STRESS INDUCED CHANGES IN ELASTIC WAVE ATTRIBUTES IN THE WATTENBERG FIELD, COLORADO, USA

by

Tom R. Bratton
A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Geophysics).

Golden, Colorado
Date ______________________

Signed: ____________________
   Tom R. Bratton

Signed: ____________________
   Tom Davis
   Thesis Advisor

Golden, Colorado
Date ______________________

Signed: ____________________
   Dr. John Bradford
   Professor and Department Head
   Department of Geophysics
ABSTRACT

I have investigated the influence of stress on elastic wave behavior. Elastic waves are rich in information content. Not only do elastic waves carry information about geologic structure, they carry geomechanical information as well. The objective of this work was to examine elastic wave behavior with rock physics and geomechanical modeling, to extract actionable information for use in engineering designs and field development decisions.

I used Techlog, a software package provided by Schlumberger to analyze all of the well-centric data. I used Computer Modeling Group’s (CMG) reservoir simulator and its geomechanics module (GEM) to compute the flow coupled geomechanical stresses.

The Fort Hays, Codell and Muddy Formations are mostly isotropic. Thin intervals within the J Sand and Carlile Formations show stress induced HTI anisotropy. The Niobrara Formation and bounding shales show strong VTI anisotropy with Thomsen’s epsilon and gamma values ranging from 0.1 to 0.5. The Niobrara Formation is overpressured. Dipole sonic data shows the top of pressure to be 1300 feet above the top of the Niobrara Formation. The sonic data also shows the Pierre Formation above the top of pressure to have a P-wave velocity change of 580 ft/s with a 1000 psi change in effective stress. However, below the top of pressure, the P-wave velocity changes by 2150 ft/s for the same 1000 psi change in effective stress. Linear slip theory (LST) was used to fit the ultrasonic velocity measurements conducted on Niobrara Formation core. These velocity measurements showed even the stress sensitivity was anisotropic; the vertical velocities showed 5 times the stress sensitivity as the horizontal velocities. The linear elastic stress models did not yield an accurate vertical stress profile; the bounding shales, the Sharon Springs Formation above, and the Carlile and Graneros Formation below, exhibit ductile strain that increased the minimum horizontal stress gradient by 0.1 psi/ft over the linear elastic stress estimates. During production, depending on the permeability anisotropy of the stimulation and the initial horizontal stress
imbalance, the horizontal stresses increase at different rates causing the stress field to rotate 90 degrees.

The entire stratigraphic column is anisotropic. In addition, the stratigraphic column exhibits significant stress-induced velocity anisotropy. This complicates time-lapse seismic measurements because both the stress sensitivity and effective stresses vary with injection and production. Numerical stress modeling is required to estimate the initial vertical stress profile and time dependent changes in effective stress due to injection and production. Engineering opportunities for improved recovery are possible based on the geomechanical insights and workflows developed in this dissertation.
TABLE OF CONTENTS

ABSTRACT .................................................................................. iii
LIST OF FIGURES ................................................................. viii
LIST OF TABLES ........................................................................ xi
ACKNOWLEDGMENTS ............................................................ xii
DEDICATION ............................................................................. xiv

CHAPTER 1 INTRODUCTION ......................................................... 1
  1.1 Background ................................................................. 2
  1.2 Objectives ................................................................. 3
  1.3 Literature Review ..................................................... 3
  1.4 Study Area ................................................................. 6
  1.5 Data and Methodology ............................................... 6

CHAPTER 2 DIPOLE SONIC PROCESSING .................................. 11
  2.1 Sonic Velocities .......................................................... 11
  2.2 Dynamic Elastic Moduli ............................................. 13
  2.3 Dispersion Analysis ................................................... 14
  2.4 Anisotropy Classification ........................................... 19
  2.5 Anisotropy Estimation ................................................. 23
  2.6 Results and Conclusions ............................................ 26

CHAPTER 3 DIPOLE STRESS INTERPRETATION ....................... 27
  3.1 Introduction ............................................................... 27
3.2 Petrophysical Model ................................................................. 30
3.3 Effective Stress Model ............................................................. 34
3.4 Stress Compensated Petrophysical Model ..................................... 36
3.5 Results .................................................................................... 37
3.6 Discussion ................................................................................ 38
3.7 Results and Conclusions ............................................................ 40
CHAPTER 4 ROCK PHYSICS ................................................................. 41
4.1 Introduction .............................................................................. 41
4.2 Linear Slip Theory ................................................................. 41
4.3 LambdaRho MuRho Analysis ...................................................... 50
4.4 LST Forward Model .................................................................. 51
4.5 Results and Conclusions ............................................................ 53
CHAPTER 5 GEOMECHANICS ............................................................. 54
5.1 Introduction .............................................................................. 54
5.2 Concepts ................................................................................... 55
  5.2.1 Stress ............................................................................... 56
  5.2.2 Strain ............................................................................... 57
  5.2.3 Moduli .............................................................................. 58
5.3 Mechanical Properties .............................................................. 59
  5.3.1 Rock Strength Parameters .................................................. 59
5.4 Earth Stresses .......................................................................... 64
  5.4.1 Horizontal Stress Magnitudes .............................................. 65
5.5 Critical Stress Analysis .............................................................. 70
5.5.1 Stresses on a Fault ........................................ 71
5.5.2 Wishbone Faults ........................................ 72
5.6 Numerical Stress Modeling ................................. 74
  5.6.1 Static Model ........................................ 74
  5.6.2 Stress Equilibration .................................... 76
  5.6.3 Dynamic Stress Behavior ................................ 82
  5.6.4 Stress Rotation ......................................... 83
5.7 Results and Conclusions ..................................... 87

CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS .......... 89
  6.1 Dipole Sonic Processing ................................. 89
  6.2 Dipole Stress Interpretation ............................. 90
  6.3 Rock Physics ............................................ 90
  6.4 Geomechanics ........................................... 92
  6.5 Future Work ............................................ 93

REFERENCES CITED ........................................ 95
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Location of Wattenberg Field</td>
<td>7</td>
</tr>
<tr>
<td>1.2</td>
<td>Location of Wishbone study area</td>
<td>8</td>
</tr>
<tr>
<td>1.3</td>
<td>Wattenberg stratigraphic column</td>
<td>9</td>
</tr>
<tr>
<td>2.1</td>
<td>Sonic velocity measurements</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>Relationship between the horizontal shear and the Stoneley measurement</td>
<td>14</td>
</tr>
<tr>
<td>2.3</td>
<td>Flexural shear dispersion curve</td>
<td>15</td>
</tr>
<tr>
<td>2.4</td>
<td>Dispersion curve behavior</td>
<td>16</td>
</tr>
<tr>
<td>2.5</td>
<td>Dispersion curves in the Codell Formation</td>
<td>18</td>
</tr>
<tr>
<td>2.6</td>
<td>Dispersion curves in the J-Sand and Carlile Formations</td>
<td>19</td>
</tr>
<tr>
<td>2.7</td>
<td>Sonic log anisotropy classification</td>
<td>20</td>
</tr>
<tr>
<td>2.8</td>
<td>Gas effect on Stoneley slowness</td>
<td>21</td>
</tr>
<tr>
<td>2.9</td>
<td>Three shear slownesses in the Codell and Fort Hays Formations</td>
<td>22</td>
</tr>
<tr>
<td>2.10</td>
<td>VTI anisotropy diagnosed in the Niobrara Formation and bounding shales</td>
<td>23</td>
</tr>
<tr>
<td>2.11</td>
<td>VTI anisotropy diagnosed in the Niobrara Formation and bounding shales</td>
<td>24</td>
</tr>
<tr>
<td>2.12</td>
<td>VTI anisotropy in the Niobrara Formation and bounding shales</td>
<td>25</td>
</tr>
<tr>
<td>3.1</td>
<td>Multi-mineral solver results</td>
<td>32</td>
</tr>
<tr>
<td>3.2</td>
<td>Normalized Vp analysis</td>
<td>34</td>
</tr>
<tr>
<td>3.3</td>
<td>Normalized Vp versus effective vertical stress</td>
<td>35</td>
</tr>
<tr>
<td>4.1</td>
<td>Niobrara core data fit with Shapiro’s model</td>
<td>43</td>
</tr>
</tbody>
</table>
Figure 4.2  Niobrara core data fit with Sayer’s linear slip theory model
Figure 4.3  Generalized velocity behavior as a function of stress
Figure 4.4  LambdaRho vs. MuRho
Figure 4.5  LambdaRho vs. MuRho with fluid overlay
Figure 4.6  MuRho vs. LambdaRho with fracture compliance overlay
Figure 4.7  Expected change in seismic trace based on reduction in velocity
Figure 5.1  Normal and shear stresses
Figure 5.2  Stress vs. strain at different confining pressures
Figure 5.3  Mohr-Coulomb diagram of a formation at failure
Figure 5.4  Overburden and pore pressure calculation
Figure 5.5  Minimum horizontal stress calculations
Figure 5.6  Faults interpreted from the regional 3D PP survey
Figure 5.7  Mapped faults in the Wishbone area
Figure 5.8  Cartesian grid showing different isotropic layers
Figure 5.9  Side view of the grid showing effective porosity and Poisson’s ratio
Figure 5.10  Pore pressure and effective stress after 30,000 years of equilibration
Figure 5.11  Vertical stress profile for the isotropic linear elastic model
Figure 5.12  Vertical stress profile for elastic-plastic numerical model
Figure 5.13  Vertical strain
Figure 5.14  Impact of plastic strain
Figure 5.15  Pore pressure reduction due to production
Figure 5.16  Increase in effective stress above and below the producing layer
Figure 5.17  Elliptical drainage of reservoir
Figure 5.18 Flipping of horizontal stress field ............................................. 85
Figure 5.19 Stress field flips east of the borehole ......................................... 86
Figure 5.20 Stress field flips north of the borehole ........................................ 86
Figure 5.21 Stress field flips in the B chalk .................................................. 87
<table>
<thead>
<tr>
<th>Table 2.1</th>
<th>Anisotropy Classification</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2.2</td>
<td>Isotropic Elastic Moduli</td>
<td>24</td>
</tr>
<tr>
<td>Table 2.3</td>
<td>Anisotropic Elastic Moduli</td>
<td>25</td>
</tr>
<tr>
<td>Table 3.1</td>
<td>Standard formation volumes and endpoint velocities</td>
<td>33</td>
</tr>
<tr>
<td>Table 3.2</td>
<td>Stress sensitivity parameters</td>
<td>37</td>
</tr>
<tr>
<td>Table 4.1</td>
<td>Stress sensitivity coefficients using the Shapiro model</td>
<td>44</td>
</tr>
<tr>
<td>Table 4.2</td>
<td>Stress sensitivity coefficients using the Sayers model</td>
<td>47</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

Fourteen years ago Tom Davis invited me to the Spring sponsor’s meeting of the Reservoir Characterization Project. Since then, I have been involved with RCP in different roles, first as a sponsor, then as a faculty member, and now as a student. Returning to school after after so many years was quite an experience. It reminded me of a well-known quote by T. S. Eliot. "We will not cease from exploration, and the end of all our exploring will be to arrive where we started and know the place for the first time.” This has been an incredible experience and I thank Tom for his encouragement, guidance, mentoring, and friendship, graciously given to me over all these years.

I am indebted to these faculty members who served on my multi-disciplinary committee, Steve Sonnenberg, Monica Prasad, Ali Tura, and Whitney Trainor-Guitton. Their guidance has been crucial in focusing my research to a productive end.

I’d like to acknowledge my Schlumberger colleagues who have greatly influenced my research. Colin Sayers has been my mentor in several branches of physics, and has helped me tackle the finer points of linear slip theory. Bikash Sinha helped me understand both monopole and dipole dispersion. Dick Plumb, John Cook, Romain Prioul, Rob Marsden, and Roberto Suarez-Rivera were my mentors in geomechanics. Valery Polyakov was my mentor in computer science. Michael Thamblynayagam was an inspiration to me in his dedication to science and continuing education. I’d also like to acknowledge Jacques Tabanou and Tom Plona for their many years mentoring me in petrophysics and borehole geophysics. Bob Will has help me understand a more encompassing interdisciplinary approach to research. I don’t know who is enriched more in a student-teacher relationship; the student or the teacher. I often find it difficult to distinguish between the two. With only their questions, Adam Donald and Shannon Higgins-Borchardt have taught me more than they can possibly imagine. Both have been a joy to work with.
I’d also like to acknowledge Sigma Cubed and my Sigma Cubed colleagues. They have supported me in this integrated endeavor and helped me better understand hydraulic fracturing and reservoir simulation.

Reservoir characterization is a collaborative effort and I’ve enjoyed working with every student in RCP. It’s a joy to see them graduate and exciting to meet their replacements.

I would like to extend my sincere appreciation to the management of Anadarko Production Company for the opportunity to work with them on the RCP Wishbone field study and publish these results. Additionally, the author extends his sincere appreciation to the sponsors of the Reservoir Characterization Project for their guidance and support for the work presented in this paper.
To Terry, Jeremy, and Michael
CHAPTER 1
INTRODUCTION

The study of Geophysics gives me the tools to investigate the behavior of elastic waves, especially at great depth. The behavior of elastic waves is interesting to me due to the considerable information content contained in these waves about the subsurface. If I’m able to understand and extract this information, then perhaps I can influence decisions that can benefit our stewardship of our hydrocarbon resources.

I’m particularly interested in the influence of stress on the behavior of elastic waves. Elastic waves are influenced by stress, so there is at least a possibility to invert observations of elastic wave behavior for the in-situ stresses in the earth. This dissertation explores ideas related to this concept and hopefully contributes to the body of knowledge and understanding.

Studies of elastic wave behavior are often conducted in conventional high porosity reservoirs dominated by large changes in fluid compressibility. Fewer studies have investigated the opposite extreme of unconventional low porosity reservoirs dominated by a multiplicity of smaller effects.

My research investigates elastic wave behavior under the influence of stress in an unconventional reservoir. Changes in elastic wave behavior are conceptualized as a product of stress sensitivity multiplied by a change in effective stress. Stress sensitivity is defined as the variation of an elastic wave property with a change in effective stress. For instance, the change in P-wave velocity is a product of the stress sensitivity of the P-wave velocity in relation to effective stress, times the change in effective stress. This concept is expressed mathematically in Equation 1.1.

\[ \Delta V_p = \frac{\partial V_p}{\partial \sigma'} \Delta \sigma' \]  \hspace{1cm} (1.1)
My investigation looked at changes in elastic wave behavior at different stress magnitudes in two ways. First, I investigated the behavior of elastic waves in a fairly consistent formation over a wide range of depths. This range of depths imposed a varying stress on the formation. Second, I developed a workflow to model in-situ stresses in the earth and predict elastic wave behavior based on time-lapse changes in effective stress.

1.1 Background

Frankly, I find geoscience for the sake of geoscience boring. What attracted me to geoscience was the impact it could have on engineering decisions and how we might take better engineering decisions based on geophysical insight and understanding.

This work addresses diverse engineering issues from drilling hazard avoidance to stimulation optimization. In drilling, boreholes can be very unstable, especially when drilling through weak faulted formations. In the Niobrara Formation, the B chalk has similar, perhaps more hydrocarbon resources than the C chalk, but most wells in this study area target the C chalk. Boreholes encounter too many instability problems in the Sharon Springs Formation when the shallower B chalk is targeted.

This work also addresses completion issues related to stimulation optimization. Hydraulic fracture stimulations can be quite ineffective when the pumped fluids and proppant enter a critically stressed fault and go out of zone.

Monitoring fluid flow in the reservoir using an understanding of elastic wave behavior is an emerging technology being researched within the Reservoir Characterization Project (RCP) at Colorado School of Mines (CSM). Previous work has concentrated on monitoring fluid flow where the elastic wave signal was dominated by the difference in compressibility between the in-situ fluids and the fluid being injected. New work is being focused on Seismic Geomechanics (Herwanger & Koutsabeloulis, 2011) and the influence of stress on elastic waves (Sayers, 2010).
1.2 Objectives

The overall goal of this research is to achieve a better understanding of geomechanics and stress related elastic wave behavior so as to influence drilling and completion decisions. The research questions to be addressed are as follows:

- In the absence of natural fractures and faults, are the different formations isotropic or anisotropic?
- Which formations exhibit stress induced elastic wave behavior?
- What rock physics model can adequately quantify the stress-sensitivity parameters needed to investigate stress dependent elastic wave behavior?
- How can the stress sensitivity of the formations be quantified?
- What is the initial state of stress in the reservoir?
- How do the in-situ stresses change with changes in pore pressure?

1.3 Literature Review

A heterogeneous material is one consisting of dissimilar or diverse constituents. It can also consist of a single matrix, but have fractures that are filled with a fluid or different material. Heterogeneity can exist at all scales. Anisotropy is often caused by heterogeneity; it is defined as the variation of a property with the direction in which it is measured.

