should be adjusted for based upon the level of risk specific to that cash flow component (Samis et al. (2006)). With this approach, real options is able to differentiate between similar assets with different cash flow structures and provide a more insightful and accurate valuation (Samis et al. (2006)).

The first step in performing a real options valuation is identifying the primary source of that risk, i.e the state variable (Guthrie (2009)). The state variable is agnostic to the project and combined with management decisions, determines the cash flows for the project (Guthrie (2009)). In a project that contains no management flexibility, the cash flows are a function of the state variable. After the state variable is selected, its behavior through time must be represented throughout the life of the project. A common approach to doing this is to build out a binomial tree that describes the values the state variable can take should it increase or decrease at each point in the project lifetime (Guthrie (2009)). As the number of time steps increases within a modeling period the finer the resolution of the model and the more representative the binomial tree will be of all the possible paths the state variable can take. Building out the binomial tree raises the apparent question: how are the magnitudes of the up and down moves determined?

### 3.3 Geometric Brownian Motion Commodity Price Movement

Establishing the magnitude of up and down moves first begins with understanding the underlying probabilistic mechanism describing the behavior of the state variable through time. Two of the widely considered mechanisms are mean reversion and Geometric Brownian Motion. Mean reverting processes are constrained by the magnitude of successive up and down steps the state variable can make before beginning to revert to the mean. This results in an envelope bounding the state variable from taking extreme values. A GBM process has
constant and equal probabilities of transitioning up or down and therefore is not limited by
an upper or lower bound envelope. Within this study, the behavior observed by the state
variable is statistically proven to not be stationary and thus follows a GBM process. To
fully understand and prove this concept, the two subprocesses underlying GBM must first
be understood, Brownian motion and the Markov Process.

A Brownian Motion process is characterized as a time continuous process that behaves
according to the Markov property (Karlin & Taylor (1975)). The Markov property states that
given a variable value at time $t$, $X_t$, the value at time $s$, $X_s$ such that $s > t$ is independent of
values $X_u$ for all $u < t$ (Karlin & Taylor (1975)). An implication of this is that for the process
characterizing commodity price movements the future probability of the movement of the
price is not altered by any knowledge concerning its past behavior (Karlin & Taylor (1975)).

Time continuous Markov Processes, like the Brownian motion process are called diffusion
processes (Karlin & Taylor (1975)). This terminology is often interchanged in literature and
it is important to understand that it refers to the same concept. A Markov Process is said
to have stationary transition probabilities if it is a function of time only. However due to the
Markov, and therefore Brownian motion process, being conditional on the present state of
the variable, it cannot be said that the process itself is stationary (Karlin & Taylor (1975)).
This concept can be exemplified in the fact that gold price is not stationary while gold price
returns are stationary. Two important definitions of Brownian motion are:

1. Every increment in between the random variable at time $X(t+s) - X(s)$ being normally
distributed with mean 0 and variance $\sigma^2t$. (Karlin & Taylor (1975))

2. $X(0) = 0$ and $X(t)$ is continues at time $t$ (Karlin & Taylor (1975))

With the above definition, an important and special version of Brownian motion can be
defined, the standard Brownian motion. This is a Brownian motion process with variance
one and the underlying mechanism describing the behavior of commodity price as shown in
Davis (1998). The Brownian motion process by itself cannot model the movements of non-
stationary commodity prices. To overcome that, consider the generalized Wiener process

\[ dx = a \, dt + b \, dz \]  

(3.1)

Where \( dx \) is the change in the state variable, \( a \) is the drift, \( b \) is the volatility of state variable and \( dz \) is the standard Brownian motion process (Hull (2012)). The final step in proving that the state variable moves according to GBM is derived in Hull (2012) and results in validating the use of the Ito process for modeling price movements. The Ito process as shown in Hull (2012) is a type of generalized Weiner process in which the parameters \( a \) and \( b \) are functions of the state variable.

\[ dS = \mu_S(S, t)dt + \sigma_S(S, t)dz \]  

(3.2)

Hull (2012) proves that the corresponding formulation of the Ito process follows the definition of GBM as defined in Karlin & Taylor (1975). Davis (1998) asserts that in real options analysis it is often assumed that the price of the project output good behaves according to the Ito process. With the understanding of the fundamental implications of a state variable moving according to GBM the process of building out the binomial tree can begin. These fundamental implications are as follows The probability of future price movements is independent of past price behavior Those price movement probabilities are stationary and normally distributed around mean zero and variance one.

3.4 Discounting

The two components of discounting presented within the scope of this study are adjusting for risk and discounting for the time value of money. While the risk adjusting methodology implements the risk free rate, it is important to define the two separately.

3.4.1 Discounting for Time Value of Money

As can be seen in the multi and single period valuation equations, time value of money is incorporated by dividing the expected value by the risk-free rate of return. The risk-free rate of return is the rate of return that investors seek without exposing themselves to
any downside risk while also sacrificing any upside potential. A convenient source for these values are the national treasury coupon less bonds called Separate Trading of Registered Interest and Principal of Securities (STRIPS). The yield of these bonds gives the risk-free rate for the corresponding periods ahead $r_f$. Alternatively stated, the 10-year STRIPS bond coupon gives the 10-year risk free rate. The risk-free rate of return is expressed as $R_f = 1 + r_f$. With the recursive nature of the multi period cash valuation equation, time value of money discounting behaves exponentially as it does in traditional static discounted cash flow analysis. This emulates the compounding returns investors seek as cash flows are further away from the present.

### 3.4.2 Risk Adjustment

From the previous section with probabilities of future price behavior being stationary and normally distributed around zero with a variance of one, the cumulative probability of a random variable being negative is 50% and the cumulative probability of a random variable being positive is 50%. This behavior gives the probabilities of true up and down movements in the state variable. Applying the true probabilities to the size of up and down steps will allow the binomial tree to be fully built out. The size of up and down steps is determined once the model is calibrated for the real-world behavior of the state variable. For now, they can be simply expressed as $X_U$ and $X_D$ respectively. An example state variable binomial tree can be seen below in Figure 3.1.

At this point the expected behavior of the state variable has been successfully been described and now must be translated into the expected cash flows of the asset in question. Being that within the scope of this study the uncertainty in the assets cash flows are a function solely of the behavior of the state variable; the cash flow tree will have a one to one correlation with the state variable binomial tree as shown below.
Figure 3.1: Example binomial tree of state variable "X" with the first index indicating the number of down moves and the second index being the period. The true probabilities of up and down moves are given as $\theta_u$ and $\theta_d$ respectively.

Figure 3.2: Cash flow binomial tree corresponding to the behavior of the state variable in Figure 3.1.
At this point, no adjustment for risk has been made. The cash flows shown in Figure 3.2 are simply those that would arise from the asset being operated at the corresponding state. It is in the next step that the risk adjustment is applied. Before applying the risk, first consider a fundamental assumption of real options valuation, that arbitrage opportunities do not exist (Guthrie (2009)). The purpose of this assumption is that given two portfolios with identical risk and identical cash flow payouts, their values are identical. This is called the law of one price and it is used to build out the relationship between value and risky cash flows. This relationship is derived in Guthrie (2009) and exemplified in valuing a one period ahead risky cash flow. Consider that the two cash flows one period from the valuation date are $Y_u$ and $Y_d$ in the respective up and down states. The value of these cash flows can be sufficiently approximated by considering a portfolio of risk-free bonds and the risky asset $X$ (Guthrie (2009)). The risky asset is referred to as the spanning asset and is the underlying physically traded entity providing value to the asset. In the case of a metals mine the spanning asset would be the metal itself. Using a linear combination of bonds and the spanning asset a replicating portfolio can be generated that produces cash flows $Y_u$ and $Y_d$ identical to the asset in question (Guthrie (2009)). The resulting value of the replicating portfolio is derived in (Guthrie (2009)) and is expressed as:

$$V = \frac{\pi_d Y_u + \pi_d Y_d}{R_f}$$  \hspace{1cm} (3.3)

