TIME AND FREQUENCY DOMAIN FINITE ELEMENT ANALYSIS OF VIBRATORY DRUM INTERACTION WITH LAYERED EARTHWORK

by

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ABSTRACT

The earthwork construction industry has been making a necessary shift from quality control/quality assurance (QC/QA) via spot based density and moisture testing to QC/QA via full coverage mechanistic testing (e.g. modulus, stiffness) to allow for performance based assessment of compaction QA/QC. As a result, vibration-based drum measurement of soil properties during compaction, known as Intelligent Compaction (IC) or Continuous Compaction Control (CCC), has gained traction in the US. Using the contact force– drum displacement ($F - z$) data, coupled with an onboard computer and GPS measurements, the operator can perform real-time QC on 100% of the compacted region, a significant improvement over current spot test methods. For vibratory drums to provide useful, mechanistic measurements, a quantitative understanding of the drum/soil system is needed.

This research contributes to advancements in Intelligent Compaction by addressing the challenge of modeling the dynamic, non-linear drum/soil system for homogenous and two-layer soil systems. This thesis focuses on time and frequency domain finite element (FE), and analytical modeling to explore the sensitivity of layer parameters such as Young’s modulus, material damping, and top layer thickness on drum response. Results from both finite element models are compared with field data and with each other, to gain insight into how the finite element implementations of Rayleigh damping in the time domain, and hysteretic damping in the frequency domain, affect the force-displacement behavior of the drum.

The time domain model shows that the roller-measured stiffness increases with lift thickness and with subgrade and base moduli, showing sensitivity to both changes in lift thickness and in soil materials commonly observed in practice. The time-varying contact area is shown to have negligible effects on the roller- measured values. This observation is justified using a plane strain analysis of a layered elastic medium subjected to a dynamic strip loading.
The frequency domain model is able to accurately capture field observed inertial and dissipative properties through its ability to model individual material dissipative parameters. The model shows a decrease in radiation damping with increases in half-space stiffness. The addition of material damping increase total system energy loss, and for constant $\eta_M$ (and $\alpha$ and $\beta$), the relative influence of material damping increases as the influence from radiation damping decreases.

Additionally, this thesis examines expertise in technology adoption process within state DOT’s to gain insights into the combinations of expertise required for successful new technology adoption.
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S-wave Velocity $v_s$

Stiffness Proportional Damping Coefficient $\beta$

Transformed Spatial Dimension $\zeta$

Transformed $x$ Normal Stress $\bar{\sigma}_x$

Transformed $x$–displacement $\bar{u}$

Transformed $xz$ Shear Stress $\bar{\sigma}_{xz}$

Transformed $z$ Normal Stress $\bar{\sigma}_z$

Transformed $z$–displacement $\bar{w}$

Young’s Modulus $E$

$w$ at $z = 0$ & $x = 0$ $w_{00}$

$x$–displacement $u$

$z$–displacement $w$
LIST OF ABBREVIATIONS

Continuous Compaction Control ............................................ CCC
Department of Transportation .................................................. DOT
Finite Element ................................................................. FE
Frequency Domain Finite Element ............................................. FE-F
Intelligent Compaction ............................................................. IC
Measured Value ................................................................. MV
Quality Assurance/Quality Control ............................................. QA/QC
Studies of Expertise and Experience .......................................... SEE
Time Domain Finite Element ..................................................... FE-T
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Additionally, I could not have done this without the support of those closest to me, and to them I say, thank you, and I love you.
Significant advances have been made in the past decade in quality assurance/quality control (QA/QC) in earthwork construction. One notable advancement is the introduction of continuous compaction control (CCC) or Intelligent Compaction (IC) technologies. CCC allows for continuous monitoring of soil compaction through vibratory roller integrated measurements, coupled with GPS mapping. By combining the roller-measured values (MV) (e.g. acceleration, contact force) with onboard GPS measurements and mapping, the operator can perform real-time QC (Figure 1.1). CCC/IC technology provides significant improvement in QA/QC over current methods by measuring soil properties for 100% of the compacted area versus the less than 1% coverage provided by current methods.

However, for CCC rollers to provide mechanistic measurements useful for engineering analysis and design, there must be a quantitative understanding of the drum/soil system and the roller MVs. Historically, roller MVs have been used to gage soil compaction via relativistic measures: for example, the compaction meter value (CMV), the harmonic content of the MV signal, was noted to increase with increased compaction [2]. More recently, roller MVs have been used to define absolute measures of soil stiffness ($k_s$) [3–6]. $k_s$ is calculated from the force–displacement loops per Figure 1.2. The relationship between $k_s$ and in situ soil response is complex.

Experimental data [5, 7–10] have shown that drum/soil interaction is highly nonlinear and dependent upon the inertial and dissipative properties of the soil, based on the vibration frequencies employed (20-35 Hz). It involves transient response with time-varying loading conditions, including decoupling between the drum and soil, chaotic behavior, and drum and frame rocking [11–14], which cannot be captured using analytic models. The drum/soil contact area ($2a$) changes throughout each cycle of vibration from a maximum area to a
minimum area, which can be zero if loss of contact is experienced. Vibratory drums provide a measure of $k_s$ that reflects a composite nature of the underlying layers for most earthwork construction situations (Figure 1.3). The composite nature of $k_s$ has led researchers to investigate potential relationships between roller-measured stiffness and individual layer moduli values to allow for a more mechanistic relationship to CCC. To achieve this end, there is a clear need to better understand the mechanics of vibratory drum interaction with layered soil.

This research addresses the challenge of modeling the dynamic, non-linear drum/soil system for homogeneous and two-layer situations. The studies focus on time and frequency
domain finite element (FE), and analytical modeling to explore the sensitivity of mechanistic and geometric parameters such as modulus ($E$), material damping ($\xi, \eta$), and top layer thickness ($h$) on drum MVs. Chapter 2 explores this relationship through the development of a time domain FE model that is able to explicitly model the contact conditions between the drum and soil. The time domain model is limited because of its inability to model individual material damping properties, as system level Rayleigh damping coefficients must be used.

The frequency domain model used in Chapter 4 overcomes this limitation by using individual layer material loss factors ($\eta_M$) as an input to the model. The effects of material damping on drum response are explored using a frequency domain FE model (with similar setup to FE model in Chapter 2). Best fits for the same sets of field data for both the time and frequency domain models are compared to better understand the effects of the different damping models, and identify the benefits and shortcomings of each model in capturing field observed behavior.

Chapter 3 explores the influence of contact area ($2a$) on surface response through the development of an analytical model of a layered halfspace subject to sinusoidal strip loading. This analysis was performed to help determine if $2a$ needs to be explicitly modeled in FE analyses to capture drum behavior. Individual simulations are run for different values of
Figure 1.3: Vibratory Roller MVs incorporate multiple layers, requiring layered forward models (from [15]).

2a and different loading situations to better understand the relationship between 2a and vibratory drum response.

Chapter 5 deviates from technical research and uses Studies of Expertise and Experience (SEE) to analyze and to better understand the process of technology adoption in the road construction industry.
CHAPTER 2
FINITE ELEMENT ANALYSIS OF VIBRATORY ROLLER RESPONSE ON LAYERED
SOIL SYSTEMS

The following chapter is modified from an article published in *Computers and Geotechnics*, and can be referenced as:


2.1 Abstract

The objective of this study is to quantify the relationships between continuous compaction control (CCC) roller soil stiffness measurements and subgrade and base lift moduli and thickness for quality control applications on fully compacted soils (e.g. proof rolls). This is done using plane strain, dynamic, time-domain finite element (FE) analyses. The FE model is calibrated against field data from two construction sites and is shown to capture the time-varying loading characteristics of the roller and the force-deflection behaviors of the underlying soil surface. The model is then used to explore the effects of subgrade and base moduli and the thickness of the compacted base layer on roller-measured stiffness values. The roller-measured stiffness increases with lift thickness and with subgrade and base moduli, showing sensitivity to both changes in lift thickness and in soil materials commonly observed in practice. The time-varying contact area is shown to have negligible effects on the roller-measured values. This observation is justified using a plane strain analysis of a layered elastic medium subjected to a dynamic strip loading.
2.2 Introduction

Continuous monitoring of soil compaction through roller measurements or continuous compaction control (CCC) has been used in the construction industry for over 30 years. By combining the roller-measured value (MV) (derived from drum accelerometer data) with onboard GPS measurements, the operator can perform real-time quality control (QC).

However, for CCC rollers to provide mechanistic measurements (e.g. individual layer Youngs moduli ($E_1$, $E_2$), density ($\rho$), Poissons Ratio ($\nu$)) useful for engineering analysis and design, there must be a quantitative understanding of the roller/soil system and the roller MV. Historically, roller MVs have been used to gage soil compaction via relativistic measures: for example, the compaction meter value (CMV), utilizes the harmonic content of the drum acceleration and it was noted to increase with increased compaction [2]. More recently, roller MVs have been used to define absolute measures of soil stiffness, ($k_s$), variously defined by individual roller manufacturers (e.g., [4–6, 16]). Vibratory drums provide a measure of $k_s$ that reflects a composite nature of the underlying layers [3, 6, 17–19] for most earthwork construction situations (Figure 2.1). The composite nature of $k_s$ has led researchers to investigate potential relationships between roller-measured stiffness and individual layer moduli values [15, 20–22] to allow for a more mechanistic relationship to CCC.

Figure 2.1: Measurement Depth of Vibratory Roller
The relationship between $k_s$ and in situ soil response is complex. Experimental data [5, 7–10] have shown that drum/soil interaction is highly nonlinear and dependent upon the inertial and dissipative properties of the soil, based on the vibration frequencies employed (20-35 Hz). It involves transient response with time-varying loading conditions, including decoupling between the drum and soil, chaotic behavior, and drum and frame rocking [11–14]. The drum/soil contact area ($2a$) changes throughout each cycle of vibration from a maximum area to a minimum area, which can be zero if loss of contact is experienced. The literature has clearly conveyed $2a$ has a strong influence on stress/strain distributions within homogeneous bodies [23, 24] and layered systems [19, 25, 26]. A simple analytic analysis is performed to determine whether or not this behavior needs to be explicitly modeled.

The majority of the published literature on vibratory drum-soil mechanics is based on the analysis of lumped parameter models [10, 27–29], and cone models [7, 30, 31]. Although van Susante and Mooney [10] are able to capture the decoupling between the drum and soil, and drum/frame rocking, and Rich [29] models a two-layer system, these aforementioned models are limited due to their inability to accurately model the inertial and dissipative properties of the soil. Since mass-spring-dashpot elements are used to describe both the roller and the soil, analysts using these models must guess at the inertial involvement via added masses. The elastic springs and dashpots cannot capture the continuum nature of the material (they have been successful in one-layer applications, e.g., Lysmers analog [32], but have not been proven for multi-layer applications).

Multiple continuum based forward models have been developed to explore the influence of surface loads and soil parameters on system response. Although plane strain dynamic, elastic analytic models exist for strip loading on the surface of a half-space [24], none exist for a two-layer system. The majority of published literature on elastic half-space, two-layer or N-layer systems uses either axisymmetric [33–36] or three-dimensional modeling techniques [37–40]. In the above analytic models, a contact pressure distribution and $2a$ must be assumed, as the drum cannot be explicitly modeled. As a result, a constant $2a$ must be assumed throughout
loading and loss of contact cannot be modeled.

The variable contact force during each cycle, combined with the dynamics and the multilayer continua preclude the adoption of analytical solutions to describe the roller-soil system. As a result, discretized computational approaches must be used to capture the complex behavior. Dynamic elastic [15, 41] and elasto-plastic FE models [42–44] have been developed that are able to capture the deformation directly below the roller. Erdmann et al. [30, 42] show the importance of dynamics in roller response but focus their analysis on modeling different types of roller excitation and do not explore the relationship between measurable drum response and layer properties. Mooney et al. [15] and Mooney and Facas [41] provide preliminary analysis on individual layer parameter sensitivity for the FE model, but their focus is on the inversion process and on sensitivity analysis of a pseudo-static BEM model. None of the above FE models examines or addresses how the contact area is modeled and its influence on results. A thorough examination of the effects of time-varying contact area on drum response and sensitivities to underlying soil parameters, is needed to gain a truly mechanistic understanding of the system. To do this a robust forward model is needed that captures the dynamic loading conditions of the drum in addition to the inertial and dissipative properties of the soil.

In this chapter the results of a study to model vibratory drum-layered soil interaction using dynamic finite element analysis are presented. A main motivation for this study is to capture the time varying loading conditions of the system. Varying loading conditions created by curved drum interaction with the ground and decoupling of the drum from the ground are modeled. Using a kinematic contact algorithm, no assumptions need to be made regarding contact, allowing for a more physically accurate contact model.

The FE model is calibrated and validated with experimental data from homogeneous and two-layer conditions and use the FE model to parametrically explore the relationship between vibratory roller response and system parameters such as elastic moduli and layer thickness. U.S. earthwork construction currently performs QA on a per lift basis (typically
15-30 cm). For each layer of earthwork (and the existing base), the roller creates a spatial map of stiffness data and of lift thickness. These data for the existing base or subgrade and each subsequent lift can be combined with this forward model, using an inversion program (per [41]), to extract individual layer moduli. This can be done simply by first performing the inversion process on the subgrade to find $E_1$. Once $E_1$ is known, we can find the subgrade modulus ($E_2$) from $E_1$ and $h$ using inversion (a full description of this process is provided in [41]). The interpretation of the results from this forward finite element model provide a foundation for the mechanistic interpretation of the composite roller-measured stiffness for the individual dynamic mechanical properties of the underlying soil layers.

2.3 Background

2.3.1 FE Model

2.3.1.1 Roller Parameters and Quantification of Soil Stiffness

Smooth drum vibratory CCC/IC rollers are nominally in the 12-15 metric ton range with drum diameters of approximately 1.5 m and drum lengths of approximately 2.1 m. Excitation is created by uni-directional or counter-rotating eccentric masses, $m_0$, located at effective moment arms of $e_0$ within the drum (see Figure 2.2); magnitudes of eccentric mass moment, $m_0 e_0$, can range from 0 to 5.0 kg-m, and excitation frequencies, $\Omega$, can range from 25 to 35 Hz. In this study, we validate our finite element model with experimental data from a Sakai SV510D (Figure 1.1) roller and accordingly summarize the key roller properties in Table 2.1.

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<thead>
<tr>
<th>Parameters</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drum Mass, $m_d$</td>
<td>4,466 kg</td>
</tr>
<tr>
<td>Frame Mass, $m_f$</td>
<td>2,534 kg</td>
</tr>
<tr>
<td>Mass Moment, $m_0 e_0$</td>
<td>1.0 - 4.25 kg-m</td>
</tr>
<tr>
<td>Excitation Frequency, $f \left( \frac{1}{2\pi} \right)$</td>
<td>20-35 Hz</td>
</tr>
<tr>
<td>Drum/Frame Stiffness, $k_{df}$</td>
<td>6.02 MN/m</td>
</tr>
<tr>
<td>Drum/Frame Damping, $c_{df}$</td>
<td>4,000 kg/s</td>
</tr>
</tbody>
</table>
Figure 2.2: (a) Free body diagram of vertical forces acting on drum. (b) Contact force vs. drum displacement response and resulting dynamic stiffness measures

Figure 2.2a illustrates the lumped parameter mechanics of the vibrating drum. In this analysis geomechanics conventions are used, where the +z direction points from the drum toward the ground (per Figure 2.2). It is commonly assumed that the drum behaves as a rigid mass with a single vertical degree of freedom, $z_d$. Since the drum is modeled as a rigid mass, $z_d$ corresponds to the vertical deflection of the soil surface when in contact. The drum is connected to the roller frame via low stiffness rubber isolation mounts. The weight of the frame on the drum is considered; however, for soil rollers, the influence of frame dynamics on the drum is insignificant [4] and thus is commonly neglected. To estimate a measure of composite ground stiffness, $k_s$, the position of the eccentric masses and the vertical drum motion, $z_d$, are continuously measured. The drum/soil contact force is determined from dynamic equilibrium (Figure 2.2, Equation (2.1)).

$$F_c = m_0 e_0 \cos \Omega t + (m_f + m_d)g - m_d \ddot{z}_d \quad (2.1)$$

where $\ddot{z}_d$ is the drum acceleration. With these data, it is possible to estimate the contact force vs. drum deflection ($F_c - z_d$) behavior and the corresponding $k_s$. $k_s$ is traditionally defined as either the secant or tangent stiffness of the force-displacement loop of the drum [45]. In this
analysis, the secant stiffness ((2.2), Figure 2.2b) is used and is denoted \( k \) for the remainder of the analysis. Figure 2.2b shows \( F_c-z_d \) response from a vibration cycle for continuous drum-soil contact (top) and for partial loss of contact, assuming no rocking. Partial loss of contact behavior is common at higher excitation force levels and/or when operating on stiff soil.

\[
k = \frac{F_c(@MAXz_d) - F_c(STATIC)}{z_d(MAX) - z_d(STATIC)}
\]  

(2.2)

2.3.1.2 FE Model Development

The interaction between the vibratory drum and the layered soil foundation is a dynamic process with a time-varying contact force during each cycle of vibration. The evolution of the contact force depends upon the operational parameters of the vibratory roller, as well as the material properties of the underlying soil layers [10, 20]. Therefore, to create a forward model of the interaction between the drum and the layered soil systems, finite element (FE) analysis was used because no prior assumptions must be made about pressure distribution, contact area, and contact force [46, 47].

A two dimensional (2D) FE model was developed using ABAQUS 6.11-2 [48]. Full three dimensional (3D) modeling is computationally expensive, and experimental results have shown that 2D plane strain conditions persist beneath the center of the 2.1 m long drum [49, 50]. The discretized soil region was modeled with overall and element dimensions as shown in Figure 2.3. Linear infinite elements were applied at the semi-infinite boundaries to capture the effects of radiation damping in both the horizontal and vertical directions. These infinite elements emulate a half-space by introducing velocity-proportional damping on the boundaries. The boundary damping constants are chosen by ABAQUS to minimize reflections of shear and dilatational wave energy back into the mesh. A vertical symmetry boundary was used directly through the center of the drum and soil to minimize computational run-time.
The vibratory drum was modeled as a rigid cylinder 1.5 m in diameter. The combined weight of the roller drum frame and the drum (16.35 kN, based upon the typical data shown in Table 2.1, modified for plane strain, and symmetric vertical boundary conditions) was modeled as a single, static vertical load acting at the center of the rigid cylinder. The eccentric loading caused by the rotating drum masses was modeled as a vertical harmonic excitation force, \( m_0 e_0 \Omega^2 \cos(\Omega t) \), also applied at the center of the rigid cylinder. The horizontal component of force is zero and therefore assumes that the mechanism involves counter-rotating eccentric masses. The excitation frequency, \( \Omega \), used for the majority of the analysis, is 30 Hz. This is the median of the range shown for a typical roller in Table 2.1. The value for the magnitude of the eccentric mass moment used for this study, \( m_0 e_0 = 3.0 \text{ kg-m} \), was chosen to be within the range shown in Table 2.1; its selection is additionally associated with overall force-deflection loop behavior patterns discussed in a following section.

The contact conditions between the drum and soil were applied as discontinuous frictionless surface boundary constraints, active only when the two adjacent surfaces are in contact. The kinematic contact algorithm in ABAQUS uses a predictor-corrector algorithm to solve the contact problem, using a master/slave relationship, that allows complete loss of contact.
between the drum and soil surface. The drum is modeled as a rigid body and is therefore automatically the master surface. At each time step, the kinematic state of the model is advanced to a predicted condition, without considering contact. The algorithm then determines which slave nodes (in the predicted configuration) penetrate the master surface (drum). The depth of penetration, time step, and associated mass are used to determine the resisting force required to oppose the penetration. Since the master surface is a rigid body, the resisting forces of the slave nodes are applied as generalized forces on the rigid body. The mass of each slave node in contact is added to the rigid body, allowing determination of the total inertial mass of the contact interface. These forces and added masses are used to calculate an acceleration correction to the drum. From the corrected motion of the rigid drum, the acceleration corrections for the soil (slave) surface nodes are then determined [48]. Since the positions of both the drum and soil surface have been determined for the given time step, the contact area can be calculated from these data. The contact conditions between the base and subgrade layers are set so that no horizontal slippage or vertical separation is allowed.

The soils constitutive properties were modeled using plane strain, linear elasticity. This approach assumes that the behaviors of the particulate materials may be modeled as continua that do not undergo plastic deformation. The continuum approximation is inherent in modeling large-scale soil infrastructure systems using FE methods; the elasticity approximation is invoked under the assumption of small-strain soil response characteristic of a fully compacted proof roll situation. In practice, proof rolls are performed using lower eccentric mass moments on fully compacted soils to verify the condition of a compacted earth structure using low vibration amplitudes. The modeling of elastic behavior is able to capture vibratory drum operation on fully compacted soil. The mass densities of the soil materials were modeled as constants determined via the experimental calibration studies discussed in the following section. Rayleigh damping is used to approximate intrinsic (material) damping of the soil as time domain modeling cannot explicitly model intrinsic damping. The Rayleigh
damping parameters ($\alpha$ and $\beta$) have units of $s^{-1}$ and $s$, respectively, and were determined through calibration with experimental data from two different field sites, which will be shown in a later section. Although real soils have elastic moduli and damping properties that are strain dependent, we use constant elastic and dissipative properties in this study to model proof roll situations.

