ADVANCED POWER THEORIES AND SIGNAL DECOMPOSITION
METHODS FOR CONTROLLING SMART CONVERTERS
IN SMART GRID APPLICATIONS

by:
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ABSTRACT

During last two decades, the enormous level of aggregation of distributed generation units DGUs (widely known as the technology of Microgrids), in addition to increasing usage of nonlinear loads in power systems has raised new mathematical-conceptual challenges, specially in power electronics. Most of the traditional power theories and concepts therein, have been defined and formulated for simple balanced and linear systems. As a result, most of them are not directly applicable in case of new system structures with a considerable amount of uncertainty in the production and nonlinearity in the consumption. Due to uncertainties injected by the dynamic behavior of the DGUs (mostly renewable-based), the power components in the traditional power theories should be redefined under highly dynamic behavior of the power signals. Moreover, corresponding justifications need to be implemented to adapt all the related control strategies and compensation techniques. Renewable-based energies, such as wind and solar, are inherently uncertain power sources which can have unpredictable unwanted impacts on power flow, voltage regulation, and result in distribution losses. Microgrids that are quickly expanded through the power networks and power theories play a critical role in all the control strategies designed for these systems. When operating in the islanded mode, low-voltage Microgrids can exhibit considerable variation of amplitude and frequency of the voltage supplied to the loads, thus affecting power quality and network stability. Limited power capability in Microgrids can cause a voltage distortion which affects measurement accuracy, and possibly cause tripping of protections. Besides, the nonlinear and unbalanced loads obscure the traditional power definitions and equations. In such contexts, a reconsideration of power theories is required, since they form the basis for supply and load characterization and accountability. Moreover, developing new control techniques for harmonic and reactive compensators are mandatory, because they operate in a strongly interconnected environment and must perform cooperatively to face system dynamics, ensure power quality, and limit distribution losses.

The main purpose of this research is to improve the quality, reliability and stability of future electrical power delivery by improving the overall performance of smart Microgrids through usage of advanced time-domain power theories (such as instantaneous power theory (PQ) and Conservative Power Theory (CPT)). Another major contribution of this work is the introduction
of new mathematical power theory concepts (termed Enhanced Instantaneous Power Theory (EIPT)) in addition to implementation of adequate new control strategies. This work specially expanded based on a specific viewpoint which says that power theories can be interpreted as advanced signal decomposition techniques which are used as the initial step in electrical power signals analysis. This signal analysis step forms the fundamental headstock for power electronic interfaces controller design procedure. After describing the mathematical fundamentals of our modified power theory, EIPT; then this method is used as a time-domain signal decomposition approach for relevant applications. Exploiting the fine levels of information revealed through analysis of the power signals with the mentioned decomposition approaches, we provide more levels of freedom in the case of control frameworks. This research also investigate the interesting application of EIPT, besides other practical power theories such as CPT, in islanding detection problems, where a new instantaneous intelligent passive islanding detection strategy will be introduced. In a nutshell, developing new time-domain power theory concepts while exploiting the inherent capacities of the pre-existing power theories, the main goal of this work will be designing a reliable and smart multifunctional control scheme that can address all the aforementioned challenges.
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NOMENCLATURE AND ABBREVIATIONS

\( v_{abc} \): instantaneous voltage vector in the \( abc \) frame

\( i_{abc} \): instantaneous current vector in the \( abc \) frame

\( T_{\alpha\beta0} \): \( \alpha\beta0 \) transformation matrix

\( v_{\alpha\beta0} \): instantaneous voltage vector in the \( \alpha\beta0 \) frame

\( i_{\alpha\beta0} \): instantaneous current vector in the \( \alpha\beta0 \) frame

\( P \): active power

\( Q \): reactive power

\( p \): instantaneous active power

\( q \): instantaneous reactive power

\( p_a, p_b \) and \( p_c \): instantaneous active power in phase \( a, b \) and \( c \) respectively

\( q_a, q_b \) and \( q_c \): instantaneous reactive power in phase \( a, b \) and \( c \) respectively

\( p_{\alpha\beta} \): instantaneous active heteropolar power component

\( \bar{p}_{\alpha\beta} \): average active balanced heteropolar power component

\( \bar{p}_{\alpha\beta} \): oscillating active heteropolar power component

\( p_0 \): instantaneous active unbalanced homopolar power component

\( q_{\alpha\beta} \): instantaneous reactive heteropolar power component

\( \bar{q}_{\alpha\beta} \): average reactive balanced heteropolar power component

\( q_{\alpha\beta} \): oscillating reactive heteropolar power component

\( q_{\alpha0} \) and \( q_{\beta0} \): instantaneous unbalanced homopolar reactive power component

\( i_p \): instantaneous active current

\( i_q \): instantaneous reactive current

\( i_{p\alpha0}, i_{p\beta0}, i_{p\gamma0} \): active unbalanced homopolar current components in phase \( a, b \) and \( c \) respectively

\( i_{p\alpha}, i_{p\beta} \) and \( i_{p\gamma} \): active balanced sinusoidal current components in phase \( a, b \) and \( c \) respectively

\( i_{p\alpha}, i_{p\beta} \) and \( i_{p\gamma} \): active oscillating current components in phase \( a, b \) and \( c \) respectively

\( i_{p\alpha a}, i_{p\alpha b}, i_{p\alpha c} \): active unbalanced heteropolar current components in phase \( a, b \) and \( c \) respectively

\( i_{p\alpha a}, i_{p\alpha b}, i_{p\alpha c} \): active harmonic current components in phase \( a, b \) and \( c \), respectively
$i_{aq_0}, i_{bq_0}, i_{cq_0}$: reactive unbalanced homopolar current components in phase $a$, $b$ and $c$

$i_{aq}, i_{bq}, i_{cq}$: reactive balanced sinusoidal current components in phase $a$, $b$ and $c$ respectively

$i_{aq}, i_{bq}, i_{cq}$: reactive current components in phase $a$, $b$ and $c$ respectively

$i_{qu_a}, i_{qu_b}, i_{qu_c}$: reactive unbalanced heteropolar current components in phase $a$, $b$ and $c$

$i_{qHA}, i_{qHB}$ and $i_{qHC}$: reactive harmonic current components in phase $a$, $b$ and $c$, respectively

$i_{puas}, i_{pubs}, i_{pucs}$: active unbalanced part of current components caused by unbalanced part of voltage source in phase $a$, $b$ and $c$

$i_{quas}, i_{qubs}, i_{qucs}$: reactive unbalanced part of current components caused by unbalanced part of voltage source in phase $a$, $b$ and $c$

$i_{pHAs}, i_{pHBS}, i_{pHCS}$: active harmonic part of current components caused by distorted part of voltage source in phase $a$, $b$ and $c$

$i_{qHAs}, i_{qHBS}, i_{qHCS}$: reactive harmonic part of current components caused by distorted part of voltage source in phase $a$, $b$ and $c$
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You showed me the right path for success with your lessons and encouragements. 
Your soul may rest in peace.

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The reason of what I become today. During all these years I missed you every second but you 
always gave me a positive energy. Your boundless love and continuous care helps me through all 
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Thanks for your patience, endless love and support during the past four years. Every single day I 
spend as your wife I realized how lucky I am to live such an amazing life. 
I love you to moon and back.
CHAPTER 1
INTRODUCTION

In this chapter, an overview on the Microgrids technology, opportunities and challenges, in addition to the motivation for and contributions of this work is presented.

1.1 Distributed Generation, Renewable Energy and Microgrids: Role of Power Electronics

Nowadays rising global electrical energy consumption is a big challenge in the electrical power delivery system. As a solution to this increasing ramp up in demand, the traditional power systems are changing globally, and many distributed generation (DG) units\(^1\), including both renewable and nonrenewable energy sources such as wind turbines, photovoltaic (PV) generators, fuel cells, combined heat and power stations, are being integrated into power systems at the distribution level.

Microgrids, also named minigrids, are becoming an important concept for integrating distributed generation and distributed storage systems. This concept has been developed to cope with the penetration of renewable energy systems, which can be realistic if the final user is able to generate, store, control, and manage part of the energy that will be consumed. Microgrids should be able to locally solve energy problems and hence increase flexibility. Based on the connection situation of the Microgrid with the main grid, there are two modes of operation for Microgrids, the “Grid Connected\(^2\) (GC)” mode and “Islanded Mode” (Stand-Alone Mode). The use of distributed generation (DG) makes no sense without using distributed storage (DS) systems to cope with the energy balances.

Power electronics, the technology of efficiently processing electric power, plays an essential role in the integration of the distributed generation units to develop efficient and high-performance power systems. Different power electronic topologies are designed for each of the DG systems; however, in all types of DG or Microgrid configurations, power electronic converters play an important role to control the flow of power and convert it into a suitable DC or AC form as

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\(^1\) In many contexts, the terms Microgrid and DGU have been used interchangeably, although, usually a Microgrid includes one or more number of DGUs.

\(^2\) In the rest of this document, we may use the following two terms interchangeably: grid connected (GC) and grid-tied (GT)
required. These converters can be intelligently designed to support a variety of fundamental functionalities which are needed to be implemented in Microgrids. The most important functionalities of control frameworks in Microgrids are mentioned below.

1.2 Microgrid: A Challenging Double Mode System

Several studies are considering Microgrids with single operational mode inverters (which can operate only in the GC [1] or only in the islanded situation [2]). However, in practice, the double-mode Microgrids are widely preferred. Another important required functionality for future smart Microgrids is grid supporting. Regarding new industrial standards, future smart Microgrids should be able to negotiate with the main grid. Specifically, they must be equipped with the capability of injecting active and reactive power to the main grid (during load peak hours) to retain the adequate frequency and stabilize the voltage magnitude. Due to different objectives in each operating mode (current control in the GC mode and voltage control in the islanded mode), the inverter’s control framework will be more complicated in double-mode Microgrids. Consequently, another desirable feature of a double-mode smart inverter is a smooth transition between the islanded and GC modes [3]. As an instance, in [3], a master-slave based control system is used to provide a smooth transition between two modes; however, because of using two distinct controllers and switching between these two controllers, the transient response is still considerable. As a result, an adequate smart Microgrid system should be equipped with fast but reliable transition methods that can quickly detect the failures of the main grid and switches between two operating modes.

Another important concern regarding aggregation of Microgrids (which is more highlighted in the case of renewable based distributed generation (RDG) Microgrids) is their output power oscillations and harmonic injection. Basically, these unwanted phenomena are injected to the grid by the switching devices used in power electronic interfaces and by nonlinear loads. They can strongly affect the quality of voltage and current waveforms. On the other hand, the presence of harmonics in the power lines causes a variety of unwanted issues such as: interference problems in communication, power losses in distribution networks, and failure in the operation of electronic devices [4]. Uncertainty in renewable generations, in addition to the harmonic distortion effect, makes the power quality a serious issue within integration of RDG-based Microgrids. To organize the related consequences with power grid stability, a variety of international standards have been introduced regarding electrical power quality in Microgrids (such as IEEE-519). According to these standards, the Microgrid should not produce and inject harmonic contents greater than
specified values to the main grid at the point of common coupling (PCC). Passive filters are the first solutions which were used to solve harmonic current problems; however, they present several disadvantages. The most significant bug with these techniques is that they are only able to filter the frequencies which are exactly tuned for, while other resonances can occur because of the interaction between the passive filters and other loads.

To cope with this drawback, active filters can be implemented as an alternative approach [5]. Similarly, several methods have been introduced for active filtering implementation for different applications. The vital step in all the active filtering methods is reference signal generation, which is used to extract a reference compensating current or voltage from the distorted waveform (widely known as current signal decomposition of power theories). Various power theories have been introduced so far [6]-[41]. Generally, we can categorize these methods into two main classes: Time domain methods and Frequency domain methods. Fast Fourier transform or FFT is the most notable method among frequency domain techniques. However, there are much more popular methods proposed based on a time-domain approach, such as dq or Synchronous reference frame theorem [16]. In other articles, new schemes and solutions have been developed based on power theories such as the FBD-based method [18] and the pq or conventional instantaneous power theory [12]. Recently another active filtering method has been introduced based on Conservative Power Theory (CPT) concepts and formulation [36]. However, in most of the presented work, an individual power electronic interface module in addition to a few extra devices is needed to implement the active filtering framework. They mostly use an extra harmonic compensation unit in parallel with the main inverter in their control scheme; these are parallel frameworks, widely known as shunt active filtering methods. Moreover, the compensation unit contains an extra battery source in addition to an extra inverter. Also, most of the proposed methods are not considering the double-mode inverter infrastructure and basically focus on single mode Microgrids (mostly grid-tied mode). Not only these multi-stages interconnected control goals are needed to be addressed simultaneously, but also the minimization of the computational and unit costs (in addition to the system size limitations) should be considered in parallel.
1.3 Overview and Contributions

In this work, we aim to improve the renewable-based Microgrid performance in power systems by introducing and developing new adequate control schemes for power electronic converters. We specifically address the following set of challenges in a double mode Microgrid: 1) Smooth transition between operational modes, 2) Control of DC-DC converters, 3) Distorted signal compensation (includes a set of sub-challenges), 4) Islanding detection and classification. In this regard, multifunctional inverters with a variety of functionalities are designed and implemented using advanced power theories as powerful signal decomposition tools. Due to inherent advantages of time domain-based signal decompositions, we limit the focus of this work to instantaneous time-dependent power theories.

Chapter 2 gives a comprehensive overview of the most important and widely acknowledged time-domain power theories, their pros and cons. The basic concepts of the most important advanced power theories such as FBD Theory [18], d-q theory [16], conventional p-q theory [12] and Conservative Power Theory (CPT) [36] are described, and their role and applicability in designing control frameworks for switching converters in Microgrid systems will be investigated.

In chapter 3, inspired by the Akagi’s and Fang Zheng Peng’s instantaneous power theory framework, an “enhanced instantaneous power theory” will be introduced. Compared to its ancestors, this new method can decompose the electrical signals into much finer levels, namely: balanced sinusoidal active, balanced sinusoidal reactive, non-sinusoidal active, non-sinusoidal reactive, unbalanced active and, unbalanced reactive components in each phase of the system. Exploiting the higher level of information revealed from such a detailed signal decomposition, it can bring a higher level of flexibility into control strategies optimization in Microgrid systems. Moreover, it might be used independently in any signal processing applications. The proposed method is compared with the aforementioned methods specially CPT from a signal decomposition perspective. The most notable advantages of this new theorem compared to CPT is that in this theory different current components are calculated independently, while in CPT the voided current definition depends on other components (in fact, voided currents are calculated from active and reactive currents). Moreover, all decomposed components are physical meaning-oriented in terms of power definitions, in contrast in CPT, there is no physical meaning for the reactive energy [48].

Chapter 4 discusses a novel possible application for time domain power theories: islanding recognition. Due to a variety of reasons, a Microgrid system may fall into the islanding mode
which can happen unintentionally or intentionally. It is expected from a smart Microgrid system to be able to recognize any faulty scenarios and decide to work under the islanding mode for the sake of system safety. A variety of islanding detection approaches have been developed within the literature so far, with all pros and cons. In this chapter, exploiting the inherent instantaneous nature of time-domain power theories, we develop an intelligent instantaneous passive islanding detection (IIPID) approach. Regarding locality advantages, we keep our focus on a passive islanding detection framework and address three major challenges associated with most of the former algorithms developed under this category. These problems are: phase-phase coupling information ignorance, time complexity, and not being informed about the reason of the fault. We discuss each of these issues in detail and make a comprehensive evaluation on the proposed IIPID method.

Mathematically speaking, the IIPID is interpreted in terms of a pattern recognition problem where advanced power theories are used to generate the most distinguishable features (from 3-phase voltage signals) between different classes of fault. Next these features are fed into a nonlinear classifier (a multilayer artificial neural network) to recognize the corresponding class of the islanding scenario. A major difference between this work versus the state of the art in literature [68] is the reason-oriented islanding scenario generation and classification. In other words, instead of labeling classes based on the fault types (such as sag, swell, flicker and so on), we label the classes with the reasons of the faults. Thus, an islanding scenario may generate a combination of fault types (for example a sag plus harmonic and oscillation), but regardless of the fault types, the classification approach should be able to recognize the reason of the faulty situations (such as transformer energizing, arc furnace, etc.). This reason-oriented classification is a more challenging.

Finally, chapter 5 presents the mathematical and practical details of our new control strategies through incorporating the advanced signal decomposition and power theory analysis. Here are the highlights:

Besides the aforementioned mandatory functionalities that a Microgrid system may have for an appropriate operation, each one of the power electronic interfaces need to be armed with adequate controllers to optimize the overall revenue. The combination of these controllers will form the final multifunctional control framework. In our final configuration, converters are supported by a variety of additional functionalities such as harmonic, reactive power or unbalanced compensation,
and even for voltage regulation in case of weak grids. Each of these functionalities, in addition to our approaches to implement them, is described in chapter 5.

Some of the controllers which have been developed in this work are listed below:

- Multifunctional unified controller for double mode inverter with capability of harmonic and unbalanced compensation by using enhanced instantaneous power theory
- Unified controller for floating interleaved buck-boost converter (FIBBC)

Matlab, Simulink, PSIM, real time simulator OPAL-RT and DSPACE modules have been used for sake of simulation, validation and performance verification of the proposed approaches as needed.
2.1 Fundamental Concepts and Definitions

Power components (active, reactive, apparent) are important tools for system evaluation, control and compensator design. Due to straightforward configuration of a typical power system, there is no ambiguity in the definition of these components under sinusoidal operating conditions in linear single-phase and linear balanced three-phase systems. However, during recent years a huge amount of Microgrids in addition to nonlinear loads such as power electronic converters and speed drives aggregated to the power system that raised new challenges for traditional power definitions and compensation techniques. A multitude of studies were performed to find a generalized power theory which is applicable for power systems under non-sinusoidal and/or unbalanced conditions [6-41]. In this sense, lots of power theories have been developed during recent years for three-phase unbalanced and nonlinear systems but usually were criticized by other researchers after a short while [43-50]. Generally, we can categorize these theories into three main classes: time domain, frequency domain and combined time-frequency domain which mostly use wavelet transformation. Figure 2.1 illustrates some of the main power theories specifying their corresponding categories. The first attempt to solve the problem of defining powers under non-sinusoidal conditions is credited to Budeanu [6] who used a frequency-domain based approach. Thus, considering just the time domain approaches, one could call attention to the theory of Depenbrock (FBD) [18], Akagi et al. (pq-Theory) [12] and CPT [36], which are strongly related to active filtering applications. Conventional \( pq \)-theory is very well-known and accepted by the power electronics community; some authors tend to consider it as a theoretical tool not only for active filter control but also for energy property definitions. In [19], Willems verifies that the \( pq \)-theory faces some problems. CPC (Current’s physical components) for three-phase four-wire systems was investigated by Czarnecki [14], [41].

---

3 This section is based on the following set of papers: [79]
Figure 2.1 The history chart of the power theories
There are lots of papers published in this area; some of them compare some existing methods and some criticize them [42-50]. Among them some have been characterized by analyzing the power theories from physical viewpoints and the others by giving some examples or showing simulation results. The example oriented approaches cannot be easily validated due to their dependency on how the methods have been implemented. In [42] authors make an informative survey over power theories by reviewing a huge body of articles; however, they did not go into enough detail.

In this chapter, the most famous power theories (with focuses on five dominant, widely implemented time-domain-based approaches) are explained using a comparative approach and show their advantages and disadvantages to understand why (after so many years working on defining power terms under nonlinear and unbalanced multiphase systems) there is not a unique method accepted by the power system community in this regard.

2.2 Different Categories of Power Theories

Power theories can be divided in three main classes: frequency domain, time domain and time-frequency domain.

2.2.1 Frequency Domain

Budeanu in [6] did the first attempt to solve the problem of defining power components under non-sinusoidal conditions by using a frequency-domain based approach. In frequency domain, the voltage and current signals are expressed as functions of multiples of the fundamental frequency. Some of the frequency domain approaches are Fourier series, Fourier transform and Fast Fourier transform (FFT)--which is the most notable method among frequency domain techniques. FFT can decompose current and voltage signals into different frequencies. The main problem of frequency domain approaches is that in case of large numbers of harmonics, specially when the signal contains inter-harmonics, the approaches cannot work properly and produce large amounts of error [42].

2.2.2 Time Domain:

After Budeanu, Fryze introduced his theory in time-domain [7]. In time domain approaches, voltage and current signals are expressed as a time function. The main advantage of time domain
approaches is instantaneous calculation of current and power terms, and their main problem is that they are not able to decompose signals into different frequencies for harmonic analysis.

2.2.3 Time-Frequency Domain:

Time-frequency can be an appropriate domain to define power and current terms. By using time-based approaches, they can calculate power terms fast, and using the frequency domain approach they can analyze current signals with more details in different frequencies. Yoon used the time-frequency approach in [29-30] by using a discrete wavelet transform (DWT) for a single-phase system. This method works for a single-phase system; however, expanding it to multiphase systems is still under discussion [37]. Thus, this method will be used after finding an appropriate time-based approach, because in this domain, the base frame is one of the time-based approaches [42].

2.3 Definition of Power Components within the Most Famous Time Domain Power Theories under Unbalanced and Nonlinear Conditions

In this section, we will explain the most acknowledged time domain-based power theories and discuss their main advantages and disadvantages.

2.3.1 Instantaneous Power Theory (PQ Theory)

In 1984, the instantaneous power theory was introduced by Akagi, and it is very well-known and accepted by the power electronics community. Some authors consider it a theoretical tool for active filter control and for energy property definitions [51-52]. The p-q theory uses the Clarke transformation which transfers three-phase voltages and currents in a-b-c coordinates to the α-β-0 coordinates.

\[
\begin{bmatrix}
  v_{\alpha} \\
v_{\beta} \\
v_{0}
\end{bmatrix} = \sqrt{3} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2 \\
  \sqrt{2}/2 & \sqrt{2}/2 & -\sqrt{2}/2
\end{bmatrix} \begin{bmatrix}
  v_a \\
v_b \\
v_c
\end{bmatrix}
\]  

(2.1)

\[\text{Since the power component definitions, under sinusoidal and balanced situation, are vastly introduced within the power engineering literature, we do not cover the details of the related mathematical formulations.}\]
Then they defined:

\[
\begin{bmatrix}
    p_0 \\
p \\
q
\end{bmatrix} =
\begin{bmatrix}
    v_0 & 0 & 0 \\
0 & v_\alpha & v_\beta \\
0 & v_\beta & -v_\alpha
\end{bmatrix}
\begin{bmatrix}
i_0 \\
i_\alpha \\
i_\beta
\end{bmatrix},
\]  

(2.3)

where, \(p\) is an instantaneous real power and corresponds to the energy which is transferred from the source to the load, and \(q\) is the instantaneous imaginary power, and it is responsible for the existence of undesirable currents which circulate between the phases of the system.