Where $\pi_U$ and $\pi_D$ are the risk-adjusted probabilities associated with realizing cash flows $Y_U$ and $Y_D$ and $R_f = (1 + r_f)$ where $r_f$ is the risk-free rate. These risk-adjusted probabilities alter the second fundamental implication of GBM commodity price movement that there is an equal chance of positive and negative movements, to probabilities that reflect the markets aversion to risk that is contained within the replicating portfolio. The implications of this are powerful, by weighting the probabilities of positive and negative cash flows to reflect investor risk aversion, no risk-adjusted discount rate is used. Expanding the replicating portfolio to
multiple period cash flows results in the following recursive equation (Guthrie (2009)):

\[ V(i, n) = \frac{\pi_u(i, n)V(i, n+1) + \pi_d(i, n)V(i+1, n+1)}{R_f} \]  (3.4)

Solving the above equation requires using the backwards induction lattice approach. This equation states that the value at a period \( n \) is the expected value one period ahead of it determined by the risk-adjusted probabilities and discounted by the one period ahead risk-free rate. This method relies on the final period value being known as a boundary condition to the backwards induction method. This will be addressed when the solution lattice is introduced, however, the risk-adjusted probabilities must be determined first. The methods associated with calculating these vary according to the nature of the state variable used and the market information available about it. However, the basic formulas for calculating the risk-adjusted probabilities are:

\[ \pi_u = \frac{ZR_f - X_d}{X_u - X_d} \]  (3.5)

And

\[ \pi_d = \frac{X_u - ZR_f}{X_u - X_d} \]  (3.6)

Where \( Z \) is the price of the spanning asset, \( X_u \) and \( X_d \) are the values of the state variable after an up or down move respectively (Guthrie (2009)). In this study, the state variable of gold price is discounted by using the certainty equivalent of futures contracts. This strategy is described by both Samis et al. (2006) and Guthrie (2009) to capture market attitudes towards risk. Forward and futures contracts are agreements between two parties to exchange an item at a future date for a specified price (Guthrie (2009)). The primary difference between forward and futures contracts is that forward contracts are an agreement between two parties while futures contracts are traded on organized exchanges. In both cases the payoffs from the contracts comes from the difference between the contract amount and the spot price on the day of the transaction. Guthrie (2009) proves that by the law of one price \( Z = \frac{F}{R_f} \) where \( Z \) is the price of the spanning asset, \( F \) is the value of the futures
contract one period ahead and $R_f$ is the risk-free rate. Substituting this into the equations to calculate the risk-adjusted probabilities yields:

$$\pi_u = \frac{F - X_d}{X_u - X_d} \quad (3.7)$$

And

$$\pi_d = \frac{X_u - F}{X_u - X_d} \quad (3.8)$$

These relationships hold for valuing single period cash flows, to expand the idea to multi period cash flows, such as those observed in most mining projects, the relationship between futures prices and the state variable value needs to be established. The rate at which the state variable is expected to grow adjusted for risk is called the risk-adjusted growth factor, $K$. What $K$ represents is the risk premium associated with the state variable subtracted from the expectation value of the growth of the state variable (Guthrie (2009)). From the futures contracts $F_{(0,1,...,T)}$, $K$ can be calculated as the ratio between the futures price one period ahead, i.e $n + 1$, and the futures price at period $n$. Intuitively this makes sense because the futures price is a risk adjusted value of the state variable, therefore the ratio would return a factor that indicates how much the state variable has grown in that one period adjusted for risk.

$$K_n = \frac{F_{n+1}}{F_n} \quad (3.9)$$

Guthrie (2009) states that the futures price at date $n$ follows the following relationship, Equation 3.10, with the spot price at time zero:

$$F_n(0,0) = K_{n-1} \ldots K_1 K_0 X(0,0) \quad (3.10)$$

Rearranging this equation $K_0$ can be solved for

$$K_0 = \frac{F_n(0,0)}{\prod_{i=1}^{(n-1)} K_i X(0,0)} \quad (3.11)$$
The equation above can be used to calculate any $K$ within the valuation dates and each is used to determine the risk adjusted probabilities for that period using the equations

$$
\pi_u = \frac{K_n - D}{U - D}
$$

and

$$\pi_d = \frac{U - K_n}{U - D}
$$

where $U$ and $D$ are the sizes of the up and down moves of the state variable. These can be calculated for a GBM process according to the equations Guthrie (2009)

$$U = e^{\hat{\sigma} \sqrt{\Delta t_m}}$$ (3.12)

And

$$D = e^{-\hat{\sigma} \sqrt{\Delta t_m}}$$ (3.13)

Where $\hat{\sigma}$ is the historical volatility of the state variable and $\Delta t_m$ is the time step.

3.4.3 The Binomial Lattice Method

Now all the tools are in place to calculate the value of the asset using a backward induction binomial lattice method. Using the multi period valuation equation and the calibrated risk adjusted probabilities the binomial lattice is generated shown in Figure 3.3. One may notice that to perform the backward induction calculation the values at the final period must be known. These values are given by the boundary condition $V(\bullet, n) = Y(\bullet, n)$ where $n$ is the final period of the project. The boundary condition holds true since the value of the asset in the final period $n$ are the expectation value of the cash flows received. The value at the root of the tree ($V(0,0)$) is the NPV of the asset.
Figure 3.3: Binomial lattice showing risk adjusted probabilities being solved by backward induction.

3.4.4 Conclusions and Remarks Related to Mine Planning

Starting from the fundamental observations regarding the GBM movement of commodity price, a structure modeling the behavior of the underlying uncertain variable is modelled and translated into cash flows. From historical data the model is calibrated to reflect the magnitudes of the up and down moves between periods as well as the risk-adjusted probabilities of those moves. Using a multi period valuation equation, the NPV of the asset can be determined through backward induction. Complications arise when the methodology is applied to mine planning in that to determine the cash flows in the $Y(\bullet, n)$ binomial tree terminal nodes requires solving the OMP-PSP for all possible paths in the tree. While this does prevent the backwards induction binomial lattice approach from being used, the other features of the real options methodology can still be applied to perform the close approximation approach proposed in this study.
3.5 Simulating the Gold Price

To approximate the lattice solution method, a small sample of representative paths of commodity price movement are used to generate production schedules which then return a NPV for the project under each price realization, or price path. Using historical data the probabilistic mechanism describing price behavior must be statistically determined before modelling activity can take place. Once the model is understood, statistical moments from the historical data provide information about the volatility of the process and potentially the mean, should the series demonstrate mean reverting behavior. From there the model will be adjusted for market risk using exchange traded futures contracts. This results in a complete risk adjusted model that honors the observed statistical behavior of the commodity in question.

3.6 Confirming Geometric Brownian Motion

As Previously stated, commodity prices are generally assumed to follow an Ito process if they behave according to GBM (Davis (1998)) or according to an Ornstein-Uhlenbeck process if it is mean reverting (Dixit & Pindyck (1994)). For a process to be mean reverting there must be statistically significant stationarity in the historical data of that price time series. Often by visual inspection the underlying behavior can be estimated, however the only certain method to confirm behavior is by applying a statistical test. The Augmented Dickey-Fuller (ADF) test is used to determine if a process is a random walk or a random walk with drift (Tsay (2010)). The test states two hypotheses: the null hypothesis, or $H_0$, is that the series is a random walk or a random walk with drift. The alternative hypothesis, or $H_a$ is that the process is stationary. There are two outputs of the test, the critical value and the $p$ value. If the $p$ value is above 0.05 then the null hypothesis of a random walk cannot be rejected. Additionally, if the critical value is less than that as calculated by Fuller (1976) the null hypothesis cannot be rejected. The time series in question for this study is the gold price from August 2010 to November 2017 as shown in Figure 3.4. This selection
was arbitrary in a sense that by visual inspection, the gold price was determined to be a random walk therefore the mean of the series is irrelevant. The subject of interest within the time series was the volatility. The series was selected based on a period where the price rose considerably quickly and then returned to lower levels. The volatility and intercept for simulation is based upon this time series.