An explicit time-integration approach with a maximum time step of 0.2 ms is used. The dynamic analyses were performed such that the static self-weight of the roller was linearly increased to its full value over the first 0.5 s. The dynamic excitation from the eccentric masses was then applied until steady state vibration was obtained, typically achieved after 5 to 10 cycles of vibration. For the purposes of this study, the vibratory drum was not modeled as translating in the horizontal direction. This may be conceptualized as the vertically vibrating drum in its steady state condition, horizontally translating over a layered soil system with horizontally homogeneous material properties. Therefore, all results presented in this study are steady state vibration.

2.3.1.3 Mesh Refinement

Standard mesh refinement tests were performed to determine appropriate element size and mesh dimensions. The first analysis was performed wherein the element size was gradually decreased until $k$, $F_c$, $z_d$, and $2a$ converged to constant values (Figure 2.4). Although $2a$ takes slightly longer to converge, these small oscillations have a negligible effect on the surface values, and an element size of 20 mm x 20 mm is chosen for the analysis.

A second test was performed to determine mesh dimensions wherein the overall dimensions of the mesh were decreased until a difference in results is seen. This allows us to choose the most computationally efficient (smallest) mesh, while retaining full accuracy. The analysis begins with both a soil width and depth of 4 m, and then keeping the depth constant, decreasing the width of the mesh until a minimum is found. Refinement in the x-direction showed less than 0.5% difference between surface values with width equal to 4 m and 0.5 m. As a result, a width of 1 m is chosen for both accuracy and computational efficiency. The
depth of the soil is based upon the measurement depth of the roller (1.0–1.2 m [51]), and therefore must be greater than 1.2 m in order to capture the full response. A depth of 2 m is chosen for the mesh and is verified through the same process described above, but in the z-direction with a width of 1 m.

2.3.1.4 Effect of Contact Area on Drum Response

To determine if the changing contact area (2a) needs to be explicitly modeled, a two-layer, plane strain, analytical model is developed in Chapter 3, to test the sensitivity of surface displacement to 2a (Figure 2.5). The classic analytical solution to dynamic strip loading on a half-space [24] is expanded here for a two-layered system. The contact force (F_c) is applied as a uniform surface load (P) over 2a. In this system, for a known E_1, E_2 and m_0e_0, F_c is constant and independent of 2a so drum response should also be independent of 2a.
To examine the vibratory drum sensitivity to $2a$, parametric sweeps were performed on $2a$, using the two-layer system presented in 3 with $E_2 = 100$ MPa, $E_1 = 50$ MPa, and $h = 15$ and 30 cm. $F_c$ was held constant at 150 kN, for all analyses. Figure 2.6 shows the normalized $z_d$ vs. $2a/h$ for $h = 15$ cm, where $z_d$ is normalized by dividing each $z_d$ value by the maximum $z_d$ value across all $2a$. The insensitivity of surface $z_d$ to changes in $2a$, when

$$F_c$$ is held constant can clearly be seen, especially when $2a \ll h$. The analytical model shows that for a constant $F_c$, $z_d$ is insensitive to changes in $2a$ (for $2a \leq h$), demonstrating that the time-varying $2a$ does not need to be explicitly modeled to capture surface behavior.
This finding also explains why previous models of the roller-soil system have been able to match field $F_{c}=z_{d}$ behavior without accounting for (or properly modeling) the contact area.

### 2.3.2 FE Model Calibration

Experimentally measured contact force vs. drum displacement data were used to calibrate and validate the finite element model. Rayleigh damping parameters ($\alpha$ and $\beta$, units of $s^{-1}$ and $s$, respectively) are used to numerically approximate material damping in the FE model (2.3) and are assumed constant for all soil types used in this analysis.

$$[C] = \alpha[M] + \beta[K] \quad (2.3)$$

Rayleigh damping does not explicitly relate to individual soil material properties, particularly in a non-homogeneous layered system. However, the Rayleigh damping formulation, based upon global system mass and stiffness matrices, is commonly used in time domain FE modeling to approximate material damping [52, 53]. In order to determine the appropriate $\alpha$ and $\beta$ parameters for the soil layers, calibration with experimental data from two different sites was performed to verify that constant $\alpha$ and $\beta$ can be assumed.

The data shown in Figure 2.7 and Figure 2.8 were obtained from an instrumented vibratory roller with characteristics as listed in Table 2.1 during testing. Figure 2.7 shows data from three test beds. Test bed 1 was comprised of silty sand (SM) that was homogeneous to a depth greater than 2 m. We assume this is therefore a homogeneous half-space of a single material because the measurement depth of this class of vibratory rollers is approximately 1.0-1.2 m [51]. Test bed 2 was comprised of 25 cm of crushed rock base material (SP-SM) overlying the silty sand subgrade of test bed 1. Test bed 3 involved 55 cm of the crushed rock base material overlying the SM subgrade. Test bed 3 was constructed by adding 30 cm of crushed rock to test bed 2. Data in Figure 2.8 are comprised of multiple test beds with different homogeneous materials. The subgrade and base course soils were fully compacted to standard Proctor and modified Proctor maximum dry densities and optimum moisture contents, respectively, prior to the collection of vibratory drum response data.
Test bed 1 was numerically modeled as a half-space, with the discretized soil region comprised of one homogeneous continuum material. Test beds 2 and 3 were modeled as horizontally stratified systems, with the modeled subgrade and the base layers thickness determined by the base lift heights, and the overall discretized depth of 2 m. Values for the soil layer elastic moduli, were varied within reasonable values to match the numerical data as closely as possible. A best fit is chosen by minimizing the percent difference ($\delta$) between field and FE results for both $F_{c-MAX}$ and $z_{d-MAX}$ (2.4)–(2.7). $\delta$ is calculated for $F_{c-MAX}$ and $z_{d-MAX}$ at both lift thicknesses individually (2.4) and then averaged across lift thickness for each parameter (2.7) and (2.6). The best fit is then found by minimizing the average of $\delta_{F_{c-MAX}}$ and $\delta_{z_{d-MAX}}$ (2.6).

\[
\%\text{Diff} = \delta = \frac{F_{c-\text{FIELD}} - F_{c-\text{FE}}}{\frac{F_{c-\text{FIELD}} + F_{c-\text{FE}}}{2}} \quad (2.4)
\]

\[
\delta_{F_{c-MAX}} = \frac{\delta_{F_{c-MAX}}^{h=25} + \delta_{F_{c-MAX}}^{h=55}}{2} \quad (2.5)
\]

\[
\delta_{z_{d-MAX}} = \frac{\delta_{z_{d-MAX}}^{h=25} + \delta_{z_{d-MAX}}^{h=55}}{2} \quad (2.6)
\]

\[
\text{best fit} =\min \left[ \frac{\delta_{F_{c-MAX}} + \delta_{z_{d-MAX}}}{2} \right] \quad (2.7)
\]

Field data were averaged over four vibration cycles. To account for drum-frame rocking, field data from both left and right accelerometers have been combined to give acceleration at the center of gravity of the drum to account for the effects of rocking (per [14]). The resulting experimental vs. numerical comparisons are shown in Figure 2.7, and the resulting $k$, $F_c$ and $z_d$ values are shown in Table 2.2 for Test beds 1, 2, and 3, respectively. These data result from values of subgrade elastic modulus ($E_1$) equal to 60 MPa, base course elastic modulus ($E_2$) equal to 161 MPa, and for both materials, $\nu = 0.3$, $\rho = 2,000 \text{ kg/m}^3$, $\alpha = 25 \text{ s}^{-1}$ and $\beta = 0.002 \text{ s}$. 

18
Table 2.2: $k$, $F_c$ and $z_d$ values from FE and Field Data from best fits for Test Beds 1-3

<table>
<thead>
<tr>
<th></th>
<th>$h = 0$</th>
<th>$h = 25$ cm</th>
<th>$h = 55$ cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>46.5</td>
<td>51.6</td>
<td>67.6</td>
</tr>
<tr>
<td>Field</td>
<td>46.4</td>
<td>59.9</td>
<td>71.1</td>
</tr>
<tr>
<td>% Diff</td>
<td>0.3</td>
<td>14.9</td>
<td>5.0</td>
</tr>
<tr>
<td>$F_{c-MAX}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>142.5</td>
<td>186.6</td>
<td>218.6</td>
</tr>
<tr>
<td>Field</td>
<td>145.3</td>
<td>188.3</td>
<td>218.7</td>
</tr>
<tr>
<td>% Diff</td>
<td>1.9</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>$z_{d-MAX}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>1.25</td>
<td>1.24</td>
<td>1.28</td>
</tr>
<tr>
<td>Field</td>
<td>1.24</td>
<td>1.29</td>
<td>1.30</td>
</tr>
<tr>
<td>% Diff</td>
<td>0.4</td>
<td>3.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The FE model best captures both field observed stiffness and $F_c-z_d$ behavior for total contact (Figure 2.7a, Figure 2.8a-d) and minimal loss of contact (Figure 2.8e). For loss of contact, the $F_c-z_d$ loops look less similar: the FE model cannot accurately capture the
unloading portion \((z_d < 0)\) of the loops, although it is able to capture behavior during loading \((z_d > 0)\) (Figure 2.7b-c). Since \(k\) is derived from the loading portion of the curve, the models inability to capture unloading behavior is acceptable.

The best fit values for homogeneous soils from the FE model also have very accurate \(k\) values (Table 2.2, \(h = 0\), Table 2.3), but for the layered systems the FE best fit gives a value of \(k_{FE}\) that is much smaller than \(k_{FIELD}\) (Table 2.2). The computation of \(k\) is very sensitive to when peak drum displacement occurs. The peak drum deflections for the FE model occur at much lower \(F_c\) values, resulting in a slight underestimation of \(k\) by the FE model (Table 2.2). This effect could be a result of the difference between Rayleigh damping and the actual material damping. Further study is needed to determine and quantify the differences between the two types of damping.

Table 2.3: \(k\) values from FE and Field Data from best fits from homogeneous test beds

<table>
<thead>
<tr>
<th>(E_1) (MPa)</th>
<th>(k_{FE})</th>
<th>(k_{FIELD})</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>21.5</td>
<td>21.2</td>
</tr>
<tr>
<td>47</td>
<td>38.5</td>
<td>38.4</td>
</tr>
<tr>
<td>60</td>
<td>46.5</td>
<td>46.8</td>
</tr>
<tr>
<td>72</td>
<td>53.1</td>
<td>53.0</td>
</tr>
<tr>
<td>81</td>
<td>56.8</td>
<td>56.8</td>
</tr>
</tbody>
</table>

2.4 Force-Deflection Behavior

As previously discussed, the magnitude of the eccentric mass moment may be varied between typical values of 1.0 to 5.0 kg-m. Higher \(m_0\varepsilon_0\) levels are used during early compaction passes while low \(m_0\varepsilon_0\) levels are used for finishing passes and proof rolling [15]. It is useful to examine the effect of this variation upon overall behavior of roller-soil interaction with respect to full contact, loss of contact, and a chattering behavior commonly known as bifurcation [4]. Figure 9 shows the evolution of the \(F_c-z_d\) response with increasing values of \(m_0\varepsilon_0\) during steady state, \(\Omega=30\) Hz dynamic vibration of an example homogeneous soil foundation.
with $E_1 = 100 \text{ MPa}$, $\nu=0.3$, and $\rho=2,000 \text{ kg/m}^3$. The drum remains in full contact with the ground for $m_0e_0 = 1.0$ and 2.0 kg-m and begins to experience periodic loss of contact from the ground when $m_0e_0 = 3.0$ kg-m. Figure 2.9e shows the force-deflection behavior for $m_0e_0 = 4.00$ kg-m, where the eccentric mass moment has increased to the point of bifurcation of the force-deflection loops.
The bifurcation can be interpreted as the splitting of the steady state behavior into two distinct, alternating force magnitudes shown in the force time history in Figure 2.9f. This can be a result either of high amplitude excitation or of behavior caused by resonance in the drum-soil system. The force-deflection behaviors seen in Figure 2.9a, b, and d, are representative of those commonly used in practice. However, the bifurcation behavior shown in Figure 2.9e is avoided in practice. If the chaotic jumping is noticed, the roller operator typically reduces the magnitude of the eccentric mass moment, as the roller becomes difficult or dangerous to operate. The frequency at which the force-deflection loop bifurcates is dependent upon both roller characteristics and the material properties of the underlying soil foundation. The value of $m_0e_0$ at which bifurcation occurs typically decreases with increasing underlying foundation moduli. As bifurcation is avoided in practice, only the force-deflection behaviors corresponding to full contact and loss of contact were considered. For the purposes of this study, a value of $m_0e_0 = 3.0 \text{ kg-m}$ was used, as it was found to satisfy the aforementioned conditions.
2.5 Parametric Studies

To better characterize anticipated drum response over a range of layered earthwork situations, three parametric studies were performed. For each of the studies, $\alpha$, $\beta$, $\rho$, $\nu$ and $m_0e_0$ values remained constant ($\alpha = 25 \text{ s}$, $\beta = 0.002 \text{ s}^{-1}$, $\rho = 2000 \text{ kg/m}^3$, $\nu = 0.3$, $m_0e_0 = 3.0 \text{ kg-m}$). The first parametric study varies $E$ for homogeneous soil. Both the second and third studies examine layered soil response, the former by varying $E_2/E_1$, and the latter by varying $h$.

2.5.1 Homogeneous Half-Space Response

Using the parameters presented above, vibratory drum response on a homogeneous half-space was investigated. This simulates roller data on a compacted subgrade or subbase with a thickness greater than 1.2 m. Figure 2.10 summarizes the results of the first parametric study, varying $E_1$ from 20 to 100 MPa. The maximum values of $F_c$, $z_d$, and $k$, obtained from the dynamic analyses are shown in Figure 12b. Despite constant mass moment and excitation frequency, $F_{c-MAX}$ increases considerably with $E$, illustrating the influence of soil modulus on contact force. The dynamic drum deflection $z_{d-MAX}$ increases accordingly with $E$ as a result of the increase in $F_{c-MAX}$. If $F_c$ were constant across all $E$, $z_d$ would decrease with increasing $E$. However, a 500% increase in $E$, resulted in a 60% increase in $F_{c-MAX}$ and a 25% increase in $z_{d-MAX}$. This shows that the more substantial increase in $F_c$ due to $E$ causes $z_d$ and $k$ to also increase with $E$.

2.5.2 Two Layer Response

Parametric studies were then performed on various two-layer situations. The first examines the influence of the ratio $E_2/E_1$ on drum response, where $E_2$ is the modulus of the top layer and $E_1$ is the modulus of the underlying layer. This ratio was varied from 1 to 3 for $E_1 = 50$, 75 and 100 MPa. These are common combinations of subgrade/subbase and base course situations. Figure 2.11 shows the resulting values of $F_c$, $z_d$, and $k$. Both $F_c$ and $k$ increase with $E_2$, but do so at a decreasing rate. Therefore at higher $E_2/E_1$ values (around
Figure 2.10: FE-simulated $F_c$-$z_d$ and $F_{c-MAX}$, $z_{d-MAX}$ and $k$ for homogeneous system where $E$ is varied from 20 to 100 MPa.

2-3), $F_c$ and $k$ become less sensitive to changes in $E_2$, a limitation that must be considered when using CCC in practice. It is also worthy to note that as $E_1$ increases, $k$ becomes insensitive at lower $E_2/E_1$ ratios. $z_d$ initially increases with $E_2$, which seems counterintuitive. However, this increase in $z_d$ is due to the large increases in $F_c$ (for low values of $E_2/E_1$) contributing more to $z_d$ than does the corresponding increase in $E_2$. As $F_c$ becomes less sensitive to increases in $E_2$ (larger values of $E_2/E_1$), $z_d$ decreases because the increase in $E_2$ contributes more to $z_d$ than does $F_c$. The percent change (8) in $F_c$, over the range of $E_2/E_1$ presented, is so much greater than the respective percent change in $z_d$, that $F_c$ influences $k$ far more than $z_d$ (Table 2.4).

Table 2.4: FE results for two-layer system, showing % change in $F_{c-MAX}$, $z_{d-MAX}$ and $k$ for the range of $E_2/E_1 = 1.0-3.0$

<table>
<thead>
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<th>$E_1$ (MPa)</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>h (cm)</td>
<td>15</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>% Change $z_{d-MAX}$</td>
<td>1.7</td>
<td>4.3</td>
<td>2.0</td>
</tr>
<tr>
<td>% Change $F_{c-MAX}$</td>
<td>21.9</td>
<td>31.6</td>
<td>20.9</td>
</tr>
<tr>
<td>% Change $k$</td>
<td>17.7</td>
<td>26.2</td>
<td>13.9</td>
</tr>
</tbody>
</table>
Figure 2.11: FE-simulated $F_{c\cdot z\cdot d}$ and $F_{c\cdot MAX}$, $z_{d\cdot MAX}$ and $k$ for two-layer system ($E_1 = 50$, 75 and 100 MPa; $h = 15$, 30 cm) where $E_2$ is varied so that $E_2/E_1$ varies from 1.0 to 3.0

The final parametric study was performed to examine the effect of top layer lift thickness ($h$) on drum response. $h$ was varied from 0 (half-space) to 1.0 m for $E_1 = 50$, and $E_2 = 100$ MPa. $F_{c\cdot MAX}$, $z_{d\cdot MAX}$ and $k$ increase with $h$, and approach the values found for a half-space of $E = 100$ MPa (dotted line, Figure 2.12), suggesting a measurement depth of around 1 m, which corresponds with measurement depths shown in [51].

2.6 Conclusion

This paper presents the results of a study to model vibratory drum-layered soil interaction using dynamic elastic FE analysis. The relationship between roller-measured $k$ and in situ soil response is complex. The roller-soil interaction is highly nonlinear and dependent upon the inertial and dissipative properties of the soil. It involves transient response with time-varying loading conditions, including decoupling between the drum and soil, chaotic behavior,
and drum and frame rocking. The time-varying loading conditions are explicitly modeled, allowing for full and partial loss of contact between drum and soil. The model is able to capture the dynamic response, including decoupling between the drum and soil. Based on the analysis performed, the following conclusions can be made:

- For constant $F_c$, changes in $2a$ do not affect surface behavior ($z_d$, $k$). As a result the time-varying $2a$ does not need to be modeled in order to accurately capture desired roller/soil behavior (on the surface). Changes in $2a$ do still affect the underlying stresses within the soil, so must be considered if more than surface behavior is desired. This conclusion may be somewhat surprising, but is analyzed and justified in Chapter 3 of this thesis.

- FE model slightly underestimates $k$ because peak drum deflections for the model occur at much lower $F_c$ values. This effect could be a result of the difference between Rayleigh damping and the actual material damping. 4 provides a thorough analysis of the two different damping models.
• $k$ is sensitive to increases in $E_2$ at low values of $E_2/E_1$ ($<2.5$) but becomes less sensitive as $E_2/E_1$ increases. As $E_1$ increases, $k$ becomes insensitive at lower values of $E_2/E_1$. Due to this sensitivity limit, very stiff over soft situations ($E_2/E_1 > 3$) should be avoided in practice.

• $k$ is sensitive to increases in $h$ up to 1 m, after which $k$ no longer sees the underlying layer, and the response is only influenced by the top layer. This is consistent with previous research that shows the measurement depth of vibratory rollers to be 1.0-1.2 m.
CHAPTER 3
INFLUENCE OF CONTACT AREA ON SURFACE RESPONSE FOR A LAYERED HALF-SPACE WITH HARMONIC STRIP LOADING

3.1 Introduction

As suggested in the previous chapter, the motivation for this chapter is to develop an analytic solution to a simplified version of the roller-soil system to determine whether contact area \( (2a) \) needs to be explicitly modeled in finite element analyses in order to capture field observed surface behavior. To examine the influence of \( 2a \) on surface response for the roller-soil system, an analytical solution to harmonic strip loading on a layered elastic half-space is developed. The ground is modeled as a homogeneous isotropic layer over an elastic half-space of a different material. This analysis expands the work of Miller and Pursey [24] from a half-space to an elastic layer upon a half-space.

Analytical solutions have been published for circular and strip footings supported by an elastic or viscoelastic halfspace [24, 35, 54, 55] and for circular foundations on layered elastic and viscoelastic soil deposits [39, 40, 56, 57]. Semi-analytical solutions have also been developed for a strip load on a layered half-space [36, 38, 58–63], but to the author’s knowledge, no analytical solutions have been published. Additionally, aforementioned layered solutions only examine the case of a soft layer overlying a stiff subgrade \((0 \leq E_2/E_1 \leq 1)\), whereas for road construction applications the opposite case is true (i.e. \( E_2/E_1 \geq 1 \)).

In the following analysis, the ground is modeled as a homogeneous isotropic layer of finite depth, \( h \), with Young’s Modulus, \( E_2 \), and damping ratio, \( \xi \), overlying a homogeneous isotropic half-space with different elastic properties \( (E_1) \), but the same damping ratio and density as the above layer. In this analysis, \( E_2 > E_1 \), to emulate material profiles found in road construction. The model has plane strain conditions and in our analysis geomechanics conventions are used where positive \( z \) is down. Figure 3.1 shows a schematic of the model.
including the coordinate system used. The force per unit length \( Q = F_c/L_d \), where \( L_d \) is the drum length in the y-direction) acts on an infinite strip on the ground surface, with a width, \( 2a \). In order to examine the effect of different values of \( 2a \) on the surface displacement directly under the center of the strip load \( (x = 0, z = 0) \), the model is run for different values of \( 2a \) with total force held constant by allowing the applied surface pressure \( (P = Q/2a) \) to change with \( 2a \).