Heterogeneous materials often exhibit velocity and amplitude variations caused by a change in stress (Sayers, 2010). Different aspects of this behavior are considered in this dissertation. For instance, a reasonable model for a fractured medium, such as the Niobrara Formation, is to consider a small scale background material superimposed with a larger scale fluid filled natural fracture. Chapter 2 deals with the processing and interpretation of a dipole sonic log to determine the small scale background properties. Chapter 3 deals with quantifying stress induced behavior in dipole sonic log data while Chapter 4 deals with
quantifying stress induced behavior in core data. Chapter 5 deals with quantifying the initial state of in-situ stresses in the earth and the changes over time due to production.

The assumption that materials are continuous, homogeneous, isotropic and linear elastic (CHILE) are ubiquitous in the oil and gas industry. Most of the rock physics models are based on a large subset of these assumptions (Mavko et al., 2009). In reality, most formations are discontinuous, inhomogeneous (heterogeneous), anisotropic, non-linear, and anelastic (DIANA). This work seeks to relax some of these assumptions and build models that are more appropriate.

Chapter 2 deals with the processing and interpretation of a dipole sonic log to determine the small scale background properties. Most papers quantifying dynamic elastic moduli prior to dipole sonic logs assumed formations to be isotropic (ISO). However, with the advent of dipole sonic logs in 1990 (Harrison et al., 1990), azimuthal anisotropy could now be quantified (Esmersoy et al., 1995). This relaxed the assumption of isotropy in favor of a transverse isotropic formation with a horizontal axis of symmetry (HTI). Naturally fractured formations could now be identified, at least at the log scale. But geomechanical interpretations for stress typically ignored HTI formations in favor of the simpler isotropic equations. Schlumberger’s Sonic Scanner was introduced in 2005 (Pistre et al., 2005) and added the diagnosis of transverse isotropy with a vertical axis of symmetry (VTI). This diagnosis had a significant impact on stress estimation in unconventional shale development. However, most geomechanical interpretations for stress now assumed all formations to be VTI (Higgins et al., 2008). My work adds an important step to the workflow; a diagnosis of anisotropy is made for every depth logged by the sonic tool. Dependent on the diagnosis, e.g. ISO, HTI, or VTI, the appropriate model is used to estimate the dynamic elastic moduli.

Chapter 3 deals with quantifying stress induced behavior in dipole sonic log data. Sonic logs from the mid 1950s (Wyllie & Gardner, 1955) have been used to interpret petrophysical properties (Raymer et al., 1980). P-wave velocity was characterized in terms of the petrophysical parameters of porosity, matrix velocity, and fluid velocity. Changes in velocity
due to stress have typically been ignored in petrophysics. In geomechanics, sonic logs have been used to quantify pore pressure (Eaton, 1975). (Bowers, 1995) extended this technique to account for mechanisms other than undercompaction. These geomechanical interpretations account for changes in P-wave velocity caused by effective stress, but fail to account for changes in P-wave velocity for changes in porosity, mineralogy and fluids. (Eberhart-Phillips et al., 1989) introduced a P-wave velocity model that combined both petrophysics and geomechanics. My work extends this concept to a more complicated multi-mineral petrophysical model. I now compensate the P-wave and S-wave velocity for changes in porosity, mineralogy, and fluids, to expose and quantify the stress related velocity behavior. In addition, the stress influence on the original sonic log is removed to improve the petrophysical analysis.

Chapter 4 deals with quantifying stress induced behavior in core data. The testing of core data has the advantage of isolating the stress behavior independent of the changing petrophysics encountered in logging formations over an extended depth interval. However, most laboratory tests only evaluate the velocity behavior over a limited stress range and post failure studies are rare. Most stress dependent velocity models characterize stress with two terms, an exponential term to account for closing of high aspect ratio pores and a linear term to account for the closing of low aspect ratio pores (Shapiro & Kaselow, 2005). While there are physical grounds for these two terms, there is no insight to be gleaned from the relationship between the stress sensitivity for the P-wave and S-wave velocities. Sayers has championed linear slip theory, a model that couples the stress sensitivity of both the P-wave and S-wave velocity with different sources of compliance (Sayers, 2010). This theory is extremely flexible and is used to help understand the elastic wave behavior of Niobrara core. In addition, model is used to create an overlay for the Lambda Rho vs. Mu Rho cross plots showing the impact of fracture compliance. It is discovered that in general, velocity variations due to changing fluids are indistinguishable from variations due to changing fracture compliance.

5
Chapter 5 deals with quantifying the initial state of in-situ stresses in the earth and the changes over time due to injection and production. The vertical stress profile as computed from analytic stress models assuming linear elasticity have had a profound positive influence in geomechanics (Plumb et al., 2000). However, most stratigraphic columns have members that are structurally weak, or loaded with a significant differential stress, that they cannot be accurately modeled with linear elastic assumptions. Thus, analytic stress models assuming linear elasticity are contrasted with a numerical model that accounts for both both linear elastic and elastic-perfectly plastic behavior. It is found that weak layers can have a significantly higher stress than that computed from the analytic models. In addition, stress loading varies with injection and production. Roussel & Sharma (2012) shows successful refracture treatments in tight gas sands. My work implements a workflow coupling an improved vertical stress profile with time dependent stresses that can indicate when and where refracture treatments can be successful.

1.4 Study Area

The study area is located in the Wattenberg Field in the Denver basin, see Figure 1.1. The Wattenberg Field is highlighted in green. The area of focus is in the south central part of the field highlighted in orange.

1.5 Data and Methodology

The area of focus is called the Wishbone section, a one square mile area shown as a yellow square in Figure 1.2. This section was the object of the "Turkey Shoot", a time-lapse, 9 component, 3D seismic survey.

Other data was provided that was important in my study. Two dipole sonic logs were available near, but not in, the Wishbone section. These data were especially important as they provided a multitude of elastic wave measurements. One dipole log recorded fewer modes but was logged from below the reservoirs of interest to nearly the surface. The other dipole logs recorded an extensive suite of modes, but was only logged over the reservoir
Figure 1.1: Location of Wattenberg Field
interval.

Twenty-three sets of petrophysical triple-combo logs were recorded inside or within 1000 foot of the Wishbone section. These logs, after normalization and integration with core, were used to build a petrophysical model of the Wishbone section. The cored wells were all within a distance of 7 miles.

I had the opportunity to analyze a set of core measurements that was also investigated by (Panfiloff, 2016). While this core data was from the Niobrara Formation, the location of the cored well was withheld from the study.

The stratigraphic column for the Wattenberg Field is shown in Figure 1.3.

The only difference in the Wishbone section is the absence of the A chalk. Of particular importance is the expansive deposition of the Pierre Formation in the overburden. It extends from the surface to within a couple of hundred feet of the top of the Niobrara Formation. The Pierre Formation is a marine deposit with similar petrophysical properties in the horizontal
Figure 1.3: Wattenberg stratigraphic column
and vertical directions.

My study began with a data audit and construction of a calibration database. The data audit identifies and compiles the available data. The calibration database reviews and compiles the data that represents the highest quality or "ground truth" data. I hesitate to use the ground truth term because no single scale of data represents ground truth. In fact, all scales of data; core, logs, in-situ tests, and seismic, must be integrated. I use the term integration to mean the synthesis of multi-scale and multivariate data required solve complex interpretation problems.

Data were analyzed and models were built to help understand the behavior of elastic waves. I used Techlog, a software package provided by Schlumberger to analyze all of the well-centric data. In particular, it was Techlog’s Python interface that allowed me to program and build the tools and models that supported this research. I used Computer Modeling Group’s (CMG) reservoir simulator and its geomechanics module (GEM) to compute the flow coupled geomechanical stresses.
CHAPTER 2
DIPOLE SONIC PROCESSING

The first step is to process the sonic waveforms for P-wave and S-wave velocities. In the first step, I processing the raw waveforms for velocity, but I also include dispersion analysis, a technique to diagnose the type of anisotropy.

I found the Fort Hays and Codell Formations to be isotropic while the Niobrara and surrounding shales exhibit VTI anisotropy. It is interesting to note that the J Sand and parts of the Carlile Formations exhibit HTI anisotropy, but the source of anisotropy is stress, not fractures.

2.1 Sonic Velocities

Sonic tools were introduced to the oil industry in the late 1950s. The original sonic tool measured the transit time, $TT$, between two receivers spanning a short interval, $L$, typically two feet. The interval transit time per foot was called $\Delta t$ and had the units of microseconds per foot, ($\mu s/ft$). $\Delta t$, is a measure of slowness which is the reciprocal of velocity, $V$.

$$\Delta t = \frac{TT}{L} = \frac{1}{V} \quad (2.1)$$

Geophysicists prefer dealing with velocities and thus convert the $\Delta t_{\text{zo}}$ compressional slowness measurement to a P-Wave velocity, $V_p$. Secondary waves, also known as shear waves, $V_s$, were also observed in the sonic recordings, but it would be the 1980s before logging tools could reliably measure an S-Wave velocity. $V_p$ in units of ($ft/sec$) is given by Equation 2.2.

$$V_p = \frac{10^6}{\Delta t} \quad (2.2)$$

In the early 1980s, sonic tools could only measure critically refracted head-waves. This limited the S-wave measurement to ”fast” formations where the shear velocity of the formation was greater than the velocity of the borehole fluid. In the 1990s, dipole logs were
introduced to overcome this limitation. Rather than measuring head-waves, dipole tools measure the vibrational modes of the borehole. This new technology allowed the S-wave to be measured in all formations regardless of the velocity of the borehole fluid. In addition, if sufficient azimuthal anisotropy existed in the formation, a dipole sonic tool could quantify both the fast and slow shear wave velocity. Geophysicists call this type of anisotropy transverse isotropy with a horizontal axis of symmetry (HTI). By 2005, the Stoneley mode could be measured with sufficient accuracy to determine the shear velocity in the horizontal plane (Pistre et al., 2005). Since 2005, the shear velocity can be determined in three orthogonal planes and diagnose the two major types of transverse isotropy, HTI as previously mentioned, and the second type of anisotropy, transverse isotropy with a vertical axis of symmetry (VTI) (Sinha et al., 2006).

While three shear velocities can now be quantified, only one compressional velocity can be determined, the P-wave velocity aligned with the borehole. In a vertical borehole, four velocities can now be quantified, the vertical P-wave velocity, two vertical S-wave velocities (called the fast and slow shear), and the horizontal shear wave velocity.
Figure 2.1 shows the stratigraphic column in track 1, correlation curves in tracks 2 and 3, sonic velocities in track 4, and sonic slownesses in track 5. In tracks 4 and 5, the P-wave is green, the fast S-wave is red, the slow S-wave is dark blue, and the horizontal S-wave is light blue. The Muddy (J-Sand) Formation is mostly isotropic as all three shear velocities approximately overlay.

2.2 Dynamic Elastic Moduli

I can now estimate four of the $C_{ij}$ stiffness parameters from estimates of the P-wave and S-wave velocities. The vertical P-wave modulus is estimated from the bulk density and the vertical P-wave velocity as given by Equation 2.3.

$$C_{33} = \rho_b V_p^2$$ (2.3)

The two vertical shear moduli are estimated from the fast and slow S-wave velocities as given in Equations 2.4 and 2.5.

$$C_{44} = \rho_b V_{s(FastShear)}^2$$ (2.4)

$$C_{55} = \rho_b V_{s(SlowShear)}^2$$ (2.5)

The horizontal shear velocity is estimated from the measured Stoneley slowness and mud parameters (Norris, 1993). There is a Pythagorean relationship between the Stoneley slowness, mud slowness, and the shear slowness perpendicular to the borehole, as shown in Figure 2.2.

For vertical boreholes, the shear slowness perpendicular to the borehole is the horizontal shear.

$$\frac{\rho_{mud}}{\rho_{bulk}} \Delta t_{HorizontalShear}^2 + \Delta t_{Mud}^2 = \Delta t_{Stoneley}^2$$ (2.6)

While the Stoneley mode can be accurately measured, it is clear that uncertainty in the mud properties impact the accuracy of the horizontal shear. I can now compute the horizontal shear modulus from the horizontal shear velocity derived from the Stoneley slowness.
Figure 2.2: Relationship between the horizontal shear and the Stoneley measurement

\[ C_{66} = \rho_b V_{\text{HorizontalShear}}^2 = \rho_{\text{mud}} (V_{\text{mud}} V_{\text{Stoneley}})^2 \]

(2.7)

2.3 Dispersion Analysis

A sonic wave is said to be dispersive when its phase velocity depends on frequency. Dispersion was not a problem before the introduction of dipole sonic logs because critically refracted head-waves are not dispersive. However, with dipole technology, and the measurement of highly dispersive borehole modes such as the flexural shear and Stoneley mode, dispersion analysis has become an important technology. The velocities extracted from flexural shear and Stoneley modes will have errors if the dispersion is not quantified and removed. In addition, dispersion analysis has become a diagnostic tool to differentiate isotropic formations from anisotropic formations. And if a formation is anisotropic, dispersion analysis can help determine the type and orientation of the anisotropy.

Figure 2.3 shows a typical dipole flexural dispersion curve. The plot on the top shows the slowness of the flexural mode, in red, as a function of frequency (Pistre et al., 2005). As frequency increases, so does the slowness. At low frequencies, the flexural mode propagates
at the slowness of the shear body wave in the formation. At high frequencies, the flexural mode propagates as a Sholte wave, a surface wave on the interface between the borehole fluid and the formation. The Sholte wave slowness is controlled by a number of factors, but is largely dominated by the slowness of the borehole fluid. The green plot on the bottom shows the energy of the flexural mode. The mode has a maximum energy at the Airy phase frequency, where the flexural slowness is an intermediate value between the shear and Sholte slowness. Herein lies the challenge of measuring modes. No dispersion correction is needed if the mode can be measured at a low enough frequency. The challenge at low frequencies is the flexural wave has a very low amplitude or energy. Thus, the noise must also be low to extract a valid signal. This can be difficult, especially in rugose boreholes. The other alternative is to characterize the dispersion where sufficient energy exists, and extrapolate the dispersion curve to zero frequency (Wang, 2006).
An application of dispersion analysis is to diagnose the source of anisotropy (Plona et al., 2002). Figure 2.4 shows four different flexural dispersion curve behaviors. Each sub-plot shows the two orthogonal flexural curves, the fast shear in red, the slow shear in blue and a modeled homogeneous isotropic (HI) dispersion curve in black. The low frequency components of the flexural wave probe deep into the formation while the high frequency components probe near the borehole. Thus, comparing the flexural dispersion curves to a HI model can indicate the radial profile in slowness (Sinha et al., 2006).

![Dispersion Curve Behavior](image)

**Figure 2.4: Dispersion curve behavior**

The top left figure shows no azimuthal anisotropy, the two orthogonal flexural waves overlay in slowness. And both flexural dispersion curves overlay the HI curve for all frequencies. This is a good diagnosis of a HI formation. The top right figure also shows no azimuthal anisotropy but both flexural dispersion curves deviate above the HI curve at high frequency with increasing slowness as frequency increases. This behavior is consistent with mechanical alteration near the borehole. The high frequencies probing near the borehole are traveling
with a greater slowness due to the higher compliance surrounding the borehole. In fact, this is the reason long-spaced sonic tools were invented, to use the extra transmitter-receiver spacing to investigate beyond the drilling damage.

The bottom left figure shows azimuthal anisotropy. The red curve is the fast shear and the blue curve is the slow shear. Both flexural curves overlay with a modeled HI dispersion curve. This means that whatever is causing the compliance difference between the fast and slow shear waves, the compliance difference exists near the borehole as well as deep into the formation. This behavior is consistent with natural fractures or bedding that crosses the borehole at an angle. The bottom right figure also shows azimuthal anisotropy but the fast and slow dispersion curves cross at an intermediate frequency. This is diagnostic of stress induced anisotropy where the high frequency slownesses near the borehole are dominated by the near borehole hoop stress and the low frequency slownesses far from the borehole are dominated by the far-field stresses.

When the source of anisotropy is open natural fractures, the fast shear azimuth derived from Alford rotation indicates the strike direction of the fracture. When the source of anisotropy is stress, the fast shear azimuth indicates the direction of the present day maximum horizontal stress.

In addition to the dipole dispersion curves, the Stoneley mode is also dispersive. Reviewing Stoneley dispersion in concert with dipole dispersion can also help diagnose anisotropy. If azimuthal anisotropy is caused by open natural fractures crossing the borehole, the Stoneley slowness will increase, especially at low frequency, the Stoneley amplitude will decrease, and the Stoneley attenuation will increase (Donald & Bratton, 2006). If the azimuthal anisotropy is caused by stress, these fracture effects on the Stoneley dispersion are absent.

If a formation is VTI, there are two additional effects. The Stoneley slowness will decrease with respect to the dipole slowness due to the increase in stiffness in the horizontal direction. In addition, the flexural dispersion will reduce at higher frequencies. Because these signatures are unique, dispersion analysis is a key tool in the diagnosis of anisotropy.
It must be noted that the anisotropy diagnosed by dipole sonic tools is at the scale of a few feet and measured along the borehole path.

Figure 2.5 shows the dispersion curves in the Codell Formation in a well about 4 miles north of the Wishbone section.

