STOCHASTIC INVERSION OF SEISMIC DATA BY IMPLEMENTING IMAGE QUILTING TO BUILD A LITHO-FACIES MODEL FOR RESERVOIR CHARACTERIZATION OF DELHI FIELD, LA

by

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ABSTRACT

In this research a new stochastic inversion approach along with an image reconstruction method is implemented to build a litho-facies model with a focus on Delhi Field, LA. The field is under $CO_2$ injection as an enhanced oil recovery (EOR) method. This makes it critical to define the $CO_2$ flow paths and flow baffles in higher resolution to plan for the EOR project. The algorithm starts at the well location by defining the litho-facies, using well logs, K-mean clustering method and core studies, and updated by the elastic properties distribution. The whole inversion approach is performed including multiple point statistics (MPS). The key element in the MPS algorithms is the training image. It is a conceptual model from the reservoir, which is built based on information from the reservoir regarding the depositional environment, structure, and any other information from the reservoir.

A 3D training image is built for the reservoir, but the inversion is performed on a 2D seismic line, therefore the training image is sub-sampled in the direction parallel to the direction of the 2D inline of interest. Then a square template is chosen of sizes of $5 \times 5$ and $7 \times 7$ and all the 2D planes are scanned with this template and the pattern database is constructed. The pattern database includes all the possible configurations of the litho-facies from the training image. At the next step, the search algorithm begins and searches for all the patterns from the database that have similar configuration to the litho-facies at the well location. A distance function is defined (here Manhattan distance) and the patterns providing the smallest distance with the patterns at the well location are stored. Multiple realizations of litho-facies from the stored patterns are generated.

The next step is to choose the realization, which provides the highest correlation or similarity to the subsurface. At this step, seismic forward modeling is implemented. Pseudo-logs of density and P-wave velocity are generated from the joint distribution of the properties at the well location that are conditioned to each litho-facies. Multiple realizations of pseudo-
logs are generated (15 in this case) and synthetic seismic traces are created, having extracted the wavelet from the seismic volume. The realization that has the highest cross correlation is chosen as the litho-facies at the well location. To continue the algorithm away from the well location, an image reconstruction method that is called image quilting is implemented. This algorithm searches for similar patterns that have some overlapping criteria with the previously accepted pattern. The distance function is defined in a way to search for the overlapping grid nodes. The algorithm continues and the seismic forward modeling is implemented in a stochastic approach to find the best elastic properties and the corresponding litho-facies realization. Multiple realizations of litho-facies for the whole 2D inline is generated and the maximum probability of multiple realizations (ten in this case) is obtained as a representative of the litho-facies of the reservoir.

The structural and depositional complexity of Delhi Field, presents a heterogeneous reservoir in the vertical and horizontal directions. Due to the fact that the field is under an EOR process, obtaining a detailed definition of litho-facies and flow paths distributions is of great importance. The method conducted in this research incorporates stochastic inversion and image reconstruction and provides a new methodology for constructing a detailed and high resolution litho-facies model by integrating multi-scale and multiple data types for complex and heterogeneous reservoirs like Delhi Field. Because of the stochastic characteristics of this methodology, equi-probable scenarios are generated and the most probable one is calculated.
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dad.
The ultimate goal of using seismic data is to predict reservoir properties and build reservoir models of lithology, porosity and fluid saturation. Rock physics is the tool that provides empirical and analytical relations between reservoir properties and their elastic responses. However, this approach does not always provide a unique solution.

Estimating physical properties of the subsurface (e.g. elastic properties) from any geophysical data is an inverse problem. In seismic methods, by inverting travel times and amplitudes of the elastic waves, one can obtain estimates of the elastic properties of the subsurface. Yet, the main objective in the application of geophysical methods is beyond estimation of the physical quantities to which the methods respond. Rather, the final goal would be to characterize the subsurface for rock properties such as lithology, fluid saturation, etc. or the physical condition like pressure and temperature that they are subjected to (Gonzalez et al., 2007).

Estimating reservoir properties from seismic data is an inversion problem with a non-unique answer. Even in the ideal case of noise-free data, the limited frequency of the seismic data is the cause of the non-unique solution. Therefore, in practice, obtaining reservoir properties from seismic data becomes more complicated, due to the presence of noise in the data and the simplification made in the forward modeling part to gain the solution in a plausible time (Gonzalez et al., 2007).

1.1 **Background and problem statement**

Defining subsurface heterogeneity is an important aspect in reliable reservoir characterization and building an accurate reservoir management plan. Different methods and approaches have been developed in order to construct a reservoir model respecting the heterogeneity of
the subsurface and accounting for the available datasets of different scales. Some of the methods applied in reservoir characterization are based on physical methods and laboratory measurements, known as deterministic solutions. Others are stochastic and based on multivariate methods. Constructing the solution based on the combination of the two approaches, will be the best strategy and provides much more accurate results, than solely each of the methods.

Estimating reservoir properties through seismic measurements is subject to uncertainty because of the inherent uncertainty in data acquisition, processing and interpretation. Therefore, it is crucial to quantify the uncertainty in the information obtained from seismic data. Statistical rock physics has an important role in considering some of the uncertainty by implementing multivariate stochastic relations linking the elastic parameters and reservoir properties. Moreover, the process of seismic inversion, per se, is a non-unique and ill-posed problem, which leads to an infinite number of elastic property models that match the seismic data. The result of a deterministic inversion is presented as a bulk elastic response that is restricted to the seismic wavelength. Provided that level of accuracy, there can be a combination of model parameters that create similar seismic response considering some tolerance.

Solution of an inverse problem in a statistical approach, is not bound to only one group of predicted parameters. Rather, it is provided by multiple realizations of equal probability of the model space. The goal of the inversion is not only to find out the best solution that matches with the model parameters, but also to define the uncertainty in the solution space (Buland and Omre, 2003).

The aim of most seismic inversion methods is to reduce the error between the predicted models and recorded data, and predict the geological heterogeneity and future reservoir performance by using a set of matched models. Since, these inversion solutions are limited to a pre-defined geologic scenario, they are unable to handle larger sources of geological uncertainty. Another shortcoming of these approaches is that often the previously matched models
have to be disregarded due to the inconsistency with the newly obtained data. The inconsist-

cency can be caused by a very small range of uncertainty because of an inaccurate geological
scenario. Therefore, modeling multiple geological scenarios is a necessary pre-required task.
In more reliable and realistic geomodeling, many different sources of uncertainty have to be
considered such as proportion of facies, size of the geobodies, stacking pattern, rock physics
relations, rock-fluid type and so forth. Stochastic seismic inversion is the approach that
facilitates implementation and addressing of previously mentioned sources of uncertainty
(Gonzalez et al., 2016).

1.2 Objective and contribution

In stochastic reservoir modeling there are two main approaches: variogram-based and
object-based. In variogram-based approaches the relation between data points is defined
through the variogram, which is calculated by pair of known values. Then by using kriging
or sequential methods, unknown values are evaluated by conditioning to well or seismic data.
These methods are incapable of capturing the sinuosity or large continuity of geobodies, such
as channels, and they may result in misinterpretation or inaccurate definition of the reservoir
that leads to poor forecasting of the reservoir performance.

However, object-based methods give this opportunity to the user to incorporate more
realistic geologic models into stochastic simulation in the form of training data. But condi-
tioning to hard well data or secondary data is a limitation of these methods. As a solution
to this shortcoming, multiple point statistics (MPS) methods have been developed (Stre-
belle, 2002). These methods bring together the ability of reproducing the shape from the
object-based methods and the straightforward data conditioning from the variogram-based
methods.

Here in this research the main objective is to build a litho-facies model for the hydrocar-
bon reservoir, at Delhi Field in Louisiana, that captures the complexity of the depositional
environment and provides a detailed definition for the reservoir. What makes it very im-
portant is that this specific reservoir is under CO₂ injection as an Enhanced Oil Recovery
(EOR) process, and understanding the complexity of the reservoir regarding the litho-facies distribution is key to design the EOR project.

The main contribution of this work is to implement multi-scale data (core studies, well logs, seismic data, geomorphology studies) in a specific multiple point statistics algorithm to build the litho-facies model in an inversion approach. The multiple point statistics algorithm is called SIMulation with PATterns (SIMPAT), which is a patch-based algorithm with a pattern reconstruction approach. The algorithm is fully coded in MATLAB, in a different approach than the original algorithm in order to provide more accurate data conditioning. The random simulation path is substituted by uni-lateral simulation path and implementation of the image quilting concept.

1.3 Thesis layout

The thesis consists of a compendium of chapters:

Chapter 1 is the introduction, which encompasses the background and problem statement, objectives, and contributions of the thesis.

Chapter 2 provides literature review, with the main focus on seismic reflectivity inversion methods and stochastic inversion approaches.

Chapter 3 provides a discussion on different multiple point statistics methods and the implementation of training images in MPS approaches.

Chapter 4 discusses the initial reservoir model of litho-facies built for the field under study, applying conventional geostatistics.

Chapter 5 involves application of the SIMPAT algorithm in building the litho-facies model for Delhi Field

Chapter 6 summarizes conclusion and recommendations
CHAPTER 2
REVIEW OF REFLECTIVITY INVERSION AND STOCHASTIC METHODS

In general, seismic inversion refers to the process of obtaining elastic properties such as density, velocities, and impedances of the subsurface layers from seismic data. Many authors have proposed different methods for seismic inversion. Cooke and Schneider (1983) proposed a method called generalized linear inversion (GLI), which could overcome the shortcomings of the recursive inversion method regarding the scale of the impedance results, resolution of the impedance boundaries, and distortion caused by residual wavelet. They also mentioned some other advantages of their method as being insensitive to noise in many cases, and providing the interpreter the ability to fix the impedance values for any number of known lithologies in the interval of interest.

In another method presented by Oldenburg et al. (1983), they examined the problem of calculating the acoustic impedance from a band-limited normal incidence seismic dataset. In their work they introduced two methods to recover the low frequency information, the first was a linear programming (LP) algorithm that would find a reflectivity function made of isolated delta functions. This was a robust method in the presence of noise and also a computationally efficient approach. Their other method was based on the fact that the Fourier transform of a reflectivity function for a layered media can be represented as an autoregressive (AR) procedure. The high and low frequencies that were missed, could be recovered by their approach.

Sen and Stoffa (1991) introduced an inversion algorithm, which used simulated annealing to find the best model parameters \( \mathbf{m} \) that minimizes the error energy between the recorded data and the synthetic seismograms. They were able to apply their method to band-limited data in the presence of random noise.

Gouveia and Scales (1998) applied Bayesian theory for inversion of seismic data. Their main
The purpose of implementing the method was to assess the errors and uncertainties present in the information about ambient noise, discretization and theoretical errors, and the prior information about the layered Earth model obtained from petrophysical analysis. They assumed that the whole model and data uncertainties can be defined by multidimensional Gaussian probability densities. They optimized the posteriori probability through a nonlinear conjugate gradient model that maximizes the a posteriori. They analyzed the uncertainty by making a Gaussian approximation of the a posteriori distribution about the peak model, and presented the results in three different forms: maximum a posteriori model associated with error bars of one standard deviation, semi-random simulation of the a posteriori probability representing the expected range of subsurface models, and marginal of this probability at specific depths. Their method could be incorporated for both the surface and the borehole seismic data.