Using inverse transformation, real and imaginary current components in \(\alpha-\beta-0\) frame will obtain:

\[
\begin{bmatrix}
i_0 \\
i_\alpha \\
i_\beta
\end{bmatrix} = \frac{1}{v_{\alpha\beta_0}^2}
\begin{bmatrix}
v_0 & 0 & 0 \\
0 & v_\alpha & v_\beta \\
0 & v_\beta & -v_\alpha
\end{bmatrix}
\begin{bmatrix}
p_0 \\
p \\
q
\end{bmatrix},
\]  

(2.4)

where:

\[
v_{\alpha\beta_0}^2 = v_\alpha^2 + v_\beta^2 + v_0^2
\]  

(2.5)

\[
i_{ap} = \frac{v_{ap}}{v_{\alpha\beta_0}^2} \\
i_{aq} = \frac{v_{aq}}{v_{\alpha\beta_0}^2}
\]  

(2.6)

\[
i_{bp} = \frac{v_{bp}}{v_{\alpha\beta_0}^2} \\
i_{bq} = -\frac{v_{aq}}{v_{\alpha\beta_0}^2}
\]  

(2.7)

Consequently, power terms (\(p\) and \(q\)) decomposed to their mean and oscillating parts as follows:

\[
p = \bar{p} + \tilde{p}
\]  

(2.8)

\[
q = \bar{q} + \tilde{q}
\]  

(2.9)

where \(\bar{p}\) and \(\tilde{p}\) are called the fixed and alternated value of the instantaneous real power respectively. Moreover, \(\bar{q}\) and \(\tilde{q}\) are the mean and alternated value of the instantaneous imaginary power respectively.

\[
i_{ap} = i_{a\bar{p}} + i_{a\tilde{p}} = \frac{v_{ap}}{v_{\alpha\beta_0}^2} + \frac{v_{a\tilde{p}}}{v_{\alpha\beta_0}^2}
\]  

(2.10)
\[ i_{aq} = i_{aq} + i_{a*} = \frac{v_{p\bar{q}}}{v_{a*0^2}} \pm \frac{v_{p\bar{q}}}{v_{a*0^2}} \] (2.11)

\[ i_{\beta p} = i_{\beta p} + i_{\beta \bar{p}} = \frac{v_{p\bar{p}}}{v_{a*0^2}} \pm \frac{v_{p\bar{p}}}{v_{a*0^2}} \] (2.12)

\[ i_{\beta q} = i_{\beta q} + i_{\beta \bar{q}} = \frac{-v_{a\bar{q}}}{v_{a*0^2}} \pm \frac{-v_{a\bar{q}}}{v_{a*0^2}} \] (2.13)

Then by using \( \bar{p}, \bar{p}, \bar{q}, \bar{q} \) and inverse Clarke transformation (2.2), currents in each phase are decomposed to different components as follows:

\[
\begin{bmatrix}
i_{a\bar{p}} \\
i_{b\bar{p}} \\
i_{c\bar{p}}
\end{bmatrix}
= \sqrt{3} \begin{bmatrix}
1 & 0 & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
i_{a\bar{p}} \\
i_{b\bar{p}} \\
i_{c\bar{p}}
\end{bmatrix}
\] (2.14)

\[
\begin{bmatrix}
i_{a\bar{p}} \\
i_{b\bar{p}} \\
i_{c\bar{p}}
\end{bmatrix}
= \sqrt{3} \begin{bmatrix}
1 & 0 & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
i_{a\bar{p}} \\
i_{b\bar{p}} \\
i_{c\bar{p}}
\end{bmatrix}
\] (2.15)

\[
\begin{bmatrix}
i_{a\bar{q}} \\
i_{b\bar{q}} \\
i_{c\bar{q}}
\end{bmatrix}
= \sqrt{3} \begin{bmatrix}
1 & 0 & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
i_{a\bar{q}} \\
i_{b\bar{q}} \\
i_{c\bar{q}}
\end{bmatrix}
\] (2.16)

\[
\begin{bmatrix}
i_{a\bar{q}} \\
i_{b\bar{q}} \\
i_{c\bar{q}}
\end{bmatrix}
= \sqrt{3} \begin{bmatrix}
1 & 0 & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{-\sqrt{3}}{2} \\
-\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
i_{a\bar{q}} \\
i_{b\bar{q}} \\
i_{c\bar{q}}
\end{bmatrix}
\] (2.17)

The corresponding algorithm which is used for calculating the harmonic compensating currents is visualized and detailed in Figure 2.2.

- Pros and cons:

This theory is formed based on instantaneous voltages and currents in three-phase power systems with or without neutral wire, and it is valid for steady-state or transient conditions. Since the p-q theory works in the time domain, it is used to design real-time and fast controllers for active filter. The averaged powers (\( \bar{p} \) and \( \bar{q} \)) are active and reactive powers, and oscillating powers (\( \bar{p} \) and \( \bar{q} \)) represent the undesirable powers due to harmonic and unbalanced components in the load current. This theory presents some interesting features, namely:
It can be applied to three-phase systems balanced/unbalanced with/without harmonics

It is based on instantaneous values and allows excellent dynamic response

Its calculations are relatively simple

It directly separates zero-sequence components because of using Clarke transformation

In [43] and [44], Willems and Czarnecki verify that the $pq$-theory faces some problems. Some of the main drawbacks of this theory that are mentioned in articles are listed here:

- This theory is confined to the three-phase systems
- For ease of calculation, the effect of zero-sequence components is not considered in the imaginary power
- It does not work properly when the source voltage becomes distorted.
- It is not able to decompose the unbalanced and harmonic parts of currents separately.

2.3.2 The FBD Theory

Several years later, in 1993, Depenbrock proposed his theory with the name of FBD (Fryze-Buchholz-Depenbrock) because his definition was an extension of Fryze’s and Buchholz’s power theories [18]. He used Fryze’s current decomposition and Buchholz’s instantaneous and RMS values. Decomposed currents were applied to controllers for application of active filtering. This method works in multiphase power systems. The voltages and current vectors are shown with $v$ and $i$ respectively:
\[ v = \begin{bmatrix} v_a \\ v_b \\ v_c \\ \vdots \\ v_n \end{bmatrix}, \quad i = \begin{bmatrix} i_a \\ i_b \\ i_c \\ \vdots \\ i_n \end{bmatrix} \quad (2.18) \]

Their instantaneous collective values \((v_\Sigma, i_\Sigma)\), are defined as:

\[ i_\Sigma = \sqrt{\sum_{k=1}^{n} i_k^2}, \quad v_\Sigma = \sqrt{\sum_{k=1}^{n} v_k^2} \quad (2.19) \]

where, \(n\) indicates the number of phases. Instantaneous power calculated from the inner product of voltage and current vectors:

\[ p = v \cdot i^T \quad (2.20) \]

where, \(i^T\) is the transpose of the current vector. The collective RMS value of currents and voltages under periodic conditions can be calculated as:

\[ I_{rms} = \frac{1}{T} \int_0^T i_\Sigma^2 dt, \quad V_{rms} = \frac{1}{T} \int_0^T v_\Sigma^2 dt \quad (2.21) \]

and collective active power is calculated from:

\[ P = \frac{1}{T} \int_0^T p dt \quad (2.22) \]

Depenbrock decomposed the instantaneous current in each phase of the system \((i_k)\) to active \((i_{ak})\) and nonactive \((i_{nk})\) currents. Active current contributes to the energy that is transferred from the source to the load.

\[ i_{ak} = \frac{p}{V_{rms}^2} v_k, \quad (2.23) \]

and nonactive currents \((i_{nk})\) contributes to the amount of energy that used in the system by disturbances and oscillations but it is not transferred from source to the load:

\[ i_{nk} = i_k - i_{ak} \quad (2.24) \]

Moreover, he decomposed \(i_k\) to power currents \((i_{pk})\) which is responsible for the instantaneous power and powerless currents \((i_{zk})\) which is not contribute to the current that is transferred between the source and load and can be easily compensated without any energy storage element in the system.
Finally, he defined variation currents \( i_{vk} \) which is responsible for the oscillation of the instantaneous power current around its average value.

\[
i_{vk} = i_{pk} - i_{ak} = i_{nk} - i_{zk}
\]  

\[ (2.27) \]

**Pros and cons:**

His definition for power components are valid and is similar to lots of famous power theories. The orthogonal current decomposition results that:

\[
i_{\text{i}} = i_{\text{p}} + i_{\text{g}} + i_{\text{y}}
\]

\[ (2.28) \]

However, some problems are associated with this theory:

- The power terms are not defined as conventional terms such as active, reactive and apparent. So here we try to compare his terms with conventional terms and explain their meanings. He defined \( i_{ak} \) as an active current, and it is similar in all power theories. \( i_{pk} \) is the instantaneous active current and contains active current in addition to harmonics and unbalanced parts which transfer from source to load. Powerless current \( i_{zk} \) is the instantaneous reactive current in addition to harmonics and unbalanced parts of current that are exchanged between phases of the system.

- His theory does not work in case of distorted or unbalanced voltage source, because his current terms are proportional with the source voltage. Thus, if the voltage contains distortion, current terms contain distortion as well.

- This theory is not able to decompose current to more detailed parts such as reactive, unbalanced, harmonics, and zero-sequence.

2.3.3 Synchronous Reference Frame Method (\( dq \))

In 1991 Batacharya proposed the dq method [16], and he used synchronous reference frame \( dq0 \) to transform load currents from the \( abc \) frame to the \( dq0 \) frame:
\[
\begin{bmatrix}
    i_d \\
    i_q \\
    i_0 \\
\end{bmatrix}
= \frac{2}{\sqrt{3}}
\begin{bmatrix}
    \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
    -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
    \sqrt{\frac{2}{3}} & \sqrt{\frac{2}{3}} & \sqrt{\frac{2}{3}} \\
\end{bmatrix}
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix}
\] (2.29)

Where, \(\theta\) is the synchronization angle which is time variant and represents the angular position of the \(dq\) frame and it is detected by a Phase Locked Loop (PLL). When \(\theta\) is calculated by PLL, the phase current transferred to \(dq0\) frame by using equation (2.29). After that the load currents in \(dq\) frame pass though high-pass filter to extract their oscillating parts representing the harmonic and unbalanced components of load currents in \(dq\) frame. Thus, the rest of current is the fundamental part of currents. Finally, by using equation (2.30) which is the inverse park transformation and transfers the currents from the \(dq0\) frame to the \(abc\) frame, those components are transferred to the \(abc\) frame.

\[
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix}
= \frac{2}{\sqrt{3}}
\begin{bmatrix}
    \cos(\theta) & -\sin(\theta) & \sqrt{\frac{2}{3}} \\
    \cos(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{2\pi}{3}) & \sqrt{\frac{2}{3}} \\
    \cos(\theta + \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) & \sqrt{\frac{2}{3}} \\
\end{bmatrix}
\begin{bmatrix}
    i_d \\
    i_q \\
    i_0 \\
\end{bmatrix}
\] (2.30)

Figure 2.3 shows the block diagram of \(dq\) method for current decomposition.

The tilde superscripted current terms (\(\tilde{i}_a, \tilde{i}_b, \tilde{i}_c\)) are oscillating parts of currents which contain unbalanced and harmonic parts.

- Pros and cons:
Since this method is using PLL, it is not instantaneous. Although in [21-22] the author introduces an instantaneous $dq$ current decomposition method without using PLL, it is criticized in [49], and it does not work properly in case of distorted currents.

Although, this method can be use in active filtering systems for compensation purposes, it is not a power theory, because it is not able to define current components according to conventional power terms.

It is not able to separate unbalanced and harmonic parts of currents separately.

2.3.4 Generalized Instantaneous Power Theory

In 1996, Fang Zheng Peng introduced a generalized instantaneous power theory which he claims works for three-phase systems in different conditions such as balanced/unbalanced, sinusoidal/nonsinusoidal, with/without zero-sequence components.

He defines instantaneous active power $p$ as an internal product of voltage and current vectors.

\[ p = \mathbf{v} \cdot \mathbf{i} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = v_a i_a + v_b i_b + v_c i_c \quad (2.31) \]

Then he defines instantaneous reactive power vector as cross product of voltage and current vectors:

\[ q = \mathbf{v} \times \mathbf{i} \quad (2.32) \]

Then by using definitions for active and reactive powers, he defines corresponding active and reactive current vectors according to these power components:

\[ i_p = \frac{p}{\mathbf{v} \cdot \mathbf{v}} \mathbf{v} \quad (2.33) \]

\[ i_q = \frac{q \times \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \quad (2.34) \]

Then he proves that the three-phase current vector ($\mathbf{i}$) is equal to sum of the active and reactive current vectors:

\[ \mathbf{i}_p + \mathbf{i}_q = \frac{p}{\mathbf{v} \cdot \mathbf{v}} \mathbf{v} + \frac{q \times \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} = \]

\[ = \frac{\mathbf{v} \cdot \mathbf{i}}{\mathbf{v} \cdot \mathbf{v}} + \frac{(\mathbf{v} \times \mathbf{i}) \times \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} = \frac{\mathbf{v} \cdot \mathbf{i}}{\mathbf{v} \cdot \mathbf{v}} - \frac{\mathbf{v} \cdot \mathbf{i}}{\mathbf{v} \cdot \mathbf{v}} + \frac{\mathbf{v} \times \mathbf{i}}{\mathbf{v} \cdot \mathbf{v}} = \mathbf{i} \]
Pros and cons:

We can show this theory is also valid in case of single phase systems:

\[ p = v \cdot i = V \cos(\theta) \]  \hspace{1cm} (2.36)
\[ q = v \times i = V \sin(\theta) \]  \hspace{1cm} (2.37)

where \( V \) and \( I \) are the RMS value of the phase voltage and current signals and \( \theta \) is their phase differences.

Conventional relation for apparent, active and reactive powers is still valid in this theory:

\[ s^2 = p^2 + q^2 = (v \cdot i_p)^2 + \|v \times i_q\|^2 = v^2 (i_p^2 + i_q^2) = v^2 i^2 \]  \hspace{1cm} (2.38)

Active and reactive current terms are perpendicular to each other.

\[ i_q \cdot i_p = 0 \]  \hspace{1cm} (2.39)

In [38], Die claims that this theory is only applied for a three-phase system.

It is not able to decompose current components, especially the zero-sequence and unbalanced parts of currents.

2.3.5 Conservative Power Theory (CPT)

The main problem associated with aforementioned power theories is that they are not appropriate for the unbalanced condition. Conservative power theory or CPT is the most recent power theory proposed by Tenti [36]. He defines power and current terms in the stationary frames and claims that this theory is an appropriate alternative in case of nonlinear and unbalanced conditions. Recently, this theory has been used as a popular method in compensation strategies for Microgrid applications. His definition for instantaneous active power is the same as other power theories:

\[ p(t) = v \cdot i = \sum_{k=1}^{M} v_k i_k \]  \hspace{1cm} (2.40)

where \( (v) \) and \( (i) \) are the voltage and current vectors, respectively, and \( k \) indicates the number of phases of the system. Like conventional power theories, the average value of \( p(t) \) is defined as active power.

\[ P = \bar{p} = \langle v \cdot i \rangle = \frac{1}{T} \int_{0}^{T} v \cdot i \ dt = \sum_{k=1}^{K} P_k \]  \hspace{1cm} (2.41)
He defines a new term which he calls instantaneous reactive energy which is calculated as follows:

\[ w(t) = \bar{\theta}.i = \sum_{k=1}^{M} \bar{\vartheta}_k i_k \]  \hspace{1cm} (2.42)

where \( \bar{\theta} \) is the unbiased integral of the voltage vector.

\[ \bar{\theta} = v_f(t) - \bar{v}_f(t) \]  \hspace{1cm} (2.43)

and \( v_f(t) \) is the time integral of voltage vector:

\[ v_f(t) = \int_0^T v(\tau)d\tau \]  \hspace{1cm} (2.44)

Moreover, he defines reactive energy as an average value of \( w(t) \):

\[ W = \bar{w} = \langle \bar{\theta}.i \rangle = \frac{1}{T} \int_0^T \bar{\theta}.i \, dt = \sum_{k=1}^{K} W_k \]  \hspace{1cm} (2.45)

Based on the above definitions the phase currents are decomposed in to three basic current components:

Active phase currents are defined by:

\[ i_{ak} = \frac{(v_k i_k)}{\|v_k\|^2} v_k = \frac{P_k}{\|v_k\|^2} v_k \]  \hspace{1cm} (2.46)

while reactive phase currents are given by:

\[ i_{rk} = \frac{(\vartheta_k i_k)}{\|\vartheta_k\|^2} \vartheta_k = \frac{W_k}{\|\vartheta_k\|^2} \vartheta_k \]  \hspace{1cm} (2.47)

where \( \|v_k\| \) and \( \|\vartheta_k\| \) are the RMS value of the phase voltage and unbiased integral of the voltage respectively.

Void phase currents are the remaining current terms:

\[ i_{vk} = i_k - i_{ak} - i_{rk} \]  \hspace{1cm} (2.48)

which is not an active or reactive current. The active and reactive phase currents can be decomposed into balanced and unbalanced terms. The balanced active currents have been defined as:

\[ i_{akb}^b = \frac{(v_l i_l)}{\|v\|^2} v_k = \frac{P}{\|v\|^2} v_k \]  \hspace{1cm} (2.49)
The balanced active currents are the portion of the phase currents that flows because of a balanced equivalent circuit and are responsible for conveying the total active power \( P \) in the circuit.

The balanced reactive currents have been defined as:

\[
i_{rk}^b = \frac{\langle \mathbf{\phi}, \mathbf{i} \rangle}{\| \mathbf{\phi} \|^2} \hat{\phi}_k = \frac{W}{\| \mathbf{\phi} \|^2} \hat{\phi}_k
\]

They represent the portion of the phase currents that flows in the system because of the balanced equivalent circuit and are responsible for conveying the total reactive energy \( W \) in the circuit.

The unbalanced active currents are calculated by:

\[
i_{ak}^u = i_{ak} - i_{ak}^b
\]

and unbalanced reactive currents are calculated as:

\[
i_{rk}^u = i_{rk} - i_{rk}^b
\]

Thus, the unbalanced three phase current defined as:

\[
i_k^u = i_{ak}^u + i_{rk}^u
\]

The current vector can be decomposed as:

\[
i = i_a^b + i_r^b + i_a^u + i_r^u + i_v
\]

This theory is based on an orthogonal decomposition so all the current components are orthogonal to each other, so their RMS value calculated as following equation:

\[
i = \sqrt{i_a^b + i_r^b + i_a^u + i_r^u + i_v^2}
\]

Pros and cons:

In [48] Czarnecki criticizes the CPT theory for the following drawbacks:

The physical meaning of the “reactive energy” as defined by (2.42) is not clear. In [48] the author shows that the “reactive energy does not account for inductive and capacitive energy stored in the load circuit”. He claims the lack of any relationship between the “reactive energy” \( W \) and the amount of the energy stored in the loads is even more visible in the case of linear and non-linear loads. To prove this, he considered a purely resistive
load with a periodic switch, made of a TRIAC, shown in Figure 2.4, supplied from an ideal source of a sinusoidal voltage [48].

![Figure 2.4 Resistive load with periodic switch](image)

when \( u(t) \) is a sinusoidal supply voltage.

\[
    u(t) = \sqrt{2} \sin \ U(\omega_1 t) \tag{2.56}
\]

The supply unbiased voltage is:

\[
    \hat{u}(t) = -\sqrt{2} \frac{U}{\omega} \cos (\omega_1 t) \tag{2.57}
\]

the reactive energy \( W \) is equal to:

\[
    W = \bar{w} = \langle \hat{\theta} \cdot i \rangle = \langle \hat{\theta} \cdot i_1 \rangle = \frac{u_{1s}}{\omega_1} \sin (\phi_1) \tag{2.58}
\]

thus, loads without any capability of energy storage could have a reactive energy. This confirms that the reactive energy does not have physical meaning and is not associated with the phenomenon of energy storage.

\( \hat{} \) In CPT theory, reactive current is proportional to reactive energy. Since the physical meaning of the reactive energy in CPT remains unclear, the same applies to the reactive current. The remaining current which is not active and not reactive is referred to as a void current. Since the physical meaning of the reactive current is unclear, the physical meaning of the void current is also unclear.

\( \hat{} \) Moreover, it is shown in [48] that the void power (distortion power) is not related to the distortion of the load current with respect to the supply voltage.

\( \hat{} \) It does not work in the case of distorted or unbalanced source voltage. The active and reactive currents in CPT theory are proportional with the source voltage. If the source voltage is distorted, then active and reactive currents contain distortion and unbalanced parts. Simulation results done by Tenti, prove this fact [53].
2.4 Discussions

In this chapter, we aim to discuss why after all the efforts by researchers during recent years to define a generalized power theory, there is still not an accepted one by the power system society. In [50], Emanuel divides power theories into two main classes:

1. Fryze class
2. Budeanu class.

He divides them into two classes not because of their different domains (time/frequency) but because of their different methodology. Fryze divides power or current terms into two main parts which are active and reactive terms, and then the distortion part is defined as a part of active or reactive components.

This is what the authors did in FBD, \( pq \), and Peng’s power theory. On the other hand, according to Budeanu’s definition the power and current terms are divided by three main terms which are named active, reactive and distortion terms. This is like what Czarnecki and Tenti suggest in their definitions, [41], and [36], respectively.

Fryze is severely opposed to define power terms based on Fourier series, because he believes that by considering Gibbs phenomenon at discontinuity points, minimization of the error which is produced by approximating a given function with Fourier series is impossible. Usatin in [5] criticizes Budeanu’s methodology in 1961. He points out the lack of physical interpretation of distortion power and unauthorized summing up of amplitudes of oscillating components of different harmonics.

Moreover, in [50], the author suggests to abolish using a distortion power in power term definitions, because distortion power is a harmonic part of active or reactive powers. If it is defined separately, it loses its physical meaning. From the physics of electrical systems, there are two main paths for transferring energy. The energy may transfer from source to the load (active power), or it is exchanged between phases of the system (reactive power).