![Dispersion curves in the Codell Formation](image)

Figure 2.5: Dispersion curves in the Codell Formation

The fast flexural dispersion curve is shown in red and the slow dispersion curve is shown in blue. Both curves are in the top half of the plot. The Stoneley dispersion curve is shown in cyan and is also in the top half of the plot. The slowness scale is on the y-axis on the left side of the plot. The three curves at the bottom of the plot are the amplitudes of the waves with the scale shown on the y-axis on the right side of the plot. The Stoneley mode has the greatest amplitude between 500 Hz and about 2 kHz. The two flexural curves have the greatest amplitude at about 4 kHz and the amplitude decreases at both lower and higher frequencies.

The fast (red) and slow (blue) flexural dispersion curves overlay indicating the Codell Formation is isotropic. It is more difficult to diagnose VTI anisotropy with dispersion plots.
but I will cover this in the next sub-section.

Figure 2.6 shows flexural dispersion curves that cross over. At low frequency, the fast flexural dispersion curve in red has a lower slowness, or faster velocity, than the slow flexural dispersion curve in blue. However, at high frequency, the ordering is reversed.

![Figure 2.6: Dispersion curves in the J-Sand and Carlile Formations](image)

Flexural dispersion curves that cross over in this manner is diagnostic of stress induced anisotropy. This 3.7% difference between the fast and slow S-wave velocities in the J-Sand is caused by the difference between the minimum and maximum horizontal stress. In addition, the fast shear azimuth of 96 degrees with respect to true north gives the present day maximum horizontal stress direction.

### 2.4 Anisotropy Classification

It is possible to diagnose and classify the two major types of transverse anisotropy, HTI and VTI, using the three orthogonal S-wave velocities. In fact, I identify five classifications based on the relationship between $C_{44}$, $C_{55}$, and $C_{66}$, see Figure 2.7 and Table 2.1.

Isotropic formations are characterized and diagnosed by $C_{44} = C_{55} = C_{66}$. A threshold is used to test for equality to account for noise in the measurement. VTI formations by $C_{44} =$
When all three moduli are different the formation is orthotropic. Rather than tackling the difficult problem of quantifying all nine components of an orthotropic formation, a simplification reduces orthotropic formations into a more manageable transverse isotropic equivalent.

### Table 2.1: Anisotropy Classification

<table>
<thead>
<tr>
<th>Class</th>
<th>Anisotropy Value</th>
<th>$C_{ij}$ Relationship</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>$C_{44} = C_{55} = C_{66}$</td>
<td>Isotropic</td>
</tr>
<tr>
<td>2</td>
<td>20 - 30</td>
<td>$C_{44} = C_{55} &lt; C_{66}$</td>
<td>VTI</td>
</tr>
<tr>
<td>3</td>
<td>10 - 20</td>
<td>$C_{44} &lt; C_{55} = C_{66}$</td>
<td>HTI</td>
</tr>
<tr>
<td>4</td>
<td>30 - 40</td>
<td>$\frac{C_{44} + C_{55}}{2} &lt; C_{66}$</td>
<td>VTI Like</td>
</tr>
<tr>
<td>5</td>
<td>0 - 10</td>
<td>$\frac{C_{44} + C_{55}}{2} &gt; C_{66}$</td>
<td>HTI Like</td>
</tr>
</tbody>
</table>

Each depth is assigned an anisotropic value depending on the class. The more the anisotropic value deviates from 20 the more anisotropic the formation.

It is important to understand that Stoneley interpretation is complicated by two factors, uncertainty in the borehole fluid properties and uncertainty in formation permeability. It is relatively easy for drillers to drill through overpressured low permeability formations un-
derbalanced. When this happens, highly compressible fluids such as oil and gas can enter the borehole and alter the compressibility of the drilling fluid. This generally increases the slowness of the mud. Everything else being equal, this would decrease $C_{66}$ making the formation appear more compliant. A similar effect happens when logging through a permeable formation. The Stoneley slowness increases with permeability especially at low frequency (Brie et al., 2000). This effect is especially pronounced when high gas saturation or open natural fractures exist near or cross the borehole.

![Figure 2.8: Gas effect on Stoneley slowness](image)

Figure 2.8 shows a gas effect on the Stoneley slowness. The stratigraphic column is shown in track 1, Gamma ray in track 2, density-neutron crossover, shown in red, in track 3, sonic slownesses in track 4, Thomsen’s anisotropic parameters in track 5 and my anisotropy classification in track 6. The J-Sand is mostly isotropic. The horizontal shear slowness shown in light blue, correlates with the neutron density gas crossover shown in red. Light hydrocarbon near the mud-borehole interface is more compressible and thus reduces the velocity of the Stoneley wave. This translates into a slower horizontal shear slowness. Outside of this effect, the three shear moduli overlay indicate the J-Sand is isotropic.
Because of this interplay between environmental and fracture effects on the Stoneley mode, it can be difficult in choosing the appropriate mud parameters to interpret the 3 shear moduli overlay. However, many formations are close to isotropic, and logging a sufficiently long depth interval gives a good chance to observe isotropic formations and help determine the best mud parameters.

Figure 2.9: Three shear slownesses in the Codell and Fort Hays Formations

The Codell and Fort Hays Formations shown in Figure 2.9 tell a similar story. Outside of the fluid effect in the Fort Hays Formation, these two layers are substantially isotropic. It is also clear that the horizontal shear velocity in the Carlile Formation is much faster than the two dipole vertical shear velocities. This is the key signature for a VTI formation.

Figure 2.10 shows a strong VTI signature in the Niobrara Formation and bounding shales. Elastic moduli are likely to be representative of non-fractured background formation properties when analyzed from either core or advanced sonic logs acquired in vertical wells. This is due to the scale of the measurement and the orientation of the borehole with respect to the faults and fractures. An advantage of sonic logs over recovered core is complete coverage over the layers of interest.
2.5 Anisotropy Estimation

There are five independent elastic moduli that characterize a VTI formation. One way to classify a VTI formation with $C_{ij}$ components is used to define $C_{11}$, $C_{13}$, $C_{33}$, $C_{44}$, and $C_{66}$. Dipole sonic logs running in vertical boreholes can only quantify a subset of these components: $C_{33}$, $C_{44}$, and $C_{66}$; $C_{11}$ and $C_{13}$ are not measured. Suarez-Rivera & Bratton (2012) patented a technique that characterizes a VTI formation using two multipliers $\xi$ and $\zeta$. The multiplier $\xi$ relates $C_{12}$ and $C_{13}$ and the multiplier $\zeta$ relates $C_{11}$ with $C_{33}$ and $C_{44}$ as shown in Equations 2.8 and 2.9.

$$C_{12} = \xi C_{13} \quad (2.8)$$

$$C_{13} = \zeta C_{33} - 2C_{44} \quad (2.9)$$

These relationships allow $C_{11}$ to be computed from the measured values of $C_{33}$, $C_{44}$, and $C_{66}$ and $\xi$ and $\zeta$ as shown in Equation 2.10.

$$C_{11} = \xi (\zeta C_{33} - 2C_{44}) + 2C_{66} \quad (2.10)$$
There are different ways to estimate $\xi$ and $\zeta$. The method used here was to equate Thomsen’s anisotropic parameters epsilon and gamma, and to set $\xi = 1.1$ (Thomsen, 1986). (Sondergeld & Rai, 2011) show that the $\epsilon = \gamma$ approximation is reasonable for shale reservoirs. The $\xi$ equal to 1.1 approximation yields reasonable values of Thomsen’s delta when compared to Niobrara Core (Panfiloff, 2016).

Thomsen’s epsilon computed by this algorithm show the epsilon value in the Niobrara chalks is less than the Niobrara marls as shown in Figure 2.11.

![Figure 2.11: VTI anisotropy diagnosed in the Niobrara Formation and bounding shales](image)

In addition, this analysis shows the epsilon value in the Niobrara Formation is less than the bounding shales as shown in Figure 2.12.

The average values of the isotropic dynamic elastic constants are given in Table 2.2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Nio B-Chalk</th>
<th>Codell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>5.78 (Mpsi)</td>
<td>4.76 (Mpsi)</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.26 (—)</td>
<td>0.21 (—)</td>
</tr>
</tbody>
</table>

The average values of the VTI anisotropic dynamic elastic constants are given in Table 2.3.
Figure 2.12: VTI anisotropy in the Niobrara Formation and bounding shales

Table 2.3: Anisotropic Elastic Moduli

<table>
<thead>
<tr>
<th>Property</th>
<th>Nio B-Chalk</th>
<th>Codell</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$</td>
<td>6.28 (Mpsi)</td>
<td>4.81 (Mpsi)</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>6.28 (Mpsi)</td>
<td>4.81 (Mpsi)</td>
</tr>
<tr>
<td>$E_{33}$</td>
<td>4.85 (Mpsi)</td>
<td>4.71 (Mpsi)</td>
</tr>
<tr>
<td>$\nu_{13}$</td>
<td>0.41 (—)</td>
<td>0.23 (—)</td>
</tr>
<tr>
<td>$\nu_{31}$</td>
<td>0.32 (—)</td>
<td>0.22 (—)</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.41 (—)</td>
<td>0.23 (—)</td>
</tr>
<tr>
<td>$\nu_{32}$</td>
<td>0.32 (—)</td>
<td>0.22 (—)</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.17 (—)</td>
<td>0.21 (—)</td>
</tr>
<tr>
<td>$\nu_{21}$</td>
<td>0.17 (—)</td>
<td>0.21 (—)</td>
</tr>
</tbody>
</table>
2.6 Results and Conclusions

In this chapter, I show results from the processing and interpretation of the dipole sonic logs. The vertical P-wave velocity and all three shear velocities; fast, slow and horizontal, were quantified.

I analyzed the dispersion in the dipole sonic data to diagnose isotropy and different types of anisotropy. The Fort Hays and Codell Formations were diagnosed isotropic.

One thin layer in both the Muddy and Carlile Formations was diagnosed to be HTI. The source of the HTI anisotropy was caused by stress, not open natural fractures.

I used the results of the dispersion analysis to classify the anisotropy as a function of depth along the borehole. Quantitative values of the $C_{ij}$ components were computed based on the type of anisotropy. I used an isotropic model to quantify the dynamic elastic moduli in the isotropic layers, an HTI model to quantify the dynamic elastic moduli in the HTI layers, and a VTI model to quantify the dynamic elastic moduli in the VTI layers.

It is interesting that the Muddy and Carlile Formations exhibit stress induced anisotropy. This means that either the stress sensitivity, horizontal stress contrast, or both, are large enough to cause a difference in the dipole shear measurements. This is not observed in the shallower formations. This would indicate that either the stress sensitivity, horizontal stress contrast, or both decrease with depth. I suspect that the horizontal stress contrast decreases in the overlying formations.

I find the Niobrara Formation and the bounding shale formations to have a significant amount of VTI anisotropy. This will impact the isotropic assumptions made in processing and interpreting the PS and SS wavefields.
I will present and explain a new method to interpret sonic velocities measured by dipole sonic logging tools. The method quantifies the influence of porosity, mineralogy, and fluids, while isolating and quantifying the influence of stress. A dipole sonic log in the Wattenberg Field of Colorado has been evaluated using this new method. Velocity variations due to changes in porosity, mineralogy, and fluids were determined using a multi-mineral petrophysical solver. Matrix and fluid substitution was then used to normalize the velocities to an idealized formation with constant porosity, mineralogy and fluids. This normalized velocity curve was used to diagnose and quantify the stress sensitivity of the Pierre Shale. The stress sensitivity of the P-wave velocity in the normally pressured interval is 580 ft/s per 1000 psi change in effective stress. In contrast, the stress sensitivity of the P-wave velocity in the overpressured interval is 3150 ft/s per 1000 psi change in effective stress. This work quantifies the stress sensitivity of the Pierre Shale and thus contributes to the interpretation of time-lapse seismic attributes for spatial changes in effective stress. In addition, the petrophysical interpretations are improved by using the stress normalized velocity curves after removing the stress contribution from the measured curves.

3.1 Introduction

A time-lapse seismic survey was recently acquired over one square mile in the Wattenberg Field located in the Denver Basin of Colorado. These data were acquired to investigate the hypothesis that time-lapse seismic measurements can quantify the spatial extent of the reservoir that has been treated by a hydraulic fracture stimulation. An analysis of these data should guide the optimum hydraulic fracture stage spacing as well as the optimum horizontal borehole spacing. However, for seismic data to indicate the spatial changes of stress with time, some sonic attribute such as velocity must vary with stress. Therefore, an important
component of the overall objective is to quantify the sensitivity of sonic velocities to changes in stress. A wireline dipole sonic log in the study area has been evaluated to quantify this stress sensitivity.

The sonic velocities measured by dipole logging tools are known to vary with stress in favorable situations. However, velocity variations are also caused by changes in porosity, mineralogy and fluids. Therefore, to isolate the velocity dependence due to changes in stress, the velocity dependence to variations in porosity, mineralogy and fluids must be independently determined and removed. If the formation normalized velocity is independent of stress, one would expect to see a constant, depth invariant normalized velocity. On the other hand, changes in normalized velocity with depth could be explained by changes in stress. For instance, one would expect to see the normalized velocity increase with depth due to an increase in stress with increasing overburden.

Borehole sonic logs measuring P-wave velocity were introduced in the late 1950s. A landmark paper published by Wyllie (Wyllie & Gardner, 1955) was instrumental in deriving porosity from these new sonic logs. The P-wave velocity is given by $V_p$.

\[
\frac{1}{V_p} = \frac{1 - \phi}{V_{ma}} + \frac{\phi}{V_f}
\]  

(3.1)

The petrophysical factors influencing $V_p$ are the porosity defined by $\phi$, the mineralogy defined by the velocity of the matrix, $V_{ma}$, and the saturating fluid defined by the velocity of the fluid $V_f$. While Wyllie observed substantial changes in velocity with stress, the petrophysics community ignored this complication in favor of the stress independent time average equation.

Later, an effort by Eaton (Eaton, 1975) was directed at predicting geopressure from sonic and other logs to aid in the safety of drilling wells penetrating overpressured formations.

\[
P_p = \sigma - \sigma_{nor}' \left[ \frac{V_p}{V_{nor}} \right]^3
\]  

(3.2)

Eaton’s model was based on the idea that a decrease in effective stress, caused by an increase in pore pressure, will decrease the measured P-wave velocity from that of an identical
formation having normal pore pressure. The stress dependence of velocity is clearly explicit in this model but it sidesteps the dependence of velocity on mineralogy and fluids by excluding these petrophysical properties from the model.

Eberhart-Phillips (Eberhart-Phillips et al., 1989) introduced a P-wave velocity model that acknowledged some of the variation of the P-wave velocity was due to petrophysics (mineralogy, porosity and clay content), and some of the variation was due to effective stress.

\[ V_p = A_0 - A_1 \phi - A_2 \sqrt{Vol_{cl}} - C_1 \sigma' - C_2 e^{-C_3 \sigma'} \]  

(3.3)

Porosity is defined by \( \phi \), the volume of clay is defined by \( Vol_{cl} \), and effective stress is defined by \( \sigma' \). The \( A_i \) coefficients are fitting parameters for the petrophysical part of the model and the \( C_i \) coefficients are fitting parameters for the stress part of the model. These stress coefficients were determined by empirical fits to the data studied.

By the 1990s, sufficient log data was becoming available to introduce multi-mineral petrophysical models. A P-wave velocity model was introduced into a conventional petrophysical solver, to account for the variety of minerals commonly encountered in the subsurface, (Quirein et al., 1986).

\[ V_p = \sum_{i=1}^{i=nMin} V_p^i (Vol_i) + \sum_{j=1}^{j=nFlu} V_p^j (Vol_j) \]  

(3.4)

The first term is the matrix or mineralogical component computed as a linear combination of the minerals P-wave velocity weighted by the mineral volume. \( V_p^i \) is the P-wave velocity of the mineral, and \( Vol_i \) is the relative volume of the mineral. The second term is the fluid-filled porosity component computed in the same way. This model has the strength that it can account for the multiple minerals and fluids commonly found in subterranean formations and is consistent with the numerous observations of P-wave velocity decreasing with porosity in an approximate linear fashion, e.g. (Han et al., 1986). Unfortunately, it does not account for stress.
The model used in this study combines a multi-mineral petrophysical model with the linear stress term of the Eberhart-Phillips stress model. A reference stress term was introduced to provide a basis to normalize the velocities with stress.

\[ V_p = \sum_{i=1}^{i=nMin} V_p^i(Vol_i) + \sum_{j=1}^{j=nFlu} V_p^j(Vol_j) - \frac{\partial V_p}{\partial \sigma'} (\sigma' - \sigma'_{ref}) \] (3.5)

This model accounts for the petrophysical changes due to multiple minerals and fluids in the first two terms, and accounts for the stress sensitivity in the last term. The focus of this work is to build a combined petrophysical and stress model that can be used to isolate and quantify the various factors influencing the sonic velocities measured along a vertical borehole. In other words, apply a multi-mineral petrophysical model to differentiate petrophysical effects from stress effects.