The solution has time dependence of \( e^{i\omega t} \), since all solutions need to be multiplied by the time dependence, it is left out (and implied) for most of the derivation. This chapter derives the solution for the forced vibration case and then presents and discusses the results directly under the center of the load \( (x = 0, z = 0) \).

### 3.2 Governing Equations

The governing equation for wave propagation in an isotropic elastic solid is

\[
(\lambda + 2\mu) \nabla \nabla \cdot \mathbf{u} - \mu \nabla \times \nabla \times \mathbf{u} = \rho \ddot{\mathbf{u}} \tag{3.1}
\]

\( \mathbf{u} \) is the displacement vector, \( \rho \) is the mass density, and \( \lambda \) and \( \mu \) are the Lamé constants given by:

\[
\lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)}
\]
where $E$ is the Young’s modulus and $\nu$ is Poisson’s Ratio.

Letting $v_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ and $v_s = \sqrt{\frac{\mu}{\rho}}$, (3.1) becomes:

$$v_p^2 \nabla \nabla \cdot \mathbf{u} - v_s^2 \nabla \times \nabla \times \mathbf{u} = \ddot{\mathbf{u}} \tag{3.2}$$

After performing a Fourier Transform on the time domain:

$$v_p^2 \nabla \nabla \cdot \tilde{\mathbf{u}} - v_s^2 \nabla \times \nabla \times \tilde{\mathbf{u}} = -\omega^2 \tilde{\mathbf{u}} \tag{3.3}$$

For plane strain we use only x and z directions, so equation (3.3) becomes:

For x-direction:

$$v_p^2 \left[ \frac{\partial^2 \ddot{u}}{\partial x^2} + \frac{\partial^2 \ddot{w}}{\partial x \partial z} \right] - v_s^2 \left[ \frac{\partial^2 \ddot{w}}{\partial z \partial x} - \frac{\partial^2 \ddot{u}}{\partial z^2} \right] + \omega^2 \ddot{u} = 0 \tag{3.4}$$

For z-direction:

$$v_p^2 \left[ \frac{\partial^2 \ddot{u}}{\partial z \partial x} + \frac{\partial^2 \ddot{w}}{\partial z^2} \right] - v_s^2 \left[ \frac{\partial^2 \ddot{w}}{\partial x \partial z} - \frac{\partial^2 \ddot{u}}{\partial x^2} \right] + \omega^2 \ddot{w} = 0 \tag{3.5}$$

Now apply Fourier Transform to x-direction, where the Fourier Transform pair used is defined as:

$$\mathcal{F}[u(x)] = \tilde{u}(\zeta) = \int_{-\infty}^{\infty} u(x) e^{-i\zeta x} \, dx \tag{3.6a}$$

$$\mathcal{F}^{-1}[\tilde{u}(\zeta)] = u(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{u}(\zeta) e^{i\zeta x} \, d\zeta \tag{3.6b}$$

and the Fourier Transform derivative property is defined as:

$$\mathcal{F} \left[ \frac{\partial^n}{\partial x^n} u(x) \right] = (i\zeta)^n \tilde{u}(\zeta)$$

So $x \Rightarrow \zeta; u \Rightarrow \tilde{u}; w \Rightarrow \tilde{w}$ and (3.4) and (3.5) become:

$$v_p^2 \left[ -\zeta^2 \ddot{u} + i\zeta \frac{\partial \ddot{w}}{\partial z} \right] - v_s^2 \left[ i\zeta \frac{\partial \ddot{w}}{\partial z} - \frac{\partial^2 \ddot{u}}{\partial z^2} \right] + \omega^2 \ddot{u} = 0 \tag{3.7a}$$

$$v_p^2 \left[ i\zeta \frac{\partial \ddot{u}}{\partial z} + \frac{\partial^2 \ddot{w}}{\partial z^2} \right] - v_s^2 \left[ i\zeta \frac{\partial \ddot{u}}{\partial z} + \zeta^2 \ddot{w} \right] + \omega^2 \ddot{w} = 0 \tag{3.7b}$$
3.3 General Solution

Now we solve for $\bar{u}$ and $\bar{w}$. If we assume solution of the form $\bar{u} = U e^{ikz}$ and $\bar{w} = W e^{ikz}$, where $U$, $W$, and $k$ are constants to be determined, then:

$$\frac{d\bar{u}}{dz} = U ike^{ikz} \quad \frac{d^2\bar{u}}{dz^2} = -U k^2 e^{ikz}$$

$$\frac{d\bar{w}}{dz} = W ike^{ikz} \quad \frac{d^2\bar{w}}{dz^2} = -W k^2 e^{ikz}$$

So equation (3.7a) becomes:

$$v_p^2 \left[ -\zeta^2 U e^{ikz} - \zeta kW e^{ikz} \right] - v_s^2 \left[ -k\zeta W e^{ikz} + k^2 U e^{ikz} \right] + \omega^2 U e^{ikz} = 0 \quad (3.8a)$$

$$v_p^2 \left[ -\zeta^2 U - \zeta kW \right] - v_s^2 \left[ -k\zeta W + k^2 U \right] + \omega^2 U = 0 \quad (3.8b)$$

$$U \left( \omega^2 - \zeta^2 v_p^2 - k^2 v_s^2 \right) + W k\zeta \left( v_s^2 - v_p^2 \right) = 0 \quad (3.8c)$$

and equation (3.7b) becomes:

$$v_p^2 \left[ -\zeta kU e^{ikz} - k^2 W e^{ikz} \right] - v_s^2 \left[ -k\zeta U e^{ikz} + \zeta^2 W e^{ikz} \right] + \omega^2 W e^{ikz} = 0 \quad (3.9a)$$

$$v_p^2 \left[ -\zeta kU - k^2 W \right] - v_s^2 \left[ -\zeta kU + \zeta^2 W \right] + \omega^2 W = 0 \quad (3.9b)$$

$$U k\zeta \left( v_s^2 - v_p^2 \right) + W \left( \omega^2 - \zeta^2 v_s^2 - k^2 v_p^2 \right) = 0 \quad (3.9c)$$

In matrix form:

$$\begin{bmatrix}
\omega^2 - \zeta^2 v_p^2 - k^2 v_s^2 & k\zeta \left( v_s^2 - v_p^2 \right) \\
k\zeta \left( v_s^2 - v_p^2 \right) & \omega^2 - \zeta^2 v_s^2 - k^2 v_p^2
\end{bmatrix} \begin{bmatrix}
U \\
W
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix} \quad (3.10)$$

We then have $Ax = 0$ and to solve for $k$, $\det(A) = 0$.

Using MATLAB symbolic toolbox to solve the polynomial for $k^2$, we find:
\[ k_1^2 = \frac{\omega^2 - \zeta^2 v_s^2}{v_s^2} \] (3.11)
\[ k_2^2 = \frac{\omega^2 - \zeta^2 v_p^2}{v_p^2} \] (3.12)

or

\[ k_1 = \frac{\pm \sqrt{\omega^2 - \zeta^2 v_s^2}}{v_s} \] (3.13)
\[ k_2 = \frac{\pm \sqrt{\omega^2 - \zeta^2 v_p^2}}{v_p} \] (3.14)

Filling Matrix \( \mathbf{A} \) (3.10) with values of \( k_1 \):

\[
\begin{bmatrix}
\zeta^2 (v_s^2 - v_p^2) & \pm \sqrt{\omega^2 - \zeta^2 v_s^2} \left( v_s - \frac{v_s^2}{v_p^2} \right) \\
\pm \sqrt{\omega^2 - \zeta^2 v_s^2} \left( v_s - \frac{v_s^2}{v_p^2} \right) \zeta & \omega^2 \left( 1 - \frac{v_s^2}{v_p^2} \right) + \zeta^2 \left( v_p^2 - v_s^2 \right)
\end{bmatrix}
\] (3.15)

and for \( k_2 \):

\[
\begin{bmatrix}
\omega^2 \left( 1 - \frac{v_s^2}{v_p^2} \right) + \zeta^2 (v_s^2 - v_p^2) & \pm \sqrt{\omega^2 - \zeta^2 v_p^2} \left( \frac{v_s^2}{v_p^2} - v_p \right) \\
\pm \sqrt{\omega^2 - \zeta^2 v_p^2} \left( \frac{v_s^2}{v_p^2} - v_p \right) \zeta & \zeta^2 \left( v_p^2 - v_s^2 \right)
\end{bmatrix}
\] (3.16)

To solve for the amplitude ratios between \( U \) and \( W \), we put (3.15) into (3.10) for \( k_1 \):

\[
\begin{bmatrix}
\zeta^2 (v_s^2 - v_p^2) & \pm \sqrt{\omega^2 - \zeta^2 v_s^2} \left( v_s - \frac{v_s^2}{v_p^2} \right) \\
\pm \sqrt{\omega^2 - \zeta^2 v_s^2} \left( v_s - \frac{v_s^2}{v_p^2} \right) \zeta & \omega^2 \left( 1 - \frac{v_s^2}{v_p^2} \right) + \zeta^2 \left( v_p^2 - v_s^2 \right)
\end{bmatrix}
\begin{bmatrix}
U_1 \\
W_1
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\] (3.17)

We find

\[
\frac{U_1}{W_1} = -\frac{\sqrt{\omega^2 - \zeta^2 v_s^2}}{\zeta v_s} = -\frac{k_1}{\zeta} = n_1
\] (3.18)
And similarly for $k_2$:

$$
\begin{bmatrix}
\omega^2 \left(1 - \frac{v_s^2}{v_p^2}\right) + \zeta^2 \left(v_p^2 - v_s^2\right) & \pm \sqrt{\omega^2 - \zeta^2 v_p^2} \left(\frac{v_s^2}{v_p} - v_p\right) \\
\pm \sqrt{\omega^2 - \zeta^2 v_p^2} \left(\frac{v_s^2}{v_p} - v_p\right) \zeta & \zeta^2 (v_p^2 - v_s^2)
\end{bmatrix}
\begin{align*}
U_2 \\
W_2
\end{align*} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

(3.19)

We find

$$W_2 \frac{U_2}{\zeta v_p} = k_2 = \eta_2$$

(3.20)

Using equations (3.17) to (3.20) $\mathbf{u}$ can be calculated

$$\begin{align*}
\begin{bmatrix}
\bar{\mathbf{u}} \\
\bar{\mathbf{w}}
\end{bmatrix} &= \begin{bmatrix} U_1 \\ W_1 \end{bmatrix} \begin{pmatrix} A & B \end{pmatrix} \left(1 e^{\pm ik_1 z} + 1 e^{\pm ik_2 z} \right) \\
\begin{bmatrix}
\bar{\mathbf{u}} \\
\bar{\mathbf{w}}
\end{bmatrix} &= \begin{bmatrix} \eta_1 \\ 1 \end{bmatrix} \begin{pmatrix} 1 & \eta_2 \end{pmatrix} \left(1 e^{\pm ik_1 z} + 1 e^{\pm ik_2 z} \right)
\end{align*}$$

(3.21)

(3.22)

(3.22) is our solution to the differential equations where $A$ and $B$, are constants to be determined from boundary conditions. This problem involves 2 layers, so we need a version of (3.22) for each layer. From this point forward subscripts of 2 will be used for material constants and variables in the top layer. For top layer $k_1 = k_{12}$ and $k_2 = k_{22}$ Constants and variables for the half space will have no subscripts.

### 3.4 Apply Boundary Conditions

Before the boundary conditions (BCs) are applied, the stressed need to be defined in terms of our displacement functions. In the transformed domain, the stresses are:

$$\bar{\sigma}_x = \rho \left[ i \zeta v_p^2 \bar{u} + (v_p^2 - 2v_s^2) \frac{d \bar{w}}{dz} \right]$$

(3.23)

$$\bar{\sigma}_z = \rho \left[ i \zeta \left(v_p^2 - 2v_s^2\right) \bar{u} + v_p^2 \frac{d \bar{w}}{dz} \right]$$

(3.24)

$$\bar{\sigma}_{xz} = v_s^2 \rho \left[ \frac{d \bar{u}}{dz} + i \zeta \bar{w} \right]$$

(3.25)
The boundary at \( z = 1 \) is at the bottom of the half space since \(+z\) is in downward direction. This BC applies to the equations for the half space.

At \( z = \infty \) the waves must be travelling in the positive \( z \)-direction. For this to be true we need the waves to be of the form \( f(z - ct) \) where \( c \) is the wave speed. Since we have time dependence of \( e^{i\omega t} \), the exponentials are of the form:

\[
e^{ikz + i\omega t} = e^{ik\left(z - \frac{c}{\omega}t\right)}
\]

For this to be true and the answer non-trivial:

\[
-\frac{\omega}{k} \text{ must be positive} \tag{3.26}
\]

Therefore we can eliminate the terms with \(+k\) since only the \(-k\) terms satisfy (3.26), so for the halfspace layer, we find:

\[
\begin{align*}
\{ \bar{u} \bar{w} \} &= \{ \eta_1 \} A e^{-ik_1z} + \{ \eta_2 \} B e^{-ik_2z} \\
&= \{ \eta_1 \} e^{ik_1z} + \{ \eta_2 \} e^{ik_2z} \\
\end{align*}
\]

and for the top layer, we have to keep both the positive and negative exponentials

\[
\begin{align*}
\{ \bar{u}_2 \bar{w}_2 \} &= \{ \eta_{12} \} (C e^{ik_{12}z} + D e^{-ik_{12}z}) + \{ \eta_{22} \} (F e^{ik_{22}z} + G e^{-ik_{22}z}) \\
\end{align*}
\]

It is worth noting that the constants are all functions of \( \zeta \) (e.g. \( A \to A(\zeta) \), \( B \to B(\zeta) \), etc.).

The BC at \( z = 0 \) applies only to equations for top layer (3.28) only, and includes a uniform harmonic surface load, \( Q \), applied over a contact area \( (2a) \) as a surface stress, \( P = Q/2a \).

At \( z = 0 \), \( \bar{\sigma}_{zz} = P(\zeta) \) and \( \bar{\sigma}_{xz} = 0 \).

From (3.25) we can solve \( \bar{\sigma}_{xz}(0) = 0 \)

\[
\begin{align*}
\bar{\sigma}_{xz} = & k_{12}\eta_{12}(C - D) + k_{22}(F - G) + \zeta [C + D + \eta_{22}(F + G)] \\
0 &= C(\zeta + k_{12}\eta_{12}) + D(\zeta - k_{12}\eta_{12}) + F(\eta_{22} + k_{22}) + G(\eta_{22} - k_{22}) \\
0 &= C(\zeta^2 - k_{12}^2) + D(\zeta + k_{12}^2) + F(k_{22} + k_{22}) + G(k_{22} - k_{22}) \\
0 &= C(\zeta^2 - k_{12}^2) + D(\zeta^2 + k_{12}^2) + F(2\zeta k_{22}) \\
&= \zeta (\zeta^2 - k_{12}^2) + \zeta (\zeta^2 + k_{12}^2) + F(2\zeta k_{22}) \\
\end{align*}
\]

34
and from (3.24):

\[
P(x, t) = \begin{cases} 
Pe^{i\omega t} & \text{if } |x| \leq a \\
0 & \text{if } |x| > a 
\end{cases}
\]

\[
\bar{P}(\zeta) = \frac{2P \sin(\zeta a)}{\zeta} = \sigma_{zz}(z = 0)
\]

where \( P = \frac{Q}{2a} \).

\[
\sigma_{zz} = P(\zeta) = \rho_2 \left[ i\zeta(v_{p2}^2 - 2v_{s2}^2)(\eta_{12}C + \eta_{12}D + F + G) + v_{p2}^2(i\eta_{12}C - i\eta_{12}D + ik_{22}\eta_{22}F - ik_{22}\eta_{22}G) \right]
\]

\[
\frac{2P \sin(\zeta a)}{\rho_2 \zeta} = C \left( i\zeta \eta_{12}(v_{p2}^2 - 2v_{s2}^2) + ik_{12}v_{p2}^2 \right) + D \left( i\zeta \eta_{12}(v_{p2}^2 - 2v_{s2}^2) - ik_{12}v_{p2}^2 \right)
+ F \left( i\zeta(v_{p2}^2 - 2v_{s2}^2) + ik_{22}\eta_{22}\eta_{p2} \right) + G \left( i\zeta(v_{p2}^2 - 2v_{s2}^2) - ik_{22}\eta_{22}\eta_{p2} \right)
\]

\[
= C(2i\zeta k_{12}v_{s2}^2) + D(2i\zeta k_{12}(v_{s2}^2 - v_{p2}^2))
+ F(iv_{p2}^2(\zeta^2 + k_{22}^2) - 2i\zeta^2 v_{s2}^2) + G(iv_{p2}^2(\zeta^2 - k_{22}^2) - 2i\zeta^2 v_{s2}^2)
\]

\[
(3.30)
\]

\( z = h \) is at the interface of the top layer and the half-space (for our study \( \rho = \rho_2 \)), therefore at \( z = h \):

\[
\bar{w}(h) = \bar{w}_2(h) \tag{3.31}
\]

\[
\bar{u}(h) = \bar{u}_2(h) \tag{3.32}
\]

\[
\bar{\sigma}_z(h) = \bar{\sigma}_{zz}(h) \tag{3.33}
\]

\[
\bar{\sigma}_{xz}(h) = \bar{\sigma}_{xxz}(h) \tag{3.34}
\]

For the displacements per (3.31):

\[
0 = A(-e^{-ik_1h}) + B(-\eta_2 e^{-ik_2h}) + Ce^{ik_1h} + De^{-ik_1h} + F\eta_{22}e^{ik_{22}h} + G\eta_{22}e^{-ik_{22}h}
\]

\[
0 = A(-\zeta e^{-ik_1h}) + B(-k_2 e^{-ik_2h}) + C\zeta e^{ik_1h} + D\zeta e^{-ik_1h} + Fk_{22}e^{ik_{22}h} + Gk_{22}e^{-ik_{22}h} \tag{3.35}
\]

and (3.32):
\[0 = A(-\eta_1 e^{-ik_1h}) + B(-e^{-ik_2h}) + C\eta_{12}e^{ik_{12}h} + D\eta_{12}e^{-ik_{12}h} + Fe^{ik_{22}h} + Ge^{-ik_{22}h}\]
\[0 = Ak_1e^{-ik_1h} + B(-\zeta e^{-ik_2h}) + C(-k_{12}e^{ik_{12}h}) + D(-k_{12}e^{-ik_{12}h}) + F\zeta e^{ik_{22}h} + G\zeta e^{-ik_{22}h}\]

(3.36)

The stresses at \(z = h\) are found per (3.33):

\[A\left[2\zeta k_1v_s^2e^{-ik_1h}\right] + Be^{-ik_2h}\left[v_p^2(\zeta^2 - k_2^2) - 2\zeta^2v_s^2\right] = C\left[2\zeta k_{12}v_s^2e^{ik_{12}h}\right] + D\left[2\zeta k_{12}(v_s^2 - v_p^2)e^{-ik_{12}h}\right] + Fe^{ik_{22}h}\left[v_p^2(\zeta^2 - k_{22}^2) - 2\zeta^2v_s^2\right] + Ge^{-ik_{22}h}\left[v_p^2(\zeta^2 - k_{22}^2) - 2\zeta^2v_s^2\right]\]

(3.37)

and (3.34):

\[Av_s^2e^{-ik_1h}(\zeta^2 + k_1^2) = Cv_s^2e^{ik_{12}h}(\zeta^2 - k_{12}^2) + Dv_p^2e^{-ik_{12}h}(\zeta^2 + k_{12}^2) + F(2v_s^2\zeta k_{22}e^{ik_{22}h})\]

(3.38)

### 3.5 Solution

Using MATLAB’s symbolic toolbox, the system of equations is solved for the six constants. The resulting values are too large to be displayed here, but the corresponding MATLAB scripts used for the solution are presented in Appendix A.

In order to solve for surface \(z\)-displacement \((z = 0)\), \(\bar{w}_2\) (3.28) must be transformed from the \((\zeta, z, \omega)\) domain to the \((x, z, \omega)\) domain using (3.6). For \(x = 0\) and \(z = 0\), (3.6) and (3.28) simplify to:

\[w(x = 0, z = 0, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{w}(\zeta)d\zeta\]
\[w_{00} = w(x = 0, z = 0, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (C + D + \eta_2F + \eta_{22}G)d\zeta\]

(3.39)

(3.40)

To make the material behavior more realistic, hysteretic damping is introduced, creating a complex modulus of the form:

\[E^* = E(1 + 2i\xi)\]
where $\xi$ is the damping ratio. A value of $\xi = 0.05$ is used in the analysis. Equation (3.40) is solved for $w_{00}$ using adaptive Gauss-Kronrod quadrature in MATLAB (A).

### 3.6 Results

To examine the effect of $2a$ on surface response, equation (3.40) is solved for values of $2a = 40$ mm to $2a = 300$ mm, with $Q$ held constant through the analysis.