The concept of geostatistical inversion was introduced by Bortoli et al. (1993) and Haas and Dubrule (1994) by incorporating sequential Gaussian simulation. Haas and Dubrule (1994) used elastic information (density and velocity logs) at the well location to calculate the impedance log. By choosing a random path, they calculated the impedance values of a desired location through sequential simulation. Then by performing depth to time conversion, calculating the reflectivity series and convolving with the extracted wavelet, they calculated the synthetic seismic. Comparing the synthetic seismic with the recorded data, if not within the accepted range of mean square errors or the cross correlation, they performed the sequential simulation on another random path until the satisfactory result is obtained. This method is pixel-based and the variogram has an important role in predicting the vertical and horizontal continuity or heterogeneity.

Mukerji et al. (2001) introduced a new method in quantifying and reducing the uncertainties in reservoir management. They proposed to extend the facies classification of wells to the whole seismic cube by implementing rock physics, statistical classification technique, and seismic inversion. Then by using the spatial correlation of data points, capture the
heterogeneity at well locations. They used rock physics and Bayesian classification to obtain the probability density function (pdf) of facies distribution at the well location. They also implemented information theory and Shannon’s information entropy to find which seismic attributes are better choices for reducing the uncertainty in reservoir property identification. They showed for example, in estimating the probability density function (pdf) of porosity when conditioned to information of only P-wave velocity, the pdf is narrower and longer compared to when only calculating the pdf of porosity itself. Also when conditioned to information of both P-wave and S-wave, the pdf is even more narrower and longer, which means more specific pdf information. Then they used the inversion seismic attributes to extend the facies classification away from the wells to the whole seismic volume. Although, this was only at seismic resolution, further they implemented geostatistical approaches to include the spatial correlation through a variogram and to capture the small scale variations, which are below seismic resolution.

In another study by Caers (2001), they applied the algorithm proposed by Mukerji et al. (2001) and used an object-based simulation method as the geostatistical simulation step. They provided a method to downscale the seismic derived probability to smaller scale, and considering the reservoir continuity through a conceptual geologic model. This conceptual model was created using a Boolean method, and the proposed geostatistical approach was pixel based.

In another approach to stochastic reservoir characterization Eidsvik et al. (2004) introduced an algorithm that used Bayesian classification and Markov Chain Monte Carlo (MCMC) sampling. The algorithm is described as follows: first define the litho-fluid-facies classes by using well log analysis, core data and geology information. Then estimate the prior pdf of reservoir properties (density, porosity, and velocities) for each litho-fluid-facies from well data and incorporate Morakov random field model to define a prior model of spatial continuity. Then define a likelihood model (forward modeling operator) to relate the reservoir
properties to the seismic data. Finally, sample from the posterior distribution using MCMC algorithm, and from the multiple realizations generated, build a maximum probability of predictions (or occurrences) for each litho-fluid-facies. Due to the stochastic nature of this method, uncertainty associated with the estimation could also be evaluated.

Bosch et al. (2007) proposed a method for stochastic simulation, where rather than conventional methods of geophysical inversion to calculate the elastic properties then petrophysical inversions to estimate the reservoir properties, was to combine the geophysical and petrophysical inversion through a Bayesian approach and jointly estimate the elastic and reservoir properties by solving the inverse problem using a Monte Carlo sampling approach to generate multiple realizations and quantifying uncertainties of different types. Their estimations considered the seismic uncertainty, deviation of the elastic properties from the values calculated through petrophysical transformations, and the nonlinear relations between geophysical and petrophysical models. They implemented this method to estimate the total porosity and the acoustic impedance in a reservoir. Their method also enabled them to calculate the marginal probability of the modeled parameter, including medium porosity, impedance and the wavelet of the seismic data.

Grana and Rossa (2010) implemented the same approach to estimate the petrophysical properties in a probabilistic fashion. The difference was that by assuming more general parametric distributions or non-parametric statistical formulations like kernel-density estimation, they could overcome some limitations on the data size, and the type of the dependencies. Grana et al. (2012) later introduced a new approach in stochastic inversion by using a probability perturbation method. They used an iterative process to generate models of reservoir properties by using sequential simulation, calculating the corresponding elastic properties through rock physics relations, and calculating the synthetic seismograms and comparing them with the recorded seismic data. Optimization of the inversion was performed through the stochastic approach of perturbing the probability distribution of the initial realization.
The probability perturbation method uses a Tau model that provides an analytic form to combine single probability information into a joint conditional probability. By this method, they were able to combine geostatistical methods and geophysical models to generate fine scale porosity and net to gross reservoir property determination.

There are also many studies in estimating uncertainty in the observed data, mainly the outcrop studies, and how to quantify them for reservoir modeling by accounting for different scenarios. In a study by Martinius and Næss (2005), they performed the uncertainty analysis for reservoir modeling by investigating the uncertainties in the outcrop data. They studied them in four major categories, as: (1) the definition of the conceptual depositional model (2) the number of observations of the sand bodies dimensions (3) accuracy of the size of the observed data for 3D sand body analysis and how representative they are, and (4) the orientation of the sand bodies. They found out that the uncertainties related to the depositional environment are the most difficult to quantify. They used a $N_0$ measure (the number of data points needed to estimate the mean within $\pm20\%$) to quantify if the number of observations for dimensions are sufficient to reduce the uncertainty to an acceptable value. Also they concluded that the uncertainty in determining the orientation of the sand bodies will affect the estimation of continuity of the structures.
CHAPTER 3
MULTIPLE POINT STATISTICS AND TRAINING IMAGE

3.1 Introduction

Considering the limitations of conventional simulation methods based on two-point statistics, this chapter describes more advanced approaches that account for multiple-point correlations of data points. Sinuousoidal channels in a fluvial depositional environment, providing reservoir characteristics, and long range features cannot be modeled using traditional two-point statistics and variograms.

The first approaches to overcome this limitation were application of Boolean object-based algorithms to incorporate random geometries over the simulation volumes (Guardiano and Srivastava, 1993). Yet, object-based algorithms had two major shortcomings: (1) every type of object has to be defined separately by its geomorphological properties, and not all of them could be defined into one single object, (2) data conditioning to local events for these random objects is difficult.

As a solution to this problem, some pixel-based methods were introduced that used multiple point information. Deutsch (1992) and Farmer (1988) used simulated annealing, Tjelmeland (1996) and Caers and Journel (1998) used an iterative algorithm in the form of Gibbs sampler, which was similar to Markov Chain Monte Carlo (MCMC). These methods used a training image to infer the multiple point information, but all were based on an iterative approach that relied on some convergence criteria. The rate of convergence usually was not known beforehand and some stopping criteria had to be implemented. The first multiple point statistics algorithm was introduced by Guardiano and Srivastava (1993). The algorithm inferred the conditional probability of each categorical variable directly through scanning the training image. The algorithm was non-iterative and therefore did not have convergence problems, but because the training image had to be scanned anew each time, it
was time intensive and CPU demanding.

3.2 Multiple point statistics algorithms

In general, multiple point statistics methods can be divided into two main categories: pixel based and patch based. Each of the categories will be discussed in further detail in this chapter. However, a generic algorithm can be described for both type of approaches as follows (Mariethoz and Caers, 2015):

Inputs to the algorithm include:

- One or more variables for simulation $Z$
- Simulation grid $SG$
- Training image $TI$
- Conditioning data points
- Parameters specific for each algorithm

Steps of the algorithm:

1. Move conditioning data points to the nearest grid node on the simulation grid
2. Generate the pattern database from the training image
3. Define a path for simulation
4. Repeat until the desired criterion is met
   - Define a node to be simulated
   - Determine the conditional distribution based on neighboring nodes information
   - Draw a value from the distribution
   - Assign the value to the corresponding location on the simulation grid
5. End

Output: Simulation grid with variable $Z$ known at all grid nodes.

### 3.2.1 Pixel-based algorithm

The first practical algorithm proposed as a multiple point statistics algorithm, was the one by Guardiano and Srivastava (1993) known as single normal equation simulation (SNESIM). This algorithm incorporated a training image to spatially correlate the data points. The main steps of the algorithm are as follows:

1. At a grid node $u$ to be simulated, search for $n$ conditioning data considering well data as hard data or previously simulated nodes. This provides a data event $d_n$ that is characterized by the geometrical configuration and the data values.

2. Search the training image with a pre-defined template. Find all the similar configurations of $d_n$ (geometry and data value).

3. For every replicate of the data event $d_n$, save the value (facies type) at the center of each node, $u$, then count the number of replicates for each of the facies appearance.

4. Calculate the conditional probability of each facies as the proportion of replicates that have the specific facies at their central node to the total number of replicates.

The major advantage of this method is that unlike conventional object-based methods, well data or other conditioning data are honored completely and relatively easily. Besides, estimating facies probabilities conditioned to multiple-point data events by obtaining multiple point statistics through the training image; the MPS algorithm follows and reproduces patterns of the training image. However, scanning the training image each time to search for the replicates of the data events at every location $u$ is a very time consuming process. To overcome this problem, Strebelle (2002) introduced a dynamic approach to save all the possibilities of the search results of conditional probability distributions in a search tree. He
also introduced the concept of multi-grid simulation, which made it possible to capture finer scale variations and various scale resolutions in a nested grid approach.

An important aspect of the SNESIM algorithm is the size of the template, which provides neighboring information of the simulation node $u$. The larger the template, the larger the size of the data event $d_n$ and the more specific and fewer the replicates over the training image would be necessary. Therefore, to infer the corresponding conditional probability, there are very few replicates available, and assigning the conditional probability to the simulating node would be doubtful. To obtain a reliable conditional probability for the simulation node, there should be *enough* replicates of the data event, approximately 10-20. To fulfil this requirement the training image should have repetitive characteristic known as stationarity. Consequently, stationarity will lead to exporting multiple point statistics from the training image that refer to the commonly found features.

Common pixel-based algorithms include SNESIM, direct sampling and simulated annealing. The core structure of the algorithm remains similar, but there are differences in details of each algorithm. In direct sampling, the algorithm is capable of handling categorical and continuous variables either separately or simultaneously. Also, the algorithm utilizes a distance function (type of the function depends on the application of the algorithm) to search for the replicates of the data event, and each replicate that has a distance smaller to some pre-defined treshold is assigned to the simulation node.

### 3.2.2 Patch-based algorithm

As a solution to the stationarity requirement of most of probabilistic pixel based algorithms, Arpat (2005) introduced an alternative approach of a patch based algorithm. The idea was to redefine the sampling problem as a direct image construction problem. The goal of the new algorithm was to directly reproduce training image patterns in a stochastic way. Such pattern reconstruction techniques are mainly implemented in computer vision, image reconstruction, and image processing subjects (Arpat, 2005).
To overcome the limitations imposed by the stationarity requirement of pixel based methods, the use of probabilities is not applied any further in patch based algorithms. Image reconstruction algorithms, instead rely on similarity concept and the final realization is constructed based on similarity between individual data events (patterns).