Table 2.1 summarizes the properties of the power theories discussed in this work, so it will be very easy to compare them and use them in an appropriate application. Table. II and Table. III show a comparison of different current and power terms, respectively, in some famous power theories which are discussed in this chapter.
Table 2.1. Properties of the most famous power theories

<table>
<thead>
<tr>
<th>Properties</th>
<th>pq</th>
<th>FBD</th>
<th>dq</th>
<th>Peng</th>
<th>CPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fryze/Budeanu</td>
<td>Time</td>
<td>Time</td>
<td>Time</td>
<td>Time</td>
<td>B</td>
</tr>
<tr>
<td>Works with nonlinear loads</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Works with unbalanced loads</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Works with unbalanced voltage</td>
<td>✓</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Works with distorted voltage</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Unbalanced current decomposition</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>Distorted current decomposition</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
</tr>
<tr>
<td>Instantaneous</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>✓</td>
<td>×</td>
</tr>
<tr>
<td>Harmonics &amp; unbalanced decomposition</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 2.2. Current components compared in power theories

<table>
<thead>
<tr>
<th>Current components</th>
<th>pq</th>
<th>FBD</th>
<th>dq</th>
<th>Peng</th>
<th>CPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active</td>
<td>$i_{k\overline{p}}$</td>
<td>$i_{ak}$</td>
<td>$i_{\alpha}$</td>
<td>×</td>
<td>$i_{ak}^b$</td>
</tr>
<tr>
<td>Reactive</td>
<td>$i_{k\overline{q}}$</td>
<td>×</td>
<td>$i_{\alpha}$</td>
<td>×</td>
<td>$i_{ak}^b$</td>
</tr>
<tr>
<td>Distorted</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>$i_{vk}$</td>
</tr>
<tr>
<td>Unbalanced</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>$i_{ak}^u + i_{rk}^u$</td>
</tr>
<tr>
<td>Harmonics + Unbalanced</td>
<td>$i_{kp} + i_{k\overline{q}}$</td>
<td>×</td>
<td>$i_{d} + i_{q}$</td>
<td>×</td>
<td>$i_{ak}^u + i_{rk}^u + i_{vk}$</td>
</tr>
<tr>
<td>Instantaneous active</td>
<td>$i_{kp}$</td>
<td>$i_{pk}$</td>
<td>$i_{d}$</td>
<td>$i_{p}$</td>
<td>×</td>
</tr>
<tr>
<td>Instantaneous reactive</td>
<td>$i_{kq}$</td>
<td>$i_{zk}$</td>
<td>$i_{q}$</td>
<td>$i_{q}$</td>
<td>×</td>
</tr>
<tr>
<td>Zero-sequence</td>
<td>$i_0$</td>
<td>×</td>
<td>$i_0$</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>
Table. 2.3. Power Terms in different power theories

<table>
<thead>
<tr>
<th>Power theory</th>
<th>Active power</th>
<th>Reactive power</th>
<th>Distortion power</th>
</tr>
</thead>
<tbody>
<tr>
<td>pq</td>
<td>P</td>
<td>Q (imaginary power)</td>
<td>-</td>
</tr>
<tr>
<td>FBD</td>
<td>P</td>
<td>Non-active power</td>
<td>-</td>
</tr>
<tr>
<td>dq</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Peng</td>
<td>P</td>
<td>Q</td>
<td>-</td>
</tr>
<tr>
<td>CPT</td>
<td>P</td>
<td>Q (reactive energy)</td>
<td>D</td>
</tr>
</tbody>
</table>

As a conclusion to this chapter, to define a valid generalized power theory, the following facts should clearly be addressed:

- It is very important that each term has an appropriate physical meaning.
- It should be able to work under distorted and unbalanced source voltage as well as distorted and unbalanced load currents.
- It must be able to decompose the unbalanced, distorted, active, reactive, and zero-sequence parts of current in each phase for different applications, such as active filtering.
- It should be able to expand to a multiphase system.
- Time-domain approaches are better than the frequency-based approaches, because they are instantaneous and thus much faster.
- Defining power theories based on Fryze class is preferred.
CHAPTER 3
ENHANCED INSTANTANEOUS POWER THEORY

In this chapter we introduce a modified power theory framework that is able to address some of the corresponding drawbacks of the former methods in terms of nonlinear, unbalanced and distorted voltage source and load conditions\(^5\).

3.1 Overview

Power theories are advancing signal decomposition approaches used for electrical power signal analysis. Nowadays they are key tools for designing reliable and efficient power electronic interface controllers. Due to uncertainty in power generation within new technology of distributed generation units such as Microgrids (accounting for renewable energy resources), most of these theories are not directly applicable in the era of new power engineering systems. On top of that the traditional formulations are not appropriate once we face with the nonlinearity issue in the consumption (accounting for power electronic interfaces) along a weak-grid that may not maintain steady-voltage or frequency. However, the ever-increasing aggregation of smart systems in addition to higher penetration of renewable energy sources in power systems is an unpreventable fact in the electrical power delivery system. Therefore, a new signal decomposition methodology with higher level of selectivity must be developed to modify the traditional power theories and improve the control strategies from designing alternation perspective.

This work describes a novel formulation for an advanced instantaneous power theory for unbalanced and non-linear three-phase power systems. It demonstrates a proper decomposition of current terms for cases of both symmetrical balanced and asymmetrical unbalanced and distorted voltage sources. In addition to mathematical analysis, this research provides simulations using PSIM and Matlab/Simulink, respectively, supporting the performance evaluation of the proposed theory in terms of active filtering method. System performance is examined under different loads and source conditions, contributing to the overall improvement of the apparent power, reactive power, power factor and many other related quality indices. Such a new comprehensive approach

\(^5\) This chapter is based on the following set of papers: [80]
is helpful in optimizing control strategies for power electronic interfaces and power quality compensators.

### 3.2 Enhanced Instantaneous Power Theory and Current Decomposition Methodology

The basic idea of this work comes from the \(pq\) theory (which has been developed by Akagi) and Peng’s power theory [12], [24]. The proposed theory is developed to decompose the electrical currents of a neutralized 3-phase power system (4-wired system) to a set of meaningful sub-components that are used for active filtering and control of power electronic converters. A three-phase power system is shown in Figure 3.1 where instantaneous voltages and currents are defined as instantaneous space vectors \((v_{abc} \text{ and } i_{abc})\).

![Figure 3.1. Three-phase power system](image)

In general, our methodology in this work is divided into three main parts. In the first part which starts on Section 3.3 entitled “Mathematical methodology for current decomposition in three-phase non-sinusoidal unbalanced system with symmetrical balanced voltage source”, we considered the voltage source to be a sinusoidal balanced symmetrical voltage, thus, all definitions are developed based on this assumption. In this situation, the source of nonlinearity and unbalance in the current terms is caused solely by the nonlinear and unbalanced loads. However, in the second part of the methodology section, Section 3.4, entitled “Methodology for current decomposition in three-phase non-sinusoidal unbalanced system with asymmetrical unbalanced voltage source”, we considered an asymmetrical and unbalanced voltage source in addition to the nonlinear and unbalanced loads. Finally, in Section 3.5 EIPT is extended for the distorted voltage source condition. From our mathematical redefinitions, the effects of unbalanced and distorted parts of voltage source has been interpreted along with new definitions and appears within the consequent formulations.
3.2.1 Concordia Transformation (αβ0 transformation)

There is an algebraic transformation known as either Clarke or Concordia transformation to map the three-phase instantaneous voltages and currents in the abc coordinates to the αβ0 coordinates. The Clarke matrix is a constant project matrix widely implemented in electrical drives for control purposes (because the magnitude of voltages and currents are preserved), while the Concordia matrix is considered a demodulated signal with power invariance used in faulty conditions and power analysis [54]. The main reason of using this later transformation is to separate the zero sequence components of the abc phase voltage and current signals. Regarding what discussed above, we use the Concordia transformation to convert instantaneous currents and voltages from abc into αβ0 frame. The Concordia transformation of three-phase voltages and currents are given by:

\[
\begin{bmatrix}
\nu_α \\
\nu_β \\
\nu_0
\end{bmatrix} =
\begin{bmatrix}
\nu_α \\
\nu_b \\
\nu_c
\end{bmatrix} = \begin{bmatrix}
T_{αβ0}
\end{bmatrix}
\begin{bmatrix}
\nu_α \\
\nu_β \\
\nu_0
\end{bmatrix},
\begin{bmatrix}
i_α \\
i_β \\
i_0
\end{bmatrix} = \begin{bmatrix}
i_α \\
i_b \\
i_c
\end{bmatrix} = \begin{bmatrix}
T_{αβ0}
\end{bmatrix}
\begin{bmatrix}
i_α \\
i_β \\
i_0
\end{bmatrix}
\tag{3.1}
\]

where, \(\nu_α\) and \(\nu_β\) are the instantaneous voltages, \(i_α\) and \(i_β\) are the instantaneous currents, in \(α\) and \(β\) axis respectively, and \(\nu_0\) and \(i_0\) are zero sequence (also termed as homopolar) components of voltages and currents. \(T_{αβ0}\) is known as \(αβ0\) transformation matrix with the following format:

\[
T_{αβ0} = \begin{bmatrix}
\frac{2}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
\sqrt{\frac{1}{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\tag{3.2}
\]

(The reconfiguration of the axes under Concordia transformation is shown in Appendix.C.) Similarly, the inverse of Concordia transformation maps the voltages and currents in \(αβ0\) frame to the instantaneous three-phase voltages and currents in \(abc\) frame:

\[
\begin{bmatrix}
\nu_α \\
\nu_b \\
\nu_c
\end{bmatrix} = \begin{bmatrix}
\frac{2}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\
-\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
\nu_α \\
\nu_β \\
\nu_0
\end{bmatrix} = \begin{bmatrix}
\nu_α \\
\nu_β \\
\nu_0
\end{bmatrix} = \begin{bmatrix}
T_{αβ0}
\end{bmatrix}^{-1}
\begin{bmatrix}
\nu_α \\
\nu_β \\
\nu_0
\end{bmatrix}
\tag{3.3}
\]
\[
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} = \begin{bmatrix}
\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}
\end{bmatrix}\begin{bmatrix}
i_\alpha \\
i_\beta \\
i_0
\end{bmatrix} = \left[T_{\alpha\beta0}\right]^{-1}\begin{bmatrix}
i_\alpha \\
i_\beta \\
i_0
\end{bmatrix}
\]

(3.4)

Considering the Concordia transformation matrix \(T_{\alpha\beta0}\), one may easily find that:

\[
T_{\alpha\beta0}T_{\alpha\beta0}^T = [I]_{3\times3}
\]

(3.5)

3.3 Mathematical Methodology For Current Decomposition In Three-Phase Non-sinusoidal Unbalanced System With Symmetrical Balanced Voltage Source

In this part, we will decompose the current components in the presence of strong grid conditions. So here the balanced voltage source with steady voltage and frequency is considered as a grid (Please find the full list of all quantities, and components in the nomenclature).

3.3.1 Fundamental Definitions

The definition of instantaneous active power, \(p\), is almost the same in most of the power theories. This electrical quantity is defined as a scalar product of instantaneous current and voltage vectors. Within the \(\alpha\beta0\) frame notation, \(p\) is calculated as:

\[
p = v_{\alpha\beta0} \cdot i_{\alpha\beta0} = v_{\alpha\beta0} i_{\alpha\beta0}^T = \begin{bmatrix}
v_\alpha \\
v_\beta \\
v_0
\end{bmatrix} \begin{bmatrix}
i_\alpha \\
i_\beta \\
i_0
\end{bmatrix} = v_a i_a + v_\beta i_\beta + v_0 i_0
\]

(3.6)

where “\(\cdot\)” denotes the dot (internal or scalar) product of two vectors. By definition, \(p\) is the instantaneous active power and corresponds to the energy per time unity which is transferred from the power supply to the load. We separate (3.6) into the following two arbitrary terms:

\[
p_{\alpha\beta} = v_a i_a + v_\beta i_\beta
\]

(3.7)

\[
p_0 = v_0 i_0
\]

(3.8)

where \(p_{\alpha\beta}\) is termed as instantaneous active heteropolar power and \(p_0\) is termed as instantaneous active unbalanced homopolar power.
**Property 3.1:** The following equality can be obtained for instantaneous active power in $\alpha\beta\gamma$ frame versus $abc$ frame:

$$p = v_{abc} \cdot i_{abc} = v_{\alpha\beta\gamma} \cdot i_{\alpha\beta\gamma}$$  \hfill (3.9)

After a quick search beyond the related articles, one may understand that similar to the active power definition, the corresponding definition of the active current, $i_p$, is also the same within different power theories:

$$i_p = \frac{p \cdot v_{abc}}{v_{abc} \cdot v_{abc}} \text{ in } \alpha\beta\gamma \text{ frame}$$

$$i_p = \frac{p \cdot v_{\alpha\beta\gamma}}{v_{\alpha\beta\gamma} \cdot v_{\alpha\beta\gamma}}$$  \hfill (3.10)

where the denominators can be expanded as follows:

$$v_{abc} \cdot v_{abc} = v_{\alpha\beta\gamma} \cdot v_{\alpha\beta\gamma} = v_a^2 + v_b^2 + v_c^2 = v_\alpha^2 + v_\beta^2 + v_\gamma^2$$  \hfill (3.11)

Now, following a similar approach as Peng (which has been developed in the $abc$ frame), using the cross product of the instantaneous voltage and the instantaneous current vectors, we define the following instantaneous space vector, $q$, as the reactive power vector in the $\alpha\beta\gamma$ frame (alternatively $q_{\alpha\beta\gamma}$):

$$q = v_{\alpha\beta\gamma} \times i_{\alpha\beta\gamma}$$  \hfill (3.12)

where “$\times$” denotes the cross product and $v_{\alpha\beta\gamma} \times i_{\alpha\beta\gamma}$ is a vector that is perpendicular to $v_{\alpha\beta\gamma}$ and $i_{\alpha\beta\gamma}$. Consider the following definition for the reactive current:

$$i_q = \frac{q \times v_{\alpha\beta\gamma}}{v_{\alpha\beta\gamma} \cdot v_{\alpha\beta\gamma}}$$,  \hfill (3.13)

We show that the following statement is valid (that verifies the definitions in (3.12) and (3.13) are physically correct):

$$i_p + i_q = i$$  \hfill (3.14)

where $q$ is the instantaneous reactive or non-active power and $i_q$ is the instantaneous reactive current. Applying (3.10) and (3.13) in (3.14) we have:

$$i_p + i_q = \frac{p \cdot v_{\alpha\beta\gamma}}{v_{\alpha\beta\gamma} \cdot v_{\alpha\beta\gamma}} + \frac{q \times v_{\alpha\beta\gamma}}{v_{\alpha\beta\gamma} \cdot v_{\alpha\beta\gamma}} = \frac{(v_{\alpha\beta\gamma} \cdot i_{\alpha\beta\gamma}) \cdot v_{\alpha\beta\gamma}}{v_{\alpha\beta\gamma} \cdot v_{\alpha\beta\gamma}} + \frac{(v_{\alpha\beta\gamma} \times i_{\alpha\beta\gamma}) \times v_{\alpha\beta\gamma}}{v_{\alpha\beta\gamma} \cdot v_{\alpha\beta\gamma}}$$,  \hfill (3.15)

utilizing the following vector product property:
\[(x \times y) \cdot z = -(y \cdot z)x + (x \cdot z)y, \quad (3.16)\]

we end up with:
\[
\rightarrow i_p + i_q = \frac{(v_{a\beta_0} \times i_0) v_{a\beta_0}}{v_{a\beta_0} \times i_0} + \frac{(v_{a\beta_0} \times i_{a\beta_0}) \times v_{a\beta_0}}{v_{a\beta_0} \times i_{a\beta_0}} = \frac{(v_{a\beta_0} \times i_{a\beta_0}) \times v_{a\beta_0}}{v_{a\beta_0} \times i_{a\beta_0}} + \frac{[-(i_{a\beta_0} \cdot v_{a\beta_0}) v_{a\beta_0} + (v_{a\beta_0} \times v_{a\beta_0}) i_{a\beta_0}]}{v_{a\beta_0} \times i_{a\beta_0}} = i_{a\beta_0} = i \quad (3.17)
\]

and as a result, the assumption for the definitions of \( q \) and \( i_q \) are validated accordingly. Now, we expand (3.12) as follows:

\[
q_{a\beta_0} = v_{a\beta_0} \times i_{a\beta_0} = \begin{bmatrix} v_0 \\ i_0 \\ v_\alpha \\ i_\alpha \\ v_\beta \\ i_\beta \end{bmatrix} = \begin{bmatrix} q_\beta_0 \\ q_{a0} \\ q_{a\beta} \end{bmatrix} \quad (3.18)
\]

where \( q_{\beta_0}, q_{a0} \) and \( q_{a\beta} \) are calculated as:

\[
q_{\beta_0} = -v_0 i_\beta + v_\beta i_0 \quad (3.19)
\]
\[
q_{a0} = v_0 i_\alpha - v_\alpha i_0 \quad (3.20)
\]
\[
q_{a\beta} = -v_\beta i_\alpha + v_\alpha i_\beta \quad (3.21)
\]

\( q_{a0} \) and \( q_{\beta_0} \) are the instantaneous unbalanced homopolar reactive power components and \( q_{a\beta} \) is the instantaneous reactive heteropolar power component.

**Property 3.2**: The following relation can be found between Peng’s reactive power vector, \( q_{abc} \), in \( abc \) frame versus ours in \( a\beta_0 \) frame:

\[
q_{a\beta_0} = v_{a\beta_0} \times i_{a\beta_0} = \left( [T_{a\beta_0}] v_{abc} \right) \times \left( [T_{a\beta_0}] i_{abc} \right) = [T_{a\beta_0}] (v_{abc} \times i_{abc}) = [T_{a\beta_0}] q_{abc} \quad (3.22)
\]

The instantaneous amount of power that is transferred between phases (reactive power) is defined as the magnitude of \( q_{a\beta_0} \), is shown with \( Q \):

\[
Q = |q_{a\beta_0}| = \sqrt{q_{\beta_0}^2 + q_{a0}^2 + q_{a\beta}^2} \quad (3.23)
\]

In a system without zero sequence components: \( v_0 = i_0 = 0, q_{\beta_0} = q_{a0} = 0 \), accordingly, as a result:
\[ q_{\alpha\beta} = q_{\alpha\beta} = v_\alpha i_\beta - v_\beta i_\alpha \] (3.24)

where \( q_{\alpha\beta} \) is equal to the imaginary power in conventional \( pq \) theory [4].

**Property 3.3:** It can be shown that the voltage vector \( v_{\alpha\beta} \) is perpendicular to the reactive currents, while it is parallel with active current:

\[ i_p \times v_{\alpha\beta} = 0 \] (3.25)

\[ i_q \cdot v_{\alpha\beta} = 0 \] (3.26)

(mathematical proof can be found in Appendix. A)

In this framework, the instantaneous apparent power and the power factor are defined as follows:

\[
3-\Phi : \begin{cases} P = |v_{\alpha\beta} \cdot i_{\alpha\beta}| \\ Q = |v_{\alpha\beta} \times i_{\alpha\beta}| \end{cases} \xrightarrow{yields} S^2 = P^2 + Q^2 \xrightarrow{yields} S = IV = |i_{\alpha\beta}| |v_{\alpha\beta}|, \quad (3.27)
\]

where \( P \) is the real power, and \( Q \) is the reactive power (the proof of (3.27) can be find in the Appendix. B). \( V \) and \( I \) are the \textit{rms} values of voltage and current and are defined as follows:

\[
V = |v_{\alpha\beta}| = \sqrt{v_\alpha^2 + v_\beta^2} \quad (3.28)
\]

\[
I = |i_{\alpha\beta}| = \sqrt{i_0^2 + i_\alpha^2 + i_\beta^2} \quad (3.29)
\]

\[
PF = \frac{P}{S}, \quad (3.30)
\]

3.3.2. EIPT Current Decomposition Approach: Concepts and Components

In what comes next, (3.31) shows the relation between different power components according to different voltage and current terms and (3.32) represents its inverse form:

\[
\begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\beta\alpha} \\ q_{\alpha\beta} \\ q_{\alpha\alpha} \\ q_{\beta\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & v_0 & 0 & 0 \\ v_\alpha & v_\beta & 0 & \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} \quad (3.31)
\]
These two equations are the basis of the formulation that we use in order to decompose the current components in $\alpha\beta\gamma$ frame according to different power components. The corresponding coefficients of the instantaneous active unbalanced homopolar power ($p_0$) in (3.31), are utilized to find the active unbalanced homopolar current terms as follows:

$$i_{\alpha p_0} = 0, i_{\beta p_0} = 0, i_{\theta p_0} = \frac{v_0 p_0}{v_0^2}.$$  \hspace{1cm} (3.33)

Applying the inverse Concordia transformation, we calculate:

$$i_{\alpha p_0} = i_{\beta p_0} = i_{\theta p_0} = \frac{1}{\sqrt{3}} \frac{v_0 p_0}{v_0^2}.$$  \hspace{1cm} (3.34)

where, $i_{\alpha p_0}$, $i_{\beta p_0}$, $i_{\theta p_0}$ are the active unbalanced homopolar current components in phase $a$, $b$ and $c$ respectively. To find the active heteropolar current components, we used the corresponding coefficients of $p_{\alpha\beta}$ in (3.32):

$$\begin{align*}
    i_{\alpha p_{\alpha\beta}} &= \frac{v_0 p_{\alpha\beta}}{v_{\alpha\beta}^2} \\
    i_{\beta p_{\alpha\beta}} &= \frac{v_0 p_{\alpha\beta}}{v_{\alpha\beta}^2} \\
    i_{\theta p_{\alpha\beta}} &= 0
\end{align*}$$ \hspace{1cm} (3.35)

we can decompose the instantaneous active power in $\alpha\beta$ axis ($p_{\alpha\beta}$) into its dc and oscillating parts as follows:

$$p_{\alpha\beta} = \bar{p}_{\alpha\beta} + \tilde{p}_{\alpha\beta}.$$ \hspace{1cm} (3.36)

where $\bar{p}_{\alpha\beta}$ is termed as the average active balanced heteropolar power and $\tilde{p}_{\alpha\beta}$ is termed as oscillating heteropolar power, which is caused by harmonics and imbalance between phases in the system.

To find the sinusoidal balanced active currents, we use the average active heteropolar power ($\bar{p}_{\alpha\beta}$):
\[
\begin{align*}
\begin{cases}
i_{a\beta} &= \frac{v_{a\beta}}{v_{a\alpha} \omega^2} i_{a\alpha} \\
i_{b\beta} &= \frac{v_{b\beta}}{v_{a\beta} \omega^2} i_{b\alpha} \\
i_{c\beta} &= 0
\end{cases}
\end{align*}
\]

where, \(i_{a\alpha}, i_{b\alpha} \) and \(i_{c\alpha} \) are active balanced sinusoidal current components in phase \(a, b \) and \(c \) respectively. To find distorted and unbalanced active current components, we need to use the oscillating active heteropolar power \(\vec{P}_{ab} \) in our calculation:

\[
\begin{align*}
\begin{cases}
i_{a\beta} &= \frac{v_{a\beta}}{v_{a\alpha} \omega^2} i_{a\alpha} \\
i_{b\beta} &= \frac{v_{b\beta}}{v_{a\beta} \omega^2} i_{b\alpha} \\
i_{c\beta} &= 0
\end{cases}
\end{align*}
\]

by implementing inverse Concordia transformation, we got \(i_{a\beta}, i_{b\beta} \) and \(i_{c\beta} \), which are the active oscillating current components in phase \(a, b \) and \(c \), respectively. These oscillating components contain both the distorted and unbalanced parts of the current. These two parts can be easily separated by implementing a band-pass filter with a notch frequency as tightly tuned over 60 Hz.

![Bandpass Filter](image)

Figure 3.2. Bandpass filter used for extraction of unbalanced active currents

where \(i_{pua}, i_{pub} \) and \(i_{puc} \) are active heteropolar unbalanced currents in phase \(a, b \) and \(c \), respectively. Moreover, we have:

\[
\begin{align*}
\begin{cases}
i_{a\beta} - i_{pua} &= i_{pHa} \\
i_{b\beta} - i_{pub} &= i_{pHb} \\
i_{c\beta} - i_{puc} &= i_{pHc}
\end{cases}
\end{align*}
\]

where \(i_{aHa}, i_{aHb} \) and \(i_{aHc} \) are active distorted currents in phase \(a, b \) and \(c \), respectively.