The above sweeps of $2a$ were performed for $E_1 = 50$ MPa and $E_2 = 100$ MPa, for $h = 0$, 150, and 300 mm. For similarity with the Vibratory Roller model, a contact force magnitude ($F_c$) of 150 kN is chosen for the analysis. To account for the plane strain conditions, $F_c$ is divided by the length of the roller in the y-direction (2.13 m), leading to $Q = 70.42$ kN/m. $Q$ is held constant for all values of $2a$, but the applied pressure changes with $2a$ per (3.41).

$$P = \frac{Q}{2a}$$  \hspace{1cm} (3.41)

Figure 3.2 shows maximum z-displacement, $w_{00}$ vs. $2a$, where $w_{00}$ is the maximum displacement at $z = 0$ and $x = 0$. The insensitivity of $w_{00}$ to changes in $2a$, can clearly be seen. As $2a \rightarrow h$ (and then exceeds $h$), the influence of $2a$ on surface response increases, but still only minimally.

### 3.7 Conclusion

In this chapter an analytical model of harmonic strip loading on a layered elastic half-space is derived from the governing equations for plane strain elasticity. This model is then used to explore the sensitivity of surface z-displacement under the center of the strip ($w_{00}$) to changes in $2a$. This analysis is performed to determine if $2a$ needs to be accurately modeled in our finite element analysis in order to capture observed surface behavior.

The results from this analysis show that $w_{00}$ is insensitive to changes in $2a$ when the total force is held constant. This is especially true when $2a < h$ as is the case in for vibratory rollers. As a result, it is not necessary to explicitly and accurately model $2a$ for the roller/soil system in order to fully capture field observed behavior, this echoes what was
Figure 3.2: $w_{00}$ vs. $2a$ for a halfspace with $E_1 = 50$ MPa ($h = 0$), and for a layer with $E_2 = 100$ MPa of $h = 150 \& 300$ mm, overlying $E_1$

observed numerically in the FE studies presented in Chapter 2.
CHAPTER 4
INFLUENCE OF FINITE ELEMENT ANALYSIS DAMPING MODELS ON VIBRATORY DRUM-SOIL RESPONSE

Modified from a paper to be submitted to *Journal of Engineering Mechanics* Bernadette Kenneally, Michael A. Mooney and Judith Wang

4.1 Abstract

The objective of this study is to analyze the differences in surface force-deflection behavior when modeling vibratory drum-soil interaction for quality control applications on fully compacted soils (e.g. proof rolls), using hysteretic damping, where material loss factors are represented by, $\eta_M$ and Rayleigh damping, where system damping coefficients are represented by ($\alpha$ and $\beta$). This is done by comparing results from time and frequency domain finite element (FE) models of vibratory drum interaction with layered earthwork systems. Both models are calibrated using field data from two construction sites, and results are compared to better understand the effects of the damping models on surface force-displacement response.

4.2 Introduction

The earthwork construction industry has been making a necessary shift from quality control/quality assurance (QC/QA) via spot based density and moisture testing to QC/QA via full coverage mechanistic testing (e.g. modulus, stiffness) to allow for performance based assessment of compaction QA/QC. Accordingly, vibration-based drum measurement of soil properties during compaction, known as Intelligent Compaction (IC) or Continuous Compaction Control (CCC), has gained traction in the US. Using the contact force–drum displacement ($F–z$) data, coupled with onboard GPS measurements, and graphical representations of these data via an onboard computer (Figure 1.1), the operator can perform real-time QC
on 100% of the compacted region, a significant improvement over current spot test methods. For vibratory drums to provide useful, mechanistic measurements (e.g. individual layer soil Youngs moduli, $E_1$, $E_2$), a quantitative understanding of the drum/soil system and of the $F-z$ behavior is needed. Historically, drum $F-z$ response has been used to gage soil compaction in a relativistic manner (e.g. compaction meter value, compaction control value) [2, 64], but more recently these have been used to define absolute measures of compaction (e.g. soil stiffness) [4–6, 16]. Estimated soil stiffness ($k$) provided by vibratory IC drums is a composite measure of ground stiffness up to a depth of 1.2 m [3, 6, 17–19]. This is much larger then a 15-30 cm thick lift of subgrade or base used in practice. Since earthwork QA in the US is performed on a per-lift basis, there is a need to measure the stiffness/modulus of each individual lift rather than just the single composite drum-measured $k$. This composite nature of $k$ has led researchers to investigate relationships between drum-measured composite $k$ and individual lift (or layer) moduli values [15, 20–22, 65].

The relationship between drum response and underlying soil response is highly nonlinear [5, 7–10] and dependent upon the drum vibration frequency (typically, 20-35 Hz), and the inertial and dissipative properties of the soil. During vibration the drum can experience decoupling from the soil, chaotic behavior (bouncing or bifurcation), and drum/frame rocking. To effectively capture this complex behavior, the use of discretized numerical approaches is required.

Intrinsic (material or hysteretic) damping is significant in soil materials and has been shown to have a significant effect on soil-structure interaction (SSI) [53, 66–69]. These previous studies focused on SSI for seismic loading using lumped parameter models, but did not explore the effects of material damping on vertical surface loading (as is the case for the vibratory drum). Additionally, radiation damping (or geometric spreading) contributes to the system response, although there is much disagreement about the contributions of radiation vs. material damping in SSI [63, 67, 70–74].
Previous finite element (FE) models have been developed for the drum-soil system, but these focus on the effects of drum excitation amplitude [42], homogeneous response [42–44] or inversion methods [15]. Kenneally et al. [65] developed a time domain FE model to explore drum sensitivity to underlying soil parameters. The aforementioned models either modeled the soil as a perfectly elastic or elasto-plastic medium with no damping, or used numerical Rayleigh damping to approximate system energy loss. Rayleigh damping, however, does not explicitly represent material-based damping properties and is used purely for mathematical convenience in approximating observed energy loss in a system. As a result, these previous models were unable to explicitly model individual material dissipative properties.

Frequency domain analyses use a hysteretic damping model (\(\eta\) or \(\xi\)) to accurately represent material damping. Although this model accurately represents physically observed behavior, the resulting mathematical expressions violate the principle of causality (i.e. for an impulse load, the response begins before the excitation) [75]. This can be easily handled by frequency domain analysis since it solves for the steady-state response. This non-causal behavior (and complex inputs) cannot be handled by explicit time domain analyses. Resultantly, time domain analyses must approximate the observed material damping using Rayleigh damping parameters.

Dissipative response reflected in observed force-displacement behavior may hold important information about the compacted state. All of the work to date has focused on elastic modulus estimation from observed \(F-z\) response. However, the type of damping assumption made by a modeler likely impacts the modulus estimation, as it affects the characteristics of the predicted cyclic \(F-z\) response. Additionally, material damping is shown to change during compaction (Figure 4.14), so the material loss factor could hold additional information about the actual compaction state. A FE model is needed that appropriately represents the dissipative properties of the soil to better understand the effects of material and radiation damping to better represent the in situ response of the drum soil system.
This paper explores the effect of material damping on vibratory drum response by developing a frequency domain finite element (FE) model that explicitly models individual layer material damping (using the loss factor, $\eta$). The results from the frequency domain model are then compared to the results from a previously developed time-domain FE model that uses Rayleigh damping [65] to better understand the effects of material damping on drum response.

### 4.3 Damping Background

Dynamic response is governed by the inertial (mass and stiffness) and dissipative (damping) properties of the system. The mass and stiffness properties govern the energy stored, and the damping properties govern the energy loss in the system. Energy loss in soil-structure systems consist of both material (hysteretic or viscoelastic) and radiation damping (geometric spreading). While there are many different damping measurement indices (e.g. damping ratio, $\zeta$), they are defined on the grounds of single degree of freedom (SDOF) rheological models, whereas the loss factor, $\eta$, is able to directly measure energy dissipation from observed response without making any assumptions about the physical system (4.1)[76]. $\eta$ is defined as the ratio of energy dissipated per cycle ($\Delta U$) to the strain (or elastic) energy ($U$) (4.1).

$$\eta = \frac{\Delta U}{2\pi U}$$  \hspace{1cm} (4.1)

$$\Delta U = \int F \cdot \text{d}z$$  \hspace{1cm} (4.2)

The choice of $U$ is non-trivial for damped systems [77] as the total energy varies throughout a loading cycle. Although there are many accepted definitions of $U$, for most engineering applications (and in this analysis) $U$ is defined as the maximum deformation energy stored relative to the non-dissipative part of the system [76], $U_{MAX}$ (4.3). Figure 4.1 shows how these are calculated from $F-z$ loops. Once an appropriate definition of $U$ is chosen, the loss
The loss factor can be calculated per (4.1).

\[ U = U_{MAX} = \frac{1}{2} k z_{MAX}^2 \]  
\[ k = \frac{F_c(z_{d-MAX}) - F_{c-STATIC}}{z_{d-MAX} - z_{d-STATIC}} \]

For most dynamic systems the loss factor \( \eta \) depends on both the amplitude and frequency of the oscillation. However, for completely linear systems, both \( \Delta U \) and \( U \) are proportional to the square of the oscillation amplitude and \( \eta \) is independent of amplitude [75]. Radiation damping is generally considered to be frequency dependent [63, 75, 78, 79], whereas the frequency dependence of the material damping depends on the chosen material model (viscoelastic vs. hysteretic).

The loss factor provides an accurate linear approximation of both the linear and non-linear observed energy loss in soil systems and is able to directly measure energy dissipation from observed response without making any assumptions about the physical system. \( \eta \) encompasses both material and radiation damping and these two damping mechanisms act independently of each other. Resultantly, the total energy loss, \( \Delta U_{TOT} \) (or the corresponding loss factor, \( \eta_{TOT} \)), is traditionally represented in engineering analysis as the linear sum of the two forms of damping, per (4.5)[63, 80], where the subscripts \( R \) and \( M \) represent radiation.
and material damping, respectively. Radiation and material damping are discussed in more detail in the following sections.

\[ \Delta U_{TOT} = \Delta U_R + \Delta U_M \] (4.5a)

\[ \eta_{TOT} = \eta_R + \eta_M \] (4.5b)

### 4.3.1 Radiation Damping

Radiation damping (or geometrical spreading) is caused by stress waves at the contact surface propagating outward in the form of body and surface waves. These waves carry away (or radiate) some of the energy transmitted onto the soil. Reissner [70] first discovered this phenomenon in his approximate solution to the response of a vertically loaded circular disk on an elastic half-space.

For soil-structure interaction, radiation damping is traditionally modeled using a viscoelastic SDOF assumption [32, 56, 63, 67, 71, 74, 78, 81] where, for vertical vibrations, the radiation damping is expressed through a damping coefficient, \( C_v \), and an equivalent stiffness, \( K_v \). These expressions were first developed by Lysmer [32] for a circular foundation vibrating on an elastic half-space, but have been expanded to include other foundation geometries. For strip footings \( C_v \) is shown in (4.6)[78], where \( \rho \) is the unit mass density of soil, \( A = 2a \) is the area per unit length, \( a \) is the contact half width of the footing, \( H_1^{(2)}(b) \) and \( H_0^{(2)}(b) \) are Hankel functions and \( \Re \) denotes taking the real part of the function. \( V_{Ly} \) is Lysmer’s velocity, which is a fictitious wave velocity that accounts for the fact that in the soil near the footing (or drum in this case), compression—extension waves propagate with at least some degree of normal straining in the lateral direction [78]. As a result, \( V_{Ly} \) is used as the compression—extension ”wave velocity” for plane strain.
Using the SDOF assumption $C_v$ can be related to the radiation loss factor $\eta_R$ by examining the energy loss of a viscous damper. For one cycle of motion a viscous damper has consumed energy equal to:

$$\Delta U_v = 4\pi \beta \frac{\omega}{\omega_n} U$$  \hspace{1cm} (4.9)

where $\beta$ is the critical viscous damping ratio, $\omega_n$ and $\omega$ are the natural and loading angular frequencies, respectively, and $k_v$ is the equivalent vertical stiffness. In the $\Delta U_v$ calculations, this study uses the dynamic stiffness per (4.4). The viscoelastic radiation loss factor, $\eta_R$ can then be computed from (4.9) per (4.11), and for the SDOF assumption $U = U_{MAX}$ (4.3)

$$\eta_R = \frac{\Delta U_v}{2\pi U_{MAX}} = 2\beta \frac{\Omega}{\omega_n}$$  \hspace{1cm} (4.11)

where $k$ is the equivalent dynamic stiffness of the soil and $z_{MAX}$ is the maximum $z$-displacement of the soil. There is much disagreement in the literature about the importance of material damping vs. radiation damping in soil-structure interaction. Gazetas [63], using a lumped parameter model, showed that for vertical foundation vibrations on a soil half-space, radiation damping dominates the system response for the vertical modes. Conversely, Sienkiewicz [71] shows that material damping is on the same order of magnitude as radiation damping. For shallow layers of soil, Wolf [74] shows that radiation damping is drastically reduced and material damping is the primary source of energy dissipation. Ambrosini [67] concludes that material damping significantly influences maximum displacement for soil-
structure interaction, and therefore must be included in analyses. Resulting from the lack of consensus in the literature, we will investigate the influence of radiation damping (implemented via infinite elements) and of material damping on the shape and size of the resulting force–displacement loops for our system.

4.3.2 Hysteretic Damping

In this analysis the soil material damping is modeled using hysteretic damping. Hysteretic damping represents material damping using a complex stiffness derived from SDOF equations with the assumption that the system can be described using an equivalent dynamic stiffness, \( k^* \) (4.12).

\[
    k^* = k (1 + i \eta_M) \tag{4.12}
\]

where \( i = \sqrt{-1} \), and \( \eta_M \) is the material loss factor, defined per (4.1). The complex nature of \( k^* \) causes a phase lag in the strain (or displacement) response, resulting in a more accurate representation of observed material damping. The material loss factor \( \eta_M \) for hysteretic materials is frequency independent and embodies both grain scale and atomic scale energy loss (e.g. friction between and damage to the individual soil grains). This is due to the fact that \( \eta_M \) comes from a continuum as opposed to a particulate material approach. Frequency domain analyses must be employed to accommodate the complex nature of \( k^* \). When using the hysteretic damping model \( \eta_M = 2 \xi \) for \( 0 \leq \eta_M \leq 0.28 \) [76]. The frequency independent hysteretic damping model (4.12) equals the viscoelastic damping model at resonance.

For a system with hysteretic material damping and viscoelastic radiation damping the total system \( \Delta U \) can be represented per (4.5b) or (4.13).

\[
    \Delta U = 2\pi \eta_M U_{MAX} + 4\pi \beta \frac{\omega}{\omega_n} U_{MAX} \tag{4.13}
\]

\[
    \frac{\Delta U}{2\pi U_{MAX}} = \eta = \eta_M + 2\beta \frac{\omega}{\omega_n} \tag{4.14}
\]
4.3.3 Rayleigh Damping

Rayleigh damping is used in time domain numerical analyses to approximate material damping and is based upon global system mass and stiffness matrices. A damping matrix, \([C]\), is formed and assumed to be linearly proportional to the velocity. The form used by ABAQUS [48] (and most commercial FEA packages) uses a combination of mass and stiffness proportional damping, and is shown in (4.15).

\[
[C] = \alpha[M] + \beta[K]
\]  
(4.15)

where \([M]\) is the mass matrix, \([K]\) is the stiffness matrix, \(\alpha\) and \(\beta\) are the Rayleigh damping parameters (units of \(s^{-1}\) and \(s\), respectively). (4.15) implies that the mass proportional term damps the lower frequencies, while the stiffness proportional term damps higher frequencies. \(\alpha\) introduces damping forces caused by the absolute velocities of the model, simulating motion through a viscous ether. \(\beta\) introduces damping proportional to the strain rate, and is interpreted in ABAQUS by creating an additional damping stress that is proportional to the total strain rate. The damping stress is added to the stress due to the constitutive response at the integration point, when the dynamic equilibrium equations are formed [48].

\(\alpha\) and \(\beta\) can be related to the modal loss factor \((\eta_i)\) using (4.16)

\[
\eta_i = 2\xi_i = \frac{\alpha}{\omega_i} + \beta\omega_i
\]  
(4.16)

where \(\xi_i\) is the modal damping ratio and \(\omega_i\) is the corresponding natural frequency (in rads/s) of the system. This system can be optimized to solve for \(\alpha\) and \(\beta\), given pairs of \(\xi_i\) and \(\omega_i\) for two or more natural frequencies of the system. In the reverse direction, given \(\alpha\) and \(\beta\), and either a natural or dominant loading frequency, the Rayleigh damping parameters can be related back to system material parameters. It should be noted this is valid only for \(\xi < 0.25\) [52].

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4.4 Vibratory Drum Mechanics

IC/CCC smooth drum vibratory drums are generally in the 12-15 metric ton range, with drum diameters and lengths of approximately 1.5 m and 2.1 m, respectively. Excitation is created by uni-directional or counter-rotating eccentric masses, $m_0$, located at effective moment arms of $e_0$ within the drum (see Figure 4.2); magnitudes of eccentric mass moment, $m_0e_0$, can range from 0 to 5.0 kg-m, and excitation frequencies, $\Omega = 2\pi f$, can range from 20–35 Hz. Field data from a Sakai SV510D drum Table 4.1 is used in the analysis to compare and validate both FE models.

Drum-measured $k$ is traditionally defined as either the secant or tangent stiffness of the force-displacement ($F-z$) loop. $F-z$ loops are derived from drum accelerometer and eccentric mass location data, using the lumped parameter mechanics of the vibrating drum Figure 4.2. The drum is assumed to behave as a rigid mass with a single vertical degree of freedom, $z_d$. The drum is attached to the frame via low stiffness isolation mounts, however the influence of frame dynamics has been shown to be negligible [4], and therefore only the static weight of the frame ($m_fg$) is considered. The drum-soil contact force, $F_c$, is then determined using force equilibrium (4.17), where $\ddot{z}_d$ is the drum acceleration.

![Free body diagram of vertical forces acting on drum.](a) Contact force vs. drum displacement response and resulting dynamic stiffness measures](b)

Figure 4.2: (a) Free body diagram of vertical forces acting on drum. (b) Contact force vs. drum displacement response and resulting dynamic stiffness measures
Table 4.1: Sakai SV510D drum Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drum Mass, $m_d$</td>
<td>4,466 kg</td>
</tr>
<tr>
<td>Frame Mass, $m_f$</td>
<td>2,534 kg</td>
</tr>
<tr>
<td>Mass Moment, $m_0e_0$</td>
<td>1.0 - 4.25 kg-m</td>
</tr>
<tr>
<td>Excitation Frequency, $f$</td>
<td>20-35 Hz</td>
</tr>
<tr>
<td>Drum/Frame Stiffness, $k_{df}$</td>
<td>6.02 MN/m</td>
</tr>
<tr>
<td>Drum/Frame Damping, $c_{df}$</td>
<td>4,000 kg/s</td>
</tr>
</tbody>
</table>

\[
F_c = m_0e_0\Omega^2 \cos \Omega t + (m_f + m_d)g - m_d \ddot{z}_d
\]  

(4.17)

\[
k = \frac{F_c(\ddot{z}_{d-MAX}) - F_c-STATIC}{z_{d-MAX} - z_{d-STATIC}}
\]  

(4.18)

$k$ can then be determined from the resultant $F-z$ loops. For our analysis, the secant stiffness is used and is defined using (4.18). Figure 4.2b shows $F-z$ response, and corresponding $k$ for continuous drum-soil contact (top) and for partial loss of contact (bottom).

4.5 FE Model

The overall setup and geometries of both the time domain (FE-T) and frequency domain (FE-F) FE models is the same, and will be outlined in this section. A two-dimensional, plane strain model was developed using the commercial package ABAQUS. Experimental results have shown that 2D plane strain conditions exist beneath the center of the 2.1 m long drum [49]. The discretized soil region was modeled using dimensions shown in Figure 4.3, with linear infinite elements applied at the semi-infinite boundaries to emulate half-space conditions and capture the effects of radiation damping. The drum of the vibratory drum is modeled as a rigid body cylinder 1.5 m in diameter. A static load of 67.68 kN (equal to the combined weight of drum and frame) is applied to the center of the drum. The eccentric mass load caused by the counter-rotating eccentric masses inside the drum then drives the dynamic analysis. The eccentric load is modeled as a vertical harmonic excitation force,
Infinite Elements
Soil Foundation
Uniform Linear Elastic Finite Element Mesh

Plan Strain
Linear Element
20 mm x 20 mm

Vertical Excitation Force:
\( m_0 e_0 \Omega^2 \cos(\Omega t) \)
Rigid Vibratory
Drum Diameter: 1.5 m
Static Weight: 68.67 kN

Figure 4.3: Schematic of FE mesh (actual mesh has vertical boundary at x=0, but full system shown for clarification)

\[ m_0 e_0 \Omega^2 \cos \Omega t, \] also applied at the center of the rigid cylinder. The excitation frequency, \( \Omega \), used for the majority of the analysis, is 188 rad/sec (30 Hz), which is within the range commonly used in practice. Drum acceleration (\( \ddot{z}_d \)) from the model is then used to calculate \( F_c, z_d \) and \( k \), in the same manner as with the field data (4.17).

The analyses are performed assuming that the particulate nature of the material can be modeled as continua that do not undergo plastic deformation. This assumption can be made for vibratory drums used in fully compacted proof-roll quality control (QC) applications. In practice proof rolls are performed for QC using low amplitude vibrations to verify the condition of fully compacted soils. The mass densities of the soil materials were kept constant and determined via experimental calibration studies.