Another advantage of these algorithms is the capability of building pattern to pattern correlation Arpat (2005). Most MPS algorithms, and more specifically pixel-based algorithms, use pattern-to-point correlations. They incorporate an entire data event to estimate the probability distribution of one single node. However, pattern-to-pattern relations have more significant impact when conditioning to high resolution seismic data. Such data are better able to capture patterns in a realization (shape of a channel regardless of the size) rather than individual point probability values.

Patch-based algorithms include SIMPAT, FILTERSIM, and CCSIM (Mariethoz and Caers, 2015). Here a description of the SIMPAT algorithm is provided, which was initially developed by Arpat (2005), modified by Gonzalez (2006) and improved and modified by myself throughout the research.

This algorithm is an image reconstruction algorithm and utilizes the patterns of the training image as building blocks, and is developed based on the similarity concepts of the patterns.

To generate the patterns from the training image, it should be scanned with a pre-defined template. Assuming Figure 3.1 as a binary training image of size $11 \times 11$, any arbitrary location $u$ on the training image is discretized as an indicator variable by:

$$ti(u) = \begin{cases} 0 & \text{if contains non-sand} \\ 1 & \text{if contains sand} \end{cases}$$  

(3.1)

This definition is for the categorical variables, but it can be extended to continuous variables as well.
**Pre-processing of the training image**

To generate the pattern database, the training image is scanned with the assigned template. Figure 3.2 shows the pattern database obtained from the training image of Figure 3.1 scanned with a $3\times3$ template. Each of the patterns is represented with a mathematical description:

$$t_{i_T}(u) = \{t_i(u + h_1), t_i(u + h_2), \cdots, t_i(u + h_\alpha), \cdots, t_i(u + h_{n_T})\} \quad (3.2)$$

where $n_T$ is the number of nodes in a template, and the $h_\alpha$'s are the vectors defining the geometry of the nodes of template $T$ and $\alpha = 1, \ldots, n_T$. The vector $h_1 = 0$ points out the center of the template. The geometrical representation of a pattern and connection between its nodes is shown in Figure 3.3. When patterns are stored in the pattern database, they are location-independent, and only the value at each node and the pattern configuration is of importance. In the pattern database each node in $t_{i_T}(u)$
Figure 3.2: Pattern database generated from the training image in Figure 3.1 by scanning the training image with a $3 \times 3$ template (Arpat, 2005).

is denoted by $\text{pat}_T^k(h_\alpha)$ where $k = 1, \ldots, n_{\text{pat}}$ and $n_{\text{pat}}$ is the number of total available patterns in the training image and is denoted by:

$$\text{pat}_T^k = \{\text{pat}_T^k(h_1), \text{pat}_T^k(h_2), \ldots, \text{pat}_T^k(h_\alpha), \ldots, \text{pat}_T^k(h_{n_T})\}$$  \hspace{1cm} (3.3)

Figure 3.3: Geometrical representation of a $3 \times 3$ template and the relations between every template node. (Arpat, 2005)
Simulation algorithm

Simulation begins and proceeds with searching for patterns similar to a data event on the simulation grid. A data event can be presented as:

\[
\text{dev}_T(u) = \{\text{dev}_T(u + h_1), \cdots, \text{dev}_T(u + h_\alpha), \cdots, \text{dev}_T(u + h_{nT})\} \tag{3.4}
\]

A similarity (or usually distance) function is defined, and the distance between a data event and all the patterns in the database is calculated. There are different distance functions available, and in this algorithm usually Manhattan distance (Arpat, 2005) is calculated:

\[
d\left(\text{dev}_T(u), \text{pat}^k_T\right) = \sum_{\alpha=0}^{n_T} \left|\text{dev}_T(u + h_\alpha) - \text{pat}^k_T(h_\alpha)\right| \tag{3.5}
\]

Patterns with the smallest distance are the most similar ones to the data event. One pattern is randomly chosen among them and pasted on the simulation grid at location \(u\). The procedure continues until all nodes on the simulation grid is visited in a random path and the most similar pattern is pasted for every node.
CHAPTER 4
CONVENTIONAL GEOSTATISTICS FOR FACIES MODELING OF DELHI FIELD

During the initial studies, I constructed a facies model for the field. I did the modeling by implementing sequential Gaussian simulation to distribute the well log measurements over the entire field. Before showing the final results of simulation, the theory of variogram modeling and sequential Gaussian is explained.

4.1 Variogram modeling and sequential methods

4.1.1 Introduction

The main goal in the first part of the research was to build the litho-facies model for Delhi Field, which captures the heterogeneity present in the field. The available data at the time were porosity and permeability (obtained by transformation of porosity and SP) logs. The proposed method to characterize the heterogeneity of the reservoir is sequential Gaussian simulation, which is a simple, flexible and reasonably efficient method (Deutsch, 2002). Stochastic methods pursue the solution of an inverse problem with respect to a conditional probability distribution which embodies the full range of possible solutions that are consistent with the prior information while accounting for its particular uncertainties (Tarantola, 2005). Sequential Gaussian simulation is a kriging method, which incorporates variogram to calculate the variance of the data. In the upcoming sections, the principles and theory of variogram calculations, kriging, and sequential Gaussian simulation will be discussed.

Initial reservoir models that were used many decades ago, were generated using "layer cake" models. In those models, homogenous layers of reservoir were given constant permeability values that did not represent smaller scale flow paths (high permeability zones) and flow barriers (low permeability zones) within these layers. Although, layer cake models were
sufficient for primary reservoir calculations such as material balance, they could not provide
detailed information for modern flow simulation applications.

Next, the estimation methods that distributed petrophysical properties within reservoir
layers by means of interpolation of well data were implemented. Those methods mainly
include nearest neighbor, inverse distance weighting, and kriging. Among these, kriging uses
a priori information for spatial correlation, known as the variogram model. The method is
still widely used in the field of mining engineering, however, its limitations when applied to
reservoir modeling are well known. The spatial distribution of kriging estimates tend to be
too smooth (Journel and Deutsch, 1993) leading to the overestimation of smaller values and
the underestimation of larger values. This characteristic of kriging can cause severe issues
when the resulting model is used for flow simulation purposes of a reservoir.

4.1.2 Calculating experimental variogram

The variogram is defined as the variance of the difference between values at two locations.
In stochastic modeling, the variogram is defined as the expected value:

\[ 2\gamma(u) = E[Z(u) - Z(u + h)]^2, \]

(4.1)

where \( Z(u) \) and \( Z(u+h) \) are known values separated by lag distance \( h \) throughout the study
area. Variogram value is \( 2\gamma(h) \) and the value for semivariogram is half of the variogram and
equals \( \gamma(h) \). Experimentally, the semivariogram for lag distance \( h \) is described as the average
squared difference of data values nearly separated by \( h \):

\[ \hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(u_i) - Z(u_i + h)]^2, \]

(4.2)

where \( N(h) \) is the number of pairs for lag \( h \). Figure 4.1 represents a typical (semi)-variogram
with the main parameters defining a variogram (Trainor-Guittton, 2016):
In practice experimental variograms are hardly helpful, hence a variogram model is built using the sampled data and models provided for variograms. Variogram modeling provides the variogram value for all the lag distances, and also smooths the fluctuations in the data. Different kinds of variogram models consist of spherical, exponential, power and Gaussian models. They have different characteristics and implementations. The choice of variogram model depends on the purpose of study and the geological information of the study area. Figure 4.2 shows a modeled variogram, the smoothing effect can be observed on this model.

Variograms provide information on measures of spatial variability and spatial correlations (covariance). These two parameters are related to each other through the sill value of the variogram. This relation is shown in Equation 4.3

\[ \gamma(h) = \text{Cov}(0) - \text{Cov}(h) \]
\[ \text{Cov}(h) = \text{Cov}(0) - \gamma(h) \]  \hspace{1cm} (4.3)

Figure 4.1: An experimental variogram showing different components (Trainor-Guitton, 2016).
In the presence of heterogeneity in the data, it is more convenient to build the variogram model in the direction of heterogeneity. An important parameter in variogram modeling is the direction of variogram continuity. As stated by Deutsch (2002), in sedimentary structures continuity in the vertical direction is less than the horizontal direction. In addition, horizontal continuity depends on the direction of deposition and diagenetic alteration. The heterogeneity in the reservoir is defined by an angle that indicates the major and minor horizontal directions of continuity. The vertical direction is assumed to be perpendicular to the plane containing major and minor horizontal directions. The minor direction is perpendicular to the major direction in the horizontal plane.

A powerful tool in defining the direction of continuity of the data is the variogram map. A variogram map is a contour map of variogram surface and it displays the value of variance between each pair of data points separated by the lag distance $h$. The variance values are calculated in both $x$ and $y$ direction for the horizontal variogram. Building this map aids in understanding the direction of heterogeneity or the direction of most continuity in the
data. By using an anisotropic model for the variogram, larger kriging weights are given to
the samples in the direction of maximum continuity and smaller weights to samples located
in the direction of minimum continuity. (Isaaks and Srivastava, 1998)

Another important parameter regarding variogram modeling is the lag distance. If the
data are spaced regularly, the distance between the data points is assumed as the lag dis-
tance. This rule can be applied for vertical lag, because the well log data have regular
spacing. The horizontal lag distance mostly depends on well spacing. Since well spacing is
not usually regularly distributed, the average well spacing should be chosen. In addition,
another criterion in choosing horizontal lag distance is that the total distance, \( n_h \cdot h \), with
\( n_h \) being the number of lags, is about one half of the dimension of the area represented by
variogram. (Isaaks and Srivastava, 1998)

4.1.3 Methodology
4.1.3.1 Kriging

Kriging is a linear estimator defined as:

\[
Z^*(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i \cdot Z(\mathbf{u}_i),
\]

where \( \lambda_i \)'s are the kriging weights, \( Z(\mathbf{u}_i) \) are the data points at locations \( \mathbf{u}_i \) and \( Z^*(\mathbf{u}) \) is the
kriging estimated value. The corresponding estimation error as the difference between the
true value and the estimated value is given by:

\[
\epsilon_0 = Z(\mathbf{u}) - Z^*(\mathbf{u}),
\]

but the true value is an unknown to us, still we can estimate it statistically by evaluating
the expected value of the error (or residual). There are some basic rules in kriging estimates;
the expected value of the errors should be zero (unbiased), the variance of the errors should
be minimized, and the stationarity holds for the area of estimation (Isaaks and Srivastava,
1998). Stationarity implies that the mean and variance of the data are the same over the
modeling region. It is required for interpolation and extrapolation from data to unknown locations. Applying these conditions, we can solve for the kriging weights and obtain the estimated value. Combining and re-arranging the Equations 4.4 and 4.5, the error is obtained as:

\[ \epsilon_0 = Z(u) - \sum_{i=1}^{n} \lambda_i \cdot Z(u_i) \quad , \quad (4.6) \]