![Figure 3.4: Gold Price from August 2nd, 2010 to November 3rd, 2017. (Slight offset in the time scale due to non equal number of trading days per year.)](image)

Using the statistical programming language R, data from the Federal Economic Reserve Data (FRED) is retrieved and processed to reflect the correct number of trading days in the period as the data is daily. From here the daily gold price data is tested for random walk behavior. Recall the implications of the Markov property that a GBM process must follow: no past information regarding the series will alter the probability of future behavior. Assuming for the sake of testing that the gold price did have some weak stationarity about a mean, an autofitting routine is run to quantify that expected behavior. Running the auto
regressive fitting function in R, the expected order of auto regression is estimated to be three lags. This means that it is estimated that three time periods prior to any given point in time contain some forecasting information regarding the future behavior of the series. Now the ADF test can be run given the expected behavior of the alternative hypothesis. The results are $ADF\ Statistic = -3.004$ and $p - value = 0.5206$. Comparing the ADF statistic to those given in the table by Fuller (1976), show that it is more negative than -2.55 for a no intercept and no trend series indicating that the null hypothesis of GBM behavior cannot be rejected. Examining the $p$ value shows that it is greater than the 0.05 needed to reject the null hypothesis. Combining these two results indicates strong statistical confidence that the time series is a random walk with no intercept and no trend. It is now appropriate to define the model forecasting gold price as a random walk.

### 3.7 Parameterizing the Model

Recall again the differential form of the Ito process

$$dS = \mu_S(S)dt + \sigma_S(S)dz$$

(3.14)

The numerical solution of this equation returns a single simulated behavior of $S$ through time and is what will be used as input into the mine planning model for final valuation. While $S$ and $dz$ are known, $\mu_s$ and $\sigma_s$ the drift and volatility of $S$ have yet to be determined. Historical data gives insight into the volatility of the series while the drift parameter will be the risk-adjusted drift parameter based on the behavior of the futures curve. Guthrie (2009) models the drift and volatility parameters in a GBM process as:

$$\hat{\mu} = \frac{\hat{\nu}}{\Delta t_d} \quad \text{and} \quad \hat{\sigma} = \frac{\hat{\phi}}{\sqrt{\Delta t_d}}$$

(3.15)

Where $\Delta t_d$ is the time step and $\hat{\nu}$ and $\hat{\phi}$ are the sample arithmetic mean of the time series and the sample standard deviation of the log changes in the price respectively. The standard deviation of the log gold price returns is $\hat{\phi} = 0.0106$ which gives a volatility of $\sigma_s = 0.179$. The final step is to estimate the risk adjusted drift. From the Chicago Mercantile Exchange
and the Wall Street Journal, gold futures contract quotes and treasury STRIPS are retrieved with the following values

Table 3.1: Futures contracts and risk-free bond yields

<table>
<thead>
<tr>
<th>Year</th>
<th>Year</th>
<th>Futures</th>
<th>STRIPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018</td>
<td>1</td>
<td>1351.30</td>
<td>1.56%</td>
</tr>
<tr>
<td>2019</td>
<td>2</td>
<td>1387.00</td>
<td>2.08%</td>
</tr>
<tr>
<td>2020</td>
<td>3</td>
<td>1424.80</td>
<td>2.20%</td>
</tr>
<tr>
<td>2021</td>
<td>4</td>
<td>1463.80</td>
<td>2.37%</td>
</tr>
<tr>
<td>2022</td>
<td>5</td>
<td>1546.30</td>
<td>2.47%</td>
</tr>
</tbody>
</table>

As is apparent, the futures contracts only extend out to five years past the present; therefore an extrapolation method is needed to generate an expected futures curve from years 6-10. This equation is given in Guthrie (2009) as:

\[
F_n = X_0 e^{(r_f - C)n} \tag{3.16}
\]

Where \(X_0\) is the spot price, \(r_f\) is the risk-free bond yield, \(C\) is the convenience yield and \(n\) is the year. Using the equation from Guthrie (2009) the convenience yield of gold is calculated based on the values in Table 1. The convenience yield is extra value that the market place on having physical gold as opposed to simply the right to that gold (i.e. the futures contract). The equation is:

\[
C = r_f - \frac{1}{n} \ln \left( \frac{F}{X_0} \right) \tag{3.17}
\]

Which are evaluated to be:

Table 3.2: Convenience yield of gold

<table>
<thead>
<tr>
<th>Year</th>
<th>Year</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018</td>
<td>1</td>
<td>1.241%</td>
</tr>
<tr>
<td>2019</td>
<td>2</td>
<td>0.617%</td>
</tr>
<tr>
<td>2020</td>
<td>3</td>
<td>0.328%</td>
</tr>
<tr>
<td>2021</td>
<td>4</td>
<td>0.291%</td>
</tr>
<tr>
<td>2022</td>
<td>5</td>
<td>-0.290%</td>
</tr>
</tbody>
</table>

And
Table 3.3: Futures contracts extrapolated from year 6 through 10.

<table>
<thead>
<tr>
<th>Year</th>
<th>Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>2023</td>
<td>$1,602.01</td>
</tr>
<tr>
<td>2024</td>
<td>$1,661.73</td>
</tr>
<tr>
<td>2025</td>
<td>$1,717.82</td>
</tr>
<tr>
<td>2026</td>
<td>$1,777.22</td>
</tr>
<tr>
<td>2027</td>
<td>$1,842.00</td>
</tr>
</tbody>
</table>

Applying the ten year futures contract values to the equation for risk adjusted growth rate models how the gold price, which is the state variable of the model, grows at a rate adjusted for investor risk aversion. Taking the average value minus one gives the drift parameter $\mu_s$ that will be used in the model: $\mu_s = 0.319$.

Table 3.4: Yearly summary of information needed to parameterize the risk adjusted gold price. Shown are the futures contracts, risk-free rate (STRIPS), convenience yield (C) and the risk adjusted growth rate (K).

<table>
<thead>
<tr>
<th>Year</th>
<th>Year</th>
<th>Futures</th>
<th>Strips</th>
<th>C</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018</td>
<td>1</td>
<td>1351.30</td>
<td>1.56%</td>
<td>1.241%</td>
<td>100.32%</td>
</tr>
<tr>
<td>2019</td>
<td>2</td>
<td>1387.00</td>
<td>2.08%</td>
<td>0.617%</td>
<td>102.64%</td>
</tr>
<tr>
<td>2020</td>
<td>3</td>
<td>1424.80</td>
<td>2.20%</td>
<td>0.328%</td>
<td>102.73%</td>
</tr>
<tr>
<td>2021</td>
<td>4</td>
<td>1463.80</td>
<td>2.37%</td>
<td>0.291%</td>
<td>102.74%</td>
</tr>
<tr>
<td>2022</td>
<td>5</td>
<td>1546.30</td>
<td>2.47%</td>
<td>-0.290%</td>
<td>105.64%</td>
</tr>
<tr>
<td>2023</td>
<td>6</td>
<td>1602.01</td>
<td>2.60%</td>
<td>-</td>
<td>103.60%</td>
</tr>
<tr>
<td>2024</td>
<td>7</td>
<td>1661.73</td>
<td>2.71%</td>
<td>-</td>
<td>103.73%</td>
</tr>
<tr>
<td>2025</td>
<td>8</td>
<td>1717.82</td>
<td>2.75%</td>
<td>-</td>
<td>103.38%</td>
</tr>
<tr>
<td>2026</td>
<td>9</td>
<td>1777.22</td>
<td>2.79%</td>
<td>-</td>
<td>103.46%</td>
</tr>
<tr>
<td>2027</td>
<td>10</td>
<td>1842.00</td>
<td>2.84%</td>
<td>-</td>
<td>103.64%</td>
</tr>
</tbody>
</table>
Figure 3.5: Futures contract prices, actual and extrapolated