The homopolar part of reactive current is calculated from the zero sequence components of reactive vector \((q_{a0} \) and \(q_{\beta0}) \), so from (3.32) we have:
\[
\begin{align*}
\begin{cases}
    i_{aq_0} &= \frac{\nu_{q_{a_0}}}{\nu_{\alpha_0}} \\
    i_{\beta q_0} &= \frac{\nu_{q_{\beta_0}}}{\nu_{\alpha_0}} \\
    i_{0q_0} &= \frac{\nu_{q_{0_0}}}{\nu_{\alpha_0}}
\end{cases}
\quad \text{in abc frame}
\end{align*}
\]
\[
\begin{bmatrix}
    i_{aq_0} \\
    i_{\beta q_0} \\
    i_{0q_0}
\end{bmatrix} = \begin{bmatrix}
    \frac{2}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
    i_{aq_0} \\
    i_{\beta q_0} \\
    i_{0q_0}
\end{bmatrix}
\]
\]
(3.40)

where \(i_{aq_0}, i_{\beta q_0}, i_{0q_0}\) are termed as homopolar reactive current components in phase \(a, b\) and \(c\), respectively. To find the heteropolar reactive components of the current, we need to use the coefficients of \(q_{a\beta}\) in (3.32):

\[
\begin{align*}
\begin{cases}
    i_{aq_{a\beta}} &= -\frac{\nu_{q_{a\beta}}}{\nu_{\alpha_0}} \\
    i_{\beta q_{a\beta}} &= \frac{\nu_{q_{a\beta}}}{\nu_{\alpha_0}} \\
    i_{0q_{a\beta}} &= 0
\end{cases}
\quad \text{in abc frame}
\end{align*}
\]
\[
\begin{bmatrix}
    i_{aq_{a\beta}} \\
    i_{\beta q_{a\beta}} \\
    i_{0q_{a\beta}}
\end{bmatrix} = \begin{bmatrix}
    \frac{2}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
    i_{aq_{a\beta}} \\
    i_{\beta q_{a\beta}} \\
    i_{0q_{a\beta}}
\end{bmatrix}
\]
(3.41)

Similar to (3.36), one may decompose the instantaneous reactive power in \(a\beta\) axis \((q_{a\beta})\) into its average and oscillating components as follows:

\[
q_{a\beta} = \bar{q}_{a\beta} + \bar{q}_{a\beta},
\]
(3.42)

where \(\bar{q}_{a\beta}\) is termed as the average balanced reactive power and \(\bar{q}_{a\beta}\) as oscillating reactive power, which is basically caused by the harmonics and unbalanced between phases in the system. The sinusoidal reactive balanced components of the current are calculated from the \(\bar{q}_{a\beta}\) as follows:

\[
\begin{align*}
\begin{cases}
    i_{aq_{a\beta}} &= -\frac{\nu_{q_{a\beta}}}{\nu_{\alpha_0}} \\
    i_{\beta q_{a\beta}} &= \frac{\nu_{q_{a\beta}}}{\nu_{\alpha_0}} \\
    i_{0q_{a\beta}} &= 0
\end{cases}
\quad \text{in abc frame}
\end{align*}
\]
\[
\begin{bmatrix}
    i_{aq_{a\beta}} \\
    i_{\beta q_{a\beta}} \\
    i_{0q_{a\beta}}
\end{bmatrix} = \begin{bmatrix}
    \frac{2}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
    i_{aq_{a\beta}} \\
    i_{\beta q_{a\beta}} \\
    i_{0q_{a\beta}}
\end{bmatrix}
\]
(3.43)

where, \(i_{aq}, i_{bq}, i_{cq}\) are sinusoidal BR current components in phase \(a, b\) and \(c\) respectively. To find distorted and unbalanced reactive current components, the oscillating reactive power component is used in our calculation:

\[
\begin{align*}
\begin{cases}
    i_{aq_{a\beta}} &= -\frac{\nu_{q_{a\beta}}}{\nu_{\alpha_0}} \\
    i_{\beta q_{a\beta}} &= \frac{\nu_{q_{a\beta}}}{\nu_{\alpha_0}} \\
    i_{0q_{a\beta}} &= 0
\end{cases}
\quad \text{in abc frame}
\end{align*}
\]
\[
\begin{bmatrix}
    i_{aq_{a\beta}} \\
    i_{\beta q_{a\beta}} \\
    i_{0q_{a\beta}}
\end{bmatrix} = \begin{bmatrix}
    \frac{2}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\
    -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
    i_{aq_{a\beta}} \\
    i_{\beta q_{a\beta}} \\
    i_{0q_{a\beta}}
\end{bmatrix}
\]
(3.44)
$i_{aq}$, $i_{bq}$, $i_{cq}$ are distorted and unbalanced reactive current components in phase $a$, $b$ and $c$ respectively. To separate these two parts, we can use bandpass filters:

![Diagram of Bandpass Filter]

Figure 3.3 Bandpass filter used for extraction of unbalanced reactive currents

where $i_{qua}$, $i_{qub}$ and $i_{quc}$ are reactive heteropolar unbalanced currents in phase $a$, $b$ and $c$, respectively. Moreover, we have:

$$\begin{align*}
    \begin{cases}
    i_{aq} - i_{qua} = i_{qHa} \\
    i_{bq} - i_{qub} = i_{qHb} \\
    i_{cq} - i_{quc} = i_{qHc}
    \end{cases}
\end{align*}$$

(3.45)

where $i_{rHa}$, $i_{rHb}$ and $i_{rHc}$ are reactive distorted currents in phase $a$, $b$ and $c$, respectively.

Now we can define the instantaneous active and reactive powers for each phase of the system as follows:

$$\begin{align*}
    \begin{cases}
    p_a = (i_{ap_0} + i_{ap} + i_{pHa} + i_{pua})v_a \\
    p_b = (i_{bp_0} + i_{bp} + i_{pHb} + i_{pub})v_b \\
    p_c = (i_{cp_0} + i_{cp} + i_{pHc} + i_{puc})v_c
    \end{cases}
\end{align*}$$

(3.46)

$$\begin{align*}
    \begin{cases}
    q_a = (i_{aq_0} + i_{aq} + i_{qHa} + i_{qua})v_a \\
    q_b = (i_{bq_0} + i_{bq} + i_{qHb} + i_{qub})v_b \\
    q_c = (i_{cq_0} + i_{cq} + i_{qHc} + i_{quc})v_c
    \end{cases}, \quad q_a(t) + q_b(t) + q_c(t) = 0
\end{align*}$$

(3.47)

3.4 Mathematical Methodology for Current Decomposition In Three-phase Non-sinusoidal Unbalanced System with Asymmetrical Unbalanced Voltage Source

In this section, we will decompose the current components in the presence of weak grid conditions where we do not have balanced voltage in the grid side. As a result, we will have the negative sequence active and reactive current components, as well as zero and positive sequence active and reactive current components.
3.4.1 A Note on the Source of Unbalance

Before going through the details of the mathematical approach we would like to clarify the following points: a) In case of a system without neutral wiring, we will only face with a single type of unbalanced which is namely the *between phase unbalanced*, we term this type as *heteropolar unbalanced phenomenon*. b) When we are working with a neutral wiring architecture we will face with two following types of imbalances in the corresponding current components:

1. The unbalanced between each phase versus neutral wire is modelled in terms of *unbalanced homopolar or zero-sequence components*, which appears in two current terms namely: unbalanced active/reactive homopolar current $i_{xp_0}, i_{xq_0}$ (where index $x$ refers to the phase name). These two components are calculated by exploiting homopolar current $i_0$ and voltage $v_0$ as detailed in section 2-2 and 2-3 over each phase (thus 6 terms over all 3 phases).

2. The unbalanced between phases which we arbitrarily term them as *unbalanced heteropolar components* and appears in two current terms namely: unbalanced active/reactive heteropolar current $i_{xp}, i_{xq}$ (where index $x$ refers to the phase name). These two components are calculated by exploiting the oscillating parts of active and reactive powers $\bar{p}_{a\beta}$ and $\bar{q}_{a\beta}$ through an auxiliary bandpass filtering step over each phase (thus 6 terms over all three phases).

The mathematical formulation under distorted voltage source condition is started from here. While the unbalanced voltage source is considered, it is natural to initially find the zero, positive and negative sequence components of the voltage on each individual phase using Fortescue transformation as follows:

$$
\begin{bmatrix}
  v_0 \\
  v_+ \\
  v_-
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
  1 & 1 & 1 \\
  1 & \gamma & \gamma^2 \\
  1 & \gamma^2 & \gamma
\end{bmatrix} \begin{bmatrix}
  v_a \\
  v_b \\
  v_c
\end{bmatrix}
$$

(3.48)

Where the phase shift operator is defined as $\gamma = 1\angle 120^\circ$. Inversely the phase voltages are calculated as:

$$
\begin{align*}
  v_a &= v_0 + v_+ + v_- \\
  v_b &= v_0 + \gamma^2 v_+ + \gamma v_- \\
  v_c &= v_0 + \gamma v_+ + \gamma^2 v_-
\end{align*}
$$

(3.49)
Using the phase shift operator $\gamma$, we represent the positive and negative sequences of the voltage in each phase within the following notation:

\[
\begin{align*}
\begin{cases}
\nu_{a+} = \nu_+ \\
\nu_{b+} = \gamma^2\nu_+ \\
\nu_{c+} = \gamma\nu_+
\end{cases},
\begin{cases}
\nu_{a-} = \nu_- \\
\nu_{b-} = \gamma\nu_- \\
\nu_{c-} = \gamma^2\nu_-
\end{cases}
\end{align*}
\] (3.50)

We now exploit this notation to generalize the definition of voltage components in terms of the Concordia transformation to the unbalanced voltage situation as follows:

\[
\begin{align*}
&\begin{bmatrix}
\nu_{a+} \\
\nu_{b+}
\end{bmatrix} =
\begin{bmatrix}
\frac{\sqrt{3}}{2} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
\nu_{a+} \\
\nu_{b+} \\
\nu_{c+}
\end{bmatrix} \\
&\begin{bmatrix}
\nu_{a-} \\
\nu_{b-}
\end{bmatrix} =
\begin{bmatrix}
\frac{\sqrt{3}}{2} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\
0 & \frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
\nu_{a-} \\
\nu_{b-} \\
\nu_{c-}
\end{bmatrix}
\end{align*}
\] (3.51)

where $\nu_{a+}$, $\nu_{a-}$, $\nu_{b+}$ and $\nu_{b-}$ are representing the positive and negative sequence components within the $\alpha - \beta$ transformation domain. Since the zero-component of the voltage will remain the same under Concordia transformation, the square of the voltage magnitude is calculated as:

\[
|\nu_{\alpha\beta0}|^2 = |\nu_{a+}^2 + \nu_{a-}^2 + \nu_{b+}^2 + \nu_{b-}^2 + \nu_0|^2
\] (3.52)

Moreover, within the $\alpha - \beta$ transformation subspace, the positive and negative components can be vectorially added to form the total $\alpha - \beta$ voltage sub-vectors. In another word, from the vector space definitions, each term can be quantized in terms of the following factorized summation:

\[
\begin{align*}
\begin{cases}
\nu_\alpha = \nu_{a+} + \nu_{a-} \\
\nu_\beta = \nu_{b+} + \nu_{b-}
\end{cases}
\end{align*}
\] (3.53)

as a result, the transformed voltage vector is decomposed as:
\[ v_{\alpha\beta 0} = \begin{bmatrix} v_{\alpha+} \\ v_{\beta+} \\ 0 \end{bmatrix} + \begin{bmatrix} v_{\alpha-} \\ v_{\beta-} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ v_0 \end{bmatrix}. \]  

(3.54)

We may now define the corresponding positive and negative power components in terms of voltage and current vectors within the \( \alpha - \beta \) transformation domain as follows:

\[
\begin{align*}
    p_{\alpha\beta}^+ &= v_{\alpha+}i_{\alpha} + v_{\beta+}i_{\beta} \\
    p_{\alpha\beta}^- &= v_{\alpha-}i_{\alpha} + v_{\beta-}i_{\beta}
\end{align*}
\]  

(3.55)

The total active power is then calculated as:

\[ p_{\alpha\beta 0} = p_{\alpha\beta}^+ + p_{\alpha\beta}^- + p_0 \]  

(3.56)

Using a simple mathematical factorization in (3.55) and considering \( v_{\alpha} \) and \( v_{\beta} \) from (3.53) we can easily see:

\[ p_{\alpha\beta 0} = v_{\alpha}i_{\alpha} + v_{\beta}i_{\beta} + v_0i_0 = p_{\alpha\beta} + p_0. \]  

(3.57)

This is a similar final notation as in case of balance voltage source with different components which are now calculated by our definitions in (3.51). Here again we decompose the instantaneous active power in terms of average and sinusoidal parts: \( p_{\alpha\beta} = \bar{p}_{\alpha\beta} + \tilde{p}_{\alpha\beta} \)

In order to find the sinusoidal positive sequence balanced active current components, we need to use the average active power component \( (\bar{p}_{\alpha\beta}) \) in our calculation. Moreover, the oscillating parts of the positive sequence balanced active current components are defined based on the \( \tilde{p}_{\alpha\beta} \) (A similar approach is used to find both the sinusoidal and oscillating current components in terms of reactive power):

\[
\begin{align*}
    i_{\alpha \bar{p}_{\alpha\beta}}^+ &= \frac{v_{\alpha+}}{v_{\alpha\beta 0}} \bar{p}_{\alpha\beta} \\
    i_{\beta \bar{p}_{\alpha\beta}}^+ &= \frac{v_{\beta+}}{v_{\alpha\beta 0}} \bar{p}_{\alpha\beta} \\
    i_{0 \bar{p}_{\alpha\beta}} &= 0
\end{align*}
\]  

(3.58)

\( i_{\alpha}, i_{\beta}, i_{c} \) are obtained which are termed as sinusoidal positive sequence balanced active current components in phase \( a, b \) and \( c \), respectively. On the other hand, negative sequence active current components are calculated from:
\[
\begin{align*}
\left\{ i_{q_{\alpha\beta}} &= \frac{v_{\alpha\beta}}{v_{\alpha\beta}} \bar{a}_{\alpha\beta} \quad \text{in abc frame} \quad \begin{bmatrix} i_{pua} \\ i_{pub} \\ i_{puc} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{a_{\alpha\beta}}^- \\ i_{b_{\alpha\beta}}^- \end{bmatrix} \\
i_{q_{\alpha\beta}}^- &= \frac{v_{\alpha\beta}}{v_{\alpha\beta}} \bar{a}_{\alpha\beta} \\
i_{q_{\alpha\beta}} &= 0
\end{align*}
\]

where \( i_{a_{\alpha\beta}}^- \), \( i_{b_{\alpha\beta}}^- \) and \( i_{c_{\alpha\beta}}^- \), are negative sequence active current components in phase \( a \), \( b \) and \( c \), respectively. Moreover, the oscillating parts of the active current components are defined as follows:

\[
\begin{align*}
\left\{ i_{\alpha_{\alpha\beta}} &= \frac{v_{\alpha\beta}}{v_{\alpha\beta}} \bar{a}_{\alpha\beta} \quad \text{in abc frame} \quad \begin{bmatrix} i_{a_{\alpha\beta}} \\ i_{b_{\alpha\beta}} \\ i_{c_{\alpha\beta}} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{a_{\alpha\beta}}^- \\ i_{b_{\alpha\beta}}^- \end{bmatrix} \\
i_{\beta_{\alpha\beta}} &= \frac{v_{\alpha\beta}}{v_{\alpha\beta}} \bar{a}_{\alpha\beta} \\
i_{\gamma_{\alpha\beta}} &= 0
\end{align*}
\]

where \( i_{pua}, i_{pub}, \) and \( i_{puc} \) are distorted and unbalanced active current components in phase \( a \), \( b \) and \( c \) respectively. To separate these two parts, we can use bandpass filters:

\[
\begin{array}{c}
\text{i}_{a\beta} \\
\text{i}_{b\beta} \\
\text{i}_{c\beta}
\end{array} \quad \text{Bandpass Filter} \quad \frac{60\text{Hz}}{} \quad \begin{array}{c}
\text{i}_{pua} \\
\text{i}_{pub} \\
\text{i}_{puc}
\end{array}
\]

Figure 3.4 Bandpass filter used for extraction of unbalanced active currents

where \( i_{pua}, i_{pub} \) and \( i_{puc} \) are active unbalanced currents in phase \( a \), \( b \) and \( c \), respectively. Moreover, we have:

\[
\begin{align*}
i_{a\beta} - i_{pua} &= i_{pHa} \\
i_{b\beta} - i_{pub} &= i_{pHb} \\
i_{c\beta} - i_{puc} &= i_{pHc}
\end{align*}
\]

where \( i_{pHa}, i_{pHb} \) and \( i_{pHc} \) are active distorted currents in phase \( a \), \( b \) and \( c \), respectively.

We will follow the same procedure for calculating the positive and negative sequences of reactive current. For the positive sequence reactive current we have:

\[
\begin{align*}
\left\{ i_{q_{\alpha\beta}}^+ &= -\frac{v_{\alpha\beta}}{v_{\alpha\beta}} \bar{a}_{\alpha\beta} \quad \begin{bmatrix} i_{a_{\alpha\beta}} \\ i_{b_{\alpha\beta}} \\ i_{c_{\alpha\beta}} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{a_{\alpha\beta}}^+ \\ i_{b_{\alpha\beta}}^+ \end{bmatrix} \\
i_{q_{\alpha\beta}}^+ &= \frac{v_{\alpha\beta}}{v_{\alpha\beta}} \bar{a}_{\alpha\beta} \\
i_{q_{\alpha\beta}} &= 0
\end{align*}
\]

where \( i_{a_{\alpha\beta}}^+, i_{b_{\alpha\beta}}^+ \) and \( i_{c_{\alpha\beta}}^+ \), are positive sequence reactive current components in phase \( a \), \( b \) and \( c \), respectively.
\( i_{aq}, i_{bq}, i_{cq} \) are sinusoidal balanced positive sequence reactive current components in phase \( a \), \( b \) and \( c \) respectively. Negative sequence reactive current components are obtained from:

\[
\begin{align*}
\begin{cases}
    i_{a\alpha_{a\beta}} &= -\frac{v_{\beta}}{v_{\alpha\beta}^2} \bar{q}_{a\beta} & \text{in abc frame} \\
    i_{\beta\alpha_{a\beta}} &= \frac{v_{\alpha}}{v_{\alpha\beta}^2} \bar{q}_{a\beta} \\
    i_0 q_{a\beta} &= 0
\end{cases}
\begin{bmatrix}
    i_{qua_s} \\
    i_{qub_s} \\
    i_{quc_s}
\end{bmatrix} =
\begin{bmatrix}
    \frac{2}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{6}} \\
    0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
    -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
    i_{a\alpha_{a\beta}}^- \\
    i_{\beta\alpha_{a\beta}}^- \\
    i_0 q_{a\beta}^-
\end{bmatrix}
\end{align*}
\]

(3.63)

where, \( i_{qua_s}, i_{qub_s}, i_{quc_s} \) are the sinusoidal negative sequence reactive current components in phase \( a \), \( b \) and \( c \), respectively. Finally, to find the unbalanced and harmonic parts of the reactive current components we need to use the oscillating reactive power component in our calculation:

\[
\begin{align*}
\begin{cases}
    i_{a\alpha_{a\beta}} &= -\frac{v_{\beta}}{v_{\alpha\beta}^2} \bar{q}_{a\beta} & \text{in abc frame} \\
    i_{\beta\alpha_{a\beta}} &= \frac{v_{\alpha}}{v_{\alpha\beta}^2} \bar{q}_{a\beta} \\
    i_0 q_{a\beta} &= 0
\end{cases}
\begin{bmatrix}
    i_{a\alpha} \\
    i_{b\alpha} \\
    i_{c\alpha}
\end{bmatrix} =
\begin{bmatrix}
    \frac{2}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{6}} \\
    0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
    -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
    i_{a\alpha_{a\beta}}^- \\
    i_{\beta\alpha_{a\beta}}^- \\
    i_0 q_{a\beta}^-
\end{bmatrix}
\end{align*}
\]

(3.64)

Then \( i_{a\alpha}, i_{b\alpha}, i_{c\alpha} \) are distorted and unbalanced reactive current components in phase \( a \), \( b \) and \( c \) respectively. To separate these two parts, we can use bandpass filters:

Figure 3.5 Bandpass filter used for extraction of unbalanced reactive currents

where \( i_{qua}, i_{qub} \) and \( i_{quc} \) are reactive unbalanced currents in phase \( a \), \( b \) and \( c \), respectively. Moreover, we have:

\[
\begin{align*}
\begin{cases}
    i_{a\alpha} - i_{qua} &= i_{qHa} \\
    i_{b\alpha} - i_{qub} &= i_{qHb} \\
    i_{c\alpha} - i_{quc} &= i_{qHc}
\end{cases}
\end{align*}
\]

(3.65)

where \( i_{qHa}, i_{qHb} \) and \( i_{qHc} \) are active distorted currents in phase \( a \), \( b \) and \( c \), respectively.

So we can add all heteropolar unbalanced terms together as follows:

\[
\begin{align*}
\begin{cases}
    i_{pua_s} + i_{qua_s} + i_{pua} + i_{qua} &= i_{ua} \\
    i_{pub_s} + i_{qub_s} + i_{pub} + i_{qub} &= i_{ub} \\
    i_{puc_s} + i_{quc_s} + i_{puc} + i_{quc} &= i_{uc}
\end{cases}
\end{align*}
\]

(3.66)

40
3.5. An alternative approach for current decomposition in three-phase non-sinusoidal unbalanced system with unbalanced and distorted voltage source

Although the proposed approach in section 3.4 can address the corresponding challenges in case of asymmetrical unbalanced voltage source conditions, we propose the following EIPT-based decomposition approach to deal with a more challenging situation where one may be faced with a non-sinusoidal and unbalanced voltage source condition. Without loss of generality, the EIPT formulation is first interpreted as a general signal decomposition algorithm to decompose the 3-phase grid voltage into its detailed components. Next, these components are incorporated in the EIPT formulation to decompose the load currents (Algorithm 1). Consider the distorted 3-phase grid voltage vector \( \mathbf{v}_{abc} = [v_a, v_b, v_c] \) at the PCC and the load current vector to be \( \mathbf{i}_{abc} = [i_a, i_b, i_c] \). Using a symmetrical large resistive load on the grid-side, the corresponding grid current behavior is recorded as \( \mathbf{i}_g = [i_{gref}, i_{bref}, i_{cref}] \).