By calculating the expected value of both sides of the above equation, we obtain:

\[ E[\epsilon_0] = E[Z(u)] - \sum_{i=1}^{n} \lambda_i \cdot E[Z(u_i)] \quad , \quad (4.7) \]

Assuming errors to be unbiased \( E[\epsilon_0] = 0 \), the Equation 4.7 is equal to zero, and considering stationarity \( E[Z(u_i)] = E[Z(u)] = E[Z] \), it will yield to:

\[ E[\epsilon_0] = 0 = E[Z] - E[Z] \sum_{i=1}^{n} \lambda_i \]

\[ E[Z] = E[Z] \sum_{i=1}^{n} \lambda_i \quad , \quad (4.8) \]

\[ \sum_{i=1}^{n} \lambda_i = 1 \quad . \]

As mentioned previously, the kriging estimate attempts to produce a set of estimations with minimum error variance. This will lead us to derive the kriging equations as shown below:

\[ \sigma^2_{\epsilon} = \text{Var}[Z(u)] - \sum_{i=1}^{n} \lambda_i \cdot Z(u_i) \quad , \quad (4.9) \]

From statistics we know the variance of difference between two variables is defined as:

\[ \text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X, Y) \quad . \quad (4.10) \]
Applying this relation in Equation 4.9, we obtain:

\[
\sigma^2 = \text{Var}[Z(\mathbf{u})] + \text{Var}\left[\sum_{i=1}^{n} \lambda_i \cdot Z(\mathbf{u}_i)\right] - 2\text{Cov}\left[Z(\mathbf{u}), \sum_{i=1}^{n} \lambda_i \cdot Z(\mathbf{u}_i)\right].
\] (4.11)

If we write the above Equation in the covariance format, we get:

\[
\sigma^2 = \text{Cov}(\mathbf{u}, \mathbf{u}) + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \text{Cov}(\mathbf{u}_i, \mathbf{u}_j) - 2 \sum_{i=1}^{n} \lambda_i \text{Cov}(\mathbf{u}, \mathbf{u}_i).
\] (4.12)

where \(\text{Cov}(\mathbf{u}, \mathbf{u})\) is the variance at the location of estimation, \(\text{Cov}(\mathbf{u}_i, \mathbf{u}_j)\) is data to data covariance matrix, and \(\text{Cov}(\mathbf{u}, \mathbf{u}_i)\) is data to the unknown covariance. In order to minimize the variance, the first derivative of the variance with respect to weights should be set equal to zero:

\[
\frac{\partial \sigma^2}{\partial \lambda_i} = 2 \sum_{j=1}^{n} \lambda_j \text{Cov}(\mathbf{u}_i, \mathbf{u}_j) - 2 \text{Cov}(\mathbf{u}, \mathbf{u}_i) = 0
\] (4.13)

\[
\sum_{j=1}^{n} \lambda_j \text{Cov}(\mathbf{u}_i, \mathbf{u}_j) = \text{Cov}(\mathbf{u}, \mathbf{u}_i),
\]

substituting Equation 4.13 into Equation 4.12, estimation of error variance would be:

\[
\sigma^2 = \text{Cov}(\mathbf{u}, \mathbf{u}) - \sum_{i=1}^{n} \lambda_i \text{Cov}(\mathbf{u}_i, \mathbf{u}).
\] (4.14)

Borrowing the definition and calculation from the variogram model;

\[
\text{Cov}(\mathbf{h}) = \text{Cov}(0) - \gamma(\mathbf{h}),
\] (4.15)

and inserting in the result of Equation4.13, the final kriging estimator would be obtained in the form of:

\[
\sum_{j=1}^{n} \lambda_j \text{Cov}[\text{Cov}(0) - \gamma(\mathbf{u}_i, \mathbf{u}_j)] = \text{Cov}(0) - \gamma(\mathbf{u}, \mathbf{u}_i),
\] (4.16)

The system of equations for ordinary kriging (OK) is provided in Equation 4.17 and can be solved directly. The benefit of ordinary kriging is that the values do not need to be centered around zero and there is no need to know the mean beforehand. The parameter \(\mu_0\) is the Lagrange parameter, which is introduced to equalize the number of parameters and
unknowns (Isaaks and Srivastava, 1998).

$$\sum_{i=1}^{n} \lambda_i Cov_{i,j} + \mu_0 = Cov_{i,0}$$

$$\sum_{i=1}^{n} \lambda_i = 1,$$

\[ (4.17) \]

4.1.3.2 Sequential Gaussian simulation

Simulation and more specifically sequential Gaussian simulation methods were invented to overcome the smoothing problem of kriging. Another disadvantage of kriging methods is that its estimates are locally accurate, and not globally. This can be favorable to mining industry ore reserve estimates, because the local estimation of particular blocks is required. However, in flow simulation problems where, representation of continuity and global heterogeneity of the subsurface is of importance, building a model that provides the global or structural accuracy is crucial (Trainor-Guitton, 2016).

The main difference between kriging methods and sequential simulation methods is that the latter uses previously estimated values as data points and brings them into calculation. In addition to that, the sequential simulation methods use a cumulative distribution function (cdf) to draw the estimated values from. In summary, the algorithm is as follows:

1. Transform all data into standard Gaussian distribution
2. Choose a location by random means
3. Calculate the kriging estimation at that location to get $Z(u)$ and $\sigma_u^2$
4. Build the cdf using $Z(u)$ and $\sigma_u^2$
5. Draw a random value from the cdf
6. Assign the value to the location and keep it for the next cdf generation
7. Move to the next random location and repeat steps 3-7
8. Transform back all the values to the original data space (range)

Other simulation methods include Direct Sequential Simulation (DSS) and Sequential Indicator Simulation (SIS). In DSS, the data are not transformed into normal (Gaussian) space and the data are directly used in estimation. In SIS, an indicator-based approach is used and the cdf is constructed using a series of $K$ threshold values, discretizing the range of variation of values. This approach is based on the definition of conditional probability. An advantage of this method is that it allows the use of multiple indicator variograms to model different categorical indicators (litho-facies).

All the sequential simulation methods that have been developed so far, follow the same generic flowchart as introduced before, the only difference between the various methods is how the draws from the cdf are made.

4.2 Application of sequential Gaussian simulation in litho-facies modeling, Delhi Field, LA

4.2.1 Introduction to Delhi Field

Delhi Field is located 30 miles east of Monroe, Louisiana in Northeastern part of the state (Figure 4.3). The field is 15 miles long by 2-2.5 miles wide covering an area of 6200 acres. The oil originally in place (OOIP) is about 357 MMbbl and has an API gravity of 43.9. The field was discovered in 1944, primary and second recovery was about 14% and 40% respectively. The peak production of the field was 18000 barrels per day (BOPD). The $CO_2$ continuous flooding program started in November 2009. The first production associated with the injection began in March 2010 and the production rate since the tertiary recovery started has reached 4458 barrels BOPD.

The primary reservoir zone is Holt Bryant that lies at depths between 3000 and 3500 ft, and consists of the Early Cretaceous Paluxy Formation sandstones unconformably overlain by Late Cretaceous Tuscaloosa Formation sandstones. The Paluxy represents a progradational deltaic depositional environment with average porosity 30%, permeability of 1000 $mD$ and it ranges in thickness from 30 to 75 ft. The Tuscaloosa can be divided into lower and
upper units. The lower unit consists of transgressive marine deposits. The upper Tuscaloosa was deposited in a fluvial depositional environment with the presence of some marine processes. The average porosity and permeability of the Tuscaloosa units is 29% and 2700 mD and it range in thickness from 35 to 85 ft (Carvajal et al., 2014). The Tuscaloosa sandstones are amalgamated and exhibit high spatial variability. A regional stratigraphic column for Delhi Field is shown in Figure 4.4. The primary trapping mechanisms include a depositional pinchout between Paluxy and Tuscaloosa sediments and an upper erosional angular unconformity against the Monroe Gas Rock, which provides a seal for the Holt Bryant reservoir zone (Klepacky, 2012).

### 4.2.2 Available data

A large amount of data are available in Delhi Field, including seismic, VSP, well log and core data.
Figure 4.4: Regional lithostratigraphic column of the field. Paluxy and Tuscaloosa sediments compose Holt Bryant zone, which separates the boundary between Lower and Upper Cretaceous strata (Klepacki, 2012).
4.2.2.1 Seismic data

There are three seismic surveys over parts of Delhi Field. The baseline survey was shot in January 2008 and it is a 3D single-component survey, covering approximately 13.25 $mi^2$ in extent. Another 3D single-component survey was acquired over the eastern portion in March 2010. The second monitor, a 3D multicomponent survey, covering four $mi^2$ was acquired over the RCP study area in June 2010.

![Figure 4.5: Timeline of activities performed over Delhi Field.](image)

The third monitor survey was acquired over the RCP study area in August 2011. A relative timeline of different activities performed over Delhi Field is presented in Figure 4.5, and the locations of the seismic surveys are shown in Figure 4.6.

4.2.2.2 Well log data

There are approximately 68 wells drilled in the 1940’s and 1950’s that contain spontaneous potential (SP), shallow, medium, and deep resistivity and induction logs. There are 18 wells in the study area, drilled within the last 15 years, that have gamma ray (GR), resistivity, microresistivity, spontaneous potential (SP), density(RHOB), neutron porosity(NPHI), caliper, photoelectric potential (PE), magnetic resonance image (MRIL), X-tended range micro image (XRMI) logs, and some additional calculated curves such as density-porosity. Four wells have P-wave sonic logs, but only two have P- and S- wave sonic logs.

There is one cored well within the survey area; producer well 159-1, in which the cored interval is approximately 120 $ft$ long and covers the extent of the Tuscaloosa and Paluxy
Figure 4.6: Location of seismic surveys of Delhi Field. The first part of the study is performed over the entire field and the second part is focused on the survey acquired in Jan 2008.
Formations sandstones. From this cored interval, 58 thin sections were made for detailed interpretation. The cores were previously analyzed by RCP students, Silvis (2011) and Cavallini (2011), to determine reservoir properties such as porosity and permeability and the depositional environments of the Holt Bryant interval.

4.2.3 Litho-facies modeling

In order to do litho-facies modeling, first by using the well log information litho-facies clustering at the wells is defined. In this part of the research a simple clustering approach by observing the probability distribution of histogram of porosity and permeability is used. In the next chapter, a more advanced clustering method is implemented and a variety of well logs are incorporated.

Histograms of porosity and permeability logs (Figure 4.7) represent a bi-modal distribution that can be related to the presence of two major litho-facies in the reservoir interval. The two categories are defined as:

- Litho-facies 1: with porosities larger that 18% and permeabilities larger than 100 mD that provide better reservoir quality rocks
- Litho-facies 2: with the cut off values of porosities less than 18% and permeabilities less than 100 mD representing lower reservoir quality rocks

To distribute the litho-facies throughout the reservoir, using sequential Gaussian simulation variogram properties should be defined. I generated the variogram map of porosity data to define the direction of continuity in the reservoir. Figure 4.8 shows the variogram map of porosity for the Paluxy Formation. A directional trend of NE-SW and azimuth of 35 -40° can be recognized on the map. This trend has been applied in variogram simulation to populate the litho-facies.