The final parameterized equation for modeling the risk adjusted gold price is

\[ \frac{dS}{S} = 0.319dt + 0.179dz \]  \hspace{1cm} (3.18)

3.7.1 Selecting a Simulation for Scheduling

Using the statistical programming language ”R,” the equation is solved numerically in batches of ten at a time. These ten will be the input prices for the mine planning software. A total of fifteen simulations of ten price paths each is done. According to the law of large numbers (Karlin & Taylor (1975)) the simulation average will converge to the mean if a sufficient number of runs are done. To emulate this and not encounter the same problem with the computation limitations in the binomial lattice method, the simulation of ten prices that is most representative of the average will be selected for production scheduling. The average of the series as the number of price paths approaches infinity is the futures curve. All simulated prices will be included in the appendix while the two best performing simulations will be studied in further detail here. As shown in Figure 3.6 and Figure 3.7, the two
simulations that were most representative of the average, Simulation 5 and Simulation 7, had deviations away from the futures curve 0.22% and 1.77% respectively. While Simulation 5 had the smallest deviation from the futures curve, it had one path that climbed to over $5000/oz in the ninth and tenth years. The rest of the simulations stayed in the $500/oz to $3500/oz range. While $5000/oz gold is a possibility afforded by the GBM nature of gold price, Simulation 7 was chosen instead due to the closer clustering of price simulations that explored price oscillating near the $1250/oz range, climbing to $4000/oz, falling to $700/oz and many different behaviors in between. The tight clustering with large variation made this simulation appealing to study how the production scheduling program would respond in seeking an optimal schedule.
Figure 3.6: Gold Price Simulation 7, deviation from the average: 1.77%
Figure 3.7: Gold Price Simulation 5, deviation from the mean: 0.22%
CHAPTER 4
GENERATING PRODUCTION SCHEDULES

There are many existing production scheduling software solutions available to solve the OPM-PSP. While there has been considerable research directed towards obtaining the optimal solution for the OMP-PSP, this is out of the scope of this study. The methodology proposed here is applicable to all scheduling solution algorithms since the focus is on the valuation methodology of the mineral asset. This study does not seek to address the improvement of production scheduling algorithms but rather the treatment of commodity price uncertainty and real options valuation methodology in the mine planning and valuation approach.

When incorporating commodity price uncertainty in the production scheduling problem, often a set of simulated prices will be compared against a production schedule that is generated either from a constant price or a combination of constant price and grade variation. This combination comparison is not valid as the production schedule would change in response to each individual price simulation. The two concepts are explored here are:

1. Real Options based mine planning versus the traditional method

2. Pairing one schedule to one price path versus evaluating production schedules against multiple price paths

To illustrate the objectives outlined here, a case study on the McLaughlin gold deposit was done.

4.1 Establishing Supporting Parameters

The McLaughlin deposit was mined by open pit from 1985 to 1996 by the Homestake Mining Company and was located in Napa and Lake counties of the state of California, USA. The block model and ultimate pit limits are shown in Figure 4.1.
Figure 4.1: Ultimate Pit Limits of the McLaughlin Deposit with gold grades color mapped to the block model as shown in Table 4.1.

Table 4.1: Legend for the color coded grades for the McLaughlin Deposit.
Several details need to be considered before the deposit is ready for production scheduling. First, the ultimate pit limit needs to be determined so that the quantity of ore and waste can be quantified. Using that quantity, the capital requirements to operate the mine can be defined.

Regarding the block model, the scheduler considers only blocks contained within the ultimate pit limit (UPL). Determining the UPL is done using the Lerchs-Grossman algorithm with the parameters shown in Table 4.2.

Table 4.2: Design parameters used to generate the ultimate pit

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Mining Cost</td>
<td>$1.2/ton</td>
</tr>
<tr>
<td>Mill Processing Cost</td>
<td>$10/ton</td>
</tr>
<tr>
<td>UPL Price</td>
<td>$1577/oz</td>
</tr>
</tbody>
</table>

Notice that the UPL is determined using a constant price as opposed to a time dependent price as is seen in the rest of this study. This is because the Lerchs-Grossman algorithm as well as other maximum flow and pseudo flow algorithms require that the block values be pre-calculated. The UPL price is determined based on the arithmetic average of the futures curve price. Once the UPL is determined, the block model is coded to the topography to correct the tonnages in the blocks that intersect the topography. This is what is processed into the scheduling algorithm. The mill capacities are determined according to the number of the ore tons within the ultimate pit limit and is sized such that all the ore tons can be processed within ten years. It may be observed that the classification of ore and waste is dependent on the price path. Because of this dynamic classification interfacing with the static requirements of having a fixed mill capacity in an assumed environment where there is no optionality to expand, the mill is sized based on the ore waste classification under the futures curve. This is chosen as it is the average of all simulations and therefore should result in a reasonable capacity. Mining capacity is chosen using a different approach than the mill capacity. Production scheduling is done in this study with the objective to operate the mill at full capacity in each period given a price path realization. As the mining fleet varies, the
capital cost associated with those variations must be captured in the project valuation.

Table 4.3: Design parameters used to generate the production schedules

| Capital Cost - Mill and General Infrastructure | $1.8 Billion |
| Mining Fleet Cost | $100 M per 10 Million Tons |
| Mill Capacity | 22.5 MTons |
| Average Mining Fleet Capacity | 48.5 MTons |

4.1.1 Cutoff Grade

The cutoff grade is an important aspect of mine planning in that it determines what material will be sent to which destinations. The study of optimum cutoff grade policy has been performed by many including Lane (1964) and Dagdelen (1992) who showed that seeking optimum cutoff grade policy adds significant value to a mining operation. The algorithm developed by Lane (1964) is an iterative approach that uses the projects NPV in an attempt to determine the optimum cutoff grade policy. While software packages exist to determine the optimum policy, the used in this study relies on user input to define the cutoff grade in each period. The scheduling package used does offer a bench scheduling option that establishes a so called optimal cutoff grade policy for the project. These cutoff grades tracked the break even cutoff grade in each period close enough that the break even cutoff grade was used in each period. The breakeven cutoff grade is determined using Equation 4.1.

\[
BECG = \frac{pc}{(P_i - s) * r}
\]  

(4.1)

Where \(pc\) is the processing cost, \(P_i\) is the gold price in year \(i\), \(s\) is the sales cost and \(r\) is the recovery. These parameters are given in Table 4.4.
Table 4.4: Cutoff grade parameters

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Period Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing Cost</td>
<td>$10/ton</td>
<td></td>
</tr>
<tr>
<td>Sales Cost</td>
<td>$0/ton</td>
<td></td>
</tr>
<tr>
<td>Recovery</td>
<td>90%</td>
<td></td>
</tr>
</tbody>
</table>

4.1.2 Destinations

From the block model, the blocks can be sent to one of five destinations. The stockpiles are defined based on the lowest cutoff grade corresponding to that price path and the mill cutoff grade.

Table 4.5: All possible destinations for blocks.