The Pierre Shale overlying the Niobrara Formation is sensitive to stress in the study area. The normalized velocity curve increases in velocity with increasing depth to a depth within the Lower Pierre Formation where a velocity reversal is observed. The normalized velocity decreases significantly below that depth. The velocity reversal is interpreted as the top of overpressure due to a complicated process that includes the maturation of the organic content in the source beds in and around the Niobrara Formation and the subsequent migration of pressure and fluids to shallower depths. If the formation is normally pressured above the velocity reversal, and a value of 4500 psi is used for pore pressure in the Niobrara Formation, the stress sensitivity of the P-wave velocity in the normally pressured interval is 580 ft/s per 1000 psi change in effective stress. In contrast, the stress sensitivity of the P-wave velocity in the overpressured interval is 3150 ft/s per 1000.0 psi change in effective stress.

3.2 Petrophysical Model

The only inputs used in the petrophysical analysis of the Pierre Shale are the nuclear measurements of gamma ray (GR curve), bulk density (RHOB curve), volumetric photoelectric factor (U curve) and thermal neutron porosity (TNPH curve). Neither deep resistivity (AT90 curve) nor sonic velocities (Vp, Vs) are used.
There are fundamental differences between these two classes of measurements; standard nuclear logs vs. resistivity and/or sonic logs. Standard nuclear measurements (e.g. RHOB, TNPH) are governed by bulk volume properties such as porosity and the volume of clay. Nuclear measurements can accurately quantify the bulk volume of porosity and clay, but cannot determine how the porosity or clay is distributed within the bulk volume. Resistivity and sonic measurements are governed not only by bulk volume properties, they are also governed by how the porosity and clay are distributed within the formation. I’ll refer to how porosity and clay are distributed in the formation as fabric.

Consider a highly saline water saturated formation of 10% porosity with 10% clay. The porosity could be completely connected yielding a low resistivity or alternatively, the porosity could be completely isolated yielding a high resistivity. In addition, the 10% clay content could be dispersed in the porosity or exist as nodules or laminations. The different clay fabric will affect the sonic velocities. Thus, fabric differences affect resistivity and sonic velocities, even when porosity and clay volume remain the same. Standard nuclear measurements are only governed by bulk volume properties. They contain no information about fabric. Resistivity and sonic measurements are governed by both bulk volume properties and the fabric of the formation. In terms of sonic velocities, pore pressure or stress could also be considered a fabric effect.

Thus, to isolate the stress effects from changes in petrophysical properties, only standard nuclear logs were used in the determination the petrophysical properties of the Pierre Shale.

Three models were built. The Upper Pierre model solved for quartz, calcite, illite, smectite and water filled porosity. The Terry/Hygiene model solved for quartz, calcite, illite, water and gas. Deep resistivity was used in this model but I ignored these two thin zones. The lower Pierre/Sharon Springs model solved for quartz, calcite, illite, kerogen and water. The results are shown in Figure 3.1.

Track 1 shows the stratigraphic column. Track 2 shows the volumetric analysis. Tracks 3, 4, 5, and 6 show the measured logs, Gr, Rhob, Tnph and U, overlaid with the reconstructed
logs. Note the excellent agreement between the measured and reconstructed logs based on the petrophysical model. Track 7 shows the Vp and Vs velocities. Note the Vs data is discontinuous due to limitations of the logging tool.

It is informative to compare the P-wave velocity with the volumetric analysis. There is a clear correlation between an increase in the P-wave velocity and an increase in the volume of quartz. There is also a clear correlation between a decrease in the P-wave velocity and an increase in the volume of illite.

Therefore, I employed a matrix and fluid substitution process to determine and compensate the variation in the P-wave velocity with the rapid variations in petrophysical volumes with depth. A normalized P-wave velocity was constructed in the upper Pierre Shale, lower Pierre Shale and the Sharon Springs Formation, accounting for the variations in mineralogy and porosity using Equation 3.6.

\[
V_p^{Norm} = \sum_{i=1}^{nMin} V_{pi}^{Std} (Vol_i^{Std} - Vol_i) + \sum_{j=1}^{nFlu} V_{pj}^{Std} (Vol_j^{Std} - Vol_j) \quad (3.6)
\]
The volumes from the petrophysical analysis are normalized to a standard formation with constant volumes with depth as given by Equation 3.7. The standard formation constant volumes were chosen to be the approximate average volumes of the shale interval.

\[ V_p^{\text{Std}} = \sum_{i=1}^{i=n_{\text{Min}}} V_p^{i(\text{Std})}(Vol_i^{\text{Std}}) + \sum_{j=1}^{j=n_{\text{Flu}}} V_p^{j(\text{Std})}(Vol_j^{\text{Std}}) \]  

Because the sonic velocities were not used in the petrophysical analysis of the shales, it was possible to determine the endpoint sonic velocities, \( V_p^i(\text{Std}) \) and \( V_p^j(\text{Std}) \) by minimizing the difference between the measured sonic log and the petrophysical model as shown in Equation 3.8.

\[ \text{min} \left\| V_p^\text{log} - \sum_{i=1}^{i=n_{\text{Min}}} V_p^{i(\text{Std})}(Vol_i) - \sum_{j=1}^{j=n_{\text{Flu}}} V_p^{j(\text{Std})}(Vol_j) \right\|^2 \]  

The results of these computations are shown in Table 3.1. Note that the petrophysical endpoints for the S-wave velocity are be determined in the same manner as the P-wave velocity.

Table 3.1: Standard formation volumes and endpoint velocities

<table>
<thead>
<tr>
<th>Component</th>
<th>Volume (v/v)</th>
<th>( V_p^{\text{Std}} ) (ft/s)</th>
<th>( V_s^{\text{Std}} ) (ft/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>0.23</td>
<td>17,000</td>
<td>9,887</td>
</tr>
<tr>
<td>Calcite</td>
<td>0.13</td>
<td>15,025</td>
<td>8,999</td>
</tr>
<tr>
<td>Illite</td>
<td>0.59</td>
<td>9,517</td>
<td>4,886</td>
</tr>
<tr>
<td>Porosity/water</td>
<td>0.05</td>
<td>-600</td>
<td>650</td>
</tr>
<tr>
<td>Sum</td>
<td>1.00</td>
<td>11,148</td>
<td>6,359</td>
</tr>
</tbody>
</table>

The normalized velocity can now be plotted as a function of depth to diagnose any variation in velocity due to stress as shown in Figure 3.2.

The stratigraphic column is shown in track 1 and the volumetric analysis is shown in track 2. Track 3 shows the measured Vp curve. The normalized Vp curve is shown in track 4. Note the variations in the red Vp curve due to changes in mineralogy and porosity have been largely removed. The blue curve superimposed on the red Vp curve is a 100
foot Gaussian filter used to smooth the rapid foot by foot petrophysical variations. Track 5 shows a magnified scale to better illustrate any influence (e.g. stress) not associated with the petrophysics. Track 6 shows the smoothed normalized Vp in blue with the stress effect superimposed in red. The final track shows the stress model used to interpret the sonic logs.

The P-wave velocity increasing with depth is representativ e of a loading behavior from the shallowest depths to a depth in the middle of the Lower Pierre Formation. Below this depth, the P-wave velocity decreases representative of an unloading curve. The velocity reversal in the Lower Pierre Formation is clearly visible in track 6.

3.3 Effective Stress Model

An initial 1D mechanical earth model (1D-MEM) was constructed from the available data. The bulk density was integrated from the surface to determine an overburden pressure gradient of 1.08 psi/ft. The pore pressure in the Niobrara Formation was determined from multiple engineering data sets to be approximately 4500 psi. Because of the obvious P-wave velocity reversal seen in the Lower Pierre Formation in Figure 3.2, the pore pressure was assumed to be normally pressured to the observed velocity reversal. The pore pressure was
set to 4500 psi in the Niobrara Formation and interpolated using a constant overpressure gradient as computed by Equation 3.9.

\[
P_{p}^{\text{over}} = P_{p}^{\text{nor}} + \frac{\partial P_{p}}{\partial D_{\text{tvd}}} (D_{\text{tvd}} - D_{\text{top}}^{\text{tvd}})
\]  

(3.9)

A value of 1.2 psi/ft was used for the pore pressure gradient, \(\frac{\partial P_{p}}{\partial D_{\text{tvd}}}\), below the velocity reversal. The vertical effective vertical stress, \(\sigma_{V}'\), is then computed as the difference between the total vertical stress and pore pressure.

\[
\sigma_{V}' = \sigma_{V} - \alpha P_{p}
\]  

(3.10)

The normalized P-wave velocity as a function of effective vertical stress is shown in Figure 3.3. The normalized P-wave velocity is approximately linear with depth for both the normal and overpressure intervals.

![Figure 3.3: Normalized Vp versus effective vertical stress](image)

These data and analyses clearly demonstrate the P-wave velocity to be a strong function of stress.
3.4 Stress Compensated Petrophysical Model

In the section I introduce a stress based petrophysical model.

\[
V_p^{model} = \sum_{i=1}^{i=nMin} V_i^i (Vol_i) + \sum_{j=1}^{j=nFlu} V_j^j (Vol_j) - \frac{\partial V_p}{\partial \sigma'} (\sigma' - \sigma'_{ref}) \tag{3.11}
\]

The first two terms capture the variability in the P-wave velocity due to the petrophysics. The third term captures the variability due to effective stress. An objective function can be built to determine both the mineral and fluid P-wave endpoints as well as the two stress sensitivity differentials, one for the normally pressured interval and one for the overpressured interval.

\[
\min ||V_p^{log} - V_p^{model}||^2 \tag{3.12}
\]

An effective stress of 3700 psi was chosen as the reference effective stress. This represents the effective stress at the top of overpressure. An overlay of the measured and modeled P-wave velocity can be seen in track 7 of Figure 3.2. The mineral and fluid P-wave velocity petrophysical endpoints for this study are given in Table 3.1. They represent values representative of a formation at the reference effective stress.

For quality control, a P-wave velocity curve can be computed for the standard, constant volume, formation where \( V_p^{Std} \) is given by 3.7. Equations 3.13 and 3.14 isolate the impact of stress on the P-wave velocity in both the normally pressured interval and the overpressured interval. This P-wave velocity curve, \( V_p^{Stress} \), is shown in red in track 6 of Figure 3.2.

\[
V_p^{nor} = V_p^{Std} + \frac{\partial V_p^{nor}}{\partial \sigma'} (\sigma' - \sigma'_{ref}) \tag{3.13}
\]

\[
V_p^{over} = V_p^{Std} + \frac{\partial V_p^{over}}{\partial \sigma'} (\sigma' - \sigma'_{ref}) \tag{3.14}
\]

A reference P-wave velocity normalized to a constant effective stress can be computed from the following two equations, Equation 3.15 for the normally pressured interval and Equation 3.16 for the overpressured interval.

\[
V_p^{ref} = V_p - \frac{\partial V_p^{nor}}{\partial \sigma'} (\sigma' - \sigma'_{ref}) \tag{3.15}
\]
\[ V_{p}^{ref} = V_{p} - \frac{\partial V_{p}^{over}}{\partial \sigma'} (\sigma' - \sigma'_{ref}) \quad (3.16) \]

Normalizing the P-wave velocity to a reference effective stress enables one to use this new P-wave velocity curve in a petrophysical analysis to provide a more accurate petrophysical determination.

I used the same workflow to study the S-wave velocity. While the extracted petrophysical and stress sensitivity parameters are different, the same sonic behavior was observed in the S-wave data as observed in the P-wave data. The S-wave results exhibit a larger uncertainty due to the quality of the input data.

### 3.5 Results

The stress sensitivity parameters inverted from this workflow are given in Table 3.2. The stress sensitivity parameters are given in units of feet per second per 1000 psi change in pore pressure.

| Regime          | Sense   | Velocity change \( V_{p} \) (ft/s per 1000 psi) | Percentage (%)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal pressure</td>
<td>loading</td>
<td>580</td>
<td>5.4</td>
</tr>
<tr>
<td>Over pressure</td>
<td>unloading</td>
<td>-3,150</td>
<td>-28</td>
</tr>
<tr>
<td>Normal pressure</td>
<td>loading</td>
<td>( V_{s} ) (ft/s per 1000 psi)</td>
<td>(%)</td>
</tr>
<tr>
<td>Over pressure</td>
<td>unloading</td>
<td>-1,808</td>
<td>-29</td>
</tr>
</tbody>
</table>

These stress sensitivity parameters are both large and important. It is not uncommon for pore pressure to change by hundreds of psi within a depth interval of interest. This change in loading causes significant changes in the P-wave velocity. This is significant for three reasons. First, the Codell and the Niobrara Formations are in pressure communication with the bottom half of the Lower Pierre Shale Formation. Therefore, what happens in the Niobrara is reflected in the lower Pierre Shale. Second, the pay intervals of the Niobrara
are seismically thin and difficult to quantify while the lower Shale Pierre is seismically thick and much easier to quantify. And third, this number represents the link, between what is measured in geoscience and the pressure and pressure changes that drive many drilling and completion decisions.

3.6 Discussion

The observations and analysis performed in this study has raised two issues. First, variations in porosity, mineralogy, and fluids have such a significant influence on the P-wave velocity, especially in unconventional reservoirs, that this influence cannot be ignored. Second, the decrease in normalized velocity below the point of maximum velocity is much greater than that observed in most laboratory studies on recovered core.

Different models are available to interpret sonic velocities as a function of porosity, mineralogy, and fluids e.g. (Wyllie & Gardner, 1955); (Raymer et al., 1980). These models are empirical. In addition, different models are available to interpret sonic velocities as a function of stress (Eberhart-Phillips et al., 1989). These are also empirical. Shapiro shows that if the porosity can be divided into two classes, a stiff part due to low aspect ratio pores and a compliant part due to high aspect ratio pores, the stress dependence utilized by Eberhart-Phillips can be derived using logical arguments (Shapiro, 2003). The motivation for this work was not to derive a combined petrophysical and stress model from first principles, but to begin to tackle the long-standing conflict between petrophysicists who typically ignore stress variations and geomechanists who typically ignore petrophysical variations.

Another remarkable observation was the rapid decrease in normalized velocity below the top of pressure. One must remember there are two opposing effects that alter sonic velocities in stress sensitive materials. An increase in effective stress generally increases sonic velocities because it forces the mineral particles comprising the formation closer together rendering a more solid connection. An increase in pore pressure generally decreases sonic velocities because it forces the mineral particles farther apart rendering a less solid connection. Total stresses are applied to the outside of a formation while pore pressure is applied to the inside.
of the pore structure. Because of formation heterogeneity, these loading mechanisms are
different; the rate of velocity increase due to loading from the outside is in general different
from loading from the inside. Loading a material from the inside by increasing pore pressure
is in fact called unloading because it reduces the stress supported by the solid mineral
particles.

In most published examples discussing effective stress pore pressure prediction methods,
the rate of velocity decrease due to unloading is smaller than the rate of velocity increase
due to loading. Examples of the change in sonic velocity with loading and unloading are
common (Sayers, 2010); (Bowers, 1995); (Ciz et al., 2005). What is interesting about the
data set reported in this paper, is the decrease in velocity due to unloading is much larger,
not smaller, than the increase in velocity due to loading; as shown in Figure 3.3.

The velocity versus effective stress reported in most papers comes from laboratory mea-
surements on recovered core where the imposed loads and loading rates are controlled to
limit damage within the specimen. Perhaps the difference between in-situ and laboratory
observations are due to the dramatically different loading and unloading rates between in-situ
geological processes and laboratory measurements.

(Surdam et al., 1997) makes similar observations of P-wave velocity versus depth in the
Cretaceous formations in Wyoming. The interpretation of overpressure in Surdam’s paper
is identical in some respects but different in others. However, regardless of the differences
in interpretation, it is asserted that overpressure can be interpreted from P-wave velocities
and used in the exploration of hydrocarbons.

It is also interesting to note that prior to hydraulic stimulation the P-wave and S-wave
velocity above the Niobrara Formation is on a steep unloading curve. The injection of
stimulation fluids can only further decrease the effective stress. If the P-wave and S-wave
velocities continue their rapid velocity decrease as observed in the pre-stimulation wireline
sonic log, the spatial extent of this stimulation should be readily observable from time-lapse
seismic. However, it is unknown whether the stress sensitivity of the sonic velocities due to
the hydraulic stimulation will be the same as that observed due to the maturation process over geologic time. It is also unknown how quickly in time the increase in pore pressure due to the hydraulic stimulation will dissipate into a stimulated Niobrara Formation and the overlying Pierre Shale.

3.7 Results and Conclusions

I have introduced a new technique to diagnose stress sensitivity in dipole data and differentiate petrophysical effects from stress effects. A petrophysical model is used to explicitly separate the variations in log measured sonic velocities due to porosity, mineralogy, and fluids from variations due to stress. With this technique, I quantify the stress sensitivity of the Pierre Formation. The top of overpressure has been determined to extend about 1300 feet above the Niobrara Formation into the overlying lower Pierre Shale. This depth correlates with the top of many faults penetrating the Niobrara Formation (see chapter 5). In addition to quantifying the stress sensitivity, this new method synthesizes the dipole logs that would have been recorded had there been no stress sensitivity. These new stress independent logs are utilized to improve the original petrophysical evaluation.