By applying the cut off values for defining the litho-facies and the information from the variogram properties, I built the facies model for the field. Figure 4.9 shows the result of the simulation for the whole extent of the reservoir for Paluxy Formation.
(a) Histogram of porosity from the well logs of the entire field. The histogram represents a bi-modal distribution.

(b) Histogram of permeability from the well logs of the entire field. The histogram represents a bi-modal distribution.

Figure 4.7: The histograms of porosity and permeability from well logs of the entire Delhi Field.
Figure 4.8: Variogram map for porosity from Paluxy Formation. The direction of most continuity is defined as NE-SW with 35 -40° azimuth.
Figure 4.9: The litho-facies model for the Paluxy Formation. Areas with light blue color are the litho-facies 1 with higher reservoir quality and mostly sedimentary depositional environment of channels, point bars and crevasse splays and the dark blue ones are litho-facies 2, representing lower quality reservoir and mainly overbank deposits.
For the initial studies and obtaining an overall understanding of the distribution of facies in a very broad context using this model can be helpful. But when more detailed definition of the reservoir is needed for designing the secondary and tertiary EOR methods, the so-called facies model is insufficient in many ways: it is affected by the variogram properties (range and direction) and the footprint of the variogram is very obvious over the facies model. Also, variogram modeling is incapable of predicting sinusoidal and meandering features in a reservoir. In addition, conventional geostatistical simulation and variogram modeling does not allow the user to incorporate multiscale data. Considering these drawbacks, I decided to use multiple point statistics simulation methods to help overcome these limitations.
Considering the depositional and geological complexity present in the fluvial deltaic systems, such as Delhi Field, application of conventional geostatistics for reservoir modeling or conventional seismic inversion approaches will not obtain a detailed and accurate litho-facies and rock property model for the field. Therefore, incorporation of advanced geostatistical approaches that can capture the complex depositional features of the reservoir is of great importance.

Delhi Field is deposited as sequences of fluvial deltaic and barrier bar deposits providing a complex stratigraphy that cannot be modeled via conventional modeling approaches. A new modeling method in the form of inversion has been implemented in this field to capture the complex depositional features and provide a detailed definition for the reservoir. The method is a multiple point statistics (MPS) algorithm, which incorporates pattern recognition and reconstruction methods along with seismic data in an inversion approach.

5.1 SIMPAT algorithm

Simulation with patterns (SIMPAT) is a pattern-based multiple point statistics algorithm that incorporates pattern reconstruction concepts and approaches. The algorithm was proposed by Arpat (2005), modified by Gonzalez (2006) to include seismic data in the forward modeling scheme. I modified the algorithm by incorporating image quilting technique (Efros and Freeman, 2001) to provide a better control on pattern reconstruction and reduce the number of random location visits.
### 5.1.1 Image quilting as a pattern reconstruction approach

In image reconstruction and computer vision applications, Efros and Freeman (2001) introduced an algorithm to improve the previously used pixel-based methods of synthesis algorithms. As a motivation to this new approach, they mentioned that in pixel-based algorithm a considerable amount of search is performed for pixels that their values are already determined through synthesizing. Therefore, the unit of synthesis should be larger than a single pixel, and a patch would be more suitable. They suggested that the synthesis process would be more similar to putting together pieces of a puzzle, and quilting them.

Quilting algorithm that they proposed was to choose a user-defined $B_i$ square block from the texture to be synthesized (input texture). Their initial attempt was to tie this block with random blocks chosen from the input texture, the results were reasonable to some extent, yet the borders of the patterns were relatively obvious in the final image (Figure 5.1(a)). The second approach, instead of choosing a random block, was to search for blocks that with some similarity measures agree with the neighboring block (Figure 5.1(b)). The resulting texture was more similar to the input one and much improved compared to the random choice of blocks. The last approach was to assume an irregular boundary for the blocks and search for the boundary, which provides the minimum error in the overlapping area (Figure 5.1(c)). The resulting image has less obvious edges at the boundaries of the blocks.

They summarized the image quilting algorithm as follows:

1. Scan the image in a raster order with a pre-defined block size.

2. At every location search for the blocks that match the overlapping criteria to some user-defined criterion.

3. Calculate the error at the overlapping surface and search for the minimum error path along this surface and make that the new boundary. Paste the block with the minimum error onto the image.

4. Repeat the steps.
Figure 5.1: Three different approaches of the image quilting method. Figure 5.1(a) shows the random choice of blocks to reconstruct the image, Figure 5.1(b) is when the overlapping images are implemented as the criteria for image reconstruction, and Figure 5.1(c) is the resulting image when the boundaries of the blocks are modified to provide the least error of the overlapping area. (Efros and Freeman, 2001)
Size of the block and the overlapping area are the user-defined parameters in the algorithm. The size of the block should be large enough to capture the feature properties of the image, and the overlapping area should provide enough control on choosing the neighboring block.

I have adapted this methodology in the SIMPAT algorithm with some modifications. Instead of modifying the boundary to provide zero distance between the two overlapping blocks, I have kept the boundary constant (fixed block size and shape), and the search algorithm searches for the patterns that provide zero distance between the reference block and the patches within the image across the overlapping area. The details of the algorithm will be provided later in this chapter.

5.2 Theory

The solutions of an inverse problem are a set of model parameter realizations that when forward modeled to generate the synthetic data, match the measured data to some predefined measure. Any inversion problem can be described as an inference problem where the prior knowledge is updated to account for the observations (Tarantola, 2005):

\[ \sigma_M(m) = c\gamma_M(m)\gamma_D(f(m)) \]  

(5.1)

where \( \sigma_M(m) \) is the posterior probability density, \( c \) is a normalization constant, and \( \gamma_M(m) \) is the prior probability density both defined in the model space (denoted by index \( M \)), and \( \gamma_D(f(m)) \) is the likelihood function, which is a measure of the fit of the model \( m \) to the data. The model parameters can be divided into two subspaces: \( m = (m_g, m_e) \). \( m_g \) refers to the parameters defining the reservoir properties (known as group properties) of interest and \( m_e \) refers to the elastic properties. The prior probability density is a joint distribution of group and elastic properties, hence by using the chain rule of conditional probability, the prior probability density function (pdf) can be written as:
\[
\gamma_M(m_g, m_e) = \gamma_M(m_e|m_g)\gamma_M(m_g), \tag{5.2}
\]

and the posterior probability density function can be expressed as:

\[
\sigma_M(m_g, m_e) = c\gamma_M(m_e|m_g)\gamma_M(m_g)\gamma_D(f(m_g, m_e)) \tag{5.3}
\]

where \(\gamma_M(m_e|m_g)\) is the conditional distribution of elastic properties (e.g. velocity and density) to reservoir properties (e.g. lithology), and \(\gamma_M(m_g)\) is the prior pdf of reservoir properties. Equation 5.3 provides the core structure of this stochastic inversion. In this methodology posterior pdf is not sampled to obtain the final solution, instead multiple realizations of the solution space are generated and the maximum occurrence of the group properties are calculated.

The proposed workflow for this methodology is presented in Figure 5.2, different data types are integrated to build the litho-facies model in a stochastic inversion approach.

![Figure 5.2: Summary of the workflow proposed for the MPS litho-facies modeling.](image)

Figure 5.2: Summary of the workflow proposed for the MPS litho-facies modeling.
5.3 Rock physics and litho-facies clustering

A pre-processing step in this inversion approach is to define the rock physics properties of the litho-facies. For this, the initial step is to define the litho-facies at the well location using log data, and then incorporate the rock physics in the litho-facies clustering. Before performing the clustering, some conditioning should be applied on the well log data.

5.3.1 Well log conditioning and normalization

Well log normalization basically is to find an interval that has been the least affected by the reservoir fluids and the drilling activities (reference interval), and then normalize the well log measurements in other intervals to the reference interval. The reference interval shows similar well log response in different wells. Usually an interval of clean shale is chosen as the reference interval. Here in Delhi Field, there is a thick interval of marine shale that can be assigned as the reference zone. Figure 5.3 shows this interval in multiple wells in the field.

![Figure 5.3: The shale interval used for well log normalization.](image)
To ensure that this interval provides the required characteristics, histograms of deep resistivity (RT90) for all the wells both in the reference zone and the total length of the well logs are plotted (Figure 5.4). These histograms obviously show that the Midway Shale represents the same response of resistivity in all the wells, therefore it can be an appropriate choice for the normalization purposes. The reason for choosing RT90 log as an indicator is that this log penetrates deep into the formation and measures the primary and un-invaded characteristics of the grains and is not affected by the composition or configuration of them.

![Histogram of deep resistivity (RT90) for the total length of the well logs and the reference zone of Midway Shale. Different wells are shown by different colors](image)

(a) RT90 for the total length of all the wells

(b) RT90 for the reference zone of Midway Shale

Figure 5.4: Histogram of deep resistivity (RT90) for the total length of the well logs and the reference zone of Midway Shale. Different wells are shown by different colors

The normalization process is performed on all other well logs and the results for gamma ray (GR) and bulk density (RHOB) before and after normalization by the Midway Shale are presented in Figure 5.5 and Figure 5.6.
Figure 5.5: Histogram of gamma ray of all the wells. The top diagram shows the plots before normalization, and the bottom diagram shows the result of normalization to the reference zone (Midway Shale).
Figure 5.6: Histogram of density of all the wells. The top diagram shows the plots before normalization, and the bottom diagram shows the result of normalization to the reference zone (Midway Shale).
Obviously, the normalization process has placed all the well log measurements within a consistent range for all the wells. The process is performed on all other well logs used in litho-facies clustering of the wells, which include neutron porosity (NPHI), photoelectric (PE), and GR and RHOB.

5.3.2 Litho-facies clustering

The normalized well logs are used for clustering the litho-facies. The clustering approach is K-mean clustering, which primarily performs principal component analysis on the data to transform them onto the independent axes that represent the highest variance in the data, and to ensure the data are functionally independent. Then the data are assigned to different clusters in a way that the inner distance between the data points in a cluster is minimized and the distance between data points of different clusters are maximized (Kung, 2014). A more detailed explanation of K-mean clustering approach is provided in Appendix A.

To perform the clustering, Heterogeneous Rock Analysis (HRA) Clustering module of Techlog® software is used, and I have tried different numbers of clusters, six, seven, eight, nine, and ten to find the optimum cluster number. There are different methods for choosing the optimum cluster number, the ones being used in this work are the silhouette plot and the fall-off plot. Figure 5.7 shows the silhouette plot for seven clusters. Each knife-blade shape represents a cluster of data points. The vertical axis is the distance ratio between the data points within a cluster and the data points of a neighboring cluster. The ideal match would be +1, and the mismatch is -1 which, in practice never happens. A negative value is an indicator that the data points can be assigned to an adjacent cluster and an additional cluster may be required.