<table>
<thead>
<tr>
<th>Destination</th>
<th>Low cutoff</th>
<th>High cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directly to the Mill</td>
<td>Period based break even cutoff grade</td>
<td>Period based break even cutoff grade</td>
</tr>
<tr>
<td>To the Mill overflow stockpile</td>
<td>Halfway between break even cutoff grade and lowest cutoff grade in period</td>
<td>Halfway between break even cutoff grade and lowest cutoff grade in period</td>
</tr>
<tr>
<td>To the high-grade stockpile</td>
<td>Halfway between break even cutoff grade and lowest cutoff grade in period</td>
<td>Break even cutoff grade</td>
</tr>
<tr>
<td>To the low-grade stockpile</td>
<td>Lowest cutoff in price path</td>
<td>Halfway between break even cutoff grade and lowest cutoff grade in period</td>
</tr>
<tr>
<td>To the waste dump</td>
<td>Below break even cutoff grade</td>
<td>Below break even cutoff grade</td>
</tr>
</tbody>
</table>

The stockpile is segmented into two parts in a user defined manner, allowing reclaimed material grades to be controlled in more detail than if one stockpile is used. Figure 4.2 shows the layout of destinations in the Evolution Origin software.

![Figure 4.2: Flowchart of destinations that a block may be sent to from the block model](image-url)
4.2 Production Scheduling Software

This study makes use of a commercially available production scheduling software package, Maptek Evolution. Evolution is a meta-heuristic production scheduling algorithm that has the capability to schedule based on operational constraints such as loader and haul fleet capacity. Evolution provides two levels of scheduling aimed at strategic and tactical scheduling. Evolution Strategy is the tactical bench level scheduler that is able to work towards optimizing a schedule to meet multiple targets. Strategy provides period by period cut off grade optimization using the algorithm developed by Lane (1964). Evolution Origin provides a direct block scheduling solution based on a genetic algorithm meta-heuristic. Origin allows the scheduler to mine from faces as opposed to a bench top-down approach. With this ability, Origin can set the number of active faces to emulate the active loading equipment available.

The genetic algorithm meta-heuristic used by Origin explores several schedules and is able to return a set that maximize the objectives set by the user. The primary objectives that must be set are either material movement or equipment hours with optional objectives being min-max limits of a variable, NPV, and blending targets. This study set the desired objectives to be material movement, where the total tonnage moved was the capacity of the mining fleet, and NPV. After processing the schedules in Origin, results are exported to a spreadsheet for comparison and post processing.

Due to the intended purpose of the Evolution scheduling software as well as the algorithms employed, a mathematical global optimum solution is not guaranteed. This introduces a source of uncertainty in the results when comparing the performance of production schedules with respect to maximizing NPV.

4.3 Traditional Method and Corresponding Sources of Modeling Error

The traditional method of production scheduling fundamentally relies on three concepts: value parameterized phases, a constant risk adjusted price, and a risk adjusted discount
rate. These concepts aim to generate a mine plan that seeks high value material first, is robust against price volatility and brings value forward. The ultimate objective of these strategies is to maximize NPV under the operating conditions throughout the scope of the project. Further examining the traditional method’s three main concepts reveals two areas of concern that may impact the production schedule and valuation of the asset.

The idea of selecting a risk adjusted price is in line with real options valuation techniques, however the method of execution is significantly different. While real options valuation techniques look to commodity futures markets for assessing investor attitudes regarding the risk associated with the commodity, the traditional method heuristically chooses a price that is assumed to likely be lower than the realized commodity price throughout the scope of the production schedule. The problem that this method poses may have significant impacts to project valuation and to production schedule generation; however, the magnitude of the error correlates to the underlying probabilistic mechanism of the commodity price in question. For mean reverting commodities, selecting a constant price may be a reasonable representation of a risk adjusted commodity price. When considering random walk commodities such as gold, an artificially lowered constant price has a much lower probability of being representative of the market risk adjusted behavior.

The second area of concern regarding accurate project valuation is the selection of a risk-adjusted discount rate (RADR). This static percentage seeks to penalize the value of cash flows further in the future due to uncertainty and time value of money. This is only appropriate if the risk of the project increases exponentially at the RADR each year, a unique scenario. If this is not the true case of project risk, the project valuation will be incorrect. By using the static risk adjusted discount rate as a parameter in the production scheduling process, later period material value will be decreased exponentially, potentially altering the way it is handled by the scheduling algorithm, resulting in a decrease in overall schedule value.
To emulate this methodology, three scenarios shown in Table 4.6 are generated from which production schedules and valuations will be generated.

Table 4.6: The three scenarios chosen to emulate traditional production scheduling methodology.

<table>
<thead>
<tr>
<th>Name</th>
<th>Price $/oz</th>
<th>Discount Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional A</td>
<td>1347</td>
<td>8%</td>
</tr>
<tr>
<td>Traditional B</td>
<td>1250</td>
<td>8%</td>
</tr>
<tr>
<td>Traditional C</td>
<td>1000</td>
<td>8%</td>
</tr>
</tbody>
</table>

The topic of maximizing project NPV by optimally parameterizing nested pits within the ultimate pit has been studied extensively. While the analysis and improvement of these methods is beyond the scope of this study, a salient concern regarding their implementation must be noted. This concern is that the algorithms used to do this do not guarantee that yearly production requirements will be satisfied. This may pose a problem to the scheduling algorithm that is processing those phases for scheduling. An attempt will be made in this study to completely bypass the value parameterized pits concept because of this significant shortfall.

### 4.4 Real Options Production Scheduling

The scheduling approach proposed in this study aims to resolve some of the modeling concerns in the traditional approach. These will attempt to be resolved by removing value parameterized phasing and taking a real options approach to risk adjustment.

#### 4.4.1 Phasing

With the potential issues posed by implementing value parameterized phasing into the production scheduling process, a new approach that bypasses value parameterization is implemented. Initially, a production schedule is generated based on an ultimate pit containing blocks that are not flagged with phases. This schedule results in the orebody being mined horizontally as opposed to sinking down into higher grade material when possible. This may
be an issue that is specific to the software used here and further work using other scheduling software may shed light on a resolution to this.

To overcome the horizontal stripping schedule, a manual heuristic fix is used to allow the scheduler to select from different phases to mine. These phases are not directly correlated to value or mining period, instead, are based on the geometric occurrence of the mineralized material within the ultimate pit. Examining the orebody in a North-South cross section view provides insight into how this fix is conceived and generated.

Figure 4.3: Cross section of the ultimate pit showing the geometric phasing on easting line 11137.5. Turquoise is the highest grade phase, green is the second highest, and blue is the lowest grade phase.

Figure 4.4: Cross section of the ultimate pit on easting line 11137.5 showing the gold grade throughout the deposit. The grades are color coded according to Table 4.1.
Figure 4.5: Plan view of the ultimate pit limits with geometrically designed phases. Starting from the initial phase shown in grey and expanding outwards, the phases have decreasing concentrations of high grade material. The red shape inside the pit is material above 0.05 oz/ton. The Yellow line intersecting the pit indicates where future cross sections will be taken.
Comparing Figure 4.5 and Figure 4.6 the first phase is centered around material that is the highest grade, consequently, phases 2, 3 and 4 contain decreasing grade material. The phases do not depict a sequence of mining, rather are necessary to allow the scheduler more freedom to produce a higher value schedule. It is apparent that this method of phasing is dependent on the geometry of the ore body and is not intended to be an optimal method to phase within the ultimate pit. Ideally, the scheduling process would not contain any phases and the deposit would be considered holistically for scheduling.