4.5.1 Time Domain Model

The FE-T model uses an explicit time-integration approach, where contact between the drum and soil is explicitly modeled using a kinematic contact algorithm. The soil is modeled
using plane strain, linear elastic elements with Rayleigh damping applied to the system, and
the drum is modeled as described above (for a complete description of this model see [65]). \( \alpha \)
and \( \beta \) are determined through experimental calibration with two different field sites discussed
in a later section. The analysis is performed in two steps: first the static weight of the drum
is applied; and second the dynamic eccentric mass load is applied, and the step is run until
steady-state vibration is obtained. All of the results presented are then for steady-state
vibration.

4.5.2 Frequency Domain Model

The FE-F uses a direct integration, steady-state dynamics approach, which solves the
model in the frequency domain, using hysteretic damping. The model is setup in a similar
manner to the FE-T model, except the eccentric mass load is given in the frequency domain,
and no explicit contact conditions or algorithms are used (ABAQUS does not support contact
models in the FE-F). \( \eta \) for each material is determined at each location through calibration
with the same field data as the FE-T model. The analysis is performed in the same steps
described above. The FE-F results are imported into MATLAB and returned to the FE-T
for analysis.

4.6 Effect of Damping on Drum Response

To examine the influence of damping on drum response, multiple studies are performed to
understand and quantify the damping implementations used by ABAQUS in both the time
and frequency domains. To verify that the \( \eta_M \) value input into ABAQUS for frequency do-
main analysis is in fact the material loss factor, a triaxial test was simulated using ABAQUS
with \( \eta_M = 0, 0.15, \) and 0.25. \( \eta \) is then calculated from the resulting \( F-z \) loops (Figure 4.4)
per (4.1), using Gauss–Kronrod quadrature to numerically evaluate the integral. This test
verifies that \( \eta_M = \eta \) for a simple triaxial test. In the following section (4.6.1) the influence
and effects of radiation damping on the resulting \( F-z \) loops for FE-F and FE-T models
is examined. Section 4.6.2 examines the effects of material damping on surface response
through parametric sweeps of $\eta_M$ using the FE-F model. Section 4.6.3 performs a similar analysis using the FE-T model, performing parametric sweeps of $\alpha$ and $\beta$. For all sections parametric sweeps are performed for a homogeneous half-space with $E = 50 \text{ MPa}$.

### 4.6.1 Effects of Radiation Damping

To compare the influence of radiation damping to material damping in the system, the FE-F is run with $\eta_M = 0$ and 0.25 for a homogeneous half-space with $E = 50$, 70, and 90 MPa. The $\eta_M$ value of 0.25 is reasonable given the range of field observed strains for the drum/soil system presented in [51]. When the model is run with $\eta_M = 0$, all energy lost per cycle can be attributed to radiation damping. The corresponding $F-z$ loops, and $U_{\text{MAX}}$ and $\Delta U/2\pi$ values are shown in Figure 4.5. With no material damping, the dependence of radiation damping on the natural frequency of the system discussed in the literature (4.9)[32, 56, 60, 67, 71, 74, 75, 78, 81] can clearly be seen in the energy results for $\eta_M = 0$ (Figure 4.5(bottom)). As the stiffness of the half-space increases, the difference between $\Delta U$ and $U_{\text{MAX}}$ increases, resulting in an decrease in radiation damping ($\eta_R$) with soil stiffness.
Figure 4.5: (top) $F-z$ loops from FE-F for $E = 50$, 70 and 90 MPa with and without material damping. (bottom) corresponding $\Delta U/2\pi$ and $U_{MAX}$ values.

The addition of material damping (green Figure 4.5) to the system clearly influences the elastic energy in the system and resultantly, the shape and size of the $F-z$ loops. Material damping causes an increase in $\Delta U$ and a decrease in $U_{MAX}$, resulting in larger values of $\eta$ as compared to the results for $\eta_M = 0$. These results clearly show that the addition of material damping reduces the elastic energy in the system.

The corresponding SDOF analytical solutions for $\Delta U_{MAX}$ from (4.9) and (4.13) are presented in Table 4.2. The analytical solutions are very dependent upon the chosen foundation area, $A$, and for an equivalent strip foundation area of $A = 150$ mm, the SDOF and FE-F results are in close agreement. This value of $A$ is only slightly larger than field-measured drum/soil contact areas (65–130 mm) found by Musimbi et al. [20].
Table 4.2: $\Delta U_{TOT}$, $\Delta U_R$ and $\Delta U_M$ for FE-F and analytical solution when $\eta_M = 0$ and 0.25 for homogeneous half-space with $E = 50$, 70 and 90 MPa.

<table>
<thead>
<tr>
<th>$\Delta U_{2\pi}$</th>
<th>$\eta_M = 0$</th>
<th>$\eta_M = 0.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FE-F</td>
<td>SDOF</td>
</tr>
<tr>
<td></td>
<td>E (MPa) 50 70 90</td>
<td>FE-F 10.5 16.2 22.4</td>
</tr>
</tbody>
</table>

Figure 4.6: (left) $F$–$z$ loops from FE-F for $E = 50$ to 100 MPa with $\eta_M = 0.25$. (right) corresponding $\Delta U/2\pi$, and $\eta$ values for total, material and radiation damping, and the relative contributions of $\Delta U_R$ and $\Delta U_M$ on $\Delta U_{TOT}$. 
To more comprehensively compare the influence of radiation damping to material damping in the system, the FE-F is run with $\eta_M = 0.25$ for $E=50$ to 100 MPa. $\Delta U_{TOT}$, $\Delta U_R$ and $\Delta U_M$ are then calculated and examined (NB: $\Delta U$ values presented and discussed are actually $\Delta U/2\pi$ but for convienence the denominator is excluded within the text). First, the values of $\Delta U$ are computed from the loops in Figure 4.6 with $\eta_M = 0.25$. $\Delta U_{TOT}$ is computed directly from the $F-z$ loops per (4.2), where $\Delta U$ is calculated for each cycle and then averaged across all cycles. To find $\Delta U_M$, $\eta_M = 0.25$ is treated as a constant (since this is the input into FE-F program) for all $E$ values, and is multiplied by $U_{MAX}$ per (4.19).

$$\frac{\Delta U_M}{2\pi} = \eta_M U_{MAX} \tag{4.19}$$

$\Delta U_R$ is found by subtracting $\Delta U_M$ from $\Delta U_{TOT}$ (4.5a). The relative influence of radiation and material damping is shown in Figure 4.6 by dividing both $\Delta U_M$ and $\Delta U_R$ by $\Delta U_{TOT}$.

Radiation damping is shown to have greater influence (60-65%) than does material damping (35-40%) the system damping response (Figure 4.6). The dominance of radiation damping decreases with stiffness, resulting in an increase in the influence of material damping. For a 100% increase in $E$, there is 285% increase in $U_{MAX}$ and only 240% increase in $\Delta U_{TOT}$ resulting in a 16% decrease in $\eta_{TOT}$. Over the same range, $\Delta U_M$ increases by exactly the same amount as $U_{MAX}$, 285% (expected since $\eta_M$ is constant), and $\Delta U_R$ increases by only 216%, resulting in a 23% decrease in $\eta_R$. For comparison with the FE-F results for no material damping, the FE-T model is run with no material damping ($\alpha = \beta = 0$) for $E = 50$–100 MPa. The $F-z$ loops and $U_{MAX}$, $\Delta U/2\pi$ and $\eta$ values from both the FE-F and FE-T are presented in Figure 4.7. The FE-T $\Delta U$ and corresponding SDOF $\Delta U$ values are presented in Table 4.3. $\Delta U$ for FE-F is computed from the $F-z$ loops per (4.5) and the SDOF $\Delta U$ is computed per (4.9).

For a 100% increase in $E$, there is a 290% increase in $U_{MAX}$ and only 180% increase in $\Delta U$. The maximum elastic potential energy ($U_{MAX}$, (4.3)) increases with $E$ at a much faster rate than does $\Delta U$, resulting in a 31% decrease in $\eta$ with no material damping. The $\Delta U$ values from the FE-T and FE-F models are very similar for cases of total contact ($E \leq$
Figure 4.7: $F-z$ loops for $E = 50$ to $100$ MPa with $\alpha = \beta = \eta_M = 0$ and corresponding $U_{MAX}$, $\Delta U/2\pi$ and $\eta$ for FE-T and FE-F models.

70 MPa, Figure 4.7), but the FE-T underestimates $\Delta U$ from the FE-F for loss of contact ($E > 70$ MPa), which is clearly evident in the corresponding $F-z$ loops (Figure 4.7). The FE-T overestimates FE-F $U_{MAX}$ for $E \leq 80$ MPa, and underestimates it for $E > 80$ MPa, resulting in an underestimation of $\eta$.

The corresponding $\Delta U$ solutions from the FE-T and SDOF models are presented in Table 4.3. In the SDOF calculation, $k$ and $U_{MAX}$ were taken from the FE-T solution and for consistency, the same $A = 150$ mm as was used in the FE-F is used here. The SDOF $\Delta U$ less closely matches the FE-T than it did the FE-F, especially for loss of contact. The SDOF
ΔU values slightly underestimate the FE-T for E ≤ 70 MPa, and overestimate FE-T ΔU for E>70 MPa. The SDOF ΔU increases with E at almost the same rate as FE-T U_{MAX}, and since U_{MAX} is constant across both analyses, this results in only a slight decrease in η for the SDOF whereas FE-T η decreases with stiffness at a greater rate (Table 4.3).

Table 4.3: ΔU_{TOT}/2π and η results for FE-T and analytical solution when α = β = 0 for homogeneous half-space with E = 50–100 MPa.

<table>
<thead>
<tr>
<th>E (MPa)</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔU_{2π} FE-T</td>
<td>8.2</td>
<td>9.9</td>
<td>11.3</td>
<td>12.9</td>
<td>14.5</td>
<td>15.3</td>
</tr>
<tr>
<td>ΔU_{2π} SDOF</td>
<td>7.0</td>
<td>8.9</td>
<td>11.0</td>
<td>13.5</td>
<td>16.5</td>
<td>19.4</td>
</tr>
<tr>
<td>η FE-T</td>
<td>0.45</td>
<td>0.42</td>
<td>0.38</td>
<td>0.35</td>
<td>0.32</td>
<td>0.29</td>
</tr>
<tr>
<td>η SDOF</td>
<td>0.38</td>
<td>0.38</td>
<td>0.37</td>
<td>0.37</td>
<td>0.37</td>
<td>0.36</td>
</tr>
</tbody>
</table>

To understand the relative influence of material and radiation damping for the Rayleigh damping implementation, the FE-T model is run with the Rayleigh damping coefficients used in the time domain analysis in 2 (α = 25 s^{-1} and β = 2 ms). To find equivalent η_M for the FE-T, (4.16) is solved with ω_i equal to the loading frequency (188 rads/s). The corresponding ΔU_M and ΔU_R values are then found per (4.19) and (4.2), respectively.

The results are presented in Figure 4.8 and an equivalent η_M of 0.51 is found using the aforementioned α and β values in (4.16). Although this is well above the range of common η_M values for soil (generally 0.01-0.4 [67, 81–83]), it is not surprising as α and β were selected by choosing values that most closely matched the shape of F–z across two field sites, and not from any physical relationship [65]. Since η_M is held constant, ΔU_M increases with E and U_{MAX}. ΔU_R increases only slightly with E, resulting in the expected decrease in radiation damping with E, especially for loss of contact. Since η_M is so large, material damping dominates the response, and this dominance increases with E (Figure 4.8).
Figure 4.8: (left) $F-z$ loops from FE-T for $E = 50$ to 100 MPa with $\alpha = 25$ s$^{-1}$ and $\beta = 2$ ms and (right) corresponding $\Delta U/2\pi$, and $\eta$ values for total, material and radiation damping, and the relative contributions of $\Delta U_R$ and $\Delta U_M$ on $\Delta U_{TOT}$.

4.6.2 $\eta$ Sweep

This section presents results from a parametric sweep of $\eta_M$ using the FE-F model to better understand the effects of material damping on response. $\eta_M$ is varied from 0 to 0.4 as soils exhibit material damping magnitudes that span this range, depending on soil type and on strain magnitude [67, 81–83]. Damping affects the phase lag between drum displacement and contact force, influencing $k$ significantly more than $F_c$ and $z_d$ (Figure 4.9). Increasing $\eta_M$ from 0 to 0.4, results in a slight increase in $F_c$ ($< 2\%$), a larger decrease in $z_d$ (6\%), and
a 10% decrease in $k$ (Figure 4.9).

$\Delta U_M$ and $\Delta U_R$ are calculated from $\Delta U_{TOT}$ in the manner described in 4.6.1, since $\eta_M$ is known. Increasing $\eta_M$ causes a decrease in $U_{MAX}$ (from the decrease in both $k$ and $z_{d-MAX}$), and an increase in energy lost per cycle, $\Delta U_{TOT}$ (Figure 4.10). For a 400% increase in $\eta_M$, $\Delta U_M$ increases from 0 to 5.21 kN-mm, $\Delta U_R$ decreases by 15%, $\Delta U_{TOT}$ increases by 154% and $U_{MAX}$ decreases by 19%. This causes an increases of 190% and 105% for $\eta_{TOT}$ and $\eta_R$, respectively Figure 4.10. As $\eta_M$ is increased, the contribution of $\Delta U_M$ to $\Delta U_{TOT}$ increases and the contribution of $\Delta U_R$ decreases. This trend is clearly shown in Figure 4.10 where $\Delta U_M \rightarrow \Delta U_R$ as $\eta_M$ is increased. When $\eta_M = 0$, 100% of $\Delta U_{TOT}$ is due to $\Delta U_R$ (e.g. $\Delta U_R = \Delta U_{TOT}$). For $\eta_M = 0.4$, the magnitude of $\Delta U_R$ is very similar to the $\eta_M = 0$ case (Figure 4.11), but since $\Delta U_{TOT}$ has increased from the addition of material damping, the relative influence of radiation damping decreases from 100% to 60%.

Figure 4.9: (left) $F-z$ loops and (right) normalized $F_{c-MAX}$, $z_{d-MAX}$ and $k$ vs. $\eta_M$ for a homogeneous half-space with $E = 50$ MPa

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4.6.3 $\alpha$ and $\beta$ Sweep

The same analysis discussed in the previous section was performed on $\alpha$ and $\beta$ using the FE-T model. In this study $\alpha$ is varied from 0 to 60 s$^{-1}$ for $\beta = 0$. Using (4.16) with $\omega_i$ equal to the loading frequency (188 rads/s) this range of $\alpha$ corresponds to the same range of $\eta_M$ used in 4.6.2 (0–0.4). $\beta$ is then varied from 0 to 2.2 ms with $\alpha = 0$ (again, corresponding to $\eta_M = 0$–0.4 per (4.16)).

Figure 4.10: $U_{MAX}$, $\Delta U/2\pi$, $\eta_{TOT}$ and $\eta_R$ vs. $\eta_M$ for a homogeneous half-space with $E = 50$ MPa
There is very little difference in observed response from increasing $\alpha$, causing increases in $F_{c-MAX}$, $z_{d-MAX}$ and $k$ of only 1.5%, 2% and 4.5%, respectively (Figure 4.12). The increase in both $k$ and $z_{d-MAX}$ results in a larger (8%) increase in $U_{MAX}$. The increases in both $U_{MAX}$ and $F_{c-MAX}$ cause the $F-z$ loops to elongate along the major axis. Conversely, $\Delta U_{TOT}$ decreases by 8%, meaning that the total area of the $F-z$ loops decreases. Therefore as the $F-z$ loops elongate along the major axis, they become skinnier along minor axis so that the total area decreases with $\alpha$. The increase in $U_{MAX}$ and decrease in $\Delta U_{TOT}$ result in a 17% decrease in $\eta_{TOT}$ (Figure 4.12).

Using the equivalent $\eta_M$ values, $\Delta U_M$ and $\Delta U_R$ are calculated using (4.19) and (4.2). As $\alpha$ is increased, the influence of material damping ($\Delta U_M/\Delta U_{TOT}$) increases and the influence of radiation damping decreases. At $\alpha \geq 40 \text{ s}^{-1}$ material damping dominates system response (Figure 4.12).

$\beta$ is varied from 0 to 2.5 ms with $\alpha = 0$, which corresponds to the same $\eta_M$ range as the $\alpha$ sweep. $\beta$ significantly affects $\Delta U$ and, in turn, the area of the $F-z$ loops Figure 4.13. The increase in $\beta$ causes a 60% increase in $\Delta U_{TOT}$, and is evident from the $F-z$ loops in Fig-
Figure 4.12: (left) $F-z$ loops for $\alpha$ sweep with $\beta = 0$, and (right) Normalized $F_{c-MAX}$, $z_{d-MAX}$ and $k$, and $\Delta U/2\pi$ and $U_{MAX}$ and $\eta$ vs. $\alpha$ for a homogeneous half-space with $E = 50$ MPa.

In Figure 4.13, $z_{d-MAX}$ and $k$ decrease by 7% and 10%, respectively, and $F_{c-MAX}$ increases by only 2%, resulting in a 22% decrease in $U_{MAX}$. The large increase of $\Delta U_{TOT}$ and corresponding decrease of $U_{MAX}$, results in a 100% increase in $\eta_{TOT}$ (Figure 4.13).

$\Delta U_R$ decreases with $U_{MAX}$, resulting in constant $\eta_R$, which is the expected behavior since $E$ is held constant across the analysis. The contribution of $\Delta U_M$ to $\Delta U_{TOT}$ increases with $\beta$ and the contribution of $\Delta U_R$ decreases. This trend is clearly shown in Figure 4.13 and it mirrors the trend seen in the FE-F damping results (Figure 4.10).

### 4.7 Field Results

The following sections present the results from matching both the FE-T and FE-F models with the field data described below. Two different criteria for best fit are shown below. The first fits the data based on matching $k$ values by minimizing the percent difference (4.20)
Figure 4.13: (left) $F$–$z$ loops for $\beta$ sweep with $\alpha = 0$, and (right) Normalized $F_{c-MAX}$, $z_{d-MAX}$ and $k$, and $\Delta U/2\pi$ and $U_{MAX}$ and $\eta$ vs. $\alpha$ for a homogeneous half-space with $E = 50$ MPa between field and FE $k$ values ($\delta_k$). The second finds a best fit for both $F_{c-MAX}$ and $z_{d-MAX}$. This best fit is found by minimizing the percent difference ($\delta$) between field and FE results for both $F_{c-MAX}$ and $z_{d-MAX}$ (4.20). For layered situations, $\delta$ is calculated for $F_{c-MAX}$ and $z_{d-MAX}$ (or $k$) at each lift thickness individually (4.20) and then averaged across lift thickness for each parameter ((4.21)-(4.22)). The best fit is then found by minimizing the average of $\delta_{F_{c-MAX}}$ and $\delta_{z_{d-MAX}}$ (4.23).

\[
%Diff = \delta = \frac{F_{c-\text{FIELD}} - F_{c-\text{FE}}}{F_{c-\text{FIELD}} - F_{c-\text{FE}}} \tag{4.20}
\]

\[
\delta_{F_{c-MAX}} = \frac{\delta_{h=25}^{F_{c-MAX}} + \delta_{h=55}^{F_{c-MAX}}}{2} \tag{4.21}
\]
\[
\delta_{zd-MAX} = \frac{\delta_{h=25}^{h=55}}{2} + \delta_{zd-MAX}^{h=55}
\]

(4.22)

\[
\text{best fit} = \min \left[ \frac{\delta_{c-MAX} + \delta_{zd-MAX}}{2} \right]
\]

(4.23)

Data from two field sites, one in Florida and the other in North Carolina, were collected using an instrumented Sakai SV 510D vibratory drum (Table 1). Selected data from these sites are presented in the following sections.

### 4.7.1 Florida Best Fits

The Florida data was extracted from a single test bed that was a 30 m long lane of single drum width (2.13 m) of homogeneous silty-sand (SM) subgrade soil. The soil was compacted over 10 passes; the odd numbered passes were performed using forward motion, low amplitude excitation force and excitation frequency of 30 Hz. Although the density changes from pass to pass, the change in density ($\rho$) from initial to final states is only about 10 – 15\%, but this difference has little influence on simulated response. As a result, $\rho$ is held constant throughout the analysis. Since the FE-F is being compared to the FE-T analysis from 2, $\alpha$ and $\beta$ are held constant ($\alpha = 25$ $s^{-1}$ and $\beta = 2$ ms) for all analyses as $\alpha$ and $\beta$ were chosen in 2 by fitting to the shape and size of the field-observed $F-z$ loops presented. In the FE-F, $\eta$ is allowed to vary with each pass. The following sections present best fits from both models for passes 3, 5, and 9. The FE-F results are presented first, followed by the FE-T results.

#### 4.7.1.1 FE-F Fits

The results for both $k$ and $F_{c-MAX}$ and $zd-MAX$ best fits of the FE-F model with the Florida data are presented in Table 4.4. The corresponding $F-z$ loops for the $k$ (Figure 4.14(a–c)) and $F_{c-MAX}$ and $zd-MAX$ (Figure 4.14(d–f)) fits are presented with the field data. Both best fits for the Florida data are plotted together, for ease of comparison, in Fig-
Figure 4.14: Best fit $F-z$ loops from the FE-F for $k$ (a–c) and $F_{c-MAX}$ and $z_{d-MAX}$ (d–f) with Florida field data. Both fits are plotted together for comparison (g–i).