The Niobrara Formation in the study area is in pressure communication with the overlying Pierre Formation through fluid and pressure migration. This geomechanical reality decreases the complexity of the seismic interpretation in two ways. First, the Pierre Shale is thousands of feet thick compared to the chalks and marls in the Niobrara Formation, which are only 30 ft thick. Intervals thicker than the dominant seismic wavelength, 200 ft, greatly reduces the uncertainty in seismic attributes. Second, the Pierre Shale has lower petrophysical variability than the Niobrara Formation lending itself to a more accurate velocity normalization. This work quantifies the stress sensitivity of the Pierre Shale and thus provides a critical input to interpret time-lapse seismic attributes for spatial changes in effective stress. And due to the pressure coupling with the Niobrara, a measurement of the velocity changes in the lower Pierre Shale can indicate pressure changes in the Niobrara Formation.
I will discuss in this chapter a model to account for changes in measured velocities with changes in fracture compliance. This model has several unique attributes. First, it can handle a background model that is fully anisotropic. Second, it couples the P-wave and S-wave velocities to the same rock physics model. And finally, it provides for the fracture compliances to be a function of stress.

This model was constructed to investigate the impact of fractures on elastic waves. For instance, the stress dependent fracture compliances can be adjusted to match ultrasonic measurements made on core data. They can also be adjusted to match changes in seismic traces acquired in 4D seismic.

4.1 Introduction

There are different rock physics models that relate velocity and stress. The simplest rock physics model is derived from Hooke’s law. This poro-elastic stress model assumes the formation is a continuous and homogeneous material and deformations due to stress loading are described by linear elasticity.

4.2 Linear Slip Theory

Since strain is impossible to measure in-situ, stress evaluation using linear elastic theory is inherently non-unique. In practice, the strain terms are chosen such that the model yields stress values consistent with in-situ measurements of stress. Thus, strain becomes a calibration knob for stress. The strain value implied from this workflow can only match reality if the linear elastic model represents the true rock behavior. This is highly questionable due to the assumptions of a continuous and homogeneous material used in the derivation of linear elastic theory. While this rock physics model is fundamental to seismic interpretation,
it is inappropriate for use in time-lapse seismic interpretation in unconventional reservoirs. Unconventional reservoirs are hydraulically stimulated and the stimulation creates cracks on many different size scales which violate the assumptions of a continuous and homogeneous material.

Empirical models exist (Shapiro & Kaselow, 2005) that capture some of the velocity behavior observed in laboratory measurements on recovered core. Shapiro popularized a dual-porosity type model that included two types of pores, compliant pores and stiff pores. This formulation gives rise to stress dependent velocities. As stress normal to the compliant pores increases, the velocities increase exponentially as these high aspect ratio, compliant pores close. As stress continues to increase, the velocities increase linearly as the low aspect ratio, stiff pores begin to close. P-wave and S-wave velocities are given by:

\[ V_p = a_1 + a_2 \sigma' - a_3 e^{-a_4 \sigma'} \]  
\[ V_s = a_5 + a_6 \sigma' - a_7 e^{-a_8 \sigma'} \]

While this empirical model can accurately predict measured velocities, 8 coefficients are needed. In addition, unlike even the most basic rock physics model, there is no coupling between the P-wave and S-wave velocities. An example of Shapiro’s model with Niobrara core is shown in Figure 4.1. This Niobrara data was graciously provided by (Panfiloff, 2016) but the location of the cored well has been kept secret from the research group.

The \( V_p \) data is shown in red while the \( V_s \) data is shown in blue. The black lines give a linear fit to the last four data points to highlight the stress sensitivity at large mean effective stresses. The seven core samples had these stress sensitivity coefficients as shown in Table 4.1.

Linear slip theory (LST) was first proposed by Michael Schoenberg and popularized by Colin Sayers (Schoenberg, 1980; Schoenberg & Sayers, 1995). It begins with the same foundation as linear elastic theory but models formation heterogeneity as excess compliance. Most geophysicists are familiar with what is called the \( C_{ij} \) stiffness matrix because they are
Figure 4.1: Niobrara core data fit with Shapiro’s model
Table 4.1: Stress sensitivity coefficients using the Shapiro model

<table>
<thead>
<tr>
<th>Sample Id</th>
<th>( V_p ) (ft/s)</th>
<th>( a_1 ) (ft/s/kpsi)</th>
<th>( a_2 ) (ft/s/kpsi)</th>
<th>( a_3 ) (ft/s/kpsi)</th>
<th>( a_4 ) (1/kbar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>12,134</td>
<td>257.5</td>
<td>366.6</td>
<td>36.18</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>11,500</td>
<td>226.2</td>
<td>238.4</td>
<td>19.51</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>12,726</td>
<td>37.3</td>
<td>257.3</td>
<td>18.14</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>11,687</td>
<td>173.1</td>
<td>228.0</td>
<td>41.70</td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>11,778</td>
<td>429.6</td>
<td>45.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>11,365</td>
<td>2.9</td>
<td>213.2</td>
<td>14.00</td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>12,249</td>
<td>94.1</td>
<td>352.9</td>
<td>30.84</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample Id</th>
<th>( V_s ) (ft/s)</th>
<th>( a_1 ) (ft/s/kpsi)</th>
<th>( a_2 ) (ft/s/kpsi)</th>
<th>( a_3 ) (ft/s/kpsi)</th>
<th>( a_4 ) (1/kbar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>7,422</td>
<td>77.8</td>
<td>146.8</td>
<td>36.7</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>7,076</td>
<td>49.9</td>
<td>178.1</td>
<td>55.4</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>7,483</td>
<td>25.5</td>
<td>61.0</td>
<td>39.7</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>6,780</td>
<td>21.1</td>
<td>199.8</td>
<td>51.9</td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>7,080</td>
<td>90.5</td>
<td>15.8</td>
<td>30.0</td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>6,321</td>
<td>52.0</td>
<td>55.6</td>
<td>40.0</td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>7,144</td>
<td>57.8</td>
<td>114.5</td>
<td>39.8</td>
<td></td>
</tr>
</tbody>
</table>

the coefficients that relate stress and strain and because they are simply related to velocity.
Consider a material that represents a fracture free background.

\[
C^B_{ij} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\tag{4.3}
\]

Geomechanists like to work in compliance space which is the reciprocal of stiffness space. The compliance is simply the reciprocal of the background stiffness.

\[
S^B_{ij} = \frac{1}{C^B_{ij}} = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\tag{4.4}
\]

Geomechanists work in compliance space for essentially the same reason as geophysicists work in stiffness space; the mathematics is simple in compliance space. Since formation het-
ergogeneity can be modeled as excess compliance, one has only to add the different compliance values together.

\[ S_{ij}^{Total} = S_{ij}^{Background} + S_{ij}^{Excess} \] (4.5)

For instance, to compute the total compliance of a heterogeneous formation, one would add the background compliance to the excess compliance. There are two important compliances in linear slip theory; the normal compliance and the tangential compliance.

\[ u_n = B_n \sigma_n; \; u_t = B_t \sigma_t \] (4.6)

The normal compliance relates a non-elastic discontinuous slip, \( u_n \), to the normal stress \( \sigma_n \), causing the normal displacement or slip. The tangential compliance relates a non-elastic discontinuous slip, \( u_t \), to the tangential (shear) stress \( \sigma_t \), causing the shear displacement or slip.

The excess compliance arising from the tangential component of stress is given by equation 4.7, a second rank tensor called \( \alpha_{ij}^{Excess} \).

\[
\alpha_{ij}^{Excess} = \begin{bmatrix}
\alpha_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_{22} + \alpha_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & \alpha_{11} + \alpha_{33} & 0 \\
0 & 0 & 0 & 0 & 0 & \alpha_{11} + \alpha_{22}
\end{bmatrix}
\] (4.7)

A similar tensor called \( \beta_{ijkl}^{Excess} \), arises from the difference between the normal compliance and the tangential compliance. In gas-filled formations \( \beta_{ijkl} \) is small with respect to \( \alpha_{ij} \), but in liquid-filled formations such as encountered in Wattenberg, \( \beta_{ijkl} \) probably cannot be ignored. Regardless, the total compliance of the formation is the sum of the background compliance, given by equation 4.4, plus the ’excess’ compliance that arises from the heterogeneity of the
formation. If $\beta_{ijkl}$ can be ignored, the total compliance is given by equation 4.8.

$$S_{ij}^\text{Total} = \begin{bmatrix}
S_{11} + \alpha_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{21} & S_{22} + \alpha_{22} & S_{23} & 0 & 0 & 0 \\
S_{31} & S_{32} & S_{33} + \alpha_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} + \alpha_{22} + \alpha_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} + \alpha_{11} + \alpha_{33} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66} + \alpha_{11} + \alpha_{22}
\end{bmatrix}$$

(4.8)

The total compliance can now inverted back to the stiffness do main where the velocities can be extracted from the $S_{ij}^\text{Total}$ components.

$$C_{ij}^\text{Total} = \frac{1}{S_{ij}^\text{Total}} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}$$

(4.9)

The $C_{ij}^\text{Total}$ components now represent the background and the excess compliance such as grain-to-grain contacts and fractures. Heterogeneity is now incorporated into the theory through excess compliance. However, the theory at this point does not explain a variation in velocity with stress; it just accounts for different forms of heterogeneity such as layering and fractures. This gives rise to anisotropy, such as VTI or HTI.

To make velocities a function of stress, one has to explicitly make excess compliances a function of stress, such as given by a simple exponential (Sayers, 2007).

$$B_n = B_0 e^{-\frac{\sigma_n'}{\sigma_0}}; B_t = B_1 e^{-\frac{\sigma_t'}{\sigma_1}}$$

(4.10)

These excess compliances can now be integrated within a unit volume to compute the alpha parameters from equation 4.7.

$$\alpha_{ij} = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} B_t(\theta, \phi)n_i n_j \sin(\theta)d\theta d\phi$$

(4.11)
The fourth rank tensor, $\beta_{ijkl}$, can also be computed that quantifies additional compliance based on the difference between the normal and tangential compliances.

$$\beta_{ijkl} = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} [B_n(\theta, \phi) - B_t(\theta, \phi)] n_i n_j n_k n_l \sin(\theta) d\theta d\phi$$

(4.12)

The total excess compliance is the sum of the $\alpha_{ij}$ and $\beta_{ijkl}$ tensors.

$$\Delta S_{ijkl} = \frac{1}{4} [\delta_{ik}\alpha_{jl} + \delta_{il}\alpha_{jk} + \delta_{jk}\alpha_{il} + \delta_{jl}\alpha_{ik}] + \beta_{ijkl}$$

(4.13)

The total compliance can now be computed by summing the background compliance with the excess compliance.

$$S_{ijkl} = S_{ijkl}^0 + \Delta S_{ijkl}$$

(4.14)

The same data as shown in Figure 4.1 can be fit with Sayers’s linear slip theory as shown in Figure 4.2. The black curves represent a best fit using the alpha tensor only. The red curve represent a best fit using the sum of the alpha and beta tensors.

The fitting coefficients are shown in Table 4.2.

<table>
<thead>
<tr>
<th>Sample Id</th>
<th>$Z_0 Tan$</th>
<th>$\sigma_c Tan$</th>
<th>$Z_0 Tan$</th>
<th>$\sigma_c Tan$</th>
<th>$Z_0 Nor$</th>
<th>$\sigma_c Nor$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_p$</td>
<td>Alpha</td>
<td>AlphaBeta</td>
<td>Alpha</td>
<td>AlphaBeta</td>
<td>Alpha</td>
<td>AlphaBeta</td>
</tr>
<tr>
<td>S1</td>
<td>0.178</td>
<td>350.7</td>
<td>0.153</td>
<td>237.6</td>
<td>0.208</td>
<td>407.3</td>
</tr>
<tr>
<td>S2</td>
<td>0.124</td>
<td>580.3</td>
<td>0.189</td>
<td>248.7</td>
<td>0.095</td>
<td>498.5</td>
</tr>
<tr>
<td>S3</td>
<td>0.067</td>
<td>621.1</td>
<td>0.032</td>
<td>322.3</td>
<td>0.101</td>
<td>640.7</td>
</tr>
<tr>
<td>S4</td>
<td>0.113</td>
<td>298.1</td>
<td>0.268</td>
<td>216.9</td>
<td>0.020</td>
<td>1418.2</td>
</tr>
<tr>
<td>S5</td>
<td>0.175</td>
<td>283.5</td>
<td>0.000</td>
<td>93.6</td>
<td>0.50</td>
<td>202.1</td>
</tr>
<tr>
<td>S6</td>
<td>0.084</td>
<td>774.2</td>
<td>0.068</td>
<td>432.6</td>
<td>0.01</td>
<td>853.5</td>
</tr>
<tr>
<td>S7</td>
<td>0.145</td>
<td>346.1</td>
<td>0.067</td>
<td>886.8</td>
<td>0.231</td>
<td>244.1</td>
</tr>
</tbody>
</table>

Different forms of the stress dependence as shown in equation 4.10, can be evaluated within this model. A linear term could be added or a power law tried in place of the exponential function. Unlike the empirical model, the P-wave and S-wave velocities are coupled to the same stress dependent parameters. In addition, acoustical velocities that

---

47
Figure 4.2: Niobrara core data fit with Sayer’s linear slip theory model
decrease with stress can also be modeled. This is particularly important when the stresses exceed the formation strength as when a formation is hydraulic fractured.

Figure 4.3 shows a generalized velocity behavior as a function of stress. The blue curve shows the stress-strain behavior as typically observed in core data. The red curve shows the velocity behavior in response to the applied stress. Four regions are identified. Region 1 shows an exponential increase in velocity as the high aspect ratio pores are closed. Region 2 shows a linear increase in velocity as the low aspect ratio pores are being closed. Region 3 shows no variation in velocity with stress because the stress loading has closed all the heterogeneity and the formation is behaving as predicted by linear elastic theory. Region 4 is of particular interest as the velocities drop significantly due to the opening of new cracks caused by the high stresses acting to fail the formation.

![Figure 4.3: Generalized velocity behavior as a function of stress](image)

Linear slip theory has the ability to model all of these acoustical behaviors with an underlying physics that models compliance as a function of stress.
4.3 LambdaRho MuRho Analysis

A popular interpretation, called lambda mu rho (LMR) uses a rock physics template that plots MuRho, $\mu\rho$, as a function of LambdaRho, $\lambda\rho$ (Goodway et al., 2010). The data plotted in Figure 4.4 is the dipole sonic data studied in chapter 2. LambdaRho and MuRho were computed from the bulk density and vertical P-wave and S-wave velocities. The fast and slow where were averaged to determine the S-wave velocity.

Figure 4.4: LambdaRho vs. MuRho

MuRho is plotted as a function of LambdaRho with the gamma ray in color. While this template assumes isotropy, it does a good job in separating different lithologies. A common use of this plot is to diagnose fluid effects. Figure 4.5 plots the chalk and marl intervals in the Niobrara Formation on the left side of the plot. The fluid overlay shown on the right side of the plot, shows the predicted fluid effect using the standard Biot-Gassmann model. An isotropic limestone formation with no natural fracturing is used to model the background formation.

The dry-frame correlation shown in red, approximates gas-filled porosity while the dark blue correlation shows the opposite extreme of salt saturated water-filled porosity. Equal
porosity lines are shown for the background limestone formation.

Figure 4.6 compares the fluid effect on the left with a fracture compliance effect on the right. An isotropic limestone formation with oil-filled porosity is used to model the background formation. The fracture compliance overlay is constructed by adding a single set of parallel fractures with increasing fracture compliance to the isotropic background.

The fluid effect on the left is computed assuming no addition compliance due to fractures or stress. The fracture compliance on the right is computed assuming a constant saturation with a variable fracture compliance. It is seen that when fracture compliance is added to a formation, there is a large effect in a highly stiff formation and a small effect in a highly compliant formation. In addition, for the chalks and marls of the Niobrara Formation, a fluid effect is indistinguishable from a fracture compliance effect.

4.4 LST Forward Model

Linear slip theory can be used to model the impact of fracture compliance on elastic wave behavior. I have created a model that starts with a non-fractured background compliance. This background is not limited to an isotropic description; it can have orthotropic symmetry.
The compliance of a fully orthotropic fracture set can be added to this background material. The fracture compliance can be independent of stress, or more likely, dependent on stress. The anisotropic velocities output from the model can be used to determine reflection coefficients and convolved with a filter. Figure 4.7 shows a test of this model.

Track 1 shows the stratigraphic column, Track 2 shows two P-wave velocities and the bulk density. The black P-wave velocity is shown in black. To test the model, I computed a time-lapse P-wave velocity using the same fracture compliance for every layer. This had the effect of reducing the P-wave velocity in every layer. This curve is green and is shown in track 2. These two P-wave velocities were used to compute reflection coefficients in track 3. The red curve is the original reflection coefficient as computed from the original logs. The green curve is the time-lapse reflection coefficients as computed from the reduced velocity. These reflection coefficients were convolved with two different wavelets, a low frequency wavelet and a high frequency wavelet, to see the impact on the seismic trace. The original seismic trace is shown in black and shaded red and blue, the time-lapse seismic trace is shown in green. Because the overall velocity was reduced by a constant amount, there is no change in the phase of the time-lapse trace. This model can now be used to adjust fracture compliances.
Figure 4.7: Expected change in seismic trace based on reduction in velocity to match the observed time-lapse changes in the seismic data.