4.4.2 Price Adjustment

In line with the real options method of risk adjustment, the commodity price used for valuing the project is derived from the futures prices for gold. These prices are received from
the Chicago Mercantile Exchange, where commodity futures are traded. These future prices give the mine planner the market expectations of a risk-free value for the commodity price in future time periods. The behavior of these so-called futures curves, also gives an indication of the behavior of the underlying probabilistic behavior mechanism of the commodity in question. The risk adjusted gold price and time-value-of-money parameters are summarized in Table 4.7.

Table 4.7: Gold commodity futures contract prices for ten years from the date of the valuation along with bond rates that account for time value of money.

<table>
<thead>
<tr>
<th>Year</th>
<th>Futures Price</th>
<th>Bond Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1351.30</td>
<td>1.56%</td>
</tr>
<tr>
<td>2</td>
<td>$1387.00</td>
<td>2.08%</td>
</tr>
<tr>
<td>3</td>
<td>$1424.80</td>
<td>2.20%</td>
</tr>
<tr>
<td>4</td>
<td>$1463.80</td>
<td>2.37%</td>
</tr>
<tr>
<td>5</td>
<td>$1546.30</td>
<td>2.47%</td>
</tr>
<tr>
<td>6</td>
<td>$1602.01</td>
<td>2.60%</td>
</tr>
<tr>
<td>7</td>
<td>$1661.73</td>
<td>2.71%</td>
</tr>
<tr>
<td>8</td>
<td>$1717.82</td>
<td>2.75%</td>
</tr>
<tr>
<td>9</td>
<td>$1777.22</td>
<td>2.79%</td>
</tr>
<tr>
<td>10</td>
<td>$1842.00</td>
<td>2.84%</td>
</tr>
</tbody>
</table>

4.4.3 Price Simulation

Work has been done by several authors exploring the effects that simulated price paths have on production schedules and their consequent valuations. An objective of this study is to examine the treatment of these price paths with respect to the production scheduling problem. An aspect of this objective is to explore the question: is there any value gained from planning based on simulated price paths? The second aspect of the objective is to assert that if price path simulations are to be used, they cannot be used to evaluate mine plans that are generated independently of the price path. Alternatively stated, each price path will yield its own mine plan, and only that mine plan.

With the objective established, the commodity price simulation shown in Figure 3.6 is used to give ten price paths from which production schedules will be generated and valued.

4.5 Production Schedules and Value Calculations

Production schedules are generated by passing the above defined parameters into Evolution Origin for each price path or valuation scenario. After being processed by the algorithm, physicals from the schedule are returned and can be used to calculate the value of that sched-
The calculation used to determine the NPV of the mine plan under a given valuation scenario is given in the following Equation 4.2. Where $i$ is the period.

$$
\text{Revenue}_i = \text{Price}_i \times \text{Ounces}_i
$$

$$
\text{Costs}_i = \text{Capital Cost}_i + \text{Processing Cost}_i + \text{Mining Cost}_i
$$

$$
\text{Cash Flow}_i = \text{Revenue}_i - \text{Cost}_i
$$

$$
\text{NPV} = \sum_{i=1}^{10} \text{Cash Flow}_i \times \text{Discount}_i
$$

Table 4.8: Example spreadsheet style NPV calculation showing the futures curve production schedule NPV calculation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Ounces - MSH</th>
<th>Ounces - SP</th>
<th>Price</th>
<th>Processing</th>
<th>Capital</th>
<th>Mining</th>
<th>Discount</th>
<th>DCF</th>
<th>NPV (Billion USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1028653.304</td>
<td>0</td>
<td>$1351.30</td>
<td>$225,008,804.95</td>
<td>$2,285,000,000.00</td>
<td>$60,017,649.65</td>
<td>1.56%</td>
<td>$5,226,000,000.00</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>924706.841</td>
<td>25678.96471</td>
<td>$1387.00</td>
<td>$225,000,008.21</td>
<td>$2,285,000,000.00</td>
<td>$62,560,319.19</td>
<td>2.08%</td>
<td>$(1,161,884,547.95)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>599222.075</td>
<td>5991.056343</td>
<td>$1424.80</td>
<td>$225,000,002.70</td>
<td>$2,285,000,000.00</td>
<td>$64,261,721.22</td>
<td>2.20%</td>
<td>$974,019,800.49</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>777057.7316</td>
<td>7770.615313</td>
<td>$1463.90</td>
<td>$225,000,000.51</td>
<td>$2,285,000,000.00</td>
<td>$66,489,953.55</td>
<td>2.37%</td>
<td>$663,326,659.09</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>841987.5299</td>
<td>8422.095783</td>
<td>$1546.30</td>
<td>$225,000,439.40</td>
<td>$2,285,000,000.00</td>
<td>$69,599,356.79</td>
<td>2.47%</td>
<td>$780,739,266.59</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>818223.4508</td>
<td>8182.234508</td>
<td>$1602.01</td>
<td>$225,000,386.60</td>
<td>$2,285,000,000.00</td>
<td>$71,459,448.90</td>
<td>2.60%</td>
<td>$893,307,831.63</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>815868.2409</td>
<td>8158.682409</td>
<td>$1661.73</td>
<td>$225,000,000.00</td>
<td>$2,285,000,000.00</td>
<td>$73,181,014.26</td>
<td>2.71%</td>
<td>$872,641,629.42</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>737833.691</td>
<td>7378.33691</td>
<td>$1717.82</td>
<td>$225,000,000.00</td>
<td>$2,285,000,000.00</td>
<td>$77,004,592.19</td>
<td>2.75%</td>
<td>$882,213,613.44</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>429083.6008</td>
<td>311623.7306</td>
<td>$1777.22</td>
<td>$225,000,000.00</td>
<td>$2,285,000,000.00</td>
<td>$80,713,182.04</td>
<td>2.79%</td>
<td>$356,636,003.11</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>$1842.00</td>
<td>$225,000,000.00</td>
<td>$2,285,000,000.00</td>
<td>$34,712,139.06</td>
<td>2.84%</td>
<td>$237,527,342.47</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9: Comparison of different economic scenarios along with their respective NPV values.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>NPV (Billion USD)</th>
<th>Price ($/oz)</th>
<th>Discount Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of Paths 1-10</td>
<td>$5.226</td>
<td>1347</td>
<td>8%</td>
</tr>
<tr>
<td>Futures Curve</td>
<td>$5.275</td>
<td>Futures Curve</td>
<td>Bond Yield</td>
</tr>
<tr>
<td>Traditional A</td>
<td>$3.173</td>
<td>1250</td>
<td>8%</td>
</tr>
<tr>
<td>Traditional B</td>
<td>$2.686</td>
<td>1250</td>
<td>8%</td>
</tr>
<tr>
<td>Traditional C</td>
<td>$1.325</td>
<td>1000</td>
<td>8%</td>
</tr>
</tbody>
</table>

Table 4.9 illustrates the main objective of this study, how the valuation of an asset changes with respect to economic assumptions. The three traditional heuristic scenarios
that use a constant price and risk adjusted discount rate yield three significantly different valuations for the McLaughlin deposit. None of these three traditional scenarios can be argued to be based on anything other than intuition alone as providing the fundamental basis for asset value. Using the futures curve and bond yield time value of money allows for a valuation that is based on market perceptions of risk towards the gold price and time value of money, accounted for separately. Irrespective that the valuation of the asset is higher due to the upward trend of the futures curve, the valuation is based on market signals instead of intuition providing a supportable basis for which to move forward with the mine planning and project development process.

With respect to the path by path based valuation, it averaged approximately the same NPV as the futures curve based mine plan. This result provides insight into expectations of using price based simulations to value a project. Since the price paths are parameterized based on the futures curve, the average of a representative sample of simulations will return the futures curve. Therefore attempting to generate mine plans based on individual price simulations is an exercise in futility as the probability of simulating the true future behavior of the commodity is almost zero. Instead, planning based on market signals and expectations of the commodity price future behavior provides a platform upon which to proceed.