The model finds very similar $E$ values for both fits, but $\eta_M$ varies considerable depending of if the loops are fitted to $k$ or to $F_{c-MAX}-z_{d-MAX}$. Fitting to $k$ results in $E = 49, 62, \text{ and } 78 \text{ MPa with } \eta = 0.28, 0.26, \text{ and } 0.22$, and for the $F_{c-MAX}-z_{d-MAX}$ fit, $E = 51, 64, \text{ and } 80 \text{ MPa and } \eta = 0.05, 0.09, \text{ and } 0.17$ (Figure 4.14, Table 4.4). The FE-F model is able to capture the general shape of the field-measured $F-z$ loops, but both fits underestimate $z_d$. 

Then compared to each other (Figure 4.14). The same energy analysis as in the damping analysis is performed and Figure 4.15 and Table 4.4 show the corresponding $U_{MAX}$, $\Delta U$ and $\eta$ values for total, material and radiation damping for each pass. The FE-F
during loading, although they are able to effectively capture field-observed contact force at $z_{d-MAX}$ (where the $k$ measurement is taken), for all passes (Figure 4.14).

The field data show that with compaction (from Pass 3 to Pass 9), $U_{MAX}$ increases by 84% and $\Delta U_{TOT}$ increases by 93% and resultantly, $\eta_{TOT}$ increases from Pass 3 to 5 (from 0.60 to 0.68), but then decreases from Pass 5 to 9 (from 0.68 to 0.63) causing an overall increase in $\eta_{TOT}$ of 5% (Figure 4.15).

The $k$ fit results show that $U_{MAX}$ doubles and $\Delta U_{TOT}$ increases by 59% causing $\eta_{TOT}$ to decrease by 21% from Pass 3 to 9. Corresponding with the increase in $\Delta U_{TOT}$, both $\Delta U_M$ and $\Delta U_R$ increase, but not as quickly as $U_{MAX}$, causing a decrease in both $\eta_M$ and $\eta_R$.
\( \eta_R \). The relative influence of both radiation and material damping stays constant across all passes, with radiation damping having a 62\% influence on \( \Delta U_{TOT} \) (Figure 4.15). The FE-F underestimates field \( U_{MAX} \) for all passes, overestimates field \( \Delta U_{TOT} \) for Passes 3 and 5, but closely matches it for Pass 9. This results in an overestimation of field \( \eta_{TOT} \) for Passes 3 and 5, and a slight underestimation for Pass 9.

Figure 4.16: Maximum Shear Strain in % from FE-F model \( k \) Fits for Florida data.

Figure 4.16 shows the maximum shear strain levels (in \%) for each pass. The chosen \( \eta_M \) values are reasonable given the strain levels in the soil. It should be noted that although the best fits show that \( \eta_M \) decreases with each pass, the strain levels in the soil are shown to increase slightly (Figure 4.16). The analysis in this thesis uses constant moduli and material loss factors for each material, and does not incorporate strain dependence.
Table 4.4: Results from FE-F best fit for Florida data

<table>
<thead>
<tr>
<th>Pass 3</th>
<th>E (MPa)</th>
<th>$\eta_M$</th>
<th>$U_{MAX}$ (kN-mm)</th>
<th>$\Delta U_{TOT}$ (kN-mm)</th>
<th>$\Delta U_M$ (kN-mm)</th>
<th>$\Delta U_R$ (kN-mm)</th>
<th>$\eta_TOT$</th>
<th>$\eta_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>31.92</td>
<td>19.04</td>
<td>0.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k fit</td>
<td>49</td>
<td>0.28</td>
<td>28.97</td>
<td>21.00</td>
<td>7.83</td>
<td>13.17</td>
<td>0.75</td>
<td>0.47</td>
</tr>
<tr>
<td>F–z Fit</td>
<td>51</td>
<td>0.05</td>
<td>33.10</td>
<td>17.06</td>
<td>1.66</td>
<td>13.30</td>
<td>0.52</td>
<td>0.28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pass 5</th>
<th>E (MPa)</th>
<th>$\eta_M$</th>
<th>$U_{MAX}$ (kN-mm)</th>
<th>$\Delta U_{TOT}$ (kN-mm)</th>
<th>$\Delta U_M$ (kN-mm)</th>
<th>$\Delta U_R$ (kN-mm)</th>
<th>$\eta_TOT$</th>
<th>$\eta_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>41.70</td>
<td>28.14</td>
<td>0.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k fit</td>
<td>62</td>
<td>0.26</td>
<td>38.62</td>
<td>27.27</td>
<td>10.04</td>
<td>17.23</td>
<td>0.71</td>
<td>0.45</td>
</tr>
<tr>
<td>F–z Fit</td>
<td>64</td>
<td>0.09</td>
<td>45.06</td>
<td>23.34</td>
<td>4.06</td>
<td>13.30</td>
<td>0.52</td>
<td>0.30</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Pass 5</th>
<th>E (MPa)</th>
<th>$\eta_M$</th>
<th>$U_{MAX}$ (kN-mm)</th>
<th>$\Delta U_{TOT}$ (kN-mm)</th>
<th>$\Delta U_M$ (kN-mm)</th>
<th>$\Delta U_R$ (kN-mm)</th>
<th>$\eta_TOT$</th>
<th>$\eta_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>58.76</td>
<td>36.70</td>
<td>0.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k Fits</td>
<td>78</td>
<td>0.22</td>
<td>56.06</td>
<td>33.27</td>
<td>12.33</td>
<td>20.94</td>
<td>0.59</td>
<td>0.37</td>
</tr>
<tr>
<td>F–z Fit</td>
<td>80</td>
<td>0.17</td>
<td>60.94</td>
<td>32.52</td>
<td>10.36</td>
<td>20.19</td>
<td>0.53</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Analysis of strain dependent material properties will provide further insight into the soil behavior, but is beyond the scope of this thesis.

In order to fit the FE-F to $F_{c-MAX}$ and $z_{d-MAX}$, a slight increase in $E$ and both a significant decrease in the magnitude of $\eta_M$ and an increase in $\eta_M$ with compaction is required (as compared to the $k$ fits). Resultantly, these fits overestimate field $U_{MAX}$ and underestimate field $\Delta U_{TOT}$, thereby underestimating field $\eta_{TOT}$. The fitted loops capture $F_{c-MAX}$ and $z_{d-MAX}$ values, the resulting small values of $\eta_M$ create narrower loops, underestimating $z_{d-FIELD}$, resulting in an overestimation of $k$. This is much less pronounced for pass 9, where material damping is significantly larger than the other passes (over three times the value for pass 3, and just under twice the value for pass 5).

The passes cause an 84% increase in $U_{MAX}$ and a 90% increase in $\Delta U_{TOT}$, although most of the increase in $\Delta U_{TOT}$ happens between Passes 5 and 9 when $\eta_M$ is doubled (Figure 4.15, Table 4.4). The $F_{c-MAX}$–$z_{d-MAX}$ fits show the slight increase in $\eta_{TOT}$ from Pass 3 to 9 exhibited in the field data, however, this corresponds to an increase in $\eta_R$ with stiffness, which does not make sense physically. The unrealistic radiation damping behavior, combined with the narrowness of the loops, suggests that for the FE-F, fitting to $k$ more accurately represents both the inertial and dissipative properties of the system.

4.7.1.2 FE-T Fits

The results for both $k$ and $F_{c-MAX}$ and $z_{d-MAX}$ best fits of the FE-T with the Florida data are presented in Figure 4.17 and Figure 4.18. The best fits of $k$ and $F_{c-MAX}$ and $z_{d-MAX}$ both result in $E = 47$ MPa for pass 3, and have nearly identical values of $E$ for passes 5 and 9 of $E = 61$ and 62 MPa and $E = 86$ and 84 MPa, respectively (Figure 4.17). Since the Rayleigh damping parameters were constant for all analyses and the fitted $E$ values are nearly identical for both fits, they will be grouped together in the following discussion.

FE-T $F-z$ loops slightly overestimate field $F_{c-MAX}$ for passes 3 and 9, but underestimate field $F_{c-MAX}$ for pass 5 ($\delta_{F_{c-MAX}} < 3\%$ for all passes) and underestimate $z_{d-MAX}$ for all passes, causing an underestimation of field $U_{MAX}$ (Figure 4.18). Even with the large underestimation
of $z_{d-MAX}$, both FE-T fits can still capture field observed $k$, due in large part to the FE-T models ability to almost perfectly capture field $\Delta U_{TOT}$ for all passes (Figure 4.18). The FE-T results show the physically realistic decrease in $\eta_R$ with stiffness, but have much smaller than the corresponding FE-F $\eta_R$ values, as $\eta_M$ is much larger. As $\eta_M$ is held constant, $\eta_{TOT}$ to decreases with compaction pass.

Figure 4.17: Best fit $F-z$ loops from the FE-T for $k$ (a–c) and $F_{c-MAX}$ and $z_{d-MAX}$ (d–f) with Florida field data. Both fits are plotted together for comparison (g–i).

The FE-T model is able to almost perfectly capture loading behavior for $0 \leq z_d \leq z_{d-MAX}$, but is unable to capture the unloading behavior, and becomes out of phase with the field results (especially for stiffer materials). The FE-T and field $F-z$ loops are essentially in phase for the cases of total contact (passes 3 and 5) but for pass 9, when loss of contact exists, the FE-T response becomes out of phase with the field response.
Figure 4.18: $\Delta U/2\pi$ and $U_{MAX}$ and $\eta$ from the FE-T model and Florida field data for each pass.

This phase difference is most likely a result of the decrease in material damping with each pass found in the FE-F fits. The FE-T model uses Rayleigh Damping (4.15), with constant coefficients for all analyses. Since $\beta$ is multiplied by the stiffness matrix, and mass is held constant, as the material increases in stiffness, the FE-T $[C]$ increases, causing an increase in material damping, which is the opposite of the observed decrease in material damping, shown in the FE-F results. This erroneous increase in material damping causes the large underestimation of field $U_{MAX}$ seen in the FE-T results.
4.7.2 North Carolina Best Fits

This section presents best fits from both FE-T and FE-F models for data from the North Carolina site. This test bed was a layered system, consisting of compacted silty-sand (SM) subgrade, upon which a 25 cm layer of crushed stone base material (SP-SM) was placed and compacted. A further 30 cm of the same crushed stone material (total layer depth of 55 cm) was placed and compacted. The subgrade and base course soils were fully compacted to standard Proctor and modified Proctor maximum dry densities and optimum moisture contents, respectively, prior to the collection of vibratory drum response data. Similarly to the Florida data, $\rho$ is held constant for all analyses, and for the FE-T, $\alpha$ and $\beta$ are held constant at the values listed in the previous section. For the FE-F model, both $\eta_M$ and $E$ are fitted for each material. In the following discussion, parameters for the top layer are denoted with the subscript 2, whereas those for the underlying layer use the subscript 1.

The processes used in the previous sections to calculate the relative influence of material and radiation damping only apply to systems with one material. Resultantly, only total $\Delta U$ and $\eta$ values will be presented for the North Carolina data.

4.7.2.1 FE-F Fits

The results for both $k$ and $F_{c-MAX}$ and $z_{d-MAX}$ best fits of the FE-F model with the North Carolina data are presented in Figure 4.19, Table 4.5, Table 4.6, Table 4.7. To fit to the layered situation, first, $E_1$ and $\eta_{M1}$ are found by fitting to the subgrade. $E_1$ and $\eta_{M1}$ are then treated as a known constant when the base material is added. $E_2$ and $\eta_{M2}$ are then fitted by assuming them to be constant for both lift thicknesses (since both lifts are of the same material). As was explained with the Florida data, if strain-dependent material properties were being used, $\eta_{M1}$ would actually decrease significantly with the addition of the top layer as this significantly decreases the strain levels in the half-space (Figure 4.20). The incorporation of strain-dependence is beyond the scope of this thesis, but is recommended for further study.
The FE-F model can closely capture field-observed behavior for all three scenarios by allowing for different $\eta_M$ values for each material. The FE-F fit to $E_1 = 63$ MPa and $\eta_1 = 0.32$ for both fits, and $E_2 = 132$ MPa and $\eta_2 = 0.27$ for the $k$ fits and $E_2 = 146$ MPa and $\eta_2 = 0.18$. This follows the same trend seen in the Florida fits wherein much lower levels of material damping, and slightly stiffer materials are needed when fitting to $F_{c-MAX}$ and $z_{d-MAX}$ versus fitting to $k$. Both fits show higher levels of material damping in the SM subgrade material than in the SP-SM base ($\eta_M = 0.32$ vs. 0.27 and $\eta_M = 0.32$ vs. 0.18).

The resulting $F_c-z_d$ loops (Figure 4.19) for $h = 0$, closely match those from the field data during the loading portion of the curve, but have trouble capturing the unloading behavior.

For the layered situations, $k$ fits underestimate both $F_{c-MAX}$ and $z_{d-MAX}$, and the underestimation of $F_{c-MAX}$ increases with top layer thickness, whereas the $F-z$ fits result in a significant overestimation of field measured $k$.

Figure 4.19: Best fits for $k$ (top) and $F_{c-MAX}$ and $z_{d-MAX}$ (bottom) with North Carolina field data from the FE-F model.
Figure 4.20: Maximum Shear Strain in % for each layer, from FE-F model k Fits for North Carolina Data.
Table 4.5: Results from FE-F best fit for North Carolina data

<table>
<thead>
<tr>
<th>h = 0</th>
<th>$E$ (MPa)</th>
<th>$\eta_M$</th>
<th>$U_{MAX}$ (kN-mm)</th>
<th>$\frac{\Delta U}{2\pi}$ (kN-mm)</th>
<th>$\eta_{TOT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Field</strong></td>
<td>36.56</td>
<td>0.72</td>
<td>26.19</td>
<td>0.72</td>
<td><strong>F - z Fits</strong></td>
</tr>
<tr>
<td><strong>k fit</strong></td>
<td>63</td>
<td>0.32</td>
<td>37.22</td>
<td>28.86</td>
<td>0.78</td>
</tr>
<tr>
<td><strong>F - z Fits</strong></td>
<td>63</td>
<td>0.32</td>
<td>37.22</td>
<td>28.86</td>
<td>0.78</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>h = 25 cm</th>
<th>$E$ (MPa)</th>
<th>$\eta_M$</th>
<th>$U_{MAX}$ (kN-mm)</th>
<th>$\frac{\Delta U}{2\pi}$ (kN-mm)</th>
<th>$\eta_{TOT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Field</strong></td>
<td>49.87</td>
<td>0.91</td>
<td>45.13</td>
<td>0.91</td>
<td><strong>F - z Fits</strong></td>
</tr>
<tr>
<td><strong>k Fits</strong></td>
<td>132</td>
<td>0.27</td>
<td>48.02</td>
<td>43.99</td>
<td>0.92</td>
</tr>
<tr>
<td><strong>F - z Fits</strong></td>
<td>146</td>
<td>0.18</td>
<td>53.14</td>
<td>46.07</td>
<td>0.87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>h = 55 cm</th>
<th>$E$ (MPa)</th>
<th>$\eta_M$</th>
<th>$U_{MAX}$ (kN-mm)</th>
<th>$\frac{\Delta U}{2\pi}$ (kN-mm)</th>
<th>$\eta_{TOT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Field</strong></td>
<td>61.86</td>
<td>0.86</td>
<td>52.90</td>
<td>0.86</td>
<td><strong>F - z Fits</strong></td>
</tr>
<tr>
<td><strong>k Fits</strong></td>
<td>132</td>
<td>0.27</td>
<td>59.21</td>
<td>49.67</td>
<td>0.84</td>
</tr>
<tr>
<td><strong>F - z Fits</strong></td>
<td>146</td>
<td>0.18</td>
<td>71.64</td>
<td>53.29</td>
<td>0.74</td>
</tr>
</tbody>
</table>
4.7.2.2 FE-T Fits

The results for both $k$ and $F_{c-MAX}$ and $z_{d-MAX}$ best fits of the FE-T model with the North Carolina data are presented in Figure 4.21, Table 4.6 and Table 4.7. The FE-T model is able to almost perfectly capture field-observed $F_{c-z_d}$ behavior for $h = 0$ (both loading and unloading): in this case, the best fit for $k$ is also the best fit for $F_{c-MAX}-z_{d-MAX}$ (Figure 4.21). It is, however, unable to match $k$ for the layered situations. The best fits for $k$ result in 13.4 and 20.6% differences in $k$ for $h = 25$ and 55 cm, respectively. The $F_{c-MAX}-z_{d-MAX}$ fits are much better at capturing field observed behavior, resulting in the same $E_1$ (60 MPa), and a higher $E_2$ (161 MPa) than the for the $k$ fit.

![Figure 4.21: Best fits for $k$ (top) and $F_{c-MAX}$ and $z_{d-MAX}$ (bottom) with North Carolina field data from the FE-T model.](image)

The FE-T loops are out of phase with the field data, and this is more pronounced as $h$ is increased. The FE-F results show that the subgrade and base have different amounts of material damping, which influences the phase (and in turn $k$) significantly. Since the FE-T
model uses constant system level Rayleigh damping ($\alpha = 25 \text{ s}^{-1}$ and $\beta = 0.002 \text{ s}$ for all analyses), it does not allow for individual damping parameters for each material, making capturing field-observed $k$ for layered situations difficult.

Table 4.6: Results from $k$ best fit for North Carolina data

<table>
<thead>
<tr>
<th>$h = 0$</th>
<th>$E_1$ (MPa)</th>
<th>$k$ (kN/mm)</th>
<th>% Diff</th>
<th>$F_{c-MAX}$ (kN)</th>
<th>% Diff</th>
<th>$z_{d-MAX}$ (mm)</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>46.36</td>
<td>145.28</td>
<td>1.242</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE-F</td>
<td>63</td>
<td>46.44</td>
<td>0.16%</td>
<td>144.48</td>
<td>0.55%</td>
<td>1.254</td>
<td>0.93%</td>
</tr>
<tr>
<td>FE-T</td>
<td>60</td>
<td>46.50</td>
<td>0.30%</td>
<td>142.26</td>
<td>2.11%</td>
<td>1.245</td>
<td>0.25%</td>
</tr>
<tr>
<td>$h = 25 \text{ cm}$</td>
<td>$E_2$ (MPa)</td>
<td>$k$ (kN/mm)</td>
<td>% Diff</td>
<td>$F_{c-MAX}$ (kN)</td>
<td>% Diff</td>
<td>$z_{d-MAX}$ (mm)</td>
<td>% Diff</td>
</tr>
<tr>
<td>Field</td>
<td>59.88</td>
<td>188.32</td>
<td>1.288</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE-F</td>
<td>132</td>
<td>60.51</td>
<td>1.05%</td>
<td>181.14</td>
<td>3.54%</td>
<td>1.248</td>
<td>3.18%</td>
</tr>
<tr>
<td>FE-T</td>
<td>150</td>
<td>52.38</td>
<td>13.4%</td>
<td>183.28</td>
<td>2.37%</td>
<td>1.241</td>
<td>3.75%</td>
</tr>
<tr>
<td>$h = 55 \text{ cm}$</td>
<td>Field</td>
<td>71.92</td>
<td>218.75</td>
<td>1.304</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE-F</td>
<td>132</td>
<td>71.88</td>
<td>0.06%</td>
<td>201.20</td>
<td>7.37%</td>
<td>1.265</td>
<td>3.00%</td>
</tr>
<tr>
<td>FE-T</td>
<td>150</td>
<td>58.52</td>
<td>20.6%</td>
<td>212.15</td>
<td>3.06%</td>
<td>1.284</td>
<td>1.50%</td>
</tr>
</tbody>
</table>

Table 4.7: Results from $F_{c-MAX}$ and $z_{d-MAX}$ best fit for North Carolina data

<table>
<thead>
<tr>
<th>$h = 0$</th>
<th>$E_1$ (MPa)</th>
<th>$k$ (kN/mm)</th>
<th>% Diff</th>
<th>$F_{c-MAX}$ (kN)</th>
<th>% Diff</th>
<th>$z_{d-MAX}$ (mm)</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>46.36</td>
<td>145.28</td>
<td>1.242</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE-F</td>
<td>63</td>
<td>46.44</td>
<td>0.16%</td>
<td>144.48</td>
<td>0.55%</td>
<td>1.254</td>
<td>0.93%</td>
</tr>
<tr>
<td>FE-T</td>
<td>60</td>
<td>46.50</td>
<td>0.30%</td>
<td>142.26</td>
<td>2.11%</td>
<td>1.245</td>
<td>0.25%</td>
</tr>
<tr>
<td>$h = 25 \text{ cm}$</td>
<td>$E_2$ (MPa)</td>
<td>$k$ (kN/mm)</td>
<td>% Diff</td>
<td>$F_{c-MAX}$ (kN)</td>
<td>% Diff</td>
<td>$z_{d-MAX}$ (mm)</td>
<td>% Diff</td>
</tr>
<tr>
<td>Field</td>
<td>59.88</td>
<td>188.32</td>
<td>1.288</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE-F</td>
<td>146</td>
<td>64.79</td>
<td>7.88%</td>
<td>189.63</td>
<td>0.69%</td>
<td>1.262</td>
<td>2.01%</td>
</tr>
<tr>
<td>FE-T</td>
<td>161</td>
<td>51.62</td>
<td>14.8%</td>
<td>188.32</td>
<td>0.89%</td>
<td>1.239</td>
<td>3.85%</td>
</tr>
<tr>
<td>$h = 55 \text{ cm}$</td>
<td>Field</td>
<td>71.92</td>
<td>218.75</td>
<td>1.304</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE-F</td>
<td>146</td>
<td>81.89</td>
<td>13.0%</td>
<td>218.27</td>
<td>0.22%</td>
<td>1.304</td>
<td>0.01%</td>
</tr>
<tr>
<td>FE-T</td>
<td>161</td>
<td>57.85</td>
<td>21.7%</td>
<td>218.60</td>
<td>0.07%</td>
<td>1.284</td>
<td>1.55%</td>
</tr>
</tbody>
</table>
4.8 Conclusion

This chapter used results from both time and frequency domain FE models to understand the effects of each damping implementation on drum response. The FE-F was then used to extract $E$ and $\eta_M$ to understand how they change with compaction, and with the addition of a stiff layer ontop of a half-space. This was done by fitting to field data from the same locations presented in 2. The results from the FE-T are also used for comparison. From the analysis the following conclusions can be drawn:

- Results from both models show a decrease in radiation damping with increases in half-space $E$. The addition of material damping increase total system energy loss, and for constant $\eta_M$ (and $\alpha$ and $\beta$), the relative influence of material damping increases as the influence from radiation damping decreases.