4.5 Results and Conclusions

I used a rock physics model, based on linear slip theory (LST), to quantify the stress sensitivity observed in the core data provided by (Panfiloff, 2016). The LST model can fit the vertical and horizontal measurements well, but not simultaneously. This confirms the stress sensitivity in these samples is anisotropic. I evaluated the beta tensor to account for possible liquids in the core sample. This improved the fit to the ultrasonic measurements.

In LMR space, there is no differentiation between a fluid effect and a fracture compliance effect.

A forward model has been built that adds fracture compliance to background compliance to investigate the impact of fractures on a seismic trace. The model is not limited to typical isotropic assumptions, but can model a fully orthotropic set of inputs.
CHAPTER 5
GEOMECHANICS

My goal in this chapter is to explain how a better understanding of geomechanics can directly influence hydraulic fracture stimulation design both in the initial stimulation as well as in subsequent re-stimulations. My secondary goal is to explain how a better understanding of geomechanics can help explain some of the observed seismic attribute changes with time. This chapter begins with a short introduction followed by an explanation of some critical concepts required to understand the geomechanics of hydraulic fracturing. A 1D mechanical earth model (1D-MEM) is then constructed using petrophysical and well log data and is calibrated to core and in-situ engineering measurements. This 1D-MEM is used to construct a 3D geocellular model. A flow coupled finite element solver is used to equilibrate the geomechanical model and understand the initial stress state prior to any drilling or completion activities. The solver is subsequently used to estimate the time dependent stress changes due to injection and production. Finally, I’ll summarize my conclusions based on this research.

5.1 Introduction

A critical input into hydraulic fracture design is the vertical stress profile, which is the variation in the minimum horizontal stress as a function of the vertical dimension. The vertical stress profile substantially controls the geometry of the induced hydraulic fracture. An optimum stimulation design would define the pumping variables so as to limit the height growth to the productive interval while extending the fracture an optimum distance from the borehole. Vertical growth above or below the productive interval is not optimal. Fracture height growth into non-productive layers above or below the pay interval reduces the fracture length in the productive interval. In addition, gravity can concentrate the proppant in the fracture below the producing interval which diminishes fracture conductivity in the producing
interval. Both mechanisms lead to a decrease in production. Therefore, to design an optimal stimulation, an accurate vertical stress profile is required.

In addition to the vertical stress profile, an understanding of the evolution of stresses with time is needed to guide field development plans for either developmental drilling or re-completions. An understanding of how in-situ stresses change with time could support a proactive strategy to manipulate the stress field to avoid drilling problems or enhance completions. Time dependent stress changes can also help explain some of the observed seismic attribute changes with time.

5.2 Concepts

I will begin by introducing some geomechanical concepts and equations. Introduced first are the basic geomechanical definitions of stress and strain. I then proceed to discuss dynamic and static moduli.

The symbol $\sigma$ will be used to describe a "normal" stress where the term normal refers to the direction perpendicular to a plane. In contrast, the symbol $\tau$ will be used to describe a "shear" stress where the term shear refers to a direction parallel to a plane. The state of stress in the earth in "simple" situations, e.g. flat layers with no uneven surface topology, is often described by a vertical and two horizontal stresses. $\sigma_V$ refers to the vertical stress, $\sigma_h$, refers to the minimum horizontal stress and $\sigma_H$, refers to the maximum horizontal stress. Sometimes numbers are used to mean direction. When this is the case, the subscript 1 will mean the minimum horizontal stress direction, the subscript 2 will mean the maximum horizontal stress direction and the subscript 3 will mean the vertical stress direction. An unprimed symbol will be used to describe total stress, e.g. $\sigma$ to mean the total normal stress. A primed symbol will be used to describe an effective stress, e.g. $\sigma'$ to mean effective normal stress. Normal strain is defined with the symbol $\epsilon$ and shear strain is defined with the symbol $\gamma$. Positive numbers are used to describe compressional stresses and negative numbers are used to describe tensile stresses. It should be noted that using positive numbers to describe compressive stresses is intuitive for engineers because the in-situ stresses in the
earth are almost always compressive. An exception to this nomenclature is in the use of finite element analysis where the numerical solvers use the opposite convention. However, most finite element packages hide this reversed convention from the user and require compressive stresses to be input as positive numbers. It is always best to check the convention for any geomechanical solver.

5.2.1 Stress

Stress is a force per unit area that acts to compress the volume or distort the shape. In general, there are two types of stress; normal stress and shear stress. Normal stresses act to change the volume without changing the shape. Shear stresses act to change the shape without changing the volume. It must be remembered that stress is a force per unit area. One would normally visualize an increase in stress with an increase in force or load. However, it is also possible to have substantial stress increase at points of contact that represents a small area, even when the applied loads are small. This is sometimes called a stress concentration.

Imagine a porous formation, that is composed of solid minerals with water filling the porosity, existing at a depth of 10,000 feet. Imagine the total vertical load, or stress, on this formation is 10,000 psi. This load is also described as the overburden pressure. One could envision that some of this load is supported by the solid minerals comprising the formation and some of the load is supported by the fluid. The load supported by the solid minerals is defined as the effective stress while the load supported by the fluid is defined as the pore pressure. The total stress is conceptualized as the sum of the effective stress and pore pressure as shown in Equation 5.1.

\[
\sigma = \sigma' + P_p
\]  

(5.1)

This concept of effective stress is critically important to quantitative geomechanics because the yield and failure of brittle rock is dependent on the effective stress, that part of the total stress acting on the rock matrix, sometimes referred to as the rock framework.
Biot (1941) derived poroelastic theory, and Nur & Byerlee (1971), demonstrated that given the assumptions of linear elasticity, the effective stress law is actually Equation 5.2 where Biot’s effective stress coefficient, $\alpha$, enters as a multiplier in the pore pressure term.

$$\sigma = \sigma' + \alpha P_p$$ \hspace{1cm} (5.2)

In linear elastic theory, effective stress is thus computed from Equation 5.3.

$$\sigma' = \sigma - \alpha P_p$$ \hspace{1cm} (5.3)

When the applied loads exceed the yield strength of a brittle formation, and inelastic processes begin to dominate, the accuracy of linear elastic predictions greatly diminish. In these situations, the common practice is to revert to Terzaghi’s effective stress law as given by Equation 5.1.

Thus, when linear elastic conditions apply, effective stress is computed by Equation 5.3. After the onset of yield, and linear elastic assumptions are violated, effective stress is computed from Equation 5.4.

$$\sigma' = \sigma - P_p$$ \hspace{1cm} (5.4)

Rock and other porous formations respond to pore pressure as well as applied loads. The behavior of the formation can be approximated by using effective stress in elastic or failure calculations.

### 5.2.2 Strain

Strain is the deformation induced in a material by the action of stress. Just as there are two types of stress, there are two types of strain. Normal strain is a change in the length, area or volume of the material without a change in shape. Shear strain is a change or distortion in the shape of a material without an accompanying change in volume. Strain can be simple or very complicated. Strain has no dimensions and can be quoted as a ratio, percent, milli-strain or micro-strain. Stress and strain are related through elastic moduli.
5.2.3 Moduli

Galileo Galilei (1564-1642) became the grandfather of elastic theory by asking questions and testing the ideas experimentally. Today we recognize that solids, e.g. a core sample, have three independent properties. The first is bulk density, a certain amount of mass per unit volume. The second is compressibility, the resistance to a change in volume using only normal stresses. The third is what might be called shear distortion, a resistance to a change in shape using shear stress, a property we quantify with what we call the shear modulus.

Robert Hooke (1635-1703) is known for his law of linear elasticity. Hooke’s law provided a partial foundation for elastic theory. However, the concept understood by Hooke, was essentially that of compressibility. He was not aware of the concept of shear distortion. That would come later.

It was Thomas Young (1773-1829) who first considered shear, the resistance to a change in shape, as an elastic strain. He called it ”detrusion”. He was the first to recognize that the elastic resistance of a body to a change in volume (compression), was in general different from its resistance to a change in shape (shear distortion).

Moduli are thought of as constants that relate stress with strain and define some intrinsic properties of a material. However, only under certain restrictive conditions are moduli invariant with stress and thus truly constant. There are two basic but very different types of moduli, dynamic and static.

Dynamic moduli relate stress and strain when the stresses acting on the material are small, typically less than the elastic limit of the material. Most materials, when subjected to a load, will deform from the materials original position or configuration. The material is considered elastic if it returns to the original position or configuration upon unloading. Elasticity implies no permanent deformation. George Green (1793-1841) was a British mathematical physicist that derived elastic theory based on the ideas of elasticity and conservation of energy. Because the stresses never exceed the elastic limit of the material, dynamic moduli are often called dynamic elastic moduli or just elastic moduli. Dynamic moduli are mea-
sured using small amplitude waves that cause an elastic disturbance to propagate through
the material. Two waves are generally observed, a primary wave, known as the P-wave, and
a secondary wave, known as an S-wave. Both waves are dependent of the dynamic elastic
properties of the material. Repeatable measurements are desired in experimental science.
In fact, repeatable measurements are often considered a tenet of science. Dynamic elastic
measurements are always repeatable and this forms a solid foundation in rock mechanics.

In contrast, static moduli relate stress and strain when the stresses acting on the material
are large, typically more than the elastic limit of the material. A mechanical loading frame
is used to stress a formation sample to failure, while measurements are made of the applied
stresses and induced strains. The stress versus strain behavior, as observed in these labora-
tory experiments, are considered to be appropriate for use in engineering practice because
the applied loads used to drill or fracture a borehole exceed the elastic limit of the formation.

5.3 Mechanical Properties

A formation’s mechanical properties is often a strong control on it’s behavior to a changing
stress field.

5.3.1 Rock Strength Parameters

Shear stress develops in a solid that is subjected to unequal orthogonal stresses. The
magnitude of the effective normal and shear stress, on a plane within the solid, varies with
the angle \( \beta \) measured from the minimum stress direction. The effective normal stress to this
plane, \( \sigma_n' \), is given by Equation 5.5 where \( \sigma_1' \) is the maximum principal stress and \( \sigma_3' \) is the
minimum principal stress.

\[
\sigma_n' = \frac{1}{2}(\sigma_1' + \sigma_3') + \frac{1}{2}(\sigma_1' - \sigma_3') \cos 2\beta \tag{5.5}
\]

The effective shear stress, \( \tau' \) is given by Equation 5.6.

\[
\tau' = -\frac{1}{2}(\sigma_1' - \sigma_3') \sin 2\beta \tag{5.6}
\]
Laboratory tests on recovered core are important in defining rock strength parameters. Formation samples are loaded to failure with the application of external loads as shown in Figure 5.1.

![Figure 5.1: Normal and shear stresses](image)

The relationship between stress and strain for most formations is quite complicated. For instance, the stress versus strain behavior is always a function of confining pressure. Confining pressure is the load applied to the side of the sample in a typical laboratory test. It is the conceptual equivalent of the horizontal stress in the earth.

Figure 5.2 shows a family of stress versus strain curves for different values of confining pressure for a porous sandstone from southwest Germany (Gowd & Rummel, 1980).

When confining pressure increases, the formation transitions from a brittle behavior to a ductile behavior. Brittle behavior is diagnosed when the stress strain curve exhibits a peak in stress or load bearing capacity. This peak in stress is followed by a sharp reduction in load bearing capacity. The peak in the stress strain curve is generally considered the failure of the sample, and is also known as the compressive strength. It is interesting to note that even
Figure 5.2: Stress vs. strain at different confining pressures
though the sample has technically failed, it can still support a reduced load. This reduced load is called the residual strength of the sample.

In contrast to brittle behavior, ductile behavior is diagnosed when the stress strain curve fails to reach a peak in stress prior to the end of the test. Most formations exhibit a brittle to ductile transition at some characteristic confining pressure. The initial, straight line slope, of the stress strain curve is known as Young’s modulus. This slope tends to increase slightly with confining pressure.

It is important to note that brittleness, or brittle behavior, is a strong function of confining pressure. It is not an intrinsic property of the formation such as Young’s modulus or Poisson’s ratio.

Formations that exhibit brittle behavior can be characterized by a number of unique points, or line segments, on a stress strain curve. Yield strength, peak strength and residual strength are parameters that characterize brittle behavior. Yield strength is the stress where the stress strain curve departs from a linear response. This occurs at about one half the value of the peak strength. Peak strength is the stress where the stress strain curve reaches it’s maximum or peak in stress. The residual strength is the stress after the peak, where the stress strain curve shows a constant stress with increasing strain. All three of these strength parameters are unique functions of confining pressure.

When a formation is subjected to a shear stress that exceeds the limit of elasticity, the formation will yield at an angle $\beta$ with the minimum in-situ normal stress. The Mohr-Coulomb criteria for shear failure is given by Equation 5.7.

$$\sigma_1' = C_0 + \sigma_3' \tan^2 \beta \tag{5.7}$$

This criterion states that the formation will fail if the maximum effective compressive normal stress $\sigma_1'$ overcomes two material properties that resist deformation. These are the unconfined compressive strength, $C_0$, and the failure angle beta, $\beta$. 62
Beta is related to the angle of internal friction by Equation 5.8.

\[
\beta = \frac{\pi}{4} + \frac{\phi}{2}
\]  

(5.8)

The Mohr-Coulomb equation is also thought of as a confined compressive strength equation. \(\sigma'_1\) is the confined compressive strength which is a sum of the unconfined compressive strength, \(C_0\), and an additional strength due to confinement. The \(\tan^2\beta\) term is often replaced with the equivalent \(N\) as shown in Equation 5.9.

\[
\sigma'_1 = C_0 + N\sigma'_3
\]  

(5.9)

\(N\) is related to the friction angle \(\phi\) by the following equation, 5.10.

\[
N = \frac{1 + \sin \phi}{1 - \sin \phi}
\]  

(5.10)

Mohr popularized a graphical representation of stresses on a plane as shown in Figure 5.3.

Figure 5.3: Mohr-Coulomb diagram of a formation at failure

His diagram, known as a Mohr’s circle plot, has had a large influence on the present day geomechanics community. The diagram provides an illustration of the effective normal and
shear stresses on a plane, and can be used to compare different stress states with a failure envelope overlay. The friction angle is given by $\phi$ and the cohesion is given by $S$. The unconfined cohesive strength, $S_0$, is related to the unconfined compressive strength, $C_0$, by Equation 5.11.

$$S_0 = \frac{C_0}{2\tan\beta} \quad (5.11)$$

### 5.4 Earth Stresses

Stress models are generally derived from Hooke’s law. This stress model is quite general, but makes assumptions about the formation’s intrinsic properties and its stress/strain behavior. Hooke’s law assumes the formation is a continuous and homogeneous material and the formation’s elastic constants relate stresses with strain. Hooke’s law is the basis of linear elastic theory.

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad (5.12)$$

As Tsvankin explains in (Tsvankin, 1997), it is possible to use the ”Voigt recipe” to condense the fourth rank tensor into a second rank tensor as shown in Equation 5.13. This reduced notation will be used in this chapter when appropriate.

$$\sigma_i = C_{ij}\epsilon_j \quad (5.13)$$

Stresses exist in the earth and originate from several different sources. Assume the principal earth stresses are in the vertical and horizontal directions. Let the number 3 describe the vertical direction with positive numbers indicating depth or down rather than up. Let the number 1 describe the minimum horizontal stress direction and the number 2 describe the maximum horizontal stress direction. These three directions are mutually orthogonal. The magnitudes of the three principal stresses are given by the minimum horizontal stress $\sigma_1$, the maximum horizontal stress, $\sigma_2$, and the vertical stress, $\sigma_3$.

The easiest stress to determine is the vertical stress. It is often assumed to be equal to the weight of the overburden. The weight of the overburden is determined by integrating the
density of the formations to the depth of interest as shown in Equation 5.14 and Figure 5.4.

\[ P_{oeb} = \int_0^z g \rho_b(z) \, dz \]  

(5.14)

Figure 5.4: Overburden and pore pressure calculation

Engineering analysis of the diagnostic formation injection test (DFIT) data gave a value for pore pressure in the Niobrara Formation of 0.6 psi/ft. The orientation of the maximum horizontal stress was taken from the analysis of microseismic data to be in a plane of 100/280 degrees with respect to true north.