4.6.2 Price Path Performance

A component of this study is to compare how mine plans perform against different price paths in an effort to expose the fallacy of testing a production schedules robustness against different price simulations in order to select an optimal plan. An initial result of the price specific production schedule testing is the apparent variation at each period. Figure 4.7 and Figure 4.8 show the variation of the production schedules in response to different price simulations and scenarios.
Figure 4.7: Cross section of the ultimate pit limits showing schedules: Path 7 (Red), Path 9 (Dark Gray), Path 10 (Light Gray), Futures Curve (Blue) and Traditional A (Yellow) at the end of period 3.

Figure 4.8: Cross section of the ultimate pit limits showing schedules: Path 7 (Red), Path 9 (Dark Gray), Path 10 (Light Gray), Futures Curve (Blue) and Traditional A (Yellow) at the end of period 5.

Figure 4.9: Combined view of Figure 4.7 and Figure 4.8 showing the how the schedules change between periods.
Further variations can be seen when inspecting the average grade processed, ore tons and waste tons mined in each year as shown in Table 4.10. The information shown in these tables is graphically depicted in Figure 4.10 Figure 4.11 and Figure 4.12.

Table 4.10: Average grade, ore tons and waste tons processed in each period (row) by each production schedule (vertical)

<table>
<thead>
<tr>
<th>Path 1</th>
<th>Path 2</th>
<th>Path 3</th>
<th>Path 4</th>
<th>Path 5</th>
<th>Path 6</th>
<th>Future Curve</th>
<th>Traditional A</th>
<th>Traditional B</th>
<th>Traditional C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waste Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Ore Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Waste Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Ore Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Waste Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Ore Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Waste Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Ore Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Waste Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Ore Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Waste Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
<tr>
<td>Ore Tons</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
<td>22,500,000</td>
</tr>
</tbody>
</table>

Figure 4.10 Figure 4.11 and Figure 4.12 show the ore tons processed, waste tons mined and strip ratio per period per schedule respectively. In Figure 4.10 almost all schedules are able to produce the required ore tons. The schedules that are unable to are generated off of economic scenarios with a higher definition of ore on average than the futures curve. The waste ton movement remains relatively consistent throughout all schedules in each period with variations shown in scenarios that deviate significantly from the futures curve. Examining scenario "Traditional C" shows a larger increase in waste tons due to the cutoff grade increasing substantially.
Figure 4.10: Surface map depicting ore tons processed by each production schedule at each period.

Figure 4.11: Surface map depicting waste tons mined by each production schedule at each period.
Figure 4.12: Surface map depicting strip ratio processed by each production schedule at each period

Figure 4.13: Average grade per period for each mine plan
With the material movement objective of Evolution Origin, the target of meeting mill feed tonnage is satisfied in all price paths for all scenarios aside from scenarios with prices significantly lower than the futures curve. In those situations, the definition of ore is at a higher cutoff grade resulting in less ore to process, shortening the mine life. As any good production scheduling package would, Origin creates balanced schedules that efficiently uses the capital of the mine in each period. By ensuring that the material movement target is consistent, the internal variation within the production schedules in response to price path or scenario changes is more pronounced as can be seen in Figure 4.14, Figure 4.15 and Figure 4.16.

![Total Tonage Per Period: Futures Curve](image.png)

Figure 4.14: Total material movement for the futures curve production schedule. Excluding stockpile reclaim to the mill, all material movement satisfies the yearly mining fleet capacity (shown as horizontal line). This is important for ensuring efficient use of capital.
Figure 4.15: Total material movement for the Price Path 1 production schedule. Excluding stockpile reclaim to the mill, all material movement satisfies the yearly mining fleet capacity (shown as horizontal line). This is important for ensuring efficient use of capital.

Figure 4.16: Total material movement for the futures curve production schedule. Due to the higher strip ratio of this schedule relative to that of the futures curve, a large mining fleet is needed to fill the mill in each period.
The fundamental implication of these results is that production schedules will vary as the definition of ore and commodity price changes year by year in an effort to maximize project NPV given those parameters. From this implication it can be extrapolated that evaluating production schedules against several price simulations is invalid as the relationship is dynamic not static. Table 4.11 illustrates the discrepancies in value when production schedules are evaluated against price paths that are independent to their generation.

Table 4.11: A color map that shows how a production schedule (column) performs against an economic scenario (row). The colors indicate how a production schedule performs with respect to NPV when evaluated against an economic scenario. Green indicates highest NPV while red indicates lowest NPV. The column labeled "Path Dependent" corresponds to production schedules that are generated based on a price path and evaluated exclusively against that price path. "Futures Curve" and "Traditional A,B,C" refer to production schedules generated based on those economic scenarios.

<table>
<thead>
<tr>
<th></th>
<th>Path Dependant</th>
<th>Futures Curve</th>
<th>Traditional A</th>
<th>Traditional B</th>
<th>Traditional C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward Curve</td>
<td>$ 5,275,629,154.48</td>
<td>$ 5,519,548,058.60</td>
<td>$ 5,612,228,430.60</td>
<td>$ 5,436,010,958.50</td>
<td></td>
</tr>
<tr>
<td>Constant Price</td>
<td>$ 4,077,916,633.88</td>
<td>$ 4,341,272,283.99</td>
<td>$ 4,356,776,154.20</td>
<td>$ 4,154,200,540.81</td>
<td></td>
</tr>
<tr>
<td>Path 1</td>
<td>$ 3,558,805,175.00</td>
<td>$ 2,915,649,519.20</td>
<td>$ 3,065,233,727.99</td>
<td>$ 3,071,734,683.92</td>
<td>$ 2,728,772,868.15</td>
</tr>
<tr>
<td>Path 2</td>
<td>$ 4,079,370,617.60</td>
<td>$ 3,875,755,248.93</td>
<td>$ 4,144,707,260.05</td>
<td>$ 4,121,595,025.15</td>
<td>$ 4,015,149,587.61</td>
</tr>
<tr>
<td>Path 3</td>
<td>$ 3,495,065,674.49</td>
<td>$ 3,238,295,447.17</td>
<td>$ 3,422,288,441.58</td>
<td>$ 3,460,544,762.24</td>
<td>$ 3,126,512,878.03</td>
</tr>
<tr>
<td>Path 4</td>
<td>$ 3,934,209,310.46</td>
<td>$ 3,672,795,058.66</td>
<td>$ 3,971,579,842.93</td>
<td>$ 3,949,370,145.58</td>
<td>$ 3,834,524,606.60</td>
</tr>
<tr>
<td>Path 5</td>
<td>$ 4,838,750,003.08</td>
<td>$ 4,687,706,285.66</td>
<td>$ 4,770,279,942.42</td>
<td>$ 4,746,386,230.86</td>
<td>$ 4,016,900,943.70</td>
</tr>
<tr>
<td>Path 6</td>
<td>$ 8,385,310,367.64</td>
<td>$ 9,003,047,908.96</td>
<td>$ 9,201,876,794.32</td>
<td>$ 9,174,547,292.82</td>
<td>$ 8,567,610,825.50</td>
</tr>
<tr>
<td>Path 7</td>
<td>$ 3,056,714,739.16</td>
<td>$ 2,317,854,984.98</td>
<td>$ 2,605,223,056.67</td>
<td>$ 2,658,276,741.74</td>
<td>$ 2,649,488,011.97</td>
</tr>
<tr>
<td>Path 8</td>
<td>$ 5,778,421,524.53</td>
<td>$ 5,579,647,034.24</td>
<td>$ 5,611,931,444.37</td>
<td>$ 5,590,195,193.33</td>
<td>$ 4,927,666,856.16</td>
</tr>
<tr>
<td>Path 9</td>
<td>$ 10,164,402,946.18</td>
<td>$ 11,134,555,032.62</td>
<td>$ 11,285,791,120.78</td>
<td>$ 11,190,619,857.14</td>
<td>$ 10,336,213,300.29</td>
</tr>
<tr>
<td>Path 10</td>
<td>$ 4,954,096,364.96</td>
<td>$ 4,485,244,765.68</td>
<td>$ 4,849,031,446.39</td>
<td>$ 4,850,255,650.53</td>
<td>$ 4,721,792,475.18</td>
</tr>
</tbody>
</table>