First, using the sinusoidal nominal grid voltage (\( \mathbf{V}_{abc} \)), we apply the mathematical formulation of the EIPT approach to the \( \mathbf{i}_g \) and perform this current decomposition over these reference currents, which results in an initial set of (namely) voltage-based current components as follows:

\[
\begin{align*}
& i_{xp\text{ref}}, i_{op\text{ref}}, i_{phx\text{ref}}, i_{pux\text{ref}}, i_{xp\text{ref}}, i_{op\text{ref}}, i_{phx\text{ref}}, i_{pux\text{ref}}.
\end{align*}
\]

Since a set of purely resistive loads are used, all the corresponding reference reactive terms (indexed by “q”) will be equal to zero. Consider a resistance \( R = 1k\Omega \) to be used, multiplying the extracted set of 3-phase active current components \( i_{xp\text{ref}}, i_{op\text{ref}}, i_{phx\text{ref}}, \) and \( i_{pux\text{ref}} \) with \( R \), the following set of voltage components are calculated respectively:

\[
\begin{align*}
& \mathbf{v}_{abc} = [\mathbf{v}_a, \mathbf{v}_b, \mathbf{v}_c], \quad \mathbf{v}_{0abc}, \\
& \mathbf{v}_{Habc} = [v_{aH}, v_{bH}, v_{cH}], \text{ and } \mathbf{v}_{Uabc} = [v_{au}, v_{bu}, v_{cu}].
\end{align*}
\]

Next, we modify the definition of the EIPT-based load current components as directed in Algorithm 3.1. The homopolar current components are still calculated as directed in section 3.3, while the balanced active/reactive components are calculated in step 3. Roughly speaking, in this case, we have two individual sources of imbalance, one related to the load itself and the other one to the voltage source (steps 4-6); the same concept is valid for the harmonic effect. What we do within algorithm 1 is that calculating the contributions of these effects by decomposing the distorted voltage into its sub-components as well (step 6), while the final (total) unbalanced or harmonic terms in the load current is calculated from the sum of each of these terms as directed in step 7.
Algorithm 3.1 Generalized EIPT-based Current Decomposition for Non-Sinusoidal Voltage Source

Inputs: $i_{abc} = [i_a, i_b, i_c], v_{abc} = [v_a, v_b, v_c], \bar{v}_{abc} = [\bar{v}_a, \bar{v}_b, \bar{v}_c], v_{0abc},$
$v_{Uabc} = [v_{ah}, v_{bh}, v_{ch}], \text{ and } v_{Uabc} = [v_{aw}, v_{bw}, v_{cw}].$

1. The Concordia transform is applied: $i_{abc} \rightarrow i_{\alpha\beta}, v_{abc} \rightarrow v_{\alpha\beta}, v_{abc} \rightarrow \bar{v}_{\alpha\beta}$

2. $i_{\alpha\beta} & v_{\alpha\beta} \rightarrow \text{calculating } \bar{p}, \bar{q}.$ (average part of the calculated powers),

3. Calculate balanced active/reactive components:

4.1. The homopolar current components are still calculated as directed in section 2.2.

4.2. Calculate oscillating active/reactive components:

5.1. Calculate load oriented active/reactive heteropolar components by apply a 60 Hz bandpass filter on $i_{\alpha\beta}, i_{\alpha\beta}$ and extract $i_{pux}, i_{qux}$ for $x \in \{a, b, c\}.$

5.2. Calculate load oriented active/reactive harmonic components such that

6.1. Calculate voltage source oriented active/reactive heteropolar components

6.2. Calculate voltage source oriented active/reactive harmonic components

7. Calculate total unbalanced and harmonic components as follows:

$$i_{pux\text{total}} = i_{pux} + i_{pux}, \quad i_{qux\text{total}} = i_{qux} + i_{qux},$$

$$i_{pHx\text{total}} = i_{pHx} + i_{pHx}, \quad i_{qHx\text{total}} = i_{qHx} + i_{qHx}, \text{ for } x \in \{a, b, c\}.$$
3.6 Orthogonality of Components

From signal processing perspective, there is a variety of reasons for a signal decomposition approach to end up with a pair-wise orthogonality property within the final components. Although most of the aforementioned power theories do not provide any evidence on orthogonality of their decomposition approach and some of them are using relaxations on such a definition (such as CPT, where while active and reactive terms are pair-wise orthogonal there is no reason for active balance vs active unbalanced terms to be orthogonal, from the formulation they are placed on the same mathematical direction), here we justify how our decomposition technique preserves the orthogonality property while transfers within different mathematical coordination.

As it can be seen from our fundamental equations since these power terms are defined as the dot and cross products the power terms are orthogonal to each other:

\[ p = v_{abc} \cdot i_{abc} = v_{\alpha\beta_0} \cdot i_{\alpha\beta_0}, \quad q = v_{\alpha\beta_0} \times i_{\alpha\beta_0} \]  \hspace{1cm} (3.67)

Moreover, while

\[ i_q \cdot v_{\alpha\beta_0} = \frac{q \times v_{\alpha\beta_0}}{v_{\alpha\beta_0} \cdot v_{\alpha\beta_0}} \cdot v_{\alpha\beta_0} = \frac{(v_{\alpha\beta_0} \times i_{\alpha\beta_0}) \times v_{\alpha\beta_0}}{v_{\alpha\beta_0} \cdot v_{\alpha\beta_0}} \cdot v_{\alpha\beta_0} = \]
\[ \frac{[-(i_{\alpha\beta_0} \cdot v_{\alpha\beta_0}) v_{\alpha\beta_0} + (v_{\alpha\beta_0} \cdot v_{\alpha\beta_0}) i_{\alpha\beta_0}]}{v_{\alpha\beta_0} \cdot v_{\alpha\beta_0}} \cdot v_{\alpha\beta_0} = 0. \]  \hspace{1cm} (3.68)

\[ i_p \times v_{\alpha\beta_0} = \frac{p \cdot v_{\alpha\beta_0}}{v_{\alpha\beta_0} \cdot v_{\alpha\beta_0}} \times v_{\alpha\beta_0} = 0. \]  \hspace{1cm} (3.69)

active and reactive current components \( i_p, i_q \) are orthogonal. In our formulation, we transform the \( abc \) frames to \( \alpha\beta0 \) which is a mathematical map from phasor-polar space to an orthogonal space so all terms will remain orthogonal.

As a result, all corresponding components which are defined within the same approach for all per-phase terms are also orthogonal to each other. Moreover, in terms of unbalanced condition, we implemented the Fortescue transformation which is an orthogonal mapping, so all terms will preserve their orthogonality properties and all the newly generated terms must be orthogonal to each other regarding the fact of orthogonality of components in symmetrical components analysis properties.
3.7 Case Studies, Discussions and Future Directions

To examine the performance of the proposed EIPT, a set of case studies have been implemented. We start with an anthology of simulations (Case. 1) where a variety of load types have been fed by a three-phase sinusoidal balanced voltage source (Figure 3.7, 3.8, 3.9, 3.10) to generate desirable distortion patterns in the currents.

In the second case study, the implementation of the proposed EIPT has been examined in case of unbalanced voltage source. Finally, in the third case study the current decomposition is performed with unbalanced and distorted voltage source and nonlinear and unbalanced loads and results compared with CPT method.

Case 1: Current decomposition under different load conditions with sinusoidal balanced voltage source

As we mentioned above, in this case study, a controlled distortion generation framework has been implemented by applying appropriate load types in PSIM software (Figure 3.6). This initial case study will illustrate the resolution of the proposed algorithm in addition to justifying the irredundant component-wise selectivity of the proposed signal decomposition framework. Once the distorted current is generated the EIPT formulation is used to decompose currents to different components in each phase.

![Figure 3.6 Three-phase four wire system with different types of loads](image)

Comprehensive simulations were done and the performance of the proposed theory was examined in different loads conditions. Figure 3.6 shows the implemented system which is a three-
phase four wire system with variety of loads. Parameters of the system are provided in appendix D. First, a three-phase linear balanced load (three-phase RL load) is connected to a balanced 3-phase voltage source, as a result, one may expect that currents will only contain active and reactive balanced components. As it has been shown in Figure 3.7, the simulation results justify our expectation, where all harmonic/unbalanced-related components are equal to zero.

Figure 3.7 Decomposed current components, in case of balanced linear load

Figure 3.8 illustrates the simulation results after aggregation of the three-phase nonlinear load (three-phase rectifier) to the previous circuit configuration. In this case, we expect to have non-zero harmonic components and simulation results indicates our argument. For sake of comparison, we illustrated the summation of all current components in each phase on the top plot as well. As it can be observed from Figure 3.8, these superposition quantities are completely fitted to the actual phase currents, without any redundancy.
Figure 3.8 Decomposed current components in phases \( a, b \) and \( c \) in case of balanced non-linear load

Figure 3.9 illustrates the performance of the proposed theory in case of unbalanced loads. In this case, we expected to have unbalanced components in active and reactive currents and simulation results prove our argument. As it has been shown in the second window, under this situation, harmonic components are zero. We can again see that the summation of all current components exactly matches the current of each phase.
To illustrate the performance of our proposed method in case of combinational unbalanced and non-linear loads, we added single-phase and three-phase rectifiers to the previous configuration. Figure 3.10 shows the simulation results. In this case, we expected to simultaneously have the harmonics and unbalanced components in active and reactive currents and simulation results are following this fact.

As it has been shown within different windows, under this situation, unbalanced and harmonic components are both nonzero. Moreover, different current components are shown in case of nonlinear unbalanced loads in three-phases of the system. We can see the aggregation of different current components exactly matches the current of each phase.
Reactive powers in different phases and their aggregations is shown in Figure 3.11. The aggregation of reactive powers (in all phases) equals to zero every time, and this shows that the
reactive powers definition in this theory has a physical meaning, and it is equal to the amount of power that is exchanged between the phases and not the one which is transferred from the source to the load.

Case 2: Current decomposition under unbalanced voltage with unbalanced and nonlinear load

To present the resolution of the corresponding signal decomposition approach which has been developed in Section 3.4, we implemented an asymmetrical unbalanced voltage as a voltage source in the presence of unbalanced and nonlinear loads and decomposed currents in different phases to their variety of components. Simulation results show a good performance of the proposed current decomposition method in this case.

Figure 3.12 Decomposed current components in case of unbalanced non-linear load and unbalanced asymmetrical voltage source
Moreover, the aggregation of different current components in each phase of the system exactly matches phase currents. Reactive powers in different phases and their aggregations under unbalanced voltage source condition are shown in Figure 3.13.

![Figure 3.13 Reactive powers in different phases and their aggregation in case of unbalanced non-linear load and unbalanced asymmetrical voltage source](image)

Case 3: Current decomposition under distorted and unbalanced voltage source conditions and comparison with CPT method

Using the idea that was comprehensively discussed in the Section 3.5, we expanded our methodology for the case of distorted voltage source as well. Here we show the performance of the proposed algorithm (Algorithm.3.1) within the following two examples, as well as its comparison versus the CPT approach.

1) In the first example, we considered a set of resistive loads with distorted (harmonic of the order 5 and amplitude of 10% of the sinusoidal voltage) and unbalanced voltage (zero sequence components 20% of positive sequence component). Since the current of resistive load is proportional to the source voltage we expect and observe the distorted-unbalance in the load currents behavior. Figure 3.14 represents the 3-phase source voltage and the corresponding load current waveforms in phases a, b and c. Because of the voltage source conditions, we would expect to have harmonic parts by the order of 5 and unbalanced zero sequence components in the corresponding decomposed current signals.
Figure 3.14 Voltage and current waveforms of the source in phases a, b and c.

Figure 3.15 active and reactive current components in EIPT and CPT

Figure 3.16 harmonic and unbalance current components in EIPT and CPT
However, Figure 3.15 and Figure 3.16 are indicating that the CPT approach is not able to decompose the current components properly. As a matter of fact, the active current component contains some harmonics and unbalanced parts while the voided and unbalanced components are zero. Considering the source-voltage-based mathematical formulation of the CPT, this was an expected phenomenon (also one may refer to the Prof. Tenti’s complementary notes, on the CPT limitations in terms of any source of voltage distortions). In contrast, our EIPT-based framework can extract harmonics and unbalanced parts with a higher level of accuracy. Figure 3.17 shows the total (heteropolar+homopolar) unbalance, homopolar and heteropolar current components in EIPT respectively.

![Graph showing EIPT unbalanced component, EIPT homopolar component, and EIPT heteropolar component](image)

Figure 3.17 Heteropolar, homopolar and the total unbalance current components in the EIPT

2) In the second case study, we considered nonlinear and unbalanced load (parameters of load are mentioned in Appendix.D) supplied by distorted (including harmonics of the order of 5 and amplitude of 10% of the sinusoidal voltage) and unbalanced voltage source (zero sequence components 20% of positive sequence component). Figure 3.18 shows the behavior of the corresponding load current waveforms in phases a, b and c.
Figure 3.18 the current waveforms of the load in phases a, b and c.

Figure 3.19 illustrates the active and reactive current components calculated by the EIPT and CPT formulations, accordingly. As it is shown, the active and reactive current components extracted by the CPT are suffering from the distorted and unbalanced voltage parts, while EIPT-based approach resulted in pure sinusoidal wave-shapes.

Figure 3.19 active and reactive current components in EIPT and CPT

Figure 3.20, indicates that the unbalanced and distorted components are well separated through the EIPT approach. Finally, Figure 3.21 shows the total (heteropolar+homopolar) unbalanced, homopolar and heteropolar current components for the EIPT, respectively.
Figure 3.20 harmonic and unbalance current components in EIPT and CPT

Figure 3.21 Heteropolar and homopolar and total (heteropolar+homopolar) unbalance current components in EIPT
4.1. Introduction and Overview

Nowadays, comprehensive awareness and online monitoring of the power system situation, besides the instantaneous fault detection and localization are among the most critical tasks and major challenges in the smart power grids [66]-[67]. By definition, islanding is referred to as a situation where a portion of the power system (mainly in the distribution level) is disconnected from the main grid, although remains energized. Such a situation was not a common issue in the conventional power networks; however, through fast growth of newly distributed generation units (especially renewable-based Microgrids) such a situation can become a serious concern for reliability of the system in future smart grids. For example, an accurate islanding detection could play a critical role in applications like static transfer switches (STS) and uninterruptible power supplies (UPS), or approaches where the utility supply must be disconnected as part of the compensation. In general, a renewable-based DGU contains a renewable micro-generation source such as PV or wind in addition to an energy back-up module such as a battery. This capability provides a semi-stable flow of power in these sub-systems. Once an outage happens in the power grid, the energy flow can remain in the system supported from micro-generators plus backup modules. As a result, the islanding situation can be considered a common issue to be addressed in DGUs and Microgrid systems.

Islanding phenomenon can be categorized into intentional and unintentional islanding. However, most of the work in this area is focusing on unintentional islanding detection (ID). On the other hand, the introduced methodologies and techniques for ID can be classified under two major classes as follows: 1) local and, 2) remote. In the remote methods, we usually need a communicational infrastructure, while in case of local techniques a collection of local information is adequate for further processing and decision making procedure. Usually, the reliability of adequate implementation of remote techniques is affected by the difficulties caused by practical issues of utilization of direct communication between the DGs and utility systems through fiber
optic and wireless communication networks. In addition to that, the high penetration of DGs in complex systems will result in an expensive and complicated practical implementation procedure for these schemes. As a result, although remote methods are more reliable, due to their simplicity and applicability, cost-effective local approaches are usually preferred. Commonly, local methods are categorized into passive and active types. In general, the basic idea behind the local islanding detection techniques is rooted in tracking the temporal behavior of the fundamental system parameters (like voltage signals) which are supposed to be steady during the grid-connected operational mode but may significantly be impacted within a state transition between grid connected and islanded modes.

Passive methods were the first of kind to be developed for sake of local islanding detection. These methods are usually analyzing the behavior of electrical signals to determine the islanding situation occurrence [71], and generally, rely on parameter thresholds. The notable advantages of passive techniques include: ease of implementation without any extra control unit, no demolition in power quality, and their inexpensiveness. A major disadvantage of passive approaches is the inappropriateness in multi-inverter systems, however, the most important drawback of these techniques is known to be their relatively large non-detection zones (NDZ) [72]-[73]. Through the conventional passive approaches, the system islanding state is determined from an indicator criterion such as under/over voltage (UVP/OVP), under/over frequency (UFP/OFP), or vector shift (VS) functions, that itself, is evaluated by exploiting the measurement of electrical quantities, such as voltage, current, or frequency.

Upon the new progresses in the electrical sensor technology, active methods have been developed as an alternative to passive approaches to overcome the aforementioned limits of passive techniques. In a typical active method, a small disturbance signal is injected at the PV inverter output for to detect the islanding status. In comparison, these approaches are providing a relatively smaller NDZ than passive strategies. The major drawbacks of active methods can be noted as their higher complexity which is enforced by the need of additional controllers, besides their unwanted effects on output power quality which may result in destabilizing the PV inverter [72]-[73]. Regarding enormous variation in grid fault types and characteristics, each of the aforementioned strategies has found its own application within a certain condition. Although both of these local approaches can be used for grid monitoring, from the power quality point of view, the passive islanding detection method is preferable. Due to the advantages of the passive ID
approach, in this chapter, we aim to introduce a novel and timely reliable passive ID framework. From now on, we focus on this specific category of the ID methods and the frontier technologies in this category to address the inherent challenges especially the risk of large NDZ, especially by incorporating advanced signal processing and artificial intelligence techniques.

4.2. State of the art in the literature in the passive islanding detection

Recently, to mitigate the NDZ problem, variety of improved techniques have been proposed in the literature. Authors in [55], proposed a comparison based approach by investigating the P–V and P–Q characteristics of inverters equipped with a constant current controller to reduce the NDZs of UVP/OVP and UFP/OFP. Another strategy which exploits the monitoring of phase differences between the inverter terminal voltage and output current was reported in [56]. The major advantage of this method is the ease of implementation while it exclusively requires the modification of the phase locked loop by inverters for utility synchronization. A harmonic measurement-based technique has been developed by Teoh and Tan in [75]. This technique tries to identify the occurrence of the islanding situation by tracking the changes in the total harmonic distortion (THD) at the point of common coupling (PCC). Once the voltage signals’ THD exceeds a certain amount (a pre-defined threshold), the inverter detaches the Microgrid from the main grid.

During last decade, the artificial intelligence and advanced signal processing has found its way through the area of islanding detection. In [57], Yin et al, tried to exploit the fast Fourier transform (FFT) with integration of the immunological principle. Since the islanding phenomena is inherently non-stationary, FFT is not an appropriate choice for ID purposes. To deal with the associated challenges of non-stationary signal behavior in the ID problem, Wavelet features in combination with artificial neural network (ANN) classifier has been used in [58]. In another try Samantaray et al. [76], tried to extract distinctive and informative features from the fault patterns, during the islanding situation, by applying the discrete wavelet transform (DWT) on the current signal recorded at DG terminal. Followed by this feature extraction procedure, they have reported the corresponding islanding detection performance for a wide range of classification techniques, such as decision tree, radial basis function (RBF), and probabilistic neural network (PNN), where PNN was reported to end up with the best overall performance.

More recently, a frontier technique named phase space method is trending as a popular approach for use in islanding detection and classification. Roughly speaking, this is a mathematical approach where the data of a time series is reconstructed within a higher dimensional space. This technique
has been initially exploited in 2008, for the power quality detection and distance relays problems [77]-[78]. It has been shown in the literature that the time complexity of this method is low enough to make it a suitable choice for the sake of real-time implementation. However, this technique has not been thoroughly applied to islanding detection. Some preliminary applications of the phase space technique in islanding detection were explored in [59].

4.3. Technical Challenges and Contributions

Roughly speaking an intelligent passive ID (IPID) method is interpreted as a 2-step voltage-dependent signal processing-based classification problem. The general pattern of the voltage signals is used to extract a set of meaningful features (signals processing step) to reveal the type and importance of an unusual deviation, in the voltage signals behavior, from a pure sinusoidal waveform (fault-type classification step). Although a variety of IPID approaches has been introduced in the literature (as mentioned above), a couple of technical challenges still remain unanswered. Besides some inherent disadvantages of most of the signal processing-based feature extraction methods (such as non-stationary features ignorance in FFT), these methods are not instantaneous. Which means that, for sake of feature extraction, the voltage signal should be first recorded for a window of time and then this segment of the data is run through a signal decomposition step to calculate the corresponding Fourier or wavelet coefficients (features) for further analysis. This will impose a delay to the processing procedure. On the other hand, these methods should be implemented separately over each phase, as a result the possible relative coupling information of the 3-phase fault behavior is missed (such as unbalance). Finally, most of the former methods do not deal with the fault reason and usually classify the fault type in standard categories such as sag, swell, and etc. without mentioning or addressing any information over the fault reason.

Motivated by these important facts, we developed a new framework that can address these challenges and uses the 3-phase coupling information in the feature extraction step. In this chapter⁶, we present an instantaneous intelligent passive islanding detection (IIPID) strategy by incorporating the aforementioned instantaneous power theories (refer to chapter 2 and 3) as a fast and reliable feature extraction framework. In another word, we are going to use the advanced power theories (which have been deeply discussed in previous chapters) as possible signal

⁶ This section is based on the following set of papers: [81].
decomposition techniques to extract the corresponding features and to model the behavior of different type of faults in the smart Microgrid systems. For this reason, we have used the DQ, CPT, and the EIPT (also a combination of these techniques) to extract the corresponding distinctive features from 3-phase electrical voltage signals under faulty situations.

From pattern recognition perspective, a good feature extraction strategy yields to a separable class-clusters formation within the data feature space, which means that any regular nonlinear classifier (like a multilayer neural network) can correctly recognize the different classes (types) of the events (faults or islanding onset in the ID problem) in the corresponding data space (dataset). We comprehensively discuss this approach in terms of feature space representation to demonstrate it suitability, as a successful 3-phase instantaneous feature extraction strategy, for islanding detection. One notable advantage of our strategy is that, while features are extracted instantaneously, there are no delays in the process described above. It is worth noting that an important function to ensure proper islanding detection is the algorithm ability to detect the disturbance immediately, regardless of the nature of the voltage disturbance. In another word, the faster the disturbance is detected, the faster is the application of subsequent actions.

Figure 4.1 General scheme of IPID approaches
4.4. Fault Scenarios

Since in a power system fault types may vary in a big range, and to make the situation (reason-oriented) more challenging we have considered the following faulty scenarios that may result in a variety of standard power quality issues within different time scales (such as voltage sag, harmonic, transient, oscillation and so on, or a combination of that). These situations may result in intentional/unintentional islanding status based on the seriousness of the fault duration and dept. The following set of importance 3-phase fault scenarios have been modelled using MATLAB/SIMULINK for 10 different conditions:

1. Capacitor Switching (possible power quality issues: transient oscillatory, harmonic)
2. Arc Furnace (possible power quality issues: flicker)
3. Induction Motor Start-up (possible power quality issues: sag, flicker)
4. Lightening (possible power quality issues: impulse, swell)
5. Line-Ground (possible power quality issues: voltage fluctuation, sag, harmonic, unbalance, swell)
6. Line-Line (possible power quality issues: voltage fluctuation, sag, harmonic, unbalance)
7. Single-phase Nonlinear Load (possible power quality issues: sag, harmonic, unbalance)
8. Three-phase Nonlinear Load (possible power quality issues: harmonic)
9. Three-phase fault (possible power quality issues: 3-phase sag)
10. Transformer Energizing (possible power quality issues: harmonic, unbalance)

These islanding scenarios have been implemented as directed in [60]. Figure 4.2 illustrates the temporal behavior of the 3-phase voltage signal under each of the aforementioned 3-phase fault scenarios.

4.5. Instantaneous 3-phase Reason-Oriented Islanding Detection and Classification

In this section, we comprehensively describe and investigate the performance of our strategy for IIPID. Back to our descriptions in Chapter 2 and 3, each of the instantaneous power theories, such as PQ, CPT to EIPT, can be interpreted as an instantaneous signal decomposition approach. By definition, a signal decomposition is a mathematical transformation that segmentizes a recorded signal into more detailed components. The major advantage of such a transform is the revealing of
some useful information hidden (unobservable) in the initial domain representation (usually time in case of electrical signals) of the behavior of the understudied phenomena.

Figure 4.2 Temporal behavior of the 3-phase voltage signal under each 10 different 3-phase fault scenarios
Fourier transform is the most usual and widely well-known approach in this regard, that transform the signal from time-domain into another mathematical domain usually known as frequency domain. The representation of the signal in the frequency domain is named power spectrum density and illustrates the power of each frequency component that is presented in the original signal. This approach is widely used in important power signals analysis such as THD calculation and so on.
In terms of fault or islanding detection automation, the importance of such a representation is that, while it may be hard to define distinctive time-domain features that are usable to design a computer-bases detector for detecting and assigning the correct class of fault to a faulty event occurrence, it may be possible to use the Fourier coefficients to more accurately recognize them from each other. This is widely known as feature extraction and optimization.