5.4.1 Horizontal Stress Magnitudes

Analytic expressions for both horizontal stresses can be derived from Hooke’s law. Hooke’s law defines three equations that describe the relationship between normal stresses and normal strains as shown in Equations 5.15 thru 5.17.

\[ \sigma_1 = C_{11} \varepsilon_1 + C_{12} \varepsilon_2 + C_{13} \varepsilon_3 \]  

(5.15)

\[ \sigma_2 = C_{21} \varepsilon_1 + C_{22} \varepsilon_2 + C_{23} \varepsilon_3 \]  

(5.16)
\[ \sigma_3 = C_{31}\epsilon_1 + C_{32}\epsilon_2 + C_{33}\epsilon_3 \quad (5.17) \]

It is desired to derive the minimum and maximum horizontal stress in terms of the vertical stress. This is done for two reasons: first, the largest contribution to the horizontal stresses is often the vertical stress and second, the vertical stress is perhaps the easiest stress to quantify. To solve for the two horizontal stresses in terms of the vertical stress, Equations 5.15 thru 5.17 are solved for \( \epsilon_3 \) as shown in Equations 5.18 thru 5.20.

\[ \epsilon_3 = \frac{\sigma_1 - C_{11}\epsilon_1 + C_{12}\epsilon_2}{C_{13}} \quad (5.18) \]
\[ \epsilon_3 = \frac{\sigma_2 - C_{21}\epsilon_1 + C_{22}\epsilon_2}{C_{23}} \quad (5.19) \]
\[ \epsilon_3 = \frac{\sigma_3 - C_{31}\epsilon_1 + C_{32}\epsilon_2}{C_{33}} \quad (5.20) \]

To determine the minimum horizontal stress as a function of the vertical stress, Equation 5.18 is set equal to Equation 5.20. Likewise, to determine the maximum horizontal stress as a function of the vertical stress, Equation 5.19 is set equal to Equation 5.20. This is shown in Equations 5.21 and 5.22.

\[ \frac{\sigma_1 - C_{11}\epsilon_1 + C_{12}\epsilon_2}{C_{13}} = \frac{\sigma_3 - C_{31}\epsilon_1 + C_{32}\epsilon_2}{C_{33}} \quad (5.21) \]
\[ \frac{\sigma_2 - C_{21}\epsilon_1 + C_{22}\epsilon_2}{C_{23}} = \frac{\sigma_3 - C_{31}\epsilon_1 + C_{32}\epsilon_2}{C_{33}} \quad (5.22) \]

Rearranging terms gives the following result.

\[ \sigma_1 = \frac{C_{13}}{C_{33}}\sigma_3 + (C_{11} - \frac{C_{13}}{C_{33}}C_{31})\epsilon_1 + (C_{12} - \frac{C_{13}}{C_{33}}C_{32})\epsilon_2 \quad (5.23) \]
\[ \sigma_2 = \frac{C_{23}}{C_{33}}\sigma_3 + (C_{21} - \frac{C_{23}}{C_{33}}C_{31})\epsilon_1 + (C_{22} - \frac{C_{23}}{C_{33}}C_{32})\epsilon_2 \quad (5.24) \]

The minimum and maximum horizontal stresses, \( \sigma_1 \) and \( \sigma_2 \) are now expressed in terms of a number of elastic constants, the \( C_{ij} \) parameters, the vertical stress, \( \sigma_3 \), and the two horizontal strains, \( \epsilon_1 \) and \( \epsilon_2 \). These two equations form the general result for the minimum and maximum horizontal stress assuming linear elasticity.
To account for poroelasticity, effective stress are substituted for total stresses. Equations 5.23 and 5.24 transform into Equations 5.25 and 5.26.

\[
\sigma_1 - \alpha_1 P_p = \frac{C_{13}}{C_{33}} (\sigma_3 - \alpha_3 P_p) + (C_{11} - \frac{C_{13} C_{31}}{C_{33}}) \epsilon_1 + (C_{12} - \frac{C_{13} C_{32}}{C_{33}}) \epsilon_2 \quad (5.25)
\]

\[
\sigma_2 - \alpha_2 P_p = \frac{C_{23}}{C_{33}} (\sigma_3 - \alpha_3 P_p) + (C_{21} - \frac{C_{23} C_{31}}{C_{33}}) \epsilon_1 + (C_{22} - \frac{C_{23} C_{32}}{C_{33}}) \epsilon_2 \quad (5.26)
\]

No assumption has been made concerning any potential symmetry of the elastic constants. However, if some symmetry exists, the $C_{ij}$ parameters can be expressed in terms of more common elastic moduli. For instance, for isotropic formations, the two horizontal stress equations are shown by Equations 5.27 and 5.28.

\[
\sigma_h - \alpha P_p = \frac{\nu}{1 - \nu} (\sigma_V - \alpha P_p) + \frac{E}{1 - \nu^2} \epsilon_h + \frac{E \nu}{1 - \nu^2} \epsilon_H \quad (5.27)
\]

\[
\sigma_H - \alpha P_p = \frac{\nu}{1 - \nu} (\sigma_V - \alpha P_p) + \frac{E}{1 - \nu^2} \epsilon_h + \frac{E}{1 - \nu^2} \epsilon_H \quad (5.28)
\]

A formation with VTI symmetry would be expressed by Equations 5.29 and 5.30.

\[
\sigma_h - \alpha_h P_p = \frac{E_h}{E_V} \frac{\nu_V}{1 - \nu_h} (\sigma_V - \alpha_V P_p) + \frac{E_h}{1 - \nu_h^2} \epsilon_h + \frac{E_h \nu_h}{1 - \nu_h^2} \epsilon_H \quad (5.29)
\]

\[
\sigma_H - \alpha_h P_p = \frac{E_h}{E_V} \frac{\nu_V}{1 - \nu_h} (\sigma_V - \alpha_V P_p) + \frac{E_h \nu_h}{1 - \nu_h^2} \epsilon_h + \frac{E_h}{1 - \nu_h^2} \epsilon_H \quad (5.30)
\]

These linear elastic horizontal stress models should only be applied where the assumptions of linear elasticity are valid. Application of these models is limited to formations that are low on the stress strain curve before the stresses reach the elastic limit of the formation. When the elastic limit of the formation is reached, the formation begins to yield, and the linear stress strain relationship is no longer valid.

When a formation is high on the stress strain curve and beyond the elastic limit, The Mohr-Coulomb failure model is more appropriate for estimating the minimum horizontal stress. The Mohr-Coulomb confined compressive strength equation, Equation 5.9, can be rewritten to solve for the minimum horizontal stress.

\[
\sigma_h = \frac{\sigma_V' - C_0}{N} \quad (5.31)
\]
The standard practice in geomechanics (Sayers, 2010) is to compute the minimum horizontal stress based on $N$ and set the cohesion, $S_0$, of the fault to zero. A cohesion of zero implies the compressive strength, $C_0$, is also zero. This calculation would represent the minimum horizontal stress of a formation where the horizontal stress is controlled by the strength of a weak formation or a fault. $N$ is determined from the geometry of mapped faults, laboratory tests on recovered core, or taken from existing rock mechanics correlations. I have used a standard rock mechanics correlation from the literature (Plumb, 1994) that matches the geometry of the faults in the Wishbone area. While Plumb’s $V_{\text{grain}}$ correlation was developed from core data in clastic formations, it matches the failure angle of the faults in the carbonate Niobrara Formation. The correlation is also valid for the weak shales above and below the Niobrara Formation.

Equations 5.27, 5.29, and 5.31 were used to compute the minimum horizontal stress as a function of depth for the Wishbone section of the study area. The results are shown in Figure 5.5 along with the in-situ engineering measurements from the two DFIT in-situ stress tests.
The anisotropy analysis showed the Fort Hays and Codell Formations to be isotropic. In addition, the strain terms in the isotropic poroelastic horizontal strain model were adjusted to match the minimum horizontal stress gradient from the Codell DFIT. The $\epsilon_h$ value was set to -0.05 while the $\epsilon_H$ value was set to 0.00. This gave a good match to the isotropic linear elastic stress model shown in green. The anisotropic linear elastic stress model shown in red, also honors the DFIT data because the anisotropic model simplifies to the isotropic model when the formation is isotropic. It is interesting that the Mohr-Coulomb failure model, shown in blue, matches the DFIT data without any calibration to the input friction angle. Thus, all three stress models fit the Codell DFIT.

The chalks, marls, and shales all exhibit VTI anisotropy. The impact of VTI anisotropy on the anisotropic poroelastic stress model is to increase the value of the horizontal stress as compared to the isotropic model. This increase is seen in every formation except the isotropic Fort Hays and Codell Formations. Interestingly, the anisotropic model does not match the DFIT data in the B Chalk while the isotropic model does. By itself, this observation would be puzzling as the chalk layers are diagnosed to be anisotropic.

The Mohr-Coulomb failure model matches both DFIT values and shows a stress barrier at the top of the Niobrara Formation. In addition, the Mohr-Coulomb failure model shows a higher stress than the isotropic stress model, an impossibility as previously discussed.

It is common practice to compute the vertical stress profile based on sonic logs using these linear elastic equations. Indeed, these equations have been coded into every hydraulic fracture simulator. However, even today, it is not a common practice to test the applicability of these equations based on possibility that the formations of interest could be close to, or in, a state of failure. This research clearly shows inconsistent calculations between two linear elastic stress equations and a simple failure model. This work led me to investigate critical stress analysis and numerical stress modeling.
5.5 Critical Stress Analysis

Yield is often described as the start of non-elastic and non-reversible deformation. Different mechanisms give rise to yield. One mechanism is microcracking and another is grain crushing and rearrangement of grains. Yield is often quantified as the first softening on a stress/strain curve. Yield generally leads to the strengthening of the formation. Yield is modeled with elastoplastic theories.

Failure is different from yield and is much more difficult to define. One might define it as the peak stress on a stress/strain diagram. It could also be defined as the loss of load-bearing capacity. To a driller, it might be defined as loss of function; as in the loss of a usable borehole. Depending on the application, there can be different valid definitions of failure.

Yield and failure in earth formations depends on the stress loading, both normal and shear, and the strength properties of the formation. Given an appropriate definition of yield and failure, a Mohr’s circle analysis can define when yield will begin on a predetermined plane and when failure will occur.

The earth is quite heterogeneous. It is composed of many different materials. Each material, while similar in composition, can have a different structure or fabric. Discontinuities, such as fractures and faults, add to the heterogeneity.

Critical stress analysis compares the stresses acting on a discontinuity with the strength of the discontinuity to better understand and predict its behavior. Discontinuities, such as faults, are critically stressed when the maximum shear stress parallel to the discontinuity exceeds the shear strength of the discontinuity.

I will discuss the geomechanics of faults, but the same analysis is appropriate for other types of discontinuities such as natural fractures or bedding planes.
5.5.1 Stresses on a Fault

If the maximum shear stress exceeds the shear strength of the fault, that fault would be considered critically stressed and unstable. It is straightforward to compute the stresses on a fault given the in-situ far-field earth stresses, the fault orientation, and the strength properties of the fault (Priest, 1993). Consider a principal effective stress tensor defined by the coordinates x, y and z, as in Equation 5.32.

\[
\sigma_{earth} = \begin{bmatrix}
\sigma_{xx} & 0 & 0 \\
0 & \sigma_{yy} & 0 \\
0 & 0 & \sigma_{zz}
\end{bmatrix}
\]  \hspace{1cm} (5.32)

These far-field in-situ principal stresses can be described in a different coordinate system, a system aligned with the fault, by doing a standard stress transformation. Consider a stress tensor aligned with the fault, defined by the coordinates x’, y’, z’, as in Equation 5.33.

\[
\sigma_{fault} = \begin{bmatrix}
\sigma_{x'x'} & \tau_{x'y'} & \tau_{x'z'} \\
\tau_{y'x'} & \sigma_{y'y'} & \tau_{y'z'} \\
\tau_{z'x'} & \tau_{z'y'} & \sigma_{z'z'}
\end{bmatrix}
\]  \hspace{1cm} (5.33)

A rotation matrix, defined by the directional cosines between the two coordinate systems, is needed to compute the stresses aligned with the fault. The rotation matrix, \([R]\), is given by Equation 5.34 where the fault orientation is given by \(\theta\), the inclination of the fault with respect to the horizontal axis and \(\psi\), the azimuth of the fault with respect to the maximum horizontal stress azimuth.

\[
[R] = \begin{bmatrix}
\cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\
-\sin \psi & \cos \psi & 0 \\
\sin \theta \cos \psi & \sin \theta \sin \psi & \cos \theta
\end{bmatrix}
\]  \hspace{1cm} (5.34)

The stress tensor aligned with the fault is given by equation 5.35.

\[
[\sigma_{fault}] = [R][\sigma_{earth}][R]^T
\]  \hspace{1cm} (5.35)

The normal and shear stresses on the fault is given by the stress components in Equation 5.33. The normal stress on the fracture is \(\sigma_{x'x'}\), the shear stress acting along the strike of the fracture is \(\tau_{x'y'}\), and the shear stress acting down dip is \(\tau_{x'z'}\). The maximum shear stress
is given by Equation 5.36.

\[ \tau_{\text{max}} = \sqrt{\tau_{x'y'}^2 + \tau_{y'z'}^2} \]  \hspace{1cm} (5.36)

The fracture strength is defined by two properties, an angle, \( a \), representing the combined effects of friction and fault roughness, and the fault cohesion. The friction angle is given by \( \phi \) and the interlocking angle by \( i \). The combined effect of these two parameters is \( a \), as is shown in Equation 5.37.

\[ a = \phi + i \]  \hspace{1cm} (5.37)

The shear strength of the fault, \( \tau_{\text{fault}} \), is given by Equation 5.38 where \( S_0 \) is the fault unconfined cohesive strength and the second term is the additional strength due to the effective normal stress.

\[ \tau_{\text{fault}} = S_0 + \sigma'_{x'y'} \tan(a) \]  \hspace{1cm} (5.38)

When the maximum shear stress, \( \tau_{\text{max}} \), exceeds the shear strength of the fault, \( \tau_{\text{fault}} \), the fracture is considered critically stressed and unstable. A measure of instability, \( R_{\text{fail}} \), is the ratio of maximum shear stress to the shear strength of the fault as given by Equation 5.39.

\[ R_{\text{fail}} = \frac{\tau_{\text{max}}}{\tau_{\text{fault}}} \]  \hspace{1cm} (5.39)

### 5.5.2 Wishbone Faults

Figure 5.6 shows faults interpreted from the regional 3D PP survey. There are numerous faults that extend from the Niobrara and Codell Formations upward into the lower Pierre shale. There are also numerous faults that penetrate the Terry and Hygiene sands.

Figure 5.7 shows the mapped faults in the Wishbone area. The mechanical earth model provides the inputs to the critical stress computation.

The results of the calculations show the faults trending east and west are critically stressed. These fault segments are shown in the red boxes.
Figure 5.6: Faults interpreted from the regional 3D PP survey

Figure 5.7: Mapped faults in the Wishbone area
5.6 Numerical Stress Modeling

A critical input into hydraulic fracture design is the vertical stress profile, which is the variation in the minimum horizontal stress as a function of the vertical dimension. The vertical stress profile substantially controls the geometry of the induced hydraulic fracture. An optimum stimulation design would define the pumping variables so as to limit the height growth to the productive interval while extending the fracture an optimum distance from the borehole. Vertical growth above or below the productive interval is not optimal. Fracture height growth into non-productive layers above or below the pay interval reduces the fracture length in the productive interval. In addition, gravity can concentrate the proppant in the fracture below the producing interval which diminishes fracture conductivity in the producing interval. Both mechanisms lead to a decrease in production. Therefore, to design an optimal stimulation, an accurate vertical stress profile is required.

In addition to the vertical stress profile, an understanding of the evolution of stresses with time is needed to guide field development plans for either developmental drilling or re-completions. An understanding of how in-situ stresses change with time could support a proactive strategy to manipulate the stress field to avoid drilling problems or enhance completions.

I will now describe some of my learnings from the numerical stress modeling.

5.6.1 Static Model

I constructed a simple static model to evaluate the initial vertical stress profile and the subsequent effective stress changes due to injection and/or production. I constructed a Cartesian grid using 17 grid cells in the x and y directions and 29 grid cells in the z-direction as shown in Figure 5.8.

I populated the grid with petrophysical and geomechanical properties. A side view of the effective porosity and Poisson’s ratio is shown in Figure 5.9. The reservoir layers from the Sharon Springs Formation down to the Graneros Formation are shown in the center of
Figure 5.8: Cartesian grid showing different isotropic layers
the figure. The vertical grid has more grid cells over the reservoir layers and less grid cells in the shales above and below the reservoir. Other properties, such as water saturation, matrix permeability, Young’s modulus, and formation strength properties were also created to complete the static model definition. I kept the static model simple, with no lateral variability, to isolate the impact of pore pressure and stress equilibration, and separate the time effects from spatial variations in the model.

![Figure 5.9: Side view of the grid showing effective porosity and Poisson’s ratio](image)

5.6.2 Stress Equilibration

I made an initial estimate of mechanical properties, pore pressure and earth stresses based on well log data and calibrated the resulting 1D mechanical earth model (1D-MEM) to core and in-situ engineering measurements. The overburden pressure was computed from the bulk density log and applied to the numerical grid as a vertical load. I used a constant strain boundary condition for the sides and bottom of the model. As might be expected, at time zero, the fluid distribution and internal strains were not in static equilibrium. The model was run for a time span of 30,000 years at which time, the fluids and stresses had
stabilized. Figure 5.10 shows the resulting pore pressure and effective stress distribution after the 30,000 year stabilization period.

The equilibrated pore pressure is shown on the left side of the figure while the effective stress, also known as the minimum horizontal stress, is shown on the right side of the figure. As expected, the pore pressure profile showed a smooth gradient. However, the minimum horizontal stress showed low values in the reservoir, and significantly higher values in the boundary shales. Upon examination, I found the reason.