- From the FE-F, it is found that increases is $\eta_M$ for a half-space with constant $E$ result in a decrease in $U_{MAX}$ and an increase in $U_{TOT}$ and $\eta_{TOT}$ as $\eta_R$ is essentially constant for constant $E$.

- From the FE-T, increases in $\alpha$ are shown to have a minimal effect on drum response, causing only a slight increase in length, and decrease in width of the $F-z$ loops. However, increasing $\beta$ has a large effect on system energy loss, causing decreases in $U_{MAX}$ and an increase in $\Delta U$

- Results from the Florida data show that in addition to increases in stiffness with compaction, $U_{MAX}$ and $\Delta U_{TOT}$ also increase keeping $\eta_{TOT}$ approximately constant, but material damping ($\eta_M$) is shown to decrease with compaction.

- The North Carolina fits show that the addition of a 25 cm thick stiffer layer with lower levels of material damping causes a greater increase in $\Delta U_{TOT}$ than in $U_{MAX}$, causing an increase in $\eta_{TOT}$. The addition of the additional 30 cm of the stiff material,
increases the influence of the top layer, causing a decrease in $\eta_{TOT}$ resulting from a smaller increase in $\Delta U_{TOT}$ than was found with the previous lift thickness.

- Across both field sites, the FE-F model is better able to capture field—observed drum force—deflection behavior through its ability to model individual material dissipative parameters. For the homogeneous system, he $k$ fits most closely capture both surface behavior and energy loss observed in practice, but for the layered system, the $F-z$ fits more closely capture the shape and size of the loops, but for the FE-F, overestimate, and for the FE-T, underestimate field $k$. 
CHAPTER 5
EXPLORING EXPERTISE IN NEW TECHNOLOGY ADOPTION IN STATE HIGHWAY CONSTRUCTION

5.1 Introduction

The road construction industry is known to be particularly averse to new technology adoption, due in large part to institutional impedance [84], although efforts have recently been undertaken by many state Departments of Transportation (DOT) to change this attitude and actively promote innovation and new technology adoption [85–87]. In this new vein of promoting new technology adoption, Collins and Evans Studies of Expertise and Experience (SEE) framework [88–90] is used in this paper to examine the adoption and diffusion of new technologies, like Intelligent Compaction (IC), within the US road construction industry. Pulling from public reports and literature, the key players and necessary combinations of expertise for successful technology adoption within state DOTs are identified and examined. The analysis of technology diffusion and adoption of IC technologies by Kimmel & Toohey [91] is expanded and re-examined using SEE in this chapter.

This study extends SEE to examine the case of new technology adoption focusing on state DOTs. The situation for IC technology adoption is unique in that the technology is in development but there is both a lack of incentive (in the form of policy or standards change) for the adoption of this new technology, and a change in culture from density based testing to stiffness based testing. As a result, those working for the adoption of IC need to have people with interactional expertise in policy making and politics, in addition to people with contributory technical expertise in order to effect the policy changes needed to implement this new technology.

The majority of SEE literature focuses on esoteric sciences [88–90, 92–94], where there is minimal or no policy change required for implementation of new technology or the focus is
on examining expertise using historical case studies that have already been resolved [95–98]. Jenkins [95] uses SEE to study the development of technology to solve the Bycatch problem in the tuna industry in response to new government regulations. In this case, policy was driving science, whereas for IC, science needs to drive policy in order to have successful adoption. This paper uses insights gained from interviews and surveys of state DOT employees and contractors in [85] and [91], to identify the combinations of expertise and key players that have historically led to successful technology adoption, and hopefully giving some insight into the necessary combination of expertise needed for future technology adoption within a state DOT.

Section 5.2 provides a brief introduction to IC and current soil compaction standards. Section 5.3 presents and explains the key concepts of SEE. Section 5.4 pulls from [85] and [91] to identify key players/roles in the adoption process and the respective expertises needed for successful adoption, and Section 5.5 provides a brief conclusion of the findings.

5.2 Soil Compaction

This paper focuses on the adoption of IC technology (used for monitoring soil stiffness, which is related to the compaction state, during soil compaction) by state DOTs. Road construction involves compaction by roller compactors (vibratory, padfoot, etc.) of first the existing soil in the ground, and then of each subsequent lift (about 15-30 cm) of material. The use of a roller allows for efficient compaction of large tracts of soil. In this section the quality control (QC) methods, specifications and standards currently employed during road construction are briefly outlined. IC technology is then briefly introduced, and the improvements to the QC process provided by IC are presented.

5.2.1 Current Quality Control Methods/Standards

In current US earthwork construction, soil compaction is specified in terms of a target density and water content (per ASTM D698 [99]), based on the assumption that there is a correlation between density and soil response. This assumption was proven incorrect as
early as 1948, when Hveem and Carmanay showed that:

the density of a granular mass is one of the least reliable and least informative
of all determinations which can be madethe internal structure of the particle ar-
angement may vary considerably without any significant change in density[100].

Regardless of these obvious flaws, this is the current national standard and is used across the
US. The most widely used technologies for evaluating soil compaction (via density and water
content measurement) are the Nuclear Density Gauge (ASTM D5195-14) and the Sand Cone
(ASTM D1556-07). Both of these devices take measurements at discrete locations across the
construction site, usually testing << 1% of the compacted area [? ]. This leads to compaction
acceptance, despite the possibility that the untested areas have insufficient compaction levels
or water content.

5.2.2 Intelligent Compaction

IC involves vibration-based roller measurement of soil properties during compaction,
using an instrumented roller. These rollers measure a value that is correlated to the soil
stiffness, allowing for mechanistic, performance-based testing. Using roller accelerometer
data, coupled with onboard GPS measurements, and graphical representations of these data
via onboard computer, the operator can perform real-time QC on 100% of the compacted
region. This allows for increased compaction accuracy and eliminates fuel and labor costs
associated with over-compaction.

The drum accelerometer data, when combined with a numerical model of the roller-soil
system, can be used to extract individual lift stiffness (or modulus) allowing for a direct,
mechanistic measurement of compaction. One drawback is that there is no standardization
of the roller measured stiffness values, and these definitions vary across manufacturers and
software products [15]. This lack of standardization across manufacturers means that new
policies (i.e., a standard for stiffness measurement across all platforms) must be created
to allow for widespread adoption of IC. State DOTs are the primary avenue for this type
of policy change, but the reported institutional inertia [91, 101] can create a considerable barrier to innovation. While most state DOTs do not actively discourage innovation, there are no incentives for developing and testing new technologies, and a lot of bureaucracy must be dealt with for approval [85–87, 101]. In the past few years, some DOTs have actively worked to remove these barriers to innovation. Most notably, MnDOT has done this by creating a designated fund to sponsor innovative projects, and by restructuring the DOT internally to allow for more communication across disciplines [87]. While these steps actively taken by DOTs are a movement in the right direction, they are recent developments, so the long-term effectiveness of these programs cannot currently be examined.

5.3 Studies of Expertise and Experience (SEE)

In the preliminary paper on SEE, Collins and Evans [88] propose the need for a “normative theory of expertise, [that] will disentangle expertise from political rights in technical decision making” [88]. The framework for this theory (SEE) is laid out in [88] and expanded in [89, 90] for categorizing and distinguishing between experts in technical decision making. One of the primary tenets of this theory is that, to become an expert in a technical domain, one must acquire the tacit knowledge pertaining to that domain [90]. To develop a normative theory of expertise, assumptions must be made (summarized from [93]):

1. Expertise is treated as ”real”

2. Expertise in a domain can be possessed by those with experience in the domain but without formal qualifications (experience-based experts).

3. Technical expertise in esoteric domains is difficult to acquire

4. Classifying expertise into different types and levels is useful.

5. The Periodic Table of Expertises (from [89]) provides some useful descriptions of types of expertise.
6. Interactional expertise is an important type of specialist expertise that is acquired more through immersion in the discourse of the hands-on experts than through active participation in the field.

7. High levels of expertise generally involve tacit knowledge acquired through embedding in specific social groups (e.g. entire societies or small specialist groups).

8. An understanding of expertise may allow one to make both prescriptive and normative statements about the way expertise has and should be used.

9. Increased public involvement in technical decision-making, although necessary, proper and at times scientifically useful, can sometimes obscure scientific and technological issues and give rise to undesirable outcomes.

5.3.1 Three Waves of Science Studies

Before getting into the details of SEE, Collins and Evans historical classification of science studies into Three Waves, must be understood. Wave I is the period of technological determinism (dominant until 1960s), Wave II (which includes present studies) is the period of social constructivism, Wave III is the proposed era focused on Studies of Expertise and Extension (SEE). Wave I dominated the post-war era (1950s & 60s) with the belief that scientists, through their scientific training and methods, were inherently in a place to speak with authority and decisiveness in technical decision-making, and this authority was generally unquestioned. Kuhns, The Structure of Scientific Revolutions [102], questioned the technological determinist viewpoint, and ushered in the second wave.

Wave II is dominated by the Sociology of Scientific Knowledge (SSK), which adopts the social constructivist viewpoint believing that science is an inherently social process [103–106]. In order to understand science, we must understand the social processes involved in its creation: Fact construction is so much a collective process that an isolated person builds only dreams, claims, and feelings, not facts [103]. In his earlier work, Collins warned against
the relativism adopted by Latour and the Paris school, claiming that if left unleashed it leads us to have nothing to say...the result is impotence [107]. The social constructivist movement was a direct refute of the technological determinism (Wave I) of the 1950s and 1960s [104]. The second wave has been able to solve The Problem of Legitimacy by showing that technical decision-making should be expanded beyond the core of certified experts, but it has been unable to address The Problem of Extension [88].

Collins and Evans [88] propose the third wave to address and to solve the problem of extension. They hope to ”hammer a piton into the ice wall of relativism with enough delicacy not to shatter the whole edifice” [88]. To do this the focus of study must shift from knowledge to expertise, and a call is made for a new area of science studies SEE. Wave III hopes to reconstruct the knowledge that was so heavily deconstructed during the second wave and draw boundaries around who is qualified to contribute to technical decision-making [88–90].

5.3.2 Types of Expertise

Collins [90] debunks the idea that everyone is a scientific expert by clearly presenting different kinds of expertise and distinguishing substantial expertise from ubiquitous expertise. An expert is one who shares the tacit knowledge of a specialist group [90]. Collins concedes that although science is a social process, it is social in the acquisition of tacit knowledge, that through sustained social contact with other experts, one can acquire tacit knowledge. This analysis focuses on the different types of substantive expertise (for more detailed definitions see [89]).

In SEE there are three main categories of substantive expertise: ubiquitous, specialist, and meta-expertise.

- Ubiquitous expertise – comes from expertises that we have acquired without putting any self-conscious effort [90], such as speaking your native language or how often to bathe in society.
• Specialist expertise – expertise in a specific field, and is the type of expertise generally associated with experts (e.g. Doctors, concert pianists, mathematicians).

• Meta-expertise – a method for choosing between experts and their expertises.

While ubiquitous expertise is valuable in life, science does not fall into this category in our society, so it is indeed something special.

5.3.2.1 Specialist Expertise

Specialist expertise can be further broken down into three categories: contributory, interactional, and referred expertise. Contributory expertise comes from being a contributory member of a specialist group. These can either traditional academic experts (researchers in the field) or experience based experts (those with hands on or non-traditional expertise in a field, e.g. farmers in pesticide debate, or contractors in IC debate). The introduction of experience-based experts does away with the incorrect and oxymoronic term lay expert. That label has often been used incorrectly to describe people with real (although non-traditional) specialist contributory expertise.

Interactional expertise is acquired by engaging in the spoken discourse of an expert community to the point of fluency but without participating in the practical activities or deliberately contributing to those activities. [90]. One can be an interactional expert without being contributory, but all contributory experts are also interactional experts. In order to have informed technical decision-making, at least one set of experts needs to have enough interactional expertise with the other group so that a combination of contributory expertise can emerge [88]. The introduction of interactional expertise broadens the boundaries of specialist expertise to include qualified persons outside the core set of scientists or researchers.

Referred expertise is unique because it allows an expert to understand how to contribute to fields of scientists they are leading at one remove (e.g. gravitational physicist leading the Thirty Meter Telescope project in [92]) and is necessary for those in management positions. To do this one needs contributory expertise in some related science, but only interactional
expertise with respect to the specific science. It is worth noting that referred expertise is also a type of non-transmuted meta-expertise (see Section 5.3.2.2).

The wider scientific community (those without the relevant specialist expertise on the issue) do not play any special part in the decision making process. They are treated as indistinguishable from the general citizen and do not have a role in the technical domain [88, 89]. Scientists do not have referred expertise about fields of science distant from their own. Claiming that they have expertise leads to an overemphasis on scientific generalization rather than specialization, when specialists are the only ones with valuable expertise in technical decision-making. In order to have informed technical decision making, at least one set of experts needs to have enough interactional expertise with the other group so that a combination of contributory expertise can emerge [88, 89]. This allows for cooperation between technical scientists and people with more local or experiential knowledge.

5.3.2.2 Meta-Expertise

Meta-expertise is a method developed for choosing between experts and their expertises [90]. It is broken down into transmuted expertise and non-transmuted expertise. The former takes a judgment of a person and transforms it into a technical decision using ones local or ubiquitous knowledge, whereas the latter involves using expertise that is not primarily about other people but is a substantive technical expertise [90]. Meta-expertise is developed to help non-experts assess the technical claims made by experts.

According to SEE, these non-technical experts can use transmuted expertise (i.e. judging the social position or performance of experts), wherein social expertise is being transmuted into technical judgments, to help differentiate between experts. Using local knowledge to inform these judgments is far more effective than just the use of ubiquitous social expertise alone [108]. Conversely, non-transmuted expertise (i.e. Technical Connoisseurship, Downward Discrimination, Referred Expertise) use technical understanding to discriminate between experts and their opinions, requiring technical expertise in at least one domain [90].
5.4 Successful Technology Adoption in State DOTs

Results from multiple interviews with state DOT employees published in the literature [85, 91] are used to identify the factors and key players for successful technology adoption. The types of specialist expertise held by each of these players, and the requisite combinations of expertise are identified. Due to the bureaucratic and risk adverse nature of many DOTs, successful adoption is difficult, requiring a specific combination of factors. Interview respondents in both [85, 91] identified the three most important factors to be:

1. a champion associated with the project
2. pilot projects and performing field demonstrations
3. upper management support

Each of these factors will be explored in the following analysis.

5.4.1 Champions

Across the board, policy entrepreneurs from [91] or champions from [85] were identified as the most important factor in successful technology adoption. Since there is a distinct lack of external motivators for new technology adoption, champions tend to be those with both an interactive ability, and an internal desire to both interact and innovate. This willingness to interact has historically been a key factor in successful technology development [95], without this internal drive, the correct combinations of experts cannot be effectively assembled. Champions were found to facilitate technology transfer by fostering ownership, recognizing future benefits, not giving up, and creating a faster buy-in with both management and workers [85]. Successful champions were able to effectively communicate across disciplines, bringing together the required combinations of contributory and interactional expertise required to effectively communicate the need for and benefits of a new technology, paving the way for policy change in favor of new technology adoption.
In successful cases, these champions were generally found to have contributory expertise in the field of the new technology and need to have interactional expertise in the policy making or technology adoption process so that they can pull together the key players (managerial support, contractors, other researchers, policy-makers), and elucidate both the problem being addressed and the solution provided by the new technology. They also need contributory or interactional expertise with actual construction site operations in order to ensure ease of use and implementation. This void can be filled through interaction with experience-based contributory experts (e.g. contractors) who can provide valuable insight into actual implementation and use. These champions can be found at all levels of the DOT, but tend to be research engineers and district engineers.

The research engineers are the technical experts of the DOT and are given funds to perform research and pilot projects. They normally have contributory expertise in topics relevant the new technology, and at least interactional expertise in its use. This expertise gives them the credentials to suggest solutions to other people within DOT, allowing them to more effectively get people on board the project.

The district engineer holds the power and political capital needed to put policy changes on the agenda, and has near autonomy over projects [91]. In order to be successful, the district engineer needs at least referred expertise (if not interactional or contributory) in the new technology to make informed decisions regarding its implementation. Additionally, interactional expertise in the policy domain is needed to effect policy change allowing for implementation of the new technology. If the champion lacks this policy expertise, he must find someone with, at minimum, interactional expertise in both the policy and the technical domains to join the cause, because without changes to standards and specifications, the new technology will not be widely adopted. The champions need either the above combination of expertises or the ability to identify expertise voids, and bring together the necessary experts to fill them.
5.4.2 Pilot Projects and Demonstrations

Pilot programs, usually led by research or district engineers, provide a venue for testing implementation of new technology and for evaluation of policy ideas relating to the technology. The goal of these pilot projects is defined by one Florida DOT research agency to be:

an information exchange mechanism that can reduce or eliminate the financial, professional, and political risk public agencies face when committing hard-to-come-by funds implementing technology when little or no practical field experience exists [85].

Successful pilot projects often result in explicit policy recommendations.

The success of pilot projects was found to be dependent on the personal characters of the key players involved [91]. One of the largest barriers to implementation found by DOTs is contractor resistance to change [85]. This reinforces importance of interactive ability shown by Jenkins [95], and of innovation proclivity shown by Kimmel & Toohey [91], for successful adoption. An interviewee in [91] even said Personal character can outweigh technical research and field validation of IC with disinterest and lack of acceptance [91]. For successful pilot project implementation, the DOT must work with a contractor that has this innovation proclivity. Champions can use meta-expertise to better inform the choice of contractor. As was discussed in champions section, the champion also needs at least interactional expertise with field implementation of the technology in order to work effectively with the contractors during the pilot project.

Field demonstrations (performed in conjunction with pilot programs) were found to be the most effective way to illuminate a problem and spread awareness. It allows politicians and other members of industry and government with no expertise in the technology to understand its importance and potential impact. One interviewee from [91] said, following an IC demonstration: "Wow, I didn't realize and I guess its a no brainer, we ought to do
this,” clearly expressing the effectiveness of the field demonstration in clearly establishing both the problem and the proposed solution.

5.4.3 Upper Management Support

DOT employees identified the support of upper management as a key factor in successful implementation. In DOTs, upper management (usually chief or division engineers) hold the most power in technical decision-making, and have the ability to implement policy change. Without their support, new standards and specifications for the technology will not become unified and reach full adoption. To be successful, these managers need both referred and interactional expertise. In some cases they also have contributory expertise, but this is less important for people in managerial positions. Collins & Sanders [92] demonstrate the importance of referred and interactional expertise for managers making technical decisions. The chief or division engineers need either interactional expertise in both the policy-making process and the technology or someone trusted with that expertise to join their team. To be successful, managers also need to use referred meta-expertise to understand the motivations behind, and the conflicting claims made by different technical experts. Using this effectively will allow them to distinguish between experts that are motivated by the facts and the science, and those who have external motivations (i.e. general stubbornness or aversion to change).

5.5 Conclusion

This chapter used Collins and Evans (SEE) framework to examine the adoption and diffusion of new technologies within state DOTs. In successful new technology adoption in DOTs, champion, pilot projects/field demonstrations and upper management support are all vital. Effective champions need have contributory expertise in the field of the technology and at least interactional expertise in the policy making or technology adoption process. Pilot programs and field demonstrations are the most effective way to educate non-experts on the benefits of the technology. The success of the programs are generally very dependent
upon the choice of contractor, so the champion must use meta-expertise to ensure that the contractor is a good fit for the project. Finally, without upper management support, none of these programs would ever be successful. These are the people with the most technical decision making power, and they need to have enough technical referred or contributory expertise in the related science to make informed decisions. Additionally, they need to have, at least, interactional expertise with the policy realm, as they hold the power to lobby the correct people to effect policy change and allow for successful adoption.
CHAPTER 6
CONCLUSION

This thesis has presented the development and evaluation of two finite element (time and frequency domain) models of the vibratory drum/soil system. These models were developed to explore the effects of individual layer parameters ($E_1$, $E_2$, and $h$) and damping on drum response for the time and frequency domain models, respectively. A semi-analytical model of harmonic strip loading on a layered half-space is also developed to explore the effects of changing contact area on surface response. The models developed here advance the understanding of the mechanics of the vibratory drum/soil system and can be used to more effectively extract stiffness and damping information for individual lifts in road construction. In addition to this technical research, this thesis performed policy research concerning successful paths and combinations of expertise that have generally led to successful new technology adoption within state DOTs. This policy research is performed to aid in the successful adoption of the CCC technologies discussed in this thesis.