While Fourier coefficients are useful features to recognize some of the fault patterns their overall performance is weak in terms of dynamic events. As a result, time frequency transforms such as short-time Fourier transform or wavelets are more popular. However, a major drawback with these usual signal decomposition (feature extraction) approaches, is their time delay. To implement these mathematical transforms, we need an enough data to be recorded first. As a result, they may not be appropriated in case on online (real-time) applications were time is a critical factor. Moreover, inherently, they can be implemented on one signal at the time and they do not directly provide any information regarding the possible correlations between two signals. Thus, in the islanding detection problem these are not the best possible candidates.

As we mentioned, power theories can be interpreted in terms of a 3-dimentional time to time signal decomposing (transform), which are surprisingly instantaneous as well! This new view, can make them a powerful set of feature extraction tools for islanding detection and classification.

4.6. 3-phase Feature Extractors: DQ, CPT and EIPT

Consider a 3-phase voltage signal at time $t$ as a 3-D vector $y(t) = \begin{bmatrix} y_a(t) \\ y_b(t) \\ y_c(t) \end{bmatrix}$. Using EIPT, CPT and $dq$ methods one can transform this 3-dimentional vector into an 8-dimensional 3-phase feature vector $(f(t))$ at each time instance as follows:

$$f(t) = \begin{bmatrix} v_{\text{harmonic}_eipt}(t) \\ v_{\text{unbalance}_eipt}(t) \\ v_{\text{pure}_eipt}(t) \\ v_{\text{harmonic}_cpt}(t) \\ v_{\text{unbalance}_cpt}(t) \\ v_{\text{pure}_cpt}(t) \\ v_d(t) \\ v_q(t) \end{bmatrix}$$ (4.1)
Now, consider a fault event is started at some time instance, we define a time-window \((\Delta t: t_0 - t_1)\) over which we would like to know what type of fault has happened. Consider a sampling rate of \(T\) sample/second, our signal is changed to a vector \(y = [y(1), \ldots, y(N)]^T\), where \(N = \frac{\Delta t}{T}\) can be any appropriate number of samples for which a fault shape is recognizable.

While our signal decomposition is instantaneous at \(t = t_1\), \(f(t)\) is an \(8 \times N\) matrix \(F\) (actually \(8 \times 3 \times N\) tensor, since each of the parameters in (4.1) is a 3-phase signal). Figure 4.3, simultaneously illustrates a time-window of size \(\Delta t = 0.05s\) from the temporal behavior of 8, 3-phase faulty voltage signals per 10 aforementioned islanding scenarios (section 4.4).

Figure 4.4 – 4.11, are illustrating the corresponding temporal behavior of each of the following components (equivalently rows of the matrix \(F\)) over the same 8, 3-phase faulty voltage signals per 10 aforementioned islanding scenarios:

4.4: \(v_{\text{harmonic}_\text{eipt}}\) for \((\Delta t: t_0 - t_1)\)

4.5: \(v_{\text{unbalance}_\text{eipt}}\) for \((\Delta t: t_0 - t_1)\)

4.6: \(v_{\text{pure}_\text{eipt}}\) for \((\Delta t: t_0 - t_1)\)

4.7: \(v_{\text{harmonic}_\text{cpt}}\) for \((\Delta t: t_0 - t_1)\)

4.8: \(v_{\text{unbalance}_\text{cpt}}\) for \((\Delta t: t_0 - t_1)\)

4.9: \(v_{\text{pure}_\text{cpt}}\) for \((\Delta t: t_0 - t_1)\)

4.10: \(v_d\) for \((\Delta t: t_0 - t_1)\)

4.11: \(v_q\) for \((\Delta t: t_0 - t_1)\)

Based on the specifications of each of the aforementioned signal decomposition approaches, each of these 8 components presents a certain detailed version or information level of the corresponding original signal. For example, DQ components are representing a kind of dc-ac map, while each of CPT and EIPT are decomposing the signal into its purely sinusoidal component, in addition to unbalance, harmonic or voided components. Now we would like to take a deeper consider the feature space of the IIPID problem.
Figure 4.3 Temporal behavior 3-phase Voltage in time window
Figure 4.3: continued
Figure 4.4 Harmonic components of EIPT in different faults
Figure 4.4: continued
Figure 4.5 Unbalanced component of EIPT in different faults
Figure 4.5: continued
Figure 4.6 Pure sinusoidal component of EIPT in different faults
Temporal Behavior Purified Component EIPT

Figure 4.6: continued
Figure 4.7 Harmonic component of CPT in different faults
Figure 4.7: continued
Figure 4.8 Unbalanced component of CPT in different faults
Temporal Behavior Unbalance Component CPT

Figure 4.8: continued
Figure 4.9 Pure sinusoidal component of CPT in different faults
Figure 4.9: continued
Figure 4.10 $V_d$ in different faults
Figure 4.10: continued
Figure 4.11 $V_d$ in different faults
Temporal Behavior Q Component DQ

Figure 4.11: continued
4.7 IIPID Feature Spaces

Take a brief look at the Figure 4.2, one may easily observe some unique characteristics within each of the fault patterns. Although visually recognizable, these temporal characteristics are needed to be quantified in a meaningful way to be understandable for a computer-based device (feature extraction). There are two major parameters identifying the general behavior of the 3-phase voltage signals in the time-domain: Amplitude and the relative Phase.

Since the major difference between each class of islanding scenarios vs normal sinusoidal waveform is mostly highlighted in terms of Amplitude, we can consider the amplitude to be the first natural distinctive feature choice. However, in terms of 3-phase voltage we need to define an amplitude-dependent feature that not only represents the superposition of all phase amplitudes at once but also have a unique value for each class of islanding events. Some initial choices would be the maximum amplitude within the time window of the fault $\Delta t$, or the average of the relative (maximum amplitude/average amplitude) per 3-phase. Another meaningful choice would be average of the line-ground rms voltage $\frac{V_{rms_a} + V_{rms_b} + V_{rms_c}}{3}$. Figure 4.12 illustrates the histogram of this quantity during the selected fault time window ($\Delta t = 0.05$), for all the eight samples of the ten simulated islanding scenarios (80 faulty signals in total).

![Distribution of the temporal 3-phase V\textsubscript{rms} in the event Window](image.png)

Figure 4.12 1-D feature space of IIPID (histogram of the $v_{rms}$)
This histogram can be interpreted as a 1-D feature space representation of the dataset. Since ten classes of islanding scenarios are effective in the total dataset, the ideal shape of the histogram would be a uniform distribution with eight events per 10 bins. However, as it can be seen, most of the samples are located within the tail of the distribution. This means that the 3-phase average $V_{rms}$ is not a valid distinctive feature for sake of islanding scenario (alternatively fault class or fault reason) classification. As we discussed before we are not interested in frequency or time-frequency features such as THD, Fourier coefficients or wavelets due to their operational time-delay enforcement.

4.7.1 IIPID Feature Spaces: DQ, CPT, and EIPT-based spaces

One may alternatively extract variety of other time-domain features however, in this work we are going to exploit the potential of the power theories as strong tools for sake of instantaneous time-domain-dependent feature extractors. As such, for each of the 80 faulty scenarios, we calculate the 3-phase average value of each of the extracted signal components in matrix $F$ as follows:

$$v_{ind_a} + v_{ind_b} + v_{ind_c} \over 3 \quad \text{where } ind \in \{v_{harmonic_{eipt}}, v_{unbalance_{eipt}}, v_{pure_{eipt}}, v_{harmonic_{cpt}}, v_{unbalance_{cpt}}, v_{pure_{cpt}}, v_d, v_q\}$$  (4.2)

Thus, we extracted eight instantaneously calculated time-domain-based features per each of the 80 simulated faulty scenarios (with eight sample of each of ten scenarios in the dataset). We may use the whole or any subset of these 8 features to form a D-dimensional feature space for the IIPID problem and illustrate the location of each of these 80 data samples within the corresponding feature space. We start with theory dependent feature spaces. Figure 4.13a-c are illustrating the generated 2D or 3D feature space scheme using: three extracted EIPT components (features), three CPT components, and two DQ components, respectively:
Figure 4.13.a) Corresponding 3D IIPID problem feature space formed out of 3 EIPT-based extracted features. b) Corresponding 3D IIPID problem feature space formed out of 3 CPT-based extracted features. c) Corresponding 2D IIPID problem feature space formed out of 2 DQ-based extracted features
As we can observe the corresponding DQ-based space does not provide a sufficiently class-cluster separated form. In another word, we would expect to see as set of ten distinguished data clusters with each include eight data samples (small-stars) to indicate that the associated power theory has extracted useful features for sake of islanding scenario detecting or alternatively fault type classification. However, Figure 4.13.b indicates that CPT-based extracted features will shape a much better feature space for the same dataset and one can observe a set of distinguishable class-clusters. This can be also related to addition of another dimension to the feature space that provides another degree of freedom, however, in our opinion this is dominantly caused by the finer and more informative level of information revealed from applying the CPT transformation on the voltage signals compare to the DQ. Finally, Figure 4.13.a illustrates the corresponding EIPT-based feature space. Within this feature space we may clearly recognize a set of ten separated class-clusters. In our opinion, this improvement is happened because of the more meaningful component-wise definition which is used in the mathematical formulation of the EIPT compare to the CPT transform (please also refer to chapter 2 for our detailed discussion on this fact). Although these results look promising, one may alternatively select any other combination of these 8 features to form new feature spaces. In the following section, we investigate an alternative feature space formation that we believe is the most meaningful feature space formation from information extraction perspective.

4.7.2 IIPID Feature Spaces: Dynamic-Static features

In electrical power signal processing, faults can be considered as unwanted waveforms appeared on top of the normal sinusoidal shape. By their nature, most of the faults will add some level of dynamic behavior to the voltage signals that may last for a duration of time. Since CPT and EIPT are decomposing the faulty signals into a pair of pure sinusoidal vs unwanted components, it would be useful to define a new feature (we named as dynamic/static ratio) as follows:

\[
    d - s_{ratiopt} = \frac{v_{\text{harmonic}_{pt}} + v_{\text{balance}_{pt}}}{v_{\text{pure}_{pt}}} \quad \& \quad pt \in \{\text{CPT, EIPT}\}. \tag{4.3}
\]

Figure 14.4 a-b illustrate the resulted 2D feature space, using \(d - s_{ratioeipt}\) and \(d - s_{ratioeipt}\) vs \(V_{rms}\), respectively.
As we expected regarding more meaningful physical-mathematical definition of power components in the EIPT theory, a more useful amount of information is revealed which finally results in a better and more separable information-based feature space representation for IIPID.
problem. For the sake of sustainable usage of the extracted information we formed the corresponding 3D feature space out of $d - s_{ratio_{eipt}}, d - s_{ratio_{cpt}}$ and $V_{rms_{3q}}$ (Figure 4.15).

We can see that using the aforementioned strategy we could form a feature space were each class-cluster is located in an isolated sub-space (it is worth noting that, this is a normalized 3D view and some of the islanding scenarios are generating data samples with big scales in terms of aforementioned features which results in a compact show up for those who are closer to the center of the feature space. If we zoom into the center area and rotate the axis we can easily see all clusters are separated from each other through the center of the plot).

**3D Feature Space $d-s_{r_{Eipt}}, d-s_{r_{CPT}}, V_{rms_{Components}}$**

![Figure 4.15. 3D dynamic-static feature space using EIPT and CPT theories](image)

4.8 IIPID Classification Using Multilayer Perceptron Neural Network

From the theory of pattern recognition, such a well separated feature space will provide an appropriate substrate for any classifier to well-perform the classification procedure. However, while not all the class-clusters are linearly separable within the feature space the nonlinear classifiers are preferred (Figure 4.16).
4.8.1 Network Architecture

A multilayer perceptron feedforward artificial neural network with 3 input neurons (accepting calculated $d - s_{ratio_{eipt}}$, $d - s_{ratio_{cpt}}$ and $V_{rms_{3\theta}}$ for each data sample), and 20 McCulloch-Pitts neurons (threshold logic unit) in the hidden layer.

Since the IIPID is a multi-class classification, we have $n = 10$, McCulloch-Pitts neurons in the final layer. A typical McCulloch-Pitts neuron is formed as a series of mathematical operations followed by each other. First, a set of synapses (i.e. connections) brings in activations from other neurons. Next, a processing unit sums the inputs, and then applies a non-linear activation function (i.e. squashing/transfer/threshold function), and finally, an output line transmits the result to other neurons.

Mathematically speaking, an ANNs can be represented as weighted directed graphs. For our purposes, we can think of it in terms of activation flowing between processing units via one-way connections. The illustrated neural network model has been implemented in Matlab for IIPID (Figure 4.17). Each of the output neurons represent an individual class if close to 1. At the end, the neuron which has the highest prediction 'wins' and that class is predicted. Although one would be
able to have completely separate networks for each class or label, there are some major advantages
to using a shared network for all classes. For example, the correlations between classes or labels
can be learned, and specifically a shared feature space can be represented by the network. This
generally leads to a drastic speed-up in the performance.

Figure 4.17. The architecture of the implemented neural network

Here are the underlying mathematical equations of the neuron outputs over the network. For the
hidden layer, we have:

$$out_{hj} = f_h(\sum_{i=1}^{3} x_i w_{i,h} - \alpha_j)$$  \hspace{1cm} (4.4)

while for the output layer we have:

$$out_{ok} = f_o(\sum_{j=1}^{15} out_{hj} w_{h,o} - \beta_k)$$  \hspace{1cm} (4.5)
where $f_h$ and $f_o$ are called activation functions and parameters $\alpha$ and $\beta$ are called activation thresholds. Variety of activation functions have been developed so far, however, it has been shown in the literature that using the tangent hyperbolic and sigmoid functions, in the hidden, and output, layer of the multilayer perceptron network will help the network to map any nonlinear relation may lie over the data (or equivalently, learn nonlinear boundaries in the corresponding feature space). These two functions are defined as follows:

$$f_h(x) = \frac{1-e^{-2x}}{1+e^{-2x}}, \quad f_o(x) = \frac{1}{1+e^{-x}}$$

(4.6)

4.8.2 Training-Test Data and the Learning Back-propagation Algorithm

The training step in any classifier can be interpreted as a repeated learning procedure where the classifier learns the location of the between-class boundaries within the feature space using a set of information fed to the classifier, named “training set or labeled data”. In general, for $n$ inputs (features), the connecting weights define a decision boundary that is an $n-1$-dimensional hyperplane in the $n$ dimensional input (feature) space. In another word, the hyperplanes are formed by calculating all of the connecting weights over the network graph, equivalently the set $W = \{w_{i,h}, w_{h,o}\}$. A variety of approaches have been developed so far to optimize the training rate of the multilayer neural networks. Beyond these methods, the Back-propagation algorithm has been selected in this work. The incorporated back-propagation algorithm has been summarized in the following section.

4.8.2.1 Cross-entropy-based backpropagation learning

Using the eight generated samples per ten islanding scenarios, we have generated a dataset of 800 faulty situations (80/each class) within ten level of SNR (10-100 dB, based on current PMU standards). As the rule of thumb in the supervised learning [61], we first, design a training set by labeling a certain amount of data manually. Consider an input vector $x = [d - s_{ratioept}, d - s_{ratioept}, V_{rms3}]$, extracted from the faulty 3-phase voltage signal as discussed in 4.7.2 in addition to a target class assigned to this input vector, $y$. The classifier was trained using 70 samples per each islanding scenario category (700 training samples total) and ten samples were kept for validation (100 test samples). Roughly speaking the training algorithm takes as input a sequence
of training examples \((x_t, y_t), t = 1:700\), and estimates a boundary-designing function \(y = f_N(W, x)\), which given a weight \(W\), maps a test input vector \(x\) into a class-label \(y\).

Within the backpropagation learning procedure, the weights are initialized with a set of random values \(W_0\). Next, they will be updated using the batch gradient descent framework through minimizing a sum-squared error function. In this work, we have used the following alternative network performance measure known as the cross-entropy error function:

\[
E(W) = -\sum_t \sum_j [y_t^j \log(out_j(x_t^i)) + (1 - y_t^j) \log(1 - out_j(x_t^i))] \tag{4.7}
\]

This definition has a couple of advantageous over regular square of sum cost function (please refer to [70] for more info on cross-entropy-based back-propagation). To minimize this function, all weights are updated in an iterative procedure (with each repetition named as an epoch) by a small shift, following a series of gradient descent weight updates:

\[
\Delta w_{ij}(l) = -\eta \frac{\delta E(W)}{\delta w_{ij}} \quad w_{ij}(l + 1) = w_{ij}(l) + \Delta w_{ij}(l) \quad l = 1:L \tag{4.8}
\]

where \(l\) stands for the epoch number. Usually after many epochs, when all the network outputs match the targets for all the training patterns, all the \(\Delta w_{ij}\) will be zero and the process of training will cease and say that the training process has converged to a solution. Considering (4.4) – (4.8), and following the partial derivative rules, the updating terms for each weight is calculated as follows:

\[
\Delta w_{ij} = \eta \sum_j [y_t - out_{ok}] \cdot out_{ih} \tag{4.9}
\]

The regularization parameter (learning rate) \(\eta\) was applied and optimized at 0.005, to reduce the risk of overfitting to the training data. Under the aforementioned network configuration for the IIPID, the proposed neural network has been trained within a K-fold \((K = 8)\) approach using the aforementioned back-propagation algorithm and ended up with a training error equal to 2.78%. 

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4.8.2.2 Results and discussion

To examine the classification performance of the proposed instantaneous approach, a set of 100 (10 per class) data samples from all classes has been given to the trained classifier, at each fold, and an average test accuracy performance equal to 82.07% was achieved.

Regarding our examinations and observations in this chapter, we can conclude that, without loss of generality, one may consider the instantaneous power theories as a $\mathbb{C}^3 \rightarrow \mathbb{R}^N$ signal decomposition approach that maps a 3-dimensional complex vector into an N-dimensional real-time space that can be further exploit in order reveal a hidden layer of instantaneous coupling information from the original 3-D signal. Such a unique functionality makes them strong tools to extract useful dynamic-static features from the 3-phase voltage signals to better understand the nature of 3-phase faults in power systems.

As it has been mentioned once the instantaneous set of features has been extracted from the voltage patterns, one may incorporate a variety of different feature generation approaches to further extract a set of more distinctive features. We illustrated that the initial instantaneous features extracted directly from power theories such as CPT or EIPT can form some separable cluster shapes. However, it seems that the EIPT can form a well-isolated class-cluster structure compare to the CPT. We believe that this fact is caused by the better definition of power terms and the more meaningful way of defining the decomposed components in EIPT vs CPT.

Although one may directly apply these initial features, since fault occurrence is usually followed by a dynamic change in the temporal behavior of the voltage signals we have extracted a new feature named dynamic-static-ratio to design a reason-oriented islanding scenario classification framework.

We have shown that the corresponding class boundaries between different islanding classes within the resulted feature space is nonlinear in general. As a result, a nonlinear classifier is needed to correctly learn and track these boundaries. As such, an entropy-based back propagation learning algorithm was applied into a multilayer perceptron neural network and our final results indicates the appropriate classification performance for the IIPID problem. Due to the model-based limitations of the MATLAB/SIMULIK for generating these faulty scenarios we obeyed to these promising results, however, we can defiantly improve this rate using more number of data samples in the training step, upon accessibility of a larger dataset.
4.9 Experimental Validation: Grid-connected inverter using instantaneous islanding detection method with Protection Design

In this section, the electronic circuits and computational hardware used for the experimental evaluation of the protection system for grid-connected inverter will be described. The system is working as a hardware-in-the-loop (based on a dSPACE). Figure 4.18 shows the schematic of the complete experimental setup. The components include a Semikron inverter (HVSKAI), a programmable DC power supply capable to emulate a photovoltaic array, a dSPACE 1104 real-time hardware, an LCL filter, protection system (including dynamic breaker, protection board, master and slave breakers, relay control circuit), current and voltage sensors to provide voltage and current feedback, one computer with the Control Desk and Matlab/Simulink software running the real-time Simulink model of the control system. The main goal of this section is designing an appropriate protection system for the existing grid connected inverter in ACEPs lab and more details related to LCL filter design, sensors, Dspace 1104 and Semikron inverter is described in [62].

Figure 4.18. The schematic of the experimental set up
The protection system for grid-connected inverter must be able to protect it in case of any faults happen in the grid side, DC side and inside the inverter. This system contains different parts as follows:

4.9.1 Master-Slave Circuit Breakers:

The master and slave circuit breakers are shown in Figure 4.19. The slave circuit breaker is the static transfer switch (STS) which connects the GC-inverter to the grid.

The master circuit breaker is a protection breaker for the slave one. It means that if the master is off, the slave will be off as well. As shown in Figure 4.19, A1 which is an energizing signal for slave circuit breaker comes from master circuit breaker. The signals X1-8 and X1-10 are coming from the relays control board. Moreover, the master circuit breaker is responsible to connect or disconnect the DC voltage source to the GC-inverter.

![Figure 4.19. Master and slave circuit breakers](image)

4.9.2 Push Buttons:

Push button is the safety button. In case that we want to turn the system off for any reason by pushing the red button, the whole system will be stopped. In this situation, the system will not work again unless we press the green button.

4.9.3 Relay Control Board:

This circuit contains three main relays. The first one in a control relay, the second one is the master relay and the third one is the slave relay.
The relay control circuit is shown in Figure 4.20 and Figure 4.21. The first relay is controlled by push-buttons and it is responsible to control the other two relays (control relays for master and slave breakers).
It means that if the red button is pushed, this control relay will not allow the master and slave relays to work and will not send the control signals to these relays. On the other hand, if the green button is pushed, this relay will send the control signals to the master and slave relays. The control signal for master is X2-1 and for slave is X2-2 and they come from the Dspace. The output of the Relay control circuit is X1-10 and X1-8 which is used to control the master and slave breakers. This circuit comprises of integrated circuits (ULN2803AG) that interface the output TTL signals of the connectors board to the voltage levels required to command the 5VDC relays (DS2E-SL2-DC5V).

4.9.4 Protection Board:

Protection board is responsible to disconnect switches of inverter from the Dspace in case of any fault happens or in the emergency case when the red button is pushed. Figure 4.22 shows the protection circuit. The error signal which is shown by red color is come from the internal error signal of the Semikron inverter (open collector signal).

4.9.5 Dynamic Breaking Circuit:

Dynamic breaking (DB) circuit regulates the voltage of the DC link and does not allow it to increase from a certain defined value in case of controller is not work properly. In this situation,
the grid send power to the DC link and causes increasing voltage in DC side and damages the system. When the DC link voltage exceeds from the reference value, the Dynamic break starts to work and using parallel resistor will dissipate power and regulates voltage.

Dynamic break contains power supply (to feed optocoupler and driver ICs), operational amplifier (for voltage comparison), Mosfet transistor, and high-power resistor for power dissipation and voltage regulation. Figure 4.24 shows the power supply circuit for DB system and it provides different DC voltages (+5V and +12V).