The other indicator is the fall-off plot. A criterion to choose the appropriate number of clusters, in multiple cluster trials is to search for the minimum sum of square of distances (SSD) between the cluster centroid and the data points in a cluster. Therefore, the Euclidean distance can be used as a metric to select the optimal number of clusters (Kung, 2014). The Euclidean distance decreases continuously as the number of iteration increases, and flattens.
out when the minimum distance of the centroid and the data points is achieved. But, if the minimum distance is reached too simply, that is, for the most part the plot is flat it means that the solution is determined easily and is very unique. In other words, it means that the distribution of inputs per class is too broad and possibly more classes should be added to the data points (Figure 5.8 is an example of such situation).

Figure 5.8: Fall-off plot for the result of K-mean clustering for seven clusters.

To find the optimal number of clusters, I have compared both the silhouette and the fall-off plots for multiple cluster numbers. The results are shown in Figure 5.9 and Figure 5.10. Comparing these plots with each other and considering the theory mentioned on analyzing
Figure 5.9: Silhouette plots for five different numbers of litho-facies clusters at the well location.
Figure 5.10: Fall-off plots for five different numbers of litho-facies clusters at the well location.
these types of plots, nine clusters are chosen as the optimal litho-facies clustering. It should be mentioned that nine clusters cover an interval much larger than the interval of interest, and this interval includes five cluster of litho-facies. In choosing the optimal number of clusters I considered the consistency of the zonation for the reservoir interval in different wells. Figure 5.11 shows the result of clustering for three of the wells and highlighted area is the reservoir zone.

Figure 5.11: Litho-facies clustering over three wells in the field. The highlighted section demonstrates the clustering for the reservoir interval.

The litho-facies clustering from the well logs is then updated using the elastic properties of P-wave velocity and density. The main consideration is the distribution of elastic properties and the acoustic impedance contrast for consecutive layers. As seen in the middle column of Figure 5.12 There are impedance contrasts between zones that have been assigned to similar clusters, specifically in the reservoir interval. Therefore, the litho-facies clustering is updated, and upcaled to a vertical interval of ten feet. The ultimate result of litho-facies
clustering at well 159-1 is shown on the far right column of Figure 5.12.

Figure 5.12: Litho-facies classification of well 159-1, the upscaled clustering and the modified litho-facies clustering based on the elastic properties.

5.4 Training image and pattern database generation

5.4.1 Training image

For modeling and interpretation of the subsurface, earth scientists need to have an image of it. The term image here is used as a general representation of processes occurred overtime on the surface and in the subsurface (Mariethoz and Caers, 2015).

As discussed, geostatistical methods generate realizations of the subsurface in the form of an image, which mimic the real world. In conventional geostatistical approaches, this goal is obtained by associating the statistical properties with the subsurface, and then building
the image using the statistical constraints. However, the subsurface represents a complex combination of random phenomena and physical laws, and representing the subsurface only with the statistics of it is a simplification of its complex nature. This complexity is described by terms such as “non-stationarity”, “non-Gaussianity”, and “non-heteroscedasticity” (Mariethoz and Caers, 2015) All these terms indicate that conventional statistical descriptions usually are not enough to apply to the subsurface.

As a solution, training images are introduced to mimic the reservoir regarding the shape, size, and any other characteristics and complexity of the depositional features. There is still a down-side to it, if the training image contains some degree of complexity to better represent the reservoir, there may not be enough replicates of the multiple-point statistics. Therefore, there is a trade-off between statistical representation and real-world representation. Using training images provides a framework to include subjectivity in the subsurface models. However, subjectivity makes it very critical to choose the model parameters for the training image, and consequently the multiple-point statistics approaches.

The training image must be representative of the expected geology, and consistent with the litho-facies defined at the well location. It has to be rich enough to contain all (or most) possible geologic patterns expected in the study area. Training images are conceptual 3D images from the reservoir, considering the depositional and geomorphological data from the reservoir. They are not directly conditioned to any data type, but are generated based on the information from the reservoir of interest. As mentioned by Mariethoz and Caers (2015), training images can be constructed based on physical principles or statistical methods or by using empirical rules, and can be categorized as:

- Object-based methods
- Process-based methods
- Process-mimicking models
Object-based methods or Boolean methods are among stochastic simulation methods that are widely used in geological modeling. The concept of this method is to define 2D or 3D features that populate a model using a set of rules, which define the specification of the objects along with the interaction between the objects. Conditioning to the data points, and non-stationarity are straightforward to deal with in this approach. In some cases, when complex structures are present in the study area, the objects may not provide enough flexibility to represent the reality. In such cases, combining object-based methods and multiple-point statistics algorithms, can address the issue. By using object-based methods, the conceptual model is built and then used as the training image of the MPS algorithms.

Process-based methods are similar to the object-based methods, except that the geological and depositional laws such as dynamics of river flooding, sedimentation, sea level changes and other similar processes, can be simulated. This is done through defining the time-space in a discretized gridded domain, assigning the corresponding equations and models of each process and solving them. The process is similar to modeling and simulation of hydrocarbon production from a reservoir using flow equations. These processes that are simulated by computers, solve the equations that define activities such as sediment deposition and erosion by time-dependent flow equations. The solutions require definition of boundary condition of rock and fluid properties (Mariethoz and Caers, 2015)

Process-mimicking methods are similar to process-based models, but unlike them, are not very CPU demanding. They are not physically realistic, but they follow the geometries of the depositional environment. They mainly generate the end result of the process-based methods but in a much smaller amount of time.

To build the training image for Delhi Field, I chose an object-based method, since it is easy to implement, and provides enough control over the parameters of the depositional environment, regarding shape, size, position in the 3D cube, and position of different objects relating to each other. Multiple sources of information and data type are integrated for generation of the training image for this field:
Core and thin section studies (Cavallini, 2011)

Sequence stratigraphic (Klepacki, 2012)

Cluster facies analysis

Sedimentary Analog Database (SAND, 2015)

According to the core description of litho-facies for the Paluxy performed by Cavallini (2011), most of the litho-facies agree with the interpretation of a near-shoreline depositional system that alters with changes in relative sea level and local uplift episodes. Also petrographic analysis done by Cavallini (2011), interprets the Paluxy as a shelf deposit pro-delta. Upward coarsening in the grain size from very fine- to fine-grained sandstone, followed by a fining upward to very fine-grained sediment, the relative increase in quartz content, the sand-sized grains and the ripple and cross-bedded structures are indicative of upper delta front or delta plain. Finally, an upward fining sequence with a corresponding decrease in quartz content is observed in the thin sections, which are interpreted as a series of delta plain deposits. Sequence stratigraphic studies done by Silvis (2011), shows that the Paluxy was deposited as a fluvial deltaic system that prograded basinward during a relative sea-level highstand to regression. Klepacki (2012) has defined Paluxy litho-facies 2 as reservoir quality deltaic sandstone, which is texturally immature. It is interpreted as a delta front distributary system depositional environment.

The Tuscaloosa is made of individual sandstone intervals that are interconnected but are discontinuous. The sandstone has been deposited in two different depositional environment of barrier bar and fluvial. Klepacki (2012) has distinguished two main cluster litho-facies for the Tuscaloosa. Litho-facies 1 based on seismic mapping is aligned in the paleo-shoreline direction. Thin section analysis shows that it is a mature litho-facies, which is an indicative of marine processes. It is interpreted as shore-face / beach/ barrier bar depositional environment. Litho-facies 4 is analyzed as mixed textural and compositional maturity, possibly due to transport of both recycled marine strata and terrigenous sediments. This litho-facies
is interpreted as a regressive phase, fluvial system deposited during overall transgression as distributary channels. Different litho-facies defined through core and stratigraphic studies, are aligned with the elastic properties distribution in the well logs, and are implemented in building the training image.

Based on the available information on the depositional environment and the estimates on the width and thickness of each sedimentary unit, I have built a 3D training image for Delhi Field using Stanford Geostatistical Modeling Software (SGeMS). The dimension of the training image is $200 \times 100 \times 100$ grids in X-Y-Z directions respectively, and the size of each grid cell is $150 \times 150 \times 10$ feet$^3$. The number of the grid cells and the size of each grid is selected in a way to approximately obtain the actual size of the reservoir of interest. The areal extent of the field is $15$ mi$^2$. Size of the training image in the vertical direction is almost equal to the reservoir interval and two formation above the reservoir. The thickness of the reservoir is 200-250 ft and the thickness of the formations above it, Clayton Chalk and Midway Shale, is 700-800 ft, which leads to 1000 ft as size of the training image in the vertical direction. Figure 5.13 shows the initial training image generated for the field. Different colors represent different litho-facies present in the reservoir interval and the top and bottom formations.

5.4.2 Pattern database generation

The multiple-point statistics algorithm that is used here, SIMPAT, generates the realizations based on patterns obtained from the training image. Therefore, we need to have the pattern database. The method for generating the pattern database is similar to what described in Section 3.2.2. The similar approach is implemented on 2D and 3D training images. In case of 3D training images, a cube or a rectangular box is used instead of the square.

The inversion algorithm is performed on an inline seismic section. Therefore, the three-dimensional training image generated before should be sub-sampled to represent two-dimensional planes from the 3D cube of the training image and consequently the subsurface. The 2D
Figure 5.13: The initial training image for the Delhi Field, constructed based on multi scale data and integrated studies over the field. Different colors represent different litho-facies in the reservoir interval and the overlying and underlying formations.
planes are the cross sections in the direction parallel to the inline of interest, which is North-South. Regarding the size of the three-dimensional training image, which is $200 \times 100 \times 100$ grids, and the direction to generate the cross section, there will be 200 planes of $100 \times 100$. These 2D planes represent the reservoir of interest in two-dimensional space. Figure 5.14 shows some cross sections from the training image.

![Figure 5.14: Three examples of cross sections built from the training image for Delhi Field. The cross sections are made parallel to the seismic inline direction. The sections are color coded similar to Figure 5.13](image)

The next step in generating the pattern database is to scan the 2D planes of the training image with a pre-defined template, as explained in Section 3.2.2. In this case study, I started the simulation with a $5 \times 5$ template, and considering the size of the training image and the template, there are $19996 \times 96$ patterns in the pattern database that are used for building the SIMPAT realizations. Figure 5.15 shows some examples of patterns generated by a $5 \times 5$ template from the training image in Figure 5.13.
5.5 Stochastic inversion

The inversion is performed to fill out the simulation grid. Figure 5.16 shows the simulation grid with the nodes at the well location filled with the information from the litho-facies clustering and the rock physics. The extent of this grid in the vertical direction is equal to the length of zones of interest at the well location and the horizontal extent is equal to the length of the seismic inline. The size of the nodes in the simulation grid are the same as those in the training image, $150 \times 10$ ft.