The finite element models were developed to better understand the relationship between vibratory drum response and the underlying soil system for real—time monitoring of soil behavior for ‘proof roll’ QC applications. The time domain model presented in Chapter 2 was calibrated and validated with experimental data from vertically homogeneous and two—layer conditions. The FE model is then used to parametrically explore the relationship between vibratory drum response and system parameters such as elastic moduli and layer thickness. The model was shown to slightly underestimate field-measured $k$, as peak deflections for the model occur at much lower $F_c$ values. $k$ was found to be sensitive to increases in subgrade modulus ($E_2$) for $E_2/E_1$ values less than 2.5. Beyond this point $k$ becomes insensitive to increases in $E_2$. Confirming the vibratory drum measurement depth of 1 m found in the literature[3, 6, 17–19], the FE model found $k$ to be sensitive to increases in top layer
thickness up to 1 m, where beyond that point, drum—measured $k$ is only influenced by the top layer.

The harmonic strip load on a layered half-space model presented in Chapter 3, was developed to determine whether the FE models needed to explicitly model contact area in order to capture drum response. Results showed that surface displacement directly under the center of the strip load is insensitive to changes in contact area when the total force is held constant. These results inform the FE model development by showing that for the drum/soil system (where for a given $E_1, E_2$ and $h$, the contact force is constant) the contact area does not need to be explicitly modeled to capture drum response.

The analysis from Chapter 2 was expanded to examine the effects of damping on drum response through the development of a frequency domain FE model, wherein individual material damping loss factors can be input as material parameters (Chapter 4). Time time domain analysis cannot handle to complex inputs associated with material hysteretic damping, therefore numerical Rayleigh damping parameters must be chosen by fitting them to the system. The analysis in Chapter 4 both the time and frequency domain models to para-metricaly explore the effects of $\eta_M, \alpha$ and $\beta$ and radiation damping on drum response and system energy loss. The FE-F model is fitted to field data from the same field sites presented in Chapter 2.

Results from the Florida data show that in addition to increases in stiffness with compaction, $U_{MAX}$ and $\Delta U_{TOT}$ also increase keeping $\eta_{TOT}$ approximately constant, but material damping ($\eta_M$) is shown to decrease with compaction. The North Carolina fits show that the addition of a 25 cm thick stiffer layer with lower levels of material damping causes a greater increase in $\Delta U_{TOT}$ than in $U_{MAX}$, causing an increase in $\eta_{TOT}$. The addition of the additional 30 cm of the stiff material, increases the influence of the top layer, causing a decrease in $\eta_{TOT}$ resulting from a smaller increase in $\Delta U_{TOT}$ than was found with the previous lift thickness. The FE-F models are shown to be more able to capture both field—measured $k$, and the observed shape and behavior of the loops than were the FE-T, which significantly
underestimate field $k$. This results from the constant system Rayleigh Damping parameters that are unable to accurately capture the dissipative behavior of materials with different levels of material damping.

As the forward models developed in this thesis apply to the geo-construction industry, investigating the policy environment surrounding the implementation and adoption of these technologies. Resultantly, an analysis of the needed combinations of expertise for successful technology adoption in state DOT’s is performed and presented in Chapter 5. The analysis used Collins and Evans’, Studies of Expertise and Experience as a framework and previous interviews and surveys with DOT employees found in the literature.

In successful cases it was found that, champions, pilot projects/field demonstrations and upper management support are all vital components. Through examination of each of these components, a combination of contributory expertise in the technology and at least interactional expertise with field practices and implementaion is needed for successful champions and pilot projects. Upper management needs to be both supportive and have members with referred or contributory expertise in a relevany technical field in addition to interactional expertise with policy makers in order to make the code and standard changes need to allow for full adoption.

6.1 Recommendations for Future Work

Continued work on Intelligent Compaction technologies for in situ measurement of soil properties would benefit both the scientific community and the condition of US highways. The energy dissipation analysis presented in Chapter 4, can be expanded to examine the contribution of individual layer material damping to total damping response. The field data from compaction pass showed that material damping decreases with compaction, indicating that there is a mechanistic relationship between material damping and compaction state. This thesis presented a preliminary analysis of energy loss in the system, but the relationships between material damping and compaction state, would benefit from further examination. Furthermore, valueable insights could be gained by expanding the analysis to identify the
individual contributions of each materials dissipative properties to total energy loss.
REFERENCES CITED


[99] ASTM International. Standard test methods for laboratory compaction characteristics of soils using standard effort (12 400 ft-lbf/ft³ (600 kn-m/m³))., 2007.


APPENDIX - MATLAB SCRIPTS FROM CHAPTER 3

This appendix presents MATLAB scripts and functions used to solve the harmonic strip loading solution derived and used in Chapter 3.

Listing A.1 is the MATLAB script used to solve for the constants from Chapter 3. Listing A.2 solves for $w_{00}$ at various values of $2a$ and calls the MATLAB function Listing A.3 to transform $\bar{w}_{00}$ to $w_{00}$.

Listing A.1: MATLAB script used to solve for the constants in the differential equations presented in Chapter 3.

```matlab
%% Solving for Coefficients for Strip Load on Layered Half-space

clear all; close all; clc;

%% Big Matrix
syms vs vs2 vp vp2 h xi k1 k2 k12 k22 A B C D F G eta1 eta2 eta12 eta22 p ii P a rho rho2
rho=rho2;

%% z = 0 BC's
%% BC 1 sigma z^2 = 0
BC1= C*(xi^2 - (k12^2)) + D*(xi^2 + (k12^2)) + F*(2*xi*k22) ==0;

%% z = h BC's

%% BC 3 w(h) = w2(h)
BC3=A*(-xi*exp(-1i*k1*h)) + B*(-k2*exp(-1i*k2*h)) + C*xi*exp(1i*k12*h)
    + D*xi*exp(-1i*k12*h) ...
    + F*k22*exp(1i*k22*h) + G*k22*exp(-1i*k22*h)==0;

%% BC 4 u(h) = u2(h)
BC4=A*k1*exp(-1i*k1*h) + B*(-xi*exp(-1i*k2*h)) + C*(-k12*exp(1i*k12*h))
    + D*(-k12*exp(-1i*k12*h)) ...
    + F*xi*exp(1i*k22*h) + G*xi*exp(-1i*k22*h)==0;
```

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% BC5 \( \sigma_{x}(h) = \sigma_{x2}(h) \)

\[
BC5 = A(2*\text{k1} \times \text{vs}^2 \times \exp(-1*i*\text{k1}*h)) + B \times \exp(-1*i*\text{k2}*h) \times (\text{vp}^2 \times (\text{xi}^2 - \text{k2}^2) - 2*\text{xi}^2 * \text{vs}^2) \ldots
\]

\[
= C(2*\text{xi} \times \text{k12} \times \text{vs}^2 \times \exp(1*i*\text{k12}*h)) + D(2*\text{xi} \times \text{k12} \times (\text{vs}^2 - \text{vp}^2)^2) \times \exp(-1*i*\text{k12}*h)) \ldots
\]

\[
+ F \times \exp(1*i*\text{k22}*h) \times (\text{vp2}^2 \times (\text{xi}^2 + \text{k22}^2) - 2*\text{xi}^2 * \text{vs}^2) \times \exp(-1*i*\text{k22}*h) + G \times \exp(-1*i*\text{k22}*h) \times (\text{vp2}^2 \times (\text{xi}^2 - \text{k22}^2) - 2*\text{xi}^2 * \text{vs}^2) \ldots
\]

% BC6 \( \sigma_{xz}(h) = \sigma_{xz2}(h) \)

\[
BC6 = A \times \text{vs}^2 \times \exp(-1*i*\text{k1}*h) \times (\text{xi}^2 + \text{k1}^2) = C \times \text{vs}^2 \times \exp(1*i*\text{k12}*h) \times (\text{xi}^2 - \text{k12}^2) \ldots
\]

\[
+ D \times \text{vs}^2 \times \exp(-1*i*\text{k12}*h) \times (\text{xi}^2 + \text{k12}^2) + F \times 2 \times \text{vs}^2 \times \text{xi} \times \text{k22} \times \exp(1*i*\text{k22}*h); \]

\text{save('coeffs_sym.mat');}

Listing A.2: MATLAB script used to solve for vertical displacement at \( x = 0 \) and \( z = 0 \).

clear all; close all; clc

\( E1 \) top layer

\( E = \) bottom layer

\text{syms} \ xi

\[
\text{h}=300*\text{e}-3;
\]

\[
\%h=150*\text{e}-3;
\]

\[
\%h=0;
\]

\[
E=50*\text{e}6+(0.1*1\times 50*\text{e}6);
\]

\[
E1=100*\text{e}6+(0.1*1\times 100*\text{e}6);
\]

\[
\text{poiRat}=0.3;
\]

\[
\mu=\text{sqrt}(2*(1-\text{poiRat})/(1-(2*\text{poiRat})))
\]

\[
\lambda\text{shearMod}=(E*\text{poiRat})/((1+\text{poiRat})*((1-(2*\text{poiRat})/2*(1+\text{poiRat})));
\]

\[
\lambda\text{shearMod1}=(E1*\text{poiRat})/((1+\text{poiRat})*((1-(2*\text{poiRat})/2*(1+\text{poiRat})));
\]

\[
\rho\text{ho}=2000;
\]

\[
\rho\text{ho2}=\rho\text{ho};
\]

\[
\omega=2*\text{pi}*30;
\]

\[
\text{vp}2=\text{sqrt}((\lambda\text{shearMod1}+2*\lambda\text{shearMod1}/\rho)/\rho);
\]

\[
\text{vs}2=\text{sqrt}(\text{shearMod1}/\rho);
\]

\[
\text{k12}=(\text{sqrt}(\omega^2-\text{xi}^2 \times 2 \times \text{vs}^2))^2/\text{vs}2;
\]

\[
\text{k22}=(\text{sqrt}(\omega^2-\text{xi}^2 \times 2 \times \text{vp}^2))^2/\text{vp}2;
\]

\[
\text{vp}=\text{sqrt}((\text{lambda}+2*\text{shearMod})/\rho);
\]
\[ \text{vs} = \sqrt{\text{shearMod}/\rho}; \]
\[ k_1 = \sqrt{\omega^2 - \xi^2 \cdot \text{vs}^2}/\text{vs}; \]
\[ k_2 = \sqrt{\omega^2 - \xi^2 \cdot \text{vp}^2}/\text{vp}; \]

\[ \pi_i = \pi; \]

\%\%
\[ \text{matrixBig} = \text{sym('matrixBig', [6, 6])}; \]
\[ \text{matrixBig} (1, 1) = 0; \]
\[ \text{matrixBig} (1, 2) = 0; \]
\[ \text{matrixBig} (1, 3) = \text{xi}^2 - k_{12}^2; \]
\[ \text{matrixBig} (1, 4) = \text{xi}^2 + k_{12}^2; \]
\[ \text{matrixBig} (1, 5) = 2\text{xi} \cdot k_{22}; \]
\[ \text{matrixBig} (1, 6) = 0; \]

\[ \text{matrixBig} (2, 1) = 0; \]
\[ \text{matrixBig} (2, 2) = 0; \]
\[ \text{matrixBig} (2, 3) = 2\text{xi} \cdot k_{12} \cdot \text{vs}^2 \cdot 2; \]
\[ \text{matrixBig} (2, 4) = 2\text{xi} \cdot k_{12} \cdot (\text{vs}^2 - \text{vp}^2 \cdot 2); \]
\[ \text{matrixBig} (2, 5) = 2\text{xi} \cdot \text{vs}^2 \cdot 2 \cdot (\text{xi}^2 + k_{22}^2) - 2\text{xi} \cdot \text{vs}^2 \cdot 2; \]
\[ \text{matrixBig} (2, 6) = 2\text{xi} \cdot \text{vs}^2 \cdot 2 \cdot (\text{xi}^2 - k_{22}^2) - 2\text{xi} \cdot \text{vs}^2 \cdot 2; \]

\[ \text{matrixBig} (3, 1) = -\text{xi} \cdot \exp(-\text{1i} \cdot k_{1} \cdot h); \]
\[ \text{matrixBig} (3, 2) = -\text{k2} \cdot \exp(-\text{1i} \cdot k_{2} \cdot h); \]
\[ \text{matrixBig} (3, 3) = \text{xi} \cdot \exp(\text{1i} \cdot k_{12} \cdot h); \]
\[ \text{matrixBig} (3, 4) = \text{xi} \cdot \exp(-\text{1i} \cdot k_{12} \cdot h); \]
\[ \text{matrixBig} (3, 5) = \text{k22} \cdot \exp(\text{1i} \cdot k_{22} \cdot h); \]
\[ \text{matrixBig} (3, 6) = \text{k22} \cdot \exp(-\text{1i} \cdot k_{22} \cdot h); \]

\[ \text{matrixBig} (4, 1) = \text{k1} \cdot \exp(-\text{1i} \cdot k_{1} \cdot h); \]
\[ \text{matrixBig} (4, 2) = -\text{xi} \cdot \exp(-\text{1i} \cdot k_{2} \cdot h); \]
\[ \text{matrixBig} (4, 3) = -\text{k12} \cdot \exp(\text{1i} \cdot k_{12} \cdot h); \]
\[ \text{matrixBig} (4, 4) = -\text{k12} \cdot \exp(-\text{1i} \cdot k_{12} \cdot h); \]
\[ \text{matrixBig} (4, 5) = \text{xi} \cdot \exp(\text{1i} \cdot k_{22} \cdot h); \]
\[ \text{matrixBig} (4, 6) = \text{xi} \cdot \exp(-\text{1i} \cdot k_{22} \cdot h); \]

\[ \text{matrixBig} (5, 1) = -2\text{xi} \cdot k_{1} \cdot \text{vs}^2 \cdot 2 \cdot \exp(-\text{1i} \cdot k_{1} \cdot h); \]
\[ \text{matrixBig} (5, 2) = \exp(-\text{1i} \cdot k_{2} \cdot h) \cdot (\text{vp}^2 \cdot 2 \cdot (\text{xi}^2 - k_{2}^2) - 2\text{xi} \cdot \text{vs}^2); \]
\[ \text{matrixBig} (5, 3) = 2\text{xi} \cdot k_{12} \cdot \text{vs}^2 \cdot 2 \cdot \exp(\text{1i} \cdot k_{12} \cdot h); \]
\[ \text{matrixBig} (5, 4) = 2\text{xi} \cdot k_{12} \cdot (\text{vs}^2 - \text{vp}^2 \cdot 2) \cdot \exp(-\text{1i} \cdot k_{12} \cdot h); \]
\[ \text{matrixBig} (5, 5) = \exp(\text{1i} \cdot k_{22} \cdot h) \cdot (\text{vp}^2 \cdot 2 \cdot (\text{xi}^2 + k_{22}^2) - 2\text{xi} \cdot \text{vs}^2); \]
\[ \text{matrixBig} (5, 6) = \exp(-\text{1i} \cdot k_{22} \cdot h) \cdot (\text{vp}^2 \cdot 2 \cdot (\text{xi}^2 - k_{22}^2) - 2\text{xi} \cdot \text{vs}^2); \]

\[ \text{matrixBig} (6, 1) = -\text{vs}^2 \cdot 2 \cdot \exp(-\text{1i} \cdot k_{1} \cdot h) \cdot (\text{xi}^2 + k_{1}^2); \]
\[ \text{matrixBig} (6, 2) = 0; \]
\[ \text{matrixBig} (6, 3) = \text{vs}^2 \cdot 2 \cdot \exp(\text{1i} \cdot k_{12} \cdot h) \cdot (\text{xi}^2 - k_{12}^2); \]
\[ \text{matrixBig} (6, 4) = \text{vs}^2 \cdot 2 \cdot \exp(-\text{1i} \cdot k_{12} \cdot h) \cdot (\text{xi}^2 + k_{12}^2); \]
\[ \text{matrixBig} (6, 5) = 2\text{vs}^2 \cdot 2 \cdot \text{xi} \cdot k_{22} \cdot \exp(\text{1i} \cdot k_{22} \cdot h); \]
matrixBig (6,6)=0;
%
tstDet=det(matrixBig);
blah=tstDet==0;
roots1=solve(blah, xi);
%p=double(real(roots1))
P=double((roots1(1)));
if p < 0
   p=p.*(-1);
end
display(p)


aa=20e-3:10e-3:150e-3;
blahh=numel(aa);
for jj=1:blahh
   tic
   Force=150e3;
a=aa(jj);
P=Force/(2.13*2*a);
   [wSum, err]=surfDispCmplx(a, p);
tmpR=abs(wSum);
theta=angle(wSum);
realX=tmpR*cos(theta);
realY=tmpR*sin(theta);
toc
   filename=strcat ( 'LayerE', int2str(E./1e6), '_E1', int2str(E1/1e6), '_h',
   int2str(h.*1e3), '_Fz', num2str(Force./1e3), '_a', int2str(a.*1e3), '_
damp1fova.mat' );
save(filename, 'a', 'Force', 'wSum', 'realX', 'realY', 'tmpR', 'err', 'E', 'E1',
'P');
end

Listing A.3: MATLAB function used to transform $w_{00}$ back into the $x$–$z$ domain

%% Solving for $w$ @ $z=0$ and $x=0$
function [wSum, err]=surfDispCmplx(aa, p)
load ('coeffs_sym.mat', 'S');
a=aa;
poiRat=0.3;
mu=sqrt(2*(1-poiRat)/(1-(2*poiRat)));

%
\[ [w_1, \text{err}(1)] = \text{quadgk} (@\text{calcQuad}, -1e3, -p, 'Waypoints', [-975, -710, -513, -507, -417, -389, -359, -p]); \]
\[ [w_2, \text{err}(2)] = \text{quadgk} (@\text{calcQuad}, -p, -\mu, 'Waypoints', [-p, -\mu]); \]
\[ [w_3, \text{err}(3)] = \text{quadgk} (@\text{calcQuad}, -\mu, -1, 'Waypoints', [-\mu, -1]); \]
\[ [w_4, \text{err}(4)] = \text{quadgk} (@\text{calcQuad}, 1, 0, 'Waypoints', [-1, 0]); \]
\[ [w_5, \text{err}(5)] = \text{quadgk} (@\text{calcQuad}, 0, 1, 'Waypoints', [0, 1]); \]
\[ [w_6, \text{err}(6)] = \text{quadgk} (@\text{calcQuad}, 1, \mu, 'Waypoints', [1, \mu]); \]
\[ [w_7, \text{err}(7)] = \text{quadgk} (@\text{calcQuad}, \mu, p, 'Waypoints', [\mu, p]); \]
\[ [w_8, \text{err}(8)] = \text{quadgk} (@\text{calcQuad}, p, 1000, 'Waypoints', [p, 359, 389, 417, 507, 513, 710, 975]); \]

\[ w\text{Tmp}=w_1+w_2+w_3+w_4; \]
\[ w\text{Sum}=(w\text{Tmp}+w_5+w_6+w_7+w_8) / (2^\pi); \]

\textbf{function } y = \text{calcQuad}(x) \textbf{ end }

\textbf{Force}=150e3; \textbf{a}=a; 

\textbf{P}=\text{Force} / (2.13^2*2*a); 
\textbf{h}=300e-3; 
\textbf{E}_1=100e6+(0.1*1i*100e6); 
\textbf{E}=50e6+(0.1*1i*50e6); 
\textbf{poiRat}=0.3; 
\textbf{lambda}=E*\text{poiRat}/((1+\text{poiRat})*(1-(2*\text{poiRat}))); 
\textbf{shearMod}=E/(2*(1+\text{poiRat})); 
\textbf{lambda1}=(E_1*\text{poiRat}/((1+\text{poiRat})*(1-(2*\text{poiRat})))) ; 
\textbf{shearMod1}=E_1/(2*(1+\text{poiRat})); 
\textbf{rho} = 2000; 
\textbf{rho2} = \text{rho}; 
\textbf{omega}=2*pi*30; 
\textbf{vp2} = \text{sqrt}((\text{lambda1}+2*\text{shearMod1})/\text{rho}); 
\textbf{vs2} = \text{sqrt}(\text{shearMod1}/\text{rho}); 
\textbf{k12} = (\text{sqrt}(\text{omega}^2-\text{xi}.*2*\text{vs2}.*2))/\text{vs2}; 
\textbf{k22} = (\text{sqrt}(\text{omega}^2-\text{xi}.*2*\text{vp2}.*2))/\text{vp2}; 
\textbf{vp} = \text{sqrt}((\text{lambda}+2*\text{shearMod})/\text{rho}); 
\textbf{vs} = \text{sqrt}(\text{shearMod}/\text{rho}); 
\textbf{k1} = (\text{sqrt}(\text{omega}^2-\text{xi}.*2*\text{vs}.*2))/\text{vs}; 
\textbf{k2} = (\text{sqrt}(\text{omega}^2-\text{xi}.*2*\text{vp}.*2))/\text{vp}; 
\textbf{pii}=\pi; 
\textbf{G.CC} = \text{vpa}(\text{subs}(\text{S.C}));
G.DD=vpa(subs(S.D));
G.FF=vpa(subs(S.F));
G.GG=vpa(subs(S.G));

y=double(G.CC+G.DD+(k22/xi).*G.FF+(k22/xi).*G.GG);

end
end