Examining Table 4.11 reveals both expected an unexpected results. The expected results are that the price dependent production schedules out perform the production schedules generated on alternative economic scenarios for each price path. The results indicate that on certain price paths, the path dependent schedules outperform the other schedules by hundreds of millions of dollars NPV. On certain schedules, particularly ones that realize high gold prices, the NPVs for the path dependent schedules are worse by up to a billion dollars. This counter-intuitive result may be the topic for future work on evaluating the performance of production scheduling algorithms on price paths in hopes of ensuring algorithms respond optimally to input parameters.
The futures curve production schedule yields consistently lower NPVs relative to the traditional production schedules and the price path specific schedules. This may be in part due to the consistently lower average grade per period of the futures curve production schedule relative to the traditional and path dependent production schedules. Further analysis may be done to examine the performance of this production schedule to ensure that it is optimally exploiting the deposit.

### 4.6.2.1 Expected Performance Paths

Production schedules generated with price paths that yielded the expected results can be seen in Figure 4.17 with corresponding NPVs shown in Table 4.12. These price paths generally maintain values near the traditional scenario price values for a majority of the ten year valuation period. Often these price paths will dip below the traditional scenario values for some time then return to higher values.

<table>
<thead>
<tr>
<th>Table 4.12: Comparative NPVs of production schedules that performed as expected.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Dependant</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Path 1</td>
</tr>
<tr>
<td>Path 3</td>
</tr>
<tr>
<td>Path 5</td>
</tr>
<tr>
<td>Path 7</td>
</tr>
<tr>
<td>Path 8</td>
</tr>
<tr>
<td>Path 10</td>
</tr>
</tbody>
</table>

The largest creation of value is found with production schedules that are generated from price paths which drop below the traditional scenario price for a significant duration of the valuation period. Comparing the average grades reveals that all production schedules follow the same slowly decreasing trend of average processed grade. However, some production schedules realize average grades that are significantly lower than the traditional scenario production schedules, yet still outperform them. One such example of this behavior is the production schedule generated from path 8.
4.6.2.2 Unexpected Performance Paths

Contrary to the expected performing production schedules, the production schedules that performed unexpectedly are generated from price paths that stay above the traditional scenario values for the entirety of the valuation period shown in Figure 4.18. With ore tons equal across all periods and average grades following similar trends to each other, the performance of the production schedules appears to be heavily influenced by either processing sequencing or the stochastic nature of the genetic algorithm meta heuristic.

Table 4.13: Comparative NPVs of production schedules that did not perform as expected.
Figure 4.18: Price paths that generate mine plans which do not perform as expected when evaluated against that price path.

The largest loss of value is where the production schedules are generated with price paths that realize the highest values in the simulation set, Path 6 and Path 9. Paths that oscillate closer to the traditional scenario price values realize smaller losses of value. The comparative NPV’s of the schedules can be seen in Table 4.13. The performance of production scheduling algorithms against prices that are significantly higher than prices at the time of this studies writing may be subject to future work.
CHAPTER 5
RESULTS AND CONCLUSIONS

5.1 Conclusions

Given the unique characteristics and challenges facing mining projects new and existing, it is important to be able to formulate a method for robust and consistent project valuation. A component of this methodology has been the slow transition from traditional discounted cash flow valuation techniques towards more modern real options valuation methods. Although thoroughly understood and developed at high level project application, the interface between real options and production scheduling is not well established. This crucial link to generating sound project valuations is difficult to model as the methodology for obtaining production schedules is computationally intensive, not yet able to guarantee optimality nor handle commodity price simulation. The methodology presented here aims to approach the application of real options valuation to a mining asset at a fundamental level; from this, certain aspects can be concluded.

The first and foremost important concept that must be applied to a real options valuation is to understand the underlying sources of uncertainty within the project. Such uncertain variables can be of technical, political, investment financing and market origins. These uncertainties must be reflected by adjusting expected cash flows on a period by period basis for each component of the cash flow where the risk is presenting itself. By doing so, projects can be distinguished with more clarity from one another than by taking a traditional discounted cash flow approach. The final stage in performing proper risk adjustment is to ensure that risk and time value of money are treated as separate entities.

In compartmentalizing sources of project uncertainty, the underlying mechanisms generating the uncertainty need to be understood and quantified to estimate future behavior as accurately as possible. In the case of this study, the uncertain variable is gold with two prob-
abilistic mechanisms considered that have been used in previous work: Geometric Brownian Motion and mean reversion. The latter was rejected by using statistical tests to determine the behavior seen in the historic series. By statistically proving the behavior of the uncertain variable in question, a more accurate behavior can be parameterized for forecasting expected outcomes and planning accordingly.

An additional method in consideration has been testing the robustness of a production schedule against simulated price paths. This study concludes that doing so is not realistic due to fact that each price path will have its own unique production schedule corresponding to it. The ability to generate a mine plan for every conceivable price path is impossible as the potential paths are infinite; as such, any number of simulated paths is only a marginal realization. If one mine plan appears ”robust” with a given set of price paths it may completely fail against another. It is clear that this process is futile and an alternative method should be implemented. The need to test for robustness against random price volatility can be eliminated by generating long range plans based on market derived certainty equivalents as is done in this study. In practice, managers respond to changing operating environments and regenerate production schedules correspondingly to optimize project value. From a valuation standpoint it is sufficient to use market derived certainty equivalents and adjust expected cash flows for risk without attempting to generate a production schedule that is optimal under several price path simulations.

It is important to note that unlike grade and geological uncertainty, few steps can be taken to reduce the risk of commodity price uncertainty. Market derivatives provide a basis for gauging market attitudes towards the predicted behavior of commodity prices and the associated risk, but in no way provide a guaranteed prediction of future outcomes. The methodology introduced in this study aimed to provided a more supportable basis for economic parameter selection into the production scheduling process; however, operators will still be exposed to the uncertainty associated with investing in a particular commodity.
5.2 Future Work

The solution methodology presented here aims to be practically implementable and approach the interface between modern valuation techniques and production scheduling. The essential nature of this study contributes to laying the foundation for future studies that incorporate more elaborate risk elements such as project technical and political risks as well as management flexibility optionality. Performing such a study would explore the implications of using multiple state variables in determining a project valuation and in generating production schedules considering that uncertainty. Additionally, performing this study on other projects where the uncertain commodity price is mean reverting may provide insights on production schedule behavior and the value difference between real options and traditional discounted cash flow valuation techniques.

The objectives in this study may be reproached using an integer programming-based direct block scheduling algorithm that guarantees global optimum solutions. This will hopefully resolve the discrepancies seen when generating production schedules based on price paths and provide a clearer answer regarding the treatment of commodity price simulations in production scheduling. An ancillary topic that may be explored is improving scheduling algorithms to plan according to varying commodity prices through time. Further work may be done on combining the solution methodology explored in this study with grade uncertainty and the cumulative effects they would have on the mine planning and project valuation process. This may provide a method to decrease the dimensionality presently encountered when attempting to generate mine plans considering both grade and commodity price uncertainty.
REFERENCES CITED