Figure 4.24 shows the comparison circuit of the DB with a pre-defined reference voltage for DC bus and hysteresis band.

\[
v_{\text{ref}} = v_{cc}R_1/(R_1 + R_2) \quad (4.10)
\]

\[
v_{TH} = ((R_3 + R_4)v_{\text{ref}} - (R_3v_{OL}))/R_4 \quad (4.11)
\]

\[
v_{TL} = ((R_3 + R_4)v_{\text{ref}} - (R_3v_{OH}))/R_4 \quad (4.12)
\]

\[
Hyst = v_{TH} - v_{TL} = R_3(v_{OH} - v_{OL})/R_4 \quad (4.13)
\]
If the voltage in DC side exceed from the defined value the output of op-amp becomes one and DB tries to regulate voltage. An optocoupler is used as an interface between control and power circuit which is shown in Figure 4.26.

![Optocoupler used as an interface for DB system](image)

**Figure 4.26** Optocoupler used as an interface for DB system

![Comparison circuit of the DB](image)

**Figure 4.25** Comparison circuit of the DB

\[
R_1 = 3.3 \text{k}\Omega, R_2 = 4.7 \text{k}\Omega, R_v = 10 \text{k}\Omega, R_3 = 12 \text{k}\Omega, R_4 = 100 \text{k}\Omega, R_p = 1.2 \text{k}\Omega
\]
Finally, the output of optocoupler is connected to the driver of mosfet which is shown in Figure 4.27. Figure 4.28 shows the implemented DB circuit.

Figure 4.28 Implemented DB circuit
5.1 Overview

Power electronic, the technology of efficiently processing electric power, plays an essential role in the integration of the distributed generation units for good efficiency and high performance of the power systems. Different power electronics topologies are designed for each of DG systems. Power Electronic converters are used in Microgrids to control the flow of power and convert it into suitable DC or AC form as required. In this chapter, a control technique for DC-DC boost and buck-boost power converters are proposed which helps to reduce transient response and improve steady state behavior. In this study, PV modules are considered as the desirable DG source. To deal with the low output voltage issue, PV cells are aggregated through a high gain FIBC. Moreover, a battery back-up module with bidirectional DC-DC FIBBC is used to improve the system reliability and dispatch ability [63]-[66]. Multi-functional smart inverter is proposed which uses enhanced instantaneous power theory for harmonics and unbalanced currents compensation and for islanding detection in three phase system [67]-[69]. The schematic of the grid connected PV system with all power electronic interfaces is shown in Figure 5.1.⁷

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⁷ This section is based on the following set of papers: [63], [69], [82]-[84]
5.2 Analysis of FIBC and FIBBC

In this study, the floating interleaved boost, and buck-boost converters (FIBC-FIBBC) are selected to connect PV-arrays and BESS to the microgrids’ DC link, respectively. The schematic of the FIBBC is shown in Figure 5.2. Incorporating eight switches, $S_1, S_2, S_3, S_4,  \hat{S}_1, \hat{S}_2, \hat{S}_3, \hat{S}_4$, with two legs. This converter operates in both the boost and buck modes. In this section, we give a short overview on FIBBC controller design and analysis for low power residential applications (The FIBC can be analyzed as the unidirectional (boost) configuration of FIBBC). One of the main challenges in controlling of the buck-boost converters is the unwanted transient which is occurred because of the switching between two different controllers in buck and boost modes. Here we will show that by using FIBBC, we are able to design a single controller for two modes of operation in this converter.

5.2.1 Analysis of FIBBC

The FIBC and FIBBC converters are a modified version of conventional Boost and buck-boost topologies that has been proposed for PV and electric vehicle applications. This new topology improves the boost and buck performance from different perspectives such as higher voltage gain, higher efficiency, lower input current ripple and smaller components size (capacitors and inductors) [63]-[66].

The schematic of the FIBBC is shown in Figure 5.2. By having 8 switches, this circuit can operate in both Boost and Buck modes.

Figure 5.2 Schematic of the FIBBC
5.2.1.1. Buck mode

During time periods of low demand, the excess power generated by the PV can be used to charge an energy storage device. Thus, the FIBBC must work in Buck mode to charge the battery. For converter to be used in Buck mode, switches $S_1 - S_4$ must work as a diode and control signals must be applied to $S_1 - S_4$ with 90-degree phase shift respect to each other.

The output voltage of the conventional Buck converter is related to the input voltage with equation (5.1).

$$V_0 = D_{\text{buck}} V_{in},$$ (5.1)

where, $D_{\text{buck}}$ is the duty cycle.

The KVL relation between input and output of the FIBBC (in Buck mode) in Figure 5.2 indicates that:

$$V_{in} = V_{DC} = V_1 + V_2 - V_{bat}$$ (5.2)

where $V_{bat} = V_o$.

Since all elements (inductors and capacitors in Figure 5.2) have the same values, we have:

$$\frac{V_{bat}}{V_1} = \frac{V_{bat}}{V_2} = D_{\text{buck}}$$ (5.3)

From equations (5.2) and (5.3) we can conclude that:

$$\frac{V_{in}}{V_o} = \frac{2V_1}{V_o} - 1 \rightarrow \frac{V_o}{V_{in}} = \frac{D_{\text{buck}}}{2-D_{\text{buck}}}$$ (5.4)

In FIBBC, state variables are four inductor’s currents ($i_k$ for $k = 1, ..., 4$) and two capacitor’s voltages ($v_k$ for $k = 1, 2$).

Applying the average state equation method after vast mathematical procedure we can reach to the following transfer function of the FIBBC in Buck mode as follows:

$$\frac{V_o(s)}{d(s)} = \frac{2 \left[ \frac{V_{in}}{2-D_{\text{buck}}} \right]^2 (1+sR_C C)}{LCs^2 + sR_C C + 1}$$ (5.5)

5.2.1.2 Boost mode

The stored energy in the battery in Buck mode can be used to provide electricity during periods of high demand. So FIBBC must change its mode to the Boost mode. In this mode, the battery must inject power to the grid, each of switches $S_1 - S_4$ is working as a diode and, control signals must be applied to switches S1-S4 with 90 degree phase shift respect to each other. The ideal Boost
converter has four basic components, namely a power semiconductor switch, a diode, an inductor and a capacitor. Output voltage of conventional Boost converter is related to the input voltage with equation (5.6).

\[ V_0 = \frac{V_{in}}{1-D_{boost}} \]  

(5.6)

where, \( D_{boost} \) is a duty cycle of Boost mode. The relation between input and output of the FIBBC can be found in equation (5.9).

\[ V_o = V_{DC} = V_1 + V_2 - V_{bat} \]  

(5.7)

where, \( V_{bat} = V_{in} \). Because of all parameters (inductors and capacitors) in Figure 5.2 is the same so we have

\[ V_1 = V_2 = \frac{V_{in}}{1-D_{boost}} \]  

(5.8)

From equations (5.7) and (5.8) we can conclude that:

\[ V_{DC} = \frac{V_{bat}}{1-D_{boost}} + \frac{V_{bat}}{1-D_{boost}} - V_{bat} = \frac{(1+D_{boost})}{1-D_{boost}} V_{bat} \]  

(5.9)

Thus, the gain of FIBBC is \((1 + D)(1+D_{boost})\) times greater than conventional Boost converter.

In FIBBC state variables are four inductor’s currents and two capacitor’s voltages so equations are as follow:

\[ L_1 \frac{di}{dt} = V_{in} - (1 - S_1)V_1 \]  

(5.10)

Using the average state equation, we can find transfer function of the FIBBC in the Boost mode:

\[ \frac{V_o(s)}{d(s)} = \frac{2 \left( \frac{V_{in}}{1-D_{boost}} \right) (1+sR_c)}{(1-s^2)(1+2sR_c+s^2R_c+1)} \]  

(5.11)

where, \( R_c \) is a capacitor resistance (ESR).

5.2.1.3 Comparison of Buck and Boost modes in FIBBC

Transfer function of the Buck and Boost modes in FIBBC is very similar to each other. Moreover, the parameters of FIBBC such as capacitor, inductors, resistors in Buck mode and Boost mode are the same and we consider output capacitor value for battery system same as output capacitor of the FIBBC. Thus, we can conclude that all parameters in both transfer functions are the same. The behaviors of the FIBBC from the open loop stability analysis viewpoint are shown in Fig .5.3 for Boost mode and in Fig .5.4 for Buck mode.
Due to similarities within working range and stability range of these two behaviors in FIBBC, and since the crossover frequencies of both two transfer functions are close, it seems that we can reach to desirable phase and gain margins at the same time using a unique type III controller without further need of two separated controllers for each working mode. This can help us to decrease the unwanted distortion effect of transient phenomena happens due to the switching between two separated controllers. We will test the reliability of this idea in future sections.

Another useful relation between Buck and Boost modes in FIBBC is as follows:

Since the output voltage in Buck mode is the input voltage in Boost mode and vice-versa, we have:
\[
\frac{V_{in-buck}}{V_{o-buck}} = \frac{V_{o-boost}}{V_{in-boost}} \quad \frac{D_{buck}}{2-D_{buck}} = \frac{(1-D_{boost})}{1+D_{boost}} \quad \rightarrow D_{buck} = 1 - D_{boost}
\]  

(5.12)

5.2.2 Controller Design

The presented control scheme contains two main objectives as follows:

5.2.2.1 Design of the voltage controller to control output voltage in Buck and Boost mode with a type III compensator

Due to its flexibility, a Type III controller (compensator) is used to control the voltage of the FIBBC in different modes. The error amplifier compares the converter output voltage with a reference voltage to produce an error signal that is used to adjust the duty ratio of the switch. Compensation associated with the amplifier determines control loop performance and provides a stable control system.

An error amplifier should have a high gain at low frequencies, a low gain at high frequencies, and an appropriate phase shift at the crossover frequency. The small-signal transfer function of this amplifier is expressed in terms of input and feedback impedances \(Z_i\) and \(Z_f\).

\[
G(s) = \frac{V_C(s)}{V_o(s)} = -\frac{Z_f}{Z_i} = -\frac{(R_2 + \frac{1}{sC_2})|| \frac{1}{sC_3}}{R_t || (R_3 + \frac{1}{sC_3})}
\]  

(5.13)

Generally, there are two known methods for designing a type III error amplifier. First method is the \(K\) factor method and the second method, which is used here, is Manual Placement of Poles and Zeros (MPPZ). In the MPPZ method, the resonant frequency of the LC filter in the converter is used.

\[
f_{LC} = \frac{1}{2\pi \sqrt{LC}}
\]  

(5.14)

<table>
<thead>
<tr>
<th>Table 5.1. Parameters of the FIBBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage</td>
</tr>
<tr>
<td>Output voltage</td>
</tr>
<tr>
<td>Inductor</td>
</tr>
<tr>
<td>Capacitor</td>
</tr>
<tr>
<td>Duty cycle</td>
</tr>
<tr>
<td>Output Capacitor ESR</td>
</tr>
<tr>
<td>Switching frequency</td>
</tr>
</tbody>
</table>
The first zero is commonly placed at 50 to 100 percent of \( f_{LC} \), the second zero is placed at \( f_{LC} \), the second pole is placed at the ESR zero in the filter transfer function \( \frac{1}{R_{ec}} \), and the third pole is placed at one-half of the switching frequency. The corresponding values for zeros and poles of the controller are as follows: \( \omega_{z1}=461 \), \( \omega_{z2}=922.5 \), \( \omega_{p1}=0 \), \( \omega_{p2}=5.19K \), \( \omega_{p3}=62.8K \).

Table 5.1 and Table 5.2 represent the parameters of the converter and compensator respectively.

**Table 5.2. Parameters of the Type-III Compensator**

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( R_1 )</th>
<th>( C_2 )</th>
<th>( R_2 )</th>
<th>( C_3 )</th>
<th>( R_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2nF</td>
<td>1kΩ</td>
<td>1.5nF</td>
<td>120kΩ</td>
<td>1uF</td>
<td>15Ω</td>
</tr>
</tbody>
</table>

The Bode diagram of the compensator and FIBBC with and without controller in Buck and Boost mode is shown in Figure 5.5 and Figure 5.6. The overall compensated phase margin of the system is 74.7 and 84.3 degree for Boost and Buck modes, respectively.

**Figure 5.5 Bode diagram in Buck mode for FIBBC with and without controller**

**Figure 5.6 Bode diagram in Boost mode for FIBBC with and without controller**
Finally Pulse-width modulation (PWM) circuit converts the output from the compensated error amplifier to a duty ratio, which is applied to switches.

5.3 Multifunctional Double Mode Inverter (MFDMI)

In this section, a grid connected multifunctional double mode inverter is proposed for microgrid applications which can inject the generated power (in the form of active/reactive) by RESs and compensate harmonics and unbalanced parts of currents caused by unbalanced and nonlinear loads, enhancing the power quality at the point of common coupling (PCC), detecting islanded situation and control the voltage at PCC in islanded mode. A unified controller approach is proposed to control the inverter in both operating modes simultaneously with an optimal and fast transition scheme. Moreover, a novel instantaneous islanding detection method which is explained in chapter 4 is used to detect the fault near real time and switch the system from the GT to islanded (and vice-versa).

Proposed enhanced instantaneous power theory (chapter 3) is used as a current decomposition method for harmonic and unbalanced current compensation. Technically, we exploit a combinational control scheme formed by $dq$ and a proportional integral resonant (PIR) controller to address the required functionalities. This inverter is designed to be used as a smart interface for microgrids, contributing for the technology of smart-grids. The system diagram of the proposed MFDMI is shown in Figure 5.1.

5.3.1 Operational Modes for MFDMI

The proposed inverter is the core part of the microgrid with main features called as operational modes. The operating modes for the proposed MFDMI are described below.

5.3.1.1 Grid-connected mode / Current control mode

The most important functionalities in GT mode includes: controlling the output current of the MFDMI at PCC to negotiate with the main grid by injecting active and reactive powers, and compensating harmonics of the nonlinear loads in addition to the power fluctuations of RESs. In this work, the Enhanced instantaneous power theory is used to adjust the reference value of the currents according to the grid’s demand.
Based on PV generated power in addition to load-grid requirements the proposed MFDMI can operate in the following (sub-modes) functionalities:

- Harmonic/unbalance compensation and reactive power injection
- Harmonic/unbalance compensation and active power injection
- Active power injection
- Reactive power injection
- Active and reactive power injection
- Active and reactive power injection and harmonic/unbalance compensation
- Harmonic compensation
- Unbalance compensation

5.3.1.2 Islanded mode / Voltage control mode

In the Islanded mode since the grid is disconnected from the microgrid, the voltage stability is considered as a main purpose. Accordingly, the main objective of the controller is to control the PCC voltage magnitude and frequency, which is called the grid forming mode.

5.3.1.3 Transition Mode

The transition procedure between two aforementioned main operational modes can be considered as a third mode, also called the transient operational mode. The main objective within this mode is to perform a smooth transition avoiding unwanted transient effects on the system operation.

In the transition phase, synchronization is an important issue. A unified control approach has been implemented in this thesis to solve the transition problem. The corresponding current controller in the GT mode and the voltage controller in the islanded mode are combined as a unified controller to provide a smooth transition for MFDMI.

5.3.2 Proposed Control Scheme

5.3.2.1 Control Scheme in GC mode

In this section, we will first describe the control strategy in the GT mode. Next, we will discuss the control strategy for the islanded mode. Finally, we will end up with describing the unified smooth transition algorithm.
5.3.2.1.1 Grid Negotiation (Active and Reactive power injection)

The main functionalities in GT mode are controlling injected active and reactive powers as well as harmonic and unbalance compensation. In this mode synchronization is very important and it is usually done by PLL. There are mainly three types of PLL systems for phase tracking:

- Zero crossing
- Stationary reference frame
- Synchronous rotating reference frame (SRF).

The SRF PLL is the one with good performance under distorted and non-ideal grid conditions, also, it is applicable for single-phase and three-phase applications.

In this study SRF PLL is used as a synchronization method to synchronize the inverter with the main grid [62]. A basic PLL configuration is depicted in Figure 5.7. The phase voltages \((V_a, V_b, V_c)\) are obtained from the sampled phase voltages. These stationary reference frame voltages are then transformed to voltages \(V_d\) and \(V_q\) (in a stationary reference frame) using d-q transformation (park transformation). The angle \(\theta^*\) used in these transformations is calculated by integrating a frequency signal \(\omega^*\) and the initial angle must be carefully setup as initial condition in this integrator. If the frequency command \(\omega^*\) is identical to the utility frequency, the voltages \(V_d\) and \(V_q\) appear as DC values depending on the angle \(\theta^*\).

![Figure 5.7 SRF PLL implementation](image)

In order to inject an specified amount of active and reactive powers named \(P_{ref}\) and \(Q_{ref}\), respectively, the corresponding reference values for the (namely) active \(i_{d1}^*\) and reactive \(i_{q1}^*\) currents are calculated from (5.15)-(5.16), where, \(P_{ref}\) and \(Q_{ref}\) can be determined by an energy management system according to the selling price of active and reactive power to the grid.
\[ P_{ref} = \frac{3}{2} \left( v_d i_{d1}^* + v_q i_{q1}^* \right) \]  \hspace{1cm} (5.15) \\
\[ Q_{ref} = \frac{3}{2} \left( v_q i_{d1}^* - v_d i_{q1}^* \right) \]  \hspace{1cm} (5.16)

where \( v_d \) and \( v_q \) are the representation of the 3-phase voltages in the rotating reference frame \((d-q)\).

5.3.2.1.2 Enhanced instantaneous power theory for harmonic and unbalance compensation in GT

In this section, the load currents are decomposed by using enhanced instantaneous power theory (EIPT) which is introduced in chapter 3. EIPT works in the time-domain so it allows the control strategy to be defined within a near real time approach.

Since we want to compensate the unbalance and distorted part of load currents, the reference current of MFDMI for compensation is calculated from:

\[
\begin{align*}
\bar{i}_{\text{comp}-a} &= i_{aq0} + i_{apo} + i_{pha} + i_{pua} + i_{qha} + i_{qua} \\
\bar{i}_{\text{comp}-b} &= i_{bq0} + i_{bpo} + i_{phb} + i_{pub} + i_{qhb} + i_{qub} \\
\bar{i}_{\text{comp}-c} &= i_{cq0} + i_{cpc} + i_{phc} + i_{puc} + i_{qhc} + i_{quc}
\end{align*}
\]  \hspace{1cm} (5.17)

where \( \bar{i}_{\text{comp}-a} \), \( \bar{i}_{\text{comp}-b} \) and \( \bar{i}_{\text{comp}-c} \) are the harmonic and unbalanced parts of the current in \( abc \) axis, that should be compensated (generated) by MFDMI. Since the current control scheme is built based on the \( d-q \) control framework, compensated current components should be converted from the \( abc \) frame to the rotating reference frame \((d-q)\) by using:

\[
\begin{bmatrix}
i_{\text{com}-a} \\
i_{\text{com}-b}
\end{bmatrix} = \sqrt{2} \begin{bmatrix}
1 & \frac{1}{2} & \frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
i_{\text{comp}-a} \\
i_{\text{comp}-b} \\
i_{\text{comp}-c}
\end{bmatrix}
\]  \hspace{1cm} (5.18)

\[
\begin{bmatrix}
i_{d2}^* \\
i_{q2}^*
\end{bmatrix} = \begin{bmatrix}
cos(\theta) & sin(\theta) \\
-sin(\theta) & cos(\theta)
\end{bmatrix} \begin{bmatrix}
i_{\text{com}-a} \\
i_{\text{com}-b}
\end{bmatrix}
\]  \hspace{1cm} (5.19)

where \( i_{d2}^* \) and \( i_{q2}^* \) are the distorted and unbalance parts of current in \( d \) and \( q \) axis, respectively.

5.3.2.1.3 d-q based PIR control scheme in GT Mode

Once the grid currents are decomposed by the EIPT theory, the distorted and unbalance part of currents are chosen as compensating currents. As mentioned before, these components are converted to a stationary domain \((d-q \text{ frame } i_{d2}^* \text{ and } i_{q2}^*)\), and added to the reference injected active
and reactive currents to the grid ($i_{d1}^*$ and $i_{q1}^*$) to make the final reference values for the current controller as shown in Figure 5.8.

$$I_{dref} = i_{d1}^* + i_{d2}^*$$  \hspace{1cm} (5.20)
$$I_{qref} = i_{q1}^* + i_{q2}^*$$  \hspace{1cm} (5.21)

A regular PI controller has a good performance in controlling DC parameters. Therefore, in case of linear loads, (since $I_d$ and $I_q$ are DC parameters) the PI controller can be used as an appropriate option. However, in case of nonlinear loads, the harmonic component of the current waveform changes the $I_d$ and $I_q$ to AC parameters. Thus, in this case the PI controller cannot operate adequately. As a result, in this work a proportional-integral-resonant (PIR) controller (which is a combination of a PI and PR controllers) is implemented to control the current loop. The most important PIR control feature is its ability to track both DC and AC quantities at the same time.

Roughly speaking, in a PIR configuration, the PI part controls DC current components in $d$-$q$ axis to follow the desired active and reactive power injection to the grid, while the resonant part controls AC current components in $d$-$q$ axis, which are caused because of the presence of harmonics and unbalance. The overall combination guarantees an accurate tracking of DC and AC currents components.

Under distorted grid current conditions, if the angular frequency of the system is $\omega$, which is calculated by a phase locked loop [62], the current in the $d$-$q$ reference frame contains DC values of the positive sequence from the fundamental component in addition to AC terms caused by harmonic components.

An important point is that the generalized AC resonant term (R) is a two-sided integrator operator. This means that having a resonant part adjusted at the angular frequency of $6\omega$, we can nullify the error for the positive sequence at both positive and negative frequencies of $6\omega$ and $-6\omega$ simultaneously.

Similarly, the zero steady state error of DC components can be achieved by the usage of the standard PI controllers. A resonant regulator can be adjusted at the $6\omega$ frequency value (in the current controller) resulting in a decreased error of the AC signal (with the frequencies of $\pm 6\omega$).
Therefore, a PIR current controller in the d-q reference frame can be developed for directly regulating both the positive sequence of the fundamental component and the harmonic components of $-5\omega$ and $7\omega$. Figure 5.9 shows a block diagram of the designated PIR controller in d and q frames. The general transfer function of a typical PIR controller is defined as follows:

$$G_{PIR} = K_p + \frac{K_I}{s} + \frac{K_R s}{s^2 + \omega_R^2},$$

(5.22)

where $K_p$ is a proportional gain and $K_I$ plays the same role as that in the regular PI controller, which basically controls the system dynamics such as bandwidth and system marginal parameters (gain and phase). Moreover, the resonant control term $K_R$ provides an infinite gain at the frequencies of $\pm 6\omega$ to address the corresponding issues within the AC components.
To design the control system some simplifications of the model can be done. For example, the capacitor and damping resistor of the LCL filter can be neglected. The LCL filter is used in this work is the same which is designed in [62]. As shown in Figure 5.8, the grid side voltages are:

\[
\begin{align*}
    v_{ga} &= A \sin(\omega t) \\
    v_{gb} &= A \sin(\omega t - \frac{2\pi}{3}) \\
    v_{gc} &= A \sin(\omega t + \frac{2\pi}{3})
\end{align*}
\]

where \(A\) is the peak value of the line-to-neutral voltages. We can describe the dynamics of the AC side of inverter system on Figure 5.8 by the following differential equations:

\[
\begin{align*}
    v_{ia} &= (L_1 + L_2) \frac{di_{ga}}{3} + R i_{ga} + v_{ga} \\
    v_{ib} &= (L_1 + L_2) \frac{di_{gb}}{3} + R i_{gb} + v_{gb} \\
    v_{ic} &= (L_1 + L_2) \frac{di_{gc}}{3} + R i_{gc} + v_{gc}
\end{align*}
\]

where \(L (L = L_1 + L_2)\) is the equivalent LCL filter inductance and \(R\) is the resistances of first and second inductors of filter. By transferring voltages to \(dq\) frame and considering cross-coupling terms \((\omega L i_q , -\omega L i_d)\) control signals will obtain:

\[
\begin{align*}
    m_{d1} &= (i_{d}^* - i_d) \left( K_p + \frac{K_i}{s} + \frac{K_R s}{s^2 + \omega R^2} \right) + \omega L i_q + v_d \\
    m_{q1} &= (i_{q}^* - i_q) \left( K_p + \frac{K_i}{s} + \frac{K_R s}{s^2 + \omega R^2} \right) - \omega L i_d + v_q
\end{align*}
\]

where \(m_{d1}\) and \(m_{q1}\) represent the output current control signals of the PIR controller. The PIR controller coefficients are as follow: \(K_p = 10.5, K_i = 25.8\), and \(K_R = 24\). Once the controlling
signals $m_{q1}$ and $m_{d1}$ are calculated, the park inverse transformation is applied to convert them back into $a$-$b$-$c$ coordinates and finally the control signals are applied to the PWM.