The main idea is to distribute the information from the well location in a stochastic way, implementing a pattern reconstruction approach and utilization of seismic data. The stochastic inversion algorithm begins at the well location by searching the pattern database for the patterns similar to the ones at the well location. To find the similar pattern a distance function is defined and the patterns with the smallest distance are chosen as similar ones. The proposed distance function is Manhattan distance, which calculates the pairwise difference between the nodes in two patterns (Section 3.2). The reason to implement Manhattan distance is, when dealing with categorical variables (litho-facies in this case), the
variables are evaluated as 0 or 1 and the intensity of the difference between the variables is not of importance and interest. Therefore, distance functions such as $L-2$ norm or least square errors are not favorable, instead $L-1$ norm can provide a reasonable meaning of the distance between the categorical variables and the corresponding patterns.

The search algorithm begins at the top of the litho-facies log at the well, however; it can start from any other position of the log, while it is multiplies of the template size (here five initially). The search criteria at the well location is bound to the patterns that the distance between their third column and the reference pattern at the well location is zero. If not enough patterns by a user-defined limit are provided at this stage of the pattern search, then the criteria is relaxed and the search continues. Considering the number of SIMPAT realizations that we want to generate, that number of random patterns from the search results are selected. In this research, based on the available computation power, ten realizations are considered. Figure 5.17 shows some of the realizations at the well location.
The inversion algorithm continues by choosing the best SIMPAT realization among all the realizations by generating synthetic seismic and forward modeling.

5.5.1 Forward modeling

The forward modeling and generating the synthetic seismic is performed through a stochastic approach as well, and then accept/reject is carried out on multiple realizations of the synthetic seismograms.

5.5.1.1 Bi-variate joint distribution

The seismic data incorporated in the forward modeling is post-stack data, therefore, the parameters required are P-wave velocity and density. The parameters are obtained from the joint distribution of the values from the well logs. Knowing the distributions from the well logs and the means and the standard deviations, bi-variate joint distribution of P-wave velocity and density are calculated. The values are drawn conditioned to each litho-facies at the well location. Figure 5.18 shows the cross plot of P-wave velocity and density from the well log data color-coded by the litho-facies and Figure 5.19 shows 300 values drawn from the distribution of the well logs, knowing mean, variance, and the covariance of the data.

For each of the litho-facies SIMPAT realizations, I draw 15 random values from the joint distribution to build the pseudo-logs of velocity and density.
Figure 5.18: Distribution of P-wave velocity and density from the well logs.

Figure 5.19: Bi-variate distribution of P-wave velocity and density from the parameters obtained from the well logs. 100 values are drawn for each litho-facies.
5.5.1.2 Seismic data preparation

As mentioned, the stochastic inversion is performed on an inline seismic section, which is shown in Figure 5.20(a). Selection of a reference horizon to assign to the top of the synthetic seismograms is important. Considering the reference litho-facies at the well location, the top of the Midway Shale is considered the reference horizon. This horizon is time shifted to represent a datum for the comparison of the synthetic seismograms and the seismic traces. Figure 5.20 is the time shifted seismic section.

Figure 5.20: The seismic inline 64 incorporated in stochastic inversion, and the time shift implemented on the section. The reference horizon is marked in blue.

(a) The seismic inline of interest before time shift

(b) Seismic inline of interest after time shift to the top of Midway Shale
5.5.1.3 Wavelet extraction

To perform the forward modeling, the wavelet should be extracted from the data. Sonic (P-wave slowness) and density logs from well 159-1 are used to generate the synthetics at the well location, and by doing the well-tie the wavelet is extracted from the seismic data. The wavelet extraction procedure is performed in Jason software. The well-tie and the extracted wavelet is shown in the Figure 5.21.

![Figure 5.21: Well tie and the wavelet extraction for well 159-1.](image)

Rather than the wavelet extracted from the data using the commercial software, I tried two other methods to extract the wavelet, conjugate gradient and the least square error. The synthetic seismograms generated by convolving the reflection coefficient and each of the wavelets, showed lower correlation coefficient with the seismic data compared to the synthetic seismograms obtained by the initial wavelet. Therefore, I used the initial wavelet extracted using the commercial software for the forward modeling purpose.
5.5.1.4 Synthetic generation and selection of the best SIMPAT realization

For every litho-facies SIMPAT realization, I have generated 15 realizations of P-wave velocity and density pseudo-logs. The acoustic impedance and the reflection coefficient is created for each of these pseudo-logs. Equation 5.4 represents the acoustic impedance, and Equation 5.5 represents the reflection coefficient for the P-wave normal incidence. The available seismic data for this research are stacked data, and I assume that the stacked data are equivalent to normal incidence P-wave reflection.

$$Z = \rho \cdot V_P$$  \hspace{1cm} (5.4)

where $\rho$ represents density of the media and $V_P$ represents the P-wave velocity in the media.

$$RC = \frac{\rho_{i+1} \cdot V_{P_{i+1}} - \rho_i \cdot V_{Pi}}{\rho_{i+1} \cdot V_{P_{i+1}} + \rho_i \cdot V_{Pi}}$$  \hspace{1cm} (5.5)

where the parameters are the same as those for the Equation 5.4, and the $i + 1$ index refers to the properties of the bottom layer and the $i$ index refers to the properties of the top layer. By calculating the reflectivity series and convolving them with the extracted wavelet, multiple realizations of the synthetic seismograms are generated for each of the litho-facies SIMPAT realization. Figure 5.22 shows one of the SIMPAT realizations at the well with the corresponding synthetic seismograms.

Selecting the best litho-facies SIMPAT realization, is performed in two steps, first the best realization of pseudo-logs of velocity and density are chosen among multiple realizations. Cross correlation of the synthetic seismogram and the recorded seismic is calculated and the synthetic that provides the highest cross correlation is assigned as corresponding elastic properties (P-wave velocity and density). Next the SIMPAT realization, which provides the highest cross correlation over the sum of the synthetic seismograms, is selected as the best match for the litho-facies SIMPAT realization.

The stochastic inversion algorithm continues by moving to the next location on the simulation grid (Figure 5.16). In moving to the next location, image quilting algorithm that was explained in Section 5.1.1 is incorporated. When moving to the right of the well
Figure 5.22: A litho-facies SIMPAT realization at the well location, and the corresponding synthetic seismograms realizations generated using pseudo-logs of velocity and density. The vertical lines mark the location of the well.
location, the two right columns of the accepted SIMPAT are used as the overlapping area to choose the next realizations. The two columns are assigned as the controlling criteria for choosing the patterns from the database. Therefore, the distance function defined for this search is the patterns that obtain vectors of zero at the distance between the two right columns of the accepted SIMPAT realization and the first two columns of the patterns in the pattern database. Figure 5.23 is a representation of the image quilting to the right of the well location.

![Image Quilting](image)

Figure 5.23: The best litho-facies SIMPAT realization at the well location, and the neighboring realization to the right of the well, implementing image quilting concept and maximum cross correlation between the synthetic seismograms and the seismic data. The vertical lines mark the location of the well.

The same approach is implemented when moving to the left of the well location. The only difference is that the distance function to minimize is defined as the distance between the two left columns of the accepted realization and the last two columns (four and five) of
the patterns in the pattern database. Figure 5.24 shows the image quilting method when moving to the left of the well location.

Figure 5.24: The best litho-facies SIMPAT realization at the well location, and the neighboring realization to the left of the well, implementing image quilting concept and maximum cross correlation between the synthetic seismograms and the seismic data. The vertical lines mark the location of the well.

The same as the well location, the best litho-facies SIMPAT realization is selected by drawing random values from the bi-variate distribution, performing the forward modeling to calculate the synthetics and choosing the SIMPAT realization with the highest cross correlation as the accepted litho-facies SIMPAT realization for that location and pasting it onto the simulation grid.

The algorithm continues and all the locations on the simulation grid are visited on a unilateral path and the simulation grid is filled out with the litho-facies realization that have provided the highest cross correlation. Figure 5.25 shows the final result of a complete SIMPAT algorithm.
Due to the stochastic nature of this approach, multiple realizations of the SIMPAT lithofacies are generated, and some of them are shown in Figure 5.26.

Multiple equi-probable realizations of the stochastic inversion algorithm are generated and each of them represent a scenario for the litho-facies distribution for the corresponding seismic section. To obtain one final figure from the reservoir (on the specific seismic section), the maximum probability map of all the realizations is generated, which presents the litho-facies with the maximum occurrence at each node on the simulation grid. Figure 5.27 shows the maximum probability map of ten realizations of litho-facies.

In this work, ten realizations of litho-facies are generated for every case study. The number of realizations is chosen due to the available computation power, in case of more powerful computers that can perform the inversion in a faster speed, more realizations can be generated that will lead to a better understanding of the litho-facies models.

5.5.1.5 Error estimation of the inversion results

The stochastic inversion algorithm is mainly focused on minimizing the error between the patterns selected from the pattern database and the patterns in the simulation grid. But,
Figure 5.26: Multiple realizations of the litho-facies as the results of the stochastic inversion.

Figure 5.27: Maximum probability map of multiple realizations generated via stochastic inversion on the seismic section.
it can be considered from another viewpoint, which is the error between the recorded and the synthetic seismic. As an example of one of the realizations, I have normalized the best synthetic traces at the end of the inversion algorithm, and normalized the corresponding recorded traces. Then, the difference between the synthetic and the recorded seismic as a conventional subtraction is calculated. The result is shown as a section in Figure 5.28.

![Figure 5.28: Difference between the recorded and the synthetic seismic traces, in one of the inversion realizations. The light yellow and the navy blue areas represent the maximum errors. The velocity and density models can be improved through iteration. The color bar represents the normalized errors between the recorded and the synthetic traces.](image)

Obviously, there are some mismatches as shown on the difference section, mainly in light yellow and navy blue areas. Different approaches can be considered at this step, which mainly involves matching the synthetic and the recorded seismic. However, the litho-facies corresponding to the synthetic section remains constant, because it has been obtained through image quilting error minimization. The approach would be to improve the velocity and density models, and if the initial velocity and density models are changed out of a user-defined range, the litho-facies realization associated with them will be disregarded as an inversion result.
5.5.2 Sensitivity to training image

To study the sensitivity of the algorithm to the training image, another scenario for the training image is considered. In the initial training image, the channels within the Paluxy were considered as meandering features, that as a result appeared as lenticular features on the inverted section. In this part, another training image is incorporated where the Paluxy channels represent less sinusoidal shapes. Figure 5.29 shows this new training image.

![Figure 5.29: The new training image for Delhi Field with modified channels for Paluxy formation](image)

The stochastic inversion algorithm is run for the new training image and multiple realizations are generated. Figure 5.30 shows some of these realizations. The maximum probability map of the realizations is shown in Figure 5.31

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Figure 5.30: Multiple realizations of the litho-facies as the results of stochastic inversion for the modified training image.

Figure 5.31: Maximum probability map of multiple realizations generated via stochastic inversion on the seismic section for the modified training image.
This inversion result shows that the algorithm is sensitive to the training image, and in the case where the channels are more continuous in the direction of simulation, the resulting inverted section represents continuous features.