The lower friction angles in the bounding shales (see Figure 5.11) equates to lower formation strength values. The stress difference between the vertical and horizontal directions was more shear stress then the shales could absorb in elastic deformation. The elastic limit was reached and the shales exist in a state of failure. This explains why the drillers experience more borehole instability problems when landing the well in the shallow B chalk. The preferred strategy is to land the well in the deeper C chalk allowing the borehole to exit the weak Sharon Springs Formation at a reduced borehole deviation thus minimizing the shear stresses on the borehole and minimizing the associated borehole instabilities. To
validate this result, the rock strength parameters from the 1D-MEM were increased by an order of magnitude and the 30,000 year equilibration pass was recomputed. As expected, the numerical results matched the analytic equations for linear elasticity as given by Equations 5.40 and 5.41.

\[
\sigma_h - \alpha P_p = \frac{\nu}{1 - \nu} (\sigma_V - \alpha P_p) + \frac{E}{1 - \nu^2} \epsilon_h + \frac{E\nu}{1 - \nu^2} \epsilon_H \quad (5.40)
\]

\[
\sigma_H - \alpha P_p = \frac{\nu}{1 - \nu} (\sigma_V - \alpha P_p) + \frac{E\nu}{1 - \nu^2} \epsilon_h + \frac{E}{1 - \nu^2} \epsilon_H \quad (5.41)
\]

The results of the analytic poroelastic horizontal stress model were identical to the numerical model to five significant digits.

Figure 5.11: Vertical stress profile for the isotropic linear elastic model

Linear elastic stress equations require a linear relationship between stress and strain. Inherent in this assumption, but seldom explicitly stated, is formation strength is infinite; strain will be linear with stress for any and all stresses. In addition, the only formation properties that control the vertical stress profile are Poisson’s ratio and Young’s modulus; formation strength is beyond the scope of the model. As can be seen in Figure 5.11, Young’s modulus and Poisson’s ratio are fairly constant over the entire interval. Thus, a consequence
of the linear elastic stress assumption is there will be minimal variations in the vertical stress profile and no significant vertical stress barrier between the Niobrara Formation and the bounding shales.

The missing stress barrier at the top of the Niobrara Formation is concerning. History matching of the stimulation treatment shows a significantly higher stress (approximately 0.1 psi/ft) in the Sharon Springs Formation. A similar stress increase is seen in the Carlile and Graneros shales below the Codell (see Figure 5.12).

Thus, the increase in stress in the bounding shales is due not to the elastic response of the shales, but the plastic strain that occurs after failure.

Figure 5.13 shows the initial vertical strain on the left (after equilibration) and a subsequent final strain on the right (after a few months of production). Note the vertical strain values in the shales (red and orange color) are nearly twice the magnitude of the strain values in the chalks and marls (green color).

Figure 5.14 gives the explanation. When a vertical load is applied to any given layer, the vertical dimension is decreased while two perpendicular dimensions are increased. This...
Figure 5.13: Vertical strain

Figure 5.14: Impact of plastic strain
is known as the Poisson effect with Poisson’s ratio relating the lateral strain to the vertical strain. As long as the two layers have the same Poisson’s ratio, and remain in the elastic domain, the lateral deformation in each layer will be the same. However, when the elastic limit is reached in the upper layer, there is additional vertical strain due to the plastic response of the formation, which would cause additional lateral expansion. But because the lateral strain is constrained by the imposed constant strain boundary condition, the upper layer has to increase in stress. Thus, the cause of the higher stress in the upper layer is due to the plastic strain in the vertical direction that cannot be redirected into the lateral direction.

This new understanding of mechanical behavior has critical importance. Most shales are inherently weak and ductile. When the shale yields and approaches failure, the linear elastic models, either isotropic or anisotropic are inappropriate, as they cannot account for the consequences of ductile behavior.

The historical success of linear elastic models is in clastic environments where there is a natural difference in Poisson’s ratio between the sands and the shales. Because the sands typically have a lower Poisson’s ratio than the shales, the elastic models show a lower stress in the sands and a higher stress in the shales. With calibration, an inappropriate model can be made to work. However, in carbonate environments, where Poisson’s ratio varies little, even in totally different formations, the calibration trick fails to work. Thus, to accurately quantify a vertical stress profile in weak, ductile and fractured shales, we must move beyond linear elasticity. A reasonable solution is to use a simple numerical model as shown here.

Thus, to optimize the stimulation design, the interplay between fracture height and fracture length must be accurately modeled. The most important input to determine fracture geometry is the vertical stress profile. Once the fracture geometry is accurately determined, a well spacing study can influence field development decisions.
5.6.3 Dynamic Stress Behavior

To understand the changes in effective stress from an initial model, injection and production is simulated to understand the spatial change in effective stress.

Figure 5.15 shows a side view of a generic grid. The grid has a high vertical resolution around the producing layer and a lower vertical resolution further away. A vertical borehole is shown in the center of the model. The initial pore pressure for every grid cell was set to 4500 psi, shown in red. Figure 5.15 shows the pore pressure after 18 months of production. The grid cell, shown in dark blue, represents the perforated zone and a drop in pore pressure to 2700 psi, representing a partial depletion of 1800 psi from the initial pore pressure of 4500 psi.

Figure 5.16 shows the corresponding change in effective stress. The effective stress surrounding the depleted zone has increased from the initial condition of 2000 psi. The initial effective stress is shown in blue while the increase due to depletion is shown in color. When
pore pressure is partially depleted, the load supported by fluid decreases, and the rock matrix above and below the producing layer must accommodate the change in load.

In addition, the redistribution of load, represented by the effective stress, is more expansive, effecting more grid cells and a larger volume of rock than the volume of rock that has undergone a change in pore pressure. This is a consequence of St. Venant’s principle: stress concentrations reduce as you move away from the source. The insight from this learning model suggests a considerable overlap in the redistribution of effective stresses due to the stimulation where pore pressure increase, and production where pore pressure decrease.

### 5.6.4 Stress Rotation

The injection or production of fluid from a petroleum reservoir causes changes in the local stress field. These changes in magnitude and direction occur not only in the layers undergoing injection or production, but in the layers above and below the targeted reservoir.
Figure 5.17: Elliptical drainage of reservoir

Figure 5.17 shows a map view of an elliptical drainage pattern that is common in hydraulically fractured reservoirs. This map view of the stimulated layer shows the decrease in pore pressure with time due to production. At time zero, the minimum horizontal stress direction is in the x-direction, which is in an east/west plane. In addition, the maximum horizontal stress direction is in a north/south plane. Hydraulic fractures are known to propagate in the maximum horizontal direction while the net pressure in the fracture increases the fracture width, working against the minimum horizontal stress. How does this decrease in pore pressure change the magnitude and direction of the two horizontal stresses?

Figure 5.18 shows a reversal in direction, or a flipping, of the minimum and maximum horizontal stresses after 3 months of production. Pore pressure and effective horizontal stress are plotted on the y-axis while time is plotted on the x-axis. When the well is turned on, and the pore pressure begins to decrease, the maximum horizontal stress is in the y-direction (blue curve) and the minimum horizontal stress is in the x-direction (red curve).
Pore pressure decreases with time causing the effective horizontal stresses to increase. Since the total loads are partially supported by the pore pressure and partially supported by the rock framework (the effective stress), the in-situ loads shift from the fluid to the rock frame increasing the effective stress. But because the permeability in the y-direction has increased due to the stimulation, some of the load in y-direction is shifted into the x-direction. Thus, at some point in time, dependent on the permeability and stress anisotropy, the horizontal stress field flips in direction.

Does the stress field flip at locations other than at the borehole? I examined this question as well.

I examined the grid cell 55 feet east of the wellbore. Figure 5.19 shows the stress field took more time to flip, approximately 2 years, but it did flip. This suggests the depletion effect on the direction of the horizontal stresses is not just a localized phenomena.

I also looked 385 foot north of the borehole. Figure 5.20 shows a flipped stress field after about 13 months of production. Can the stress field rotate in layers above the producing layer? For instance, I’m interested if the stress field in the B chalk will rotate due to the
Figure 5.19: Stress field flips east of the borehole

Figure 5.20: Stress field flips north of the borehole
production in the C chalk.

Figure 5.21: Stress field flips in the B chalk

Figure 5.21 shows this can also be the case. Because the propped hydraulic fracture couples the depletion in multiple layers, it is possible for the stress field to flip in layers above and below the perforated interval.

These simulations suggest that if this formation is re-stimulated by injecting fluid into partially depleted rock, a new hydraulic fracture will propagate in the x-direction. Of course, when this new fracture propagates outside of the depleted region, it has to rotate and align parallel with the original hydraulic fracture as it continues to propagate. The end result is this new hydraulic fracture contacts virgin rock and the subsequent production is increased.

5.7 Results and Conclusions

I have used a flow-coupled geomechanical simulator to determine the vertical stress profile in the Wishbone area. The model assumed a linear elastic behavior until the formation fails and then a perfectly plastic behavior post failure. This model showed a 0.1 psi/ft increase in minimum horizontal stress in the shales bounding the Niobrara Formation due to the ductile response of the shales.
The flow-coupled simulator was also used to determine the change in the horizontal stress magnitudes and direction caused by changes in pore pressure. Both horizontal stress magnitudes increased with time, but not at the same rate. The consequence of this behavior is at some time, dependent on the stimulation altered permeability, the horizontal stress direction flips in orientation.

Two important learnings have resulted from this research. First, a limitation of linear-elastic theory has been highlighted and an alternative workflow has been proposed to more accurately define the vertical stress profile. And second, injection and production of fluids cause stress changes in both the reservoir and boundary layers. Understanding these changes provides an opportunity to re-stimulate layers to increase the ultimate recovery of hydrocarbons.
Unlike most students, I have been involved with Colorado School of Mines (CSM) for nearly two decades. I was Schlumberger’s representative to the several consortia including the Reservoir Characterization Project (RCP) from 2004 until my retirement in 2013. I was a visiting professor in the Petroleum Engineering department from 2013 until 2015. I started my advanced studies in Geophysics in 2014. Thus, I have been involved with the Anadarko Wattenberg project from the beginning of Phase 15 in 2013, first as a sponsor, second as a faculty member, and now as a student. This unique experience has given me a broad perspective.

6.1 Dipole Sonic Processing

I identified different types of anisotropy using dispersion analysis and the three-shear moduli overlay technique. The Fort Hays, Codell and Muddy Formations were diagnosed as mostly isotropic. However, short intervals of the J-sand and Carlile Formations exhibited HTI anisotropy. The source of anisotropy was diagnosed to originate not from open natural fractures, but from the differential stress between the two horizontal directions. Indeed, no other HTI behavior was observed between the top of Carlile Formation and the top of the logged interval, about 1000 feet above the Niobrara Formation. This observation suggests that the differential horizontal stress decreases with depth.

The remaining formations from the lower Pierre to the Muddy Formations exhibited strong VTI anisotropy.

My recommendations are:

- Acquire wireline sidewall core samples at different depths to confirm anisotropy in the bounding shales
• Acquire dipole sonic data from below the formations of interest to the surface such that log scale anisotropy can be determined throughout the overburden

• Insure the advanced logs are processed using vendor software and the advanced results shared with the students

6.2 Dipole Stress Interpretation

I developed a new technique to interpret dipole sonic data. The technique differentiates and quantifies variations in sonic velocities caused by mineralogy, porosity, and fluids from variations caused by stress. The variations due to stress give important information such as the location of overpressure. In addition, either pore pressure or the stress sensitivity can be quantified. Additionally, velocity variations due to stress can be removed to eliminate the stress bias and improve the petrophysical analysis.

What was interesting about the data set reported in this investigation, is the decrease in velocity due to unloading is much larger, not smaller, than the increase in velocity due to loading. One explanation is the expected increase in the number and size of fractures due to the maturation process.

My recommendations are:

• Use this technique to interpret dipole sonic data in different basins to better quantify the geomechanics and petrophysics.

• Extend the method by normalizing to two standard formations; a clay-supported formation as shown here, and a grain-supported facies such as the Niobrara formation

6.3 Rock Physics

I quantified the stress sensitivity of the Niobrara Formation core data using linear slip theory. The background model can have arbitrary anisotropy up to and including orthotropic. The dipole sonic data suggests the background anisotropy is mostly VTI. The sensitivity of
both P-wave and S-wave velocities to effective stress are much greater in the vertical direction than in the horizontal direction. Thus, any reasonable rock physics model must account for at least orthotropic anisotropy.

While linear elastic theory has been fundamental to seismic interpretation, its application to time-lapse studies of hydraulically fractured unconventional reservoirs is questioned. The hydraulic fracture stimulation increases the effective permeability of a nanodarcy formation by orders of magnitude with the creation of multi-scaled fractures and cracks. These new fractures alter the continuity and homogeneity of the formation and violate the strict assumptions of a continuous, homogeneous, and linear elastic material. It is thus concluded that rock physics models based on linear elasticity are inappropriate, and models based on linear slip theory are preferred.

The first monitor survey is expected to be complicated by stress effects due to the induced hydraulic fractures extending into the Sharon Springs. The induced velocity variations will extend into the lower Pierre Formation. This will impact the standard cross-equilibration technique used in time-lapse studies.

When pore pressure in the reservoir decreases below the bubble point, gas saturation will increase. An increase in gas saturation will add compliance to the seismic signal. When a formation is stimulated, and fractures are created in the formation, this will also add compliance to the seismic signal. The compliance in each case is similar. Thus, there is no differentiation between a fluid effect and a fracture effect. Either could be the cause of an excess compliance on the seismic signal.

My recommendations are:

- Use linear slip theory as the base method to characterize stress sensitivity
- Characterize the stress sensitivity of all formations
- Revisit the cross-equilibration method for time-lapse studies
• Model the impact of stress dependent fracture compliance and changing stresses on the seismic trace; alter the fracture compliance to fit the observed changes in the time-lapse seismic

6.4 Geomechanics

I have used a flow-coupled geomechanical simulator to determine the vertical stress profile in the Wishbone area. I assumed a linear elastic behavior until the formation fails and then a perfectly plastic behavior post failure. I used the gravity force along with strain boundary conditions to load the model. I simulated the geomechanics over a time period of 30,000 years, the time it took for the fluids and stresses to equilibrate. This model showed the increase in stress in the bounding shales is due not to the elastic response of the shales, but the plastic strain that occurs after failure. This stress showed a 0.1 psi/ft increase in minimum horizontal stress in the shales bounding the Niobrara Formation due to the ductile response of the shales.

I used the equilibrated stresses and the strength properties of the different formations to determine the shear stresses on the faults. Faults striking in the direction of the maximum horizontal stress are critically stressed. However, faults striking by more than 15 degrees in either direction of the maximum horizontal stress will be stable.

The flow-coupled simulator was also used to determine the change in the horizontal stress magnitudes and direction caused by changes in pore pressure. Both horizontal stress magnitudes increased with time, but not at the same rate. The consequence of this behavior is at some time, dependent on the stimulation altered permeability, the horizontal stress direction flips in orientation.

Two important learnings have resulted from this research. First, a limitation of linear-elastic theory has been highlighted and an alternative workflow has been proposed to more accurately define the vertical stress profile. And second, injection and production of fluids cause stress changes in both the reservoir and boundary layers. Understanding these changes provides an opportunity to re-stimulate layers to increase the ultimate recovery of
hydrocarbons.

My recommendations are:

- Use numerical modeling to determine the vertical stress profile anytime weak ductile shales exist within a distance of two times the fracture height achieved in the stimulation, a distance both above and below the productive formation.

- Use the calibrated stress model and formation strength properties to determine the maximum shear stress on any mapped faults in the area of interest. Consider the impact drilling and completion operations might have on the critically stressed faults.

- From reservoir simulation results, quantify the increase in permeability caused by the stimulation. Use these results to map the dynamic stress changes, both magnitude and direction, caused by production.

- Use the rock physics model to predict changes in seismic reflectivity.

- Look for opportunities to re-stimulate to improve recovery and test the forward model.

6.5 Future Work

I have built the basic foundation for using seismic reflectivity to understand stress changes in the earth caused by the injection and production. Future work should include refining the workflow and testing the predictions.

My recommendations are:

- Acquire more core data to establish yield and failure behavior over a range of different shale properties. In the laboratory, determine the elastic wave response as a function of loading the sample through yield and failure.

- Investigate the pore pressure and stress changes in the Niobrara Formation due to the hydraulic fracturing stimulation, and the diffusion of the stimulation into the overlying Sharon Springs and Lower Pierre Formations.
• Adjust the fracture compliances in a few discrete layers to best match the seismic traces.

• Acquire high end log data in the same well that is cored. The logging suite should include formation images, nuclear spectroscopy, and dipole sonic logs. These data need advanced processing with the vendor software to insure students have high quality input data.

• A better understanding is needed to accurately apply the advanced learning of core and in-situ stress measurements acquired in one location to some other location.

• Future time-lapse studies should consider a remote area that is surveyed, but sufficiently removed from operations, to provide a control on changes in instruments and surface conditions.
REFERENCES CITED