5.3.2.2 Control Scheme in islanded mode

5.3.2.2.1 Islanded detection

Islanded mode detection is classified as a challenging issue in microgrid technology. Due to system sensitivity (in case of critical loads), an instantaneous fault (islanded situation) detection is a critical task in microgrids. Two major factors should be considered in Islanded detection procedure: 1) detection time and 2) serious level of the fault.

Once a fault occurs in the grid, the grid’s voltage becomes distorted. Distorted voltage can happen in different formats (such as, voltage sag, voltage swell, over voltage, under voltage and interruption). The islanding detection method used for the proposed MFDMI is based on EIPT which is explained in chapter 4. Once the inverter recognizes that the fault in the grid voltage or a sever power quality event is happened, the static transfer switch (STS) is opened and finally the operating mode is changed to the islanded mode.

5.3.2.2.2 d-q based PI Control scheme in Islanded Mode

The proposed voltage control block in islanded mode has been shown in Figure 5.10. In the islanded mode, the microgrid is disconnected from the main grid, thus the main objective of the controller is to adjust the voltage magnitude and frequency in adequate levels for local loads. Since $d$ and $q$ components of voltages appear as DC components, the PI controller is adequate for stabilization in this mode. The corresponding $d$ and $q$ components are shown by $m_{d2}$ and $m_{q2}$, respectively. The PI controller coefficients are as follow: $K_p = 11$, and $K_i = 120$.

![Figure 5.10 Voltage control diagram in the islanded mode.](image-url)
5.3.2.3 Unified control Scheme for Smooth Transition

Figure 5.11 illustrate the proposed smooth transition algorithm between the GT and islanded modes. Based on the defined control rules in this algorithm, once the operating mode is changed to Islanded mode the system dominant control rule should be changed into voltage control. In the Islanded mode, the reference values for voltages are set to $v_d = 1pu$ and $v_q = 0pu$, respectively. In this condition since the STS is open the actual injected current to the grid is zero, as a result, by setting the reference values of current components ($i_d$ and $i_q$) to zero, the output control signal of the current controller will be zero ($m_{d1} = m_{q1} = 0$) as well, and the voltage controller becomes the dominant controller. Once the grids’ problem is solved, the STS is closed so the microgrid is reconnected to the main grid. Synchronization is the most important issue in this case. In this work, the synchronization is provided by a conventional synchronous reference frame PLL [62].

In the GT mode, injected power to the grid is controlled by the quality and quantity of the grid current. In this mode since the PCC voltage is equal to the grid voltage (which is the reference voltage), the output of the voltage controller is zero ($m_{d2} = m_{q2} = 0$). Thus, the current controller is the dominant controller. Finally, adding the output of the current controller with the output of the voltage controller, and applying the resulted control signal into the PWM, a very smooth transition between the two modes can be achieved.

Figure 5.11 Unified Control algorithm for MFDMI with a smooth transition between two main operational modes
Figure 5.11. shows the proposed unified control algorithm for the MFDMI, where $m_d^*$ and $m_q^*$ are the final output control signals of the unified controller that are inserted into PWM by using $dq/abc$ transformation.

5.3.3 Simulation Results

This section contains simulation results in Matlab/Simulink for the proposed MFDMI under variety of practical conditions. Parameters of the inverter are listed in Table 5.3. To verify the performance of the designed system, different case studies were considered. We will discuss these situations using the following case studies.

All the corresponding results have been illustrated for phase a (similar results were observed for phase b and c). The reference values for injected active and reactive powers are basically defined based on MPPT or energy management system in microgrids.

Table 5.3: Parameters of the inverter

| Input voltage | $V_{dc} = 400$ |
| Output voltage (1Φ) | $V_{out} = 120\text{Vrms}$ |
| Inductor of LCL | $L_1 = 2.33\text{mH}$ (inverter side), $L_2 = 0.045\text{mH}$ (grid side) |
| Capacitor of LCL | $C = 5e-6\text{F}$ |
| Switching frequency | $f_s = 12\text{KHz}$ |
| Frequency of the grid | $f_g = 60\text{Hz}$ |
| Inverter nominal power | $5 \text{kVA}$ |

**Case 1:** In this case, at $t=0.1s$, a nonlinear load (a 3-phase diode rectifier with 5A DC current source) is added to the PCC. The corresponding current waveform of the nonlinear load is shown in Figure 5.12a.

The compensating current term (calculated by $E IPT$) is shown in Figure 5.12b. The compensated grid current, in presence of the nonlinear load is shown in Figure 5.12c. We can see that, in the presence of the nonlinear load in the system, the controller compensates harmonic terms, and the THD of the grid current remains under 2%. After 0.3s, the nonlinear load is removed, where the controllers’ fast response to this load change is observed from Figure 5.12c.
Figure 5.12 a) Current of nonlinear load b) Compensating current calculated by EIPT c) Compensated grid current after adding nonlinear load at 0.1, and removing nonlinear load at 0.3s

**Case 2:** Using the proposed instantaneous islanded detection approach, a fault is detected in the grid at $t=0.4s$. As a result, the protection system changes the STS status to the open position and the inverter switches to the islanded mode.

Figure 5.13 a) PCC voltage after inverter goes to the islanded mode from the GT mode at 0.4s. b) PCC voltage after inverter goes from the islanded mode to the GT mode at 0.6s
In this case, the voltage controller adjusts the PCC voltage and does not have any control on the injected power to the grid.

![Figure 5.14 Injected current to grid in different operational conditions of MFDMI](image)

Figure 5.14 and Figure 5.13a show the grids’ current and PCC voltage for the islanded mode duration (0.4-0.6s), respectively. As it can be seen, the controller has a good performance with an appropriate response to the changes in the operating modes. Since the STS is open, logically, the injected current to the grid goes to zero.

**Case 3:** At 0.6s, the grid’s problem is solved and its voltage goes back to the acceptable range. As a result, the STS is closed (GT mode) and the system is reconnected and synchronized to the main grid using a synchronous reference frame PLL.

![Figure 5.15 a) Comparison between $I_d$ and $I_{d_{ref}}$ and, b) Comparison between $I_q$ and $I_{q_{ref}}$.](image)

Figure 5.15 a) Comparison between $I_d$ and $I_{d_{ref}}$ and, b) Comparison between $I_q$ and $I_{q_{ref}}$. 
In this condition, the reference value of the voltage controller is dictated by the voltage value at PCC. According to the smooth transition algorithm, the current controller will be the dominant controller. Figure 5.14 and Figure 5.13b are representing the corresponding grids’ current and the PCC voltage waveforms when the inverter’s mode is changed from islanded to GT mode. These results indicate a smooth transition between these two operational modes.

Figure 5.15 shows the reference versus actual values of the \( I_d \) and \( I_q \) in GT mode. Optimizing the PIR controller parameters, the reference values are perfectly followed by the actual signals.

This section contains the real-time simulation for the proposed MFDMI. The overall system was modelled using a real-time simulator OPAL-RT (32 port OP5312). Figure 5.16a represents the voltage and injected current to the grid at PCC without harmonic compensation (where \( i_{d2}^* \) and \( i_{q2}^* \) are not considered in the controlling signal). Figure 5.16b and Figure 5.16c illustrate the same quantities with the harmonic compensation and unbalanced compensation respectively.

Figure 5.16 Voltage and injected currents \((pu)\) to the grid at PCC a) without compensation, b) with harmonic compensation, c) with imbalance compensation, and d) with full compensation.
$i_a$ vs $i_b$ with harmonic compensation

(i)

$\bar{i}_a$ vs $\bar{i}_c$ with harmonic compensation

(ii)

$\bar{i}_a$ vs $\bar{v}_a$ with full, reactive, harmonic, and imbalance compensation

(iii)

$\bar{i}_a$ vs $\bar{i}_b$ with full, reactive, harmonic, and imbalance compensation

(iv)

$\bar{i}_a$ vs $\bar{i}_c$ with full reactive, harmonic, and imbalance compensation

(v)

$\bar{i}_a$ vs $\bar{i}_b$ with imbalance compensation

$\bar{i}_a$ vs $\bar{i}_b$ with full reactive, harmonic, and imbalance compensation

$\bar{i}_a$ vs $\bar{i}_c$ with imbalance compensation

Figure 5.16 continued
Figure 5.17 the real-time simulation realization using OPAL-RT-32 port OP5312

Figure 5.18a and Figure 5.18b represent the MFDMI performance once the operational mode is changed from GT to Islanded and vice-versa, respectively.

Figure 5.18 Voltage and injected current (\( \nu \)) at the PCC after changing control method from a) GT to islanded, b) islanded to GT mode.
We can see the fast and smooth transition between two modes while the voltage and current signals (pu) are adjusted to the new modality of the system quickly and stably. Finally, we have shown the behavior of the power signals under GT mode (at PCC) versus their reference values in Figure 5.19. Using the proposed control approach, both the active and reactive powers can follow their reference values within a suitably stable and reliable behavior.

![Figure 5.19 Output (pu) a) active and b) reactive powers at PCC vs their ref value.](image)

**Case 4:** In this case study, we considered nonlinear and unbalanced load (parameters of load are mentioned in Appendix. D) Supplied by distorted (including harmonics of the order of 5 and amplitude of 10% of the sinusoidal voltage) and unbalanced voltage source (zero sequence components 20% of positive sequence component).

Figure 5.20 shows the output current injected to the grid and voltage of the system at PCC, without any compensation:

![Figure 5.20 Voltage and current waveforms of the source in phases a, b and c.](image)
Figure 5.21 illustrates the active and reactive current components calculated by the EIPT formulations. As it is shown EIPT-based approach resulted in pure sinusoidal wave-shapes.

Figure 5.21 active and reactive current components in EIPT

Figure 5.22 harmonic and unbalance current components in EIPT

Figure 5.22 indicates that the unbalanced and distorted components are well separated through the EIPT approach. Figure 5.23a and b are illustrating the compensated currents of the grid with MFDMI system using EIPT and CPT decomposition methods, respectively.

Figure 5.23 Grid currents after compensation (a) using EIPT (b) using CPT
CHAPTER 6
CONCLUSION AND FUTURE RESEARCH

Power theories are strong mathematical tools, which have been specifically developed to define and model the characteristics of the fundamental power signals and concepts, and their interactions. A major drawback with traditional versions is conceptual and physical definition-dependency on the pre-assumed pure sinusoidal behavior of the power signals. Regarding the uncertainties injected by the aggregation of renewable energy sources in addition to harmonic effects caused by incorporation of nonlinear loads, the general behavior of the power signals may significantly deviate from a normal sinusoidal waveform. This fact imposes a serious challenge to be addressed in terms of design and control strategy selection in the modern power systems. Motivated by this important issue, various modified and generalized power theories came onto the scene during the last couple of years. Some expanded in time domain while others are rooted into other mathematical domains such as frequency domain, time or time-frequency domain. Due to their instantaneous calculation ability, the time domain-based analysis has piqued much interest. Widely acknowledged theories such as instantaneous power theory (PQ) or conservative power theory are currently used in the most novel power electronic literature. A major application of power theories is in electrical converters control (active filters). Nowadays, inverters are the heart of any power electronic interface and the critical components of Microgrid systems.

6.1 Conclusions

In this study, we explored the applications of advanced instantaneous power theories for operation improvement of power electronic interfaces in the case of renewable-based Microgrids. We specifically utilized the power theories in terms of typical signal decomposition methods to extract the fine levels of information from the electrical signals and use this revealed information to optimize the corresponding control strategies. The following set of desired specifications have been considered, and the inherent challenges have been comprehensively explored. Frontier methodologies were developed and introduced to address each in a typical Microgrid.

1. The investigated system is composed of a PV-based generation unit in addition to a battery backup module and two FIBC and FIBBC DC-DC converters to connect these two modules into
the DC bus. The system is supposed to operate in two operational modes: GC and islanding, where the major control preference is on voltage in islanding and current in the GC mode at the PCC. A unified control signal scheme was developed to guarantee the smooth transition between two major operational modes.

2. Due to various advantages, floating interleaved schemes have been used in the system. A few novel techniques for controlling these power converters in photovoltaic (PV)-based Microgrids were proposed, and an improved control framework for PV-based microgrids was presented, termed as the short transient recovery for the low voltage-grid-tied DC distributed generation.

3. Inverters are the most important part of the power electronic interfaces which are used for integration of distributed energy sources (solar, wind, and others) in the Microgrids and play the most important role in improving Microgrid optimization and functionality. Thus, in this work a variety of pre-provided power theories and methods such as instantaneous power theory has been used for the sake of control and design of a smart inverter to appropriately connect the Microgrid to the main grid while preserving all grid standards and requirements. This module was named a multifunctional inverter with a set of functionalities noted in chapter 5. A variety of case studies have been tested using both simulation-based and real-time oriented software.

4. While adequate to some extent, most of the usual time domain instantaneous power theories suffer from a set of weaknesses especially in terms of distorted voltage sources. In the last few years, several articles and papers have discussed or presented solutions and methodologies for power quality and weak grid conditions; however, a more solid approach must be taken for non-sinusoidal and unbalanced waveform conditions in electrical circuits. In the third chapter of this thesis, we proposed the idea of a new Enhanced instantaneous power theory (EIPT) for smart grid applications by considering unbalanced and non-linear three-phase power systems. The mathematical framework can decompose the electrical signals into balanced sinusoidal active, balanced sinusoidal reactive, non-sinusoidal active, non-sinusoidal reactive, homopolar and heteropolar unbalanced active and reactive components in each phase of the system. We hope that this new detailed decomposition approach is helpful in optimizing the control strategies for power electronic interfaces of Microgrid systems with higher flexibility. The proposed framework is valid for both asymmetrical and unbalanced conditions. Moreover, it might be used independently in any power signal processing application. Figure 6.1 shows the proposed multifunctional GC-inverter for hardware implementation as a future work.
5. Finally, as another contribution associated with this research, we discuss one of the most important issues in Microgrid applications widely known as islanding detection and classification. A variety of remote and local methods have been developed to address this issue. However, most of them suffer from high cost, high time complexity and low detection performance. Moreover, most of these methods have been developed in the case of unintentional islanding, and no clear strategies have been included for intentional islanding situations. Aggregating the artificial intelligence methods with aforementioned instantaneous power theories, we have defined a new intelligent instantaneous islanding detection scheme. We modelled and analyzed a set of major islanding scenarios with respect to electrical signal behavior under this regime. We have shown that using advanced power theories a set of instantaneous features named “dynamic-static ratios” can be extracted from the 3-phase voltages signals. These features can then be exploited to form
and prepare a well-separated feature space for classification methods. In this work, we used ANN as powerful nonlinear classifier. Our approach was developed under a much more challenging reason-oriented situation, in contrast with most of the usual fault-type frameworks.

6.2 Future Directions

In general, our proposed EIPT in chapter 3 can be further exploited in terms of any 3-channel signal decomposition application. As a possible application, we have implemented this method for sake of feature extraction in the islanding detection problem, however, one may further investigate the corresponding potentials of the EIPT (instantaneous power theories) within other power engineering and signal processing fields.

Beside our proposed alternative approach for case of distorted voltages one may develop a more mathematical-oriented strategy by incorporating the idea of “Generalized symmetrical components for periodic non-sinusoidal three-phase signals which has been developed within the proposed approach of [86]. In this framework, the resulting sequence components do preserve both the symmetry and orthogonality properties. In such an approach, the signal is decomposed within the following orthogonal space. Given a set of generic three-phase variables:

\[
x = \begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix},
\]

we decompose them in the orthogonal form:

\[
x = x^z + x^h = x^z + x^p + x^n + x^r
\]

where:

- \( x^z \) are zero-sequence (homopolar) components
- \( x^h \) are non-zero-sequence (heteropolar) components
- \( x^p \) are positive-sequence components
- \( x^n \) are negative-sequence components
- \( x^r \) are residual components

The final symmetrical decomposition would have the following form:

\[
x = \begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} (x^z(t) + x^p(t) + x^n(t) + x^r(t)) \\ \frac{1}{3} (x^z(t) + x^p(t - \frac{\tau}{3}) + x^n(t + \frac{\tau}{3}) + x^r(t)) \\ \frac{1}{3} (x^z(t) + x^p(t - \frac{2\tau}{3}) + x^n(t + \frac{2\tau}{3}) + x^r(t)) \end{bmatrix};
\]
such that:

\[
x^r = \begin{bmatrix}
x^r_a(t) \\
x^r_b(t) \\
x^r_c(t)
\end{bmatrix} = \begin{bmatrix}
\frac{1}{3} (x^h_a(t) + x^h_a(t - \frac{T}{3}) + x^h_a(t - \frac{2T}{3})) \\
\frac{1}{3} (x^h_b(t) + x^h_b(t - \frac{T}{3}) + x^h_b(t - \frac{2T}{3})) \\
\frac{1}{3} (x^h_c(t) + x^h_c(t - \frac{T}{3}) + x^h_c(t - \frac{2T}{3}))
\end{bmatrix}.
\]

This can be addressed as a future work.

On the other hand, the proposed feature transformation in chapter 4, is just one alternative way of thinking of instantaneous-based feature extraction. It is worthy to form and try a set of different possible combination of features with more amount of real data to improve the IIPID results. Moreover, one may apply a variety of other types of classifiers to end up with a higher overall performance. Classification techniques such as support vector machines or fuzzy-based classifiers are beyond the possibilities.

Finally, we should mention that, recently nonlinear and model-based control approaches [85] have found lots of attentions in power electronics. Specifically, we have developed a sliding mode controller in addition to a model predictive control approach in [64]-[65]. These approaches have been already examined successfully in terms of power factor correction while raised up promising results in terms of power electronic converters control, and can be investigated in other applications in the future.
REFERENCES CITED


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APPENDIX

A. Proof of Property.2:

1. \( i_q \cdot v_{\alpha \beta 0} = \frac{q \times v_{\alpha \beta 0}}{v_{\alpha \beta 0} \cdot v_{\alpha \beta 0}} \cdot v_{\alpha \beta 0} = \frac{(v_{\alpha \beta 0} \times i_{\alpha \beta 0}) \times v_{\alpha \beta 0}}{v_{\alpha \beta 0} \cdot v_{\alpha \beta 0}} \cdot v_{\alpha \beta 0} = \frac{[-(i_{\alpha \beta 0} \cdot v_{\alpha \beta 0}) v_{\alpha \beta 0} + (v_{\alpha \beta 0} \cdot v_{\alpha \beta 0}) i_{\alpha \beta 0}]}{v_{\alpha \beta 0} \cdot v_{\alpha \beta 0}} \cdot v_{\alpha \beta 0} = 0 \). (A.1)

2. \( i_p \times v_{\alpha \beta 0} = \frac{p \cdot v_{\alpha \beta 0}}{v_{\alpha \beta 0} \cdot v_{\alpha \beta 0}} \times v_{\alpha \beta 0} = 0 \). (A.2)

B. Proof of Property.27:

Proof:

The cross product is defined as:

\[ \mathbf{I} \times \mathbf{V} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ i_1 & i_2 & i_3 \\ v_1 & v_2 & v_3 \end{bmatrix} = <i_2 v_3 - i_3 v_2, i_3 v_1 - i_1 v_3, i_1 v_2 - i_2 v_1> \]

Thus, the magnitude squared of the cross product will be calculated as:

\[ |\mathbf{I} \times \mathbf{V}|^2 = (\mathbf{I} \times \mathbf{V}) \cdot (\mathbf{I} \times \mathbf{V}) = i_2^2 v_3^2 - 2i_2i_3 v_2 v_3 + i_3^2 v_2^2 + i_3^2 v_1^2 - 2i_1i_3 v_1 v_3 + i_1^2 v_2^2 + i_1^2 v_2^2 - 2i_1i_2 v_1 v_2 + i_2^2 v_1^2 \] (B.1)

On the other hand:

\[ (|\mathbf{I}| |\mathbf{V}|)^2 = \sqrt{i_1^2 + i_2^2 + i_3^2 \sqrt{v_1^2 + v_2^2 + v_3^2}} = i_1^2 v_1^2 + i_2^2 v_2^2 + i_3^2 v_3^2 + i_2^2 v_1^2 + i_2^2 v_2^2 + i_3^2 v_1^2 + i_3^2 v_2^2 + i_3^2 v_3^2 \] (B.2)

Moreover:

\[ (\mathbf{I} \cdot \mathbf{V})^2 = i_1^2 v_1^2 + i_2^2 v_2^2 + i_3^2 v_3^2 + 2i_1i_2 v_1 v_2 + 2i_1i_3 v_1 v_3 + 2i_2i_3 v_2 v_3 \] (B.3)

Have a close look at these star-signed equations, we can easily observe that (B.1) = (B.2)-(B.3). In mathematic literature, this is widely known as the Lagrange’s Identity. As a result, one may alternatively write:

\[ |\mathbf{I} \times \mathbf{V}|^2 = (|\mathbf{I}| |\mathbf{V}|)^2 - (\mathbf{I} \cdot \mathbf{V})^2 \] (†)

Using † in (27) we have

\[ S^2 = P^2 + Q^2 = |\mathbf{I} \times \mathbf{V}|^2 + (\mathbf{I} \cdot \mathbf{V})^2 = (|\mathbf{I}| |\mathbf{V}|)^2 - (\mathbf{I} \cdot \mathbf{V})^2 + (\mathbf{I} \cdot \mathbf{V})^2 = \]
\[(|I||V|)^2 = |I|^2 |V|^2 \longrightarrow S = |I||V| \quad \square.\]

C. The Schematic of axis formation in Concordia transformation:

![Schematic diagram](image)

Figure C.1. The axes for transformation of a three-phase into a two-phase system

D. Parameters Related Case Study. A (Figure 3.6):

Table D. Grid and Load Parameters

| Grid Voltage | \(V_{a, rms} = 120\) | \(L_1\) | 300\(\mu\)H |
| Grid Frequency(f) | 60Hz | \(L_2\) | 100\(\mu\)H |
| \(R_1\) | 500\(\Omega\) | \(L_3\) | 1mH |
| \(R_2\) | 100\(\Omega\) | \(C_1\) | 470\(\mu\)F |
| \(R_3\) | 50\(\Omega\) | \(C_2\) | 100\(\mu\)F |
| \(R_4\) | 50\(\Omega\) |