5.5.3 Sensitivity to template size

Template size is a parameter that the user has control over. As mentioned in Section 5.1.1, the size of the block for reconstructing the image is a user-defined parameter. However, Arpat (2005); Honarkhah (2011) have proposed a method to find out the optimum template size based on the entropy of the patterns with different template sizes. Their method is to calculate the Shannon’s entropy as defined in:

\[
H = -\sum_{i=1}^{K} p_i \log(p_i)
\]

where \(K\) is the number of possible outcomes for the random variable (litho-facies present in every pattern in this case), and \(p_i\) is the probability mass function. High entropy relates to randomness and as the template size increases the entropy of the patterns of a training image increases. As the template size reaches the optimum value, the entropy becomes relatively constant.

The algorithm for selecting the optimal size of the template starts with scanning the training image with different template sizes and calculating the mean entropy value of all the patterns in the training image for the every template size. Possible sizes for a square template can be defined as:

\[
\mathcal{T} = \{(3 \times 3), (5 \times 5), (7 \times 7), ..., (n' \times n')\}
\]

and \(n' = 0.4 \times \min(N_x, N_y)\), where \(N_x\) and \(N_y\) are the dimensions of the training image. The mean entropy of template \(\mathbf{T}\) with the number of patterns \(n_{\mathbf{T}}\) is calculated by the formula given in Equation 5.8:
ME(ω) = \frac{1}{n_T} \sum_{k=1}^{n_T} H(\text{pat}_T^k). \tag{5.8}

I have implemented the algorithm for multiple template sizes for the training image. The plot of mean entropy for different template sizes is shown in Figure 5.32. Contrary to our expectation, the plot does not flatten at any template size. The main reason is, as proposed by Honarkhah (2011), Shannon’s entropy is applicable on stationary images, whereas, the training image constructed for the Delhi Field is not stationary. Hence, their proposed method may not provide an accurate analysis on the template size selection and other approaches should be considered.

Figure 5.32: Plot of Shannon’s entropy for different template sizes to choose the optimal template size. The plot should become flat when it reaches the optimum template size.

Here, another template size, which is 7×7 is tested with the inversion algorithm. Figure 5.33 provides the results of the stochastic inversion with the initial training image and template size 7×7:

As before, the algorithm is run multiple times, but due to limitations in computation power, for this case only seven realizations are generated. Figure 5.34 shows the maximum
Figure 5.33: Multiple realizations of the litho-facies for template size $7 \times 7$, and the results of stochastic inversion.
probability map for seven realizations of template size $7 \times 7$.

![Image: Maximum probability map of litho-facies from multiple realizations for template size.](image)

Figure 5.34: Maximum probability map of litho-facies from multiple realizations for template size.

Observing the inversion result for the template size $7 \times 7$, shows that a larger template results in capturing large features and disregards the details in the training image. Each of the realizations represent a more continuous shape for the channelized features. Even though, the final image may not represent the same characteristic that might be due to the lack of enough realizations.
A new stochastic inversion method has been developed and implemented to build a litho-facies model for the reservoir at Delhi Field, LA. This method incorporates an image reconstruction approach, which is known as image quilting, to control the selection of patterns in the multiple point statistics algorithm. Application of image quilting reduces the number of random visits to each location, and the search algorithm operates on a unilateral path instead of a random path.

Multiple point statistics methods, and specifically the SIMPAT algorithm provides a detailed definition of the facies in the reservoir, compared to conventional geostatistical approaches. They incorporate training images as conceptual figures from the reservoir to distribute the properties in the subsurface.

In this research, the SIMPAT algorithm as a multiple point statistics method is implemented for building a litho-facies model in an inversion approach. By gathering the stratigraphic and depositional information of the field, I constructed the training image for the field using an initial $5 \times 5$ template to scan the training image and build the pattern database. I incorporated seismic data, and image reconstruction and generated multiple equi-probable litho-facies realizations from the seismic section. The litho-facies model provides a detailed definition and captures more complexities from the reservoir.

I investigated the sensitivity of the algorithm to different parameters, such as the training image and the template size. In the initial training image, the distributary channels of the Paluxy were considered as sinusoidal, undulating features. In the second training image, the channels were assumed as more straight channels with less sinusoidal characteristics. In the first case, the resulting litho-facies cross section presented more discrete features of the channels similar to the lenticular bodies. Whereas, due to the features extracted from the
seismic section, in the direction of North-South the channels are continuous, elongated features. The result of the inversion with the second training image, provided more continuous channels for the Paluxy. This shows that the algorithm is dependent on the shape, size, and characteristics of the objects within the training image. Therefore, the process of building the training image should be performed with attention to the stratigraphic and structural characteristics of the reservoir.

In a second attempt, a larger template size is chosen to generate the pattern database. Application of a larger template led to higher computation time, because the number of calculations between pair of patterns have increased. On the other hand, a larger template covers a larger area of the training image and would recover larger features. Therefore, it is expected that the larger template would provide a more continuous feature of the channels and other depositional units. This is to some extent observed in the results of the inversion, however, when generating the maximum probability map, the final image does not represent these characteristics. It may be due to the fewer number of realizations compared to the smaller template size.

In this research, the seismic data incorporated is post stack data. The multiple point statistics algorithm can be implemented on any other data type that can be defined by a forward operator, such as angle stacks. That should provide more control on the accept/reject process, due to more parameters involved in the forward modeling operator (P-wave velocity, S-wave velocity, density, and incident angle). Application of time-lapse seismic data would improve our understanding from a reservoir under EOR processes. The stochastic inversion algorithm would be implemented on time-lapse seismic volume with the application of rock-physics equations and empirical relations in the forward modeling.

Availability of multiple data types is very important in this problem, as example presence of horizontal wells in the area would improve our understanding of geology and would contribute in building a more accurate training image. Moreover, incorporation of bandwidth extended seismic data is to be considered for higher resolution data in the forward modeling.
Also, other image reconstruction algorithms rather than image quilting can be incorporated in the search algorithm. Template size is one of the user-defined parameters that has an impact on the final litho-facies model. In this research two arbitrary template sizes are tested on the data, and a method is implemented to calculate the optimum template size. Due to the non-stationarity of the training image, the Shannon’s entropy method does not provide a robust solution, and other methods should be studied to find an appropriate algorithm.

This algorithm involves analyzing and storing big data, which necessitates application of data analytics, for example different methods of dimensionality analysis and reduction are interesting topics that can be incorporated in this stochastic inversion method.
REFERENCES CITED


APPENDIX

THEORY AND METHODOLOGY OF CLUSTERING METHOD

A.1 Introduction

The application of clustering algorithm in this research belongs to defining litho-facies in the wells. Multiple well logs are used as input to define the clustering. The clustering algorithm is implemented by Heterogeneous Rock Analysis (HRA) clustering module of Techlog®. To perform the clustering, first Principal Component Analysis (PCA) is implemented to transform the data onto independent axes that represent the highest variance in the data. Next, the principal components are used in the k-means clustering algorithm to generate the classifications.

A.2 Principal Component Analysis (PCA)

PCA is one of the major methods in dimensionality reduction of data when analyzing them. Dimensionality reduction is the process of mapping data from a high dimension space and mapping it into a new space, which has much smaller dimension. There are several purposes for reducing the dimensionality of data, first high dimension data are computationally expensive to work with, next, in some situations high dimensionality may lead to weak generalization capability for learning algorithms. Finally, dimensionality reduction can help in finding the structure in the data for interpretation and illustration purposes.

PCA in general applies a linear transformation on the original data. That means if the original data is in $\mathbb{R}^n$ and we want to transform it into $\mathbb{R}^p$ with $p < n$, we should find a matrix $W \in \mathbb{R}^{p,n}$ that performs the mapping $x \mapsto Wx$. The criterion to choose $W$ is that it enables a reasonable back transformation of $x$. However, the exact recovery of $x$ is impossible.

Assuming $x_1 \cdots x_m$ are $m$ vectors in $\mathbb{R}^n$, and $Wx$ is the projection of $x$ into $\mathbb{R}^p$, the lower dimensionality space. There should be a second matrix $U \in \mathbb{R}^{n,p}$, which can transforms back $Wx$ into the original space. if we assume $y = Wx$ as the compressed form of the vectors
in $\mathbb{R}^p$, we can construct $\tilde{x} = Uy$, such that $\tilde{x}$ is the recovered form of $x$ in the original high dimensional space $\mathbb{R}^n$. The compression matrix $W$ and the recovery matrix $U$ should be constructed in a way that the sum of squared errors between the two original and the recovered matrices are minimum:

$$\arg\min_{W \in \mathbb{R}^{p,n}, U \in \mathbb{R}^{n,p}} \sum_{i=1}^{m} \|x_i - UWx_i\|_2^2 .$$

(A.1)

It is proved that the matrices $W$ and $U$ are unique, therefore there exists solution to Equation A.1 and is unique.

### A.3 K-means clustering

In exploratory data analysis, clustering is one of the techniques that is vastly used. In most disciplines, the user tries to discover an initial understanding of the data by by dividing them into separate meaningful groups. Clustering, intuitively is performed to assign a set of similar objects in the same group and dissimilar objects into different groups.

A common approach to clustering is to define a cost function for a set of possible clusterings and the goal of the clustering algorithm is to find a set of clusters that minimizes the cost. Under this definition, clustering turns into an optimization problem. The problem is defined with pair of $(\chi, d)$ as the input parameters. Where $\chi$ is the set of parameters to be clustered and $d$ is the distance function such that $d(x, x) = 0$. The set of $C = (C_1, \cdots, C_k)$ are the proposed clustering solutions. The goal of the objective function $F$ is to find the clustering $C$ for a given input so that $F((\chi, d), C)$ is minimized. In k-means clustering the data is grouped into separate sets $C_1, \cdots, C_k$ and each $C_i$ is represented by a centroid $\mu_i \in \chi'$, where $\chi \subseteq \chi'$. The k-means objective function is the measure of squared distance between each point in $\chi$ and the centroid of its cluster. The centroid of cluster $C_i$ is defined as:

$$\mu_i(C_i) = \arg\min_{\mu \in \chi'} \sum_{x \in C_i} d(x, \mu)^2 .$$

(A.2)
The k-means objective function is given by:

$$F_{k\text{-means}}((\chi, d), C_1, \cdots, C_k) = \sum_{i=1}^{k} \sum_{x \in C_i} d(x, \mu_i(C_i))^2 .$$  \hspace{1cm} (A.3)

It can be also written as:

$$F_{k\text{-means}}((\chi, d), C_1, \cdots, C_k) = \min_{\mu_1, \cdots, \mu_k \in \chi'} \sum_{i=1}^{k} \sum_{x \in C_i} d(x, \mu_i)^2 .$$  \hspace{1cm} (A.4)

The algorithm is an iterative process and at each iteration the centroid is updated. The algorithm stops when updating the centroids does not decrease the value of the objective function (Kung, 2014).